Heterogeneous Innovations, Firm Creation and Destruction, and Asset Prices[*]

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Abstract

We study the implications of the creative destruction lifecycle of innovation for asset prices. We develop a general equilibrium model of endogenous firm creation and destruction where "incremental" innovations by incumbents and "radical" innovations by entrants drive productivity improvements. Micro-founded incentives of firms to innovate lead to the joint equilibrium determination of time-varying economic growth and countercyclical economic uncertainty. The model matches well the properties of consumption and asset prices in the data as well as novel stylized facts on the rate of radical innovation in the U.S. economy that we document from a comprehensive sample of patents over the 1975–2013 period. These findings show that the interplay between incumbent and entrants that is at the core of the creative destruction process, through its effect on the fluctuations of long-run growth and economic uncertainty, is an important determinant of risk that is priced in financial markets.

1 Introduction

Sustained economic growth occurs through the interplay of innovations by established companies and entering entrepreneurs in the process of creative destruction. A key feature of creative destruction is the heterogeneity of innovation efforts. Established companies aim to grow their value by improving their goods and production methods, while entrepreneurs strive to invent new goods and production methods destroying the value of established companies.[1] This dynamic process brings about new goods and markets, changes in production processes, productivity improvements, as well as volatility of firms' valuations. Through the randomness of successes and failures in innovation, creative destruction leads to the randomness of economic change. Since the variations in the growth prospects of the economy and the fluctuations of economic uncertainty are important for explaining the level and time-variation of risk premia,[2] studying creative destruction can help us understand the properties of asset prices.

In this paper we develop a dynamic stochastic general equilibrium model of endogenous firm creation and destruction, where existing firms (incumbents) enjoy monopoly profits, but face the threat of being displaced by new firms (entrants). Both incumbents and entrants invest in R&D, but their innovations are heterogenous. Incumbents' R&D results in incremental improvements of their existing products and higher profits. Entrants undertake R&D in order to create radically better products, displace incumbents, and capture their profits. In the competitive equilibrium of this economy, R&D investments of incumbents and entrants endogenously drive a small, persistent component in productivity which generates long-run uncertainty about economic growth. We use this model to analyze the implications of the presence of heterogeneous innovations by incumbents and entrants for aggregate asset prices.

At the core of our model is the mechanism through which R&D investments of incumbents and entrants are jointly determined in equilibrium. When responding to aggregate productivity shocks, both R&D expenditures adjust in the direction of the shock and therefore incumbents' and entrants' R&D expenditures are complementary in equilibrium. The magnitudes of R&D adjustments in response to shocks depend on the efficiency with which entrants' and incum-

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[1] Schumpeter (1934, 1942) emphasizes the importance of both creative destruction by new firms and innovations by large firms for economic growth. When describing the nature of the technological innovation process, Scherer (1984) and Freeman and Soete (1997) highlight the importance of new ventures for infrequent major advances in science and technology as well as the dominance of large firms in commercialization and continued development. See also Anderson and Tushman (1990) and Pennings and Buitendam (1987).

[2] See, for example, Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2010), and Ai and Kiku (2013).

bents' R&D expenditures are converted into innovation success. In equilibrium, the household allocates total R&D expenditure between incumbents and entrants so that the benefits of the marginal dollar spent in R&D is the same. The ability to invest in two technologies with heterogenous characteristics affects the volatility of aggregate R&D expenditures and therefore that of expected consumption growth.

We first show that, despite featuring a single aggregate productivity shock, our model endogenously generates time variation in both the level and conditional volatility of expected consumption growth. The presence of heterogeneous innovations by incumbents and entrants has two contrasting effects on the overall level of economic uncertainty in the economy. On the one hand, the complementarity of incumbents' and entrants' R&D expenditures induces a positive correlation between incumbents' and entrants' successes and failures in innovation, which increases economic uncertainty. On the other hand, the heterogeneity in incumbents' and entrants' R&D technologies allows the household to achieve better consumption smoothing in the presence of aggregate productivity shocks, which decreases economic uncertainty. Our numerical calibrations show that this dual role of innovation heterogeneity is quantitatively important. The overall level of economic uncertainty in our heterogeneous innovations economy is higher than in an analogous economy in which only entrants innovate, but it is lower than in an analogous economy in which only incumbents innovate. The differences in economic uncertainty across these economies lead to corresponding differences in the levels of consumption risk premia.

Furthermore, when R&D technologies exhibit decreasing marginal productivity, we show that the conditional volatility of expected consumption growth is higher in recessions and lower in expansions. The fluctuations in economic uncertainty are therefore countercyclical. Since time-variations in expected growth and fluctuating economic uncertainty are important determinants of asset prices (Bansal and Yaron (2004)), our structural model thus allows to study the relationship between the process of creative destruction in the economy and asset prices.

Next, we explore the ability of our model to match novel stylized facts on the process of creative destruction in the U.S. economy as well as conventional empirical evidence on long-run economic growth and aggregate asset returns. We collect data on the universe of patents applied for at the United States Patent and Trademark Office (USPTO) in 1975–2013. Using this data, we construct an empirical measure of the relative importance of entrants' radical innovations

in the economy—the rate of radical innovation—as well as proxies for the innovation intensities of incumbents and entrants. In the sample period we consider, we find that the rate of radical innovation is about 11% on average, suggesting that incumbents account for the majority of technological innovations, as measured by patents, and that their contribution to productivity growth is large.[3] We use the proxies for the innovation intensities of incumbents and entrants and the structural equation for equilibrium productivity growth from our model to estimate the magnitudes of incumbents' and entrants' innovations. Consistent with Akcigit and Kerr (2010), who find that large firms engage more in exploitative R&D while small firms pursue exploratory R&D, we show that, on average, the size of technological innovations by entrants is about twice as big compared to incumbents. When calibrated to match statistics on the rate of radical innovation, the innovation intensities of incumbents and entrants, the magnitudes of incumbents' and entrants' innovations, and long-run economic growth, our model can generate an equity premium as in the data and a low and stable risk-free interest rate.

Last, we generalize our model by introducing an exogenous shock to the barriers to entry in innovation. We show that such an extension, while preserving the ability of our baseline model in matching asset pricing moments, is also able to account for the time-series variation and autocorrelation of the rate of radical innovation and of incumbents' and entrants' innovation intensities observed in the data.

Our paper fits into a growing literature that studies the asset pricing implications of technological innovation. Using a model with heterogeneous firms, households, and imperfect risk sharing, Kogan, Papanikolaou, and Stoffman (2013) show that technological innovations embodied in new capital displace existing firms and thus benefit new cohorts of shareholders at the expense of existing ones. Gârleanu, Kogan, and Panageas (2012) argue that innovation introduces an unhedgeable displacement risk due to lack of intergenerational risk sharing. Gârleanu, Panageas, and Yu (2012) examine how infrequent technological shocks embodied in new capital vintages can explain excess return predictability and other stylized cross-sectional return patterns. Pástor and Veronesi (2009) show how technology adoption can explain the rise of stock

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[3]According to U.S. Manufacturing Census data in recent years, annual product creation, by existing firms and new firms, accounts for 9.3 percent of output, and the lost value from product destruction, by existing and exiting firms, accounts for 8.8 percent of output. About 70 percent of product creation and destruction occurs within existing firms (see Bilbiie, Ghironi, and Melitz (2012), Bernard, Redding, and Schott (2010), and Broda and Weinstein (2010)). Bartelsman and Doms (2000) and Foster, Haltiwanger, and Krizan (2001) show that most total factor productivity growth comes from existing as opposed to new establishments.

price bubbles. Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) link the stock market drop in the 1970s and its rebound in the 1980s to the information technology revolution.

We differ from this literature by embedding aggregate risk into a canonical Schumpeterian quality ladder growth model.[4] The Schumpeterian approach allows us to endogenize the arrival of innovations through R&D investments, which then endogenously determines equilibrium growth, fluctuation in economic uncertainty, and firm dynamics. Since, in our model, R&D is performed by both existing firms and entrepreneurs, and R&D investments are motivated by the prospect of future monopoly rents, we preserve key competitive aspects of the innovation process in the economy as formalized by the industrial organization literature. Our model thus provides a structural link between the process of creative destruction, the uncertainty about economic growth, and asset prices.

Our asset pricing results operate through the long-run risk mechanism of Bansal and Yaron (2004). Similar to us, Kung and Schmid (2013) study a production economy whose long-term growth prospects are endogenously determined by R&D. Since our model is based on Schumpeterian growth and we allow for heterogeneous innovations by incumbents and entrants, we study how the presence of heterogenous innovations affects the properties of economic uncertainty and asset prices. Loualiche (2013) shows that differential exposure to exogenous shocks to entrants' productivity across sectors explains differences in incumbent firms' expected returns. While Loualiche (2013) studies competitive threat of entry in product markets, we focus on competition in innovation among the firms on the technological frontier. Furthermore, in our model, innovation success of entrants leads to displacement of incumbents, which lowers incumbents' valuations and changes the incentives to invest in R&D of both entrants and incumbents. This equilibrium feedback between R&D incentives and valuations does not occur in Kung and Schmid (2013) and Loualiche (2013) as they are based on the expanding product variety model of Romer (1990). Through this feedback effect, Schumpeterian models can feature high competition and high economic growth, while competition always lowers growth in the expanding product variety models, which is counterfactual.[5] Ai and Kiku (2013) and Ai, Croce, and Li (2013) develop general equilibrium models with tangible and intangible capital to show that

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[4]Our model follows Acemoglu and Cao (2011) and Klette and Kortum (2004). See also seminal contributions of Grossman and Helpman (1991) and Aghion and Howitt (1992) to endogenous growth theory. Aghion and Howitt (1998, 2009) and Acemoglu (2010) provide a summary of this literature.

[5]See evidence by Nickell (1996) and Blundell, Griffith, and Reenen (1995, 1999).

growth options are less risky than assets in place. Our model also has two types of capital, but has no optionality features.

The paper is structured as follows. In Section 2, we describe our model. In Section 3, we present the qualitative analysis of the model. In Section 4, we quantitatively examine the asset pricing implications of our model and present model extensions. Section 5 concludes.

2 Model

We develop a Schumpeterian model of growth in which R&D activities are carried out by both existing firms (incumbents) and new firms (entrants). The innovation process is based on the model of Acemoglu and Cao (2011) which we embed into a standard macroeconomics setting with physical capital, aggregate uncertainty, and recursive preferences.[6] The economy admits a representative final good sector firm producing the unique good consumed by an infinitely-lived representative household. The production of the consumption good requires labor, physical capital, and a continuum of intermediate goods (inputs). The baseline model features a single aggregate shock affecting the productivity of the final good sector firm.

Each incumbent is a monopolist in the production of its own input and has access to an innovation technology that stochastically improves its input's quality. For each input, there is an infinite supply of atomistic entrants deploying R&D to radically increase the input's quality. Upon success, the entrant displaces the incumbent in the production of the input and captures its monopoly position. Economic growth arises endogenously and is driven by the speed of quality improvements of inputs, i.e., by the rate of growth of "technology capital". The relative contributions of incumbents and entrants to growth are determined in equilibrium through their decisions to invest in R&D.

2.1 Representative household

The representative household has Epstein-Zin-Weil preferences over the final consumption good

$$U_t = \left\{ (1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

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[6]For a textbook treatment of the model of innovation, see also Chapter 14.3 of Acemoglu (2010).

where β is the subjective time preference parameter, γ is the coefficient of relative risk aversion, and ψ is the elasticity of intertemporal substitution (EIS). The household chooses consumption C_t to maximize (1) taking wage w_t, aggregate dividend distributed by all firms in the economy D_t^{A}, and entrants' R&D expenditure S_t^{E} as given

$$\max_{\{C_s\}_{s=t}^{\infty}} U_t \qquad \text{s.t.} \qquad C_t \leq w_t L_t + D_t^{\mathrm{A}} - S_t^{\mathrm{E}}. \tag{2}$$

Since we do not model the consumption-leisure tradeoff, labor L_t is supplied inelastically, and we thus normalize it to be $L_t = 1$ for all t. The one-period stochastic discount factor (SDF) at time t is

$$\mathbb{M}_{t,t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\theta/\psi} R_{\mathrm{C},t+1}^{-(1-\theta)}, \tag{3}$$

where $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $R_{\mathrm{C},t+1}$ is the return on the consumption claim.[7]

2.2 Final good sector

The production of the unique final good requires labor, capital, and a continuum of measure one of intermediate goods denoted "inputs" $i \in [0,1]$. The production function is

$$Y_t = \left(K_t^{\alpha} (A_t L_t)^{1-\alpha} \right)^{1-\xi} G_t^{\xi} \quad \text{with} \quad G_t = \left[\int_0^1 q(i,t)^{1-\frac{1}{\nu}} x(i,t|q)^{\frac{1}{\nu}} di \right]^{\nu}. \tag{4}$$

In (4), K_t and L_t denote capital and labor, respectively, $\alpha \in (0,1)$ is the capital share, $\xi \in (0,1)$ is the share of inputs in the final output. Quantity G_t defines the composite intermediate good obtained by weighting the quantity $x(i,t|q)$ of each input i by its quality $q(i,t)$ through a constant elasticity aggregator.[8] The parameter ν captures the elasticity of substitution between any two inputs. The production process (4) implies that, for each input $i \in [0,1]$, only the highest quality type is used. In the next section, we discuss the dynamics of the quality of inputs. Aggregate risk originates from an exogenous shock $A_t = e^{a_t}$, where a_t is a stationary AR(1) process

$$a_t = \rho \, a_{t-1} + \varepsilon_t, \;\; \varepsilon_t \sim \mathcal{N}(0, \sigma_a^2). \tag{5}$$

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[7]Specifically, $R_{\mathrm{C},t+1} = \frac{W_{t+1}}{W_t - C_t}$ is the return on household's wealth W_t, defined as the present value of future consumption, $W_t = \mathbb{E}_t[\sum_{s=1}^{\infty} \mathbb{M}_{t,t+s} C_{t+s}]$.

[8]The choice of the functional form (4) for the composite intermediate good G_t implies that, under balanced growth, the relative size of incumbent firms is described by the ratio of its quality relative to the average quality of inputs. See Appendix A for details.

The firm's dividend D_t is

$$D_t = Y_t - I_t - w_t L_t - \int_0^1 p(i, s|q) x(i, t|q) di. \tag{6}$$

The final good firm takes wage w_t, the prices $p(i, t|q)$ of each input i and the SDF $\mathbb{M}_{t,t+1}$ as given, and chooses labor L_t, investment I_t, and the quantity $x(i, t|q)$ of each input to maximize its value

$$\max_{\{I_s, K_{s+1}, L_s, x(i,s|q)\}_{s=t}^{\infty}} \mathbb{E}_t \left[\sum_{s=t}^{\infty} \mathbb{M}_{t,s} D_s \right], \tag{7}$$

where the next period capital stock K_{s+1} is

$$K_{s+1} = (1 - \delta) K_s + \Lambda \left(\frac{I_s}{K_s} \right) K_s, \tag{8}$$

with δ the capital depreciation rate and $\Lambda(\cdot)$ is a convex adjustment cost function.[9]

2.3 Intermediate goods sector

The intermediate goods sector consists of a continuum of firms each producing a single input $i \in [0, 1]$. At each time t, input i is characterized by quality $q(i, t)$. Economic growth arises due to the growth of inputs' quality achieved by innovation successes by incumbents and entrants.

2.3.1 Incumbents

At each time t, each input belongs to an incumbent that holds a patent on the input's current quality. Incumbents are thus monopolists in the production of the input with current quality. Taking as given the demand schedule $x(i, t|q)$, incumbent i sets price $p(i, t|q)$ by maximizing its profit at each time t

$$\pi(i, t|q) = \max_{p(i,t|q)} p(i, t|q) \, x(i, t|q) - \mu \, x(i, t|q), \tag{9}$$

where μ is the marginal cost of producing one unit of input i.

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[9]We follow Jermann (1998) and define $\Lambda \left(\frac{I_s}{K_s} \right) = \frac{\alpha_1}{1 - \frac{1}{\zeta}} \left(\frac{I_s}{K_s} \right)^{1 - \frac{1}{\zeta}} + \alpha_2$, where $\alpha_1 = (\overline{\alpha} + \delta - 1)^{\frac{1}{\zeta}}$, $\alpha_2 = \frac{1}{1-\zeta} (\overline{\alpha} + \delta - 1)$. When solving the model numerically in Section 4, we choose the constant $\overline{\alpha}$ such that there are no adjustment costs in the deterministic steady state. The parameter ζ is the elasticity of the investment rate. The limiting cases $\zeta \to 0$ and $\zeta \to \infty$ represent infinitely costly adjustment and frictionless adjustment, respectively.

Each incumbent has access to a stochastic quality-improving innovation technology for its own input. If the incumbent spends $s^{\mathrm{I}}(i,t)\,q(i,t)$ units of the consumption good on R&D toward its input with quality $q(i,t)$, over a time interval Δt, the quality increases to $q(i, t + \Delta t) = \kappa_{\mathrm{I}}\,q(i,t)$, $\kappa_{\mathrm{I}} > 1$, with probability $\phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t))\Delta t$, where $\phi_{\mathrm{I}}(\cdot)$ is a strictly increasing and concave function satisfying Inada-type conditions $\phi_{\mathrm{I}}(0) = 0$ and $\phi'_{\mathrm{I}}(0) = \infty$.[10] If R&D does not result in innovation, we assume that the quality "depreciates" by a factor $\kappa_{\mathrm{D}} < 1$, i.e., $q(i, t + \Delta t) = \kappa_{\mathrm{D}}\,q(i,t)$. The parameter κ_{D} captures patent expiration and obsolescence of inputs over time.

Investing in R&D is an intertemporal decision that affects the accumulation of quality $q(i,t)$, which is the source of future profits. Patent protection of the input, however, does not prevent entrants to invest in R&D in order to invent a higher-quality input. Upon entrant's success, incumbent's input with quality $q(i,t)$ becomes obsolete and the entrant "displaces" the incumbent in the production of input i. Since incumbents' innovation success as well as the likelihood with which any incumbent is displaced by an entrant are uncertain, the evolution of inputs' quality is stochastic.

2.3.2 Entrants

Entrants deploy R&D in order to leapfrog incumbents in increasing inputs' quality and steal rights to produce inputs from them. If an entrant spends one unit of the consumption good on R&D toward input i with quality $q(i,t)$, its rate of success is $\frac{\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))}{q(i,t)}$, where $\hat{s}^{\mathrm{E}}(i,t)$ is the *total* amount of R&D by all entrants toward input i at time t. Since each input-i entrant is atomistic, all entrants take the technology $\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))$ as given. The function $\phi_{\mathrm{E}}(\cdot)$ is strictly decreasing to capture the fact that when many entrants are undertaking R&D to replace the same input, they are likely to try similar ideas leading to some amount of external diminishing returns.

Therefore, if all input-i entrants spend $\hat{s}^{\mathrm{E}}(i,t)\,q(i,t)$ units of the consumption good on R&D, over a time interval Δt, the quality increases to $q(i, t + \Delta t) = \kappa_{\mathrm{E}}\,q(i,t)$, with probability $\hat{s}^{\mathrm{E}}(i,t)\,\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))\Delta t$. We assume that $\hat{s}^{\mathrm{E}}(i,t)\,\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))$ is increasing in $\hat{s}^{\mathrm{E}}(i,t)$ to insure that larger aggregate R&D toward a particular input increases the overall probability of discovery by entrants for this input, and that Inada-type conditions $\lim_{\hat{s}^{\mathrm{E}}(i,t) \to 0} \hat{s}^{\mathrm{E}}(i,t)\,\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t)) = 0$ and $\lim_{\hat{s}^{\mathrm{E}}(i,t) \to 0} \phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t)) = \infty$ hold. Upon innovation success, the entrant acquires a patent

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[10]The conditions ensure that, for any interval $\Delta t > 0$, the probability of one innovation success is $\phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t))\Delta t$, while the probability of more than one innovation successes is $o(\Delta t)$ with $o(\Delta t)/\Delta t \to 0$ as $\Delta t \to 0$.

on quality $\kappa_{\mathrm{E}}\, q(i,t)$ of input i and becomes a new incumbent producing the input. We assume that $\kappa_{\mathrm{E}} > \kappa_{\mathrm{I}}$ to capture the fact that entrants' innovation technology is more "radical" than that of incumbents.[11] We use the following constant elasticity forms for $\phi_{\mathrm{I}}(\cdot)$ and $\phi_{\mathrm{E}}(\cdot)$:[12]

$$\phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t)) = \eta_{\mathrm{I}}(s^{\mathrm{I}}(i,t))^{\omega_{\mathrm{I}}} \text{ and } \phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t)) = \eta_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))^{\omega_{\mathrm{E}}-1}, \ \ 0 < \omega_{\mathrm{I}}, \omega_{\mathrm{E}} < 1 \text{ and } \eta_{\mathrm{I}}, \eta_{\mathrm{E}} > 0, \tag{10}$$

where η_{I} and η_{E} are productivity shift parameters and ω_{I} and ω_{E} are elasticities of innovation intensity with respect to R&D expenditure. Since $\hat{s}^{\mathrm{E}}(i,t)\, \phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))$ is the intensity of the Poisson process that drives the arrival of entrants' innovations, this intensity has the same functional form as the intensity of incumbents' innovations, i.e., $\hat{s}^{\mathrm{E}}(i,t)\, \phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t)) = \eta_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))^{\omega_{\mathrm{E}}}$. Bigger η_{I} increases the intensity of incumbents' innovations, while bigger ω_{I} decreases the intensity of incumbents' innovations if $s^{\mathrm{I}}(i,t) < 1$. Similarly for entrants.

2.4   R&D expenditure by incumbents and entrants

The value $V(i,t|q)$ of incumbent i at time t is the present value of its future net profits. Since the incumbent can be replaced by an entrant, the time at which the incumbent's stream of net profits ends is a random variable $\mathcal{T}(i,t) > t$. Incumbent's value is

$$V(i,t|q) = \max_{\{s^{\mathrm{I}}(i,\tau)\}_{\tau=t}^{\mathcal{T}(i,\tau)}} \mathbb{E}_t \left[\sum_{\tau=t}^{\mathcal{T}(i,\tau)} \mathbb{M}_{t,\tau}\, (\, \pi(i,\tau|q) - s^{\mathrm{I}}(i,\tau)q(i,\tau)) \right]. \tag{11}$$

The innovation technologies of incumbents and entrants described in Section 2.3 imply that, over the next Δt time period, the incumbent is displaced with probability $\hat{s}^{\mathrm{E}}(i,t)\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))\, \Delta t$ and survives otherwise. The incumbent takes entrants' R&D expenditure $\hat{s}^{\mathrm{E}}(i,t)$ and the SDF process (3) as given. In case of displacement, incumbent's value drops to zero. In case of survival, its value depends on whether or not incumbent's R&D expenditure $s^{\mathrm{I}}(i,t)$ results in a quality improvement. With probability $\phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t))\, \Delta t$, quality increases to $q(i,t+\Delta t) = \kappa_{\mathrm{I}}\, q(i,t)$, while with probability $(1 - \phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t))\, \Delta t - \hat{s}^{\mathrm{E}}(i,t)\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t)))\, \Delta t$, quality depreciates to $q(i,t+\Delta t) = \kappa_{\mathrm{D}}\, q(i,t)$. Assuming that Δt is sufficiently small, the future incumbent's value $V(i,t+\Delta t|q')$

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[11]Although the technology for radical innovation could also be accessed by incumbents, they have no incentive to use it due to Arrow's replacement effect. Incremental innovation technology of incumbents is not available to entrants.

[12]The functional forms for R&D technology is similar to the one used by Comin, Gertler, and Santacreu (2009).

can be written as a random variable with the following distribution

$$V(i, t + \Delta t|q') = \begin{cases} 0 & \text{with probability} \quad \hat{s}^{\mathrm{E}}(i,t)\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))\,\Delta t, \\ V(i, t + \Delta t|\kappa_{\mathrm{I}}\,q) & \text{with probability} \quad \phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t))\,\Delta t, \\ V(i, t + \Delta t|\kappa_{\mathrm{D}}\,q) & \text{otherwise.} \end{cases} \quad (12)$$

To simplify notation, in the sequel, we refer to time "$t + \Delta t$" as "$t+1$", with the understanding that the time lapse between two adjacent periods is close enough for the above approximation to be valid. Using (12), the stopping time problem (11) can then be rewritten as the following Bellman equation

$$V(i,t|q) = \max_{s^{\mathrm{I}}(i,t)} \{\pi(i,t|q) - s^{\mathrm{I}}(i,t)q(i,t) + \mathbb{E}_t\,[\mathbb{M}_{t,t+1}\,\{\phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t)) \times V(i,t+1|\kappa_{\mathrm{I}}\,q)$$
$$+ (1 - \phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t)) - \hat{s}^{\mathrm{E}}(i,t)\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))) \times V(i,t+1|\kappa_{\mathrm{D}}\,q)\}]\}. \quad (13)$$

We interpret $\pi(i,t|q) - s^{\mathrm{I}}(i,t)q(i,t)$ as the dividend distributed by the incumbent firm, the term $\hat{s}^{\mathrm{E}}(i,t)\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}(i,t))$ as the probability with which a radical innovation by an entrant occurs in input i, and the term $\phi_{\mathrm{I}}(s^{\mathrm{I}}(i,t))$ as the probability with which incumbent i innovates improving its input.

In Appendix A.1 we show that input quantities $x(i,t)$ and profits $\pi(i,t|q)$ are linear in $q(i,t)$ (see equations (A7) and (A10)). Under balanced growth conditions, this implies that the incumbent's value $V(i,t|q) = v_t\,q(i,t)$ for all t and $i \in [0,1]$, where v_t and s^{I}_t solve the following Bellman equation

$$v_t = \max_{s^{\mathrm{I}}_t} \{\pi_t - s^{\mathrm{I}}_t + \mathbb{E}_t\,[\mathbb{M}_{t,t+1}\,v_{t+1}\,(\phi_{\mathrm{I}}(s^{\mathrm{I}}_t)\,\kappa_{\mathrm{I}} + (1 - \phi_{\mathrm{I}}(s^{\mathrm{I}}_t) - \hat{s}^{\mathrm{E}}_t\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}}_t))\,\kappa_{\mathrm{D}})]\}. \quad (14)$$

The quantities π_t, s^{I}_t, $\hat{s}^{\mathrm{E}}_t$, and v_t are functions of the state variables K_t and A_t, which we omit to ease notation. The aggregate value of all incumbents is $V_t = \int_0^1 V(i,t|q)di = v_t\,Q_t$, where technology capital Q_t, defined as

$$Q_t = \int_0^1 q(i,t)di, \quad (15)$$

denotes the aggregate quality of inputs. The optimal choice of incumbents' R&D expenditure s^{I}_t is determined by the first order condition for problem (14)

$$1 = \phi'_{\mathrm{I}}(s^{\mathrm{I}}_t)(\kappa_{\mathrm{I}} - \kappa_{\mathrm{D}})\mathbb{E}_t\,[\mathbb{M}_{t,t+1}v_{t+1}]. \quad (16)$$

Entrants maximize the present value of future net profits achieved if they become incumbents

$$\max_{\hat{s}_t^{\mathrm{E}}} \hat{s}_t^{\mathrm{E}} \, \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}) \, \kappa_{\mathrm{E}} \, \mathbb{E}_t[\mathbb{M}_{t,t+1} v_{t+1}] - \hat{s}_t^{\mathrm{E}}. \tag{17}$$

Since they are atomistic, each entrant takes $\phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})$ as given. This assumption means that entrants do not internalize the fact more R&D reduces the probability of success or other entrants. Solving (17) under this assumption leads to the following free entry condition that implicitly determines the optimal level of entrants' R&D expenditure

$$1 = \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}) \, \kappa_{\mathrm{E}} \, \mathbb{E}_t[\mathbb{M}_{t,t+1} v_{t+1}]. \tag{18}$$

Equations (16) and (18) show that R&D decisions of incumbents and entrants depend on the same equilibrium value v_t given in (14).

2.5 Equilibrium

An equilibrium allocation in this economy consists of (i) time paths of consumption levels, physical capital, investment, aggregate expenditure on inputs and aggregate R&D expenditure $\{C_t, K_t, I_t, X_t, S_t\}_{t=0}^{\infty}$, (ii) time paths of R&D expenditures by incumbents and entrants $\{s^{\mathrm{I}}(i,t), \hat{s}^{\mathrm{E}}(i,t)\}_{t=0,\,i\in[0,1]}^{\infty}$, (iii) time paths of prices and quantities for each input, and values of each incumbent $\{p(i,t|q), x(i,t), V(i,t|q)\}_{t=0,\,i\in[0,1]}^{\infty}$, and (iv) time paths for wages and SDF $\{w_t, \mathbb{M}_{t,t+1}\}_{t=0}^{\infty}$ such that (a) the representative household maximizes lifetime utility (2), (b) the final good firm maximizes the present value of future dividends (equations (7) to (8)), (c) incumbents and entrants maximize present values of their future net profits (equations (14), (16), and (18)), (d) the labor market clears (i.e., $L_t = 1$), and (e) the final good market clears (i.e., resource constraint (A18) holds).[13]

Since incumbents' and entrants' R&D expenditures are not functions of inputs' specific qualities, technology capital Q_t evolves according to

$$\frac{Q_{t+1}}{Q_t} = \phi_{\mathrm{I}}(s_t^{\mathrm{I}}) \, \kappa_{\mathrm{I}} + \hat{s}_t^{\mathrm{E}} \, \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}) \, \kappa_{\mathrm{E}} + (1 - \phi_{\mathrm{I}}(s_t^{\mathrm{I}}) - \hat{s}_t^{\mathrm{E}} \, \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})) \, \kappa_{\mathrm{D}}. \tag{19}$$

---

[13]In Appendix A.1, we define the aggregate expenditure on inputs and aggregate R&D expenditure, and determine the equilibrium quantity $x(i,t|q)$ and price $p(i,t|q)$ of inputs. The solution of the final good firm's maximization problem (7)–(8) is standard and is described in Appendix B.

The growth of technology capital is thus due to a combination of heterogenous innovations by incumbents and entrants and depends on the level of their R&D expenditures. Over a short period of time, a fraction $\phi_{\mathrm{I}}(s_t^{\mathrm{I}})$ of inputs experience an innovation by incumbents who increase quality by factor κ_{I}, and a fraction $\hat{s}_t^{\mathrm{E}} \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})$ of inputs experience displacement by entrants who increase quality by factor κ_{E}, and the remaining inputs see their quality depreciate by factor κ_{D}.

Due to the homogeneity property discussed in the previous section, the economy is described by two endogenous state variables: physical capital K_t evolving according to (8), technology capital Q_t evolving according to (19), and the exogenous state variable $A_t = e^{a_t}$, where a_t evolves according to (5). By rescaling all growing variables by the average technology capital Q_t we make the problem stationary and can solve for the deterministic steady state growth.[14]

3 Economic growth and uncertainty

In this section, we present a qualitative analysis of the model. We show that R&D expenditures drive a slow moving component of productivity growth and thus give rise to economic uncertainty. We then study how the interplay between incumbents' and entrants' innovations affects the properties of economic uncertainty and asset prices.

3.1 Economic growth

In Appendix A we show that, under balanced growth, the equilibrium output is given by

$$Y_t = \left(\frac{\xi}{\nu\mu}\right)^{\frac{\xi}{1-\xi}} K_t^{\alpha}(A_t L_t)^{1-\alpha} Q_t^{1-\alpha} \tag{20}$$

The above expression indicates that the productivity of labor is

$$Z_t \equiv \left(\frac{\xi}{\nu\mu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}}, A_t\, Q_t, \tag{21}$$

where the evolution of the forcing process A_t is given in (5) and the evolution of the technology capital Q_t is endogenously determined by R&D expenditures of incumbents and entrants according to (19). If we assume that A_t is a fairly persistent process, from (21) we can approximate

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[14]Details of the rescaled problem and of the steady state conditions are in Appendix B.

the conditional expected productivity growth as

$$\mathbb{E}_t[\Delta \log Z_{t+1}] \approx \mathbb{E}_t[\log(Q_{t+1}/Q_t)] = \log(Q_{t+1}/Q_t), \tag{22}$$

where we use the fact that Q_{t+1}/Q_t is known at t. The technology capital growth Q_{t+1}/Q_t can be thought of as the expectation of the random variable

$$\tilde{\kappa} = \begin{cases} \kappa_{\mathrm{E}} & \text{with probability } \hat{s}_t^{\mathrm{E}} \, \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}), \\ \kappa_{\mathrm{I}} & \text{with probability } \phi_{\mathrm{I}}(s_t^{\mathrm{I}}), \\ \kappa_{\mathrm{D}} & \text{otherwise.} \end{cases} \tag{23}$$

The realizations of $\tilde{\kappa}$ are the magnitudes by which technology capital can change and the probabilities are given by the innovation intensities of incumbents and entrants.

Equation (22) shows that the growth of technology capital is the key determinant of the conditional expected productivity growth and, therefore, conditional expected consumption growth. We define the conditional expected consumption growth as

$$e_t \equiv \phi_{\mathrm{I}}(s_t^{\mathrm{I}})\Delta_{\mathrm{I}}\kappa + \hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})\Delta_{\mathrm{E}}\kappa + \kappa_{\mathrm{D}}, \tag{24}$$

where $\Delta_{\mathrm{I}}\kappa \equiv \kappa_{\mathrm{I}} - \kappa_{\mathrm{D}}$, $\Delta_{\mathrm{E}}\kappa \equiv \kappa_{\mathrm{E}} - \kappa_{\mathrm{D}}$, and $\hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}) \equiv \hat{s}_t^{\mathrm{E}} \, \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})$. The volatility of e_t represents the level of total economic uncertainty in the economy. Through technology capital growth, the stochastic properties of innovation intensities of incumbents and entrants determine the evolution of the conditional expected consumption growth and therefore asset prices.

The key economic mechanism underlying the investment in innovation by incumbents and entrants can be understood from combining the first-order conditions (16) and (18). In equilibrium, R&D expenditures of incumbents and entrants are set to equalize their marginal benefits

$$\phi_I^{'}(s_t^{\mathrm{I}})(\kappa_{\mathrm{I}} - \kappa_{\mathrm{D}}) = \phi_E(\hat{s}_t^{\mathrm{E}})\kappa_{\mathrm{E}}. \tag{25}$$

In what follows we use condition (25) to study the effect of heterogeneous innovations on the level (Section 3.2) and fluctuations (Section 3.3) of economic uncertainty.

3.2   Level of economic uncertainty

From the definition of conditional expected consumption growth in (24), the time series volatility of e_t is the volatility of a mix of two random variables $\phi_I(s_t^I)$ and $\hat{\phi}_E(\hat{s}_t^E)$ with weights $\Delta\kappa_I$ and $\Delta\kappa_E$, respectively,

$$\sigma(e_t) \equiv \sqrt{\sigma_{\phi_I}^2 \, \Delta_I\kappa^2 + \sigma_{\hat{\phi}_E}^2 \, \Delta_E\kappa^2 + 2\,\Delta_I\kappa \,\Delta_E\kappa \,\sigma_{\phi_I}\,\sigma_{\hat{\phi}_E}\,\rho_{\phi_I,\hat{\phi}_E}}, \tag{26}$$

where σ_{ϕ_I} is the unconditional volatility of $\phi_I(s_t^I)$, $\sigma_{\hat{\phi}_E}$ is the unconditional volatility of $\hat{\phi}_E(\hat{s}_t^E)$, and $\rho_{\phi_I,\hat{\phi}_E}$ is the unconditional correlation between the two quantities.

To analyze the effect of the presence of heterogeneous innovations on $\sigma(e_t)$, we first impose the equilibrium relation between incumbents' and entrants' R&D investments (25). Using the functional forms for the innovation intensities (10), condition (25) implies the following relationship between entrants' and incumbents' R&D

$$\hat{s}_t^E = (s_t^I)^{\frac{\omega_I-1}{\omega_E-1}} H \quad \text{with} \quad H \equiv \left(\frac{\eta_I}{\eta_E} \frac{\kappa_I - \kappa_D}{\kappa_E} \omega_I \right)^{\frac{1}{\omega_E-1}} > 0. \tag{27}$$

From (27), we note that s_t^I and $\hat{s}_t^E$ are positively related, indicating that R&D of incumbents and entrants are complementary in equilibrium. To gain some intuition, let us assume that incumbents' and entrants' innovation technologies have similar elasticities with respect to R&D expenditures, i.e., $\omega_I \approx \omega_E$. Under this assumption, the relationship between the two R&D expenditures is approximately linear, implying that $\rho_{\phi_I,\hat{\phi}_E} \approx 1$ in (26), and therefore

$$\sigma(e_t) \approx \sigma_{\phi_I} \, \Delta_I\kappa + \sigma_{\hat{\phi}_E} \, \Delta_E\kappa. \tag{28}$$

All else being equal, i.e., for the same model parameter values, equation (28) suggests that the presence of the heterogeneity in innovation increases the level of total economic uncertainty, compared to model economies in which only incumbents or only entrants are active in innovation. We refer to this effect as the 'composition effect.'

Such ceteris paribus comparison, however, is incomplete as it ignores the equilibrium effect of the presence of the heterogeneity in innovation on the volatilities σ_{ϕ_I} and $\sigma_{\hat{\phi}_E}$. Specifically, for the same unconditional level of economic growth, σ_{ϕ_I} and $\sigma_{\hat{\phi}_E}$ are lower in our heterogeneous innovations model compared to models in which only incumbents or only entrants are

active in innovation. This obtains because the representative household, by having access to two technologies for growth, can better smooth consumption. When responding to aggregate productivity shocks, both R&D expenditures adjust in the direction of the shock. The magnitudes of R&D adjustments in response to the shock depend on the efficiency with which entrants' and incumbents' R&D expenditures are converted into innovation success. By condition (25), in equilibrium, the household allocates total R&D expenditure between incumbents and entrants so that the benefits of the marginal R&D dollar in each technology is the same. This decreases the volatility of aggregate R&D expenditure and of expected consumption growth. We refer to this effect as the 'smoothing effect.' Since the composition and the smoothing effects work in the opposite way, the overall effect of heterogeneous innovation on the level of total economic uncertainty (28) is ambiguous.

In Section 4.2, we quantitatively compare the level of total economic uncertainty in our model economy with those in which only incumbents or only entrants are active in innovation keeping the level of unconditional economic growth constant. This comparison reveals that volatilities $\sigma_{\phi_{\mathrm{I}}}$ and $\sigma_{\hat{\phi}_{\mathrm{E}}}$ are indeed substantially lower in the heterogeneous innovations model, highlighting the presence of the smoothing effect. However, we also show that, because of the composition effect, the level of total economic uncertainty in our heterogenous innovations model is higher compared to the model in which only entrants are active in innovation.

3.3 Economic uncertainty fluctuations

To understand how economic uncertainty varies over the business cycle, we provide a heuristic derivation of the underpinnings of economic uncertainty fluctuations in our model. The evolution of the expected consumption growth in (24) over time can be written as

$$e_{t+1} = e_t + [\phi_{\mathrm{I}}(s_{t+1}^{\mathrm{I}}) - \phi_{\mathrm{I}}(s_t^{\mathrm{I}})]\,\Delta_{\mathrm{I}}\kappa + [\hat{\phi}_{\mathrm{E}}(\hat{s}_{t+1}^{\mathrm{E}}) - \hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})]\,\Delta_{\mathrm{E}}\kappa. \tag{29}$$

Using a first-order Taylor expansion centered at time t values, we can approximate (29) as

$$e_{t+1} \approx e_t + \underbrace{\left[\phi_{\mathrm{I}}' \cdot s_a^{\mathrm{I}}\,\Delta_{\mathrm{I}}\kappa + \hat{\phi}_{\mathrm{E}}' \cdot \hat{s}_a^{\mathrm{E}}\,\Delta_{\mathrm{E}}\kappa\right]}_{\equiv \sigma_t} \varepsilon_{t+1}, \tag{30}$$

where ε_{t+1} is the shock to the exogenous component of aggregate productivity a_t, ϕ'_{I} and $\hat{\phi}'_{\mathrm{E}}$ are the derivatives of incumbents' and entrants' innovation intensities, respectively, and s^{I}_a and $\hat{s}^{\mathrm{E}}_a$ denote, respectively, partial derivatives of time-homogeneous policy functions for incumbents' and entrants' R&D expenditures with respect to the forcing process a_t.[15]

We refer to the quantity σ_t in equation (30) as the (approximate) level of economic uncertainty at t. Since e_t and σ_t depend on incumbents' and entrants' equilibrium R&D investments through the innovation probabilities ϕ_{I} and $\hat{\phi}_{\mathrm{E}}$, in our model time-variation in expected economic growth and fluctuating economic uncertainty arise endogenously.

Time-variations in expected growth and fluctuating economic uncertainty are important determinants of asset prices. For example, in the long-run risks model of Bansal and Yaron (2004), the price-consumption ratio z_t is approximately given by

$$z_t \approx A_0 + A_1\, e_t + A_2\, \sigma_t^2, \tag{31}$$

where, if the EIS of the representative agent is larger than 1, $A_1 > 0$ and $A_2 < 0$. An increase in e_t leads to higher valuations, while an increase in σ_t lowers asset prices and leads to higher risk premia. Furthermore, time-varying σ_t is useful for explaining the time variation and predictability of risk premia. The relationship between e_t and σ_t is therefore crucial for determining properties of aggregate asset prices. Since our structural model allows to explicitly study how heterogeneous activities of incumbents and entrants in innovation jointly determine e_t and σ_t, it allows to study the effect of the process of creative destruction on asset prices.

Differentiating the expression for economic uncertainty in (30) with respect to a_t, we obtain

$$\frac{\partial \sigma_t}{\partial a_t} = \phi''_{\mathrm{I}} \cdot (s^{\mathrm{I}}_a)^2 + \phi'_{\mathrm{I}} \cdot s^{\mathrm{I}}_{aa} + \hat{\phi}''_{\mathrm{E}} \cdot (s^{\mathrm{E}}_a)^2 + \hat{\phi}'_{\mathrm{E}} \cdot s^{\mathrm{E}}_{aa}. \tag{32}$$

Economic uncertainty is countercyclical if $\partial \sigma_t / \partial a_t < 0$. Since the innovation intensities are increasing and concave functions of R&D, $\phi'_{\mathrm{I}} > 0$, $\hat{\phi}'_{\mathrm{E}} > 0$, $\phi''_{\mathrm{I}} < 0$, and $\hat{\phi}''_{\mathrm{E}} < 0$. From (32), it thus follows that, in our model, the economic uncertainty can be pro- or countercyclical depending on the sign and magnitude of terms s^{I}_{aa} and s^{E}_{aa}. The calibration of our model presented in Section 4.2 shows that R&D policy functions are increasing and concave in the state variable a_t, meaning that $s^{\mathrm{I}}_{aa} < 0$ and $s^{\mathrm{E}}_{aa} < 0$. This implies that our model features

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[15]For ease of notation, we ignore the explicit dependence of the policy functions from the endogenous state variable K_t. We do account for this state variable in the numerical implementation.

countercyclical economic uncertainty. Furthermore, equation (32) suggests that, all else being equal, the economic uncertainty is more likely to be countercyclical when the degrees of concavity of the function ϕ_{I} and $\hat{\phi}_{\mathrm{E}}$, a measure of "congestion externalities" in R&D, are stronger.

All else being equal, i.e., for the same model parameter values, equation (32) suggests that the presence of the heterogeneity in innovation makes the economic uncertainty more counter-cyclical compared to model economies in which only incumbents or only entrants are active in innovation. As argued in Section 3.2, such ceteris paribus comparison is incomplete because it ignores the equilibrium effect of the presence of the heterogeneity in innovation on the values of the derivatives of the functions present in equation (32).

To illustrate the effect of the presence of heterogeneous innovations on economic uncertainty fluctuations, we impose condition (25) on the expression for σ_t given in (30). Using the fact that $\hat{\phi}_{\mathrm{E}}'(\hat{s}_t^{\mathrm{E}}) = \omega_{\mathrm{E}}\phi_{\mathrm{E}}(\hat{s}^{\mathrm{E}})$, we have

$$
\begin{aligned}
\sigma_t &= \phi_{\mathrm{I}}'(s_t^{\mathrm{I}}) \left[s_a^{\mathrm{I}} + \omega_{\mathrm{E}}\hat{s}_a^{\mathrm{E}} \frac{\Delta_{\mathrm{E}}\kappa}{\kappa_{\mathrm{E}}} \right] \Delta_{\mathrm{I}}\kappa \\
&= \phi_{\mathrm{I}}'(s_t^{\mathrm{I}}) \left[s_a^{\mathrm{I}} + \omega_{\mathrm{E}}H \frac{\omega_{\mathrm{I}}-1}{\omega_{\mathrm{E}}-1}(s_t^{\mathrm{I}})^{\frac{\omega_{\mathrm{I}}-\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}-1}} s_a^{\mathrm{I}} \frac{\Delta_{\mathrm{E}}\kappa}{\kappa_{\mathrm{E}}} \right] \Delta_{\mathrm{I}}\kappa,
\end{aligned}
\tag{33}
$$

where the second equality follows from the relationship between s^{I} and $\hat{s}^{\mathrm{E}}$ given in (27). Differentiating the above expression with respect to a_t, we obtain

$$
\begin{aligned}
\frac{\partial \sigma_t}{\partial a_t} &= \phi_{\mathrm{I}}'' \cdot (s_a^{\mathrm{I}})^2 \left[1 + \omega_{\mathrm{E}}H \frac{\omega_{\mathrm{I}}-1}{\omega_{\mathrm{E}}-1}(s_t^{\mathrm{I}})^{\frac{\omega_{\mathrm{I}}-\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}-1}} \frac{\Delta_{\mathrm{E}}\kappa}{\kappa_{\mathrm{E}}} \right] \Delta_{\mathrm{I}}\kappa + \\
&\quad \phi_{\mathrm{I}}' \cdot \left[s_{aa}^{\mathrm{I}} + \omega_{\mathrm{E}}H \left(\frac{\omega_{\mathrm{I}}-1}{\omega_{\mathrm{E}}-1} \right) \left(\frac{\omega_{\mathrm{I}}-\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}-1} \right) (s_t^{\mathrm{I}})^{\frac{\omega_{\mathrm{I}}-2\omega_{\mathrm{E}}+1}{\omega_{\mathrm{E}}-1}} (s_a^{\mathrm{I}})^2 \frac{\Delta_{\mathrm{E}}\kappa}{\kappa_{\mathrm{E}}} \right. \\
&\quad \left. + \omega_{\mathrm{E}}H \frac{\omega_{\mathrm{I}}-1}{\omega_{\mathrm{E}}-1}(s_t^{\mathrm{I}})^{\frac{\omega_{\mathrm{I}}-\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}-1}} s_{aa}^{\mathrm{I}} \frac{\Delta_{\mathrm{E}}\kappa}{\kappa_{\mathrm{E}}} \right] \Delta_{\mathrm{I}}\kappa.
\end{aligned}
\tag{34}
$$

The first term in the above expression is negative because $\phi_{\mathrm{I}}'' < 0$ and $0 < \omega_{\mathrm{I}}, \omega_{\mathrm{E}} < 1$. The sign of the second term depends on the properties of the R&D technologies of incumbents and entrants. In particular, the term $\frac{\omega_{\mathrm{I}}-\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}-1}$ is negative if $\omega_{\mathrm{I}} > \omega_{\mathrm{E}}$, and therefore $\omega_{\mathrm{I}} > \omega_{\mathrm{E}}$ is a sufficient condition for generating countercyclical volatility if the policy function s^{I} is concave. In Section 4.2, we calibrate our model and find that the economic uncertainty in our model is indeed countercyclical.

4 Quantitative analysis

We now calibrate our model and explore its ability to replicate key moments of macroeconomic quantities, corporate innovation activity, and asset returns in the U.S. economy. In Section 4.1, we construct empirical measures of the process of creative destruction. Section 4.2 presents the calibration of our model. In Section 4.3, we highlight the implications of the presence of heterogenous innovations for economic uncertainty and asset prices. In Section 4.4, we study the relationship between model parameters and the steady state level of economic growth, economic uncertainty, and the rate of radical innovation. Finally, in Section 4.5, we generalize the model to allow for stochastic barriers to entry.

4.1 Empirical measures of creative destruction

To capture the intensity of creative destruction in the economy, we use the innovation intensities of incumbents and entrants to define the 'rate of radical innovation'

$$\Gamma_t \equiv \frac{\hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})}{\hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}) + \phi_{\mathrm{I}}(s_t^{\mathrm{I}})}, \tag{35}$$

where $\phi_{\mathrm{I}}(s_t^{\mathrm{I}})$ is the rate at which incumbents improve their technology and $\hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})$ is the rate at which innovation by entrants occurs. The rate of radical innovation Γ_t measures the relative importance of entrants' radical innovations in the economy. To obtain empirical proxies for $\phi_{\mathrm{I}}(s_t^{\mathrm{I}})$, $\hat{\phi}_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})$, and Γ_t, we rely on patent data.

We obtain the universe of awarded patents applied for at the United States Patent and Trademark Office (USPTO) from January 1975 to June 2013. For each patent, we identify patent assignees listed on the patent grant document and keep only utility patents awarded to U.S. and non-U.S. corporations.[16] We proceed by splitting patents with application dates during each quarter t into those applied for by incumbents and the residual, which we take to be patents applied for by entrants in innovation. Specifically, the set of 'incumbents at t' is the set of patent assignees that applied for at least one patent with application date during a τ-quarter period ending with quarter $t-1$. Because in our model there is a continuum of incumbents, each of which is successful with probability $\phi_{\mathrm{I}}(s_t^{\mathrm{I}})$, as discussed in Section 2.5, in a small interval Δt, there will be a fraction $\phi_{\mathrm{I}}(s_t^{\mathrm{I}})\Delta t$ of incumbents that innovate. We use this property of the model

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[16]Using only patents awarded to U.S. corporations leads to quantitatively very similar results.

to construct a proxy of $\phi_\mathrm{I}(s_t^\mathrm{I})$ as the ratio of the number of incumbents at t that also applied for at least one patent with application date during t, to the total number of incumbents at t.[17] To proxy Γ_t, we compute the ratio of the number of patents applied for by patent assignees that are not incumbents at t to the total number of patents with application dates during t. Using the values of $\phi_\mathrm{I}(s_t^\mathrm{I})$ and Γ_t, we then recover a proxy for $\hat{\phi}_\mathrm{E}(\hat{s}_t^\mathrm{E})$. We set the threshold τ to 40 quarters, so the time series of our proxies start in the first quarter of 1985. Since we do not know whether patents applied for in the recent years will be awarded, we stop the time series in the last quarter of 2008.

Figure 1 displays the quarterly time series of the rate of radical innovation Γ_t together with the number of entrants in innovation. The ratio of radical innovations is steadily declining over time, while the number of entrants peaks in the late 1990s. This means that, while the number of entrants was increasing until the end of 1990s, these entrants were accounting for a decreasing fraction of patents in the economy.

We calibrate our model to match the sample mean of the rate of radical innovation of 10.67%. We estimate parameters κ_D, κ_I, and κ_E of the innovation production functions of incumbents and entrants using the structural link between the technology capital growth, given in (19), and productivity growth from equation (21). Assuming a persistent exogenous process A_t, productivity growth is approximately

$$
\begin{aligned}
\Delta \log Z_{t+1} &= \log(Q_{t+1}/Q_t) + \log(A_{t+1}/A_t) \\
&\approx \log(Q_{t+1}/Q_t) + \varepsilon_{t+1} \\
&= \log\left(\kappa_\mathrm{D} + \phi_\mathrm{I}(s_t^\mathrm{I})\,\Delta_\mathrm{I}\kappa + \hat{\phi}_\mathrm{E}(\hat{s}_t^\mathrm{E})\,\Delta_\mathrm{E}\kappa\right) + \varepsilon_{t+1}.
\end{aligned}
\tag{36}
$$

We measure productivity growth as the quarterly TFP growth in non-equipment business output from Fernald (2012).[18] We estimate equation (36) using non-linear least squares. Standard errors are adjusted for autocorrelation using the Newey-West estimator with 10 lags. The coefficient estimates (standard errors reported in parentheses) are: $\widehat{\kappa_\mathrm{D}} = 0.966$ (0.031), $\widehat{\kappa_\mathrm{I}} = 1.355$ (0.547), and $\widehat{\kappa_\mathrm{E}} = 2.890$ (2.118). The estimates are consistent with our assumptions that $\kappa_\mathrm{D} < 1$ and

---

[17]To identify incumbents at t that also applied for at least one patent with application date during t, we match the set of firm name strings of patent assignees of patents with application dates during t to the set of firm name strings of patent assignees of patents with application dates during a τ-quarter period ending with quarter $t-1$. We standardize firm name strings before matching.

[18]We obtain the data from http://www.frbsf.org/economic-research/total-factor-productivity-tfp/.

$\kappa_{\mathrm{E}} > \kappa_{\mathrm{I}} > 1$, and are in line with the prior literature (see, e.g., Acemoglu and Cao (2011)). In our calibrations, we set the parameter values to these point estimates.

4.2   Calibration

In this section, we present the calibration of the heterogenous innovations model of Section 2 and describe its asset pricing implications. We calibrate the model to match (i) the long run annual consumption growth rate $\mathbb{E}[\Delta C] = 1.89\%$, (ii) the short run (business-cycle frequency) consumption volatility $\sigma(\Delta C) = 2.21\%$ annually, and (iii) the average rate of radical innovation $\mathbb{E}[\Gamma_t] = 10.67\%$ as obtained from our empirical analysis in Section 4.1. The empirical macroeconomic moments correspond to the U.S. sample in 1929-2008 as reported by Benzoni, Collin-Dufresne, and Goldstein (2011). Our preference parameters are the same as in Kung and Schmid (2013). We set the parameters that govern the magnitudes of innovations of incumbents and entrants, i.e., κ_{I}, κ_{E}, and κ_{D}, to be equal to the point estimates obtained in Section 4.1. We set the parameter that governs the elasticity of substitution between inputs to $\nu = 1.25$, which implies 25% markup for incumbents. We set the values of R&D productivity shift parameters to $\eta_{\mathrm{I}} = 1.50$ and $\eta_{\mathrm{E}} = 0.18$, and the values of the elasticities of the innovation intensities with respect to R&D to $\omega_{\mathrm{I}} = 0.7483$ and $\omega_{\mathrm{E}} = 0.7808$. The model is calibrated at a quarterly frequency and is solved using third-order perturbation methods around the stochastic steady state. We simulate 2,500 instances of the model economy, each characterized by a time series of 550 quarters. The moments we report are computed by averaging across simulations, after eliminating the observations in the first 50 quarters. The details of the model solution and parameter choices used in the calibration are discussed in Appendix C.

Table 2 contains aggregate consumption moments and statistics describing the process of creative destruction. We show that the annual unconditional volatility of conditional expected consumption growth, i.e., the level of total economic uncertainty is $\sigma(\mathbb{E}_t[\Delta C_{t+1}]) = 0.43\%$. Our model features a volatile slow-moving component in expected consumption growth which is consistent with the long-run risk mechanism of Bansal and Yaron (2004). We also find that the impulse response function of the conditional volatility given in (32) to a positive shock to a_t is $\partial\sigma_t/\partial a_t = -0.52$, meaning that the economic uncertainty is higher in recessions and lower in expansions at business cycle frequency. As we discussed in Section 3, the countercyclicality of volatility is primarily due to decreasing marginal productivity of the R&D technology of

incumbents and entrants. In summary, our model thus endogenously generates substantial uncertainty about expected growth, this uncertainty fluctuates, and is countercyclical.

The second part of Table 2 contains statistics describing the process of creative destruction in our model economy and compares them to their empirical counterparts. We report: (i) the mean, volatility, and autocorrelation of the rate of radical innovation Γ_t, and (ii) the means, volatilities, and correlation between innovation intensities of incumbents and entrants. While $\mathbb{E}[\Gamma_t]$ is exactly targeted in our calibration, our model closely matches the means of $\phi(s^{\mathrm{I}})$ and $\hat{\phi}_{\mathrm{E}}(\hat{s}^{\mathrm{E}})$, as well as the volatility of $\phi(s^{\mathrm{I}})$ and the autocorrelation of Γ_t. Since $\omega_{\mathrm{E}} \approx \omega_{\mathrm{I}}$ in our calibration, the equilibrium condition (27) implies that s^{I} and $\hat{s}^{\mathrm{E}}$ are approximately linearly related, as shown in Section 3.2, and therefore the success rates of incumbents and entrants are close to perfectly correlated $\rho_{\phi_{\mathrm{I}},\hat{\phi}_{\mathrm{E}}} = 1$. The correlation of incumbents' and entrants' innovation intensities is very high in the data as well. Finally, we show that, in our model, the contribution of entrants' innovations to growth is 34.3%. While an analogous empirical moment does not exist, this statistic is broadly consistent with arguments in Bilbiie, Ghironi, and Melitz (2012) that about 70 percent of product creation and destruction occurs within existing firms, as well as with evidence in Bartelsman and Doms (2000) and Foster, Haltiwanger, and Krizan (2001) who show that most total factor productivity growth in the U.S. comes from existing, as opposed to new, establishments. Our model is thus able to capture the key features of the structure of the corporate innovation process in the economy. In our model, the volatility of the rate of radical innovation σ_Γ and the volatility of $\hat{\phi}_{\mathrm{E}}(\hat{s}^{\mathrm{E}})$ are counterfactually low. To account for this, in Section 4.5, we generalize our model by introducing the stochastic barriers to entry.

Table 3 reports the mean and volatility of (i) the risk-free rate; (ii) the excess return on the consumption claim, defined as the security whose dividend is the aggregate consumption; and (iii) the excess return on the market portfolio, i.e., the security whose dividend D_t^{A} is the sum of the dividend D_t distributed by the final good sector firm and profits of incumbents $\Pi_t - S_t^{\mathrm{I}}$, where $S_t^{\mathrm{I}} = s_t^{\mathrm{I}} Q_t$ denotes the aggregate level of resources spent in incremental R&D by incumbents. As discussed in Appendix A.1, the resource constraint implies that the aggregate dividend is $D_t^{\mathrm{A}} = C_t + S_t^{\mathrm{E}} - w_t L_t$ (se equation (A19)) where $S_t^{\mathrm{E}} = s_t^{\mathrm{E}} Q_t$ denotes the aggregate level of resources spent in radical R&D technologies. Therefore, the wedge between the dividend on the market portfolio and on the consumption claim is due to entrants' R&D expenditures and aggregate wages. Details of the derivation of the price of risk and risk premia in the model

are given in Appendix A.2. The empirical asset pricing moments correspond to the U.S. sample in 1929-2008 as reported by Benzoni, Collin-Dufresne, and Goldstein (2011). We find that our model matches exactly the level of market risk premium and achieves a low and stable risk-free rate. Similarly to other production-based models, our model generates counterfactually low volatilities of market returns and the risk-free rate.

Finally, Table 3 shows that the average excess return on final good sector stocks, $\mathbb{E}[r_d - r_f]$, is significantly higher than the average excess return return on incumbents stocks, $\mathbb{E}[r_I - r_f]$ (see Appendix A.2 for definition of these quantities). Interpreting the former as return on physical capital and the latter as return on intangible capital, this spread is consistent with the magnitude of the value premium in the data.

4.3 Heterogenous versus homogenous innovations

To highlight the effect of heterogenous innovations on economic uncertainty and asset prices, we quantitatively compare our model to two model economies in which innovations are homogenous. In the first model, we assume that there is no entry and R&D is carried out only by incumbents. In this case, the optimal level of incumbents' R&D is determined by the Bellman equation

$$v_t = \max_{s_t^{\mathrm{I}}} \left\{ \pi_t - s_t^{\mathrm{I}} + \mathbb{E}_t \left[\mathbb{M}_{t,t+1} \, v_{t+1} \left(\phi_{\mathrm{I}}(s_t^{\mathrm{I}}) \, \kappa_{\mathrm{I}} + (1 - \phi_{\mathrm{I}}(s_t^{\mathrm{I}})) \, \kappa_{\mathrm{D}} \right) \right] \right\}, \tag{37}$$

which is analogous to equation (14) where we ignore the presence of entrants. The optimal level of R&D is determined by FOC (16). In the second model, we assume that R&D is carried out only by entrants. In this case, the value of incumbents is given by Bellman equation

$$v_t = \pi_t + \mathbb{E}_t \left[\mathbb{M}_{t,t+1} \, v_{t+1} \left((1 - \hat{s}_t^{\mathrm{E}} \phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})) \, \kappa_{\mathrm{D}} \right) \right], \tag{38}$$

where the optimal level of entrants' R&D is determined by the free entry condition (18).

To make the comparison between different economies meaningful, we calibrate all models to the long run annual consumption growth rate $\mathbb{E}_t[\Delta C_{t+1}] = 1.89\%$ and the short run consumption volatility $\sigma(\Delta C) = 2.21\%$ annually. To this end, we vary the volatility of the forcing process σ_a, R&D productivity shift parameters η_{I} and η_{E}, and the elasticity parameters ω_{I} and ω_{E}. All other parameters are as in our heterogenous innovations model. Specifically, in the two columns under the heading 'Only incumbents innovate' ('Only entrants innovate') of Table 2, we report

two calibrations of the model in which R&D is carried out only by incumbents (entrants). In the 'Only incumbents innovate' ('Only entrants innovate') case, in the left column, η_I (η_E) changes, which we highlight using a box, while ω_I (ω_E) remains the same. The opposite applies to the right column. This means that, in each case, the long run growth and its short term growth volatility is achieved using two different parameterizations of R&D technology.

In the only-incumbents-innovate model, η_I is smaller while ω_I is bigger compared to our heterogenous innovations model. Given the functional forms for innovation technology (10), smaller η_I and bigger ω_I both imply a less efficient R&D technology. This means that without displacement threat by entrants, the same level of growth is achieved with less efficient R&D technology. We can interpret this finding as evidence that the presence of displacement threat introduces inefficiency in the economy due to dissipative R&D efforts. In contrast, in the only-entrants-innovate model, η_E is bigger while ω_E is smaller compared to our heterogenous innovations model, meaning that the same level of growth is achieved with a more efficient R&D technology. This is because, in the heterogenous innovations model, upon success in innovation, entrants become incumbents, and thus entrants indirectly benefit from having access to R&D technology of incumbents. Since this mechanism does not operate in the only-entrants-innovate model, entrants need to have a more productive R&D technology to achieve the same level of growth as in the heterogeneous innovation model.

As discussed in Section 3.2 there are two forces that determine the level of total economic uncertainty $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$ in our heterogeneous innovations model: the smoothing effect, which lowers the uncertainty, and the composition effect, which increases the uncertainty. The comparison of volatilities of innovation intensities σ_{ϕ_I} and σ_{ϕ_E} reported in Table 2 across models highlights the presence of the smoothing effect. Specifically, we find that both volatilities in the heterogenous innovations model are about half of the corresponding values in the homogeneous innovations economies. Since, in the only-incumbents-innovate model, the volatility of σ_{ϕ_I} is large, the absence of the smoothing effect leads to larger $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$ and thus to a larger consumption risk premium compared to the one observed in the heterogenous innovations model. Since, in the only-entrants-innovate model, the volatility of σ_{ϕ_E} is small and there is no composition effect, this economy has lower $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$ and thus consumption risk premium compared to our heterogenous innovations model.

The comparison of asset pricing moments of Table 3 across different models shows that the level and volatility of the market risk premium are considerably lower in the only-incumbents-innovate model compared to our heterogenous innovations model, despite higher level of economic uncertainty. This results is a consequence of the definition of aggregate dividend. In the only-incumbents-innovate model, $S_t^{\mathrm{E}} = 0$, and therefore the aggregate dividend in (A19) is $D_t^{\mathrm{A}} = C_t - w_t L_t$. Since wages are procyclical, they act as a hedge agains the volatility of consumption, lowering risk premium and volatility of the market portfolio, compared to the corresponding values for the consumption claim. In contrast, the only-entrants-innovate model has level and volatility of the market risk premium that are comparable to those in our heterogenous innovations model, despite lower level of economic uncertainty. The reason for this result lies in the volatility of entrants' R&D expenditure. In the only-entrants-innovate model, the aggregate dividend is $D_t^{\mathrm{A}} = C_t + S_t^{\mathrm{E}} - w_t L_t$. Since R&D expenditures S_t^{E} are procyclical, they compensate for the hedging effect of procyclical wages. From the values of volatility of entrants' innovation intensity $\sigma_{\hat{\phi}_{\mathrm{E}}}$ reported in Table 2, we infer that the volatility of entrant's R&D expenditure is large, which is then reflected in higher market risk premia and volatility compared to corresponding values for the consumption claim.

4.4   Comparative statics

In order to understand the effect of our parameter choices on the solution of the solution of the benchmark model with heterogeneous innovation, in this section we perform a comparative statics analysis. We alter the values of the parameters in Table 1 and analyze the effect of these changes on (i) consumption growth $\mathbb{E}[\Delta C]$, (ii) the rate of radical innovation $\mathbb{E}[\Gamma_t]$, and (iii) the level of total economic uncertainty $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$. The results are reported in Table 4. In the interest of space, we focus our comparative statics analysis on three set of parameters: preference parameters (Panel A), R&D technology parameters (Panel B), and the degree of market power (markup) of incumbents (Panel C). The numbers in bold correspond to the benchmark calibration of Table 1.

With the exception of the time preference parameter β, changing preferences has a minor effect on growth, the rate of radical innovation, and economic uncertainty. A larger value of β means that the representative household values future consumption relatively more and hence is willing to consume less now and invest more. This implies a higher growth which is achieved

through a higher rate of radical innovation. The level of total economic uncertainty is also slightly bigger when β is bigger. Although risk aversion γ does not have any effect on the deterministic steady state growth, it does affect the stochastic steady state growth. In particular, a larger value of γ leads to a more conservative growth, less radical innovation, and a lower level of uncertainty.

As Panel B shows, R&D technology parameters significantly affect growth, the rate of radical innovation, and economic uncertainty. Higher shift parameters η_I and η_E imply more efficient innovation technologies of incumbents and entrants, respectively. Interestingly, while making incumbents more efficient results in higher growth and higher economic uncertainty, the opposite is true if entrants become more efficient. This is because, if entrants become more efficient, the household diverts resources from consumption toward R&D by entrants. In a decentralized economy, as discussed in Section 2.3.2, entrants do not internalize the fact that one more unit of R&D reduces the probability of success of other entrants, i.e., ϕ_E is decreasing with R&D.[19] Similar results obtain when we change the elasticities of R&D technologies ω_I and ω_E. Note that, because of the functional forms assumed in (10), a higher level of the elasticity parameters implies less efficient R&D technologies.

Finally, Panel C shows that an increase in markup level ν has two opposite effects. First, it increases monopoly profits of incumbents leading to bigger incumbents' and entrants' R&D expenditures. Second, it reduces the demand for intermediate goods, which decreases the profitability of R&D investments. In our calibrations, the first effect dominates and higher markup is thus associated with higher growth, more radical innovation, and higher economic uncertainty. This feature highlights the essence of Schumpeterian models: stronger market power, driven by bigger product differentiation, for example, leads to stronger incentives to innovate and higher growth.

4.5 Model with stochastic barriers to entry

As illustrated in Figure 1, the rate of radical innovation Γ_t exhibits substantial variation over the sample period we consider (the quarterly volatility is $\sigma_\Gamma = 2.19\%$). In our benchmark calibration reported in Table 2, the quarterly volatility of the rate of radical innovation is only

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[19]This fact is emphasized in the free entry condition (18), where we assume that entrants take $\phi_E(\hat{s}^E)$ as given when choosing the optimal level of R&D $\hat{s}^E$.

0.14% quarterly. This fact suggests that the single productivity shock that drives the dynamics of the benchmark model is not sufficient to generate the variation in the entry process observed in the data. In this section we generalize the benchmark model of Section 2 by allowing the R&D technology of entrants to be subject to exogenous shocks, different from the TFP shocks considered in our benchmark case. Because this technology-specific shock directly affects the efficiency of entrants' R&D, in this economy there is an exogenous force that determines the level of barriers to entry in this economy.

To model time variation in the barrier to entry in a convenient way, we assume that the elasticity ω_{E} of the entrants' R&D technology is stochastic and evolves overtime as a stationary AR(1) process

$$\omega_{\mathrm{E},t+1} = \overline{\omega_{\mathrm{E}}}(1 - \rho_{\omega_{\mathrm{E}}}) + \rho_{\omega_{\mathrm{E}}}\omega_{\mathrm{E},t} + \varepsilon_{t+1}^{\omega_{\mathrm{E}}}, \quad \varepsilon_{t+1}^{\omega_{\mathrm{E}}} \sim \mathcal{N}(0, \sigma_{\omega_{\mathrm{E}}}^2). \tag{39}$$

The resulting model, therefore, features two exogenous shocks, the original shocks ε_{t+1} driving the forcing process (5) and the shock $\varepsilon_{t+1}^{\omega_{\mathrm{E}}}$ driving the elasticity of the entrant's R&D technology. In our numerical solution we consider different levels of correlations between these two shocks. We also consider the case in which the scale parameter η_{E}, instead of ω_{E}, is stochastic and evolves according to an AR(1) process similar to (39). Specifically,

$$\eta_{\mathrm{E},t+1} = \overline{\eta_{\mathrm{E}}}(1 - \rho_{\eta_{\mathrm{E}}}) + \rho_{\eta_{\mathrm{E}}}\eta_{\mathrm{E},t} + \varepsilon_{t+1}^{\eta_{\mathrm{E}}}, \quad \varepsilon_{t+1}^{\eta_{\mathrm{E}}} \sim \mathcal{N}(0, \sigma_{\eta_{\mathrm{E}}}^2). \tag{40}$$

Recall that, from the functional form (10), the efficiency of entrants' technology is higher for lower levels of ω_{E} and higher levels of η_{E}.

Tables 5 and 6 are the equivalent of Tables 2 and 3 for the case of a stochastic barrier to entry. The first three columns refer to the case in which ω_{E} evolves according to the process (39). The remaining three columns consider the case in which η_{E} evolves according to the process (40).

For both the stochastic ω_{E} and η_{E} cases, we calibrate the models that features shocks orthogonal to ε_{t+1}, i.e., the columns $corr(\varepsilon_{t+1}, \varepsilon_{t+1}^{\omega_{\mathrm{E}}}) = 0$ and $corr(\varepsilon_{t+1}, \varepsilon_{t+1}^{\eta_{\mathrm{E}}}) = 0$, by targeting the long run consumption growth to 1.89% annual, the short-run volatility to 2.21% annual, the rate of radical innovation to 10.67% and the volatility of the rate of radical innovation to the observed level of 2.19%. The parameters that allow us to match these quantities are reported

in Table 1. We then keep these parameter fixed and alter the correlation between shocks by considering two additional correlation values: -0.4 and $+0.4$.

Table 5 and 6 shows that time variation in the barriers to entry allows to closely match the key statistics of the process of creative destruction, while preserving the ability of our benchmark model to replicate the key asset pricing moments. In particular, from the zero-correlation columns in Table 5 we note that, besides matching, by construction, the volatility σ_Γ of the rate of radical innovation, the models with stochastic barriers to entry produce first order autocorrelation of the rate of radical innovation and innovation intensity volatilities $\sigma_{\phi_{\mathrm{I}}}$ and $\sigma_{\phi_{\mathrm{E}}}$ that are close to the values observed in the data.

An evident drawback of modelling stochastic barriers to entry through shocks to the R&D technology of entrants is that this implies a negative correlation $\rho_{\phi_{\mathrm{I}},\hat{\phi}_{\mathrm{E}}}$ between incumbents and entrants intensity of innovation, while it is positive (0.760) in the data. The failure of the stochastic entry barrier model to match the correlation is mechanical. In fact, allowing only ω_{E} to be subject to shocks implies that a positive shock to ω_{E} increases barriers to entry and makes incumbent relatively more productive. As a consequence s^{I} responds positively to a shock to ω_{E} and s^{E} responds negatively. Figure 2 illustrates this point by reporting the impulse response functions of s^{I} and $\hat{s}^{\mathrm{E}}$ to a one-standard deviation shock ε_{t+1} and to the barrier to entry, $\varepsilon_{t+1}^{\omega_{\mathrm{E}}}$. This result suggests that to achieve a level of correlation $\rho_{\phi_{\mathrm{I}},\hat{\phi}_{\mathrm{E}}}$ consistent with the data, one would need to introduce uncertainty also in the R&D technology of incumbents.

Comparing across different levels of correlations between shocks to R&D technology and productivity, Table 5 shows that in the case of stochastic ω_{E}, average growth $\mathbb{E}[\Delta C]$ and economic uncertainty $\sigma(\mathbb{E}_t[\Delta C])$ are higher when shocks to ω_{E} are negatively correlated to productivity shocks but the short run volatility of consumption $\sigma(\Delta C)$ is lower. When correlation is negative, lower ω_{E}, i.e., more productive entrants, tend to be associated with high productivity shocks. This has an amplifying effect on growth. Note that this amplifying effect is absent in the model with constant ω_{E} where, as illustrated in the comparative statics of Table 4 show, a lower ω_{E} implies *higher* growth.

To understand the effect of correlation on the short run volatility $\sigma(\Delta C)$ it is helpful to think of the budget constraints of the representative household's problem (2). The household allocates resources (wages and dividends from owning final good firms and incumbents) between consumption C_t and entrant R&D expenses S_t^{E}. When $corr(\varepsilon_{t+1}, \varepsilon_{t+1}^{\omega_{\mathrm{E}}}) < 0$, a positive shock

to productivity is associated with an increase in the efficiency of entrants' technology. Because EIS > 1, the agents tend to consume less and do more R&D. This effect lowers short run volatility of consumption and increases the volatility of entrants' R&D expenses. Note in fact that the volatility $\sigma_{\hat{\phi}_\mathrm{E}}$ is higher when $corr(\varepsilon_{t+1}, \varepsilon_{t+1}^{\omega_\mathrm{E}}) < 0$.

As in the case of constant barriers to entry, in the economies with stochastic barriers to entry the volatility of expected consumption growth is still countercyclical, as indicated by the negative numbers reported in the row $\partial\sigma_t/\partial a_t$. The correlation between the innvovation-specific shock and shocks to the exogenous productivity A_t does not affect in a substantial way the fluctuating nature of economic uncertainty. Note finally that the case in which stochastic barriers to entry are modelled through a stochastic η_E is similar to the case of stochastic ω_E, because a higher η_E has a similar effect as a lower ω_E on the entrant's probability of success ϕ_E.

Table 6 reports key asset pricing moments for the case of stochastic barrier to entry. Overall the asset pricing moments from the stochastic barrier to entry economy are similar to those of the constant barrier to entry economy. Note that the market risk premium $\mathbb{E}_t[r_m - r_f]$ is unaffected by the correlation between shocks to productivity and shocks to the elasticity of entrants' R&D technology. Market volatility is lower for the case of positive correlation between these shocks while it is higher for the return r_c on the consumption claim, and for the return r_d on final good sector firms. To understand this fact, note that, the aggregate dividend is defined as the sum of consumption and entrants' R&D expenditures, net of wages (see equation (A17) in Section A.1 of Appendix A). From the results in Table 5, as noted above, when the correlation between shocks to productivity and shocks to ω_E is positive, the volatility of entrants' R&D is lower, as can be inferred from the values of $\sigma_{\hat{\phi}_\mathrm{E}}$. The lower volatility of entrant's R&D expenditures implies a higher price of risk (Sharpe ratio) in an economy in which the shocks to productivity and entrants' elasticity are positively correlated. The volatilities of the consumption claim and of the final good sector stock inherit the patterns of the volatility $\sigma(\Delta C)$ and, as discussed earlier, are higher for high level of correlation between ε_{t+1} and $\varepsilon_{t+1}^{\omega_\mathrm{E}}$. The asset pricing properties of the model with stochastic η_E are equivalent to the case of stochastic ω_E after noticing, that high η_E are equivalent to lower ω_E and vice versa.

In summary, the extension of our baseline model to the case of stochastic barriers to entry allows us to match more closely the key features of the process of creative destruction we

documented in Section4.1 while preserving the ability of our benchmark model to reproduce the average level of risk free rate and risk premia observed in the data.

5   Conclusion

In this paper, we study the implications of the creative destruction process in innovation for aggregate asset prices. We embed a multi-sector general equilibrium Schumpeterian model with incumbent and entrants in a fairly standard macroeconomics model of business cycle fluctuation and show that the interplay between incremental innovation of incumbents and radical innovation of entrants is an important determinant of long run growth and of fluctuations in economic uncertainty.

In equilibrium, R&D expenses of incumbents and entrants are complementary and their volatility increase the level of economic uncertainty. However, the ability to perform innovation through two channels with different efficiencies provides better opportunities for consumption smoothing. Furthermore, we show that when marginal productivity of R&D is decreasing, the level of economic uncertainty is countercyclical.

The model reproduces fairly accurately properties of aggregate asset prices and, most important, is capable of matching key stylized fact on the process of creative destruction that we document from the universe of patents applied for at the United States Patent and Trademark Office (USPTO) in 1975-2013. In particular, heterogeneous innovation is key in generating the empirically observed level of radical innovation, i.e., the relative importance of entrants' radical innovations in the economy. This quantity has been declining over our sample period, despite an increase in the number of entrants in the economy, suggesting an increased role of incumbents' innovation, especially in the later part of our sample. Finally, we explore the role of time-variation in the barrier to entry as potential explanation of the volatility of the rate of radical innovation in the data.

By highlighting the effect of the interplay between incumbents and entrants on their incentives to innovate, our model provides a micro-foundation of long run growth and time variation in economic uncertainty that are important for explaining the time variation and predictability of aggregate risk premia in the data. A natural important direction of inquiry is the study of

the effect of creative destruction on the cross sectional properties of asset prices, a task that we leave for future research.

A   Appendix. Model details

In this appendix we provide details of the model discussed in Section 2.

A.1   Quantity and price of intermediate goods

We denote by X_t the total amount of expenditure on the production of the intermediate goods

$$X_t = \mu \int_0^1 x(i,t)di, \tag{A1}$$

by S_t^{I} the total amount of R&D expenditure by incumbent firms

$$S_t^{\mathrm{I}} = \int_0^1 s^{\mathrm{I}}(i,t)\, q(i,t)di, \tag{A2}$$

and by S_t^{E} the total amount of R&D expenditure by entrants

$$S_t^{\mathrm{E}} = \int_0^1 \hat{s}^{\mathrm{E}}(i,t)q(i,t)di. \tag{A3}$$

Aggregate R&D expenditure in the economy is $S_t = S_t^{\mathrm{I}} + S_t^{\mathrm{E}}$. Since the labor market is competitive, the wage satisfies

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)(1-\xi)Y_t. \tag{A4}$$

The final good firm's demand $x(i,t|q)$ for input i arises from an intra-temporal decision where the final good firm maximizes its dividend D_t defined in (6) at each time t. Using the definition of Y_t in (4), this maximization yields the following demand for input i

$$x(i,t|q) = \xi^{\frac{\nu}{\nu-1}} \left(K_t^\alpha (A_t L_t)^{1-\alpha} \right)^{\frac{(1-\xi)\nu}{\nu-1}} G_t^{\frac{\xi\nu-1}{\nu-1}} \left(p(i,t|q) \right)^{\frac{\nu}{1-\nu}} q(i,t). \tag{A5}$$

Using (A5) in incumbent's problem (9) leads to markup pricing

$$p(i,t|q) = \nu\,\mu. \tag{A6}$$

The result that the profit maximizing price is a markup over marginal cost obtains because demand (A5) is isoelastic. Higher degree of substitutability across inputs (i.e., lower ν) leads to

a smaller markup. Using the markup price (A6), incumbent's profit is

$$\pi(i,t|q) = (\nu - 1)\,\mu\,x(i,t|q). \tag{A7}$$

Substituting (A5) and (A6) into (4) gives the following expression for the composite input

$$G_t = \left(\frac{\xi}{\nu\mu}\right)^{\frac{1}{1-\xi}} K_t^{\alpha}(A_tL_t)^{1-\alpha}Q_t^{\frac{\nu-1}{1-\xi}}, \tag{A8}$$

where

$$Q_t = \int_0^1 q(i,t)di \tag{A9}$$

is the aggregate quality of inputs, which we denote as the technology capital. Expressions (A6) and (A8) allow us to rewrite the equilibrium quantity of input $x(i,t|q)$ given in (A5) as a linear function of quality

$$x(i,t|q) = \left(\frac{\xi}{\nu\mu}\right)^{\frac{1}{1-\xi}} K_t^{\alpha}(A_tL_t)^{1-\alpha}\,Q_t^{\frac{\xi\nu-1}{1-\xi}}\,q(i,t). \tag{A10}$$

Linearity of $x(i,t|q)$ in $q(i,t)$ is convenient as it allows us to easily obtain aggregate quantities. Specifically, using (A1), (A7), and (A8), the equilibrium aggregate expenditure on inputs X_t, aggregate incumbents' profits Π_t, and output Y_t are

$$X_t = \mu \int_0^1 x(i,t)di = \mu \left(\frac{\xi}{\nu\mu}\right)^{\frac{1}{1-\xi}} K_t^{\alpha}(A_tL_t)^{1-\alpha}\,Q_t^{\frac{(\nu-1)\xi}{1-\xi}}, \tag{A11}$$

$$\Pi_t = \int_0^1 \pi(i,t|q)di = (\nu-1)\,X_t, \tag{A12}$$

$$Y_t = \left(\frac{\xi}{\nu\mu}\right)^{\frac{\xi}{1-\xi}} K_t^{\alpha}(A_tL_t)^{1-\alpha}Q_t^{\frac{(\nu-1)\xi}{1-\xi}}. \tag{A13}$$

Since technology capital Q_t is a growing process driven by R&D expenditures by incumbents and entrants, to insure balanced growth, we impose the following parametric restriction

$$\frac{(\nu-1)\xi}{1-\xi} = 1 - \alpha. \tag{A14}$$

Under this condition, output and aggregate expenditures on inputs can be written as

$$Y_t = \left(\frac{\xi}{\nu\mu}\right)^{\frac{\xi}{1-\xi}} K_t^\alpha (A_t Q_t L_t)^{1-\alpha}, \tag{A15}$$

$$X_t = \frac{\xi}{\nu} Y_t. \tag{A16}$$

From (A15), technology capital acts as an endogenous "labor augmenting" productivity factor. Note that, imposing the balance growth condition (A14) on (A10) and (A11), the incumbent's relative firm size is $\tilde{x}(i, t|q) = x(i, t|q)/X_t = q(i, t)/Q_t$.

From (A2) and (A3), the aggregate R&D expenditures of incumbents and entrants are, respectively, $S_t^{\mathrm{I}} = s_t^{\mathrm{I}} Q_t$ and $S_t^{\mathrm{E}} = \hat{s}_t^{\mathrm{E}} Q_t$. The aggregate dividend D_t^{A} is the sum of the dividend distributed by the final good firm and by all incumbents, i.e.,

$$\begin{aligned} D_t^{\mathrm{A}} &= D_t + \int_0^1 (\pi(i, t|q) - s_t^{\mathrm{I}} q(i, t)) di \\ &= Y_t - I_t - w_t L_t - X_t - S_t^{\mathrm{I}}, \end{aligned} \tag{A17}$$

where we use the definition of the final good firm's dividend D_t given in (6), equilibrium input prices $p(i, t|q)$ given in (A6), equilibrium incumbents' profits $\pi(i, t|q)$ given in (A7), and the definition of aggregate expenditure on inputs X_t given in (A1). Using the resource constraint

$$Y_t = C_t + I_t + X_t + S_t^{\mathrm{I}} + S_t^{\mathrm{E}}, \tag{A18}$$

we can express the aggregate dividend as

$$D_t^{\mathrm{A}} = C_t + S_t^{\mathrm{E}} - w_t L_t. \tag{A19}$$

A.2 Asset prices

To study asset pricing implications of our model, in this appendix, we first define the market price of risk for the shock ε_{t+1} to the exogenous component of aggregate productivity A_t defined in (5). Next, we define securities that are exposed to this shock and derive risk premia demanded in equilibrium for holding those securities.

Projecting the log of the SDF process (3) on the space spanned by these shocks gives

$$\mathbb{m}_{t,t+1} = \log(\mathbb{M}_{t,t+1}) = \mathbb{E}_t[\mathbb{m}_{t,t+1}] - \gamma_{t+1}^\varepsilon \frac{\varepsilon_{t+1}}{\sigma_a}. \tag{A20}$$

The quantity γ_{t+1}^ε is the market prices of risk for shock ε_{t+1}. To see this, consider a projection of the log return $r_{j,t+1}$ of a generic asset j on the space spanned by the shocks

$$r_{j,t+1} = \mathbb{E}_t[r_{j,t+1}] + \beta_{j,t+1}^\varepsilon \varepsilon_{t+1}, \tag{A21}$$

where $\beta_{j,t+1}^\varepsilon = Cov(\varepsilon_{t+1}, r_{j,t+1})/\sigma^2$. With the Jensen's inequality adjustment, the log risk premium of asset j can be written as

$$\mathbb{E}_t[r_{j,t+1} - r_{f,t+1} + \sigma_j^2/2] = -Cov(\mathbb{m}_{t,t+1}, r_{j,t+1}) = \beta_{j,t+1}^\varepsilon \sigma_a \gamma_{t+1}^\varepsilon, \tag{A22}$$

where $r_{f,t+1}$ is the log risk-free rate from t to $t+1$, σ_j is the volatility of asset j's log returns, and the second equality follows from (A20) and (A21). If asset j is perfectly correlated with shock ε_{t+1}, $\beta_{j,t+1}^\varepsilon = \sigma_j/\sigma_a$. Hence, from (A22), the Sharpe ratio of this asset is

$$\frac{\mathbb{E}_t[r_{j,t+1} - r_{f,t+1} + \sigma_j^2/2]}{\sigma_j} = \frac{\beta_{j,t+1}^\varepsilon \sigma_a \gamma_{t+1}^\varepsilon}{\beta_{j,t+1}^\varepsilon \sigma_a} = \gamma_{t+1}^\varepsilon, \tag{A23}$$

proving that γ_{t+1}^ε in (A20) is the market price of risk for shock ε_{t+1}, i.e., the risk premium per unit volatility of the shock. From (A20), the market price of risk is

$$\gamma_{t+1}^\varepsilon = -\sigma_a \frac{\partial \mathbb{m}_{t,t+1}}{\partial \varepsilon_{t+1}}. \tag{A24}$$

The market price of risk is positive (negative) if a positive shock $\varepsilon_{t+1} > 0$ causes a decrease (increase) in the marginal utility of consumption of the representative household.

To analyze risk premia of securities exposed to shock ε_{t+1}, let $R_{j,t+1}$ be the return of a claim on a dividend stream $D_{j,t}$ and let $V_{j,t}$ be the value of this claim. The log return of this asset j is

$$r_{j,t+1} = \log(R_{j,t+1}) = \log\left(\frac{V_{j,t+1}}{V_{j,t} - D_{j,t}}\right). \tag{A25}$$

From (A21), the loading of the returns of asset j on shock ε_{t+1} is

$$\beta_{j,t+1}^{\varepsilon} = \frac{\partial r_{j,t+1}}{\partial \varepsilon_{t+1}} = \frac{\partial \log(V_{j,t+1})}{\partial \varepsilon_{t+1}}. \tag{A26}$$

Using the risk premium definition (A22), we see that the risk premium of asset j is

$$\lambda_{j,t+1}^{\varepsilon} = \beta_{j,t+1}^{\varepsilon} \sigma_a \gamma_{t+1}^{\varepsilon}, \tag{A27}$$

where $\beta_{j,t+1}^{\varepsilon}$ is given in (A26) and the market price of risk $\gamma_{t+1}^{\varepsilon}$ is given in (A24).

We consider four securities: (i) the consumption claim asset, defined as the claim on aggregate consumption C_t whose value we denote by $V_{c,t}$; (ii) the market, defined as the claim on aggregate dividend D_t^{A} given in (A17) whose value we denote by $V_{m,t}$; (iii) the stock of the final good firm, defined as the claim on dividend D_t given in (6) whose value we denote $V_{d,t}$; and (iv) the portfolio that holds all incumbent firms, defined as the claim on the aggregate dividend of incumbent firms $D_{I,t} = \Pi_t - S_t^{\mathrm{I}}$ (see equations (A11) and (A12)) whose value we denote $V_{I,t} = v_t Q_t$, where v_t is a solution to equation (14). The loadings of the returns of these assets on shock ε_{t+1} and their risk premia are given in (A26) and (A27), respectively, with $j = \{c, m, d, I\}$.

B   Appendix. Model solution

In this Appendix, we provide conditions that characterize the solution of the model described in Section 2. In Appendix B.1, we state the first order conditions for the original, non-stationary, formulation of the model. Appendix B.2 presents the equivalent conditions for the rescaled stationary version of the model. Appendix B.3 describes how to solve for the deterministic steady state. In this Appendix, variable λ_t refers to the lagrangian multiplier with respect to the capital accumulation constraint (8), i.e., Tobin's marginal Q.

B.1   Original problem

$$\text{(DEF\_U)} \qquad U_t = \left\{ (1-\beta)C_t^{1-\rho} + \beta \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right)^{1-\rho} \right\}^{\frac{1}{1-\rho}}, \qquad \rho = 1/\text{EIS} \tag{B1}$$

$$\text{(FOC\_I)} \qquad \lambda_t = \frac{1}{\Lambda_t'(I_t/K_t)}, \qquad \text{where} \quad \Lambda_t(\cdot) = \frac{a_1}{1-\zeta^{-1}}(\cdot)^{1-\zeta^{-1}} + a_2 \tag{B2}$$

$$\text{(FOC\_L)} \qquad w_t = (1-\alpha)(1-\xi)\frac{Y_t}{L_t} \tag{B3}$$

$$\text{(FOC\_K)} \qquad \lambda_t = \mathbb{E}_t \left[\mathbb{M}_{t,t+1} \left\{ \alpha(1-\xi)\frac{Y_{t+1}}{K_{t+1}} + \lambda_{t+1} \left((1-\delta) - \Lambda_{t+1}'\frac{I_{t+1}}{K_{t+1}} + \Lambda_{t+1} \right) \right\} \right] \tag{B4}$$

$$\text{(FOC\_X)} \qquad p_t = \nu\mu \tag{B5}$$

$$\text{(FOC\_}\lambda\text{)} \qquad K_{t+1} = K_t(1-\delta) + \Lambda_t K_t \tag{B6}$$

$$\text{(DEF\_Y)} \qquad Y_t = \left(\frac{\xi}{\nu\mu} \right)^{\frac{\xi}{1-\xi}} K_t^\alpha (A_t L_t)^{1-\alpha} Q_t^{1-\alpha} \tag{B7}$$

$$\text{(DEF\_X)} \qquad X_t = \frac{\xi}{\nu}Y_t \tag{B8}$$

$$\text{(DEF\_}\Pi\text{)} \qquad \Pi_t = (\nu-1)X_t \tag{B9}$$

$$\text{(DEF\_}v\text{)} \qquad v_t = \pi_t - s_t^I + \left(\phi_I\left(s_t^I\right)\kappa_I + \left(1 - \phi_I\left(s_t^I\right) - \hat{s}_t^E \phi_E\left(\hat{s}_t^E\right)\right)\kappa_D \right)\mathbb{E}_t\left[\mathbb{M}_{t,t+1}v_{t+1}\right],$$
$$\text{where } \phi_I(\cdot) = \eta_I(\cdot)^{\omega_I}, \quad \phi_E(\cdot) = \eta_E(\cdot)^{\omega_E-1}, \quad \omega_I, \omega_E < 1. \tag{B10}$$

$$\text{(FOC\_}s^I\text{)} \qquad 1 = \phi_I'\left(s_t^I\right)(\kappa_I - \kappa_D)\mathbb{E}_t\left[\mathbb{M}_{t,t+1}v_{t+1}\right] \tag{B11}$$

$$\text{(FOC\_}s^E\text{)} \qquad 1 = \phi_E\left(\hat{s}_t^E\right)\kappa_E\mathbb{E}_t\left[\mathbb{M}_{t,t+1}v_{t+1}\right] \tag{B12}$$

$$\text{(DEF\_Q)} \qquad Q_{t+1} = Q_t\left(\kappa_D + (\kappa_I - \kappa_D)\phi_I\left(s_t^I\right) + (\kappa_E - \kappa_D)\hat{s}_t^E\phi_E\left(\hat{s}_t^E\right)\right) \tag{B13}$$

$$\text{(MCC\_C)} \qquad C_t = Y_t - I_t - X_t - S_t^I - S_t^E \tag{B14}$$

$$\text{(DEF\_SDF)} \quad \mathbb{M}_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\rho-\gamma}{1-\gamma}} = \beta^{\frac{1-\gamma}{1-\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho\frac{1-\gamma}{1-\rho}}\left(R_{t+1}^W\right)^{\frac{\rho-\gamma}{1-\rho}} \tag{B15}$$

B.2 Rescaled problem

We scale all aggregate growing variables by Q_t, and denote the rescaled variables using lower-case letters, e.g., $k_t = \frac{K_t}{Q_t}$, etc. We define $g_{q,t+1} = \frac{Q_{t+1}}{Q_t}$. With some abuse of notation, we define $u_t = \frac{U_t}{C_t} = \frac{u_t Q_t}{c_t Q_t}$.

$$
\text{(DEF\_U)} \qquad u_t = \left\{ 1 - \beta + \beta \left(\mathbb{E}_t \left[\left(u_{t+1} \frac{c_{t+1}}{c_t} g_{q,t+1} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right)^{1-\rho} \right\}^{\frac{1}{1-\rho}} \tag{B16}
$$

$$
\text{(FOC\_I)} \qquad \lambda_t = \frac{1}{\Lambda_t'(i_t/k_t)}, \qquad \text{where} \quad \Lambda_t(\cdot) = \frac{a_1}{1 - \zeta^{-1}} (\cdot)^{1-\zeta^{-1}} + a_2 \tag{B17}
$$

$$
\text{(FOC\_L)} \qquad w_t = (1-\alpha)(1-\xi)\frac{y_t}{L_t} \tag{B18}
$$

$$
\text{(FOC\_K)} \qquad \lambda_t = \mathbb{E}_t \left[\mathrm{m}_{t,t+1} \left\{ \alpha(1-\xi)\frac{y_{t+1}}{k_{t+1}} + \lambda_{t+1} \left((1-\delta) - \Lambda_{t+1}'\frac{i_{t+1}}{k_{t+1}} + \Lambda_{t+1} \right) \right\} \right] \tag{B19}
$$

$$
\text{(FOC\_X)} \qquad p_t = \nu\mu \tag{B20}
$$

$$
\text{(FOC\_}\lambda) \qquad k_{t+1} g_{q,t+1} = k_t(1-\delta) + \Lambda_t k_t \tag{B21}
$$

$$
\text{(DEF\_Y)} \qquad y_t = \left(\frac{\xi}{\nu\mu} \right)^{\frac{\xi}{1-\xi}} k_t^\alpha (A_t L_t)^{1-\alpha} \tag{B22}
$$

$$
\text{(DEF\_X)} \qquad x_t = \frac{\xi}{\nu} y_t \tag{B23}
$$

$$
\text{(DEF\_}\pi) \qquad \pi_t = (\nu - 1)x_t \tag{B24}
$$

$$
\text{(DEF\_}v) \qquad v_t = \pi_t - s_t^I + \left[(\kappa_I - \kappa_D)\phi_I\left(s_t^I\right) + \kappa_D \left(1 - \hat{s}_t^E \phi_E\left(\hat{s}_t^E\right)\right) \right] \mathbb{E}_t \left[\mathrm{m}_{t,t+1} v_{t+1} \right],
$$
$$
\text{where } \phi_I(\cdot) = \eta_I(\cdot)^{\omega_I}, \quad \phi_E(\cdot) = \eta_E(\cdot)^{\omega_E - 1}, \quad \omega_I, \omega_E < 1. \tag{B25}
$$

$$
\text{(FOC\_}S^I) \qquad 1 = \phi_I'\left(s_t^I\right)(\kappa_I - \kappa_D)\mathbb{E}_t\left[\mathrm{m}_{t,t+1}v_{t+1}\right] \tag{B26}
$$

$$
\text{(FOC\_}S^E) \qquad 1 = \phi_E\left(\hat{s}_t^E\right)\kappa_E\mathbb{E}_t\left[\mathrm{m}_{t,t+1}v_{t+1}\right] \tag{B27}
$$

$$
\text{(DEF\_Q)} \qquad g_{q,t+1} = \kappa_D + (\kappa_I - \kappa_D)\phi_I\left(s_t^I\right) + (\kappa_E - \kappa_D)\hat{s}_t^E \phi_E\left(\hat{s}_t^E\right) \tag{B28}
$$

$$
\text{(MCC\_C)} \qquad c_t = y_t - i_t - x_t - s_t^I - \hat{s}_t^E \tag{B29}
$$

$$
\text{(DEF\_SDF)} \qquad \mathrm{m}_{t,t+1} = \beta \left(\frac{c_{t+1}}{c_t} g_{q,t+1} \right)^{-\rho} \left(\frac{(u_{t+1}c_{t+1})^{1-\gamma}}{\mathbb{E}_t\left[(u_{t+1}c_{t+1})^{1-\gamma}\right]} \right)^{\frac{\rho-\gamma}{1-\gamma}} \tag{B30}
$$

B.3 Steady state

$$(\text{DEF\_}U) \qquad u^{1-\rho} \;=\; 1 - \beta + \beta(u\,g_q)^{1-\rho} \quad \Rightarrow \quad u^{1-\rho} = \frac{1-\beta}{1-\beta g_q^{1-\rho}} \tag{B31}$$

$$(\text{FOC\_}I) \qquad \lambda \;=\; 1 \tag{B32}$$

$$(\text{FOC\_}L) \qquad w \;=\; (1-\alpha)(1-\xi)y \tag{B33}$$

$$(\text{FOC\_}K) \qquad \lambda \;=\; \mathbbm{m}\left\{\alpha(1-\xi)\frac{y}{k} + \lambda(1-\delta)\right\} \tag{B34}$$

$$(\text{FOC\_}X) \qquad p \;=\; \nu\mu \tag{B35}$$

$$(\text{FOC\_}\lambda) \qquad kg_q \;=\; k(1-\delta) + i \tag{B36}$$

$$(\text{DEF\_}Y) \qquad y \;=\; \left(\frac{\xi}{\nu\mu}\right)^{\frac{\xi}{1-\xi}} k^\alpha \tag{B37}$$

$$(\text{DEF\_}X) \qquad x \;=\; \frac{\xi}{\nu}y \tag{B38}$$

$$(\text{DEF\_}\pi) \qquad \pi \;=\; (\nu-1)x \tag{B39}$$

$$(\text{DEF\_}v) \qquad v \;=\; \frac{\pi - s^{\mathrm{I}}}{1 - \mathbbm{m}\left((\kappa_{\mathrm{I}} - \kappa_{\mathrm{D}})\phi_I\left(s^{\mathrm{I}}\right) + \kappa_{\mathrm{D}}(1 - \hat{s}^{\mathrm{E}}\phi_E\left(\hat{s}^{\mathrm{E}}\right))\right)} \tag{B40}$$

$$(\text{FOC\_}S^I) \qquad 1 \;=\; \phi_I'\left(s^{\mathrm{I}}\right)(\kappa_{\mathrm{I}} - \kappa_{\mathrm{D}})\,\mathbbm{m}\,v \tag{B41}$$

$$(\text{FOC\_}S^E) \qquad 1 \;=\; \phi_E\left(\hat{s}^{\mathrm{E}}\right)\kappa_{\mathrm{E}}\mathbbm{m}\,v \tag{B42}$$

$$(\text{DEF\_}Q) \qquad g_q \;=\; \kappa_{\mathrm{D}} + (\kappa_{\mathrm{I}} - \kappa_{\mathrm{D}})\,\phi_I\left(s^{\mathrm{I}}\right) + (\kappa_{\mathrm{E}} - \kappa_{\mathrm{D}})\,\hat{s}^{\mathrm{E}}\,\phi_E\left(\hat{s}^{\mathrm{E}}\right) \tag{B43}$$

$$(\text{MCC\_}C) \qquad c \;=\; y - i - \mu x - s^{\mathrm{I}} - \hat{s}^{\mathrm{E}} \tag{B44}$$

$$(\text{DEF\_SDF}) \qquad \mathbbm{m} \;=\; \beta g_q^{-\rho} \tag{B45}$$

Using (B32), (B34), (B37) and (B45), we can express k as a function of s^{I} and $\hat{s}^{\mathrm{E}}$

$$k\left(s^{\mathrm{I}}, \hat{s}^{\mathrm{E}}\right) = \left[\frac{1}{(1-\xi)\alpha}\left(\frac{\xi}{\nu\mu}\right)^{\frac{\xi}{\xi-1}}\left(\delta - 1 + \frac{1}{\beta}g_q\left(s^{\mathrm{I}}, \hat{s}^{\mathrm{E}}\right)^\rho\right)\right]^{\frac{1}{\alpha-1}}, \tag{B46}$$

where $g_q\left(s^{\mathrm{I}}, \hat{s}^{\mathrm{E}}\right)$ is given by (B43). Using (B38) and (B46) in (B39) we have that $\pi = \pi\left(s^{\mathrm{I}}, \hat{s}^{\mathrm{E}}\right)$. Hence, solving the steady state involves solving for v, s^{I} and $\hat{s}^{\mathrm{E}}$ from the equations (B40), (B41) and (B42). Once v, s^{I} and $\hat{s}^{\mathrm{E}}$ are determined, all the other quantities can be obtained directly.

C   Appendix. Calibration

The model is solved via third-order perturbation around the stochastic steady state. Statistics we report in the tables and figures are computed based on 2,500 paths of quarterly simulated data. Each path is 500 quarters long after excluding the initial 50 quarters. Growth rates and returns are in logs. The innovation moments are reported quarterly only. All other moments are annualized. Growth rates and returns are annualized by summing up 4 consecutive quarterly observations. Standard deviations of quantities in levels and expected growth rates are annualized by multiplying quarterly standard deviation by $\sqrt{4}$.

Parameter values we use in simulations of our model are summarized in Table 1. We set the preference parameters to standard values used in the finance literature that employs recursive preferences (Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2010)). In particular, we closely follow Kung and Schmid (2013) who show that augmenting a standard endogenous growth model with aggregate risk and applying recursive preferences can jointly capture the dynamics of aggregate quantities and asset markets. We target consumption growth volatility and market risk premium to the values reported by Benzoni, Collin-Dufresne, and Goldstein (2011) for the sample including the Great Depression, i.e., the period 1929–2008.

In our empirical analysis, we estimate the sizes of entrant and incumbent innovation to be $\kappa_{\mathrm{E}} = 2.890$, and $\kappa_{\mathrm{I}} = 1.355$, respectively. Furthermore, our estimate of the depreciation rate of technology capitalis $\kappa_{\mathrm{D}} = 0.966$, consistent with the value of the patent protection in Kung and Schmid (2013). We check that the limit-pricing condition $\kappa_{\mathrm{E}} \geq \nu^{\frac{1}{\nu-1}}$ is satisfied in all our calibrations. The intermediate goods share is chosen to satisfy the balanced growth condition (A14), hence $\xi = \frac{1-\alpha}{\nu-\alpha}$ for given parameters α and ν.

We target the mean rate of radical innovation to a level of 10.67 percent, which is the value we compute using the universe of patents awarded by the The United States Patent and Trademark Office (USPTO) from January 1975 to December 2013. To measure the rate of radical innovation, we compute the ratio of (i) the number of US patents applied for, in a given quarter, by firms that did not patent prior to the beginning of this quarter (i.e., by "successful entrants" in innovation as of the beginning of this quarter) to (ii) the total number of US patents applied for by all firms in the same quarter. The resulting quarterly time series of the rate of radical innovation starts with the first quarter of 1985 and ends with the last quarter of 2008.

We start in 1985 because data on awarded patents are available since 1976 and, for the first quarter of 1985, we define the successful entrants in innovation based on at least 10 years of data prior to this quarter. For all consecutive quarters, we gradually expand the window over which we define successful entrants until the beginning of the respective quarter. We stop in 2008, because many patents applied in 2009 and later are still in the patent prosecution process and it is not clear whether they will be awarded. The time-average of our quarterly rate of radical innovation is 10.67 percent over the 1985-2008 period.

To achieve the targeted mean quarterly rate of radical innovation $\Gamma_t = 10.67\%$ and a mean consumption growth rate of 1.89%, we change parameters ω_{E} and ω_{I} so that consumption growth is equal to 0.4725 percent quarterly (i.e., annual growth rate of 1.89 percent) and that the rate of radical innovation is 10.67%. Specifically, we use the following two conditions to restrict parameter values in the deterministic steady state system B.3

$$1 + 0.00425 = \kappa_{\mathrm{I}}\,\phi_{\mathrm{I}}(s_t^{\mathrm{I}}) + \kappa_{\mathrm{E}}\,\hat{s}_t^{\mathrm{E}}\,\phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}}) + \kappa_{\mathrm{D}}(1 - \phi_{\mathrm{I}}(s_t^{\mathrm{I}}) - \hat{s}_t^{\mathrm{E}}\,\phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})), \tag{C1}$$

$$0.1067 = \frac{\hat{s}_t^{\mathrm{E}}\,\phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})}{\phi_{\mathrm{I}}(s_t^{\mathrm{I}}) + \hat{s}_t^{\mathrm{E}}\,\phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})}, \tag{C2}$$

where $\phi_{\mathrm{I}}(s_t^{\mathrm{I}})$ and $\phi_{\mathrm{E}}(\hat{s}_t^{\mathrm{E}})$ are given in (10).

Table 1: Calibration

This table reports the parameters used in the quarterly calibration of the benchmark model of Section 2 and of the model with stochastic barriers to entry discussed in Section 4.5.

Parameter	Description	Constant barriers to entry	Stochastic barriers to entry	
			ω_{E}	η_{E}
β	Time preference parameter	$\sqrt[4]{0.984}$	$\sqrt[4]{0.984}$	$\sqrt[4]{0.984}$
γ	Risk aversion	10	10	10
ψ	Elasticity of intertemporal substitution	1.85	1.85	1.85
α	Capital share	0.35	0.35	0.35
ξ	Intermediate goods share	0.72	0.72	0.72
ν	Markup	1.25	1.25	1.25
μ	Marginal cost of producing an intermediate good	1	1	1
δ	Depreciation rate of physical capital	0.02	0.02	0.02
ζ	Investment adjustment costs parameter	0.7	0.7	0.7
ρ	Autocorrelation of A_t	$\sqrt[4]{0.95}$	$\sqrt[4]{0.95}$	$\sqrt[4]{0.95}$
σ_a	Volatility of exogenous shock ε_{t+1}	2.31%	2.28%	2.28%
κ_{I}	Size of incumbents' incremental innovation	1.355	1.355	1.355
κ_{E}	Size of entrants' radical innovation	2.890	2.890	2.890
κ_{D}	Depreciation rate of technology capital	0.966	0.966	0.966
η_{I}	Incumbents' R&D shift parameter	1.50	1.74	1.74
η_{E}	Entrants' R&D shift parameter	0.18	0.225	—
ω_{I}	Incumbents' R&D elasticity parameter	0.7483	0.7897	0.7897
ω_{E}	Entrants' R&D elasticity parameter	0.7808	—	0.8324
$\overline{\omega_{\mathrm{E}}}$	Long run mean of $\omega_{\mathrm{E},t}$ in (39)	—	0.8324	—
$\rho_{\omega_{\mathrm{E}}}$	Autocorrelation of $\omega_{\mathrm{E},t}$ in (39)	—	0.9642	—
$\sigma_{\omega_{\mathrm{E}}}$	Volatility of exogenous shock $\varepsilon^{\omega_{\mathrm{E}}}_{t+1}$ in (39)	—	0.296%	—
$\overline{\eta_{\mathrm{E}}}$	Long run mean of $\eta_{\mathrm{E},t}$ in (40)	—	—	0.225
$\rho_{\eta_{\mathrm{E}}}$	Autocorrelation of $\eta_{\mathrm{E},t}$ in (40)	—	—	0.9642
$\sigma_{\eta_{\mathrm{E}}}$	Volatility of exogenous shock $\varepsilon^{\eta_{\mathrm{E}}}_{t+1}$ in (40)	—	—	0.270%

Table 2: Macroeconomic quantities

In the "Consumption" panel we report statistics for consumption growth dynamics. $\mathbb{E}[\Delta C]$ denotes expected consumption growth; $\sigma(\Delta C)$ is short run volatility of consumption; $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$ denotes volatility of expected consumption growth, and $\partial\sigma_t/\partial a_t$ is a measure of the degree of countercyclicality of economic uncertainty, calculated as the change of economic uncertainty in percentage points after a 1 standard deviation shock to the forcing process a_t. In the "Creative destruction" panel we report statistics for the dynamic of innovation. $\phi_I(s^I)$ and $\hat{\phi}_E(\hat{s}^E)$ are the innovation intensities of incumbents and entrants, respectively; Γ is the rate of radical innovation defined in (35); σ_{ϕ_I} and $\sigma_{\hat{\phi}_E}$ are the volatilities of the innovation intensities; $\rho_{\phi_I,\hat{\phi}_E}$ is the correlation between incumbents' and entrants' innovation intensities, and σ_Γ and $AC1(\Gamma)$ denote the volatility and first order autocorrelation of the rate of radical innovation. The entrants' growth share is computed from equation (C1) as $s^E\phi_E(\kappa_E - \kappa_D)/(1 + \mathbb{E}[\Delta C]/4 - \kappa_D)$. The moments have been obtained by simulating the model of Section 2. To match the short run volatility $\sigma(\Delta C)$ in the homogenous innovation economies, we vary the volatility σ_a of the shock ε_{t+1} from its benchmark value of Table 1. The moments have been obtained by simulating 2,500 economies each containing a time series of 500 quarters after excluding the initial 50 quarters. Details of the calibration and simulations are described in Appendix C. Moments in the Consumption panel are annual. Moments in the Innovation panel are quarterly.

	Data	Heterogenous innovations	Only incumbents innovate		Only entrants innovate	
		$\eta_I = 1.5000,\ \eta_E = 0.1800$ $\omega_I = 0.7483,\ \omega_E = 0.7808$	$\eta_I = 1.3455$ $\omega_I = 0.7483$	$\eta_I = 1.5000$ $\omega_I = 0.7807$	$\eta_E = 0.2985$ $\omega_E = 0.6341$	$\eta_E = 0.1800$ $\omega_E = 0.6334$
Consumption						
$\mathbb{E}[\Delta C]$	1.89%	1.89%	1.89%	1.89%	1.89%	1.89%
$\sigma(\Delta C)$	2.21%	2.21%	2.21%	2.21%	2.21%	2.21%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	—	0.428%	0.556%	0.595%	0.354%	0.230%
$\partial\sigma_t/\partial a_t$	—	-0.52	-0.69	-0.62	-0.40	-0.58
Creative destruction						
$\mathbb{E}[\Gamma_t]$	10.67%	10.67%	100.00%	100.00%	100.00%	100.00%
$\mathbb{E}[\phi_I(s_t^I)]$	5.58%	6.21%	9.94%	9.91%	—	—
$\mathbb{E}[\hat{\phi}_E(\hat{s}_t^E)]$	0.68%	0.74%	—	—	1.99%	1.99%
σ_{ϕ_I}	0.50%	0.45%	0.98%	0.93%	—	—
$\sigma_{\hat{\phi}_E}$	0.18%	0.06%	—	—	0.14%	0.14%
$\rho_{\phi_I,\hat{\phi}_E}$	0.76	1.00	—	—	—	—
σ_Γ	2.19%	0.14%	0.00%	0.00%	0.00%	0.00%
$AC1(\Gamma_t)$	0.969	0.977	—	—	—	—
Entrants' growth share	—	34.28%	0.00%	0.00%	100.00%	100.00%

Table 3: Asset pricing implications

This table reports asset pricing statistics: the mean, $\mathbb{E}[r_f]$, and volatility, $\sigma(r_f)$, of the risk-free rate; the mean, $\mathbb{E}[r_c - r_f]$, and volatility, $\sigma(r_c - r_f)$, of the excess return on the consumption claim; and the mean, $\mathbb{E}[r_m - r_f]$, and volatility, $\sigma(r_m - r_f)$, of the excess return on the market portfolio, i.e., the claim to aggregate dividends, defined in (A17); the mean, $E[r_d - r_f]$, and volatility, $\sigma(r_d - r_f)$, of the excess return on the claim to final good firm's dividends, defined in (6), and the mean, $\mathbb{E}[r_I - r_f]$, and volatility, $\sigma(r_I - r_f)$ of the incumbent firm's excess returns, defined in Appendix A.2. Calibration parameters are in Table 1. The moments have been obtained by simulating 2,500 economies each containing a time series of 500 quarters after excluding the initial 50 quarters. Details of the calibration and simulations are contained in Appendix C. The model statistics correspond to annualized populations moments.

	Data	Heterogenous innovations $\eta_{\rm I} = 1.5000, \eta_{\rm E} = 0.1800$ $\omega_{\rm I} = 0.7483, \omega_{\rm E} = 0.7808$	Only incumbents innovate $\eta_{\rm I} = 1.3455$ $\omega_{\rm I} = 0.7483$	$\eta_{\rm I} = 1.5000$ $\omega_{\rm I} = 0.7807$	Only entrants innovate $\eta_{\rm E} = 0.2985$ $\omega_{\rm E} = 0.6341$	$\eta_{\rm E} = 0.1800$ $\omega_{\rm E} = 0.6334$
$\mathbb{E}[r_f]$	0.6%	0.8%	0.4%	0.3%	1.1%	1.5%
$\sigma(r_f)$	3.9%	0.5%	0.6%	0.7%	0.4%	0.2%
$\mathbb{E}[r_c - r_f]$	—	6.6%	10.2%	11.0%	5.2%	3.3%
$\sigma(r_c - r_f)$	—	6.3%	6.6%	6.6%	6.0%	5.5%
$\mathbb{E}[r_m - r_f]$	6.7%	6.7%	4.8%	5.5%	5.5%	3.4%
$\sigma(r_m - r_f)$	20.5%	6.5%	4.8%	5.3%	6.5%	5.7%
$E[r_d - r_f]$	—	5.2%	6.9%	7.3%	4.2%	3.0%
$\sigma(r_d - r_f)$	—	7.2%	7.9%	8.0%	6.6%	6.0%
$\mathbb{E}[r_I - r_f]$	—	1.3%	2.1%	2.0%	0.9%	1.0%
$\sigma(r_I - r_f)$	—	1.9%	2.5%	2.2%	1.4%	2.0%

Table 4: Comparative statics

This table reports comparative statics analysis of the growth rate $\mathbb{E}[\Delta C]$, level of radical innovation $\mathbb{E}[\Gamma_t]$ and long run risk $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$, around the benchmark calibration of Table 1. Panel A reports the effect of varying preference parameters, Panel B reports the effect of varying technology parameters, and Panel C reports the effect of varying the market power of incumbents.

	Panel A: Preferences parameters				
β	0.9950	0.9955	**0.9960**	0.9965	0.9970
$\mathbb{E}[\Delta C]$	1.31%	1.61%	**1.90%**	2.23%	2.58%
$\mathbb{E}[\Gamma_t]$	10.60%	10.64%	**10.67%**	10.71%	10.75%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.42%	0.43%	**0.44%**	0.45%	0.46%
ψ	1.70	1.80	**1.85**	1.90	2.00
$\mathbb{E}[\Delta C]$	1.89%	1.90%	**1.90%**	1.89%	1.89%
$\mathbb{E}[\Gamma_t]$	10.67%	10.67%	**10.67%**	10.67%	10.67%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.44%	0.44%	**0.44%**	0.44%	0.44%
γ	6	8	**10**	12	14
$\mathbb{E}[\Delta C]$	1.90%	1.90%	**1.90%**	1.89%	1.89%
$\mathbb{E}[\Gamma_t]$	10.67%	10.67%	**10.67%**	10.67%	10.67%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.45%	0.44%	**0.44%**	0.44%	0.44%
	Panel B: Innovation technology parameters				
η_{I}	1.4000	1.4500	**1.5000**	1.5500	1.6000
$\mathbb{E}[\Delta C]$	0.66%	1.26%	**1.90%**	2.58%	3.35%
$\mathbb{E}[\Gamma_t]$	13.75%	12.10%	**10.67%**	9.43%	8.35%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.39%	0.41%	**0.44%**	0.47%	0.50%
η_{E}	0.1600	0.1700	**0.1800**	0.1900	0.2000
$\mathbb{E}[\Delta C]$	2.26%	2.06%	**1.90%**	1.79%	1.76%
$\mathbb{E}[\Gamma_t]$	6.75%	8.57%	**10.67%**	13.04%	15.67%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.47%	0.45%	**0.44%**	0.43%	0.43%
ω_{I}	0.7350	0.7450	**0.7483**	0.7550	0.7650
$\mathbb{E}[\Delta C]$	2.92%	2.15%	**1.90%**	1.42%	0.77%
$\mathbb{E}[\Gamma_t]$	9.12%	10.25%	**10.67%**	11.58%	13.14%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.46%	0.45%	**0.44%**	0.43%	0.41%
ω_{E}	0.7400	0.7600	**0.7808**	0.8000	0.8200
$\mathbb{E}[\Delta C]$	1.75%	1.76%	**1.90%**	2.14%	2.53%
$\mathbb{E}[\Gamma_t]$	18.70%	14.43%	**10.67%**	7.79%	5.34%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.41%	0.42%	**0.44%**	0.46%	0.48%
	Panel C: Market power				
ν	1.2000	1.2250	**1.2500**	1.2600	1.2700
$\mathbb{E}[\Delta C]$	-1.25%	0.34%	**1.90%**	2.52%	3.18%
$\mathbb{E}[\Gamma_t]$	10.27%	10.48%	**10.67%**	10.74%	10.82%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	0.33%	0.38%	**0.44%**	0.46%	0.48%

Table 5: Macroeconomic quantities: Stochastic barriers to entry

In the "Consumption" panel we report statistics for consumption growth dynamics. $\mathbb{E}[\Delta C]$ denotes expected consumption growth; $\sigma(\Delta C)$ is short run volatility of consumption; $\sigma(\mathbb{E}_t[\Delta C_{t+1}])$ denotes volatility of expected consumption growth, and $\partial \sigma_t / \partial a_t$ is a measure of the degree of countercyclicality of economic uncertainty, calculated as the change of economic uncertainty in percentage points after a 1 standard deviation shock to the forcing process a_t. In the "Creative destruction" panel we report statistics for the dynamic of innovation. $\phi_{\mathrm{I}}(s^{\mathrm{I}})$ and $\hat{\phi}_{\mathrm{E}}(\hat{s}^{\mathrm{E}})$ are the innovation intensities of incumbents and entrants, respectively; Γ is the rate of radical innovation defined in (35); $\sigma_{\phi_{\mathrm{I}}}$ and $\sigma_{\hat{\phi}_{\mathrm{E}}}$ are the volatilities of the innovation intensities; $\rho_{\phi_{\mathrm{I}}, \hat{\phi}_{\mathrm{E}}}$ is the correlation between incumbents' and entrants' innovation intensities and σ_Γ and $AC1(\Gamma)$ denote the volatility and first order autocorrelation of the rate of radical innovation. The entrants' growth share is computed from equation (C1) as $s^{\mathrm{E}} \phi_{\mathrm{E}} (\kappa_{\mathrm{E}} - \kappa_{\mathrm{D}}) / (1 + \mathbb{E}[\Delta C]/4 - \kappa_{\mathrm{D}})$. The moments have been obtained simulating the extended model of Section 4.5 which incorporates stochastic barriers to entry. Details of the calibration and simulations are in Appendix C. Calibration parameters are in Table 1. The moments have been obtained by simulating 2,500 economies each containing a time series of 500 quarters after excluding the initial 50 quarters. Moments in the Consumption panel are annual. Moments in the Innovation panel are quarterly.

| | | Stochastic ω_{E} | | | Stochastic η_{E} | | |
| | | $corr(\varepsilon_a, \varepsilon_{\omega_{\mathrm{E}}})$ | | | $corr(\varepsilon_a, \varepsilon_{\eta_{\mathrm{E}}})$ | | |
	Data	-0.4	0.0	0.4	-0.4	0.0	0.4
Consumption							
$\mathbb{E}[\Delta C]$	1.89%	2.03%	1.89%	1.75%	1.75%	1.89%	2.03%
$\sigma(\Delta C)$	2.21%	2.01%	2.21%	2.39%	2.41%	2.21%	2.01%
$\sigma(\mathbb{E}_t[\Delta C_{t+1}])$	—	0.497%	0.467%	0.435%	0.431%	0.473%	0.505%
$\partial \sigma_t / \partial a_t$	—	-0.41	-0.42	-0.42	-0.42	-0.42	-0.41
Creative destruction							
$\mathbb{E}[\Gamma_t]$	10.67%	10.68%	10.67%	10.64%	10.64%	10.67%	10.70%
$\mathbb{E}[\phi_{\mathrm{I}}(s_t^{\mathrm{I}})]$	5.58%	6.31%	6.24%	6.18%	6.16%	6.22%	6.28%
$\mathbb{E}_t[\hat{\phi}_{\mathrm{E}}(\hat{s}^{\mathrm{E}})]$	0.68%	0.74%	0.74%	0.73%	0.73%	0.74%	0.75%
$\sigma_{\phi_{\mathrm{I}}}$	0.50%	0.55%	0.69%	0.81%	0.80%	0.69%	0.55%
$\sigma_{\hat{\phi}_{\mathrm{E}}}$	0.18%	0.15%	0.13%	0.10%	0.10%	0.13%	0.15%
$\rho_{\phi_{\mathrm{I}}, \hat{\phi}_{\mathrm{E}}}$	0.76	-0.25	-0.28	-0.34	-0.34	-0.28	-0.25
σ_Γ	2.19%	2.27%	2.19%	2.12%	2.12%	2.19%	2.28%
$AC1(\Gamma_t)$	0.969	0.960	0.959	0.959	0.959	0.960	0.960

Table 6: Asset pricing implications. Stochastic barriers to entry

This table reports asset pricing moments: the mean, $\mathbb{E}[r_f]$, and volatility, $\sigma(r_f)$, of the risk-free rate; the mean, $\mathbb{E}[r_c - r_f]$, and volatility, $\sigma(r_c - r_f)$, of the excess return on the consumption claim; and the mean, $\mathbb{E}[r_m - r_f]$, and volatility, $\sigma(r_m - r_f)$, of the excess return on the market portfolio, i.e., the claim to aggregate dividends, defined in (A17); the mean, $E[r_d - r_f]$, and volatility, $\sigma(r_d - r_f)$, of the excess return on the claim to final good firm's dividends, defined in (6), and the mean, $\mathbb{E}[r_I - r_f]$, and volatility, $\sigma(r_I - r_f)$ of the incumbent firm's excess returns, defined in Appendix A.2. The moments have been obtained simulating the extended model of Section 4.5 which incorporates stochastic barriers to entry. Details of the calibration and simulations are in Appendix C. Calibration parameters are in Table 1. The moments have been obtained by simulating 2,500 economies each containing a time series of 500 quarters after excluding the initial 50 quarters. Details of the calibration and simulations are contained in Appendix C. The model statistics correspond to annualized populations moments.

	Data	Stochastic ω_{E} $corr(\varepsilon_a, \varepsilon_{\omega_{\mathrm{E}}})$			Stochastic η_{E} $corr(\varepsilon_a, \varepsilon_{\eta_{\mathrm{E}}})$		
		-0.4	0.0	0.4	-0.4	0.0	0.4
$\mathbb{E}[r_f]$	0.6%	1.0%	0.9%	0.7%	0.7%	0.9%	1.0%
$\sigma(r_f)$	3.9%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%
$\mathbb{E}[r_c - r_f]$	—	6.2%	6.6%	7.1%	7.0%	6.6%	6.2%
$\sigma(r_c - r_f)$	—	6.1%	6.3%	6.5%	6.5%	6.3%	6.0%
$\mathbb{E}[r_m - r_f]$	6.7%	6.7%	6.7%	6.8%	6.8%	6.7%	6.7%
$\sigma(r_m - r_f)$	20.5%	6.7%	6.5%	6.3%	6.3%	6.5%	6.7%
$E[r_d - r_f]$	—	5.1%	5.2%	5.4%	5.4%	5.2%	5.1%
$\sigma(r_d - r_f)$	—	7.1%	7.2%	7.3%	7.3%	7.2%	7.1%
$\mathbb{E}[r_I - r_f]$	—	0.51%	1.09%	1.67%	1.65%	1.08%	0.52%
$\sigma(r_I - r_f)$	—	2.06%	2.62%	3.08%	3.07%	2.58%	2.07%

Figure 1: Creative destruction

The Figure plots the quarterly time series of the number of entrant in innovation (left axis) and the rate of radical innovation Γ_t (right axis). Entrants at time t are firms that first applied for a patent at that time. The rate of radical innovation is defined in in (35). Patent data are from the United States Patent and Trademark Office (USPTO).
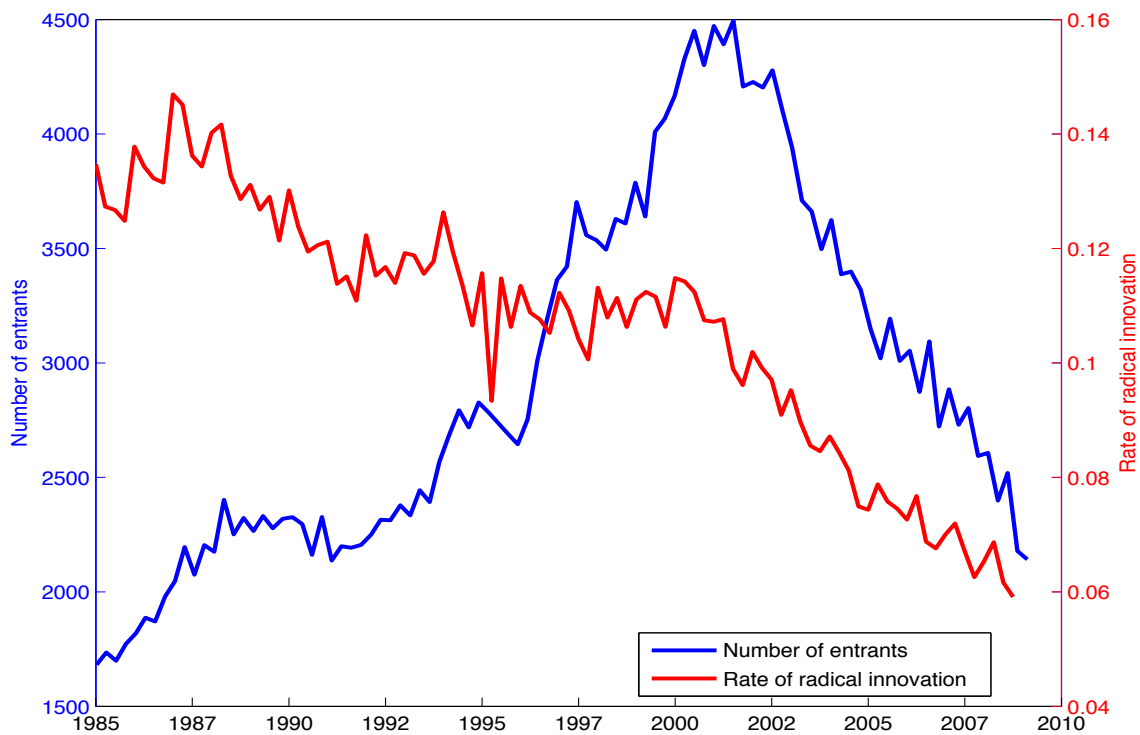
Figure 2: Incumbents' and entrants' R&D. Stochastic barrier to entry

The figure reports impulse response functions of incumbents' R&D, s^{I}, and entrants R&D, $\hat{s}^{\mathrm{E}}$, with respect to a one standard deviation shock in the forcing process, a_t, and in the elasticity ω_{E} of entrants' R&D technology. The value reported are log deviations from the steady state, in percent units.

Panel A: Incumbents' R&D
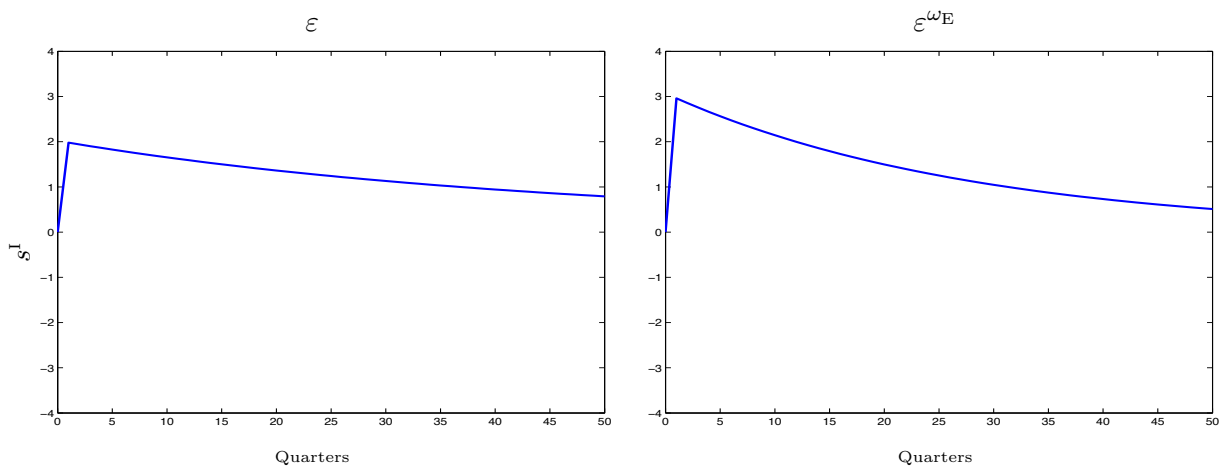
Panel B: Entrants' R&D
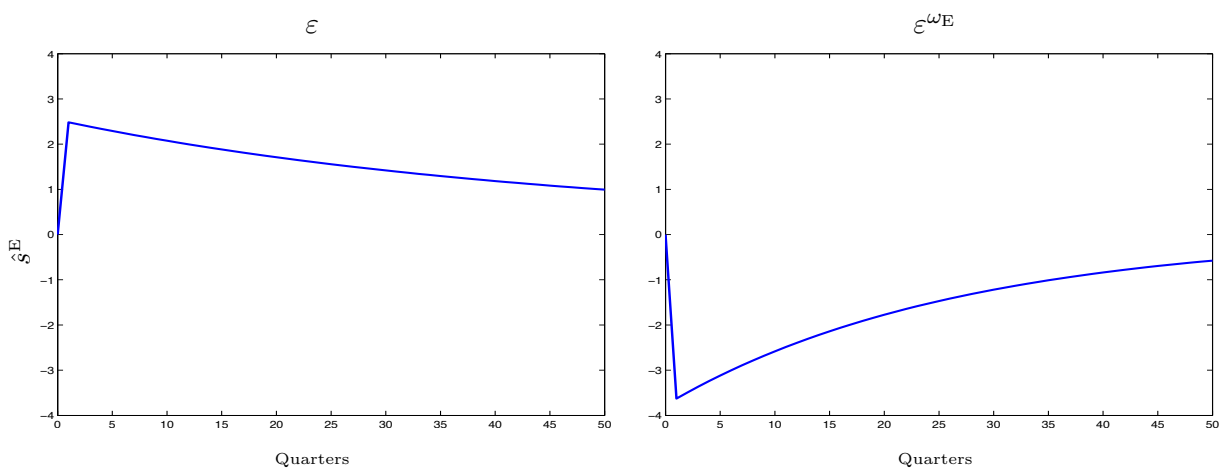
References

Acemoglu, Daron, 2010, *Introduction to Modern Economic Growth* (Princeton University Press).

———— , and Dan V. Cao, 2011, Innovation by Entrants and Incumbents, Working Paper.

Aghion, Philippe, and Peter Howitt, 1992, A Model of Growth Through Creative Destruction, *Econometrica* 60, 323–351.

———— , 1998, *Endogenous Growth Theory* (MIT Press).

———— , 2009, *The Economics of Growth* (MIT Press).

Ai, Hengjie, Mariano Massimiliano Croce, and Kai Li, 2013, Toward a quantitative general equilibrium asset pricing model with intangible capital, *Review of Financial Studies* 26, 491–530.

Ai, Hengjie, and Dana Kiku, 2013, Growth to value: Option exercise and the cross section of equity returns, *Journal of Financial Economics* 107, 325–349.

Akcigit, Ufuk, and William R Kerr, 2010, Growth through heterogeneous innovations, Discussion paper National Bureau of Economic Research.

Anderson, Philip, and Michael L Tushman, 1990, Technological discontinuities and dominant designs: A cyclical model of technological change, *Administrative science quarterly* pp. 604–633.

Bansal, Ravi, Dana Kiku, and Amir Yaron, 2010, Long run risks, the macroeconomy, and asset prices, *American Economic Review* 100, 542–546.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Bartelsman, Eric J., and Mark Doms, 2000, Understanding Productivity: Lessons from Longitudinal Microdata, *Journal of Economic Literature* 38, 569–594.

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Goldstein, 2011, Explaining asset pricing puzzles asssociated with the 1987 market crash, *Journal of Financial Economics* 101, 552–573.

Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott, 2010, Multiple-Product Firms and Product Switching, *American Economic Review* 100, 70–97.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz, 2012, Endogenous entry, product variety and business cycles, *Journal of Political Economy* 120, 304–345.

Blundell, Richard, Rachel Griffith, and John Van Reenen, 1995, Dynamic Count Data Models of Technological Innovation, *Economic Journal* 105, 333–344.

———— , 1999, Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms, *Review of Economic Studies* 66, 529–554.

Broda, Christian, and David E. Weinstein, 2010, Product Creation and Destruction: Evidence and Price Implications, *American Economic Review* 100, 691–723.

Comin, Diego A, Mark Gertler, and Ana Maria Santacreu, 2009, Technology innovation and diffusion as sources of output and asset price fluctuations, Discussion paper National Bureau of Economic Research.

Fernald, John, 2012, A quarterly, utilization-adjusted series on total factor productivity, *Manuscript, Federal Reserve Bank of San Francisco.*

Foster, Lucia, John C Haltiwanger, and Cornell John Krizan, 2001, Aggregate productivity growth. Lessons from microeconomic evidence, in *New developments in productivity analysis* . pp. 303–372 (University of Chicago Press).

Freeman, Christopher, and Luc LG Soete, 1997, *The economics of industrial innovation* (Psychology Press).

Gârleanu, Nicolae, Leonid Kogan, and Stavros Panageas, 2012, Displacement risk and asset returns, *Journal of Financial Economics* 105, 491–510.

Gârleanu, Nicolae, Stavros Panageas, and Jianfeng Yu, 2012, Technological growth and asset pricing, *Journal of Finance* 67, 1265–1292.

Greenwood, Jeremy, and Boyan Jovanovic, 1999, The information-technology revolution and the stock market, *American Economic Review (Papers and Proceedings)* 89.

Grossman, Gene M., and Elhanan Helpman, 1991, Quality Ladders in the Theory of Growth, *Review of Economic Studies* 58, 43–61.

Hobijn, Bart, and Boyan Jovanovic, 2001, The Information-Technology Revolution and the Stock Market: Evidence, *American Economic Review* 91, 1203–1220.

Jermann, Urban J., 1998, Asset Pricing in production economies, *Journal of Monetary Economics* 41, 257–275.

Klette, Tor J., and Samuel Kortum, 2004, Innovating Firms and Aggregate Innovation, *Journal of Political Economy* 112, 986–1018.

Kogan, Leonid, Dimitris Papanikolaou, and Noah Stoffman, 2013, Technological innovation: Winners and losers, Discussion paper National Bureau of Economic Research.

Kung, Howard, and Lukas Schmid, 2013, Innovation, growth, and asset prices, *Journal of Finance,* forthcoming.

Loualiche, Erik, 2013, Asset Pricing with Entry and Imperfect Competition, Working Paper.

Nickell, Stephen J., 1996, Competition and Corporate Performance, *Journal of Political Economy* 104, 724–746.

Pástor, Lubos, and Pietro Veronesi, 2009, Technological Revolutions and Stock Prices, *American Economic Review* 99, 1451–1483.

Pennings, Johannes M, and Arend Buitendam, 1987, *New technology as organizational innovation: The development and diffusion of microelectronics* (Ballinger Pub Co).

Romer, Paul M., 1990, Endogenous Technological Change, *Journal of Political Economy* 98, 71–102.

Scherer, Frederic M, 1984, *Innovation and growth: Schumpeterian perspectives* . , vol. 1 (The MIT Press).

Schumpeter, Joseph, 1942, Creative destruction, *Capitalism, socialism and democracy.*

Schumpeter, Joseph A, 1934, *The Theory of Economic Development* (Harvard University Press).