Markets, Banks, and Shadow Banks

David Martinez-Miera  
UC3M and CEPR

Rafael Repullo  
CEMFI and CEPR

December 2018

Abstract

We analyze the effect of bank capital requirements on the structure and risk of a financial system where markets, regulated banks, and shadow banks coexist. Banks face a moral hazard problem in screening entrepreneurs’ projects, and they choose whether to be regulated or not. If regulated, a supervisor certifies their capital; if not, they have to rely on more expensive private certification. Under both risk-insensitive and risk-sensitive requirements, safer entrepreneurs borrow from the market and riskier entrepreneurs borrow from banks. But risk-insensitive (sensitive) requirements are especially costly for relatively safe (risky) entrepreneurs, which may shift from regulated to shadow banks.

JEL Classification: G21, G23, G28

Keywords: Bank regulation, bank supervision, capital requirements, credit screening, credit spreads, loan defaults, optimal regulation, market finance, shadow banks.

We thank Jason Allen, Andres Almazan, Frederic Boissay, Philip Bond, Jean-Edouard Colliard, Hans Degryse, Jose Fillat, Mark Flannery, Florian Heider, Julian Kolm, Tim Landvoigt, David Miles, George Pennacchi, Jean-Charles Rochet, Javier Suarez, and Anna Toldra for very useful comments. Financial support from the Spanish Ministry of Economy and Competitiveness, Grants No. ECO2014-59262-P (Repullo) and ECO2017-82385-P (Martinez-Miera), and from Banco de España (Martinez-Miera) is gratefully acknowledged. The work of Martinez-Miera has been prepared under the Lamfalussy Fellowship Program of the European Central Bank (ECB). The views expressed are those of its authors and do not represent those of the Banco de España, the ECB, or the Eurosystem. E-mail: david.martinez@uc3m.es, repullo@cemfi.es.
“While higher capital and liquidity requirements on banks will no doubt help to insulate banks from the consequences of large shocks, the danger is that they will also drive a larger share of intermediation into the shadow banking realm.”

S. Hanson, A. Kashyap, and J. Stein (2011)

1 Introduction

The aftermath of the 2007-2009 financial crisis resulted in a widespread adoption of tougher bank regulation, exemplified by the 2010 Accord of the Basel Committee on Banking Supervision, known as Basel III. However, a concern has emerged about the possibility that the effectiveness of the new regulation may be hindered by a shift of intermediation away from regulated banks and into the shadow banking system.

This paper contributes to this debate by proposing an analytical framework to assess the effects of bank capital requirements on the structure and risk of the financial system. In particular, we address issues such as (i) what is the difference between regulated and shadow banks, and how do they differ from direct market finance, (ii) what type of borrowers are funded by them, (iii) how does bank capital regulation affect lending through these channels, and (iv) how does the existence of shadow banks affect the effectiveness of this regulation.

Our model has a set of heterogeneous entrepreneurs that need to raise funds to undertake their risky investment projects, and a set of financial intermediaries that can reduce the probability of default of their loans to entrepreneurs by screening them at a cost.\footnote{Our notion of screening is in line with Vanasco (2017), where “originators exert screening effort by hiring better employees (e.g. loan officers), by devoting time to understand the pool of available projects, and (...) by improving the technology used to verify the information content of loan applications.”} There are three possible modes of funding entrepreneurs’ projects: they may be directly funded by the market, or through perfectly competitive intermediaries that can choose to comply with bank capital regulation, which we call regulated banks, or not, which we call shadow banks. Market finance differs from intermediated finance in that entrepreneurs are not (privately) screened.\footnote{All public screening, for example by credit rating agencies, is included in the observable heterogeneity of entrepreneurs.} Both regulated and shadow banks fund themselves by raising (uninsured) debt
and (costly) equity from a set of risk-neutral investors.

The shadow banking system has been broadly defined by the Financial Stability Board (FSB) as credit intermediation involving entities and activities outside the regular banking system. Our screening-based notion of shadow banking is closer to the narrow definition put forward by the FSB, in which non-bank financial institutions (excluding insurance corporations and pension funds), are classified with reference to five economic functions, each of which involves non-bank credit intermediation. The quantitatively most important of these functions is defined as the management of collective investment vehicles, which includes, in order of importance, fixed income funds, mixed (equity and credit) funds, money market mutual funds, and credit hedge funds.\(^3\) One common feature of these institutions is that, as in our model, they actively select (screen) the assets in their portfolios.

A key financial friction that is at the core of our approach is that screening is not observed by investors. Hence, there is a moral hazard problem in intermediated finance. In this situation, (inside) equity capital provides “skin in the game,” serving as a commitment device for screening borrowers. For this reason, banks may be willing to use more expensive equity in order to ameliorate the moral hazard problem and reduce the cost of uninsured debt.\(^4\)

However, for this channel to operate, the capital structure has to be observable to investors. Given the incentives of banks to save on costly equity, we assume that capital has to be certified by an external (private or public) agent. Public certification is done by a bank supervisor that verifies whether banks that choose to be regulated comply with the regulation. The capital of banks that choose not to be regulated is not certified by the supervisor, so they have to resort to private certification, which we assume to be more expensive. Thus, (cheaper) public certification is tied to complying with a regulation that might be very tough.

\(^3\)At end-2016, this function amounted to 71.6% of the total narrow measure of $45.2 trillion in the 29 jurisdictions covered in the report; see Financial Stability Board (2018). The other functions are securitization-based credit intermediation and funding of financial entities (9.6%), intermediation of market activities that is dependent on short-term funding or on secured funding of client assets (8.4%), loan provision that is dependent on short-term funding (6.4%), and facilitation of credit creation (0.4%).

\(^4\)In this moral hazard setup, if capital were not more expensive than debt, banks would be 100% equity financed. In such a case, there would be no informational friction and no rationale for regulation.
at least for banks financing certain types of entrepreneurs. For this reason, intermediaries might prefer not to comply with the regulation and resort to private certification, giving rise to shadow banks. Hence, in this setup the emergence of shadow banks is linked to a trade-off between the costs (in terms of higher cost of capital) and benefits (in terms of lower certification costs) of being subject to capital regulation.

An alternative trade-off, which is complementary to the one analyzed in our main setup and can also give rise to shadow banks, derives from the assumption of underpriced deposit insurance for regulated banks. In Appendix A we show that replacing lower certification costs by underpriced deposit insurance yields essentially the same results on the equilibrium structure of the financial system.

We first consider two types of bank capital regulation, namely risk-insensitive (or flat) and risk-sensitive capital requirements. The former broadly correspond to the 1988 Accord of the Basel Committee (Basel I), while the latter correspond to the 2004 (Basel II) and 2010 (Basel III) Accords. We follow the Basel II and III approach of using a Value-at-Risk (VaR) criterion to determine the risk-sensitive requirements. We highlight the different effects that these regulations have on the equilibrium market structure, with especial emphasis on whether they will shift some types of lending from regulated banks into shadow banks or direct market finance, and their impact on the overall risk of the financial system.

Specifically, under both regulations, safer entrepreneurs borrow from the market and riskier entrepreneurs borrow from intermediaries. This is explained by the fact that screening is not that useful for lending to safer entrepreneurs. The difference between both regulations is that flat requirements are especially costly for relatively safe entrepreneurs, that may be better off borrowing from shadow banks, while VaR requirements are especially costly for risky entrepreneurs, that may be better off borrowing from shadow banks. Hence, with flat capital requirements the equilibrium market structure is such that regulated banks always fund the riskiest projects, while if shadow banks operate they fund projects that are safer than those of the regulated banks. With VaR capital requirements the equilibrium market

---

5 This also corresponds to the regulation advocated by Admati and Hellwig (2013).
6 Although Basel III combines risk-sensitive capital requirements with a risk-insensitive leverage ratio.
structure is such that regulated banks always fund the intermediate risk projects, while if shadow banks operate they fund the riskiest projects. Thus, the type of capital requirements leads to very different structures of the financial sector.

Our results illustrate how the possibility of unregulated finance affects the effectiveness of the different types of regulation, with some interesting empirical implications. In particular, tightening flat (VaR) capital requirements increases the screening incentives of banks for which the regulation is binding at the cost of driving some safer (riskier) entrepreneurs to the shadow banking system, where they will have lower screening and higher default risk. Hence, a tightening of capital requirements can lead to a reduction in the risk of loans to entrepreneurs that stay with the regulated banks, but at the same time lead to an increase in the risk of loans to those that shift out of the regulated banks, which may result (if the second effect is large enough) in an increase in the overall risk of the financial system.

After analyzing the effect of these regulations, we characterize the second-best optimal capital requirements, that is, those that maximize social welfare when the regulator is subject to the same informational frictions as banks. We show that (in general) optimal capital requirements are risk-sensitive and binding, in the sense that they force banks to have more capital than they would in the absence of the regulation, resulting in higher screening and lower investment.\textsuperscript{7} We also show that the presence of direct market and shadow bank finance imposes a constraint on the regulator that leads to lower capital requirements for low and high risk entrepreneurs, respectively.

Finally, we propose two extensions of our basic setup. In a first extension we analyze how the equilibrium structure and risk of the financial system change with two key parameters of the model, namely the expected return required by investors (the safe rate) and the excess cost of bank capital. We find that for both types of capital requirements a higher safe rate and/or a lower cost of capital expand the range of entrepreneurs financed by regulated banks. According to these results, the shadow banking system will thrive when the safe rate is low (due, for example, to a savings glut) and the cost of bank capital is high (due, for example,\textsuperscript{7}Interestingly, although optimal capital requirements are risk-sensitive, they have a lower slope than those derived from a VaR criterion.)
to the relative scarcity of bank capital following a bubble-driven expansion of banks’ balance sheets). In a second extension we analyze a variation of the model in which the cost of capital is endogenously derived from a fixed supply of bank capital. In this case, tightening capital requirements affects all banks in the economy through the increase in the equilibrium cost of capital, which can lead to lower capital and higher risk of those (regulated and shadow) banks not constrained by the regulation.

**Literature review** This paper is related to a long standing strand of research analyzing the role of capital requirements on banks’ risk-taking decisions; see Koehn and Santomero (1980), Rochet (1992), Hellmann et al. (2000), and Repullo (2004), among many others. Our main departure from this strand of literature is related to the fact that, as Hanson et al. (2011) and Flannery (2016) highlight, we analyze relevant trade-offs that appear when the existence of unregulated financial intermediaries (shadow banks) is taken into account. Understanding the shifts of lending from regulated to shadow banks is crucial when, as highlighted by Admati (2013) and Thakor (2014), the main objective of tightening capital requirements is to reduce the risk of the financial system. Another departure from this strand of research is that we endogenize the return structure of financial intermediaries in a perfectly competitive environment. As our results highlight, the endogenous response of loan rates is crucial to understand the effect of capital requirements on the structure and risk of the financial system.

Recent empirical studies such as Buchak et al. (2017), analyzing the mortgage market, and Irani et al. (2018), analyzing the (syndicated) corporate loan market, find how, as predicted by our model, stricter capital requirements are linked to an expansion of the shadow banking system. Buchak et al. (2017) also find that shadow banks specialize in lending to riskier households, which is in line with our results for the VaR requirements of Basel II and Basel III.

Our focus on understanding the emergence of shadow banks relates our research to a

---

8See for example Harris et al. (2017) or Martinez-Miera (2009) for other papers analyzing how capital regulation can shape the endogenous return of financial assets and by doing so impact banks’ risk-taking decisions.
recent strand of the theoretical banking literature. In contrast to Plantin (2015) that focuses on the different ability of shadow banks to issue money-like liabilities, or Bengenau and Landvoigt (2017) that focus on the impact of bailouts as a key difference between regulated and shadow banks, we consider a novel trade-off between the costs and benefits of regulation, where the latter are linked to the savings on certification costs by bank supervision. In contrast to Fahri and Tirole (2017) that link intermediaries’ decisions to become shadow banks to the existence of deposit insurance and a lender of last resort, our main setup does not rely on the existence of such safety net.9

It is important to highlight that in contrast to the arguments in Acharya et al. (2013), our paper does not build on regulatory arbitrage and implicit subsidies received by shadow banks through their linkages to regulated banks, as in our setup regulated and shadow banks are separate entities without any direct linkages.10 Also it is important to highlight that we focus on a situation in which banks are ex-ante homogenous, which differentiates our setup from that of Ordoñez (2018), in which the existence of shadow banks can be beneficial when regulators are constrained in their knowledge of banks’ types.

Our moral hazard setup is obviously connected to the seminal paper of Hölmstrom and Tirole (1997). They show how in a laissez-faire economy financial intermediaries can use capital to ameliorate moral hazard problems and how (when capital is costly) this gives rise to intermediated and market finance. In contrast to having entrepreneurial wealth as key determinant of the financing mode, we follow Martinez-Miera and Repullo (2017) in considering ex-ante differences in the risk of entrepreneurial projects. In contrast with our earlier work, that focuses on ex-post monitoring of entrepreneurial projects, this paper focuses on ex-ante screening of these projects. Although both approaches lead to similar results, the screening setup seems more suitable to encompass shadow banks.

9However, we also acknowledge (and analyze in Appendix A) the relevance of underpriced deposit insurance for the regulated banks as a possible factor explaining the emergence of shadow banks.

10See Fahri and Tirole (2017) for a more extensive review on different elements that can explain the emergence of shadow banks. These elements include, but are not limited to, the role of financial intermediaries as producers of safe assets (Hanson et al., 2015) and of liquidity services (Moreira and Savov, 2017).
Structure of the paper Section 2 presents the model of bank lending under moral hazard in which banks are not regulated and have to pay a cost to certify their capital. Section 3 introduces minimum capital requirements and analyzes the effects of flat and VaR requirements on the structure and risk of the financial system. Section 4 characterizes optimal capital requirements. Section 5 considers the effect of changes in funding costs and of endogenizing the cost of capital. Appendix A shows that the qualitative results remain unchanged when the advantage of regulated banks relative to shadow banks comes from the existence of underpriced deposit insurance. Appendix B contains the proofs of the analytical results.

2 The Model

Consider an economy with two dates \( t = 0, 1 \), a large set of penniless entrepreneurs with observable types \( p \in [0, 1] \), and a large set of risk-neutral investors characterized by an infinitely elastic supply of funds at an expected return equal to \( R_0 \) (the safe rate). Entrepreneurs have investment projects that require external finance. Such finance may come directly from investors (market finance) or may be intermediated by banks (bank finance).

Intermediated finance differs from direct market finance in two respects. First, banks screen borrowers, which reduces their default risk. Second, banks raise funds from investors, in the form of uninsured deposits, and also from (inside) shareholders. We assume that bank capital is costly. Specifically, there is an infinitely elastic supply of bank capital at an expected return equal to \( R_0 + \delta \), where the excess cost of bank capital \( \delta \) is positive.\(^{11}\)

Each entrepreneur of type \( p \) has a project that requires a unit investment at \( t = 0 \) and yields a stochastic return at \( t = 1 \) given by

\[
\tilde{A}_p = \begin{cases} 
A(x_p), & \text{with probability } 1 - p + s_p, \\
0, & \text{with probability } p - s_p, 
\end{cases}
\]

where the success return \( A(x_p) \) is a decreasing function of the aggregate investment \( x_p \) of entrepreneurs of type \( p \), and \( s_p \in [0, p] \) is the screening intensity of the entrepreneur’s lender.\(^{12}\)

\(^{11}\)Appendix A analyzes a setup in which deposits of regulated banks are insured.

\(^{12}\)The important function that financial intermediaries perform is to reduce the informational asymmetries between entrepreneurs and investors. This can be done by screening the quality of entrepreneurs’ projects...
When \( s_p = 0 \) we have direct market finance, and when \( s_p > 0 \) we have bank finance. Thus, the safest type \( p = 0 \) will always be funded by the market. Screening is not observed by the investors, so there is a moral hazard problem.

Screening increases the probability of success of entrepreneurs’ projects but entails a cost \( c(s_p) \). The screening cost function \( c(s_p) \) satisfies \( c(0) = c'(0) = 0 \), and \( c'(s_p) > 0 \), \( c''(s_p) > 0 \), and \( c'''(s_p) > 0 \), for \( s_p > 0 \). A special case that satisfies these assumptions and will be used for our numerical results is the quadratic function

\[
c(s_p) = \frac{\gamma}{2} (s_p)^2,
\]

where \( \gamma > 0 \).

To simplify the presentation we assume that (i) the returns of the projects of entrepreneurs of each type \( p \) are perfectly correlated, and (ii) for each type \( p \) there is a single bank that specializes in funding entrepreneurs of this type. The perfect correlation assumption is made for convenience, and could be easily relaxed. The assumption that a single bank lends to each type of entrepreneurs is not restrictive, since we will assume that the loan market is contestable. The key simplifying assumption is that no bank lends to more than one type of entrepreneur, since otherwise we would have to model bank competition across types.\(^\text{13}\)

The assumption \( A'(x_p) < 0 \) may be rationalized by introducing a representative consumer with a utility function over the continuum of goods produced by entrepreneurs of types \( p \in [0, 1] \). Specifically, suppose that one unit of investment produces (if successful) one unit of output, and consider the utility function

\[
U(q, x) = q + \sigma \left( \frac{\sigma - 1}{\sigma} \int_0^1 (x_p)^{\sigma-1} \, dp \right),
\]

where \( q \) is the consumption of a composite good, \( x = \{x_p\} \), and \( \sigma > 1 \). The budget constraint of the representative consumer is

\[
q + \int_0^1 A_p x_p \, dp = I,
\]

ex-ante or by monitoring them ex-post. Although the interpretation of what intermediaries do is different, both models yield similar results.

\(^{13}\)It should be noted that this assumption is not restrictive in a model with deposit insurance, since in this case competitive banks would want to specialize in a single type of loans; see Lemma 1 in Repullo and Suarez (2004).
where $A_p$ is the unit price of the good produced by entrepreneurs of type $p$, and $I$ is her (exogenous) income. Maximizing (3) subject to (4) gives a first-order condition that implies

$$A_p = A(x_p) = (x_p)^{-1/\sigma}.$$  

(5)

Thus, the higher the investment $x_p$ of entrepreneurs of type $p$ the lower the equilibrium price $A_p$, if the investment is successful. If it is not, output will be zero and the representative consumer will not consume this good.\(^{14}\)

We assume free entry of entrepreneurs in the loan market. Hence, if the lowest loan rate for entrepreneurs of type $p$ offered by either markets or banks is $R_p$, then a measure $x_p$ of these entrepreneurs will enter until $A(x_p) = R_p$. Thus, entrepreneurs will only be able to borrow at a rate that leaves them no surplus.

Since investors are characterized by an infinitely elastic supply of funds at an expected return equal to $R_0$, the equilibrium loan rate $R_p^*$ for entrepreneurs of type $p$ under direct market finance will be the rate that satisfies the participation constraint

$$(1 - p)R_p^* = R_0.$$  

(6)

Computing the equilibrium loan rate under bank finance is more complicated because one has to derive banks’ decision on capital and screening. To do this, we assume that the loan market is contestable. Thus, although there is a single bank that lends to each type, the incumbent would be undercut by another bank (or by the market) if it were profitable to do so.

Despite the assumption that bank capital is more expensive than debt, banks may be willing to use equity finance in order to ameliorate the moral hazard problem and reduce the cost of debt. But this requires that banks’ capital structure be observable to outside investors. Given the incentives of banks to save on costly equity, we assume that capital has to be certified by an external agent at a cost $\eta$ per unit of capital.\(^{15}\)

\(^{14}\)An alternative rationalization may be derived from the demand of a set of final good producers that use entrepreneurs’ output as an intermediate input; see Martínez-Miera and Repullo (2017).

\(^{15}\)Alternatively, we could assume a certification cost per unit of loans, that is proportional to the banks’ balance sheet. This setup is analytically less tractable than simply adding $\eta$ to the excess cost of capital $\delta$.  

9
The bank lending to entrepreneurs of type \( p \) will raise \( 1 - k_p \) funds per unit of loans from investors at a rate \( B_p \) (the rest will be funded with capital), set a loan rate \( R_p^* \), and choose a screening intensity \( s_p \in [0, p] \). By contestability, the equilibrium loan rate \( R_p^* \) for entrepreneurs of type \( p \) will be the lowest feasible rate.

Formally, an equilibrium for entrepreneurs of type \( p \) under bank finance is an array \((k_p^*, B_p^*, R_p^*, s_p^*)\) that minimizes the loan rate \( R_p \) subject to the bank’s incentive compatibility constraint

\[
s_p^* = \arg \max_{s_p} \left[ (1 - p + s_p)[R_p^* - (1 - k_p^*)B_p^*] - c(s_p) \right],
\]

the shareholders’ participation constraint

\[
(1 - p + s_p^*)[R_p^* - (1 - k_p^*)B_p^*] - c(s_p^*) \geq (R_0 + \delta + \eta)k_p^*,
\]

and the investors’ participation constraint

\[
(1 - p + s_p^*)B_p^* \geq R_0.
\]

The incentive compatibility constraint (7) characterizes the bank’s choice of screening \( s_p^* \) given that the bank gets \( R_p^* \) and pays \((1 - k_p^*)B_p^* \) with probability \( 1 - p + s_p \) (and with probability \( p - s_p \) gets zero, by limited liability). The participation constraints (8) and (9) ensure that shareholders and investors get the required expected return on their investments.

Note that the assumption that project returns are perfectly correlated implies that the bank’s return per unit of loans is identical to the individual project return, which is given by (1). It also implies that the loans’ probability of default equals the bank’s probability of failure.

The following result characterizes the range of entrepreneurs’ types that borrow from the market and from banks.

**Proposition 1** There exists a marginal type

\[
\hat{p} = 1 - \sqrt{\frac{R_0(R_0 + \delta + \eta)}{(\delta + \eta)c'(0)}},
\]

such that entrepreneurs of types \( p \leq \hat{p} \) borrow from the market and entrepreneurs of types \( p > \hat{p} \) borrow from banks.
To ensure that market and bank finance coexist in equilibrium, we assume that the screening cost function is sufficiently convex. In particular,

$$c''(0) > \frac{R_0(R_0 + \delta)}{\delta} \tag{11}$$

which implies $\hat{\rho} \in (0, 1)$.

The sketch of the proof is as follows. Consider a type $p$ for which the equilibrium screening intensity $s_p^*$ satisfies $0 < s_p^* < p$. Then the bank’s incentive compatibility constraint (7) reduces to the first-order condition

$$R_p^* - (1 - k_p^*)B_p^* = c'(s_p^*). \tag{12}$$

From here it can be shown (see the formal proof in Appendix B) that both the shareholders’ participation constraint (8) and the investors’ participation constraint (9) are binding. Solving for $R_p^* - (1 - k_p^*)B_p^*$ in the shareholders’ participation constraint (8) (written as an equality), substituting it into the first-order condition (12), and solving for $k_p^*$ gives

$$k_p^* = \frac{(1 - p + s_p^*)c'(s_p^*) - c(s_p^*)}{R_0 + \delta + \eta}. \tag{13}$$

By the properties of the screening cost function $c(s_p)$ this equation implies that $k_p^* > 0$ if and only if $s_p^* > 0$. In other words, banks will always have a positive amount of capital.

Next, solving for $B_p^*$ in the investors’ participation constraint (9) (written as an equality), substituting it into the first-order condition (12), and rearranging gives

$$R_p^* = \frac{(1 - k_p^*)R_0}{1 - p + s_p^*} + c'(s_p^*). \tag{14}$$

The equilibrium loan rate $R_p^*$ is found by minimizing (14) with respect to $s_p$ and $k_p$, subject to (13). Finally, we show that for entrepreneurs of types $p \leq \hat{\rho}$, the loan rate (14) is minimized by setting $s_p^* = k_p^* = 0$, so they will borrow from the market, and for entrepreneurs of types $p > \hat{\rho}$, the loan rate (14) is minimized by setting $s_p^* > 0$ and $k_p^* > 0$, so they will borrow from banks.

---

16Note that the convexity of the screening cost function implies $s_p^*c'(s_p^*) > c(s_p^*)$, for $s_p^* > 0$. 
It should be noted that in the absence of certification, there would be a second moral hazard problem related to the choice of capital. In this case, the definition of equilibrium requires a second incentive compatibility constraint, namely

$$k_p^* = \arg \max_{k_p} \left[ (1 - p + s_p^*) [R_p^* - (1 - k_p) B_p^*] - c(s_p^*) - (R_0 + \delta) k_p \right].$$

Differentiating the expression in the right-hand side with respect to $k_p$, and using the investors’ participation constraint (9) (written as an equality), gives

$$(1 - p + s_p^*) B_p^* - (R_0 + \delta) = -\delta < 0.$$

Hence, certification is essential for banks to have any capital, since in its absence they will be entirely funded with debt.

The relevant question then is: will a bank that does not certify its capital (and hence has zero capital) have an incentive to choose a positive level of screening and be able to undercut banks that certify their capital? The following result provides a negative answer to this question.

**Proposition 2** Entrepreneurs will not borrow from banks that do not certify their capital.

This result implies that the possibility of capital certification, even if it is costly, ensures that all intermediated finance is channeled through institutions that have a certified amount of capital. In other words, intermediaries that do not certify their capital cannot successfully compete with those that do so.

We next introduce two possible institutions that may certify banks’ capital. One is a private auditor that charges a rate $\eta_1$ per unit of capital. The other is a public auditor that charges a rate $\eta_0$ per unit of capital. The existence of a public auditor may be justified by introducing bank capital requirements and associating the public auditor to a bank supervisor that verifies whether the bank complies with the regulation. We assume that private certification is costlier than public certification, so $\eta_1 > \eta_0$. This may be rationalized by assuming that supervisors have lower agency problems or better access to relevant bank information than private auditors.
But if private auditors are more expensive than the public auditor, why would banks want to resort to them? The answer is that using the public auditor is tied to complying with a regulation that might be very tough, at least for banks financing certain types of entrepreneurs. These (shadow) banks might then prefer not to comply with the regulation and resort to private certification. In this manner, the emergence of shadow banks is linked to a trade-off between the costs (in terms of higher cost of capital) and benefits (in terms of lower certification costs) of being subject to capital regulation.

Bank capital regulation will be introduced in the next section. Here we present, for future reference, the comparative statics of the certification cost.

**Proposition 3** An increase in the certification cost $\eta$ expands the range $[0, \hat{\eta}]$ of market finance, and for types $p > \hat{p}$ reduces banks’ equilibrium capital and screening.

Figure 1 illustrates this result for the quadratic screening cost function (2) and two values of the certification cost, $\eta_0$ and $\eta_1$, corresponding respectively to a public and a private auditor. To simplify the presentation, in what follows we will normalize to zero the cost of the public auditor ($\eta_0 = 0$), and drop the subindex for the cost of the private auditor ($\eta_1 = \eta$). Panel A shows that an increase in the certification cost shifts to the right from $\hat{\eta}_0$ to $\hat{\eta}_1$ the marginal type that is indifferent between market and bank finance. As capital becomes more expensive, due to the higher certification costs, banks reduce their capital per unit of loans. Panel B shows the effect on the probability of failure $p - s_p$. Under market finance $s_p = 0$, so the probability of failure coincides with the 45° line. The reduction in the level of capital under high certification costs worsens the banks’ moral hazard problem and leads to an increase in the probability of failure.

Summing up, we have presented a model in which a heterogeneous set of entrepreneurs seek funding from either banks or the market. The difference between bank and market finance is that banks screen their borrowers, which leads to a reduction in the probability of default. Bank screening is subject to a moral hazard problem that can be ameliorated by equity capital. However, capital is costlier than deposits, and using capital also requires paying a certification cost. We have shown that safer entrepreneurs borrow from the market
Figure 1. Public and private capital certification

This figure shows the equilibrium of the model with public and private capital certification. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure. Solid (dashed) lines represent equilibrium values with public (private) certification.

while riskier entrepreneurs borrow from banks, and that banks will always want to fund part of their lending with capital. Higher certification costs shift some entrepreneurs from bank to market finance, and increase banks’ probability of failure.

3 Bank Capital Regulation

This section introduces a bank regulator that sets minimum capital requirements and a bank supervisor that verifies whether banks that choose to be regulated comply with the regulation, in which case their capital is certified at a cost that is normalized to zero. Banks that choose not to be subject to the regulation will be called shadow banks. Since their capital is not certified by the supervisor, they will have to resort to more expensive private certification.

Two types of minimum capital requirements, flat and risk-based, will be analyzed. A flat requirement does not vary with the bank’s risk, whereas a risk-based requirement is increasing in the bank’s risk. The risk-insensitive regulation broadly corresponds to the 1988 Basel Capital Accord (Basel I), while the risk-sensitive regulation corresponds to the
2004 (Basel II) and 2010 (Basel III) Accords.\textsuperscript{17} We follow the Basel II and III approach of using a Value-at-Risk (VaR) criterion to determine the risk-sensitive requirements. We highlight the different impact that these regulations have on the equilibrium market structure of our model, with especial emphasis on whether they will shift some types of lending from regulated banks into shadow banks or direct market finance.

3.1 Flat capital requirements

Suppose that regulated banks are required to fund at least a proportion $k$ of their lending with capital, independently of their type $p$. In this case, we show that when the requirement $k$ is low, safer entrepreneurs borrow from the market while riskier entrepreneurs borrow from regulated banks. However, when the requirement $k$ raises above a threshold the equilibrium of the model changes, with safer entrepreneurs borrowing from the market, medium risk entrepreneurs borrowing from shadow banks, and higher risk entrepreneurs borrowing from regulated banks.

To characterize the equilibrium under flat capital requirements, consider a bank lending to entrepreneurs of type $p \geq \hat{p}_0$, where $\hat{p}_0$ denotes the marginal type that is indifferent between market and bank finance under zero capital requirements and zero certification costs.\textsuperscript{18} Clearly, if the bank would like to have more capital than the minimum required by the regulation, that is if $k_p \geq k$, the capital requirement would not have any effect, with the bank keeping a capital buffer $k_p - k$. If $k_p$ is below but close to $k$, then complying with the regulation has low costs so these entrepreneurs are funded by regulated banks. But when this is not the case, complying with the regulation has high costs so these entrepreneurs shift to either market or shadow bank finance. Specifically, when the capital requirement $k$ is below a threshold $\hat{k}$, there is a marginal type $p_m$ that switches from market to regulated bank finance. And when $k$ is above the threshold $\hat{k}$, shadow banks can profitably enter the market, so there is a marginal type $p_s > p_m$ that switches from market to shadow bank finance and a marginal type $p_s > p_m$ that switches from shadow to regulated bank finance.

\textsuperscript{17}See Basel Committee on Banking Supervision (2015). Note that Basel III combines risk-sensitive capital requirements with a risk-insensitive leverage ratio.

\textsuperscript{18}By Proposition 1, $\hat{p}_0$ is given by (10) for $\eta = \eta_0 = 0$. 

\hspace{1cm}
Figure 2 illustrates the result for the case of a low minimum capital requirement ($\bar{k} \leq \hat{k}$). Panel A shows equilibrium bank capital. Two regions may be distinguished. To the left of the marginal type $p_m$ entrepreneurs borrow from the market. To the right of the marginal type $p_m$ entrepreneurs borrow from regulated banks, with the safer ones borrowing from banks for which the capital requirement is binding ($k_p = \bar{k}$) and the riskier ones borrowing from banks for which the capital requirement is not binding ($k_p > \bar{k}$). Panel B shows the corresponding probabilities of failure $p - s_p$, which jump down at $p_m$ because of the effect of
the binding capital requirements.

Figure 2 also illustrates the result for the case of a high minimum capital requirement ($k > \hat{k}$). Panel C shows equilibrium bank capital. Three regions may be distinguished. To the left of the marginal type $p_m$ entrepreneurs borrow from the market, between $p_m$ and $p_s$ entrepreneurs borrow from shadow banks, and to the right of the marginal type $p_s$ entrepreneurs borrow from regulated banks.\textsuperscript{19} Panel D shows the corresponding equilibrium probabilities of failure $p - s_p$, which bend down at $p_m$ where shadow banks start to operate, and jump down at $p_s$ because of the effect of the binding capital requirements.

Thus, although flat capital requirements reduce the probability of failure of relatively safe banks in the regulated banking system, this comes at the cost of pushing some entrepreneurs toward alternative sources of funding (market or shadow bank finance), which reduces screening and increases the probability of failure. With these requirements the equilibrium market structure of the financial system is such that regulated banks always fund the riskiest projects, while if shadow banks operate they fund projects that are ex-ante safer than those of the regulated banks (although not necessarily ex-post, given their different screening incentives).

### 3.2 VaR capital requirements

The risk-sensitive minimum capital requirements for credit risk of Basel II and Basel III are based on the criterion that capital should cover the losses of a sufficiently diversified loan portfolio with a confidence level $1 - \alpha = 0.999$ (99.9%). To translate this Value-at-Risk (VaR) criterion to our model setup, in which loan defaults are perfectly correlated, we define a capital requirement $\bar{k}_p$ such that the probability of default $p - s_p^*$ of the loans to entrepreneurs of type $p$ is equal to $\alpha$.

By Proposition 1, there is an equilibrium relationship between capital and screening given by (13). Solving for $s_p^*$ in the condition $p - s_p^* = \alpha$, and substituting it into (13), and setting

\textsuperscript{19}Notice that in this case $p_m$ coincides with the marginal type $\hat{p}_1$ that is indifferent between market and bank finance under zero capital requirements and positive certification costs. By Proposition 1, $\hat{p}_1$ is given by (10) for $\eta = \eta_1 > 0.$
the certification cost $\eta = 0$ then gives the model-equivalent of the Basel formula

$$\bar{k}_p = \begin{cases} 0, & \text{if } p \leq \alpha, \\ \frac{(1 - \alpha) c' (p - \alpha) - c(p - \alpha)}{R_0 + \delta}, & \text{otherwise.} \end{cases}$$

(15)

Notice that for $p > \alpha$ we have

$$\frac{d\bar{k}_p}{dp} = \frac{(1 - \alpha) c'' (p - \alpha) - c'(p - \alpha)}{R_0 + \delta}.$$ 

Thus, riskier banks will be required to have more capital if $(1 - \alpha) c'' (p - \alpha) - c'(p - \alpha) > 0$, which holds by the properties of the screening cost function.\textsuperscript{20}

As in the case of flat requirements, if the bank would like to have more capital than the minimum required by the regulation, that is if $k_p \geq \bar{k}_p$, the capital requirement would not have any effect, with the bank keeping a capital buffer $k_p - \bar{k}_p$. If $k_p$ is below but close to $\bar{k}_p$, then complying with the regulation has low costs so these entrepreneurs are funded by regulated banks. But when this is not the case, complying with the regulation has high costs so these entrepreneurs shift to shadow bank finance. Specifically, when the confidence level $1 - \alpha$ is below a threshold $1 - \bar{\alpha}$, there is a marginal type $p_m$ that switches from market to regulated bank finance. And when $1 - \alpha$ is above the threshold $1 - \bar{\alpha}$, shadow banks can profitably enter the market, so there is a marginal type $p_m$ that switches from market to regulated bank finance and a marginal type $p_s > p_m$ that switches from regulated to shadow bank finance. Thus, in contrast with the equilibrium under flat capital requirements, here if shadow banks operate they fund projects that are ex-ante (and ex-post) riskier than those of the regulated banks.

Figure 3 illustrates the result for the case of a low confidence level ($\alpha > \bar{\alpha}$). Panel A shows equilibrium bank capital. Two regions may be distinguished. To the left of the marginal type $p_m$ entrepreneurs borrow from the market.\textsuperscript{21} To the right of the marginal type $p_m$ entrepreneurs borrow from regulated banks, with the safer ones borrowing from banks for which the capital requirement is not binding ($k_p > \bar{k}$) and the riskier ones borrowing from

\textsuperscript{20}To see this notice that $(1 - \alpha) c'' (p - \alpha) - c'(p - \alpha) > (p - \alpha) c'' (p - \alpha) - c'(p - \alpha) \geq 0$. For the quadratic monitoring cost function (2) the condition simplifies to $\gamma (1 - p) > 0$.

\textsuperscript{21}Notice that $p_m$ coincides with the marginal type $\bar{p}_0$ that is indifferent between market and bank finance under zero capital requirements and zero certification costs.
This figure shows the equilibrium of the model with VaR capital requirements. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure with low VaR requirements. Panel C exhibits equilibrium capital and Panel D equilibrium probabilities of failure with high VaR requirements. Solid (dashed) lines represent equilibrium values with (without) VaR capital requirements.

banks for which the capital requirement is binding \((k_p = \bar{k})\). Panel B shows the corresponding probabilities of failure \(p - s_p\), which are equal to \(\alpha\) for high-risk banks.

Figure 3 also illustrates the result for the case of a high confidence level \(\alpha < \tilde{\alpha}\). Panel C shows equilibrium bank capital. Three regions may be distinguished. To the left of the marginal type \(p_m\) entrepreneurs borrow from the market, between \(p_m\) and \(p_s\) entrepreneurs borrow from regulated banks, and to the right of the marginal type \(p_s\) entrepreneurs borrow from shadow banks. Panel D shows the corresponding probabilities of failure \(p - s_p\), which
jump up at $p_s$ when lending switches to shadow banks.

Thus, although VaR capital requirements reduce the probability of failure of relatively risky banks in the regulated banking system, this comes at the cost of pushing the riskiest entrepreneurs to the shadow banking system, which reduces screening and increases the probability of failure. With these requirements the equilibrium market structure of the financial system is such that regulated banks always fund the medium risk projects, while if shadow banks operate they always fund the riskiest (ex-ante and ex-post) projects.

4 Optimal Capital Requirements

This section characterizes the second-best optimal capital requirements, that is those that maximize social welfare when the regulator is subject to the same informational frictions as banks. We show that optimal requirements are such that the regulator requires banks to have more capital than they would have in the absence of regulation. The reason is that, in the absence of regulation, bank competition reduces spreads to a level that, given the banks’ moral hazard problem, implies too little screening. By imposing capital requirements the regulator is able to increase screening in a way that compensates the higher cost of the additional equity capital.

We also show that the presence of direct market and shadow bank finance imposes a constraint to the regulator that leads to lower capital requirements in order to prevent shifting of some lending outside of the regulated banking system, where, as previously argued, there is either no screening (direct market finance) or too low screening (shadow bank finance).\footnote{Moreover, in the case of shadow bank finance higher (private and social) costs of certification are incurred.}

The implicit assumption is that the regulator cannot prevent the issuing of debt securities in the market or the activity of nonregulated financial intermediaries.

To derive the social welfare function, we first note that by the proof of Proposition 1 the shareholders’ participation constraint (8) and the investors’ participation constraint (9) are satisfied with equality, which means that they exactly receive the opportunity cost of their funds. Moreover, by the assumption of free entry of entrepreneurs, they will only be able to
borrow at a rate that leaves them no surplus.

Hence, social welfare reduces to the utility of the representative consumer. Substituting the budget constraint (4) into the utility function (3), and taking into account the fact that the goods of entrepreneurs of type $p$ are produced with probability $1 - p + s_p$, yields

$$W(x) = I + \frac{\sigma}{\sigma - 1} \int_0^1 (1 - p + s_p) (x_p)^{\frac{\sigma - 1}{\sigma}} dp - \int_0^1 (1 - p + s_p) A_p x_p dp,$$

where $x = \{x_p\}$ denotes an investment allocation. Substituting the first-order condition (5) into this expression then gives the social welfare function

$$W(x) = I + \frac{1}{\sigma - 1} \int_0^1 (1 - p + s_p) (x_p)^{\frac{\sigma - 1}{\sigma}} dp. \quad (16)$$

By our previous results, any minimum capital requirement $\bar{k}_p$ for banks’ lending to entrepreneurs of type $p$ implies a corresponding equilibrium loan rate $R_p$. Since entrepreneurs of type $p$ will enter the market until $R_p = A(x_p) = (x_p)^{-1/\sigma}$, the equilibrium aggregate investment $x_p$ of these entrepreneurs will be

$$x_p = (R_p)^{-\sigma}. \quad (17)$$

Hence, we can compute the social welfare $W(x)$ associated with any bank capital regulation $\bar{k} = \{\bar{k}_p\}$.

Optimal capital requirements are defined by

$$\bar{k}^* = \arg\max_{\bar{k}} W(x(\bar{k})),$$

where $x(\bar{k})$ denotes the equilibrium investment allocation corresponding to the regulation $\bar{k}$.

Figure 4 shows the optimal capital requirements, together with the corresponding probabilities of failure $p - s_p$. The dashed line in Panel A shows the capital that banks would have in the absence of regulation, and in Panel B the corresponding probabilities of failure. The dotted lines in Panel A show the optimal unconstrained capital requirements (i.e. in the absence of direct market or shadow bank finance), and in Panel B the corresponding probabilities of failure. Finally, the solid lines in Panel A represent the optimal capital requirements in the presence of direct market finance, which reduce the feasible requirements
Figure 4. Optimal capital requirements

This figure shows the equilibrium of the model with optimal capital requirements. In Panel A the dashed line exhibits the capital that banks would have in the absence of regulation, the dotted line the optimal unconstrained capital requirements, and the solid line the optimal capital requirements in the presence of direct market finance and shadow banks. Panel B exhibits the corresponding equilibrium probabilities of failure.

For lending to intermediate risk entrepreneurs, and shadow banks, which reduce the feasible requirements for lending to high risk entrepreneurs. The solid lines in Panel B show the corresponding probabilities of failure, which are lower than in the absence of regulation, but higher for intermediate and high risk loans in the presence of direct market and shadow bank finance.

It should be noted that optimal (constrained and unconstrained) capital requirements are risk-sensitive, and higher in general than the capital that banks would have in the absence of regulation. However, they are not VaR requirements, since the probability of failure of the regulated banks is not constant. In fact, the implicit confidence level goes down for riskier loans.\(^{23}\)

Finally, we would like to address the question of why is regulation welfare improving? Specifically, we show that the welfare associated to lending to any particular type \(p\) of entrepreneur, evaluated at the solution to the bank’s problem in the absence of regulation,

\(^{23}\)Note that the effect of a lower confidence level goes in the same direction as the reduction in the correlation parameter \(\rho\) for riskier loans in the Basel II and III capital charge formula.
is increasing in \( k_p \), that is
\[
\frac{d}{dk_p} \left[ \frac{1}{\sigma - 1} (1 - p + s_p) (x_p)^{\frac{\sigma - 1}{\sigma}} \right] > 0.
\]

To prove this, note that equation (13), which is derived from the shareholders’ participation constraint (8) and first-order condition (12), implies
\[
\frac{ds_p}{dk_p} = \frac{R_0 + \delta + \eta}{(1 - p + s_p)c''(s_p)} > 0,
\]
so higher capital implies higher screening. Also, note that differentiating equation (14), which is derived from the investors’ participation constraint (9) and the shareholders’ first-order condition (12), and using (18) gives
\[
\frac{dR_p}{dk_p} = \left[ - \frac{(1 - k_p)R_0}{(1 - p + s_p)^2} + c''(s_p) \right] \frac{ds_p}{dk_p} - \frac{R_0}{1 - p + s_p} \left[ - \frac{(1 - k_p)R_0}{(1 - p + s_p)^2} + \frac{(\delta + \eta)c''(s_p)}{R_0 + \delta + \eta} \right] \frac{ds_p}{dk_p}.
\]

By the proof of Proposition 1, the last term in brackets, evaluated at the solution to the bank’s problem in the absence of regulation, is equal to zero; see equation (22) in Appendix B. Since \( x_p \) is decreasing in \( R_p \), and we have just shown that \( R_p \) does not vary with \( k_p \), we conclude that
\[
\frac{d}{dk_p} \left[ \frac{1}{\sigma - 1} (1 - p + s_p) (x_p)^{\frac{\sigma - 1}{\sigma}} \right] = \frac{1}{\sigma - 1} (x_p)^{\frac{\sigma - 1}{\sigma}} \frac{ds_p}{dk_p} > 0.
\]
Thus, requiring banks to have more capital than they would otherwise choose to have is welfare improving.

The intuition for this result is that the assumption that the loan market is contestable leads to low equilibrium interest rates and spreads that, given the banks’ moral hazard problem, induce too little screening. In this context, minimum capital requirements (which will be binding) induce higher screening through two channels, namely a direct channel linked to the positive incentive (or “skin in the game”) effect of capital, and an indirect effect linked to the higher spreads that are required to compensate shareholders for their additional contribution to funding the bank. Obviously, the higher cost of equity acts as a drag on welfare, so there is a point beyond which further raising the capital requirements would be welfare decreasing. In other words, and as shown in Figure 4, a positive level of leverage maximizes social welfare.
5 Extensions

This section discusses how the equilibrium structure and risk of the financial system evolves when (i) there are exogenous changes in the safe rate or in the excess cost of capital, and (ii) the excess cost of capital is endogenously determined in equilibrium. The first extension shows that the regulated banking sector will shrink when the safe rate is low and the excess cost of bank capital is high. The second extension shows that bank capital regulation affects all financial intermediaries in the economy (both regulated and shadow banks) through its impact on the equilibrium cost of capital.

5.1 Changes in funding costs

We consider the effects of changing two key parameters of the model, namely the expected return required by investors $R_0$ (the safe rate) and the excess cost of capital $\delta$, which may be linked, respectively, to the scarcity of debtholders’ and shareholders’ wealth. The results illustrate the implications of the model for the structure and risk of the financial system along the business cycle, as funding costs are a key variable that evolves with the cycle.

We first use the result in Proposition 1 to show the effects of changes in $R_0$ and $\delta$ on the marginal type of entrepreneurs $\hat{\rho}$ that is indifferent between market and bank finance under zero capital requirements. Differentiating (10) it is immediate to show that $\hat{\rho}$ is decreasing in the safe rate $R_0$ and increasing in the excess cost of bank capital $\delta$. Hence, in the absence of regulation a decrease (increase) in the the safe rate $R_0$ (the excess cost of bank capital $\delta$) results in an expansion of the range of entrepreneurs funded by the market.

We next analyze how the equilibrium structure of the financial system varies with funding costs in the presence of capital regulation. We show that for both flat and VaR capital requirements a lower safe rate $R_0$ and a higher excess cost of capital $\delta$ expands the range of entrepreneurs financed by markets and shadow banks, and reduce range of entrepreneurs financed by regulated banks. According to these findings, our model predicts that, in the presence of capital requirements of either type, the regulated banking sector will shrink and the unregulated sector (markets and shadow banks) will expand when the safe rate is low.
and the excess cost of bank capital is high.

The top panels of Figure 5 illustrate the effect of an increase in the safe rate $R_0$ (from solid to dashed lines) and in the excess cost of capital $\delta$ (from solid to dotted lines) under sufficiently high flat capital requirements, so shadow banks fund intermediate risk entrepreneurs. Panel A shows that for those (regulated and shadow) banks for which capital regulation is not binding a higher safe rate $R_0$ increases their capital, while for those banks for which the regulation is binding capital remains unchanged. The intuition is that the increase in the safe rate makes debt finance relatively more expensive than equity finance. Panel B shows that an increase in the safe rate results in a lower probability of failure of all banks. For those banks for which the capital requirement is binding, the effect is explained by the increase in loan rates and spreads, which increases screening incentives.

Panel A of Figure 5 also shows that for those (regulated and shadow) banks for which capital regulation is not binding a higher excess cost of bank capital $\delta$ reduces their capital, while for those banks for which the regulation is binding capital remains unchanged. Panel B shows the differential effects on probabilities of default: entrepreneurs that move out of the regulated banking system have higher probabilities of default, those funded by regulated banks with zero capital buffers have lower probabilities of default, due to the higher screening incentives associated with higher loan rates, and those funded by (regulated and shadow) banks with capital buffers have higher probabilities of default, due to the lower screening incentives associated with lower capital. Hence, a change in the excess cost of capital has a differential impact on the risk of financial institutions, reducing (increasing) the probability of failure of those for which the regulation is (not) binding.

The bottom panels of Figure 5 illustrate the effect of an increase in the safe rate $R_0$ (from solid to dashed lines) and in the excess cost of capital $\delta$ (from solid to dotted lines) under sufficiently high VaR capital requirements, so shadow banks fund high risk entrepreneurs. As in the case of flat requirements, an increase in the safe rate $R_0$ results in higher incentives to raise capital. Panel C shows that a higher safe rate increases the capital of (regulated and shadow) banks for which regulation is not binding, but reduce it for those for which it is binding. This is due to the fact that higher funding costs translate into higher loan rates.
Figure 5. Changes in funding costs with flat and VaR capital requirements

This figure shows the effect of changes in funding costs. Panels A and C exhibit equilibrium capital with, respectively, high flat and VaR capital requirements. Panels B and D exhibit the corresponding equilibrium probabilities of failure. Dotted (dashed) lines represent equilibrium values with a higher excess cost of capital (safe rate). Solid lines represent the initial equilibrium.

and hence higher incentives to screen borrowers, which under a VaR regulation reduces capital requirements.\textsuperscript{24} Panel D shows that an increase in the safe rate results in lower probabilities of failure for those banks for which the regulation is not binding, but have no effect for those for which the regulation is binding.\textsuperscript{25}

\textsuperscript{24}Notice that $k_p$ in (15) is decreasing in $R_0$.
\textsuperscript{25}Notice that when the regulation is binding the probability of failure is, by construction, equal to $\alpha$. 

26
This figure shows the effect of changes in funding costs with optimal capital requirements in the presence of direct market finance and shadow banks. Panel A exhibits optimal capital and Panel B the corresponding equilibrium probabilities of failure. Dotted (dashed) lines represent equilibrium values with a higher excess cost of capital (safe rate). Solid lines represent the initial equilibrium.

Panel C of Figure 5 also shows that a higher excess cost of capital reduces the capital of (regulated and shadow) banks for which regulation is not binding, as well as for those for which it is binding. As before, this is due to the fact that higher funding costs translate into higher loan rates and hence higher incentives to screen borrowers, which under a VaR regulation reduces capital requirements.\textsuperscript{26} Panel D shows that an increase in the excess cost of bank capital results in higher probabilities of failure for those banks for which the regulation is not binding, but as before has no effect for those for which the regulation is binding.

Finally, we analyze how the optimal capital requirements characterized in Section 4 vary with the safe rate $R_0$ and the excess cost of capital $\delta$. Figure 6 shows the results. An increase in the safe rate leads to higher optimal capital requirements, an expansion of the range of intermediated finance, and consequently a safer financial system. This effect is explained by the fact that the increase in $R_0$, for a given $\delta$, reduces the relative cost of bank capital. Not surprisingly, an increase in the excess cost of capital leads to the opposite effects, reducing optimal capital requirements and the range of intermediated finance, and increasing the risk of the financial system.\textsuperscript{27}

\textsuperscript{26}Notice that $k_p$ in (15) is decreasing in $\delta$.

\textsuperscript{27}These results provide a rationale for the cyclical adjustment of capital requirements. In particular, when
5.2 Endogenous cost of capital

We next consider the implications of replacing an exogenous by an endogenous excess cost of capital \( \delta \), determined by the intersection of an exogenous supply of bank capital \( K \) with a downward sloping demand for bank capital \( K(\delta) \), derived from our previous analysis. In this case, bank capital regulation affects all financial intermediaries in the economy and not only the ones directly affected by the regulation. More specifically, entrepreneurs funded by regulated banks with capital buffers and entrepreneurs funded by shadow banks are also affected by changes in capital requirements through its impact on the equilibrium cost of capital. Thus, we identify spillover effects of regulation across the whole spectrum of regulated and shadow banks, even in the absence of direct linkages between them.

Following Holmström and Tirole (1997), suppose that there is fixed aggregate supply of bank equity capital \( \bar{K} \).\(^{28}\) As explained in Section 3, for any given bank capital regulation \( \bar{k}_p \) and any given excess cost of capital \( \delta \) there is a corresponding equilibrium investment \( x^*_p \) and capital \( k^*_p \) for regulated and shadow banks, so the aggregate demand for bank capital is given by

\[
K(\delta) = \int_0^\bar{p} x^*_p k^*_p \, dp.
\]

The equilibrium capital \( k^*_p \) for banks lending to entrepreneurs of type \( p \) is constrained to be at least \( \bar{k}_p \) for regulated banks, is unconstrained (and lower than \( \bar{k}_p \)) for shadow banks, and it is equal to zero in the case of direct market finance. By our previous results, an increase in the cost of capital \( \delta \) expands the market finance and the shadow banking regions. Moreover, shadow banks and regulated banks with capital buffers will reduce their capital. Finally, the higher cost of capital for both regulated and shadow banks will translate into higher loan rates, and consequently by (17) lower aggregate investment. All in all, it follows that an increase in the cost of capital \( \delta \) will reduce the aggregate demand for bank capital \( K(\delta) \).

The equilibrium cost of bank capital \( \delta^* \) is then obtained by solving the equation \( K(\delta) = \bar{K} \).

A tightening of (either flat or VaR) capital requirements that produces an upward shift...
in the demand for bank capital leads to an increase in the equilibrium cost of capital $\delta^*$. While regulated banks for which the regulation is binding will be safer, regulated banks with buffers and shadow banks will have an incentive to save on costly capital, and hence they will be riskier. The indirect effect of tightening capital requirements through the equilibrium cost of capital may lead to overall increase in the risk of the financial system. Thus, it is not only the case that, as noted by Hanson et al. (2011), high requirements will drive a larger share of intermediation into the shadow banking realm (with lower capital and higher risk-taking), it is also the case that some regulated banks (those for which the regulation is not binding) and all shadow banks can become riskier.

Summing up, when the cost of capital is endogenous, tightening bank capital regulation can have a negative effect on the risk-taking behavior of (regulated and shadow) banks that are not directly constrained by the regulation. This effect is a novel source of risk that should be taken into account when analyzing the costs and benefits of bank capital regulation.

6 Concluding Remarks

This paper presents a model that studies the effects of bank capital regulation on the structure and risk of the financial system where direct market finance, regulated banks, and shadow banks coexist. The model builds on the idea that financial intermediaries can reduce the probability of default of their loans by screening their borrowers at a cost. We assume that screening is not observed by debtholders, and therefore there is a moral hazard problem. Intermediaries may be willing to use (more expensive) equity finance in order to ameliorate the moral hazard problem and reduce the cost of debt. One of the novelties in the paper is that we assume that for this channel to operate, the capital structure has to be certified by an external (public or private) agent. Public certification is done by a bank supervisor that verifies whether those intermediaries that choose to be regulated (called regulated banks) comply with the regulation. Intermediaries that do not comply with the regulation (called shadow banks) have to resort to more expensive private certification.

We consider two different types of regulation, namely risk-insensitive (or flat) and risk-
sensitive (Value-at-Risk based) capital requirements, which broadly correspond to, respectively, the Basel I and the Basel II and III Accords of the Basel Committee on Banking Supervision. We show that regardless of the risk-sensitivity of the capital requirements, different types of financing can coexist. In particular, safer projects are always funded by the market, while riskier projects are funded by intermediaries. Depending on the risk-sensitivity of the requirements two different market structures can emerge. With flat requirements the equilibrium market structure is such that regulated banks always fund the riskiest projects, while if shadow banks operate they fund projects that are safer than those of the regulated banks. With Value-at-Risk requirements the equilibrium market structure is such that regulated banks always fund the intermediate risk projects, while if shadow banks operate they fund the riskiest projects.\(^{29}\)

We also examine an alternative to the certification model, in which the advantage of regulated banks relative to shadow banks comes from the existence of underpriced deposit insurance. Although the main results remain unchanged, there are some interesting differences. In particular, in the model with deposit insurance regulated banks never want to have capital buffers.

Our results imply that reducing the gap between the costs of private and public certification, say by charging banks for the cost of bank supervision\(^{30}\) or by increasing deposit insurance premia, would lead to an expansion of the shadow banking system.

The paper also contains a characterization of optimal capital requirements, which are less risk-sensitive than the those based on a Value-at-Risk criterion à la Basel II and III. It also discusses what happens when there are exogenous changes in the safe rate or in the cost of bank capital, showing that the regulated banking sector will shrink and the unregulated sector will expand when the safe rate is low and the excess cost of bank capital is high.

\(^{29}\)Our setup can also serve to analyze a situation in which regulated banks can choose between standardized (less risk-sensitive) and VaR (more risk-sensitive) capital requirements, as well as a situation in which banks are subject to both a (risk-insensitive) leverage ratio and (risk-sensitive) VaR requirements.

\(^{30}\)In the US, neither the Federal Reserve System nor the Federal Deposit Insurance Corporation (FDIC) charged their supervised banks the cost of supervisory oversight. The Fed’s supervision expenses are funded with the revenue generated from monetary policy operations, while the FDIC allocates a portion of deposit insurance premia for operations, including supervision. In contrast, the European Central Bank charges supervisory fees that amounted to €425 million in 2017.
Finally, it analyzes what happens when we endogenize the cost of capital, showing that in this case a tightening of capital requirements has a negative effect on the risk-taking behavior of (regulated and shadow) banks that are not directly constrained by the regulation, via the higher cost of bank capital.

We would like to conclude with a few remarks. First, we have assumed that screening reduces the loans’ probability of default, but we could also consider other effects on the quality of the pool of loan applicants, say reducing the loss given default. Second, a thorough discussion of the financial stability implications of our results would require introducing a more realistic correlation structure of project returns within and across types of entrepreneurs. Third, although the model is set in terms of entrepreneurial finance it could also be interpreted in terms of household finance, with different types corresponding to borrowers with, for example, different loan-to-values.
Appendix

A Deposit Insurance

This Appendix shows that our main qualitative results remain unchanged when we replace the assumption that private certification of capital (of shadow banks) is costlier than public certification (of regulated banks) by the assumption that regulated banks (but not shadow banks) are able to raise insured deposits with an underpriced deposit insurance premium that is normalized to zero. In our original setup, shadow banks enter the market when the higher cost of resorting to private certification is compensated by the lower cost of not complying with the regulation. In this setup, shadow banks enter the market when the higher cost of uninsured deposits is compensated by the lower cost of not complying with the regulation.

Clearly, the equilibrium loan rate $R^*_p$ for entrepreneurs of type $p$ under direct market finance will be the rate that satisfies the participation constraint (6). Similarly, the equilibrium loan rate $R^*_p$ for entrepreneurs of type $p$ under shadow bank finance will be the minimum rate that satisfies the bank’s incentive compatibility constraint (7), the shareholders’ participation constraint (8), and the investors’ participation constraint (9) for a certification cost $\eta = 0$. So the only loan rate that needs to be determined is the one corresponding to the regulated banks.

One important difference with the model with certification costs is that with underpriced deposit insurance the capital constraint for the regulated banks is always binding. To see this, notice that with deposit insurance the investors’ participation constraint (9) becomes $B^*_p = R_0$. Substituting this result into the first-order condition (12) gives

$$R^*_p - (1 - k^*_p)R_0 = c'(s^*_p).$$  \hspace{1cm} (19)

Substituting this expression into the shareholders’ participation constraint gives

$$(1 - p + s^*_p)c'(s^*_p) - c(s^*_p) = (R_0 + \delta)k^*_p.$$  \hspace{1cm} (20)

Differentiating this expression we get

$$\frac{ds^*_p}{dk^*_p} = \frac{R_0 + \delta}{(1 - p + s^*_p)c''(s^*_p)} > 0.$$
Finally, differentiating (19) we conclude
\[
\frac{dR_p^*}{dk_p^*} = -R_0 + c''(s_p^*) \frac{ds_p^*}{dk_p^*} = \frac{R_0(p - s_p^*) + \delta}{1 - p + s_p^*} > 0.
\]
Since \(R_p^*\) is increasing in \(k_p^*\), contestability implies that regulated banks will choose the lowest possible capital that complies with the regulation, that is \(k_p^* = \bar{k}_p\). The intuition is that with deposit insurance regulated banks have no incentive to have capital buffers, since they have no effect on their borrowing costs.

Substituting \(k_p^* = \bar{k}_p\) into (20) and solving for \(s_p^*\) gives the equilibrium screening intensity of regulated banks lending to entrepreneurs of type \(p\), and from here (19) gives the equilibrium loan rate \(R_p^*\).

In the case of a flat capital requirement we would have that \(\bar{k}_p = \bar{k}\) for all \(p\), while in the case of a VaR requirement \(\bar{k}_p\) would be given by (15). Figure 7 shows the equilibrium structure and the risk of the financial system under flat and VaR capital requirements. We focus on the case of sufficiently high requirements, so shadow banks operate. As in the certification model, high flat (VaR) requirements move intermediate (high) risk entrepreneurs to shadow banks. In contrast to the certification model, the probability of default of entrepreneurs funded by regulated banks need not be lower than the one that would obtain in a laissez-faire economy (with no capital regulation and no deposit insurance), since without deposit insurance banks could choose to have a higher level of capital than the one required by the regulation.

Thus, our qualitative predictions regarding the emergence of shadow banks when capital requirements tighten are robust to changing the nature of the positive effects of being subject to the regulation, from lower certification costs to lower cost of deposit funding via deposit insurance. In both cases the nature of the shadow banks that appear depend on the form of regulation, with flat (VaR) requirements inducing the entry in the financial system of intermediate (high) risk shadow banks.
This figure shows the equilibrium of the model with deposit insurance. Panel A exhibits equilibrium capital and Panel B equilibrium probabilities of failure with high flat requirements. Panel C exhibits equilibrium capital and Panel D equilibrium probabilities of failure with high VaR requirements. Solid (dashed) lines represent equilibrium values with (without) capital requirements.
B Proofs

Proof of Proposition 1 Suppose that the equilibrium screening intensity \( s_p^* \) satisfies \( s_p^* \in (0, p) \). Then, by the convexity of the screening cost function \( c(s_p) \), the bank’s incentive compatibility constraint (7) reduces to the first-order condition (12).

To show that in this case the investors’ participation constraint (9) is binding, note that if it were not we could slightly reduce the borrowing rate \( B_p^* \) and the loan rate \( R_p^* \) so that (12) would hold for the same \( s_p^* \); in which case the shareholders’ participation constraint (8) would still be satisfied, which contradicts the definition of equilibrium. To show that the shareholders’ participation constraint (8) is also binding, note that if it were not we could slightly increase the bank’s capital \( k_p^* \) and reduce the loan rate \( R_p^* \) so that (12) would hold for the same \( s_p^* \); in which case the investors’ participation constraint (9) would still be satisfied, which contradicts the definition of equilibrium.

Solving for \( R_p^* - (1 - k_p^*)B_p^* \) in the shareholders’ participation constraint (8) (written as an equality), substituting it into the first-order condition (12), and solving for \( k_p^* \) gives (13). And solving for \( B_p^* \) in the investors’ participation constraint (9) (written as an equality), substituting it into the first-order condition (12), and rearranging gives (14).

The equilibrium loan rate \( R_p^* \) is then given by

\[
R_p^* = \min_{s_p, k_p} \left[ \frac{(1 - k_p)R_0}{1 - p + s_p} + c'(s_p) \right]
\]

subject to (13). The first-order condition that characterizes the solution to this problem is

\[
\frac{dR_p^*}{ds_p^*} = - \frac{(1 - k_p^*)R_0}{(1 - p + s_p)^2} + \frac{\delta + \eta)c''(s_p^*)}{R_0 + \delta + \eta} = 0.
\]

The second-order condition is

\[
\frac{d^2R_p^*}{ds_p^*} = \frac{2(1 - k_p^*)R_0}{(1 - p + s_p)^3} + \frac{(\delta + \eta)c'''(s_p^*)}{R_0 + \delta + \eta} + \frac{R_0c''(s_p^*)}{(R_0 + \delta + \eta)(1 - p + s_p^*)} > 0,
\]

which holds by our assumptions on the screening cost function \( c(s_p) \).

The first-order condition (22) implies

\[
\frac{dR_p^*}{ds_p^*} \bigg|_{s_p = k_p = 0} = - \frac{R_0}{(1 - p)^2} + \frac{(\delta + \eta)c''(0)}{R_0 + \delta + \eta} < 0
\]
if and only if \( p > \hat{p} \), where \( \hat{p} \) is defined in (10). From here it follows that riskier entrepreneurs of types \( p > \hat{p} \) will borrow from banks, while safer entrepreneurs of types \( p \leq \hat{p} \) will borrow from the market. Note that \( \hat{p} < 1 \), and since \( \hat{p} \) is increasing in \( \eta \) assumption (11) ensures that \( \hat{p} > 0 \).

It only remains to show that \( s^*_p < p \) for \( p > \hat{p} \). Since \( s^*_p = 0 \) for \( p = \hat{p} \), it suffices to show that \( ds^*_p/dp \leq 1 \) for \( p > \hat{p} \). Differentiating the first-order condition (22), and taking into account (13), gives

\[
\frac{ds^*_p}{dp} = \frac{2(1-k^*_p)R_0}{(1-p+s^*_p)^3} + \frac{R_0 c'(s^*_p)}{(R_0+\delta+\eta)(1-p+s^*_p)^2}.
\]

Since \( c''(s^*_p) \geq 0 \), \( ds^*_p/dp \leq 1 \) if

\[
\frac{R_0 c'(s^*_p)}{(R_0+\delta+\eta)(1-p+s^*_p)^2} \leq \frac{R_0 c''(s^*_p)}{(R_0+\delta+\eta)(1-p+s^*_p)},
\]

which simplifies to \( c'(s^*_p) \leq (1-p+s^*_p)c''(s^*_p) \), which holds by the properties of the screening cost function \( c(s_p) \).

**Proof of Proposition 2** Let \( \tilde{s}_p \) denote the equilibrium screening intensity of a bank lending to entrepreneurs of type \( p \) that does not certify its capital, and assume that \( \tilde{s}_p \in (0, p) \). Then, by the convexity of the screening cost function \( c(s_p) \), the bank’s incentive compatibility constraint reduces to the first-order condition

\[
\tilde{R}_p - \tilde{B}_p = c'(\tilde{s}_p).
\]

By the same argument as in the proof of Proposition 1, the equilibrium loan rate \( \tilde{R}_p \) is given by

\[
\tilde{R}_p = \min_{s_p} \left[ \frac{R_0}{1-p+s_p} + c'(s_p) \right].
\]

Comparing this expression with (21), it follows that \( \tilde{R}_p > R^*_p \) for all \( p \) for which \( k^*_p > 0 \), that is for all \( p > \hat{p} \).

The first-order condition that characterizes the solution to this problem is

\[
\frac{d\tilde{R}_p}{d\tilde{s}_p} = -\frac{R_0}{(1-p+\tilde{s}_p)^2} + c''(\tilde{s}_p) = 0.
\]

36
Hence, we have
\[ \frac{d\tilde{R}_p}{ds_p} \bigg|_{s_p=0} = -\frac{R_0}{(1-p)^2} + c''(0) < 0 \]
if and only if
\[ p > \tilde{p} = 1 - \sqrt{\frac{R_0}{c''(0)}}. \]
But since \( \tilde{p} > \hat{p} \), where \( \hat{p} \) is defined in (10), we conclude that noncertifying banks that choose a positive screening intensity will only lend to entrepreneurs of types for which certifying banks will undercut them, which proves the result. \( \square \)

**Proof of Proposition 3** Differentiating (10) with respect to \( \eta \) implies \( d\tilde{p}/d\eta > 0 \). Differentiating the first-order condition (22), and taking into account (13), gives
\[ \frac{\partial s_p^*}{\partial \eta} = \frac{R_0}{(R_0 + \delta + \eta)^2(1-p+s_p^*)^2} \left[ (1 - p + s_p^*)c'(s_p^*) - c(s_p^*) - (1 - p + s_p^*)^2 c''(s_p^*) \right] \]
\[ \times \frac{2(1-k_p^*)R_0}{(1-p+s_p^*)^3} + \frac{(\delta + \eta)c''(s_p^*)}{R_0 + \delta + \eta} + \frac{R_0c''(s_p^*)}{(R_0 + \delta + \eta)(1-p+s_p^*)}. \]
Since the denominator of this expression is positive by the second-order condition (23), the result follows from the properties of the screening cost function \( c(s_p) \), which imply \( c'(s_p^*) < (1 - p + s_p^*)c''(s_p^*) \). Finally, differentiating (13) gives
\[ \frac{\partial k_p^*}{\partial \eta} = -\frac{(1 - p + s_p^*)c'(s_p^*) - c(s_p^*)}{(R_0 + \delta + \eta)^2} + \frac{(1 - p + s_p^*)c''(s_p^*)}{R_0 + \delta + \eta} \frac{\partial s_p^*}{\partial \eta} < 0, \]
by the properties of the screening cost function \( c(s_p) \) and our previous result. \( \square \)
References


