Dynamic Investment, Capital Structure, and Debt Overhang[*]

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Abstract

We model dynamic investment, financing and default decisions of a firm, which begins its life with a collection of growth options. The firm exercises them optimally over time, and finances the costs of investment by adjusting its capital structure, which trades off the tax benefits with the distress cost of debt and the agency cost of investment distortions from potential debt overhang. Conflicts of interests between equityholders and various classes of debtholders are managed through optimal choice of investment triggers, capital structure, and default triggers. We show that (*i*) existing debt may significantly distort investment decisions (debt overhang and risk shifting); (*ii*) anticipating distortions induced by debt, firms with more growth options on average have lower leverages, consistent with empirical evidence; (*iii*) the priority structure of debt has significant effects on the firm's default, leverage, and investment decisions, when existing debt is *exogenously* given; (*iv*) when the future growth options are perfectly anticipated, the firm optimally chooses its initial investment, default triggers and capital structure decisions, so as to mitigate the anticipated *endogenous* debt overhang. In this case, financial contracting plays a less prominent role.

Keywords: Real options, default, leverage, debt overhang, debt priority

JEL Classification: E22, G1, G3

1 Introduction

Corporations make intertemporal investment and financing decisions jointly. Corporate debt and equity are issued to finance investment, and hence their values depend on the firm's investment, financing and default decisions. This paper provides an analytically tractable framework to analyze dynamic corporate investment, financing, and default decisions.[1] The firm starts as a collection of growth options and optimally exercises the growth options over time. It finances the exercising costs of these growth options by adjusting its capital structure via sequential issuances of debt and equity. This naturally generates multiple classes of debt with different seniorities and priorities. As a result, the firm must confront the issue of debt overhang, which is an *endogenous* outcome, and evolves over the life cycle of the firm.

The conventional wisdom of debt overhang (Myers (1977)) is that (*i*) the pre-existing debt discourages the firm from investing because part of the value increase from new investment accrues to the existing debtholders due to the priority structure of the payoff and (*ii*) anticipating this debt overhang, the firm lowers its initial debt issuance. Hennessy (2004) studies the effect of pre-existing debt on firm's investment by injecting a consol debt into a neoclassical inter-temporal capital accumulation model of Abel and Eberly (1994). Empirically, he finds a significant debt overhang effect due to the pre-existing debt. Our model predicts that the pre-existing debt not only affects future growth option exercising, but also discourages default on the current debt. We further endogenize the initial investment and capital structure decision. We show that the endogenous coupon decision at the first stage significantly mitigates the *ex post* debt overhang effect. Our model shows that investment, leverage and default decisions are fundamentally linked in an intertemporal setting.

Our paper provides a natural bridge between structural credit risk/capital structure models, and the dynamic irreversible investment theory.[2] We find that even for firms with only one growth option, integrating investment and financing decisions generates important new insights, not captured by either the standard irreversible investment models such as McDonald and Siegel (1986), or credit risk/capital structure models such as Leland (1994). For

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[1] See Stein (2003) for a survey on corporate investment, agency conflicts, and information. See Caballero (1999) for a survey on aggregate dynamic investment. See Harris and Raviv (1991) for a survey on theories of capital structure.

[2] McDonald and Siegel (1985, 1986) and Brennan and Schwartz (1985) are fundamental contributions to modern real options approach to investment under uncertainty. Dixit and Pindyck (1994) is a standard textbook reference on real options approach towards investment. Abel and Eberly (1994) provide a unified framework integrating the neoclassical adjustment cost literature with the literature on irreversible investment. Grenadier (2002) shows that strategic interactions among agents may substantially erode the option value of waiting.

example, Leland (1994) shows that the default threshold decreases in volatility for the standard (put) option argument, in a contingent claim framework based on the standard trade-off theory of Modigliani and Miller (1963). However, the default threshold in our model may either decrease or increase in volatility. The intuition is as follows: (i) a higher volatility raises the investment threshold in our model for the standard (call option) value of waiting argument; (ii) a higher investment threshold naturally leads to a greater amount of debt issuance. That is, the firm issues more debt (but at a later time), when volatility is higher. Larger debt issuance raises the default threshold, *ceteris paribus*. As a result, unlike Leland (1994), we have two opposing effects of volatility on the default threshold due to endogenous investment in our model. We also find that debt financing has potentially quantitatively important effects on firm value, when the firm can take advantage of tax benefits of debt.

Now consider a firm which makes sequential investment and financing decisions. In order to sharpen our intuition, we proceed our analysis in two steps. First, we analyze the impact of existing debt on future investment, leverage and default decisions. Then, we endogenize the initial investment and leverage.

First, hold the initial investment and leverage decisions fixed. Provided that the amount of pre-existing debt for the firm is not too high, the firm rationally delays its next investment by increasing the future investment threshold.[3] The intuition is as follows. After collecting the proceeds from (earlier) debt issuance, the firm no longer behaves in the seasoned debtholders' interests. Equityholders and new debtholders pay for the exercising cost, but the benefits from investment first go to seasoned debtholders, under the absolute priority rule (APR).[4] This *ex post* wealth transfer effect discourages the firm from investing. However, once the existing debt is sufficiently high, the firm starts to take excessive risks (*risk shifting*) by prematurely exercising growth options. This risk shifting incentive is another widely studied form of conflicts of interests between equityholders and debtholders (Jensen and Meckling (1976)). When the firm engages in risk shifting, the seasoned debtholders bear more default risks. Intuitively, when outstanding debt is too high, equityholders are better off by unloading some credit risks to senior debtholders, even under the strict debt priority structure. Not surprisingly, we also find that financial contracting plays a significant role for the degree of debt overhang and risk shifting. We show that the debt overhang distortions and risk shifting incentives are more severe, under APR than under the *pari passu* structure.

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[3]Numerical work that studies the effect of pre-existing debt on investment includes Mello and Parsons (1992), Mauer and Triantis (1994), Parrino and Weisbach (1999), and Moyen (2006).

[4]By APR, we refer to the priority structure that more seasoned debt has strict priority over the newer debt in default.

Next, we endogenize the initial investment and leverage decisions. Because the firm anticipates future conflicts of interests between debtholders and equityholders once debt is in place, the firm takes a lower leverage to finance its first growth option exercising, *ceteris paribus*. This explains why firms with more growth options may take a lower leverage (Smith and Watts (1992) and Rajan and Zingales (1995)). Moreover, our model predicts that the more attractive the firm's future growth options are, the lower the firm's current leverage is, *ceteris paribus*. When the firm fully anticipates agency conflicts induced by debt, the firm manages to stay within the region of moderate levels of debt, and hence avoids *ex post* risk shifting in *endogenous* debt overhang region. Intuitively, equityholders do not want to issue too much debt in the first stage and then behave opportunistically *ex post* via risk shifting. Finally, we find that financial contracting, such as debt priority structure (APR versus *pari passu*), has smaller effects on *ex ante* firm value, under a wide range of specifications for various structural parameters. Intuitively, the firm may use the initial investment and leverage decisions to mitigate the anticipated conflicts between debtholders and equityholders. We also provide a real investment/agency theory based (structural) pricing model with multiple classes of debt, by extending Black and Cox (1976), who study debt pricing with exogenously specified seniority and priority structure for debt in a contingent claim framework.[5]

Recently, there is a growing body of literature that extends Leland (1994) to allow for strategic debt service,[6] and dynamic capital structure decisions. Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), and Strebulaev (2006) formulate dynamic trade-off decisions for leverage with exogenously specified investment policies.[7] Leary and Roberts (2005) empirically find that firms rebalance their capital structure infrequently in the presence of adjustment costs. Following Leland (1994), most contingent claims models of credit risk/capital structure assume that the firm's cash flows are exogenously given and focus on the firm's financing and default decisions.[8] Unlike these work, our model endogenizes growth option exercising decisions and induces dynamic leverage decisions via motives of

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[5]While we focus on the seniority of debt, there are studies which differentiate the priority structure between market debt and bank debt. For example, Hackbarth, Hennessy, and Leland (2005) study the optimal mixture and priority structure of bank and market debt using the tradeoff theory. They focus on the strategic debt service motives and do not model investment decisions.

[6]Anderson and Sundaresan (1996) use a binomial model to study the effect of strategic debt service on bond valuation. See Mella-Barra and Perraudin (1997), Hua and Sundaresan (2000), and Lambrecht (2001) for continuous-time contingent claim treatment.

[7]Early important contributions towards building dynamic capital structure models include Kane, Marcus, and McDonald (1984, 1985).

[8]Leland (1998) extends Leland (1994) by incorporating risk management with capital structure, and also allows the firm to engage in asset substitution by selecting volatility of the project.

financing investment. Titman and Tsyplakov (2005) also build a model that allows dynamic adjustment of both investment and capital structure. Their model is based on continuous investment decisions, while our model focuses on the irreversibility of growth option exercising.[9] We solve the model in closed form (up to a few nonlinear equations), while their model has three state variables and is numerically solved. Ju and Ou-Yang (2006) show that the firm's incentive to increase firm risk *ex post* is mitigated if the firm wants to issue debt periodically.

Our work is closely related to Hennessy and Whited (2005, 2006). Our paper complements their analysis, in that we also study dynamic investment and optimal capital structure with tradeoff, debt overhang, and endogenous default. Our work differs from theirs in the following several aspects. First, motivated by the desire to deliver a parsimonious framework to integrate dynamic investment with capital structure decisions, we derive a closed form characterization for default thresholds, investment thresholds, and optimal capital structure. In contrast, they aim to capture realism (such as tax codes, distribution policy, and wedge between internal and external financing) and focus on the model's fit to the data. Second, we model irreversibility of investment explicitly, and generate endogenous action/inaction regions for investment. They assume that investment can be made continuously. Third, we assume that the debt financing is accomplished through the issuance of perpetual debt as in Leland (1994) and Hennessy (2004), while they use one period debt.

Our work also relates to a large and growing literature on financial constraints and investment. Gomes (2001) studies the investment behavior of financially constrained firms. He finds that standard investment regressions may produce misleading results. Cooley and Quadrini (2001) analyze an industry dynamics model of investment where the firm may issue defaultable debt and also faces costly external financing.

We abstract away from other frictions that may affect capital structure and investment decisions, such as conflicts between managers and shareholders. Using the empire building/free cash flow theory of Jensen (1986), Zwiebel (1996) develops a model of dynamic capital structure in which the manager trades off the benefits from empire building with the need to ensure sufficient efficiency to avoid control challenges.[10] Broadly speaking, our work also relates to the growing literature on dynamic capital structure using recursive contracting methodology. DeMarzo and Fishman (2005), and DeMarzo and Sannikov (2006) derive opti-

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[9]Brennan and Schwartz (1984) is an early important contribution, which allows for the interaction between investment and financing.

[10]Building on Jensen (1986), Stulz (1990), and Zwiebel (1996), Morellec (2004) develops a contingent claim model with manager-shareholder conflicts and shows that this agency conflict lowers leverage ratio.

mal dynamic contracts and implement the contracts with capital structure (using credit line, long term debt and equity) in discrete time and continuous time formulations, respectively.

The remainder of the paper is organized as follows. Section 2 introduces the model setup and summarizes the results for the benchmark with equity financing. Section 3 solves for the firm's interdependent investment, default and leverage decisions via backward induction. Section 4 derives closed-form solutions for the investment, financing, and default decisions, and analyzes the interactions among these decision rules, for firms with only one growth option. In Section 5, we first study the effects of existing debt on investment, default and leverage decisions; and then solve for the initial investment and leverage decisions, taking into account the anticipated debt overhang problem in the future. Section 6 concludes.

2 Model setup and all-equity benchmark

We first set up the model that allows for joint determination of sequential investment, financing, and default decisions. Then, we solve for optimal investment decisions when the firm is all equity financed. We later use this all-equity setting as a natural benchmark to assess the impact of debt financing on investment.

2.1 Setup

Assume that the firm behaves in the interests of equityholders. The firm starts with two sequentially ordered growth options, with no initial assets in place. Suppose that the second growth option can only be exercised after the first asset (obtained from exercising the first growth option) is in place. That is, the second growth option may be viewed as an expansion option once the first asset is in place.[11] The costs of exercising each growth option are I_1 and I_2, respectively. The firm may issue a mixture of debt and equity to finance the exercising costs. Assume that debt has tax advantage. The firm faces a constant tax rate $\tau > 0$ on its income after servicing interest payments on debt.

The firm observes the demand shock X for its product, where X is given by the following geometric Brownian motion (GBM) process:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW_t, \tag{1}$$

where W is a standard Brownian motion.[12] Assume that the risk-free interest rate is constant

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[11]See Abel and Eberly (2005) for a model on investment and valuation with growth options.

[12]Let W be a standard Brownian motion in $\mathbb{R}$ on a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and fix the standard filtration $\{\mathcal{F}_t : t \geq 0\}$ of W. Since all securities are traded here, we directly work under the risk-neutral probability

and is equal to r. For convergence, we assume $r > \mu$. Let $\Pi_1(x)$ and $\Pi_2(x)$ denote the after-tax (all-equity-financed) values of assets in place generated from exercising of the first and the second growth options, respectively. Under all-equity financing, the asset in place from exercising the k-th growth option is given by

$$\Pi_k(x) = \frac{1-\tau}{r-\mu} Q_k x, \quad k = 1, 2, \tag{2}$$

where $Q_1 > 0$ and $Q_2 > 0$. The k-th asset in place has revenue rate given by $Q_k X$, where Q_k is the (constant) quantity produced from the k-th asset in place and X is the stochastic price process for the output. There is no variable production cost, and hence revenue flow is equal to profit flow. For analytical tractability, we have intentionally chosen to model the firm as one with two sequentially ordered growth options. While our model of the firm has a stylized capital accumulation process, it captures the repeated interactions between the firm's investment and financing decisions.

Insert Figure 1 here.

Figure 1 describes the decision making process of the firm over its life cycle. The firm may exercise its first growth option by paying the fixed cost I_1 at endogenously chosen time T_1^i as in McDonald and Siegel (1986). When exercising the first growth option at T_1^i, the firm may issue a perpetual debt with coupon c_1 to finance the exercising cost I_1. Here, we follow Leland (1994) to assume that the firm will issue debt with infinite maturity. This assumption simplifies the analysis substantially. The remaining amount is contributed by equityholders.

After the first asset is in place, the firm has the (second) growth option and the (first) default option. Let T_1^d and T_2^i denote the endogenously chosen time for firm's first default and the second investment, after the exercise of the first growth option ($t \geq T_1^i$). Assume that the firm recovers a fraction of residual values from the first asset in place and also from the second (unexercised) growth option, upon default at T_1^d. Extending Leland (1994), we assume that the firm's total value $V_1(\cdot)$ at T_1^d is given by a fraction $(1-\alpha)$ of the sum of (i) the "un-levered" value of (first) asset in place $\Pi_1(X(T_1^d))$ and (ii) $\omega\Pi_2(X(T_1^d))$, in that

$$V_1(X(T_1^d)) = (1-\alpha)\left(\Pi_1(X(T_1^d)) + \omega\Pi_2(X(T_1^d))\right), \tag{3}$$

where $\Pi_1(x)$ and $\Pi_2(x)$ are given by (2), and $0 \leq \omega < 1$. As in Leland (1994), we interpret $0 \leq \alpha < 1$ as a measure of inefficiency due to default. The firm loses value both because of

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measure $\mathbb{Q}$. That is, the stochastic process (1) is under this risk neutral measure $\mathbb{Q}$. Under the infinite horizon, additional technical conditions such as uniform integrability are assumed here.

distress cost ($\alpha > 0$) and also loss of tax shelters. Intuitively, the residual value from the (unexercised) growth option is lower than the (first) asset in place ($0 \leq \omega < 1$). For example, the growth option may be potentially less tangible than the asset in place, and the firm can only sell the growth option at a discount, compared with the first asset in place. The inalienability of the manager's human capital for the growth option may be more significant than for the asset in place. As a result, it may be harder to sell the unexercised growth option, *ceteris paribus*. Because the firm receive a scrap value $(1 - \alpha) \, \omega \Pi_2(X(T_1^d))$ at default time T_1^d without paying the cost I_2, we need to make sure ω is sufficiently low to ensure that the (second) growth option has a lower scrap value in default than the (first) asset in place.

If the demand shock X is sufficiently high, then it is optimal to exercise its second growth option. By paying the fixed investment cost I_2 and exercising its second growth option at endogenously chosen time T_2^i, the firm generates an additional stream of cash flows $Q_2 X$, in addition to the stream of cash flows $Q_1 X$ from the first asset in place. Therefore, the total cash flow is given by $(Q_1 + Q_2) X$, after T_2^i and before the firm exercises its second default option at T_2^d. Let $Q = Q_1 + Q_2$. As at the first investment time T_1^i, the firm issues the second perpetual debt with coupon c_2 at T_2^i. We assume that the firm cannot call back its first perpetual debt.

After both assets are in place and both types of debt are outstanding, the firm may still default at endogenously chosen time T_2^d. As in the standard tradeoff theory, assume that debt may potentially cause distress at default, and hence is also costly to the equityholders *ex ante*. Let $\Pi(x)$ denote the total "un-levered" firm value (with positive tax rate τ):

$$\Pi(x) = \Pi_1(x) + \Pi_2(x) = \frac{1 - \tau}{r - \mu} Q x, \tag{4}$$

where $Q = Q_1 + Q_2$. If the the firm defaults at T_2^d, then the firm's default value is given by $(1 - \alpha) \, \Pi(X(T_2^d))$, where $\Pi(x)$ is given by (4), and $0 \leq \alpha < 1$ is a measure of inefficiency due to default as in Leland (1994).

The long maturity of debt allows us to generate debt overhang in a convenient way (Myers (1977) and Hennessy (2004)) . We leave the modeling of debt maturity for future research. Because debt is perpetual and not callable, the first debt continues to exist even after exercising the second growth option. Let $D_2^s(x)$ and $D_2^n(x)$ denote the market values of the first (seasoned) debt, and of the second debt issued at the second investment time T_2^i,

respectively. These debt values (after the second growth option is exercised) are given by

$$D_2^s(x) = E_t^x \left[\int_t^{T_2^d} e^{-r(s-t)} c_1 ds + e^{-r(T_2^d - t)} D_2^s(X(T_2^d)) \right], \quad T_2^i \le t \le T_2^d, \quad (5)$$

$$D_2^n(x) = E_t^x \left[\int_t^{T_2^d} e^{-r(s-t)} c_2 ds + e^{-r(T_2^d - t)} D_2^n(X(T_2^d)) \right], \quad T_2^i \le t \le T_2^d. \quad (6)$$

The residual values of the first and second debt, $D_2^s(X(T_2^d))$ and $D_2^n(X(T_2^d))$ are given by the debt priority and payoff structure to be discussed later. We assume that the debt structure is always respected in distress and there is no deviation from the covenants. The majority of this paper focuses on a commonly observed debt priority structure: The more seasoned debt has absolute priority over debt issued later (absolute priority rule (APR)). In Section 5, we will also consider an alternative debt priority structure, where all debt has equal priority regardless of the issuance date, i.e. *pari passu*. Since these two debt structures have different implications on the residual values of debt, they also have implications on investment and financing decisions. The total market value of the two debt issuances is then given by $D_2(x) = D_2^n(x) + D_2^s(x)$.

Let $D_1(x)$ denote the market value of the first debt after the exercise of the first growth option, but before the exercise of the second growth option. We have

$$D_1(x) = E_t^x \left[\int_t^{T_1^d \wedge T_2^i} e^{-r(s-t)} c_1 ds + e^{-r(T_1^d - t)} D_1(X(T_1^d)) \mathbf{1}_{T_1^d < T_2^i} + e^{-r(T_2^i - t)} D_2^s(X(T_2^i)) \mathbf{1}_{T_1^d > T_2^i} \right]. \quad (7)$$

Before delving into the details on the interactions between sequential investment and financing, we first propose a benchmark, where the firm is all equity financed. This benchmark helps us to understand the impact of debt financing on investment, financing decisions and firm value.

2.2 Benchmark: All equity financing

By definition, there is no debt ($c_1 = c_2 = 0$) under all equity financing. Since the firm's demand shock follows a GBM process (1), its cash flow is always positive, and hence it always has incentives to invest. The firm chooses its first investment time T_1^i, and its second investment time $T_2^i \ge T_1^i$ to maximize its value given below:

$$E^x \left[\int_{T_1^i}^{\infty} e^{-rs} (1 - \tau) Q_1 X(s) ds - e^{-rT_1^i} I_1 + \int_{T_2^i}^{\infty} e^{-rs} (1 - \tau) Q_2 X(s) ds - e^{-rT_2^i} I_2 \right]. \quad (8)$$

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Throughout the paper, we will focus on the parameter regions under which the firm finds optimal to exercise the growth options sequentially. Under all equity financing, the following condition ensures that sequential exercising of the growth options is optimal.

Condition 1 *Investment benefits and costs satisfy the following inequality:*

$$\frac{Q_2}{I_2} < \frac{Q_1}{I_1}. \tag{9}$$

The above condition gives a notion for decreasing returns to scale (Grenadier (1996)). Intuitively, the second growth option is less attractive than the first growth option. Hence, the firm continues to wait (at least for an instant) after exercising the first growth option.

Let $E_0(x)$ and $E_1(x)$ denote the equity value before exercising the first growth option $(t \leq T_1^i)$, and the equity value before exercising the second growth option but after exercising the first growth option $(T_1^i \leq t \leq T_2^i)$, respectively. For expositional convenience, the following proposition summarizes the known results (Grenadier (1996)) when the firm is all equity financed and is subject to a corporate tax at rate τ.

Proposition 1 *The investment decisions under all equity financing are characterized by the threshold strategies:* $T_1^i = \inf\{t \geq 0 : X(t) = x_1^i\}$ *and* $T_2^i = \inf\{t \geq 0 : X(t) = x_2^i\}$. *Under Condition 1, we have* $x_1^i = x_1^{ae}$ *and* $x_2^i = x_2^{ae}$, *where*

$$x_k^{ae} = \frac{1}{1-\tau} \frac{r-\mu}{Q_k} \frac{\beta}{\beta-1} I_k, \quad k = 1, 2, \tag{10}$$

and $\beta > 1$ *is given by*

$$\beta = \frac{1}{\sigma^2} \left[-\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2} \right]. \tag{11}$$

Equity values $E_0(x)$ *and* $E_1(x)$ *are given in Appendix A.1.*

When Condition 1 holds, it is optimal for the firm to sequentially exercise its two growth options. Intuitively, the exercising decisions for the two growth options are effectively independent. That is, both x_1^i and x_2^i are equal to the respective threshold in a setting with only one growth option (and the same set of parameters). We may strengthen our intuition for this result by noting that the joint maximization problem given in (8) may be separated into two independent one-growth-option exercising problems with parameters (I_k, Q_k), provided that Condition 1 holds. Intuitively, the technological constraint that the second growth option can only be exercised after the first growth option is exercised $\left(T_2^i \geq T_1^i\right)$, is not binding.

Second, taxes lower the benefits from investing under all equity financing. This explains the factor $1/(1-\tau)$ for the investment thresholds x_1^i and x_2^i given in (10). Finally, both x_1^i and x_2^i increase in volatility, as in standard real options model such as those of McDonald and Siegel (1986).

When Condition 1 does not hold, in that $Q_1/I_1 \leq Q_2/I_2$, simultaneous exercising of both growth options is optimal. Intuitively, the second growth option is immediately worth exercising after the exercise of the first growth option. The firm rationally chooses the optimal exercising strategy by treating the two sequentially ordered growth options as a combined growth option with exercise cost $I = I_1 + I_2$, and $Q = Q_1 + Q_2$. The optimal investment threshold is then given by $x_1^i = x_2^i = x^{ae}$, where x^{ae} is given by (10), with the exercising cost $I = I_1 + I_2$ and $Q = Q_1 + Q_2$.

For future comparisons, let x_1^* and x_2^* denote the first and second investment threshold without taxes $(\tau = 0)$ when Condition 1 holds. We have

$$ x_k^* = \frac{r-\mu}{Q_k} \frac{\beta}{\beta-1} I_k, \quad k = 1, 2. \tag{12} $$

Let x^* denote the corresponding optimal investment threshold with investment cost I and output parameter Q. For example, when Condition 1 does not hold, in that $Q_1/I_1 \leq Q_2/I_2$, simultaneous exercising of both growth options is optimal. Under such a setting, x^* denote the corresponding optimal investment threshold with $I = I_1 + I_2$ and $Q = Q_1 + Q_2$.

Having described the decision making process over the life-cycle of the firm and summarized the all-equity benchmark, we now solve the two investment thresholds $\left(x_1^i, x_2^i\right)$, two default thresholds $\left(x_1^d, x_2^d\right)$, and the two coupon decisions (c_1, c_2), via backward induction.

3 Sequential investment, default and financing

First consider the situation after exercising the second growth option $\left(t \geq T_2^i\right)$. Equityholders have incentives to default after debt is in place as in Black and Cox (1976). Equityholders choose the default time T_2^d to maximize

$$ E_t^x \left[\int_t^{T_2^d} e^{-r(s-t)} (1-\tau)(QX(s) - c)\, ds \right], \quad t \geq T_2^i. \tag{13} $$

Under the assumption that equity is junior to debt, equityholders receive nothing at default. Let $E_2(x)$ denote equity value from the above optimization problem, and x_2^d denote the endogenous (second) default threshold.

Now consider the equityholders' decision problem after the exercise of the first growth option $\left(t \geq T_1^i\right)$. Equityholders choose either to default in which case they receive nothing, or to exercise the second growth option. If choosing to exercise the second growth option at T_2^i, they will also choose the amount of the second debt issuance at the second investment time T_2^i, where c_2 is the selected coupon payment.

Let $V_2^n(x)$ denote the sum of equity value and (newly issued) debt value after the second investment is made, in that $V_2^n(x) = E_2(x) + D_2^n(x)$. The net gain for the equityholders is thus given by $E_2(X(T_2^i)) - \left(I_2 - D_2^n(X(T_2^i))\right) = V_2^n(X(T_2^i)) - I_2$. Equityholders choose the first default time T_1^d, the second investment time T_2^i and the coupon on the second debt c_2 to maximize the following objective function:

$$E_t^x \left[\int_t^{T_1^d \wedge T_2^i} e^{-r(s-t)} (1-\tau) (Q_1 X(s) - c_1) \, ds + e^{-r(T_2^i - t)} \left(V_2^n(X(T_2^i)) - I_2\right) \mathbf{1}_{T_1^d > T_2^i} \right]. \quad (14)$$

Let $E_1(x)$ denote the value function from the above optimization problem, and x_1^d and x_2^i denote the endogenous default threshold, and the investment threshold, respectively. As we naturally anticipate, the default decision (the default time T_2^d and the default threshold x_2^d) solved from the last stage optimization problem (13) enters into the objective function (14) because $V_2^n(x)$ depends on the second default threshold x_2^d.

Finally, consider equityholders' first growth option exercising decision and debt financing decision $\left(t \leq T_1^i\right)$. Since equityholders will issue debt with market value $D_1(X(T_1^i))$ when investing, the net amount needed from equityholders will be $I_1 - D_1(X(T_1^i))$. Note that equityholders internalize both tax benefits and the distress cost from debt issuance. They choose its first investment time T_1^i and the coupon c_1 on the first debt issued at T_1^i to maximize equity value given below:

$$E_t^x \left[e^{-r(T_1^i - t)} \left(V_1(X(T_1^i)) - I_1\right)\right], \quad t \leq T_1^i. \quad (15)$$

where $V_1(X(T_1^i)) = E_1(X(T_1^i) + D_1(X(T_1^i))$. Let $E_0(x)$ denote the value function from the above optimization problem, and x_1^i denote the endogenous first investment threshold.

First, we solve for the default decision x_2^d and value functions such as equity value $E_2(x)$ and firm value $V_2(x)$ after the second growth option is exercised $\left(t \geq T_2^i\right)$.

3.1 After the exercise of the second growth option $\left(t \geq T_2^i\right)$

After both growth options are converted into assets in place, the firm generates total cash flows at the rate of Qx, where $Q = Q_1 + Q_2$. The total coupon rate is then $c = c_1 + c_2$. The

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firm has only the default decision (characterized by the default threshold x_2^d) to make after both growth options are exercised. Failure to pay either debtholders immediately triggers default. As in Leland (1994), the following value-matching and smooth-pasting conditions hold at the endogenous default boundary x_2^d:

$$E_2(x_2^d) = 0, \tag{16}$$

$$E_2'(x_2^d) = 0. \tag{17}$$

Note that when $x \leq x_2^d$, equity is worthless $(E_2(x) = 0)$.

Leland (1994) shows that the equity value $E_2(x)$ may be written as follows:

$$E_2(x) = \Pi(x) - \frac{(1-\tau)c}{r} - \left[\Pi(x_2^d) - \frac{(1-\tau)c}{r}\right]\left(\frac{x}{x_2^d}\right)^\gamma, \quad x \geq x_2^d, \tag{18}$$

where the optimal default threshold x_2^d is given by

$$x_2^d = \frac{r-\mu}{Q}\frac{\gamma}{\gamma-1}\frac{c}{r}, \tag{19}$$

and γ is the negative root of the fundamental quadratic equation and is given by

$$\gamma = -\frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]. \tag{20}$$

Equity value $E_2(x)$ is given by (i) the "un-levered" firm value $\Pi(x)$, subtracting (ii) the present value of the tax shields $(1-\tau)c/r$, and adding (iii) the value of the default option, which is given by the product of (a) the present discounted value $(x/x_2^d)^\gamma$ for a unit payoff at the default boundary x_2^d and (b) the present value of savings from default, $-\left(\Pi(x_2^d) - (1-\tau)c/r\right)$. At the chosen default threshold x_2^d given in (19), the inequality $\Pi(x_2^d) < (1-\tau)c/r$ reflects the positive value of waiting before default. As in Black and Cox (1976) and Leland (1994), the standard option value argument implies that the default threshold x_2^d decreases with volatility σ, and the equity value $E_2(x)$ is convex in x.

We now may define various value functions, given the default threshold x_2^d and the coupon rates c_1 and c_2. Before the firm defaults, equityholders make the promised payments. When the firm defaults, debt priority structure gives the recovery value for various debt claims: $D_2^s(x_2^d)$ and $D_2^n(x_2^d)$. Assume that the debt covenants will be strictly enforced without any violation. Given these values at the endogenous default boundary x_2^d, we may write the market values of the seasoned debt issued at T_1^i and of the debt issued at T_2^i, before default

at T_2^d, as follows:

$$D_2^s(x) = \frac{c_1}{r} - \left[\frac{c_1}{r} - D_2^s(x_2^d) \right] \left(\frac{x}{x_2^d} \right)^\gamma, \quad x \geq x_2^d, \tag{21}$$

$$D_2^n(x) = \frac{c_2}{r} - \left[\frac{c_2}{r} - D_2^n(x_2^d) \right] \left(\frac{x}{x_2^d} \right)^\gamma, \quad x \geq x_2^d. \tag{22}$$

The total debt value is $D_2(x) = D_2^s(x) + D_2^n(x)$. The total debt value at default $D_2(x_2^d)$ is equal to the total firm's liquidation value at default, since equity is worthless at default. Using the standard argument in option pricing, we note that $D_2^s(x)$, $D_2^n(x)$, and $D_2(x)$ are all concave in x because of default.

Firm value $V_2(x) = E_2(x) + D_2(x)$ is then given by

$$V_2(x) = \Pi(x) + \frac{\tau c}{r} - \left[\alpha \Pi(x_2^d) + \frac{\tau c}{r} \right] \left(\frac{x}{x_2^d} \right)^\gamma, \quad x \geq x_2^d. \tag{23}$$

Firm value $V_2(x)$ is given by the "unlevered" (after-tax) firm value $\Pi(x)$, plus $\tau c / r$, the perpetuity value of tax shield τc from both coupon payments c_1 and c_2 (assuming no default), minus the expected loss given default (the last term). The expected loss given default is given by the product of (i) the present discounted value $(x/x_2^d)^\gamma$ for a unit payoff at the default boundary x_2^d and (ii) the loss given default $\alpha \Pi(x_2^d) + \tau c / r$, which includes both liquidation cost $\alpha \Pi(x_2^d)$ and the perpetuity value of forgone tax shields $\tau c / r$. As in Leland (1994), firm value $V_2(x)$ is concave in x. Intuitively, the firm is long the "unlevered" firm and the tax shield perpetuity $\tau c / r$, and short in a liquidation option.

Recall that $V_2^n(x)$ is the sum of equity value $E_2(x)$ and debt value $D_2^n(x)$ issued when exercising the second growth option: $V_2^n(x) = E_2(x) + D_2^n(x)$. Using (18) and (22), we have

$$V_2^n(x) = \Pi(x) + \frac{\tau c - c_1}{r} + \nu_2^n \left(D_2^n(x_2^d) - \Pi(x_2^d) + \frac{c_1 - \tau c}{r} \right) \left(\frac{x}{x_2^d} \right)^\gamma, \quad x \geq x_2^d. \tag{24}$$

The distinction between $V_2(x)$ and $V_2^n(x)$ is essential for our analysis. Equityholders no longer care about the payoffs to the seasoned debtholders after collecting the proceeds from the debt issuance at T_2^i. This creates conflicts of interests between equityholders and seasoned debtholders. Equityholders choose the investment threshold x_2^i and the coupon policy c_2 to maximize $V_2^n(x)$, not $V_2(x)$. The seasoned debt issued at T_1^i to finance the exercise of the first growth option generates a debt overhang problem and distorts the exercising decision for the second growth option. Of course, debtholders anticipate the equityholders' incentives and price the debt accordingly. Equityholders eventually bear the cost of this debt overhang. Unlike most papers in the literature on debt overhang, the amount of pre-existing debt and

hence the severity of debt overhang in our model will be determined endogenously. We show that the significance of debt overhang is quite different, depending on whether debt is pre-specified or endogenously determined. Moreover, different debt priority structure affects the debt overhang problem in different ways as we show later in Section 5.

Now consider the coupon policy c_2 on the second debt issuance. The first debt issued at T_1^i is already in place when the firm exercises its second growth option at T_2^i. Equityholders choose c_2 to maximize $V_2^n(x)$ and then evaluate at the investment threshold x_2^i, for given c_1.

3.2 After the exercise of the first growth option $\left(T_1^i \leq t \leq T_2^i \wedge T_1^d\right)$

When investing at the threshold x_2^i, equityholders need to finance the exercise cost I_2. Immediately after investing, the equity value is worth $E_2\left(x_2^i\right)$ after paying the part of the exercise cost $\left(I_2 - D_2^n\left(x_2^i\right)\right)$ not financed by debt. The value matching condition at the investment threshold x_2^i is then given by

$$E_1\left(x_2^i\right) = E_2\left(x_2^i\right) - \left(I_2 - D_2^n\left(x_2^i\right)\right) = V_2^n\left(x_2^i\right) - I_2. \tag{25}$$

When the equityholders choose the investment threshold x_2^i optimally, the following smooth pasting condition also holds:

$$E_1'\left(x_2^i\right) = V_2^{n\prime}\left(x_2^i\right). \tag{26}$$

The value matching condition (25) and the smooth pasting condition (26) reflect the debt overhang problem. The equityholders make the second investment decision without taking into account the interests of the seasoned debt. Therefore, $V_2^n(x_2^i)$, not $V_2(x_2^i)$, enters the right sides of the boundary conditions (25) and (26). Now turn to the default decision. Using the same arguments as those for the value matching and smooth pasting conditions (16) and (17) for equity value $E_2(x)$, equityholders choose the first default threshold x_1^d to satisfy the value-matching condition $E_1(x_1^d) = 0$ and the smooth pasting condition $E_1'(x_1^d) = 0$.

Let $\Phi_i(x)$ denote the present discounted value of receiving a unit payoff at T_2^i if the firm invests at T_2^i, namely, $T_2^i < T_1^d$. Similarly, let $\Phi_d(x)$ denote the present discounted value of receiving a unit payoff at T_1^d if the firm defaults at T_1^d, namely $T_1^d < T_2^i$. The closed-form expressions for $\Phi_i(x)$ and $\Phi_d(x)$ are given by (A.7) and (A.8) in the appendix, respectively.[13] Using these formulae, we may write equity value $E_1(x)$ as follows:

$$E_1(x) = \Pi_1(x) - \frac{(1-\tau)\,c_1}{r} + e_1^i \Phi_i(x) + e_1^d \Phi_d(x), \quad x_1^d \leq x \leq x_2^i, \tag{27}$$

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[13]Formally, $\Phi_i(x) = E_t^x\left[e^{-r\left(T_2^i - t\right)}\mathbf{1}_{T_1^d > T_2^i}\right]$, and $\Phi_d(x) = E_t^x\left[e^{-r\left(T_1^d - t\right)}\mathbf{1}_{T_1^d < T_2^i}\right]$, where $\mathbf{1}_{T_1^d > T_2^i}$ and $\mathbf{1}_{T_1^d > T_2^i}$ are the indicator functions. If $T_1^d > T_2^i$, we have $\mathbf{1}_{T_1^d > T_2^i} = 1$. Otherwise, $\mathbf{1}_{T_1^d > T_2^i} = 0$.

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where

$$e_1^i = V_2^n(x_2^i) - I_2 - \left(\Pi_1(x_2^i) - \frac{(1-\tau)c_1}{r}\right) > 0, \tag{28}$$

$$e_1^d = -\left[\Pi_1(x_1^d) - \frac{(1-\tau)c_1}{r}\right] > 0. \tag{29}$$

Equity value $E_1(x)$ is given by the sum of the "un-levered" equity value (with neither default nor growth options) and two option values: the growth option and the default option. The un-levered equity value (without growth/default options) for $E_1(x)$ is given by the difference between the un-levered value of the asset in place converted from the first growth option $\Pi_1(x)$ and the perpetual value of tax shields from the first debt issuance, $(1-\tau)c_1/r$. The third term in (27) measures the present value of the growth option, which is given by the product of $\Phi_i(x)$, and the *net* payoff e_1^i from exercising the option. The *net* payoff e_1^i is the difference between the payoff from option exercise $V_2^n(x_2^i) - I_2$ and $\left(\Pi_1(x_2^i) - (1-\tau)c_1/r\right)$, the forgone un-levered equity value when investing at the threshold x_2^i. Note that the forgone "un-levered" equity value appears as an additional cost term in the net payoff e_1 because the option payoff $V_2^n(x_2^i) - I_2$ contains cash flows from the first asset in place. Similarly, the fourth term in (27) is the present value of the default option, which is given by the product of $\Phi_d(x)$ and the *net* payoff e_1^d upon default. Since equityholders receive nothing at default, the *net* payoff e_1^d is given by the savings, $-\left(\Pi_1(x_1^d) - (1-\tau)c_1/r\right) > 0$, from avoiding the loss of running the "un-levered equity value" at the default threshold x_1^d.

Given the default threshold x_1^d and the investment threshold x_2^i, we may write firm value $V_1(x)$ as follows:

$$V_1(x) = \Pi_1(x) + \frac{\tau c_1}{r} + v_1^i \Phi_i(x) + v_1^d \Phi_d(x), \quad x_1^d \le x \le x_2^i, \tag{30}$$

where

$$v_1^i = V_2(x_2^i) - I_2 - \left(\Pi_1(x_2^i) + \frac{\tau c_1}{r}\right) > 0, \tag{31}$$

$$v_1^d = -\left[\alpha \Pi_1(x_1^d) - (1-\alpha)\omega \Pi_2(x_1^d) + \frac{\tau c_1}{r}\right] < 0. \tag{32}$$

In addition to (*i*) the "unlevered" (after-tax) value of the asset in place $\Pi_1(x)$ from exercising the first growth option, and (*ii*) the perpetuity of the tax shield $\tau c_1/r$, firm value $V_1(x)$ also includes (*iii*) a long position in the growth option (the third term in (30)) and (*iv*) a short position in the liquidation option (the fourth term in (30)). When the firm exercises its second growth option at T_2^i, it generates a net gain $v_1^i = V_2(x_2^i) - I_2 - (\Pi_1(x_2^i) + \tau c_1/r)$.

Note that $V_2(x_2^i)$ includes the cash flows generated form the first and the second assets in place. When equityholders default at T_1^d, the firm loses $|v_1^d|$ where v_1^d is given in (32) because default induces distress cost and also loses tax shields as in our model.

Now, we have used backward induction to solve for the second default threshold x_2^d, second coupon c_2, the second investment threshold x_2^i and the first default threshold x_1^d. We now turn to the investment and financing decisions for the first growth option.

3.3 Before the exercise of the first growth option $(t \leq T_1^i)$

First, consider the region for the initial value x_0 where it is optimal for equityholders to wait. Conjecture that the investment decision takes the threshold form as in previous sections. That is, for $x_0 \leq x_1^i$, the firm will until $T^i = \inf\{t : X(t) \geq x_1^i\}$ to exercise the first growth option.

Because equityholders internalize the tax benefits, distress costs, and agency costs of debt, the net payoff to equityholders from exercising the first growth option is equal to $V_1(x) - I_1$. We thus have the following value matching and smooth pasting conditions:

$$E_0(x_1^i) = V_1(x_1^i) - I_1, \tag{33}$$

$$E_0'(x_1^i) = V_1'(x_1^i). \tag{34}$$

Equity value $E_0(x)$ is then given by

$$E_0(x) = \left(\frac{x}{x_1^i}\right)^\beta \left(V_1\left(x_1^i\right) - I_1\right), \quad x \leq x_1^i, \tag{35}$$

where the first investment threshold x_1^i satisfies the following implicit equation:

$$x_1^i = \frac{1}{1-\tau} \frac{r-\mu}{Q_1} \frac{\beta}{\beta-1} \left[\left(I_1 - \frac{\tau c_1}{r}\right) + \frac{\beta-\gamma}{\beta\Delta}(x_1^i)^\gamma \left((x_1^d)^\beta v_1^i - (x_2^i)^\beta v_1^d\right)\right], \tag{36}$$

and Δ is a strictly positive constant given in (A.9). Unlike in the standard equity-based real options models, the payoff from investment in our model is $V_1(x)$, the sum of debt value $D_1(x)$ and equity value $E_1(x)$, which includes the present values of cash flows from both operations and financing.

Now turn to the first coupon policy c_1. Equityholders choose c_1 to maximize $E_0(x)$ and then evaluate $E_0(x)$ at $x = x_1^i$. By the value matching condition (33) and the smooth pasting condition (34) at x_1^i, it is equivalent for equityholders to choose c_1 to maximize $V_1(x)$ and evaluate at x_1^i. This reflects that equityholders internalize both the tax benefits, distress and agency costs of debt when choosing c_1.

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So far, we have presented the solution methodology for the firm's optimization problem, when the initial value x_0 is below x_1^i, the optimal first investment threshold. Now suppose that the initial value x_0 is above the optimal investment threshold x_1^i from the above optimization problem $(x_0 \geq x_1^i)$, then the firm shall immediately exercise its first growth option. We thus have $E_0(x_0) = V_1(x_0) - I_1$. As in earlier discussions, we will continue to use the backward induction to find optimal default thresholds x_1^d and x_2^d, the second investment threshold x_2^i, and the coupon c_2. Equityholders then choose c_1 to maximize $V_1(x_0)$, taking into account the dependence of the thresholds x_1^d, x_2^d, x_2^i and c_2 on c_1. If the initial value x_0 is really high, then the firm will find that simultaneously exercising both growth options is valuable.[14]

3.4 Debt priority structure and coupon policies

For expositional simplicity and concreteness, we assume that the APR holds unless otherwise noted. Smith and Warner (1979) document that 90.8% of their sampled covenants contain some restrictions on future debt issuance. As in Black and Cox (1976), at default, the junior debtholders will not get paid at all until the senior debtholders are completely paid off. At the second default threshold x_2^d, the senior debtholders collect

$$D_2^s(x_2^d) = \min \left\{ F_1, (1 - \alpha) \, \Pi(x_2^d) \right\}, \tag{37}$$

where F_1 is the par value of the first debt and is equal to $F_1 = D_1(x_1^i)$. The payoff function (37) states that either the senior debtholders get paid F_1 at T_2^d, or the senior debtholders collect the total recovery value of the firm $(1 - \alpha) \, \Pi(x_2^d)$ at T_2^d. It is immediate to see that under APR, the junior debt value at default time T_2^d is given by

$$D_2^n(x_2^d) = \max \left\{ (1 - \alpha) \, \Pi(x_2^d) - F_1, 0 \right\}. \tag{38}$$

Let F_2 denote the par value of the second debt issued at T_2^i. Because the debt is issued at par, we thus have $F_2 = D_2^s(x_2^i)$. Equityholders receives nothing at default, hence, we have $(1 - \alpha) \, \Pi(x_2^d) \leq F_1 + F_2$. Note that even when the senior debtholders receive par F_1 at default time T_2^d, senior debtholders would still prefer collecting coupons. This is intuitive, because the par value $F_1 < c_1/r$.

Debt priority structure matters not only for payoffs at default boundaries x_2^d as in Black and Cox (1976), but also for the real investment and financial leverage decisions. The costs and benefits of issuing debt depend on the priority and payoff structures. Moreover, the

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[14]This case is effectively one when the firm is a cash flow generating machine and effectively faces no growth option exercising decisions. The analysis is essentially Leland (1994).

equityholders' interests and incentives also change over time and after each financing and investment decisions. How equityholders' incentives change over time naturally depends on the debt priority structure.

Before studying sequential interactions among default, investment, and financing decisions, we first analyze a setting where the firm only issues one class of debt to finance the exercising cost of a single growth option. This one growth option setting provides useful insights for understanding the setting when the firm has multiple growth options.

4 Investment, default and financing: One growth option

When the firm has only one growth option, we have closed-form formulae for the joint investment, leverage, and default decisions. The explicit formulae help us to understand economic intuition on the interactions between investment and financing without getting involved in complications due to issues such as seniority of debt and associated distortions on investment. With one growth option, the firm only has one investment threshold x^i, default threshold x^d and one optimal coupon c decisions. Therefore, the subscript k for the optimal decision rules x_k^i, T_k^i, x_k^d, T_k^d, c_k all refer to the one growth option setting with corresponding investment cost I_k and the cash flow multiple Q_k, where $k = 1, 2$. The next proposition summarizes the main results.[15]

Proposition 2 *The firm's investment decision follows a stopping time rule $T_k^i = \inf\{t : X(t) \geq x_k^i\}$, where the investment threshold x_k^i is given by*

$$x_k^i \;=\; \frac{\psi}{1-\tau} \frac{r-\mu}{Q_k} \frac{\beta}{\beta-1} I_k = \psi x_k^{ae}, \tag{39}$$

x_k^{ae} is all-equity investment threshold given in (10), and

$$\psi \;=\; \left[1 + \frac{1}{h}\left(\frac{\tau}{1-\tau}\right)\right]^{-1} \leq 1, \tag{40}$$

$$h \;=\; \left[1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tau}\right)\right]^{-1/\gamma} > 1. \tag{41}$$

The corresponding default time T_k^d is given by $T_k^d = \inf\{t > T_k^i : X(t) \leq x_k^d\}$, where the default threshold x_k^d is given by $x_k^d = x_k^i/h < x_k^i$. The optimal coupon c_k on the perpetual debt issued at the investment time T_k^i is given by

$$c_k \;=\; \frac{r}{1-\tau}\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\beta}{\beta-1}\right)\left(h + \frac{\tau}{1-\tau}\right)^{-1} I_k, \quad k = 1, 2. \tag{42}$$

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[15]Mauer and Sarkar (2005) derive similar results under one growth option setting. Their focus on the results and economic interpretations is very different.

Equity value before investing at T_1^i, $E_0(x)$, is given in Appendix A.3.

Let $L_k(x)$ and $P_k(x)$ denote the debt (loan) value and firm (project) value for the setting with one growth option in order to avoid confusion with debt values $D_k(x)$ and firm values $V_k(x)$ in settings with more than one growth option. Equations (A.20) and (A.21) give the explicit formulae for $P_k(x)$ and $L_k(x)$, respectively. The investment threshold x_k^i, the default threshold x_k^d, and the optimal coupon policy c_k are all proportional to the investment cost I_k. Intuitively, if we double the investment cost I_k, the firm will double its investment threshold x_k^i, its default threshold x_k^d, and the optimal coupon c_k accordingly. Therefore, equity value before investment $E_0(x_k^i)$, loan value $L_k(x_k^i)$, and firm (project) value $P_k(x_k^i)$ all double. This leaves leverage at the moment of investment, $L_k(x_k^i)/P_k(x_k^i)$, independent of the size of the investment cost I_k.

Next turn to the model's predictions on the comparative statics with respect to volatility.

Proposition 3 *The investment threshold x_k^i given in (39), increases with volatility σ, in that $dx_k^i/d\sigma > 0$. The credit spread cs_k and the ratio h between the investment and the default threshold also increase with volatility σ, in that $dh/d\sigma > 0$.*

In Leland (1994), the default threshold is given by $x^d = x_0/h$, where x_0 is exogenously given initial value and $h > 1$ is given in (41). Since h increases with volatility, and x_0 is constant, the default threshold x^d in Leland (1994) decreases with volatility. This captures the intuition that equityholders have a default option, whose value increases with volatility and hence the threshold x^d decreases with volatility. Unlike Leland (1994), in our model, the default threshold x_k^d is no longer monotonic in volatility σ. Note that $x_k^d = x_k^i/h$, where both x_k^i and h increase with volatility. For low levels of volatility, x_k^d decreases with volatility, because the positive effect of volatility on $\log h$ is greater than the positive effect of volatility on $\log x_k^i$. For higher levels of volatility, the x_k^d increases with volatility, because the positive effect of volatility on $\log h$ is weaker than the positive effect of volatility on $\log x_k^i$.

Now turn to our model's predictions on pricing. Let cs_k denote the credit spread: $cs_k = c_k/L_k(x_k^i) - r$. Using the debt pricing formula, we have

$$cs_k = r\frac{\xi}{1-\xi}, \tag{43}$$

where ξ is given in (A.24). It is immediate to see that $cs_k > 0$, because $h > 1$ and $\gamma < 0$ imply $0 < \xi < 1$. Our model generates the same prediction on the credit spread as Leland (1994) if we condition on the time of debt issuance (time 0 in Leland (1994) and T_k^i in our model, respectively). In both models, the credit spread increases with volatility σ.

The following proposition summarizes the results on the ordering of investment thresholds and also characterize the payoff functions at the moment of investment under equity financing or optimal financing. Since there is only one growth option, one default option and one financing decision, we drop the subscript for notational simplicity.

Proposition 4 *The investment threshold x^i under optimal financing given in (39) is lower than x^{ae} given in (10), the investment threshold under all equity financing in the presence of taxes, but is higher than x^* given in (12), the investment threshold under all equity financing without taxes. That is, we have $x^{ae} > x^i > x^*$. Moreover, equity payoff values when exercising the growth option under all three scenarios are equal, in that*

$$\Pi(x^{ae}) = V(x^i) = \frac{Qx^*}{r - \mu}. \tag{44}$$

First, consider the impact of financing on the investment threshold. Debt provides tax benefits but induces distress costs. Positive debt issuance implies that tax benefits outweigh financial distress costs, as in standard trade-off models. Hence, the firm is more valuable under optimal financing than under all equity financing. Since the payoff is higher under optimal financing, the firm has greater incentives to invest *ceteris paribus*, which in turn implies that the investment threshold x^i is lower than the threshold x^{ae} under all-equity financing with taxes. Second, compared with the benchmark setting without taxes, the firm's payoff from investment (even under optimal financing) is lower when the tax rate is positive. Therefore, the firm has weaker incentives to invest, relative to the case where the firm faces no taxes. Therefore, the investment threshold x^i under optimal financing is lower than the the optimal threshold x^* under all-equity financing without taxes.

Insert Figure 2 here.

Next, turn to the payoff values for equityholders when the firm exercises its growth options. Recall that the investment timing decisions are different under different forms of financing as discussed earlier. However, the payoffs to equityholders are all equal, if evaluated at respective growth option exercising times T^i for both all equity financing (with or without taxes) and optimal financing. Our intuition relies on the following observation. First, the present discounted value of receiving a unit payoff contingent on hitting the investment threshold x^i is $\Phi_i(x; x^i) = (x/x^i)^\beta$ for $x < x^i$ because $x^d = 0$, where x is the current value of the demand shock. It is immediate to see that $\Phi_i(px; px^i) = \Phi_i(x; x^i)$ for any constant $p > 0$. Therefore, as long as the gross payoff upon exercising the growth option at the

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threshold level x^i is proportional to x^i, say, px^i, the optimal investment threshold is given by $px^i = \beta/(\beta - 1)I$ as shown in Appendix A.5. This relies on the scale-invariance property of $\Phi_i(x; x^i)$ for the GBM process (1).

Figure 2 illustrates the predictions of the above proposition. We see that the three thresholds, x_1^*, x_1^i, and x_1^{ae} are ordered sequentially from the left to the right. The payoff values to equityholders at these investment thresholds are equal, as seen from the (dashed) horizontal line. It is immediate to see from Figure 2 that $\Pi(x) < V(x) < Qx/(r - \mu)$. First, tax benefits of debt imply $V(x) > \Pi(x)$. However, taxes in net lower firm value, because forgone revenues are greater than the net tax benefits of debt in excess of financial distress costs in our model. This gives $V(x) < Qx/(r - \mu)$. Recall that the net payoff function from exercising the option is $V(x) - I$, which is concave in x. The concavity of $V(x)$ arises from the fact that the firm as a whole is short a default option *ex ante*. Like in standard real option models, equity value $E_0(x)$ before exercising the growth option is increasing and convex in x.

Next, we analyze the feedback effects between investment and financing when the firm has two growth options. Importantly, future growth option exercising and current default decision become intertwined.

5   Model Analysis

Recall that our optimization problem has six decision variables: two investment, two leverage, and two default decisions. In order to sharpen the intuition behind the working mechanism of our model, we first freeze the initial investment and leverage decisions by fixing x_1^i, the coupon c_1 and the implied face value on the first debt F_1. Intuitively, imagine a new manager is just hired to run the firm. He finds that the firm has existing perpetual debt with coupon c_1 and face value F_1 from the investment and leverage decisions made in the past. Behaving in equityholders' interests, he has four decisions to make: the default decisions $\left(x_1^d, x_2^d\right)$, the investment threshold x_2^i, and the second coupon policy c_2. Without loss of generality, let the face value F_1 be a fraction of the corresponding risk-free debt value c_1/r, in that $F_1 = mc_1/r$, where the *ex ante* default risk of the debt implies that $m < 1$.

The newly hired manager takes the first debt as given, and analyzes his optimization problem in three steps. Section 5.1 solves the special case without pre-existing debt ($c_1 = 0$) in closed form. This case gives us a natural benchmark to analyze the effect of existing debt on future decisions and value functions. Section 5.2 analyzes the impact of pre-existing debt on firm's default, growth option exercising, and leverage decisions, when the amount

of existing debt is moderate. Section 5.3 shows that when the amount of existing debt is sufficiently large, risk shifting incentives as in Jensen and Meckling (1976) in addition to debt overhang will arise.

In Section 5.4, we endogenize the initial investment and leverage decisions, when the firm anticipates conflicts of interests after debt is in place. Finally, we study the effect of alternative debt priority structure on investment, financing decisions and equity value in Section 5.5.

First, consider the special case without pre-existing debt ($c_1 = 0$).

5.1 First asset in place with no debt overhang: $c_1 = 0$

When $c_1 = 0$, the firm has the first asset in place generating a perpetual stream of positive cash flow $Q_1 x$, and the (second) growth option. Therefore, the firm never defaults before exercising the growth option ($T_1^d = \infty$). Moreover, we have closed form solutions for both value functions, and the decision rules x_1^d, x_2^i, x_2^d, and c_2. The following proposition states the main results.

Proposition 5 *The firm's optimal investment decision follows a stopping time rule $T_2^i = \inf\{t : X(t) \geq x_2^i\}$, where the investment threshold x_2^i is given by*

$$x_2^i = \frac{1}{1-\tau} \frac{r-\mu}{Q_2} \frac{\beta}{\beta-1} I_2 \left(1 + \frac{\tau}{1-\tau} \frac{Q}{Q_2} \frac{1}{h}\right)^{-1}. \tag{45}$$

The optimal default time T_2^d is given by $T_2^d = \inf\{t > T_2^i : X(t) \leq x_2^d\}$, where the default threshold x_2^d is given by $x_2^d = x_2^i/h < x_2^i$. The optimal coupon c_2 on the perpetual debt issued at the investment time T_2^i is given by

$$c_2 = \frac{r}{1-\tau} \left(\frac{\gamma-1}{\gamma}\right) \left(\frac{\beta}{\beta-1}\right) \left(h\frac{Q_2}{Q} + \frac{\tau}{1-\tau}\right)^{-1} I_2. \tag{46}$$

Equity value $E_1(x)$ and firm value $V_2(x)$ are given in Appendix A.6.

The firm's investment incentive is greater than the case where the firm has only one growth option and no asset in place as in Section 4. Intuitively, the existence of the asset in place (from previous exercising of the first growth option) enhances the firm's ability to issue debt. This additional tax benefits (netting out the financial distress cost), supported by the (first) asset in place, further encourage the firm to exercise the (second) growth option sooner, *ceteris paribus*. To summarize, we have the following two results: (i) the optimal coupon c_2 given in (46) is higher than the coupon c_2 given in (42) for the case with one growth option,

evaluated at investment cost I_2 and the cash flow multiple Q_2; (ii) the (second) investment threshold x_2^i given in (45) is lower than the corresponding investment threshold x_2^i given in (39), evaluated with investment cost I_2 and Q_2. Finally, as in one growth option setting of Section 4, the ratio between the investment threshold x_2^i and the default threshold x_2^d is equal to h: $x_2^i/x_2^d = h$. This is an outcome from the optimal coupon and default decisions after the firm invests and issues the debt at T_2^i.

Next turn to the case where c_1 is not too high (to be made precise later). We show that the existing debt induces the classic debt overhang effect (Myers (1977)) and is reflected via default, investment and leverage decisions.

5.2   Debt overhang: "not too high" first debt coupon c_1

By c_1 being not too high, we refer to the equilibrium outcome where the following endogenous condition is satisfied:

Condition 2 $F_1 < (1-\alpha)\,\Pi(x_2^d)$.

Under the above condition, the senior debtholders receive the face value F_1 at default time T_2^d. While senior debtholders collect the par value F_1 at T_2^d, this does not mean that the senior debt has no risk after investing at T_2^i. After the firm invests at T_2^i, senior debtholders prefer a longer coupon collecting period (a higher value of T_2^d and a lower value x_2^d), *ceteris paribus*. Intuitively, the senior debtholders can always collect F_1, but only for a finite (stochastic) period for the coupon under Condition 2. The junior debt is subject to both the risk from loss given default (compared with its par F_2) and the timing at which the firm default (T_2^d turns out to be too early).

Consider the effect of increasing c_1 on various decision rules. Start with the effect on the first default threshold x_1^d. In the appendix, we show that

$$x_1^d < \frac{\gamma}{\gamma-1}\frac{r-\mu}{Q_1}\frac{c_1}{r}, \tag{47}$$

where the right side of the above inequality is the default threshold as in Leland (1994) in the absence of the second growth option. The intuition is straightforward. Because default loses future growth options and tax benefits from increasing leverage in the future, equityholders are less willing to exercise the first default option, compared with the setting without the second growth option. Note that debtholders still collect a scrap value $(1-\alpha)\,\omega\Pi_2(x_1^d)$ upon default from the (unexercised) second growth option, however, equityholders do not internalize this value *after* the first debt is issued. That is, $E_1(x)$ does not depend on the

scrap value from the second growth option at default time T_1^d. The top left panel in Figure 3 shows that for all levels of c_1, the default threshold x_1^d is below the corresponding Leland default threshold, given by the right side of (47). Also, the default threshold x_1^d naturally increases with c_1.

Next turn to the effect of c_1 on the (second) investment threshold x_2^i. The seniority structure of the debt weakens equityholders' investment incentive after the first debt is in place. This is the standard debt overhang result (Myers (1977)). Equityholders, not existing debtholders, pay the cost of exercising the second growth option by issuing securities. Given that the new debt is fairly priced, equityholders internalize the tax benefits and the distress cost from new security issuance. However, the value created from exercising the (second) growth option first goes to the senior debtholders, then new debtholders and finally equityholders. This *ex post* wealth transfer effect discourages equityholders from investing. Therefore, a larger c_1 leads to a higher investment threshold x_2^i, *ceteris paribus*. The top right panel in Figure 3 shows that the second investment threshold x_2^i increases in c_1, up to a switching point, at which Condition 2 is no longer satisfied. We will return to discuss this switching point and the discontinuous downward jump in Section 5.3.

Now consider the model's predictions after the second investment is made ($t \geq T_2^i$). First, recall that equityholders' (second) default threshold x_2^d as a function of the total coupon $c = c_1 + c_2$ is given by (19), the same as in Leland (1994) and in the one-growth option setting of Section 4. Taking this dependence of x_2^d on the total coupon c into account, the firm chooses its second coupon c_2 (equivalently, total coupon $c = c_1 + c_2$, because c_1 is predetermined) to maximize $V_2^n(x_2^i)$. Using the same insight and analysis as the ones in Section 4, equityholders choose the total coupon c to trade off the tax benefits with distress costs. As in Leland (1994) and Section 4, the second default threshold x_2^d is proportional to the value of x at which the (second) investment is taken, i.e. the investment threshold x_2^i. That gives $x_2^d = x_2^i/h$, where h is a constant given in (41) in Section 4. The middle left panel of Figure 3 shows that indeed the ratio x_2^i/x_2^d is equal to the constant h, provided that c_1 is below the "switching" point.

More interestingly, there are two opposing effects of a higher c_1 on c_2. On one hand, fixing the level of the default threshold x_2^d, a unit increase of c_1 crowds out a unit of c_2. On the other hand, the investment threshold x_2^i and the default threshold x_2^d both increase with c_1 due to the debt overhang argument. Hence, the net impact of c_1 increase on c_2 is ambiguous.

Next, consider the impact of increasing c_1 on the credit spreads at the second investment time T_2^i. The credit spread at T_2^i for the first debt is constant and independent of c_1.

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Intuitively, there is no loss given default for the first debt under Condition 2 (The senior debtholders receive the par value F_1 at default time T_2^d). Moreover, the ratio between the second investment threshold and second default threshold, x_2^i/x_2^d, is constant and equal to h. Hence, the credit spread at T_2^i for the first debt reflects only the risk due to the stochastic default timing, which *ex ante* is captured by $x_2^i/x_2^d = h$. Using the same insight as in the one growth option setting of Section 4, we know that the credit spread of the first debt at T_2^i is independent of the level of c_1. Because the junior debt bears all the residual risk from loss given default at T_2^d, a higher coupon c_1 induces a higher loss given default for junior debt, *ceteris paribus*. The bottom left panel of Figure 3 illustrates the impact on debt overhang on credit spreads for the first and the second debt at the firm's second growth option exercising time T_2^i.

Finally, turn to the market leverage of (combined first and second) debt at T_2^i. Building on the insights from Section 4 (the setting with one growth option), we find that the market leverage at T_2^i is constant and independent of c_1. Intuitively, conditioning on investing at T_2^i, the existing debt c_1 crowds out debt capacity for the second issuance, but the total market leverage at T_2^i is equal to the setting with one growth option setting, which is independent of the coupon level c_1. The bottom right panel of Figure 3 shows that market leverage at T_2^i is independent of c_1, for c_1 lower than the switching point.

Insert Figure 3 here.

So far, we have assumed that the outstanding debt is not too high. Intuitively, equityholders react to the debt overhang problem by postponing the second investment (by increasing the threshold x_2^i), and deferring the default decision (by decreasing the threshold x_1^d). Conditioning on the level of x_2^i, when c_1 is not too high, the firm's second default threshold x_2^d and the second coupon c_2 (implied by the total coupon c) are effectively the same as in one growth option setting and the Leland (1994) setting.

Next, turn to the case where c_1 is sufficiently high.

5.3 Debt overhang and risk shifting: High first debt coupon c_1

Intuitively, when c_1 is sufficiently high, postponing the second investment to the extent that senior debt does not face any default risk at T_2^d becomes too costly from equityholders' perspective. That is, Condition 2 no longer holds in equilibrium. Rather than delaying investment, servicing the senior debt, and repaying the par F_1 on the senior debt at T_2^d, equityholders may have incentives to engage in risk shifting. This is in the spirit of asset

substitution and risk shifting argument in Jensen and Meckling (1976). The discontinuous downward jump of the investment threshold x_2^i in the top right panel of Figure 3 reflects this additional effect of risk shifting on the debt overhang argument discussed earlier. Equity-holders trade off the cost of risk shifting against the excessively delayed investment from the debt overhang problem. Doing so makes the senior debtholders bear additional default risk. Now the senior debt is not only exposed to the default *timing* risk, but also to the risk due to loss given default at T_2^d. The senior debtholders collect $(1 - \alpha) \Pi(x_2^d) < F_1$ and the junior debtholders receive nothing at T_2^d. This is precisely when Condition 2 does not hold.

After the downward jump, the investment threshold x_2^i continues to increase with the coupon level c_1 for the standard debt overhang argument as in the previous subsection. Finally, we emphasize that where c_1 is sufficiently high, equityholders have both debt overhang and risk shifting incentives. The investment threshold x_2^i is chosen to reflect both the debt overhang and the risk shifting incentives.

5.4 Initial investment and coupon policy

The analysis in previous subsections have shown that the effects of existing debt (pre-determined c_1) on future default, investment and leverage decisions are significant. However, the firm anticipates its debt policy on its future decisions, which in turn affects its current value. We show that the effect of debt overhang is significantly mitigated when the firm anticipates the distortions of current debt issuance on future decisions. Intuitively, the firm adjusts its initial investment and leverage decisions to mitigate conflicts of interests between debtholders and equityholders in the future.

One immediate prediction of the above analysis is that the firm with more growth options has a lower leverage, which is among the most important empirical findings in the capital structure literature (Smith and Watts (1992) and Rajan and Zingales (1995)). The left panel of Figure 4 shows that for all levels of the exercising cost I_2 of the second growth option, the market leverage at the first investment time T_1^i is lower than the market leverage at the second investment time T_2^i. Intuitively, firms with more growth options save more debt capacity for future growth option exercising in order to avoid the potential debt overhang effect on investment and induced value reduction. Second, the left panel of Figure 4 shows that the more attractive the second growth option is, (a lower I_2, or more intuitively, a smaller $(I_2/Q_2) / (I_1/Q_1)$), the lower the firm' market leverage is at the first investment time T_1^i. Third, the left panel of Figure 4 confirms that the market leverage at the second investment time T_2^i is independent of the investment cost I_2. This is the scale invariance

property that we have seen in Section 4 for the setting with one growth option.

Insert Figure 4 here.

The right panel of Figure 4 compares the market leverages at T_1^i and T_2^i in settings with two growth options to the respective stand-alone one growth option settings. To be more precise, the solid line in the right panel of Figure 4 plots the ratio between (i) the market leverage when exercising the first growth option at T_1^i and (ii) the market leverage when the firm exercises its only growth option (with exercise cost I_1 and Q_1). We show that the ratio is less than one and increases with I_2. This provides additional insights that future growth options lowers current leverage. The dashed line in the right panel of Figure 4 plots the ratio between (i) the market leverage when exercising the second growth option at T_2^i and (ii) the market leverage when the firm exercises its only growth option (with exercise cost I_2 and Q_2). Since there is no future growth options, the ratio is equal to unity. Intuitively, with no future debt overhang, the firm has no further incentives to deviate leverage decisions. The firm anticipates with too much debt issued at T_1^i, its future investment decisions will be substantially distorted as we have shown earlier. Therefore, the firm issues mild amount of debt in order to take advantage of current tax benefits without generating too much *ex post* investment inefficiency. In equilibrium, the firm faces moderate amount of debt overhang, but no incentives to risk shift. That is, anticipating the debt overhang, the firm will situate itself in the region with moderate amount of c_1 as in Section 5.2, and do not get into the region with very high c_1 as discussed in Section 5.3.

Because debt priority structure determines the payoffs for various debtholders at default, we naturally anticipate that alternative debt priority structure, such as *pari passu*, will have different implications on investment, leverage and default decisions. This is to which we now turn. We analyze both (i) the case with exogenously specified coupon c_1 and also (ii) the case with optimal initial investment and leverage decisions.

5.5 An alternative debt priority structure: *Pari passu*

Now suppose that debt issued at T_1^i and the one issued at T_2^i have equal priority in default at time T_2^d. Debt payoffs at default $\left(x = x_2^d\right)$ are proportional to $(1 - \alpha)\,\Pi(x_2^d)$, the total recovery value of the firm. Since both types of debt are perpetual, the residual values at the

default threshold x_2^d are thus given by

$$D_2^n(x_2^d) = \frac{c_2}{c_1 + c_2}(1 - \alpha)\Pi(x_2^d), \tag{48}$$

$$D_2^s(x_2^d) = \frac{c_1}{c_1 + c_2}(1 - \alpha)\Pi(x_2^d). \tag{49}$$

Here, we assume that the payments to debtholders are based on the debt values at the second investment time T_2^i. This assumption captures the key feature of the *pari passu* structure, and substantially simplify the analysis.[16]

Equityholders choose c_2 (at T_2^i) to maximize $V_2^n(x)$ given in (24), the sum of equity value $E_2(x)$ and newly issued debt value $D_2^n(x)$. The following implicit function characterizes the optimal coupon c_2 for a given level of the first coupon c_1:

$$c_2 = -c_1 + \frac{r}{r-\mu}\left(\frac{\gamma-1}{\gamma}\right)\frac{1}{h}\left[1 - \frac{\gamma}{\gamma-1}\left(\frac{\tau^{-1} - \gamma(1-\alpha+\alpha/\tau)}{1-\gamma(1-\alpha+\alpha/\tau)}\right)\frac{c_1}{c_1+c_2}\right]^{1/\gamma}Qx_2^i. \tag{50}$$

Insert Figure 5 here.

Figure 5 graphs the effects of existing debt on future decisions, credit spread and market leverages. Other than the first default threshold x_1^d, Figure 5 shows that decision rules, credit spreads and leverages are drastically different, depending on whether the firm has APR or *pari passu* debt structure. The impact of c_1 on the (second) investment threshold x_2^i is much less significant under *pari passu* than under APR. Intuitively, the *pari passu* debt structure makes debt overhang much less concern for equityholders. Indeed, Myers (1977) noted that firms may mitigate the debt overhang problem by issuing secured debt, or debt with equal or more senior priority over the existing debt. This is exactly what *pari passu* priority structure does relative to APR. However, the less distorted investment threshold decision comes at a cost of lowering the equityholders' ability to raise debt for the first growth option exercising. This suggests that the credit spread[17] and market leverage are much higher and more sensitive to c_1 under *pari passu* than under APR. The bottom panels of Figure 5 confirm this intuition.

Insert Table 1 here.

Based on the above analysis, we find that financial contracting (debt priority structure) plays an important role in the firm's real investment, leverage decisions and market values of

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[16]Under this assumption, we do not need to carry the face values F_1 and F_2 for both classes of debt. A more realistic way to model *pari passu* priority structure is to make the payment at default proportional to the face values F_1 and F_2.

[17]In our *pari passu* structure, both the first and second debt have the same credit spread.

corporate debt. This is because financial contracting affects the magnitude of the conflicts of interests between equityholders and debtholders. So far, we have held the initial coupon c_1 fixed. Now, we endogenize the initial investment and leverage decisions. That is, we analyze the complete optimization problem under *pari passu*, which is the counterpart to the complete optimization analysis in Section 5.4 under APR. Table 1 shows that the equity value $E_0(x)$ before exercising the first growth option under *pari passu* is very close to the equity value $E_0(x)$ under APR. For a wide range of drift μ, distress cost α, and volatility σ, these two value functions are within 2% difference. Intuitively, the additional adjustment margins via the initial investment threshold x_1^i and the initial coupon policy c_1 that equityholders have substantially mitigate the agency cost of debt arising from *ex post* debt overhang. Therefore, financial contracting plays relatively a minor role in determining the equity value $E_0(x)$.

6 Conclusions

We have provided an analytically tractable framework for studying a firm's joint dynamic investment, capital structure and default decisions. The model features endogenous investment and default triggers, optimal capital structure and dynamic feedback effects between investment and financing. Our model integrates irreversible investment literature with dynamic capital structure theory. The induced conflicts of interests between equityholders and debtholders naturally evolve over time. Integrating investment and financing decisions enriches the predictions for both decisions. Ignoring one may potentially leads to misleading results and intuition.

We show that in the presence of pre-existing debt, the firm defers its default decisions in order to preserve future growth options. Moreover, the firm delays its growth option exercising decisions due to the standard debt overhang argument. When the firm anticipates future growth options perfectly, the firm uses its initial investment and leverage decisions as adjustment margins to mitigate the debt overhang problem. As a consequence, the role of financial contracting becomes less relevant. However, when the firm faces new growth options unexpectedly, and with excessive debt outstanding, debt priority structure plays an important role.

For the purpose of illustrating intuition in a tractable way, we have intentionally left some potentially important issues outside of our model. For example, we simplify the the dividend payout policy, and abstract away from issues of internal cash balance and risk management.

Appendices

A Proofs

A.1 Proof of Proposition 1

Using the standard present value formulae (e.g. Dixit and Pindyck (1994)), equityholder's objective function (8) under all equity financing may be written as follows:

$$E_0(x) = \left(\frac{x}{x_1^{ae}}\right)^{\beta} \left[\left(\frac{1-\tau}{r-\mu}\right) Q_1 x_1^{ae} - I_1\right] + \left(\frac{x}{x_2^{ae}}\right)^{\beta} \left[\left(\frac{1-\tau}{r-\mu}\right) Q_2 x_2^{ae} - I_2\right], \quad x \leq x_1^{ae} \quad \text{(A.1)}$$

for given x_1^{ae} and $x_2^{ae} \geq x_1^{ae}$. Equityholders choose x_2^{ae} and x_1^{ae} to maximize (A.1). First, suppose that the constraint $x_2^{ae} \geq x_1^{ae}$ does not bind. Then, we obtain the candidate optimal thresholds x_1^{ae} and x_2^{ae}, given by (10), and $E_0(x)$ given below:

$$E_0(x) = \left(\frac{x}{x_1^{ae}}\right)^{\beta} (E_1(x_1^{ae}) - I_1), \quad x \leq x_1^{ae}, \quad \text{(A.2)}$$

where $E_1(x)$ given by

$$E_1(x) = \Pi_1(x) + \left(\frac{x}{x_2^{ae}}\right)^{\beta} (\Pi_2(x_2^{ae}) - I_2), \quad x \leq x_2^{ae}. \quad \text{(A.3)}$$

Now check if the constraint binds. We conclude that the constraint does not bind if and only if Condition 1 holds.

When the constraint $x_2^{ae} \geq x_1^{ae}$ binds, i.e. $Q_1/I_1 < Q_2/I_2$, we have $x_2^{ae} = x_1^{ae}$. That is, simultaneous exercising of both options are optimal. We thus have

$$x_1^{ae} = x_2^{ae} = \frac{1}{1-\tau} \frac{r-\mu}{Q} \frac{\beta}{\beta-1} I, \quad \text{(A.4)}$$

where $I = I_1 + I_2$ and $Q = Q_1 + Q_2$. Equity value is then given by

$$E_0(x) = \left(\frac{x}{x^{ae}}\right)^{\beta} (\Pi(x) - I), \quad x \leq x^{ae}, \quad \text{(A.5)}$$

and $E_0(x) = \Pi(x) - I$, if $x \geq x^{ae}$.

A.2 Derivation for decision rules and value functions in Section 3

We proceed our analysis in several steps as in the main text.

After the exercise of the second growth option $(t \geq T_2^i)$. We conjecture that equity value $E_2(x)$ solves the following ordinary differential equation (ODE):

$$rE_2(x) = (1 - \tau)(Qx - c) + \mu x E_2'(x) + \frac{\sigma^2}{2} x^2 E_2''(x), \quad x \geq x_2^d, \tag{A.6}$$

subject to the endogenous default boundary conditions (16) and (17), and the standard no-bubble condition for $E_2(x)$ as $x \to \infty$. Solving this (standard) default problem gives the equity value $E_2(x)$ given in (18) and default threshold x_2^d given in (19).

After the exercise of the first growth option $(T_1^i \leq t \leq T_2^i \wedge T_1^d)$. Given the first default threshold x_1^d and the second investment threshold x_2^i, we may write down equity value (before exercising the second, but after exercising the first growth option) $E_1(x)$ as in (27), using the present discounted value of receiving a unit payoff contingent on the second growth option is exercised in finite time or not, ($\Phi_i(x)$ and $\Phi_d(x)$), respectively. (Alternatively, we may also derive $E_1(x)$ by writing down the HJB equation with the corresponding four boundary conditions. This procedure also yields (27).)

Using the standard results on hitting time distributions for GBM processes (Harrison (1985)), we have the following explicit formulae for $\Phi_i(x)$ and $\Phi_d(x)$:

$$\Phi_i(x) = E_t^x \left[e^{-r(T_2^i - t)} \mathbf{1}_{T_1^d > T_2^i} \right] = \frac{1}{\Delta} \left[(x_1^d)^\gamma x^\beta - (x_1^d)^\beta x^\gamma \right], \tag{A.7}$$

$$\Phi_d(x) = E_t^x \left[e^{-r(T_1^d - t)} \mathbf{1}_{T_1^d < T_2^i} \right] = \frac{1}{\Delta} \left[(x_2^i)^\beta x^\gamma - (x_2^i)^\gamma x^\beta \right], \tag{A.8}$$

and

$$\Delta = (x_1^d)^\gamma (x_2^i)^\beta - (x_1^d)^\beta (x_2^i)^\gamma > 0. \tag{A.9}$$

It is immediate to see that $\Phi_d(x_1^d) = \Phi_i(x_2^i) = 1$, $\Phi_d(x_2^i) = \Phi_i(x_1^d) = 0$, and $\Phi_d(x) > 0$, $\Phi_i(x) > 0$, for $x_1^d < x < x_2^i$.

Using the equity value formula (27), we have $xE_1'(x) = \Pi_1(x) + e_1^i \Phi_i'(x)x + e_1^d \Phi_d'(x)x$. The smooth pasting condition $E_1'(x_2^i) = V_2^{n\prime}(x_2^i)$ implies

$$\Pi_2(x_2^i) + \gamma \nu_2^n \left(\frac{x_2^i}{x_2^d} \right)^\gamma = \frac{\beta}{\Delta} (x_2^i)^\beta \left[e_1^i (x_1^d)^\gamma - e_1^d (x_2^i)^\gamma \right] - \frac{\gamma}{\Delta} (x_2^i)^\gamma \left[e_1^i (x_1^d)^\beta - e_1^d (x_2^i)^\beta \right], \tag{A.10}$$

where ν_2^n is given by

$$\nu_2^n = D_2^n(x_2^d) - \Pi(x_2^d) + \frac{c_1 - \tau c}{r}. \tag{A.11}$$

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Similarly, the smooth pasting condition $E_1'(x_1^d) = 0$ gives

$$0 = \Pi_1(x_1^d) + \frac{\beta}{\Delta}(x_1^d)^\beta \left[e_1^i(x_1^d)^\gamma - e_1^d(x_2^i)^\gamma \right] - \frac{\gamma}{\Delta}(x_1^d)^\gamma \left[e_1^i(x_1^d)^\beta - e_1^d(x_2^i)^\beta \right]. \quad (A.12)$$

For given x_1^d and x_2^i, debt value $D_1(x)$ is then given by

$$D_1(x) = \frac{c_1}{r} + \left(D_2^s(x_2^i) - \frac{c_1}{r} \right) \Phi_i(x) + \left[(1-\alpha)\,\Pi_1(x_1^d) - \frac{c_1}{r} \right] \Phi_d(x). \quad (A.13)$$

Before the exercise of the first growth option $(t \leq T_1^i)$. We conjecture that the equity value $E_0(x)$ solves the following ODE:

$$rE_0(x) = \mu x E_0'(x) + \frac{\sigma^2}{2} x^2 E_0''(x), \quad x \leq x_1^i. \quad (A.14)$$

The above ODE is solved subject to the endogenous default boundary conditions (33) and (34) given in the main text, and also the standard absorbing barrier for $E_0(x)$ at the origin, in that as $E_0(x) \to 0$, when $x \to 0$.

Substituting the conjectured equity value (35) into the ODE (A.14) and applying the endogenous default boundary conditions (33) and (34) give the following implicit equation for the first investment threshold x_1^i:

$$\Pi_1(x_1^i) = \frac{\beta}{\beta-1} \left[I_1 - \frac{\tau c_1}{r} + \frac{\Phi_i'(x_1^i)x_1^i - \beta\Phi_i(x_1^i)}{\beta} v_1^i + \frac{\Phi_d'(x_1^i)x_1^i - \beta\Phi_d(x_1^i)}{\beta} v_1^d \right]. \quad (A.15)$$

Simplifying the above gives (36).

A.3   Proof of Proposition 2

With one growth option, we have $x_2^i = \infty$. Therefore, $\Phi_i(x) = 0$ and $\Phi_d(x) = (x/x_1^d)^\gamma$, for $x \geq x_1^d$. Equation (36) thus implies

$$x_1^i = \frac{1}{1-\tau} \frac{r-\mu}{Q_1} \frac{\beta}{\beta-1} \left[\left(I_1 - \frac{\tau c_1}{r} \right) + \frac{\beta-\gamma}{\beta} \left(\alpha\Pi_1(x_1^d) + \frac{\tau c_1}{r} \right) \left(\frac{x_1^i}{x_1^d} \right)^\gamma \right]. \quad (A.16)$$

The optimal coupon policy c is given by

$$c_1 = \frac{r}{r-\mu} \frac{\gamma-1}{\gamma} \frac{1}{h} Q_1 x_1^i. \quad (A.17)$$

Re-arranging and simplifying (A.16) gives the following implicit equation for the investment threshold:

$$
\begin{aligned}
(\beta-1)\,\Pi_1(x^i) &= \beta I_1 - \beta\frac{\tau c_1}{r} + (\beta-\gamma)\frac{c_1}{r} \left[\alpha\,(1-\tau)\left(\frac{\gamma}{\gamma-1} \right) + \tau \right] h^\gamma, \\
&= \beta I_1 - \beta\frac{\tau c_1}{r} + (\beta-\gamma)\frac{\tau c_1}{r} \left(\frac{h^{-\gamma}}{1-\gamma} \right) h^\gamma \\
&= \beta I_1 - (\beta-1)\,\tau\frac{1}{h}\frac{Q_1 x_1^i}{r-\mu}, \quad (A.18)
\end{aligned}
$$

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where the first, second, and third line uses the explicit formulae for x^d given in (19), h given in (41), and coupon c given in (A.17), respectively. Finally, re-arranging the last expression gives x_1^i in (39). Substituting (39) into (A.17) gives the coupon policy (42) and the default threshold $x_1^d = x_1^i/h$. The same naturally analysis applies when the firm has investment cost I_2 and the cash flow multiple Q_2.

If the initial value x_0 is below the investment threshold x_k^i given in (39), the firm will wait to invest. Equity value before investment $E_0(x)$ is given by

$$E_0(x) = \left(\frac{x}{x_k^i}\right)^\beta \left(P_k(x_k^i) - I_k\right), \quad x \leq x_k^i, \tag{A.19}$$

where project value $P_k(x)$ after investment and before default $(T_k^i \leq t \leq T_k^d)$ is given by

$$P_k(x) \;=\; \Pi_k(x) + \frac{\tau c_k}{r} - \left(\alpha \Pi_k(x_k^d) + \frac{\tau c_k}{r}\right)\left(\frac{x}{x_k^d}\right)^\gamma, \quad x \geq x_k^d. \tag{A.20}$$

When $x \leq x_k^d$, project is worthless, in that $P_k(x) = 0$. The loan value $L_k(x)$ issued to finance the project is then given by

$$L_k(x) \;=\; \frac{c_k}{r} - \left[\frac{c_k}{r} - (1-\alpha)\,\Pi_k(x_k^d)\right]\left(\frac{x}{x_k^d}\right)^\gamma, \quad x \geq x_k^d. \tag{A.21}$$

The difference $P_k(x) - L_k(x)$ is the residual equity value.

A.4 Proof of Proposition 3

Recall that $\Phi_d(x)$ denote the present discounted value of receiving a unit payoff contingent on the event that the process X hits x^d, the default threshold for the firm after investing at the threshold x^i at time T^i (Note that the upper boundary in this case for the calculation $\Phi_d(x)$ is ∞). (A.8) implies $\Phi_d(x) = (x/x^d)^\gamma$ for $x \geq x^d$. Hence,

$$\Phi_d(x^i) = h^\gamma = \left[1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tau}\right)\right]^{-1}. \tag{A.22}$$

It is immediate to see that $\Phi_d(x^i)$ increases with volatility σ, increases with tax rate τ, and decreases with financial distress cost α. Now consider the credit spread at issuance/investment time T^i:

$$cs = \frac{c}{L(x^i)} - r = r\frac{\xi}{1 - \xi}, \tag{A.23}$$

where $L(x)$ is loan value given in (A.21) and

$$\xi \;=\; \left(1 - (1-\alpha)(1-\tau)\frac{\gamma}{\gamma - 1}\right)\Phi_d(x^i). \tag{A.24}$$

Note $0 < \xi < 1$, because $h > 1$, $\gamma < 0$, $0 \le \alpha < 1$, and $0 \le \tau < 1$. We also note that $d\xi/d\sigma^2 > 0$ because $d\gamma/d\sigma^2 > 0$ and $d\Phi_d(x^i)/d\sigma^2 > 0$. Therefore, credit spread increases with volatility σ.

Note that $h = (1 - \gamma B)^{-1/\gamma}$ satisfies $1 < h < e^B$, where $B = 1 + \alpha(1 - \tau)/\tau > 1$. Using the chain rule, we have

$$\frac{dh}{d\sigma^2} = \frac{dh}{d\gamma}\frac{d\gamma}{d\sigma^2} = h\frac{d\log h}{d\gamma}\frac{d\gamma}{d\sigma^2}, \tag{A.25}$$

where

$$\frac{d\log h}{d\gamma} = \frac{1}{\gamma^2}\left(\log(1 - \gamma B) + \frac{\gamma B}{1 - \gamma B}\right) = \frac{1}{\gamma^2}G(\gamma), \tag{A.26}$$

and

$$G(\gamma) = \log(1 - \gamma B) + \frac{1}{1 - \gamma B} - 1, \quad \text{for } \gamma < 0. \tag{A.27}$$

It is immediate to see that $G(0) = 0$ and $G'(\gamma) < 0$, over the region $\gamma < 0$. Therefore, we have $d\log h/d\gamma > 0$. Note that $d\gamma/d\sigma^2 > 0$. Therefore, using (A.25), we have $dh/d\sigma^2 > 0$.

The sign of $d\psi/d\sigma^2$ is the same as the sign of $dh/d\sigma^2$. We thus have $d\psi/d\sigma^2 > 0$. Since $x^i = \psi x^{ae}$, and both ψ and x^{ae} given in (10) increase with volatility, we thus have $dx^i/d\sigma^2 > 0$.

A.5   Proof of Proposition 4

First, we show that as long as the (gross) payoff to equityholders when exercising the growth option at the threshold x^i is proportional to x^i, then the payoff values when investing are identical and independent of financing arrangements. Suppose that the gross payoff when investing is given by px, where $p > 0$ is a constant.

Because the equity value $E_0(x)$ (for $x \le x^i$) is given by product of (i) the present discounted value of a unit payoff at the investment threshold x^i, $(x/x^i)^\beta < 1$, and (ii) the net payoff at the investment threshold, $px^i - I$. Therefore, equityholders choose x^i to maximize $(x/x^i)^\beta (px^i - I)$. Solving gives

$$px^i = \frac{\beta}{\beta - 1}I. \tag{A.28}$$

Therefore, equity value is given by $(x/x^i)^\beta I/(\beta - 1)$.

Now, we show that for both equity financing and optimal financing, we have linear payoff value. Under all equity financing with taxes, the gross payoff value when investing is given

by $\Pi(x^{ae}) = (1 - \tau) Q x^{ae} / (r - \mu)$. Under all equity financing without taxes, the gross payoff value when investing is given by $\Pi(x^*) = Q x^* / (r - \mu)$. Finally, under optimal financing, we have

$$V(x^i) = \Pi(x^i) + \left[\tau - \left(\alpha(1 - \tau)\frac{\gamma}{\gamma - 1} + \tau\right)h^\gamma\right]\frac{\gamma - 1}{\gamma}\frac{1}{1 - \tau}\frac{1}{h}\Pi(x^i) = \Pi(x^i)/\psi, \quad \text{(A.29)}$$

using expressions for h given in (41), ψ given in (40), x^d given in (19), and c given in (A.17). Note that $\Pi(x^i)$ is linear in x^i.

A.6 Proof of Proposition 5

Using the standard pricing argument, we have that equity value before exercising the (second) growth option, $E_1(x)$, is given by

$$E_1(x) = \Pi_1(x) + \left(V_2(x_2^i) - I_2\right)\left(\frac{x}{x_2^i}\right)^\beta, \quad x \leq x_2^i, \quad \text{(A.30)}$$

where firm value $V_2(x)$ after investment and before default $(T_2^i \leq t \leq T_2^d)$ is given by

$$V_2(x) = \Pi(x) + \frac{\tau c_2}{r} - \left[\alpha\Pi(x_2^d) + \frac{\tau c_2}{r}\right]\left(\frac{x}{x_2^d}\right)^\gamma, \quad x \geq x_2^d. \quad \text{(A.31)}$$

The optimal coupon policy c_2 that maximizes $V_2(x_2^i)$ is given by

$$c_2 = \frac{r}{r - \mu}\frac{\gamma - 1}{\gamma}\frac{1}{h}Q x_2^i, \quad \text{(A.32)}$$

and $x_2^d = x_2^i/h$. Using the smooth pasting condition $E_1'(x_2^i)x_2^i = V_2'(x_2^i)x_2^i$, we have

$$x_2^i = \frac{1}{1 - \tau}\frac{r - \mu}{Q_2}\frac{\beta}{\beta - 1}\left[\left(I_2 - \frac{\tau c_2}{r}\right) + \frac{\beta - \gamma}{\beta}\left(\alpha\Pi(x_2^d) + \frac{\tau c_2}{r}\right)\left(\frac{x_2^i}{x_2^d}\right)^\gamma\right]. \quad \text{(A.33)}$$

Re-arranging and simplifying (A.33) gives the following implicit equation for x_2^i:

$$(\beta - 1)\Pi_2(x_2^i) = \beta I_2 - \beta\frac{\tau c_2}{r} + (\beta - \gamma)\frac{c_2}{r}\left[\alpha(1 - \tau)\left(\frac{\gamma}{\gamma - 1}\right) + \tau\right]h^\gamma,$$

$$= \beta I_2 - (\beta - 1)\tau\frac{Q x_2^d}{r - \mu}\frac{\gamma}{\gamma - 1}. \quad \text{(A.34)}$$

Using $x_2^d = x_2^i/h$ and re-arranging the last equation gives (45).

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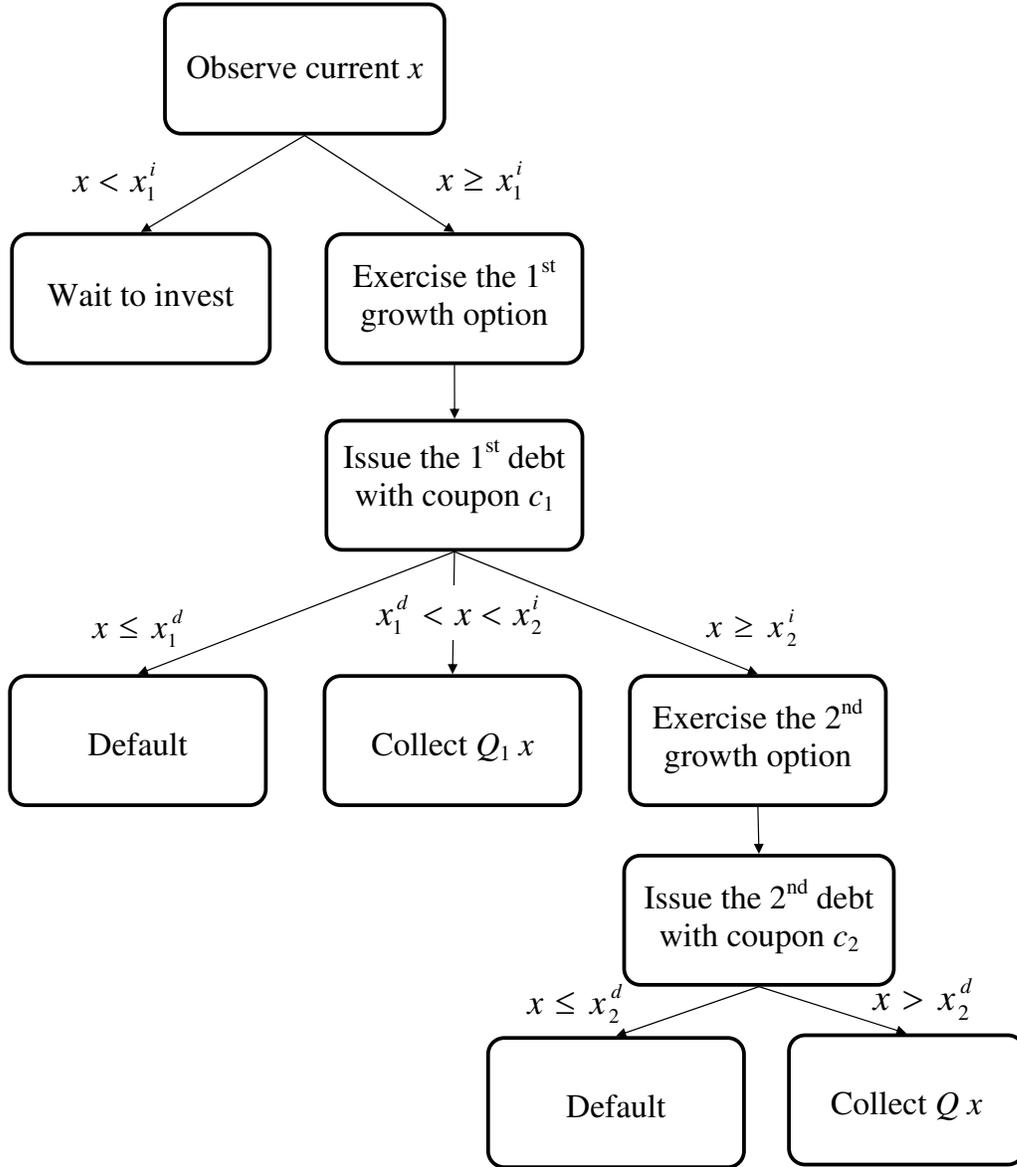
Figure 1: This flowchart describes the firm's decision making process over its life cycle. The firm starts with two sequentially ordered growth options. It exercises its first growth option when $x \geq x_1^i$ and waits otherwise. When exercising, the firm issues the first perpetual debt with coupon c_1, and generates EBIT $Q_1 x$, provided that $x_1^d < x < x_2^i$. When $x \leq x_1^d$, the firm defaults. When $x \geq x_2^i$, the firm exercises its second growth option, and issues the second perpetual debt with coupon c_2. After both options are exercised, the firm generates EBIT $Q x$, where $Q = Q_1 + Q_2$, for $x \geq x_2^d$. It defaults when $x \leq x_2^d$. The two investment thresholds (x_1^i, x_2^i), two default thresholds (x_1^d, x_2^d), and two coupon policies (c_1, c_2) are endogenously determined.

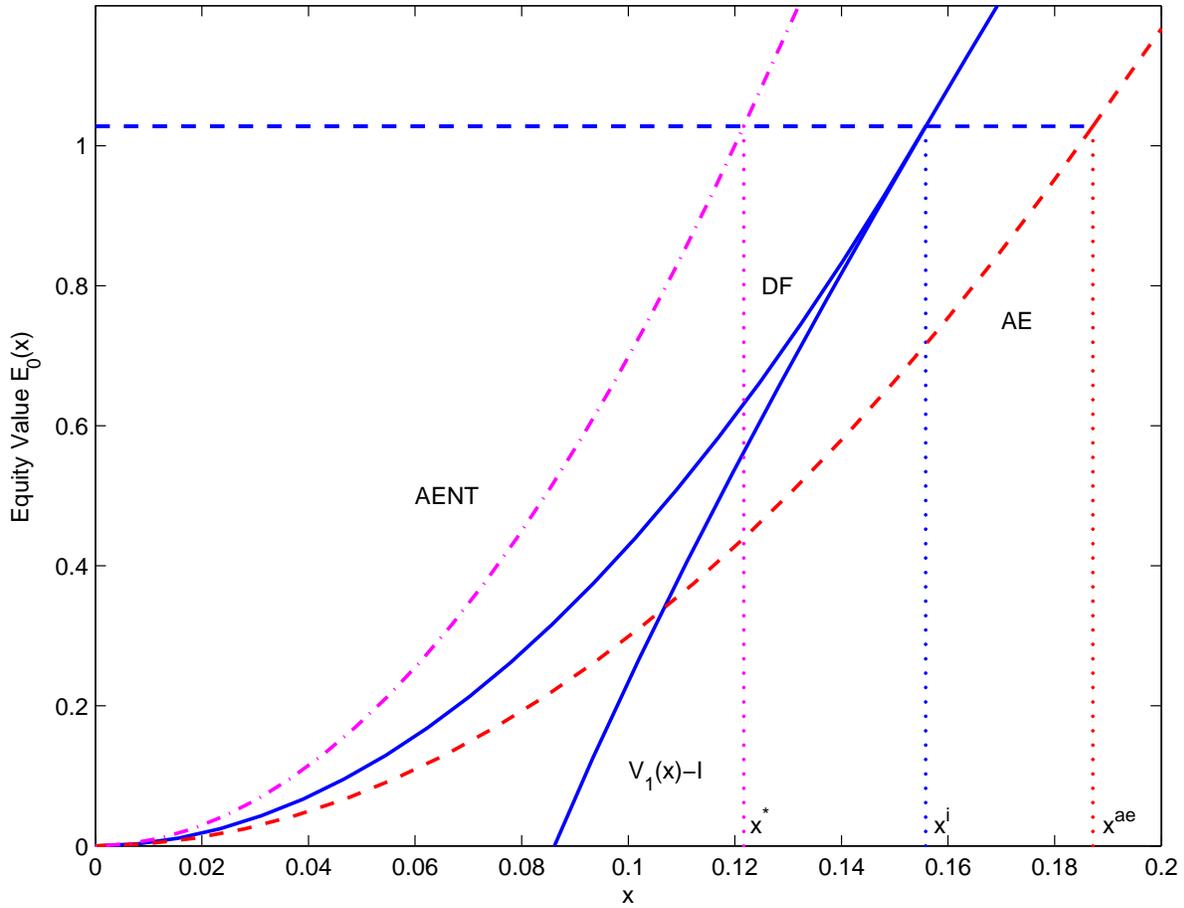Figure 2: This graph plots equity values $E_0(x)$ under all equity financing ($\tau > 0$), optimal financing, and all equity financing (with $\tau = 0$). The respective investment thresholds are ordered sequentially: $x^{ae} > x^i > x^*$. The payoff at (different) exercising thresholds are equal under the three settings, as seen from the horizonal dashed line. Equity value $E_0(x)$ under all equity financing (with $\tau = 0$) is highest (labeled 'AENT'); Equity value $E_0(x)$ under all equity financing (with $\tau > 0$) is the lowest (labeled 'AE'); Equity value $E_0(x)$ under optimal (debt) financing (with $\tau > 0$), solid convex curve (starting at the origin) lies between the two equity values under equity financing (with $\tau = 0$ and with $\tau > 0$). The concave curve $V_1(x) - I$ is the payoff from exercising, where $V_1(x)$ is the firm value after investing. Parameter values: $\alpha = 25\%$, $r = 6\%$, $\tau = 35\%$, $\mu = 0\%$, $\sigma = 25\%$, $I = 1$, $Q = 1$.
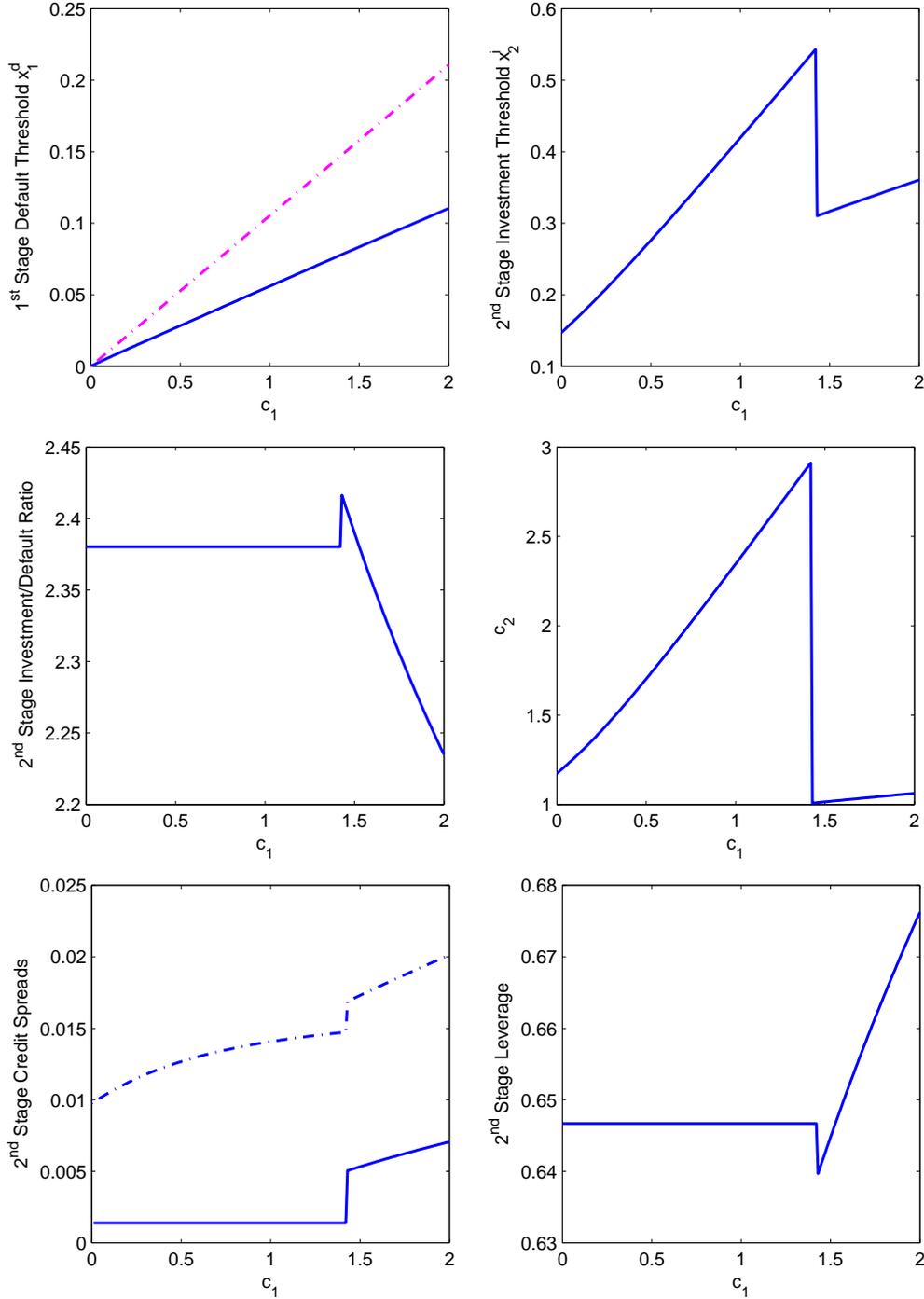
Figure 3: The solid line in the top left panel gives the default threshold as a function of c_1 in our model (with the second growth option). The dash-dotted line in the top left panel gives the (Leland) default threshold for given c_1 (without future growth options). The wedge between the solid line and the dash-dotted line in the top left panel measures the preference for continuation in our model. The top right panel plots the second investment threshold x_2^i as a function of c_1. The mid-left panel plots the ratio x_2^i/x_2^d as a function of c_1. The mid-right panel plots the second coupon c_2. The solid and dash-dotted lines in the bottom left panel give the credit spreads at T_2^i, for the first debt, and for the second debt, respectively. The bottom right panel plots the total market leverage at T_2^i. Parameter values: $\alpha = 25\%$, $r = 6\%$, $\tau = 20\%$, $\mu = 5\%$, $\sigma = 25\%$, $I_2 = 1.5$, $Q_2 = Q_1 = 1$. The credit spread for the first debt when originally issued is $c_1/F_1 - r = 0.67\%$.
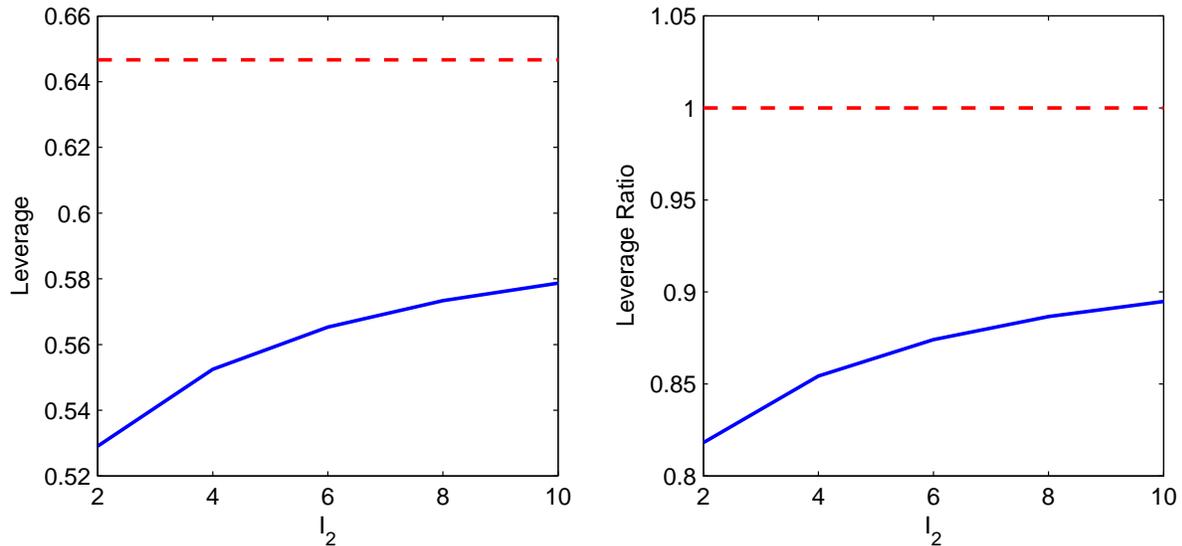
Figure 4: This figure is under "full" optimization, in that both investment thresholds, both default thresholds, and both coupon decisions are all endogenously chosen. The solid and dashed lines in the left panel correspond to the total market leverage at T_1^i and T_2^i, respectively, as functions of the exercise cost for the second growth option I_2. This figure shows that the market leverage at T_1^i is lower than the market leverage at T_2^i. Moreover, the market leverage at T_1^i is higher when the growth option is less attractive (higher I_2), consistent with our intuition on debt overhang. The solid line in the right panel plots the ratio of the total market leverage at T_1^i (in the two growth option setting), scaled by the corresponding stand-alone one-growth option with I_1 and Q_1 (as in Section 4), as a function of the exercise cost for the second growth option I_2. The dashed line in the right panel plots the ratio of the total market leverage at T_2^i (in the two growth option setting), scaled by the corresponding stand-alone one-growth option with I_2 and Q_2 (as in Section 4), as a function of the exercise cost for the second growth option I_2. The horizon dashed line indicates that there is no debt overhang in the second stage. Parameter values: $\alpha = 25\%$, $r = 6\%$, $\tau = 20\%$, $\mu = 5\%$, $\sigma = 25\%$, $I_1 = 1$, $I_2 = 1.5$, $Q_2 = Q_1 = 1$.
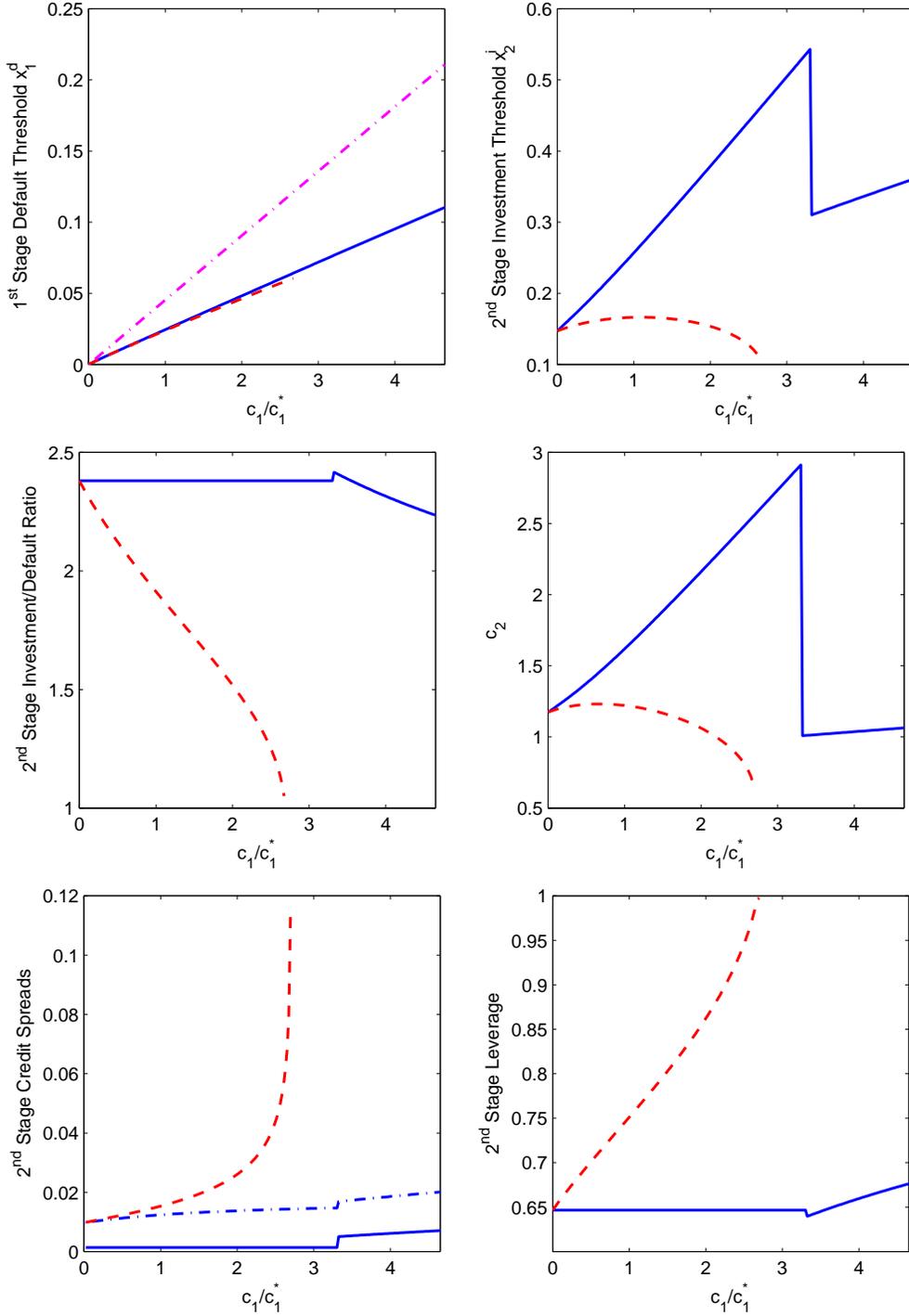
Figure 5: This graph extends Figure 3 to compare the model predictions under APR and *pari passu*. Unlike Figure 3, the horizonal axis is c_1/c_1^*, where c_1^* is the optimal coupon level from the full optimization framework under APR (See Section 5.4). Scaling c_1 by c_1^* gives us a notion about how much the potential debt overhang/risk shifting distortions are. The dashed lines depict the results under *pari passu*. Parameter values: $\alpha = 25\%$, $r = 6\%$, $\tau = 20\%$, $\mu = 5\%$, $\sigma = 25\%$, $I_2 = 1.5$, $Q_2 = Q_1 = 1$.

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μ	1%	2%	3%	4%	5%
$\dfrac{E_0^{apr}(x)}{E_0^{pp}(x)} - 1$	2.40%	1.96%	1.65%	1.43%	1.18%
σ	15%	23%	30%	38%	45%
$\dfrac{E_0^{apr}(x)}{E_0^{pp}(x)} - 1$	0.24%	0.93%	1.64%	1.89%	1.69%
α	10%	30%	50%	70%	90%
$\dfrac{E_0^{apr}(x)}{E_0^{pp}(x)} - 1$	1.91%	1.06%	0.70%	0.52%	0.41%
τ	5%	15%	25%	35%	45%
$\dfrac{E_0^{apr}(x)}{E_0^{pp}(x)} - 1$	0.20%	1.02%	1.17%	0.75%	0.15%
I_2	2	4	6	8	10
$\dfrac{E_0^{apr}(x)}{E_0^{pp}(x)} - 1$	0.77%	0.21%	0.08%	0.03%	0.01%

Table 1: This table reports $E_0^{apr}(x)/E_0^{pp}(x) - 1$, the equity value ratio (minus unity) under APR and under *pari passu* for initial values of x such that the firm is willing to wait under both APR and *pari passu*. We show that $E_0^{apr}(x)/E_0^{pp}(x) - 1$ is close to zero for various levels of μ, σ, α, τ, and I_2. This table shows that financial contracting matters little in terms of $E_0(x)$ when the firm may adjust its initial investment and leverage decisions to mitigate anticipated debt overhang. Benchmark parameter values: $\alpha = 25\%$, $r = 6\%$, $\tau = 20\%$, $\mu = 5\%$, $\sigma = 25\%$, $I_1 = 1$, $I_2 = 1.5$, $Q_2 = Q_1 = 1$. For example, for the first comparative statics with respect to μ, we are using all the benchmark parameter values other than $\mu = 5\%$.