

Securities Auctions under Moral Hazard: Theory and Experiments

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Abstract

We study, both theoretically and in the lab, the performance of open outcry debt and equity auctions in the presence of both private information and hidden effort in an independent private value setting. We characterize symmetric equilibrium bidding strategies and show that these lead to efficient allocation. More interestingly, the revenue ranking between the debt and equity auctions depends on the returns to entrepreneurial effort. When returns are either very low or very high, the equity auction leads to higher expected revenues to the seller than does the debt auction. When the returns to effort are intermediate, we show that debt auctions can outperform equity auctions. We then test these predictions in a controlled laboratory setting and find broad support for the comparative predictions of the model.

1 Introduction

The cornerstone of auction theory, the revenue equivalence theorem (Myerson 1981), suggests that in an ex ante symmetric independent private values setting, the form of the auction does not affect a seller's revenues. That is, a first-price auction, an English auction, or a host of other ways of auctioning off a particular item have no effect on either the allocation of the object or the expected revenues to the seller. The practical applications of this theorem are vast and well-documented (see, for instance, Klemperer 2003). One possible area where auction theory has been fruitfully applied is in the area of competition among firms to acquire a valuable asset. Here, the key issue is how the auction affects the acquiring firm's balance sheet. Hansen (1985) points out that, in a symmetric independent private values setting, a seller running an English auction obtains strictly higher revenues by selling the object through bidding in the form of equity rather than bidding in the form of cash or debt. In a recent important paper, De Marzo, *et al.* (2005) consider a more general structure of auctions for debt, equity, and so on while varying auction forms. However, in all of the extant literature, the fundamental problem is one of private information—the buyer has a better view of his or her value of acquiring the asset than does the seller.

Yet, in many situations, the problem of the seller is not simply one of hidden information, but also hidden action—the effect of the auction on the acquiring firm's balance sheet affects managerial incentives and these, in turn, affect the value of the asset and, consequently, the returns to the seller in conducting the auction. An important area where the joint problem of private information and managerial effort both affect the returns to the seller is in the area of venture capital firms negotiating with several interested entrepreneurs to determine which, if any, of the entrepreneurs to both finance and offer expertise. In this case, the revenue ranking identified by Hansen becomes more problematic. While conducting an English auction where bids are in the form of equity is superior at extracting the surplus from the private values component of the project, it undermines the incentives of the entrepreneur to exert effort possibly leading to a worse outcome than if debt contracts were offered. Indeed, this observation appears in a purely hidden action context dating back to the seminal results of Jensen and Meckling (1976). In addition, Hansen abstracts away from the fact that the winning bidder typically enjoys protection in the form of limited liability; therefore, the “payment” of winning bidder in the form of a debt contract does not represent the actual financial return to the seller. In this paper, we examine, both theoretically and through controlled laboratory experiments, how the choice between auctions for debt and equity affect the returns to the seller.

Our theory model differs from the standard independent private values model of auction theory in the following ways: (1) Bidders can exert unobservable effort which affects the valuation of the asset. As we will show, the amount of effort exerted typically depends on the outcome of the auction; meanwhile, bidding in the auction depends on the valuation of the asset. Equilibrium, of course, factors both effects

in simultaneously. (2) The protection afforded by limited liability differs depending on the structure of the auction and the riskiness of the future cash flows derived from the asset being sold. This in turn affects equilibrium bidding behavior. As we show, the presence of limited liability has a qualitative effect on equilibrium bidding in debt auctions compared to the standard case of unlimited liability. This is not the case with equity auctions. To the best of our knowledge, we are the first paper to analyze equilibrium bidding in securities in the presence of unobservable effort, private information, and limited liability. We are also the first to test such models in controlled laboratory settings.

To fix ideas, it is useful to consider the following setting: Successfully obtaining financing from venture capital firms is critical to the success of the business idea of an entrepreneur. The problem, of course, is that this financing, and the managerial expertise that accompanies it, is a scarce resource over which entrepreneurs must compete. Viewed from the perspective of a venture capital firm, the problem of choosing among competing business ideas presented by entrepreneurs is daunting. Often these ideas are sufficiently novel and the markets these businesses propose to serve sufficiently undeveloped, that gaining an accurate picture of the quality of the business idea of a given entrepreneur is tremendously difficult.

In practice, venture capital firms try to solve the problem of determining “which horse to back” in a variety of ways: by listening to presentations, by reading business and marketing plans, by undertaking independent studies of the markets the entrepreneurs propose to serve, and so on. But there is an additional important source of information available to venture capital firms in making funding decisions—the negotiation with and competition among entrepreneurs to secure scarce funding opportunities. Specifically, we consider competition for scarce financing resources under two sorts of contractual schemes: offers of debt financing and offers of equity financing. Under the debt financing scheme, the competing entrepreneurs “bid” against one another in the form of debt contracts to obtain scarce financing. We model this bidding competition as an open outcry auction and show that the equilibrium in weakly dominant strategies leads the company with the more valuable business plan to always be selected. However, in practice, debt financing schemes are rarely used in venture capital financing arrangements. A more common scheme is one where venture capital financing is offered in exchange for an equity share of the company. Accordingly, we also model competition among entrepreneurs where the “bids” consist of offers of control percentages of the company to the venture capital firm in exchange for financing. Again, we model this as an open outcry auction and find an equilibrium in weakly dominant strategies. In this equilibrium the higher valued project is always selected.

So why is it that equity schemes are common while debt schemes are not? Clearly, both are equally efficient in determining the quality of the business ideas of competing entrepreneurs. Moreover, the inherent quality of the business idea is not the only determinant of a successful business. Entrepreneurial effort is often critical to the value

of a start-ups financed by venture capital firms. Accordingly, we enrich the model by adding entrepreneurial effort, which cannot be directly observed nor contracted upon, to the model and examine the performance of debt and equity auctions where both private information about the quality of the business idea as well as the level of entrepreneurial effort affect the value of the company. Our main findings are as follows:

1. Regardless of the effect of entrepreneurial effort, both debt and equity auctions succeed in selecting the higher quality business idea with probability one.
2. When the returns to entrepreneurial effort are either very high or very low, the equity auction leads to strictly higher payoffs to the venture capital firm.
3. When the returns to entrepreneurial effort are intermediate the debt auction can yield higher expected payoff to the venture capital firm.

We then test these results in a controlled laboratory experiment. Our main treatments are to vary the structure of the auction and the returns to entrepreneurial effort. To the best of our knowledge, we are the first to compare the performance of debt versus equity auctions in the lab. Overall, under a variety of performance metrics, we find broad support for the comparative predictions of the theory. Specifically,

1. In the treatment where the returns to entrepreneurial effort are low, the revenues in the equity auction are statistically significantly higher than those in a debt auctions.
2. In the treatment where the returns to entrepreneurial effort are intermediate (and where, for the calibrated values, the theory predicts that debt auctions will outperform equity auctions), the revenues in the equity auction are statistically significantly higher than those in a debt auctions.
3. In all auctions, the higher quality business idea is funded with probability close to 100%.

While the comparative static predictions are largely supportive of the theory, the market level results suggest systematic deviations from the theory level predictions. Deriving and estimating a structural model for revenues, we show that winners' bids reflect too high weighting on the highest private value and too low weighting on the second highest private value - all relative to the theoretical predictions. To explore the effect of erroneous bidding behavior on these results, we test differences in allocation efficiencies - the probability that the winner is the entrepreneur with the highest project value - across treatment cells and rounds. While we find that overall allocation efficiency is high, systematic deviations related to auction form and learning are observed.

The remainder of the paper proceeds as follows: In section 2 we sketch the model and derive a characterization of equilibrium bidding behavior in debt and equity auctions with both private information and moral hazard. Section 3 outlines the design of the experiment. Section 4 reports the experiment results as they relate to the comparative static predictions of the theoretical model while section 5 discusses level prediction tests of the model. The structural model used in some of the estimations is contained in an appendix.

2 Theory

Consider a setting in which there are two entrepreneurs competing for resources from a venture capital firm to fund a risky project. Each entrepreneur currently operates a small business that has a commonly known and identical value of m . Each entrepreneur has access to a risky project which requires financing (and other inputs) from a venture capital firm. A venture capital firm possesses this package of resources in sufficient quantity to finance exactly one project. If an entrepreneur receives a package of resources from the VC, it then undertakes the project. The payoff from the project of entrepreneur i depends on its inherent quality (v_i) and the degree of entrepreneurial effort, $e_i \in \{0, 1\}$. In particular, suppose with probability p a project succeeds and produces cash equivalent to $v_i(1 + \delta e_i)$, where δ denoted the returns to effort. Otherwise, a project fails and pays zero to all parties.¹ Thus, when entrepreneur i undertakes a project of quality v_i and exerts effort e_i , then the payoff from the project is

$$\pi(v_i, e_i) = \begin{cases} v_i(1 + \delta e_i) & \text{with Pr} = p \\ 0 & \text{with Pr} = 1 - p \end{cases}$$

Let the cost of entrepreneurial effort be equal to the effort. Suppose entrepreneur i is privately informed about the quality of his or her business idea, v_i . Suppose, however, that it is commonly known that for all i , v_i is drawn from the atomless distribution F on $[\underline{v}, \bar{v}]$. In addition, an entrepreneur privately undertakes entrepreneurial effort that is personally costly. Entrepreneurial effort is not directly observable nor contractible by any outside party. Finally, suppose that the entrepreneur is protected by limited liability.

Notice that there is a trade-off between undertaking the project (even on the most favorable possible terms) and risking a failure versus retaining the “safe” outside option, m , and avoiding the costs associated with failed projects. Since our focus is on how the investment decision is affected by the structure of the negotiation between the entrepreneurs and the VC rather than whether to undertake any investment at all, we assume that the quality of any of the ideas is such that it is socially optimal

¹We assume that the costs of a failure strictly exceed m .

to undertake the risky project. Formally, this amounts to the condition:

$$m \leq p(v + m) \tag{1}$$

Suppose that an entrepreneur obtains VC financing on the following terms: the entrepreneur retains a fraction α_i of the company and has debt service D_i . In that case, the expected payoff to the entrepreneur is

$$EU_i = p\alpha_i(v_i(1 + \delta e_i) + m - \min(D_i, v_i(1 + \delta e_i) + m)) - e_i$$

In this case, the entrepreneur should optimally exert effort ($e_i = 1$) provided that

$$p\alpha_i(v_i(1 + \delta) + m - \min(D_i, v_i(1 + \delta) + m)) - 1 \geq p\alpha_i(v_i + m - \min(D_i, v_i + m))$$

which we may then simplify to

$$p\alpha_i(v_i\delta - (\min(D_i, v_i(1 + \delta) + m) - \min(D_i, v_i + m))) \geq 1 \tag{2}$$

That is, the entrepreneur's net expected return to effort, $p\alpha_i(v_i\delta - C)$ exceeds her cost of effort, 1, where C denotes the change in debt liability associated with a successful project under high effort.

Absent the support of the VC, the value of entrepreneur i 's company is simply $EU_i = m$, and the optimal amount of entrepreneurial effort is zero.

Since neither the entrepreneurs' quality of ideas nor their effort is directly observable nor contractible by the VC, the key problem faced by the VC is in designing a contractual scheme with an entrepreneur to "solve" the combined adverse selection and moral hazard problems. Of course, the objective of the manager of the VC is to maximize the expected return of the investors subject to some constraints described below: Suppose that if the resources of the VC are put to neither of the two projects, then the investors of the VC withdraw their funds and the manager of the VC firm suffers infinite negative utility from suddenly becoming unemployed. Therefore, the VC cannot credibly commit not to fund one of the entrepreneurs.

We shall consider the following schemes:

1. **Equity "auction"**: We will compare the above procedure with an alternative. In an equity auction, entrepreneurs compete by offering the VC fractional ownership of the company in exchange for the VC's resources. We model this as an open outcry auction—the entrepreneur offering the larger ownership share is the "winner" of the auction at the bid amount.

2. **Debt "auction"**: Suppose that the entrepreneurs compete with one another by offering the VC debt contracts in exchange for VC support. Again, we model this process as an open outcry auction. The "bidder" offering the higher amount of debt repayment in exchange for the resources of the VC is the "winner" of the auction at the bid amount.

Analysis of Equilibrium in Equity Auctions

First, we consider the equity auction and determine when it is optimal for the winning entrepreneur to undertake effort.

Lemma 1 *Winning entrepreneur i should undertake effort if and only if the winning price is less than $\frac{v_i p \delta - 1}{v_i p \delta}$*

Proof. Suppose that the current “price” in the auction is $1 - \alpha$. Then, if entrepreneur i won at this price, it would be optimal to undertake effort if and only if

$$p\alpha(v_i(1 + \delta) + m) - 1 \geq p\alpha(v_i + m)$$

or

$$1 - \alpha \leq \frac{v_i p \delta - 1}{v_i p \delta}$$

■

We are now in a position to reason backwards in the auction to determine equilibrium bidding strategies. As we show below, these depend on the parameter values pertaining to the returns to effort.

Case 1. $p v_i \delta \leq 1$.

Clearly, when $v_i p \delta \leq 1$, the returns to effort are never sufficient to justify any effort “investment” on the part of the entrepreneur. In that case, the weakly dominant strategy for the entrepreneur is to bid up to the point where expected value of the company under no effort is equal to the outside option in the event no financing is obtained. Specifically, let $1 - \alpha_i^0$ denote the “drop-out price; then

$$\alpha_i^0 p(v_i + m) = m$$

or

$$\alpha_i^0 = \frac{m}{p(v_i + m)}$$

which is a well-defined bidding strategy by equation (1).

Case 2. $p v_i \delta > 1$.

Now, for prices that are sufficiently low, the entrepreneur is willing to undertake effort. Again, consider the strategy where the entrepreneur bids up to the point where expected value of the company under positive effort is equal to the outside option in the event no financing is obtained. Specifically, let $1 - \alpha_i^1$ denote the “drop-out price; then

$$\alpha_i^1 p(v_i(1 + \delta) + m) - 1 = m$$

or

$$\alpha_i^1 = \frac{m + 1}{p(v_i(1 + \delta) + m)}$$

which is a well-behaved bidding strategy since $p v_i \delta > 1$.

Provided that $1 - \alpha_i^1 \leq \frac{v_i p \delta - 1}{v_i p \delta}$, then the above drop-out strategy is weakly dominant. That is, when

$$\begin{aligned} 1 - \frac{m + 1}{p(v_i(1 + \delta) + m)} &\leq \frac{v_i p \delta - 1}{v_i p \delta} \\ \frac{p(v_i(1 + \delta) + m) - m - 1}{p(v_i(1 + \delta) + m)} &\leq \frac{v_i p \delta - 1}{v_i p \delta} \end{aligned}$$

Cross-multiplying

$$(p(v_i(1+\delta)+m) - m - 1)v_i p\delta \leq p(v_i(1+\delta)+m)(v_i p\delta - 1)$$

Rewriting

$$p(v_i(1+\delta)+m)(v_i p\delta - 1) - (p(v_i(1+\delta)+m) - m - 1)v_i p\delta \geq 0$$

Simplifying

$$(m\delta - 1)v_i - m \geq 0$$

Therefore, we have shown that if $(m\delta - 1)v_i - m \geq 0$, then an equilibrium in weakly dominant strategies is to bid up to a price $1 - \alpha_i^1$ and exert high effort conditional on winning.

Suppose that $(m\delta - 1)v_i - m < 0$, in that case, $1 - \alpha_i^1 > \frac{v_i p\delta - 1}{v_i p\delta}$. Therefore, the bidding strategy must change once the price $\frac{v_i p\delta - 1}{v_i p\delta}$ is exceeded. In particular, for all prices $1 - \alpha > \frac{v_i p\delta - 1}{v_i p\delta}$. It is not optimal to exert effort. In that case, the drop out condition is exactly as in case 1. It may be readily shown that $1 - \alpha_i^0 \geq \frac{v_i p\delta - 1}{v_i p\delta}$ if and only if $(m\delta - 1)v_i - m < 0$. To summarize

Proposition 1 *In an equity auction, an equilibrium in weakly dominant strategies is for bidder i to drop out at price $1 - \alpha_i$ where*

$$\alpha_i = \begin{cases} \frac{m+1}{p(v_i(1+\delta)+m)} & \text{if } pv_i\delta > 1 \text{ and } (m\delta - 1)v_i - m \geq 0 \\ \frac{m}{p(v_i+m)} & \text{otherwise} \end{cases}$$

Together with the effort strategy in Lemma 1, this comprises a symmetric subgame perfect equilibrium in undominated strategies in an equity auction.

We now argue that the equity auction has the property that the higher valued idea is funded with probability one. To see this, suppose that $v_1 > v_2$. There are two cases to consider:

Case 1. $pv_1\delta \leq 1$ or $(m\delta - 1)v_1 - m < 0$.

In that case α_1 and α_2 are identical and strictly decreasing functions of v_i ; hence entrepreneur 1's project is funded.

Case 2. $pv_1\delta > 1$ and $(m\delta - 1)v_1 - m \geq 0$.

If $(m\delta - 1)v_2 - m \geq 0$ and $pv_1\delta > 1$, then α_1 and α_2 are identical and strictly decreasing functions of v_i ; hence entrepreneur 1's project is funded. Otherwise, entrepreneur 2 drops out at price

$$\begin{aligned} \alpha_2 &= 1 - \frac{m}{p(v_2+m)} \\ &< 1 - \frac{m}{p(v_1+m)} \\ &\leq 1 - \frac{m+1}{p(v_1(1+\delta)+m)} \\ &= \alpha_1 \end{aligned}$$

where the strict inequality follows from the fact $v_1 > v_2$ and the weak inequality follows from the fact that $(m\delta - 1)v_1 - m \geq 0$. Therefore, entrepreneur 1's project is funded.

Hence, we have shown that

Proposition 2 *In an equity auction under the equilibrium in weakly dominant strategies given in Proposition 1, the higher valued idea is funded with probability one.*

What is the expected return to the VC under this auction? There are three possibilities to consider, either (i) the winning entrepreneur exerts high effort and the price is set as though high effort will be undertaken; (ii) the winning entrepreneur exerts low effort and the price is set as though low effort will be undertaken; or (iii) the winning entrepreneur exerts high effort and the price is set as though low effort will be undertaken. The fourth possibility, a price set as though high effort will be undertaken followed by a low effort choice from the winning entrepreneur is inconsistent with subgame perfect equilibrium. The expected return in each of the possibilities is as follows:

(i)

$$\begin{aligned} ER_{equity} &= \left(1 - \frac{m+1}{p(v_2(1+\delta) + m)}\right) p((v_1(1+\delta)) + m) \\ &= \frac{v_1(1+\delta) + m}{v_2(1+\delta) + m} \times (pv_2(1+\delta) - (1-p)m - 1) \end{aligned}$$

(ii)

$$\begin{aligned} ER_{equity} &= \left(1 - \frac{m}{p(v_2 + m)}\right) (p(v_1 + m)) \\ &= \left(\frac{p(v_2 + m) - m}{p(v_2 + m)}\right) (p(v_1 + m)) \\ &= \frac{v_1 + m}{v_2 + m} \times (pv_2 - (1-p)m) \end{aligned}$$

(iii)

$$\begin{aligned} ER_{equity} &= \left(1 - \frac{m}{p(v_2 + m)}\right) p((v_1(1+\delta)) + m) \\ &= \frac{v_1(1+\delta) + m}{v_2 + m} \times (pv_2 - (1-p)m) \end{aligned}$$

Using Proposition 1, we now determine the parameter values in which each of three possibilities arise.

Remark 1 If $pv_2\delta > 1$ and $(m\delta - 1)v_2 - m \geq 0$, then the winning entrepreneur exerts high effort and the price is set in anticipation of high effort.

If either (a) $pv_1\delta \leq 1$; or (b) $pv_1\delta > 1$, $(m\delta - 1)v_1 - m < 0$, and $1 - \frac{m}{p(v_2+m)} > \frac{v_1p\delta-1}{v_1p\delta}$, then the winning entrepreneur exerts low effort and the price is set in anticipation of low effort

Otherwise, the winning entrepreneur exerts high effort and the price is set in anticipation of low effort.

Case 1: $pv_1\delta \leq 1$ then $\langle \text{low}, \text{low} \rangle$

Case 2: $pv_1\delta > 1$

Case 2a: $pv_2\delta > 1$ and $(m\delta - 1)v_2 - m \geq 0$ then $\langle \text{high}, \text{high} \rangle$

Case 2b: $(m\delta - 1)v_1 - m < 0$ and $1 - \frac{m}{p(v_2+m)} > \frac{v_1p\delta-1}{v_1p\delta}$ then $\langle \text{low}, \text{low} \rangle$

Case 2c: $(m\delta - 1)v_1 - m < 0$ and $1 - \frac{m}{p(v_2+m)} \leq \frac{v_1p\delta-1}{v_1p\delta}$ then $\langle \text{high}, \text{low} \rangle$

Case 2d: $(m\delta - 1)v_1 - m \geq 0$ and $(m\delta - 1)v_2 - m < 0$ or $pv_2\delta \leq 1$ then $\langle \text{high}, \text{low} \rangle$

Analysis of Equilibrium in Debt Auctions

Next, we turn to debt auctions. Before proceeding, it is useful to establish several preliminary facts about equilibrium bidding in a debt auction. As above, define D_i^1 to be an equilibrium bid from an entrepreneur who expects to undertake positive effort if awarded the financing. Let D_i^0 be likewise defined.

Lemma 2 In any equilibrium of a debt auction, $D_i^1 \leq v_i(1 + \delta)$ and $D_i^0 \leq v_i$.

Proof. Suppose to the contrary that $D_i^1 > v_i(1 + \delta)$. In that case, the expected payoff to the entrepreneur in the event that she “wins” and is awarded the financing is

$$\begin{aligned} E\pi_i &= p(v_i(1 + \delta) + m - \min(D_i^1, v_i(1 + \delta) + m)) - 1 \\ &\leq m - 1 \\ &< m \end{aligned}$$

Therefore, it is a profitable deviation for the entrepreneur simply to stop bidding before reaching this price. An identical argument establishes the claim for $D_i^0 \leq v_i$.

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Thus, for a given level of effort, it is a weakly dominant strategy to bid up to a debt level that leaves the entrepreneur indifferent between obtaining funding and not. That is, D_i^1 solves

$$p(v_i(1 + \delta) + m - D_i^1) - 1 = m$$

Or, equivalently

$$D_i^1 = v_i(1 + \delta) - \frac{1}{p} - \frac{1-p}{p}m \quad (3)$$

Likewise D_i^0 solves

$$D_i^0 = v_i - \frac{1-p}{p}m \quad (4)$$

Lemma 3 *Undertaking effort is optimal if and only if $D_i^1 \geq D_i^0$.*

Proof. Suppose to the contrary that $D_i^1 \geq D_i^0$ and undertaking effort was not optimal. In that case, the expected payoff from undertaking effort for a bid D_i^0 is at least m ; whereas, by construction, it is exactly equal to m in the case of undertaking no effort. This is a contradiction. Suppose to the contrary that $D_i^1 < D_i^0$ and undertaking effort is optimal. In that case, the expected payoff undertaking no effort and bidding D_i^1 is strictly greater than m while undertaking effort and bidding D_i^1 , by construction, produces expected payoff equal to m . This is a contradiction. ■

It can be shown that $D_i^1 \geq D_i^0$ is and only if

$$v_i \geq \frac{1}{\delta p}$$

To summarize

Proposition 3 *In a debt auction, an equilibrium in weakly dominant strategies is for bidder i to bid according to*

$$D_i = \begin{cases} v_i(1+\delta) - \frac{1}{p} - \frac{1-p}{p}m & \text{if } v_i \geq \frac{1}{\delta p} \\ v_i - \frac{1-p}{p}m & \text{otherwise} \end{cases}$$

We now argue that the debt auction has the property that the higher valued idea is funded with probability one. To see this, suppose that $v_1 > v_2$. There are three cases to consider:

Case 1. $v_2 \geq \frac{1}{\delta p}$.

In that case, D_1 and D_2 are identical and strictly decreasing functions of v_i ; hence entrepreneur 1's project is funded.

Case 2. $v_1 < \frac{1}{\delta p}$.

In that case, D_1 and D_2 are identical and strictly decreasing functions of v_i ; hence entrepreneur 1's project is funded.

Case 3. $v_1 \geq \frac{1}{\delta p}$ and $v_2 < \frac{1}{\delta p}$.

In that case, $D_1 > D_1^0 > D_2^0 = D_2$. Hence entrepreneur 1's project is funded.

Hence, we have shown that

Proposition 4 *In a debt auction under the equilibrium in weakly dominant strategies given in Proposition 1, the higher valued idea is funded with probability one.*

What is the expected return to the VC under this auction? The expected payoff to the VC when $v_1 > v_2$ is as follows:

Case 1. $v_2 \geq \frac{1}{\delta p}$.

$$ER_{debt} = p \left(v_2 (1 + \delta) - \frac{1}{p} - \frac{1-p}{p} m \right)$$

Case 2. $v_2 < \frac{1}{\delta p}$.

$$ER_{debt} = p \left(v_2 - \frac{1-p}{p} m \right)$$

2.1 Revenue Comparisons

Proposition 5 *Suppose that (i) $\delta p v_1 \leq 1$ or (ii) $\delta p v_1 > 1$ and $\delta p v_2 < 1$. Then, for all realizations, $v_1 > v_2$, the equity auction yields greater revenues to the VC than does the debt auction.*

Proof. Notice that under either condition, entrepreneur 2 bids in the debt and equity auctions anticipating undertaking zero effort. Hence,

$$ER_{debt} = p v_2 - (1-p) m$$

while

$$ER_{equity} \geq \frac{v_1 + m}{v_2 + m} \times (p v_2 - (1-p) m)$$

Differencing these expressions one obtains

$$\begin{aligned} ER_{equity} - ER_{debt} &\geq \frac{v_1 + m}{v_2 + m} \times (p v_2 - (1-p) m) - (p v_2 - (1-p) m) \\ &> (p v_2 - (1-p) m) - (p v_2 - (1-p) m) \\ &= 0 \end{aligned}$$

■

Proposition 6 *Suppose that $\delta p v_1 > 1$ and $(m\delta - 1) v_2 - m \geq 0$. or (ii) $\delta p v_2 < 1$. Then, for all realizations, $v_1 > v_2$, the equity auction yields greater revenues to the VC than does the debt auction.*

Proof. Notice that, when $\delta p v_2 > 1$ and $(m\delta - 1) v_2 - m \geq 0$, then under both the equity and debt auctions, high effort is undertaken and the price is set anticipating high effort. Hence

$$ER_{debt} = p v_2 (1 + \delta) - (1-p) m - 1$$

while

$$ER_{equity} = \frac{v_1 (1 + \delta) + m}{v_2 (1 + \delta) + m} \times (p v_2 (1 + \delta) - (1-p) m - 1)$$

Differencing these two expressions, one obtains

$$\begin{aligned}
ER_{equity} - ER_{debt} &= \frac{v_1(1+\delta) + m}{v_2(1+\delta) + m} \times (pv_2(1+\delta) - (1-p)m - 1) - (pv_2(1+\delta) - (1-p)m - 1) \\
&> (pv_2(1+\delta) - (1-p)m - 1) - (pv_2(1+\delta) - (1-p)m - 1) \\
&= 0
\end{aligned}$$

■

The main lesson from the proposition is that, when the returns to effort are either sufficiently high that undertaking high effort is still profitable in an equity auction or so low that undertaking effort is not optimal in either auction, then the equity auction always outperforms the debt auction.

Finally, we consider the intermediate cases. Here, the trade-off is more complicated. Entrepreneur 2 will bid as though high effort will be undertaken in the debt auction and as though low effort will be undertaken in the equity auction. For the equity auction, the expected revenues also depend on whether entrepreneur 1 undertakes effort. That is, whether $\delta \geq \frac{1}{v_1} + \frac{1}{m}$ or not.

As we shall see below, the revenue ranking in this case depends heavily on the gap between v_1 and v_2 . For future reference define

$$\Delta = \frac{(v_2 + m)(p\delta v_2 - 1)}{(pv_2 - (1-p)m)}$$

Proposition 7 *Suppose that $\delta pv_2 > 1$, $(m\delta - 1)v_1 - m < 0$, and $1 - \frac{m}{p(v_2+m)} > \frac{v_1 p \delta - 1}{v_1 p \delta}$. Then*

If v_1 and v_2 are “close”; that is $v_1 - v_2 < \Delta$, the debt auction yields greater revenues to the VC than does the equity auction.

If v_1 and v_2 are not close, that is, $v_1 - v_2 \geq \Delta$, the equity auction yields greater revenues to the VC than does the debt auction.

Proof. Under the above conditions, the price in the debt auction is set in anticipation of high effort and high effort is undertaken while the price in the equity auction is set in anticipation of low effort and low effort is undertaken. Thus, the revenue comparison is as follows:

$$ER_{debt} = (pv_2(1+\delta) - (1-p)m - 1)$$

while

$$ER_{equity} = \frac{v_1 + m}{v_2 + m} \times (pv_2 - (1-p)m)$$

Differencing these two expressions

$$ER_{equity} - ER_{debt} = \frac{v_1 + m}{v_2 + m} \times (pv_2 - (1-p)m) - (pv_2(1+\delta) - (1-p)m - 1)$$

The sign of this expression depends on whether

$$v_1 \geq v_2 + \Delta$$

■

Proposition 8 *Suppose that $\delta p v_2 > 1$, $(m\delta - 1)v_2 - m < 0$, and either (a) $(m\delta - 1)v_1 - m \geq 0$, or (b) $1 - \frac{m}{p(v_2 + m)} \geq \frac{v_1 p \delta - 1}{v_1 p \delta}$. Then*

If v_1 and v_2 are “close”; that is $v_1(1 + \delta) - v_2 < \Delta$, the debt auction yields greater revenues to the VC than does the equity auction.

If v_1 and v_2 are not close, that is, $v_1(1 + \delta) - v_2 \geq \Delta$, the equity auction yields greater revenues to the VC than does the debt auction.

Proof. Under the above conditions, the price in the debt auction is set in anticipation of high effort and high effort is undertaken while the price in the equity auction is set in anticipation of low effort and high effort is undertaken. Thus, the revenue comparison is as follows:

$$ER_{debt} = (p v_2 (1 + \delta) - (1 - p) m - 1)$$

while

$$ER_{equity} = \frac{v_1 (1 + \delta) + m}{v_2 + m} \times (p v_2 - (1 - p) m)$$

Differencing these two expressions

$$ER_{equity} - ER_{debt} = \frac{v_1 (1 + \delta) + m}{v_2 + m} \times (p v_2 - (1 - p) m) - (p v_2 (1 + \delta) - (1 - p) m - 1)$$

The sign of this expression depends on whether

$$v_1 (1 + \delta) \geq v_2 + \Delta$$

■

Despite the complexity of the parameter conditions, the results of the last two propositions are intuitive: When v_1 is sufficiently high relative to v_2 , the equity auction outperforms the debt auction simply by linking the payment received by the VC to v_1 . In the case where only low effort is undertaken, the required gap for this effect to dominate is Δ whereas, when the incentive diluting effect of the equity auction is not a concern (as in Proposition 4), the required gap between v_1 and v_2 falls in proportion to the returns to effort—to $\Delta - \delta v_1$. In contrast, the VC obtains little benefit from linking its payment received to the value of v_1 when the valuations of the two entrepreneurs are relatively equal. In that case, the superior incentive effects of the debt auction dominate.

3 Experimental Design

3.1 General

The experiment consisted of 14 sessions conducted at the University of California at Berkeley Experimental Social Sciences Laboratory (XLab) during the Spring 2004 semester. Eight subjects participated in each session, and no subject appeared in more than one session. Subjects were recruited from a distribution list comprised of primarily economics, business and engineering undergraduate students. Participants received a show-up fee of \$3 and an additional performance based pay of \$0-\$40 for a session lasting around 2 hours.

All sessions started with subjects being seated in front of a computer terminal and given a set of instructions, which were then read aloud by the experimenter. Throughout the session, no communication between subjects was permitted and all choices and information were transmitted via the computer terminal.

The session then consisted of three phases of 12 periods each. During the first and last phase subject participating as "entrepreneurs" bid with debt while in the second phase "entrepreneurs" bid with equity. Thus, the sequence of sessions is Debt, equity, Debt.

At the beginning of each period, subjects were randomly assigned to groups of four. Within each group a single unit of funding was sold at an English auction. Each subject received an independently and identically draw from a uniform distribution with a support of 0 to 100, which corresponded to the value of project (if it is funded) to the entrepreneur. Each entrepreneur then submitted bids in a computerized outcry process subject to improvement rule (this mechanism mirrors the one used by large art auction houses as Christie's and Sotheby's). The period ended if no new bids arrived in a period of 15 seconds, during which subjects received a "going, going, gone" warning message. Each bid included two elements – a price and an effort decision. While the former is standard, the later denotes entrepreneur's decision whether or not they would opt to increase the value of the project (i.e. exert effort) by incurring a known cost. While the benefit resulting from exerting effort accrued to the project being financed, the cost was borne completely by the bidder. The terminal provided a calculator which allowed subjects to compute their earnings given different inputs of winning bids and effort decisions.

At the start of each period subjects were endowed with ten points each. During the debt auctions, bids were interpreted as points. Thus, winning bid earnings were equal to ten points plus private and effort values minus bid and effort cost. During the equity auction, bids were interpreted as *percentage* points. Thus, winning bid earnings were equal to 100 minus percentage point bid times 10 points plus private value, effort value, minus effort cost². Losing bid earned ten points. At the end of each period, subjects' earnings were calculated and displayed on their interface.

²Notice that effort costs are borne solely by the entrepreneur.

3.2 Discussion of the design

The experiment was designed around two treatments: security type (debt / equity) and returns to effort (low / high). The main purpose of this design is to test the revenue ranking predictions. When effort returns are low, the moral hazard problem is immaterial and equity auctions yield higher revenues to the seller than debt auctions. On the other hand, when effort returns are high, the moral hazard problem becomes sizable and debt auctions yield higher revenues to the seller. The auction type treatment was implemented across subjects so that some sessions were parameterized with low returns to effort while other sessions were parameterized with high returns to effort. The auction type treatment was implemented within subjects so that each subject participated in both debt and equity auctions.

One contribution of the study is to model the auction in the lab as an ascending bid English (or open outcry) auction. While laboratory implementations of this auction typically use a so-called clock auction design where bidders need only decide at what price to drop out of the auction, we use a more natural form of the English auction. This mechanism has a number of advantages over the commonly used first and second price sealed bid auctions. First, it is familiar to subjects and thus easy to understand. Since the securities with which subjects bid are somewhat non-standard, we believed that an intuitive mechanism was important. Second, while English auction is theoretically equivalent to the canonical sealed bid auctions, the strategies in the former are substantially simpler, making it less prone to potential cognitive biases. Third, this auction mechanism is invariant to risk preferences (see for example Maskin and Riley (1984)). Previous studies suggested that deviations from risk-neutrality may be consequential for results obtained in under sealed bid auctions (see Kagel (1995) for a review of this literature).

We parametrized the experiment such that in the "low returns" sessions the effort value was low enough to make it unprofitable for player, in either the debt or equity auctions, to exert effort. In the "high returns" sessions, effort was optimally exerted by the winning bidder in all debt auction instances but only in small fraction of equity auctions. For simplicity, we kept the cost of effort the same for all sessions. The specific return-to-effort values were determined so as to generate a powerful test of the revenue ranking predictions while making bidders decisions manageable in terms of their complexity. Given the real-time nature of the auction, we wanted to avoid cases where theoretical effort choice switches during the bidding process.

To summarize, each session was conducted using one of the two effort conditions ("low returns" or "high returns"). Under both treatments, outside value, m , was equal to 10, private value of the project, v_i , was drawn from a uniform distribution with support of $[0, 100]$, and the cost of effort, c , was equal to 20^3 . Returns to effort, δ , were set to 0.1 in the "low" case, and to 1.3 in the "high" case. These parameters

³The experiment tests the deterministic version of the model discussed in the Theory section; that is, probability of the high node state is 1.

were chosen such that the expected loss from socially inefficient effort choice in the equity auctions outweighed the expected benefits arising from linking the revenues to the highest private value. The effort returns needed to be sufficiently high to induce effort exertion in the debt case but not high enough to induce effort exertion in the equity bidding case.

The equilibrium predictions for each type of auction under each treatment is given in table 1. The table provides mean predictions of sellers' revenues (in points), normalized revenues and effort decisions, which are defined below:

- **Revenues:** This is simply a measure of the revenues obtained by the seller in a given auction (measured in the experimental points).
- **Normalized Revenues:** Since the valuations of each of the bidders are drawn randomly, there may be variations in revenues that are purely driven by the realizations of the draws. A more useful measure of the performance of an auction is the fraction of the maximum theoretically possible surplus captured by the seller. To take a simple example, suppose that the surplus available in auction A was \$10 and the seller received \$7. In auction B, the available surplus was \$5 and the seller obtained \$4. Then, even though the revenues from auction B, measured in levels, are lower than those under auction A, the percentage of surplus captured by the seller is higher. Thus, given the variation across auctions in the available surplus, this measure of auction performance seems useful.
- **Effort choices:** The measure for effort indicates whether the winning bidder chose to pay the costs required to “upgrade” the asset. We code this as “1” if effort was exerted in a given round of the experiment and "0" otherwise.

Table 1: Theoretical predictions for Revenue, Normalized Revenues, and Effort Choices

Revenues		Standardized Revenues		Effort choices							
		Security									
		Debt	Equity								
Returns to effort	Low	60.21	79.94	Returns to effort	Low	0.78	0.88	Returns to effort	Low	0%	0%
	High	117.17	80.00		High	0.70	0.55		High	100%	14%

4 Comparative Static Results

4.1 Overview

We start by presenting descriptive statistics from the experiment, which are provided in table 2. The table is divided into four columns reflecting the four different treatment “cells” in the experiment. The first two columns correspond to the low returns cases – under the debt and equity bidding. The next two columns correspond to the high return cases under both security types.

Table 2: Descriptive statistics

	Low returns		High returns	
	Debt	Equity	Debt	Equity
Number of sessions		9		5
Number of participants		72		40
Total number of rounds played	216	108	108	60
Total number of market instances	416	216	209	120
Total number of bids	3,132	1,855	1,442	690

There are roughly twice as many rounds under the equity columns as there are under the debt columns⁴. This is because of our ABA design where debt auctions occur both at the beginning and at the end of each experimental session.

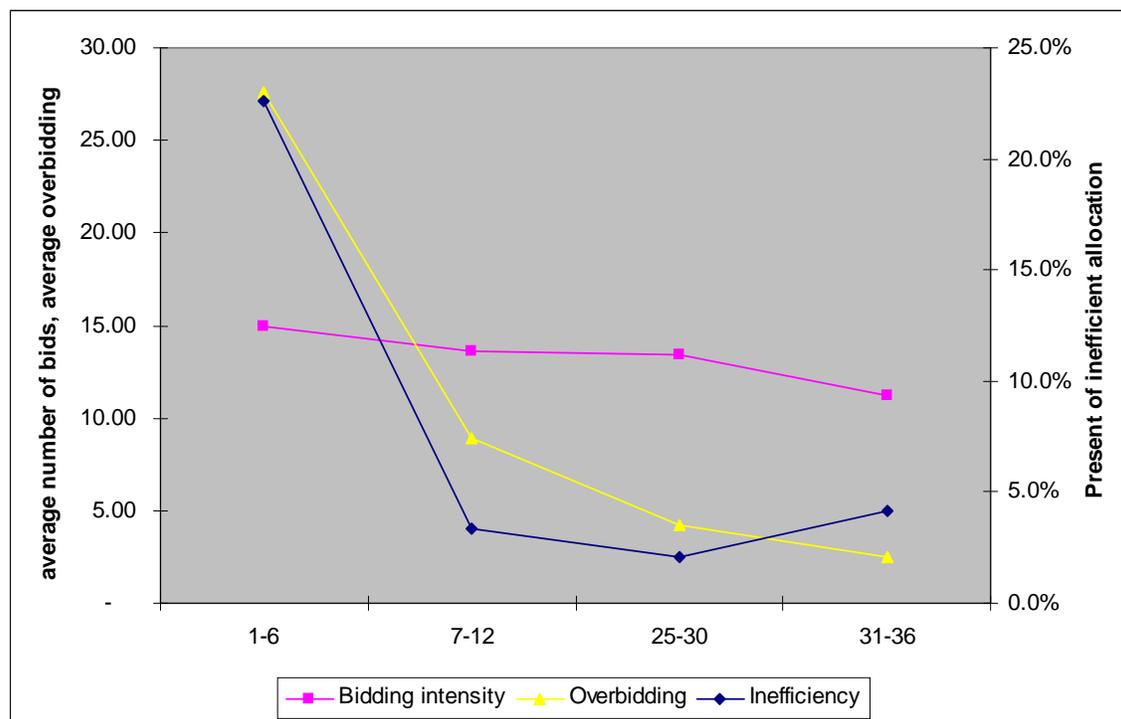
The rationale behind this design is as follows. Pilot studies suggested that subjects’ learning was much easier in going from debt to equity auctions than vice-versa. Since we are interested in equilibrium behavior, we decided to start the sessions with rounds of debt auctions that serve to familiarizing subjects with the bidding process. The results suggest that most of the learning process is completed by round six. To illustrate that, we split all debt rounds into four groups of six rounds each: 1-6, 7-12, 25-30 and 31-36,⁵. We construct a number measures that intend to capture the dynamics of bidding activity: bidding intensity (average number of bids per round), overbidding (average amount by which winning bidder overbid relative to the theoretical predictions), and inefficiency (the fraction of times the funding was provided to the highest venture value). The results are presented separately for the low and high return sessions in figures 1 and 2.

In both the low and the high return variants we find dramatic decrease in inefficiency and overbidding, from the initial rounds (1-6) to the subsequent rounds (7-12 and 25-36), is observed. We do not find similar changes when comparing the first and second half of the third phase rounds (rounds 25-30 vs. 31-36).

⁴It is not exactly twice because of a technical problem that forced early termination of one of the high return sessions.

⁵Recall that in rounds 13 through 24 we use share auctions.

Figure 1: Average bidding intensity, overbidding and inefficiency (high returns to effort)



Further, the intensity of bidding seems to be fairly stable across rounds in the both variants, while there is a downward (upward) trend in the high (low) returns variant. These results suggest that presentation effects are immaterial since the debt auction rounds conducted just *before* the equity auction rounds appear to be indistinguishable from the debt auction rounds conducted immediately *after* the equity auction rounds. To summarize, it appears that learning takes place during the initial rounds but the process stabilizes halfway into the first phase of rounds.

4.2 Comparative Static Predictions

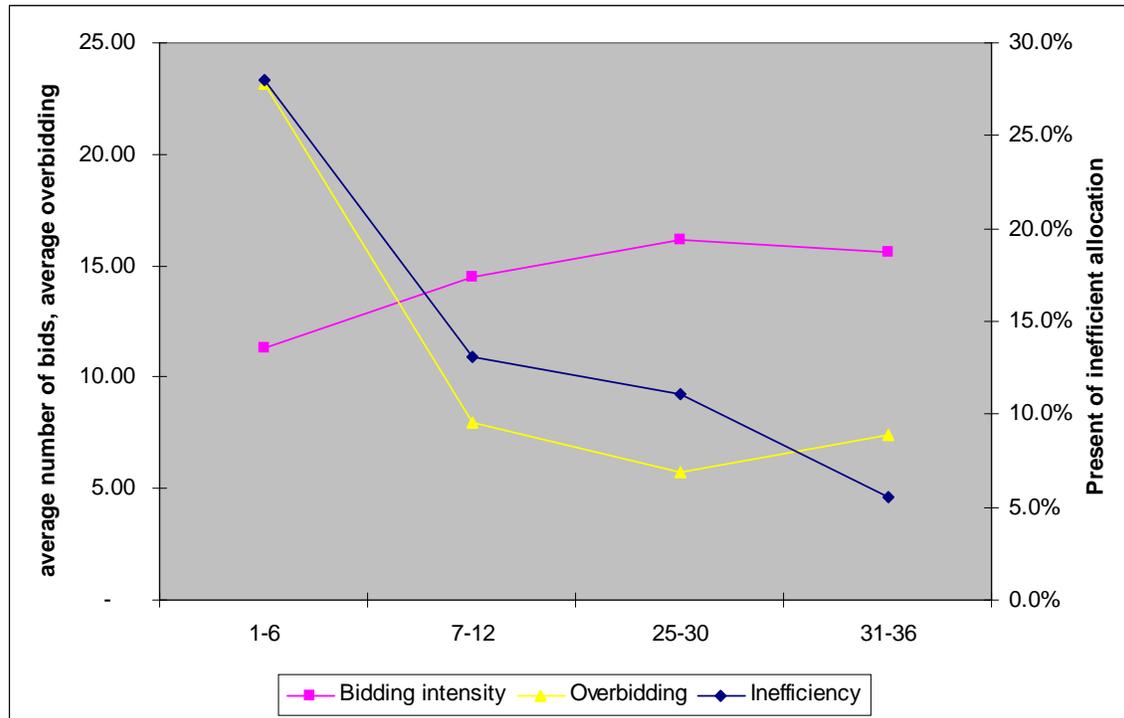
As we saw in table 1, for the parameter values presented in the experiment, the theory model suggests that we test the following four hypotheses about comparative static effects on revenues and effort choices:

Hypothesis 1: When returns to effort are low, revenues and normalized revenues are higher in equity auction than in a debt auction.

Hypothesis 2: When returns to effort are high, revenues and normalized revenues are higher in a debt auction than in an equity auction.

Hypothesis 3: When returns to effort are low, the effort choice is the same under debt as well as equity auctions.

Figure 2: Average bidding intensity, overbidding and inefficiency (low returns to effort)



Hypothesis 4: When returns to effort are high, more effort is undertaken under a debt auction than under a equity auction.

We examine these hypotheses under a variety of specifications and ways of handling the data and find strong support for all four hypotheses regardless of the handling of the data or the particular specification employed.

Session Level Analysis First, we examine the four hypotheses using the *session* as the unit of observation. The justification for this handling of the data is that, since subjects participated in multiple rounds, interacted with one another, and learned over the course of the experiment, arguably the observations should not be treated as independent. Thus, an extremely conservative view of the data is that each session constitutes a unit of observation. In terms of our experiments, this leaves us with only 14 data points (9 obtained in the low returns condition and 5 obtained in the high returns condition).⁶

Since we used a within-subjects design to compare auction forms, we can examine how changing the auction form affects each of the performance measures by differ-

⁶Because of the learning effects highlighted in the previous section, we omit the first twelve rounds of data in constructing observations at the session level. The exception is session 1 where, due to a computer glitch, rounds 25-36 were not completed. For that session, we used rounds 1-12 instead.

Table 3: Session Level Results

Session	Type	Change in Revenues	Change in Normalized Revenues	Change in Effort Choice
1	High Returns	-38.65	-16.5%	-75.0%
2	High Returns	-24.03	-9.6%	-58.3%
3	High Returns	-21.50	-14.5%	-66.7%
4	High Returns	-31.54	-21.0%	-95.8%
5	High Returns	-27.21	-15.5%	-83.3%
<i>Sign test (p-value)</i>		<i>0.031</i>	<i>0.031</i>	<i>0.031</i>
6	Low Returns	-3.31	-2.7%	0.0%
7	Low Returns	12.14	12.8%	0.0%
8	Low Returns	10.66	8.1%	0.0%
9	Low Returns	14.84	18.5%	-8.3%
10	Low Returns	4.20	4.6%	12.5%
11	Low Returns	12.81	24.9%	0.0%
12	Low Returns	1.97	5.1%	4.2%
13	Low Returns	17.95	14.9%	0.0%
14	Low Returns	6.72	7.6%	-4.2%
<i>Sign test (p-value)</i>		<i>0.020</i>	<i>0.020</i>	<i>0.688</i>

In this table we subtract the average levels (within session) of revenues, normalized revenues and fraction of effort choice in the debt rounds (25-36) from the average levels in the shares rounds (13-24)

encing the average revenues, normalized revenues and efforts for shares versus debt auctions session by session. The results of this are reported in Table 3 above. In that table, we test the null hypothesis that each of the three performance measures are equal the same across auction forms against the one-sided alternative implied by hypotheses 1-4 using a Mann-Whitney sign test.

According to hypothesis 1, the shares auction should produce higher revenues (or normalized revenues) compared to debt auctions in the low returns sessions. As Table 3 shows, in 8 of the 9 sessions, the average revenues were in the predicted direction. The differences are statistically significant at the 2 percent level.

Hypothesis 2 predicted that the revenue ranking will reverse in the high returns sessions. As the table shows, average revenues were higher under debt auctions compared to shares auctions in all 5 sessions. Once again, the difference in revenues is statistically significant—this time at the 3% level.

Hypothesis 3 suggests that there should be no difference in effort choices across the two auction forms for the low returns sessions. Notice that, in 2 of the sessions, higher average effort is undertaken in a shares auction than in a debt auction. The reverse is true for 2 sessions as well, while for the remaining 5 sessions, average effort is exactly the same under the two auction forms. Taken together, this suggests no difference in average effort undertaken across auction forms. Formally, we fail to reject the null hypothesis of a zero treatment effect at the 68 percent level.

Hypothesis 4, however, predicts that in high returns treatments, shares auctions will undermine effort choices relative to debt auctions. The data in Table 3 strongly supports this prediction. In all 5 sessions, average effort is lower under a shares auction than under a debt auction and the differences are considerable. Formally, we find the differences in effort are statistically significant at the 3% level.

Market Level Analysis In the preceding analysis, we excluded the first twelve rounds owing to learning effects and treated the session as the unit of observation. Yet, this leaves unanswered the question of how important these learning effects (or their exclusion) are to the conclusions with respect to hypotheses 1-4. Moreover, the preceding analysis examined the results effectively pairwise across auction forms for a given high or low returns treatment. It is of some interest to examine the strength of the interaction terms against the level effects of the high or low returns treatment itself. For these reasons, we now examine the four hypotheses using the interaction of a group of subjects in a particular “market” as the unit of observation. Since these markets took place over time during the experiment, this lets us isolate some learning effects on market outcomes. Moreover, by pooling across auction type and returns treatment, we are able to separately identify level from interaction effects present in the data.

While the differences shown are significant using parametric and non-parametric tests that use mean levels across samples obtained from the two security types, we examine the robustness of the four hypotheses using regression analysis. In what will

follow we take a slightly less conservative view of the data and treat each “round” of the experiment as an observation, while explicitly incorporating the fact that errors are possibly subject to autocorrelation and/or heteroskedasticity within each session. Indeed, it is precisely this sort of worry about session effects that suggested pooling by session in the first place. However, clustering by session might, theoretically, allay this concern somewhat. Further, using regression analysis allows us to explicitly control for various types of learning effects (and which motivated omitting the first six rounds from the session level analysis above).

Specifically, we run the following regression:

$$measure_{st} = \beta (auction\ form_t \times agency\ effects_s) + \gamma X_{st} + \varepsilon_{st} \quad (5)$$

where $measure_{st}$ denotes one of the three measures of auction performance given above for round t of session s . The variable $auction\ form_t$ is equal to one if a equity auction occurred in period t and zero if a debt auction occurred in that period. The variable $agency\ effect_s$ is equal to zero if returns to effort are low and it is equal to one if returns to effort are high, in a given session s . The matrix X_{st} is a matrix of controls for learning effects over the course of a session. Specifically, we add a linear and squared time trends. Additionally, the matrix X_{st} includes a control, $learning_{st}$, which is equal to the number of previous rounds conducted within the same security type, in period t , session s . For instance, if period t were the k th period in which a equity auction was run, then the value of the learning control would be equal to k (rather than t). This accounts for the fact that learning may occur at different rates for different auction forms. Thus,

$$\gamma X_{st} \equiv \gamma_1 t + \gamma_2 t^2 + \gamma_3 learning_{st} \quad (6)$$

Of course, we continue to be concerned that past market interactions could affect current market interactions as subjects in a given session repeatedly interact. To allow for possible heteroskedasticity and autocorrelation of market outcomes in a given session, we Regressing the various measures of revenues and effort on the X variables and clustering by session to account for possible autocorrelation and heteroskedasticity resulting from subject interactions within each session, we obtain the following coefficient estimates summarized in Table 4.⁷

To see how the regression coefficients bear on the hypotheses listed above, it is helpful to write out the interaction term explicitly. That is, all else equal,

$$measure_{st} = \beta_0 + \beta_1 auction\ form_t + \beta_2 agency\ effects_s + \beta_3 auction\ form_t \times agency\ effects_s + \varepsilon_{st} \quad (7)$$

⁷We also ran an alternative specification where we included session level fixed effects and used robust standard errors. The results of this specification yield quantitatively similar estimates and precisions. The results are available upon request to the authors.

Table 4: Round Level Regression Results: Revenues, Standardized Revenues and Effort Choice

	β_0	β_1	β_2	β_3	γ_1	γ_2	γ_3	R^2
Revenues	61.770 (10.79)	10.794 (2.18)	53.344 (21.97)	-33.881 (-7.78)	-.2994 (-0.37)	.01116 (0.56)	.00346 (0.01)	0.3195
Normalized revenues	.8120 (20.51)	.1267 (3.13)	-.0786 (-4.16)	-.2456 (-6.31)	-.0068 (-1.43)	.00019 (1.75)	.00032 (0.08)	0.1768
Effort choice	.168144 (2.67)	.0126 (0.33)	.8831 (35.35)	-.6895 (-9.82)	-.0101 (-1.42)	.00019 (1.19)	-.00593 (-1.32)	0.6838
$N = 961$; (t values are provided in parenthesis)								

There are four cases we need to consider, {debt, low returns}, {equity, low returns}, {debt, high returns}, and {equity, high returns}. Since *auction form* takes on the value of zero in the case of debt auction and *agency effects* takes on the value of zero when returns to effort are low, we obtain that in the:

- {debt, low returns}, $\overline{measure} = \beta_0$
- {equity, low returns}, $\overline{measure} = \beta_0 + \beta_1$
- {debt, high returns}, $\overline{measure} = \beta_0 + \beta_2$
- {equity, high returns}, $\overline{measure} = \beta_0 + \beta_1 + \beta_2 + \beta_3$

Therefore, the differences in average levels of the dependent measure when comparing equity and debt auctions in the low returns case, $\overline{measure}_{\{\text{equity, low returns}\}} - \overline{measure}_{\{\text{debt, low returns}\}}$, is equal to β_1 . Likewise, the difference in between the equity and debt auctions in the high returns case, $\overline{measure}_{\{\text{equity, high returns}\}} - \overline{measure}_{\{\text{debt, high returns}\}}$, is equal to $\beta_1 + \beta_3$.

According to Hypothesis 1, when returns to effort are low, equity auction should yield higher revenues and normalized revenues than debt auctions. Thus, β_1 is predicted to be positive when the dependent variables are revenues or normalized revenues. Indeed, we find that these coefficient are estimated to be positive (61.77 for revenues and 0.812 for normalized revenues) and statistically different from zero (at the 1% level). Hypothesis 2 suggests that in the high returns case, debt auctions should yield higher revenues and normalized revenues than do equity auction, implying that $\beta_1 + \beta_3 < 0$. We find that this sum is negative for both revenues (-23.087) and normalized revenues (-0.1189) with statistical significance of 1%.

Hypothesis 3 predicts that effort decisions should be the same across the auction forms when returns to effort are low. That is, estimated β_1 in the effort choice regression should not be significantly different from zero. Indeed, the results suggest that the value of this coefficient (.0126) is indistinguishable from zero at conventional significance level. According to hypothesis 4, effort choices should be significantly different across the security forms when returns to effort are high. The results strongly support the hypothesis. We find that estimated $\beta_1 + \beta_3$ is negative (-9.49) and significant at confidence level of 1%.

The coefficients that capture across rounds and within security form learning do not appear to be statistically different from zero. Nonetheless, the sign of the linear round trend coefficient in the revenues and normalized revenues regressions appear to be positive. This is consistent with the intuition that learning decreases overbidding, resulting in lower revenues to the seller. The effect of within-auction-form seem to be negligible in the presence of time trend variables. The results suggest that while learning probably takes place, the process' effects are not significant when considering the complete set of rounds.

Summary

The session and market level data strongly supports the comparative static implications of the theory model. When returns to effort are low, shares auctions significantly outperform debt auctions; however, the reverse is true when the returns to effort are high. The key distinction in the revenue ranking is that competition in shares auctions undermines effort incentives and, as we saw above, leads to significant reductions in effort levels of the winning bidder.

5 Level Predictions of the Theory

While it is reassuring that the comparative static predictions of the model are borne out, the model also offers more detailed predictions about the levels of winning bids, the distribution of scarce venture capital across firms, and about effort choices as a function of the current bid level. We investigate these questions in this section. To examine these questions, we derive a structural model of revenue and effort choices using the tractable expressions for these obtained in the theory.

5.1 Structural Estimation

We start by deriving a structural model for revenues under both debt and equity auctions and in the two effort return conditions. Recall that the auctions were run under four different treatments: debt versus equity interacted with high versus low returns to effort.

In the case of a debt auction under the low returns treatment, it is never optimal for a bidder to undertake effort. Therefore, revenues are simply given by

$$ER_{debt}^{low} = v_2 \quad (8)$$

In the case of a debt auction under the high returns treatment, the theory predicts that the price will be set by the value of the object to the bidder with the second highest valuation. This, in turn, depends on whether that bidder's valuation is sufficiently high that there is a positive return to effort. For the parameter specification in the experiment, this then implies. In case two, it is always optimal to exert effort in the debt auction when returns to effort are high. Therefore revenues are given by

$$ER_{debt}^{high} = \begin{cases} v_2 & \text{if } v_2 < 9 \\ v_2(1 + \delta) - 20 & \text{if } v_2 \geq 9 \end{cases} \quad (9)$$

In the case of an equity auction under the low returns treatment, it is never optimal for a bidder to undertake effort. Therefore, revenues are simply given by

$$ER_{equity}^{low} = \frac{v_1 + m}{v_2 + m} \times v_2 \quad (10)$$

To next this expression with the other treatments, we can linearize equation 10 by using first-order Taylor approximation around $v_1 = v_2 = m$ to obtain

$$\begin{aligned} ER_{equity}^{low} &= \frac{v_1 + m}{v_2 + m} \times v_2 \approx \frac{v_1 + m}{v_2 + m} \times v_2 + \frac{m}{m + m}(v_1 - m) + m \frac{m + m}{(m + m)^2}(v_2 - m) \\ &= m + \frac{1}{2}(v_1 - m) + \frac{1}{2}(v_2 - m) = 0.5v_1 + 0.5v_2 \end{aligned} \quad (11)$$

The case of an equity auction under the high returns treatment is more complex. As noted above, it is sometimes optimal to undertake effort and sometimes not depending on the realizations of v_1 and v_2 . At the same time, given the parameterization of the experiment only a subset of cases is attainable. Recall that we set $p = 1, m = 10, E = 20$ (cost of exerting effort), $v_i \in [0, 100]$ and $\delta = 1.3$ (in the high return session). Therefore, $pv_i\delta < m$ for all v_i meaning that bidders' dominant strategy would always be to bid in anticipation of low effort choice. The relevant expected revenues cases are represented by two sub-cases

$$ER_{equity}^{high} = \begin{cases} \left(1 - \frac{m}{v_2 + m}\right)(v_1 + m) & \text{if } 1 - \frac{m}{(v_2 + m)} > \frac{v_1\delta - E}{v_1p\delta} \\ \left(1 - \frac{m}{v_2 + m}\right)(v_1(1 + \delta) + m) & \text{if } 1 - \frac{m}{(v_2 + m)} \leq \frac{v_1\delta - E}{v_1\delta} \end{cases}$$

After simplifying we get that

$$ER_{equity}^{high} = \begin{cases} \frac{v_1 + m}{v_2 + m} \times v_2 & \text{if } \frac{E(v_2 + m)}{m\delta} > v_1 \\ \frac{v_1(1 + \delta) + m}{v_2 + m} \times v_2 & \text{if } \textit{otherwise} \end{cases} \quad (12)$$

Using once again first order Taylor approximation we get that:

$$ER_{equity}^{high} \approx \begin{cases} \frac{1}{2}v_1 + \frac{1}{2}v_2 & \text{if } \frac{20}{13}v_2 + \frac{20}{1.3} > v_1 \\ 1.15v_1 + 0.825v_2 - 3.25 & \text{if } otherwise \end{cases} \quad (13)$$

Since the top case above is identical to the low returns to effort in equity auctions we will limit our analysis to the bottom case, where the winner exerts effort. This formulation allows us to clearly identify the driving forces behind revenues under all relevant conditions. In debt auctions, revenues are a function of the second highest value *only*, while in equity auction revenues dependent on the second *and* the first highest value. We also see that revenues become more sensitive to the second highest value when moving from low to high returns settings in both debt and equity auctions. At the same time, the sensitivity of equity revenues to the highest private value goes down when in high returns to effort condition. Thus, the linkage principal weakens as result of the moral hazard problem.

All cases can be represented in a linear form

$$ER = \alpha + \beta v_1 + \gamma v_2$$

While each implies different values for $\alpha, \beta,$ and γ . Case one implies $\{\alpha = 0, \beta = 0, \gamma = 1\}$, case two implies $\{\alpha = -20, \beta = 0, \gamma = 2.3\}$, case three implies $\{\alpha = 0, \beta = \frac{1}{2}, \gamma = \frac{1}{2}\}$, and case four implies $\{\alpha = -3.25, \beta = 1.15, \gamma = 0.825\}$. To represent all three in a single regression equation, we include dummies for auction form, D^{equity} , which takes on the value of 1 in the case of equity auction, and D^{high} , which takes on the value of 1 in the case of high returns to effort.

$$\begin{aligned} Revenues = & \alpha_1 + \alpha_2 D^{high} + \alpha_3 D^{equity} + \alpha_4 D^{equity} D^{high} \\ & + \beta_1 v_1 + \beta_2 v_1 D^{high} + \beta_3 v_1 D^{equity} + \beta_4 v_1 D^{high} D^{equity} \\ & + \gamma_1 v_2 + \gamma_2 v_2 D^{high} + \gamma_3 v_2 D^{equity} + \gamma_4 v_2 D^{high} D^{equity} \end{aligned} \quad (14)$$

In this framework, we get that

$$ER_{debt}^{low} = \alpha_1 + \beta_1 v_1 + \gamma_1 v_2 \quad (15)$$

$$ER_{debt}^{high} = \alpha_1 + \alpha_2 + (\beta_1 + \beta_2) v_1 + (\gamma_1 + \gamma_2) v_2 \quad (16)$$

$$ER_{equity}^{low} = \alpha_1 + \alpha_3 + (\beta_1 + \beta_3) v_1 + (\gamma_1 + \gamma_3) v_2 \quad (17)$$

$$ER_{equity}^{high} = \sum_{i=1}^4 (\alpha_i + \beta_i v_1 + \gamma_i v_2) \quad (18)$$

Combing these cases with the restrictions described above we generate predictions for sets of $\{\vec{\alpha}, \vec{\beta}, \vec{\gamma}\}$. We present the estimation results of equation 14 using robust

cluster regression in table 5. Columns two and three list the various sets of parameters for which the model makes predictions and their corresponding values. Column four provides the estimated values of these sets and column five contains the probability that each of the sets is equal to the theoretical predictions. In this framework, lack of statistical significance indicates that the results *confirm* the theoretical predictions.⁸

Table 5: Structural Estimation Results: Revenues

Case	Parameter	Hypothesis	Estimate	<i>s.e.</i>
{Debt, Low}	α_1	0	4.1915*	1.8778
	β_1	0	.1186**	.04236
	γ_1	1	.8042**	.04122
{Debt, High}	$\alpha_1 + \alpha_2$	-20	-15.7865	5.8631
	$\beta_1 + \beta_2$	0	0.1597*	.07742
	$\gamma_1 + \gamma_2$	2.3	2.0138**	.07402
{Equity, Low}	$\alpha_1 + \alpha_3$	0	-4.5950*	1.8778
	$\beta_1 + \beta_3$	0.5	0.7942**	.02850
	$\gamma_1 + \gamma_3$	0.5	0.2880**	.03001
{Equity, High}	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$	-3.25	-27.509	34.855
	$\beta_1 + \beta_2 + \beta_3 + \beta_4$	1.15	1.6432	.60266
	$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4$	0.825	.15160	.91066
$N = 720, R^2 = 0.8775$				

Statistical significance is denoted by *for 5% level and by **for 1% level.

We see that the support for the *level* predictions of the theory are mixed. First, looking at the results under low returns to effort, we find that in line with the theoretical predictions, debt auctions load more on the second highest valuation and less on the highest valuation; the opposite happens in equity auctions. Second, debt auction results suggest that the sensitivity of revenues to the *second* highest valuation increases when going from low returns environment (.804) to high returns environment (2.01), once again supporting the theory's predictions. At the same time we see that in debt auctions, revenues are not sensitive enough to the second highest valuation and are too sensitive to the highest valuation (though with marginal statistical significance). Last, the theory predicts that in the equity auction, sensitivity to highest private value relative to the second highest value should increase when going from

⁸For the high returns to effort in under equity auctions we include observations in which the highest private value is greater than 15.38 plus 1.53 times the second highest private value.

Table 6: Probit Estimates of Allocation Efficiency

Parameter	Estimate	z-value
Shares Auction Dummy	-.1563	-2.81**
High Returns dummy	.0840	2.54*
Share Auction×High Returns dummy	-.0758	-1.35
Round number	.0203	2.87**
Round number squared	-.0003	-1.96
Within-auction form round number	-.0036	-0.72
Baseline probability of efficient allocation: 0.8549		
$N = 961, Pseudo R^2 = 0.0741$		

Statistical significance is denoted by *for 5% level and by **for 1% level.

low to high returns settings. The pattern we observe concur with this prediction; the coefficient on the highest private (second highest) value is 0.79 (0.29) in the low returns to effort and it is 1.64 (0.15) in the high returns to effort.

5.2 Efficiency

Recall that we have a number of theorems that indicate that the adverse selection problem is perfectly solved by either debt or shares auctions, irrespective of the returns to effort. In this section, we use a probit model to estimate the probability that the higher quality project is funded across auction forms and effort returns conditions. To estimate this model, we once again use the specification in the right-hand side of equation (5). We use a binary left-hand side variable for $measure_{it}$, which we code as "1" when the winner of the auction is the bidder with the highest v_i in the market and "0" otherwise. The coefficient estimates of the marginal effects of each of the factors on the probability of an efficient allocation for this specification are reported in Table 6 below.

As Table 6 shows, the baseline probability of an efficient allocation is quite high (85%) across all treatments and rounds. The coefficients for learning effects indicate that, as subjects gain experience over the course of a session, efficiency improves at about a 2 percent rate per round of the experiment. Turning to the treatment effects, recall that, according to Propositions 2 and 4, the theory predicts no differences across

treatments in the efficiency of allocations. However, one might expect that, given the cognitive complexity of bidding in a shares auction, especially in the high returns treatment, compared to bidding in a debt auction, that the former will perform less well by the efficiency metric. As Table 6 shows, the marginal effect of using a shares auction is to decrease the probability of an efficient allocation by almost 16 percent, which reduces to 15 percent in the case of high returns. Thus, the complexity of bidding in shares does have a significant adverse effect on efficiency, but the added complexity of the effort choice in the high returns treatment does not appear to exacerbate this effect. Indeed, in debt auctions under the high returns treatment, efficiency increases by about 8 percent.

Why might it be the case that efficiency improves under the high returns treatments? One possible explanation is that bidders make zero mean mistakes and hence, when the values of the highest and second highest bidders are relatively close to one another, these mistakes lead to inefficiency. Under the high returns treatment, especially under a debt auction, the average gap in the valuation of the highest and second highest bidder is magnified by the multiplier on effort. Therefore, one would expect that, adding a control for the magnitude of the gap between v_1 and v_2 interacted with a dummy for whether the treatment is high returns or not would drastically reduce the magnitude of the effect of the high returns treatment alone. We modified the specification in equation (5) to allow for this possibility and found that it had little effect on the magnitude or significance of the effect of the high returns treatment on efficiency. Thus, random mistakes alone cannot account for differences in efficiency between the high and low returns treatments in debt auctions.

While the economic magnitudes of the marginal effects appear large, it is important to note that, given a pseudo- R^2 of only 0.07, much of the variation in efficiency across auctions is driven by non-systematic factors. Moreover, taken together with estimates of revenue differences across treatments, one sees that the adverse effect on allocations in shares auctions is more than offset by revenue gains from “linkage” of the winning bidder’s payment to the underlying value of the asset. In other words, equity auctions continue to outperform debt auctions in low returns treatments despite being less efficient in their allocations.

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6 Appendix

6.1 Structural model

In developing the structural model, we claim that

$$ER_{equity}^{high}(effort) \approx 1.15v_1 + 0.825v_2 - 3.25$$

To see that, use first order approximation around $v_1 = v_2 = m$.

$$\begin{aligned} ER_{equity}^{high}(effort) &= \frac{v_1(1+\delta) + m}{v_2 + m} v_2 \approx \\ \frac{v_1(1+\delta) + m}{v_2 + m} v_2 + \frac{\partial}{\partial v_1} \left(\frac{v_1(1+\delta) + m}{v_2 + m} v_2 \right) (v_1 - m) + \frac{\partial}{\partial v_2} \left(\frac{v_1(1+\delta) + m}{v_2 + m} v_2 \right) (v_2 - m) &= \\ \frac{v_1(1+\delta) + m}{v_2 + m} v_2 + v_2 \frac{\delta + 1}{m + v_2} (v_1 - m) + \frac{m}{(m + v_2)^2} (m + v_1 + \delta v_1) (v_2 - m) &= \\ \frac{m(1+\delta) + m}{m + m} m + m \frac{\delta + 1}{m + m} (v_1 - m) + \frac{m}{(m + m)^2} (m + m + \delta m) (v_2 - m) &= \\ 1.15v_1 + 0.825v_2 - 3.25 \end{aligned}$$