Skewness and the Bubble*

Jennifer Conrad[†] **Robert F. Dittmar**[‡] **Eric Ghysels**[§]

First Draft: March 2007

This Draft: April 2008

Preliminary and incomplete.
Please do not quote or cite.

Abstract

We use a sample of option prices, and the method of Bakshi, Kapadia and Madan (2003), to estimate the *ex ante* higher moments of the underlying individual securities' risk-neutral returns distribution. We find that individual securities' volatility, skewness and kurtosis are strongly related to subsequent returns. Consistent with Ang, Hodrick, Xing and Zhang (2006), we find a negative relation between cross-sectional volatility and returns. We also find a significant relation between skewness and returns, with more negatively (positively) skewed returns associated with subsequent higher (lower) returns, while kurtosis is positively related to returns. We analyze the extent to which these returns relations represent compensation for risk. We use data on index options and the underlying market return to estimate the stochastic discount factor over the 1996-2005 sample period, and allow the stochastic discount factor to include higher moments. We find evidence that, even after controlling for differences in co-skewness, individual securities' skewness matters.

*All errors are the responsibility of the authors. We thank Robert Battalio, Patrick Dennis, and Stewart Mayhew for providing data and computational code. We thank seminar participants at Babson College, Boston College, Cornell University, National University of Singapore, Queen's University Belfast, and the Universities of Arizona, Michigan, Texas, and Virginia for helpful comments.

[†]Department of Finance, Kenan-Flagler Business School, University of North Carolina at Chapel Hill

[‡]Department of Finance, Stephen Ross School of Business, University of Michigan, Ann Arbor, MI 48109

[§]Department of Finance, Kenan-Flagler Business School, and Department of Economics, University of North Carolina at Chapel Hill

1 Introduction

Models suggesting that investors consider higher moments in returns have a long history in the literature. For example, Rubinstein (1973), and Kraus and Litzenberger (1976, 1983) consider models of expected returns which incorporate skewness. More recent empirical work provides evidence that higher moments of the return distribution are important in pricing securities. For example, Harvey and Siddique (2000) test whether skewness is priced, and Dittmar (2002) tests whether a security's skewness and kurtosis might influence investors' expected returns. In these papers, although additional restrictions are imposed on investors' utility functions (e.g., that of decreasing absolute prudence), investors are still maximizing expected utility, holding optimal portfolios, and evaluating risk in a portfolio context. Consequently, the higher moments which are relevant for individual securities in these models are co-moments with the aggregate market portfolio; the tests in these papers ask whether a security's co-skewness or co-kurtosis with the market is priced, and use historical returns data to measure these co-moments.

Other recent papers have suggested that additional features of individual securities' payoff distribution may be relevant for understanding differences in assets' returns. For example, Ang, Hodrick, Xing, and Zhang (2006a, 2006b) document that firms' idiosyncratic return volatility contains important information about future returns. The work of Barberis and Huang (2004), Brunnermeier, Gollier and Parker (2007), and the empirical evidence presented in Mitton and Vorkink (2007) and Boyer, Mitton and Vorkink (2008) suggest that the skewness of individual securities may also influence investors' portfolio decisions.

In this paper, we examine the importance of higher moments using a different approach. We exploit the fact that if option and stock prices reflect the same information, then it is possible to use options market data to extract estimates of the higher moments of the securities' (risk-neutral) probability density function. Our method has several advantages. First, option prices are a market-based estimate of investors' expectations. Authors such as Bates (1991), Rubinstein (1985, 1994) and Jackwerth and Rubinstein (1996) have argued that option market prices appear to efficiently capture the information of market participants. Second, the use of option prices eliminates the need of a long time-series of returns to estimate the moments of the return distribution; this is especially helpful when trying to forecast the payoff distribution of relatively new firms (such as Internet companies) or during periods where expectations, at least for some firms, may change relatively quickly. Third, options reflect a true *ex ante* measure of expectations; they do not give us, as Battalio and Schultz (2006) note, the "unfair advantage of hindsight." As Jackwerth and Rubinstein (1996) state, "not only can the nonparametric method reflect the possibly complex logic used by market participants to consider the significance of extreme events, but it also implicitly brings a much larger set of information ... to bear on the formulation of probability distributions." Finally, the use of the data available in *index* options, and underlying index returns, may allow us to separate the relation between higher (co)moments and the stochastic discount factor from the effect of idiosyncratic higher moments on prices.

We begin with a sample of options on individual stocks, and test whether cross-sectional differences in estimates of the higher moments of an individual security's payoff extracted from options are related to subsequent returns. Consistent with Ang, Hodrick, Xing, and Zhang (2006a, 2006b), we find a negative relation between volatility and subsequent returns.[1] We also document a significant negative relation between firms' risk-neutral skewness and subsequent returns-that is, more negatively skewed securities have higher subsequent returns. In addition, we find a significant positive relation between firms' risk-neutral kurtosis and subsequent returns. These relations persist after controlling for firm characteristics, such as beta, size, and book-to-market ratios, and adjustment for the Fama and French (2003) risk factors.

Evidence that other features of an individual security's return distribution are relevant for stock prices seems particularly intriguing in the context of anomalies such as the Internet bubble. Given that the rest of the market appeared relatively unaffected during this period– in fact, Siegel (2006) argues that if one removes technology and telecommunication stocks from the S&P 500, the remaining stocks had depressed prices in early 2000–it seems unlikely that the pricing of the 'bubble' stocks is due to cross-sectional differences in the covariances of return distributions with the aggregate portfolio; the dispersion in the covariances, and the risk premium associated with the exposure, would have to be implausibly high. However, if the characteristics of individual securities' payoff distribution are important, then the concentration of the 'bubble' in particular segments of the market, which have large idiosyncratic differences in return distributions, may be less puzzling.

The relation between higher moments and subsequent returns for which we find evidence may be a combination of differences in co-moments, and differences in idiosyncratic features

[1]Spiegel and Wang (2006) also find a significant relation between idiosyncratic volatility and subsequent returns, although the sign of the relation is reversed from the Ang, et al results.

of the return distribution. We return to the information contained in option prices to form an estimate of the stochastic discount factor so that we can separate the two effects.

To estimate the stochastic discount factor, we estimate the risk-neutral density of the aggregate market portfolio using index option prices. We also form an estimate of the physical probability distribution using historical returns on the index. We use these estimates to construct stochastic discount factors, and use these to risk-adjust the returns that are related to higher moments. Our results suggest that, after controlling for the significant effect of higher moments on the stochastic discount factor, only idiosyncratic skewness has residual predictive power for returns.

Our results are consistent with models such as Brunnermeier, Gollier and Parker (2007), and Barberis and Huang (2004) which suggest that investors will trade off the benefits of diversification and skewness, holding more concentrated positions in skewed securities, and resulting in a negative relation between idiosyncratic skewness and expected returns. These results are also consistent with the empirical evidence in Mitton and Vorkink, who examine the choices of investors in a sample of discount brokerage accounts and find that investors appear to hold relatively undiversified portfolios and accept lower Sharpe ratios for positively skewed portfolios and securities.

The remainder of the paper is organized as follows. In section 2, we detail the method we employ for recovering measures of volatility, skewness, and kurtosis, following Bakshi, Kapadia, and Madan (2003). Section 3 discusses the data used in our analysis and presents results of empirical tests performed on portfolios formed on the basis of the volatility, skewness, and kurtosis measures. In Section 4, we use data on the market portfolio, and its options, to estimate a stochastic discount factor which includes the information in higher moments, and use this stochastic discount factor to risk-adjust the raw returns related to higher moments. In Section 5, we discuss the estimation of implied physical distributions for individual securities, and present these estimates for industry portfolios. We conclude in Section 6.

2 Computing Risk-neutral moments

Our focus in the first part of the paper is on testing whether estimates of the higher moments of the payoff distribution obtained from options data are related to the subsequent returns of the underlying security. Under the assumption that no-arbitrage rules hold between the options market and the underlying security prices, the information set contained in both sets of prices should be the same. Several authors have shown that information in option prices can provide valuable forecasts of features of the payoff distributions in the underlying market. For example, Bates (1991) examines option prices (on futures contracts) prior to the market crash of 1987 and concludes that the market anticipated a crash in the year, but not the two months, prior to the October market decline. He also shows that fears of a crash increased immediately after the crash itself. Jackwerth (2000) uses option market data to estimate an empirical measure of risk-aversion, and finds that these estimates are also affected by events in the underlying market.

To estimate the higher moments of the (risk-neutral) density function of individual securities, we use the results in Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003). Bakshi and Madan (2000) show that any payoff to a security can be constructed and priced using a set of option prices with different strike prices on that security. Bakshi, Kapadia, and Madan (2003) demonstrate how to express the risk-neutral density moments in terms of quadratic, cubic, and quartic payoffs.

In particular, Bakshi, Kapadia, and Madan (2003) show that one can express the τ-maturity price of a security that pays the quadratic, cubic, and quartic return on the base security as

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2(1-ln(K/S(t)))}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{2(1+ln(K/S(t)))}{K^2} P(t,\tau;K)dK \quad (1)$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6ln(K/S(t)) - 3(ln(K/S(t)))^2)}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{6ln(K/S(t)) + 3(ln(K/S(t)))^2}{K^2} P(t,\tau;K)dK \quad (2)$$

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12(ln(K/S(t)))^2 - 4(ln(K/S(t)))^3)}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{12(ln(K/S(t)))^2 + 4(ln(K/S(t)))^3}{K^2} P(t,\tau;K)dK \quad (3)$$

where $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$ are the quadratic, cubic, and quartic contracts, respectively, and $C(t,\tau;K)$ and $P(t,\tau;K)$ are the prices of European calls and puts written on the underlying stock with strike price K and expiration τ periods from time t. As equations (1), (2) and (3) show, the procedure involves using a weighted sum of (out-of-the-money) options across varying strike prices to construct the prices of payoffs related to the second, third and fourth moments of returns.

Using the prices of these contracts, standard moment definitions suggest that the risk-neutral moments can be calculated as

$$VAR(t,\tau) = e^{r\tau}V(t,\tau) - \mu(t,\tau)^2 \tag{4}$$

$$SKEW(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2\mu(t,\tau)^3}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^{3/2}} \tag{5}$$

$$KURT(t,\tau) = \frac{e^{r\tau}X(t,\tau) - 4\mu(t,\tau)W(t,\tau) + 6e^{r\tau}\mu(t,\tau)^2V(t,\tau) - \mu(t,\tau)^4}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^2} \tag{6}$$

where

$$\mu(t,\tau) = e^{r\tau} - 1 - e^{r\tau}V(t,\tau)/2 - e^{r\tau}W(t,\tau)/6 - e^{r\tau}X(t,\tau)/24 \tag{7}$$

and r represents the risk-free rate. We follow Dennis and Mayhew (2002), and use a trapezoidal approximation to estimate the integrals in expressions (1)-(3) above using discrete data.[2] In the next section, we discuss the data employed in this estimation in detail.

3 Data and Empirical Results

We wish to examine the relation, if any, between features of the risk-neutral density function and the pricing of stocks. In this section we report a first set of empirical results that answers this question. We start with a description of the data, and how individual firms' daily estimates of higher moments are aggregated. Next, we examine the relation between moment-based portfolios and subsequent return measures.

[2] We are grateful to Patrick Dennis for providing us with his code to perform the estimation.

3.1 Data

Our data on option prices are provided (through Wharton Research Data Services) from Optionmetrics. We begin with daily option price data for all out-of-the-money calls and puts for all stocks from 1996-2005.[3] Closing prices are constructed as midpoint averages of the closing bid and ask prices.

Some researchers have argued that option prices and equity prices diverged during our sample period. For example, Ofek and Richardson (2003) propose that the Internet bubble is related to the 'limits to arbitrage' argument of Shleifer and Vishny (1997). This argument requires that investors could not, or did not, use the options market to profit from mispricing in the underlying market, and, in fact, Ofek and Richardson (2003) also provide empirical evidence that option prices diverged from the (presumably misvalued) prices of the underlying equity during this period. However, Battalio and Schultz (2006) use a different dataset of option prices than Ofek and Richardson (2003), and conclude that shorting synthetically using the options market was relatively easy and cheap, and that short-sale restrictions are not the cause of persistently high Internet stock prices. A corollary to their results is that option prices and the prices of underlying stocks did not diverge during the 'bubble' period and they argue that Ofek and Richardson's results may be a consequence of misleading or stale option prices in their data set. Note that if option and equity prices do not contain similar information, then our tests should be biased against finding a systematic relation between estimates of higher moments obtained from option prices and subsequent returns in the underlying market.[4] However, motivated by the Battalio and Schultz results, we employ a number of filters to try to ensure that our results are not driven by stale or misleading prices. We eliminate option prices below 50 cents, as well as options with less than one week to maturity. At the outset, we require that an option has a minimum of ten days of quotes during any month; in later robustness checks, we impose additional constraints on the liquidity in the option. We also eliminate days in which closing quotes on put-call pairs violate no-arbitrage restrictions.

[3]We do not adjust for early exercise premia in our option prices. As Bakshi, Kapadia and Madan (2003) note, the magnitude of such premia in OTM calls and puts is very small, and the implicit weight that options receive in the estimation declines as they get closer to at-the-money. In their empirical work, BKM show that, for their sample of OTM options, the implied volatilities from the Black-Scholes model and a model of American option pricing have negligible differences.

[4]Robert Battalio graciously provided us with the OPRA data used in their analysis; unfortunately, these data, provided by a single dealer, do not have a sufficient cross-section of data across calls and puts to allow us to estimate the moments of the risk-neutral density function in which we are interested.

In estimating equations (4) - (6), we use equal numbers of out-of-the-money (OTM) calls and puts for each stock for each day. Thus, if there are n OTM puts with closing prices available on day t we require n OTM call prices. If there are $N > n$ OTM call prices available on day t, we use the n OTM calls which have the most similar distance from stock to strike as the OTM puts for which we have data. We require a minimum n of 2; we perform robustness checks on our results when this minimum data constraint is increased.[5] The resulting set of data consists of 3,722,700 daily observations across firms and maturities over the 1996-2005 sample period.

Data on individual stock returns is provided (through Wharton Research Data Services) from the Center for Research in Security Prices. We employ daily and monthly returns from 1996-2005 for all securities covered by CRSP with common shares outstanding. In addition, we collect option prices on the S&P 500 index and utilize daily returns on the index in our estimation, also obtained from CRSP. The index data are sampled from 1993-2005 to provide a sufficiently long time series for estimates of physical moments, employed in our later analysis of the time series of risk aversion.

3.2 Raw and Characteristic-Adjusted Returns

We begin by selecting out-of-the-money calls and puts on individual securities which have maturities closest to 1 month, 3 months, 6 months and 12 months. In each maturity bin, we estimate the moments of the risk-neutral density function for each individual security on a daily basis. Following Bakshi, Kapadia and Madan (2003), we average the daily estimates for each stock over time (in our case, the trading quarter.) For each maturity bin, we further sort options into tercile portfolios based on the moment estimates (volatility, skewness or kurtosis); the 'extreme' portfolios contain 30% of the sample, while portfolio 2 contains 40% of the sample. We re-form portfolios each quarter. In each quarter, we also remove firms that are in the top 1% of the cross-sectional distribution of volatility, skewness or kurtosis to mitigate the effect of outliers.

[5]Dennis and Mayhew (2006) examine and estimate the magnitude of the bias induced in Bakshi-Kapadia-Madan estimates of skewness which is due to discreteness in strike prices. For $ 5 ($2.50) differences in strike prices, they estimate the bias in skewness is approximately -0.07 (-0.05). Since most stocks have the same differences across strike prices, however, the relative bias should be approximately the same across securities, and should not affect either the ranking of securities into portfolios based on skewness, or the nature of the cross-sectional relation between skewness and returns which we examine.

In Table 1, we report results for securities sorted on the basis of estimated volatility (Panel A), estimated skewness (Panel B) and estimated kurtosis (Panel C). Specifically, we report the subsequent raw returns of these moment-ranked portfolios over the next month in the first column of data. In the next column, we report the value-weighted return, followed by a characteristic-adjusted return over that same month. To calculate the characteristic-adjusted return, we perform a calculation similar to that in Daniel et al(1997). For each individual firm, we assess to which of the 25 Fama-French size- and book-to-market ranked portfolios the security belongs. We subtract the return of that Fama-French portfolio from the individual security return and then average the resulting excess or characteristic-adjusted 'abnormal' return across firms in the moment-ranked portfolio. In the next three columns, we report the average risk-neutral volatility, skewness and kurtosis estimates for each of the ranked portfolios, as well as the average number of securities which appear in each portfolio over the sample period. Finally, we report average betas, average (log) market value and average book-to-market equity ratios of the securities in the portfolio.

Summary statistics in Panel A of Table 1 suggest a strong negative relation between volatility and subsequent raw returns; for example, in the shortest maturity options (maturity bin 1), the returns differential between high volatility (Portfolio 3) and low volatility (Portfolio 1) securities is -32 basis points per month; longer maturities have differentials between 50 and 55 basis points per month. The columns of data which report the average characteristics of securities in the portfolio show sharp differences in beta, size and book-to-market equity ratios across these volatility-ranked portfolios. Low (high) volatility portfolios tend to contain low (high) beta firms, larger (smaller) firms and somewhat higher (lower) book-to-market firms. We adjust for these differences in size and book-to-market equity ratio in the characteristic-adjusted return column. After adjusting for the differences in size and book-to-market observed across the volatility portfolios, the return differentials are somewhat attenuated in all four maturities. However, although the differential is reduced, it remains significant, with lowest volatility portfolios earning between 10 and 23 basis points per month more than the highest volatility portfolios in all three maturity bins.

Panel A also indicates that there is a weak negative relation between volatility and skewness; in all three maturity bins, skewness has a tendency to decline as volatility increases, although the effect is not monotonic in the longest two maturity bins. The relation between volatility and kurtosis in Panel A is much stronger: as average volatility increases in the portfolio, kurtosis declines in all three maturity bins. Thus, the relation between volatility and returns may be confounded by the effect, if any, of other moments on returns; we examine this possibility in later sections of the paper. Finally, the average number of securities in each portfolio indicates that the portfolios should be relatively well-diversified. The minimum number of securities, in the shortest maturity bin, is 85; combined with the fact that we are sampling securities which have publicly traded options, this should reduce the effect of outlier firms on our results.

Panel B of Table 1 sorts securities into portfolios on the basis of estimated skewness. Interestingly, we see significant differences in returns across skewness-ranked portfolios. The raw returns differential is negative for all four maturities, at 26, 43, 47 and 44 basis points per month, respectively. That is, on average, in each maturity bin the securities with lower skewness earn higher returns in the next month, while securities with less negative, or positive, skewness earn lower returns. The differentials in raw returns are of the same order of magnitude or larger than that observed in the volatility-ranked portfolios in Panel A. Compared to the volatility-ranked portfolios, the skewness-ranked portfolios show relatively little difference in their average market capitalization and betas, although differences in book-to-market equity ratios remain. When we adjust for the size- and book-to-market characteristics of securities, the characteristic-adjusted returns are reduced only slightly, and average 28, 43, 39 and 40 basis points per month, respectively, across the maturity bins.[6]

In addition to the differences in returns, the table indicates that there is a negative relation between skewness and both volatility and kurtosis. That is, both volatility and kurtosis decline as we move across skewness-ranked portfolios. As in Panel A, interactions between other moments and returns could be masking or exacerbating the relation between skewness and returns. Consequently, in later tests, we control for the relation of other higher moments to returns in estimating their effect.

Finally, Panel C of Table 1 reports the results when securities are sorted on the basis of estimated kurtosis. Generally, we see a positive relation between kurtosis and subsequent raw returns; the return differential is economically significant, at 12, 31, 35 and 37 basis points per month across the four maturities. As with the other moment-ranked portfolios, the effect

[6] In a different application, Xing, Zhang and Zing (2007) find a positive relation between a skewness metric taken from option prices and the next month's returns. Their measure of skewness is the absolute value of the difference in implied volatilities in out-of-the-money call option contracts, where the strike price is constrained to be within the range of 0.8S to S, and preferably in the range of 0.95S to S. Thus, their skewness measure is related to the slope of the volatility smile over a smaller range of strike prices.

is reduced after adjusting for book-to-market and market capitalization differences, but the differences are very slight and the effect remains highly significant, at 14, 30, 35 and 36 basis points per month across maturity bins. We also observe patterns in the other estimated moments, with both volatility and skewness decreasing as kurtosis increases. Again, this emphasizes the need to estimate the relation of higher moments to returns simultaneously.

The results in Table 1, Panels A-C, suggest that, on average, higher moments in the distribution of securities' payoffs are related to subsequent returns. Consistent with the evidence in Ang, Hodrick, Xing and Zhang (2006a), we see that securities with higher volatility have lower subsequent returns. We also find that securities with higher skewness have lower subsequent returns, while higher kurtosis is related to higher subsequent returns. As a robustness check, in the next section we use a factor-adjustment method which controls for other characteristics of the firms.

3.3 Factor-Adjusted Returns

In Table 1, we adjust for the differences in characteristics across portfolios, following Daniel, et al(1997), by subtracting the return of the specific Fama-French portfolio to which an individual firm is assigned. However, Fama and French (1993) interpret the relation between characteristics and returns as evidence of risk factors. Consequently, we also adjust for differences in characteristics across our moment-sorted portfolios by estimating a time-series regression of the 'factor-mimicking' portfolio returns on the three factors proposed in Fama and French (1993). The dependent variable in these regressions is the monthly return from t to t+1 on portfolios formed at time t, where the portfolios consist of a long position in the portfolio of securities with the highest estimated moments, and a short position in the portfolio of securities with the lowest estimated moments. The three factors used as independent variables in the regressions are the value-weighted market portfolio return (MRP), a portfolio return based on market capitalization (SMB), and a portfolio return based on book-to-market equity (HML). As in Table 1, firms are grouped by maturity and sorted into portfolios on the basis of estimated moments (volatility, skewness and kurtosis). We report intercepts, slope coefficients for the three factors, and adjusted R-squareds. Standard errors for the coefficients are presented in parentheses, and are adjusted for serial correlation and heteroskedasticity using the Newey and West (1987) procedure.

Panels A-D of Table 2 present results for options closest to one, three, six, and twelve months to maturity, respectively. The first row of each panel contains the results for the long-short portfolio constructed from volatility-sorted portfolios. Consistent with the results in Panel A of Table 1 for characteristic-adjusted returns, we observe negative alphas in our "high-low" portfolio in all three maturity bins. The absolute magnitude of the alphas declines from 57 to 41 basis points per month across maturity bins, with t-statistics of -1.77, -1.65, -1.54 and -1.16, respectively. These results are consistent with those of Ang, Hodrick, Xing and Zhang (2006), who show that firms with high idiosyncratic volatility relative to the Fama-French model earn "abysmally low" returns.

The patterns in the intercepts for skewness-sorted portfolios (row 2 of Panels A-D of Table 2) are also consistent with that observed in Panel B of Table 1. Alphas are negative in all four maturities, significant at the 10 % level for the one month maturity and at the 5% level or better in the other three maturities. The alphas remain roughly constant in magnitude as we move from short-maturity options to long-maturity options, at 58, 67, 64 and 63 basis points per month, respectively. The negative alphas still suggest a 'low skewness' premium; that is, securities with more negative skewness earn, on average, higher returns in the subsequent months, while securities with less negative, or positive skewness, earn lower returns in subsequent months.

In the third rows of Panels A-D of Table 2, we report the results for kurtosis-sorted portfolios. Consistent with the results in Table 1, we see positive intercepts in portfolios that are long kurtosis. Alphas are positive and both economically and statistically significant, at 55, 62, 56 and 62 basis points per month, respectively, across the four maturities. Similar to the characteristic-adjusted returns in Table 2, there is no discernible trend in them across maturity bins. The magnitude of the alphas with respect to kurtosis is comparable to that observed in the skewness and volatility sorted portfolios.

There is one other noteworthy feature of Table 2. The explanatory power of the Fama-French three factors is, on average, lower for the kurtosis-sorted High-Low portfolios, and much lower for the skewness-sorted portfolios, than the volatility-sorted portfolios. Some of this difference is likely due to the fact that, as Table 1 shows, skewness and kurtosis-sorted portfolios exhibit much lower differences in size, book-to-market ratios and beta than do the volatility-sorted portfolios. However, it is also possible that there are features of the returns on moment-sorted portfolios that are not captured well by the usual firm characteristics.

The evidence that skewness in individual securities is negatively related to subsequent returns is consistent with the models of Barberis and Huang (2004), and Brunnermeier, Gollier and Parker (2005). In their papers, they note that investors who prefer positively skewed distributions may hold concentrated positions in (positively skewed) securities–that is, investors may trade off skewness against diversification, since adding securities to a portfolio will increase diversification, but at the cost of reducing skewness. The preference for skewness will increase the demand for, and consequently the price of, securities with higher skewness and consequently reduce their expected returns. This evidence is also consistent with the empirical results in Boyer, Mitton and Vorkink (2008), who generate a cross-sectional model of expected skewness for individual securities and find that portfolios sorted on expected skew generate a return differential of approximately 67 basis points per month.

3.4 Additional robustness checks

We perform several additional robustness checks on our results to examine the possibility that return differentials are driven by liquidity issues, either in the underlying equity returns or by stale or illiquid option prices. To examine the latter possibility, we add an additional filter to our sample, and eliminate the observation if there is no trading in any of the out-of-the-money options on a particular day. These results are presented in Appendix Table 1. Note that the additional data requirement reduces our sample. Although the magnitude of the differential across moment-ranked portfolios declines, we continue to find that returns are negatively related to volatility and skewness, and positively related to kurtosis.

Second, we add the liquidity factor of Pastor and Stambaugh (2003) to our time-series regression and re-estimate the factor-adjusted returns. These results are presented in Appendix Table 2. The basic results change very little. The intercepts retain negative signs for volatility and skewness and positive signs for kurtosis across all three maturity bins. Statistical significance declines slightly; the alpha for the volatility portfolio is statistically significant only in the shortest maturity bin, and the alpha for the second maturity bin kurtosis portfolio loses significance. However, the overall conclusions are similar: high volatility and high skewness stocks earn negative excess returns, and high kurtosis stocks earn positive excess returns.

Overall, both the characteristic-adjusted returns in Table 1 and the regression results in Table 2 provide evidence that higher moments in the returns distribution are associated with differences in subsequent returns, and that not all of the return differential observed can be explained by differences in the size, book-to-market, beta or liquidity differentials of the moment-sorted portfolios. That is, on average, we see some relation between the higher moments of risk-neutral returns distributions of individual securities and subsequent returns on these stocks in the underlying market. In the next section, we test whether incorporating the information in (sequential) higher moments has a significant incremental impact on the opportunity sets of investors.

4 Risk-adjusting the returns of moment-sorted portfolios

The factor regressions presented in the previous section suggest that firm characteristics do a good job of explaining the cross-sectional differences in volatility-sorted portfolios, but they perform substantially less well in explaining the returns of skew- and kurtosis-sorted portfolios. Of course, some portion of the dispersion in the returns of moment-sorted portfolios, which are not explained by characteristics, presented in Tables 1 and 2 may represent compensation for co-moments, rather than idiosyncratic effects. In this section, we return to the options market, and more particularly to the index options market, to generate an estimate of the stochastic discount factor which specifically takes higher moments into account.

4.1 Estimating an implied stochastic discount factor

We begin by extracting an estimate of the stochastic discount factor from a benchmark portfolio. If we assume that the market portfolio and its options are priced correctly, then the relation between the risk-neutral and physical density functions for the market for each state, s, can be expressed as:

$$M(s,t,t+\tau) * P(s,t,t+\tau) = e^{-r(t,t+\tau)}Q(s,t,t+\tau) \quad (8)$$

where $M(s,t,t+\tau)$ is the stochastic discount factor from time t to $t+\tau$ implied by the options market, $P(s)$ is the physical density function for the market portfolio over the same period, and $Q(s)$ is the risk-neutral density function for the market portfolio. To estimate

$M(s)$, we follow a 3-step procedure.

First, we estimate the first four moments of the market's risk-neutral and physical density. For the risk-neutral moments, we use the identical method as for the individual securities, and estimate equations (4) - (6) on OTM S&P 500 index options.

For the physical probability density, we use historical data to generate sample analogues of the physical variance, skewness, and kurtosis of the underlying market return distribution.

$$\begin{aligned}
\text{Var}(R) &= E\left[(R-\mu)^2\right] \\
\text{Skew}(R) &= \frac{E\left[(R-\mu)^3\right]}{(\text{Var}(R))^{1.5}} \\
\text{Kurt}(R) &= \frac{E\left[(R-\mu)^4\right]}{(\text{Var}(R))^{2}}
\end{aligned}$$

where μ represents the expected return.

Foster and Nelson (1996) address the question of optimal sample estimators for time-varying volatility, and suggest that reasonable estimates can, under most circumstances, be obtained with a calendar year of past daily returns data. However, later papers on integrated volatility suggest that longer samples are necessary when estimating higher moments, and in their empirical work, Bakshi, Kapadia and Madan (2003) also note that skew and kurtosis may be underestimated using shorter sample periods. We use a four-year period that begins on 2/28/93 and extends through 1/31/96. Note that this historical period ends prior to the first risk-neutral moments that we estimate. This sample period is also consistent in length to the four years of historical returns used in Jackwerth (2000) and Brown and Jackwerth (2001) to estimate the physical distribution of returns. To estimate the physical mean of the market, we follow Jackwerth (2000) and add a risk premium of 8% to the risk-free rate. The risk-free rate is the (annualized) yield on a 90-day Treasury bill, obtained from the Federal Reserve H.15 report.

Once moments for both the risk-neutral and physical distribution are generated, the second step of the procedure involves estimating the density functions of both distributions using the method described in Eriksson, Forsberg and Ghysels (2004). This procedure uses the Normal Inverse Gaussian (NIG) family to estimate an unknown distribution of random variables. As they note, the appeal of the NIG family of distributions is that they can be completely characterized by the first four moments. As a consequence, given the first four moments, one can "fill in the blanks" to obtain the entire distribution and, as they show, the method is particularly well-suited when the distribution exhibits skewness and fat tails, as it does in the returns distributions which we examine in this application.

Once the entire risk-neutral and physical distributions for the market are approximated with an NIG distribution, we use two methods to estimate the stochastic discount factor. In the first method, we simply use equation 8 to solve for $M(s)$ as the discounted ratio of the risk-neutral probability density function to the physical density function. We call the resulting stochastic discount factor $M*$.

In the second method, we begin with $M*$ and employ an additional step. We follow the method in Dittmar (2000) and parameterize the stochastic discount factor from the first step by projecting it onto a polynomial. By controlling the form of the polynomial, we can force the stochastic discount factor to include (or exclude) sequential higher moments, allowing us to examine their incremental effect on the calculation of risk-adjusted returns. For example, the stochastic discount factor $M(VAR)$ includes only linear returns, while $M(SKEW)$ includes linear and squared terms (similar to that used in Harvey and Siddique (2000)) and $M(KURT)$ includes linear, squared and cubic terms.

Finally, using each of the four stochastic discount factors above, we calculate alphas or residual returns from Table 1 by substituting realizations of the market return into $M*$ or the polynomial approximations and estimate

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \hat{M}(t, t+\tau) r(t, t+\tau) \tag{9}$$

with Newey-West (1987) standard errors. The variable $r(t, t+\tau)$ represents overlapping τ-period returns on the factor mimicking portfolios for volatility, skewness, and kurtosis, and $r_m(t, t+\tau)$ is overlapping τ-period returns on the S&P 500 index. The data cover the period June, 1996 through December, 2005; consequently, we have 115 monthly observations for the three-month stochastic discount factor, 112 observations for the 6-month stochastic discount factor and 106 observations for the 12-month stochastic discount factor. The number of Newey-West lags used to compute standard errors reflects the number of overlapping months in each sample; for example, 3 lags are used in computing standard errors for the three-month stochastic discount factor.

4.2 Comparing stochastic discount factors

The four stochastic discount factors which we estimate are presented, over the range of possible market returns and the entire sample period, in Figure 1. For brevity, we focus on options closest to twelve months to maturity; results are qualitatively similar for other maturities. In the top figure, we present the four pricing kernels over the full support; in the bottom figure, we present the three polynomial approximations $M(VAR)$, $M(SKEW)$ and $M(KURT)$ over a partial support to better illustrate the differences over this range.

The linear stochastic discount factor $M(VAR)$ is downward sloping throughout its range; over the partial support, as is $M(SKEW)$. The cubic stochastic discount factor, $M(KURT)$, declines through most of its support, deviating only at extremely high and low values for the return on the market portfolio. These results are generally consistent with the behavior of investors who have declining relative risk-aversion. In contrast, note that the non-parametric stochastic discount factor presented in the top graph $M*$ has a segment which is sharply increasing. An upward sloping segment of the stochastic discount factor implied from option prices is consistent with the evidence in Jackwerth (2000) and Brown and Jackwerth (2001) and is, as these papers point out, a puzzle–it suggests that the representative investor may be risk-seeking over the upward sloping range. Brown and Jackwerth (2001) examine several possibilities for this behavior. Although we do not focus specifically on this puzzle in the current paper, it is worth noting that we obtain a similar result despite the fact that our sample period does not overlap with the sample used in the Brown and Jackwerth (2001) paper, and the estimation methods used to estimate both the risk-neutral distribution and physical distribution are different. Thus, the empirical evidence suggests that the observation of an upward sloped segment in the non-parametric stochastic discount factor implied by option prices is robust to both sample and method. In addition, the range over which the sharpest increase in the stochastic discount factor is observed is quite similar to the range over which Brown and Jackwerth observe their upward-sloping segment, at approximately 0.97 to 1.03.

Although the behavior of the polynomial approximations of M exhibit clear differences from the non-parametric discount factor $M*$, the 'fit' of the polynomial approximations is statistically significant; the average R^2 of $M(VAR)$, $M(SKEW)$ and $M(KURT)$ is 5.5%, 13.2% and 16.3%, respectively. Moreover, the inferences on residual returns presented in the next section are relatively unaffected by the particular stochastic discount factor chosen.

In the next section, we examine the implications of the estimated empirical stochastic discount factors for investors' expectations of the payoffs to individual securities, and consequently to the moment-sorted portfolios in Table 1.

4.3 Risk-adjusted returns

In Table 3, we report estimates of alphas calculated from each of the stochastic discount factors estimated above. For one-month maturity options, the assumptions underlying the NIG approximation were violated at several points in the time-series; as a consequence, we report results only for 3-month, 6-month and 12-month maturity bins.[7] The alphas are calculated for each of the Hi-Lo moment-sorted portfolios (volatility, skewness and kurtosis) using equation (**??**).

The results suggest that idiosyncratic skewness is important, even after allowing for the effects of higher moments on the stochastic discount factor. Specifically, the alphas for the skewness sorted portfolios have p-values of approximately 12% for 3-month options, and at the 5% level or better for 6- and 12-month options. The alphas related to skewness are economically significant as well, ranging from 39 to 64 basis points per month. In contrast, the alphas related to volatility are not statistically significant in any maturity bin, for any specification of the stochastic discount factor. The alphas for the kurtosis-sorted portfolio are marginally significant in the shortest maturity bin, but are not significant in the samples of either 6- and 12-month options. As with volatility, these results are not sensitive to the stochastic discount factor used to calculate alphas.

The residual importance of idiosyncratic skewness is consistent with models, such as Barberis and Huang (2004), and Brunnermeier, Gollier and Parker (2007), which suggest that investors have a preference for skewness in individual securities above and beyond their contribution to the co-skewness of the portfolio. It is also consistent with the empirical evidence in Mitton and Vorkink (2007), who find a relation between the skewness in individual securities in individuals' brokerage accounts and subsequent returns.

[7] The NIG approximation could always be calculated for 3- and 12-month options; the assumptions were violated for only one month using the sample of 6-month options.

Our results do not necessarily imply that the alpha, or residual return, is an arbitrage profit. Our estimate of the stochastic discount factor controls only for non-diversifiable risk, including the risk of higher co-moments. If investors have a preference for individual skewness and, as a result, push up the price of securities which are perceived to have a higher probability of an extremely good outcome, the lower subsequent returns of high-skew stocks may represent an equilibrium result.

5 Implied physical probability distributions

To this point, we have focused on the estimation of risk-neutral moments, and the relation of these moments to returns. However, the models that consider the effects on expected returns of skewness and fat tails in individual securities' distributions deal with investors' estimates of the physical distribution. Given an estimate of the stochastic discount factor, and risk-neutral distributions of individual securities, we can construct a market-based estimate of individual securities' physical distributions that does not rely on historical data.

Specifically, we take the stochastic discount factors $M*$, $M(VAR)$, $M(SKEW)$ and $M(KURT)$ constructed from the market portfolio and its options, and, using equation 8, apply this to the risk-neutral distribution constructed from individual securities' options data to reverse engineer an estimate of the underlying security's physical probability distribution. We examine the differences in this measure of investors' expectations across securities and across time.

We aggregate securities into industry portfolios, using the ten industry groupings available on Kenneth French's website. We assign every individual security for which we can estimate risk-neutral moments into one of these industry groupings. The Utilities portfolio had very few firms in our sample, and some months which were missing observations; consequently, we eliminate that industry portfolio as well as "Other", and report results for eight out of ten of the industry portfolios. As results across the three polynomial approximations and the NIG approximation exhibit little difference, we present results only for the stochastic discount factor calculated using the NIG approximation, or $M*$.

For each industry, we present the equal-weighted imputed physical distribution in Figure 2 at intervals in our sample period. The intervals presented are selected to accord roughly with interesting economic events (the Asian crisis, the 'bubble' period, the recession of 2000-

2001 and the recovery); although the intervals we present are chosen with perfect hindsight, recall that the risk-neutral moments at any time t are *ex ante* in nature. For comparison, at each interval we present an estimate of the distribution taken from four years of historical data ending at time t for that industry portfolio.

The implied physical distributions constructed from options market data, and the estimated pricing kernel, appear much more stable than the distributions estimated from rolling historical data. For example, note that using historical data generates negative mean returns for three out of eight industries in the fourth subperiod (Q103-Q405). This is clearly an artifact of the inclusion of the market downturn in the historical time-series. In addition, skewness estimates based on historical data are much more variable, and the distributions tend to be left-skewed; approximately 70% (23/32) of the skewness estimates across subperiods and industries are negative. Although we do not report Sharpe ratios calculated from historical distributions, they are extremely variable, ranging from -0.36 to 1.32. Overall, using historical data to generate estimates of investors' subjective probabilities generates distributions which are highly sensitive to prior events, and have very different implications for investors' opportunity sets.

In Table 4, we present estimates of the first four moments of the implied physical distribution for each industry and for the same intervals presented in Figure 2; these estimates are constructed by integrating over market states. We also present estimates of the Sharpe ratio for each industry portfolio, as well as correlations across industries between the 4 moments and the Sharpe ratio.

There are several striking results in Table 4. First, the Sharpe ratios are comparatively stable, ranging from 0.07 to 0.26. We do, however, observe a sharp increase in the *ex ante* estimates of the Sharpe ratio through our sample period. In the last interval (03Q1-05Q4), Sharpe ratios in every industry grouping are at least double what they are in the earliest interval (96Q2-98Q2). Thus, Sharpe ratios in the pre-crash period are significantly lower than Sharpe ratios in the recovery. Second, we observe a strong negative correlation between skewness and the Sharpe ratio across industries–that is, industries which are expected to have higher extreme outcomes also have lower *ex ante* Sharpe ratios. This correlation is very strongly negative throughout all four intervals examined, varying between -0.75 and -0.91, and averages -0.83 over the entire sample period. This result suggests that investors are trading off traditional risk-reward ratios for the likelihood of extreme 'good news', and is consistent with a preference for idiosyncratic skewness. However, the second surprising result in Table 4 is that firms in the Technology portfolio have the *lowest* skew (and the highest Sharpe ratio); this relation also holds throughout the sample period. In contrast, firms in the Durables portfolio have the highest skew and the lowest Sharpe ratio. And, the difference in skewness across these portfolios is large–for example, the percentage increase in skewness from the Tech portfolio to the other industry groupings varies from 15% to 72%.

Thus, we find little evidence in imputed physical probability distributions that the high prices of technology firms during the Internet bubble period is related to investors' expectations that these firms had a relatively high chance of extremely good outcomes. In fact, the relatively high *ex ante* Sharpe ratios of these firms compared to firms in other industries suggests that investors believed that these firms were good 'mean variance' bets. Of course, it may be that the requirement that firms have options traded on them limits our ability to examine younger, more skewed firms. However,while the technology portfolio we analyze does include 'established' firms such as IBM, Cisco and Microsoft, it also includes relative newcomers such as Amazon, Iomega, JDS Uniphase, Real Networks, Xilinx, etc.

6 Conclusion

We explore the possibility that higher moments of the returns distribution are important in explaining security returns. Using a sample of option prices from 1996-2005, we estimate the moments of the risk-neutral density function for individual securities using the methodology of Bakshi, Kapadia and Madan (2003). We analyze the relation between volatility, skewness and kurtosis and subsequent returns.

We find a strong relation between these moments and returns. Specifically, we find that high (low) volatility firms are associated with lower (higher) returns over the next month. This result is consistent with the results of Ang, Hodrick, Xing and Zhang (2006). We also find that skewness has a strong negative relation with subsequent returns; firms with lower negative skewness, or positive skewness, earn lower returns. That is, investors seem to prefer positive skewness, and the returns differential associated with skewness is both economically and statistically significant. We also find a positive relation between kurtosis and returns. These relations are robust to controls for differences in firm characteristics, such as firm size, book-to-market ratios and betas, as well as liquidity and momentum.

We use index returns and index options to estimate an empirical stochastic discount factor, as well as polynomial approximations of the stochastic discount factor. We use these stochastic discount factors to calculate risk-adjusted returns to portfolios sorted on the basis of volatility, skewness and kurtosis, where the risk-adjustment explicitly takes higher co-moments into account. After controlling for higher co-moments, we find weak evidence that idiosyncratic kurtosis matters for short maturities, and strong evidence that idiosyncratic skewness has significant residual predictive power for subsequent returns across maturities. This suggests that investors have a preference for the skewness in individual securities, which is consistent with the models of Barberis and Huang (2004) and Brunnermeier, Gollier and Parker (2007).

Finally, we use the estimated stochastic discount factors, and the risk-neutral distributions calculated for individual securities, to estimate implied physical distributions for securities. We find several interesting results. First, our results suggest that implied physical distributions are much more stable than those constructed using historical data. Second, in implied physical distributions, we find evidence of a trade-off between skewness in industry portfolios and *ex ante* estimates of the Sharpe ratios for the industry. That is, our results suggest a trade-off between expected returns and higher moments, with higher (lower) traditional risk-reward measures associated with lower (higher) skewness. However, we also find that the portfolio containing technology firms has low *ex ante* physical skew and kurtosis, and a high Sharpe ratio. Consequently, while we find *both* that higher moments matter, and that investors' expectations of higher moments change through time, our results do not appear to be an explanation of bubble pricing in the Internet period.

References

[1] Abreau, D. and M. Brunnermeier, "Synchronization risk and delayed arbitrage", 2002, *Journal of Financial Economics* 66, pp. 341-360.

[2] Ait-Sahalia, Y. and M. Brandt, "Consumption and Portfolio Choice with Option-Implied State Prices", 2007, Working Paper, Princeton University.

[3] Ang, A., R. Hodrick, Y. Xing and X. Zhang, "The cross-section of volatility and expected returns," 2006, *Journal of Finance* 51, pp. 259-299.

[4] Ang, A., R. Hodrick, Y. Xing and X. Zhang, "High idiosyncratic volatility and low returns: International and further U.S. evidence," 2006, forthcoming, *Journal of Financial Economics.*

[5] Bakshi, G. and D. Madan, "Spanning and derivative-security valuation", 2000, *Journal of Financial Economics* 55, pp. 205-238.

[6] Bakshi, G., N. Kapadia and D. Madan, "Stock return characteristics, skew laws and the differential pricing of individual equity options", 2003, *Review of Financial Studies* 16, pp. 101-143.

[7] Barberis, N. and M. Huang, "Stocks as lotteries: The implications of probability weighting for security prices", 2004, Unpublished manuscript, Yale University.

[8] Bates, D., "The Crash of '87: Was it expected? The evidence from options markets", 1991, *Journal of Finance* 46, pp. 1009-1044.

[9] Battalio, R. and P. Schultz, "Options and the bubble", 2006, *Journal of Finance* 61, pp. 2071-2102.

[10] Bekaert, G. and J. Liu, "Conditioning Information and Variance Bounds on Pricing Kernels", 2004, *Review of Financial Studies* 17, 339-378.

[11] Bekaert, G. and M. Urias, "Diversification, Integration, and Emerging Market Closed-End Funds", 1996, *Journal of Finance* 51, pp. 835-869.

[12] Boyer, B., T. Mitton and K. Vorkink, "Expected Idiosyncratic Skewness", 2008, Working Paper, Brigham Young University.

[13] Campbell, J.Y., M. Lettau, B.G. Malkiel and Y. Xu,"Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Ris", 2001, *Journal of Finance* 56, pp. 1-43.

[14] Carhart, M., 1997, "On persistence in mutual fund performance," *Journal of Finance* 52, pp. 57-82.

[15] Chabi-Yo, F., "Conditioning Information and Variance Bounds on Pricing Kernels with Higher-Order Moments: Theory and Evidence", 2007, *Review of Financial Studies* (forthcoming).

[16] Chabi-Yo, F., R. Garcia and E. Renault, "The Stochastic Discount Factor: Extending the Volatility Bound and a New Approach to Portfolio Selection with Higher-Order Moments", 2005, Bank of Canada Discussion Paper 2005-2.

[17] Chabi-Yo, F., E. Ghysels and E. Renault, "Disentangling the Effects of Heterogeneous Beliefs and Preferences on Asset Prices", 2006, Discussion Paper UNC.

[18] Chernov, M. and E. Ghysels, "A study towards a unified approach to the joint estimation of objective and risk neutral measures for the purpose of options valuation", 2000, *Journal of Financial Economics* 56, pp. 407-458.

[19] Daniel, K., M. Grinblatt, S. Titman, and R. Wermers, 1997, "Measuring mutual fund performance with characteristic based benchmarks," *Journal of Finance* 52, 1035-1058.

[20] Dennis, P. and S. Mayhew, "Risk-neutral skewness: Evidence from stock options", 2002, *Journal of Financial and Quantitative Analysis* 37, pp. 471-493.

[21] DeSantis, G., 1995, "Volitality bounds for stochastic discount factors: Tests and implications from international stock returns," working paper, University of Southern California.

[22] Dittmar, R.F., 2002, "Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns," *Journal of Finance* 57, pp. 369-403.

[23] Fama, E. and K. French, 1993, "Common risk factors in the returns on stocks and bonds," *Journal of Financial Economics* 33, pp. 3-56.

[24] Fama, E. and K. French, 1997, "Industry costs of equity," *Jouranl of Finance* 52.

[25] Ferson,W. and A.F. Siegel, "The Efficient Use of Conditioning Information in Portfolios", 2001 *Journal of Finance* 56, pp. 967-982.

[26] Ferson,W. and A.F. Siegel, "Stochastic Discount Factor Bounds with Conditioning Information", 2003, *Review of Financial Studies* 16, 567-595.

[27] Foster, D.F. and D.B. Nelson,"Continuous Record Asympttics for Rolling Sample Variance Estimators", 1996, *Econometrica* 64, 139-174.

[28] Gallant, R., L.P. Hansen, and G. Tauchen, "Using Conditional Moments of Asset Payoffs to Infer the Volatility of Intertemporal Marginal Rates of Substitution", 1990, *Journal of Econometrics* 45, 141-179.

[29] Geczy, C., D. Musto and A. Reed, "Stocks are special too: An analysis of the equity lending market," 2002, *Journal of Financial Economics* 66, pp. 241-269.

[30] Gomes, J., L. Kogan, and M. Yogo, 2007, "Durability of output and expected stock returns," working paper, Wharton School, University of Pennsylvania.

[31] Hansen, L.P. and R. Jagannathan, "Implications of Security Market Data for Models of Dynamic Economies," 1991, *Journal of Political Economy* 91, 249-265.

[32] Harvey, C. and A. Siddique, "Conditional skewness in asset pricing tests," 2000, *Journal of Finance* 55, pp. 1263-1296.

[33] Huberman, G. and E. Kandel, "Mean-Variance Spanning", 1987, *Journal of Finance* 42, pp. 873-888.

[34] Jackwerth, J.,"Recovering Risk Aversion from Option Prices and Realized Returns", 2000, *Review of Financial Studies* 13, pp. 433-451.

[35] Jackwerth, J. and M. Rubinstein, "Recovering probability distributions from option prices", 1996, *Journal of Finance* 51, pp. 1611-1631.

[36] Mitton, T. and K. Vorkink, "Equilibrium Underdiversification and the Preference for Skewness", 2007, *Review of Financial Studies* 20, pp. 1255-1288.

[37] Ofek, E. and M. Richardson, 2003, "DotCom mania: The rise and fall and fall of internet stocks," forthcoming, *Journal of Finance*.

[38] Pastor, L. and R. Stambaugh, 2003, "Liquidity risk and expected stock returns," *Journal of Political Economy* 111, pp. 642-685.

[39] Rubinstein, M., "Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978," 1985, *Journal of Finance* 40, pp. 455-480.

[40] Rubinstein, M., "Implied binomial trees," 1994, *Journal of Finance* 49, pp. 771-818.

[41] Shleifer, A. and R. Vishny, "The limits of arbitrage", 1997, *Journal of Finance* 52, pp. 35-55.

[42] Siegel, J., "Irrational exuberance, reconsidered," December 6, 2006, *Wall Street Journal*.

[43] Spiegel, M. and X. Wang, "Cross-sectional variation in stock returns: Liquidity and Idiosyncratic Risk," 2006, Unpublished manuscript, Yale University.

[44] Tversky, A., and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," 1992, *Journal of Risk and Uncertainty* 5, pp. 297-323.

[45] Xing, Y., X. Zhang and R. Zhao, "What Does Individual Option Volatility Smirk Tell Us about Future Equity Returns?", 2007, Working Paper, Rice University.

Table 1: Descriptive Statistics

Table 1, Panels A-C, present moment characteristics of portfolios sorted into tertiles on the basis of volatility, skewness and kurtosis. The first column presents mean returns in the month subsequent to portfolio classification. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama-French 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average beta, log market value and book-to-market equity ratio of the portfolio, while the next three columns present the average volatility, skewness and kurtosis of the portfolio. The last column presents the average number of securities in the portfolio. Moment estimates are separated into three maturity bins: the first maturity bin covers options with less than 3 months to expiration, the second with expiration between 3 and 6 months, and the third contains options with expirations greater than 6 months. Monthly return data cover the period 2/96 through 12/05, for a total of 119 monthly observations.

Panel A: Volatility-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.213	0.268	16.104	-1.363	12.888	0.890	15.703	0.368
2	0.963	0.128	24.994	-0.968	8.842	1.281	14.304	0.393
3	0.893	0.172	44.033	-1.171	6.041	1.772	13.619	0.417
3-1	-0.320	-0.096	27.929	0.192	-6.847	0.883	-2.084	0.049

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.237	0.273	17.139	-1.180	10.812	0.837	15.675	0.386
2	1.061	0.190	26.458	-0.934	8.166	1.290	14.299	0.391
3	0.738	0.062	45.890	-1.203	5.993	1.828	13.648	0.402
3-1	-0.499	-0.211	28.751	-0.023	-4.819	0.990	-2.028	0.016

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.215	0.227	18.770	-0.713	6.621	0.816	15.617	0.397
2	1.137	0.266	28.656	-0.576	5.480	1.287	14.336	0.393
3	0.659	0.002	47.734	-0.749	4.148	1.861	13.658	0.386
3-1	-0.556	-0.225	28.964	-0.036	-2.473	1.045	-1.959	-0.012

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.237	0.227	19.353	-0.692	6.416	0.824	15.453	0.402
2	1.060	0.195	29.541	-0.615	5.544	1.291	14.350	0.391
3	0.739	0.098	49.892	-0.826	4.259	1.844	13.807	0.384
3-1	-0.498	-0.129	30.539	-0.134	-2.156	1.020	-1.646	-0.018

Table continued on next page...

Panel B: Skewness-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.233	0.391	26.626	-2.642	16.231	1.274	15.318	0.343
2	0.886	0.088	30.095	-0.975	6.847	1.365	14.351	0.398
3	0.975	0.110	26.699	0.116	5.365	1.228	13.961	0.436
3-1	-0.257	-0.281	0.074	2.758	-10.866	-0.046	-1.357	0.093

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.281	0.421	29.444	-2.530	14.303	1.262	15.283	0.354
2	0.944	0.146	31.132	-0.923	6.303	1.366	14.377	0.395
3	0.849	-0.009	27.345	0.131	4.992	1.242	13.961	0.429
3-1	-0.432	-0.430	-2.099	2.661	-9.311	-0.020	-1.322	0.076

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.341	0.476	32.812	-1.717	8.891	1.260	15.276	0.378
2	0.886	0.042	31.157	-0.527	4.181	1.318	14.444	0.390
3	0.867	0.085	30.353	0.190	3.614	1.311	13.874	0.412
3-1	-0.474	-0.391	-2.459	1.907	-5.277	0.051	-1.402	0.034

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.325	0.487	35.105	-1.835	9.206	1.242	15.364	0.374
2	0.888	0.027	32.013	-0.524	4.008	1.326	14.398	0.392
3	0.881	0.090	30.842	0.195	3.522	1.319	13.847	0.413
3-1	-0.444	-0.397	-4.263	2.030	-5.684	0.077	-1.518	0.039

Table continued on next page...

Panel C: Kurtosis-Sorted Portfolios

1 Month to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	1.020	0.172	35.337	-0.358	2.864	1.387	13.665	0.461
2	0.925	0.110	27.185	-0.909	6.976	1.293	14.388	0.394
3	1.137	0.309	21.876	-2.254	18.556	1.217	15.556	0.325
3-1	0.117	0.138	-13.461	-1.896	15.691	-0.170	1.891	-0.136

3 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.938	0.109	35.972	-0.346	2.775	1.418	13.650	0.452
2	0.901	0.075	28.714	-0.865	6.482	1.310	14.401	0.391
3	1.251	0.410	24.049	-2.129	16.280	1.169	15.550	0.338
3-1	0.313	0.301	-11.923	-1.782	13.505	-0.249	1.899	-0.115

6 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.860	0.029	37.226	-0.189	2.155	1.449	13.661	0.427
2	0.987	0.149	30.722	-0.499	4.249	1.324	14.426	0.386
3	1.213	0.383	26.522	-1.375	10.259	1.124	15.507	0.370
3-1	0.353	0.354	-10.704	-1.186	8.104	-0.325	1.847	-0.057

12 Months to Maturity

Rank	Mean	Char Adj	Vol	Skew	Kurt	Beta	ln MV	BM
1	0.857	0.019	37.729	-0.179	2.097	1.465	13.669	0.424
2	0.980	0.158	32.052	-0.497	4.103	1.330	14.394	0.388
3	1.226	0.383	28.165	-1.496	10.502	1.102	15.542	0.369
3-1	0.369	0.364	-9.564	-1.316	8.404	-0.363	1.873	-0.055

Table 2: Time Series Regressions

Table 2 presents the results of time-series regressions of excess return differentials (Hi-Lo) between portfolios ranked on risk neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), and HML (the difference in returns on a portfolio of high and low book equity to market equity stocks). The moment-sorted portfolios are equally-weighted, formed on the basis of terciles and re-formed each month. The table presents point estimates of the coefficients and standard errors in parentheses. Data cover the period January 1996 through December 2005 for 119 monthly observations.

Panel A: 1 Month to Maturity

	α	β_{MRP}	β_{SMB}	β_{HML}	R^2
Vol	-0.57	0.51	0.83	-0.55	0.76
	-1.77	5.34	9.23	-5.29	
Skew	-0.58	0.14	-0.05	0.55	0.27
	-1.71	1.66	-0.37	4.23	
Kurt	0.55	-0.20	-0.30	-0.51	0.28
	2.49	-3.49	-3.17	-6.32	

Panel B: 3 Months to Maturity

	α	β_{MRP}	β_{SMB}	β_{HML}	R^2
Vol	-0.56	0.56	0.89	-1.03	0.84
	-1.65	5.00	10.21	-8.92	
Skew	-0.67	0.19	-0.13	0.37	0.17
	-1.99	2.34	-0.83	2.63	
Kurt	0.62	-0.30	-0.23	-0.16	0.21
	2.45	-4.42	-1.97	-1.37	

Panel C: 6 Months to Maturity

	α	β_{MRP}	β_{SMB}	β_{HML}	R^2
Vol	-0.55	0.59	0.90	-1.22	0.85
	-1.54	5.06	10.37	-9.73	
Skew	-0.64	0.18	0.00	0.14	0.05
	-2.43	2.48	0.00	1.10	
Kurt	0.56	-0.35	-0.26	0.10	0.44
	2.38	-4.25	-2.15	0.95	

Panel D: 12 Months to Maturity

	α	β_{MRP}	β_{SMB}	β_{HML}	R^2
Vol	-0.41	0.54	0.83	-1.29	0.85
	-1.16	4.70	10.20	-10.49	
Skew	-0.62	0.20	0.06	0.11	0.07
	-2.42	2.83	0.42	0.88	
Kurt	0.62	-0.38	-0.31	0.10	0.50
	2.65	-4.65	-2.51	0.96	

Table 3: Stochastic Discount Factor Risk Adjustments

The table presents risk adjustments for the volatility, skewness, and kurtosis factor mimicking portfolios using stochastic discount factors implied by the S&P 500 risk neutral and physical densities. The table presents returns in excess of those implied by discounting using the stochastic discount factor, calculated as

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T} r_t m_t$$

where r_t is the return on the factor-mimicking portfolio at time t, and m_t is the stochastic discount factor. Columns "Linear," "Quad," and "Cubic" represent discount factors obtained by projecting the density-implied discount factor onto a linear, quadratic, and cubic polynomial, respectively. Panel A presents results using overlapping quarterly returns and the discount factor implied by 3 month maturity options; Panels B and C present similar results using 6 month and 12 month horizons. Point estimates are scaled to the monthly frequency. Newey-West standard errors are presented in parentheses below the point estimates. Data in Panel A extend from June, 1996 through December, 2005 for 115 monthly observations. Data in Panel B cover the period September, 1996 through December, 2005 for 112 monthly observations. Data in Panel C cover the period March, 1996 through December, 2005 for 106 monthly observations.

Panel A: 3 Months

	NIG	Linear	Quad	Cubic
Vol	-0.405	-0.358	-0.437	-0.192
SE	(0.844)	(0.893)	(0.895)	(0.797)
Skew	-0.594	-0.620	-0.619	-0.597
SE	(0.381)	(0.401)	(0.402)	(0.377)
Kurt	0.479	0.456	0.512	0.450
SE	(0.254)	(0.254)	(0.263)	(0.265)

Panel B: 6 Months

	NIG	Linear	Quad	Cubic
Vol	-0.081	-0.285	-0.089	0.645
SE	(1.180)	(1.182)	(1.265)	(1.205)
Skew	-0.631	-0.626	-0.636	-0.631
SE	(0.283)	(0.281)	(0.293)	(0.303)
Kurt	0.345	0.403	0.367	0.118
SE	(0.321)	(0.327)	(0.335)	(0.294)

Panel C: 12 Months

	NIG	Linear	Quad	Cubic
Vol	-0.130	-0.283	-0.228	0.371
SE	(0.557)	(0.561)	(0.570)	(0.528)
Skew	-0.555	-0.544	-0.543	-0.619
SE	(0.278)	(0.265)	(0.277)	(0.321)
Kurt	0.411	0.477	0.473	0.160
SE	(0.304)	(0.302)	(0.315)	(0.290)

Table 4: Imputed Physical Moments

The table presents moments of imputed physical distributions of eight industry portfolios. Distributions are imputed by letting the physical distribution, $f^P(x,s,\tau)$ be related to the risk neutral distribution, $f^Q(x,s,\tau)$ by

$$f^P(x,s,\tau) = e^{-r_f\tau}\frac{f^Q(x,s,\tau)}{m(x,s,\tau)}$$

where $m(x,s,\tau)$ is the stochastic discount factor implied by the S&P 500 index. The risk neutral distribution is the equally-weighted risk neutral distribution across firms implied by risk neutral moments retrieved from option prices and the NIG probability density. We calculate the moments for four subperiods: 1996 Q2 - 1998 Q2, 1998 Q3 - 2000 Q1, 2000 Q2 - 2002 Q4, and 2003 Q1 - 2005 Q4.

Panel A: Mean

Subperiod	NonDur	Dur	Mfg	Energy	Tech	Telecom	Wh/Ret	Health
I	0.0739	0.0753	0.0786	0.0770	0.0917	0.0859	0.0845	0.0882
II	0.0702	0.0720	0.0792	0.0859	0.0982	0.0940	0.0818	0.0893
III	0.0825	0.0860	0.0882	0.0886	0.1093	0.0947	0.0897	0.0988
IV	0.0834	0.0747	0.0884	0.0918	0.1004	0.0858	0.0863	0.0982

Panel B: Volatility

Subperiod	NonDur	Dur	Mfg	Energy	Tech	Telecom	Wh/Ret	Health
I	0.2724	0.2787	0.2856	0.2852	0.3422	0.3119	0.3222	0.3348
II	0.2730	0.2742	0.2801	0.2959	0.3294	0.3109	0.3079	0.3221
III	0.2811	0.2925	0.2936	0.2925	0.3625	0.3282	0.3063	0.3345
IV	0.2684	0.2601	0.2799	0.2729	0.3266	0.3019	0.2905	0.3205

Panel C: Skewness

Subperiod	NonDur	Dur	Mfg	Energy	Tech	Telecom	Wh/Ret	Health
I	0.4785	0.4918	0.4696	0.4861	0.3589	0.4339	0.3871	0.3544
II	0.6171	0.5555	0.4997	0.3823	0.3612	0.3905	0.4705	0.4238
III	0.4007	0.4507	0.4061	0.3690	0.2869	0.4062	0.3901	0.3613
IV	0.4328	0.5507	0.2784	0.2019	0.2135	0.3667	0.2824	0.2511

Panel D: Kurtosis

	NonDur	Dur	Mfg	Energy	Tech	Telecom	Wh/Ret	Health
I	6.0171	5.6818	5.3311	5.2650	2.9991	4.3173	3.6724	3.1631
II	6.8163	6.3679	6.0763	4.9900	3.6533	4.4124	4.6471	4.0329
III	5.6132	5.3446	5.1082	5.0510	2.5129	3.7166	4.4912	3.4917
IV	6.7235	7.3218	5.1764	5.7343	3.2445	4.7178	4.7419	3.5187

Sharpe Ratio

	NonDur	Dur	Mfg	Energy	Tech	Telecom	Wh/Ret	Health
I	0.0770	0.0807	0.0899	0.0859	0.1136	0.1065	0.0975	0.1056
II	0.0707	0.0784	0.1008	0.1186	0.1444	0.1400	0.1010	0.1193
III	0.1721	0.1777	0.1850	0.1848	0.2054	0.1830	0.1811	0.1926
IV	0.2353	0.2059	0.2392	0.2605	0.2414	0.2109	0.2227	0.2388

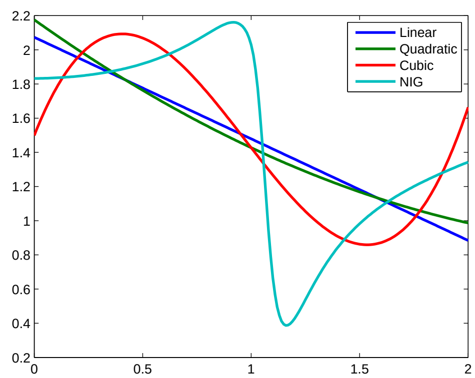
Figure 1: Stochastic Discount Factors

Figure 1 depicts stochastic discount factors formed using risk neutral moments of S&P 500 index options. The plot labeled 'NIG' represents stochastic discount factors, $m(x, s, tau)$, formed as

$$m(x,s,\tau) = e^{-r_f \tau} \frac{f^Q(x,s,\tau)}{f^P(x,s,\tau)}$$

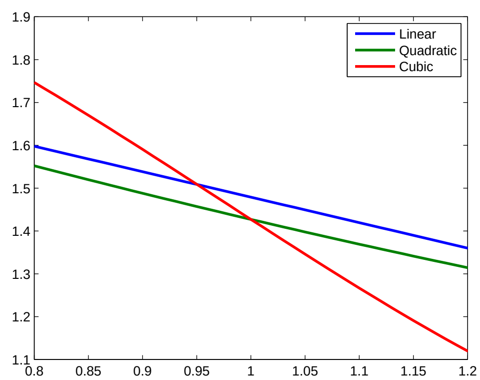
where $f(\cdot)$ is the NIG probability density function, Q denotes the risk-neutral probability measure, and P denotes the physical measure. The risk neutral measure is calculated using risk neutral moments retrieved from option prices and the physical measure using the historical moments of the S&P 500 index from 1992 through 1995. 'Linear,' 'Quadratic,' and 'Cubic' represent linear, quadratic, and cubic polynomial fits to the NIG kernel. Subfigure A depicts plots of the average stochastic discount factor for all four kernels; Subfigure B depicts the polynomial kernels over a smaller range.

Figure A



..

Figure B

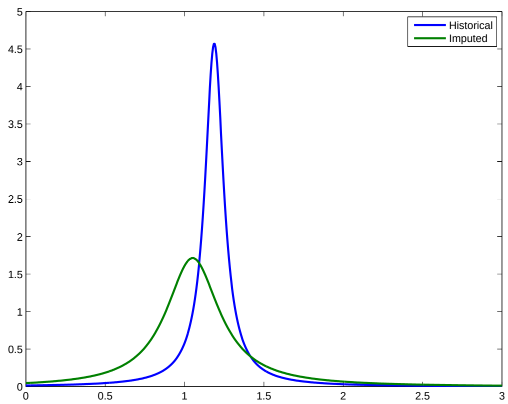


..

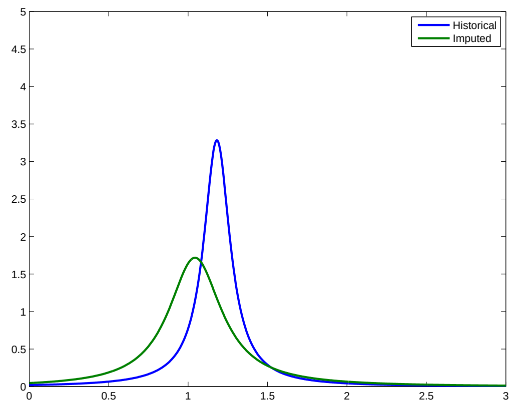
Figure 2: Imputed and Historical Probability Densities

Figure 2 depicts the probability densities for eight industry portfolios implied by historical and imputed moments. Historical moments are calculated from equally-weighted daily returns on each industry portfolio over the past four years, updated quarterly. Imputed moments are obtained by imputing the physical probability density for the industry portfolio using its risk neutral probability measure and the stochastic discount factor obtained from the S&P 500 index. Averages of moments over the relevant time periods are then used to calculate the NIG density function, evaluated at these moments. For each industry, we examine densities over four subperiods: 1996 Q2 - 1998 Q2, 1998 Q3 - 2000 Q1, 2000 Q2 - 2002 Q4, and 2003 Q1 - 2005 Q4.

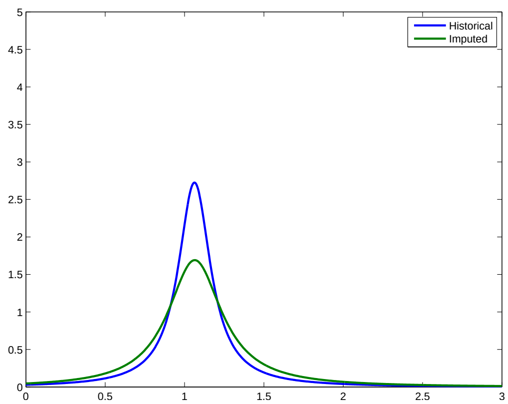
Nondurables



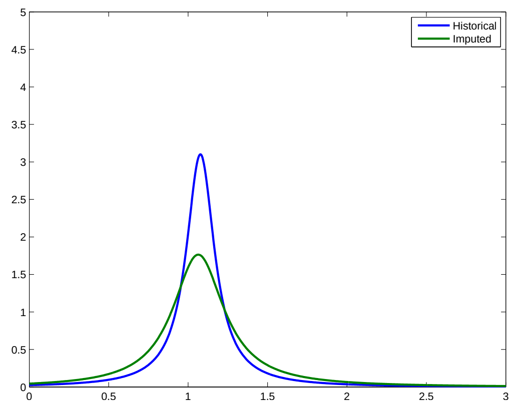
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



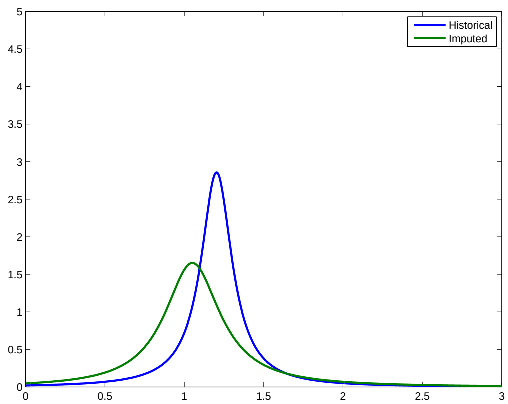
(c) 2000 Q2 - 2002 Q3



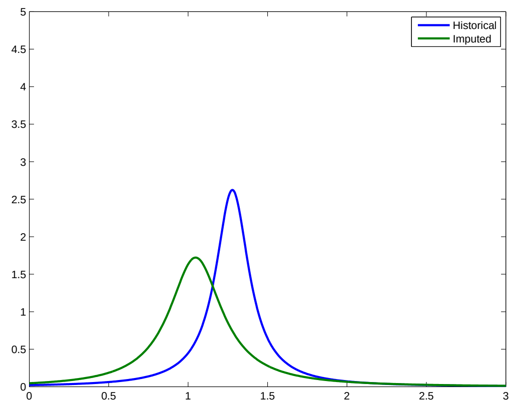
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

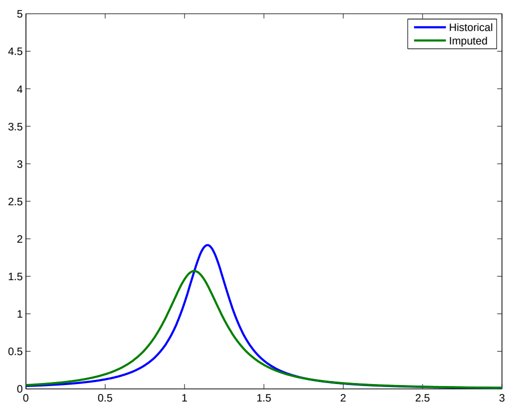
Durables



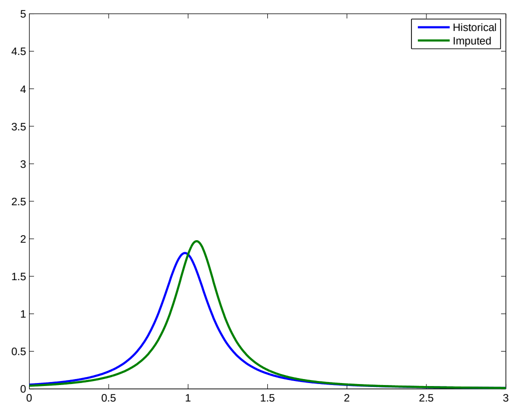
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



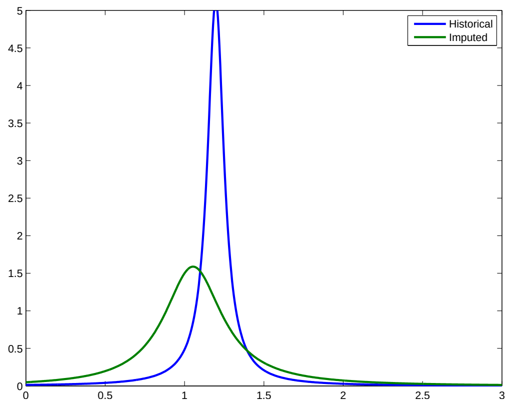
(c) 2000 Q2 - 2002 Q3



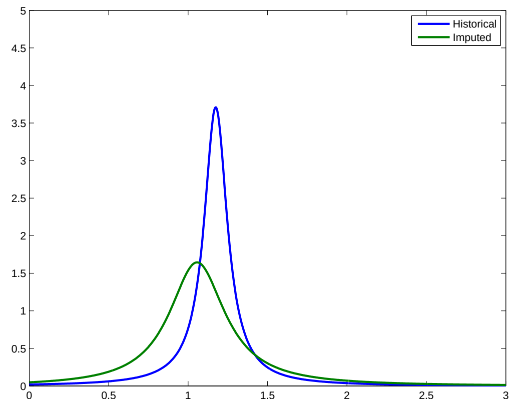
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

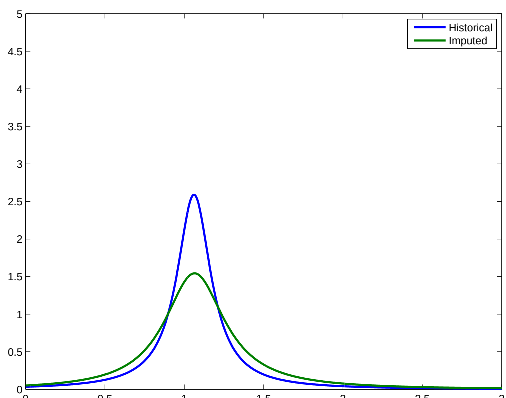
Manufacturing



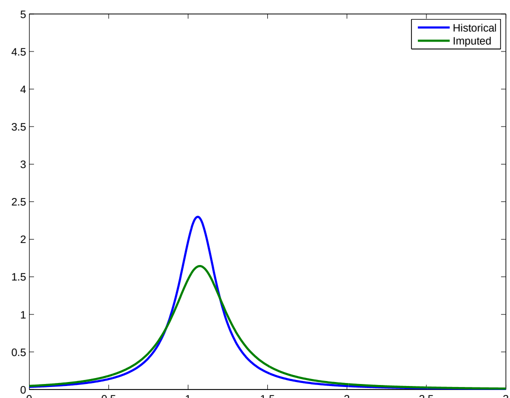
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



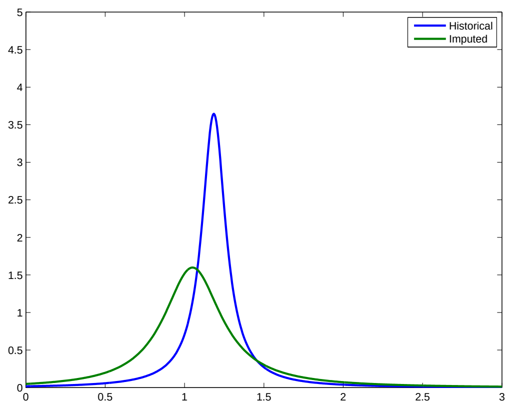
(c) 2000 Q2 - 2002 Q3



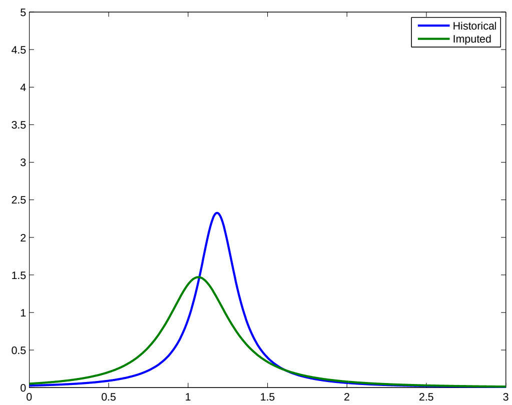
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

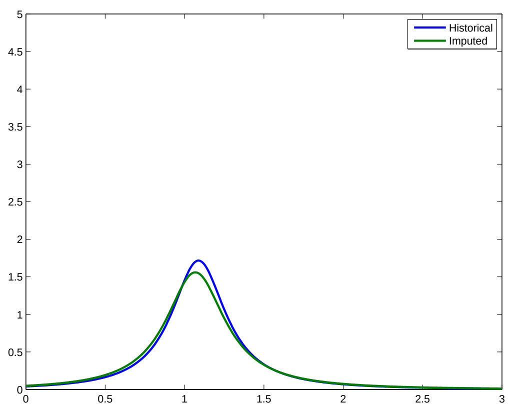
Energy



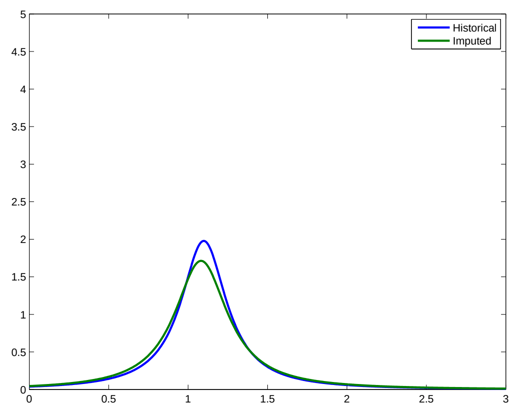
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



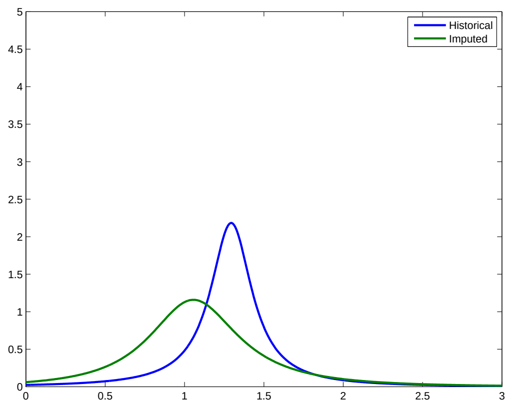
(c) 2000 Q2 - 2002 Q3



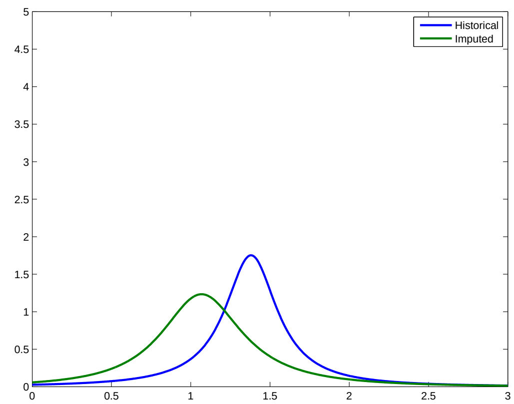
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

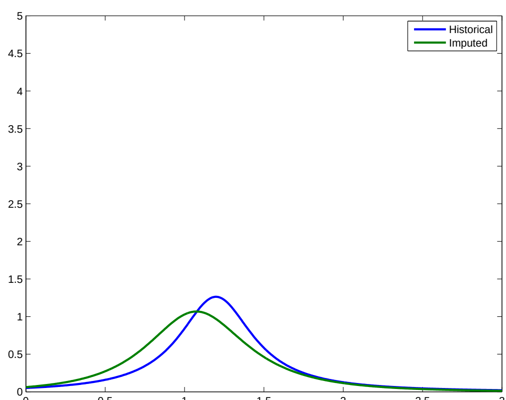
Tech



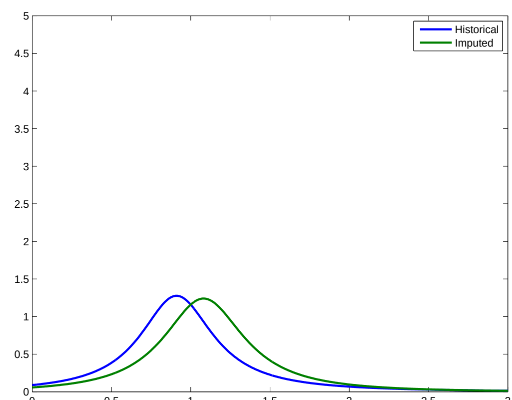
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



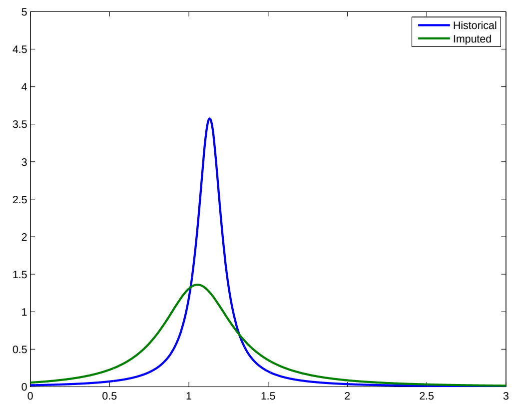
(c) 2000 Q2 - 2002 Q3



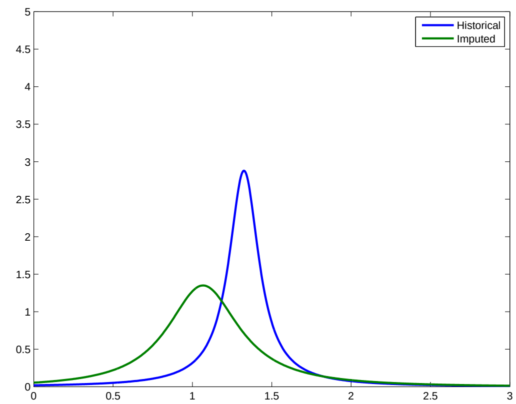
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

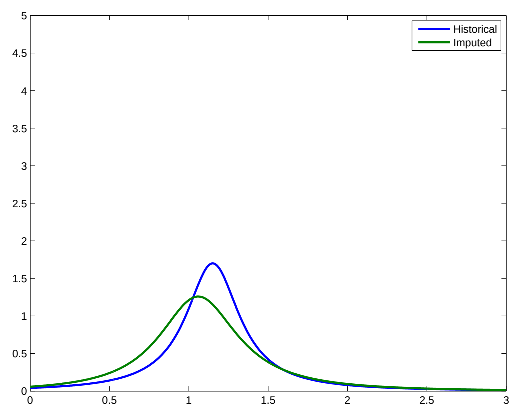
Telecom



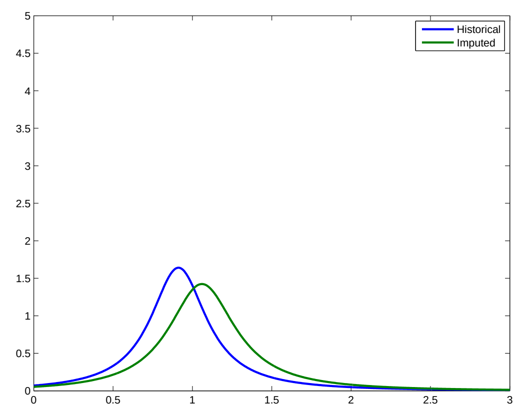
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



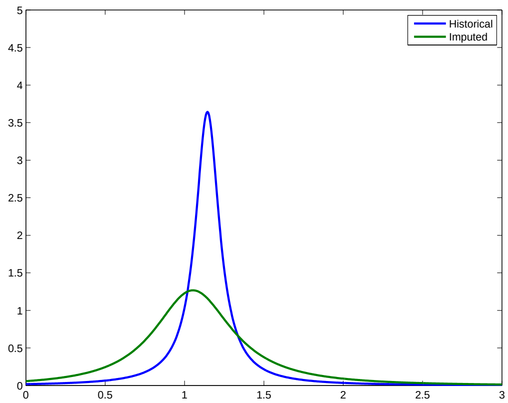
(c) 2000 Q2 - 2002 Q3



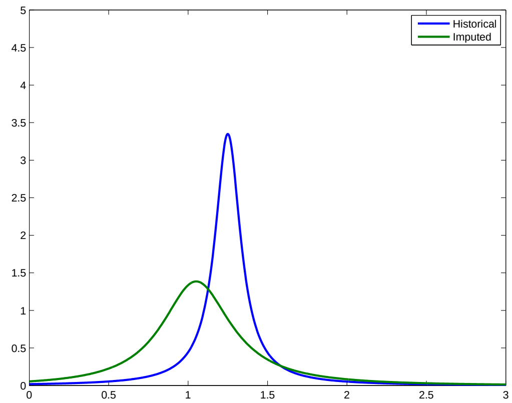
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

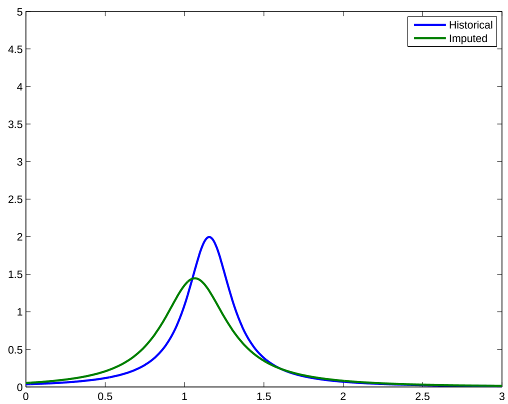
Wholesale/Retail



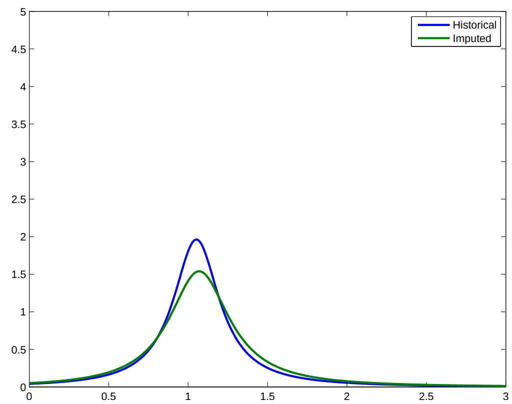
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



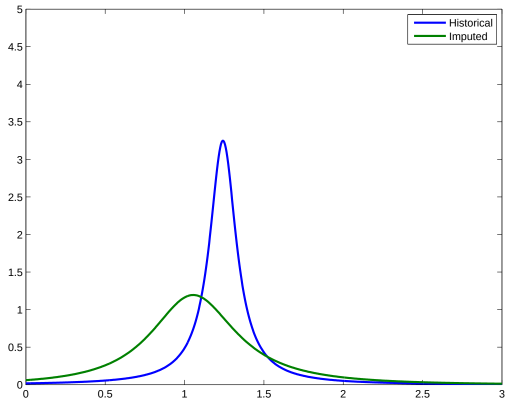
(c) 2000 Q2 - 2002 Q3



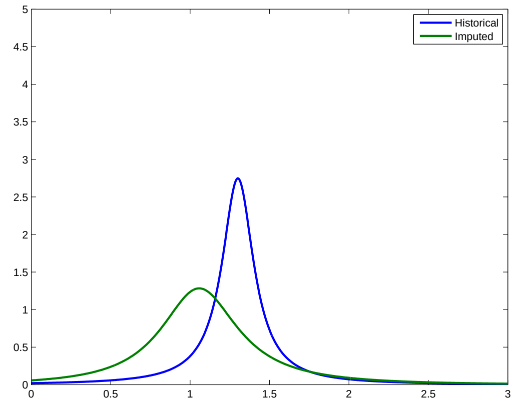
(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

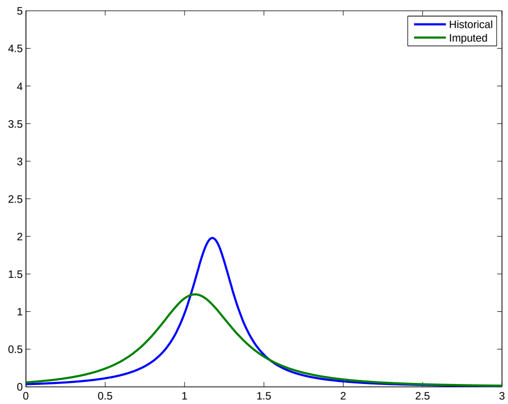
Healthcare



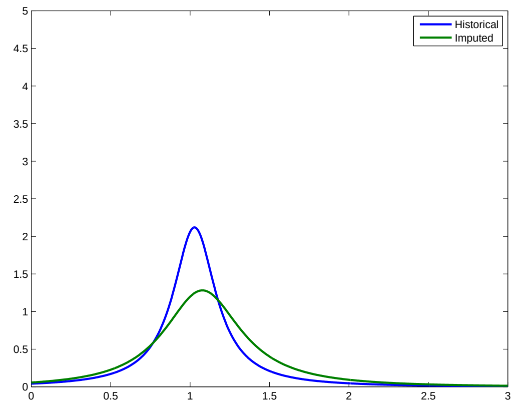
(a) 1996 Q2 - 1998 Q2



(b) 1998 Q3 - 2000 Q1



(c) 2000 Q2 - 2002 Q3



(d) 2003 Q1 - 2005 Q4

Figure continued on next page...

Appendix

Table A1: Summary Statistics with Volume Screens

Table A1, Panels A-C, present moment characteristics of portfolios sorted into terciles on the basis of volatility, skewness and kurtosis. The first column presents mean returns in the month subsequent to portfolio classification. The second column presents characteristic-adjusted returns, calculated by determining, for each firm, the Fama-French 5X5 size- and book-to-market portfolio to which it belongs and subtracting that return. The next three columns present the average beta, log market value and book-to-market equity ratio of the portfolio, while the next three columns present the average volatility, skewness and kurtosis of the portfolio. The last column presents the average number of securities in the portfolio. Moment estimates are separated into two maturity bins: the first maturity bin covers options with less than or equal to 3 months to expiration, the second with expiration greater than 3 months. Options are eliminated from consideration if they have no trading volume in the month of observation. Monthly return data cover the period 2/96 through 12/05, for a total of 119 monthly observations.

Panel A: Volatility-Ranked Portfolios

Rank	Bin	Mean	Char-Adj	Vol	Skew	Kurt	Beta	ln MV	BM	N
1	1	1.080	0.119	13.057	-1.936	23.600	0.978	16.014	0.340	31
2	1	1.272	0.596	20.679	-2.158	21.429	1.533	14.933	0.287	42
3	1	0.075	-0.777	35.517	-2.298	14.335	1.964	14.236	0.290	31
1	2	1.250	0.273	18.479	-1.022	9.021	0.837	16.037	0.361	53
2	2	0.973	0.120	28.995	-0.961	7.552	1.345	14.784	0.347	71
3	2	0.712	0.040	49.000	-1.117	5.573	1.882	14.229	0.353	53

Panel B: Skewness-Ranked Portfolios

Rank	Bin	Mean	Char-Adj	Vol	Skew	Kurt	Beta	ln MV	BM	N
1	1	1.410	0.503	22.983	-4.219	30.010	1.408	15.609	0.304	31
2	1	0.868	0.124	23.875	-2.030	14.684	1.562	14.930	0.298	42
3	1	0.325	-0.476	21.300	-0.202	17.460	1.423	14.676	0.311	31
1	2	1.548	0.521	32.866	-2.351	12.464	1.228	15.812	0.354	53
2	2	0.685	-0.069	32.610	-0.879	5.668	1.402	14.875	0.345	71
3	2	0.807	0.040	29.792	0.098	4.675	1.347	14.351	0.364	53

Panel C: Kurtosis-Ranked Portfolios

Rank	Bin	Mean	Char-Adj	Vol	Skew	Kurt	Beta	ln MV	BM	N
1	1	0.355	-0.400	27.195	-1.005	6.932	1.614	14.212	0.310	31
2	1	0.632	-0.089	22.503	-1.972	14.974	1.501	15.028	0.294	42
3	1	1.687	0.696	18.960	-3.473	39.958	1.312	15.909	0.310	31
1	2	0.632	-0.083	37.365	-0.353	2.772	1.559	14.057	0.370	53
2	2	0.904	0.025	31.571	-0.828	5.844	1.341	14.932	0.350	71
3	2	1.417	0.497	26.700	-1.967	14.124	1.111	16.015	0.342	53

Table A2: Time Series Regressions

Table A2 presents the results of time-series regressions of excess return differentials (Hi-Lo) between portfolios ranked on risk neutral volatility, skewness, and kurtosis on the three Fama and French (1993) factors MRP (the return on the value-weighted market portfolio in excess of a one-month T-Bill), SMB (the difference in returns on a portfolio of small capitalization and large capitalization stocks), HML (the difference in returns on a portfolio of high and low book equity to market equity stocks), UMD (the difference in returns on past winners and losers), and LIQ (the difference in returns on low and high liquidity stocks). The moment-sorted portfolios are equally-weighted, formed on the basis of terciles and re-formed each month. The table presents point estimates of the coefficients and standard errors in parentheses. Data cover the period January 1996 through December 2004 for 107 monthly observations.

Panel A: Volatility

Bin		α	β_{mrp}	β_{smb}	β_{hml}	β_{umd}	β_{liq}	R^2
1	Coeff	-0.855	0.354	0.822	-1.332	-0.253	0.002	0.831
	SE	(0.418)	(0.108)	(0.110)	(0.123)	(0.096)	(0.063)	
2	Coeff	-0.563	0.391	0.910	-1.350	-0.301	0.089	0.884
	SE	(0.382)	(0.106)	(0.114)	(0.093)	(0.082)	(0.067)	
3	Coeff	-0.237	0.375	0.879	-1.379	-0.412	0.179	0.858
	SE	(0.405)	(0.122)	(0.131)	(0.104)	(0.072)	(0.075)	

Panel B: Skewness

Bin		α	β_{mrp}	β_{smb}	β_{hml}	β_{umd}	β_{liq}	R^2
1	Coeff	-0.742	0.078	0.064	0.135	-0.236	-0.134	0.194
	SE	(0.356)	(0.105)	(0.154)	(0.124)	(0.115)	(0.129)	
2	Coeff	-0.704	0.184	0.166	0.202	-0.090	-0.272	0.233
	SE	(0.328)	(0.103)	(0.153)	(0.126)	(0.104)	(0.143)	
3	Coeff	-1.367	0.226	0.536	0.013	-0.052	-0.260	0.243
	SE	(0.396)	(0.119)	(0.153)	(0.134)	(0.119)	(0.139)	

Panel C: Kurtosis

Bin		α	β_{mrp}	β_{smb}	β_{hml}	β_{umd}	β_{liq}	R^2
1	Coeff	1.406	-0.095	-0.264	0.099	0.317	0.013	0.278
	SE	(0.315)	(0.112)	(0.129)	(0.104)	(0.074)	(0.112)	
2	Coeff	0.509	-0.198	-0.324	0.246	0.289	0.046	0.365
	SE	(0.360)	(0.123)	(0.165)	(0.132)	(0.111)	(0.134)	
3	Coeff	1.069	-0.321	-0.679	0.079	0.054	0.168	0.509
	SE	(0.396)	(0.099)	(0.106)	(0.110)	(0.108)	(0.066)	