

The Levered Equity Risk Premium and Credit Spreads: A Unified Framework*

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Abstract

Much empirical work indicates that there are common factors that drive the equity risk premium and credit spreads. In this paper, we embed a structural model of credit risk inside a dynamic consumption-based asset pricing model. That allows us to price equity, default-risky debt and study the co-movement of stock and bond price variables in a single framework. This paves the way for a unified understanding of what drives the equity risk premium and credit spreads. Our key economic assumptions are that the first and second moments of macroeconomic variables, such as earnings and consumption growth, depend on the state of the economy which switches randomly; agents prefer uncertainty about the future state of the economy to be resolved sooner rather than later; they optimally choose capital structure and default times. Under these assumptions the model generates co-movement between aggregate stock return volatility and credit spreads, which is quantitatively consistent with the data, and resolves the equity risk premium and credit spread puzzles. For relative risk aversion of 10 and elasticity of intertemporal substitution of 1.5, the model implies a levered equity risk premium of about 4.5%, credit spreads for Baa debt of 180 basis points, and a model-implied 4-year actual default probability of about 1.5%, which is realistically small.

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A growing body of empirical work indicates that common factors may affect both the equity risk premium and credit spreads on corporate bonds. In particular, there is now substantial evidence that stock returns can be predicted by credit spreads,¹ and that movements in stock-return volatility can explain movements in credit spreads. For example, Tauchen and Zhou (2006) show from a regression of the Moody's Baa bond spread index against the jump-component in the volatility of returns on the S&P500, that a 1 percentage point increase in jump volatility raises spreads by about 190 basis points. In essence, all these results demonstrate that there is “*overlap between the stochastic processes for bond and stock returns*” (Fama and French (1993, p. 26, our emphasis)).

The aim of this paper is to investigate two important ramifications of these results. First, the existence of common factors indicate that the two well-known puzzles, the equity risk-premium puzzle and the credit risk puzzle, are inherently linked. Much research has been devoted to finding an explanation of the equity premium puzzle since the seminal paper of Mehra and Prescott (1985) and the credit risk puzzle has been the subject of a great deal of attention since the first empirical evidence that contingent-claim models of defaultable debt underpredict credit spreads,² but there has been limited research on linking the two puzzles. Second, there is evidence suggesting that common factors are likely to be related to fundamental macroeconomic risks. For example, Bansal and Yaron (2004) show that a substantial fraction of the equity risk premium could stem from exposure to long-run fluctuations in macroeconomic growth rates, but do not study credit spreads. In credit risk, Collin-Dufresne, Goldstein, and Martin (2001) show that credit spread changes across firms are driven by a single factor.

In this paper, we exploit an economically intuitive macroeconomic mechanism related to the business cycle to generate a common factor linking *both* stock returns and credit spreads. We then use this mechanism to explain why stock-return volatility co-moves with credit spreads and resolve both the equity risk premium and credit spread puzzles.

In a nutshell, two main ideas underpin our approach. The first is that *any claim*, equity and debt alike, can be priced in a consumption-based asset pricing model. The second is *intertemporal macroeconomic risk*: the expected values and volatilities (first and second moments) of fundamental economic growth rates vary with the business cycle, which is modeled by a regime-switching process.

We use the first idea to price corporate bonds in a consumption-based asset pricing model with a representative agent. In particular, we assume aggregate consumption consists of wages paid to labor and firms' earnings, and the division between wages and earnings is exogenous. Earnings are divided into coupon

¹See Chen, Roll, and Ross (1986), Keim and Stambaugh (1986), Campbell (1987), Harvey (1989), Fama and French (1989), Fama and French (1993), Ferson and Harvey (1991), Campbell and Ammer (1993), Whitelaw (1994), Jagannathan and Wang (1996), Ferson and Harvey (1999) and Cremers (2002).

²The credit risk puzzle refers to the finding that structural models of credit risk generate credit spreads smaller than those observed in the data when calibrated to observed default frequencies. Recent evidence is presented in Eom, Helwege, and Huang (1999), Ericsson and Reneby (2003) and Huang and Huang (2003).

payments to bondholders and dividends to equityholders. Capital structure is chosen optimally by equityholders to maximize firm value which implies the endogeneity of both coupons and dividends. In addition, equityholders choose a default boundary to maximize equity value so that the default boundary is also endogenous. Thus, in our model, the prices of equity and debt are not only linked by a common state-price density, but they are also affected by the optimal leverage and default decisions. Essentially, we embed the contingent-claim models of Fischer, Heinkel, and Zechner (1989) and Leland (1994) inside an equilibrium consumption-based model.³ We call the resulting framework a *structural-equilibrium* model.⁴

We then use the second idea and introduce intertemporal macroeconomic risk into our structural-equilibrium model to capture a common macroeconomic factor that underlies both expected excess stock returns and credit spreads. Modelling of intertemporal macroeconomic risk hinges on several critical and intuitive features. Firstly, the properties of firms' earnings growth change with the state of the economy, with expected growth lower in recessions and volatility lower in booms. Secondly, the properties of consumption growth also change with the state of the economy. As expected, first moments are lower in recessions, whereas second moments are higher. We model switches in the state of the economy via a Markov chain.⁵ Thirdly, we assume that the representative agent cares about the intertemporal composition of risk. In particular, she prefers uncertainty about the future to be resolved sooner rather than later.⁶ In essence, she is averse to uncertainty about the future state of the economy. We model this by assuming that the representative agent has Epstein-Zin-Weil preferences.

The representative agent, of course, does not use actual probabilities to compute prices. Instead, she uses risk-neutral probabilities. It is well-known that for a risk-averse agent, the risk-neutral probability of a bad event occurring exceeds its actual probability. In the context of our model, asset prices will depend on the risk-neutral probability (per-unit time) of the economy moving from boom to recession. Increasing the risk-neutral probability of entering a recession increases the average duration of recessions in the risk-neutral world. When the average time spent in recessions in the risk-neutral world increases, it is intuitive that risk premia will go up. If the agent prefers earlier resolution of uncertainty, the risk-neutral probability of

³Since in contingent-claim models the state-price density is not linked to consumption, the asset prices they produce are completely divorced from macroeconomic variables, such as aggregate consumption. Consequently, these models alone cannot be used to find a macroeconomic explanation for a common factor behind stock returns and credit spreads.

⁴The germ of this idea is contained in within Goldstein, Ju, and Leland (2001). They state that their EBIT-based model can be embedded inside a consumption-based model, where the representative agent has power utility, though they do not investigate how credit spreads depend on the agent's risk aversion.

⁵See Hamilton (1989). For applications which study the stock market see Cecchetti, Lam, and Mark (1993), Whitelaw (2000), Calvet and Fisher (2005a), Calvet and Fisher (2005b) and Hansen, Heaton, and Li (2006). For option pricing applications see Jobert and Rogers (2006). The Markov chain approach to modelling intertemporal macroeconomic risk is a variant of the long-risk model of Bansal and Yaron (2004). We summarize the key economic differences in Section I.C.

⁶Kreps and Porteus (1978, p. 186) explain the intuition for modelling preferences in this way via a coin-flipping example: "If . . . the coin flip determines your income for the next two years, you probably prefer to have the coin flipped now, so that you are better able to budget your income for consumption purposes."

entering a recession exceeds the actual probability. Consequently, the agent prices assets as if recessions last longer than is actually the case, which raises risk premia.

The same mechanism delivers high credit spreads. To see the intuition, observe that the credit spread on default risky debt can be written as

$$s = r \frac{lq_D}{1 - lq_D},$$

where r is the risk-free rate, l is the loss ratio for the bond (which gives the proportional loss in value if default occurs) and q_D is the price of the Arrow-Debreu security which pays out 1 unit of consumption at default. Empirically, both the risk-free rate and loss ratio are too low to explain credit spreads. Thus, any economic channel which generates realistic credit spreads must raise the value of the Arrow-Debreu default claim. One of the novel results of this paper shows how q_D can be decomposed into three factors, each with an economically intuitive meaning:

$$q_D = \mathcal{T} \mathcal{R} p_D,$$

where p_D is the actual probability of default, \mathcal{T} is a downward adjustment for the time value of money and \mathcal{R} is an adjustment for risk. Actual default probabilities are small. Our decomposition then tells us that the value of the Arrow-Debreu default claim will be high if the risk-adjustment, \mathcal{R} , and the time-adjustment, \mathcal{T} , are high. So, why are they high in our model?

It is well known from Weil (1989) that using Epstein-Zin-Weil preferences makes it possible to obtain a low risk-free rate, simply by increasing the elasticity of intertemporal substitution. When the risk-free rate is low, the discount factor associated with the time-value of money will be high. Therefore, the time-adjustment factor, \mathcal{T} , is high. This happens even if there is no intertemporal macroeconomic risk. Combining intertemporal macroeconomic risk with Epstein-Zin-Weil preferences increases the risk-neutral probability of entering a recession, which increases the risk-adjustment factor, \mathcal{R} . Thus, our model can generate high credit spreads, while keeping the actual probability of default low, as observed in the data.

Since the same economic mechanism increases both credit spreads and the risk premium, co-movement arises naturally between equity and corporate bond market values. In particular, our model generates co-movement between credit spreads and stock return volatility as observed by Tauchen and Zhou (2006).

We now preview our quantitative results. For the benchmark case of relative risk aversion equal to 10 and an elasticity of intertemporal substitution of 1.5, the model delivers a credit spread of between 180 and 220 basis points (depending on the initial and current states of the economy), when the model-implied 4-year actual default probability is realistically low (slightly less than 1.5%, see Huang and Huang (2003)). As macroeconomic conditions change, the levered equity risk premium varies between 3% and 6.5%, and risk-free rate between 1.7% and 3.6%, which are close to empirical estimates (see Mehra and Prescott (1985), Weil (1989) and Hansen and Jagannathan (1991)). Finally, the optimal static leverage ratio is between 37% and 48%, lower than in most models of static capital structure (e.g. Leland (1994)).

Our framework also delivers a number of testable implications. From an asset pricing perspective, the most important implication concerns the cyclicity of the default boundary. When expressed in cash flow terms the default boundary is countercyclical. But for values of risk aversion and elasticity of intertemporal substitution which generate a realistic equity premium and credit spread, the *asset-value* default boundary is procyclical. In contrast, Chen, Collin-Dufresne, and Goldstein (2006) show that in asset value terms, a habit formation model with i.i.d. consumption growth must have a countercyclical default boundary to generate a realistic credit spread. Thus, empiricists can study the default boundary to determine whether a model with intertemporal macroeconomic risk and Epstein-Zin-Weil preferences (essentially a variant of Bansal and Yaron’s long-run risk model) or a model with i.i.d. consumption and habit-formation preferences offers the more plausible framework for jointly resolving the equity risk premium and credit spread puzzles.

There are also myriad implications for corporate financing. For example, defaults cluster and also can occur simply because of worsening macroeconomic conditions, despite there being no change in earnings. Financing decisions are subject to hysteresis effects, i.e. the timing of past financing decisions influences default and leverage decisions, even though our model is fully rational. When capital structure is chosen in recessions, the optimal leverage ratio is lower. However, once leverage has been chosen, market leverage is lower in booms as in Korajczyk and Levy (2003). Taken together this implies that capital structures across firms co-move and the macroeconomic conditions at previous financing dates are cross-sectional determinants of current leverage ratios.

In the remainder of the introduction we discuss the relationship between our paper and the existing literature. To provide a bird’s eye view of the field, we present an “anthropological” table showing how the line of descent runs from previous papers to our paper (see Table I). On one side, our paper inherits features of structural models of credit risk (Merton (1974), Fischer, Heinkel, and Zechner (1989), Leland (1994), Goldstein, Ju, and Leland (2001), and Hackbarth, Miao, and Morellec (2006)). On the other side, our model is deeply indebted to a different set of forebears: consumption-based asset pricing models (Lucas (1978) and Bansal and Yaron (2004)).

We now discuss several papers with which our paper is particularly close. The contingent-claims, structural model that our paper is most closely related to is Hackbarth, Miao, and Morellec (2006).⁷ They also study the influence of macroeconomic factors on credit spreads. Importantly, they are the first to show that macroeconomic factors imply a countercyclical earnings default boundary. There are several key differences between our models. Firstly, Hackbarth, Miao, and Morellec do not use a state-price density linked to consumption and therefore do not study the equity risk premium and its relation to credit spreads. Also, their model does not allow them to check the size of actual default probabilities. Finally, while we study

⁷We have recently become aware of contemporaneous, but independent, work by Chen (2007), who uses a similar modelling framework to this paper. Chen (2007) seeks to resolve the low-leverage and credit spread puzzles, but does not address the issues of co-movement between bond and stock markets and the equity premium puzzle.

the impact of macroeconomic factors on both cash flows and discount rates, Hackbarth, Miao, and Morellec focus purely on the cash flow channel by assuming that firms' earnings levels jump down in recessions.

A second closely related paper is Chen, Collin-Dufresne, and Goldstein (2006). They study a pure consumption-based model and use two distinct mechanisms to resolve the equity risk premium and credit spread puzzles. The first mechanism is habit formation, which makes the marginal utility of wealth high enough in bad states so that the equity risk premium puzzle is resolved. This does not resolve the credit spread puzzle, because actual default probabilities and thus credit spreads are procyclical. To remedy this, Chen, Collin-Dufresne, and Goldstein use a second mechanism: they force the asset-value default boundary to be *exogenously countercyclical*. There are several key difference between our models. First, we only need one economic mechanism to generate a realistic credit spread and risk premium, as outlined above. Second, the asset-value default boundary in our model is *endogenously procyclical*. Third, the risk premium in our model is directly affected by default risk creating a *levered* risk premium and capital structure is endogenous. Finally, we obtain closed-form solutions for asset prices, which are natural extensions of the formulae in Leland (1994) and the Breeden-Lucas model (see Lucas (1978) and Breeden (1979)).

Another related paper on credit spreads is David (2007). David prices corporate debt in a model where the expected earnings growth rates and expected inflation follow a Markov switching process and are unobservable. This framework generates realistic credit spreads. Since David (2007) focuses on corporate bonds alone, he does not study the equity risk premium or co-movement between stock and bond markets.⁸ Furthermore, he does not endogenize corporate financing decisions.⁹

Our paper is not the first to consider default in a consumption-based model (see e.g. Alvarez and Jermann (2000), and Kehoe and Levine (1993)). These papers focus on default from the viewpoint of households. They assume households have identical preferences, but are subject to idiosyncratic income shocks. Households can default on payments in the same way that people cannot always pay back credit card debt or a mortgage. Chan and Sundaresan (2005) consider the bankruptcy of individuals in a production framework, looking at its impact on the equity risk premium and the term structure of risk-free bonds. Unlike the above papers, which look at personal bankruptcy, we look at firm bankruptcy and the pricing of corporate debt.

The remainder of the paper is organized as follows. Section I describes the structural-equilibrium model with intertemporal macroeconomic risk and Epstein-Zin-Weil preferences. Section II explores the implications of the model for pricing corporate debt and levered equity and develops an intuitive decomposition for the Arrow-Debreu default claim. Section III builds on Section II by calibrating the model. In Section IV,

⁸Since David (2007) restricts the state-price density to one that can be obtained from a representative agent with power utility, the shifts in growth rates are not priced. Clearly, it is possible to use this framework to study the equity risk premium. But it would be only be possible to generate a realistic premium with very high risk aversion.

⁹Tan and Yan (2006) use the same framework as David (2007), but with an observable mean-reverting growth rate for firm earnings and without explicitly accounting for inflation.

we strip down the model to see which assumptions drive which results. We conclude in Section V. Proofs and other additional material are contained in the Appendices.

I Model

In this section we introduce the structural-equilibrium model with intertemporal macroeconomic risk. The basic idea is simple: we embed a structural model inside a representative agent consumption-based model. That allows us to price debt and levered equity using the state-price density of the representative agent.

Two consequences of this modelling approach are worth noting. Credit spreads depend on the agent's preferences and aggregate consumption, which is not the case in pure structural models. The equity risk premium is affected by default risk, which is not the case in pure consumption-based models.

It is also important to mention what our structural-equilibrium model does not do. It does not account for the impact of default on consumption, because we model consumption as an exogenous process. Furthermore, our model ignores the impact of agency conflicts on the state-price density, because the state-price density in our model is the marginal utility of wealth of a representative agent. Incorporating these two important effects is beyond the scope of this paper.

We start by describing how firm earnings and aggregate consumption are modeled. Then we explain how we introduce intertemporal macroeconomic risk by making the first and second moments of firm earnings and aggregate consumption growth stochastic. We also give a brief description of the state-price density, which arises from our choice to use a representative agent with Epstein-Zin-Weil preferences. The main result of this section is Proposition 1, which explains how intertemporal macroeconomic risk combined with Epstein-Zin-Weil preferences causes the agent to price securities as if recessions were of longer duration than is actually the case. Intuitively, one would expect risk premia to be larger in an economy where recessions last longer, so Proposition 1 provides a natural explanation of how our model can generate reasonable risk premia. Finally, we give a precise quantitative definition of long-run risk.

I.A Aggregate Consumption and Firm Earnings

There are N firms in the economy. The output of firm n , Y_n , is divided between earnings, X_n , and wages, W_n , paid to workers. Aggregate consumption, C , is equal to aggregate output. Therefore,

$$C = \sum_{n=1}^N Y_n = \sum_{n=1}^N X_n + \sum_{n=1}^N W_n.$$

We model aggregate consumption and individual firm earnings directly, and aggregate wages, $\sum_{n=1}^N W_n$, are just the difference between aggregate consumption and aggregate earnings.¹⁰

Aggregate consumption, C , is given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_{C_t} dB_{C,t}, \quad (1)$$

where g is expected consumption growth, σ_C is consumption growth volatility and $B_{C,t}$ is a standard Brownian motion.

The earnings process for firm n is given by

$$\frac{dX_{n,t}}{X_{n,t}} = \theta_{n,t} dt + \sigma_{X,n}^{id} dB_{X,n,t}^{id} + \sigma_{X,n,t}^s dB_{X,t}^s, \quad (2)$$

where θ_n is the expected earnings growth rate of firm n , and $\sigma_{X,n}^{id}$ and $\sigma_{X,n}^s$ are, respectively, the idiosyncratic and systematic volatilities of the firm's earnings growth rate. Total risk, $\sigma_{X,n}$, is given by $\sigma_{X,n} = \sqrt{(\sigma_{X,n}^{id})^2 + (\sigma_{X,n}^s)^2}$. The standard Brownian motion $B_{X,t}^s$ is the systematic shock to the firm's earnings growth, which is correlated with aggregate consumption growth:

$$dB_{X,t}^s dB_{C,t} = \rho_{XC} dt, \quad (3)$$

where ρ_{XC} is the constant correlation coefficient. The standard Brownian motion $B_{X,n,t}^{id}$ is the idiosyncratic shock to firm earnings, which is correlated with neither $B_{X,t}^s$ nor $B_{C,t}$.

Importantly, to study credit spreads, we consider corporate bonds issued by individual firms, but to study the aggregate equity premium we consider the levered equity claim for the aggregate firm, whose earnings is equal to aggregate firm earnings. In Appendix A we state conditions under which we can obtain the aggregate premium simply by setting $\sigma_{X,n}^{id} = 0$. For ease of notation, we omit the subscript n in the remainder of the paper.

I.B Modelling Intertemporal Macroeconomic Risk

To introduce intertemporal macroeconomic risk into the structural-equilibrium model we assume that the first and second moments of macroeconomic growth rates are stochastic. Specifically, we assume that g , θ_t , $\sigma_{C,t}$ and $\sigma_{X,t}^s$ depend on the state of the economy, which follows a 2-state continuous-time Markov chain.¹¹ Hence, the conditional expected growth rate of consumption, g_t , can take two values, g_1 and g_2 , where g_i is the expected growth rate when the economy is in state i . Similarly for θ_t , $\sigma_{C,t}$ and $\sigma_{X,t}^s$.¹² State 1 is a

¹⁰In assuming so we follow such papers as Kandel and Stambaugh (1991), Cecchetti, Lam, and Mark (1993), Campbell and Cochrane (1999), Brennan and Xia (2001) and Bansal and Yaron (2004).

¹¹The extension to $L > 2$ states does not provide any further economic intuition and is straightforward.

¹²To ensure idiosyncratic earnings volatility, $\sigma_{X,t}^{id}$, is truly idiosyncratic, we assume it is constant and thus independent of the state of the economy.

recession and state 2 is a boom. Since the first moments of fundamental growth rates are procyclical and second moments are countercyclical, we assume that $g_1 < g_2$, $\theta_1 < \theta_2$, $\sigma_{C,1} > \sigma_{C,2}$ and $\sigma_{X,1}^s > \sigma_{X,2}^s$.

The above set of assumptions introduces time variation into the expected values and volatilities of cash flow and consumption growth rates. Random switches in the moments of consumption growth will only impact the state price density if the representative agent has a preference for how uncertainty about future growth rates is resolved over time. To ensure this, we assume the representative agent has the continuous-time analog of Epstein-Zin-Weil preferences.¹³ Consequently, the representative agent's state-price density at time- t , π_t , is given by

$$\pi_t = (\beta e^{-\beta t})^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \quad (4)$$

where β is the rate of time preference, γ is the coefficient of relative risk aversion (RRA), and ψ is the elasticity of intertemporal substitution under certainty.¹⁴ Unlike the power-utility representative agent, the Epstein-Zin-Weil representative agent's state price density depends on the value of the claim to aggregate consumption per unit consumption, i.e. the price-consumption ratio, p_C .

I.C Booms and Recessions

Define the state of the economy as ν_t which is equal to 1 in a recession and 2 in a boom. The evolution of ν_t is given by a 2-state Markov chain. The Markov chain is defined by λ_i , $i = \{1, 2\}$ where λ_1 is the probability per unit time of switching from recession to boom and λ_2 is the probability per unit time of switching from boom to recession. This implies that the average duration of a recession is $1/\lambda_1$ and the average duration of a boom is $1/\lambda_2$. Since empirically, recessions are shorter than booms ($1/\lambda_1 < 1/\lambda_2$), the probability per unit time of switching from recession to boom must be higher than the probability per unit time of switching from boom to recession ($\lambda_1 > \lambda_2$). This is in contrast with Bansal and Yaron (2004). Bansal and Yaron assume growth rates and volatilities follow AR(1) processes. But the AR(1) process and its continuous-time counterpart, the Ornstein-Uhlenbeck process, have symmetric transition probabilities. That forces the probability of switching from a recession to a boom to equal the probability of switching from a boom to a recession, implying booms and recessions are of equal duration, which increases risk premia.

When recessions are of lower duration, it is of course harder to generate a realistic equity risk premium. We now provide some intuition for why our model can still generate realistically high risk premia. The switching probabilities per unit time, λ_1 and λ_2 , are not directly relevant for valuing securities. Since we must account for risk, we use the *risk-neutral* switching probabilities per unit time, which we denote by $\widehat{\lambda}_1$ and $\widehat{\lambda}_2$. Intuitively, one would expect the risk-neutral probability per unit time of switching from a boom to

¹³The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1990) is known as stochastic differential utility, and is derived in Duffie and Epstein (1992).

¹⁴Schroder and Skiadas (1999) provide a proof of existence and uniqueness for an equivalent specification of stochastic differential utility.

a recession to be higher than the actual probability, i.e. $\hat{\lambda}_2 > \lambda_2$. Similarly, when considering the probability of moving from recession to boom, $\hat{\lambda}_1 < \lambda_1$. Using risk-neutral probabilities instead of actual probabilities means securities are priced as if recessions last longer and booms finish earlier than they actually do, which leads to a significant increase in credit spreads and the equity risk premium.

To compute $\hat{\lambda}_1$ and $\hat{\lambda}_2$ from λ_1 and λ_2 , we need a state-price density to define a mapping from the actual measure, \mathbb{P} , to the risk-neutral measure, \mathbb{Q} . When the representative agent has Epstein-Zin-Weil utility, her state-price density is given by (4). When the economy changes state, the price-consumption ratio jumps because the expected growth rate and volatility of consumption change. Therefore from (4), the state-price density jumps. The size of this jump links the risk-neutral to the actual switching probabilities. We show this in the proposition below. To distinguish between the state of the economy before and after the jump, we denote the time just before the jump occurs by $t-$, and the time at which the jump occurs by t .¹⁵

Proposition 1 *The risk-neutral switching probabilities per unit time are related to the actual switching probabilities per unit time by the risk-distortion factor, ω ,*

$$\hat{\lambda}_1 = \lambda_1 \omega^{-1}, \quad (5)$$

$$\hat{\lambda}_2 = \lambda_2 \omega, \quad (6)$$

where ω measures the size of the jump in the state-price density when the economy shifts from boom to recession, i.e.

$$\omega = \frac{\pi_t}{\pi_{t-}} \Big|_{\nu_{t-}=2, \nu_t=1}. \quad (7)$$

The size of the risk-distortion factor depends on the representative agent's preferences for resolving intertemporal risk:

1. $\omega > 1$, if the agent is averse to intertemporal macroeconomic risk ($\gamma > 1/\psi$),
2. $\omega < 1$, if the agent likes intertemporal macroeconomic risk ($\gamma < 1/\psi$), and
3. $\omega = 1$, if the agent is indifferent to intertemporal macroeconomic risk ($\gamma = 1/\psi$).

Proposition 1 tells us that when the representative agent prefers intertemporal macroeconomic risk to be resolved sooner than later (she has Epstein-Zin-Weil preferences with $\gamma > 1/\psi$), then $\omega > 1$, and the

¹⁵To be more precise, suppose that during the small time-interval $[t - \Delta t, t)$, the economy is in state i and that at time t , the state changes, so that during the next small time interval $[t, t + \Delta t)$, the economy is in state j . To distinguish between the state of the economy before and after the jump, we define the left-limit of ν at time t as

$$\nu_{t-} = \lim_{\Delta t \rightarrow 0} \nu_{t-\Delta t},$$

and the right-limit as

$$\nu_t = \lim_{\Delta t \rightarrow 0} \nu_{t+\Delta t}.$$

Therefore $\nu_{t-} = i$, whereas $\nu_t = j$, so the left and right-limits are not equal.

duration of recessions under the risk-neutral measure is longer than their actual duration. Since the state-price density jumps up in recessions, asset returns contain a premium for jump-risk. The premium for jump risk is present in both credit spreads and equity risk premia. The presence of jump components forces the stochastic processes for bond and stock returns to overlap, a feature of the data observed by Fama and French (1993). As long as jump-risk is priced, there is a jump-risk component in credit spreads, which comoves with the jump component in stock-return volatility, as documented in Tauchen and Zhou (2006).

The risk-distortion factor, ω , is the solution of the nonlinear equation (C4) in Appendix C. We can compute the locally risk-free rate, the price-consumption ratio and the market price of risk in terms of the risk-distortion factor (see Equations (C3), (C10) and (C16) in Appendix C).

When $\gamma = 1/\psi$, the risk-distortion factor does not impact the price-consumption ratio, but the price-consumption ratio still jumps when the economy changes state. Because the agent is indifferent about when she receives information concerning the future state of the economy, these jumps are not priced so jump-risk premia vanish. But there are still jump components in volatilities, because volatility is a measure of total risk not just priced risk.

We can also give a quantitative measure of long-run risk in our model. As time tends to infinity, the Markov chain we use to model the state of the economy converges exponentially to a long-run distribution, i.e. the probability of being in a given state becomes constant. We can show that the long-run risk-neutral probability of being in state i is given by $\hat{f}_i = \frac{\hat{\lambda}_j}{\hat{\lambda}_1 + \hat{\lambda}_2}$, $j \neq i$, i.e. $\lim_{s \rightarrow \infty} \widehat{\Pr}(\nu_{t+s} = i | \nu_t = j) = \hat{f}_i$, $j \in \{1, 2\}$. Observe that the long-run probabilities do not depend on the initial state. The parameter $\hat{p} = \hat{\lambda}_1 + \hat{\lambda}_2$ tells us how quickly the risk-neutral distribution of the Markov chain approaches its long-run risk-neutral distribution. To be precise, convergence to the long-run is exponential at a rate of \hat{p} . The slower the convergence (under the risk-neutral measure), the more long-run risk there is in the economy.¹⁶

II Asset Valuation

In this section we derive the prices of all assets in the economy and investigate the properties of credit spreads and the equity premium.

We start by noting the following immediate implication of Proposition 1. Expressions for asset prices in an economy where the first and second moments of consumption growth *do* switch can be obtained from expressions for asset prices in an economy where the first and second moments of consumption growth *do not* switch, without any further computations. All one must do is merely *adjust* the probability that the economy changes state by the risk-distortion factor to get risk-neutral probabilities and replace the constant expected consumption growth rate and volatility by the relevant state-dependent quantities. More

¹⁶Details of how this measure of long-run risk is related to the long-run risk model in Bansal and Yaron (2004) are available upon request.

formally, if $P_i(\lambda_1, \lambda_2, g, \sigma_C)$ is the price of an asset when the economy is in state i and the first and second moments of consumption do not switch, the corresponding asset price when the first and second moments of consumption do switch is given by $P_i(\widehat{\lambda}_1, \widehat{\lambda}_2, g_i, \sigma_{C,i})$. This observation demonstrates how Proposition 1 considerably simplifies the computation of asset prices.

We now use the state-price density in (4) to value the corporate debt and equity issued by firms. As in EBIT-based models of capital structure (see Goldstein, Ju, and Leland (2001) and Strebulaev (2007)), the earnings (or EBIT cash flow), X , of a firm is split between a constant coupon, c , paid to debtholders and a risky dividend, $X - c$, paid to equityholders. Because of taxes paid at the rate η , equityholders actually receive the amount $(1 - \eta)(X - c)$. Default occurs at the moment earnings drop below a certain threshold. The default boundary is state-dependent: default occurs in state i if $X \leq X_{D,i}$, $i \in \{1, 2\}$. Further discussion of the default boundary is given in Section II.D. If the firm defaults, bondholders receive what can be recovered of the firm's assets in lieu of coupons, i.e. a fraction α_t of the after-tax value of the firm's earnings at default. The recovery rate α_t is assumed to be procyclical: $\alpha_t \in \{\alpha_1, \alpha_2\}$, where $\alpha_1 < \alpha_2$, consistent with empirical findings in Thorburn (2000), Altman, Brady, Resti, and Sironi (2002) and Acharya, Bharath, and Srinivasan (2002).

The debt and levered equity values for a firm can be written in terms of the prices of a set of Arrow-Debreu default claims and unlevered firm value (after-tax). We now proceed to derive closed-form expressions for these values.

II.A Arrow-Debreu Default Claims

The Arrow-Debreu default claim denoted by $q_{D,i,j,t}$ is the value of a unit of consumption paid if default occurs in state j and the current state is i . There are four such claims in our economy: $\{q_{D,i,j}\}_{i,j \in \{1,2\}}$. In other words, if the current date is t and earnings hit the boundary $X_{D,j}$ from above for the first time in state j , one unit of consumption will be paid that instant. Since each Arrow-Debreu default claim is effectively a perpetual digital put, their values can be derived by solving a system of ordinary differential equations, derived from the standard equations

$$E_t^{\mathbb{Q}}[dq_{D,i,j} - r_i q_{D,i,j} dt] = 0, \quad i, j \in \{1, 2\}. \quad (8)$$

Closed-form solutions are given in (C28) in Appendix C.

To link the Arrow-Debreu default claims to the actual probability of default, we decompose the value of the claims into three factors: a time-adjustment, a risk-adjustment and the actual default probability, as shown in the proposition below.

Proposition 2 *The price of the Arrow-Debreu default claim, which pays out one unit of consumption if default occurs in state j and the current state is i , is given by*

$$q_{D,i,j} = p_{D,i,j} \mathcal{I}_{ij} \mathcal{R}_{ij}, \quad (9)$$

where $p_{D,ij}$ is the actual probability of default occurring in state j , conditional on the current state being i , \mathcal{T}_{ij} is a time-adjustment factor and \mathcal{R}_{ij} is a risk-adjustment factor.

The risk-neutral probability of default occurring in state j , conditional on the current state being i , $\widehat{p}_{D,ij}$, is given by

$$\widehat{p}_{D,ij} = p_{D,ij} \mathcal{R}_{ij}. \quad (10)$$

Proposition 2 tells us that the price of the Arrow-Debreu default claim is not equal to the risk-neutral default probability, a fact not explicitly noted in the previous literature. Chen, Collin-Dufresne, and Goldstein (2006) note that to resolve the credit spread puzzle, risk-neutral default probabilities must be high while actual default probabilities are low. In fact, *Arrow-Debreu default claims* must have high prices, while actual default probabilities are low. The decomposition in (9) tells us this can be achieved when the time- and risk-adjustments factors are sufficiently high.

The time and risk adjustments are computed by solving for the actual and risk-neutral default probabilities, because

$$\mathcal{R}_{ij} = \frac{\widehat{p}_{D,ij}}{p_{D,ij}}, \quad (11)$$

and

$$\mathcal{T}_{ij} = \frac{q_{D,ij}}{\widehat{p}_{D,ij}}. \quad (12)$$

The set of actual default probabilities can also be found by solving (8), but with the risk-free rate set equal to zero to eliminate time effects and the risk-distortion factor set equal to one to eliminate risk effects. Similarly, risk-neutral default probabilities are the solution of (8), but with just the risk-free rate set to zero.

II.A.1 No Intertemporal Macroeconomic Risk

To gain more intuition about the decomposition in (9), we compute the actual default probability, the time- and risk-adjustments for the case when there is no intertemporal macroeconomic risk. (The case with intertemporal macroeconomic risk is algebraically tedious and outlined in the proof of Proposition 2.) We can show that the actual default probability is

$$p_D(X_t) = \left(\frac{X_D}{X_t} \right)^{\frac{\theta - \frac{1}{2}\sigma_X^2}{\frac{\sigma_X^2}{2}}}, \quad (13)$$

where σ_X is total earnings volatility, given by $\sigma_X = \sqrt{(\sigma_X^{id})^2 + (\sigma_X^s)^2}$. The risk-adjustment factor is

$$\mathcal{R}(X_t) = \left(\frac{X_D}{X_t} \right)^{\frac{-\gamma \rho_{XC} \sigma_X^s \sigma_C}{\sigma_X^2/2}}, \quad (14)$$

and the time-adjustment factor is

$$\mathcal{T}(X_t) = \left(\frac{X_D}{X_t} \right)^{\left(\widehat{\theta} - \frac{1}{2} \sigma_X^2 \right) \sqrt{\frac{1 + \frac{2\bar{r}\sigma_X^2}{\left[\widehat{\theta} - \frac{1}{2} \sigma_X^2 \right]^2} - 1}}{\sigma_X^2}}, \quad (15)$$

where \bar{r} is the risk-free rate (given in Appendix C, Equation (C20)) and $\widehat{\theta} = \theta - \gamma \rho_{XC} \sigma_X^s \sigma_C$ is the risk-neutral earnings growth rate.

The risk-adjustment factor is always greater than or equal to one and, as expected, is increasing in relative risk aversion, γ . In particular, when the representative agent is risk-neutral ($\gamma = 0$), the risk associated with not knowing the time of default is no longer priced, and the risk-adjustment factor reduces to unity. The risk-adjustment also increases with systematic earnings volatility, and the volatility of consumption growth. However, the risk-adjustment factor is lower when idiosyncratic risk is higher. The intuition is simple: with more idiosyncratic earnings growth volatility, the actual default probability is raised. Because the risk-neutral default probability must be less than one, it cannot rise by a proportionate amount. Thus the risk-adjustment is squeezed downwards.

The time-adjustment factor is a downward adjustment, reflecting the time value of money. In particular, it accounts for the distribution of default times in the risk-neutral world. Consistent with this interpretation, the time-adjustment factor is decreasing in the risk-free rate and is equal to one when the risk-free rate is zero.¹⁷ Consequently, the time-adjustment factor increases with both the EIS (consumption smoothing) and relative risk aversion (precautionary savings). Higher earnings volatility, whether it be systematic or idiosyncratic, makes earlier default more likely, which increases the time-adjustment factor. Also, the time-adjustment factor decreases in the risk-neutral earnings growth rate, $\widehat{\theta}$, since an increase in $\widehat{\theta}$ delays default.

The above discussion implies that the value of the Arrow-Debreu default claim is high when relative risk aversion is high (and therefore the risk adjustment is high) and when the EIS is high (and therefore the time adjustment is high). We now analyze the credit spread puzzle in a way analagous to Mehra and Prescott (1985)'s analysis of the equity risk premium puzzle by keeping the actual default probability realistically low and varying preference parameters in the absence of intertemporal macroeconomic risk. To do this, we consider Baa bonds, which according to Huang and Huang (2003) have spreads over treasuries of 158 bp. We choose the coupon, c , and idiosyncratic earnings growth volatility to match the historical 4-year default probability of 1.24% and leverage of 43% (as reported in Huang and Huang (2003)), for given relative risk aversion, γ , and EIS, ψ . The resulting model-implied credit spread shows the same comparative statics behavior with respect to relative risk aversion and the EIS as the risk-free rate. When $\gamma \neq 1/\psi$, the credit spread falls as risk aversion and the EIS increase. For the power utility case, $\gamma = 1/\psi$, spreads are initially increasing in risk aversion, but then start to decrease when the precautionary savings effect becomes

¹⁷The standard condition, $\widehat{\theta} - \frac{1}{2} \sigma_X^2 > 0$, ensures that $\mathcal{T}(X_t)$ is less than one when $\bar{r} > 0$.

larger. Hence, even for very high risk aversion, the model-implied Baa credit spread is much less than in the data, never above 30 b.p. when the actual default probability is realistically small. Using Epstein-Zin-Weil preferences instead of power utility increases credit spreads, but not by enough. Separating relative risk aversion from the EIS makes it possible to get a lower risk-free rate, and hence a larger time-adjustment factor. But in the absence of intertemporal macroeconomic risk, the risk-adjustment factor is not large enough to simultaneously produce a high value for the Arrow-Debreu default claim and a small actual default probability.

II.A.2 Intertemporal Macroeconomic Risk

Introducing intertemporal macroeconomic risk allows us to increase the risk-adjustment factor without lowering the time-adjustment. When there is a greater aversion to intertemporal macroeconomic risk, the risk-distortion factor, ω , increases as Proposition 1 shows, which leads to an increase in the risk-adjustment factor. In Section III, we actually show that this can account for a large fraction of observed credit spreads.

Recent empirical work by Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) focuses on estimating the size of risk-neutral default probabilities relative to actual default probabilities, i.e. the risk-adjustment factor. The decomposition in (9) emphasizes the importance of understanding not only the risk-adjustment factor, but also the time-adjustment factor. Furthermore, by virtue of possessing analytical expressions for risk- and time-adjustments, we can provide some theoretical underpinning for topics which the empirical literature has not yet touched upon. For example, we can study how macroeconomic and firm-level factors impact the risk and time adjustments. Our decomposition also applies for finite-horizon Arrow-Debreu default claims and closed-form expressions can be obtained using the approach of Leland (1998), which we check by simulation. In the discussion below, for concreteness we assume the time-horizon is 4 years.

The macroeconomic factors we focus on are the moments of consumption growth. Figure 1 shows that increasing the expected consumption growth in the low state, g_1 , decreases $\mathcal{R}_{11}(4)$ and $\mathcal{R}_{21}(4)$, while $\mathcal{R}_{12}(4)$ and $\mathcal{R}_{22}(4)$ increase. The intuition is that increasing g_1 decreases the jump in the state-price density that occurs when the economy moves into a recession, thus decreasing the risk-distortion factor ω . Therefore, the risk-neutral probability of switching into the low state falls, whereas the risk-neutral probability of switching into the high state rises. Consequently, the risk-adjustments associated with defaulting in the low state, i.e. $\mathcal{R}_{11}(4)$ and $\mathcal{R}_{21}(4)$, decrease, whereas risk-adjustments associated with defaulting in the high state, i.e. $\mathcal{R}_{12}(4)$ and $\mathcal{R}_{22}(4)$, increase. Note also that $\mathcal{R}_{11}(4), \mathcal{R}_{21}(4) > 1$, while $\mathcal{R}_{12}(4), \mathcal{R}_{22}(4) < 1$. Importantly, risk-adjustments associated with defaulting in the low state dominate those associated with defaulting in the high state, in the following sense. The risk-adjustment factors which are independent of the state in which default occurs are both greater than 1, i.e. $\mathcal{R}_i(4) > 1, i \in \{1, 2\}$, where $\mathcal{R}_i(4) = \widehat{p}_{D,i}(4)/p_{D,i}(4)$, $p_{D,i}(4) = p_{D,i1}(4) + p_{D,i2}(4)$ is the 4-year actual probability of default when the current state is i and $\widehat{p}_{D,i}(4)$ is the corresponding risk-neutral default probability. Conversely, as consumption growth volatility in the low

state, $\sigma_{C,1}$, rises, the distortion factor increases. Therefore the risk adjustments associated with defaulting in the low state increase and the risk adjustments associated with defaulting in the high state decrease. The intuition behind the change in time-adjustments as g_1 increases is straightforward: the risk-free rate in the low state increases, thus decreasing all the time-adjustments.

At the firm level, we are interested in how expected earnings growth, systematic and idiosyncratic earnings growth volatility and leverage impact the risk- and time-adjustments. It is intuitive that risk-adjustments fall as the impact of the bad state of the economy lessens, i.e. expected earnings growth in the low state increases or systematic earnings growth volatility in the low state falls. When there is an increase in idiosyncratic earnings growth volatility, risk-adjustments behave the same way as they did in the absence of intertemporal macroeconomic risk—they fall. Increasing leverage raises the optimal default boundary, which increases the actual default probability without a proportionate increase in the risk-neutral default probability. Consequently, risk-adjustments fall as leverage increases, as shown in Figure 2. Time-adjustments show the same qualitative behavior with respect to the first and second moments of earnings growth as for the case with no intertemporal macroeconomic risk. Increasing leverage increases the time-adjustments by raising the optimal default boundary and hence making earlier default more likely, as shown in Figure 3.

II.B Abandonment Value

The firm's state-conditional liquidation, or abandonment value, denoted by $A_{i,t}$, is the after-tax value of the future unlevered firm's earnings, when the current state is i :

$$A_{i,t} = (1 - \eta)X_t E_t \left[\int_t^\infty \frac{\pi_s X_s}{\pi_t X_t} ds \middle| \nu_t = i \right], \text{ for } i \in \{1, 2\}. \quad (16)$$

The liquidation value in (16) is a function of the current earnings level and is time-independent, $A_{i,t} = A_i(X_t)$. The next proposition derives the value of A_i in terms of fundamentals of the economy.

Proposition 3 *The liquidation value in state $i \in \{1, 2\}$ is given by*

$$A_i(X_t) = \frac{(1 - \eta)X_t}{r_{A,i}}, \quad (17)$$

where

$$r_{A,i} = \bar{\mu}_i - \theta_i + \frac{(\bar{\mu}_j - \theta_j) - (\bar{\mu}_i - \theta_i)}{\hat{p} + \bar{\mu}_j - \theta_j} \hat{p} f_j, \quad j \neq i, \quad (18)$$

and

$$\bar{\mu}_i = r_i + \gamma \rho_{XC,i} \sigma_{X,i} \sigma_{C,i}, \quad (19)$$

is the discount rate in the standard Gordon growth model.

To understand the intuition behind (17), suppose the economy is currently in state i . Then, the risk-neutral probability of the economy switching into a different state during a small time interval Δt is $\widehat{\lambda}_i \Delta t$ and the risk-neutral probability of not switching is $1 - \widehat{\lambda}_i \Delta t$. We can therefore write the unlevered firm value in state i as

$$A_i = (1 - \eta)X \Delta t + e^{-(\bar{\mu}_i - \theta_i) \Delta t} \left[(1 - \widehat{\lambda}_i \Delta t) A_i + \widehat{\lambda}_i \Delta t A_j \right], \quad i, j \in \{1, 2\}, j \neq i. \quad (20)$$

The first term in (20) is the after-tax cash flow received in the next instant and the second term is the discounted continuation value. The continuation value is the average of A_i and A_j , weighted by the risk-neutral probabilities of being in states i and $j \neq i$ a small instant Δt from now. For example, with risk-neutral probability $\widehat{\lambda}_i \Delta t$ the economy will be in state $j \neq i$ and the abandonment value will be A_j . The continuation value is discounted back at a rate reflecting the discount rate $\bar{\mu}_i$ and the expected earnings growth rate over that instant which is θ_i .

To gauge the intuition behind the discount rate, note that if the economy stays in state i forever, the discount rate in the perpetuity formula (17) reduces to the standard expression

$$r_{A,i} = \bar{\mu}_i - \theta_i. \quad (21)$$

To be concrete, assume we are in state 1. The discount rate in (18) is obtained by adjusting $\bar{\mu}_1 - \theta_1$ downwards by the amount, $\frac{(\bar{\mu}_2 - \theta_2) - (\bar{\mu}_1 - \theta_1)}{\widehat{p} + \bar{\mu}_2 - \theta_2} \widehat{p} \widehat{f}_2 < 0$, to account for time spent in state 2 (a boom) at future times. The magnitude of the adjustment increases with the growth rate in the boom state, θ_2 , and the risk-neutral probability per unit time of switching into state 2, $\widehat{\lambda}_1$. Note that as the economy is more likely to switch to another state (i.e. \widehat{p} becomes bigger), the adjustment approaches $[(\bar{\mu}_2 - \theta_2) - (\bar{\mu}_1 - \theta_1)] \widehat{f}_2$ and the discount rate approaches $(\bar{\mu}_1 - \theta_1) \widehat{f}_1 + (\bar{\mu}_2 - \theta_2) \widehat{f}_2$, which is the long-run risk-neutral mean of the difference between the discount rate and the expected earnings growth rate.

II.C Credit Spreads and the Levered Equity Risk Premium

Turning now to corporate debt, the generic value of debt at time t , conditional on the state being i is denoted by $B_{i,t}$ and given by

$$B_{i,t} = E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} cds \middle| \nu_t = \nu_i \right] + E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D} \middle| \nu_t = i \right], \quad i \in \{1, 2\}. \quad (22)$$

The first term in (22) is the present value of a perpetual coupon stream until default occurs at a random stopping time τ_D . The second term is the present value at time t of the asset recovery value the debtholders successfully claim upon default, where $\alpha_t \in \{\alpha_1, \alpha_2\}$ is the date t recovery rate. We show (see Proof of Proposition 4, Appendix C) that (22) reduces to

$$B_{i,t} = \frac{c}{r_{P,i}} \left(1 - \sum_{j=1}^2 l_{ij,t} q_{D,ij,t} \right), \quad (23)$$

where

$$l_{ij,t} = \frac{\frac{c}{r_{P,j}} - \alpha_j A_j (X_{D,j})}{\frac{c}{r_{P,i}}}$$

is the loss ratio at default, when the current state is i and default occurs in state j . The first factor in (23) is the price of the equivalent riskless consol bond, $c/r_{P,i}$, and the second factor is a downward adjustment for default risk, where $l_{ij,t}q_{D,ij,t}$ is present value of the loss ratio. The discount rate for a riskless perpetuity when the current state is i is given by

$$r_{P,i} = r_i + \frac{r_j - r_i}{\widehat{p} + r_j} \widehat{p} f_j, j \neq i. \quad (24)$$

$r_{P,i}$ is not equal to the risk-free rate in state i , r_i , because the risk-free rate is expected to change in the future whenever the state of the economy switches.¹⁸

The next proposition gives the corporate bond spread and its volatility in terms of the discount rate for a risk-free perpetuity, loss ratios and Arrow-Debreu default claims.

Proposition 4 *The credit spread in state i , $s_{i,t}$, is given by*

$$s_{i,t} = \frac{c}{B_{i,t}} - r_{P,i} = r_{P,i} \frac{\sum_{j=1}^2 l_{ij,t} q_{D,ij,t}}{1 - \sum_{j=1}^2 l_{ij,t} q_{D,ij,t}}. \quad (25)$$

The volatility of the credit spread in state i , $\sigma_{s,i}$, is given by

$$\sigma_{s,i} = \sqrt{\left((s_i + r_{P,i}) \frac{\partial \ln B_i}{\partial \ln X} \right)^2 \sigma_{X,i}^2 + \lambda_i (s_j - s_i)^2}, i \in \{1, 2\}, j \neq i, \quad (26)$$

where, in state i , the elasticity of the bond price with respect to earnings is given by

$$\frac{\partial \ln B_i}{\partial \ln X} = -\frac{s_i + r_{P,i}}{r_{P,i}} \sum_{j=1}^2 \left(\frac{\partial \ln q_{D,ij}}{\partial \ln X} \right) q_{D,ij} l_{ij}, i \in \{1, 2\}, j \neq i. \quad (27)$$

The above proposition tells us that we can generate realistic credit spreads in three ways: a high risk-free rate, high loss rates or high prices for Arrow-Debreu default claims. Empirically, we know both risk-free rates and loss rates are too low. Therefore, the only way to generate high credit spreads is via high prices for Arrow-Debreu default claims. But we know from Proposition 2 that Arrow-Debreu default claims are just actual default probabilities adjusted for time and risk. Since actual default probabilities are low, the time- and risk-adjustments factors need to be large for spreads to be large.

¹⁸Note that (24) can be obtained from the formula for the discount rate for a stochastically growing cash flow, (18), by replacing the Gordon growth model discount rate, $\bar{\mu}_i$, with the risk-free rate, r_i and setting the expected growth rate of earnings, θ_i , equal to zero.

Current levered equity value is given by the expected present value of future cashflows less coupon payments up until bankruptcy, conditional on the current state:

$$S_{i,t} = (1 - \eta) E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} (X_s - c) ds \middle| \nu_t = i \right], i \in \{1, 2\}.$$

We can show (see Proof of Proposition 5, Appendix C) that the above equation simplifies to give

$$S_{i,t} = A_i(X_t) - (1 - \eta) \frac{c}{r_{P,i}} + \sum_{j=1}^2 q_{D,ij} \left[(1 - \eta) \frac{c}{r_{P,j}} - A_j(X_{D,j}) \right], i \in \{1, 2\}. \quad (28)$$

The first two terms in the above equation are the present after-tax value of future cashflows less coupon payments, if the firm were never to default. The last term accounts for the fact that upon default shareholders no longer have to pay coupons to bondholders and at the same time they lose the rights to any future cash flows from owning the firm's assets.

In the next proposition we derive the levered equity risk premium and levered stock-market return volatility of an individual firm.

Proposition 5 *The conditional levered equity risk premium in state i is*

$$\mu_{R,i} - r_i = \gamma \rho_{XC,i} \sigma_{R,i}^{B,s} \sigma_{C,i} + \Lambda_i, i \in \{1, 2\}, \quad (29)$$

where Λ_i is the jump risk-premium in state i , given by

$$\Lambda_i = \begin{cases} (1 - \omega^{-1}) \sigma_{R,1}^P \lambda_1, & i = 1 \\ (1 - \omega) \sigma_{R,2}^P \lambda_2, & i = 2 \end{cases}, \quad (30)$$

and

$$\sigma_{R,i}^P = \frac{S_j}{S_i} - 1, i \in \{1, 2\}, j \neq i \quad (31)$$

is the volatility of stock returns caused by Poisson shocks. $\sigma_{R,i}^{B,s}$ is the systematic volatility of stock returns caused by Brownian shocks, given by

$$\sigma_{R,i}^{B,s} = \frac{\partial \ln S_{i,t-}}{\partial \ln X_t} \sigma_{X,i}^s, i \in \{1, 2\} \quad (32)$$

where

$$\frac{\partial \ln S_{i,t-}}{\partial \ln X_t} = \frac{\frac{A_i(X_t)}{X_t} + \sum_{j=1}^2 q'_{D,ij} \left[(1 - \eta) \frac{c}{r_{P,j}} - A_j(X_{D,j}) \right]}{S_{i,t-}/X_t}, i \in \{1, 2\} \quad (33)$$

is the elasticity of levered equity with respect to earnings.

Conditional levered stock return volatility in state i is

$$\sigma_{R,i} = \sqrt{\left(\sigma_{R,i}^{B,id} \right)^2 + \left(\sigma_{R,i}^{B,s} \right)^2 + \lambda_i \left(\sigma_{R,i}^P \right)^2}, i \in \{1, 2\}, \quad (34)$$

where

$$\sigma_{R,i}^{B,id} = \frac{\partial \ln S_{i,t-}}{\partial \ln X_t} \sigma_{X,i}^{id}, \quad i \in \{1, 2\} \quad (35)$$

is the idiosyncratic volatility of stock returns caused by Brownian shocks.

First we discuss how leverage affects the equity risk-premium of a firm. At first blush, one might expect the levered equity risk-premium to be larger than the unlevered risk-premium, simply because the act of paying coupons leaves behind less dividends for equity holders. But introducing leverage into a firm does not simply reduce dividend payments; it also brings in default risk. As Equation (33) tells us, default risk decreases the risk premium because the Arrow-Debreu default claim is a put and thus decreases in earnings. Intuitively, default risk increases the value of the option to default, which increases equity value and hence decreases the risk premium. When we calibrate the model, we revisit this issue (see Section III.C) to see which effect dominates. The same argument applies to stock-return volatility, because both the premium and volatility depend on the elasticity of levered equity with respect to earnings.

II.D Optimal Default Boundary and Optimal Capital Structure

Equityholders maximize the value of their default option by choosing when to default and also choose optimal capital structure. Intuitively, the endogenous default boundary depends on the current state of the economy, i.e. there is a set of default boundaries $X_{D,i}$, $i \in \{1, 2\}$, where $X_{D,i}$ is the default boundary when the economy is in state i . The default boundaries satisfy the following two standard smooth-pasting conditions:

$$\left. \frac{\partial S_i(X)}{\partial X} \right|_{X=X_{D,i}} = 0, \quad i \in \{1, 2\}. \quad (36)$$

In Appendix C we show that the default boundary is weakly countercyclical, i.e. $X_{D,1} \geq X_{D,2}$.

Equityholders choose the optimal coupon to maximize firm value at date 0. There are two important features to note. First, by maximizing firm value equityholders internalize debtholders' value at date 0. However, in choosing default times they ignore the considerations of debtholders. This feature creates the basic conflict of interest between equity and debtholders, which is standard in the optimal capital structure literature. Second, the optimal coupon depends on the state of the economy at date 0. To make this clear, we denote the date 0 coupon choice by $c_{i,0}$, where i is the state of the economy at date 0. Therefore equityholders choose the coupon to maximize date 0 firm value, $F_{i,0} = B_{i,0} + S_{i,0}$. The choice of optimal default boundaries will depend on the coupon choice. This implies hysteresis in the sense that the default boundaries not only depend on the current state of the economy, but also on its initial state. For simplicity we omit this in the notation for the default boundaries. We discuss the empirical implications of our model for leverage decisions and default timing in Section III.E.

III Calibration and Model Implications

In this section we analyze the quantitative implications of the model. We start by obtaining conditional estimates of parameter values. Then, in Sections III.B and III.C, we see whether our model can resolve the credit spread and equity risk premium puzzles, respectively. We also look at cross-market co-movement in Section III.D and the implications of our model for corporate financing in Section III.E

III.A Calibration

To calibrate parameter values we use aggregate US data at quarterly frequency for the period from 1947Q1 to 2005Q4. Consumption is real non-durables plus service consumption expenditures from the Bureau of Economic Analysis. Earnings data are from S&P and provided on Robert J. Shiller’s website. We delete monthly interpolated values and obtain a time-series at quarterly frequency. The personal consumption expenditure chain-type price index is used to deflate the earnings time-series. Unconditional parameter estimates are summarized in Table II. In the presence of intertemporal macroeconomic risk, we need conditional estimates, and their calibrated values are given in Panel A of Table III. We now discuss the calibration exercise in more detail.

We obtain estimates of $\lambda_1, \lambda_2, g_1, g_2, \theta_1, \theta_2, \sigma_{C,1}, \sigma_{C,2}, \sigma_{X,1}^s, \sigma_{X,2}^s$ and ρ_{XC} by maximum likelihood. The approach is based on Hamilton (1989) and details specific to our implementation are summarized in Appendix A. Our estimates are similar to those obtained by other authors who jointly estimate consumption and dividends with a state-dependent drift and volatility, as opposed to consumption and earnings (see Bonomo and Garcia (1996)). We calibrate idiosyncratic earnings volatility so that the model-implied 4-year default probability is consistent with the observed 4-year cumulative default probability of 1.24% for Baa bonds reported in Huang and Huang (2003), assuming a relative risk aversion of 10 and an EIS of 1.5. This gives us an idiosyncratic earnings volatility of 32%. Andrade and Kaplan (1998) report default costs of about 20% of asset value. Since the recovery rate for corporate bond holders is higher in a boom than in a recession, we assume $\alpha_1 = 0.7$ and $\alpha_2 = 0.9$. The annualized rate of time preference β is 0.01.

III.B Corporate Bond Market

Proposition 4 links the credit spread and its volatility to the prices of Arrow-Debreu default claims. Therefore we first focus on understanding their behavior and then use that understanding to explain the implications of our model for corporate bond prices and corporate financing decisions, as shown in Table IV.

To highlight the role of the distortion factor, ω , the table uses three values of the EIS parameter ψ 0.1, 0.75, and 1.5. It is also important to be clear about the importance of the size of ψ for our model’s implications, because empirical estimates of its magnitude differ widely (Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen (2002) and Guvenen (2006) estimate that $\psi > 1$, whereas Hall (1988)

estimates that ψ is much less than one). As shown in Table V, when $\psi = 0.1$, $\omega = 1$, because the representative agent is indifferent to whether intertemporal uncertainty is resolved sooner rather than later. When $\psi = 0.5$, $\omega = 1.377$ and for $\psi = 1.5$, $\omega = 1.424$, reflecting an increasing preference for the early resolution of intertemporal uncertainty. This is reflected in more long-run risk as measured by \hat{p} , because \hat{p} decreases as ω increases. Observe that this is purely a preference based effect, because while \hat{p} falls, the convergence rate of the Markov chain under the actual probability measure, p , is constant.¹⁹

For simplicity, relative risk aversion is kept constant in Table IV, although we briefly describe the impact of increasing it within the following discussion.

III.B.1 Arrow-Debreu Default Claims

The Arrow-Debreu default claims, $q_{D,i} = q_{D,i1} + q_{D,i2}$, which pay 1 unit of consumption if default occurs conditional on the current state being i , increase both with risk aversion and the EIS.

Unsurprisingly the risk adjustments, $\mathcal{R}_i(4)$, increase with relative risk aversion, which drives up the Arrow-Debreu default claim prices. Table IV also shows that the risk-adjustments increase with the EIS. The intuition is that as ψ increases, the agent then has a stronger preference for earlier resolution of intertemporal risk, which is reflected in a larger distortion factor, leading to higher state prices in recessions and more long-run risk. When the EIS is 0.1, then $\omega = 1$, and risk-adjustments are just above 1. Increasing the EIS to 1.5 increases risk-adjustments to around 1.4. This increase is large enough to counteract the decreasing actual default probabilities and ensure that the Arrow-Debreu default claims increase with the EIS.

The risk-adjustment is procyclical with respect to the current state of the economy. While this may initially seem surprising, this is a direct consequence of the countercyclicality of the actual default probabilities and our definition of the risk-adjustment factor, $\mathcal{R}_i(4) = \hat{p}_{D,i}(4)/p_{D,i}(4)$, as a ratio. Higher systematic earnings volatility in recessions implies that the actual default probability is countercyclical. For example, Table IV shows that when $\psi = 1.5$ actual default probabilities more than double in recessions. While risk-neutral default probabilities also rise in recessions, they must increase by a lesser proportion to ensure that they remain less than 1. Therefore, the ratio $\hat{p}_{D,i}(4)/p_{D,i}(4)$, which defines the risk-adjustment is lower in recessions. Furthermore, the risk-adjustment depends on the optimal default boundary, which depends on optimal leverage. Therefore, risk-adjustments depend on the state of the economy at the moment leverage is chosen. Table IV shows that risk adjustments are higher when leverage is chosen in recessions. In particular when $\psi = 1.5$, the risk-adjustment when the initial state was 1 (recession) and the current state is 2 (boom) is 1.529, whereas when the initial state and current states are 2 (boom), the risk-adjustment is 1.437. This dependence on the initial state stems from the effect of leverage on the default boundary. When optimal leverage is chosen in a boom it is higher than when chosen in a recession. Consequently, when leverage is

¹⁹Note that $\hat{p} = p(\omega^{-1}f_1 + \omega f_2)$, where $f_i = \frac{\lambda_j}{p}$, $j \neq i$ and $p = \lambda_1 + \lambda_2$. It follows that $\hat{p} < p$ and \hat{p} is decreasing in ω provided that the average duration of recessions is less than booms, i.e. $f_1 < f_2$.

chosen in a boom, default boundaries are higher, which makes early default more likely and risk-adjustment factors larger.

The time-adjustments change very little as risk aversion changes, but Table IV shows that they do increase as the EIS rises. When $\psi = 0.1$ the time adjustments are close to 0.5. Note also that they are countercyclical, because the risk-free rate is procyclical. Increasing ψ to 1.5 ensures the time adjustments are around 0.9. We can also see that time-adjustments are lower when leverage is chosen in recessions. This reason is that when chosen in recessions, leverage is lower. Thus the default boundary is lower and default is likely to occur later, implying that the time-adjustment must be smaller.

While the actual default probabilities change very little with risk aversion, Table IV shows that they drop steeply as the EIS falls. When $\psi = 0.1$, actual 4-year default probabilities are between 12% and 32%, which is unrealistically high. For ψ to 1.5, we obtain realistic default probabilities, between 0.674% and 2.804%. This effect is driven by leverage. As the EIS increases, so does the risk-adjustment, which is a measure of default risk. Therefore Arrow-Debreu default claims become more expensive. Equations (23) and (28) show that when Arrow-Debreu default claims rise in price, value is shifted from debtholders to equityholders, which lowers leverage. In other words, the increase in default risk negatively impacts debt, but not equity. Consequently, it is optimal to reduce the coupon and hence leverage. Lowering leverage decreases the default boundary, which lowers actual default probabilities.

III.B.2 Credit Spreads

The credit spread increases with relative risk aversion. This is caused by larger risk-adjustment factors, which drive up the prices of Arrow-Debreu default claims. In contrast, and perhaps surprisingly, Table IV shows that increasing the EIS decreases the credit spread. When $\psi = 0.1$ credit spreads are in the range 260 to 530 b.p. With $\psi = 1.5$ spreads are between 180 and 223 b.p. This effect occurs because the risk-free perpetuity discount rate and loss ratios fall as the EIS increases. Since the behavior of the risk-free perpetuity discount rates, $r_{P,i}$, is the same as the locally risk-free rate, r_i , raising the EIS decreases $r_{P,i}$. Equation (33) implies that loss ratios are increasing in the optimal coupon and hence optimal leverage. Raising the EIS decreases optimal leverage as discussed above. Thus loss ratios are decreasing in the EIS.

III.B.3 Credit Spread Volatilities

Credit spread volatility is barely changed by risk aversion. To see why, observe that credit spread volatility is the instantaneous conditional standard-deviation of changes in the credit spread level, and while credit spreads do go up slightly with risk aversion, they do not become more sensitive to changes in earnings or the state of the economy.

However, as the EIS increases, credit spread volatility falls substantially. This occurs because the credit spread level decreases rapidly with the EIS. For larger values of the EIS credit spread volatility is closer to its historical average of 39 b.p. for the period from 1946 to 2004.

III.B.4 Term Structure of Credit Spreads

We can extend our model to incorporate finite-maturity debt, based on the approach of Leland (1998) and similar to Hackbarth, Miao, and Morellec (2006). Our model implies non-trivial credit spreads even for short-maturity debt. In fact, as shown in Figure 4, when leverage is set equal to 43% and parameter values are as in Table III, then with a risk aversion of 10 and an EIS of 1.5 the range of 4-year model-implied credit spreads is 101-147 b.p. The 4-year Baa-treasury spread reported in Huang and Huang (2003) is 158 b.p. Some portion of this spread is not default related (e.g. De Jong and Driessen (2005) find evidence for a liquidity component). Since it is thought that the Aaa-treasury spread is predominately due to non default factors, a reasonable estimate of the default portion of the Baa-treasury spread is the Baa-Aaa spread. Huang and Huang (2003) report the 4-year Baa-Aaa spread to be 103 b.p., consistent with our model-implied range. The qualitative behavior of finite-maturity spreads with respect to macroeconomic conditions is the same as for the infinite horizon case.

III.C Equity Market

Table VI shows the implications of our model for the equity risk premium, stock-market return volatility, and the risk-free rate. We focus on the impact of leverage on the risk premium and understanding the comparative statics of levered equity return volatility with respect to the EIS.

III.C.1 Leverage and the Risk Premium

Leverage has two effects on the risk premium. First, the dividend payment to equity holders is reduced by the coupon. Second, the probability of default shifts value from debtholders to equityholders. To distinguish between these two effects we compute not only the levered equity risk premium, but also the default-free levered equity risk premium. The latter is obtained by setting the values of Arrow-Debreu default claims in the levered premium equal to zero.

The act of paying coupons increases the risk premium, which can be seen by comparing the unlevered premium (1.2 to 1.9%) with the default-free levered premium (3.5 to 8.7%). Default risk, however, increases the value of equity via introducing a default option (which is the last term in (28)) and thus decreases the premium (3.1 to 6.4%). Therefore, the coupon effect dominates the default risk effect and leverage increases the risk premium.

III.C.2 The Risk-Distortion Factor, Long-Run Risk and the Risk Premium

As is well-known, increasing risk aversion increases risk premia. Table VI shows that increasing the EIS also increases the risk premia. This is a consequence of a stronger preference for earlier resolution of uncertainty, which leads to a larger distortion factor and hence more long-run risk.

III.C.3 Levered Stock-Market Return Volatility

Volatility measures total risk, not just priced risk, so its behavior with respect to the EIS is quite different from risk premia. Table VI shows that increasing the EIS decreases levered stock-market return volatility. The effect is due to leverage. Equation (34) shows that levered stock return volatility depends on the elasticity of equity with respect to earnings, given in (33). This elasticity equals unity in the absence of leverage. However, in the presence of leverage this is not so. Crucially, the elasticity of equity with respect to earnings is increasing in $q'_{D,ij}$, the deltas of the Arrow-Debreu default claims. Because these claims are digital puts, their deltas decrease as they get more in the money. Since increasing the EIS raises the values of the Arrow-Debreu default claims it must also decrease their deltas. Therefore, the elasticity of equity with respect to earnings and hence levered stock return volatility falls as the EIS rises.

III.D Cross-Market co-movement

Both earlier work by Fama and French (1993) and more recent work by Campbell and Taksler (2003), Zhang, Zhou, and Zhu (2005) and Tauchen and Zhou (2006) finds evidence of co-movement between bond and stock market variables. For instance, Campbell and Taksler find a 1 percentage point increase in the standard deviations of excess returns increases corporate bond spreads by 14 b.p. Tauchen and Zhou regress Moody's Aaa and Baa credit spread on the jump-component of aggregate stock-return volatility and find significant regression coefficients of 1.4998 and 1.9181, respectively. To replicate Tauchen and Zhou's findings for Baa credit spreads we simulate 100 panels each containing 3000 firms using parameter values as in Table III. We simulate 5 years of monthly data and then regress the equally-weighted credit spread on the value-weighted jump component of stock-return volatility for each panel in line with Tauchen and Zhou's empirical exercise. When relative risk aversion is 10 and the EIS is 1.5, Table IX shows that our model generates mean regression coefficients of 1.786 and 2.296, depending on the initial state of the economy. These coefficients are very close to the result Tauchen and Zhou obtained from the actual data.²⁰

Our model's results hinge crucially on intertemporal macroeconomic risk, which creates a jump-factor in both stock-return volatility and credit spreads. Because we model intertemporal macroeconomic risk via jumps in the drift and diffusion components of consumption and earnings growth, both equity and debt

²⁰The regression coefficients we obtain from our simulated data are expressed in daily units so they can be compared directly with the empirical estimates in Tauchen and Zhou.

values jump. Jumps in equity value always create a jump component in stock-return volatility, but jumps in debt value are not always priced in the credit spread. The jumps are priced if and only if there are correlated jumps in the state-price density, i.e. the risk-distortion factor, ω , is not equal to one. If we want credit spreads to move up at the same time as volatility, the state-price density must jump up when the economy shifts into recession, i.e. $\omega > 1$.

III.E Empirical Corporate Finance Implications

Our framework gives rise to a number of readily testable empirical implications on corporate default and capital structure decisions. While both default and financing decisions are affected to some extent along similar lines, for clarity in stating empirical predictions we start with the effect of macroeconomic conditions on default decisions.

III.E.1 Optimal Default Timing

The model gives rise to the following four empirical predictions regarding the optimal default boundary and macroeconomic conditions.

First and most importantly, the optimal earnings default boundary is higher in a recession. The intuition is that in a boom the default option is larger and equityholders are ready to default only under particularly bad earnings outcomes. Therefore default is more likely in bad times. As a result, the distribution of future cash flow growth rates in a recession is wider and there is a noticeable time-variation in default propensity as macroeconomic conditions change. Interestingly, when preference parameters are chosen to generate a realistic risk premium and credit spread, discount rates, $r_{A,i}$, are countercyclical, implying that the asset-value default boundary is procyclical, i.e. $A_1(X_{D,1}) < A_2(X_{D,2})$. Chen, Collin-Dufresne, and Goldstein (2006), however, demonstrate that habit formation preferences with i.i.d. consumption growth can only generate a realistic spread with a countercyclical asset-value default boundary. Our result for the asset-value default boundary thus offers empiricists in corporate finance a definitive way of contributing to the asset pricing literature by determining which of the long-run risk or habit-formation classes of models are more suitable for the joint valuation of debt and equity.

Second, when the economy switches into a recession, firms can default even if their earnings have not changed recently. To see why, suppose that we are in state 2 (boom), with default boundary $X_{D,2}$ and $X_{D,2} < X < X_{D,1}$. If the economy stays in a boom, the firm will stay solvent. But as soon as the economy goes into recession, the firm will go bankrupt. Thus, in the presence of intertemporal macroeconomic risk defaults cluster, consistent with empirical facts. The effect is similar to the one in Hackbarth, Miao, and Morellec (2006), where defaults cluster because of jumps in cash flows. In our model, defaults cluster not only because of a cash flow effect, but also because of jumps in the state-price density. Importantly, we will observe a greater number of simultaneous defaults when the economy switches into a recession.

Third, the decision to default is affected by the decreased present value of future cash flows, driven by changes in growth rates, in contrast with Hackbarth, Miao, and Morellec (2006), where the earnings level switches. Modelling the effect of future growth rates on the decision to default may partly explain why defaults are observed on occasion to happen “too late” relative to a supposedly efficient timing outcome.

Last, but not least, default decisions are subject to a hysteresis effect: the propensity to default depends on the macroeconomic conditions at the dates of past financing events. Firms which decide on their capital structure in booms optimally choose higher leverage and are more likely to default, especially when the boom gives way to a recession. Table IV demonstrates that the default probabilities of firms choosing leverage in a boom are about twice those of firms choosing leverage in a recession.

III.E.2 Optimal Leverage

The relation between leverage and macroeconomic conditions has been investigated in recent empirical studies (see Korajczyk and Levy (2003)), which find leverage is countercyclical. Our results show two effects, both of which are testable empirically. First, when the economy switches from a boom into a recession, the market value of equity drops more than the market value of debt, thus making leverage countercyclical. Second, if a firm *chooses* its financing at the time of a recession, its choice will be driven more by financial flexibility considerations and thus optimal leverage will be lower. Naturally, the latter effect will be more pronounced for financially constrained firms, since they are more likely to suffer in a recession. It would be interesting to study the relation between these two effects by considering historical evidence on capital structure over the business cycle.

Importantly, there is substantial hysteresis in the model-implied leverage ratios.²¹ Table IV shows that firms with identical cash flow processes, but subject to different macroeconomic conditions will optimally choose different leverage ratios. One would also find that firms which choose lower leverage in a boom are more likely to have lower debt capacity (e.g. they have higher volatility) and thus by choosing lower leverage they protect themselves against adverse recession jumps. To sum up, we hypothesize that empirically one would find that macroeconomic conditions during past financing events are an important determinant of current capital structure.

At the same time, firms that decide to lever up in a recession for the first time are likely to be less affected by negative economy-wide shocks. While the option to delay refinancing is not explicitly considered in our model, it is intuitive that firms may prefer to delay financing decisions till a boom. Our results also point to the co-movement of capital structure across firms over the business cycle. Moreover, this co-movement is expected to be higher in bad times when equity values are affected by higher systematic volatility.

²¹Recent empirical evidence (see Lemmon, Roberts, and Zender (2006)) points to the existence of hysteresis consistent with dynamic capital-structure models of infrequent refinancing (see Strebulaev (2007)).

An empirical implication which is perhaps less easy to test because of data constraints, is the impact of preferences on capital structure decisions. We find that if the economy consists of agents who prefer the earlier resolution of temporal uncertainty, firms will issue lower leverage. While our setting employs a representative agent, it is natural to expect that in a heterogeneous agent economy, firms run by managers who prefer the earlier resolution of temporal uncertainty will opt for more conservative financial policies. This observation adds a new cross-sectional determinant of capital structure. Since we find the effect to be quantitatively important, we expect empiricists to be interested in investigating this issue further.

IV Stripping Down the Model: What Causes What?

In this section, we show how each of our modelling assumptions impacts the credit spread and the equity risk premium. We strip down the model by removing intertemporal macroeconomic risk in the first and second moments of earnings and consumption growth and the representative agent's preference about the resolution of that risk over time. This leaves us with Model 1a, where aggregate consumption growth and earnings growth is i.i.d. and the representative agent has power utility. We then rebuild the model piece-by-piece. In Model 1b, we introduce Epstein-Zin-Weil preferences. In Model 2, we add Markov switching in the first and second moments of earnings growth, but not to consumption growth. And finally, in Model 3, we rebuild the model fully by having Markov switching in the first and second moments of consumption growth. To get a fair comparison across models, we calibrate them all to the same data on earnings and consumption and use an optimal default boundary with a coupon chosen so leverage is the same in each model. One consequence of this assumption is that we cannot choose idiosyncratic earnings volatility to get a realistically low actual default probability across all models. Tables VII and VIII summarize our results for the corporate bond and equity markets, respectively.

IV.A Credit Spread Levels

When there is no intertemporal macroeconomic risk, using Epstein-Zin-Weil preferences instead of power utility (moving from Model 1a to Model 1b) increases credit spreads. This increase stems purely from an increase in the time-adjustment factor in the Arrow-Debreu default claim: separating relative risk aversion from the EIS allows us to reduce the risk-free rate.

Adding Markov switching to the first and second moments of earnings growth, but not consumption growth (moving from Model 1b to Model 2) does not impact the size of the credit spread much at all, because these switches are not correlated with the state-price density, and are hence not priced. This is summarized by the fact that the risk-distortion factor, ω , is 1.

Introducing switching in the moments of consumption growth (moving from Model 2 to Model 3) leads to a significant increase in the spread. Switches in the moments of earnings growth are now correlated with the

state-price density ($\omega > 1$, so the state-price density jumps up whenever expected earnings growth/earnings growth volatility jumps down/up), so they are priced into credit spreads. This is reflected by an increase in the risk-adjustment factor.

In summary, two assumptions lie behind the model's ability to generate high prices for Arrow-Debreu default claims, without increasing actual default probabilities. The first is the use of Epstein-Zin-Weil preferences, which increases the time-adjustment factor by lowering the risk-free rate. The second is the assumption of switching in the first and second moments of both earnings and consumption growth rates (intertemporal macroeconomic risk). When the representative agent is averse to the delayed resolution of intertemporal risk ($\gamma > 1/\psi$), she dislikes intertemporal macroeconomic risk, which is reflected in an increased risk-adjustment factor.

IV.B The Cyclicalities of Credit Spreads

First, we note that in models 1a and 1b, credit spreads are not cyclical since there is no intertemporal macroeconomic risk. Introducing switches in the first and second moments of earnings growth alone (Model 2) leads to countercyclical credit spreads. This countercyclicity is driven by countercyclical Arrow-Debreu default claims, which are in turn countercyclical since actual default probabilities are. It is intuitive that in the low state, when systematic earnings volatility is high, the actual default probability will also be high. This underlines the importance of making volatilities change with macroeconomic conditions. Introducing switching in the moments of consumption growth (Model 3) makes the time-adjustment countercyclical and the risk-adjustment procyclical. Overall, the Arrow-Debreu default claims remain countercyclical. This is in contrast with Chen, Collin-Dufresne, and Goldstein (2006), where the credit spread is procyclical, unless the asset-value default boundary is countercyclical. The reason for this stems from the time-adjustment factor. Chen, Collin-Dufresne, and Goldstein (2006) use Campbell-Cochrane habit formation to model preferences. Therefore, the risk-free rate is constant, implying that the time-adjustment is constant. Furthermore, the risk-adjustment is very procyclical. Consequently, the Arrow-Debreu default claims and hence the credit spread are procyclical, unless an exogenously countercyclical asset-value default boundary is imposed.

IV.C Equity Market Variables and the Risk-Free Rate

Introducing Epstein-Zin-Weil preferences when there is no intertemporal macroeconomic risk decreases the risk-free rate. This, however, has no impact on risk premia (see Kocherlakota (1990)). While switching in the moments of earnings growth makes the price-earnings ratio procyclical, which implies risk premia are countercyclical, risk premia remain small. Finally, introducing switching in the first and second moments of consumption growth increases risk premia since upward jumps in the state-price density are now correlated with downward jumps in price-earnings ratios, creating a jump-risk premium. We can see this clearly in

the behavior of the risk-distortion factor, which is now greater than 1. That implies that the risk-neutral probability of switching from a boom to a recession is now higher than the actual probability, which is reflected in a higher risk premium.

V Conclusion

We develop a theoretical framework that jointly prices corporate debt and equity in order to deliver a unified understanding of what drives the equity risk premium, credit spreads, and optimal financing decisions. To this end, we embed a structural model of credit risk with optimal financing decisions inside a representative agent consumption-based model. To study a common economic mechanism that affects both credit and equity markets, we introduce intertemporal macroeconomic risk by allowing the first and second moments of consumption and earnings growth processes to switch randomly. Furthermore, we ensure the representative agent dislikes these regime shifts, by assuming she has Epstein-Zin-Weil preferences and prefers uncertainty to be resolved sooner rather than later.

Intertemporal macroeconomic risk combined with an aversion to it makes the state-price density jump upward in recessions, leading to jump-risk premia in asset returns. Jump risk impacts both credit spreads and stock returns, generating co-movement between credit spreads and the jump component of stock-market return volatility. The stock-market risk premium increases as the agent's dislike for regime shifts increases. The model can generate realistically high credit spreads without raising actual default probabilities and leverage. This is crucial, because in the data expected default frequencies are very small and leverage is relatively low. In essence, the framework can drive a wedge between the value of the Arrow-Debreu default claim and actual default probabilities. To show this, we develop a novel understanding of the intuition behind the Arrow-Debreu default claim. We decompose the claim into three components: the actual default probability, a risk adjustment and a time adjustment. To increase credit spreads both the risk and time adjustments must be larger than in standard structural models. We show how incorporating intertemporal macroeconomic risk achieves this goal.

Our model also generates a number of testable corporate finance implications relating to the effect of macroeconomic conditions on default and optimal financing decisions and can give rise to co-movement in the time-variation of capital structure and default clustering. Importantly, for parameter values which generate a realistic equity premium and credit spread, the asset-value default boundary is procyclical. This is in contrast with Chen, Collin-Dufresne, and Goldstein (2006), who show that in a habit-formation model with i.i.d. consumption growth, the asset value default boundary has to be countercyclical to obtain realistic credit spreads. Hence, empirical studies of the default boundary would offer an appealing way to judge whether long-run risk or habit-formation models are more promising for jointly pricing debt and equity.

This paper is only a first step towards the development of a fully-fledged consistent framework for pricing corporate equity and debt and the unification of existing asset pricing and corporate finance paradigms. Interesting possibilities for further research include studying the effects of default on consumption and introducing heterogeneous agents to distinguish between equity and debtholders.

A Appendix: Calibration Details

To estimate a Markov switching model as described in Hamilton (1989), we start by assuming that aggregate consumption, C , is given by

$$c_{t+1} = \Delta \log C_{t+1} = g_i - \frac{1}{2}\sigma_{C,i}^2 + \sigma_{C,i}\varepsilon_{C,t+1}, \quad (\text{A1})$$

and aggregate earnings, $X = \sum_{n=1}^N X_n$, by

$$x_{t+1} = \Delta \log X_{t+1} = \theta_i - \frac{1}{2}(\sigma_{X,i}^s)^2 + \sigma_{X,i}^s \varepsilon_{X,t+1}, \quad (\text{A2})$$

where shocks to earnings growth and consumption growth are normally distributed with zero mean and unit variance and correlation of ρ_{XC} . Equation (A1) is simply a discretized version of (1). To justify (A2), we prove that if $\theta_{n,i} = \theta_i$ and $\sigma_{X,n,i}^s = \sigma_{X,i}^s$ for all $i \in \{1, 2\}$ and $n \in \{1, \dots, N\}$, and there exists some $\epsilon > 0$ independent of N , such that $\frac{X_n}{X} \leq \frac{\epsilon}{N}$ for all $n \in \{1, \dots, N\}$, then

$$\lim_{N \rightarrow \infty} \frac{dX}{X} = \theta_i dt + \sigma_{X,i}^s dB_X^s. \quad (\text{A3})$$

The proof proceeds by noting that

$$\frac{dX}{X} = \theta_i dt + \sigma_{X,i}^s dB_X^s + \sum_{n=1}^N \frac{X_n}{X} \sigma_{X,i,n}^{id} dB_{X,n}^{id},$$

from which it follows that

$$\text{Var}_t \left(\frac{dX}{X} \right) = (\sigma_{X,i}^s)^2 + \sum_{n=1}^N \left(\frac{X_n}{X} \right)^2 (\sigma_{X,i,n}^{id})^2 \leq (\sigma_{X,i}^s)^2 + \frac{1}{N} \epsilon^2 (\max_n \sigma_{X,i,n}^{id})^2,$$

which implies that

$$\lim_{N \rightarrow \infty} \text{Var}_t \left(\frac{dX}{X} \right) = (\sigma_{X,i}^s)^2.$$

Equation (A3) follows. Thus, (A2) is justified under the given assumptions if the number of firms N is large. As a consequence, we can obtain the aggregate levered equity risk premium by setting $\sigma_{X,n}^{id} = 0$. The joint normal distribution for earnings growth and consumption growth denoted by Φ

$$\Phi(x_t, c_t | \nu_t = i, \Omega_{t-1}; \Gamma) = \frac{1}{2\pi\sigma_{X,i}^s\sigma_{C,i}\sqrt{1-\rho_{XC}^2}} \exp\{-1/2(\varepsilon_{X,t}^2 + \varepsilon_{C,t}^2 - 2\rho_{XC}\varepsilon_{X,t}\varepsilon_{C,t})/(1-\rho_{XC}^2)\}.$$

where Ω_t denotes the set of all observations up to time t and Γ the set of unknown parameters. Having obtained our parameter estimates by maximizing the log-likelihood, we must obtain the parameters of the continuous-time Markov chain from the estimated discrete-time transition matrix

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix},$$

where $P_{ij} = P(\nu_{t+1} = \theta_j | \nu_t = \theta_i)$ is the probability of switching from state i to j within a quarter. To do this note that the matrix of quarterly transition probabilities, P , is related to the generator of the continuous-time chain

$$\Lambda = \begin{pmatrix} -\lambda_1 & \lambda_1 \\ \lambda_2 & -\lambda_2 \end{pmatrix}$$

by

$$e^{\Lambda \frac{1}{4}} = P.$$

Using standard techniques from linear algebra, we can show that

$$e^{\Lambda \frac{1}{4}} = \begin{pmatrix} f_1 & f_2 \\ f_1 & f_2 \end{pmatrix} + \begin{pmatrix} f_2 & -f_2 \\ -f_1 & f_1 \end{pmatrix} e^{-\frac{1}{4}p}, \quad (\text{A4})$$

Where $p = \lambda_1 + \lambda_2$ and $f_i = \frac{\lambda_i}{p}$, $i \in \{1, 2\}$. Equating (A4) with P implies after some algebra:

$$\begin{aligned} f_1 &= \left(1 + \frac{P_{12}}{P_{21}}\right)^{-1} \\ p &= -4 \ln \left(1 - \frac{P_{12}}{1 - f_1}\right). \end{aligned}$$

B Appendix: Derivation of the State Price Density

First, we introduce some notation. If some function E depends on the current state of the economy i.e. $E_t = E(\nu_t)$, then E is jump process which is right continuous with left limits, i.e. RCLL. If a jump from state i to $j \neq i$ occurs at date t , then we abuse notation slightly and denote the left limit of E at time t by E_i , where i is the index for the state. i.e. $E_{t-} = \lim_{s \uparrow t} E_s = E_i$. Similarly $E_t = \lim_{s \downarrow t} E_s = E_j$. We shall use the same notation for all processes that jump, because of their dependence on the state of the economy.

Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

$$f(c, v) = \beta (h^{-1}(v))^{1-\gamma} u(c/h^{-1}(v)),$$

where

$$\begin{aligned} u(x) &= \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0, \\ h(x) &= \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1. \\ \ln x, & \gamma = 1. \end{cases} \end{aligned}$$

The representative agent's value function is given by

$$J_t = E_t \int_t^\infty f(C_t, J_t) dt. \quad (\text{B1})$$

Theorem 1 *The state-price density of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by*

$$\pi_t = \begin{cases} (\beta e^{-\beta t})^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_{C,t} e^{\int_0^t p_{C,s}^{-1} ds} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1 \\ \beta e^{-\beta \int_0^t [1+(\gamma-1) \ln(V_s^{-1})] ds} C_t^{-\gamma} V_t^{-(\gamma-1)}, & \psi = 1 \end{cases}. \quad (\text{B2})$$

When $\psi \neq 1$, the price-consumption ratio in state i , $p_{C,i}$, satisfies the nonlinear equation system:

$$p_{C,i}^{-1} = \bar{r}_i + \gamma \sigma_{C,i}^2 - g_i - \left(1 - \frac{1}{\psi}\right) \lambda_i \left(\frac{(p_{C,j}/p_{C,i})^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1-\gamma} \right), \quad i \in \{1, 2\}, j \neq i. \quad (\text{B3})$$

where

$$\bar{r}_i = \beta + \frac{1}{\psi} g_i - \frac{1}{2} \gamma \left(1 + \frac{1}{\psi} \right) \sigma_{C,i}^2, \quad i \in \{1, 2\}. \quad (\text{B4})$$

When $\psi = 1$, define V via

$$J = \ln(CV). \quad (\text{B5})$$

Then V_i satisfies the nonlinear equation system:

$$\beta \ln V_i = g_i - \frac{\gamma}{2} \sigma_{C,i}^2 + \lambda_i \frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma}, \quad i \in \{1, 2\}, j \neq i. \quad (\text{B6})$$

Proof of Theorem 1

Duffie and Skiadas (1994) show that the state-price density for a *general* normalized aggregator f is given by

$$\pi_t = e^{\int_0^t f_v(C_s, J_s) dt} f_c(C_t, J_t), \quad (\text{B7})$$

where $f_c(\cdot, \cdot)$ and $f_v(\cdot, \cdot)$ are the partial derivatives of f with respect to its first and second arguments, respectively, and J is the value function given in (B1). The Feynman-Kac Theorem implies

$$f(C_t, J_{t-})|_{\nu_{t-}=i} dt + E_t [dJ_t | \nu_{t-}=i] = 0, \quad i \in \{1, 2\}.$$

Using Ito's Lemma we rewrite the above equation as

$$0 = f(C, J_i) + C J_{i,C} g_i + \frac{1}{2} C^2 J_{i,CC} \sigma_{C,i}^2 + \lambda_i (J_j - J_i), \quad (\text{B8})$$

for $i, j \in \{1, 2\}$, $j \neq i$. We guess and verify that

$$J = h(CV), \quad (\text{B9})$$

where V_i satisfies the nonlinear equation system

$$0 = \beta u(V_i^{-1}) + g_i - \frac{1}{2} \gamma \sigma_{C,i}^2 + \lambda_i \left(\frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma} \right), \quad i, j \in \{1, 2\}, j \neq i. \quad (\text{B10})$$

Substituting (B5) into (B7) and simplifying gives

$$\pi_t = \beta e^{-\beta \int_0^t [1 + (\gamma - \frac{1}{\psi}) u(V_s^{-1})] dt} C_t^{-\gamma} V_t^{-\left(\gamma - \frac{1}{\psi}\right)}. \quad (\text{B11})$$

When $\psi = 1$, the above equation gives the second expression in (B2). We rewrite (B10) as

$$\beta \left[1 + \left(\gamma - \frac{1}{\psi} \right) u(V_i^{-1}) \right] = \bar{r}_i - \left(\gamma - \frac{1}{\psi} \right) \lambda_i \left(\frac{(V_j/V_i)^{1-\gamma} - 1}{1-\gamma} \right) - \left[\gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], \quad i, j \in \{1, 2\}, j \neq i, \quad (\text{B12})$$

where \bar{r}_i is given in (B4). Setting $\psi = 1$ in (B12) gives (B6).

To derive the first expression in (B2) from (B11) we prove that

$$V_i = (\beta p_{C,i})^{\frac{1}{1-\frac{1}{\psi}}}, \quad \psi \neq 1. \quad (\text{B13})$$

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption (C^*) and risky asset portfolio (φ) to maximize her expected utility

$$J_t^* = \sup_{C^*, \varphi} E_t \int_t^\infty f(C_t^*, J_t^*) dt.$$

Observe that J^* depends on optimal consumption-portfolio choice, whereas the J defined previously in (B5) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, C^* , invests a proportion, φ , of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth, W , evolves according to the dynamic budget constraint:

$$\frac{dW_t}{W_{t-}} = \varphi_{t-} (dR_{C,t} - r_{t-}dt) + r_{t-}dt - \frac{C_{t-}^*}{W_{t-}}dt,$$

where $dR_{C,t}$ is the cumulative return on the claim to aggregate consumption. We define $N_{i,t}$ as the Poisson process which jumps upward by one whenever the state of the economy switches from i to $j \neq i$. The compensated version of this process is the Poisson martingale

$$N_{i,t}^P = N_{i,t} - \lambda_i t.$$

It follows from applying Ito's Lemma to $P = p_C C$, that the cumulative return on the claim to aggregate consumption is

$$dR_{C,t} = \frac{dP_t + C_t dt}{P_{t-}} = \mu_{R_C,t-} dt + \sigma_{C,t-} dB_{C,t} + \sigma_{R_C,t-}^P dN_{\nu_{t-},t}^P,$$

where

$$\begin{aligned} \mu_{R_C,t-} |_{\nu_{t-}=i} &= \mu_{R_C,i} = g_i + \frac{1}{2} \sigma_{C,i}^2 + \lambda_i \left(\frac{p_{C,j}}{p_{C,i}} - 1 \right) + \frac{1}{p_{C,i}}, \\ \sigma_{C,t-} |_{\nu_{t-}=i} &= \sigma_{C,i}, \\ \sigma_{R_C,t-}^P |_{\nu_{t-}=i} &= \sigma_{R_C,i}^P = \frac{p_{C,j}}{p_{C,i}} - 1, \end{aligned}$$

for $i \in \{1, 2\}$, $j \neq i$. The total volatility of returns to holding the consumption claim, when the current state is i , is given by

$$\sigma_{R_C,i} = \sqrt{\sigma_{C,i}^2 + \lambda_i \left(\sigma_{R_C,i}^P \right)^2}.$$

Note that C^* is the consumption to be chosen by the agent, i.e. it is a control, and at this stage we cannot rule out the possibility that it jumps with the state of the economy. In contrast, C is aggregate consumption, i.e. the dividend received by an investor who holds the claim to aggregate consumption. Because aggregate consumption, C , is continuous, its left and right limits are equal, i.e. $C_{t-} = C_t$.

The system of Hamilton-Jacobi-Bellman partial differential equations for the agent's optimization problem is

$$\sup_{C^*, \varphi} f(C_{t-}^*, J_{t-}^*) |_{\nu_{t-}=i} dt + E_t [dJ_t^* | \nu_{t-} = i] = 0, \quad i \in \{1, 2\}.$$

Applying Ito's Lemma to $J_t^* = J^*(W_t, \nu_t)$ allows us to write the above equation as

$$\begin{aligned} 0 &= \sup_{C_i^*, \varphi_i} f(C_i^*, J_i^*) + W_i J_{i,W}^* \left(\varphi_i (\mu_{R_C,i} - r_i) + r_i - \frac{C_i^*}{W_i} \right) + \frac{1}{2} W_i^2 J_{i,WW}^* \varphi_i^2 \sigma_{R_C,i}^2 + \\ &+ \lambda_i (J_j^* - J_i^*), \quad i \in \{1, 2\}, j \neq i. \end{aligned}$$

We guess and verify that

$$J_t^* = h(W_t F_t),$$

where F_t satisfies the nonlinear equation system

$$0 = \sup_{C_i^*, \varphi_i} \beta u \left(\frac{C_i^*}{W_i F_i} \right) + \left(\varphi_i (\mu_{R_C,i} - r_i) + r_i - \frac{C_i^*}{W_i} \right) - \frac{1}{2} \gamma \varphi_i^2 \sigma_{R_C,i}^2 + \lambda_i \left(\frac{(F_j/F_i)^{1-\gamma} - 1}{1-\gamma} \right), \quad i \in \{1, 2\}, j \neq i.$$

From the first order conditions of the above equations, we obtain the optimal consumption and portfolio policies:

$$\begin{aligned} C_i^* &= \beta^\psi F_i^{-(\psi-1)} W_i, \quad i \in \{1, 2\}, \\ \varphi_i &= \frac{1}{\gamma} \frac{\mu_{RC,i} - r_i}{\sigma_{RC,i}^2}, \quad i \in \{1, 2\}. \end{aligned}$$

The market for the consumption good must clear, so $\varphi_i = 1$, $W_i = P_i$, $C_i^* = C$ (and thus $J = J^*$). Note that this forces the optimal portfolio proportion to be one and the optimal consumption policy to be continuous. Hence

$$\mu_{RC,i} - r_i = \gamma \sigma_{RC,i}^2,$$

and

$$p_{C,i} = \beta^{-\psi} F_i^{1-\psi}. \quad (\text{B14})$$

The above equation implies that for $\psi = 1$, $p_{C,i} = 1/\beta$. The equality, $J = J^*$, implies that $CV_i = WF_i$. Hence, $F_i = p_{C,i}^{-1} V_i$. Using this equation to eliminate F_i from (B14) gives (B13). Substituting (B13) into (B11) and (B12) gives the expressions in (B2) for $\psi \neq 1$ and (B3), after some algebra.

C Appendix: Proofs

Proof of Proposition 1

We start by proving that the state-price density satisfies the stochastic differential equation

$$\frac{d\pi_t}{\pi_{t-}} \Big|_{\nu_{t-}=i} = -r_i dt + \frac{dM_t}{M_{t-}} \Big|_{\nu_{t-}=i}, \quad (\text{C1})$$

where M is a martingale under \mathbb{P} such that

$$\frac{dM_t}{M_{t-}} \Big|_{\nu_{t-}=i} = -\Theta_i^B dB_t + \Theta_i^P dN_{i,t}^P, \quad (\text{C2})$$

r_i is the risk-free rate in state i given by

$$r_i = \begin{cases} \bar{r}_1 + \lambda_1 \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{-\frac{\gamma-1}{\psi}} - 1 \right) - (\omega^{-1} - 1) \right], & i = 1 \\ \bar{r}_2 + \lambda_2 \left[\frac{\gamma - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{\frac{\gamma-1}{\psi}} - 1 \right) - (\omega - 1) \right], & i = 2 \end{cases}, \quad (\text{C3})$$

and

$$\omega_2 = \omega_1^{-1} = \omega,$$

where ω is the solution of

$$G(\omega) = 0, \quad (\text{C4})$$

and

$$G(x) = \begin{cases} x^{-\frac{1-\frac{1}{\psi}}{\gamma-\frac{1}{\psi}}} - \frac{\bar{r}_2 + \gamma \sigma_{C,2}^2 - g_2 + \lambda_2 \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{\frac{\gamma-1}{\psi}} - 1 \right)}{\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1-\frac{1}{\psi}}{\gamma-1} \left(x^{-\frac{\gamma-1}{\psi}} - 1 \right)}, & \psi \neq 1 \\ \ln x^{\frac{1}{\gamma-1}} - \frac{g_2 - \frac{1}{2} \gamma \sigma_{C,2}^2 + \lambda_2 (x-1)}{g_1 - \frac{1}{2} \gamma \sigma_{C,1}^2 + \lambda_1 (x^{-1}-1)}, & \psi = 1 \end{cases}. \quad (\text{C5})$$

Θ_i^B is the market price of risk due to Brownian shocks in state i , given by

$$\Theta_i^B = \gamma \sigma_{C,i}. \quad (C6)$$

and Θ_i^P is the market price of risk due to Poisson shocks when the economy switches out of state i :

$$\Theta_i^P = \omega_i - 1. \quad (C7)$$

We begin the proof by noting that if we define

$$\omega_i = \frac{\pi_t}{\pi_{t-}} \Big|_{\nu_{t-}=i, \nu_t=j}, j \neq i, \quad (C8)$$

then (B2) implies that

$$\omega_i = \begin{cases} \left(\frac{p_{C,j}}{p_{C,i}} \right)^{-\frac{\gamma-1}{\psi}}, & \psi \neq 1. \\ \left(\frac{V_j}{V_i} \right)^{-(\gamma-1)}, & \psi = 1. \end{cases} \quad (C9)$$

The above equation implies that $\omega_2 = \omega_1^{-1}$, so we can set $\omega_2 = \omega_1^{-1} = \omega$, where ω is determined below. Using (C9) we can rewrite (B3) and (B6) as

$$p_{C,i} = \frac{1}{\bar{r}_i + \gamma \sigma_{C,i}^2 - g_i + \lambda_i \frac{1-\frac{1}{\psi}}{\gamma-1} \left(\omega_i^{\frac{\gamma-1}{\gamma-1/\psi}} - 1 \right)}, i \in \{1, 2\}, \quad (C10)$$

and

$$\beta \ln V_i = g_i - \frac{1}{2} \gamma \sigma_{C,i}^2 + \lambda_i \frac{\omega_i - 1}{1 - \gamma}, i \in \{1, 2\}, \quad (C11)$$

respectively. Therefore, from (C9) and the above two equations it follows that ω is the solution of Equation (C4). Ito's Lemma implies that the state-price density evolves according to

$$\begin{aligned} \frac{d\pi_t}{\pi_{t-}} &= \frac{1}{\pi_{t-}} \frac{\partial \pi_{t-}}{\partial t} dt + \frac{1}{\pi_{t-}} C_t \frac{\partial \pi_{t-}}{\partial C_t} \frac{dC_t}{C_t} + \frac{1}{2} \frac{1}{\pi_{t-}} C_t^2 \frac{\partial^2 \pi_{t-}}{\partial C_t^2} \left(\frac{dC_t}{C_t} \right)^2 \\ &+ \lambda \nu_{t-} \frac{\Delta \pi_t}{\pi_{t-}} dt + \frac{\Delta \pi_t}{\pi_{t-}} dN_{\nu_{t-}, t}^P, \end{aligned} \quad (C12)$$

where

$$\Delta \pi_t = \pi_t - \pi_{t-}.$$

The definition (C8) implies

$$\frac{\Delta \pi_t}{\pi_{t-}} \Big|_{\nu_{t-}=i, \nu_t=j} = \omega_i - 1, j \neq i.$$

Together with some standard algebra that allows us to rewrite (C12) as

$$\frac{d\pi_t}{\pi_{t-}} \Big|_{\nu_t=\nu_i} = - \left(\kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \lambda_i (1 - \omega_i) \right) dt - \gamma \sigma_{C,i} dB_{C,t} + (\omega_i - 1) dN_{i,t}^P.$$

Comparing the above equation with (C1), which is standard in an economy with jumps, gives (C6) and (C7), in addition to

$$r_i = \kappa_i + \gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 + \lambda_i (1 - \omega_i),$$

where

$$\kappa_i = \begin{cases} \beta \left[1 + \left(\gamma - \frac{1}{\psi} \right) \frac{(\beta p_{C,i})^{-1} - 1}{1 - \frac{1}{\psi}} \right], & \psi \neq 1 \\ \beta \left[1 + (\gamma - 1) \ln \left(V_i^{-1} \right) \right], & \psi = 1 \end{cases}. \quad (\text{C13})$$

We use Equations (C10) and (C11) to eliminate $p_{C,i}$ and V_i from (C13) to obtain

$$\kappa_i = \begin{cases} \bar{r}_i - \left(\gamma - \frac{1}{\psi} \right) \lambda_i \left(\frac{\omega_i^{\frac{\gamma-1}{\psi}} - 1}{1 - \gamma} \right) - \left[\gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], & \psi \neq 1 \\ \bar{r}_i + \lambda_i (\omega_i - 1) - \left[\gamma g_i - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,i}^2 \right], & \psi = 1 \end{cases}, \quad (\text{C14})$$

so

$$r_i = \begin{cases} \bar{r}_i - \left(\gamma - \frac{1}{\psi} \right) \lambda_i \left(\frac{\omega_i^{\frac{\gamma-1}{\psi}} - 1}{1 - \gamma} \right) + \lambda_i (1 - \omega_i), & \psi \neq 1. \\ \bar{r}_i, & \psi = 1. \end{cases} \quad (\text{C15})$$

Taking the limit of the upper expression in the above equation gives the lower expression, so (C3) follows. The total market price of consumption risk in state i accounts for both Brownian and Poisson shocks, and is thus given by

$$\Theta_i = \sqrt{(\Theta_i^B)^2 + \lambda_i (\Theta_i^P)^2}, \quad i \in \{1, 2\}. \quad (\text{C16})$$

Because the Poisson and Brownian shocks in (C2) are independent and their respective prices of risk are bounded, M is a martingale under the actual measure \mathbb{P} . Thus M defines the Radon-Nikodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}}$ via

$$M_t = E_t \left[\frac{d\mathbb{Q}}{d\mathbb{P}} \right].$$

It is a standard result (see Elliott (1982)) that

$$\hat{\lambda}_i = \lambda_i E_t \left[\frac{M_t}{M_{t-}} \mid \nu_{t-} = i, \nu_t = j \right], \quad j \neq i.$$

The jump component in $d\pi$ comes purely from dM . Thus, using (C8), we can simplify the above expression to obtain

$$\hat{\lambda}_i = \lambda_i \omega_i,$$

which implies (5) and (6).

We deduce the properties of the risk distortion factor, ω , from the properties of the function g defined in (C5). We restrict the domain of G to $x > 0$. First we consider the case where $\psi \neq 1$. We assume that the price-consumption ratios, $p_{C,i}$, $i \in \{1, 2\}$ are strictly positive. Therefore, G is continuous. We observe that if G is monotonic, then by continuity, $G(1)$ and $G'(1)$ are of the same sign iff $\omega < 1$ and $G(1)$ and $G'(1)$ are of different signs iff $\omega > 1$. Clearly, in both cases, ω is unique. To establish monotonicity note that

$$\begin{aligned} G'(x) &= -\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} \left[x^{-\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} - 1} \right. \\ &+ \frac{1}{\left(\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1 - \frac{1}{\psi}}{\gamma - 1} \left(x^{-\frac{\gamma-1}{\psi}} - 1 \right) \right)^2} \left(\bar{r}_1 + \gamma \sigma_{C,1}^2 - g_1 + \lambda_1 \frac{1 - \frac{1}{\psi}}{\gamma - 1} \left(x^{-\frac{\gamma-1}{\psi}} - 1 \right) \right) \lambda_2 x^{\frac{1/\psi-1}{\gamma-1/\psi}} \\ &\left. + \left(\bar{r}_2 + \gamma \sigma_{C,2}^2 - g_2 + \lambda_2 \frac{1 - \frac{1}{\psi}}{\gamma - 1} \left(\omega^{\frac{\gamma-1}{\psi}} - 1 \right) \right) \lambda_1 x^{-\frac{1/\psi-1}{\gamma-1/\psi} - 2} \right] \end{aligned}$$

The above equation implies that for $x > 0$, if $p_{C,1}$ and $p_{C,2}$ are strictly positive, then $G'(x)$ does not change sign. Therefore, G must be monotonic. Now we use the following expressions:

$$G(1) = 1 - \frac{\bar{r}_2 + \gamma\sigma_{C,2}^2 - g_2}{\bar{r}_1 + \gamma\sigma_{C,1}^2 - g_1},$$

and

$$G'(1) = -\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} \left[1 + \frac{(\bar{r}_1 + \gamma\sigma_{C,1}^2 - g_1)\lambda_2 + (\bar{r}_2 + \gamma\sigma_{C,2}^2 - g_2)\lambda_1}{(\bar{r}_1 + \gamma\sigma_{C,1}^2 - g_1)^2} \right],$$

to relate the signs of $G(1)$ and $G'(1)$ to the properties of the agent's preferences. Note that $G'(1) < 0$, ($G'(1) > 0$) iff $\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} > 0$, ($\frac{1 - \frac{1}{\psi}}{\gamma - \frac{1}{\psi}} < 0$). We assume that $\bar{r}_i + \gamma\sigma_{C,i}^2 - g_i > 0$ for $i \in \{1, 2\}$, which is equivalent to assuming that if the economy were always in state i , then the price-consumption ratio would be positive. Simple algebra tells us that $\bar{r}_i + \gamma\sigma_{C,i}^2 - g_i = \beta + \left(\frac{1}{\psi} - 1\right) \left(g_i - \frac{1}{2}\gamma\sigma_{C,i}^2\right)$. We know that $g_1 - \frac{1}{2}\gamma\sigma_{C,1}^2 < g_2 - \frac{1}{2}\gamma\sigma_{C,2}^2$. Therefore $G(1) < 0$, ($G(1) > 0$) iff $\psi > 1$, ($\psi < 1$). Consequently, $G(1)$ and $G'(1)$ are of the same sign iff $\gamma < 1/\psi$ and $G(1)$ and $G'(1)$ are of different signs iff $\gamma > 1/\psi$. It then follows that $\omega > 1$ iff $\gamma > 1/\psi$ and $\omega < 1$ iff $\gamma < 1/\psi$, assuming that $\psi \neq 1$.

Similarly, when $\psi = 1$, if we assume that $V_i > 0$ for $i \in \{1, 2\}$, then we can prove that: $\omega > 1$ if $\gamma > 1$ and $g_i - \frac{1}{2}\gamma\sigma_{C,i}^2$, $i \in \{1, 2\}$ are of the sign and $\omega > 1$ if $\gamma < 1$ and $g_i - \frac{1}{2}\gamma\sigma_{C,i}^2$, $i \in \{1, 2\}$ are of opposite sign. Now, if $\gamma < 1$, then $\bar{r}_1 + \gamma\sigma_{C,1}^2 - g_1 > 0$ implies $g_1 - \frac{1}{2}\gamma\sigma_{C,1}^2 > 0$, which means $g_i - \frac{1}{2}\gamma\sigma_{C,i}^2$, $i \in \{1, 2\}$ cannot be of opposite sign. Therefore, $\omega > 1$ iff $\gamma > 1$.

So, for $\psi > 0$, $\omega > 1$ iff $\gamma > 1/\psi$ and $\omega < 1$ iff $\gamma < 1/\psi$. It follows that $\omega = 1$ iff $\gamma = 1/\psi$.

Derivations of Equations (13), (14) and (15)

The actual probability of default occurring within the time interval $[t, T]$, if current earnings are X_t , is given by

$$p_{D,T-t}(X_t) = \mathbb{P} \left(\inf_{s \in [t, T]} X_s \leq X_D \right).$$

It is a standard result that

$$p_{D,T-t}(X_t) = N(-d_+(\theta)) + \left(\frac{X_D}{X_t}\right)^{\frac{\theta - \frac{1}{2}\sigma_X^2}{\frac{\sigma_X^2}{2}}} N(-d_-(\theta)), \quad (\text{C17})$$

where

$$\sigma_X = \sqrt{(\sigma_X^{id})^2 + (\sigma_X^s)^2},$$

and

$$d_{\pm}(\theta) = \frac{\ln\left(\frac{X_t}{X_D}\right) \pm (\theta - \frac{1}{2}\sigma_X^2)(T-t)}{\sigma_X\sqrt{T-t}}. \quad (\text{C18})$$

The risk-neutral probability of default occurring within the time interval $[t, T]$, if current earnings are X_t , is given by

$$\hat{p}_{D,T-t}(X_t) = \mathbb{Q} \left(\inf_{s \in [t, T]} X_s \leq X_D \right).$$

Therefore,

$$\hat{p}_{D,T-t}(X_t) = N\left(-d_+(\hat{\theta})\right) + \left(\frac{X_D}{X_t}\right)^{\frac{\hat{\theta} - \frac{1}{2}\sigma_X^2}{\frac{\sigma_X^2}{2}}} N\left(-d_-(\hat{\theta})\right), \quad (\text{C19})$$

Taking the limits of (C17) and (C19) as $T \rightarrow \infty$ gives the actual and risk-neutral default probabilities

$$p_D(X_t) = \left(\frac{X_D}{X_t} \right)^{\frac{\theta - \frac{1}{2}\sigma_X^2}{\sigma_X^2}},$$

and

$$\hat{p}_D(X_t) = \left(\frac{X_D}{X_t} \right)^{\frac{\hat{\theta} - \frac{1}{2}\sigma_X^2}{\sigma_X^2}},$$

respectively. It is a standard result that the price of the Arrow-Debreu default claim is given by

$$q_D(X_t) = \left(\frac{X_D}{X_t} \right)^{\frac{1}{\sigma_X^2} \left[\hat{\theta} - \frac{1}{2}\sigma_X^2 + \sqrt{(\hat{\theta} - \frac{1}{2}\sigma_X^2)^2 + 2\bar{r}\sigma_X^2} \right]}.$$

where \bar{r} is the risk-free rate when there is no intertemporal macroeconomic risk, i.e.

$$\bar{r} = \beta + \frac{1}{\psi}g - \frac{1}{2}\gamma \left(1 + \frac{1}{\psi} \right) \sigma_C^2. \quad (\text{C20})$$

The above result is a special case of (C15). The ratio of the risk-neutral to actual default probability, \hat{q}_D/p_D , is the risk-adjustment factor, $\mathcal{R}(X_t)$, i.e. Equation (14). The time-adjustment factor is given by the ratio q_D/\hat{q}_D , i.e. Equation (15).

Proof of Proposition 2

No-arbitrage principle gives (8), which using Ito's Lemma can be rewritten as the following ordinary differential-equation system:

$$\frac{dq_{D,ij,t}}{dX_t} \hat{\theta}_i X_t + \frac{1}{2} \frac{d^2 q_{D,ij,t}}{dX_t^2} \sigma_{X,i}^2 X_t^2 + \hat{\lambda}_i (q_{D,kj,t} - q_{D,ij,t}) = r_i q_{D,ij,t}, \quad i, j \in \{1, 2\}, k \neq i, \quad (\text{C21})$$

where

$$\sigma_{X,i} = \sqrt{(\sigma_{X,i}^{id})^2 + (\sigma_{X,i}^s)^2} \quad (\text{C22})$$

is total earnings growth volatility in state i and

$$\hat{\theta}_i = \theta_i - \gamma \rho_{XC,i} \sigma_{X,i}^s \sigma_{C,i}$$

is the risk-neutral earnings growth rate in state i . The definitions of the payoffs of the Arrow-Debreu default claims give us the following boundary conditions:

$$q_{D,ij}(X) = \begin{cases} 1, & j = i, X \leq X_{D,i} \\ 0, & j \neq i, X \leq X_{D,i} \end{cases}. \quad (\text{C23})$$

Value-matching and smooth-pasting give us the remaining boundary conditions: for $j \in \{1, 2\}$

$$\begin{aligned} \lim_{X \downarrow X_{D,j}} q_{D,2j} &= \lim_{X \uparrow X_{D,j}} q_{D,2j}, \\ \lim_{X \downarrow X_{D,j}} q'_{D,2j} &= \lim_{X \uparrow X_{D,j}} q'_{D,2j}. \end{aligned}$$

Expressing (C21) in matrix form gives:

$$\left(\frac{1}{2} \begin{bmatrix} \sigma_{1,X}^2 & 0 \\ 0 & \sigma_{2,X}^2 \end{bmatrix} X^2 \frac{d^2}{dX^2} + \begin{bmatrix} \hat{\theta}_1 & 0 \\ 0 & \hat{\theta}_2 \end{bmatrix} X \frac{d}{dX} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} + \begin{bmatrix} -\hat{\lambda}_1 & \hat{\lambda}_1 \\ \hat{\lambda}_2 & -\hat{\lambda}_2 \end{bmatrix} \right) \begin{bmatrix} q_{D,11} & q_{D,12} \\ q_{D,21} & q_{D,22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (\text{C24})$$

From (C23) it follows that

$$q_{D,ij}|_{X=X_{D,i}} = \begin{cases} 1, & j=i \\ 0, & j \neq i \end{cases}. \quad (\text{C25})$$

We first solve (C24) subject to the conditions above for the region $X > X_{D,1}$. We seek solutions of the form

$$q_{D,ij} = h_{ij} X^k, j \in \{1, 2\}.$$

Hence,

$$\left(\frac{1}{2} \begin{bmatrix} \sigma_{X,1}^2 & 0 \\ 0 & \sigma_{X,2}^2 \end{bmatrix} k(k-1) + \begin{bmatrix} \hat{\theta}_1 & 0 \\ 0 & \hat{\theta}_2 \end{bmatrix} k + \begin{bmatrix} -\hat{\lambda}_1 - r_1 & \hat{\lambda}_1 \\ \hat{\lambda}_2 & -\hat{\lambda}_2 - r_2 \end{bmatrix} \right) \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (\text{C26})$$

A solution of the above equation exists if

$$\det \left(\frac{1}{2} \begin{bmatrix} \sigma_{X,1}^2 & 0 \\ 0 & \sigma_{X,2}^2 \end{bmatrix} k(k-1) + \begin{bmatrix} \hat{\theta}_1 & 0 \\ 0 & \hat{\theta}_2 \end{bmatrix} k + \begin{bmatrix} -\hat{\lambda}_1 - r_1 & \hat{\lambda}_1 \\ \hat{\lambda}_2 & -\hat{\lambda}_2 - r_2 \end{bmatrix} \right) = 0.$$

Therefore, k is a root of the quartic polynomial

$$\left[\frac{1}{2} \sigma_{X,1}^2 k(k-1) + \hat{\theta}_1 k + (-\hat{\lambda}_1 - r_1) \right] \left[\frac{1}{2} \sigma_{X,2}^2 k(k-1) + \hat{\theta}_2 k + (-\hat{\lambda}_2 - r_2) \right] - \hat{\lambda}_2 \hat{\lambda}_1 = 0, \quad (\text{C27})$$

which is the characteristic function of (C24). The above quartic has 4 distinct real roots, two of which are positive, provided that $\sigma_{X,i}, r_i, \hat{\lambda}_{ij} > 0$ for $i \in \{1, 2\}$ and $j \neq i$ (see Guo (2001)). Therefore the general solution of is

$$q_{D,ij} = \sum_{m=1}^4 h_{ij,m} X^{k_m},$$

where k_m is the m 'th root (ranked in order of increasing size, accounting for sign) of (C27). To ensure that $q_{D,ij}, i, j \in \{1, 2\}$ are finite as $X \rightarrow \infty$, we set $h_{ij,3} = h_{ij,4} = 0, i, j \in \{1, 2\}$, so we use only the two negative roots: $k_1 < k_2 < 0$. From equation (C26), it follows that

$$\frac{h_{21,m}}{h_{11,m}} = \frac{h_{22,m}}{h_{12,m}} = \epsilon(k_m), m \in \{1, 2\},$$

where

$$\epsilon(k) = -\frac{\hat{\lambda}_2}{\frac{1}{2} \sigma_{X,2}^2 k(k-1) + \hat{\theta}_2 k + (-\hat{\lambda}_2 - r_2)} = -\frac{\frac{1}{2} \sigma_{X,1}^2 k(k-1) + \hat{\theta}_1 k + (-\hat{\lambda}_1 - r_1)}{\hat{\lambda}_1}.$$

Therefore

$$\begin{aligned} q_{D,1j} &= \sum_{m=1}^2 h_{1j,m} X^{k_m}, j \in \{1, 2\}, \\ q_{D,2j} &= \sum_{m=1}^2 h_{1j,m} \epsilon(k_m) X^{k_m}, j \in \{1, 2\}. \end{aligned}$$

We now solve (C24) subject to the relevant boundary conditions for the region $X_2 < X \leq X_1$. We know

$$\begin{aligned} q_{D,11} &= 1, \\ q_{D,12} &= 0. \end{aligned}$$

Therefore

$$\left(\frac{1}{2} \sigma_{X,2}^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d^2}{dX^2} + \hat{\theta}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{d}{dX} - (\hat{\lambda}_2 + r_2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} q_{D,21,t} \\ q_{D,22,t} \end{bmatrix} + \begin{bmatrix} \hat{\lambda}_2 \\ 0 \end{bmatrix} = 0.$$

We can show (using the same method we used to solve (C24)) that the general solution of the above equation is

$$\begin{aligned} q_{D,21} &= \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2} + s_{1,1}X^{j_1} + s_{1,2}X^{j_2}, \\ q_{D,22} &= s_{2,1}X^{j_1} + s_{2,2}X^{j_2}, \end{aligned}$$

where $j_i, i \in \{1, 2\}$ are the roots of the quadratic

$$\frac{1}{2}\sigma_{X,2}^2 j(j-1) + \widehat{\theta}_2 j - (\widehat{\lambda}_2 + r_2) = 0,$$

such that $j_1 < j_2$. In summary

$$\begin{aligned} q_{D,11} &= \begin{cases} \sum_{m=1}^2 h_{11,m} X^{k_m}, & X > X_{D,1}, \\ 1, & X_{D,2} < X \leq X_{D,1}, \\ 1, & X \leq X_{D,2}. \end{cases} & q_{D,12} &= \begin{cases} \sum_{m=1}^2 h_{12,m} X^{k_m}, & X > X_{D,1}, \\ 0, & X_{D,2} < X \leq X_{D,1}, \\ 0, & X \leq X_{D,2}. \end{cases} \\ q_{D,21} &= \begin{cases} \sum_{m=1}^2 h_{11,m} \epsilon(k_m) X^{k_m}, & X > X_{D,1}, \\ \frac{\widehat{\lambda}_2}{r_2 + \widehat{\lambda}_2} + \sum_{m=1}^2 s_{1,m} X^{j_m}, & X_{D,2} < X \leq X_{D,1}, \\ 0, & X \leq X_{D,2}. \end{cases} & q_{D,22} &= \begin{cases} \sum_{m=1}^2 h_{12,m} \epsilon(k_m) X^{k_m}, & X > X_{D,1}, \\ \sum_{m=1}^2 s_{2,m} X^{j_m}, & X_{D,2} < X \leq X_{D,1}, \\ 1, & X \leq X_{D,2}. \end{cases} \end{aligned} \quad (\text{C28})$$

To find the 8 constants: $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$, we use the following 8 boundary conditions:

$$\begin{aligned} q_{D,11}|_{X=X_{D,1}} &= 1, \quad q_{D,12}|_{X=X_{D,1}} = 0, \\ \lim_{X \uparrow X_{D,1}} q_{D,21} &= \lim_{X \downarrow X_{D,1}} q_{D,21}, \quad \lim_{X \uparrow X_{D,1}} q_{D,22} = \lim_{X \downarrow X_{D,1}} q_{D,22}, \\ \lim_{X \uparrow X_{D,1}} q'_{D,21} &= \lim_{X \downarrow X_{D,1}} q'_{D,21}, \quad \lim_{X \uparrow X_{D,1}} q'_{D,22} = \lim_{X \downarrow X_{D,1}} q'_{D,22}, \end{aligned}$$

and

$$q_{D,21}|_{X=X_{D,2}} = 0, \quad q_{D,22}|_{X=X_{D,2}} = 1.$$

The first set being applied at $X = X_{D,1}$ and the second set at $X = X_{D,2}$. The 8 boundary conditions give 8 linear equations, which can be solved in closed-form (using a computer algebra package such Mathematica or Maple) to give $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$.

We obtain $\{\widehat{p}_{D,ij}\}_{i,j \in \{1,2\}}$ and $\{p_{D,ij}\}_{i,j \in \{1,2\}}$, by setting $r_1 = r_2 = 0$, and $r_1 = r_2 = 0, \gamma = 1/\psi = 0$, respectively. Then we can compute the risk- and time-adjustments via Equations (11) and (12).

Proof of Proposition 3

We take the limits of (20) as $\Delta t \rightarrow 0$, to obtain

$$0 = (1 - \eta) X - (\bar{\mu}_i - \theta_i) A_i + \widehat{\lambda}_i (A_j - A_i), \quad i \in \{1, 2\}, j \neq i.$$

To obtain the solution of the above linear equation system, we define

$$p_i = \frac{1}{(1 - \eta)} \frac{A_i}{X},$$

the before price-earnings ratio in state i . Therefore

$$\left(\text{diag}(\bar{\mu}_1 - \theta_1, \bar{\mu}_2 - \theta_2) - \widehat{\Lambda} \right) \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (\text{C29})$$

where $\text{diag}(\bar{\mu}_1 - \theta_1, \bar{\mu}_2 - \theta_2)$ is a 2×2 diagonal matrix, with the quantities $\bar{\mu}_1 - \theta_1$ and $\bar{\mu}_2 - \theta_2$ along the diagonal and

$$\widehat{\Lambda} = \begin{pmatrix} -\widehat{\lambda}_1 & \widehat{\lambda}_1 \\ \widehat{\lambda}_2 & -\widehat{\lambda}_2 \end{pmatrix}$$

is the generator matrix of the Markov chain under the risk-neutral measure. Solving (C29) gives (18), if $\det(\text{diag}(\bar{\mu}_1 - \theta_1, \bar{\mu}_2 - \theta_2) - \hat{\Lambda}) \neq 0$. We now define $P_i^X = p_i X$, the before-tax value of the claim to the earnings stream X in state t . Hence, from the basic asset pricing equation

$$E_t \left[\frac{dP^X + X dt}{P^X} - r dt \middle| \nu_{t-} = i \right] = -E_t \left[\frac{dM}{M} \frac{dP^X}{P^X} \middle| \nu_{t-} = i \right],$$

we obtain the unlevered risk premium:

$$E_t \left[\frac{dP^X + X dt}{P^X} - r dt \middle| \nu_{t-} = i \right] = \gamma \rho_{XC} \sigma_{X,i}^s \sigma_{C,i} dt - (\hat{\lambda}_i - \lambda_i) \left(\frac{p_j}{p_i} - 1 \right) dt, i \in \{1, 2\}, j \neq i.$$

Applying Ito's Lemma,

$$dP^X = p_i dX_t + \lambda_i (p_j - p_i) dt + (p_j - p_i) dN_{i,t}^P, i \in \{1, 2\}, j \neq i.$$

Thus, the unlevered volatility of returns on equity in state i is given by

$$\sigma_{R,i} = \sqrt{\sigma_{X,i}^2 + \lambda_i \left(\frac{p_j}{p_i} - 1 \right)^2}, j \neq i,$$

where $\sigma_{X,i}$ is defined in (C22).

Proof of Proposition 4

First we show that (23) holds. The central part of our proof consists of proving that

$$E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] = \frac{1}{r_{P,i}} - \sum_{j=1}^2 \frac{q_{D,ij}}{r_{P,j}}, \quad (\text{C30})$$

where

$$r_{P,i} = \left(E_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] \right)^{-1}, \quad (\text{C31})$$

and

$$E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D} (X_{\tau_D}) \middle| \nu_t = i \right] = \sum_{j=1}^2 \alpha_j A_j (X_{D,j}) q_{D,ij}. \quad (\text{C32})$$

Using the above result, (23) follows immediately from (22). First, we observe that

$$q_{D,ij,t} = E_t \left[\Pr(\nu_t = i | \nu_{\tau_D} = j) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i, \nu_{\tau_D} = j \right]. \quad (\text{C33})$$

To prove (C30), we note that

$$E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] = E_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] - E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i \right],$$

and conditioning on the event $\{\nu_{\tau_D} = j\}$, we obtain

$$E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i \right] = \sum_{j=1}^2 E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i, \nu_{\tau_D} = j \right].$$

What happens from date τ_D onwards is independent of what happened before, so

$$\begin{aligned} & E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_t = i, \nu_{\tau_D} = j \right] \\ = & E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i, \nu_{\tau_D} = j \right] E_t \left[\int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_{\tau_D} = j \right]. \end{aligned}$$

Therefore

$$\begin{aligned} E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] &= E_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] \\ &- \sum_{j=1}^2 E_t \left[\Pr(\nu_{\tau_D} = j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i, \nu_{\tau_D} = j \right] E_t \left[\int_{\tau_D}^{\infty} \frac{\pi_s}{\pi_{\tau_D}} ds \middle| \nu_{\tau_D} = j \right]. \end{aligned} \quad (C34)$$

Conditional on being in state i , the value of a claim which pays one risk-free unit of consumption in perpetuity is $E_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right]$, so the discount rate for this perpetuity, $r_{P,i}$, is given by (C31). Consequently, (C34) implies

$$E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} ds \middle| \nu_t = i \right] = \frac{1}{r_{P,i}} - \sum_{j=1}^2 \frac{E_t \left[\Pr(\nu_t = i | \nu_{\tau_D} = j) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i, \nu_{\tau_D} = j \right]}{r_{P,j}}. \quad (C35)$$

Using the definition of the Arrow-Debreu default claim, $q_{D,ij}$, given in (C33), (C30) follows. We do not have to evaluate $r_{P,i}$ from scratch based on (C31), because we can infer its value from (18), by setting $\theta_i = \sigma_{X,i} = \rho_{XC,i} = 0$, $\forall i \in \{1, 2\}$ to obtain (24). To prove (C32), we condition on the event $\{\nu_{\tau_D} = j\}$ to obtain

$$E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha_{\tau_D} A_{\tau_D}(X_{\tau_D}) \middle| \nu_t = i \right] = \sum_{j=1}^2 \alpha_j A_j(X_j) E_t \left[\Pr(\nu_{\tau_D} = \nu_j | \nu_t = i) \frac{\pi_{\tau_D}}{\pi_t} \middle| \nu_t = i, \nu_{\tau_D} = j \right].$$

Using (C33) to simplify the above expression we obtain (C32). The credit spread in state i is

$$s_i = y_i - r_i = \frac{c}{B_i} - r_{P,i}. \quad (C36)$$

Substituting (23) into the above equation and simplifying gives (25). Equation (26) follows from applying Ito's Lemma to (C36) and identifying the diffusion term. From (23), we can show that, in state i , the elasticity of the bond price with respect to earnings is given by (27).

Proof of Proposition 5

Using the same approach we used to derive (23), we can derive (28). Applying Ito's Lemma to (28), we obtain

$$dR_t |_{\nu_{t-}=i} = \frac{dS_t + (1-\eta)(X_t - c)dt}{S_{t-}} \bigg|_{\nu_{t-}=i} = \mu_{R,i} dt + \sigma_{R,i}^{B,id} dB_{X,t}^{id} + \sigma_{R,i}^{B,s} dB_{X,t}^s + \sigma_{R,i}^P dN_{i,t}^P,$$

where

$$\begin{aligned} \mu_{R,i} &= \frac{A_i(X) + \sum_{j=1}^2 \left[X q'_{D,ij} \theta_i + \frac{1}{2} X^2 q''_{D,ij} \sigma_{X,i}^2 \right] \left[(1-\eta) \frac{c}{r_{P,j}} - A_j(X_{D,j}) \right]}{S_i} \\ &+ \left(\frac{S_k}{S_i} - 1 \right) \lambda_i + \frac{A_i(X)}{S_i}, \quad k \neq i, \end{aligned}$$

and $\sigma_{R,i}^{B,id}$, $\sigma_{R,i}^{B,s}$ and $\sigma_{R,i}^P$ are given in (32), (35) and (31), respectively. Therefore,

$$-E_t \left[dR \frac{d\pi}{\pi} \middle| \nu_{t-} = i \right] = \left\{ \gamma \rho_{XC,i} \sigma_{R,i}^{B,s} \sigma_C - \sigma_{R,i}^P (\omega_i - 1) \lambda_i \right\} dt,$$

and because the levered equity risk premium is given by

$$E_t [dR - r dt | \nu_{t-} = i] = -E_t \left[dR \frac{d\pi}{\pi} \middle| \nu_{t-} = i \right],$$

we obtain (29). Overall levered stock return volatility in state i is given by combining the variances from Brownian and Poisson shocks to obtain (34).

Proof of Weakly Countercyclical of the Default Boundary

Suppose the default boundary is strongly procyclical, i.e. $X_{D,1} < X_{D,2}$. Then the 8 linear equations used to find $h_{11,1}, h_{11,2}, h_{12,1}, h_{12,2}, s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}$ are not linearly independent (this is easy to see by writing the linear equations in matrix form and checking that the determinant is zero), implying that the Arrow-Debreu default claims are not unique. Hence, by reductio ad absurdum the default boundary must be weakly countercyclical.

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Figure 1: Risk-Adjustment Factors and Expected Consumption Growth

This figure shows the 4-year risk-adjustment factors, $\mathcal{R}_{ij}(4)$, $i, j \in \{1, 2\}$, as a function of the expected consumption growth rate in state 1, g_1 . State 1 is a recession and state 2 is a boom. Leverage is held constant at 43% and the default boundaries are chosen optimally. Risk aversion is equal to 10 and the elasticity of intertemporal substitution is 1.5. All remaining parameters are as in Table III.

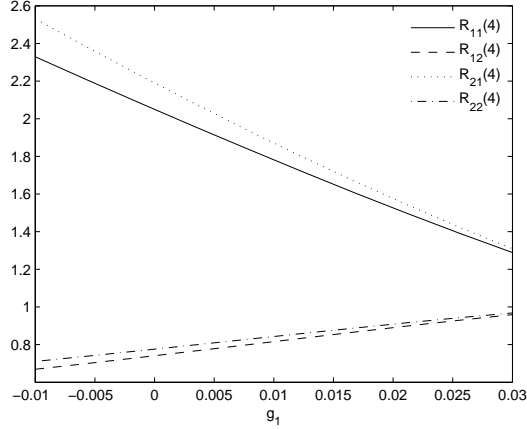


Figure 2: Risk-Adjustment Factors and Leverage

This figure shows the 4-year risk-adjustment factors, $\mathcal{R}_{ij}(4)$, $i, j \in \{1, 2\}$, as a function of leverage, where the default boundaries are chosen optimally. State 1 is a recession and state 2 is a boom. Risk aversion is equal to 10 and the elasticity of intertemporal substitution is 1.5. All remaining parameters are as in Table III.

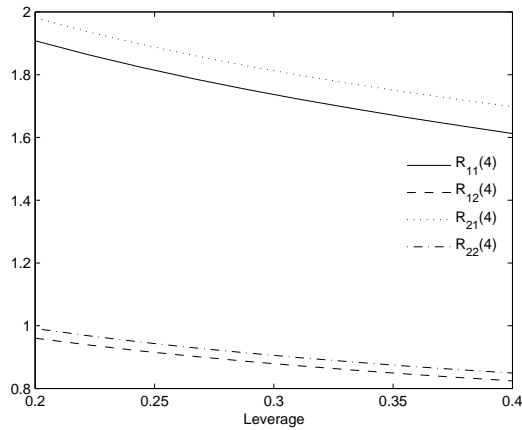


Figure 3: Time-Adjustment Factors and Leverage

This figure shows the 4-year time-adjustment factors, $\mathcal{T}_{ij}(4)$, $i, j \in \{1, 2\}$, as a function of leverage, where the default boundaries are chosen optimally. State 1 is a recession and state 2 is a boom. Risk aversion is equal to 10 and the elasticity of intertemporal substitution is 1.5. All remaining parameters are as in Table III.

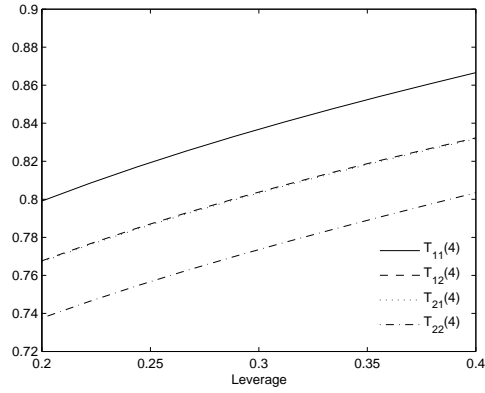


Figure 4: The Term Structure of Credit Spreads

This figure shows the term structure of credit spreads. The default boundaries are chosen optimally and leverage is exogenously set equal to 43%. State 1 is a recession and state 2 is a boom. Risk aversion is equal to 10 and the elasticity of intertemporal substitution is 1.5. All remaining parameters are as in Table III.

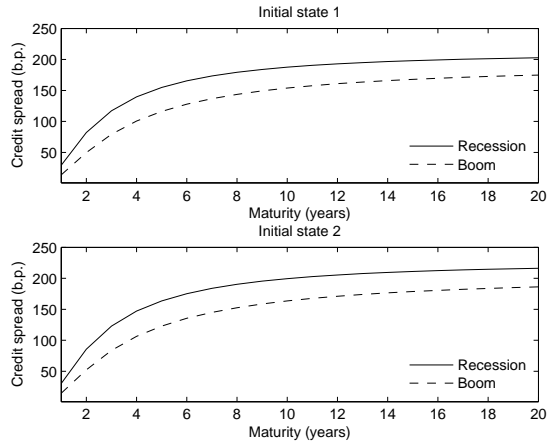


Table I Summary of Models in the Literature

	Leland	GJL	HMM	Lucas	CC	BY	CF	CDG	This paper
Structural Model Features									
default risky corporate debt	✓	✓	✓					✓	✓
endogenous default boundary	✓	✓	✓						✓
dynamic capital structure		✓	✓						✓
stochastic earnings growth rates			✓						✓
Consumption-Based Model									
recursive preferences						✓	✓		✓
habit-formation preferences					✓			✓	
stochastic dividend growth rate						✓	✓		✓
stochastic consumption growth rate						✓	✓		✓
Pricing Implications									
credit spread	✓	✓	✓					✓	✓
equity risk premium				✓	✓	✓	✓	✓	✓
impact of default risk on equity risk premium									✓
cross-market predictability								✓	✓

This table compares features of structural models which are used to price corporate debt: Leland (1994), Goldstein, Ju, and Leland (2001) (GJL), Hackbarth, Miao, and Morellec (2006) (HMM), consumption-based models which are used to price the aggregate stock market: Lucas (1978), Campbell and Cochrane (1999) (CC), Bansal and Yaron (2004) (BY), Calvet and Fisher (2005a) (CF), and models which are used to price both corporate debt and the aggregate stock market: Chen, Collin-Dufresne, and Goldstein (2006) (CDG) and this paper. The comparison table is divided into 3 panels. The first panel focuses on the features of structural models, the second on the features of consumption-based models, while the third focuses on the pricing implications of the various models.

Table II
Aggregate Parameter Estimates

To calibrate the model to the aggregate US economy, we use quarterly real non-durable plus service consumption expenditure from the Bureau of Economic Analysis and quarterly earnings data from Standard and Poor's, provided by Robert J. Shiller. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. All estimates are annualized and based on quarterly log growth rates for the period from 1947 to 2005.

	Mean	Std. dev.
Real consumption growth	0.0333	0.0099
Real earnings growth	0.0343	0.1072

Table III
Calibrated Parameter Values

This table lists parameter values used in the empirical analysis. State 1 is a recession and state 2 is a boom.

Parameter	Symbol	State 1	State 2
Consumption growth rate	g_i	0.0141	0.0420
Consumption growth volatility	$\sigma_{C,i}$	0.0114	0.0094
Earnings growth rate	θ_i	-0.0401	0.0782
Earnings growth volatility	$\sigma_{X,i}^s$	0.1334	0.0834
Correlation	ρ_{XC}	0.1998	0.1998
Actual long-run probabilities	f_i	0.3555	0.6445
Actual convergence rate to long-run	p	0.7646	0.7646
Annual discount rate	β	1%	1%
Tax rate	η	15%	15%
Recovery rate	α_i	70%	90%
Idiosyncratic earnings growth volatility	σ_X^{id}	32%	32%

Table IV
Corporate Bond Market

This table reports the results for the corporate bond market implied by the model. The coupon and default boundary are chosen optimally at date zero and depend on the initial state of the economy. The first two columns are generated when the economy is initially in state one and the last two columns when it is initially in state two. Three values of the elasticity of intertemporal substitution, ψ , are considered.

	Initial state/Current state			
	1/1	1/2	2/1	2/2
CREDIT SPREAD, s_i (b.p)				
$\psi = 0.1$	317.478	259.892	529.501	426.123
$\psi = 0.75$	215.606	195.157	254.773	228.639
$\psi = 1.5$	196.708	179.997	222.986	202.745
CREDIT SPREAD VOLATILITY, $\sigma_{s,i}$ (b.p)				
$\psi = 0.1$	229.747	174.493	410.035	301.901
$\psi = 0.75$	64.120	52.522	80.360	64.648
$\psi = 1.5$	46.545	38.446	55.850	45.536
ARROW-DEBREU DEFAULT CLAIM, $q_{D,i}$ (%)				
$\psi = 0.1$	20.044	10.645	30.808	17.302
$\psi = 0.75$	42.691	36.769	47.627	41.044
$\psi = 1.5$	52.523	47.946	56.296	51.398
4-YEAR ACTUAL DEFAULT PROBABILITY, $p_{D,i}(4)$ (%)				
$\psi = 0.1$	18.771	12.396	32.496	23.319
$\psi = 0.75$	2.986	1.533	5.711	3.185
$\psi = 1.5$	1.455	0.674	2.804	1.447
4-YEAR TIME-ADJUSTMENT, $\mathcal{T}_i(4)$				
$\psi = 0.1$	0.604	0.444	0.649	0.482
$\psi = 0.75$	0.903	0.871	0.906	0.872
$\psi = 1.5$	0.939	0.923	0.941	0.923
4-YEAR RISK-ADJUSTMENT, $\mathcal{R}_i(4)$				
$\psi = 0.1$	1.021	1.014	1.013	1.013
$\psi = 0.75$	1.288	1.389	1.268	1.297
$\psi = 1.5$	1.348	1.529	1.348	1.437
OPTIMAL LEVERAGE (%)				
$\psi = 0.1$	59.259	54.893	70.848	66.549
$\psi = 0.75$	45.566	40.746	52.316	47.065
$\psi = 1.5$	41.796	37.280	47.578	42.642
OPTIMAL DEFAULT BOUNDARIES, $X_{D,i}$				
$\psi = 0.1$	0.382	0.371	0.486	0.472
$\psi = 0.75$	0.213	0.193	0.258	0.234
$\psi = 1.5$	0.175	0.157	0.210	0.189

Table V
Long-Run Risk

This table contains the risk-distortion factor ω , the convergence rate of the Markov chain to its long-run risk-neutral distribution \hat{p} , and the long-run risk-neutral distribution (\hat{f}_1, \hat{f}_2) for risk aversion of 10 and three values of the elasticity of intertemporal substitution, ψ .

	ω	\hat{p}	\hat{f}_1	\hat{f}_2
$\psi = 0.1$	1.000	0.765	0.355	0.645
$\psi = 0.75$	1.377	0.732	0.511	0.489
$\psi = 1.5$	1.424	0.733	0.528	0.472

Table VI
Equity Market

This table reports the results of the model for the equity market. The coupon and default boundaries are chosen optimally at date zero and depend on the initial state of the economy. The first two columns are generated when the economy is initially in state one and the last two columns when it is initially in state two. All equity market values are computed with zero idiosyncratic volatility. The default-free levered equity premium is computed by assuming debt is risk-free. Three values of the elasticity of intertemporal substitution, ψ , are considered.

	Initial state/Current state			
	1/1	1/2	2/1	2/2
UNLEVERED EQUITY PREMIUM (%)				
$\psi = 0.1$	0.304	0.157	0.304	0.157
$\psi = 0.75$	1.860	1.217	1.860	1.217
$\psi = 1.5$	2.458	1.632	2.458	1.632
DEFAULT-FREE LEVERED EQUITY PREMIUM (%)				
$\psi = 0.1$	1.034	0.423	2.191	0.677
$\psi = 0.75$	5.617	3.034	8.306	3.979
$\psi = 1.5$	6.288	3.480	8.715	4.359
LEVERED EQUITY PREMIUM (%)				
$\psi = 0.1$	0.966	0.415	1.589	0.624
$\psi = 0.75$	4.892	2.744	6.227	3.286
$\psi = 1.5$	5.372	3.104	6.470	3.566
LEVERED EQUITY VOLATILITY (%)				
$\psi = 0.1$	42.63	22.30	71.68	34.73
$\psi = 0.75$	36.72	20.30	45.94	24.16
$\psi = 1.5$	35.30	19.69	42.20	22.63
LOCALLY RISK-FREE RATE (%)				
$\psi = 0.1$	14.385	42.514	14.385	42.514
$\psi = 0.75$	2.807	6.564	2.807	6.564
$\psi = 1.5$	1.744	3.650	1.744	3.650

Table VII
Model Comparison—Corporate Bond Market

This table provides a comparison between stripped down versions of our model. In Models 1a and 1b there is no intertemporal risk, the first and second moments of consumption and earnings growth rates do not switch. In Model 2 the first and second moments of earnings growth switch but the first and second moments of consumption growth do not. In Model 3 the first and second moments of both earnings and consumption growth switch. In Model 1a the representative agent has power utility whereas in Models 1b, 2 and 3 she has Epstein-Zin-Weil utility.

	Initial state	Current state	1a	1b	2	3
RISK AVERSION, γ			10.00	10.00	10.00	10.00
EIS, ψ			0.10	0.75	0.75	0.75
CREDIT SPREAD, s_i (b.p.)	1	1	144.36	178.87	187.59	243.88
	1	2			159.46	221.21
	2	1			219.05	280.09
	2	2			184.88	252.18
ARROW-DEBREU DEFAULT CLAIM	1	1	5.80	30.85	31.69	41.45
$q_{D,i}$ (%)	1	2			28.07	35.89
	2	1			35.45	45.24
	2	2			31.41	39.18
4-YEAR ACTUAL DEFAULT PROBABILITY	1	1	5.75	2.39	2.20	1.88
$p_{D,i}(4)$ (%)	1	2			0.72	0.58
	2	1			3.41	2.96
	2	2			1.28	1.34
4-YEAR RISK-ADJUSTMENT, $\mathcal{R}_i(4)$	1	1	1.02	1.04	1.06	1.36
	1	2			1.06	1.45
	2	1			1.04	1.30
	2	2			1.03	1.28
4-YEAR TIME-ADJUSTMENT, $\mathcal{T}_i(4)$	1	1	0.39	0.85	0.85	0.91
	1	2			0.84	0.87
	2	1			0.86	0.91
	2	2			0.85	0.87
LEVERAGE (%)	1	1	43.00	43.00	43.00	43.00
	1	2			38.39	38.39
	2	1			47.96	47.97
	2	2			43.00	43.00
DEFAULT BOUNDARY, $X_{D,i}$	1	1	0.28	0.22	0.22	0.21
	1	2			0.19	0.18
	2	1			0.25	0.24
	2	2			0.22	0.22
COUPON, c	1		0.45	1.27	1.58	0.83
	2				1.83	0.97
DISTORTION FACTOR, ω	1	1	1.00	1.00	1.00	1.38

Table VIII
Model Comparison—Equity Market

This table provides a comparison between stripped down versions of our model. In Models 1a and 1b there is no intertemporal risk, the first and second moments of consumption and earnings growth rates do not switch. In Model 2 the first and second moments of earnings growth switch but the first and second moments of consumption growth do not. In Model 3 the first and second moments of both earnings and consumption growth switch. In Model 1a the representative agent has power utility whereas in Models 1b, 2 and 3 she has Epstein-Zin-Weil utility. All equity market values are computed with zero idiosyncratic volatility.

	Initial state	Current state	1a	1b	2	3
RISK AVERSION, γ			10.00	10.00	10.00	10.00
EIS, ψ			0.10	0.75	0.75	0.75
UNLEVERED EQUITY PREMIUM (%)	1	1	0.21	0.21	0.26	1.86
	1	2			0.17	1.22
	2	1			0.26	1.86
	2	2			0.17	1.22
	2	2			0.17	1.22
LEVERED EQUITY PREMIUM, (%)	1	1	0.36	0.42	0.52	4.63
	1	2			0.29	2.63
	2	1			0.61	5.57
	2	2			0.33	3.03
LOCALLY RISK-FREE RATE, (%)		1	33.79	5.33	5.33	2.81
		2			5.33	6.56

Table IX
Cross-Market Comovement

We simulate 100 panels each containing 3000 firms and 5 years of daily data. For each panel, we regress the equally-weighted credit spread on the value-weighted jump component of levered equity volatility. Standard errors are reported in parenthesis.

EIS, ψ	0.75	0.75	1.5	1.5
Initial state	1	2	1	2
Coefficient	1.447	1.903	1.786	2.296
	(0.052)	(0.061)	(0.055)	(0.079)
R^2	0.476	0.546	0.645	0.645