

The *Idealized Electoral College* Voting Mechanism and Shareholder Power

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Abstract

Increasing concern over corporate governance has led to calls for more shareholder influence over corporate decisions, but allowing shareholders to vote on more issues, such as executive compensation, may not affect the quality of governance. We should expect instead that, under current rules, shareholder voting will implement the preferences of the majority of large shareholders and management. This is because majority rule offers little incentive for small shareholders to vote. I offer a potential remedy in the form of a new voting rule, the *Idealized Electoral College* (IEC), modeled on the American electoral college, that significantly increases the expected impact that a given shareholder has on election. The benefit of the mechanism is that it induces greater turnout, but the cost is that it sometimes assigns a winner that is not preferred by a majority of voters. Therefore, for issues on which management and small shareholders are likely to agree, majority rule is a superior mechanism for shareholder voting. For issues on which they are likely to disagree, the IEC is superior.

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In order to improve corporate governance, academics and policy makers often suggest increasing the number of things upon which shareholders can vote. For example, a “say on pay” rule gives shareholders a binding or non-binding vote on CEO compensation. Governance experts hope these rules can rein in excessive pay packages but, with some notable exceptions,¹ early evidence seems to run contrary to these aspirations.

In 2009:

So far during this spring’s annual meeting season, there have been few examples of investors fighting back. Shareholders have yet to vote down a single executive pay plan at US companies and only a handful of corporate directors have lost investor backing. Support for corporate management is still the status quo. “It turns out that (US) shareholders may be more accepting of how things work than the perception really is,” said Charles Elson, director of the Weinberg Center for Corporate Governance at the University of Delaware.²

In 2012:

Except in fairly extraordinary circumstances, [shareholders] don’t much care about how much people get paid...We saw it last year, the first year say-on-pay votes were required by the new Dodd-Frank financial reform law, and we’re seeing it again this year. Last year, only 36 of 2,225 companies said shareholders voted down their compensation plans.³

Are shareholders really accepting of these pay packages? I show below that, under majority rule, we should expect that small shareholders will abstain from voting: majority rule will implement the will of the majority of *large* shareholders, not the majority of *all* shareholders. Many large

¹For example, in April, 2012, shareholders rejected Citigroup CEO Vikram Pandit’s \$15 million payday.

²Rachel Beck (May 23, 2009). “All Business: Investors still support CEO pay”, Associated Press.

³http://www.huffingtonpost.com/2012/05/07/say-on-banker-pay_n_1496133.html

shareholders, especially financial firms, receive private benefits from management. For example, insurance firms may manage employee life or health insurance plans, mutual funds may manage employee retirement accounts, and investment banks may underwrite bond and equity offerings. Such shareholders are therefore likely to vote with management (Gordon and Pound, 1993; Gillan and Starks, 2000; Davis and Kim, 2007; Brickley, Lease and Smith, 1988, 1994). Moreover, management itself is often a large shareholder. Because small shareholders are inclined to abstain, majority rule will implement the will of management, not shareholders. This is especially true when, as is the case in the United States, brokers are allowed to vote the shares of their clients if the clients do not submit votes.⁴

In attempting to improve corporate governance, therefore, we must either abandon the shareholder voting route, or we must reconsider the use of the majority rule mechanism. I offer in this paper an alternative mechanism, the *Idealized Electoral College* (IEC), that may induce small shareholders to vote. The benefit of the IEC is that it significantly increases the likelihood that a given voter will affect the outcome of the election, relative to majority rule, thus increasing turnout. The cost of the IEC is that the side receiving fewer votes sometimes wins the election. This means that even if all shares are voted, the majority's will may not be implemented. Therefore, the IEC is superior to majority rule if and only if the preferences of large shareholders/management differ from those of shareholders overall.

The IEC mechanism is a randomized and stylized version of the American Electoral College, in which votes for or against a proposal are organized into groups, and majority rule determines each group's choice. Groups are formed into super-groups and majority rule is again applied. This process is repeated until all votes are aggregated to a single decision. I show that individual votes are almost always far more likely to affect the outcome of an election under the IEC mechanism

⁴For routine votes, at least, brokers may vote shares for clients (Bethel and Gillan, 2002). This fact reconciles the apparently contrary facts that even though small shareholders do not vote, 80% of shares are typically voted on proposals at large US corporations (Maug and Rydquist, 2009).

than under majority rule.⁵ I then introduce a model of shareholder preferences and voting, and derive properties of two important equilibria, one in which all shareholders vote (a “universal voting” equilibrium), and one in which only large shareholders vote (a “universal abstention” equilibrium). If parameters are such that a universal voting equilibrium exists for majority rule, then one exists for the IEC as well. The converse, however, is not true. There are cases in which the IEC induces all voters to vote, but majority rule does not. Moreover, of the set of parameter values such that the IEC allows a universal voting equilibrium, the fraction for which majority rule also induces universal voting goes to zero as the number of votes goes to infinity.

Universal abstention equilibria are much more likely to exist under majority rule. Indeed, I would argue that these equilibria are the norm in practice. Burch, Morgan and Wolf (2004) find that, in votes at acquiring firms concerning large stock-for-stock mergers—those mergers in which value is most likely to be destroyed—the fraction of shares voted in favor is 95%-98%. They do not find a single failed vote in their sample, spanning 1990-2000. Nearly all shareholders who vote side with management, which is peculiar because the sample specifically contains only proposals that are likely to harm shareholders. It seems that shareholders lacking a private benefit to siding with management are simply abstaining. As I show, under majority rule this is to be expected; under the IEC it is less likely.

I also analyze what conditions of an electorate make the IEC or majority rule superior. Majority rule is superior when large and small shareholders have similar preferences, but the IEC is superior when they have significantly different preferences. There is therefore reason to believe that the IEC could be selectively implemented for certain types of votes, such as those regarding executive pay.

There are alternative mechanisms that have been developed that could assign special power to small or minority shareholders. For example, dual class voting, introduced in Maug and Yilmaz

⁵I often call a voter more “powerful” in this paper if her vote, on average, has a higher expected impact on the outcome of an election. This definition does not refer to the power of voters, in general, vis-a-vis management.

(2002), could be applied to the problem in several ways, two of which I discuss. Either classes could be defined by the ownership stake of the voter, with small shareholders (e.g. $< 3\%$) constituting one class and large shareholders (e.g. $> 3\%$) and management constituting another, or classes could be defined by connection to management, where shareholders with a business relationship with the firm constitute one class and shareholders with no business relationship constitute another. In either case, majority rule would determine each class's choice, and each class would have veto power over a proposal. One could reasonably ask why the IEC is useful when these alternatives exist.

The IEC mechanism is inferior to these alternatives under the assumption that the class assignment is not manipulable, but manipulation may be feasible in practice. As one example, large shareholders can mimic small shareholders by splitting stakes among shell funds or corporations, each of which has a small ownership stake.⁶ For another example, firms with a business relationship could legally split the investment side of the business from the pension management side. Regardless of the details, *any rule where voting rights depend upon characteristics of the voter may be subject to manipulation*. The IEC is not subject to manipulation because each share has the same rights, regardless of its ownership. In a finance setting, where voting identities can be masked, faked or otherwise manipulated, an anonymous rule is important.

There are three lines of literature relevant for this research, the first concerning voting rules in corporate governance, the second concerning the effect of voting costs on the efficacy of a voting mechanism, and the third concerning the electoral college.

First, there has been periodic discussion in the corporate governance literature concerning optimal voting rules for corporations. Harris and Raviv (1988) and Grossman and Hart (1988) provide arguments in favor of the standard one share-one vote/majority rule policy in place at most companies. Maug and Yilmaz (2002), on the other hand, show that it can be optimal to

⁶Similarly, small shareholders could use repo agreements to combine shares into larger holdings, in order to vote as part of the "large shareholder class".

have separate classes of voter, with a majority rule mechanism operating within each class. While they focus on the ability of a mechanism to aggregate private information, which may not be fully revealed if the interests of different classes of voter are sufficiently different, I focus on the ability of the mechanism to aggregate preferences, which voters may not know or express if the likelihood of affecting the outcome of an election is too low relative to the cost of voting.

Second, in much of the voting literature, the cost of voting is zero, so voters do not abstain. Because I focus on the inability of majority rule to induce small shareholders to vote, the problem of abstention is critical to this paper. The costly voting literature began with Downs (1957) and Tullock (1968), who noted that voting appears to be irrational: because voters have a vanishingly low likelihood of affecting the outcome, even a minor cost associated with voting should induce much lower turnouts than we see in practice. Following this observation, political scientists, beginning with Riker and Ordeshook (1968), developed explanations in the so-called “calculus of voting” literature.⁷⁸

A more recent line of research integrates costly and strategic voting. Borgers (2004) shows that voluntary participation can lead to excessive turnout because voters do not take into account the negative externality of their vote on others—namely, that their vote reduces the expected impact of others’ votes. Krasa and Polborn (2009) show that turnout may also be less than optimal, so the impact of voting costs on the number of voters is model-specific.

By adding heterogeneity in voting costs, Taylor and Yildirim (2010) show that as elections become large, the voting population becomes exclusively composed of the lowest cost voters. Interestingly, this point was also made by Hegel in 1821:⁹

As for popular suffrage, it may be further remarked that especially in large states it leads inevitably to electoral indifference, since the casting of a single vote is of no

⁷Riker and Ordeshook (1968) formalize voter preferences and assume a positive reward from the act of voting, R , and a positive cost to voting, C . If $R \geq C$, then even if the likelihood of affecting the outcome is zero, then voters will vote.

⁸Amartya Sen also discusses the calculus of voting in his 1970 work, *Collective Choice and Social Welfare*.

⁹Hegel’s discussion of costly voting was pointed out in Buchanan (1974).

significance where there is a multitude of electors. Even if a voting qualification is highly valued and esteemed by those who are entitled to it, they still do not enter the polling booth. Thus the result of an institution of this kind is more likely to be the opposite of what was intended; election actually falls into the power of a few, of a caucus, and so of the particular and contingent interest which is precisely what was to have been neutralized.

I consider a simple, stylized model of shareholder voting that produces results largely consistent with this view, but then ask the question of whether changing from majority rule to an alternative mechanism can help solve the problem.

Third, the IEC mechanism is a stylized version of the American electoral college, in which voters are arranged into tiers and votes are aggregated at each tier.¹⁰ While this particular mechanism is new, there has been some work done in the political economy literature to highlight benefits of the existing American Electoral College. Natapoff (1996), for example, shows that a combination of careful districting and assigning weights to districts based on vote count (rather than population) can increase voter power beyond standard majority voting. This literature, however, has not analyzed randomized or more tiered versions of the electoral college like the one outlined in this paper.

The remainder of the paper proceeds as follows: Section 1 describes the IEC mechanism and provides some simple comparisons to majority rule. Section 2 introduces the model. Section 3 shows conditions under which universal abstention and universal voting equilibria are present under each mechanism. Section 4 analyzes what conditions of an electorate make the IEC or majority rule superior. Section 5 provides some numerical analysis suggesting that voter power

¹⁰In the electoral college system, voters vote for a presidential candidate, and those votes are aggregated at the state level. If a majority of voters in, say, Ohio vote for candidate A, then all of the votes in Ohio are voted in favor of A. The differences between the actual electoral college and the IEC are that (a) voters are assigned to groups based on geographic location in the former case, and randomly in the latter, (b) groups are of different sizes in the former case and are equally sized in the latter, and (c) there are two tiers in the former case and more than two in the latter.

increases monotonically as we incrementally adjust the mechanism from majority rule to the IEC. Section 6 discusses applications of the model and results to a shareholder voting setting, and Section 7 concludes. Figures and, where omitted, proofs are found in the Appendix, as is a general description of the mechanism.

1 The *Idealized Electoral College* voting mechanism

The *idealized electoral college* voting mechanism arranges votes into tiers and uses results from each tier as votes for the next tier. To be precise, consider an election where a vote is either for or against a proposal (there are only two choices). The IEC mechanism randomly collects all votes into groups/sets of size η , η being an odd integer greater than one. Assume that the number of votes cast is $N = \eta^z$ where z is a positive integer greater than or equal to one. Within each set, the votes for each choice are added, and the choice with a greater number of votes is declared the choice for that set.

These sets are then randomly arranged into super-sets comprised of η sets each, and the “votes” of each original set are aggregated to get a “vote” for the super-set. Super-sets are arranged into super-super-sets etc. This is repeated until a choice is made. Figure 1 offers an example of a two-tier 3-pyramid: $\eta = 3$, $z = 2$, so the number of voters equals $\eta^z = 3^2 = 9$. In the example, there are six votes for A and three for B. These votes are randomly distributed into three sets of three votes each. In the example shown in Figure 1 the first set features two votes for A and one for B. The first set therefore “votes” for A. Similarly, the second “votes” for A and the third “votes” for B. These three sets are then arranged into a super-set. In this super-set two votes are for A and one for B so the super-set “votes” for A. Because this super-set contains all votes, A is the winner of this election.

η can be any odd number between 3 and N , inclusive. At one extreme, η equals the number of votes N , and $z = 1$: the number of tiers is minimized and the number of votes per tier is

maximized. This extreme is also known as majority rule. At the other extreme, where $\eta = 3$ and $z = \log_3 N$, the number of tiers is maximized and the number of votes per tier is minimized. In this paper I focus on a comparison of these two extreme cases, and in Section 5 show that results appear to be monotonic in between these two extremes. Unless otherwise noted, the term “IEC” is henceforth used to refer to the case where $\eta = 3$.

1.1 A simple example of voter power under the IEC

Let the number of votes be $N = 3^z$ where z is an integer greater than or equal to two.¹¹ Define tier i to be the i^{th} tier from the bottom. Tier 0 has N members, tier 1 has $N/3$ members, etc., so tier z has only one member. Consider a choice between two options, A and B. Let each voter be independently in favor of A with probability $t_0 \in (0, 1)$ and in favor of B with probability $1 - t_0$.

A group chooses A if either two or three votes within the group are in favor of A. This occurs with probability

$$t_1 = t_0^3 + 3t_0^2(1 - t_0).$$

That is, the probability that a group in tier 1 “votes” for A is t_1 . This same calculation can be made for each tier: the probability that a group in tier i chooses A is the probability that either two or three of its sub-groups in tier $i - 1$ “vote” for A. Therefore,

$$t_i = t_{i-1}^3 + 3t_{i-1}^2(1 - t_{i-1}) \tag{1}$$

for integers $i > 0$, where t_0 is given. The probability that a group is pivotal in the i^{th} tier equals the probability that this group is pivotal in tier $i - 1$, multiplied by the probability that the other

¹¹If $z = 1$, then majority rule and the IEC mechanism are equivalent.

two groups in tier i split their votes.¹² This equals

$$\xi_i^{IEC} = \xi_{i-1}^{IEC} \times 2t_i(1 - t_i)$$

for all integers $i > 0$. ξ_{-1}^{IEC} is defined to equal 1. We then have

$$\xi_z^{IEC} = 2^z \prod_{i=0}^{z-1} t_i(1 - t_i). \quad (2)$$

The probability that a given vote is pivotal clearly has two standard properties of voting mechanisms: this probability is decreasing as t_0 moves away from $\frac{1}{2}$ and decreasing as the number of votes increases.

The binomial theorem gives the probability that a vote is pivotal under majority rule:

$$\xi_z^{maj} = \binom{N-1}{(N-1)/2} (t_0(1-t_0))^{(N-1)/2}. \quad (3)$$

The following two lemmas introduce the powerful aspect of the IEC mechanism: as the number of votes increases and as elections are expected to be less even, the IEC rule offers much more power to each vote than majority rule.

Lemma 1 *As the number of votes tends to infinity, the probability of a given vote being pivotal under the IEC rule is almost always infinitely higher than under majority rule: $\lim_{z \rightarrow \infty} \frac{\xi_z^{IEC}}{\xi_z^{maj}} = \infty$ for $t_0 \in (0, 1) \setminus \frac{1}{2}$. This limit equals 0 for $t_0 = \frac{1}{2}$.*

Lemma 2 *As the expected competitiveness of the contest decreases, the relative power of a vote under the IEC rule increases: $\frac{\partial}{\partial t_0} \frac{\xi_z^{IEC}}{\xi_z^{maj}} < 0$ for $t_0 < \frac{1}{2}$ and $\frac{\partial}{\partial t_0} \frac{\xi_z^{IEC}}{\xi_z^{maj}} > 0$ for $t_0 > \frac{1}{2}$.*

Lemma 1 establishes that so long as there are many shares outstanding, and so long as the election is not expected to be *exactly* even, the IEC mechanism provides a vote a far greater

¹²A voter is defined to be pivotal if the outcome of the vote depends upon her vote. As we will see, once voters may abstain from voting, we will need to expand this definition somewhat.

probability of being pivotal than does majority rule. Figure 1 highlights how this can be true: the two left-most votes are both pivotal, in that if either had voted for B, then B would have won the election. This is in spite of the fact that, under majority rule, no vote would be pivotal. The IEC mechanism offers some opportunity for votes to affect elections even if the numbers of votes for each side are not very close.

While Lemma 1 is a limiting statement, the number of votes need not be particularly large for the IEC mechanism to offer much more power than majority rule. Figure 2 displays the ratio of the probability that a given vote is pivotal under the IEC mechanism to the probability that it is pivotal under majority rule, for a variety of population sizes and values of t_0 . As is immediately clear, except for the knife-edge case of $t_0 = \frac{1}{2}$, the IEC mechanism offers far more power even for small numbers of votes. For example, if there are 729 votes and the expected probability of any given vote going for A is 35%, then a vote is 60 billion times more likely to be pivotal under the IEC mechanism than under the majority rule mechanism. Even in close elections, the IEC rule gives votes far more power: if there are roughly 60,000 shareholders, each holding one share, and the probability of any given shareholder voting for A is 49%—a very close election and a very small corporation—then the probability of any given shareholder being pivotal is approximately 14,035 times as high under the IEC mechanism as under the majority rule mechanism.

Lemma 2 establishes analytically what is clear in Figure 2: as the contest is increasingly expected to be close (i.e., $t_0 \rightarrow \frac{1}{2}$ from either direction), the relative power of the IEC mechanism decreases. That said, even for very close elections, the power afforded votes by the IEC is much greater than that afforded by majority rule.

It is worth a brief discussion of why $t_0 = 1/2$ is special. Holding constant the probability that A wins an election, the probability that a given vote is pivotal is slightly higher under the majority rule mechanism. On the other hand, the probability that A wins the election decreases very fast as N increases for majority rule relative to the IEC mechanism. The second effect quickly overrides the first, so the IEC mechanism offers more power as the number of votes increases. When $t_0 = 1/2$,

the probability of A winning the election is $1/2$ regardless of N for both the IEC and majority rule mechanisms, so the slight power advantage offered by majority rule is always present.

2 The Model

Voting entails a cost including, at least, the time to fill out a ballot and the price of a stamp. If a voter is interested in making an informed decision, then this cost also includes time and effort to understand the issues. Voting also entails a benefit, insofar as the outcome of an election is more likely to go in one's favor if one votes. Large shareholders therefore have a greater benefit from voting, but probably a similar or lesser cost, and are therefore more inclined to vote.

I model voters as differing in their cost of voting and in their preference for a proposed policy. I therefore abstract away from issues of information acquisition and, therefore, strategic voting, in setting up the model. While this is very useful in simplifying the analysis, it is also useful in highlighting the point that even when shareholders *know* that management is advancing proposals against their interest, allowing them to vote on such proposals does not necessarily solve the governance problem. The model can also be interpreted as one in which the cost of voting is zero for all voters, but some voters know their preferences and some must pay a cost to learn them perfectly. If prior beliefs are such that voters that do not pay the cost would prefer not to vote, then all results from this model would be identical.

2.1 The voters

Let there be a vote on a proposal and let the voters be divided along two dimensions. Some prefer that the vote succeed (S) and some that it fail (F). Some have a voting cost equal to $C > 0$ and some have a voting cost equal to 0. A voter's type will therefore be one of the following four: SC , $S0$, FC or $F0$. It will sometimes be convenient to refer to type $S0$ and SC voters collectively as type S , and type $F0$ and FC voters as type F . Similarly, it will sometimes be convenient to

refer to type $S0$ and $F0$ voters collectively as type 0 and type SC and FC voters as type C .

The number of voters in each group is denoted N_{ix} , where $i \in \{S, F\}$ represents whether the voters prefer success or failure, and $x \in \{0, C\}$ represents the voting cost. For ease of exposition, I will sometimes aggregate the number of voters along either or both dimensions, in which case we will have only one or zero subscripts on N , respectively. Then $N_S \equiv N_{SC} + N_{S0}$ people prefer that the vote succeed and $N_F \equiv N_{FC} + N_{F0}$ people prefer that it fail. $N_0 \equiv N_{S0} + N_{F0}$ have voting cost of 0 and $N_C \equiv N_{SC} + N_{FC}$ have voting cost C . The total number of voters is $N \equiv N_S + N_F = N_C + N_0 = N_{SC} + N_{FC} + N_{S0} + N_{F0}$.

Voters can vote yes, no or abstain. Each voter's utility is given by $U = VI - xJ/2$. I takes a value of one if the outcome of the vote is consistent with the voter's preferences and zero otherwise, J takes a value of one if she votes and zero otherwise, and $x \in \{0, C\}$ is the voter's voting cost.

2.2 The voting game

The voting game has three parts. First, nature chooses a distribution of voter types which is not observed by voters. Next, voters choose whether to abstain, vote in favor, or vote against the proposal. Finally, the voting mechanism assigns a winner based, perhaps randomly, on those votes. I now describe each step in more detail.

For each of N voters, nature assigns the voter to be type $ix \in \{S0, F0, SC, FC\}$ with probability p_{ix} , where $\sum_{i \in \{S, F\}, x \in \{0, C\}} p_{ix} = 1$. Draws are independent. The vector of the number of voters of each type, $\vec{N} = \{N_{S0}, N_{SC}, N_{F0}, N_{FC}\}$, is therefore a random variable. The number of voters N , and the probability that each voter is assigned to each type, $\vec{p} = \{p_{S0}, p_{SC}, p_{F0}, p_{FC}\}$, is common knowledge, but the actual draw \vec{N} is not observed.

Note that in practice, shareholders own different numbers of shares, and the preferences of large shareholders (e.g., management) may be known. The former implies that preferences of each "voter" (where a voter is a share) are not independent. The latter implies that, for some

voters, type is not a random, unobserved draw. I abstract from these issues in the name of analytical tractability but, in stating Propositions 3 and 4, I will change these assumptions so that large shareholders' stakes and preferences are known, and only small shareholders' preferences are independent, random and unknown.

I make the following assumption in order to be able to solve for properties of the IEC analytically.

Assumption 1: $N = 3^z$ where z is an integer.

After nature chooses the distribution of voters, voters choose whether to abstain, vote in favor of, or vote against the proposal.¹³ Because voters may abstain, the number of actual voters may not equal the number of potential voters, N . Because the number of voters of each type is a random variable, and because a voter's decision to vote may be randomized, the number who actually turn out to vote is a random variable. Let the number of votes in favor be denoted $T_S \leq N_S$ and the number of votes against be $T_F \leq N_F$. A voting rule $\Gamma : \{T_S, T_F\} \rightarrow [0, 1]$ is a mechanism that aggregates votes and assigns a winner, Success or Failure. The rule may assign a winner randomly, with Success winning with some probability and Failure winning with the complementary probability.

We will be particularly interested in the effect that an arbitrary voter has on an election. In classic voting theory, without abstentions, this is the probability of a voter being pivotal, interacted with the choice that the voter makes. In majority rule with an odd number of voters, for example, the probability that a given voter is pivotal is the probability that, without her vote, there is a tie. In a random dictator election, in which all votes are put in a proverbial hat and one is selected at random, she is pivotal if her vote is picked. The change in the probability of victory for her

¹³I implicitly assume here that voters may not buy or sell votes. Unlike political voting, it is relatively easy to buy votes in shareholder elections. To acquire votes, simply borrow shares on the record date for a vote and return them the next day. The owner of the shares may or may not charge a rent in this exchange. Christoffersen, Geczy, Musto and Reed (2007) investigate this practice, and find an average spike in loaned shares from 0.22% of shares outstanding to 0.275% on record dates. In a restricted sample of only the Russell 3000, they find a change in shares lent of 0.1%. While this practice is therefore possible, it does not appear to be important for the vast majority of shareholder votes. I ignore the practice in this model.

preferred side if she votes in favor versus against equals the probability of a tie, under majority rule, or $1/N$ in a random dictator election.

If abstention is permitted, however, we must put the question somewhat differently. As I show in Lemma 3 below, voters never vote against their types: the choice is to vote one's type or else abstain. When the voter decides whether to vote, the relevant parameter is the difference in the likelihood that her preferred choice wins if she votes versus if she does not vote. We define this difference as the *expected effect* of a vote.¹⁴

Let f_S and f_F be the distributions of T_S and T_F , respectively, given voter strategies.

Definition 1 *The expected effect $\phi_i(f_S, f_F)$ that a voter of type $i \in \{S, F\}$ has on an election equals the change in the probability that her preferred choice wins if she votes her type rather than abstains: $\phi_S(f_S, f_F) = E_{T_S, T_F}[\Gamma(T_S + 1, T_F) - \Gamma(T_S, T_F) \mid f_S, f_F]$ and $\phi_F(f_S, f_F) = E_{T_S, T_F}[(1 - \Gamma(T_S, T_F + 1)) - (1 - \Gamma(T_S, T_F)) \mid f_S, f_F] = E_{T_S, T_F}[\Gamma(T_S, T_F) - \Gamma(T_S, T_F + 1) \mid f_S, f_F]$.*

Definition 2 *A voting rule is monotonic if $\phi \geq 0$.*

Any standard voting rule is monotonic, including majority rule and the IEC. Voting in line with your preferences cannot hurt your cause.

3 Equilibrium

An equilibrium in the voting game is (potentially randomized) choices for each of N players of whether to vote yes or no, or abstain, such that, given accurate beliefs about the (potentially randomized) choices of other players, each player is maximizing her utility. I restrict attention to symmetric equilibria:

¹⁴Note also that vote mechanisms where the outcome is probabilistic, with a probability of success equal to some $\Gamma(T_S, T_F) \in (0, 1)$, no vote is pivotal, but every vote may affect the probability of each outcome. We therefore abandon pivotality in our discussion of the impact of voting.

Definition 3 *An equilibrium is symmetric if all voters of the same type play the same randomization probabilities $q_{ix}^j \in [0, 1], i \in \{S, F\}, x \in \{0, C\}, j \in \{S, F, A\}$.*

The subscript refers to the type of the player and the superscript refers to the choice of voting for success or failure, or abstaining. Because the probabilities for a given type must sum to unity, the probability of abstaining for type ix is $q_{ix}^A = 1 - q_{ix}^S - q_{ix}^F$.

Assumption 2: Players do not play weakly dominated strategies.

This assumption reduces the set of potential equilibria by excluding cases where the vote is deterministic ($\Gamma = 0$ or 1 , given voter behavior) but type 0 voters may be playing a variety of strategies. These equilibria certainly are not unreasonable in a literal sense, but they add little to the discussion at hand while adding many special cases to the proofs.¹⁵ Given that it is common to exclude the play of weakly dominated strategies on the grounds of “reasonableness”, I make the exclusion here.

This immediately yields the following result:

Lemma 3 *In an election with a monotonic voting rule:*

- (a) *Voters never vote against their types.*
- (b) *Type 0 voters always vote in favor of their types, i.e., do not abstain.*

Proof. If the voting rule is monotonic, the probability of a type’s preferred choice winning if she votes for that choice is weakly greater than if she votes against. Because the cost of voting is equal either way, part (a) of Lemma (3) follows immediately. Now restrict attention to type 0 voters. Voting in favor yields expected utility $E(U) = V \times (\Gamma + \phi_S)$ and abstaining yields utility

¹⁵The most interesting equilibria that result when we remove this assumption are those where one type of type 0 voter abstains (or votes against her preferences) and the other votes in accordance with her preferences. In this case, the will of the type voting in accordance with its preferences is implemented. This is true even if the number of type zero voters in this group is smaller than in the group abstaining/voting against its preferences. We therefore exclude with this assumption one interesting equilibrium where a potentially very small group of voters imposes its will. The plausibility of this equilibrium is open for debate.

$E(U) = V \times \Gamma$. Because $\phi \geq 0$ when a voting rule is monotonic, it is therefore a weakly dominant strategy for type 0 voters to vote their types. By Assumption 2, they will do so. ■

Therefore, type 0 players always vote their types: $q_{S0}^S = 1$ and $q_{F0}^F = 1$. Also, voters with voting cost C never vote against their types: $q_{SC}^F = q_{FC}^S = 0$. For clarity, then, let $q_S \equiv q_{SC}^S$ and $q_F \equiv q_{FC}^F$.

Definition 4 *An equilibrium is stable if, for both types S and F , a feasible increase (decrease) in the probability of voting by a single voter weakly decreases (increases) the benefit of voting for other voters of the same type.*

An equilibrium is stable, therefore, when a change in voting behavior by a single individual would be countered by opposing changes in behavior by other individuals of the same type. I now establish the conditions under which there exist equilibria where (a) everybody votes, (b) only type 0 voters vote, or (c) some type C voters play a mixed strategy. We will see that, regardless of the voting rule used, when voting costs are low, a universal voting equilibrium exists, and when voting costs are high, a universal abstention equilibrium exists. On the other hand, when we have intermediate voting costs, a universal voting equilibrium exists only under the IEC and a universal abstention equilibrium exists only under majority rule. We will also see that mixed-strategy equilibria are not stable, so universal voting and universal abstention are the only plausible equilibria.

3.1 The existence of universal voting equilibria

A universal voting equilibrium, in which all voters vote, can only obtain if the voters with a positive cost of voting find that the expected effect of a vote is greater than or equal to C/V . Under majority rule, if voter v believes that all other voters will vote, then the expected effect equals the probability of a tie, had she not voted, multiplied by $1/2$, since her vote replaces a coin

flip.¹⁶

$$\phi^{maj} = \frac{1}{2} \times \binom{N-1}{(N-1)/2} (p_F(1-p_F))^{(N-1)/2}.$$

Note that this is the same value for all types of voter, regardless of \bar{p} . As previously shown in Equations (1) and (2), the expected effect of a voter of type $j \in \{S, F\}$ under the IEC rule equals $\frac{1}{2} \times 2^z \prod_{j=0}^{z-1} t_j(1-t_j)$ where $t_j = t_{j-1}^3 + 3t_{j-1}^2(1-t_{j-1})$, and $t_0 = p_j$. Let \bar{C}^k represent the maximum value of C such that a universal voting equilibrium exists for voting rule k . Then $\bar{C}^{maj} = \frac{V}{2} \times \binom{N-1}{(N-1)/2} (p_F(1-p_F))^{(N-1)/2}$ and $\bar{C}^{IEC} = \frac{V}{2} \times 2^z \prod_{j=0}^{z-1} t_j(1-t_j)$. We then have our primary result:

Proposition 1 *Universal voting equilibria exist for majority rule and the IEC so long as $0 < C \leq \bar{C}^{maj}$ in the former case and $0 < C \leq \bar{C}^{IEC}$ in the latter. \bar{C}^{maj} and \bar{C}^{IEC} have the following properties:*

1. $\bar{C}^{IEC}, \bar{C}^{maj} > 0$.
2. $\lim_{N \rightarrow \infty} \bar{C}^{IEC} = \lim_{N \rightarrow \infty} \bar{C}^{maj} = 0$.
3. $\lim_{N \rightarrow \infty} \left[\bar{C}^{IEC} / \bar{C}^{maj} \right] = \infty$ for $p_F \neq 1/2$ and $\lim_{N \rightarrow \infty} \left[\bar{C}^{IEC} / \bar{C}^{maj} \right] = 0$ for $p_F = 1/2$.

To state these results in words, there always exist universal voting equilibria so long as voting costs are sufficiently low. The important question here is, how low? When the number of votes is large, voting costs must be very low in order for a universal voting equilibrium to exist, and in fact must approach zero as the number of votes grows to infinity. The set of values of C such that a universal voting equilibrium exists is $[0, \bar{C}^k]$, and the size of this set depends upon the voting rule. The ratio of the sizes of these sets under the IEC versus majority rule is given by $\bar{C}^{IEC} / \bar{C}^{maj}$ and Proposition 1 establishes that this goes to infinity as the number of voters grows. One becomes

¹⁶Recall that N is known and odd.

much “more likely” to find a universal voting equilibrium under the IEC than majority rule for large elections.¹⁷

Results 2 and 3 in Proposition 1 are limiting results. For this paper to be practically relevant, the difference in voter power for the IEC versus majority rule must be large even with a finite number of voters. Figure 2 shows that with even several hundred to several thousand votes, $\overline{C}^{IEC} / \overline{C}^{maj}$ becomes an enormous number. Majority rule offers voters a very low probability of affecting the outcome of the election, and therefore induces universal voting very rarely. The IEC offers an individual voter considerably more power, inducing universal voting millions, billions, etc. times as often. While all voting rules afford individual voters with little power, majority rule is particularly bad, while the IEC is considerably better.

3.2 The existence of universal abstention equilibria

A universal abstention equilibrium obtains if, given that no other high cost voters are expected to vote, it is not in the interest of a single high cost voter to vote. This will occur for any voting rule where p_{S0} and p_{F0} are sufficiently different and C/V is sufficiently high. As with our results concerning universal voting equilibria, because the IEC gives individual voters so much more power to affect the outcome of an election, they will be willing to vote even when p_{S0} and p_{F0} are more different and when C/V is higher. Let \underline{C}^k be the lower bound on C for mechanism k such that a universal abstention equilibrium exists.

Proposition 2 *There exists a value $\underline{C}^{maj}(N, p_{S0}, p_{F0}) > 0$ such that a universal abstention equilibrium exists for majority rule iff $C \geq \underline{C}^{maj}(N, p_{S0}, p_{F0})$. \underline{C}^{maj} is decreasing in $|p_{S0} - p_{F0}|$ and N , and $\lim_{N \rightarrow \infty} \underline{C}^{maj} = 0$.*

I cannot show analytical results concerning the existence of universal abstention equilibria for the IEC because N_0 is a random variable that does not generally equal an integer power of three.

¹⁷To make this concrete, if we apply Lebesgue measure to these sets, then the ratio of the measures of the sets goes to infinity as the number of voters goes to infinity.

It seems likely that the power of the IEC relative to majority rule, shown for the case of $N = 3^z$, applies more generally.

However, two changes in our assumptions about nature's initial assignment of preferences yield analytical results. We have so far assumed, effectively, that each shareholder controls one share and that her preferences are unknown. Instead, it may be reasonable to assume that there are both large and small shareholders, and that the preferences of large shareholders are observable. In this case, N_{S0} and N_{F0} would be fixed and known. For the remaining propositions, we make the following assumption:

Assumption 3: N_{S0} and N_{F0} are fixed and known, and $N_0 = 3^y - 1$ for some integer $y < z$.

Proposition 3 *If assumption 3 holds, then there exists a value $\underline{C}^{IEC}(N_0, p_{S0}, p_{F0}) > 0$ such that a universal abstention equilibrium exists for the IEC iff $C \geq \underline{C}^{IEC}(N_0, p_{S0}, p_{F0})$. \underline{C}^{IEC} is decreasing in $|p_{S0} - p_{F0}|$ and N_0 , and $\lim_{N_0 \rightarrow \infty} \underline{C}^{IEC} = 0$.*

Just as the IEC supports universal voting equilibria by increasing the expected effect of a vote, the IEC also prevents universal abstention equilibria. The bound on voting costs C , below which a universal abstention equilibrium does not exist, is higher for the IEC than majority rule. This means that, given voting costs, the IEC often will not permit a universal abstention equilibrium while majority rule will.

The ratio of these bounds goes to infinity as the number of voters goes to infinity:

Proposition 4 *If assumption 3 holds, then $\lim_{y \rightarrow \infty} \underline{C}^{IEC} / \underline{C}^{maj} = \infty$.*

3.3 The existence of mixed strategy equilibria

Mixed strategy equilibria, in which one or both types of voter randomize whether to vote or abstain with a probability strictly between zero and one, are not typically stable. The intuition can be seen in Figure 3. This figure displays the set of potential equilibria in the $E(T_S), E(T_F)$ space.

The curves represent the set of points where $\phi = C/V$, with the assumption that ϕ is the same for each type of voter for the sake of clarity.¹⁸

For a standard voting rule, ϕ is increasing as $E(T_F)$ approaches the 45° line from the left or $E(T_S)$ approaches the 45° line from below. This is because closer elections make each vote have a larger expected effect on the outcome of the election.¹⁹ Therefore, between these curves, $\phi > C/V$, so all voters strictly prefer to vote. Outside the curves, $\phi < C/V$ so all voters strictly prefer to abstain.

A proposed equilibrium is a point $\{E(T_S), E(T_F)\}$, where $E(T_i) = E(N_{i0}) + q_{iC} \times E(N_{iC})$. Because all type 0 voters vote their types, and the randomization probability for a type C voter, q_{iC} , must lie between zero and one, $E(N_{i0}) \leq E(T_i) \leq E(N_i), i \in \{S, F\}$. For a point to be an equilibrium, then either both types are playing pure strategies, or the point is on the curves representing $\phi = C/V$.²⁰

To see why mixed strategy equilibria are unstable, consider a potential equilibrium in highlighted section E. In this section, both types are playing mixed strategies. Suppose one type S voter chooses to randomize with a higher probability of voting. Then $E(T_S)$ increases and ϕ_S increases. This means that $\phi > C/V$ for type S voters, so all would strictly prefer to vote, rather than to randomize. Similarly, if one type S voter were decrease her probability of voting, then $\phi < C/V$ and type S voters would strictly prefer to abstain. Therefore, points in section E are not stable equilibria. A similar argument follows for type F voters and section F.

¹⁸Drawing curves for both type S and F voters would muddle the figure without providing significant additional intuition.

¹⁹We can see this for both majority rule and the IEC. Under majority rule, if the number of votes is odd, a vote only affects the outcome of an election if the other voters split their votes. Therefore, as the expected number of votes for each side is more similar, the expected effect of a vote is greater. The same goes for the IEC. As the expected number of votes for each side is closer, it is more likely that a voter will find herself in a group with a split vote. It is also more likely that that group will find itself in a super-group with a split vote, etc. Therefore, as the vote is expected to be closer, the expected effect of a vote is greater. This principle is true for most voting rules.

²⁰Note that if there are many type 0 voters, then certain sections of these curves are infeasible because type 0 voters always vote (Lemma 3). For example, if $E(N_{F0}) = E(N_{F0}^2)$, then the part of section E to the left of $E(N_{F0}^2)$ is not feasible. Also note that if C/V or the number of voters is low, then certain sections of these curves are infeasible because the number of type $i \in \{S, F\}$ voters cannot exceed N_i . For example, if $E(N_S) = E(N_S^1)$, then the part of section F above $E(N_S^1)$ is not feasible.

The arrows in Figure 3 show the direction in which there is “pressure” to change voting probabilities. When a proposed equilibrium lies within the curves, $\phi > C/V$, so voters would like to increase their likelihood of voting. When a proposed equilibrium lies outside the curves, $\phi < C/V$, so voters would like to decrease their likelihood of voting. On the boundary where $\phi = C/V$, the equilibrium is stable with respect to changes in behavior by the type of voter for which the arrows point at each other. The equilibrium is unstable with respect to changes in behavior by the type of voter for which the arrows point away from each other. There always exists one of type for whom the latter case holds, so mixed strategy equilibria are always unstable.

4 Voting rules and the will of the majority

Taken together, these results imply that which mechanism is more effective at implementing the majority’s will is a question of voting costs, and of whether the zero cost voters generally share the preferences of voters overall. When the voting cost C is low, or elections are very small, both majority rule and the IEC feature universal voting equilibria. In this case, majority rule is slightly better than the IEC. For intermediate values of C , the IEC is considerably better, as it implements a universal voting equilibrium while majority rule implements a universal abstention equilibrium. When C is high, both majority rule and the IEC implement universal abstention equilibria, so neither is particularly good at implementing the majority’s will.

In this section, I use figures to evaluate when the IEC is useful, and precisely how it achieves its goals. First, I display properties of proposed equilibria in the $\{E(T_S), E(T_F)\}$ space, and show graphically why proposed pure strategy equilibria will or will not exist. Second, I graph, for each mechanism, which equilibrium exists as we increase C/V . This figure highlights that, when the majority and minority disagree, the IEC is usually better, and often much better, than majority rule in implementing the will of the majority. Third, I show in the $\{E(N_{S0}/N_{F0}), E(N_S/N_F)\}$ space what types of equilibrium will exist for majority rule and the IEC. The figure highlights

that $E(N_S/N_F)$ determines whether a universal voting equilibrium exists, $E(N_{S0}/N_{F0})$ determines whether a universal abstention equilibrium exists, and $E(N_{S0}/N_{F0})/E(N_S/N_F)$ determines whether the minority's and majority's preferences coincide.

4.1 Why the IEC better supports universal voting equilibria and eliminates universal abstention equilibria

As discussed in Section 3.3, Figure 3 displays the properties of potential equilibria graphically. When a proposed equilibrium lies between the curves representing $\phi = C/V$, voters of both types S and F would strictly prefer to vote rather than abstain. When a proposed equilibrium lies outside these curves, both types of voter would strictly prefer to abstain.

We can use Figure 3 to evaluate whether universal abstention and universal voting equilibria exist. A potential universal abstention equilibrium, in which $E(T_i) = E(N_{i0})$ for $i \in \{S, F\}$, may lie inside the curves (point A) or outside the curves (point B), depending upon the vote mechanism and the values of $E(N_{i0})$. Suppose that $E(N_{F0}) = E(N_{F0}^1)$, so that all type C voters abstaining is represented by point A. At point A, type C voters would prefer to vote, so this point does not represent an equilibrium. If, however, $E(N_{F0}) = E(N_{F0}^2)$, so that all type C voters abstaining is represented by point B, type C voters would, in fact, prefer to abstain, so point B is an equilibrium.

A potential universal voting equilibrium, in which $E(T_i) = E(N_i)$ for $i \in \{S, F\}$, may also lie either inside the curves (point C) or outside the curves (point D) depending on the vote mechanism and the values of $E(N_i)$. Along the same lines as in the preceding paragraph, point C is an equilibrium because voters prefer to vote. Point D is not an equilibrium because voters wish to decrease their vote probabilities and are, in fact, able to do so. This also shows graphically that, when they exist, universal voting and universal abstention equilibria are stable.

This analysis immediately yields a graphical interpretation of what the IEC does when compared to majority rule. The IEC significantly increases the expected effect of a vote, ϕ , shifting

outward the curves representing $\phi = C/V$. Thus, there are fewer values of \bar{p} such that universal abstention is an equilibrium and more such that universal voting is an equilibrium. We see this in Figure 4, which plots curves for $\phi = C/V$ for both voting mechanisms, holding constant \bar{p} . Point A is a potential universal abstention equilibrium and point B is a potential universal voting equilibrium. Because points A and B are outside the curves representing $\phi = C/V$ for majority rule, voters wish to abstain. This means that point B is not an equilibrium and point A is an equilibrium. Because points A and B are inside the curves representing $\phi = C/V$ for the IEC mechanism, voters wish to vote. This means that point B is an equilibrium and point A is not an equilibrium. In this figure, then, the IEC permits only universal voting as an equilibrium, while majority rule permits only universal abstention. It is also possible for point(s) A and/or B to lie outside or inside both sets of curves, in which case the types of equilibria are the same under both mechanisms, but it is almost never the case that a universal voting equilibrium obtains for majority rule and not the IEC. This case, in fact, requires $p_S/p_F \approx 1$, in which case there is no majority will to implement, in expectation.

Finally, note that as C/V decreases, the curves representing $\phi = C/V$ (for the IEC mechanism) shift outward. This means that, for sufficiently low costs of voting, or sufficiently high values from implementing a preferred outcome, universal voting is an equilibrium and universal abstention is not. I evaluate the effect of the cost of voting in the next section.

4.2 The cost/benefit of the IEC versus majority rule as a function of the voting cost

Figure 5 plots the existence of universal voting and universal abstention equilibria as a function of C/V for both the IEC and majority rule, under the assumption that the type 0 voters prefer (on average) a different choice than the overall population. It is possible that $\underline{C}^k < \bar{C}^k, k \in \{maj, IEC\}$, in which case intermediate values of C/V allow both the universal voting and universal abstention

equilibria. It is also possible that $\underline{C}^k > \overline{C}^k$, in which case intermediate values of C/V do not allow any stable equilibrium. The left panel displays the former case and the right panel the latter.

Beginning with the left panel, in section A, when $C/V < \underline{C}^{maj}$, both majority rule and the IEC feature unique universal voting equilibria. Because the IEC sometimes assigns as winner the side receiving fewer votes, majority rule is slightly better than the IEC at implementing the majority's will. In section B, where $\underline{C}^{maj} < C/V < \overline{C}^{maj}$, majority rule features both universal voting and universal abstention equilibria. Without an equilibrium selection rule, we cannot say whether majority rule or the IEC is superior, but because the IEC almost always selects the winner according to the majority's preferences, and majority rule may induce universal abstention, we can say that the IEC is at least a safer bet. In section C, where $\overline{C}^{maj} < C/V < \underline{C}^{IEC}$, the IEC is clearly better, as it induces universal voting rather than universal abstention. In section D, where $\underline{C}^{IEC} < C/V < \overline{C}^{IEC}$, the IEC supports both universal voting and universal abstention equilibria. Either one, however, is better than the unique universal abstention equilibrium associated with majority rule, which is assured to make the choice less preferred by the majority. In section E, where $\overline{C}^{IEC} < C/V$, the IEC is slightly better because it chooses the majority-preferred choice at least sometimes.

In the right panel, the difference is that, rather than each mechanism sometimes supporting both types of equilibrium, each sometimes supports *no* stable equilibrium. Sections A, C, and E have identical properties as in the left hand panel. In section B, where $\overline{C}^{maj} < C/V < \underline{C}^{maj}$, majority rule does not allow a stable equilibrium. As the IEC almost always implements the will of the majority, it is clearly superior. In section D, where $\overline{C}^{IEC} < C/V < \underline{C}^{IEC}$, the IEC does not allow a stable equilibrium. However, as majority rule certainly does not implement the majority's will for these values of C/V , any outcome induced by the IEC must be weakly better.

Taken together, we see that if the preferences of type 0 voters are counter to the preferences of voters overall, the IEC is superior to majority rule in implementing the majority's will for many values of voting costs C/V . Even if voting costs are low, and majority rule is better than the IEC,

it is only slightly better. If type 0 voters share the preferences of the overall population, however, majority rule would be superior, as it always implements the majority’s will with probability one.

4.3 The existence of equilibria versus disagreement among voters

Figure 6 shows whether one or both types of equilibrium exist for each mechanism, as we vary the expected ratio of voters preferring S to F . In each panel, N and C/V are fixed. Spaces labeled “A” support only universal abstention equilibria: the vote share of type 0 voters is expected to be so lopsided that it is not worthwhile for any type C voter to vote. Spaces labeled “B” support only universal voting equilibria: the vote is expected to be close enough if everybody votes that it is worth it for each voter to vote. Spaces labeled “C” support both types of equilibrium: while the vote is expected to be close enough if everybody votes that it is worth it for each voter to vote, the vote is also expected to be so lopsided if only type 0 voters vote that, if type C voters expect that to occur, they will not vote. Spaces labeled “D” do not support stable equilibria: the vote is close enough if only type 0 voters vote that type C voters will prefer to vote. However, if all of them do so, then the vote is lopsided enough that it is no longer worth them voting. Note that area B is larger for the IEC and area A is larger for majority rule. In fact, the ratio of these sizes may be millions or billions to one.

As C/V or N rise, area B would shrink in each figure, with the left- and right-most borders approaching the center. Area B would be eliminated for the IEC before majority rule because majority rule offers voters slightly more power if $p_S \approx p_F$. Areas labeled A would grow, with their borders approaching the center line where $E(N_{S0}/N_{F0}) = 1$.

The first and third quadrants (top right and bottom left) represent situations where type 0 voters and the overall population prefer the same choice, and the second and fourth quadrants represent situations where type 0 voters and the overall population disagree. As should be immediately clear, whether type 0 voters and the overall population agree is unrelated to the conditions

allowing each type of equilibrium.

5 The power of the IEC for alternative values of η

We have seen that the IEC mechanism yields a much greater probability that an individual voter is pivotal than standard majority voting in the case where three elements are assigned to each set ($\eta = 3$), so long as there are many voters. In fact, reducing η apparently increases pivotality monotonically, though there are two difficulties in showing this analytically. First, analytical results are intractable unless the number of voters N is a power of η , so when we compare the probability of being pivotal with tiers of size η to tiers of size η' , there is not generally a value of N that is a power of both. In fact, analytical solutions are only available if η and η' share a common divisor. For example, $\eta = 3$ and $\eta = 9$ can be compared since they share the divisor of 3,²¹ while $\eta = 3$ and $\eta = 5$ could not be compared since no power of three is also a power of five. This limits the set of comparisons we can make. Second, even when such comparisons are possible, the formulae for the probability of being pivotal become un-workably complex.

I maintain the definitions of tiers, etc., but add superscripts that identify the number of elements in a tier for the mechanism in question. The probability that a vote in tier i goes in favor, p_i , equals the probability that a majority of voters in tier $i - 1$ vote in favor, yielding

$$p_i^\eta = \sum_{s=0}^{\frac{\eta-1}{2}} (p_{i-1}^\eta)^{\eta-s} (1 - p_{i-1}^\eta)^s.$$

Let the probability of being pivotal in tier i be represented ϕ_i^η . The probability of a voter being pivotal in tier i equals the probability that this voter is pivotal in tier $i - 1$ multiplied by the

²¹One need not be a power of the other. We could compare $\eta = 25$ and $\eta = 125$ since they share the common divisor of 5.

probability that the other two groups in tier i split their votes. This equals:

$$\phi_i^\eta = \phi_{i-1}^\eta \times \binom{\eta-1}{\frac{\eta-1}{2}} (p_i^\eta)^{\frac{\eta-1}{2}} (1-p_i^\eta)^{\frac{\eta-1}{2}}$$

for all integers $i > 0$. ϕ_{-1}^η is defined to equal 1. We then have

$$\phi_z^\eta = \binom{\eta-1}{\frac{\eta-1}{2}}^z \prod_{i=0}^{z-1} (p_i^\eta)^{\frac{\eta-1}{2}} (1-p_i^\eta)^{\frac{\eta-1}{2}}.$$

I wish to show that as η gets larger, holding N constant, the probability of being pivotal at the highest tier is decreasing. Since the definition of p_i^η is recursive, proving properties of ϕ_z^η is difficult for $\eta > 3$. Though analytical solutions are unavailable, I present two sets of results that strongly suggest that ϕ is monotonically decreasing in η .

First, I compare the probability of being pivotal when $\eta = 3$ to the probability of being pivotal when $\eta = 9$ for numbers of voters that are powers of 9. I display this ratio in Figure 8. The ratio of the former to the latter clearly possesses the same qualitative properties as the ratio of the $\eta = 3$ case to the majority rule case presented in Figure 2. So long as the fraction of votes for A is not, in expectation, exactly 50%, the relative probability of being pivotal when $\eta = 3$ versus $\eta = 9$ goes to infinity as the number of voters increases. While I prove this analytically only when comparing the case of $\eta = 3$ to majority rule, the charts look strikingly similar. Similar comparisons for $\eta = 5$ vs. $\eta = 25$ look qualitatively identical.

Second, I calculate the probability a vote of being pivotal for $\eta = 3$, $\eta = 5$, $\eta = 7$ and $\eta = 9$ for a variety of values of N . For each value of η I choose values of N that are successive powers of η , so the N s associated with each η are different. It can be difficult to see trends in data presented this way so I plot these points in a scatter shown in Figure 9. Points within each η series are connected to highlight more clearly which points are associated with which values of η . Note that

the curves connecting these points are merely visual aids and are not numerically estimated. As should be quite clear, so long as the number of voters is sufficiently large, larger tiers result in lower probabilities of a voter being pivotal.

6 Application to shareholder elections

Applying these results to the shareholder voting setting, we take type 0 voters to be management and institutions, and type C voters to be small/individual shareholders. In fact, institutions may have a small positive cost of voting, no cost, or even a negative cost (stemming from private benefits of siding with management). We set their cost to zero for simplicity. Small voters have heterogeneous costs: larger holdings would be associated with a smaller per-share voting cost, and shareholders that closely follow corporate events may have a smaller cost of becoming informed. For simplicity, we take all small shareholders to be type C , and assume all hold one share. I also assume no vote trading. Each of these simplifications is clearly an abstraction from reality, but they are necessary to derive analytical results.

Universal voting implies that all shareholders, large and small, vote their preferences. Suppose that institutions will always vote with management. If management offers a proposal that is clearly bad for shareholders, then we have $N_{F0} = N_{SC} = 0$: all type 0 shareholders would be in favor and all type C shareholders would be against. If $N_C > N_0$, then in a universal voting equilibrium, the proposal would fail with certainty, under majority rule, and with near certainty under the IEC. In a universal abstention equilibrium, the proposal would succeed with certainty, under majority rule, and near certainty under the IEC. Under this assumption, then, the first order effect of a voting rule is whether it supports either type of equilibrium. As was shown in Proposition 1, neither majority rule nor the IEC support universal voting equilibria when the number of voters goes to infinity. However, for finite numbers of voters, both support universal voting equilibria if the cost of voting is sufficiently low. The maximum value of voting costs that permit universal

voting under the IEC is several orders of magnitude higher under the IEC versus majority rule. Therefore, the IEC is a considerably better rule than majority rule when small shareholders and management disagree.

In a less extreme case, where institutions are not biased completely in favor of management, then some type 0 voters would vote against the proposal, making it easier to vote down. In this case, $E(N_{S0}), E(N_{F0}) > 0$. Note, however, that even if the fraction of shares held by institutions not affiliated with management is “close” to the number of shares held by management and affiliated institutions, a universal voting equilibrium may not exist if $E(N_{S0}) - E(N_{F0})$ is too large relative to C/V . The maximum value of C/V that is consistent with universal voting, given affiliations of institutions, is again many orders of magnitude higher under the IEC relative to majority rule.

The existence of universal abstention equilibria follows the same lines. The minimum value of C/V such that a universal abstention equilibrium exists under the IEC is many orders of magnitude higher than under majority rule. That is, universal abstention, in which no small shareholders vote, requires much higher voting costs under the IEC than majority rule.

All of the above discussion has assumed that management and shareholders disagree about the quality of a proposal. If they agree, then both universal voting and universal abstention produce good outcomes: choosing a rule to induce one or the other is unnecessary. In this case, majority rule is somewhat better than the IEC, but not much.

7 Conclusion

I attempt in this paper to show that, with standard majority voting rules, we should not expect expanded shareholder voting to solve problems of corporate governance. There is little incentive for small shareholders to vote, because the likelihood that they affect the outcome is too small to make voting worthwhile, *even if they know that a proposal is harmful*. I therefore argue that an alternative voting mechanism is a potential solution to the problem. Such a mechanism must

significantly increase the likelihood that a given shareholder's vote affects the outcome of an election while simultaneously ensuring that the majority's will is usually implemented.

The Idealized Electoral College voting mechanism induces turnout, but at the cost of sometimes assigning as winner the side that receives a minority of votes. When voters of varying voting costs have similar preferences, this cost slightly exceeds the benefit, and majority rule is a superior mechanism. When voters of varying voting costs have systematically differing preferences, however, the benefit greatly exceeds the cost and the IEC is preferable.

As applied to shareholder elections, large shareholders and management have low voting costs (per share owned) and small shareholders have high voting costs. There are arguments for and against the claim that small and large shareholders have similar preferences. On the one hand, all shareholders prefer the firm to deliver higher dividends and capital gains. This homogeneity of shareholder preferences has, in fact, been argued to be a major factor in the rise of shareholder owned firms in the last two hundred years (Hansmann 2000). On the other hand, many large shareholders receive private benefits from management like the opportunity to lead debt or equity offerings, to manage pension funds, etc. Moreover, management themselves often own a large fraction of shares, and receive private benefits from decisions concerning mergers, executive compensation, board composition, etc. This suggests that the preferences of management/large shareholders and the majority may well differ. Therefore, inducing turnout via an alternative to majority rule could significantly benefit shareholders.

8 References

1. Beck, Rachel (2009). "All Business: Investors still support CEO pay", Associated Press
2. Bethel, Jennifer E. and Stuart L. Gillan (2002). "The Impact of the Institutional and Regulatory Environment on Shareholder Voting", *Financial Management* 31, 4, pp. 29-54

3. Borghers, Tilman (2004). "Costly Voting", *American Economic Review* 94, pp.57-66
4. Brickley, James A, Ronald C. Lease and Clifford W. Smith (1988). "Ownership structure and voting on anti-takeover amendments", *Journal of Financial Economics* 20, pp. 267-291
5. Brickley, James A, Ronald C. Lease and Clifford W. Smith (1988). "Corporate Voting: Evidence from Charter Amendment Proposals", *Journal of Corporate Finance* 1, pp. 5-30
6. Buchanan, James M. (1974). "Hegel on the Calculus of Voting", *Public Choice* 17, pp. 99-101
7. Burch, Timothy R., Angela G. Morgan and Jack G. Wolf (2004). "Is Acquiring-Firm Shareholder Approval in Stock-for-Stock Mergers Perfunctory?" *Financial Management* 33, 4, pp. 45-69
8. Christoffersen, Susan E. K., Christopher C. Geczy, David K. Musto, and Adam V. Reed (2007). "Vote Trading and Information Aggregation", *Journal of Finance* 62, 6, pp. 2897-2929
9. Davis, Gerald F. and E. Han Kim (2007). "Business Ties and Proxy Voting by Mutual Funds", *Journal of Financial Economics* 85, pp. 552-570
10. de Marzo, Peter (1993). "Majority Voting and Corporate Control: The Rule of the Dominant Shareholder", *Review of Economic Studies* 60, 3, pp. 713-734
11. Downs, Anthony (1957). *An Economic Theory of Democracy*, New York: Harper
12. Easterbrook, Frank H. and Daniel R. Fishel (1983). "Voting in Corporate Law", *Journal of Law and Economics* 26, 2, pp. 395-427
13. Gillan, Stuart L. and Laura Starks (2000). "Corporate Governance Proposals and Shareholder Activism: The Role of Institutional Investors", *Journal of Financial Economics* 57, pp. 275-305

14. Gordon, Lilli A. and John Pound (1993). "Information, Ownership Structure, and Shareholder Voting: Evidence from Shareholder Sponsored Corporate Governance Proposals", *Journal of Finance* 48, 2, pp. 697-718
15. Grossman, Sanford J. and Oliver D. Hart (1988). "One-Share-One-Vote and the Market for Corporate Control", *Journal of Financial Economics* 20, pp. 175-205
16. Krasa, Stefan and Mattias Polborn (2009). "Is Mandatory Voting Better Than Voluntary Voting?" *Games and Economic Behavior* 66,1, pp. 275-291
17. Harris, Milton and Artur Raviv (1988). "Corporate Governance: Voting Rights and Majority Rules", *Journal of Financial Economics* 20, pp. 203-235
18. Hansmann, Henry (2000). "The Ownership of Enterprise", Belknap Press of Harvard University Press
19. G. W. F. Hegel (1821). "The Philosophy of Right", translated with Notes by T. M. Knox, Vol. 46, *Great Books of the Western World* (Chicago: Encyclopedia Britannica, (1952), pp. 104
20. Maug, Ernst and Kristian Rydqvist (2009). "Do Shareholders Vote Strategically? Voting Behavior, Proposal Screening, and Majority Rules", *Review of Finance* 13, 1, pp. 47-79
21. Maug, Ernst and Bilge Yilmaz (2002). "Two-Class Voting: A Mechanism for Conflict Resolution", *American Economic Review* 92, pp. 1448-1471
22. Morck, Randall, Daniel Wolfenzon and Bernard Yeung (2005). "Corporate Governance, Economic Entrenchment, and Growth", *Journal of Economic Literature* 43, 3, pp. 655-720
23. Natapoff, Alan (1996). "A Mathematical One-Man One-Vote Rationale for Madisonian Presidential Voting Based on Maximum Individual Voting Power", *Public Choice* 88, 3/4, pp.259-273

24. Persico, Nicola (2004). "Committee Design with Endogenous Information", *Review of Economic Studies* 71, pp.165-191
25. Sen, Amartya K (1970). *Collective Choice and Social Welfare*, San Francisco: Holden-Day, pp. 195-196
26. Taylor, Curtis R. and Huseyin Yildirim (2010). "A Unified Analysis of Rational Voting with Private Values and Group-Specific Costs", *Games and Economic Behavior* 70, 2, pp.457-471
27. Tullock, Gordon (1967). *Toward a Mathematics of Politics*, Ann Arbor: University of Michigan Press

9 Appendix

9.1 Appendix A – Rules of the IEC mechanism when $N \neq 3^z$ for some integer z

The rules of the IEC mechanism when the number of voters is not a power of three are as follows. Define \underline{z} so that $3^{\underline{z}} \leq N \leq 3^{\underline{z}+1}$ where \underline{z} is an integer greater than or equal to two. Normally the lowest tier would have $3^{\underline{z}+1}$ votes and the second lowest would have $3^{\underline{z}}$ elements. Because the number of voters is between these two numbers, they must be arranged so that some are in the lowest tier and some in the second lowest. A fraction $(1 - x)$ are placed in the lowest tier and a fraction x in the next tier up, but no votes are placed directly above other votes in the pyramid. The arrangement must satisfy the following rules:

1. Voters are no more than one tier apart in initial placement
2. No more than one group in the second lowest tier is comprised both of votes directly from voters and “votes” from groups in the lowest tier.

3. No more than one group on the lowest tier has fewer than three voters in it. If one group has fewer than three votes then the remaining votes are assigned to either choice with probability $1/2$.²²
4. If a group with fewer than three votes is present in the lowest tier (call it y) and if a group in the second lowest tier is comprised both of votes directly from voters and “votes” from groups in the lowest tier (call it w) then y is a subgroup of w .

The intuition of these rules is simple. Randomly order the votes cast and place them in a tier with $N^{\underline{z}+1}$ spaces, starting at the left. They will not fill that tier, by definition of \underline{z} . Starting at the right, move votes one by one into the next tier up, shifting the right-most remaining vote in the lowest tier to the right-most remaining space in the second lowest tier. Eventually there is no way to move votes up into an empty space, and the process is complete.

The unique fraction of votes placed in the second lowest tier, x , can then be derived:

$$N = (1 - x)3^{\underline{z}+1} + x3^{\underline{z}} = 3^{\underline{z}}(3 - 2x).$$

Only in one third of cases will this value of x be such that xN is an integer (so that all voters are arranged in groups of three even though some are initially placed on the lowest tier and some on the second lowest). Let $\bar{N} = rd(x3^{\underline{z}})$ where the function rd rounds the argument down to the nearest integer. Then \bar{N} is the number of voters arranged into the second lowest tier. $N - \bar{N}$ voters are arranged into the lowest tier. Note that in the case where N is a power of three, $x = 0$ or $x = 1$ and the rules 1-4 above are satisfied for the mechanism as previously described.

Figure 7 displays an example with 16 votes. Solving for x :

$$16 = 3^2(3 - 2x)$$

²²The randomization probability is nearly irrelevant: as the number of voters gets large the probability that this vote affects the outcome goes to zero.

$$\Rightarrow x = \frac{11}{18}.$$

Then

$$\bar{N} = rd\left(\frac{11}{18} \times 9\right) = 5.$$

Therefore five votes are put into tier 0 and 11 are put into tier 0*, where tier 0 is the second lowest tier and tier 0* is the lowest. To follow rule three, the 11 votes in tier 0* are arranged into three groups of three and one group of two. To follow rule two, the three groups of three in tier 0* are collected into one super-group in tier 0. To follow rule 4, the group of two in tier 0* (plus one random vote) is added to a group in tier 0 with two other voters.

9.2 Appendix B – Proofs

Proof of Lemma 1. Let $M_i = t_i(1 - t_i)$. Then

$$\begin{aligned} M_i &= (t_{i-1}^3 + 3t_{i-1}^2(1 - t_{i-1}))((1 - t_{i-1})^3 + 3t_{i-1}(1 - t_{i-1})^2) \\ &= t_{i-1}^2(1 - t_{i-1})^2(3 - 2t_{i-1})(1 + 2t_{i-1}) \\ &= M_{i-1}^2(3 + 4t_{i-1} - 4t_{i-1}^2) \\ &= 3M_{i-1}^2 + 4M_{i-1}^3. \end{aligned}$$

We then have

$$\frac{\phi_z^{IEC}}{\phi_z^{maj}} = \frac{2^z \prod_{i=0}^{z-1} M_i}{\binom{N-1}{(N-1)/2} M_0^{(N-1)/2}} \quad (4)$$

$$\begin{aligned} &= \frac{2^z M_0 M_1 \dots M_{z-1}}{\binom{N-1}{(N-1)/2} M_0^{(3^z-1)/2}} \\ &= \frac{2^{z+1} M_0 (3M_0^2 + 4M_0^3) (3(3M_0^2 + 4M_0^3)^2 + 4(3M_0^2 + 4M_0^3)^3) \dots}{\binom{N-1}{(N-1)/2} M_0^{(3^z-1)/2}}. \end{aligned} \quad (5)$$

The power of the lowest power term in the numerator increases according to 2^i so for the numerator of (5) we have:

$$\begin{aligned}
& 2^{z+1} M_0 (3M_0^2 + 4M_0^3) (3(3M_0^2 + 4M_0^3)^2 + 4(3M_0^2 + 4M_0^3)^3) \dots M_{z-1} \\
> & 2^{z+1} 3^{\sum_{i=0}^{z-1} i} M_0^{\sum_{i=0}^{z-1} 2^i} \\
= & 2^{z+1} 3^{z(z-1)/2} M_0^{(2^z-1)/2}.
\end{aligned}$$

Turning to the denominator, because we are focusing on large values of z , we can apply Stirling's formula. The limit of $\binom{N-1}{(N-1)/2}$ is $\frac{2^{N-1}}{\sqrt{\pi(N-1)/2}} = \frac{2^{3^z-1}}{\sqrt{\pi(3^z-1)/2}}$. For large z we therefore have that

$$\begin{aligned}
\frac{\phi_z^{IEC}}{\phi_z^{maj}} &> \frac{2^{z+1} 3^{z(z-1)/2} M_0^{(2^z-1)/2}}{\frac{2^{3^z-1}}{\sqrt{\pi(3^z-1)/2}} M_0^{(3^z-1)/2}} \\
&= \sqrt{\pi(3^z-1)/2} \times 2^{z+2-3^z} \times 3^{z(z-1)/2} \times M_0^{(2^z-3^z)/2}.
\end{aligned}$$

As $z \rightarrow \infty$ we can select the term in each exponent that dominates the others. We then get

$$\begin{aligned}
\lim_{z \rightarrow \infty} \frac{\phi_z^{IEC}}{\phi_z^{maj}} &> \lim_{z \rightarrow \infty} \sqrt{\pi(3^z-1)/2} \times 2^{z+2-3^z} \times 3^{z(z-1)/2} \times M_0^{(2^z-3^z)/2} \\
&= \lim_{z \rightarrow \infty} \sqrt{\frac{\pi}{2}} \times 2^{-3^z} \times 3^{z^2/2} \times M_0^{-3^z/2} \\
&= \lim_{z \rightarrow \infty} \sqrt{\frac{\pi}{2}} \times \left(\frac{1}{4M_0}\right)^{3^z} \times 3^{z^2/2}.
\end{aligned}$$

If $M_0 < \frac{1}{4}$ then $\frac{1}{4M_0} > 1$. This limit is then clearly infinite. For $M_0 = \frac{1}{4}$, however, the procedure of selecting only the term in each exponent that dominates the others is inappropriate because the exponents on 2 and M_0 cancel. We must therefore include the second order terms if $M_0 = \frac{1}{4}$. Because this is a single point at which to evaluate the ratio, we can do this directly. If $M_0 = \frac{1}{4}$

them $M_i = \frac{1}{4}$ for all i . The relative power becomes

$$\begin{aligned}
\frac{\phi_z^{IEC}}{\phi_z^{maj}} &= \frac{2^z \prod_{i=0}^{z-1} M_i}{\binom{N-1}{(N-1)/2} M_0^{(N-1)/2}} \\
&= \frac{2^z \prod_{i=0}^{z-1} \frac{1}{4}}{\binom{N-1}{(N-1)/2} \frac{1}{4}^{(N-1)/2}} \\
&= \frac{2^z \frac{1}{4}^{z-1}}{\binom{N-1}{(N-1)/2} \frac{1}{4}^{(N-1)/2}} \\
&= \frac{\frac{1}{2}^{z-2}}{\binom{N-1}{(N-1)/2} \frac{1}{2}^{(3^z-1)}}.
\end{aligned}$$

Applying Stirling's rule once again,

$$\begin{aligned}
\lim_{z \rightarrow \infty} \frac{\phi_z^{IEC}}{\phi_z^{maj}} &= \lim_{z \rightarrow \infty} \sqrt{\pi(3^z - 1)/2} \times 2^{2-z} \\
&= \lim_{z \rightarrow \infty} \sqrt{\frac{\pi}{2}} \times \frac{3^{z/2}}{2^z} \\
&= \lim_{z \rightarrow \infty} \sqrt{\frac{\pi}{2}} \times \left(\frac{\sqrt{3}}{2}\right)^{z/2}.
\end{aligned}$$

Because $\frac{\sqrt{3}}{2} < 1$, this limit is 0 as z goes to infinity. ■

Proof of Lemma 2. From equation (4), we have that

$$\frac{\phi_z^{IEC}}{\phi_z^{maj}} = \frac{2^{z+1} \prod_{i=0}^{z-1} M_i}{\frac{(3^z-1)!}{\frac{(3^z-1)!}{2}} M_0^{(3^z-1)/2}} = \frac{2^{z+1} \prod_{i=0}^{z-1} M_i}{\frac{(3^z-1)!}{\frac{(3^z-1)!}{2}} M_0^{(3^z-1)/2}}.$$

The polynomial in the numerator is defined recursively by $\prod_{i=0}^{z-1} M_i$, where $M_i = 3M_{i-1}^2 + 4M_{i-1}^3$.

The highest order term in a given M_i is 3^i . Therefore, the highest order in the polynomial is $\sum_{i=0}^{z-1} 3^i$.

Multiplying this sum by $(3 - 1)$ yields

$$(3 - 1) \times \sum_{i=0}^{z-1} 3^i = \sum_{i=0}^z 3^i - \sum_{i=0}^{z-1} 3^i = 3^z - 1.$$

Therefore, $\sum_{i=0}^{z-1} 3^i = \frac{3^z - 1}{2}$. We can then write our ratio from (4) as

$$\frac{\phi_z^{IEC}}{\phi_z^{maj}} = \frac{2^{z+1} \sum_{i=0}^{\frac{3^z-1}{2}} a_i M_0^i}{\frac{(3^z-1)!}{(\frac{3^z-1}{2})!^2} M_0^{(3^z-1)/2}},$$

for some positive integers a_i . Recalling that the highest order term in this polynomial is $M_0^{(3^z-1)/2}$, we can divide the numerator and denominator by $M_0^{(3^z-1)/2}$ to get

$$\frac{\phi_z^{IEC}}{\phi_z^{maj}} = \frac{2^{z+1}}{\frac{(3^z-1)!}{(\frac{3^z-1}{2})!^2}} \sum_{i=0}^{\frac{3^z-1}{2}} a_i M_0^{-i}.$$

Taking a derivative with respect to M_0 we get

$$\frac{\partial}{\partial M_0} \frac{\phi_z^{IEC}}{\phi_z^{maj}} = \frac{2^{z+1}}{\frac{(3^z-1)!}{(\frac{3^z-1}{2})!^2}} \sum_{i=0}^{\frac{3^z-1}{2}} -i a_i M_0^{-i-1}.$$

The leading term is a positive constant, and within the sum each a_i is positive. M_0 is positive as well implying that M_0^{-i-1} is positive for all i . $-i$ is clearly negative so this derivative is negative.

■

Proof of Proposition 1.

1. Since $\binom{N-1}{(N-1)/2} > 0$ and $p_F \in (0, 1)$, $\phi^{maj} > 0$. Because $t_j = t_{j-1}^3 + 3t_{j-1}^2(1 - t_{j-1})$ implies that when $t_{j-1} \in (0, 1)$, $t_j \in (0, 1)$, and a product of positive numbers is positive, we also have that $\phi^{IEC} > 0$. A universal voting equilibrium is maintained when $\phi^k > C/V$, so $\bar{C}^k = \phi^k V > 0$.

2. Since $\frac{V}{2} \times 2^z \prod_{j=0}^{z-1} t_j(1-t_j) < V \left(\frac{1}{2}\right)^z$ and $\lim_{z \rightarrow \infty} V \left(\frac{1}{2}\right)^z = 0$, $\lim_{z \rightarrow \infty} \frac{V}{2} \times 2^z \prod_{j=0}^{z-1} t_j(1-t_j) \leq 0$.

Since this last term is weakly positive, it must equal zero, so $\lim_{N \rightarrow \infty} \overline{C}^{IEC} = 0$. The limit of

$V\phi^{maj}$ can be found using Stirling's formula, and is equal to $\frac{2^{3^z-1}V}{\sqrt{\pi(3^z-1)/2}}(p_F(1-p_F))^{(3^z-1)/2} < \frac{2^{3^z-1}V}{\sqrt{\pi(3^z-1)/2}}(1/4)^{(3^z-1)/2} = \frac{V}{\sqrt{\pi(3^z-1)/2}}$, whose limit is clearly zero.

3. Letting $p_F = t_0$, this follows immediately from Lemma 1.

■

Proof of Proposition 2. If only type 0 voters vote, then a type C voter, if she votes, would affect the outcome if either $T_{S0} = T_{F0}$ or, given that she is type j , if $T_{j0} = T_{-j0} - 1$. In either case, the probability of her side winning increases by $1/2$ if she votes. Therefore,

$$\begin{aligned} \phi_F^{maj} &= \frac{1}{2} \times \sum_{k=0}^{(N-1)/2} \binom{N-1}{k}^2 (p_{F0} \times p_{S0})^k [(1-p_{F0})(1-p_{S0})]^{N-1-k} \\ &\quad + \frac{1}{2} \times \sum_{k=0}^{(N-1)/2} \binom{N-1}{k-1} \binom{N-1}{k} p_{F0}^{k-1} p_{S0}^k (1-p_{F0})^{N-k} (1-p_{S0})^{N-1-k}, \end{aligned}$$

while a similar expression follows for ϕ_S^{maj} , with p_{S0} and p_{F0} switched. for large N , this value can be approximated as the area under the standard normal density between limits of integration whose upper and lower bounds converge to equal values of 0 in each case. Therefore, $\underline{C}_j^{maj} = V\phi_j^{maj} \rightarrow 0$ as N goes to infinity. ■

Proof of Propositions 3 and 4. In a proposed universal abstention equilibrium where the number of voters is known to be $3^y - 1$, the calculation the expected impact of a voter is exactly the same as in a universal voting equilibrium, replacing N with N_0 . A universal abstention equilibrium will exist if the voter would prefer not to vote, so the threshold is calculated precisely the same way as in Proposition 1, but with the reverse sign. The proof of comparative statics also follows the same lines, but with the reverse sign. ■

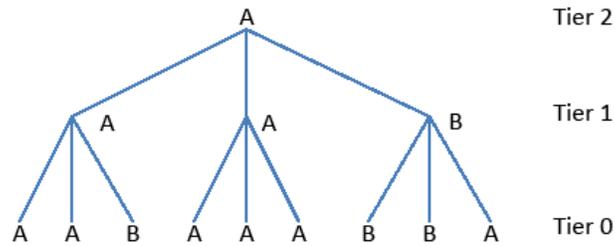


Figure 1: This is an example of a two tier pyramid. There are nine voters arranged into three groups of three. In the first group two out of three choose A; in the second group all three choose A; in the third group two out of three choose B. Therefore the first and second groups “vote” for A and the third “votes” for B. Moving up to tier one, there are two “votes” for A and one for B so this tier “votes” for A. A is therefore the choice of this set of voters. Note that both voters who vote for A in the left-most group are pivotal and other voters are not.

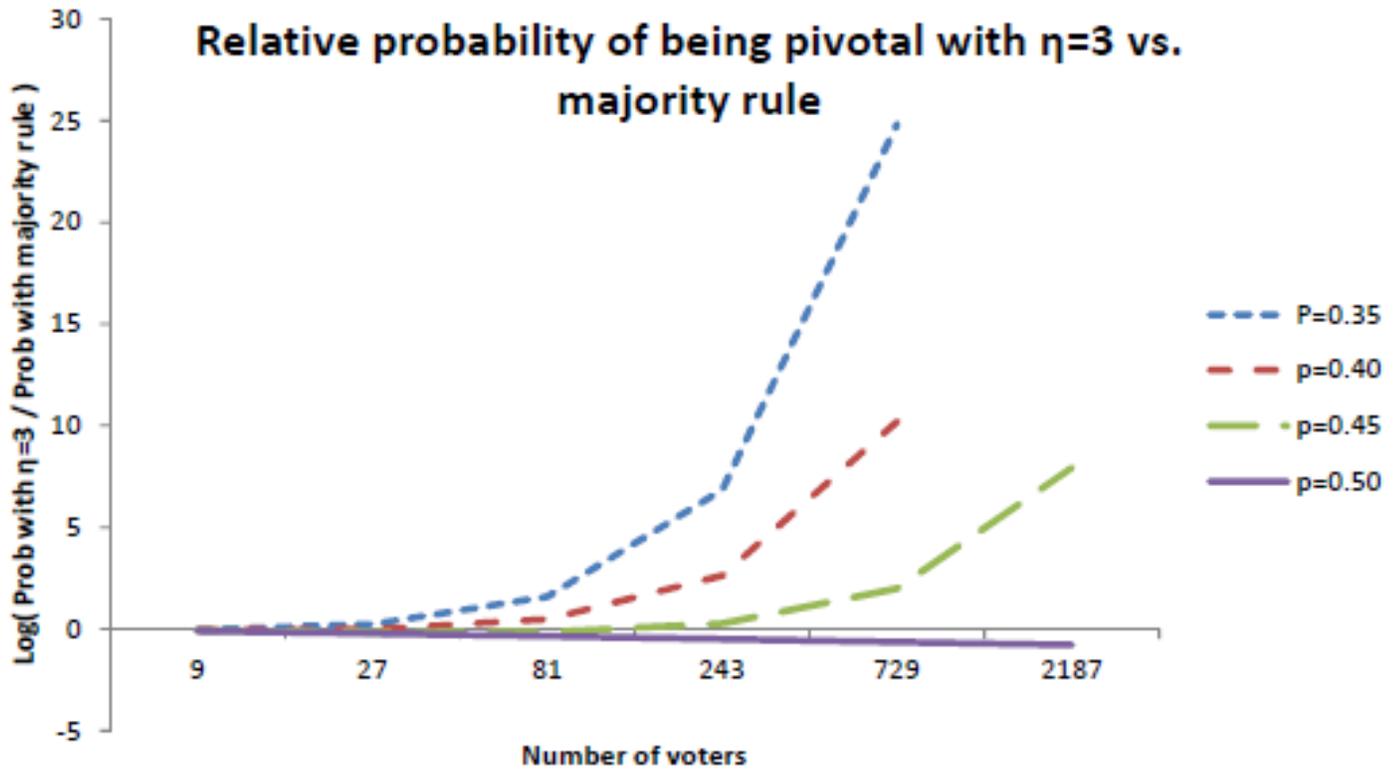


Figure 2: This figure displays the relative probability of a voter being pivotal under the IEC and majority mechanisms as a function of population size for four values of p_0 . As the vote is expected to be increasingly close, p_0 tends to $\frac{1}{2}$ and the IEC rule is less powerful. As the number of voters increases, the IEC rule becomes infinitely more powerful than the majority rule except for exactly equal choices.

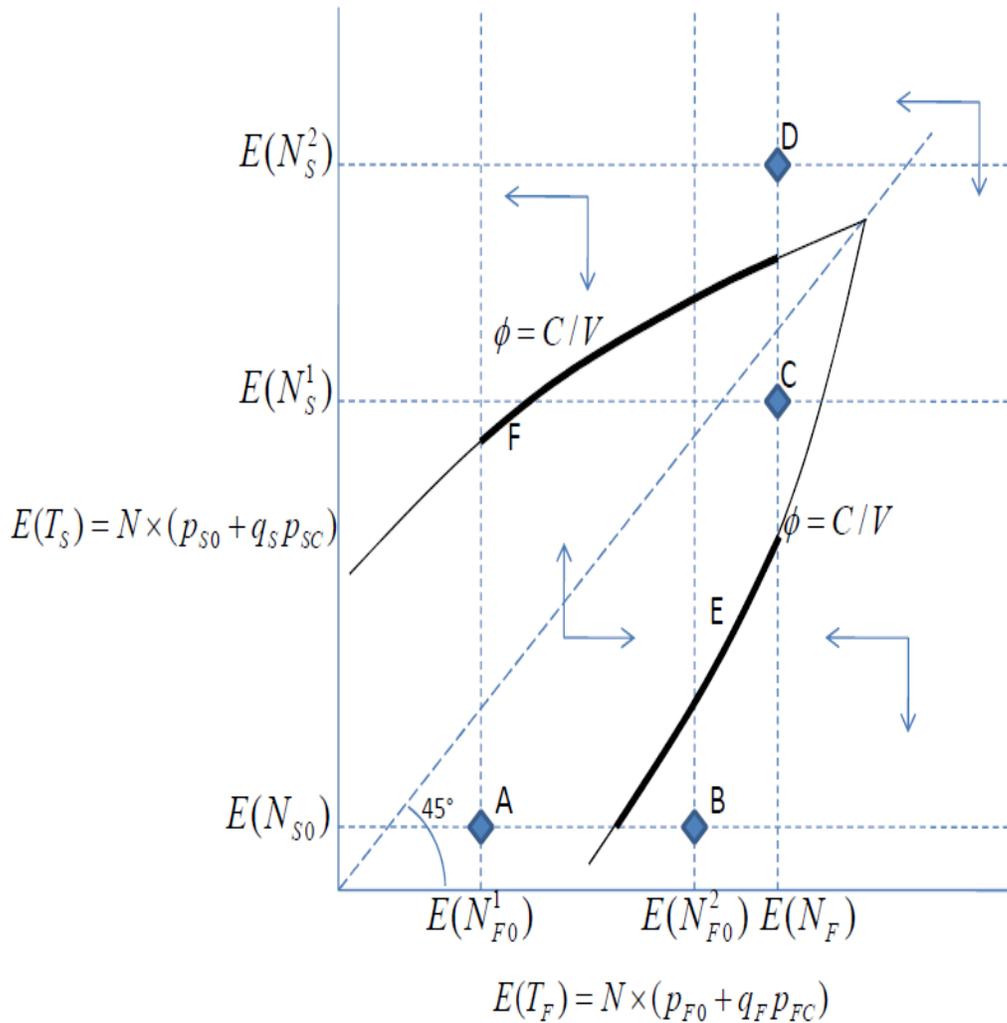


Figure 3: This figure displays the set of potential pure and mixed strategy equilibria of the game for an arbitrary monotonic voting rule. The curves represent the set of points where a vote's expected effect is equal to C/V , making a voter indifferent between voting and abstaining. Between the curves, the expected effect is greater than C/V so voters would prefer voting, and outside the curves, the expected effect is less than C/V so voters would prefer to abstain. For clarity, the expected effect is drawn as equal for voters of each type. Depending on the expected values of N_{ix} , there are a variety of potential equilibria. A candidate equilibrium must lie either on a curve (so that voters are indifferent to voting and abstaining) or at a boundary. An equilibrium is only stable if the arrows from all feasible directions point toward the point. In this figure, for example, A is not a pure strategy equilibrium because type C voters would prefer to vote rather than abstain, but at point A they are expected to abstain (i.e., $q_S = q_F = 0$). If $E(N_S) = E(N_S^1)$, then point C is an equilibrium. All voters are voting (i.e., $q_S = q_F = 1$), and all strictly prefer to vote, as shown by the arrows between the curves pointing up and to the right. Candidate equilibrium points on bold curves E and F are not stable because the up-down and left-right arrows, respectively, point away from each other. This implies that small changes in the likelihood of voting for type S and F voters, respectively, would prompt all voters of the same type to make the same change.

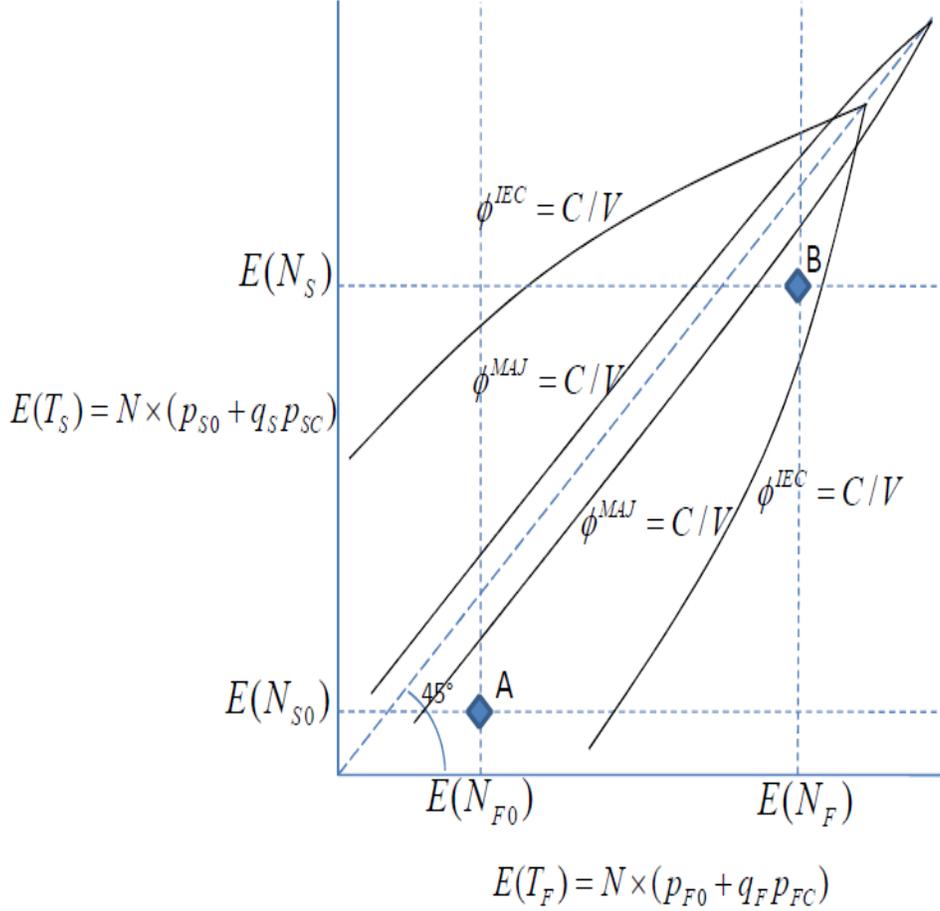


Figure 4: The IEC increases the expected effect of a vote for both types of voter, relative to majority rule, unless the expected number of voters for each side is nearly equal (in the limit, exactly equal). This means that, given a number of voters and a probability vector \vec{p} , the curves representing $\phi = C/V$ for majority rule lie within those for the IEC. Inside the curves, type C voters would strictly prefer to vote. Outside the curves, they would strictly prefer to abstain. Point A is a candidate universal abstention equilibrium (i.e., $q_S = q_F = 0$). As it lies outside the curves for majority rule, it is indeed an equilibrium, as voters prefer to abstain. As it lies inside the curves for the IEC, it fails to be an equilibrium, as type C voters would prefer to vote. Similarly, the proposed universal voting equilibrium B lies outside the curves for majority rule, so it is not an equilibrium, while it lies within the curves for the IEC and therefore is an equilibrium. It is also possible for point A to lie within the curves for both mechanisms, or for point B to lie outside the curves for both. In those cases, the same type of equilibria exist for both mechanisms. For a universal voting equilibrium to lie within the curves for majority rule and not the IEC, it must be the case that $p_S \approx p_F$, with strict equality in the limit as $N \rightarrow \infty$. It is not possible for the IEC to permit a universal abstention equilibrium while majority rule does not.

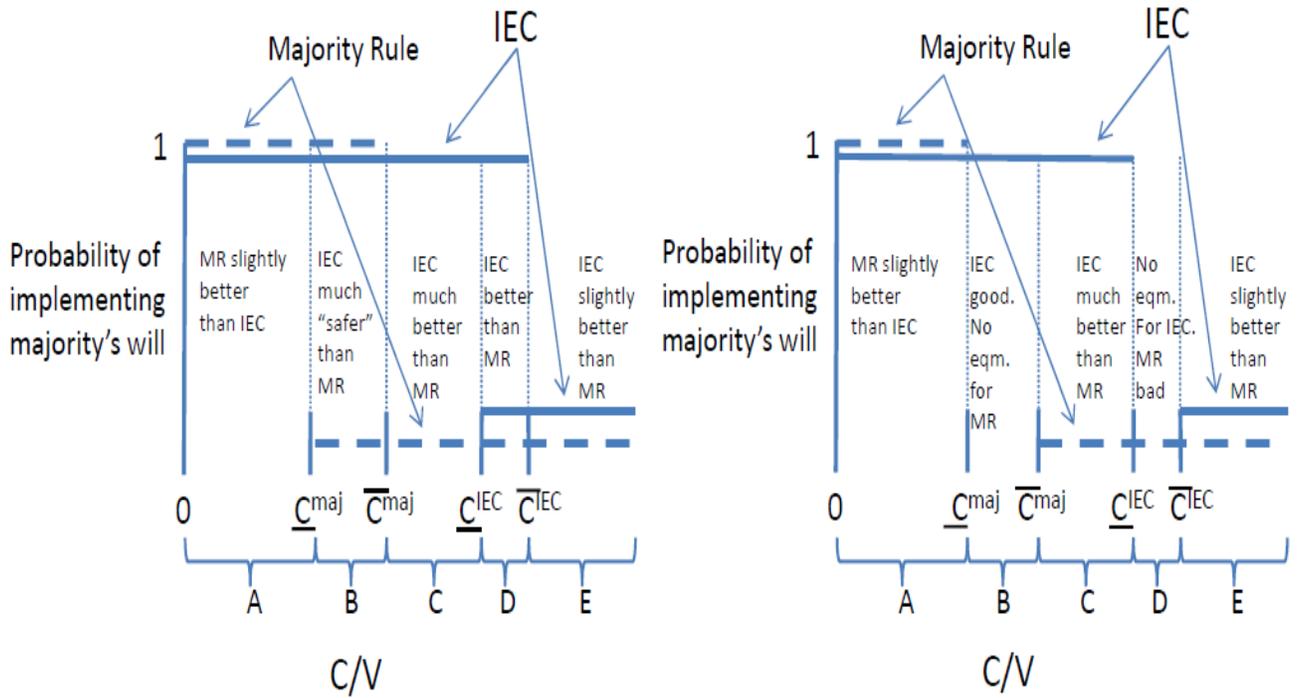


Figure 5: This figure plots the existence of universal voting and universal abstention equilibria as a function of C/V for both the IEC and majority rule, under the assumption that type 0 voters prefer (on average) a different choice than the overall population. It is possible that $\underline{C}^k < \overline{C}^k$, $k \in \{maj, IEC\}$, in which case intermediate values of C/V allow both universal voting and universal abstention equilibria. It is also possible that $\underline{C}^k > \overline{C}^k$, in which case intermediate values of C/V do not allow any stable equilibrium. The left panel displays the former case and the right panel the latter. Beginning with the left panel, in section A, when $C/V < \underline{C}^{maj}$, both majority rule and the IEC feature unique, universal voting equilibria. Because the IEC sometimes assigns as winner the side receiving fewer votes, majority rule is slightly better than the IEC at implementing the majority's will. In section B, where $\underline{C}^{maj} < C/V < \overline{C}^{maj}$, majority rule features both universal voting and universal abstention equilibria. Without an equilibrium selection rule, we cannot say whether majority rule or the IEC is superior, but because the IEC almost always selects the winner according to the majority's preferences, and majority rule may induce universal abstention, we can say that the IEC is at least a safer bet. In section C, where $\overline{C}^{maj} < C/V < \underline{C}^{IEC}$, The IEC is clearly better, as it induces universal voting rather than universal abstention. In section D, where $\underline{C}^{IEC} < C/V < \overline{C}^{IEC}$, the IEC supports both universal voting and universal abstention equilibria. Either one, however, is better than the unique universal abstention equilibrium associated with majority rule, which is assured to make the choice less preferred by the majority. In section E, where $\overline{C}^{IEC} < C/V$, the IEC is slightly better because it chooses the majority-preferred choice at least sometimes. In the right panel, the difference is that, rather than each mechanism sometimes supporting both types of equilibrium, they both sometimes support no stable equilibrium. Sections A, C, and E have identical properties as in the left hand panel. In section B, where $\overline{C}^{maj} < C/V < \underline{C}^{maj}$, majority rule does not allow a stable equilibrium. As the IEC almost always implements the will of the majority, it is clearly superior. In section D, where $\overline{C}^{IEC} < C/V < \underline{C}^{IEC}$, the IEC does not allow a stable equilibrium. However, as majority rule certainly does not implement the majority's will for these values of C/V , any outcome induced by the IEC must be weakly better.

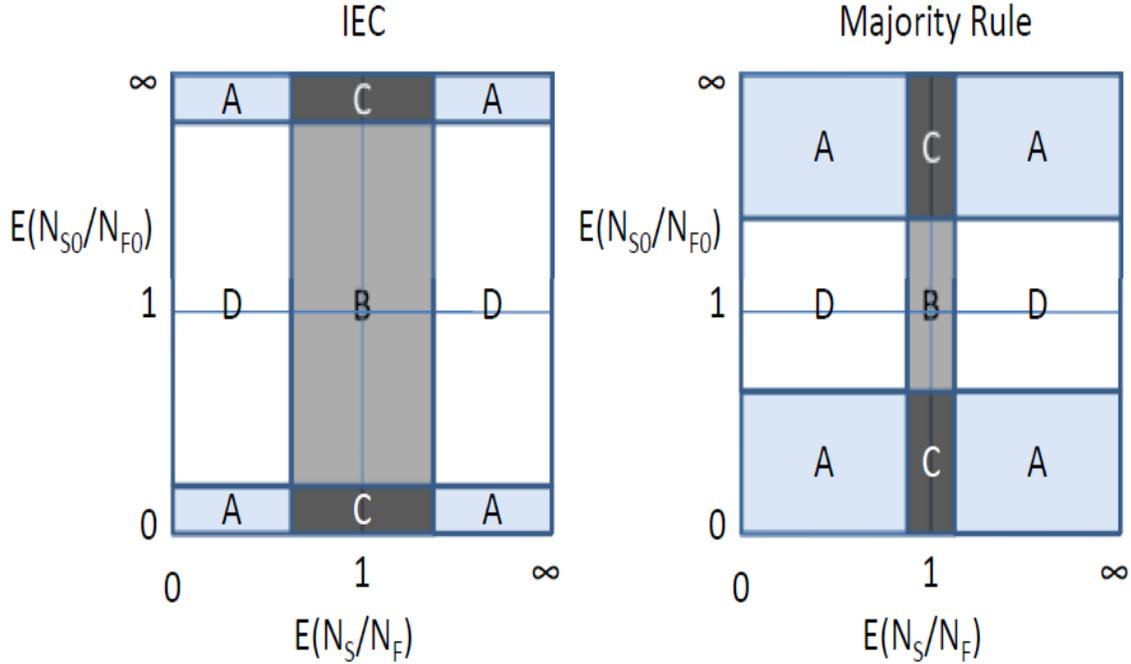


Figure 6: These panels display whether one or both types of equilibrium exist for each mechanism, as we vary the expected difference in voter preferences. In each case, N and C/V are fixed. Spaces labeled “A” support only universal abstention equilibria. The vote share of type 0 voters is expected to be so lopsided that it is not worthwhile for any type C voter to vote. Spaces labeled “B” support only universal voting equilibria. The vote is expected to be close enough if everybody votes that it is worth it for each voter to vote. Spaces labeled “C” support both types of equilibrium. While the vote is expected to be close enough if everybody votes that it is worth it for each voter to vote, the vote is also expected to be so lopsided if only type 0 voters vote that, if type C voters expect that to occur, they will not vote. Spaces labeled “D” do not support stable equilibria. The vote is close enough if only type 0 voters vote that type C voters will prefer to vote. However, if all of them do so, then the vote is lopsided enough that it is no longer worth them voting. Note that area B is larger for the IEC than majority rule. In fact, it may be millions or billions of times as large for any reasonable number of voters. Also, area A is smaller for the IEC. Note that as C/V or N rise, area B would shrink in each figure, with the left- and right-most borders approaching the center. Area B would be eliminated for the IEC before majority rule because majority rule offers voters slightly more power if $p_S = p_F$.

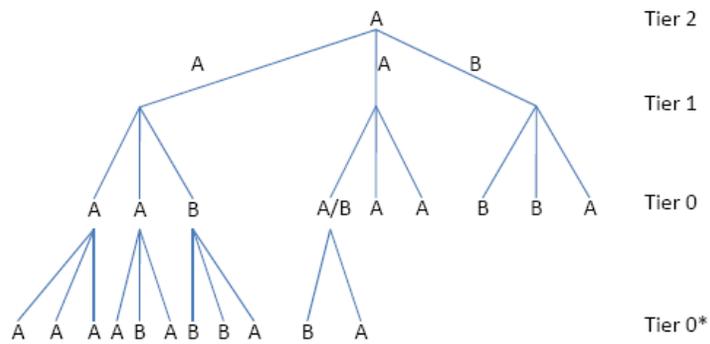


Figure 7: This would be the pyramid structure for 16 votes. Because 16 is not a power of three all votes cannot be put into a single base tier. Instead, five go in tier 0 and eleven in tier 0*. One vote in the middle group in tier 0 could go either to A or B, depending on the randomization outcome for the third vote in tier 0*.

Relative probability of being pivotal with $\eta=3$ vs. $\eta=9$

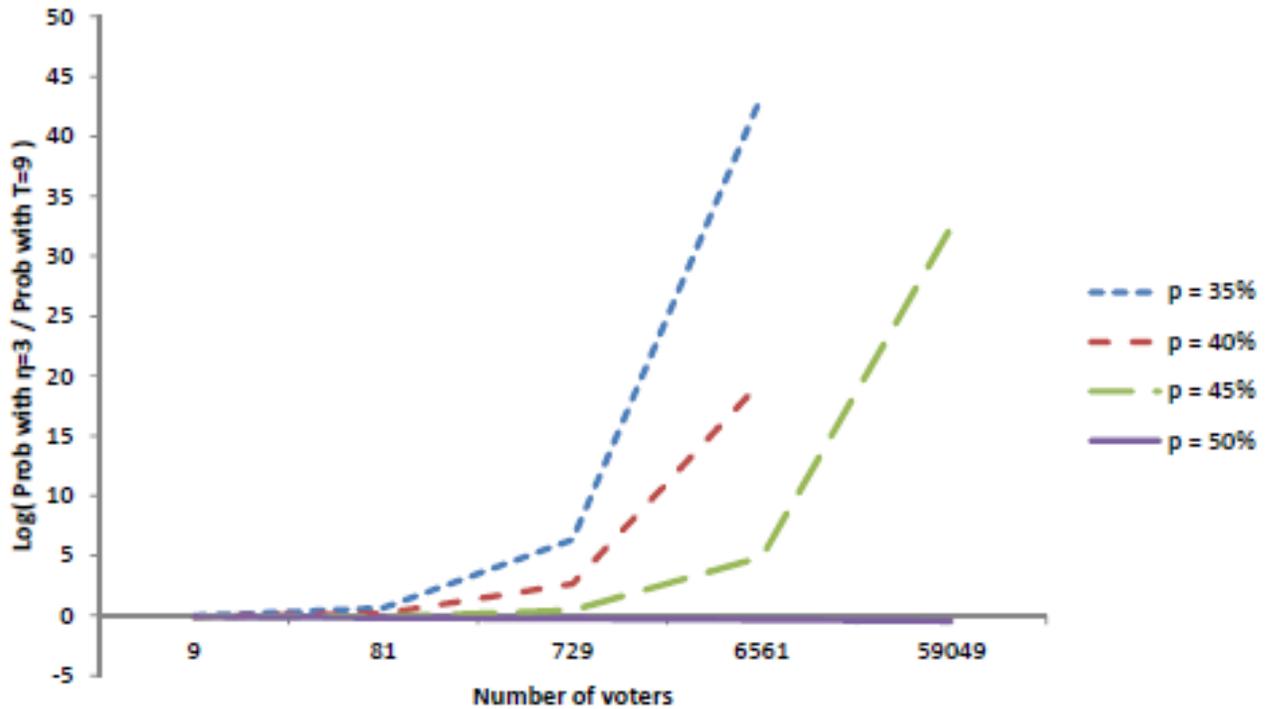


Figure 8: This figure displays the ratio of the expected effect of a vote under the IEC mechanism with $\eta = 3$ versus $\eta = 9$, as a function of population size N , for four values of p_0 . As the vote is expected to be less close (p_0 moves away from $\frac{1}{2}$), the IEC rule with $\eta = 3$ offers voters greater power relative to the IEC rule with $\eta = 9$. As the number of voters increases, the IEC rule with $\eta = 3$ offers infinitely more power than with $\eta = 9$.

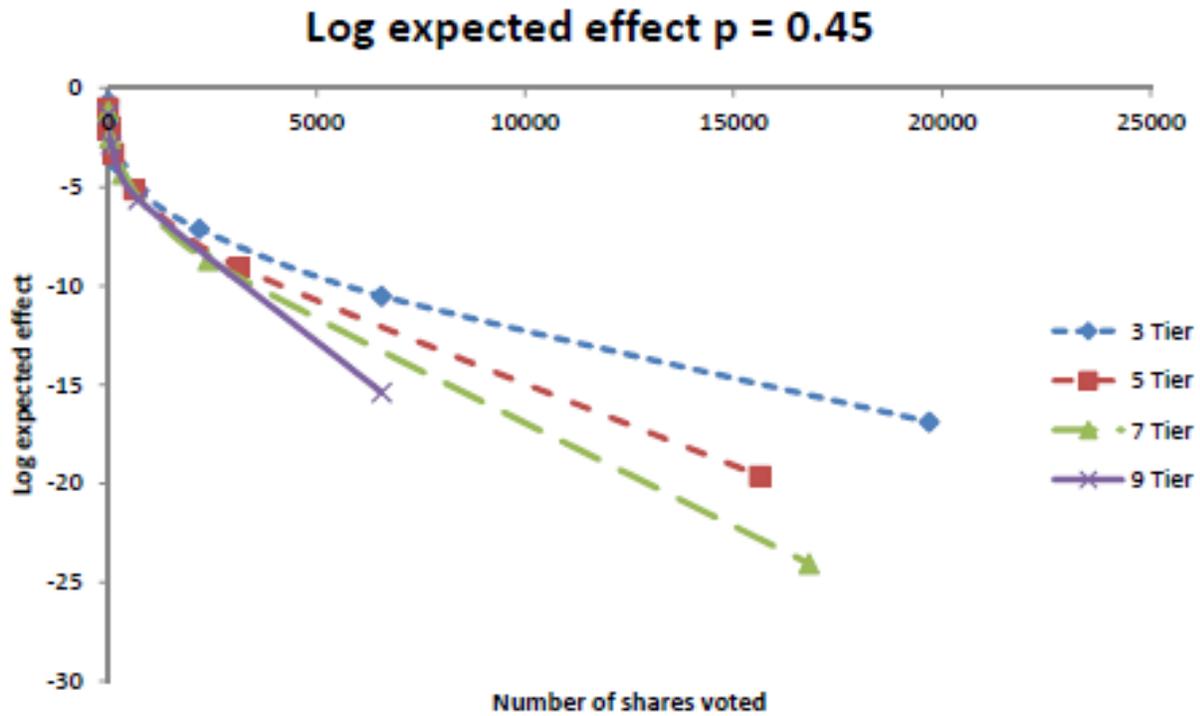


Figure 9: This chart shows the log expected effect for various tier sizes and numbers of voters, N . The expected fraction in favor of A is assumed to be 45%. Within each tier size T , the values of N chosen are successive powers of T . Spaces between points in the same sequence are filled in so the trends are more clearly visible.