Capital Structure with Endogenous Liquidation Values*

Antonio E. Bernardo and Ivo Welch
UCLA Anderson School of Management

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Abstract

Asset liquidation values are an important determinant of distress costs and therefore optimal capital structure. Capital structure theories typically assume liquidation values are exogenous even though they may be determined in part by the debt choices of firms in the industry (Shleifer and Vishny, 1992; Pulvino, 1998). We develop a model in which high industry debt leads to a greater supply of assets for sale by distressed firms but also lower demand for assets from relatively healthy firms because of debt overhang. Thus, high industry debt lowers expected asset liquidation values and provides an incentive for individual firms to take on less debt to take advantage of attractive future buying opportunities. The indirect effect of equilibrium asset prices tempers, and sometimes reverses, the effect of parameters on optimal capital structure choices compared to models with exogenous prices. For example, we show that firms may choose lower debt ratios when assets are more redeployable, contrary to standard intuition (Williamson, 1988).

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Empirical studies have shown that financially distressed firms often liquidate assets at discounts to fundamental value (see, e.g., Pulvino, 1998; Asquith, Gertner, and Scharfstein, 1994). Such asset fire sales are an important determinant of financial distress costs and therefore optimal capital structure (Shleifer and Vishny, 1992). Theories of optimal capital structure (e.g., Harris and Raviv, 1990; Leland, 1994; Leland and Toft, 1996) typically assume that asset liquidation values are exogenously determined, but empirical studies have also shown that asset fire sales are more severe when peer firms have less financial slack (Franks and Sussman, 2005; Acharya, Bharath, and Srinivasan, 2007). These findings suggest that industry debt choices have a feedback effect on asset liquidations values that are not accounted for in capital structure theories.

In this paper, we develop a model in which debt choices and asset liquidation values are jointly determined in industry equilibrium.[1] In our model, firms choose debt, which confers a tax benefit, prior to learning their value as a going concern. If the realized value falls below a strategic default threshold, management (acting in the interest of shareholders) defaults on the firm's debt obligations. In the event of default, management must decide whether to liquidate the firm's assets or continue operations. Higher debt results in an increase in the supply of liquidated assets because it leads to larger dissipative costs from continuing operations.[2] Higher debt also reduces the demand for liquidated assets because the natural buyers of these assets – higher productivity firms in the industry – are limited in their ability to raise financing due to debt overhang (Myers, 1977).[3] The price of liquidated assets is then endogenously determined in our model by the number of available sellers and buyers, both of which depend on the amount of debt that firms take on at the outset.

We show that the endogeneity of liquidation values often dampens, and sometimes reverses, the effects of key parameters on a firm's optimal capital structure. Consider, for example, the effect of debtor rights on optimal capital structure. Strong debtor rights, as granted in the U.S. bankruptcy code, gives shareholders significant bargaining power over bondholders in the event of default. This creates incentives for management to default strategically and to choose to continue operations in default when liquidation may be more efficient. Consequently, the direct effect of strong debtor rights is to increase distress costs and lower optimal debt. However, excessive continuation by firms in default reduces

[1] We extend the analysis in Shleifer and Vishny (1992) to fully describe equilibrium prices and debt policies.

[2] These costs include, for example, management's incentives for risk shifting (Jensen and Meckling, 1976) or damaged relationships with key stakeholders (Titman, 1983).

[3] For example, Pulvino (1998) has shown that financially unconstrained airlines increase their buying activity when aircraft prices are depressed, while constrained airlines do not.

the supply of liquidated assets, raises their prices, and eliminates some opportunities for profitable asset re-allocations from weak firms to healthy firms. This indirect effect reduces the availability of distressed asset buying opportunities and, hence, lowers debt overhang costs. We show that the indirect effect tempers the incentives to lower debt and may even dominate the direct effect so that stronger debtor rights may result in higher optimal debt ratios. We also show that stronger debtor rights may increase or decrease credit spreads – stronger debtor rights reduce bondholder recovery rates in default, but may increase or decrease debt and hence the likelihood of default.

It is also commonly argued that firms with more redeployable (less specific) assets have smaller distress costs and therefore should have higher debt ratios (Williamson, 1988; Harris and Raviv, 1990). However, when assets are less specific there is a larger potential surplus in reallocating assets from poorly performing firms to healthy firms. In the presence of debt overhang, firms have an incentive to take on less debt to take advantage of these opportunities. In some cases (e.g., when debt overhang is severe) firms may optimally choose lower debt ratios when assets are less specific.

We also examine an extension of the model in which firms face both idiosyncratic and industry-level shocks to firm value. As argued in Shleifer and Vishny (1992), we show that liquidation values are lower *holding cash flows fixed* in industry recessions because debt overhang limits the ability of healthy firms to buy liquidated assets. Consequently, the probability of liquidation conditional on financial distress is lower in industry recessions. We also develop new insights. For example, optimal debt is lower, and expected equilibrium liquidation values are higher, when there is greater uncertainty about industry profitability.

Although simple, the model offers a rich set of implications about leverage ratios, liquidation values, firm values, credit spreads, and conditional (and unconditional) probabilities of liquidation and distress.

I The Model

Our model considers an industry with a unit mass of risk-neutral, competitive firms. At time 0, each firm acquires one unit of a productive asset which generates a random payoff at time 1. The payoff depends on the firm's productivity, denoted $\tilde{V}_i$, and on its capital structure policy in ways described below. All firms are ex ante identical and it is common knowledge that $\tilde{V}_i$ is distributed uniformly on the interval $[0,\gamma]$. γ

At time 0, firms choose the amount of debt, **D**, to take on in order to finance the one unit of the asset. We assume that there are immediate net benefits of debt. That is, debt confers a proportional benefit of $\tau \cdot D$, which the firm receives immediately (for simplicity) and which is not dissipated by subsequent events.[4] For simplicity, we sometimes refer to τ as the tax benefit of debt even though we have a much broader concept in mind. The parameter τ can reflect the ability of debt to allow financially-constrained firms to take on more productive projects, any positive incentive effects from debt, lower fund-raising costs, debt's preferred tax treatment (which may or may not be socially valuable), and so on, all net of debts' unmodelled up-front costs. D τ

At time 1, each firm's management learns its productivity, V_i. At this time, management (acting in the interest of shareholders) chooses whether to default on the firm's debt obligations. We allow for strategic default – management may choose to default even though the firm has sufficient assets to meet its debt obligations (see, e.g., Hart and Moore, 1998; Fan and Sundaresan, 2000; Davydenko and Strebulaev, 2007). The tradeoffs associated with this choice are described below.

Let $\hat{D}$ represent the management's optimal threshold for choosing default. If $V_i > \hat{D}$, the management chooses to honor the firm's debt obligations, the firm operates without impediment, and its value is given by V_i. However, if $V_i \leq \hat{D}$, management chooses to default. In the event of default, management must decide whether to liquidate the firm's assets at the (endogenously determined) price, **P**, or to continue operations. We assume that there is a dissipative cost to the firm from continuation in the event of default. This could occur, for example, because management has an incentive to take risky, negative NPV projects (from the firm's perspective) that generate upside potential for shareholders who would otherwise receive nothing (Jensen and Meckling, 1976). We also assume the net cost of continuation is linear in the shortfall ($\phi \cdot (\hat{D} - V_i)$) and there is limited liability, therefore, P

[4] This is innocuous for our qualitative predictions about debt choices and liquidation values.

firm value under continuation is given by the maximum of zero and $V_i - \phi \cdot (\hat{D} - V_i)$. The parameter ϕ is higher when the firm faces larger distress costs. ϕ

We also assume that management can extract benefits for shareholders when it chooses to continue operations in the event of default. Again, these benefits could be due to risk shifting incentives. In particular, we assume management can extract $\beta \leq 1$ proportion of the firm's continuation value , $\beta[V_i - \phi(\hat{D} - V_i)]$, in default. The parameter β can be interpreted as the strength of debtor rights in the bankruptcy code. For example, small values of β represent strong creditor rights, as in the United Kingdom, while large values of β represent strong debtor rights, as in France and the United States (Davydenko and Franks, 2008).

Consequently, management will always prefer to continue operations rather than liquidate even though liquidation may be in the bondholders interest. Bondholders, understanding this, will renegotiate their claims to motivate management to liquidate. We consider a simple renegotiation process: bondholders make a take-it-or-leave it offer to pay $x \geq 0$ to shareholders in the event of liquidation. Management will accept the offer because shareholders are at least as well off as they would be in the absence of renegotiation. Since the shareholders' value in continuation is increasing in V_i, this implies that management will choose to liquidate for low levels of productivity below some threshold, $V_i \leq \Lambda(x)$. Importantly, we show below that the optimal payment, x^*, results in a higher liquidation threshold when firm debt is higher. Therefore, in a symmetric equilibrium, the supply of liquidated assets is increasing when industry debt is higher.[5]

We also show that the demand for liquidated assets depends on industry debt. As in Shleifer and Vishny (1992), we argue that the natural buyers of these assets are other firms in the industry. We make two assumptions about the redeployment of these assets. First, we assume that an asset with productivity V_i to owner i has productivity $\eta \cdot V_j$ to buyer j where $\eta \leq 1$. Thus, holding productivity fixed across firms, the asset is worth more to the owner than a potential buyer. This could be because the owner customized the asset to its own needs so that repurposing the asset requires, for example, retraining of workers or other complementary assets. The parameter η, therefore, is a measure of asset redeployability: higher values of η imply that assets can be redeployed at small cost. Although assets are firm-specific, some firms may find it worthwhile to buy liquidated

[5]We show below that the liquidation vs. continuation threshold is a function of the asset's liquidation value, the firm's debt choice, and the strength of debtor rights, $\Lambda(P, D; \beta)$. The optimal payment, x^*, mitigates but does not eliminate the management's incentive to continue operations in the event of default when $\beta > 0$. It is possible that more realistic renegotiation mechanisms would lead to more efficient continuation decisions in the event of default. This is the subject of ongoing work.

assets if they are sufficiently productive. In particular, acquiring liquidated assets at price P is positive NPV for all firms with $V_j > P/\eta$. Second, a potential buyer of the asset may be limited in its ability to raise new financing because of its initial debt choice. In particular, we assume that a firm can purchase $\ell(D)$ units of the liquidated asset where $\ell(D)$ is decreasing in its debt choice, D, because of the debt overhang problem (Myers, 1977). Therefore, in a symmetric equilibrium, the demand for liquidated assets is decreasing when industry debt is higher.

The key novel feature of our model is that the supply of and demand for liquidated assets is affected by industry debt choices. In contrast to prior work, the model parameters (such as the tax benefit, reorganization costs, asset redeployability, debtor rights) can now affect capital structure choices not only through their direct effects on firm value, but also through their effects on the equilibrium price of liquidated assets.

Table 1 summarizes the key variables in our model and Figure 1 illustrates the timing.

A Time 1: Resolution of Financial Distress

We solve the model by backward induction. At time 1, management learns their firm's productivity V_i. Firms are competitive, therefore, they take the price of liquidated assets, P, as given and for now we also take the management's strategic default trigger, $\hat{D}$, as given. If $V_i \leq \hat{D}$ the firm defaults on its debt obligations and bondholders make a take-it-or-leave-it offer to pay x to shareholders if the firm is liquidated. Shareholders accept the offer because it makes them at least as well off as they would be in the absence of renegotiation.

Given the payment x, management will choose to liquidate if and only if:

$$x \geq \beta \cdot [V_i - \phi(\hat{D} - V_i)] \quad \Leftrightarrow V_i \leq \frac{x}{\beta(1+\phi)} + \frac{\phi \hat{D}}{(1+\phi)} \equiv \Lambda(x).$$

The expression $\Lambda(x)$ represents the threshold value (as a function of the payment $x \geq 0$) below which management will choose to liquidate.[6] Increasing the payment x increases the likelihood management will choose liquidation, thus, helping to mitigate agency conflicts.

In the event of default, the bondholders know that V_i is distributed uniformly on the interval $[0, \hat{D}]$ but do not know the realized value of V_i. Furthermore, bondholders

[6]The threshold is always at least as great $\phi\hat{D}/(1+\phi)$ because with limited liability the value of continuing operations in the event of default is zero when $V_i \in [0, \phi\hat{D}/(1+\phi)]$.

understand that management will choose to liquidate for all $V_i \leq \Lambda(x)$, therefore, their problem is:

$$\max_x \quad \frac{1}{\hat{D}} \cdot \int_0^{\Lambda(x)} (P - x) dV + \frac{1}{\hat{D}} \int_{\Lambda(x)}^{\hat{D}} (1 - \beta) \cdot [V - \phi(\hat{D} - V)] dV$$

which implies

$$x^* = \frac{\beta \cdot (P - \phi \beta \hat{D})}{1 + \beta},$$

and, therefore,

$$\Lambda^* = \frac{P + \phi \hat{D}}{(1 + \beta)(1 + \phi)}.$$

Importantly, the optimal threshold Λ^* is increasing in the asset liquidation value, P, and the strategic default boundary, $\hat{D}$. This is intuitive. Bondholders will renegotiate with management to make liquidation more likely when liquidation values are higher and when the dissipative costs associated with continuation are higher. We show below that the strategic default boundary $\hat{D}$ is increasing in the firm's choice of debt, D; therefore, higher debt results in a greater supply of the liquidated asset.

It is straightforward to show that (conditional on dissipative continuation costs) the efficient liquidation threshold is given by:

$$\Lambda^{eff} = \frac{P + \phi \hat{D}}{(1 + \phi)}.$$

Therefore, for the special case $\beta = 0$ (no debtor rights) the management chooses the efficient threshold, however, management continues too often ($\Lambda^* < \Lambda^{eff}$) when shareholders have bargaining power in default ($\beta > 0$) .

The optimal transfer, x^*, can be interpreted as a partial debt-for-equity swap. The bondholders give up the share $\beta/(1+\beta)$ of the liquidation value, P, for partial forgiveness of its debt obligations. Interestingly, efficient liquidation could be achieved if bondholders offered β share of the sale proceeds from liquidation because shareholders would then receive the identical share (β) of value in liquidation and continuation. However, bondholders offer a smaller share of the proceeds in liquidation because the benefit of reducing transfers to shareholders for low values of V_i dominates the benefits of efficient liquidation.

We now characterize the optimal default trigger, $\hat{D}$.

Lemma 1 *Management chooses to default if and only if* $V_i \leq \hat{D} = D/(1-\beta)$.

The proof is in the Appendix.

Debtor rights in bankruptcy proceedings ($\beta > 0$) create an incentive for management to default even though the firm has sufficient value to meet its debt obligations. For example, management may choose to strategically default if bankruptcy allows the firm to rewrite onerous labor contracts in which case the continuation value for shareholders is higher in default than out of default. In the limiting case $\beta = 0$, management has no incentive to default strategically, $\hat{D} = D$.[7]

B Time 0: Optimal Debt Choice

Firms are competitive, therefore, they take as given the price of liquidated assets at time 1. The firms' objective at time 0 is to choose debt D to maximize firm value:

$$V(D) \equiv \frac{1}{\gamma} \cdot \left\{ \int_0^{\Lambda(\hat{D})} P\, dV + \int_{\Lambda(\hat{D})}^{\hat{D}} [V - \phi \cdot (\hat{D} - V)]\, dV + \int_{\hat{D}}^{\gamma} V\, dV \right\} \tag{1}$$

$$+ \frac{1}{\gamma} \cdot \left\{ \ell(D) \cdot \int_{P/\eta}^{\gamma} (\eta \cdot V - P)\, dV \right\} + \tau \cdot D \ .$$

where $\hat{D} = D/(1-\beta)$ is the strategic default threshold as given in Lemma 1.

The first term reflects the payoff, P, if the firm is eventually liquidated ($V \leq \Lambda(\hat{D})$). The second term represents the payoff to distressed firms that choose to continue ($V \in [\Lambda(\hat{D}), \hat{D}]$) and receive V less the dissipative costs of continuation $\phi \cdot (\hat{D} - V)$. The third term represents the value of non-distressed firms ($V \in [\hat{D}, \gamma]$) that continue unimpeded. The fourth term represents the expected surplus for firms choosing to acquire liquidated assets ($V \in [P/\eta, \gamma]$). Note that the number of units of the asset these firms can purchase $\ell(D)$ depends on the firm's choice of debt, D, rather than the default trigger.[8] Finally, the

[7] He and Milbradt (2013) examine a model in which bond illiquidity and the strategic default boundary are jointly determined because of the effect of illiquidity on the costs of rolling over debt. We don't consider these costs in our model.

[8] We will assume below that individual firms cannot separate themselves from the pool of all healthy firms when they seek financing for the liquidated asset. That is, $\ell(D)$ does not depend on an individual

fifth term represents the tax benefits of debt which also depends on the firm's choice of debt, D, rather than the default trigger.

A critical quantity in our model is the expected surplus from acquiring liquidated assets after firm types are learned at time 1. In our formulation, the expected surplus (per unit of the asset) is given by:

$$\frac{1}{\gamma} \cdot \int_{P/\eta}^{\gamma} (\eta \cdot V - P)\, dV = \frac{\eta}{2\gamma}(\gamma - P/\eta)^2$$

Importantly, the expected surplus is increasing when the assets are more redeployable (η is higher) and when asset liquidation values are lower (P is lower). Because of debt overhang, firms will strategically choose to take on less debt at time 0 when this surplus is large.[9]

C Market Clearing

All firms with $V_i \leq \Lambda(\hat{D})$ choose to liquidate so the aggregate supply of the liquidated asset is given by:

$$\frac{\Lambda(\hat{D})}{\gamma}.$$

The demand for liquidated assets is $\ell(D)$ units for firms that are sufficiently healthy ($V \geq P/\eta$) so aggregate demand is given by:

$$\ell(D) \cdot \left(\frac{\gamma - P/\eta}{\gamma} \right).$$

Therefore, market clearing requires:

$$\Lambda(\hat{D}) = \ell(D) \cdot (\gamma - P/\eta).$$

firm's productivity but only on the aggregate productivity of all firms that seek financing. This assumption is made only for computational simplicity (it yields a linear equilibrium) and does not impact the qualitative implications of the model.

[9] This feature of our model has a parallel in the banking literature. The strategic choice by financial institutions to hold liquid assets to take advantage of future fire sale buying opportunities is considered in, for example, Allen and Gale (2004a, 2004b), Gorton and Huang (2004), and Acharya, Shin, and Yorulmazer (2011). Simutin (2013) shows that some mutual funds use large cash holdings to take advantage of fire sales in equities.

D Equilibrium

Firms are identical ex ante, therefore, we focus on symmetric equilibria for the choice of debt, D, at time 0.

Definition 1 *Let $\hat{D}$ represent the strategic default trigger at time 1; $x^*(P, \hat{D})$ represent the transfer from bondholders to shareholders if the firm is liquidated in the event of default at time 1; and $\Lambda^*(x^*)$ represent the liquidation threshold that maximizes shareholder value at time 1. A symmetric equilibrium for the choice of debt at time 0 is defined by:*

- *Given $\{P, \hat{D}, x^*(P, \hat{D}), \Lambda^*(x^*)\}$, firms choose debt D to maximize firm value at time 0; and*
- *The price P clears the market for liquidated assets at time 1.*

II A Benchmark Model with Unlimited Capital and No Debtor Rights ($\beta \to 0$)

As a benchmark for comparison we consider a special case of our model in which firms can raise unlimited capital (no debt overhang problem) to buy liquidated assets. We also assume that $\beta \to 0$ which implies, in the limit, that there is no strategic default, $\hat{D} = D$, the bondholders choose the transfer $x^* \to 0$, and managers choose the liquidation threshold, $\Lambda(D)$, efficiently conditional on reorganization costs:

$$\Lambda(D) \equiv \frac{P + \phi \cdot D}{1 + \phi} \ .$$

The equilibrium in this benchmark model can be constructed as follows. Suppose firms conjecture that all liquidated assets will be purchased by the firm with the highest possible value, γ, at price $P = \eta \cdot \gamma$. In this case, there is no marginal surplus at time 1 from buying liquidated assets, therefore, the optimal debt choice at time 1 maximizes:

$$V(D) \equiv \frac{1}{\gamma} \cdot \left\{ \int_0^{\Lambda} P \, dV \ + \ \int_{\Lambda}^{D} [V - \phi \cdot (D - V)] \, dV \ + \ \int_{D}^{\gamma} V \, dV \right\} + \tau \cdot D \ ,$$

under parameter restrictions assuring that the integrals are sensible ($0 \leq \Lambda^* \leq D^* \leq \gamma$) and that the leverage ratio is less than one.[10] Substituting $P^* = \gamma \cdot \eta$ into $\Lambda(D)$ and solving the FOC with respect to D yields

$$D^* = \gamma \cdot \left(\eta + \tau + \tau/\phi\right) \ .$$

Substituting back yields $\Lambda(D^*) = (\eta + \tau) \cdot \gamma$.

This outcome is supported in equilibrium because the highest valued buyer is indifferent between buying and not buying all liquidated assets. For all prices $P < \eta \cdot \gamma$ the highest-valued buyer with unlimited capital would have further incentive to buy liquidated assets, eventually driving the price up to $P = \eta \cdot \gamma$.

The optimized firm value is then given by:

$$V^* \equiv V(D^*) = \frac{\gamma \cdot \left[1 + (\eta + \tau)^2 + \tau^2/\phi\right]}{2} \ .$$

The firm value is increasing in tax benefits (τ), however, there are attenuating costs: as τ increases it encourages the firm to take on more debt which leads to (i) excessive liquidation compared to the efficient outcome ($\Lambda > \eta\gamma$) and (ii) dissipative reorganization costs.

We summarize our benchmark equilibrium in the following theorem.

Theorem 1 *There is a unique symmetric equilibrium $\{D^*, P^*\}$ in the benchmark model where*

$$\begin{aligned} D^* &= \left(\eta + \tau + \tau/\phi\right) \cdot \gamma \ , \\ P^* &= \eta \cdot \gamma \ . \end{aligned}$$

The optimal debt, D^*, is increasing in η (asset redeployability), τ (tax benefits), and γ (maximum value); and decreasing in ϕ (distress costs).

The optimal debt-to-value, D^*/V, is increasing in η (asset redeployability) and τ (tax benefits); decreasing in ϕ (distress costs); and independent of γ (maximum value).

The equilibrium price of liquidated assets, P^*, in increasing in η (asset redeployability) and γ (maximum value); and independent of τ (tax benefits) and ϕ (distress costs).

[10] Parameter restrictions are presented in the Appendix.

The comparative statics for the benchmark model are illustrated in the following table for comparison to later generalizations of our model.

Exog →	Reorganization Costs	Tax Benefits	Asset Redeployability
Endog ↓	ϕ	τ	η
D^*	$-$	$+$	$+$
V^*	$-$	$+$	$+$
D^*/V^*	$-$	$+$	$+$
$P^* = \eta \cdot \gamma$			$+$

The benchmark model yields the standard intuition of extant models with exogenous liquidation values: Firms take on more debt and are worth more when the tax benefits to debt (τ) are higher; when the costs of distressed reorganization (ϕ) are lower; and when assets are easier to redeploy (η). The optimal leverage ratio has the same comparative statics in all cases.

III A Model with Debt Overhang and No Debtor Rights ($\beta \to 0$)

In reality, there may be financing constraints that limit competition for the liquidated asset. In the next version of our model, we assume that firms have limited access to funding and that these funding constraints are more severe when the firm has taken on higher levels of debt at time 0 (debt overhang). In particular, we assume that firms may purchase up to

$$\ell(D) = k - \alpha \cdot \frac{D}{\bar{V}}$$

units of the liquidated asset at time 1, where $\bar{\bar{V}}$ is the expected NPV per unit of the asset conditional on it being positive NPV for acquiring firms,

$$\bar{V} \equiv E[\eta \cdot V - P \mid \eta \cdot V \geq P] = \int_{P/\eta}^{\gamma} (\eta \cdot V - P)\, dV = \frac{\eta}{2} \cdot \left(\gamma - \frac{P}{\eta}\right) .$$

This specification assumes that lenders do not fully appreciate specific firm types but instead provide financing depending on the pooled attributes of acquiring firms.[11] Higher values of the parameter α represent increasingly severe debt overhang problems and the parameter k represents the general availability of financing in the economy. When $k \to \infty$, the model simplifies to the benchmark model in the previous section.

We continue to consider the limiting case $\beta \to 0$ so that there is no strategic default, $\hat{D} = D$, the transfer $x^* \to 0$, and the liquidation threshold, $\Lambda(D)$, is again efficient conditional on reorganization costs:

$$\Lambda(D) \equiv \frac{P + \phi \cdot D}{1 + \phi} \ .$$

Firms choose debt at time 0 to maximize

$$V(D) \ \equiv \ \frac{1}{\gamma} \cdot \left\{ \int_0^{\Lambda} P \, dV \ + \ \int_{\Lambda}^{D} [V - \phi \cdot (D - V)] \, dV \ + \ \int_D^{\gamma} V \, dV \right\} \tag{2}$$

$$+ \ \frac{1}{\gamma} \cdot \left\{ \ell(D) \cdot \int_{P/\eta}^{\gamma} (\eta \cdot V - P) \, dV \right\} \ + \ \tau \cdot D \ .$$

Differentiating with respect to D and setting equal to zero, we see that the optimal debt choice is linear in P,

$$D^* \ = \ \frac{\gamma \cdot (1 + \phi) \cdot (\tau - \alpha)}{\phi} \ + \ \left[\frac{\phi \cdot (\alpha + \eta) + \alpha}{\eta \cdot \phi} \right] \cdot P \ . \tag{3}$$

In addition, the aggregate supply of liquidated assets must equal the aggregate demand. The aggregate supply of assets from liquidating firms is

$$\text{Supply:} \ \ \frac{\Lambda(D)}{\gamma} \ = \ \frac{P + \phi \cdot D}{\gamma \cdot (1 + \phi)} \ . \tag{4}$$

[11] This turns out to be a simplification that generates similar qualitative implications as obtained in a model in which firms' abilities to raise funding depend not on the average value but on their own value. In this more realistic setup, the most valuable firms would be able to purchase more assets, the least valuable firms would be able to purchase less. Our simplified model produces closed-form solutions at a modest compromise.

The aggregate demand for assets by healthy firms is

$$\text{Demand:} \quad \ell(D) \cdot \int_{P/\eta}^{\gamma} (1/\gamma)\, dV \;=\; \frac{k(\gamma - P/\eta)}{\gamma} - \frac{2 \cdot \alpha \cdot D}{\gamma \cdot \eta} \,. \tag{5}$$

The following result describes the unique symmetric equilibrium.

Theorem 2 *There is a unique symmetric equilibrium $\{D^*, P^*\}$ where*

$$D^* \;=\; \gamma \cdot \left[\frac{\frac{k(1+\phi)\cdot\left(1+\frac{\alpha(1+\phi)}{\phi\eta}\right)}{\left(1+k(1+\phi)/\eta\right)} + \frac{(1+\phi)(\tau-\alpha)}{\phi}}{1+\left(1+\frac{\alpha(1+\phi)}{\phi\eta}\right)\cdot\left(\frac{\phi+2\alpha(1+\phi)/\eta}{1+k(1+\phi)/\eta}\right)} \right] \tag{6}$$

$$P^* \;=\; \left[\frac{k(1+\phi)}{1+k(1+\phi)/\eta}\right] \cdot \gamma - \left[\frac{\phi+2\alpha(1+\phi)/\eta}{1+k(1+\phi)/\eta}\right] \cdot D \tag{7}$$

The optimal debt, D^*, is increasing in τ (tax benefits), η (asset redeployability), γ (maximum value), and k (availability of financing); and decreasing in ϕ (distress costs) and α (debt overhang).

The equilibrium price of liquidated assets, P^*, is increasing in η (asset redeployability), γ (maximum value), and k (availability of financing); decreasing in τ (tax benefits) and ϕ (distress costs); and ambiguous in α (debt overhang).

We illustrate the comparative statics results in the following table for easy comparison to the benchmark model.

Exog →	Reorganization Costs	Tax Benefits	Asset Redeployability	Funding Availability	
Endog ↓	ϕ	τ	η	k	α
D^*	−	+	+	+	−
V^*	−	+	+	+	−
D^*/V^*	−	−/+	−/+	+	−
P^*	+	−	+	+	−/+

Red indicates different comparative statics.

The most important difference between this model (with limited capital and debt overhang) and the benchmark model is that greater asset redeployability (η) can now

lower optimal debt ratios. In the benchmark model, as in standard models with exogenous liquidation values, higher asset reployability reduces the costs of financial distress and firms respond by increasing debt and debt ratios. In our model, higher asset redeployability also increases the expected surplus associated with buying liquidated assets. Because of debt overhang, firms with lower debt can raise more money to take advantage of these opportunities, tempering the firm's incentive to increase debt. For some parameter values, the "deep pockets" motivation sufficiently tempers the effect of distress costs so that firms choose lower debt ratios.[12] Importantly, even for parameter values where debt ratios (and levels) are positively related to asset redeployability, the opportunity to buy highly transferable assets attenuates the firm's motivation to increase debt when assets are less specific.

Furthermore, the model now predicts that debt ratios may decrease when the tax benefits of debt increase. The direct effect of greater tax benefits leads to higher debt levels; however, there is an opposing indirect effect that works through equilibrium prices: higher industry debt leads to low liquidation values which motivates firms to take low debt in order to take advantage of future distressed asset purchase opportunities. Although the former dominates the latter with respect to the choice of debt levels, the debt ratio may decrease in our model.

Some qualitative predictions of the model with limited capital are unchanged. As in the benchmark model, higher reorganization costs (ϕ) lead to lower debt levels (and ratios); however, with endogenous prices, there is an opposing indirect effect: low industry debt levels lead to higher liquidation values (because supply is reduced) which lowers the expected surplus from buying distressed assets, thereby providing an incentive to increase debt. Although the latter effect attenuates the former, the standard comparative statics intuition remains true.

The effect of capital availability (k) in our model is straightforward. Greater capital availability implies more demand for the liquidating asset and thus higher prices for liquidated assets. These higher asset prices reduce the expected surplus from buying liquidated assets, giving firms less incentive to adopt conservative debt policies. Greater capital availability also increases firm valuations because it enables more efficient asset transfers between low-valued and high-valued firms.

[12] Morellec (2001) shows that greater asset redeployability can lead to lower debt in the absence of bond covenants imposing restrictions on the disposition of assets. Unlike our model, this paper assumes exogenously specified liquidation values.

More severe debt overhang (higher α) motivates firms to take on less debt for obvious reasons and leads to lower valuations. However, the effect of α on the liquidation value, P, can be ambiguous.

IV Debtor Rights ($\beta > 0$) and Inefficient Continuation

We have assumed until now that shareholders can extract no value in continuation ($\beta \to 0$). We now consider the case $\beta > 0$ which results in (i) the possibility of strategic default ($\hat{D} > D$), and (ii) inefficient liquidation according to the threshold,

$$\Lambda(\hat{D}) = \frac{P + \phi \cdot \hat{D}}{(1+\beta)(1+\phi)} \ .$$

As we described earlier, the parameter β represents debtor rights in the bankruptcy code. When $\beta > 0$ management continues operations too often ($\Lambda < \Lambda^{eff}$) because it benefits shareholders at the expense of bondholders. Moreover, higher values of β result in larger deviations from the efficient continuation threshold.

The firms' objective at time 0 is to choose debt D to maximize firm value:

$$V(D) \equiv \frac{1}{\gamma} \cdot \left\{ \int_0^{\Lambda(\hat{D})} P \, dV + \int_{\Lambda(\hat{D})}^{\hat{D}} [V - \phi \cdot (\hat{D} - V)] \, dV + \int_{\hat{D}}^{\gamma} V \, dV \right\} \tag{8}$$

$$+ \frac{1}{\gamma} \cdot \left\{ \ell(D) \cdot \int_{P/\eta}^{\gamma} (\eta \cdot V - P) \, dV \right\} + \tau \cdot D \ .$$

where $\hat{D} = D/(1-\beta)$ is the strategic default threshold as given in Lemma 1.

The first-order condition for the optimal debt choice is:

$$\Lambda_{\hat{D}}(\hat{D}) \cdot [P - (\Lambda - \phi(\hat{D} - \Lambda))] - \phi \cdot (\hat{D} - \Lambda) + (1-\beta) \cdot \ell'(D) \cdot \int_{P/\eta}^{\gamma} (\eta \cdot V - P) \, dV + (1-\beta) \cdot \tau \cdot \gamma = 0.$$

It is straightforward to show the second-order condition is satisfied with the functional forms for $\Lambda(\hat{D})$ and $\ell(D)$.

An increase in debt at time 0 now impacts firm value in four ways: (i) it changes the management's liquidation threshold ($\Lambda_{\hat{D}}(\hat{D})$); (ii) it increases reorganization costs (ϕ); (iii) it reduces the ability for healthy firms to acquire liquidated assets because of debt overhang ($\ell'(D)$); and (iv) it provides a tax shield benefit (τ).

Finally, the aggregate supply of liquidated assets must equal the aggregate demand. The aggregate supply of assets from liquidating firms is

$$\text{Supply:} \quad \frac{\Lambda(\hat{D})}{\gamma} = \frac{P + \phi \cdot \hat{D}}{\gamma \cdot (1+\beta)(1+\phi)} . \tag{9}$$

The aggregate demand for assets by healthy firms is

$$\text{Demand:} \quad \ell(D) \cdot \int_{P/\eta}^{\gamma} (1/\gamma)\, dV = \frac{k(\gamma - P/\eta)}{\gamma} - \frac{2 \cdot \alpha \cdot D}{\gamma \cdot \eta} . \tag{10}$$

In this version of the model we have a unique symmetric equilibrium.

Theorem 3 *There is a unique symmetric equilibrium* $\{x^*, D^*, \hat{D}^*, P^*\}$ *where*

$$\begin{aligned}
x^* &= \frac{\beta(P^* - \phi\beta\hat{D}^*)}{1+\beta}, \\
\hat{D}^* &= \frac{\left[(1-\beta)(1+\beta)^2(1+\phi)(\tau-\alpha)\right]}{\phi \cdot \left[(1+\beta)^2 + \phi\beta^2\right]} \cdot \gamma + \frac{\left[2\beta + 1 + \alpha(1-\beta)(1+\beta)^2(1+\phi)/\eta\phi\right]}{\left[(1+\beta)^2 + \phi\beta^2\right]} \cdot P^*, \\
P^* &= \left[\frac{k(1+\beta)(1+\phi)}{1 + k(1+\beta)(1+\phi)/\eta}\right] \cdot \gamma - \left[\frac{\phi + 2\alpha(1-\beta^2)(1+\phi)/\eta}{1 + k(1+\beta)(1+\phi)/\eta}\right] \cdot \hat{D}^*, \\
D^* &= (1-\beta) \cdot \hat{D}^*.
\end{aligned}$$

Numerical simulations of the model reveal the following comparative statics results.

Exog →	Reorganization Costs	Tax Benefits	Asset Redeployability	Funding Availability		Debtor Rights
Endog ↓	ϕ	τ	η	k	α	β
D^*	−	+	+	+	−	−/+
V^*	−	+	+	+	−	−
D^*/V^*	−	−/+	−/+	+	−	−/+
P^*	+	−	+	+	−/+	+

The equilibrium price of liquidated assets is unambiguously higher when debtor rights are stronger because this reduces management's incentive to liquidate assets in default. Firm values are unambiguously lower when debtor rights are stronger because this leads to less efficient continuation decisions. Interestingly, greater debtor rights has an ambiguous effect on debt ratios (and levels). When debtor rights are strong, management chooses to continue when liquidation may be optimal. The higher distress costs associated with excessive continuation leads firms to reduce debt levels. However, increasing debt has the benefit of moving the management's liquidation threshold closer to the efficient threshold. This latter effect may or may not dominate the former effect. This may help to explain why cross-country empirical analyses do not find an unambiguous effect of debtor rights on debt financing (Rajan and Zingales, 1995).

A Credit Spreads

For what follows, we assume that the benefits of debt (τ) accrue to shareholders and, for simplicity, we normalize the expected rate of returns to be zero. In this case, we can compute (implicitly) the equilibrium promised rate of return, denoted r, that leaves creditors indifferent between providing funding and not providing funding:

$$\frac{D^*}{1+r} = \frac{1}{\gamma} \cdot \left\{ \int_0^{\Lambda^*} (P^* - x^*)\, dV \;+\; \int_{\Lambda^*}^{\hat{D}^*} (1-\beta) \cdot [V - \phi \cdot (\hat{D}^* - V)]\, dV \;+\; \int_{\hat{D}^*}^{\gamma} D^*\, dV \right\}$$

For ease of comparison, the comparative statics for the credit spread are:

Exog →	Reorganization Costs	Tax Benefits	Asset Redeployability	Funding Availability		Debtor Rights
	ϕ	τ	η	k	α	β
Credit Spread	−/+	+	−/+	−/+	−/+	−/+

We assumed shareholders immediately receive the tax benefits of debt, therefore, an increase in the tax benefits of debt unambiguously increases the observed credit spread because it encourages firms to take on more debt, resulting in a greater likelihood of default. As in Leland (1994), credit spreads may increase or decrease when the costs of distress are higher. Higher distress costs lead to lower recovery rates in the event of default, but also causes firms to respond by choosing lower levels of debt which reduces the likelihood

of default. Similarly, there is no unambiguous relationship between credit spreads and asset redeployability (η), financing constraints (k and α), and debtor rights (β) because the direct effect of these parameters on the probability of default and recovery rates (for a given level of debt) may be counteracted by the indirect effect of these parameters via the optimal choice of debt.

V Industry Recessions and Expansions

We now extend the model to incorporate industry recessions and expansions. We introduce a state L that represents a recession and a state H that represents an expansion and assume, for simplicity, that these obtain with equal probabilities $p_H = p_L = 1/2$. At time 0 it is common knowledge that in a recession the firm productivity, $V_{i,L}$, is distributed uniformly over the interval $[0, \gamma - \Delta]$, and in an expansion firm productivity, $V_{i,H}$, is distributed uniformly over the interval $[0, \gamma + \Delta]$. Therefore, state H has higher potential outcomes and less mass below any given fixed value, but the support of V in both cases is bounded below by zero. Returning to the case where $\beta \to 0$ so there is no strategic default we have the following result.

Theorem 4 *There is a unique equilibrium $\{D^*_{HL}, P^*_H, P^*_L\}$ where*

$$\begin{aligned} D^*_{HL} &= \left(1 - \frac{\Delta^2}{\gamma^2}\right) \cdot D^* \\ P^*_H &= \left[\frac{k(1+\phi)}{1+k(1+\phi)/\eta}\right] \cdot (\gamma + \Delta) - \left[\frac{\phi + 2\alpha(1+\phi)/\eta}{1+k(1+\phi)/\eta}\right] \cdot D^*_{HL} \\ P^*_L &= \left[\frac{k(1+\phi)}{1+k(1+\phi)/\eta}\right] \cdot (\gamma - \Delta) - \left[\frac{\phi + 2\alpha(1+\phi)/\eta}{1+k(1+\phi)/\eta}\right] \cdot D^*_{HL} \end{aligned}$$

where D^* is as in Theorem 2.

There are several results that follow immediately. First, optimal debt is lower than in the single-state case.[13] In our specification, optimal debt is decreasing in the uncertainty about the industry state (Δ). Second, holding future cash flows constant, the liquidation price is lower in the industry recession state because there are fewer healthy buyers of the distressed assets (as in Shleifer and Vishny, 1992). Third, the probability of liquidation

[13] This is true for general distributions for the industry state because of Jensen's inequality.

conditional on distress is lower in the recession state than in the expansion state, again because the liquidation price is lower which encourages management to continue. This has been empirically documented by Asquith, Gertner, and Scharfstein (1994) and Acharya, Bharath, and Srinivasan (2007). Fourth, the unconditional probability of distress and the unconditional probability of liquidation are independent of uncertainty about the industry state. On one hand, greater uncertainty has the direct effect of increasing distress and liquidation probabilities because downside risk is greater; however, greater uncertainty has the indirect effect of reducing optimal debt which lowers distress and liquidation probabilities. These two effects exactly offset in our model.

VI Conclusion

Our primary interest in this paper concerns how liquidation values and capital structure choices are jointly determined in an industry equilibrium. Asset liquidation values are an important source of distress costs and therefore affect capital structure choices; however, capital structure choices affect the supply of and demand for liquidated assets because they change managerial incentives to liquidate assets and impact the availability of financing for potential buyers. We show that parameters often have competing effects on capital structure choices: a direct effect (holding liquidation prices fixed) and an indirect effect that works through equilibrium prices for liquidated assets. Although indirect effects sometimes overwhelm direct effects, the general insight here is that endogenous liquidation prices can significantly dampen the effect of changes in the net benefit or cost of debt.

We made several simplifying assumptions to obtain closed-form solutions for optimal debt choices and equilibrium prices. In particular, we assumed that lenders consider only the pooled characteristics of healthy firms in the industry when deciding how much financing to make available to firms. We also assumed a limited renegotiation framework that leads to inefficient continuation decisions in the event of default. In future work, we will relax these assumption to assess the robustness of our results.

A Proofs

Proof of Lemma 1: The following is a proof sketch. If $\Lambda^* \leq \hat{D}$ then shareholders receive $\beta \cdot [V_i - \phi(\hat{D} - V_i)]$ if they choose to default and continue operations compared to $\max\{0, V_i - D\}$ if they choose not to default and continue operations. Therefore, the optimal default trigger, $V_i = \hat{D}$, is given by:

$$\beta \cdot [\hat{D} - \phi(\hat{D} - \hat{D})] = \hat{D} - D \quad \Leftrightarrow \hat{D} = \frac{D}{1-\beta}.$$

Although the optimal transfer x^* depends on the threshold for strategic default, $\hat{D}$, it does not affect the choice of $\hat{D}$. The reason is that management knows V_i, therefore, management knows whether it will choose to liquidate (low V_i) but the strategic default boundary involves only a comparison of the benefits to shareholders of continuing in default versus continuing without default.

Proof of Theorem 1: If $\beta \to 0$ then the strategic default trigger, $\hat{D} = D$, and the threshold $\Lambda^*(\hat{D}) = (P + \phi D)/(1+\phi)$ as shown in Section I.A. The equilibrium D^* and P^* solve the FOC for optimal debt and market clearing conditions. The comparative statics for D^* and P^* are obvious by inspection. The comparative statics for D^*/V^* follow immediately from the parameter restrictions ensuring $D^* < \gamma$.

Proof of Theorem 2: If $\beta \to 0$ then the strategic default trigger, $\hat{D} = D$, and the threshold $\Lambda^*(\hat{D}) = (P + \phi D)/(1+\phi)$ as shown in Section I.A. The equilibrium D^* and P^* solve the FOC for optimal debt and market clearing conditions. Totally differentiating the FOC for optimal debt and the equilibrium pricing condition yields:

$$\begin{aligned}
dD - dP - \frac{\alpha(1+\phi)}{\eta\phi}dP &= \left[\frac{(1+\phi)(\tau-\alpha)}{\phi}\right]d\gamma + \left[\frac{(1+\phi)\gamma}{\phi}\right]d\tau \\
&- \left[\frac{\alpha P(1+\phi)}{\eta^2\phi}\right]d\eta - \left[\frac{(1+\phi)(\gamma - P/\eta)}{\phi}\right]d\alpha \\
&+ \left[\frac{\alpha(\gamma - P/\eta) - \gamma\tau}{\phi^2}\right]d\phi \\
dP + \phi dD + \frac{k(1+\phi)}{\eta}dP + \frac{2\alpha(1+\phi)}{\phi}dD &= \left[\frac{kP(1+\phi) + 2\alpha(1+\phi)D}{\eta^2}\right]d\eta - \left[\frac{2(1+\phi)D}{\eta}\right]d\alpha \\
&+ \left[k(\tau - P/\eta) - (1 + 2\alpha/\eta)D\right]d\phi \\
&+ \left[(1+\phi)k(\gamma - P/\eta)\right]dk.
\end{aligned}$$

The comparative statics results follow from an application of Cramer's Rule. Note that the comparative statics with respect to ϕ follow from (i) $D^* \geq P^*$ which implies $\alpha(\gamma - P/\eta) - \gamma\tau \leq 0$, and (ii) $k(\gamma - P/\eta) - 2\alpha D/\eta = (P + \phi D)/(1+\phi)$.

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Table 1: Variables

Exogenous Parameters		
γ	$\gamma > 0$	Highest firm productivity in uniform distribution
V_i	$V_i \sim U[0, \gamma]$	Unlevered firm value
β	$0 \leq \beta \leq 1$	Strength of debtor rights
ϕ	$0 \leq \phi \leq 1$	Distress impairment $\phi(\hat{D} - V_i)$ for firms continuing in default.
$\ell(D)$	$\ell(D^*) > 0$	Funding available to healthy firms, $k - \alpha \cdot D/\bar{\bar{V}}$
		where $\bar{\bar{V}}$ is the average surplus for healthy firms
k	$k \geq 0$	Industry-wide scale of funding
α	$0 \leq \alpha \leq 1$	Debt overhang parameter
τ	$0 \leq \tau \leq 1$	Tax and other (net) benefits of debt

Table 2: Overview of Constraints

$$V(D) \equiv \frac{1}{\gamma} \cdot \left\{ \int_0^{\Lambda} P \, dV \; + \; \int_{\Lambda}^{\hat{D}} [V - \phi \cdot (\hat{D} - V)] \, dV \; + \; \int_{\hat{D}}^{\gamma} V \, dV \; + \; \ell(D) \cdot \int_{P/\eta}^{\gamma} (\eta \cdot V - P) \, dV \right\} \; + \; \tau \cdot D \; .$$

	Interpretation
$P^* \geq 0$	Free Disposal
$D^* \leq V^*$	Leverage ratio less than 1
$\Lambda^* \geq 0$	First Integral: Sometimes liquidation is better than continuation
$\Lambda^* \leq \hat{D}$	Second Integral: Sometimes continuation is better than liquidation
$D^* \leq \gamma$	Third Integral: Sometimes firm is not in distress
$P^* \cdot \eta \leq \gamma$	Fourth Integral: Distressed asset buyers exist
$V(0) \leq V(D^*)$	Solution is better than taking no debt
$\ell(D) \geq 0$	Capital availability is non-negative

Figure 1: Game Tree