Structural Models and the Credit Crisis

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Outline of Talk

1) **Advances in Structural Modeling**

2) **Structural Model Performance during the Crisis:**
   Some Preliminary Findings

3) **Potential Explanations of Findings**

4) **Some Thoughts About the Credit Crisis**

Structural Models of Credit Risk

Structural models are widely used in finance, to determine the

- ***Value of a firm's debt*** (and other securities)
  - Dating back to early work by Merton (1974)
  - "Risk neutral" pricing methodology
- ***Likelihood of default***
  - Requires physical measure of firm growth rate
  - Moody's KMV ratings service
- ***Optimal Amount of Leverage*** ("capital structure")
  - ***How much debt should a firm have?***
  - A key question in corporate finance; my focus

Structural Models Depend Upon Several Parameters:

- **Value of Assets (or current cash flows)**
- **Stochastic Process of Assets (or cash flows)**
  - Growth rate of assets (or cash flows)
  - Volatility, possible jumps
  - Risk neutral stochastic process for values
  - Actual (physical) process for default probabilities
- Recovery rates upon default
- Capital structure: Leverage, debt maturity

- Similar to variables cited by ***ratings agencies***, but explicit modeling, formulas
- Based on *market* values, not *book* values

BUT. . . early ***Structural models show poor pricing abilities!*** They

1) Underestimate spreads of investment-grade debt
2) Predict zero spreads for very short-term debt

Also underestimate default rates over short horizons

Examples: Merton (1974); Leland-Toft ("LT") (1996)*

. . .If debt is mispriced, leverage advice will be wrong!

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* Note: Eom, Huang, & Helwege (*RFS* 2004) claim LT model ***overestimates*** credit spreads, particularly for short maturities. I think their results are incorrect, since I cannot replicate them with the LT model and Lando (2005) shows that spreads *must* go to zero as maturity shortens, in diffusion-only models.

In my **Princeton Lectures** (2006), I show both problems can be addressed *by* ***extending structural models in two ways:***

> Assuming an (*il)liquidity premium* for debt returns
(use Longstaff, Mithal, & Neis (2005) estimate: 60 bps)

> Allowing a *mixed jump-diffusion process*
(rather than pure diffusion) for asset value

. . .simple Poisson jump to default with large loss
(e.g. Barro 2006)

The model has ***closed form solutions*** for ***debt value, equity value, endogenous diffusion default boundary, and default probabilities*****.**
(see Appendix)

The model also suggests ***finite-maturity debt is optimal***
(for example, about 7.5 years for Baa-rated firms)

Structural Model Performance During the Crisis

- Earlier evidence suggests the model does OK, on average, for debt of several different ratings, during 1985-2005 period
- But does it price corporate securities consistently through the crisis?
- Our test: **Goldman Sachs and JP Morgan**
  - Initial parameters in Appendix to the lecture
  - Compared with industrials, financials have
    - Low Asset Volatility
    - High Leverage
    - Shorter debt maturity
  - Difficulties in initial calibration
    - Accounting for repos (not included as debt)
    - Assumed duration of deposits (longer maturity)

Data

(i) 1-yr. and 5-yr. Treasury and swap rates (daily)
(ii) Equity values (daily)
(iii) Option-implied equity volatility (6 mo. OTM put options) (Bloomberg, daily)
(iv) CDS rates (Bloomberg & DataStream, daily)
(v) Long and short term debt (book values; quarterly)

Data Problems:

- ***CDS***: Bloomberg data frequently revised weeks after; often disagrees substantially with DataStream
- ***Equity Volatility***: BBG Implied volatilities using the Black-Scholes model
- ***Debt***: Face value must be interpolated; deposit duration?

Our Methodology: Steps for Figure 1

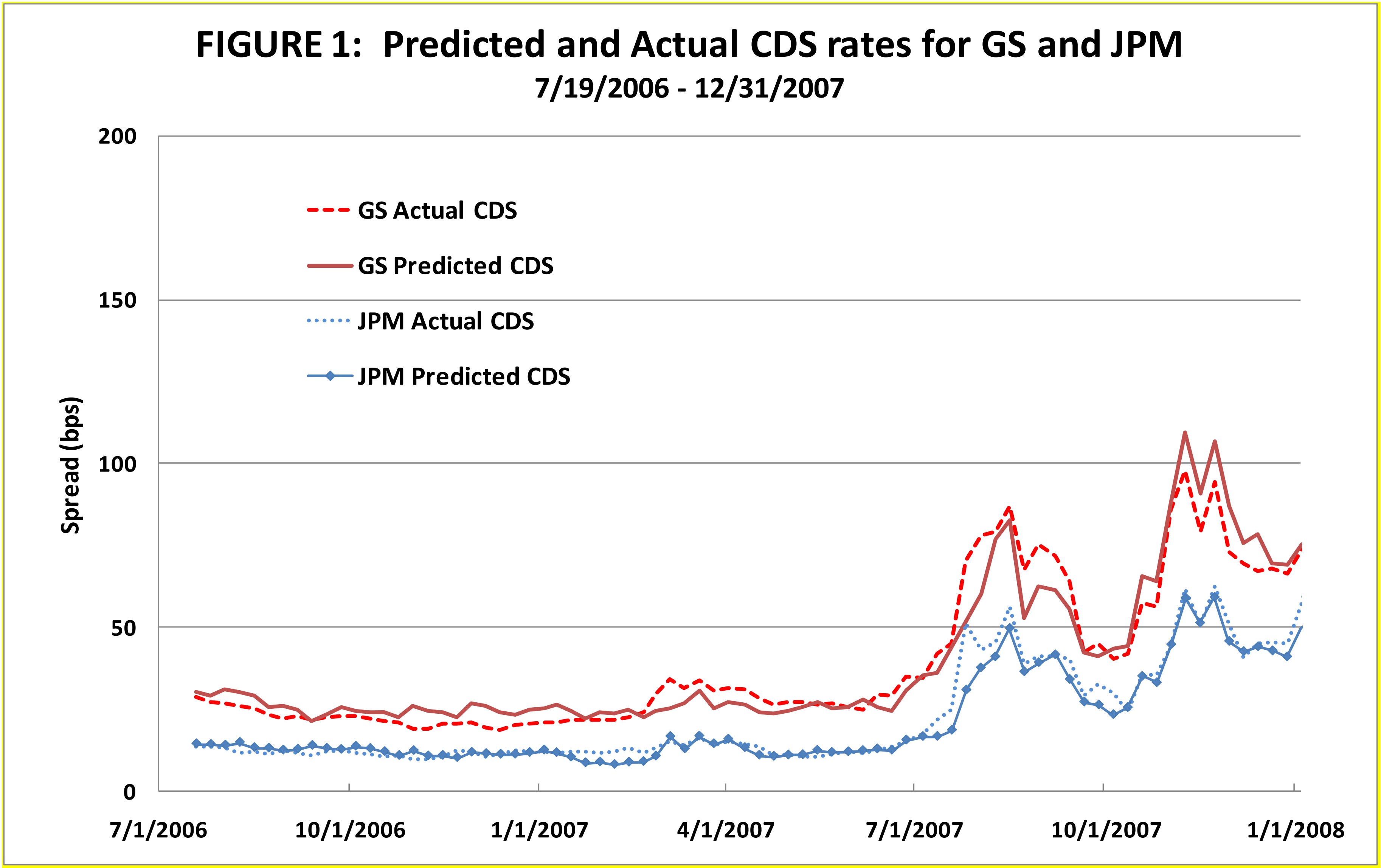
1) Set the values for basic parameters (see Appendix), including
CDS liquidity premium = 0 as observed by Duffie, others.
Bond liquidity premium = 60 bps (Longstaff et al. (2005))

2) Set riskless rate and debt face value ***equal to their observed values*** *at each date*

3) Then determine the **asset values and asset volatilities** consistent with observed **equity values and volatilities** at each date, given other parameters (like M-KMV)

4) Use the structural model with these inputs to "predict" the CDS rate at each date

- Compare with actual CDS rate
- "Out of sample" in that no *ex-post data is used*
- Really a test of model's *consistency of pricing*



GS_JPM_Graphs11 (Alternate)

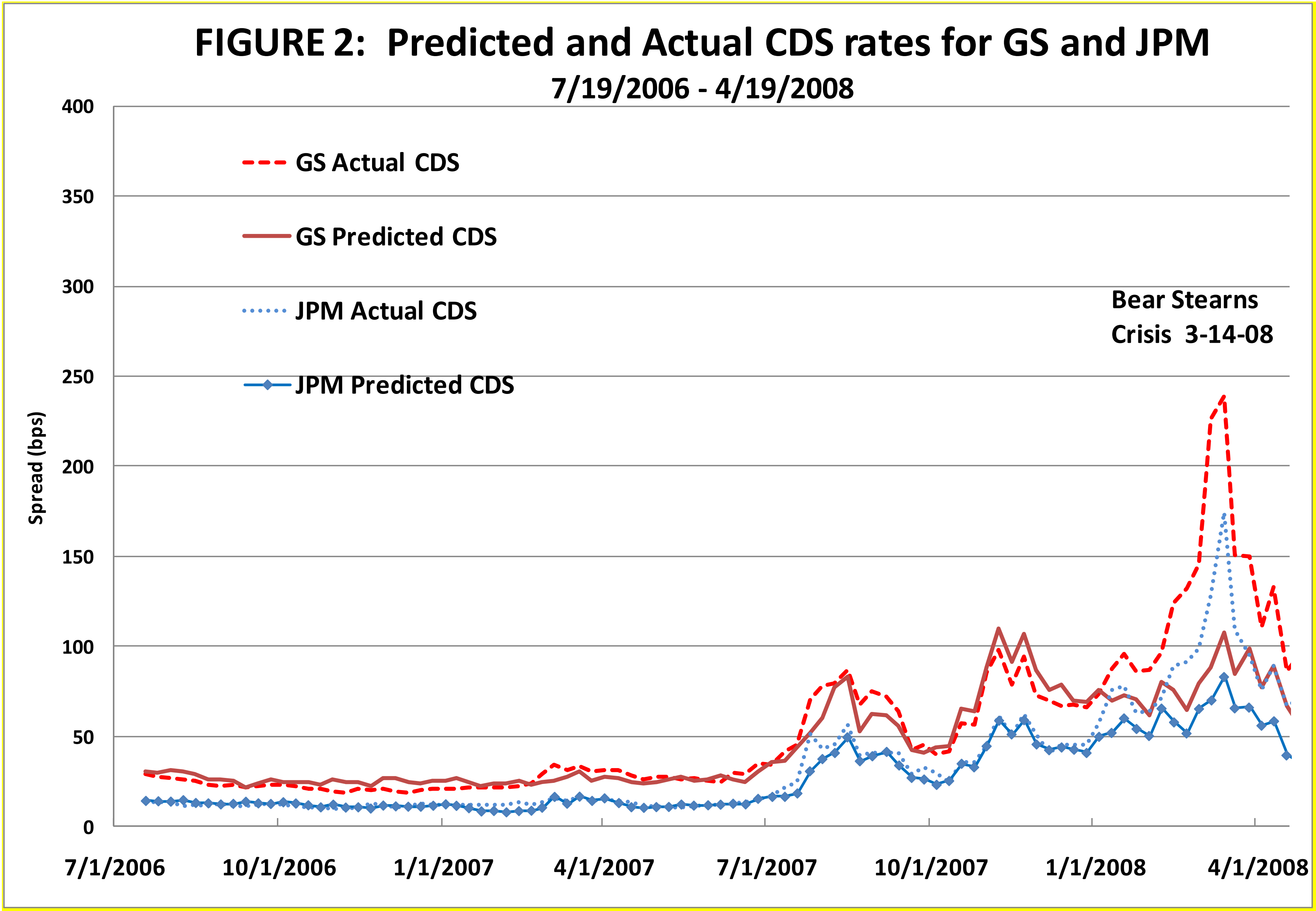
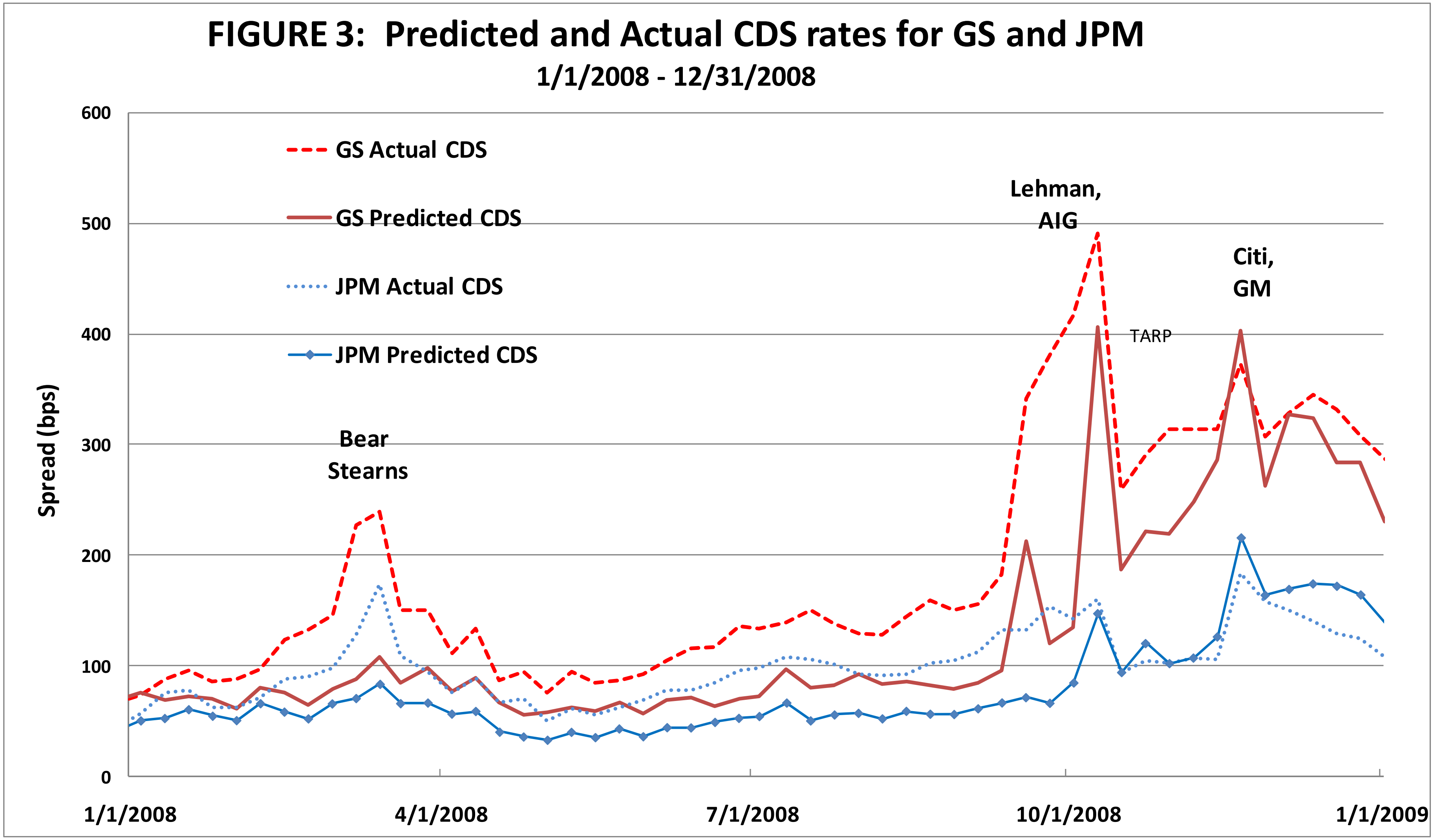
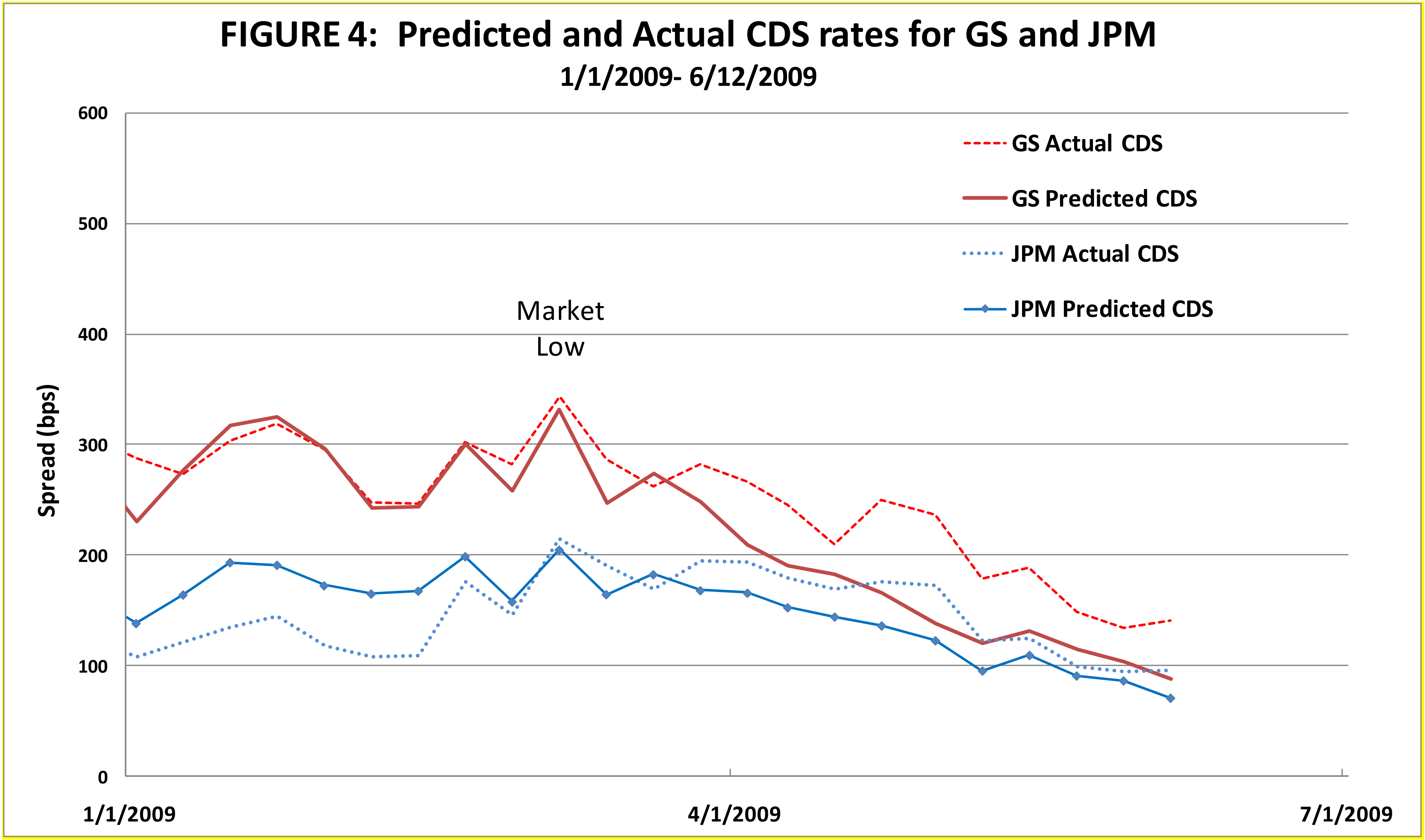


FIGURE 2: Predicted and Actual CDS rates for GS and JPM
7/19/2006 - 4/19/2008
GS Actual CDS
GS Predicted CDS
JPM Actual CDS
JPM Predicted CDS
Bear Stearns
Crisis 3-14-08
Spread (bps)
400
350
300
250
200
150
100
50
0
7/1/2006
10/1/2006
1/1/2007
4/1/2007
7/1/2007
10/1/2007
1/1/2008
4/1/2008

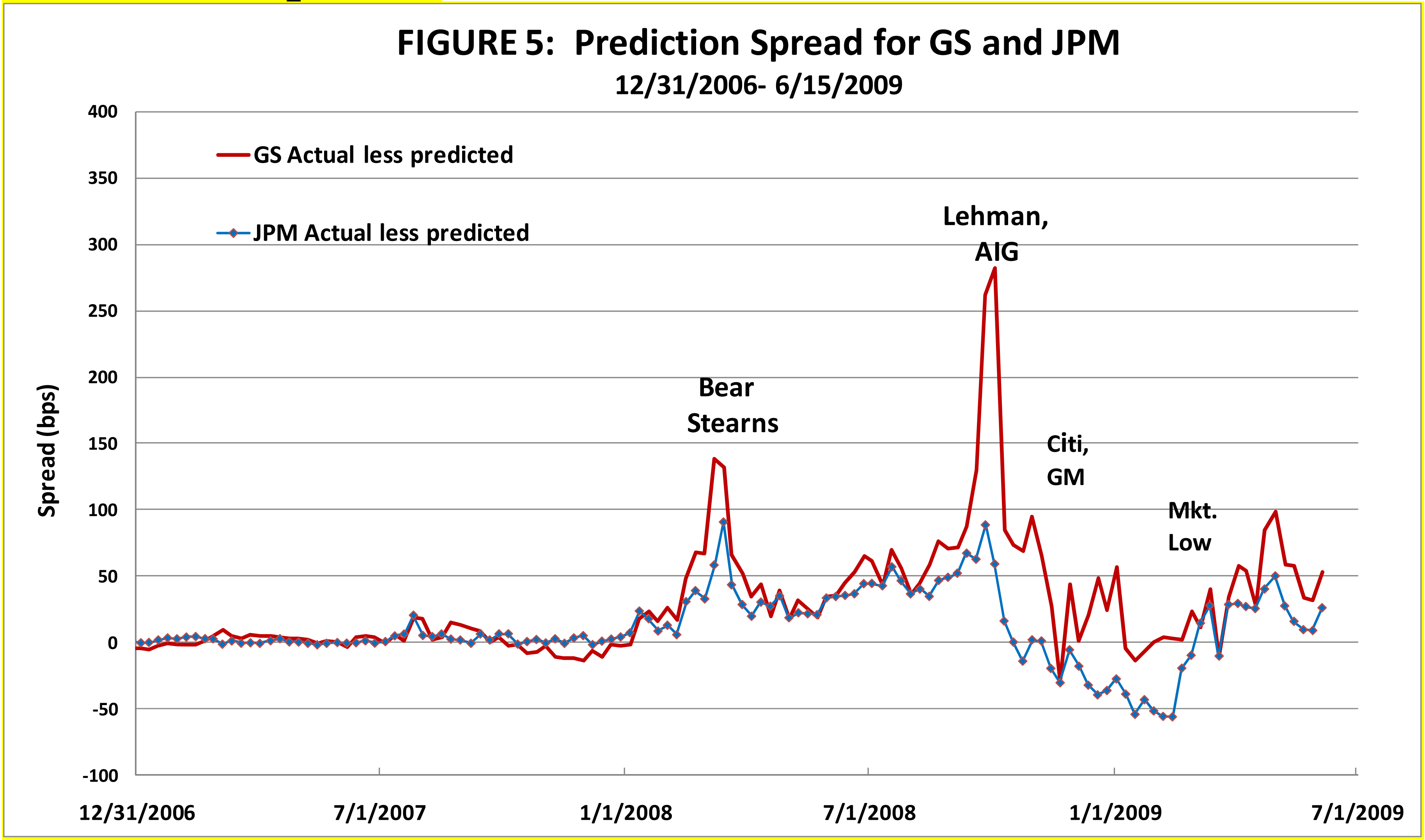
Through 2008…



Through 6/15/09



Prediction Spreads



Some Observations:

1) **Pre-Crisis,** the *prediction spread is close to zero*, indicating that markets are equilibrated in terms of risk

2) **At Beginning of Crisis** (through 2007), markets continue to be in equilibrium, despite significant rise in volatility, CDS rates

3) **Starting in 2008 (and particularly with Bear Stearns)**, *Positive prediction spread* ➔ CDS rates high relative to option prices
   - Equivalently, IVols from CDS > IVols from options
   - Conclusion is on ***relative prices***, not ***which one is mispriced***

4) **Following bailouts of AIG, Citi, and GM, relative prices reverse**
   - For JPM particularly, options now high relative to CDS
   - Lasted until market recovery begins (after March 4, 2009)

Common Factors Affect Differences

Co-movement of difference suggest *omitted market factors*

- Random noise in data could explain either GS or JPM *separately*, but not *jointly [see Goldstein et al. 2004]*
- Not counter-party risk of writers, which *decreases* CDS price

Unobserved (but correlated parameters) may have changed during crisis. Any could potentially explain prediction error:

1) **Jump risk** may have increased (general perception)

2) Expected **recovery rates** may have fallen (Altman et al.)

3) CDS **liquidity premia** may have increased (Tang & Yan)

4) **Supply** of CDSs may have become restricted (Duffie)

Model allows *all* to be explored except last (which ➔ arbitrage)

We examine these latter possibilities in order, and conclude

1) Higher jump risk has little effect on prediction spread

- ***Higher jump risk*** is must be compensated by a ***lower diffusion risk***, since calibration is to a given option price
  - Net effect is to change predicted CDS rates only slightly
- Further evidence of *small effect* of change in jump risk:
  - Regress the *prediction spread on* a measure of *jump size*: difference in OTM vs. ATM implied volatilities
    - R^2 is 0.0004 for GS, 0.0286 for JPM

2) Fall in expected recovery rates

- As stock prices fall, we assume *recovery rate falls*
  - ***GS*:** Default losses α can rise from 20% to max 35%
  - *Shifts down* the average level of prediction spread slightly
    - But ***variation in spread*** through time is little changed

3) Change in the CDS Liquidity Premia

- This premium can be chosen to exactly match CDS rates
  - Prediction error reduced to zero

4) Supply Constraints (effect similar to liquidity premia)

- Exit of MBAA, Ambac, AIG
- Shortage of bank capital to write credible CDS contracts

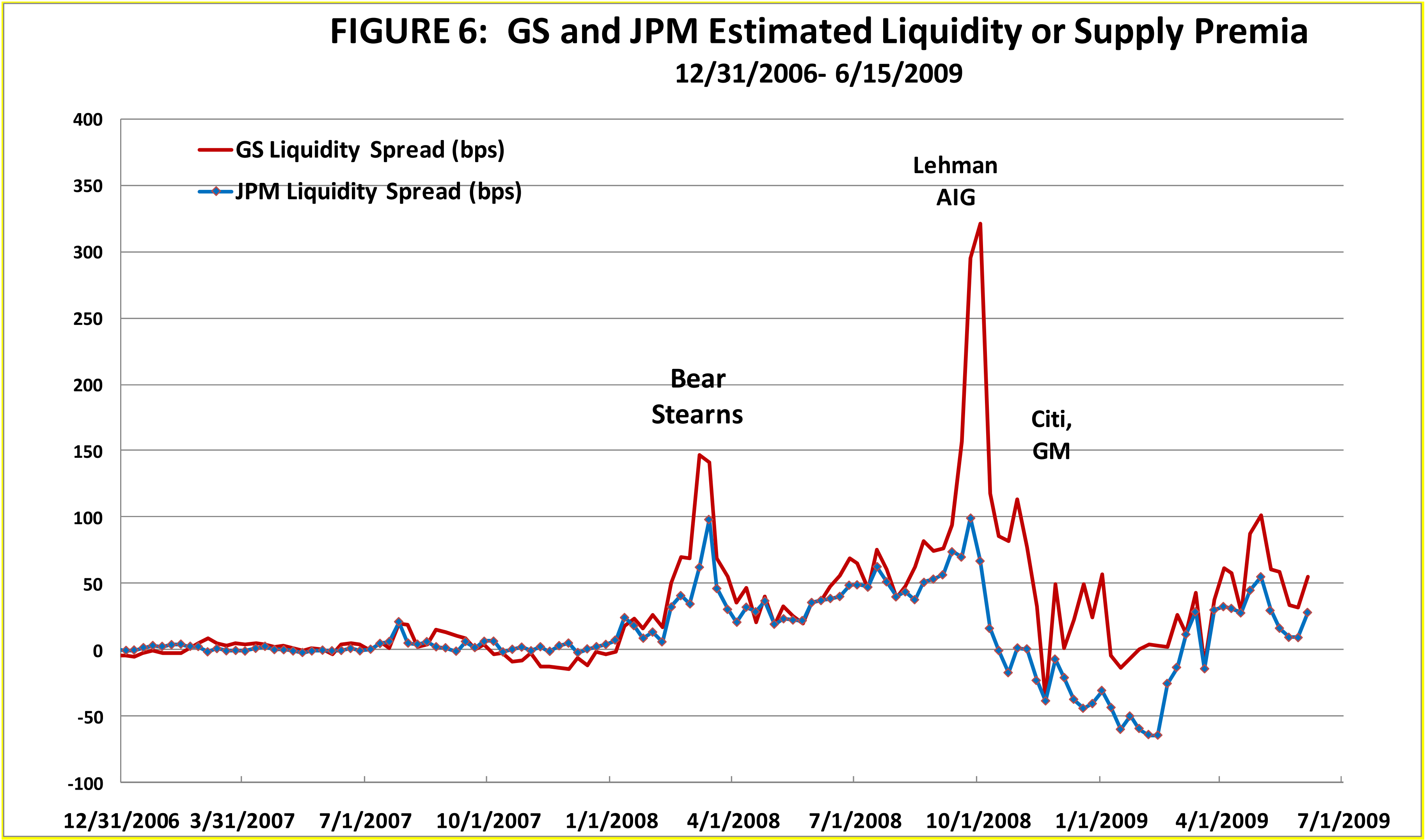


FIGURE 6: GS and JPM Estimated Liquidity or Supply Premia
12/31/2006- 6/15/2009
GS Liquidity Spread (bps)
JPM Liquidity Spread (bps)
Lehman
AIG
Bear
Stearns
Citi,
GM
400
350
300
250
200
150
100
50
0
-50
-100
12/31/2006
3/31/2007
7/1/2007
10/1/2007
1/1/2008
4/1/2008
7/1/2008
10/1/2008
1/1/2009
4/1/2009
7/1/2009

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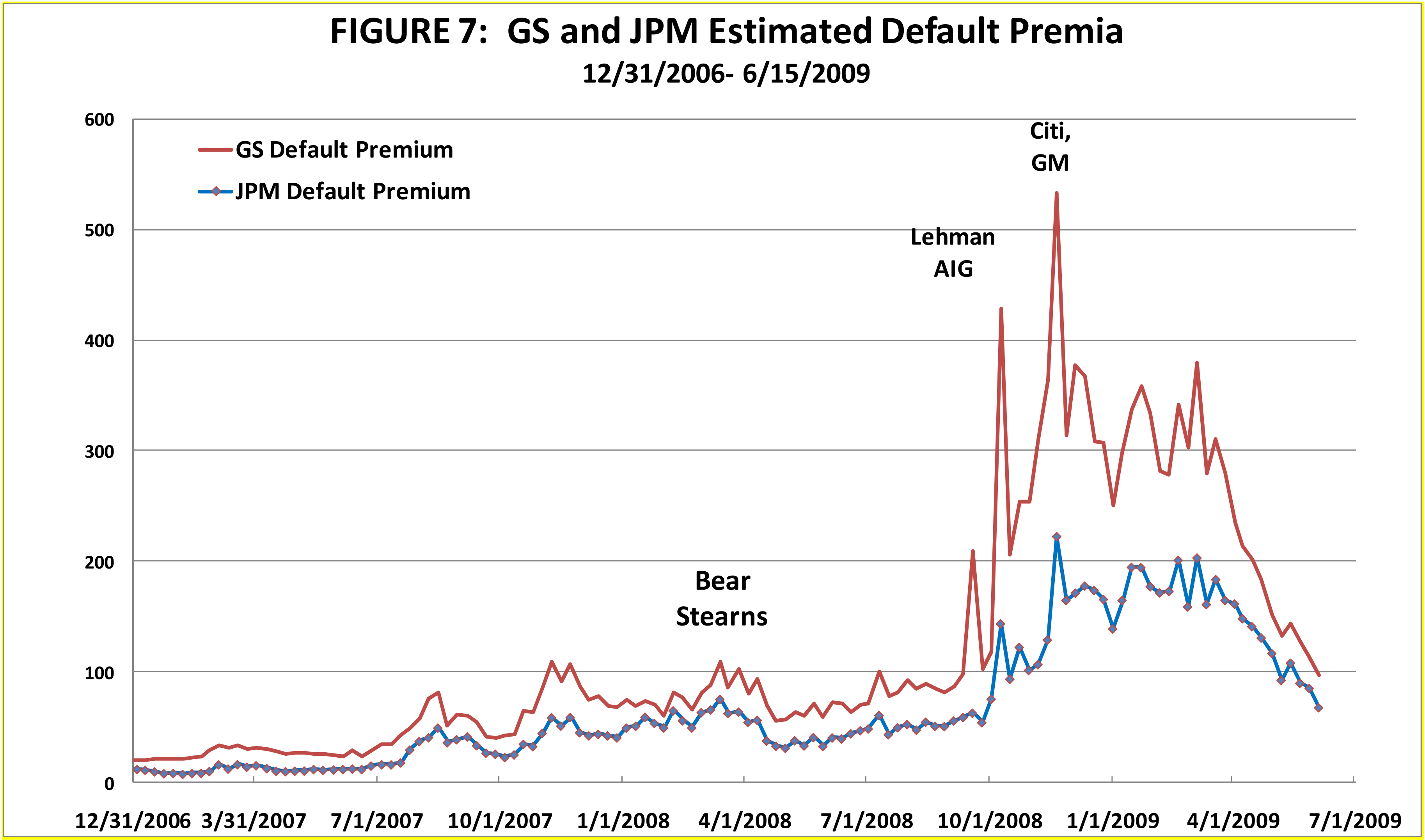


FIGURE 7: GS and JPM Estimated Default Premia
12/31/2006- 6/15/2009
GS Default Premium
JPM Default Premium
Citi,
GM
Lehman
AIG
Bear
Stearns
600
500
400
300
200
100
0
12/31/2006
3/31/2007
7/1/2007
10/1/2007
1/1/2008
4/1/2008
7/1/2008
10/1/2008
1/1/2009
4/1/2009
7/1/2009

Preliminary Conclusions of Study:

- *Structural model does well in the* **pre-crisis** period
  - Perhaps not surprising, since that was a stable period

- **During the crisis,** *considerable prediction spread* ***if the unobservable inputs remain constant***

- ***Correlation of prediction spreads*** (two firms only!) ***suggest*** *common market factors*

  - Changes in ***counter-party risk can't explain data***
  - Changes in ***jump risk can't explain data***
  - Change in recovery rates (with cycle, policy) ***can explain magnitude but not variation in CDS rates***

So What Remains As Explanation?

- **Liquidity changes in CDS Markets**
  - Tang and Yang (2007) and others document that CDS spreads on average may be positive (13 bps pre-crash)
  - Volume and info. asymmetry affect (but not bid-ask spread)
  - However: Duffie claims CDS trading remains liquid throughout

- **Changes in Market Supply: the most compelling?**
  - Duffie (2007) suggests ***temporary capital shortages*** important
  - Exit of Bear Stearns, Ambac, MBAA, and particularly AIG
    - Supply of credit-worthy writers much diminished
    - Banks' capital much reduced
    - But how to predict in advance??

The Curious Reversal in Prediction Spreads (11/14/08 – 3/04/09)

- Options are high-priced relative to CDS rates
  - This was a period of rapidly declining equity prices—the "worst of the crisis" for stock averages
  - Put option-writers had huge losses, depleted capital
  - This drove up prices of options relative to CDS
- Since March, recovering stock market has restored some balance to risk capital in options, CDS markets

But we're still a long ways from pre-crisis default levels!

SOME FURTHER THOUGHTS ON THE CRASH (if time!)

Were Structural Models (or Modelers) Partially to Blame?

Structural Models were *widely used to assess pricing & risk* by ratings agencies, investment banks, hedge funds, and others

- The numbers they came up with were very low… by historical levels as well as with hindsight

Most models didn't include jump risk, liquidity factors.

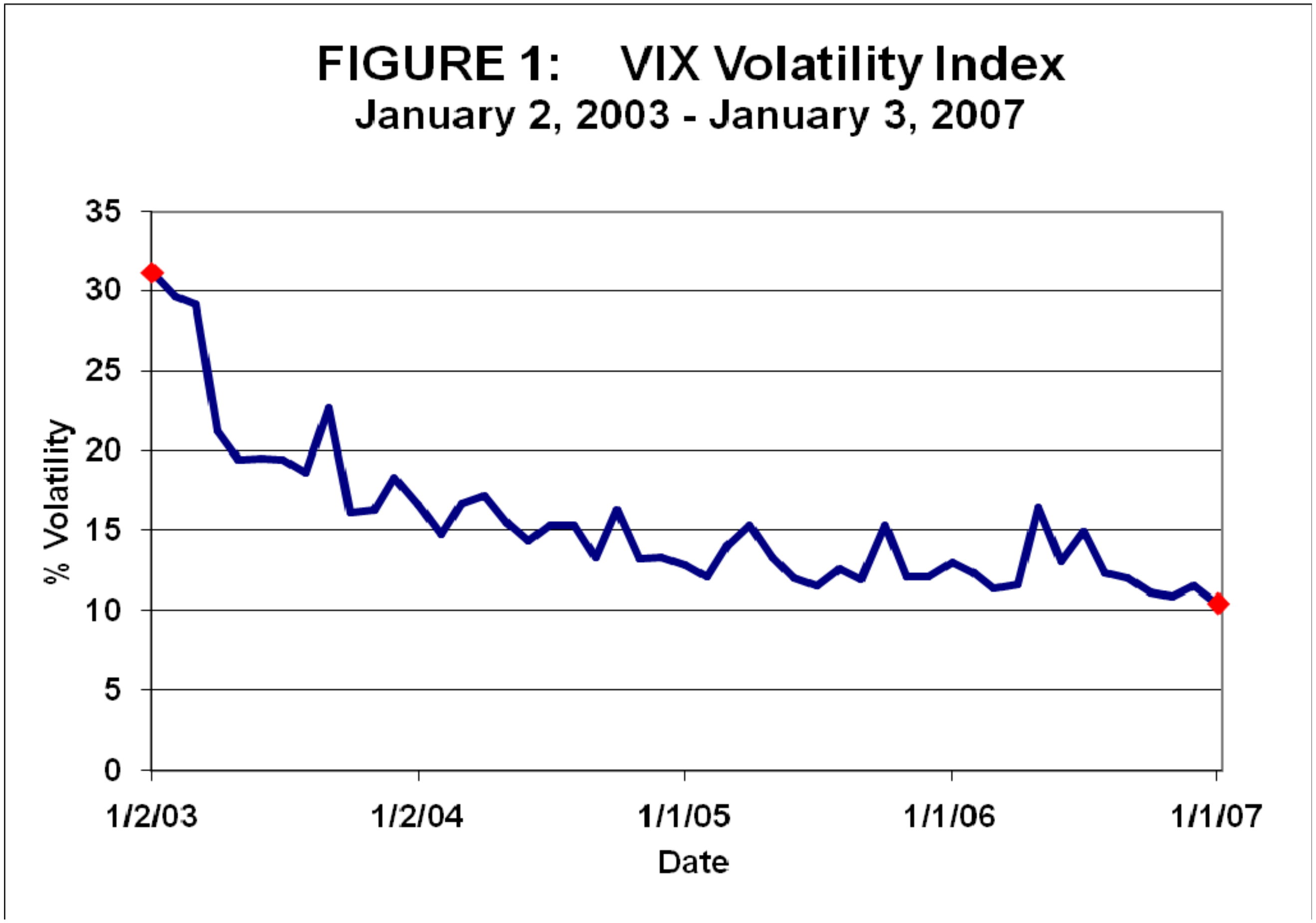
- May have led to *too low pricing of credit risk.*

But I don't think this was the major problem.

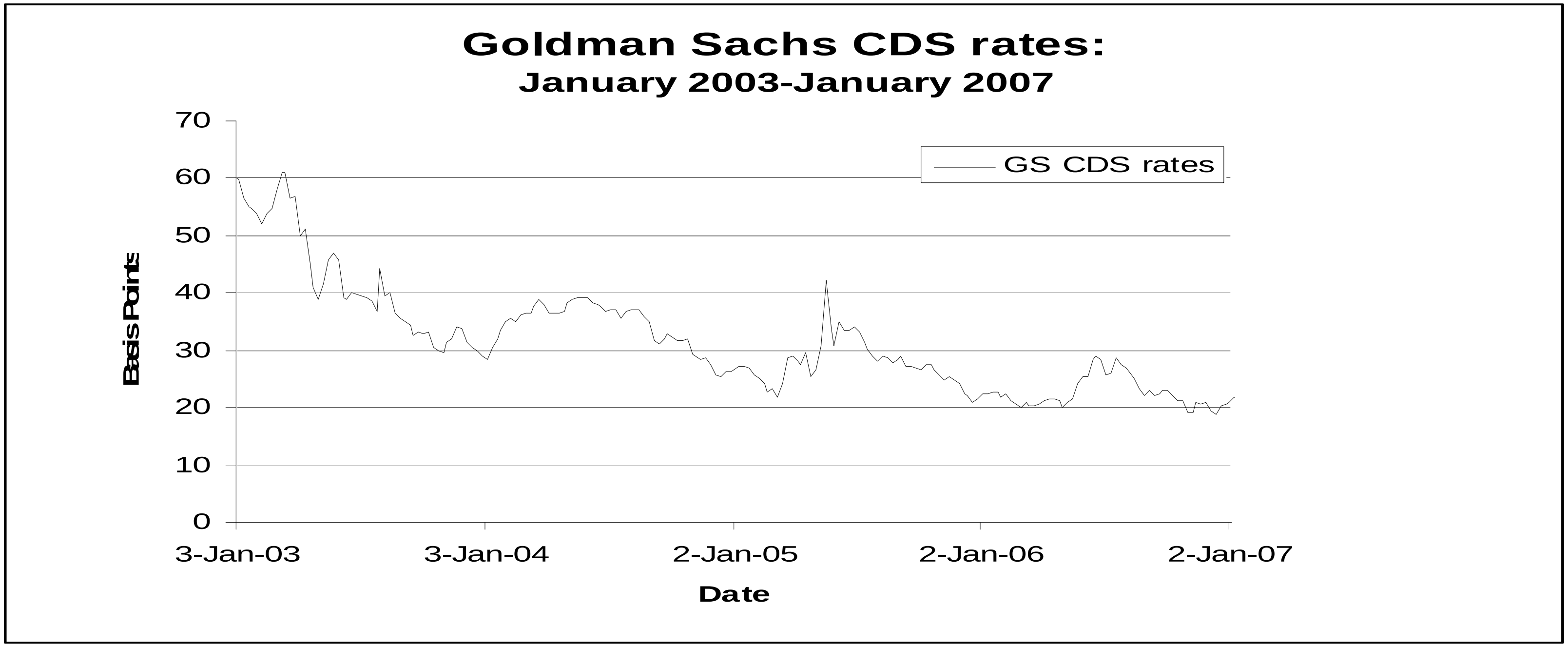
Rather, it was the ***volatility assumptions*** put into the models

The Credit Crisis: Precursors 2004 - 2007

- Lower **volatility**, **credit spreads**: ***"The Greenspan Put"***



Decline in 5-Yr. Default Protection cost (CDS), 1/2003 - 1/2007



Projections of continued low volatility led to:

- **High debt ratings and low credit spreads**
- **High leverage by banks, financial intermediaries**

THE CRISIS

- ***In mid-2007, falling asset prices plus high leverage led to***
  - ***Forced asset liquidations*** ➔ ***lower prices, higher volatility***
  - Lower optimal leverage ➔ further asset sales to reduce debt

- As in Gennotte and Leland ("Market Liquidity, Hedging, and Crashes", *AER* 1990), this led to
  - **"Negative feedback": selling leads to further selling**
  - **Equilibrium at radically lower prices** (there yet?)
  - Similar to margin sales (1929), portfolio insurance (1987)
  - TARP makes some sense to break forced selling

WHO'S TO BLAME? *Many potential culprits*

- Incentives of mortgage originators
- Complexity of new leveraged instruments (e.g. CDOs)
- Naivete of investors (who blame ratings agencies)
- Ratings agencies
- Accounting rules (mark-to-market)
- Failures by regulators (e.g. CDS markets)

But the market's *overconfidence that macro-risks had been conquered* by the Fed and Alan Greenspan was important.

Irrational? Volatility clearly wasn't low before or after 2007!

Not the first time we've been fooled into thinking the world has fundamentally changed (e.g. the internet bubble)

Going Forward: A Proposal for Data Provision

- ***Proposals call for some OTC products to be exchange-traded …but many will continue to exist as OTC products***
- ***In OTC markets, the total size and risk of industry positions are hard to verify—yet pose systemic risk***
    - But to price properly, individual sellers need to *judge risks of forced liquidations by all participants*

      . . .similar to when amount of portfolio insurance was unknown to market participants, liquidity risk high

      > Proposal: SEC or Fed gather *OTC positions* from IBs, hedge funds and the *total made public*

      . . .better knowledge of systemic risks, better pricing

APPENDIX: Valuation

Stochastic Process: Assume *CF*(*t*) is current (after tax) cash flow, paid out to security holders, with <u>risk-neutral</u> diffusion and jump components:

$$dCF(t) = \mu CF(t)dt + \sigma CF(t)\, dZ(t) \quad \text{if no jump at or prior to } t$$

$$= -kCF(t_-) \quad \text{if jump at } t$$

where k is the fractional loss of cash flow if a jump occurs at *t.*

The jump is a Poisson process with constant risk-neutral intensity λ; thus the probability of no jump before time t is $e^{-\lambda t}$.

The expected growth rate of cash flow is: $E[dCF(t)/ CF(t)] = (\mu - \lambda k)dt$

Recall: default occurs at *t* if the diffusion value $V(t)$ hits barrier V_B, or if a jump occurs. If so, debt is in default and receives value (1- α) if the barrier V_B is hit, or $(1 - k)V(t)$ if there is a jump.

WITHOUT LOSS OF GENERALITY, let current time t = 0, V = V(0)

Riskfree rate: r

V, the value of unlevered firm at t= 0 $V = CF/(r - \mu + \lambda k)$,

V(t), *excluding a jump*, has a risk-neutral process $dV/V = gdt + \sigma dZ$

where to give a risk-neutral return *r*, $g = r - \delta + \lambda k$

Dividend rate (fraction of pre-jump value): $\delta = CF/V \; = r - \mu + \lambda k$

Combining results above, we note that $g \; = \; \mu$

Bankruptcy costs: α

Cumulative default frequency at t: $F[t; V, V_B]$ (or F)

First passage density $f[t; V, V_B]$ (or f)

Clearly these latter functions depend on growth rate g and σ. I have suppressed these arguments here.

Let h denote the liquidity premium, implying debt holders discount expected cash flows at rate $r + h$. Given h, the

VALUE OF DEBT

$$D(h) = \int_0^\infty e^{-(r+h)t}(C + mP)e^{-mt}(1 - F)e^{-\lambda t}dt + (1-\alpha)V_B \int_0^\infty e^{-(r+h)t}e^{-\lambda t}e^{-mt} f \, dt$$

$$+ (1-k)\int_0^\infty e^{-(r+h)t}e^{-mt}(e^{gt}V)\lambda e^{-\lambda t}(1-F)dt$$

The first term is the discounted coupon plus principal payments, which decline exponentially at the rate m as debt is retired. Note that coupons are paid only if (i) the default barrier has not been reached, with probability $1 – F$, and that no jump has occurred, which is with probability $e^{-\lambda t}$. The second term is discounted payoffs if the barrier is reached at time t, times the probability that a jump has not occurred. Note e^{-mt} appears in this term and the next because current debt only has claim to fraction e^{-mt} of value. The final term is the value if the jump occurs at time t, which occurs with probability $\lambda e^{-\lambda t}$, reduced by $(1 – F)$, the probability the boundary V_B is reached before the jump. Note that default by jump gives expected value $(1 – k)V(t)$, where the expected value of $V(t) = V e^{gt}$ and V is the current firm value. Conditional on no prior jumps, $V(t)$ grows at rate g, whereas inclusive of expected jump loss, $V(t)$ grows at rate r.

Integrating the first term and last terms by parts gives:

$$D(h) = \frac{C+mP}{r+m+\lambda+h}(1-\int_0^\infty e^{-(r+m+\lambda+h)t} f\,dt) + (1-\alpha)V_B \int_0^\infty e^{-(r+m+\lambda+h)t} f\,dt$$
$$+ \frac{\lambda(1-k)V}{r+m+\lambda+h-g}(1-\int_0^\infty e^{-(r+m+\lambda+h-g)t} f\,dt)$$

We now make use of a key result on first passage times $f(t; V_0, V_B)$ where dV/V follows a log Brownian motion with drift rate g:

$$dV/V = gdt + \sigma dZ$$

$$q(z,V,V_B) \equiv \int_0^\infty e^{-zt} f(t;V,V_B)dt = \left(\frac{V}{V_B}\right)^{-y(z)},$$

where

$$y(z) = \frac{(g-.5\sigma^2)+((g-.5\sigma^2)^2+2z\sigma^2)^{0.5}}{\sigma^2}$$

Note that we have suppressed the arguments (g, σ) of the stochastic process in the definitions of h and y, which we continue to do hereafter. Recalling $g = \mu$, and we can rewrite the debt value function as

$$D(h) = \frac{C+mP}{r+m+\lambda+h}(1-\left(\frac{V}{V_B}\right)^{-y1}) + (1-\alpha)V_B\left(\frac{V}{V_B}\right)^{-y1} + \frac{\lambda(1-k)V}{r+m+\lambda+h-g}(1-\left(\frac{V}{V_B}\right)^{-y2})$$

where

$$y_1(h) = y(r+m+\lambda+h) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2+2((r+m+\lambda+h)\sigma^2]^{0.5}}{\sigma^2}$$

$$y_2(h) = y(r+m+\lambda+h-\mu) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2+2((r+m+\lambda+h-g)\sigma^2]^{0.5}}{\sigma^2}$$

VALUE OF CASH FLOWS TO EQUITY HOLDERS OF A LEVERED FIRM:

Equity holders discount cash flows without an additional risk premium.[1] The value of equity in a levered firm will reflect the value of the unlevered firm V_0, plus the value of tax savings provided by deductibility of coupon payment, less the value of default costs, less the value (to shareholders) of the cash flows to debt. These cash flows are discounted at rate *r* rather than $r + h$, and will have value $D(0)$. Thus equity has value

$$E = V + TS - DC - D(0),$$

where tax savings provide a constant cash flow τC when the firm is solvent, and zero otherwise.

The value of ***tax savings*** is

$$TS = \int_0^\infty e^{-rt}\tau C(1-F)e^{-\lambda t}\,dt$$

$$= \frac{\tau C}{r+\lambda}(1-\left(\frac{V}{V_B}\right)^{-y3})$$

where

$$y_3 = y(r+\lambda) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2+2((r+\lambda)\sigma^2]^{0.5}}{\sigma^2}$$

Default costs (incurred by default from diffusion) are given by

[1] Alternatively, equity holders could also discount at a rate including a risk premium. Our rate *r* could be viewed as including such an equity premium (although *r* would exceed Treasury rates, assuming equity is less liquid than Treasuries). In this case, *h* would be the incremental liquidity premium for debt relative to equity, which could in fact be negative.

$$DC = \alpha V_B \int_0^{\infty} e^{-(r+\lambda)t} f \, dt$$

$$= \alpha V_B \left(\frac{V}{V_B} \right)^{-y3}$$

OPTIMAL DEFAULT LEVEL V_B:

Equity value for arbitrary V is given by

$$E = V + TS - DC - D(0)$$

$$= V + \frac{\tau C}{r+\lambda}(1 - \left(\frac{V}{V_B}\right)^{-y3}) - \alpha V_B \left(\frac{V}{V_B}\right)^{-y3}$$

$$- \frac{C+mP}{r+m+\lambda}(1 - \left(\frac{V}{V_B}\right)^{-y4}) - (1-\alpha)V_B \left(\frac{V}{V_B}\right)^{-y4} - \frac{\lambda(1-k)V}{r+m+\lambda-\mu}(1 - \left(\frac{V}{V_B}\right)^{-y5})$$

NOTE we need to assume h = 0 here, since equity value does not discount bond payments as bondholders do.

Default occurs at the optimal (smooth pasting) level of V where $dE(V)/dV \,|_{V=VB} = 0$, implying

$$V_B = \frac{\dfrac{(C+mP)y_4}{(r+m+\lambda)} - \dfrac{TCy_3}{(r+\lambda)}}{1 + (1-\alpha)y_4 + \alpha\, y_3 - \dfrac{\lambda(1-k)}{(r+\lambda+m-\mu)} y_5}$$

where

$$y_4 = y(r+m+\lambda) = \frac{(g-.5\sigma^2) + [(g-.5\sigma^2)^2 + 2((r+m+\lambda)\sigma^2]^{0.5}}{\sigma^2}$$

$$y_5 = y(r+m+\lambda-\mu) = \frac{(g-.5\sigma^2) + [(g-.5\sigma^2)^2 + 2((r+m+\lambda-g)\sigma^2]^{0.5}}{\sigma^2}$$

Model Calibration

Unobservable Parameters and Empirical Justification

(a) Jump risk λ = .003, consistent with A-rated debt default rates (Moody's data)

(b) Bond liquidity premium h_0 = 60 bps (consistent with Longstaff, Mithal, and Neis (2005))

(c) CDS liquidity premium h_1 = 0 (CDS rate = credit risk only) (Blanco, Brennan, and Marshall (2005), Zhu (2006))

(d) Corporate tax (net) τ = 25%

(e) Default cost (GS = 20%, JPM = 5%), consistent with ratings recovery rates

Jump risk, default costs, and CDS liquidity premia are subsequently permitted to vary.