Valuation Risk and Asset Pricing*

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Abstract

Standard representative-agent models have difficulty in accounting for the weak correlation between stock returns and measurable fundamentals, such as consumption and output growth. This failing underlies virtually all modern asset-pricing puzzles. The correlation puzzle arises because these models load all uncertainty onto the supply side of the economy. We propose a simple theory of asset pricing in which demand shocks play a central role. These shocks give rise to valuation risk that allows the model to account for key asset pricing moments, such as the equity premium, the bond term premium, and the weak correlation between stock returns and fundamentals.

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1. Introduction

In standard representative-agent asset-pricing models, the expected return to an asset reflects the covariance between the asset's payoff and the agent's stochastic discount factor. An important challenge to these models is that the correlation and covariance between stock returns and measurable fundamentals, especially consumption growth, is weak at both short and long horizons. Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2001) call this phenomenon the *correlation puzzle*. More recently, Lettau and Ludvigson (2011) document this puzzle using different methods. According to their estimates, the shock that accounts for the vast majority of asset-price fluctuations is uncorrelated with consumption at virtually all horizons.

The basic disconnect between measurable macroeconomic fundamentals and stock returns underlies virtually all modern asset-pricing puzzles, including the equity-premium puzzle, Hansen-Singleton (1982)-style rejection of asset-pricing models, violation of Hansen-Jagannathan (1991) bounds, and Shiller (1981)-style observations about excess stock-price volatility. It is also at the root of the high estimates of risk aversion and correspondingly large amounts that agents would pay for early resolution of uncertainty in long-run risk models of the type proposed by Bansal and Yaron (2004) (see Epstein, Farhi, and Strzalecki (2014)).

A central finding of modern empirical finance is that variation in asset returns is overwhelmingly due to variation in discount factors (see Campbell and Ammer (1993) and Cochrane (2011)). A key question is: how should we model this variation? In classic asset-pricing models, all uncertainty is loaded onto the supply side of the economy. In Lucas (1978) tree models, agents are exposed to random endowment shocks, while in production economies they are exposed to random productivity shocks. Both classes of models abstract from shocks to the demand for assets. Not surprisingly, it is very difficult for these models to simultaneously resolve the equity premium puzzle and the correlation puzzle.

We propose a simple theory of asset pricing in which demand shocks, arising from stochastic changes in agents' rate of time preference, play a central role in the determination of asset prices. These shocks amount to a parsimonious way of modeling the variation in discount rates stressed by Campbell and Ammer (1993) and Cochrane (2011). Our model implies that the law of motion for these shocks plays a first-order role in determining the equilibrium behavior of variables like the price-dividend ratio, equity returns and bond re-

turns. So, our analysis is disciplined by the fact that the law of motion for time-preference shocks must be consistent with the time-series properties of these variables.

In our model, the representative agent has recursive preferences of the type considered by Kreps and Porteus (1978), Weil (1989), and Epstein and Zin (1991). When the risk-aversion coefficient is equal to the inverse of the elasticity of intertemporal substitution, recursive preferences reduce to constant-relative risk aversion (CRRA) preferences. We show that, in this case, time-preference shocks have negligible effects on key asset-pricing moments such as the equity premium.

We consider two versions of our model. The benchmark model is designed to highlight the role played by time-preference shocks per se. Here consumption and dividends are modeled as random walks with conditionally homoscedastic shocks. While this model is very useful for expositional purposes, it suffers from some clear empirical shortcomings, e.g. the equity premium is constant. For this reason we consider an extended version of the model in which the shocks to the consumption and dividend process are conditionally heteroskedastic. We find that adding these features improves the performance of the model.[1]

We estimate our model using a Generalized Method of Moments (GMM) strategy, implemented with annual data for the period 1929 to 2011. We assume that agents make decisions on a monthly basis. We then deduce the model's implications for annual data, i.e. we explicitly deal with the temporal aggregation problem.[2]

It turns out that, for a large set of parameter values, our model implies that the GMM estimators suffer from substantial small-sample bias. This bias is particularly large for moments characterizing the predictability of excess returns and the decomposition of the variance of the price-dividend ratio proposed by Cochrane (1992). In light of this fact, we modify the GMM procedure to focus on the plim of the model-implied small-sample moments rather than the plim of the moments themselves. We show that this modification makes an important difference in assessing the model's empirical performance.

We show that time-preference shocks help explain the equity premium as long as the risk-aversion coefficient and the elasticity of intertemporal substitution are either both greater than one or both are smaller than one. This condition is satisfied in the estimated benchmark and extended models.

[1] These results parallel the findings of Bansal and Yaron (2004) who show that allowing for conditionally heteroskedasticity improves the performance of long-run risk models.

[2] Bansal, Kiku, and Yaron (2012) pursue a similar strategy in estimating a long-run risk model. They estimate the frequency with which agents make decision and find that it is roughly equal to one month.

Allowing for sampling uncertainty, our model accounts for the equity premium and the volatility of stock and bond returns, even though the estimated degree of agents' risk aversion is very moderate (roughly 1.5). Critically, the extended model also accounts for mean, variance and persistence of the price-dividend ratio and the risk-free rate. In addition, it accounts for the correlation between stock returns and fundamentals such as consumption, output, and dividend growth at short, medium and long horizons. Finally, the model also account for the observed predictability of excess returns by lagged price-dividend ratios.

We define *valuation risk* as the part of the excess return to an asset that is due to the volatility of the time-preference shock. According to our estimates, valuation risk is a much more important determinant of asset returns than conventional covariance risk. We show that valuation risk is an increasing function of an asset's maturity. So, a natural test of our model is whether it can account for bond term premia and the return on stocks relative to long-term bonds. We pursue this test using stock returns as well as ex-post real returns on bonds of different maturity and argue that the model's implications are consistent with the data. We are keenly aware of the limitations of the available data on real-bond returns, especially at long horizons. Still, we interpret our results as being very supportive of the hypothesis that valuation risk is a critical determinant of asset prices.

There is a literature that models shocks to the demand for assets as arising from time-preference or taste shocks. For example, Garber and King (1983) and Campbell (1986) consider these types of shocks in early work on asset pricing. Tesar and Stockman (1995), Pavlova and Rigobon (2007), and Gabaix and Maggiori (2013) study the role of taste shocks in explaining asset prices in an open economy model. In the macroeconomic literature, Eggertsson and Woodford (2003), Eggertsson (2004), model changes in savings behavior as arising from time-preference shocks that make the zero lower bound on nominal interest rates binding.[3] Hall (2014) stresses the importance of variation in discount rates in explaining the cyclical behavior of unemployment.

Time-preference shocks can also be thought of a simple way of capturing the notion that fluctuations in market sentiment contribute to the volatility of asset prices, as emphasized by authors such as in Barberis, Shleifer, and Vishny (1998) and Dumas, Kurshev and Uppal (2009). Finally, in independent work, contemporaneous with our own, Maurer (2012) explores the impact of time-preference shocks in a calibrated continuous-time representative

[3] See also Huo and Rios-Rull (2013), Correia, Farhi, Nicolini, and Teles (2013), and Fernandez-Villaverde, Guerron-Quintana, Kuester, Rubio-Ramírez (2013).

agent model with Duffie-Epstein (1992) preferences.[4]

Our paper is organized as follows. In Section 2 we document the correlation puzzle using U.S. data for the period 1929-2011 as well as the period 1871-2006. In Section 3 we present our benchmark model and extended models. We discuss our estimation strategy in Section 4. In Section 5 we present our empirical results. In Section 6 we study the empirical implications of the model for bond term premia, as well as the return on stocks relative to long-term bonds. Section 7 concludes.

2. The correlation puzzle

In this section we examine the correlation between stock returns and fundamentals as measured by the growth rate of consumption, output, dividends, and earnings.

2.1. Data sources

We consider two sample periods: 1929 to 2011 and 1871 to 2006. For the first sample, we obtain nominal stock and bond returns from Kenneth French's website. We use the measure of consumption expenditures and real per capita Gross Domestic Product constructed by Barro and Ursua (2011), which we update to 2011 using National Income and Product Accounts data. We compute per-capita variables using total population (POP).[5] We obtain data on real S&P500 earnings and dividends from Robert Shiller's website. We use data from Ibbotson and Associates on the real return to one-month Treasury bills, the nominal yield on intermediate-term government bonds (with approximate maturity of five years), and the nominal yield on long-term government bonds (with approximate maturity of twenty years).

[4] Normandin and St-Amour (1998) study the impact of preference shocks in a model similar to ours. Unfortunately, their analysis does not take into account the fact that covariances between asset returns, consumption growth, and preferences shocks depend on the parameters governing preferences and technology. As a result, their empirical estimates imply that preference shocks reduce the equity premium. In addition, they argue that they can explain the equity premium with separable preferences and preference shocks. This claim contradicts the results in Campbell (1986) and the theorem in our Appendix B.

[5] This series is not subject to a very important source of measurement error that affects another commonly-used population measure, civilian noninstitutional population (CNP16OV). Every ten years, the CNP16OV series is adjusted using information from the decennial census. This adjustment produces large discontinuities in the CNP16OV series. The average annual growth rates implied by the two series are reasonably similar: 1.2 for POP and 1.4 for CNP16OV for the period 1952-2012. But the growth rate of CNP16OV is three times more volatile than the growth rate of POP. Part of this high volatility in the growth rate of CNP16OV is induced by large positive and negative spikes that generally occur in January. For example, in January 2000, 2004, 2008, and 2012 the annualized percentage growth rates of CNP16OV are 14.8, -1.9, -2.8, and 8.4, respectively. The corresponding annualized percentage growth rates for POP are 1.1, 0.8, 0.9, and 0.7.

We convert nominal returns and yields to real returns and yields using the rate of inflation as measured by the consumer price index.

For the second sample, we use data on real stock and bond returns from Nakamura, Steinsson, Barro, and Ursua (2010) for the period 1870-2006. We use the same data sources for consumption, expenditures, dividends and earnings as in the first sample.

As in Mehra and Prescott (1985) and the associated literature, we measure the risk-free rate using realized real returns on nominal, one-year Treasury Bills. This measure is far from perfect because there is inflation risk, which can be substantial, particularly for long-maturity bonds. An alternative is to assume a stochastic process for inflation and use it to compute expected inflation. We did not follow this route because our sample encompasses very different monetary regimes, including the Gold Standard, Bretton Woods, as well as the Volcker period and its aftermath.

2.2. Empirical results

Table 1, panel A presents results for the sample period 1929-2011. We report correlations at the one-, five- and ten-year horizons. The five- and ten-year horizon correlations are computed using five- and ten-year overlapping observations, respectively. We report Newey-West (1987) heteroskedasticity-consistent standard errors computed with ten lags.

There are three key features of Table 1, panel A. First, consistent with Cochrane and Hansen (1992) and Campbell and Cochrane (1999), the growth rates of consumption and output are uncorrelated with stock returns at all the horizons that we consider. Second, the correlation between stock returns and dividend growth is similar to that of consumption and output growth at the one-year horizon. However, the correlation between stock returns and dividend growth is substantially higher at the five and ten-year horizons than the analogue correlations involving consumption and output growth. Third, the pattern of correlations between stock returns and dividend growth are similar to the analogue correlations involving earnings growth.

Table 1, panel B reports results for the longer sample period (1871-2006). The one-year correlation between stock returns and the growth rates of consumption and output are very similar to those obtained for the shorter sample. There is evidence in this sample of a stronger correlation between stock returns and the growth rates of consumption and output at a five-year horizon. But, at the ten-year horizon the correlations are, once again, statistically insignificant. The results for dividends and earnings are very similar across the

two subsamples.

Table 2 assesses the robustness of our results for the correlation between stock returns and consumption using three different measures of consumption for the period 1929-2011, obtained from the National Product and Income Accounts. With one exception, the correlations in this table are statistically insignificant. The exception is the five-year correlation between stock returns and the growth rate of nondurables and services which is marginally significant.

In summary, there is remarkably little evidence that the growth rates of consumption or output are correlated with stock returns. There is also little evidence that dividends and earnings are correlated with stock returns at short horizons.

We have focused on correlations because we find them easy to interpret. One might be concerned that a different picture emerges from the pattern of covariances between stock returns and fundamentals. It does not. For example, using quarterly U.S. data for the period 1959 to 2000, Parker (2001) argues that one would require a risk aversion coefficient of 379 to account for the equity premium given his estimate of the covariance between consumption growth and stock returns.

Observing that there is a larger covariance between current stock returns and the cumulative growth rate of consumption over the next 12 quarters, Parker (2001) argues that, even with this covariance measure, one would require a risk aversion coefficient of 38 to rationalize the equity premium (see also Grossman, Melino and Shiller (1987)).[6]

Viewed overall, the results in this section serve as our motivation for introducing shocks to the demand for assets. Classic asset-pricing models load all uncertainty onto the supply-side of the economy. As a result, they have difficulty in simultaneously accounting for the equity premium and the correlation puzzle. This difficulty is shared by the habit-formation model proposed by Campbell and Cochrane (1999) and the long-run risk models proposed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). Rare-disaster models of the type proposed by Rietz (1988) and Barro (2006) also share this difficulty because all shocks, disaster or not, are to the supply side of the model. A model with a time-varying disaster probability, of the type considered by Wachter (2012) and Gourio (2012), might be able to rationalize the low correlation between consumption and stock returns

[6] Consistent with Parker (2001) and Campbell (2003) we find, in our sample, a somewhat higher correlation consumption growth and one-year lagged stock returns. The correlation between output growth and one-year lagged stock returns is still essentially zero.

as a small sample phenomenon. The reason is that changes in the probability of disasters induces movements in stock returns without corresponding movements in actual consumption growth. This force lowers the correlation between stock returns and consumption in a sample where rare disasters are under represented. This explanation might account for the post-war correlations. But we are more skeptical that it accounts for the results in Table 1, panel B, which are based on the longer sample period, 1871 to 2006.

Below, we focus on demand shocks as the source of the low correlation between stock returns and fundamentals, rather than the alternatives just mentioned. We model these demand shocks in the simplest possible way by introducing shocks to the time preference of the representative agent. Consistent with the references in the introduction, these shocks can be thought of as capturing changes in agents' attitudes towards savings or more generally investor sentiment.

3. The model

In this section, we study the properties of a representative-agent endowment economy modified to allow for time-preference shocks. The representative agent has the constant-elasticity version of Kreps-Porteus (1978) preferences used by Epstein and Zin (1991) and Weil (1989). The life-time utility of the representative agent is a function of current utility and the certainty equivalent of future utility, U_{t+1}^*:

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \tag{3.1}$$

where C_t denotes consumption at time t and δ is a positive scalar. The certainty equivalent of future utility is the sure value of $t+1$ lifetime utility, U_{t+1}^* such that:

$$\left(U_{t+1}^* \right)^{1-\gamma} = E_t \left(U_{t+1}^{1-\gamma} \right).$$

The parameters ψ and γ represent the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively. The ratio λ_{t+1}/λ_t determines how agents trade off current versus future utility. We assume that this ratio is known at time t.[7] We refer to λ_{t+1}/λ_t as the time-preference shock.

[7] We obtain similar results with a version of the model in which the utility function takes the form: $U_t = \left[C_t^{1-1/\psi} + \lambda_t \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$. The assumption that the agents knows λ_{t+1} at time t is made to simplify the algebra and is not necessary for any of the key results.

3.1. The benchmark model

To highlight the role of time-preference shocks, we begin with a very simple stochastic process for consumption:

$$\log(C_{t+1}/C_t) = \mu + \sigma_c \varepsilon^c_{t+1}. \tag{3.2}$$

Here, μ and σ_c are non-negative scalars and ε^c_{t+1} follows an i.i.d. standard-normal distribution.

As in Campbell and Cochrane (1999), we allow dividends, D_t, to differ from consumption. In particular, we assume that:

$$\log(D_{t+1}/D_t) = \mu + \pi_{dc}\varepsilon^c_{t+1} + \sigma_d \varepsilon^d_{t+1}. \tag{3.3}$$

Here, ε^d_{t+1} is an i.i.d. standard-normal random variable that is uncorrelated with ε^c_{t+1}. To simplify, we assume that the average growth rate of dividends and consumption is the same (μ). The parameter $\sigma_d \geq 0$ controls the volatility of dividends. The parameter π_{dc} controls the correlation between consumption and dividend shocks.[8]

The growth rate of the time-preference shock evolves according to:

$$\log\left(\lambda_{t+1}/\lambda_t\right) = \rho \log\left(\lambda_t/\lambda_{t-1}\right) + \sigma_\lambda \varepsilon^\lambda_t. \tag{3.4}$$

Here, ε^λ_t is an i.i.d. standard-normal random variable. In the spirit of the original Lucas (1978) model, we assume, for now, that ε^λ_t is uncorrelated with ε^c_t and ε^d_t. We relax this assumption in Subsection 3.4.

The CRRA case In Appendix A we solve this model analytically for the case in which $\gamma = 1/\psi$. Here preferences reduce to the CRRA form:

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C^{1-\gamma}_{t+i}, \tag{3.5}$$

with $V_t = U_t^{1-\gamma}$.

The unconditional risk-free rate depends on the persistence and volatility of time-preference shocks:

$$E\left(R_{f,t+1}\right) = \exp\left(\frac{\sigma^2_\lambda/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu - \gamma^2\sigma^2_c/2).$$

[8] The stochastic process described by equations (3.2) and (3.3) implies that $\log(D_{t+1}/C_{t+1})$ follows a random walk with no drift. This implication is consistent with our data.

The unconditional equity premium implied by this model is proportional to the risk-free rate:

$$E\left(R_{c,t+1}-R_{f,t+1}\right)=E\left(R_{f,t+1}\right)\left[\exp\left(\gamma\sigma_c^2\right)-1\right]. \tag{3.6}$$

Both the average risk-free rate and the volatility of consumption are small in the data. Moreover, the constant of proportionality in equation (3.6), $\exp\left(\gamma\sigma_c^2\right)-1$, is independent of σ_λ^2. So, time-preference shocks do not help to resolve the equity premium puzzle when preferences are of the CRRA form.

3.2. Solving the benchmark model

We define the return to the stock market as the return to a claim on the dividend process. The realized gross stock-market return is given by:

$$R_{d,t+1}=\frac{P_{d,t+1}+D_{t+1}}{P_{d,t}}, \tag{3.7}$$

where $P_{d,t}$ denotes the ex-dividend stock price.

It is useful to define the realized gross return to a claim on the endowment process:

$$R_{c,t+1}=\frac{P_{c,t+1}+C_{t+1}}{P_{c,t}}. \tag{3.8}$$

Here, $P_{c,t}$ denotes the price of an asset that pays a dividend equal to aggregate consumption. We use the following notation to define the logarithm of returns on the dividend and consumption claims, the logarithm of the price-dividend ratio, and the logarithm of the price-consumption ratio:

$$\begin{aligned} r_{d,t+1} &= \log(R_{d,t+1}),\\ r_{c,t+1} &= \log(R_{c,t+1}),\\ z_{dt} &= \log(P_{d,t}/D_t),\\ z_{ct} &= \log(P_{c,t}/C_t). \end{aligned}$$

In Appendix B we show that the logarithm of the stochastic discount factor (SDF) implied by the utility function defined in equation (3.1) is given by:

$$m_{t+1}=\log(\delta)+\log(\lambda_{t+1}/\lambda_t)-\frac{1}{\psi}\Delta c_{t+1}+(1/\psi-\gamma)\log(U_{t+1}/U_{t+1}^*). \tag{3.9}$$

It is useful to rewrite this equation as:

$$m_{t+1}=\theta\log\left(\delta\right)+\theta\log\left(\lambda_{t+1}/\lambda_t\right)-\frac{\theta}{\psi}\Delta c_{t+1}+\left(\theta-1\right)r_{c,t+1}, \tag{3.10}$$

where θ is given by:

$$\theta = \frac{1-\gamma}{1-1/\psi}. \tag{3.11}$$

When $\gamma = 1/\psi$, the case of CRRA preferences, the value of θ is equal to one and the stochastic discount factor is independent of $r_{c,t+1}$.

We solve the model using the approximation proposed by Campbell and Shiller (1988), which involves linearizing the expressions for $r_{c,t+1}$ and $r_{d,t+1}$ and exploiting the properties of the log-normal distribution.[9]

Using a log-linear Taylor expansion we obtain:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}, \tag{3.12}$$

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}, \tag{3.13}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ and $\Delta d_{t+1} \equiv \log(D_{t+1}/D_t)$. The constants κ_{c0}, κ_{c1}, κ_{d0}, and κ_{d1} are given by:

$$\begin{aligned} \kappa_{d0} &= \log[1+\exp(z_d)] - \kappa_{d1} z_d, \\ \kappa_{c0} &= \log[1+\exp(z_c)] - \kappa_{c1} z_c, \end{aligned}$$

$$\kappa_{d1} = \frac{\exp(z_d)}{1+\exp(z_d)}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1+\exp(z_c)},$$

where z_d and z_c are the unconditional mean values of z_{dt} and z_{ct}.

The Euler equations associated with a claim to the stock market and a consumption claim can be written as:

$$E_t[\exp(m_{t+1} + r_{d,t+1})] = 1, \tag{3.14}$$

$$E_t[\exp(m_{t+1} + r_{c,t+1})] = 1. \tag{3.15}$$

We solve the model using the method of undetermined coefficients. First, we replace m_{t+1}, $r_{c,t+1}$ and $r_{d,t+1}$ in equations (3.14) and (3.15), using expressions (3.12), (3.13) and (3.10). Second, we guess and verify that the equilibrium solutions for z_{dt} and z_{ct} take the form:

$$z_{dt} = A_{d0} + A_{d1} \log(\lambda_{t+1}/\lambda_t), \tag{3.16}$$

$$z_{ct} = A_{c0} + A_{c1} \log(\lambda_{t+1}/\lambda_t). \tag{3.17}$$

[9] See Hansen, Heaton, and Li (2008) for an alternative solution procedure.

This solution has the property that price-dividend ratios are constant, absent movements in λ_t. This property results from our assumption that the logarithm of consumption and dividends follow random-walk processes. We compute A_{d0}, A_{d1}, A_{c0}, and A_{c1} using the method of undeterminate coefficients.

The equilibrium solution has the property that $A_{d1}, A_{c1} > 0$. We show in Appendix B that the conditional expected return to equity is given by:

$$\begin{aligned} E_t\left(r_{d,t+1}\right) &= -\log\left(\delta\right) - \log\left(\lambda_{t+1}/\lambda_t\right) + \mu/\psi \\ &\quad + \left[\frac{(1-\theta)}{\theta}\left(1-\gamma\right)^2 - \gamma^2\right]\sigma_c^2/2 + \pi_{dc}\left(2\gamma\sigma_c - \pi_{dc}\right)/2 - \sigma_d^2/2 \\ &\quad + \left\{(1-\theta)\left(\kappa_{c1}A_{c1}\right)\left[2\left(\kappa_{d1}A_{d1}\right) - \left(\kappa_{c1}A_{c1}\right)\right] - \left(\kappa_{d1}A_{d1}\right)^2\right\}\sigma_\lambda^2/2. \end{aligned} \tag{3.18}$$

Recall that κ_{c1} and κ_{d1} are non-linear functions of the parameters of the model.

Using the Euler equation for the risk-free rate, $r_{f,t+1}$,

$$E_t\left[\exp\left(m_{t+1} + r_{f,t+1}\right)\right] = 1,$$

we obtain:

$$\begin{aligned} r_{f,t+1} &= -\log\left(\delta\right) - \log\left(\lambda_{t+1}/\lambda_t\right) + \mu/\psi - (1-\theta)\left(\kappa_{c1}A_{c1}\right)^2\sigma_\lambda^2/2 \\ &\quad + \left[\frac{(1-\theta)}{\theta}\left(1-\gamma\right)^2 - \gamma^2\right]\sigma_c^2/2. \end{aligned} \tag{3.19}$$

Equations (3.18) and (3.19) imply that the risk-free rate and the conditional expectation of the return to equity are decreasing functions of $\log\left(\lambda_{t+1}/\lambda_t\right)$. When $\log\left(\lambda_{t+1}/\lambda_t\right)$ rises, agents value the future more relative to the present, so they want to save more. Since risk-free bonds are in zero net supply and the number of stock shares is constant, aggregate savings cannot increase. So, in equilibrium, returns on bonds and equity must fall to induce agents to save less.

The approximate response of asset prices to shocks, emphasized by Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička and Hansen (2011), can be directly inferred from equations (3.18) and (3.19). The response of the return to stocks and the risk-free rate to a time-preference shock corresponds to that of an AR(1) with serial correlation ρ.

Using equations (3.18) and (3.19) we can write the conditional equity premium as:

$$\begin{aligned} E_t\left(r_{d,t+1}\right) - r_{f,t+1} &= \pi_{dc}\left(2\gamma\sigma_c - \pi_{dc}\right)/2 - \sigma_d^2/2 \\ &\quad + \kappa_{d1}A_{d1}\left[2\left(1-\theta\right)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}\right]\sigma_\lambda^2/2. \end{aligned} \tag{3.20}$$

We define the compensation for *valuation risk* as the part of the one-period expected excess return to an asset that is due to the volatility of the time preference shock, σ_λ^2. We refer to the part of the one-period expected excess return that is due to the volatility of consumption and dividends as the compensation for *conventional risk*.

The component of the equity premium that is due to valuation risk, v_d, is given by the last term in equation (3.20). Since the constants A_{c1}, A_{d1}, κ_{c1}, and κ_{d1} are all positive, $\theta < 1$ is a necessary condition for valuation risk to help explain the equity premium (recall that θ is defined in equation (3.11)). It is useful to consider the case in which the stock is a claim on consumption. In this case, that term reduces to:

$$v_d = (1 - 2\theta)\left(\frac{\kappa_{c1}}{1 - \rho\kappa_{c1}}\right)^2 \sigma_\lambda^2/2.$$

This expression is positive as long as $\theta < 1/2$, which holds as long as one of the following conditions holds:[10]

$$\begin{array}{ll} \gamma > 0.5\,(1 + 1/\psi) & \text{and } \psi > 1, \\ \gamma < 0.5\,(1 + 1/\psi) & \text{and } \psi < 1. \end{array} \tag{3.21}$$

As it turns out, this condition is always satisfied in the estimated versions of our model.

The intuition for why valuation risk helps account for the equity premium is as follows. Consider an investor who buys the stock at time t. At some later time, say $t + \tau$, $\tau > 0$, the investor may get a preference shock, say a decrease in $\lambda_{t+\tau+1}$, and want to increase consumption. Since all consumers are identical, they all want to sell the stock at the same, so that the price of equity will fall. Bond prices also fall because consumers try to reduce their holdings of the risk-free asset. Since stocks are infinitely-lived compared to the one-period risk-free bond, they are more exposed to this source of risk. So, valuation risk, v_d, leads to a larger equity premium. In the case where $\gamma = 1/\psi$, we are in the CRRA case and the net effect on the equity premium is very small (see equation 3.6)).

It is interesting to highlight the differences between time-preference shocks and conventional sources of uncertainty, which pertain to the supply-side of the economy. Suppose that there is no risk associated with the physical payoff of assets such as stocks. In this case, standard asset pricing models would imply that the equity premium is zero. In our model,

[10] The condition $\theta < 1/2$ is different from the condition that guarantees preference for early resolution of uncertainty: $\gamma > 1/\psi$, which is equivalent to $\theta < 1$. As discussed in Epstein et al (2014), the latter condition plays a crucial role in generating a high equity premium in long-run risk models. Because long-run risks are resolved in the distant future, they are more heavily penalized than current risks. For this reason, long-run risk models can generate a large equity premium even when shocks to current consumption are small.

there is a positive equity premium that results from the differential exposure of bonds and stocks to valuation risk. Agents are uncertain about how much they will value future dividend payments. Since λ_{t+1} is known at time t, this valuation risk is irrelevant for one-period bonds. But, it is not irrelevant for stocks, because they have infinite maturity. In general, the longer the maturity of an asset, the higher is its exposure to time-preference shocks and the large is the valuation risk.

We conclude by considering the case in which there are supply-side shocks to the economy but agents are risk neutral ($\gamma = 0$). In this case, the component of the equity premium that is due to valuation risk is always positive as long as ψ is finite. The intuition is as follows: stocks are long-lived assets whose payoffs can induce unwanted variation in the period utility of the representative agent, $\lambda_t C_t^{1-1/\psi}$. Even when agents are risk neutral, they must be compensated for the risk of this unwanted variation.

3.3. Relation to the long-run risk model

In this subsection we briefly comment on the relation between our model and the long-run-risk model pioneered by Bansal and Yaron (2004). Both models emphasize low-frequency shocks that induce large, persistent changes in the agent's stochastic discount factor. To see this point, it is convenient to re-write the representative agent's utility function, (3.1), as:

$$U_t = \left[\tilde{C}_t^{1-1/\psi} + \delta \left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}$$

where $\tilde{C}_t = \lambda_t^{1/(1-1/\psi)} C_t$. Taking logarithms of this expression we obtain:

$$\log\left(\tilde{C}_t\right) = 1/\left(1 - 1/\psi\right)\log(\lambda_t) + \log\left(C_t\right)$$

Bansal and Yaron (2004) introduce a highly persistent component in the process for $\log(C_t)$, which is a source of long-run risk. In contrast, we introduce a highly persistent component into $\log(\tilde{C}_t)$ via our specification of the time-preference shocks. From equation (3.9), it is clear that both specifications can induce large, persistent movements in m_{t+1}. Despite this similarity, the two models are *not* observationally equivalent. First, they have different implications for the correlation between observed consumption growth, $\log(C_{t+1}/C_t)$, and asset returns. Second, the two models have very different implications for the average return to long-term bonds, and the term structure of interest rates. We return to these points when we discuss our empirical results in Section 6.

3.4. The extended model

The benchmark model just described is useful to highlight the role of time-preference shocks in affecting asset returns. But its simplicity leads to two important empirical shortcomings.[11] First, since consumption is a martingale, the only state variable that is relevant for asset returns is λ_{t+1}/λ_t. This property means that all asset returns as well as the price-dividend ratio are highly correlated with each other. Second, and related, the model displays constant risk premia and cannot be used to address the evidence on predictability of excess returns.

In this subsection, we address the shortcomings of the benchmark model by allowing for a richer model of consumption growth and dividend growth. We assume that the stochastic processes for consumption and dividend growth are given by:

$$\log(C_{t+1}/C_t) = \mu + \alpha_c \left(\sigma_{t+1}^2 - \sigma^2\right) + \pi_{c\lambda}\varepsilon_{t+1}^{\lambda} + \sigma_t \varepsilon_{t+1}^{c}, \tag{3.22}$$

$$\log(D_{t+1}/D_t) = \mu + \alpha_d \left(\sigma_{t+1}^2 - \sigma^2\right) + \sigma_d \sigma_t \varepsilon_{t+1}^{d} + \pi_{d\lambda}\varepsilon_{t+1}^{\lambda} + \pi_{dc}\sigma_t \varepsilon_{t+1}^{c}, \tag{3.23}$$

and

$$\sigma_{t+1}^2 = \sigma^2 + v\left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1}, \tag{3.24}$$

where ε_{t+1}^{c}, ε_{t+1}^{d}, $\varepsilon_{t+1}^{\lambda}$, and w_{t+1} are mutually uncorrelated standard-normal variables. Relative to the benchmark model, equations (3.22)-(3.24) incorporate two main new features. First, as in Kandel and Stambaugh (1990) and Bansal and Yaron (2004), we allow for conditional heteroskedasticity in consumption. This feature generates time-varying risk premia: when volatility is high the stock is risky, its price is low and its expected return is high. High volatility leads to higher precautionary savings motive so that the risk-free rate falls, reinforcing the rise in the risk premium.

The second main new feature in equations (3.22)-(3.24) is that we allow for a correlation between time-preference shocks and the growth rate of consumption and dividends. In a production economy, time-preference shocks would generally induce changes in aggregate consumption. For example, in a simple real-business-cycle model, a persistent increase in λ_{t+1}/λ_t would lead agents to reduce current consumption and raise investment in order to consume more in the future. Taken literally, an endowment economy specification does not allow for such a correlation. Importantly, only the innovation to time-preference shocks

[11] The shortcomings of our benchmark model is shared by other simple models like the one in Bansal and Yaron (2004) that abstract from conditional heteroskedasticity in consumption and dividends.

enters the law of motion for $\log(C_{t+1}/C_t)$ and $\log(D_{t+1}/D_t)$. So, equations (3.22)-(3.24) do not introduce any element of long-run risk into consumption or dividend growth.

Since the price dividend ratio and the risk free rate are driven by a single state variable in the benchmark model, they will have the same degree of persistence. A straightforward way to address this shortcoming is to assume that the time-preference shock is the sum of a persistent shock and an i.i.d. shock:

$$\begin{aligned} \log(\lambda_{t+1}/\lambda_t) &= x_t + \sigma_\eta \eta_{t+1}, \\ x_{t+1} &= \rho x_t + \sigma_\lambda \varepsilon^\lambda_{t+1}. \end{aligned} \tag{3.25}$$

Here $\varepsilon^\lambda_{t+1}$ and η_{t+1} are uncorrelated, i.i.d. standard normal shocks. We think of x_t as capturing low frequency changes in the growth rate of the discount rate. In contrast, η_{t+1} can be thought of high-frequency changes in investor sentiment that affect the demand for assets (see, for example, Dumas (2009)). If $\sigma_\eta = 0$ and $x_1 = \log(\lambda_1/\lambda_0)$ we obtain the specification of the time-preference shock used in the benchmark model. Other things equal, the larger is σ_η, the lower is the persistence of the time-preference shock.

4. Estimation methodology

We estimate the parameters of our model using the Generalized Method of Moments (GMM). Our estimator is the parameter vector $\hat{\Phi}$ that minimizes the distance between a vector of empirical moments, Ψ_D, and the corresponding model moments, $\Psi(\hat{\Phi})$. Our estimator, $\hat{\Phi}$, is given by:

$$\hat{\Phi} = \arg\min_{\Phi} \left[\Psi(\Phi) - \Psi_D\right]' \Omega_D^{-1} \left[\Psi(\Phi) - \Psi_D\right].$$

We found that, for a wide range of parameter values, the model implies that there is small sample bias in terms of various moments, especially the predictability of excess returns. We therefore focus on the plim of the model-implied small-sample moments when constructing $\Psi(\Phi)$, rather than the plim of the moments themselves. For a given parameter vector, Φ, we create 500 synthetic time series, each of length equal to our sample size. On each sample, we calculate the sample moments of interest. The vector $\Psi(\Phi)$ that enters the criterion function is the average value of the sample moments across the synthetic time series.[12] In addition, we assume that agents make decisions at a monthly frequency and derive the

[12] As the sample size grows, our estimator becomes equivalent to a standard GMM estimator so that the usual asymptotic results for the distribution of the estimator applies.

model's implications for variables computed at an annual frequency. We estimate Ψ_D using a standard two-step efficient GMM estimator with a Newey-West (1987) weighting matrix that has ten lags. The latter matrix corresponds to our estimate of the variance-covariance matrix of the empirical moments, Ω_D.

When estimating the benchmark model, we include the following 19 moments in Ψ_D: the mean and standard deviation of consumption growth, the mean and standard deviation of dividend growth, the correlation between the one-year growth rate of dividends and the one-year growth rate of consumption, the mean and standard deviation of real stock returns, the mean, standard deviation and autocorrelation of the price-dividend ratio, the mean, standard deviation and autocorrelation of the real risk-free rate, the correlation between stock returns and consumption growth at the one, five and ten-year horizon, the correlation between stock returns and dividend growth at the one, five and ten-year horizon. We constrain the growth rate of dividends and consumption to be the same. In practice, when we estimate the benchmark model we found that the standard deviation of the point estimate of the risk free across the 500 synthetic time series is very large. So here we report results corresponding to the case where we constrain the mean risk free rate to exactly match the value in the data. When we estimate the extended model with conditional heteroskedasticity we add the following moments to Ψ_D: the slope coefficients on predictive regressions of 1-year, 3-year and 5-year excess stock returns on the price-dividend ratio.

For the benchmark model, the vector Φ includes nine parameters: γ, the coefficient of relative risk aversion, ψ,the elasticity of intertemporal substitution, δ, the rate of time preference, σ_c, the volatility of innovation to consumption growth, π_{dc},the parameter that controls the correlation between consumption and dividend shocks, σ_d, the volatility of dividend shocks, ρ, the persistence of time-preference shocks, and σ_λ, the volatility of the innovation to time-preference shocks, and μ, the mean growth rate of dividends and consumption. In the extended model, the vector Φ includes an additional seven parameters: α_c and α_d,which control the effect of volatility on mean consumption and mean dividends, respectively, $\pi_{c\lambda}$ and $\pi_{d\lambda}$, which control the effect of time preference shock innovations on consumption and dividends, respectively, ν,, which governs the persistence of volatility, σ_w, the volatility of innovations to volatility), and σ_η, the volatility of transitory shocks to time preference.

5. Empirical results

Tables 3 through 7 report our parameter estimates along with standard errors. Several features are worth noting. First, the coefficient of risk aversion is quite low, 1.6 and 1.2, in the benchmark and extended models, respectively. We estimate this coefficient with reasonable precision. Second, for both models, the intertemporal elasticity of substitution is somewhat larger than one. Third, for both models, the point estimates satisfy the necessary condition for valuation risk to be positive, $\theta < 1/2$. Fourth, the parameter ρ that governs the serial correlation of the growth rate of λ_t is estimated to be close to one in both models, 0.991 and 0.997 in the benchmark and extended model, respectively. Fifth, the parameter ν, which governs the persistence of consumption volatility in the extended model, is also quite high (0.962). The high degree of persistence in both the time-preference and the volatility shock are the root cause of the small-sample biases in our estimators.

Table 4 compares the small-sample moments implied by the benchmark and extended models with the estimated data moments. Recall that in estimating the model parameters we impose the restriction that the unconditional average growth rate of consumption and dividends are the same. To assess the robustness of our results to this restriction, we present two versions of the estimated data moments, with and without the restriction. With one exception, the constrained and unconstrained data moment estimates are similar, taking sampling uncertainty into account. The exception is the average growth rate of consumption, where the constrained and unconstrained estimates are statistically different.

Implications for the equity premium Table 4 shows that both the benchmark and extended model give rise to a large equity premium, 5.75 and 3.24, respectively. This result holds even though the estimated degree of risk aversion is quite moderate in both models. In contrast, long-run risk models require a high degree of risk aversion to match the equity premium.

Epstein, Farhi, and Strzalecki (2014) assess the plausibility of a high risk-aversion coefficient in the context of an economy with Epstein-Zin preferences and the endowment processes in Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012). They ask the question: at time zero, what fraction of consumption in each period would an agent give up to have all uncertainty resolved at time one. The answer to this question is what they call the "timing premium." For $\gamma = 10$ and $\psi = 1.5$ (the parameters used by Bansal and Yaron

(2004) and Bansal, et al (2012)) the timing premium is roughly 30 percent of consumption in each period. In contrast for $\gamma = 2$ and $\psi = 1.5$ the timing premium is roughly 5 percent of consumption. Since our estimates of γ are 1.6 and 1.2, and our value of ψ is close to 1.5, the timing premium is lower than 5 percent.

Recall that in order for valuation risk to contribute to the equity premium, θ must be less than one. This condition is clearly satisfied by both our models: the estimated value of θ is -2.00 and -0.74 in the benchmark and extended model, respectively (Table 3). In both cases, θ is estimated quite accurately. The model with the larger absolute value of θ generates a larger equity premium. Taking sampling uncertainty into account, the benchmark model easily accounts for the equity premium, while the extended model does so marginally. We can easily reject the null hypothesis of $\theta = 1$, which corresponds to the case of constant relative risk aversion.

The basic intuition for why our model generates a high equity premium despite a low coefficient of relative risk aversion is as follows. From the perspective of the model, stocks and bonds are different in two ways. First, the model embodies the conventional source of an equity premium, namely bonds have a certain payoff that does not covary with the SDF while the payoff to stocks covaries negatively with the SDF (as long as $\pi_{dc} > 0$). Since γ is close to one, this traditional covariance effect is very small. Second, the model embodies a compensation for valuation risk that is particularly pronounced for stocks because they they have longer maturities than bonds. Recall that, given our timing assumptions, when an agent buys a bond at time t, the agent knows the value of λ_{t+1}, so the only source of risk are movements in the marginal utility of consumption at time $t+1$.[13] In contrast, the time-t stock price depends on the value of λ_{t+j}, for all $j > 1$. So, agents are exposed to valuation risk, a risk that is particularly important because time-preference shocks are very persistent.

In Table 5 we decompose the equity premium into the valuation risk premium and the conventional risk premium. We calculate these premia using the estimated parameters of the two models but varying the value of ρ, which controls the persistence of the time-preference shock. Two key results emerge from this table.

The first result is that the conventional risk premium is always close to zero. For the benchmark model, this finding is consistent with Kocherlakota's (1996) discussion of why the equity premium is not explained by endowment models in which the representative

[13] Changing our timing convention so that agents do not know λ_{t+1} at time t does not have substantial implications for our quantitative analysis.

agent has recursive preferences and consumption follows a martingale. Consider next our results for the extended model. We know from Kandel and Stambaugh (1990) that a model with conditional heteroskedasticity in consumption can give rise to large equity premia. However, our estimation criterion does not chooses values of the parameters of the conditional heteroskedasticity process that generate a sizable conventional risk premia. The reason is that such a parameterization would generate implausibly high correlations between stock returns and fundamentals.

The second result that emerges from Table 5 is that the valuation risk premium and the equity premium are increasing in ρ. The larger is ρ, the more exposed are agents to large movements in stock prices induced by time-preference shocks.

Implications for the risk-free rate Recall that the benchmark model is restricted to match the average risk-free rate exactly. From Table 4 we see that the 'unconstrained' extended model implies an average risk-free rate that comes close to matching the average risk-free rate.

A problem with some explanations of the equity premium is that they imply counterfactually high levels of volatility for the risk-free rate (see e.g. Boldrin, Christiano and Fisher (2001)). Table 4 shows that the volatility of the risk-free rate and stock market returns implied by our model are similar to the estimated volatilities in the data.

An empirical shortcoming of the benchmark model is its implication for the persistence of the risk-free rate. Recall that, according to equation (3.19), the risk-free rate has the same persistence as the growth rate of the time-preference shock. Table 4 shows that the AR(1) coefficient of the risk free rate, as measured by the ex-post realized real returns to one-year treasury bills, is only 0.61, with a standard error of 0.11, which is substantially smaller that our estimate of ρ of 0.90 in the benchmark model. The extended model does a much better job at accounting for the persistence of the risk-free rate (0.62). In this model there are both transitory and persistent shocks to the risk free rate. The former account for roughly 70 percent of the variation in λ_t.

Implications for the correlation puzzle Table 6 reports the model's implications for the correlation of stock returns with consumption and dividend growth. Recall that in the benchmark model consumption and dividends follow a random walk. In addition, the estimated process for the growth rate of the time-preference shock is close to a random walk.

So, the correlation between stock returns and consumption growth implied by the model is essentially the same across different horizons. A similar property holds for the correlation between stock returns and dividend growth.

In the extended model persistent changes in the variance of the growth rate of consumption and dividends can induce persistent changes in the conditional mean of these variables. As a result, this model produces correlations between stock returns and fundamentals that vary across different horizons.

The benchmark model does well at matching the correlation between stock returns and consumption growth in the data, because this correlation is similar at all horizons. In contrast, the empirical correlation between stock returns and dividend growth increases with the time horizon. The estimation procedure chooses to match the long-horizon correlations and does less well at matching the yearly correlation. This choice is dictated by the fact that it is harder for the model to produce a low correlation between stock returns and dividend growth than it is to produce a low correlation between stock returns and consumption growth. This property reflects the fact that the dividend growth rate enters directly into the equation for stock returns (see equation (3.12)).

The extended model does better at capturing the fact that the correlations between equity returns and dividend growth rises with the horizon increases because of two reasons. When volatility is high, the returns to equity are high. Since $\alpha_d < 0$ the growth rate of dividends is low. As a result, the one-year correlation between dividend growth and equity returns is negative. The variance of the shock to the dividend growth rate is mean reverting. So, the effect of a negative value of α_d becomes weaker as the horizon extends. The direct association between equity returns and dividend growth (see (3.12), which induces a positive correlation eventually dominates as the horizon gets longer.

An additional force that allows the extended model to generate a lower short-term correlation between equity returns and dividend growth is that the estimated value of $\pi_{d\lambda}$ is negative. The estimation algorithm chooses parameters to allow the model to do reasonably well in matching the one and five year correlation, at the cost of doing less well at matching the ten-year correlation. Presumably, this choice reflects the greater precision with which the one-year and five-year correlations are estimated relative to the ten-year correlation.

Taking sampling uncertainty into account, the extended model matches the correlation between stock returns and consumption growth at different horizons. Interestingly, the

correlation between stock returns and consumption growth increases with the horizon. This increase is less pronounced than the corresponding increase in the correlation between stock returns and dividend growth. The reason is that the effect of volatility on the mean growth of consumption is smaller ($\alpha_d < \alpha_c < 0$) and $\pi_{c\lambda}$ is small and positive.

To document the relative importance of the correlation puzzle and the equity premium puzzle, we re-estimate the model with conditional heteroskedasticity subject to the constraint that it matches the average equity premium and the average risk-free rate. We report our results in Tables 3 through 7. Even though the estimates of γ and ψ are similar to those reported before, the implied value of θ goes from -0.74 to -2.34, which is why the equity premium implied by the model rises. This version of the model produces quite low correlations between stock returns and consumption growth. However, the one-year correlation between stock returns and dividend growth implied by the model is much higher than that in the data (0.64 versus 0.08). The one-year correlation between stock returns and dividend growth is estimated much more precisely than the equity premium. So, the estimation algorithm chooses parameters for the extended model that imply a lower equity premium in return for matching the one-year correlation between stock returns and dividend growth.

We conclude by highlighting an important difference between our model and long-run risk models. For concreteness, we focus on the recent version of the long-run risk model proposed by Bansal, Kiku, and Yaron (2012). Working with their parameter values, we find that the correlation between stock returns and consumption growth are equal to 0.66, 0.88, and 0.92 at the one-, five- and ten-year horizon, respectively. Their model also implies correlations between stock returns and dividend growth equal to 0.66, 0.90, and 0.93 at the one-, five- and ten-year horizon, respectively. Our estimates reported in Table 1 imply that both sets of correlations are counterfactually high. The source of this empirical shortcoming is that all the uncertainty in the long-run risk model stems from the endowment process.

Implications for the price-dividend ratio In Table 4 we see that both the benchmark and the extended models match the average of the price-dividend ratio very well. The benchmark model somewhat underpredicts the persistence and volatility of the price-dividend ratio. The extended model does much better at matching those moments. The moments implied by this model are within two standard error of their sample counterparts.

Table 7 presents evidence reproducing the well-known finding that excess returns are predictable based on lagged price-dividend ratios. We report the results of regressing excess-

equity returns over holding periods of 1, 3 and 5 years on the lagged price-dividend ratio. The slope coefficients are -0.09, -0.26, and -0.39, respectively, while the R-squares are 0.04, 0.13, and 0.23, respectively.

The analogue results for the benchmark model are shown in the top panel of Table 7. In this model, consumption is a martingale with conditionally homoscedastic innovations. So, by construction, excess returns are unpredictable in population at a monthly frequency. Since we aggregate the model to annual frequency, temporal aggregation produces a small amount of predictability (see column titled "Model (plim)").

Stambaugh (1999) and Boudoukh et al. (2008) argue that the predictability of excess returns may be an artifact of small-sample bias and persistence in the price-dividend ratio. Our results are consistent with this hypothesis. The column labeled "Model (median)" reports the plim of the small moments implied by our model. Note that the slope coefficients for the 1, 5, and 10 year horizons, are -0.05, -0.14 and -0.21, respectively. In each case, the median Monte Carlo point estimates are contained within a two-standard deviation band of the respective data estimates.

Table 7 also presents results for the extended model. Because of conditional heteroskedasticity in consumption, periods of high volatility in consumption growth are periods of high expected equity returns and low equity prices. So, in principle, the model is able to generate predictability in population. At our estimated parameter values this predictability is quite small. But, once we allow for the effects of small sample bias, the extended model does quite well at accounting for the regression slope coefficients.

Cochrane (1992) proposes a decomposition of the variation of the price-dividend ratio into three components: excess returns, dividend growth, and the risk-free rate.[14] While this decomposition is not additive, authors like Bansal and Yaron (2004) use it to compare the importance of these three components in the model and data.

In our sample, the point estimate for the percentage of the variation in price–dividend ratio due to excess return fluctuations is 102.2 percent, with a standard error of 30 percent. Dividend growth accounts for -14.5 with a standard error of 13 percent. Finally, the risk-free rate accounts for -20.4 percent with a standard error of 14.8 percent. These results are similar to those in Cochrane (1992) and Bansal and Yaron (2004).

[14] The fraction of the volatility of the price-dividend ratio attributed to variable x is given by $\sum_{j=1}^{15} \Omega^j cov(z_{dt}, x_{t+j})/var(z_{dt})$, where $\Omega = 1/(1 + E(R_{d,t+1}))$. See Cochrane (1992) for details.

Based on the small sample moments implied by the extended model, the fraction of the variance of the price-dividend ratio accounted for by excess returns, dividend growth, and the risk-free rate is 34.2, -2.8, and 54.6, respectively. So, this model clearly overstates the importance of the risk-free rate and understates the importance of excess returns in accounting for the variance of the price-dividend ratio.

The extended model does substantially better than the benchmark model. Based on the small-sample moments implied by the extended model, the fraction of the variance of the price-dividend ratio accounted for by excess returns, dividend growth, and the risk-free rate is 41.2, -8.2, and 32.4. The fraction attributed to excess returns is just within two standard errors from the corresponding point estimate. The fraction attributed to dividend growth is well within two standard errors of the point estimate. Still, the model overstates the fraction attributed to the risk-free rate.[15]

It is interesting to contrast these results to those in Bansal and Yaron (2004). Their model also attributes a large fraction of the variance of the price-dividend ratio to excess returns. At the same time, their model substantially overstates the role of dividend growth in accounting for the variance of the price-dividend ratio.

6. Bond term premia

As we emphasize above, the equity premium in our estimated models results primarily from the valuation risk premium. Since this valuation premium increases with the maturity of an asset, a natural way to assess the plausibility of our model is to evaluate its implications for the slope of the real yield curve.

Table 8 reports the mean and standard deviation of ex-post real yields on short-term (one-month) Treasury Bills, intermediate-term government bonds (with approximate maturity of five years), and long-term government bonds (with approximate maturity of twenty years). A number of features are worth noting. First, consistent with Alvarez and Jermann (2005), the term structure of real yields is upward sloping. Second, the real yield on long-term bonds is positive. This result is consistent with Campbell, Shiller and Viceira (2009) who report that the real yield on long-term TIPS has always been positive and is usually above two

[15] We find substantial evidence of small-sample bias in these statistics. For the benchmark model the fraction of the variance of the price-dividend ratio accounted for, in population, by excess returns, dividend growth, and the risk-free rate is -3.1, -0.4, and 81.0, respectively. The analogue numbers for the extended model are 5.7, 0.3, and 54.2, respectively.

percent.

Our model implies that long-term bonds command a positive risk premium that increases with the maturity of the bond (see Appendix C for details on the pricing of long term bonds). The latter property reflects the fact that longer maturity assets are more exposed to valuation risk. Table 8 shows that, taking sampling uncertainty into account, the benchmark and extended models are consistent with the observed mean yields, except that the former model generates slightly larger yields for long-term bonds than in the data. The table also shows that the estimated models account for the volatility of the yields on short-, intermediate-, and long-term bonds. So, our model can account for key features of the intermediate and long-term bond returns, even though these moments were not used to estimate the model.

Piazzesi and Schneider (2007) and Beeler and Campbell (2012) argue that the bond term premium and the yield on long-term bonds are useful for discriminating between competing asset pricing models. For example, they stress that long-run risk models, of the type pioneered by Bansal and Yaron (2004), imply negative long-term bond yields and a negative bond term premium. The intuition is as follows: in a long-run risk model agents are concerned that consumption growth may be dramatically lower in some future state of the world. Since bonds promise a certain payout in all states of the world, they offer insurance against this possibility. The longer the maturity of the bond, the more insurance it offers and the higher is its price. So, the term premium is downward sloping. Indeed, the return on long-term bonds may be negative. Beeler and Campbell (2012) show that the return on a 20-year real bond in the Bansal, Kiku and Yaron (2012) model is -0.88.

Standard rare-disaster models also imply a downward sloping term structure for real bonds and a negative real yield on long-term bonds. See, for example the benchmark model in Nakamura, Steinsson, Barro, and Ursúa (2010). According to these authors, these implications can be reversed by introducing the possibility of default on bonds and to assume that probability of partial default is increasing in the maturity of the bond.[16] So, we cannot rule out the possibility that other asset-pricing models can account for bond term premia and the rate of return on long-term bonds. Still, it seems clear that valuation risk is a natural explanation of these features of the data.

According to Table 8, the benchmark and extended models imply that the difference

[16] Nakamura et al (2010) consider a version of their model in which the probability of partial default on a perpetuity is 84 percent, while the probability of partial default on short-term bonds is 40 percent. This model generates a positive term premium and a positive return on long-term bonds.

between stock returns and long-term bond yields is roughly 1 percent. This value is substantially lower than our point estimate (4.5 percent). But the standard error associated with the point estimate is quite large (1.8). So, taking sampling uncertainty into account, the benchmark model is consistent with the data. The extended model's implications for this statistic is marginally outside of the two standard deviation band.

In our model, the positive premium that equity commands over long-term bonds reflects the difference between an asset of infinite and twenty-year maturity. Consistent with this perspective, Binsbergen, Hueskes, Koijen, and Vrugt (2011) estimate that 90 (80) percent of the value of the S&P 500 index corresponds to dividends that accrue after the first 5 (10) years.

It is important to emphasize that the equity premium in our model is not solely driven by the term premium. One way to see this property is to consider the results of regressing the equity premium on two alternative measures of excess bond yields. The first measure is the difference between yields on bonds of 20 year and 1 year maturities. The second measure is the difference between yields on bonds of 5 year and 1 year maturities. Table 9 reports our results. For both models, the slope coefficients are quite close to the point estimates. Also, both models are consistent with the fact that the R^2 in these regressions are quite low.

We conclude with an interesting observation made by Binsbergen, Brandt, and Koijen (2012). Using data over the period 1996 to 2009, these authors decompose the S&P500 index into portfolios of short-term and long-term dividend strips. The first portfolio entitles the holder to the realized dividends of the index for a period of up to three years. The second portfolio is a claim on the remaining dividends. Binsbergen et al (2012) find that the short-term dividend portfolio has a higher risk premium than the long-term dividend portfolio, i.e. there is a negative stock term premium. They argue that this observation is inconsistent with habit-formation, long-run risk models and standard of rare-disaster models.[17] Our model, too, has difficulty in accounting for the Binsbergen et al (2012) negative stock term premium. Of course, our sample is very different from theirs and their negative stock term premium result is heavily influenced by the recent financial crisis (see Binsbergen et al (2011)). Also, Boguth, Carlson, Fisher, and Simutin (2012) argue that the Binsbergen et al (2012) results may be significantly biased because of the impact of small pricing frictions.

[17] Recently, Nakamura et al (2012) show that a time-vaying rare disaster model in which the component of consumption growth due to a rare disaster follows an AR(1) process, is consistent with the Binsbergen et al (2012) results. Belo et al. (2013) show that the Binsbergen et al. (2012) result can be reconciled in a variety of models if the dividend process is replaced with processes that generate stationary leverage ratios.

7. Conclusion

In this paper we argue that allowing for demand shocks in substantially improves the performance of standard asset-pricing models. Specifically, it allows the model to account for the equity premium, bond term premia, and the correlation puzzle with low degrees of estimated risk aversion. According to our estimates, valuation risk is by far the most important determinant of the equity premium and the bond term premia.

The recent literature has incorporated many interesting features into standard asset-pricing models to improve their performance. Prominent examples, include habit formation, long-run risk, time-varying endowment volatility, and model ambiguity. We abstract from these features to isolate the empirical role of valuation risk. But they are, in principle, complementary to valuation risk and could be incorporated into our analysis. We leave this task for future research.

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8. Appendix

8.1. Appendix A

In this appendix, we solve the model in Section 3 analytically for the case of CRRA utility. Let $C_{a,t}$ denote the consumption of the representative agent at time t. The representative agent solves the following problem:

$$U_t = \max E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} \frac{C_{a,t+i}^{1-\gamma}}{1-\gamma},$$

subject to the flow budget constraints

$$W_{a,i+1} = R_{c,i+1} \left(W_{a,i} - C_{a,i} \right),$$

for all $i \geq t$. The variable $R_{c,i+1}$ denotes the gross return to a claim that pays the aggregate consumption as in equation (3.8), financial wealth is $W_{a,i} = \left(P_{c,i} + C_i \right) S_{a,i}$, and $S_{a,i}$ is the number of shares on the claim to aggregate consumption held by the representative agent. The first-order condition for $S_{a,t+i+1}$ is:

$$\delta^i \lambda_{t+i} C_{a,t+i}^{-\gamma} = E_t \left(\delta^{i+1} \lambda_{t+i+1} C_{a,t+i+1}^{-\gamma} R_{c,i+1} \right).$$

In equilibrium, $C_{a,t} = C_t$, $S_{a,t} = 1$. The equilibrium value of the intertemporal marginal rate of substitution is:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{8.1}$$

The Euler equation for stock returns is the familiar,

$$E_t \left[M_{t+1} R_{c,t+1} \right] = 1.$$

We now solve for $P_{c,t}$. It is useful to write $R_{c,t+1}$ as

$$R_{c,t+1} = \frac{\left(P_{c,t+1}/C_{t+1} + 1 \right)}{P_{c,t}/C_t} \left(\frac{C_{t+1}}{C_t} \right).$$

In equilibrium:

$$E_t \left[M_{t+1} \left(\frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \left(\frac{C_{t+1}}{C_t} \right) \right] = \frac{P_{c,t}}{C_t}. \tag{8.2}$$

Replacing the value of M_{t+1} in equation (8.2):

$$E_t \left[\delta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \left(\frac{C_{t+1}}{C_t} \right) \right] = \frac{P_{c,t}}{C_t}.$$

Using the fact that λ_{t+1}/λ_t is known as of time t we obtain:

$$\delta \frac{\lambda_{t+1}}{\lambda_t} E_t \left[\exp\left(\mu + \sigma_c \varepsilon_{t+1}^c\right)^{1-\gamma} \left(\frac{P_{ct+1}}{C_{t+1}} + 1 \right) \right] = \frac{P_{ct}}{C_t}.$$

We guess and verify that P_{ct+1}/C_{t+1} is independent of ε_{t+1}^c. This guess is based on the fact that the model's price-consumption ratio is constant absent time-preference shocks. Therefore,

$$\delta \frac{\lambda_{t+1}}{\lambda_t} \exp\left[(1-\gamma)\mu + (1-\gamma)^2 \sigma_c^2/2\right] E_t \left(\frac{P_{c,t+1}}{C_{t+1}} + 1 \right) = \frac{P_{c,t}}{C_t}. \tag{8.3}$$

We now guess that there are constants k_0, k_1,..., such that

$$\frac{P_{c,t}}{C_t} = k_0 + k_1 (\lambda_{t+1}/\lambda_t) + k_2 (\lambda_{t+1}/\lambda_t)^{1+\rho} + k_3 (\lambda_{t+1}/\lambda_t)^{1+\rho+\rho^2} + ... \tag{8.4}$$

Using this guess,

$$\begin{aligned} & E_t \left(\frac{P_{c,t+1}}{C_{t+1}} + 1 \right) \\ = \; & E_t \left(k_0 + k_1 \left((\lambda_{t+1}/\lambda_t)^\rho \exp\left(\sigma_\lambda \varepsilon_{t+1}^\lambda\right) \right) + k_2 \left((\lambda_{t+1}/\lambda_t)^\rho \exp\left(\sigma_\lambda \varepsilon_{t+1}^\lambda\right) \right)^{1+\rho} + ... + 1 \right) \\ = \; & k_0 + k_1 (\lambda_{t+1}/\lambda_t)^\rho \exp\left(\sigma_\lambda^2/2\right) + k_2 (\lambda_{t+1}/\lambda_t)^{\rho(1+\rho)} \exp\left((1+\rho)^2 \sigma_\lambda^2/2\right) + ... + 1. \end{aligned} \tag{8.5}$$

Substituting equations (8.4) and (8.5) into equation (8.3) and equating coefficients leads to the following solution for the constants k_i:

$$\begin{aligned} k_0 &= 0, \\ k_1 &= \delta \exp\left[(1-\gamma)\mu + (1-\gamma)^2 \sigma_c^2/2\right], \end{aligned}$$

and for $n \geq 2$

$$k_n = k_1^n \exp\left\{ \left[1 + (1+\rho)^2 + \left(1+\rho+\rho^2\right)^2 + ... + \left(1 + ... + \rho^{n-2}\right)^2 \right] \sigma_\lambda^2/2 \right\}.$$

We assume that the series $\{k_n\}$ converges, so that the equilibrium price-consumption ratio is given by equation (8.4). Hence, the realized return on the consumption claim is

$$R_{c,t+1} = \frac{C_{t+1}}{C_t} \frac{k_1 (\lambda_{t+2}/\lambda_{t+1}) + k_2 (\lambda_{t+2}/\lambda_{t+1})^{1+\rho} + ... + 1}{k_1 (\lambda_{t+1}/\lambda_t) + k_2 (\lambda_{t+1}/\lambda_t)^{1+\rho} + ...}. \tag{8.6}$$

The equation that prices the one-period risk-free asset is:

$$E_t \left[M_{t+1} R_{f,t+1} \right] = 1.$$

Taking logarithms on both sides of this equation and noting that $R_{f,t+1}$ is known at time t, we obtain:

$$r_{f,t+1} = -\log E_t\left(M_{t+1}\right).$$

Using equation (8.1),

$$\begin{aligned} E_t\left(M_{t+1}\right) &= \delta\frac{\lambda_{t+1}}{\lambda_t}E_t\left[\exp\left(-\gamma\left(\mu+\sigma_c\varepsilon_{t+1}^c\right)\right)\right] \\ &= \delta\frac{\lambda_{t+1}}{\lambda_t}\exp\left(-\gamma\mu+\gamma^2\sigma_c^2/2\right). \end{aligned}$$

Therefore,

$$r_{f,t+1} = -\log\left(\delta\right) - \log\left(\lambda_{t+1}/\lambda_t\right) + \gamma\mu - \gamma^2\sigma_c^2/2.$$

Using equation (3.4), we obtain

$$E\left[\left(\lambda_{t+1}/\lambda_t\right)^{-1}\right] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right).$$

We can then write the unconditional risk-free rate as:

$$E\left(R_{f,t+1}\right) = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu-\gamma^2\sigma_c^2/2).$$

Thus, the equity premium is given by:

$$E\left[\left(R_{c,t+1}\right) - R_{f,t+1}\right] = \exp\left(\frac{\sigma_\lambda^2/2}{1-\rho^2}\right)\delta^{-1}\exp(\gamma\mu-\gamma^2\sigma_c^2/2)\left[\exp\left(\gamma\sigma_c^2\right)-1\right],$$

which can be written as:

$$E\left[\left(R_{c,t+1}\right) - R_{f,t+1}\right] = E\left(R_{f,t+1}\right)\left[\exp\left(\gamma\sigma_c^2\right)-1\right].$$

8.2. Appendix B

This appendix provides a detailed derivation of the equilibrium of the model economy where the representative agent has Epstein-Zin preferences and faces time-preference shocks. The agent solves the following problem:

$$U\left(W_t\right) = \max_{C_t}\left[\lambda_t C_t^{1-1/\psi} + \delta\left(U_{t+1}^*\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}, \tag{8.7}$$

where $U_{t+1}^* = \left[E_t\left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}$. The optimization is subject to the following budget constraint:

$$W_{t+1} = R_{c,t+1}\left(W_t - C_t\right).$$

The agent takes as given the stochastic processes for the return on the consumption claim $R_{c,t+1}$ and the preference shock λ_{t+1}. For simplicity, we omit the dependence of life-time utility on the processes for λ_{t+1} and $R_{c,t+1}$.

The first-order condition with respect to consumption is,

$$\lambda_t C_t^{-1/\psi} = \delta \left(U_{t+1}^*\right)^{-1/\psi} \left[E_t \left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1} E_t \left(U\left(W_{t+1}\right)^{-\gamma} U'\left(W_{t+1}\right) R_{c,t+1}\right),$$

and the envelope condition is

$$U'\left(W_t\right) = U\left(W_t\right)^{1/\psi} \delta \left(U_{t+1}^*\right)^{-1/\psi} \left[E_t \left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1} E_t \left(U\left(W_{t+1}\right)^{-\gamma} U'\left(W_{t+1}\right) R_{c,t+1}\right).$$

Combining the first-order condition and the envelope condition we obtain:

$$U'\left(W_t\right) = U\left(W_t\right)^{1/\psi} \lambda_t C_t^{-1/\psi}. \tag{8.8}$$

This equation can be used to replace the value of $U'\left(W_{t+1}\right)$ in the first order condition:

$$\lambda_t C_t^{-1/\psi} = \delta \left(U_{t+1}^*\right)^{-1/\psi} \left[E_t \left(U\left(W_{t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)-1} E_t \left(U\left(W_{t+1}\right)^{1/\psi-\gamma} \lambda_{t+1} C_{t+1}^{-1/\psi} R_{c,t+1}\right).$$

Using the expression for U_{t+1}^* this last equation can be written compactly after some algebra as,

$$1 = E_t \left(M_{t+1} R_{c,t+1}\right). \tag{8.9}$$

Here, M_{t+1} is the stochastic discount factor, or intertemporal marginal rate of substitution, which is given by:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \frac{U\left(W_{t+1}\right)^{1/\psi-\gamma}}{\left(U_{t+1}^*\right)^{1/\psi-\gamma}} \frac{C_{t+1}^{-1/\psi}}{C_t^{-1/\psi}}.$$

We guess and verify the policy function for consumption and the form of the utility function. As in Weil (1989) and Epstein and Zin (1991), we guess that:

$$\begin{aligned} U\left(W_t\right) &= a_t W_t, \\ C_t &= b_t W_t. \end{aligned}$$

Replacing these guesses in equation (8.8) and simplifying yields:

$$a_t^{1-1/\psi} = \lambda_t b_t^{-1/\psi}. \tag{8.10}$$

Substitute the guess also in the Hamilton-Jacobi-Bellman equation (8.7) and simplifying we obtain:

$$a_t = \left[\lambda_t b_t^{1-1/\psi} + \delta \left(\left[E_t \left(\left(a_{t+1} \frac{W_{t+1}}{W_t}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}.$$

Finally, use the budget constraint to replace W_{t+1}/W_t and get

$$a_t = \left[\lambda_t b_t^{1-1/\psi} + \delta\left(\left[E_t\left(\left(a_{t+1}\left(1-b_t\right)R_{c,t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}\right)^{1-1/\psi}\right]^{1/(1-1/\psi)}. \tag{8.11}$$

Equations (8.10) and (8.11) give a solution to a_t and b_t.

Combining equations (8.10) and (8.11) gives:

$$\lambda_t b_t^{-1/\psi}\left(1-b_t\right) = \delta\left(\left[E_t\left(\left(a_{t+1}\left(1-b_t\right)R_{c,t+1}\right)^{1-\gamma}\right)\right]^{1/(1-\gamma)}\right)^{1-1/\psi},$$

which we can replace in the expression for the stochastic discount factor together with (8.10) to obtain:

$$M_{t+1} = \left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right)^{(1-\gamma)/(1-1/\psi)}\left(\frac{b_{t+1}}{b_t}\left(1-b_t\right)\right)^{-((1/\psi-\gamma)/\psi)/(1-1/\psi)}\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left(R_{c,t+1}\right)^{1/\psi-\gamma}.$$

Now note that $\theta = (1-\gamma)/(1-1/\psi)$, and that

$$\frac{\frac{C_{t+1}}{C_t}}{R_{c,t+1}} = \frac{b_{t+1}R_{c,t+1}\left(W_t - C_t\right)/\left(b_t W_t\right)}{R_{c,t+1}} = \frac{b_{t+1}\left(1-b_t\right)}{b_t},$$

to finally get,

$$M_{t+1} = \left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right)^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\left(R_{c,t+1}\right)^{\theta-1}.$$

Taking logarithms on both sides and equating the consumption of the representative agent to aggregate consumption yields equation (3.10).

The rest of the equilibrium derivation solves for $r_{c,t+1}$ and $r_{d,t+1}$ as well as for the risk free rate $r_{f,t+1}$. Up to now, we did not need to specify the process for the time-preference shock, the process for consumption growth or the process for dividend growth. We solve the rest of the model assuming the general processes of Subsection 3.4 given in equations (3.22) through (3.25). Recovering the equilibrium values for the benchmark model is easily done by setting $\pi_{c\lambda} = \pi_{d\lambda} = \sigma_\eta = 0$ and $\nu = \sigma_w = \alpha_c = \alpha_d = 0$.

To price the consumption claim, we must solve the pricing condition:

$$E_t\left[\exp\left(m_{t+1} + r_{c,t+1}\right)\right] = 1.$$

Guess that the log of the price consumption ratio, $z_{ct} \equiv \log\left(P_{c,t}/C_t\right)$, is

$$z_{ct} = A_{c0} + A_{c1}x_t + A_{c2}\eta_{t+1} + A_{c3}\sigma_t^2,$$

and approximate

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct} + \Delta c_{t+1}. \tag{8.12}$$

Replacing the approximation and the guessed solution for z_{ct} on the pricing condition gives

$$E_t\left[\exp\left(\theta\log\left(\delta\right)+\theta\log\left(\lambda_{t+1}/\lambda_t\right)+\left(1-\gamma\right)\Delta c_{t+1}+\theta\kappa_{c0}+\theta\kappa_{c1}z_{ct+1}-\theta z_{ct}\right)\right]=1.$$

Calculation of the expectation requires some algebra and yields the equation

$$\begin{aligned}
0 \;=\;& \theta\log\left(\delta\right)+\left(1-\gamma\right)\mu-\left(1-\gamma\right)\alpha_c\sigma^2+\theta\kappa_{c0}-\theta A_{c0}+\theta\kappa_{c1}A_{c0}+\left(\left(1-\gamma\right)\alpha_c+\theta\kappa_{c1}A_{c3}\right)\left(1-\nu\right)\sigma^2\\
&+\left(\left(1-\gamma\right)\pi_{c\lambda}+\theta\kappa_{c1}A_{c1}\sigma_\lambda\right)^2/2+\left(\theta\kappa_{c1}A_{c2}\right)^2/2+\left(\left(\left(1-\gamma\right)\alpha_c+\theta\kappa_{c1}A_{c3}\right)\sigma_w\right)^2/2\\
&+\theta\left(\kappa_{c1}A_{c1}\rho+1-A_{c1}\right)x_t+\theta\left(\sigma_\eta-A_{c2}\right)\eta_{t+1}\\
&+\left(\left(1-\gamma\right)^2/2+\left(\left(1-\gamma\right)\alpha_c+\theta\kappa_{c1}A_{c3}\right)v-\theta A_{c3}\right)\sigma_t^2.
\end{aligned}$$

In equilibrium, this equation must hold in all possible states resulting in the restrictions:

$$A_{c1}=\frac{1}{1-\kappa_{c1}\rho},$$
$$A_{c2}=\sigma_\eta,$$
$$A_{c3}=\frac{\left(1-\gamma\right)/2+\alpha_c v}{1-\kappa_{c1}v}\left(1-1/\psi\right),$$

and

$$\begin{aligned}
A_{c0} \;=\;& \frac{\log\left(\delta\right)+\left(1-1/\psi\right)\mu-\left(1-1/\psi\right)\alpha_c\sigma^2+\kappa_{c0}+\left(\left(1-1/\psi\right)\alpha_c+\kappa_{c1}A_{c3}\right)\left(1-\nu\right)\sigma^2}{1-\kappa_{c1}}\\
&+\frac{\theta\left(\left(1-1/\psi\right)\pi_{c\lambda}+\kappa_{c1}A_{c1}\sigma_\lambda\right)^2/2+\theta\left(\kappa_{c1}A_{c2}\right)^2/2+\theta\left(\left(\left(1-1/\psi\right)\alpha_c+\kappa_{c1}A_{c3}\right)\sigma_w\right)^2/2}{1-\kappa_{c1}}.
\end{aligned}$$

To solve for the risk free rate, we again use the stochastic discount factor to price the risk free asset. In logs, the Euler equation is

$$\begin{aligned}
r_{f,t+1} \;=\;& -\log\left(E_t\left(\exp\left(m_{t+1}\right)\right)\right)\\
=\;& -\log\left(E_t\left(\exp\left(\theta\log\left(\delta\right)+\theta\log\left(\lambda_{t+1}/\lambda_t\right)-\frac{\theta}{\psi}\Delta c_{t+1}+\left(\theta-1\right)r_{c,t+1}\right)\right)\right).
\end{aligned}$$

Using equation (8.12), we get:

$$r_{f,t+1}=-\log\left(E_t\left(\exp\left(\theta\log\left(\delta\right)+\theta\log\left(\lambda_{t+1}/\lambda_t\right)-\gamma\Delta c_{t+1}+\left(\theta-1\right)\left(\kappa_{c0}+\kappa_{c1}z_{ct+1}-z_{ct}\right)\right)\right)\right).$$

Substituting in the consumption process and the solution for the price-consumption ratio, and after much algebra, we obtain,

$$\begin{aligned}
r_{f,t+1} \;=\;& -\theta\log\left(\delta\right)+\gamma\mu-\gamma\alpha_c\sigma^2-\left(\left(\theta-1\right)\kappa_{c1}A_{c2}\right)^2/2-\left(\left(\theta-1\right)\kappa_{c1}A_{c1}\sigma_\lambda-\gamma\pi_{c\lambda}\right)^2/2\\
&-\left(\theta-1\right)\kappa_{c0}-\left(\theta-1\right)\kappa_{c1}A_{c0}-\left(\left(\theta-1\right)\kappa_{c1}A_{c3}-\gamma\alpha_c\right)\left(1-\nu\right)\sigma^2+\left(\theta-1\right)A_{c0}\\
&-\left(\left(\left(\theta-1\right)\kappa_{c1}A_{c3}-\gamma\alpha_c\right)\sigma_w\right)^2/2-\log\left(\lambda_{t+1}/\lambda_t\right)\\
&-\left(\gamma^2/2+\left(\left(\theta-1\right)\kappa_{c1}A_{c3}-\gamma\alpha_c\right)v-\left(\theta-1\right)A_{c3}\right)\sigma_t^2.
\end{aligned}$$

Setting $\pi_{c\lambda} = \pi_{d\lambda} = \sigma_\eta = 0$ and $\nu = \sigma_w = \alpha_c = \alpha_d = 0$, we get the benchmark-model value of the risk free rate (3.19).

Finally, we price a claim to dividends. Again, we assume the price dividend ratio is given by

$$z_{dt} = A_{d0} + A_{d1}x_t + A_{d2}\eta_{t+1} + A_{d3}\sigma_t^2,$$

and approximate the log linearized return to the claim to the dividend:

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1}z_{dt+1} - z_{dt} + \Delta d_{t+1}. \tag{8.13}$$

The pricing condition is

$$E_t\left[\exp\left(m_{t+1} + r_{d,t+1}\right)\right] = 1.$$

Substituting in for m_{t+1}, $r_{c,t+1}$ and $r_{d,t+1}$,

$$\begin{aligned} 1 &= E_t\Bigg(\exp\Bigg(\theta\log(\delta) + \theta\log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)\left(\kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct} + \Delta c_{t+1}\right) \\ &\quad + \kappa_{d0} + \kappa_{d1}z_{dt+1} - z_{dt} + \Delta d_{t+1}\Bigg)\Bigg). \end{aligned}$$

Further substitution of the consumption growth and dividend processes and of the price consumption and price dividend ratios, and after significant algebra, we get that in equilibrium

$$A_{d1} = \frac{1}{1-\kappa_{d1}\rho},$$

$$A_{d2} = \sigma_\eta,$$

$$A_{d3} = \frac{-\left(1/\psi - \gamma\right)(1-\gamma) + 2\left(\alpha_d - 1/\psi\alpha_c\right)v + \left(\pi_{dc} - \gamma\right)^2 + \sigma_d^2}{2\left(1-\kappa_{d1}v\right)},$$

and

$$\begin{aligned} &\theta\log(\delta) + (1-\gamma)\mu + (\theta-1)\kappa_{c0} + (\theta-1)\kappa_{c1}A_{c0} + \kappa_{d0} - (\alpha_d - \gamma\alpha_c)\sigma^2 - (\theta-1)A_{c0} \\ &+ \left((\theta-1)\kappa_{c1}A_{c3} + \kappa_{d1}A_{d3} + (\alpha_d - \gamma\alpha_c)\right)(1-\nu)\sigma^2 + \left((\theta-1)\kappa_{c1}A_{c2} + \kappa_{d1}A_{d2}\right)^2/2 \\ &+ \left(\pi_{d\lambda} - \gamma\pi_{c\lambda} + \left((\theta-1)\kappa_{c1}A_{c1} + \kappa_{d1}A_{d1}\right)\sigma_\lambda\right)^2/2 + \left((\theta-1)\kappa_{c1}A_{c3} + \kappa_{d1}A_{d3} + (\alpha_d - \gamma\alpha_c)\right)^2\sigma_w^2/2 \\ = \;& A_{d0}\left(1-\kappa_{d1}\right) \end{aligned}$$

Having solved for these constants, we can compute the expected return on the dividend claim $E_t\left(r_{d,t+1}\right)$. Setting the relevant parameters to zero, we obtain the benchmark-model value of $E_t\left(r_{d,t+1}\right)$ given in equation (3.18). We now derive the expression for the conditional risk premium in the benchmark model:

$$E_t\left(r_{d,t+1}\right) - r_{f,t+1} = E_t\left(\kappa_{d0} + \kappa_{d1}z_{dt+1} - z_{dt} + \Delta d_{t+1}\right) - r_{f,t+1}.$$

Substituting the values of z_{dt} and Δd_{t+1} and computing expectations,

$$E_t\left(r_{d,t+1}\right) - r_{f,t+1} = \kappa_{d0} + \left(\kappa_{d1} - 1\right) A_{d0} - x_t + \mu - r_{f,t+1}.$$

Substituting the values of A_{d0} and $r_{f,t+1}$ and simplifying, we obtain expression (3.20).

8.3. Appendix C

In this appendix we solve for the prices of zero coupon bonds of different maturities. Let $P_t^{(n)}$ be the time-t price of a bond that pays one unit of consumption at $t+n$, with $n \geq 1$. The Euler equation for the one-period risk-free bond price $P_t^{(1)} = 1/R_{f,t+1}$ is

$$P_t^{(1)} = E_t\left(M_{t+1}\right).$$

The price for a risk free bond maturing $n > 1$ in the future can be written recursively as

$$P_t^{(n)} = E_t\left(M_{t+1} P_{t+1}^{(n-1)}\right).$$

In Appendix C we derived the value of the risk free rate:

$$r_{f,t+1} = -\ln\left(P_t^{(1)}\right).$$

It is useful to write the risk free rate as

$$r_{f,t+1} = -\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) - p^1 - q^1 \sigma_t^2,$$

where

$$\begin{aligned} p^1 \;=\; & \theta \log(\delta) - \gamma\mu + \gamma\alpha_c\sigma^2 + (\theta - 1)\kappa_{c0} + (\theta - 1)(\kappa_{c1} - 1) A_{c0} \\ & + \left((\theta - 1)\kappa_{c1} A_{c3} - \gamma\alpha_c\right)(1 - \nu)\sigma^2 \\ & + \left((\theta - 1)\kappa_{c1}\right)^2 \sigma_\eta^2/2 + \left((\theta - 1)\kappa_{c1} A_{c1}\sigma_\lambda - \gamma\pi_{c\lambda}\right)^2/2 \\ & + \left(\left(-\gamma\alpha_c + (\theta - 1)\kappa_{c1} A_{c3}\right)\right)^2 \sigma_w^2/2 \end{aligned}$$

and

$$q^1 = \gamma^2/2 + \left((\theta - 1)\kappa_{c1} A_{c3} - \gamma\alpha_c\right) v - (\theta - 1) A_{c3}.$$

Let $p_t^{(n)} = \ln\left(P_t^{(n)}\right)$. Therefore,

$$\begin{aligned} p_t^{(1)} &= -r_{f,t+1} \\ &= p^1 + x_t + \sigma_\eta \eta_{t+1} + q^1 \sigma_t^2. \end{aligned}$$

We now compute the price of a risk free bond that pays one unit of consumption in two periods:

$$p_t^{(2)} = \ln E_t \left(\exp \left(m_{t+1} + p_{t+1}^{(1)} \right) \right).$$

Using the expression for m_{t+1} and the solution for $r_{c,t+1}$ and z_{ct} we obtain, after much algebra,

$$p_t^{(2)} = p^2 + (1+\rho)\, x_t + \sigma_\eta \eta_{t+1} + (1+\nu)\, q^1 \sigma_t^2,$$

with

$$\begin{aligned} p^2 \;=\; & \theta \log(\delta) - \gamma\mu + \gamma\alpha_c\sigma^2 + (\theta-1)\,\kappa_{c0} + (\theta-1)(\kappa_{c1}-1)\,A_{c0} \\ & + ((\theta-1)\,\kappa_{c1}A_{c3} - \gamma\alpha_c + q_1)(1-\nu)\,\sigma^2 \\ & + ((\theta-1)\,\kappa_{c1}+1)^2\,\sigma_\eta^2/2 + (((\theta-1)\,\kappa_{c1}A_{c1}+1)\,\sigma_\lambda - \gamma\pi_{c\lambda})^2/2 \\ & + (-\gamma\alpha_c + (\theta-1)\,\kappa_{c1}A_{c3} + q_1)^2\,\sigma_w^2/2 + p^1. \end{aligned}$$

Continuing similarly, we obtain the general formula for $n \geq 2$:

$$p_t^{(n)} = p^n + \left(1+\rho+...+\rho^{n-1}\right) x_t + \sigma_\eta \eta_{t+1} + \left(1+v+...+\nu^{n-1}\right) q_1 \sigma_t^2,$$

where

$$\begin{aligned} p^n \;=\; & \theta \log(\delta) - \gamma\mu + \gamma\alpha_c\sigma^2 + (\theta-1)\,\kappa_{c0} + (\theta-1)(\kappa_{c1}-1)\,A_{c0} \\ & + \left((\theta-1)\,\kappa_{c1}A_{c3} - \gamma\alpha_c + \left(1+\nu+...+\nu^{n-2}\right) q_1\right)(1-\nu)\,\sigma^2 \\ & + ((\theta-1)\,\kappa_{c1}+1)^2\,\sigma_\eta^2/2 + \left(\left((\theta-1)\,\kappa_{c1}A_{c1} + \left(1+\rho+...+\rho^{n-2}\right)\right)\sigma_\lambda - \gamma\pi_{c\lambda}\right)^2/2 \\ & + \left(-\gamma\alpha_c + (\theta-1)\,\kappa_{c1}A_{c3} + \left(1+\nu+...+\nu^{n-2}\right) q_1\right)^2 \sigma_w^2/2 + p^{n-1}. \end{aligned}$$

Finally, we define the yield on an n-period zero coupon bond as $y_t^{(n)} = -\frac{1}{n} p_t^{(n)}$.

Table 1

Correlation Between Stock Returns and Per Capita Growth Rates of Fundamentals

Panel A, 1929-2011

Horizon	Consumption	Output	Dividends	Earnings
1 year	-0.05	0.05	0.05	0.10
	(0.12)	(0.10)	(0.11)	(0.10)
5 years	0.001	0.00	0.30	0.20
	(0.14)	(0.12)	(0.13)	(0.13)
10 years	-0.11	-0.09	0.59	0.30
	(0.20)	(0.14)	(0.14)	(0.11)

Panel B, 1871-2006

Horizon	Consumption	Output	Dividends	Earnings
1 year	0.09	0.14	-0.04	0.13
	(0.09)	(0.10)	(0.10)	(0.10)
5 years	0.40	0.25	0.38	0.44
	(0.18)	(0.14)	(0.15)	(0.18)
10 years	0.25	0.00	0.64	0.41
	(0.18)	(0.11)	(0.17)	(0.13)

Table 2

Correlation Between Stock Returns and Per Capita Growth Rates of Fundamentals

NIPA measures of consumption, 1929-2011

Horizon	Durables	Non-durables	Non-durables and services
1 year	-0.04	0.05	0.27
	(0.13)	(0.14)	(0.14)
5 years	0.07	-0.08	0.18
	(0.11)	(0.09)	(0.08)
10 years	0.21	-0.2	0.06
	(0.15)	(0.13)	(0.14)

Table 3
Parameter Estimates and Standard Errors

	Benchmark Model	Extended Model	Extended Model (match equity premium)
γ	1.636 (0.033)	1.205 (0.029)	1.957 (0.032)
ψ	1.466 (0.043)	1.382 (0.004)	1.694 (0.053)
δ	0.9978 (0.0024)	0.9979 (0.0325)	0.9981 (0.0083)
σ_c	0.0069 (0.0002)	0.0065 (0.0003)	0.0067 (9.701e-05)
μ	1.52E-03 (0.00006)	7.94E-04 (5.152e-05)	2.51E-03 (6.375e-05)
$\pi_{c\lambda}$	0.00	0.0002 (0.0002)	-0.0024 (0.0002)
σ_d	0.0159 (0.0005)	2.2757 (0.1290)	1.3978 (0.20)
π_{dc}	0.0019 (0.0005)	0.1157 (0.0227)	0.3908 (0.1639)
$\pi_{d\lambda}$	0.00	-1.07E-02 (0.0007)	5.25E-03 (0.0003)
σ_η	0.00	5.93E-03 (0.0003)	1.40E-02 (0.0004)
ρ	0.99145 (0.0004)	0.997 (0.0001)	0.99891 (0.0002)
σ_λ	5.42E-04 (1.527e-05)	2.80E-04 (1.006e-05)	1.82E-04 (6.690e-06)
Implied value of θ	-2.00 (0.23)	-0.74 (0.10)	-2.34 (0.13)
ν	0.00	0.962 (0.005)	0.979 (0.032)
σ_ω	0.00	8.03E-06 (1.412e-06)	3.97E-06 (8.383e-06)
α_c	0.00	-11.538 (1.064)	-25.834 (24.449)
α_d	0.00	-125.02 (1.301)	-25.37 (17.078)

Table 4
Moments Matched in Estimation

Selected moments	Data (Constrained)	Data (Unconstrained)	Benchmark Model	Extended Model	Extended Model (match equity premium)
Average growth rate of consumption	1.44 (0.32)	2.24 (0.23)	1.837	0.953	3.017
Average growth rate of dividends	1.44 (0.32)	-0.12 (0.75)	1.837	0.953	3.017
Standard deviation of the growth rate of consumption	2.08 (0.38)	2.15 (0.31)	2.387	2.336	2.491
Standard deviation of the growth rate of dividends	6.82 (1.35)	7.02 (1.31)	5.486	7.327	3.834
Contemporaneous correlation between consumption growth and dividend growth	0.17 (0.12)	0.16 (0.09)	0.120	0.083	0.087
Average return to equities	7.55 (1.74)	6.20 (1.87)	6.106	3.625	7.549
Standard deviation of return to equities	17.22 (1.31)	17.49 (1.39)	15.964	18.059	15.794
Average risk-free rate	0.36 (0.81)	0.06 (0.83)	0.358	0.387	0.358
Standard deviation of the risk-free rate	3.19 (0.80)	3.47 (0.80)	3.993	3.482	5.426
First-order serial correlation of the risk-free rate	0.61 (0.11)	0.60 (0.08)	0.899	0.615	0.142
Equity premium	7.19 (1.77)	6.13 (1.84)	5.748	3.238	7.190
Average price-dividend ratio	3.41 (0.15)	3.38 (0.15)	3.157	3.568	3.183
Standard deviation of price-dividend ratio	0.47 (0.08)	0.45 (0.08)	0.284	0.488	0.383
First-order serial correlation of price dividend ratio	0.95 (0.03)	0.93 (0.04)	0.84	0.92	0.92

Table 5
Equity and Valuation Risk Premiums

Benchmark Model				Extended Model				Extended Model (match equity premium)			
ρ	Equity premium	Conventio-nal Risk Premium	Valuation Risk Premium	ρ	Equity premium	Conventio-nal Risk Premium	Valuation Risk Premium	ρ	Equity premium	Conventio-nal Risk Premium	Valuation Risk Premium
0.000	-0.0014	-0.0014	0.0000	0.000	0	0	0.0000	0.000	0.0061	0.0062	0.0000
0.100	-0.0014	-0.0014	0.0000	0.100	0	0	0.0000	0.100	0.0061	0.0062	0.0000
0.300	-0.0014	-0.0014	0.0000	0.300	0.0005	0.0005	0.0000	0.300	0.0061	0.0062	0.0000
0.500	-0.0014	-0.0014	0.0000	0.500	-0.0003	-0.0003	0.0000	0.500	0.0061	0.0062	-0.0001
0.700	-0.0013	-0.0014	0.0001	0.700	0.0001	0.0001	0.0000	0.700	0.0062	0.0063	-0.0001
0.900	-0.0007	-0.0016	0.0009	0.900	0.0002	0.0001	0.0002	0.900	0.0063	0.0064	-0.0001
0.950	0.0017	-0.0016	0.0033	0.950	0.0007	0.0001	0.0006	0.950	0.0069	0.0067	0.0002
0.960	0.0033	-0.0017	0.0051	0.960	0.0006	-0.0003	0.0009	0.960	0.0072	0.0068	0.0005
0.970	0.0067	-0.0019	0.0087	0.970	0.0013	-0.0001	0.0015	0.970	0.008	0.0068	0.0012
0.980	0.0159	-0.002	0.0179	0.980	0.0018	-0.0012	0.003	0.980	0.0095	0.0062	0.0033
0.990	0.0475	-0.0024	0.0499	0.990	0.0056	-0.0043	0.0099	0.990	0.0162	0.0021	0.0141
0.991	0.0539	-0.0025	0.0564	0.991	0.0062	-0.0055	0.0117	0.991	0.0179	0.0009	0.0171
0.992	0.0619	-0.0018	0.0638	0.992	0.0076	-0.0064	0.014	0.992	0.0203	-0.0006	0.0209
0.993	0.0711	-0.0012	0.0723	0.993	0.0095	-0.0075	0.017	0.993	0.0232	-0.0029	0.026
0.994	0.0812	-0.0008	0.082	0.994	0.0124	-0.0085	0.0209	0.994	0.0269	-0.0059	0.0328
0.995	0.0932	0.0002	0.093	0.995	0.0150	-0.0113	0.0262	0.995	0.0317	-0.0102	0.042
0.996	0.1067	0.0014	0.1053	0.996	0.0198	-0.0136	0.0335	0.996	0.0386	-0.0156	0.0542
0.997	0.1212	0.0023	0.119	0.997	0.0275	-0.0158	0.0434	0.997	0.0471	-0.0233	0.0704
0.998	0.1392	0.0052	0.134	0.998	0.0435	-0.0133	0.0567	0.998	0.059	-0.0323	0.0913
0.999	0.1606	0.0104	0.1503	0.999	0.0647	-0.0097	0.0744	0.999	0.0732	-0.0441	0.1173

Table 6
Correlation Between Stock Returns and Per Capita Growth of Consumption and Dividends

Moments	Data (Constrained)	Data (Unconstrained)	Benchmark Model	Extended Model	Extended Model (match equity premium)
1-year correlation between equity returns and consumption growth	-0.03 (0.12)	-0.05 (0.12)	0.047	0.062	-0.265
5-year correlation between equity returns and consumption growth	0.07 (0.17)	0.00 (0.14)	0.053	0.105	-0.183
10-year correlation between equity returns and consumption growth	-0.02 (0.30)	-0.11 (0.20)	0.061	0.127	-0.159
1-year correlation between equity returns and dividend growth	0.08 (0.12)	0.05 (0.11)	0.345	-0.149	0.639
5-year correlation between equity returns and dividend growth	0.27 (0.14)	0.3 (0.13)	0.325	0.024	0.562
10-year correlation between equity returns and dividend growth	0.51 (0.22)	0.59 (0.14)	0.386	0.100	0.588

Table 7

Predictability of Excess Returns by Price-dividend Ratio at Various Horizons

Benchmark Model

	Data	Model (median)	Model (plim)	Data	Model (median)	Model (plim)
	Slope Coefficient			R-square (% of values larger than R-square in data)		
1 year	-0.09	-0.05	0.005	0.04	0.01	0.0001
	(0.03)				(0.12)	
3 years	-0.26	-0.14	0.021	0.13	0.03	0.0006
	(0.07)				(0.09)	
5 years	-0.39	-0.21	0.025	0.23	0.04	0.0006
	(0.11)				(0.06)	

Extended Model

	Data	Model (median)	Model (plim)	Data	Model (median)	Model (plim)
	Slope Coefficient			R-square (% of values larger than R-square in data)		
1 year	-0.09	-0.05	-0.01	0.04	0.02	0.001
	(0.03)				(0.23)	
3 years	-0.26	-0.14	-0.02	0.13	0.05	0.002
	(0.07)				(0.19)	
5 years	-0.39	-0.22	-0.03	0.23	0.08	0.004
	(0.11)				(0.16)	

Extended Model (match equity premium)

	Data	Model (median)	Model (plim)	Data	Model (median)	Model (plim)
	Slope Coefficient			R-square (% of values larger than R-square in data)		
1 year	-0.09	-0.07	-0.01	0.04	0.03	0.002
	(0.03)				(0.33)	
3 years	-0.26	-0.19	-0.02	0.13	0.07	0.004
	(0.07)				(0.30)	
5 years	-0.39	-0.31	-0.04	0.23	0.12	0.007
	(0.11)				(0.25)	

Table 8
Term Structure of Bond Yields

Moments	Data (Unconstrained)	Benchmark Model	Extended Model	Extended Model (match equity premium)
Average yield				
Long-term bond	1.66	5.14	2.82	4.13
	(0.85)			
Intermediate-term bond	1.06	2.29	1.39	2.30
	(0.90)			
Short-term bond	0.36	0.36	0.387	0.36
	(0.81)			
Return to equity minus long-term bond yield	4.54	1.07	0.78	3.82
	(1.84)			
Standard deviation				
Long-term bond	3.54	1.73	2.11	1.85
	(0.76)			
Intermediate-term bond	3.65	3.19	2.64	2.05
	(0.82)			
Short-term bond	3.20	3.97	3.48	5.44
	(0.80)			
Return to equity minus long-term bond yield	16.7	15.61	18.08	15.33
	(1.48)			

Table 9
Regressions of Excess Stock Returns on Long Term Bon Yields in Excess of Short Rate

	Data 1929-2011	Data 1939-2011	Benchmark Model	Extended Model	Extended Model (match equity premium)
	Long Term Gov. Bond (20 years)				
R-square (% of values larger than R-square in 1939 sample)	0.04	0.04	0.13 (0.96)	0.02 (0.27)	0.1 (0.88)
Slope	3.49 (1.52)	2.83 (1.72)	3.44	1.16	1.08
Constant	-0.72 (3.67)	1.63 (3.49)	-0.05	0.01	0.05
	Intermediate Term Gov. Bond (5 years)				
R-square (% of values larger than R-square in 1939 sample)	0.02	0.03	0.07 (0.79)	0.02 (0.30)	0.1 (0.91)
Slope	3.89 (2.41)	3.85 (2.91)	3.33	1.07	1.08
Constant	0.87 (3.93)	2.29 (4.18)	-0.01	0.02	0.06