Liquid Capital and Market Liquidity

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Abstract

It is widely believed that the resilience of the stock market and its ability to accurately set prices are affected by credit conditions in the economy. A scarcity of deployable capital may cause market-makers to become financially constrained, leading to a breakdown in intermediation. This paper describes another channel by which the supply of available capital affects secondary market liquidity. When agents hold more wealth in *technologically liquid* investments, a marginal adjustment to portfolio holdings alters discount rates less, causing a smaller price impact. The stock of liquid wealth also buffers income shocks, leading to lower risk premia and lower volatility when savings are high. The theory thus implies that, even without intermediaries or frictions, the degree of liquid capital may be a crucial determinant of asset market conditions.

KEYWORDS: LIQUIDITY RISK, CREDIT CONDITIONS, SAVINGS, ASSET PRICING.
JEL CLASSIFICATIONS: D91, E21, E44, E52, G12

1 Introduction

The relationship between credit conditions and asset market conditions is a central topic in finance. This paper is specifically concerned with the interaction between the level of deployable capital (or liquid wealth) in an economy and the resilience (or liquidity) of its securities markets.

This interaction is a crucial one during times of economic turbulence when preserving the orderly functioning of securities markets becomes a central policy objective. In such situations, market illiquidity is the linchpin triggering financial fragility. In the absence of market depth, small trades may induce steep price concessions. Artificially depressed prices raise the specter of systemic real effects if collateral repricing leads to binding solvency constraints and hence to broader asset disposals. Such "fire sales" would not be problematic if secondary markets could absorb them without further distortion. Maintaining market resilience is a necessary condition for avoiding a downward spiral.

To combat market illiquidity, central banks will often signal that they stand ready to "provide liquidity" to the financial system. Here the word "liquidity" is being used in a very different sense. The authorities do not themselves stand ready to make two-way prices in risky securities. Rather, the liquidity they provide is cash. More specifically, the Federal Reserve can expand the real monetary base by lending reserves directly or by open market operations (or by signalling that it is prepared to do both). Implicitly, then, central banks use one type of liquidity to affect the other. Moreover, there is empirical evidence that the strategy works.[1] The question is: why? What does the stock market's willingness to accommodate trade have to do with the supply of liquid capital?

A natural and widely held view of the mechanics of this "liquidity transmission" is that, in varying credit conditions, central banks are tightening or loosening the financing constraints that enable intermediaries to make markets for risky securities.[2] This intuition presupposes the existence of a credit channel through which central banks can affect real financing conditions. It also implicitly relies on some type of "inventory cost" model of price setting, whereby market depth is determined by a financially constrained sector whose cost of capital (a shadow cost when their constraint binds) differs from that of the economy as a whole. To the extent that intervention is viewed as beneficial, it must be that these constraints themselves are inefficient, perhaps resulting from agency problems or asymmetric information.[3]

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[1]Chordia, Sarkar, and Subrahmanyam (2005) report that bid/ask spreads in stock and bond markets were negatively correlated with measures of monetary easing during three high-stress periods from 1994 to 1998. In monthly data from 1965 through 2001, Fujimoto (2004) finds that several measures of aggregate illiquidity are significantly lower during expansionary monetary regimes than during contractionary ones. Vector autoregressions also indicate a significant response of illiquidity to monetary or Fed Funds rate innovations, at least in the first half of the sample.

[2]Describing this mechanism, the president of the New York Federal Reserve recently observed, "Although the two concepts [of liquidity] are distinct, they are closely related and often mutually reinforcing. Fundamentally this is so because when funding liquidity is abundant traders have the resources with which to finance trading positions that smooth price shocks and make markets liquid." Geithner (2007).

[3]Among the large literature studying the real effects of financial intermediation, the works of Allen

This paper describes an alternative mechanism linking capital availability to market liquidity in which intermediaries play no role and in which the role of liquid capital is not to loosen borrowing constraints. Instead, in a frictionless setting in which agents can choose to hold wealth in accessible, liquid form, the role of that liquid wealth is to buffer shocks to consumption. When this stock of assets is low, a marginal purchase of risky shares will require a relatively large sacrifice of current consumption, which will raise discount rates and depress prices.

Formally, the notion of market illiquidity studied here corresponds to the steepness of a representative agent's demand curve for risky shares. This definition is due to Pagano (1989). The precise computation follows Johnson (2006), in which the definition is generalized to arbitrary endowment economies. The present paper further extends the applicability of this measure to an economy with an endogenously varying capital stock. The argument shows that the elasticity of asset prices with respect to a marginal perturbation in share holdings is higher when such a trade induces greater intertemporal substitution in consumption. That, in turn, occurs more readily when liquid asset holdings are low because the marginal propensity to consume is higher.

This explanation for the connection between market liquidity and liquid balances is grounded in the primitive properties of a well-understood, frictionless model. While reaching the same conclusions as a financial constraint-type model, it suggests that the role of contracting frictions in the supply of credit to intermediaries may not be the whole story. In particular, the results here imply that empirical evidence that there is *some* linkage between the two types of liquidity cannot necessarily be interpreted as evidence of the existence (or severity) of such frictions. More simply, limited willingness to trade in a crisis does not imply systemic failure of credit markets. It may be an equilibrium phenomenon.

The theory presented here contributes to the evolving understanding of the dynamics of market liquidity and liquidity risk. This is a topic of significant concern for investors as well as policy makers. Liquidity risk affects any participant who needs to implement

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and Gale are specifically concerned with effects on market resilience of imperfect access to capital. See especially Allen and Gale (2004), and Allen and Gale (2005) for an overview. Diamond (1997) and Diamond and Rajan (2001) investigate the interaction between the liquidity available to intermediaries (banks) and the liquidity of non-intermediated markets. Financially constrained intermediaries are also the crucial ingredient in the limits-to-arbitrage literature stemming from Shleifer and Vishny (1997). Gromb and Vayanos (2002) formally model the liquidity provision decision of arbitrageurs subject to a positive wealth financing constraint. Brunnermeier and Pedersen (2005) study the effect of a value-at-risk type constraint (whose tightness they call funding illiquidity) on price concessions demanded by risk-neutral market makers (a measure of market illiquidity) in a two-period game.

a dynamic portfolio strategy since illiquidity represents a real cost to such a strategy. The paper's model constitutes an explicit and tractable quantification of time-varying liquidity, providing a complete description of its relation to the underlying state variables.

Beyond liquidity, a secondary contribution of the paper is to delineate the broader consequences of liquid savings for asset pricing. Much popular attention, and numerous models, have focused on the likely effects of increases or decreases in the supply of liquidity on asset markets. The model directly addresses the connection between liquidity and both volatility and expected returns. In particular, I show that, because of its consumption buffering role, a rising stock of liquid wealth also implies that volatility and risk premia should endogenously decline. As with market liquidity, these associations are not due to any irrational distortion or agency problems of intermediaries.

The outline of the paper is as follows. In the next section, I introduce the economic setting. This is among the simplest economies one can study in which agents choose their relative holdings of liquid assets. I describe the economy formally and discuss equilibrium properties of consumption and savings. Section 3 describes the determination of asset prices, expected returns, and volatility. In Section 4, I define the concept of market liquidity and show how to compute it in this model. I analyze the determinants of this quantity and highlight the intuition behind them. The primary result is that market liquidity increases with the level of cash holdings. Section 5 shows how to augment the model to consistently analyze the effects of interventions or changes in the supply of risky capital. A final section summarizes the paper's contribution and concludes by highlighting the key differences and similarities between models focusing on intermediaries and the approach taken here.

2. An Economy with Time-Varying Liquid Capital

This section develops a standard Lucas (1978) type model in which agents choose how much of their wealth to hold in liquid form. Liquid capital, as defined here, is characterized by a technological property: it is freely and immediately convertible into consumption. Although there is no fiat money in the model, wealth held in this form is cash-like in the sense that it can be exchanged *directly* for needed goods and services. By contrast, non-liquid capital is costly (or impossible) to physically adjust. While *claims* to such capital may be traded, and thus converted to consumable form, this operation is contingent on secondary market prices.

As in colloquial usage, an agent is said to be more liquid when money-like assets constitute a higher percentage of wealth. In solving the model, this degree of real (or technological) liquidity becomes the main state variable driving consumption and savings. The goal of this section is to characterize the dynamics of that variable, and hence of consumption.

The setting is as follows. Time is discrete and an infinitely lived representative agent has constant relative risk aversion (CRRA) preferences over consumption of a single good. The agent receives a risky stream, D_t, of that good in each period from an endowment asset. In addition, the agent has access to a second investment technology whose capital stock can be altered freely each period, and which returns a constant gross rate, $R \equiv e^{r \, \Delta t}$. The capital stock of the endowment asset can be neither increased nor decreased. Agents can only alter their savings via the liquid investment.

Models with an elastically supplied storage technology are common in asset pricing. This investment opportunity is then usually interpreted as (one-period) government bonds. It should be noted, however, that here the risklessness of the asset's return is not its important feature. The main results of the paper are all preserved if the second technology yields an uncertain R (with constant returns to scale). What matters instead is that the capital stock is completely fungible with current output for purposes of consumption.

An alternative, perhaps better, interpretation of this technology, then, is simply as a reduced form depiction of the banking system. Like demand deposits (plus cash), the stock of wealth held in this form represents the freely consumable component of household wealth, which can be accessed without any disinvestment in physical capital or sale of securities. Aggregating liquid assets and liabilities across households and corporations, the net supply of capital in excess of physical investment can be measured by narrow money holdings. (Indeed, stored goods in the model are formally equivalent to commodity money.) This observation justifies viewing these holdings as a measure of monetary conditions, and thereby of credit conditions.

An empirically equivalent measure of the supply of liquidity would be central bank assets, as opposed to their liabilities (i.e., narrow money). The global sum of central bank holdings of reserve assets – in particular, of U.S. government bonds – is one popular gauge of worldwide liquidity. Again, as in the model, this quantity represents total savings in excess of (non-transformable) investment.

It is not necessary, however, to invoke financial technology to distinguish between sectors with relatively transformable and non-transformable capital stocks. The present

model takes the distinction between these two to the extreme for expositional simplicity. One could view its dynamics as describing the short- to medium-term adjustment of an economy in which all physical investment can be altered, but some at a longer time scale than others. The model generalizes the pure endowment economy in which *no* sector's capital can be adjusted, and fluctuations in savings demand are reflected only in the riskless rate.

The asset market implications of a variable stock of liquid savings in a CRRA economy have not, to my knowledge, been previously analyzed.[4] Such buffer-stock models have been extensively studied in the consumption literature, however. (See Deaton (1991) and Carroll (1992).) The focus in the present work is on the price of the endowment stream, which I interpret as dividends, but which in that literature is interpreted as labor income. Since labor income is not traded, its price is not a primary object of study. Labor models also sometimes impose the constraint that investment in the savings technology must be positive, ruling out borrowing. This is intuitively sensible as a property of aggregate savings as well, but need not be imposed here, because it will hold endogenously anyway in the cases considered below.[5] Moreover, it is worth pointing out that there are also no financial constraints in the model. Agents in this economy may write any contracts and trade any claims with one another.

For parsimony, I have not included any other stochastic shocks to agent's supply of goods which might be interpreted as a direct "liquidity shock." A natural way of doing so would be to make these proportional to dividends but with a transitory component. (This specification would then look exactly like the process used for total risky income in the buffer stock model of Carroll (1997)). Separate idiosyncratic shocks are not needed, however, to deduce the effects of time-varying liquid balances on consumption and prices.

Setting the notation, the investor's problem is to choose a consumption policy, C_t, to maximize

$$J_t = \mathrm{E_t} \left[\sum_{k=1} \beta^k \frac{C_{t+k}^{1-\gamma}}{1-\gamma} \right],$$

where $\beta \equiv e^{-\phi}$ is the subjective discount factor, and I have set the time interval to unity for simplicity. A key variable is the total amount of goods, G_t, that the agent could consume at time t, which is equal to the stock of savings carried into the period plus new

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[4]The literature typically employs riskless storage technology in conjunction with constant absolute risk aversion (CARA) utility. CARA utility is unsuitable here because it necessarily implies that demand for the risky asset is independent of the level of savings or liquid wealth.

[5]With lognormal dividend shocks and CRRA preferences, agents will never borrow in a finite horizon economy. The policies below are limits of finite horizon solutions. When these limits exist, and appropriate transversality conditions are satisfied, they are also solutions to the infinite horizon problem.

dividends received:

$$G_t = R(G_{t-1} - C_{t-1}) + D_t.$$

The end-of-period stock of goods invested in the transformable asset will be denoted $B_t \equiv G_t - C_t$. It is also useful to define the income each period $I_t \equiv (R - 1)B_{t-1} + D_t = (R - 1)(G_t - D_t)/R + D_t$. Although this quantity plays no direct role, it helps in understanding savings decisions.

To take the simplest stochastic specification, I assume D_t is a geometric random walk:

$$D_{t+1} = D_t \, \widetilde{R}_{t+1}, \quad \log \widetilde{R}_{t+1} \sim \mathcal{N}(\mu - \frac{\sigma^2}{2}, \sigma^2)$$

so that dividend growth is *i.i.d.* with mean e^μ. Including a transient component (as would be appropriate in labor models) will not alter the features of the model under consideration here. As it is, the model is defined by five parameters: R, β, γ, μ, and σ.

The state of the economy is characterized by G_t and D_t which together determine the relative value of the endowment stream. It is easy to show that the optimal policy must be homogeneous of degree one in either variable. So it is convenient to define

$$v_t = D_t/G_t$$

which takes values in $(0, 1)$. This variable can also be viewed as summarizing the real liquidity of the economy. As $v_t \to 0$, the endowment stream becomes irrelevant and all the economy's wealth is transformable to consumption whenever desired, and income fluctuations can be easily smoothed. As $v_t \to 1$ on the other hand, all income effectively comes from the endowment asset whose capital stock cannot be adjusted. Since cash holdings are small, agents have little ability to dampen income shocks.

The agent's decision problem is to choose how much of his available goods to consume at each point in time. Intuition would suggest that he will optimally consume less of the total, G_t, when v is low than when v is high, since, in the former case consuming the goods amounts to eating the capital base, whereas in the latter case, G_t is mostly made up of the income stream, D_t, which can be consumed with no sacrifice of future dividends. This property is, in fact, true very generally.

Proposition 2.1 *Assume the infinite-horizon problem has a solution policy $h \equiv C/G = h(v)$ such that $h(v) < 1$. Then,*

$$h' > 0.$$

Note: all proofs appear in Appendix A.

This result can also be understood by noting that $\partial C(D, G)/\partial D = h'$ so that the assertion is only that consumption increases with (risky) income, which is unsurprising because shocks to D are permanent. Moreover, the result holds for much more general preferences. This follows from the results of Carroll and Kimball (1996) who show that $\partial^2 C/\partial G^2 < 0$ whenever $u''' \, u'/[u'']^2 > 0$. Concavity implies that $0 < C - G \, \partial C/\partial G$ and the latter quantity also equals h'.

That h rises with v is essentially the only feature of the model that is necessary for the subsequent results. However further useful intuition about the dynamics of the model can be gained by considering how consumption behaves at the extreme ranges of the state variable v_t.

In the limit as $v_t \to 1$ the economy would collapse to a pure endowment one (Lucas, 1978) if the agent consumed all his dividends, i.e. if $h(1) = 1$. This will not happen if the riskless savings rate available exceeds what it would be in that economy. That is, if

$$\frac{1}{R} < \mathrm{E}_t \left[\beta \left[\frac{C_{t+1}}{C_t} \right]^{-\gamma} | h(1) = 1 \right] = \mathrm{E}_t \left[\beta \left[\frac{G_{t+1} h(v_{t+1})}{G_t h(1)} \right]^{-\gamma} | h(1) = 1 \right] = \mathrm{E}_t \left[\beta \left[\widetilde{R}_{t+1} \right]^{-\gamma} \right] \tag{1}$$

then the agent's marginal valuation of a one-period riskless investment, assuming no savings, exceeds the cost of such an investment. So not saving cannot be an equilibrium, and the conclusion is that the inequality (1) implies $h(1) < 1$. In the lognormal case, this is equivalent to $r > \phi + \gamma\mu - \gamma(1 + \gamma)\sigma^2/2$, the right-hand expression being the familiar interest rate in the Lucas (1978) economy.[6]

As dividends get small relative to cash, $v_t \to 0$, the economy begins to look like one with only the riskless asset. For such an economy, it is straightforward to show that the optimal consumption fraction is $h_0 \equiv (R - (R\beta)^{1/\gamma})/R$. Now if the agent's consumption fraction approached this limit (which it does, $h()$ is continuous at zero[7]), his cash balances would grow at rate approaching $(R\beta)^{1/\gamma}$. If this rate exceeds the rate of dividend growth, then v_t will shrink further. However, in the opposite case, the agent is dissaving sufficiently fast to allow dividends to catch up. Hence v_t will tend to rebound.

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[6] The opposite inequality to (1) is sometimes imposed in the buffer stock literature to ensure dissavings as $v_t \to 0$. In that case, the specification of D_t is altered to include a positive probability that $D_t = 0$ each period. This assures that the agent will never put $h = 1$.

[7] This is shown in Carroll (2004) under slightly different conditions. Modification of his argument to the present model is straightforward.

Thus the more interesting case is when

$$(R\beta)^{1/\gamma} < \mathrm{E}_t \left[\widetilde{R}_{t+1} \right] \tag{2}$$

or $r < \phi + \gamma\mu$ under lognormality. A somewhat stronger condition would be that the agent actually dissaves as $v_t \to 0$. This would mean consumption $G_t h_0$ exceeds income $(R-1)(G_t - D_t)/R + D_t$ or, at $v = 0$, $h_0 < (R-1)/R$, which implies $(R\beta)^{1/\gamma} < 1$ or simply $r < \phi$.

With (1) and (2), then, the state variable v_t is mean-reverting.[8] This requirement is not necessary for any of the results on prices or liquidity. However it makes the equilibrium richer. As an illustration, I solve the model for the parameter values shown in Table 1.
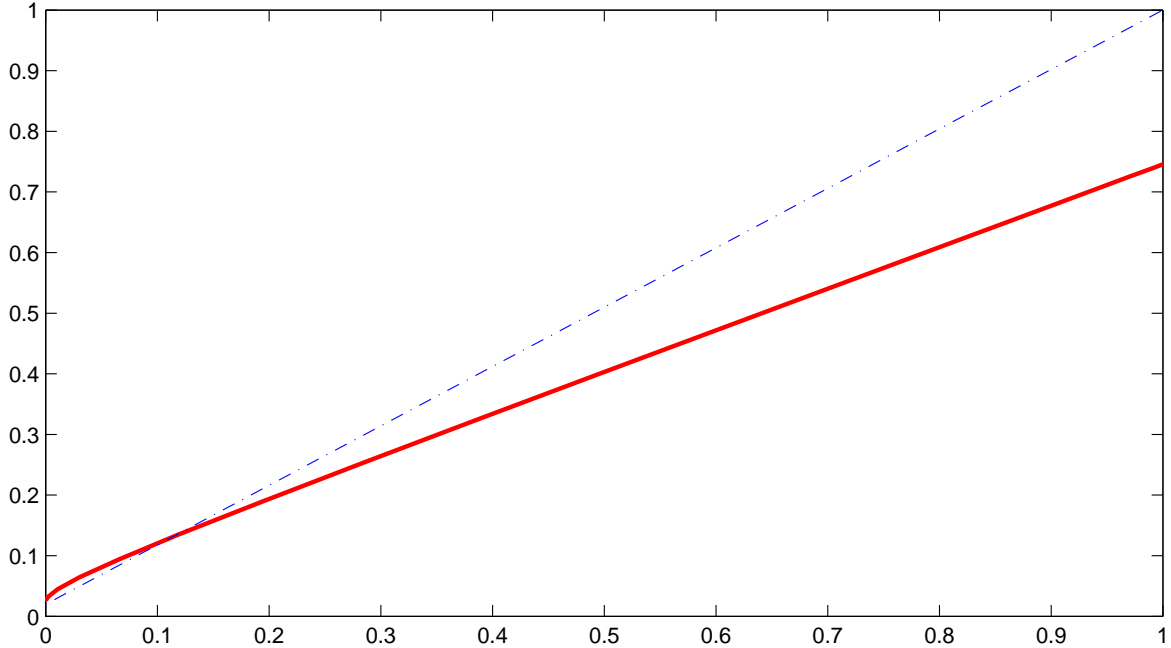
Table 1: *Baseline Parameters.*

PARAMETER	NOTATION	VALUE
coefficient of relative risk aversion	γ	6
subjective discount rate	$\phi = -\log\beta$	0.05
return to cash	$r = \log R$	0.02
dividend growth rate	μ	0.04
dividend volatility	σ	0.14
time interval	Δt	1 year

The parameters are fairly typical of calibrations of aggregate models when the endowment stream is taken to be aggregate dividends (with R approximating the real interest rate), and the preference parameters are all in the region usually considered plausible. Figure 1 plots the solution for the consumption function for these parameters. Analytical solutions are not available. However, as the proposition above indicated, h is increasing and concave. Also plotted is income (as a fraction of G), which is $(R-1+v)/R$. This shows the savings behavior described above: for small values of v agents dissave, whereas they accumulate balances whenever v exceeds about 0.12.

The graph also indicates that, away from the origin the function h is actually quite

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[8]Stationarity of v implies stationarity of consumption as a fraction of available wealth $C/G = h(v)$. Both are general properties under the model of Caballero (1990) who considers CARA preferences. Clarida (1987) provides sufficient conditions under CRRA preferences when dividends are *i.i.d.* Szeidl (2002) generalizes these results to include permanent shocks. No assumption about the long-run properties of the model are used below.

Figure 1: *Optimal Consumption*

The dark line is the optimal consumption function $\check{h}(v) \equiv C/G$ plotted against the ratio of dividends to total goods-on-hand $v \equiv D/G$. Also shown is income as a fraction of G plotted as a dashed line. All parameter settings are as in Table 1.

close to linear in v. Numerical experimentation suggests that this is a robust qualitative feature of solutions. With it, some of the dynamic properties of the endogenous state variables become easier to understand.

Analysis of the variable $w_t \equiv (G_t - D_t)/D_t = RB_{t-1}/D_t$ is more tractable than of v_t. This is the ratio of beginning-of-period cash balances (i.e. before dividends and consumption) to dividends, and the two are related by $w = \frac{1}{v} - 1$. So the consumption function $h(v)$ can be expressed equivalently as $h(w)$. Then we can write w_{t+1} in terms of time-t quantities as follows:

$$w_{t+1} = \frac{R([G_t - D_t] + D_t - C_t)}{D_t \, \widetilde{R}_{t+1}} = \frac{R}{\widetilde{R}_{t+1}}(1 - h(w_t))(w_t + 1). \tag{3}$$

Let the random ratio $R/\widetilde{R}$ define a new mean-zero variable $\widetilde{Z}$:

$$\frac{R}{\widetilde{R}_{t+1}} \equiv \widetilde{Z}_{t+1} + \bar{Z}$$

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with $\bar{Z} \equiv E(R/\widetilde{R})$ which is $e^{(r-\mu)}$. Subtracting w_t from w_{t+1} gives

$$\Delta w_{t+1} = \bar{Z}[(1 - h(w_t))(w_t + 1)] - w_t \ + \ [(1 - h(w_t))(w_t + 1)] \ \widetilde{Z}_{t+1}. \qquad (4)$$

This expression isolates the first and second moments of the w innovations. Their forms become clearer if we invoke the linear approximation for the function h. Specifically,

$$h(v) \doteq h_0 + (h_1 - h_0)v \Rightarrow h(w) \doteq \frac{h_0 w + h_1}{w + 1}.$$

Plugging this expression into equation (4) gives

$$\Delta w_{t+1} \doteq \left(\bar{Z}[(1 - h_1) + (1 - h_0)w_t] - w_t \right) \ + \ [(1 - h_1) + (1 - h_0)w_t] \ \widetilde{Z}_{t+1}.$$

$$= \left(\bar{Z}(1 - h_1) - [1 - \bar{Z}(1 - h_0)]w_t \right) \ + \ [(1 - h_1) + (1 - h_0)w_t] \ \widetilde{Z}_{t+1}. \qquad (5)$$

Here we see that w_t is approximately an affine process. The coefficient on $-w_t$ in the deterministic part of this specification can be interpreted as the speed of mean reversion. Using the expression given above for h_0 and the definition of $\bar{Z}$, the coefficient becomes

$$1 - (R\beta)^{1/\gamma} E(1/\widetilde{R}) = 1 - e^{\frac{(r-\phi)}{\gamma} - \mu} \doteq \mu - \frac{(r - \phi)}{\gamma}.$$

This shows that the inequality (2) imposed earlier also implies (up to an approximation) that the process w_t mean-reverts, and, indeed, characterizes the degree of mean reversion by the degree to which that inequality holds. Taking the parameter values given above, the coefficient in the expression evaluates to 0.045, which corresponds to a characteristic time scale (or half-life) of 15.4 years. In this economy, then, this is the "business cycle frequency" that governs the endogenous changes in liquidity, which, in turn, determine the consumption and saving behavior.

Having (approximately) characterized the dynamics of the state variable w_t (and hence v_t), we can directly infer the consumption dynamics by again appealing to the linear approximation of $h()$. Since

$$C_t/D_t = h(v_t)/v_t = h(w_t)(w_t + 1) \doteq h_0 w_t + h_1,$$

consumption growth has two sources. The first is simply the growth rate of dividends, which is $i.i.d.$ The second is the growth in the linear transformation of w_t which itself is an affine process. This second term's expected growth will be high when w_t is itself low

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or when v is high. Intuitively, when v is high, current consumption is less than income. But since v is expected to mean revert, agents expect to be saving less in the future. That is, their consumption growth will benefit from the cyclical boost that will come from freer spending in more liquid times.

Consumption volatility varies in a similar, though more complicated, fashion. Dividend innovations are perfectly negatively correlated with the shocks $\widetilde{Z}$ driving w. So the two components of consumption growth work in opposite directions. And, from equation (5), the component $h_0 w_t + h_1$ will have percentage changes equal to

$$\frac{(1 - h_1) + (1 - h_0)w_t}{\frac{h_1}{h_0} + w_t}$$

times those of $\widetilde{Z}_{t+1}$. This fraction is increasing in w and typically less than unity.[9] It follows then that consumption volatility decreases as w increases (or as v decreases). While the mathematical analysis is somewhat opaque, the intuition is simple. The consumption stream is necessarily mostly made up of dividends when liquid wealth is low. And dividends are more volatile than cash.
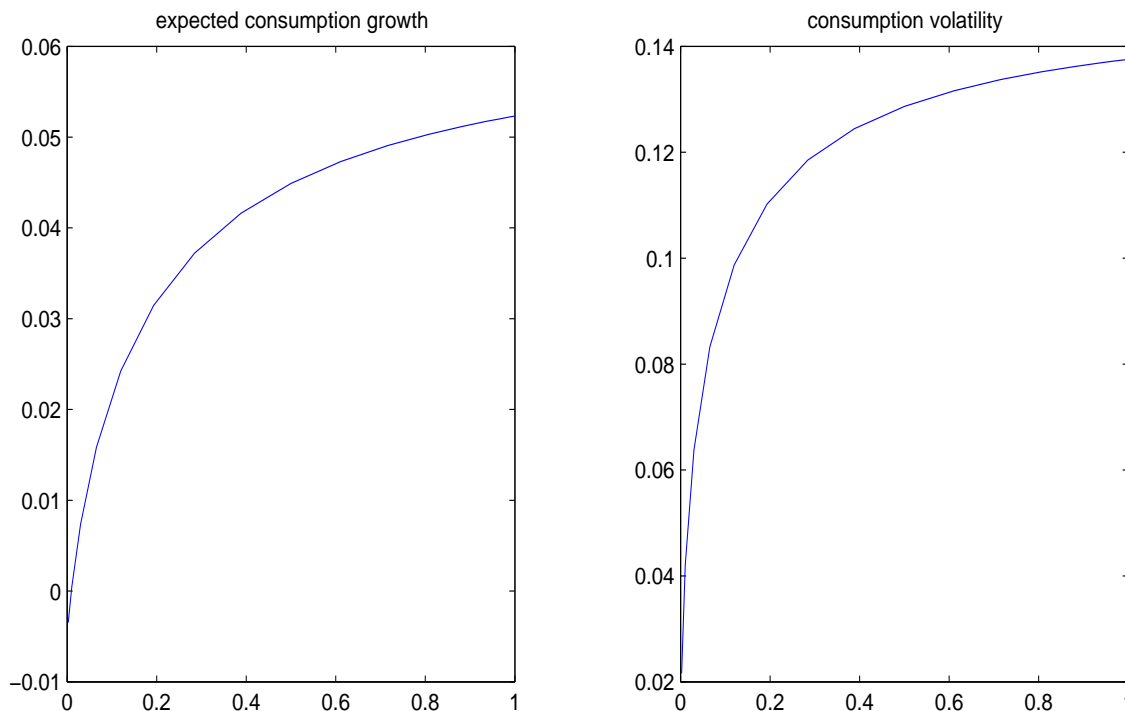
To get a sense of the degree of variation of the consumption moments, I first evaluate them numerically for the baseline parameter values. The results are shown in Figure 2. Since agents smooth consumption, the standard deviation is below that of the dividends themselves. More notably, the moments vary significantly as the liquidity of the economy varies.

How much does that liquidity vary? Figure 3 shows the unconditional distribution of v, calculated by time-series simulation. Its mean and standard deviation are 0.204 and 0.065 respectively, implying a plus-or-minus one standard deviation interval of (0.139, 0.269). Using this distribution to integrate the conditional consumption moments, the unconditional mean and volatility of the consumption process are 0.031 and 0.110. In terms of dynamic variation, the standard deviation of the conditional mean and volatility are 0.0051 and 0.0079, respectively.

To recap, this section has shown some important, basic properties of consumption in this model. The propensity to consume current goods, $h()$, is increasing in the liquidity ratio, v, the percentage of current wealth coming from dividends. Subject to some parameter restrictions, this percentage (or equivalently w) is stationary, with a degree of mean-reversion determined by μ, r, ϕ, and γ. The consumption ratio, h, and consumption

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[9]As $w \to \infty$ it goes to $(1 - h_0)R = (R\beta)^{1/\gamma}$ which could exceed unity. In that case, positive dividend shocks would lower consumption.
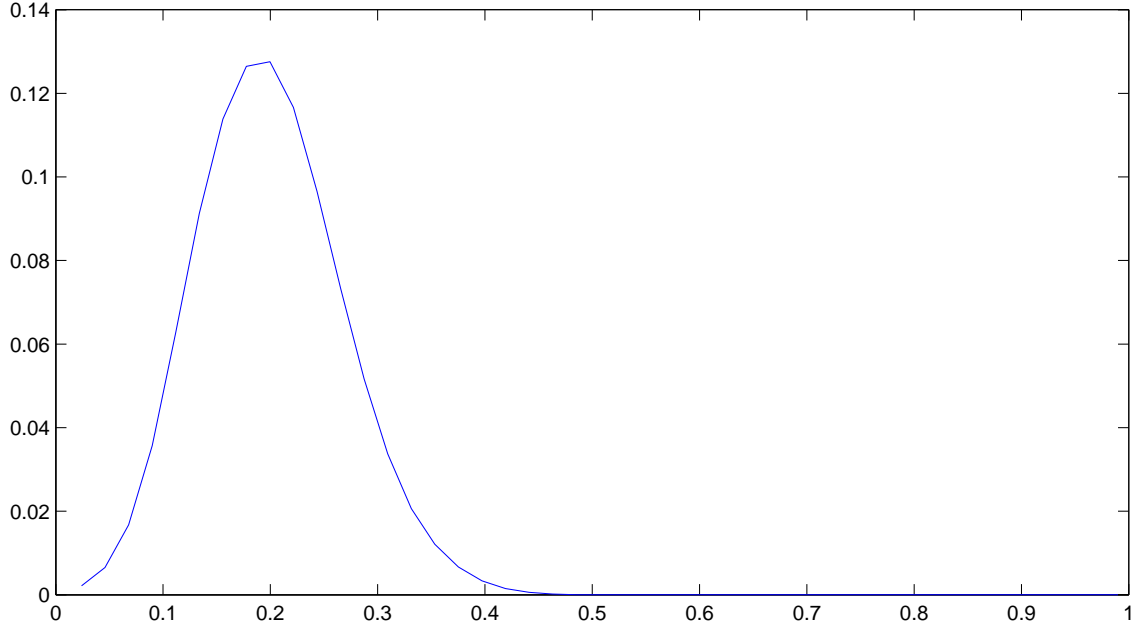
Figure 2: *Consumption Moments*

The left panel plots the conditional mean of log consumption growth, and the right hand panel plots the conditional standard deviation. The horizontal axis is v, the liquid balances ratio. All parameter settings are as in Table 1.

growth are then also stationary with the same characteristic time-scale. Remarkably, although the exogenous environment is *i.i.d.*, states with high and low levels of liquidity (or accumulated savings) seem very different. When liquidity is low, consumption is more volatile and income is saved; when liquidity is high, agents dissave and, though expected consumption growth is lower, it is smoother. These consumption dynamics lead to the main intuition needed to understand asset pricing in this economy.

3. Liquid Wealth and Asset Prices

Having solved the model to characterize consumption, one can immediately compute the price, expected return, and volatility of a claim on the risky dividend stream. This section outlines those computations and discusses the results in the context of what is known empirically and theoretically about the connection between the supply of liquid capital and asset prices. Interestingly, even though there is neither a monetary authority or an

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Figure 3: *Unconditional Distribution of Dividend-Cash Ratio*

The figure shows the unconditional distribution of v, the dividend-to-cash ratio, as computed from 40,000 realizations of a time-series simulation. The first 500 observations are discarded and a Gaussian kernel smoother has been applied. All parameter settings are as in Table 1.

intermediary sector, the economy behaves as if there were a "credit channel" affecting the stock market.

The computation starts by solving for the price-dividend ratio $g = g(v) \equiv P/D$, from investors' first-order condition. With the notation of the last section, this condition is

$$g(v_t) = \beta \mathrm{E}_t \left[\left(\frac{v_t h(v_{t+1})}{v_{t+1} h(v_t)} \widetilde{R}_{t+1} \right)^{-\gamma} [1 + g(v_{t+1})] \, \widetilde{R}_{t+1} \right].$$

Recall that the risky dividend innovation, $\widetilde{R}_{t+1}$, is the only source of uncertainty here, and that changes to the liquidity ratio, v, are driven by the same shocks, via

$$v_{t+1} = \frac{v_t \widetilde{R}_{t+1}}{(1 - h(v_t))R + v_t \widetilde{R}_{t+1}},$$

which is equivalent to (3). The function $g(v)$ can be found by iterating this mapping on the unit interval.[10] Once g is obtained, the distribution of excess returns to the claim

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[10]It is actually simpler to solve for the price-goods ratio $f(v) \equiv P/G = vg(v)$ which is not singular at the origin.

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can be evaluated from

$$\frac{(P_{t+1} + D_{t+1})}{P_t} = \widetilde{R}_{t+1} \left[1 + g(v_{t+1})\right]/g(v_t).$$

Since analytical expressions are again not attainable, it is important to highlight the intuition behind the numerical results. This is straightforward. Having seen that consumption is less volatile when liquid balances are high, one can immediately infer that discount rates will be lower in these states since marginal utility is smoother and hence the economy is less risky. Further, when liquid balances are high, dividends comprise a smaller component of consumption. Thus a claim on the dividend stream has less fundamental risk when G is high relative to D. This implies that prices will be higher and risk premia lower when the economy is more liquid.
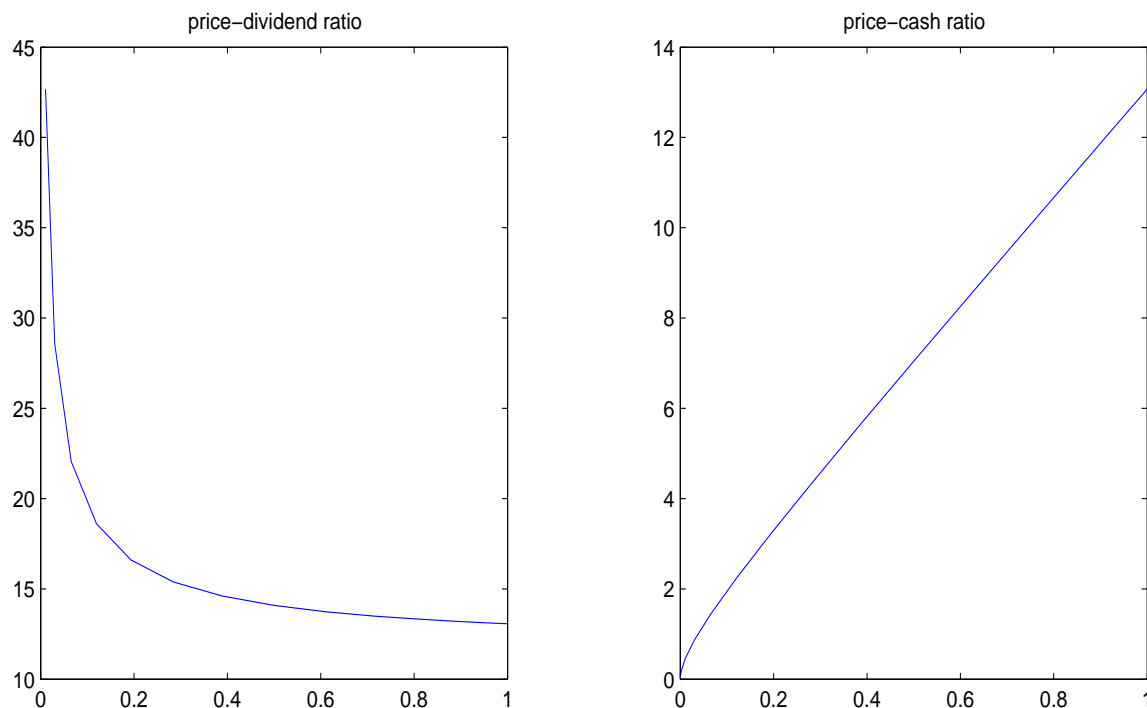
Figure 4 plots both the price-dividend ratio, g, and the price-goods ratio, f, for the parameter values in Table 1. The first function affirms the intuition above that the dividend claim must be more valuable when v is lower. In fact, g becomes unbounded as v approaches zero. This is not troublesome, however, because it increases slower than $1/v = G/D$. The plot of $f(v) = vg(v)$ goes to zero at the origin, indicating that the total value of the equity claim is not explosive. Indeed, f is monotonically increasing in v, which lends support to the interpretation of v as measuring the illiquidity of the economy, since this function is the ratio of the *value* of the non-transformable asset to the value of the transformable one.

In light of the lack of closed-form results, it is worth mentioning here the features of the pricing function that will matter when analyzing market resilience. Referring again to the figure, the fact that g explodes while f does not essentially bounds the convexity of g to be no greater than that of v^{-1} in the neighborhood of the origin. While not visually apparent, a similar convexity bound holds on the entire unit interval.[11] While the generality of this property is conjectural, numerical experimentation suggests that it is robust. The curvature of the asset pricing function as a function of v is the key determinant of how much exogenous shocks to asset supplies will affect prices. As will be shown below in Section 4, that is tantamount to determining the liquidity of the securities market.

Turning to return moments, Figure 5 evaluates the risk premium and volatility for the risky asset as a function of v for the same parameter values. The plot verifies

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[11] Technically, the requirement is $g'' \leq 2(g')^2/g$ which holds for functions of the form $Av^{-\alpha}$ as long as $\alpha \leq 1$.
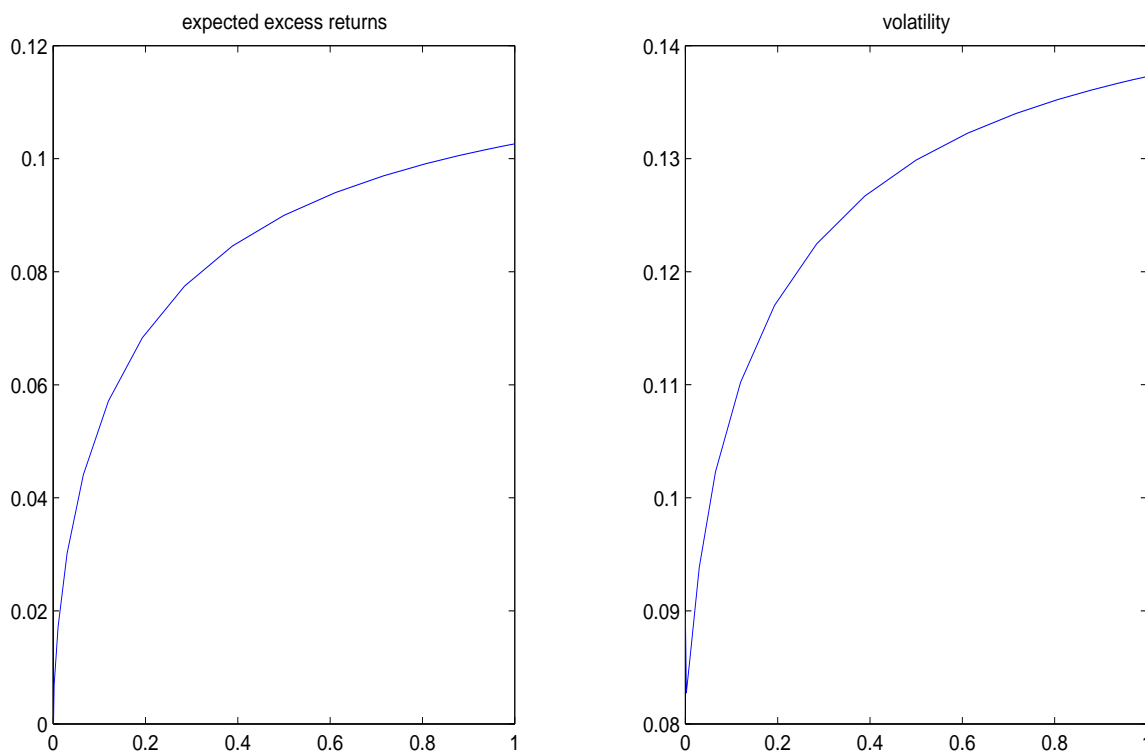
Figure 4: *Asset Pricing Functions*

The left panel plots the ratio $g \equiv P/D$, and the right hand panel plots P/G. The horizontal axis is v, the dividends-to-cash ratio. All parameter settings are as in Table 1.

the intuition about risk premia. Volatility dynamics are also straightforward: while dividend innovations are homoskedastic, the volatility of marginal utility tracks that of consumption. As seen in the last section, consumption volatility is low when buffer stock savings are high.

The implications of the model for stock prices are interesting. There is both widespread conventional agreement and solid empirical evidence that increases in financial liquidity drive increases in stock prices. But there is much less agreement on why this should be so. The paper's perspective on this question is new.

As discussed in Section 2, the paper's liquidity variable can viewed as direct proxy for monetary conditions because it captures the stock of savings in excess of physical capital. A positive relation between stock prices and monetary aggregates has been established by Thorbeck (1997) and Baks and Kramer (1999), among others. This type of association can be explained in monetary asset pricing models (for example Marshall (1992)) via the effect of an appropriately specified process for inflation risk. Another explanation, the focus of the "credit channel" literature, suggests the effect works through the real interest rate via the relaxation of borrowing constraints. Finally, it is worth

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Figure 5: *Asset Return Moments*

The left panel plots the conditional mean of continuously compounded excess returns, and the right hand panel plots their conditional standard deviation. The horizontal axis is v, the dividend-cash ratio. All parameter settings are as in Table 1.

mentioning the popular view of the relationship, which is simply "more money chasing fewer investments."

In the present model, none of these lines of explanation apply. There is no inflation risk. There are no financial constraints. The riskless interest rate and risk aversion are constant. Asset markets are fully rational. Here the reason stock prices are higher when there is more liquid capital is that risk premia are lower because liquid savings enables agents to better insulate their consumption from shocks to their illiquid wealth. Recent tests by Bernanke and Kuttner (2005) support the idea that the mechanism through which monetary liquidity affects stock prices is through changes in expected excess returns. The model here suggests one way that could happen, tying liquid savings to consumption volatility.

Having shown that the level and volatility of stock prices can be affected by the supply of deployable capital, we now return to the topic of the resilience of the stock market. Are securities markets more liquid when the economy is more liquid? If so, why?

4. Changes in Market Liquidity

In what sense can financial claims in this paper's economy be said to be illiquid? After all, no actual trade in such claims takes place in the model, and, if it did, there are no frictions to make transactions costly.

Nevertheless, these observations do not mean that the market's demand curve for risky securities is flat. In fact, in general, this will not be the case: marginal perturbations to a representative agent's portfolio will marginally alter his discount rates, altering prices. This paper uses the magnitude of this price effect – essentially the slope of the representative agent's demand curve – as the definition of a claim's secondary market illiquidity. It measures the price impact function that would be faced by an investor who did wish to trade with the market (i.e. with the representative agent) for whatever reason.[12] Likewise, it measures the willingness of a typical investor (who has the holdings and preferences of the representative agent) to accommodate small perturbations to his portfolio.

Formally, the definition proposed in Johnson (2006) views the value and price functions of the representative agent as functions of his holdings, $X^{(0)}$ and $X^{(1)}$, of any two of the available assets. Illiquidity of asset one with respect to asset zero is then defined by the change in that agent's marginal valuation of asset one following a value-neutral exchange of the two assets, holding all other asset supplies fixed.

Definition 4.1 *The illiquidity $\mathcal{I} = \mathcal{I}^{(1,0)}$ of asset one with respect to asset zero is the elasticity*

$$\mathcal{I} = -\frac{X^{(1)}}{P^{(1)}} \frac{dP^{(1)}(\Theta(X^{(1)}), X^{(1)})}{dX^{(1)}} = -\frac{X^{(1)}}{P^{(1)}} \left(\frac{\partial P^{(1)}}{\partial X^{(1)}} - \frac{P^{(1)}}{P^{(0)}} \frac{\partial P^{(1)}}{\partial X^{(0)}} \right), \qquad (1)$$

where $(\Theta(x), x)$ is the locus of endowment pairs satisfying $J(\Theta(x), x) = J(X^{(0)}, X^{(1)})$.

The definition stipulates that the derivative be computed along isoquants of the value function (parameterized by the curve Θ) and the second equality follows from the observation that value neutrality implies

$$\frac{d\Theta(X^{(1)})}{dX^{(1)}} = \frac{dX^{(0)}}{dX^{(1)}} = -\frac{P^{(1)}}{P^{(0)}}.$$

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[12]There is a long history of price impact and trading cost models in the market microstructure literature. To my knowledge, Pagano (1989) was the first to identify the slope of the aggregate demand function – as opposed to the demand of an *ad hoc* market maker – as an equilibrium measure of liquidity.

While the definition depends on the choice of asset zero, in many contexts there is an asset which it is natural to consider as the medium of exchange. In the model of Section 2 above, the storable asset is the obvious unit since its relative price in terms of goods is clearly constant, $P^{(0)} = 1$. This is one of the properties that justifies the interpretation of this asset as cash.[13]

Like the market price itself, the elasticity $\mathcal{I}$ can be defined and computed whether or not trade actually takes place in equilibrium. If the economy were disaggregated, and some subset of agents experienced idiosyncratic demand shocks forcing them to trade, they would incur costs proportional to $\mathcal{I}$ as prices moved away from them for each additional share demanded. (An application which demonstrates the equivalence of individual price impact and $\mathcal{I}$ in a setting with explicit trade appears in Johnson (2007).) Equivalently, $\mathcal{I}$ is proportional to the percentage bid/ask spread (scaled by trade size) that would be quoted by competitive agents in the economy were they required to make two-way prices. Thus, it captures two familiar notions of illiquidity from the microstructure literature.

Two simple examples may help illustrate the concept, and also clarify the analysis of illiquidity in the full model.

First, consider pricing a claim at time t to an asset whose sole payoff is at time T in a discrete-time CRRA economy. As usual, $P_t^{(1)} = \mathrm{E}_t \left[\beta^{T-t} u'(C_T) D_T / u'(C_t) \right]$. Further suppose the asset is the sole source of time-T consumption: $C_T = D_T = D_T^{(1)} X^{(1)}$, and let the numeraire asset be any other claim not paying off at T or t. Then, differentiating,

$$\frac{dP_t^{(1)}}{dX^{(1)}} = \frac{1}{u'(C_t)} \mathrm{E}_t \left[\beta^{T-t} u''(C_T) (D_T^{(1)})^2 \right] = -\gamma \mathrm{E}_t \left[\beta^{T-t} \frac{u'(C_T)}{u'(C_t)} \frac{D_T^{(1)}}{X^{(1)}} \right] = -\gamma \frac{P^{(1)}}{X^{(1)}}$$

or $\mathcal{I} = \gamma$. In this case, the effect of asking the agent to substitute away from time-T consumption causes him to raise his marginal valuation of such consumption by the percentage γ, which is also the inverse elasticity of intertemporal substitution under CRRA preferences. If this elasticity were infinite, the claim would be perfectly liquid. Notice that the effect is not about risk bearing: no assumption is made in the calculation about the risk characteristics of the *other* asset involved. So the exchange could either increase or decrease the total risk of the portfolio.

Now consider a similar exchange of asset one for units of the consumption good.

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[13]Technically, one should distinguish between the quantity of claims to a unit of the physical asset and the capital stock, G, of that asset. But since the exchange rate is technologically fixed at unity, I make no distinctions below and use G and $X^{(0)}$ interchangeably.

Then, in the computation of $\mathcal{I}$, there is an extra term in $dP^{(1)}/dX^{(1)}$ which is

$$\mathrm{E_t}\left[\beta^{T-t}u'(C_T)D_T\right]\frac{d}{dX^{(1)}}\left(\frac{1}{u'(C_t)}\right) = -P^{(1)}\frac{u''(C_t)}{u'(C_t)}\frac{dC_t}{dX^{(1)}} = \gamma\frac{P^{(1)}}{C_t}\frac{dC_t}{dX^{(1)}}.$$

Now the value neutrality condition implies $dC_t/dX^{(1)} = -P^{(1)}$ so that $\mathcal{I}$ becomes

$$\gamma\left[1 + \frac{P^{(1)}X^{(1)}}{C_t}\right].$$

It is easy to show that this is the illiquidity with respect to consumption for a general (i.e. not just one-period) consumption claim as well. Here the intertemporal substitution effect is amplified by a (non-negative) term equal to the percentage impact of the exchange on current consumption: $P^{(1)}X^{(1)}/C_t = \left|(X^{(1)}/C_t)\ dC_t/dX^{(1)}\right|$. This term may be either large or small depending on the relative value of future consumption. In a pure endowment economy with lognormal dividends and log utility, for example, current consumption is $C_t = X^{(1)}D_t^{(1)}$ and the extra term is price dividend ratio, which is $1/(1-\beta)$, which would be big. The intuition for this term is that marginally reducing current consumption (in exchange for shares) raises current marginal utility. So, if the representative agent is required to purchase $\Delta X^{(1)}$ shares and forego current consumption of $P^{(1)}\Delta X^{(1)}$, his discount rate rises (he wants to borrow) and he would pay strictly *less* than $P^{(1)}$ for the *next* $\Delta X^{(1)}$ shares offered to him.

In what follows, it will be useful to think of the mechanism in these examples as two separate liquidity effects. I will refer to that of the first example, captured by the term $\gamma \cdot 1$, as the future consumption effect, and that of the second, captured by $\gamma \cdot P^{(1)}X^{(1)}/C_t$, as the current consumption effect.

Returning to the model of Section 2, such explicit forms of $\mathcal{I}$ in terms of primitives are not available. (Direct differentiation of the discounted sum of future dividends is intractable because future consumption depends in a complicated way on the current endowments.) However it is simple to express $\mathcal{I}$ in terms of the functions g (the price-dividend ratio) and f (the value ratio), which are both functions of v, the dividend-liquid balances ratio.
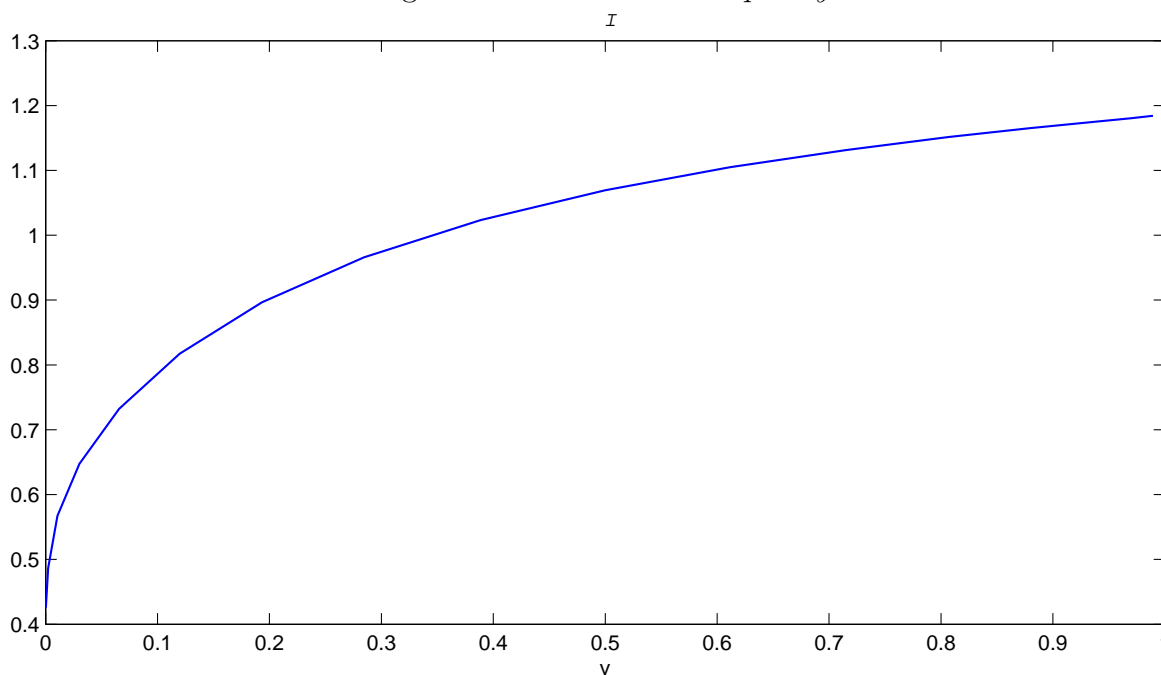
Proposition 4.1 *In the model described in Section 2,*

$$\mathcal{I} = -v(1 + vg(v))\frac{g'(v)}{g(v)} = (1 - v\frac{f'(v)}{f(v)})(1 + f(v)).$$

Illiquidity is positive in this economy because the price-dividend ratio is a declining function of v. Adding shares in exchange for cash mechanically shifts v to the right. As discussed above, when shares make up a larger fraction of the consumption stream their fundamental risk increases and their value declines.

Figure 6 plots illiquidity using the parameter values from Table 1. Notice first the most basic features, the level and variation of the function. The magnitude of illiquidity is both significant and economically reasonable. An elasticity of unity implies a one percent price impact for a trade of one percent of outstanding shares. This is the order of magnitude typically found in empirical studies of price pressure for stocks. Further, market liquidity is time-varying in this model. It is not a distinct state variable, of course, yet it is still risky in the sense of being subject to unpredictable shocks. While the current parameters restrict v, and hence $\mathcal{I}$, to a rather narrow range, even so, it is possible for illiquidity to more than double.

Figure 6: *Stock Market Illiquidity*

The figure shows the elasticity $\mathcal{I}$ as a function of v for the model of Section 2. All parameter settings are as in Table 1.

This brings us to the topic of how and why market liquidity changes here. The figure clearly provides the fundamental answer: illiquidity rises when v does. Or, to stress the main point, the stock market is more liquid when liquid assets are in greater relative supply. This is the heart of the paper's results.

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To understand why this occurs, consider the role that the availability of a savings technology plays in the determination of price impact. In effect, it dampens both of the illiquidity mechanisms in the pure-endowment examples above. When the liquidity provider (the representative agent) chooses to use some cash savings to purchase additional risky shares – instead of being forced to forego consumption – current marginal utility does not rise as much. In addition, marginally depleting savings today raises the expected marginal utility of future income, which then raises the valuation of future dividends. Hence the impact of foregone consumption (illustrated in the earlier examples) on both the numerator and denominator of the marginal rate of substitution are buffered by the use of savings. But now recall from the last section that the propensity to save when given an extra unit of G (or to dissave when required to give up a unit) falls with v. Because h rises with v (for the very general reasons discussed previously), discount rates are less affected by portfolio perturbations when v is low.

Appendix B analyzes the effects in more detail in the two-period case which corresponds to the previous examples. Even in this case, a formal proof that $\mathcal{I}$ is increasing is unobtainable. However it is possible to isolate the individual terms in $\mathcal{I}'$ and to see how each rises with the propensity to consume.

Mechanically, differentiating the expression in the proposition shows that $\mathcal{I}'$ will be positive as long as $g(v)$ is not too convex. (See note 11.) In fact, it is sufficient that $\log g$ is concave. And concavity is equivalent to the assertion that g'/g gets bigger (more negative) as v increases, meaning the percentage price impact of an increase in v is increasing.

Summarizing, markets are illiquid in this economy ($\mathcal{I}(v) > 0$) because discount rates rise with the proportion of risky asset holdings. Markets are *increasingly* illiquid as cash balances decline ($\mathcal{I}'(v) > 0$) because this impact on discount rates itself rises. Discount rates are affected more strongly by portfolio perturbations when cash is low because consumption – not savings – absorbs a higher percentage of the adjustment.

Because the liquid balances ratio determines all dynamic quantities in this model, all the covariances of market liquidity immediately follow from the positive relation between $\mathcal{I}$ and v. In particular, as the economy wanders into the low cash region, not only does it become more difficult to trade, but also stocks become cheap (the price dividend ratio falls) and risk premia and volatility go up.
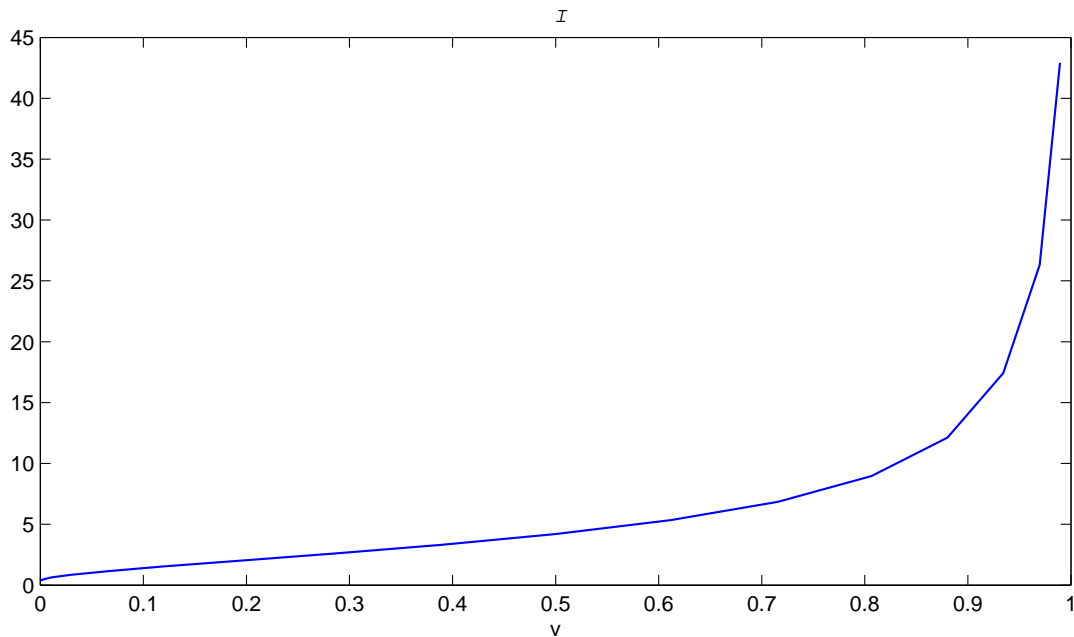
In the stationary economy described by the baseline parameters in Table 1 the steady state distribution of v is not very diffuse, and the variations in liquidity are not particularly dramatic. But consider, instead, the parameter set shown in Table 2.

Table 2: *Nonstationary Model.*

PARAMETER	NOTATION	VALUE
coefficient of relative risk aversion	γ	2
subjective discount rate	$\phi = -\log\beta$	0.02
return to cash	$r = \log R$	0.02
dividend growth rate	μ	0.03
dividend volatility	σ	0.10

This version of the model has insufficient risk aversion to induce agents to save, even as their cash balances dwindle. As a consequence, the model is not stationary. This does not present any problem for the definition of $\mathcal{I}(v)$, however. Figure 7 shows that, in fact, there is a dramatic deterioration in market liquidity with v for this economy, with $\mathcal{I}(v)$ rising by almost an order of magnitude between $v = 0.25$ and $v = 0.95$, which is approximately the range v will experience over 20 years.

Figure 7: *Stock Market Illiquidity: Nonstationary Economy*

The figure shows the illiquidity, $\mathcal{I}$, as a function of v using the parameter settings shown in Table 2.

5. Intervention

In assessing the preceding example, in which agents consume "too much" and exhaust their savings, an observer of this economy might well wonder about the case for intervention. Along any observed history, the economy would feature a steady drift of v towards one. As we have seen, this would entail rising volatility, falling asset prices, and – from the last section – a sharp spike in market illiquidity.

Thus, even though the external uncertainty facing agents is the same in all states (dividend growth is *i.i.d.*) and though the economy is always in first-best equilibrium (there are no friction or deadweight losses), it is hard to resist the conclusion that the economy approaches something that looks like a "'liquidity crisis." Interestingly, as this section demonstrates, it is also the case that a feasible intervention strategy could appear to "resolve" the crisis.

While there is no formal role for a government sector in the model, one can still consider the effect of policy actions that alter the real quantity of liquid balances. In particular, consider an open-market operation (e.g. by the central bank) aimed at adjusting the economy's real liquidity. It turns out that such interventions can be consistently incorporated in the model without altering savings or pricing decisions.

Specifically, let today be t and assume that at some $\tau > t$ a random process will dictate a positive quantity ΔG to be added to the representative agent's cash holdings, G_τ, in exchange for a number of shares $\Delta X^{(1)}$ of the endowment stream, that quantity to be determined so that the agent is indifferent to the exchange, i.e. it leaves his value function unchanged. In other words, the central bank engages in an open-market purchase of risky assets at the competitive market price. What would such an intervention accomplish?

From a comparative static point of view, the answer is immediate. A purchase by the central bank simply shifts the ratio v to the left, as the numerator decreases and the denominator increases. To be careful, the previous notation needs to be augmented to reflect the variable number of shares of the endowment claim (so far implicitly set to one). So write

$$v_t \equiv \frac{D_t}{G_t} = \frac{D_t^{(1)} X_t^{(1)}}{G_t}.$$

That is, the superscript will denote per-share quantities. Thus, also, $P^{(1)} = D^{(1)} g(v)$ will be the per-share price of an endowment claim. If the representative agent sells shares, then, his stream of dividends is lowered, which is what v measures. The per-share dynamics of the D process is not changed.

Thus, other than the perturbation to v_τ, the economy is unaltered by the intervention. Its effect, according the analysis above, would be to increase the price-dividend ratio and lower the risk premium, as well as to increase consumption relative to income and to reduce the volatility of consumption and of the stock market. This is perhaps a surprising result: a feasible intervention (i.e. involving no net transfer of wealth, by assumption) succeeds in altering the real economy in a non-trivial and perhaps desirable manner.

Is this conclusion justified from a dynamic point of view? Or would rational anticipation of the intervention at τ alter the equilibrium at t, rendering the comparative statics invalid? As the following proposition shows, the analysis is actually robust to interventions quite generally.

Proposition 5.1 *Let $\{\tau_k\}_{k=1}^{K}$ be an increasing sequence of stopping times $t < \tau_1 \ldots \tau_K$, and let $\{\delta_k\}_{k=1}^{K}$ be a sequence of random variables on $\mathcal{R}^+$. Suppose that at each stopping time an amount $\Delta G_{\tau_k} = G_{\tau_k} (\delta_k - 1)$ of goods are added to the representative agent's holdings in exchange for an amount of shares $\Delta X_k^{(1)}$ that leaves his value function unchanged. (If no such quantity exists, no exchange takes place.) Then, the value function, $J = J(v, D)$, consumption function, $h(v)$, and pricing function, $P(v, D)$, at time t are identical functions to those in the economy with no interventions when the endowments are fixed at their time-t amounts.*

The underlying logic of the proposition is simple: since the agent knows the intervention won't alter his value function at the time it occurs, the *ex ante* probability distribution of future value functions is unchanged. Hence today's optimal policies are still optimal, regardless of the intervention, which means the value function today is unaltered.

The proposition tells us that we can consistently augment any version of the model of Section 2 to include an arbitrarily specified policy rule for open-market purchases and sales, and ignore the effects of those future exchanges in computing prices and liquidity. The assumption that the exchanges are value-neutral is equivalent to imagining that agents compete perfectly in an auction (or tender) for the risky shares and that the intervening entity then acts as a price taker.

While I have described the intervention in terms of a monetary authority injecting reserves, the proposition could equally well apply to the actions of a corporate sector selling risky shares and thus altering the capital stock of the endowment asset, or of new production opportunities arising stochastically. Note that the proposition does not rule

out that the timing and amounts of the sales could depend on the state of the economy, such as the price of equity.[14]

Either interpretation is of course a reduced-form depiction of a fuller model in which the exchanges would be specified in terms of technological primitives. The intervening entity is viewed as possessing real production possibilities – in the form of the claims it sells – but as not affecting allocations in the economy apart from the interventions. The question of why the interventions occur is not addressed.

Returning to the numerical example above, however, we have already seen that a quasi-motivation for intervention exists in the sense that, were the central bank to add real liquidity to the economy when it gets "too low," it might seem to improve asset market conditions. The proposition tells us that the comparative static conclusion is, in fact, dynamically consistent. The only effect of a value-neutral open-market operation is to re-set the current value of v.
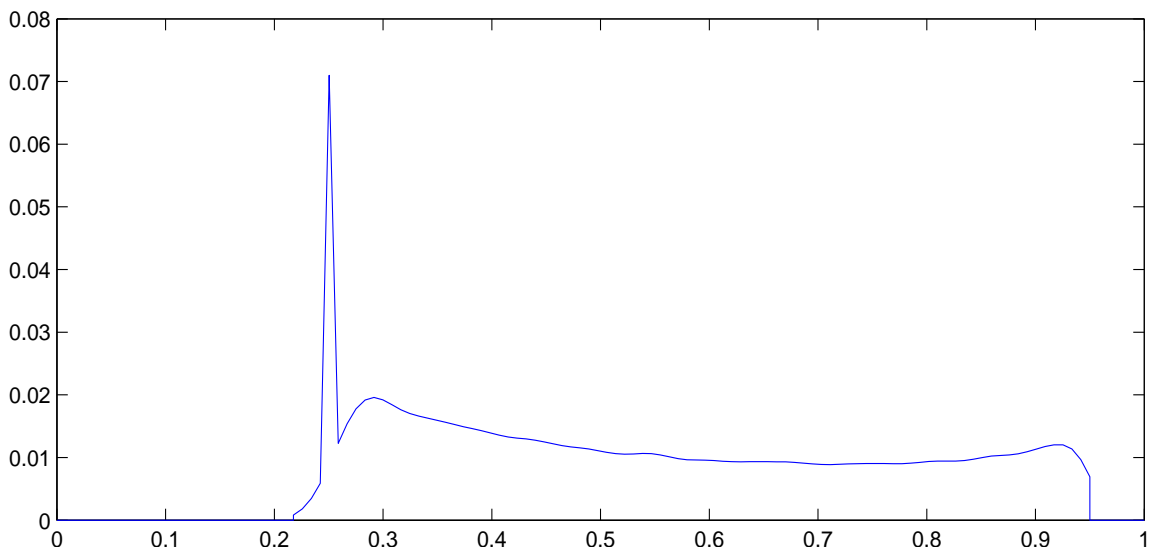
Moreover, one could make this non-stationary version of the model effectively stationary by imagining periodic "rescues" by the authorities when v approaches some higher limit. For example, suppose whenever v exceeds 0.95, an open-market operation is undertaken to re-start it at 0.25. This would induce the distribution shown in Figure 8. Here the injections of liquid reserves by the central bank in the extreme (high v) states would seem to alleviate stressful conditions. The severe sensitivity of prices to volume would be dampened, the price-dividend ratio would rise, and stock volatility would be quelled.

While incorporating intervention enriches the descriptive range of the model, the point of the example is to emphasize the ambiguity of the conclusions one might draw from observing the effects of such an action. To an observer of this economy, it could well appear that the periodic deterioration of prices and increases in volatility occurred *because* of the lack of market liquidity. The apparent success of the intervention could seem to support the idea that the scarcity of deployable capital caused a decline in intermediation, causing the rise in market liquidity, and leading to the seemingly distressed state.

The model here shows that neither of the above inferences need follow from the observed linkages. Market illiquidity and risk premia may rise simultaneously without the former having anything to do with the latter. A decrease in liquid balances can cause both, but without operating through the constraints of intermediaries. The "success" of the intervention is, in fact, illusory. The loosening of real constraints or the alleviation of

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[14]There is an implicit assumption that the intervention does not alter agents' information sets by conveying information about future values of D.

Figure 8: *Unconditional Distribution of Dividend-Cash Ratio*

The figure shows the unconditional distribution of v, the dividend-to-cash ratio, as computed from 40,000 realizations of a time-series simulation. An intervention rule is applied, restarting the process at 0.25 whenever 0.95 is hit or exceeded. The first 500 observations are discarded and a Gaussian kernel smoother has been applied. All parameter settings are as in Table 2.

frictions would result in a welfare gain. Yet the interventions in the model are irrelevant from a welfare standpoint since, by assumption, agents' value functions are unaffected. The asset market outcomes do not imply that some sort of market failure was averted.

6. Conclusion

Understanding the fundamental factors driving market illiquidity is a crucial issue for both investors and policy makers. It is widely believed that one such factor is the availability of liquid funds in the economy.

This paper describes an economic mechanism linking the two types of liquidity: when cash balances are low agents are less willing to accommodate others' trade demands because doing so entails more adjustment to current consumption. This is a primitive economic effect that is not driven by segmented markets, contracting frictions, information asymmetry, details of microstructure, or irrationality.

More broadly, the paper highlights the real, technological role of adjustable capital in affecting asset market conditions. Liquid assets serve to buffer consumption from production shocks, leading to lower volatility, risk premia, and discount rates. These

observations have important implications for our understanding of how financial liquidity affects market risks.

Much recent attention has focused on the possible effects of the stock of liquid wealth on asset markets. Popular commentary has drawn a causal connection between a global "savings glut" and the rising price of risky assets, leading many to question whether the scale of investible wealth has encouraged investors to irrationally disregard risk. Others have cited the increasing capital of the financial sector (hedge funds in particular) as an important factor in recent declines in volatility. These issues are the mirror image of the concerns that arise during "liquidity crises," when an apparent dearth of easily deployable capital is linked by many observers with illiquid asset markets, excessive volatility, and "fire sale" prices.

The model presented here speaks to both sets of observations. All of them are captured within a simple, frictionless model incorporating savings decisions. The model allows the stock of liquid capital to vary endogenously or exogenously. Despite the model's sparse structure, quantitative results show that variations in market liquidity and asset return moments can be large.

While I have contrasted the paper's theory with alternative explanations based on the capital of intermediaries, it is worth pointing out that there are, in fact, certain similarities.

This paper views investors as having access to a real technology for storing wealth in liquid form, which I have described as occurring through money or bank deposits. This view of the banking sector is similar in spirit to that of Diamond (1997), where the role of banks is to create liquid assets despite the absence of short-term projects in the economy.

Going further, while I have stressed that there are no frictions in the model, one could argue that the inability to transform liquid capital to illiquid physical investment and the inability to hold negative liquid capital do represent constraints on the savings technology. If the savings technology is viewed as a reduced-form depiction of the financial system then it is not clear that these constraints differ so much from those in the alternative models. Recent dynamic equilibrium models incorporating explicitly segmented intermediary sectors by Chen (2001) and He and Krishnamurthy (2006) attain some similar results to those found here.

This discussion raises the question of whether the current model can be empirically distinguished from models based on intermediary financial constraints. The answer is yes.

First, under the present model, the two types of illiquidity (market liquidity and

monetary liquidity) would be always linked, not just in times of stress, i.e. when intermediaries' constraints bind. Second, the model suggests that, in explaining market conditions (including liquidity), the economy-wide level of real liquid balances is a key state variable. However, controlling for the financial position of the economy as a whole, there should be no role for the financial position of securities dealers and brokerage firms in explaining aggregate fluctuations. Limits-to-arbitrage type models suggest that liquidity and prices are driven by market-maker capital alone.

More fundamentally, and perhaps controversially, the model makes the point that time-varying market liquidity – and even the occurrence of liquidity crises – need not be evidence of market failure or inefficiency. Even if intervention in credit markets is successful in promoting market stability, this need not imply any welfare gains if the mechanism is not through alleviating financing frictions.

This is not to say that such frictions cannot have an amplifying effect on risk and risk premia, nor that understanding the details behind the operation of such constraints is not important for managing financial systems. However, at a minimum, one must be cautious in inferring that the response of market conditions to credit conditions is a direct gauge of the importance of constraints.

Appendix

A. Proofs

This appendix collects proofs of the results in the text.

Proposition 2.1

Proof.   This proof will restrict attention to policy solutions in the class of limits of solutions to the equivalent finite-horizon problem. So consider the finite-horizon problem with terminal date T. Let h_t denote the optimal consumption-to-goods ratio at time t.

Clearly h_t cannot exceed one, since this would lead to a positive probability of infinitely negative utility at T. The assumption of the proposition is then that, at each v, we have an interior solution for h_t (at least for $T - t$ sufficiently large). In that case, h_t must satisfy the first order condition

$$h_t^{-\gamma} = R\beta\mathrm{E}_{\mathrm{t}} \left[\left[\left(R(1 - h_t) + v_t\widetilde{R}_{t+1} \right) \, h_{t+1}(v_{t+1}) \right]^{-\gamma} \right]$$

where $v_{t+1} = v_t\widetilde{R}_{t+1}/(R(1 - h_t) + v_t\widetilde{R}_{t+1})$. I will assume a $\mathcal{C}^1$ solution exists for all t.

An implication of the first order condition is that the expectation

$$\mathrm{E}_{\mathrm{t}} \left[\left[\frac{h_t}{\left(R(1 - h_t) + v_t\widetilde{R}_{t+1} \right) \, h_{t+1}(v_{t+1})} \right]^{+\gamma} \right] \tag{1}$$

must not be a function of v_t. I use this fact to prove the following successive properties:

(*i*) $h_t' \leq h_t/v_t \quad \forall \, v_t, t$.

(*ii*) $-(1 - h_t)/v_t \leq h_t' \quad \forall \, v_t, t$.

(*iii*) $0 < h_t' \quad \forall \, v_t, t$.

For the first point, assume the property holds for h_{t+1} but fails to hold for h_t. Write the expectation, above as

$$\mathrm{E}_{\mathrm{t}} \left[\left[\frac{\frac{h_t(v_t)}{v_t}}{\frac{h_{t+1}(v_{t+1})}{v_{t+1}}} \right]^{+\gamma} \widetilde{R}_{t+1}^{-\gamma} \right]. \tag{2}$$

The hypothesis implies that the derivative with respect to v_t of the numerator in the inner brackets is positive and the derivative with respect to v_{t+1} of the denominator is negative. Also, the derivative $\frac{dv_{t+1}}{dv_t}$ is

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$$\frac{R\widetilde{R}_{t+1}}{(R(1-h_t)+v_t\widetilde{R}_{t+1})^2}\ (1-h_t+v_th_t').$$

The hypothesis on h_t implies that $(1-h_t+v_th_t')\geq 1$. So $\frac{dv_{t+1}}{dv_t}$ is positive. Together, these observations imply that an increase in v_t will raise the numerator and lower the denominator of square bracket term in equation (2) for all values of the random variable $\widetilde{R}_{t+1}$. Hence the expectation cannot be constant. The contradiction, combined with the fact that the final optimal policy is $h_T=1$, which satisfies the induction hypothesis, proves $h_t'\leq h_t/v_t$ for all t.

Next, assume $-(1-h_{t+1})/v_{t+1}\leq h_{t+1}'$ but that the reverse holds for h_t. Differentiate the denominator of equation (1) to get

$$\frac{1}{(R(1-h_t)+v_t\widetilde{R}_{t+1})}\left((\widetilde{R}_{t+1}-Rh_t')(R(1-h_t)+v_t\widetilde{R}_{t+1})\,h_{t+1}+\widetilde{R}_{t+1}\ R\ (1-h_t+v_th_t')h_{t+1}'\right)$$

Using the result just shown, $h_{t+1}'\leq h_{t+1}/v_{t+1}$. And, by the induction hypothesis, $(1-h_t+v_th_t')<0$. So the smallest the term in large parentheses can be is

$$(R(1-h_t)+v_t\widetilde{R}_{t+1})\,h_{t+1}v_t^{-1}\left((\widetilde{R}_{t+1}-Rh_t')v_t+R\ (1-h_t+v_th_t')\right)$$

$$=(R(1-h_t)+v_t\widetilde{R}_{t+1})\,h_{t+1}v_t^{-1}\ \left(R(1-h_t)+v_t\widetilde{R}_{t+1}\right)>0.$$

These observations imply that an increase in v_t will lower the numerator and raise the denominator of the bracketed term in (1) for all values of $\widetilde{R}_{t+1}$. Hence the expectation cannot be constant. The contradiction, combined with the fact that the final optimal policy satisfies the induction hypothesis, proves $-(1-h_t)/v_t\leq h_t'$ for all t.

The third step proceeds similarly: assume the inequality (iii) holds for $t+1$ but not t. By the previous point (ii), we now have $(1-h_t+v_th_t')\geq 0$ even though $h_t'<0$. This means $\frac{dv_{t+1}}{dv_t}$ is always positive. So an increase in v_t must increase the denominator and decrease the numerator of (1), contradicting the constancy of the expectation. Given that $h_T=1$, the constancy of the expectation at $T-1$ immediately implies that h_{T-1} must be strictly increasing. Hence h_{T-1} satisfies the induction hypothesis (iii). So we conclude $h_t'>0$ for all $t<T$.

Now the limit of the discrete time maps: $h\equiv\lim_{t\to-\infty}h_t$ must also satisfy the condition that

$$E_t\left[\left[\frac{h(v_t)}{\left(R(1-h)+v_t\widetilde{R}_{t+1}\right)\ h(v_{t+1})}\right]^{+\gamma}\right]$$

is constant. The limit of increasing functions cannot be decreasing. However it can be flat. But if $h()$ is constant, then an increase in v_t would still raise $(R(1-h_t)+v_t\widetilde{R}_{t+1})$ and change the

expectation. So we must also have $h' > 0$.

<div align="right">QED</div>

Proposition 4.1

Proof. The elasticity $\mathcal{I}$ can be computed from direct differentiation of $P^{(1)}$ in terms of f or g using the value-neutral condition $dG_t/dX^{(1)} = -P^{(1)}$ and

$$\frac{dv_t}{dX^{(1)}} = \frac{v_t}{X^{(1)}}(1 + v_t g(v_t)) = \frac{v_t}{X^{(1)}}(1 + f(v_t))$$

which follows from the definition $v = D^{(1)}X^{(1)}/G$. *QED*

Proposition 5.1

Proof. Let us distinguish, at each intervention date, between the times immediately before and immediately after the exchange, writing these as e.g., τ_k- and τ_k+. (It is immaterial whether allocation and consumption decisions are made before or after.) Also write the value function of the representative agent as $J_t = J(D_t^{(1)}, X_t^{(1)}, G_t)$. Recall the superscript denotes per-share values, so that the total dividend income of the agent is $D_t = D_t^{(1)} X_t^{(1)}$.

Let J_t^o be the value function of the equivalent economy in which no further interventions will take place, that is, in which $X_s^{(1)} = X_t^{(1)}$ for all $s \geq t$, as in the original model. Similarly, let J_t^k be the value function under the assumption that the kth exchange does *not* take place. Then the assumption of the proposition that agents are indifferent to each exchange can be expressed as $J_{\tau_k-}^k = J_{\tau_k+}$.

Now consider the value function at any date t such that $\tau_{K-1} < t \leq \tau_K$. This must satisfy

$$J_t = \max_{\{C_{t+n}\}_{n=0}^{\infty}} \mathrm{E_t}\left[\sum_{n=0}^{\infty} \beta^n \, u(C_{t+n})\right].$$

The expectation can be written

$$\sum_{j=0}^{\infty} \mathrm{E_t}\left[\sum_{n=0}^{j-1} \beta^n \, u(C_{t+n}) + \sum_{n=j}^{\infty} \beta^n \, u(C_{t+n})|\tau_K = t+j\right] \mathrm{P}\left(\tau_K = t+j\right)$$

and the inner, conditional expectation is

$$\mathrm{E_t} \left[\sum_{n=0}^{j-1} \beta^n \ u(C_{t+n}) + \mathrm{E_{t+j}} \left(\sum_{n=j}^{\infty} \beta^n \ u(C_{t+n}) | \tau_K = t+j \right) | \tau_K = t+j \right].$$

Define the conditionally optimal policy $\left\{ C_{t+n}^j \right\}_{n=1}^{\infty}$ to be the one that maximizes this latter expectation. But, by assumption, the solution to

$$\max_{\{C_{t+n}\}_{n=j}^{\infty}} \mathrm{E_{t+j}} \left(\sum_{n=j}^{\infty} \beta^n \ u(C_{t+n}) | \tau_K = t+j \right)$$

coincides with the same value in the absence of the exchange, which is

$$\max_{\{C_{t+n}\}_{n=j}^{\infty}} \mathrm{E_{t+j}} \left(\sum_{n=j}^{\infty} \beta^n \ u(C_{t+n}) | \text{no trade at any date } s \geq t+j \right).$$

This solution is what was called J_{t+j}^K (times β^{j-1}).

Hence the conditionally optimal policy solves

$$\max_{\{C_{t+n}\}_{n=0}^{\infty}} \mathrm{E_t} \left[\sum_{n=0}^{j-1} \beta^n \ u(C_{t+n}) | \text{no trade at any date } s < t+j \right] + \beta^{j-1} \ J_{t+j}^K$$

which is the same as J_t^K. Since this is true regardless of the conditioning index j, it follows that $J_t = J_t^K$ and also that the optimizing policies of the two problems coincide. Since K is the last exchange date, we also have that $J_t = J_t^o$ by definition of the latter.

We have shown that the value function and optimal policies are the same at $\tau_{K-1} < t \leq \tau_K$ as they would be if there were no intervention. By backward induction, the same argument applies at $\tau_{K-2} < t \leq \tau_{K-1}$, and so on. Since the optimal policies and value function are the same functions as in the no-intervention economy, it follows that the pricing function, and its derivatives must also be the same.

QED

B. $\mathcal{I}$ as a Function of v: The Two-period Case.

This appendix analyzes the dependence of $\mathcal{I}$ on v in a two-period CRRA economy analogous to the two-period pure-endowment example in Section 4. This illustrates how the availability of savings dampens both the current consumption effect and the future

consumption effect.

Suppose the representative agent has G_0 goods today and will receive a dividend XD_1 next period which he consumes and then dies. (I suppress the per-share superscript here to lighten the notation.) If the economy does not have a savings technology, so that $C_0 = G_0$, then, as in the example above, we have

$$\mathcal{I} = \gamma \left[1 + \frac{XP}{C_0}\right] = \gamma \left[1 + \frac{XP}{G_0}\right].$$

Now add the possibility of investing in cash and, for simplicity, fix the agent's savings to be $(1-\lambda)G_0$ or $C_0 = \lambda G_0$ for some fraction λ. Ignore the determination of the optimal λ and view the price per share today as $P(\lambda)$. The algebra in calculating $\mathcal{I}$ is a little messy, but worthwhile.

First,

$$P = P(G_0, X; \lambda) = \lambda^\gamma \, G_0^\gamma \, \beta \mathrm{E}[(XD_0\widetilde{R}_1 + (1-\lambda)RG_0)^{-\gamma} \, D_0\widetilde{R}_1]$$

where I have written $D_1 = D_0\widetilde{R}_1$, as before. Differentiating this with respect to X subject to $dG_0/dX = -P$ and scaling by X/P produces three terms in $\mathcal{I}$.

$$
\begin{aligned}
\text{Term I}: \quad & \gamma \, D_0 X \lambda^\gamma \, G_0^{\gamma-1} \, \beta \mathrm{E}(XD_0\widetilde{R}_1 + (1-\lambda)RG_0)^{-\gamma}\widetilde{R}_1 \\
= \quad & \gamma \, v_0 \lambda^\gamma \, \beta \mathrm{E}(v_0\widetilde{R}_1 + (1-\lambda)R)^{-\gamma}\widetilde{R}_1
\end{aligned}
$$

$$
\begin{aligned}
\text{Term II}: \quad & -\gamma \, D_0 X \lambda^\gamma \, G_0^\gamma (1-\lambda)R \, \beta \mathrm{E}(XD_0\widetilde{R}_1 + (1-\lambda)RG_0)^{-\gamma-1}\widetilde{R}_1 \\
= \quad & -\gamma \, v_0 \lambda^\gamma \, (1-\lambda)R \, \beta \mathrm{E}(v_0\widetilde{R}_1 + (1-\lambda)R)^{-\gamma-1}\widetilde{R}_1
\end{aligned}
$$

$$
\begin{aligned}
\text{Term III}: \quad & \gamma \, D_0^2 X \lambda^\gamma \, G_0^\gamma \, \beta \mathrm{E}(XD_0\widetilde{R}_1 + (1-\lambda)RG_0)^{-\gamma-1}\widetilde{R}_1^2/P \\
= \quad & \gamma \, v_0 \, \frac{\mathrm{E}(v_0\widetilde{R}_1 + (1-\lambda)R)^{-\gamma-1}\widetilde{R}_1^2}{\mathrm{E}(v_0\widetilde{R}_1 + (1-\lambda)R)^{-\gamma}\widetilde{R}_1}. \quad (3)
\end{aligned}
$$

The first term can also be written as simply $\gamma XP(\lambda)/G_0$, which shows that it corresponds to the current consumption effect. But now it is also clear, from the second line, that this term declines as λ does. As the agent saves more, the impact of the value-neutral trade on current consumption is, of course, smaller.

The second term is another contribution from the effect of altering current consumption, which arises because now (unlike in the endowment model) an increase in consumption goods today produces interest income next period. Comparing the second line with

the fourth, term II is strictly smaller than term I, because the only difference is an extra factor in the expectation of term II equal to $\Psi \equiv (1-\lambda)R/(v_0\widetilde{R}_1 + (1-\lambda)R) \leq 1$. Calling the term I integrand Γ, the two terms can also be combined, giving

$$\gamma \frac{XP}{G_0} \left[1 - \frac{\mathrm{E}\Psi\,\Gamma}{\mathrm{E}\Gamma} \right]$$

$$= \gamma\ \lambda^\gamma\ \beta\mathrm{E}\Gamma\Theta = \gamma\ v_0^2\lambda^\gamma\ \beta\mathrm{E}(v_0\widetilde{R}_1 + (1-\lambda)R)^{-\gamma-1}\widetilde{R}_1^2$$

where $\Theta \equiv (1-\Psi) = v_0\widetilde{R}_1/(v_0\widetilde{R}_1 + (1-\lambda)R)$. And the last equation clearly still decreases as λ does, vanishing at $\lambda = 0$. As one would expect, the total impact of the current consumption terms is smaller when the current consumption ratio is lower.

Finally, term III is what I referred to as the future consumption effect in the earlier examples. It can be reexpressed as

$$\gamma\ \frac{\mathrm{E}\Theta\,\Gamma}{\mathrm{E}\Gamma} \leq \gamma.$$

Again, the ability to store goods lowers this term relative to the endowment economy cases. Intuitively, the presence of positive savings at date zero lowers the percentage impact of a change in dividends on future marginal utility. Somewhat less obviously, term III also decreases as λ does, regardless of the parameter values.[15] Loosely, this is due to the extra term in the numerator, Θ, which behaves like $(1-\lambda)^{-1}$.

Hence all the terms in $\mathcal{I}$ are less than their counterparts in the corresponding endowment economy, and more so as λ declines. Now recall that the *optimal* consumption ratio is $\lambda = h(v)$, which is an increasing function of v. This reveals the mechanism that causes the liquidity of the market for asset one to increase as the level of liquid balances in the economy does.[16] The reason this happens is because the propensity to consume current wealth increases as liquid wealth declines, and this propensity, in turn, determines how big an impact a change in risky asset holdings has on marginal utilities. That impact dictates the willingness of agents to accommodate trades, or the rigidity of prices.

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[15]This can be shown using the result that two monotonically related random variables must be positively correlated. The proof is available upon request.

[16]I have not proven, even in the two-period case, that $\mathcal{I}(v)$ must be increasing in v. The argument above does not consider either the variation of λ with v or the direct effect, i.e. not through the savings term, of v on $\mathcal{I}$. What the argument shows is that, whichever direction these other terms go, the savings channel always makes illiquidity rise with v.

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