Financial Markets and Wages[*]

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PRELIMINARY AND INCOMPLETE

Abstract

We study the optimal long-term wage contract between financially constrained firms and workers. To alleviate the financial restrictions, firms promise an increasing wage profile, that is, they pay lower wages today in exchange of higher future wages once they become unconstrained and operate at a larger scale. In equilibrium financially constrained firms are on average smaller. Because they also pay lower wages on average, the model generates a positive relation between firm size and wages. The model also captures other empirical regularities such as the lower wages paid by fast growing firms.

JEL classification: G31, J31, E24

Key words: Investment financing, long-term contracts, wages

1   Introduction

The fact that large firms pay higher wages is a well-known stylized fact. Brown and Medoff (1989) and Oi and Idson (1999) provide a review of the empirical studies. In this paper we ask whether financial factors—in addition to other considerations proposed in the theoretical literature—can contribute to explaining the dependence of wages on the size of the employer.

Our interest in understanding the importance of financial factors for the firm size-wage relation is motivated by a set of regularities about the link between the financial characteristics of firms and their size. In general, the view that emerges from the financial literature is that smaller firms face tighter financial constraints, either in the form of lower ability to raise funds or in the form of higher cost of funds. In spite of these regularities, the claim that financial market imperfections may be relevant for explaining the firm size-wage relation is new. Oi and Idson (1999) conjecture that financial market imperfections can induce a greater cost of capital for small firms and as a consequence of that they choose a mode of production with lower capital intensity. This, coupled with some form of rent sharing, may imply that these firms pay lower wages. This argument, however, has not been fully developed theoretically. Moreover, it emphasizes only one possible channel whereby financial factors affect wages, which is different from the one we characterize in this paper.

We develop a model in which firms sign optimal long-term contracts with workers as in Burdett and Coles (2003). Due to limited enforceability, external investors are willing to finance the firm only in exchange of collateralized capital. If the investment that the firm can finance with external investors is limited—that is, the firm is financially constrained—the optimal wage contracts offered by the firm to the workers is characterized by an increasing wage profile. By paying lower wages today, the firm is able to generate higher cash-flows in the current period which relax the tightness of the financial constraints. Because firms with tighter constraints operate at a sub-optimal scale—which then they gradually expand until they become unconstrained—small firms pay on average lower wages than large firms. Therefore, the model generates a positive relation between the size of the firm and the average wages it pays to workers (the firm size-wage relation). At the same time, because constrained firms grow in size, the model also captures the empirical regularity that fast growing firms pay lower wages.

There are two features in the model that explain why firms are able to

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implicitly borrow from workers beyond what they can borrow from external investors. First, if a worker quits, the firm looses some sunk investment. This could derive from recruiting costs or training expenses that enhance the job specific human capital of the worker. The firm's loss of valuable investment endows the worker with a punishment tool which is not available to external investors. Second, a worker provides effort in the working place only if he or she believes that the effort will be rewarded by the firm. But when the firm reneges its wage promises, worker's confidence is lost, and the worker prefers to quit, since he or she expects the firm to renege the wage promises also in future periods. These punishment mechanisms, in conjunction with the risk that a worker may quit in the event of repudiation, guarantee that the firm will never renege the long-term wage contract.

The plan of the paper is as follows. In the next section we review the main empirical and theoretical contributions to the study of the firm size-wage relation. Section 3 describes the basic theoretical framework and characterizes some of the firm's policy and dynamics. Section 4 extends the model to allow for firm's entrance and exit and derives the industry equilibrium. The properties of the model are then studied numerically in Section 5. Section 6 describes how the optimal long-term contract can be sustained as a subgame perfect equilibrium of the strategic interaction between the firm and each individual worker. Section 7 discusses the robustness of some simplifying assumptions and Section 8 concludes.

2   Empirical regularities and existing theories

Before describing our theoretical framework, we briefly review the main empirical regularities and theoretical contributions to the study of the firm size-wage relation. The review of the theoretical literature shows that the effect of firm size on wages is still an unresolved puzzle while some of the empirical findings suggest that financial factors could play an important role.

2.1   Empirical regularities

Figure 1 plots the payroll per-worker for different size classes of firms, which is increasing in the size of firms. This is the typical pattern in almost all industries and is robust to the introduction of several controls for worker's and firm's characteristics. See Brown and Medoff (1989) and Oi and Idson (1999).

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Figure 1: Firm size and wages in 2001.
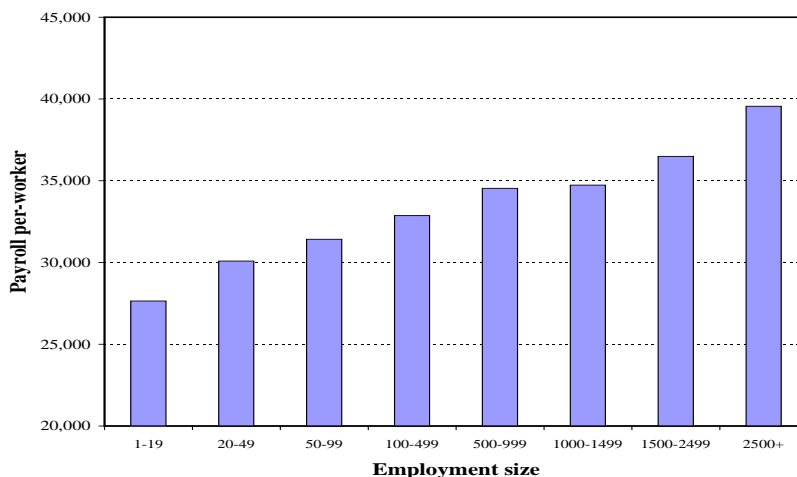
There many factors that could generate the positive relation between firm size and wages. For instance, the fact that larger firms employ more skilled workers. However, using panel data, several studies have reached the conclusion that the effect of firm size on wages is mostly explained by variation in firms' characteristics rather than workers' characteristics. In particular Abowd and Kramarz (2000) report that both in France and in the US, variation in firms' characteristics explains about 70 per cent of the firm size-wage differential. In addition to this result, there are other important findings in the empirical literature that are relevant for our paper. We summarize them below.

1. *Fast growing firms pay lower wages.* Bronars and Famulari (2001) report that employment growth has a negative effect on wages in a regression that controls for several workers and firms' characteristics. See also Hanka (1998).

2. *Firms that are in financial distress have lower employment and pay lower wages.* Nickell and Wadhwani (1991) document the negative relation between debt and employment. Nickell and Nicolitsas (1999), Hanka (1998), Blanchflower, Oswald, and Garrett (1990) provide some evidence that indicators of financial pressure are associated with lower wages.

3. *In some studies firm size is no longer significant after controlling for the*

financial conditions of the firm. Hanka (1998) finds that size (as measured by total assets) ceases of being significant after controlling for productivity (ROA and assets per employee) and financial distress variables (debt over assets ratio).

4. *The link between firm age and wages is not clear-cut.* Doms, Dunne, and Troske (1997) find that the effect of firm age on wages is positive if we do not control for worker's characteristics but it becomes negative (albeit not significant) if we control for worker's experience. The same pattern is documented by Troske (1999) and Brown and Medoff (2001).

5. *Indirect indicators point out that small firms tend to be more financially constrained.* Small firms pay fewer dividends and have higher value of Tobin's q. They rely more on bank financing and their growth is sensitive to cash flows. See for example Fazzari, Hubbard, and Petersen (1988), Gilchrist and Himmelberg (1996), Ross, Westerfield, and Jordan (1993) and Smith (1977). See also Kumar, Rajan, and Zingales (1999) for cross-countries evidence on how financial factors affect the size of firms.

These empirical results are important to evaluate our theoretical contribution to the explanation of the firm size-wage relation. Before presenting the theoretical model, however, let's summarize the existing theoretical contributions and how they relate to the empirical findings.

2.2   Existing theories

There are several explanations in the theoretical literature for the firm size-wage relation but none of them are entirely satisfactory. This view is clearly stated in Troske (1999) who concludes: *"After testing several possible explanations we are still left with the question: why do large firms pay higher wages?"*. Following is a brief description of the main theoretical contributions and their limits.

1. *Sorting of high quality workers in large firms.* If this was the basic mechanism, then the firm size-wage relation should become insignificant if we control for workers' quality. However, after controlling for several workers' characteristics, the effect of firm's characteristics remains large. The model studied in Kremer and Maskin (1996) emphasizes the complementarity that arises from matching high quality workers in the same firm. In this way, the effect of sorting on wages could possibly translate into a firm's fixed effect

that any single worker's characteristic can fail to capture. Yet, the inclusion of measures of average workers' quality into a standard wage regression does not reduce the size of the firm size-wage effect (see Bayard and Troske (1999)). Thus, sorting of high quality workers in large firms can explain only part of the size effect.

2. *Efficiency wages.* In an efficiency wage model a la Shapiro and Stiglitz (1984), large firms may pay higher wages because detecting shirking is more difficult. Some empirical evidences is not fully consistent with this explanation. For example, there are no differences in the magnitude of the firm size-wage effect between production and non-production workers (see Brown and Medoff (1989)) or supervisory and non supervisory workers (see Troske (1999)). Moreover, the magnitude of the effect does not changed after conditioning on the number of workers receiving incentive pay (again, see Brown and Medoff (1989)).

3. *Wage bargaining.* In bargaining models, wages increase with the net surplus generated by the job and with the bargaining power of workers. This theory can explain why wages are positively related to the size of the firm only if either the bargaining power of workers or the value of the job increases with the firm's size. However, the inclusion of variables that proxy for the bargaining power of workers, such as union-density or union-coverage, or the inclusion of variables that proxy for the value of the job such as firm's profit, firm's capital or severance payments, do not eliminate the significance of the firm size-wage effect (see Brown and Medoff (1989)).

4. *Burdett and Mortensen's model*: In Burdett and Mortensen (1998) firms face a trade-off between paying high wages to attract and retain a large number of workers or paying low wages but with fewer workers hired and more workers quitting. In equilibrium there are firms that pay low wages and remain small and firms that pay high wages and become large. This model, however, does not seem to capture the fact that fast growing firms tend to pay lower wages. In fact, firms that grow faster are the ones that pay higher wages. It should be point out, however, that this is only a conjecture since the firm dynamics generated by this model has not been fully explored. Similar considerations apply to the model studied in Burdett and Coles (2003).

5. *Financial factors*: Although the importance of financial factors for the firm size-wage relation has not been investigated theoretically, Oi and Idson (1999) hint a potential link. They conjecture that financial market imperfections can

lead to a greater cost of capital for small firms, which induce them to choose lower capital intensity. In a model in which workers have some bargaining power over the surplus of the firm, this would imply lower wages paid by smaller (constrained) firms. This mechanism, however, does not resolve the puzzle because the firm size effect remains significant even if we control for the capital intensity and the productivity of the firm. In our paper we propose an alternative mechanism through which financial factors affect the wage policy of the firm which does not rely on the capital intensity of the firm.

3 The basic model

We start describing a simple model in which firms face a deterministic problem and they live forever. This model allows us to emphasize some of the key features of the general model studied in Section 4.

Consider a risk-neutral infinitely lived entrepreneur with initial wealth a_0 and with lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t$$

where β is the intertemporal discount factor and d_t is consumption.

The entrepreneur has the managerial skills to run an investment project that generates revenues $y = A \cdot N$. The variable N denotes the number of hired workers and A is a constant. The project is subject to the capacity constraint $N \leq \overline{N}$. In the general model studied in Section 4, the capacity constraint $\overline{N}$ is allowed to differ across entrepreneurs or firms.

The employment of each worker requires two types of fixed investment: fungible investment κ_f and worker-specific investment κ_w. The first type of investment, κ_f, has an external value and can be resold at no cost. The second type, κ_w, is worker specific and it is lost if the worker quits or is fired. The worker-specific investment is interpreted as the cost incurred by the firm for recruiting and training a new worker. We will denote by $\kappa = \kappa_f + \kappa_w$ the sum of the two components. The total capital accumulated at the end of time t by a firm created at time zero is $\kappa \sum_{\tau=0}^{t} n_\tau$, where n_τ is the number of workers hired at time τ (and starting production at time $\tau + 1$). The output produced by the firm at $t + 1$ is $A \sum_{\tau=0}^{t} n_\tau$.

Workers are infinitely lived with lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[U(c_t) + \ell_t \Big], \qquad U(c_t) = \frac{c^{1-\sigma}}{1-\sigma}$$

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where β is the discount factor, c_t is consumption, and $\ell_t \in \{0, \bar{\ell}\}$ denotes the utility of leisure which is forgone when the worker provides working effort. The assumption that there is some forgone utility is relevant only for the analysis of renegotiation studied in Section 6. In equilibrium, workers always provide effort. Therefore, in the analysis that precedes Section 6 we simply impose $\ell_t = 0$.

We assume that workers initially do not have any assets and they can not borrow by pledging their future labor income. The idea is that the worker would always renege such promises. For the moment we also assume that workers cannot save, and therefore, consumption is simply equal to their wages. In Section 7 we will discuss the conditions under which workers would not save even if they were allowed to.

Funds are provided by *investors* who are risk-neutral and discount future payments at rate r. We assume that $1/(1 + r) \geq \beta$. The supply of funds of each investor is infinitesimal, but the aggregate number of investors is large enough to guarantee that the aggregate supply of funds is perfectly elastic at rate r. This guarantees that financial markets are perfectly competitive and the equilibrium interest rate is r. Given the assumption $1/(1 + r) \geq \beta$, firm borrowing is not dominated by internal financing.

The capital investment κ necessary to employ a worker is what creates the financial need. Using the renegotiation idea of Hart and Moore (1994) and Kiyotaki and Moore (1997), we assume that the entrepreneur can borrow only the amount that can be collateralized. In case of liquidation, investors can seize only the fungible capital. Therefore, κ_f is the only capital that can be used as a collateral. Since the collateral must also guarantee the interests on the loan, the firm can borrow at most $\bar{\kappa}_f = \kappa_f/(1 + r)$, per each worker. The borrowing limit, then, can be written as $b_t \leq \bar{\kappa}_f \sum_{\tau=0}^{t} n_\tau$, where b_t denotes the debt level contracted between time t and $t + 1$.

When a new worker is hired, the firm signs a long-term contract that specifies the whole sequence of wages. By assuming that the labor market is *competitive*, the initial promised utility provided by the contract to the worker is equal to the reservation utility $q_{res} > \bar{\ell}\beta/(1 - \beta)$. This is the utility that the worker would earn by re-entering the labor market. For the moment we assume that the firm cannot renegotiate the wage contract in future periods. In Section 6 we will characterize the conditions under which the firm never reneges on its promises and the contract can be supported as a sub-game perfect equilibrium of the repeated game played by the firm with each individual worker.

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3.1 The firm's problem

If the initial wealth of the entrepreneur a_0 is small, the firm starts with an initial employment that is smaller than $\overline{N}$. Over time the firm hires more workers and eventually it reaches the optimal scale $\overline{N}$. Let $\{w_{t,t+j}\}_{j=1}^{\infty}$ be the sequence of wages that the firm promises to the workers hired at time t. Here $w_{t,t+j}$ denotes the wage paid at time $t+j$ to workers hired at time t. Given the contracts signed with the workers of the various cohorts, the total wage payments at time $t+1$ are $\sum_{\tau=0}^{t} n_\tau w_{\tau,t+1}$. Also let a_t denote the *net worth* that appears on the balance sheet of the firm at the end of period t—that is, after production and after the payments of wages and interests. The sum of firm's net worth, a_t, and debt, b_t, equals the sum of firm's capital, $\kappa \sum_{\tau=0}^{t} n_\tau$, and dividend payments, d_t. Thus $d_t = a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau$.

Given the initial assets a_0, the firm maximizes the discounted value of the entrepreneur's consumption, which always equals dividends since the entrepreneur is at least as impatient as the market, $\beta \leq 1/(1+r)$. Thus, at time zero, the firm chooses the whole sequence of debt, employment and wages, that is $\left\{ b_t, n_t, \{w_{t,t+j}\}_{j=1}^{\infty} \right\}_{t=0}^{\infty}$, to solve the problem:

$$V(a_0) = \max \sum_{t=0}^{\infty} \beta^t \left(a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau \right) \tag{1}$$

subject to

$$a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau \geq 0, \tag{2}$$

$$b_t \leq \bar{\kappa}_f \sum_{\tau=0}^{t} n_\tau, \tag{3}$$

$$\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) \geq q_{res} \tag{4}$$

$$a_{t+1} = (\kappa + A) \sum_{\tau=0}^{t} n_\tau - \sum_{\tau=0}^{t} n_\tau w_{\tau,t+1} - (1+r)b_t,. \tag{5}$$

The firm's policy is subject to several constraints. Constraint (2) imposes the non-negativity of dividends. This results from the limited liability of the entrepreneur together with the non-negativity of consumption. Constraint (3) imposes the borrowing limit and (4) is the worker's participation constraint.

This imposes that the sequence of wages offered to each cohort of workers cannot be smaller than their reservation value q_{res}. Finally, constraint (5) defines the law of motion for the end-of-period net worth.

Let γ_t and $\lambda_t n_t$ be the lagrange multipliers associated with the constraints (2) and (4), respectively. Then Appendix A shows that the first order conditions imply that

$$U_c(w_{\tau,t}) = 1 + \gamma_t, \tag{6}$$

where U_c denotes the marginal utility of consumption. The variable λ_τ is the marginal cost to the firm of providing one unit of utility to a worker hired at time τ. The term $1 + \gamma_t$ is the value of one additional unit of internal funds. Therefore, equation (6) says that the optimal wage policy of the firm is such that the marginal cost of providing a certain utility to each worker must be equal to the marginal value of internal funds. Another way to interpret this condition is to say that the firm borrows from a worker until the cost of borrowing from the worker is equal to the marginal value of internal funds.

The multiplier γ_t captures the tightness of financial constraints and depends on the firm's net worth a_t. If a_t is small, the financial needs of the firms are high which imply that the value of an extra unit of internal funds is large. As the firm retains its earnings, the assets of the firm increase over time and the variable γ_t converges to zero. Then, equation (6) implies that:

Property 1 *The wage received by each worker grows over time until the firm becomes unconstrained, that is, $\gamma_t = 0$.*

Equation (6) also implies that the ratio of marginal utilities between workers of different cohorts remain constant over time. Indeed, if we evaluate (6) for two different cohorts indexed by τ_1 and τ_2, and we divide side by side we obtain

$$\frac{U_c(w_{\tau_1,t})}{U_c(w_{\tau_2,t})} = \frac{n_{\tau_1}\lambda_{\tau_2}}{n_{\tau_2}\lambda_{\tau_1}},$$

Since the right-hand-side does not depend on t, this condition implies that:

Property 2 *The ratios of marginal utilities between different cohorts of workers remain constant over time, that is, $U_c(w_{\tau_1,t})/U_c(w_{\tau_2,t}) = \zeta_{\tau_1,\tau_2}$, for all t, where ζ_{τ_1,τ_2} is a constant for given $\tau_1, \tau_2 < t$.*

In the next section we take advantage of this property to rewrite the problem recursively with a limited number of state variables.

3.2 Recursive formulation of the firm's problem

Let $q_{\tau,t} = E \sum_{j=1}^{\infty} \beta^j U(w_{\tau,t+j})$ be the lifetime utility promised at the end of time t to a worker hired at time τ, with $t \geq \tau$. Notice that $q_{\tau,t}$ follows the recursive form

$$q_{\tau,t} = \beta \Big[U(w_{\tau,t+1}) + q_{\tau,t+1} \Big] \tag{7}$$

with $q_{\tau,\tau} = q_{res}$.

With the utility function $U(c) = c^{1-\sigma}/(1-\sigma)$, Property 2 implies that the ratios of wages paid to workers of different cohorts remain constant over time. In particular, the ratio between the wage paid at time $t+1$ to the workers hired at time t and the wage paid to the first cohort of workers hired at time zero, $w_{t,t+1}/w_{0,t+1}$, is constant over time. But then the ratio of utilities promised at the end of period t is also constant over time, which implies that

$$\frac{q_{t,t}}{q_{0,t}} = \left(\frac{w_{t,t+1}}{w_{0,t+1}} \right)^{1-\sigma} = \frac{q_{res}}{q_{0,t}},$$

where the last equality uses the fact that $q_{t,t} = q_{res}$. Inverting the second equality provides an expression for the wage ratio between the cohort hired at time τ and the cohort hired at time zero, which reads as

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{q_{res}}{q_{0,t}} \right)^{\frac{1}{1-\sigma}} = \psi(q_{0,t})$$

From now on we omit the subscript 0 to identify the first cohort of workers. Therefore, w_t and q_t denote the time-t wage and promised utility of the first cohort of workers. The total wage payments paid by the firm at time t can be written as $H_t w_t$, where

$$H_t = \sum_{\tau=0}^{t-1} \psi(q_\tau) n_\tau,$$

which evolves as

$$H_{t+1} = H_t + \psi(q_t) n_t. \tag{8}$$

Notice that these considerations imply that, once H_t is known, and given the utility promised to the first cohort of workers, q_t, the firm just need to decide w_{t+1} to determine the optimal wage payments at time $t+1$. Given the laws of motion for q_t and H_t—equations (7) and (8)—this allows us to

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write the firm's problem recursively by making use of a limited number of state variables. Specifically, the firm's problem can be written as:

$$V(a, q, N, H) = \max_{b, w', q', N' \leq \bar{N}} \left\{ a + b - \kappa N' + \beta V(a', q', N', H') \right\} \quad (9)$$

subject to

$$a + b - \kappa N' \geq 0, \quad (10)$$
$$b \leq \bar{\kappa}_f N', \quad (11)$$
$$q = \beta \left[U(w') + q' \right], \quad (12)$$
$$a' = \kappa N' + A N' - H' w' - (1 + r) b, \quad (13)$$
$$H' = H + \psi(q)(N' - N), \quad (14)$$

where N denotes the current employment of the firm and the prime denotes the next period value. Thus $N' - N$ is the change in employment, that is, the number of workers hired in the current period (and starting production in the next period).

Let γ and $\lambda H'$ denote the lagrange multipliers associated with constraints (10) and (12), respectively. Then Appendix B shows that the first order conditions of the problem imply that

$$1 + \gamma' = \lambda U_{w'}, \quad (15)$$
$$\lambda' = \lambda, \quad (16)$$

where the first condition is analogous to (6), and the second says that the lagrange multiplier of the worker's participation constraint is constant over time.

These two conditions characterize the wage dynamics of the firm. As observed in the previous section, the lagrange multiplier γ decreases over time until it becomes zero. From equation (15) we can see that the wage rate paid to the first cohort of workers increases over time until $\gamma' = 0$. Because the wages paid to all other cohorts of workers are proportional to the wage paid to the first cohort, we also have that the average wages increase over time until $\gamma' = 0$. Wages differ across workers of different cohorts. In fact, because all workers start with $q = q_{res}$, after which the promised utility grows over time, older workers receive higher wages than younger workers.

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One of the predictions of the model is that the wage profile of constrained (young) firms is steeper than the wage profile of mature (old) firms.

Once the firm becomes unconstrained, that is, $\gamma = 0$, the firm would like to expand its capacity beyond $\overline{N}$, but the constraint $N' \leq \overline{N}$ binds.

Figure 2 shows some of the properties of the model with a numerical example based on the following parameter values: $r = 0.05$, $\beta = 0.9$, $\sigma = 1.5$, $q_{res} = U(0.58)/(1-\beta)$, $\overline{N} = 1,000$, $A = 1$, $\kappa = 3.5$, $\kappa_f/\kappa = 0.4$ and $a_0 = 330$. The numerical example considered here is provided only for illustrative purposes. A formal calibration exercise will be conducted in Section 5 after the specification of the general model.

Figure 2: Employment dynamics and wage patterns over age and size.
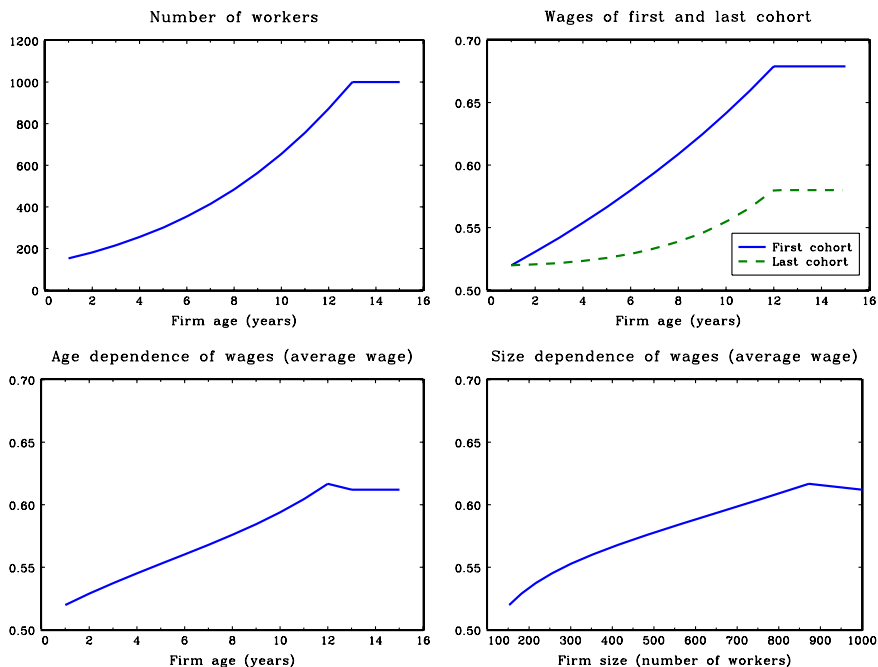

The first panel of Figure 2 plots the employment dynamics. The firm starts with an initial employment of almost 150 workers and then gradually grows over time until it reaches the optimal size of 1,000 workers. The transition takes place in 12 periods. The second panel plots the wage profile of the first cohort of workers (those hired at time 0) and the initial wage

paid to newly hired workers. The wage profile of the first cohort of workers (continuous line) is increasing until the firm reaches the unconstrained status. The dashed line shows the wage earned in the first period of employment by workers of different cohorts. As the firm gets closer to the optimal scale, it offers higher initial wages, and therefore, the wage profile of newer workers is less steep overall.

The third panel plots the average wage paid by the firm as a function of its age and the fourth panel the average wage as a function of its size (number of employees). The average wage increases with the size and age of the firm. This is a direct consequence of the fact that, when the firm is young and constrained, it operates at a suboptimal scale and offers an increasing profile of wages.

An important parameter that affects the magnitude of the size dependence of wages is σ, that is, the curvature of the utility function. As shown in Figure 3, the lower is the value of σ, the stronger is the size dependence. In the extreme case in which $\sigma = 0$, that is, the utility function is linear, workers will receive zero wages initially until the firm becomes unconstrained.

Figure 3: Firm size and wages for different curvatures of the utility function.
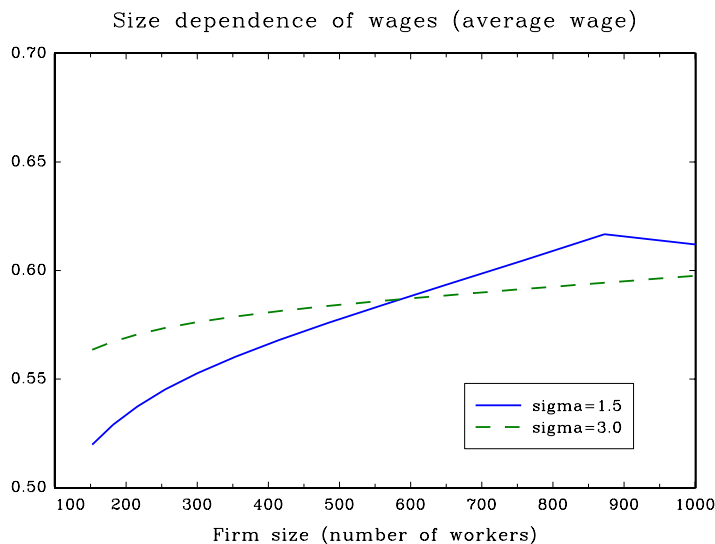
In this simple model the profile of wages is fully captured by the age of the

firm. In other words, once we control for age, the size of the firm is irrelevant because there is a one-to-one mapping between size and age. However, in a cross section of firms, size will have an independent effect. This is because firms may have different capacities $\overline{N}$ and they can start with different initial assets a_0. In order to capture the firm size effect on wages in a cross-section of firms, we need to specify the whole industry structure, including entrance and exit. Let's turn then to the specification of the general model.

4 General model and labor market equilibrium

In the analysis of the previous sections we made several simplifying assumptions including the absence of entrance and exit. In that environment, all firms will eventually become unconstrained and at some points financial frictions would no longer be relevant. We now extend the model to allow for entrance and exit. This will insure that at any point in time there is always a fraction of firms that are financially constrained.

We assume that there is a probability $1 - p$ that an investment project becomes obsolete and the firm exits. Exiting firms are replaced by new entrant firms managed by new entrepreneurs. New entrepreneurs draw the project capacity $\overline{N}$ from the distribution $\Gamma(\overline{N})$.

The initial wealth of new entrepreneurs could be correlated with the project capacity. For instance, entrepreneurs with more promising projects may be able to raise higher initial funds by pooling a larger number of funders. Alternatively, we can think that the probability of drawing large capacity projects increases with the ability of the entrepreneur, which in turn is related to the initial wealth. To formalize this idea in a simple manner, we assume that there is a unique relation between the project capacity $\overline{N}$ and the initial wealth of the entrepreneur, which takes the form $a_0 = \alpha \cdot \overline{N}^\rho$. By choosing the parameters α and ρ we can impose different degrees of financial tightness for new firms, as a function of projects capacity. Lower values of α increases the financial tightness for all new firms while the parameter ρ affects the distribution across different types of firms. When $\rho = 0$, all firms start with the same initial assets.

Given the initial assets a_0 and the project capacity $\overline{N}$, the problem solved by an active entrepreneur is very similar to the problem studied in the previous section although now we have to specify what happens to the wage contract when the project becomes obsolete. In this case workers lose their job. By re-entering the labor market the continuation utility for the worker

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is q_{res}. In the implementation analysis conducted in Section 6 we show that this is the only feasible outcome.[1] The promise-keeping constraint can then be written as:

$$q_{\tau,t} = \beta\Big[U(w_{\tau,t+1}) + p \cdot q_{\tau,t+1} + (1-p) \cdot q_{res}\Big]$$

Here the assumption is that the viability of the project is observed after paying the current wage (but before the new investment). Consequently, the current wage is not renegotiated.

For the analysis that follows it is convenient to rescale the promised utility $q_{\tau,t}$ by the constant term $(1-p)\beta q_{res}/(1-p\beta)$ so as to define

$$z_{\tau,t} = q_{\tau,t} - \frac{(1-p)\beta q_{res}}{1-p\beta}$$

which evolves as

$$z_{\tau,t} = \beta\Big[U(w_{\tau,t+1}) + p\, z_{\tau,t+1}\Big]$$

The advantage of using on z, rather than q, is that the wage ratio between the last and first cohorts takes the simple form

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left(\frac{z_{res}}{z_{0,t}}\right)^{\frac{1}{1-\sigma}} = \psi(z_t),$$

where, as in the previous analysis, we omit the zero subscript to identify the first cohort of workers—i.e. z_t denotes the re-scaled utility promised to the first cohort of workers. The recursive representation becomes similar to that of section 3.2, once we use z as a state variable. The firm's problem is formally described in Appendix C, that also characterizes the relevant first order conditions.

We are now able to define a steady state labor market equilibrium.

Definition 1 *A steady state labor market equilibrium is defined by: (i) A distribution (measure) of firms $M(a, z, N, H, \overline{N})$; (ii) A reservation utility*

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[1]In principle the entrepreneur could promise extra future payments to the worker if the firm is liquidated. However, the promises of these payments are not credible. Indeed, when the technology becomes obsolete, the sunk investment is inevitably lost and there is no cost for the firm from renegotiating the contract. Consequently, the worker's continuation utility becomes q_{res}.

q_{res}; (iii) A transition function for the distribution of firms. Such that: (a) The transition function is consistent with the firm policies and the probability distribution of initial project capacity $\Gamma(\overline{N})$ and wealth $a_0 = \alpha \overline{N}^\rho$; (b) The demand of labor $\int N \cdot dM(a, z, N, H, \overline{N})$ equals the fixed supply of workers; (c) The next period distribution generated by the transition function is equal to the current distribution.

Notice that this is still a partial equilibrium: although the reservation value q_{res} is endogenously derived to clear the labor market, the interest rate r is exogenous in the model.

5   Numerical analysis and simulated regression

We first calibrate the model and then report the results from running some wage regressions similar to those considered in the empirical literature.

Calibration: We start with the following baseline parametrization. The interest rate is set to $r = 0.05$, the intertemporal discount rate to $\beta = 0.9$, and the inverse of the elasticity of substitution to $\sigma = 1.5$. The per-worker investment κ is chosen to have a capital-output ratio of 3.5. With the normalization $A = 1$, this requires $\kappa = 3.5$. The non-sunk fraction of capital κ_f/κ determines the leverage of the firm. We set $\kappa_f/\kappa = 0.4$ which is consistent with the average leverage of Compustat companies. The survival probability is set to $p = 0.95$. This is a compromise between the low exit rate of large firms and the high exit rate of small firms.

The initial wealth of the entrepreneur is given by $a_0 = \alpha \cdot \overline{N}^\rho$. Given the linearity of the production function and of the borrowing limit, the financial tightness of a new firm is captured by the ratio:

$$FTI \equiv \frac{(\kappa - \bar{\kappa}_f) \cdot \overline{N}}{a_0} = \frac{(\kappa - \bar{\kappa}_f) \cdot \overline{N}^{1-\rho}}{\alpha}.$$

where FTI stands for Financial Tightness Index. The numerator is the total capital that must be financed internally when the firm operates at the optimal scale $\overline{N}$. The denominator is the value of internal funds. When this ratio is greater than 1 the firm is financially constrained. Lower values of α increases the financial tightness for all new firms while the parameter ρ affects the distribution across different types of firms. When $\rho = 1$, the

tightness is independent of the firm's capacity. When $0 < \rho < 1$, firms with larger capacity face tighter constraints. In the baseline model we set $\rho = 0.8$ but then we will conduct a sensitivity analysis. After fixing ρ, we choose the parameter α so that the initial assets of the smallest capacity firms are half the value required to operate at the maximum scale. Given the minimum project capacity (specified below), the required value is $\alpha = 1.495$. We will also conduct a sensitivity analysis with respect to this parameter.

Table 1: Size distribution of firms in the U.S. economy, 2001.

Firm size (Employees)	Firms	Employees	$\frac{Employees}{Firms}$
1-19	87.46%	17.90%	4.7
20-49	7.94%	10.27%	30.0
50-99	2.53%	7.43%	68.4
100-499	1.72%	14.26%	192.4
500-999	0.17%	5.13%	689.0
1,000-1,499	0.06%	3.02%	1,217.4
1,500-2,499	0.05%	3.84%	1,915.8
2,500+	0.07%	38.13%	12,074.1
Total	100.00%	100.00%	23.2

Source: SBA, Office of Advocacy, data from U.S. Census Bureau, Statistics of U.S. Businesses.

The probability distribution of new projects capacity $\Gamma(\overline{N})$ is parameterized to replicate the main moments of the distribution of firms observed in the U.S. economy. Using data from the Small Business Administration, Table 1 reports the percentage of firms and employment for eight size classes of firms in the year 2001. To replicate the same moments of the U.S. distribution, we assume that the employment capacity $\overline{N}$ can take eight values. These values and the corresponding probabilities are reported in Table 2.[2] The table also reports the Financial Tightness Index (FTI) discussed above. For firms running the largest project, the initial assets are about 10 percent the value required to operate at the optimal scale.

The last parameter of the model that needs to be pinned down is the total mass of workers, that is, the labor supply. The larger is the supply of labor, the lower is the market clearing value of q_{res}, which in turn affects

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[2]By properly choosing the eight project capacities and the corresponding probabilities, we can replicate exactly all the moments of the distribution reported in Table 1.

Table 2: Probability distribution of new projects and financial tightness.

$\overline{N}$	$\Gamma(\overline{N})$	*FTI*
5.0	0.85883	*2.0*
30.7	0.08532	*2.9*
68.7	0.02937	*3.4*
197.2	0.02171	*4.2*
698.1	0.00236	*5.4*
1,219.7	0.00091	*6.0*
1,926.9	0.00064	*6.6*
16,638.7	0.00086	*10.1*

average wages. We choose the supply of labor to have a labor income share of 60 percent.

Simulated regression: From the steady state distribution, we extract a random sample of firms and estimate the following regression model:

$$
\begin{aligned}
\ln(\text{Wage}_{i,j}) \;=\; & \bar{\alpha} + \alpha_T \cdot \text{WorkerTenure}_{i,j} + \alpha_{T^2} \cdot \text{WorkerTenure}^2_{i,j} + \\
& \alpha_A \cdot \text{FirmAge}_j + \alpha_{A^2} \cdot \text{FirmAge}^2_j + \\
& \alpha_S \cdot \ln(\text{FirmSize}_j) + \alpha_G \cdot \text{FirmGrowth}_j
\end{aligned}
$$

where the index i identifies the worker and j the firm where the worker is employed. This specification is similar to the one used in the empirical literature although we include a smaller set of control variables consistent with the structure of our model. The estimation results are reported in Table 3 with the t-statistics in parenthesis.

The first column reports the coefficient estimates when all variables are included in the regression. All the estimates are statistically significant. Of special interest are the coefficients of firm's size and growth. The estimates for these two parameters are consistent with the findings of the empirical literature. In particular, while the size of the firm has a positive effect on wages, firm's grow has a negative effect.

These results have a simple intuition. Let's start with the positive effect of firm's size. Firms that are large in the current period are those that experienced tight financial constraints in the past. Therefore, they were operating at suboptimal (smaller) scales. In order to accelerate their grow, these firms paid low wages in the past in exchange of higher future wages. Now that

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Table 3: Wage equation estimation from model-generated data.

Description	(1)	(2)	(3)	(4)
Constant	-0.5755	-0.6308	-0.5750	-0.5292
	(-234.0)	(-280.1)	(-180.7)	(-403.7)
Worker tenure	0.0094	0.0104	-	-
	(38.2)	(50.4)	-	-
Worker tenure^2	0.0000	-0.0002	-	-
	(4.0)	(-45.7)	-	-
Firm age	-0.0049	-	0.0040	-
	(-19.3)	-	(16.2)	-
Firm age^2	-0.0001	-	-0.0001	-
	(-14.9)	-	(-16.0)	-
Firm log-size	0.0102	0.0042	0.0034	0.0049
	(57.4)	(26.2)	(16.5)	(26.6)
Firm growth	-0.2106	0.0878	-0.1343	-0.3019
	(-19.7)	(8.7)	(-9.9)	(-40.0)
R-square	0.538	0.366	0.211	0.190
Observations	10,000	10,000	10,000	10,000

Notes: t-statistics in parenthesis.

they are unconstrained and larger, they pay higher wages in fulfillment of their previous promises. This generates a positive correlation between firm's size and wages. In quantitative terms the effect of the firm's size is important and comparable to those found in the empirical literature. For example Bronars and Famulari (2001) report a coefficient of log-firm-size that ranges from 0.01 to 0.02 (see their Table 4).[3] If we compare firms that are in the size class 1-19 with firms that employ more than 2,500 employees, then the average wage paid by the second group of firms is about 8 percent bigger than the wage paid by the first group of firms.

The intuition for the negative effect of firm's growth arises naturally from the discussion above: firms that grow are the ones that experience binding

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[3]Bronars and Famulari (2001) measure size by sales rather than employment. Since in our model the production function is linear and there is no heterogeneity in productivity, the coefficients of our regression (aside the constant) are unaffected by the choice of the size variable.

financing constraints. Because of these constraints, growing firms pay lower wages today in exchange of higher future wages when they are unconstrained and do not grow. Quantitatively, the coefficient is not very different from those estimated in the empirical literature. Bronars and Famulari (2001) report a coefficient of firm growth that ranges from -0.4 to -0.35 (see their Table 4).

The other two variables included in the regression is the worker's tenure and the age of the firm. The positive effect of the worker's tenure derives from the fact that the wages paid by constrained firms increase over time, and therefore, with the tenure of workers. The return to tenure is small. It is around one fifth the effect estimated by Topel (1991), but comparable to the effect estimated by Altonji and Shakotko (1987), where however the coefficient is not statistically significant. The estimated coefficient for firm's age is negative. However, the sign of this coefficient depends on the variables we include in the regression. For instance, if we exclude worker's tenure, firm's age has a positive effect as shown in the third column of Table 3. With the addition of other variables, and especially tenure, age turns out to have a negative effect. This seems consistent with the results of the empirical literature that finds ambiguous effects of firm's age on wages. If we exclude firm's age from the regression, worker's tenure remains statistically significant.

Sensitivity analysis   Table 4 reports the estimates for alternative parameters of the model. We first change the concavity of the utility function σ. When $\sigma = 0.8$ (low concavity), the firm-size wage effect more than double. In this case, the wages of firms with more than 2,500+ employees are about 17 percent higher than the wages paid by firms in the first size class 1-19. This derives from the fact that the cost of offering an increasing wage profile is smaller when the intertemporal elasticity of substitution for workers is high. Consequently, firms offer a steeper wage profile and the firm size-wage relation and the firm growth-wage relation are stronger. The opposite is true when $\sigma = 3.0$. In the limit case in which $\sigma = \infty$ all firms would pay a constant wage and the model would not generate any wage differential.[4]

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[4]There is a limit to how small σ can be. If this parameter is very small, then the wage profile becomes so steep that large-unconstrained firms pay much higher wages than the ones offered to new workers. This implies that the firm may have an incentive to renegotiate the wage contract: the gains from replacing the worker (and paying lower wages) could exceed the loss in sunk capital. However, with $\sigma = 0.8$ the non-renegotiation condition is still satisfied, as we will show in Section 6.

Table 4: Sensitivity analysis

Description	(1) $\sigma = 0.8$	(2) $\sigma = 3.0$	(3) $\rho = 0.7$	(4) $\rho = 0.9$	(5) $\rho = 1.0$	(6) $\alpha = 0.76$	(7) $\alpha = 2.22$
Constant	-0.5727	-0.5499	-0.6257	-0.5260	-0.5032	-0.6507	-0.5589
	(-110.9)	(-480.3)	(-207.7)	(-300.9)	(-534.0)	(-174.3)	(-325.8)
Worker tenure	0.0197	0.0044	0.0115	0.0086	0.0068	0.0104	0.0093
	(38.0)	(38.9)	(42.2)	(40.1)	(42.9)	(40.0)	(40.4)
Worker tenure^2	0.0001	0.0000	-0.0000	-0.0000	-0.0001	0.0001	-0.0000
	(5.3)	(3.7)	(-1.4)	(-3.6)	(-19.7)	(11.7)	(-2.7)
Firm age	-0.0162	-0.0015	-0.0027	-0.0073	-0.0063	0.0029	-0.0076
	(-29.2)	(-13.1)	(-9.6)	(-34.3)	(-40.2)	(9.3)	(-35.6)
Firm age^2	-0.0001	-0.0000	-0.0001	-0.0000	0.0001	-0.0002	0.0000
	(-7.1)	(-20.2)	(-21.0)	(-4.5)	(26.0)	(-42.0)	(2.1)
Firm log-size	0.0216	0.0047	0.0139	0.0051	0.0002	0.0095	0.0106
	(61.3)	(55.1)	(56.4)	(46.7)	(3.0)	(44.2)	(69.3)
Firm growth	-0.6967	-0.0643	-0.1366	-0.2566	-0.1844	-0.0797	-0.2431
	(-34.2)	(-12.5)	(-10.9)	(-32.3)	(-42.3)	(-5.9)	(-28.2)
R-square	0.549	0.541	0.524	0.510	0.459	0.597	0.513
Observations	10,000	10,000	10,000	10,000	10,000	10,000	10,000

Notes: t-statistics in parenthesis.

The second type of sensitivity analysis is with respect to the financial tightness of newly created firms. The initial assets of a new firm is related to its capacity according to the function $a_0 = \alpha \cdot \overline{N}^\rho$. The parameter α determines the tightness for all newly created firms, while the parameter ρ determines the relative tightness of firms with different capacity $\overline{N}$. If $\rho = 1$, all newly created firms face the same tightness. If $\rho < 1$, firms with larger capacity face tighter constraints. For the baseline model we have chosen $\alpha = 1.495$ and $\rho = 0.8$. As shown in the third, fourth and fifth columns of Table 4, the firm size effect on wages is inversely related to the value of ρ. In other words, the higher is the tightness of large capacity firms *relative* to the tightness of small capacity firms, the stronger is the effect of firm size on wages. When all newly created firms face the same tightness ($\rho = 1$), the effect of firm size is very small, almost insignificant. On the other hand, if we change the tightness for all firms, that is, we change α, the firm size effect does not change significantly (see the last two columns of Table 4).

This finding highlights the basic mechanism underlying our results. What is important for generating a firm size-wage relation is not simply the presence of financial constraints but the assumption that these constraints are tighter for firms with higher potential to grow. If all new firms face the same financial tightness, then the differences in wages are fully captured by the age of the firm. This explains why in this case the size of the firm becomes statistically insignificant.

Figure 3 illustrates this point with an example. Suppose that there are only two types of firms: low capacity and high capacity firms. We will refer to the first type of firms as "Small" and to the second type as "Large". Suppose that firms live for two periods. When young they are financially constrained. When old they are unconstrained and operate at the optimal scale. This implies that young firms pay lower wages and operate at a smaller scale. Figure 4 plots the wages and size for these two types of firms, when young and when old. The top panels are for the case in which all firms face the same financial tightness when young. The bottom panels are for the case in which high capacity firms face tighter constraints when young.

Consider first the case in which all firms face the same financial tightness (top panels). In this case, the differential in wages between young and old firms is the same for small and large firms. Therefore, a dummy variable that differentiates young firms from old firms would be sufficient to explain the wage differential. In other words, once we net out the wage differential explained by age, there is no relation between firm size and wages. This is shown in the right-hand-side panel of Figure 4.

Now let's assume that large capacity firms face tighter constraints. The bottom panels of Figure 4 show this case. Because of the tighter constraints, the wage differential between young and old firms is bigger for large capacity firms (steeper wage profile). In this case, the age dummy is unable to fully capture the wage differential. As shown by the right-hand-side panel, even if we net out the effect of age, there still remains a positive correlation between firm size and wages.

Firm size and wage-tenure profile: The theoretical analysis has shown that average wages increase with workers' tenure. An important question is whether the wage-tenure profile differs across firms of different sizes. Using data from the Benefits Supplement to the Current Population Survey (CPS), Hu (2003) shows that the wage-tenure profile for white collar workers is

Figure 4: Financial tightness and firm size-wage relation.
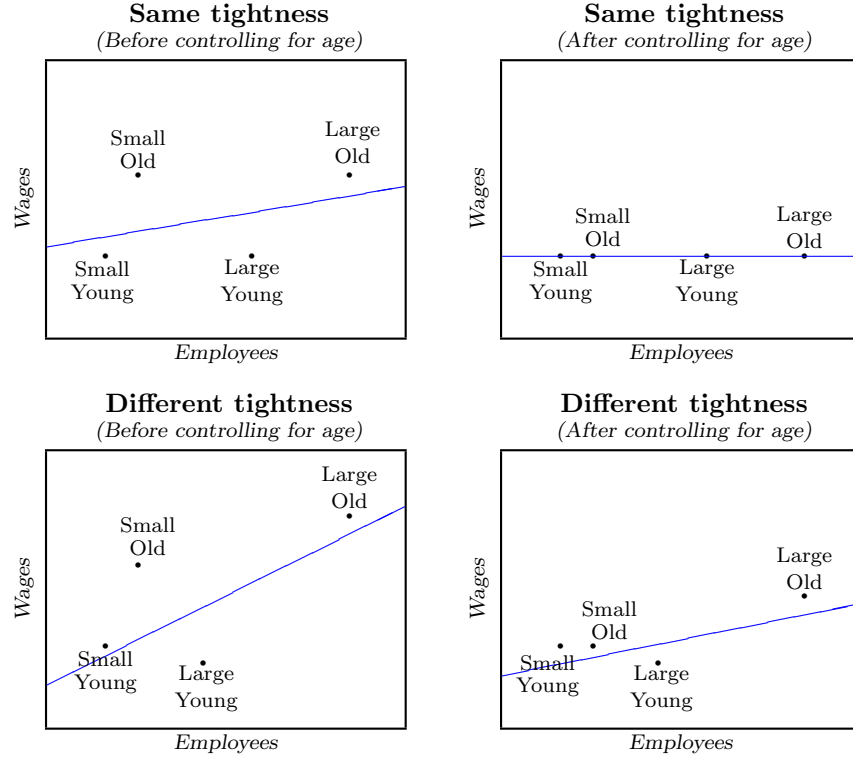
steeper in large firms. Our theoretical model is fully consistent with this finding. Following Hu (2003), we estimate a regression equation that relates the log-wage to a quadratic polynomial of worker's tenure, for four size classes of firms: less than 25 employees, 25-99 employees, 100-999 employees, 1,000 and more employees. Formally, the regression equation is:

$$\ln(\text{Wage}) = \sum_{j=1}^{4} \gamma_j \cdot \text{SizeDummy}_j + \sum_{j=1}^{4} \delta_j \cdot (\text{SizeDummy}_j \cdot \text{WorkerTenure})$$
$$+ \sum_{j=1}^{4} \lambda_j \cdot (\text{SizeDummy}_j \cdot \text{WorkerTenure}^2)$$

where the index j identifies the four size classes of firms described above.[5]

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[5]Hu also includes other workers' characteristics which we do not include because in our model all workers are alike.

Figure 5: Wage-Tenure profile for different size classes of firms.
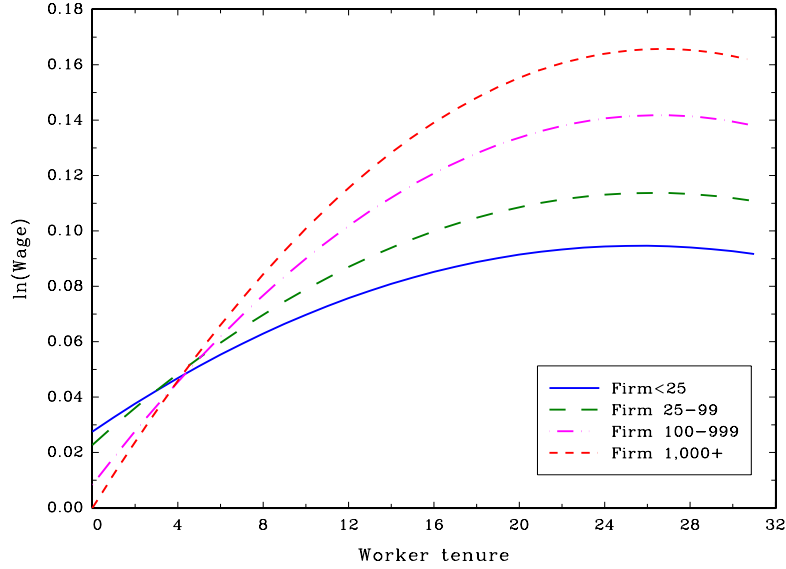
The estimated wage-tenure profiles for the four size classes of firms are plotted in Figure 5. The wage-tenure profile of workers in large firms is steeper than in small firms, which is consistent with the data. This result may appear conterintuitive at first: because firms that pay increasing wages are those with binding financial constraints—and therefore, they operate at a sub-optimal scale—we may have inferred that small firms have steeper wage profiles. This would be the case if all firms have the same capacity $\overline{N}$. But in the model the size of firms depends not only on the financial status, but also on their technological capacity. Given the parametrization of ρ, large capacity firms are on average more constrained than small capacity firms. This implies that large capacity firms pay a steeper wage profile. Because they are on average larger than small capacity firms, the model generates a positive relation between the slope of the wage-tenure profile and the size of firms.

6 Contracts implementation

In the analysis of the long-term contract we have assumed that the firm never reneges the promised wages. This could be problematic because wages and promised utilities increase until the firm becomes unconstrained. More specifically, a new hired worker starts with $q_t = q_{res}$ but then he or she receives $q_{t+j} \geq q_{res}$, for all $j > 0$. Because new workers can be hired with initial utility q_{res}, the firm may have an incentive to renege promises that exceed q_{res}. The goal of this section is to discuss the conditions that prevent the firm from renegotiating the long-term contract. We then discuss why collateralized debt is the only form of external financing for the firm.

Before continuing, it will be convenient to summarize the timing of the model. First workers decide whether to provide effort—which has a cost $\bar{\ell}$ in forgone utility—and whether to quit the firm. Then production takes place and the firm observes whether the worker has provided effort. At this point the firm could renege its wage promises. Afterwards, the firm decides whether to renegotiate the debt. Renegotiation entitles the investors to seize the firm's assets.

6.1 Worker-firm relationship

If both the worker and the entrepreneur cooperate (the worker by exerting effort and the entrepreneur by paying the promised wage), output is produced and the worker earns the promised income. The only Nash Equilibrium of each period sub-game is the one in which the firm reneges its promises and pays zero wages and the worker, anticipating that, withdraws effort and quits. In the repeated game, however, cooperation can be sustained through trigger strategies, provided that replacing the worker is sufficiently costly for the firm. Specifically, suppose that the worker and the firm follow these strategies (which for simplicity are specified independently of the investors' past history):

- **Worker**: The worker provides effort as long as the firm pays the contracted wages. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), the worker withdraws effort and quits.

- **Firm**: The firm pays the contracted wages as long as the worker provides effort. If one of the two parties has reneged sometimes in the past

(either the worker has shirked or the firm has paid a wage different from the one contracted), it sets the wage to zero.

The equilibrium associated with these strategies is sub-game perfect. To see this, let's consider first the worker. Providing low effort would trigger a wage cut which forces the worker to quit the firm and be left with the reservation value q_{res} starting from the next period. But the utility from doing so, $U(0) + \bar{\ell} + q_{res}$, is not bigger than the utility obtained from providing effort, that is, $U(w_t) + pq_t + (1-p)q_{res}$. Thus, along the equilibrium path, the worker never shirks and quits. If the firm has sometimes paid a different wage from the one contracted, quitting is optimal since the firm would pay a zero wage both today and in the future.

Consider now the firm. When the firm expects the worker to quit tomorrow, setting the wage to zero today is always the firm's best response. Thus, given each worker's strategy, paying zero wages is optimal when the worker has sometimes shirked. Along the equilibrium path, the firm never finds optimal to deviate from the promised long-term contract because, if the firm reneges its wage promises, the worker quits and the firm looses the sunk investment κ_w. Therefore, the assumptions that part of the investment is worker-specific, is key to prevent the firm from renegotiating the contract.

The fact that the replacement of an existing worker is costly for the firm, creates an indirect form of "collateral" for workers. This allows the firm to borrow from the workers beyond what it can formally borrow from external investors. Of course, there is a limit to this. If the worker's utility becomes very large, then the loss of the sunk investment may be smaller than the gains from reducing the wage obligations (by reneging the long-term contract and hiring a new worker). This may happen if $\bar{\kappa}_f/\kappa$ is close to 1 and the initial assets of the firm, a_0, are small. In this paper we have implicitly assumed that $\bar{\kappa}_f/\kappa$ is sufficiently small and a_0 sufficiently large so that this never arises in equilibrium. This condition, in particular, is satisfied in the numerical analysis of Section 5.

To show that the non-renegotiation condition is satisfied in the numerical exercises conducted in this paper,

Table 5 reports the maximum gains that can be obtain by replacing an existing worker (and paying lower wages). The maximum possible gains can be obtained by an unconstrained firm with the largest capacity $\overline{N}$. This firm is paying the highest wages to the first cohort of workers, that is, the workers that were hired when the firm was first created. Denote the wage paid to

these worker by w_{max}. The firm could replace these workers with new hired workers to whom the firm pays a constant wage w_{res}. This is the wage that gives to the worker the lifetime utility q_{res}. By doing so, the firm would save $w_{max} - w_{res}$ in wage payments. The expected discounted value of these payments are:

$$RG(\mathbf{P}) \equiv \frac{\beta p(w_{max} - w_{res})}{1 - \beta p}$$

where RG stands for Renegotiation Gains and $\mathbf{P}$ are the model's parameters. The renegotiation gains are compared with the loss of sunk capital κ_w. As shown in Table 5, for all curvatures of the utility function used in the quantitative section of the paper, the renegotiation gains are smaller than the loss in sunk capital $\kappa_w = 2.1$.

Table 5: Renegotiation gains for different curvatures of the utility function.

	$\sigma = 0.8$	$\sigma = 1.5$	$\sigma = 3.0$
$RG(\mathbf{P})$	1.812	0.821	0.387

The table shows that the renegotiation gains decrease as we reduce the curvature of the utility function. This is because the profile of wages becomes steeper. From this we can infer that for very small values of σ the non-renegotiation condition is no longer satisfied.

6.2 Investors-firm relationship

Suppose that in the case in which the entrepreneur renegotiates the debt contract (or defaults), investors have the ability to liquidate the assets of the firm but cannot exclude the entrepreneur from participating in financial markets. In other words, the entrepreneur can get new financing from other investors. Furthermore, let's assume that if the firm is able to refinance investment, the firm is also able to retain the hired workers. This implies that the investment in recruitment and training is not lost.

Under the above conditions, collateralized debt is the only form of external financing for the firm. To see this, suppose on the contrary that the firm is able to borrow above the value of the collateral. If so, after receiving the loan, the entrepreneur would always renegotiate down the part of the debt in

excess of the collateral and then obtain a new (identical) financial contract from other investors. Anticipating this, only secured loans will be offered.

7   Workers' savings

In studying the optimization problem of the firm we have made the extreme assumption that workers cannot save. We now discuss the plausibility of this assumption.

The possibility that workers could loose their existing jobs and the continuation utility drops to q_{res} creates an incentive for accumulating assets. However, if the workers' return from savings is sufficiently small relative to the intertemporal discount factor β, and the likelihood of a negative shock $1 - p$ is small, then the worker will not save. To show this, suppose that workers can accumulate assets with return r but they cannot borrow against future wages, that is, their wealth cannot be negative. The optimal saving decision of an individual worker is characterized by the following first order condition:

$$U_c(c_t) \geq \beta(1 + r)\Big[p \cdot U_c(c_{t+1}) + (1 - p) \cdot EU_c(\tilde{c}_{t+1})\Big]$$

where c_t and c_{t+1} are consumptions when the worker is employed and $\tilde{c}_{t+1}$ is consumption when the worker re-enters the labor market after loosing the job. Notice that $\tilde{c}_{t+1}$ is not known at time t because it depends on the financial status of the new employer, which explains the expectation operator. The condition holds with the inequality sign if the next period assets chosen by the worker are zero, that is, if the borrowing limit is binding.

The worker will not accumulate any assets if the above condition is always satisfied with the inequality sign. This is the case in the numerical exercises conducted in Section 5. To show this, let's consider the case of a worker that is currently receiving the highest possible wage, which we denote by w_{max}. This is the oldest worker of an unconstrained large-capacity firm. Because the firm is unconstrained, the wage paid to the worker is constant as long as he or she remains employed. Therefore, $c_t = c_{t+1} = w_{max}$. If the worker looses the job, the new wage depends on the financial status of the new employer, which is unknown ex-ante. The probability distribution is given by the economy-wide distribution of starting wages offered by hiring firms. Using the functional form of the utility, the worker's first order condition can

be rewritten as:

$$S(\mathbf{P}) \equiv 1 - \beta(1 + r)\left[p + (1 - p) \cdot E\left(\frac{\tilde{c}_{t+1}}{w_{max}}\right)^{-\sigma}\right] \geq 0$$

where $\mathbf{P}$ denotes the parameters of the model. The condition $S(\mathbf{P}) > 0$ guarantees that all workers do not save, independently of their current status.

Table 6 reports the value of $S(\mathbf{P})$ for alternative values of σ. As shown in the table, the no-saving condition is always satisfied. Therefore, given the parameters used in the paper, the assumption that workers cannot save is simply a reduced form of the environment in which they are allowed to save but they deliberately decide not to do so. The conditions that guarantee this result is the assumption that the return on savings r is small relative to the intertemporal discounting β, and the assumption that workers cannot hold negative assets.

Table 6: No-saving condition for different curvatures of the utility function.

	$\sigma = 0.8$	$\sigma = 1.5$	$\sigma = 3.0$
$S(\mathbf{P})$	0.027	0.030	0.032

8 Conclusion

This paper has studied how financial constraints affect the compensation structure of workers. Firms that are financially constrained find optimal to offer an upward profile of wages in order to alleviate their financial restrictions. Because large firms are more likely to have experienced a history of financial tightness during which they have paid low wages, after becoming unconstrained, they pay high wages. This mechanism can generate a positive correlation between firm size and wages (the firm size-wage relation). Our theory is also consistent with other empirical observations. In particular, the fact that fast growing firms—which in our model are those financially constrained—pay lower wages.

In offering an upward profile of wages, firms are implicitly borrowing from workers. This rises the question of why firms are able to borrow from

workers beyond what they can borrow from external investors. In our model this is possible because workers can use punishment mechanisms that are not available to external investors. An external investor can punish the debtor only by confiscating the firm's physical assets, which represents the only collateral that the firm can use to raise funds in financial markets. But the firm can expand its debt capacity by using another form of implicit "collateral" in the hands of workers. If a worker withdraws his effort and quits, the firm looses the job-specific investment. This gives the worker a credible punishment tool in the event of repudiation that is not available to investors. The cost of replacing the worker—due to the sunk nature of the investment—guarantees that the long-term wage contract between the worker and the firm is never reneged and allows the firm to use the wage policy to finance its growth.

A Characterization of the firm's problem

Let γ_t, μ_t, $\lambda_t n_t$ and θ_t denote the lagrange multipliers associated with the constraints (2), (3), (4) and (5) respectively. Then the Lagrangian can be written as:

$$
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \left(a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau \right) + \right.
$$
$$
\gamma_t \left[a_t + b_t - \kappa \sum_{\tau=0}^{t} n_\tau \right] +
$$
$$
\mu_t \left[\bar{\kappa}_f \sum_{\tau=0}^{t} n_\tau - b_t \right] +
$$
$$
\lambda_t n_t \left[\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) - q_{res} \right] +
$$
$$
\left. \theta_t \left[\sum_{\tau=0}^{t} (\kappa + A - w_{\tau,t+1}) n_\tau - (1+r) b_t - a_{t+1} \right] \right\}.
$$

The first order conditions with respect to $w_{\tau,t}$ and a_t, for $t \geq 1$, are

$$
\lambda_\tau U_c(w_{\tau,t}) = \theta_{t-1}, \quad \forall \tau \leq t \tag{17}
$$

and

$$
\theta_{t-1} = (1 + \gamma_t), \tag{18}
$$

respectively. Using (18) to substitute for θ_t into (17) immediately yields (6) in the text.

B First order conditions for the recursive formulation of the basic model

The Lagrangian can be written as:

$$
\begin{aligned}
\mathcal{L} &= a + b - \kappa N' + \beta V(a', q', N', H') \\
&+ \gamma \left[a + b - \kappa N' \right] \\
&+ \mu \left[\bar{\kappa}_f N' - b_t \right] \\
&+ \lambda H' \left[\beta (U(w') + q') - q \right]
\end{aligned}
$$

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where γ, μ and $\lambda H'$ are lagrange multipliers. The problem is also subject to the laws of motion for the next period value of a and H, that is, constraints (13) and (14), respectively.

The first order conditions are:

$$b: \quad 1 + \gamma - \mu = \beta(1+r)V_{a'} \tag{19}$$

$$w': \quad V_{a'} = \lambda U_{c'} \tag{20}$$

$$q': \quad V_{q'} + \lambda H' = 0 \tag{21}$$

$$N': \quad \beta\Big[\big(\kappa + A - \psi(q)w'\big)V_{a'} + V_{N'} + \psi(q)V_{H'}\Big] \geq (1+\gamma)\kappa - \mu\bar{\kappa}_f \tag{22}$$

where the last condition is satisfied with equality if $N' < \overline{N}$. The envelope conditions are:

$$V_a = 1 + \gamma \tag{23}$$

$$V_q = -\beta\psi_q(N'-N)\Big[w'V_{a'} - V_{H'}\Big] - \lambda H' \tag{24}$$

$$V_N = \beta\psi(q)\Big[w'V_{a'} - V_{H'}\Big] \tag{25}$$

$$V_H = -\beta\Big[w'V_{a'} - V_{H'}\Big] \tag{26}$$

Equation (15) in the text comes from using (23) to substitute for V_a in (20). We now show that the above conditions also imply that $\lambda = \lambda'$.

By substituting (23) in (26) we get:

$$-V_H = \beta\Big[(1+\gamma')w' - V_{H'}\Big] \tag{27}$$

From (20) we have that $(1+\gamma')w' = \lambda(w')^{1-\sigma} = \lambda(1-\sigma)U(w')$, which substituted in (27) yields

$$-V_H = \beta\Big[(1-\sigma)\lambda U(w') - V_{H'}\Big]. \tag{28}$$

Now consider the promise-keeping constraint $q = \beta[U(w') + q']$. Multiplying the left and right-hand side by $(1-\sigma)\lambda$ we get:

$$(1-\sigma)\lambda q = \beta\Big[(1-\sigma)\lambda U(w') + (1-\sigma)\lambda q'\Big] \tag{29}$$

Equations (28) and (29) imply:

$$-V_H = (1-\sigma)\lambda q \tag{30}$$
$$-V_{H'} = (1-\sigma)\lambda q' \tag{31}$$

Updating the first term we also have that:

$$-V_{H'} = (1-\sigma)\lambda' q' \tag{32}$$

Condition (31) and (32) then imply that $\lambda = \lambda'$.

C Recursive formulation of the general model

The problem solved by a firm with capacity $\overline{N}$ can be written recursively as follows:

$$V(a,z,N,H) = \max_{b,w',z',N'\leq\overline{N}} \left\{ d + \beta\Big[p\cdot V(a',z',N',H') + (1-p)\cdot a' \Big] \right\} \tag{33}$$

subject to

$$d = a + b - \kappa N' \geq 0 \tag{34}$$
$$b \leq \bar{\kappa}_f N' \tag{35}$$
$$z = \beta\Big[U(w') + p\,z'\Big] \tag{36}$$
$$a' = \kappa N' + A N' - H'w' - (1+r)b \tag{37}$$
$$H' = H + \psi(z)(N' - N) \tag{38}$$

Let γ, μ and $\lambda H'$ be the lagrange multipliers associated with the first three constraints, respectively. Following the same steps as in Appendix B we obtain the first order conditions:

$$1 + \gamma - \mu = \beta(1+r)(1+p\gamma') \tag{39}$$

$$1 + p\gamma' = \lambda U_{c'} \tag{40}$$

$$\beta(1+p\gamma')\Big[\kappa + A - \psi(z)w'\Big] + p\beta\Big[\psi(z) - \psi(z')\Big]V_{H'} \geq (1+\gamma)\kappa - \mu\bar{\kappa}_f \tag{41}$$

$$\lambda' = \lambda \tag{42}$$

$$V_H = -\beta(1 + p\gamma')w' + p\beta V_{H'} \tag{43}$$

D Computation of the equilibrium

Solving for the firm's problem: For given $\overline{N}$ and q_{res}, the firm problem is solved backward starting from the state in which the firm is unconstrained. Let's assume that the firm takes T periods to become unconstrained. Therefore, we know that $N_{T+1} = \overline{N}$ and $\gamma_T = \gamma_{T+1} = 0$.

We start by guessing the value of w_{T+1} and H_{T+1}. Using the first order condition $1 = \lambda U_c(w_{T+1})$, we determine the lagrange multiplier λ. Using the promise-keeping constraint $z_{T+1} = \beta[U(w_{T+1}) + pz_{T+1}]$, and imposing $z_T = z_{T+1}$, we determine the (transformed) promised utility at time $T + 1$. Using condition (43) with the terminal condition $V_{H,T} = V_{H,T+1}$, we determine the partial derivative of the value function with respect to H. Finally, we determine b_T using the borrowing limit $b_T = \bar{\kappa}_f N_{T+1}$ and μ_T using the first order condition $\mu_T = 1 + \gamma_T - \beta(1+r)(1+p\gamma_{T+1})$. At this point we have all the final conditions to solve the problem backward at each point $t = T, T-1, ..., 0$. The solution at each point t is determined as follows:

1. Using the budget constraint with $d_t = 0$, we determine the firm's assets:

$$a_t = \kappa N_{t+1} - b_t$$

2. The wage w_t is determined using the first order condition:

$$1 + p\gamma_t = \lambda U_c(w_t)$$

3. We now determine the variables N_t, H_t and b_{t-1} using the laws of motion for a_t, H_{t+1}, and the borrowing limit:

$$
\begin{aligned}
a_t &= \kappa N_t + A N_t - H_t w_t - (1+r)b_{t-1} \\
H_{t+1} &= H_t + \psi(q_t)(N_{t+1} - N_t) \\
b_{t-1} &= \bar{\kappa}_f N_t
\end{aligned}
$$

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4. The values of $V_{H,t}$ and z_{t-1} are determined using condition (43) and the promise-keeping constraint, that is:

$$
\begin{aligned}
V_{H,t} &= -\beta(1 + p\gamma_{t+1})w_{t+1} + \beta p V_{H,t+1} \\
z_{t-1} &= \beta[U(w_t) + pz_t]
\end{aligned}
$$

5. The values of μ_{t-1} and γ_{t-1} are then determined using the first order conditions for debt and employment, that is:

$$
1 + \gamma_{t-1} - \mu_{t-1} = \beta(1 + r)(1 + p\gamma_t)
$$

$$
\beta(1 + p\gamma_t)\left[\kappa + A - \psi(q_{t-1})w_t\right] + p\beta\left[\psi(q_{t-1}) - \psi(q_t)\right]V_{H,t} = \\
(1 + \gamma_{t-1})\kappa - \mu_{t-1}\bar{\kappa}_f
$$

After solving for all $t = T, T-1, ..., 0$, we check two conditions: Whether $z_0 = z_{res}$ and $H_1 = N_1$. The second condition implies that $N_0 = H_0 = 0$. If these two conditions are not satisfied, we change the guesses for w_{T+1} and H_{T+1} until convergence.

Labor market equilibrium: To compute the labor market equilibrium we start by guessing the equilibrium value of z_{res}. Given this value we solve for the firm's problem for all values of $\overline{N}$. The procedure to solve for the firm's problem has been described above. After finding the invariant distribution of firms, we find the aggregate demand of labor and we check the clearing condition in the labor market. We update z_{res} until the labor market clears.

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