Should Derivatives be Privileged in Bankruptcy?*

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Abstract

Derivative contracts, swaps, and repos enjoy "super-senior" status in bankruptcy: they are exempt from the automatic stay and, if collateralized, they are effectively senior to virtually all other claims. We propose a simple corporate finance model to assess the effect of this exemption on a firm's cost of borrowing and incentives to engage in derivative transactions. Our model suggests that, while derivatives are value-enhancing risk management tools, effective seniority for derivatives can lead to inefficiencies because it shifts credit risk to the firm's creditors, even though this risk could be borne more efficiently by derivative counterparties. In addition, because senior derivatives dilute existing creditors, firms may take on derivative positions that are too large from a social perspective.

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Derivatives enjoy special status in bankruptcy under current U.S. law. Derivative counterparties are exempted from the automatic stay, and through netting, closeout, and collateralization provisions, they are generally able to immediately collect payment from a defaulted counterparty.[1] Taken together, these provisions effectively make derivative counterparties senior to almost all other claimants in bankruptcy. The costs and benefits of this special treatment are an open question and the subject of a recent debate among legal scholars.[2] The fact that the special treatment does not hold universally in all jurisdictions indicates that there is considerable disagreement among lawmakers about the consequences of these provisions.[3]

In this paper, we provide a first formal analysis of the economic consequences of the privileged treatment of derivatives in bankruptcy, using a standard corporate finance framework. Our main argument is that super-seniority provisions for derivatives cannot be seen in isolation, but must be evaluated taking into account their effect on a firm's other obligations, in particular debt. We argue that while derivatives are generally value-enhancing through their role as risk management tools, the super-senior status of derivatives may be inefficient. The reason is that collateralization and (effective) seniority of derivative contracts does not eliminate risk, but only shifts risk from a firm's derivative counterparties onto the firm's creditors. However, under fairly general conditions, it is more efficient if this credit risk is borne by derivative counterparties rather than creditors. In addition, we show that the super-senior status of derivative contracts may induce firms to take on derivative positions that are excessively large from a social perspective (i.e., strictly larger than what is needed to hedge cash flow risk).

In our model a firm finances a positive NPV investment with debt. Due to operational

[1] Similarly, under the current FDIC resolution process there is essentially no stay on derivative contracts. If not transferred to a new counterparty by 5pm EST on the business day after the FDIC has been appointed receiver, derivative, swap, and repo counterparties can close out their positions and take possession of collateral. See, for example, Summe (2010, p.66).

[2] See, e.g., Edwards and Morrison (2005); Bliss and Kaufman (2006); Roe (2010); Skeel and Jackson (2011); Duffie and Skeel (2012).

[3] For example, under current bank resolution law in the U.K. and Germany, closeout and netting provisions may not always be enforceable (see Hellwig (2011)).

cash flow risk, the firm may not have sufficient funds to make required debt payments at an intermediate date. Because the firm cannot pledge all future cash flows, it is then forced into default and liquidation, even though continuation would be efficient. We begin our analysis by showing that in this setting derivatives are valuable hedging tools: by transferring resources from high cash-flow states to low cash-flow states, derivatives can reduce, or even eliminate, costly default. Hence, the introduction of derivative markets generally raises surplus relative to the benchmark case in which no derivatives are available. This result is in line with the existing literature on corporate risk management: When firms face external financing constraints and may be forced into inefficient liquidation, they generally benefit from hedging cash flow risk (see, e.g., Smith and Stulz, 1985; Froot, Scharfstein, and Stein, 1993).

The main novelty of our analysis is to consider how the bankruptcy treatment of derivatives affects these benefits from hedging. The conventional wisdom is that super-seniority provisions for derivatives lower a firm's cost of hedging and should thus be beneficial overall. We show that this argument is flawed. The reason is that super-seniority does not eliminate risk, it just transfers risk to other claimants on the firm's assets. In particular, while reducing counterparty risk in derivative markets, super-seniority increases the credit risk for the firm's creditors. In our model, this shift in risk from derivative markets to debt markets is generally inefficient and results in a loss of overall surplus. The intuition for this result is simple and surprisingly robust. By increasing the firm's cost of debt and thus the required promised debt repayments, super-seniority for derivatives has the indirect effect of raising the firm's leverage and thus the derivative position required to hedge the firm's default risk. When derivative (or debt) markets are not completely frictionless this produces greater deadweight costs. We first illustrate this result by comparing the two polar cases of *senior* and *junior* derivatives, and then show that the same intuition also holds in a more general setup that allows for partial collateralization of derivative positions.

We then go on to show that under the status quo of senior derivatives, firms may have an incentive to take on derivative positions that are excessively large from a social perspective. This is the case whenever the payoff from the derivative contract is not perfectly correlated with the operational risk of the firm (in other words, when there is '*basis risk*'). The reason is that, in the presence of basis risk, an increase in the firm's senior derivative position dilutes existing debtholders. The benefits from a unit increase in derivatives exposure fully accrue to the firm, while some of the cost of the derivative position is borne by existing creditors: in the event of default, derivative counterparties get paid before ordinary creditors, so that an increase in the firm's derivative position can leave existing creditors worse off. Effectively, the senior status of derivatives gives firms an incentive to speculate in derivatives markets over and above what is warranted for hedging purposes. This incentive to speculate disappears if the special treatment for derivatives in bankruptcy were removed.

To the extent that the favorable bankruptcy treatment of derivatives leads to inefficiencies, an important question is whether firms can 'undo the law', for example by committing not to collateralize derivative contracts, thus stripping them of their effective seniority. In this context, our model suggests that the super-seniority provisions for derivatives might have particular bite for financial institutions. While it may be possible to shield physical collateral from derivative counterparties (for example by granting collateral protection over plant and equipment to secured creditors), it is generally harder to shield unassigned cash from collateral calls by derivative counterparties that occur, for example, when a financial institution approaches financial distress. In fact, by the very nature of their business, financial institutions cannot assign cash as collateral to all depositors and creditors because, by definition, this would eliminate their value added as financial intermediaries. To the extent that firms are unable to contractually undo the effective super-seniority of derivatives, a change in the bankruptcy code that eliminates the special treatment of derivatives may be welfare-enhancing. As we show, by automatically staying collection actions by derivatives counterparties along with creditors, bankrupt firms would be protected against such inefficient collateral calls (or runs on collateral).

Although several legal scholars have informally argued that there may be costs associated with the effective seniority of derivatives (e.g. Edwards and Morrison, 2005; Bliss and Kaufman, 2006; Roe, 2010; Skeel and Jackson, 2011; Duffie and Skeel, 2012), our paper offers the first formal *ex ante* and *ex post* analysis of this issue.[4] In addition to the law literature on the bankruptcy exemption for derivatives and the literature on hedging (see the papers mentioned above), our model is closely related to the literature on debt dilution and short-term debt. In particular, in our model excessively large derivatives positions can result because the bankruptcy code allows firms to dilute their creditors by taking on derivative positions that are effectively senior. This dilution is related to the other classic forms of debt dilution, through risk shifting (e.g., Jensen and Meckling (1976)), the issuance of additional senior or short-term debt (e.g., Fama and Miller (1972), Diamond (1993), Brunnermeier and Oehmke (2012)), or by granting security interest to some creditors (e.g., Bebchuk and Fried (1996)). In addition, the fine line between hedging and speculation that we highlight in our paper is echoed in a recent paper by Biais, Heider, and Hoerova (2010), who show that when derivatives positions move out of the money for one of the parties involved, this may adversely affect this counterparty's incentive to manage risk, resulting in endogenous counterparty risk.

The remainder paper is organized as follows. Section 1 briefly summarizes the special status of derivative securities under U.S. bankruptcy law. Section 2 introduces the model. Section 3 analyzes a benchmark case without derivatives. Section 4 discusses the effect of the bankruptcy treatment of derivatives in the case where the derivative has no basis risk. Section 5 extends the analysis to allow for basis risk and presents the main findings of our analysis. Section 6 concludes.

[4] Edwards and Morrison (2005) argue that one potential adverse consequence of the exemption of the automatic stay is that a firm in financial distress may fall victim to a run for collateral by derivative counterparties. Roe (2010) argues that fully protected derivative counterparties have no incentive to engage in costly monitoring of the firm. In addition, commentators have pointed out that under the current rules firms may have an incentive to inefficiently masquerade their debt as derivatives, for example by structuring debt as total return swaps. In this article, we mostly abstract away from ex-post inefficient runs (except in subsection 5.4.2) or inefficient substitution of debt (subject to the automatic stay) for another instrument like debt masquerading as a derivative exempt from the automatic stay.

1 The Special Status of Derivatives

In this section we briefly summarize the special status of derivatives in bankruptcy and explain why derivatives are often referred so as 'super-senior' claims.[5] Strictly speaking, derivatives are not senior in the formal legal sense.[6] However, derivatives, swaps and repo counterparties enjoy certain rights that set them apart from regular creditors. While not formally senior, these rights make derivatives *effectively* senior to regular creditors, at least to the extent that they are collateralized.

The most important advantages a derivative, repo or swap counterparty has relative to a regular creditor pertain to closeout, collateralization, netting, and the treatment of eve of bankruptcy payments, eve of bankruptcy collateral calls, and fraudulent conveyances. First, upon default, derivative counterparties have the right to terminate their position with the firm and collect payment by seizing and selling collateral posted to them. This differs from regular creditors who cannot collect payments when the firm defaults, because, unlike derivative counterparties, their claims are subject to the automatic stay. In fact, even if they are collateralized, regular creditors are not allowed to seize and sell collateral upon default, since their collateral, in contrast to the collateral posted to derivative counterparties, is subject to the automatic stay. Hence, to the extent that a derivative counterparty is collateralized at the time of default, collateralization and closeout provisions imply that the derivative counterparty is *de facto* senior to all other claimants.[7]

Second, when closing out their positions with the bankrupt firm, derivative counterparties have stronger netting privileges than regular creditors. Because they can net out offsetting positions, derivative counterparties may be able to prevent making payments to a bankrupt

[5] The discussion in this section is kept intentionally brief and draws mainly on Roe (2010). For more detail on the legal treatment of derivatives, see also Edwards and Morrison (2005) and Bliss and Kaufman (2006).

[6] As pointed out by Roe (2010, p.5), "The Code sets forth priorities in §§ 507 and 726, and those basic priorities are unaffected by derivative status."

[7] If after selling all the posted collateral a derivative counterparty still has a claim on the firm, this remaining claim becomes a regular unsecured claim in Chapter 11. Hence, collateralization is key to the effective seniority of derivative contracts.

firm that a regular debtor would have to make, thus strengthening the position of derivative counterparties vis-à-vis regular creditors in bankruptcy.[8]

Finally, derivative counterparties have stronger rights regarding eve of bankruptcy payments or fraudulent conveyances. For example, while regular creditors often have to return payments made or collateral posted within 90 days before bankruptcy, derivative counterparties are not subject to those rules. Any collateral posted to a derivative counterparty at the time of a bankruptcy filing is for the derivative counterparty to keep.

Taken together, this special treatment of derivative counterparties puts them in a much stronger position than regular creditors. While they do not have priority in the strict legal sense, their special rights relative to other creditors make derivative counterparties effectively senior, at least to the extent that they are collateralized. In practice, this collateralization is usually ensured via regular marking to market and collateral calls. While for most of the remainder of the paper we will loosely refer to derivatives as being senior to debt, this should be interpreted in the light of the special rights end effective priority of derivative counterparties discussed in this section.

2 Model Setup

We consider a firm that can undertake a two-period investment project. This firm can be interpreted as an industrial firm undertaking a real investment project, or as a bank or financial institution that invests in risky loans. The investment requires an initial outlay F at date 0 and generates cash flows at dates 1 and 2. At date 1 the project generates high

[8] The advantages from netting are best illustrated through a simple example. Suppose that a firm has two counterparties, A and B. The firm owes $10 to A. The firm owes $10 to B, and, in another transaction, B owes $5 to the firm. Suppose that when the firm declares bankruptcy there are $10 of assets in the firm. When creditor B cannot net its claims, he has to pay $5 into the firm. The bankruptcy mass is thus $15. A and B have remaining claims of $10 each, such that they equally divide the bankruptcy mass and each receive $7.5. The net payoff to creditor B is $7.5-$5 = $2.5. When creditor B can net his claim, he does not need to make a payment to the firm at the time of default. Rather he now has a net claim of $5 on the bankrupt firm. As before, A has a claim on $10 on the firm. There are now $10 to distribute, such that A receives 2/3*$10 = $6.66 and creditor B receives 1/3*$10 = $3.33. Hence, with netting B receives a net payoff of $3.33, while without netting he only receives $2.5.

cash flow C_1^H with probability θ, and low cash flow $C_1^L < C_1^H$ with probability $1 - \theta$. At date 2 the project generates cash flow C_2. Following the realization of the first-period cash flow, the project can be liquidated for a liquidation value L. We assume that $0 \leq L < C_2$, implying that early liquidation is inefficient. Unless we explicitly state otherwise, for most of our analysis we also normalize the firm's date 1 liquidation value to $L = 0$. After the realization of C_2 the firm fully depreciates, such that the liquidation value at date 2 is zero.

The firm has no initial wealth and finances the project by issuing debt.[9] A debt contract specifies a contractual repayment R at date 1.[10] If the firm makes this contractual payment, it has the right to continue the project and collect the date 2 cash flows. If the firm fails to make the contractual date 1 payment, the creditor has the right to discontinue the project and liquidate the firm. Liquidation can be interpreted as outright liquidation, as in a Chapter 7 cash auction, or as forcing the firm into Chapter 11 reorganization. In the latter interpretation L denotes the expected payment the creditor receives in Chapter 11. Both the firm and the creditor are risk neutral, and the riskless interest rate is zero.

The main assumption of our model is that the firm faces a limited commitment problem when raising financing for the project, similar to Hart and Moore (1994, 1998) and Bolton and Scharfstein (1990, 1996). More specifically, we assume that only the minimum date 1 cash flow C_1^L is verifiable, and that all other cash flows can be diverted by the borrower. This means that even if the high cash flow C_1^H realizes at date 1, the firm can always claim to have received the low cash flow, default and pay out C_1^L instead of R. We also assume that at date 0 none of the date 2 cash flows can be contracted upon. One interpretation of this assumption is that, seen from date 0, the timing of date 2 cash flows is too uncertain and too complicated to describe to be able to contract on when exactly payment is due. Finally, to make financing choices non-trivial, we assume that $C_1^L < F$, such that the project cannot be financed with risk-free debt.

[9] In the case of a bank, this means that beyond the minimum equity capital requirement, which we normalize to zero, the bank must raise the entire amount needed for the loan in the form of deposits. In what follows, when we interpret the firm as a bank we also take it that the *creditor* is then a bank *depositor*.

[10] In the case of a bank R denotes the gross interest payment on deposits of size F.

Next, we introduce derivative contracts into the analysis. As with debt contracts, we do this in the simplest possible way. Formally, a derivative contract specifies a payoff that is contingent on the realization of a *verifiable* random variable $Z \in \{Z^H, Z^L\}$. For example, Z could be a financial index or a similar variable that is observable to both contracting parties and verifiable by a court. Verifiability is the crucial defining characteristic of a derivative contract in our model: the ability to verify the derivative payoff means that in contrast to cash flows generated through the firms real operations, cash flows from derivatives positions can be contracted on without any commitment or enforceability problems.

A derivative contract of a notional amount X is a promise by the derivative counterparty to pay X to the firm if $Z = Z^L$, against a premium x that is payable from the firm to the derivative counterparty when $Z = Z^H$.[11] For simplicity, we assume that Z^L is realized with the same probability as C_1^L, i.e., $\Pr\left(Z = Z^L\right) = 1 - \theta$. Hence, a long position in the derivative contract pays off with the same probability as receiving the low cash flow C_1^L. The derivative's usefulness for hedging the low cash flow outcome is then determined by the correlation of the derivative payoff with the low cash flow state. We parametrize this correlation through γ. Specifically, we assume that Z^L is realized conditional on $C_1 = C_1^L$ with probability γ:

$$\Pr\left(Z = Z^L | C_1 = C_1^L\right) = \gamma. \tag{1}$$

Hence, if $\gamma = 1$ the derivative is a perfect hedge for the low cash flow state, since it pays out in exactly the same states in which the firm receives the low cash flow. When $\gamma < 1$, on the other hand, a long position in the derivative only imperfectly hedges the low cash flow state; with probability $(1 - \theta)(1 - \gamma)$ the derivative does not pay out X even though $C_1 = C_1^L$.[12]

When the firm enters a derivative position, the other side of the contract is taken by

[11] The derivative thus has payoffs that are equivalent to a swap contract, one of the most common derivatives used for hedging purposes in practice: It has value zero when entered, and then moves in favor of the firm or the counterparty, depending on the realization of Z.

[12] We have chosen the unconditional payoff probability of the derivative to coincide with the probability that the low cash flow obtains (both are equal to $1 - \theta$). This is not necessary for the analysis. We could more generally assume that the derivative pays off with probability $1 - p$. Our setup has the convenient feature that when $\gamma = 1$, the derivative is a perfect hedge: it pays if, and only if, the firm's cash flow is low.

what we will loosely refer to as the derivative counterparty. This derivative counterparty could be a financial institution, an insurance company, or a hedge fund that is providing hedging services to the firm. Typically, providing this type of insurance is not free of costs for the derivative counterparty. For example, faced with a notional exposure of X, the counterparty may face costs as it has to post collateral or set aside capital in order to fulfill capital requirements. In addition, if not all of the exposure created by the derivative is fully hedgeable, (or if it is only hedgeable at a cost) the derivative counterparty incurs a deadweight cost for each unit of notional protection that it writes to the firm. We capture these costs in the simplest possible way, by assuming that when entering a derivative contract with a notional amount of X, the derivative writer incurs a deadweight *hedging cost* of $\rho(X)$, where $\rho(0) = 0$ and $\rho'(\cdot) > 0$.[13] We will explicitly illustrate most of our findings for a linear hedging cost function $\rho(X) = \delta X$. However, qualitatively none of our main findings will depend on this particular functional form, in fact our main results continue to hold as long as $\rho(\cdot)$ is increasing.[14]

The firm enters the derivative contract after it has signed the debt contract with the creditor. Moreover, we assume that at the initial contracting stage the firm and the creditor cannot condition the debt contract on a particular realization of Z. This assumption reflects the idea that at the ex ante contracting stage it may not be known which business risks the firm needs to or can hedge in the future, and what derivative positions will be required to do so. Essentially, this assumption rules out a fully state-contingent contract between the creditor and the firm that 'bundles' financing and hedging at date 0, which is in line with the literature on incomplete contracting.[15]

[13] While we take this cost of hedging as exogenous, the hedging cost could be derived from first principles. For example, in the model of demand-based option pricing of Gârleanu, Pedersen, and Poteshman (2009), the hedging cost arises endogenously because not all of the risk in the derivatives position can be hedged. The literature on hedging pressure has emphasized the costs (see, e.g., Hirshleifer (1990) and the references therein). In addition to the direct costs of hedging to the derivative writer, $\rho(X)$ may also contain the cost of potential systemic risk created by the derivative writer.

[14] The implications of our model are robust to introducing a similar deadweight cost also in debt markets. Please see the discussion on robustness following Proposition 6.

[15] For a more formal justification of this assumption, assume that there is a continuum of Z-variables that may potentially be used to hedge the firm's business risk, but that at the ex-ante contracting stage it is not

Derivatives have economic value in our setting, since the correlation between the derivative payoff and the firm's operational risk can be used to reduce the firm's default risk. In particular, the derivative can be used to decrease the variability of the firm's cash flow at date 1. This effectively raises the verifiable cash flow the firm has available at date 1. From a welfare perspective this is beneficial, because by raising the low date 1 cash flow, the derivative may allow the firm to reduce the probability of default at date 1. When the derivative is a perfect hedge, it may even allow the firm to completely eliminate the default such that it can finance the project using risk-free debt. This reduction in (or elimination of) the probability of default is socially beneficial, because it reduces the probability that the firm is terminated at date 1. Hence, in the presence of derivatives, the date 2 cash flow C_2 is lost less often. Derivatives increase surplus whenever the gains from reducing date 1 bankruptcy costs outweigh the cost of using derivatives, which is captured by the deadweight hedging cost $\rho(\cdot)$.

Note that our formal description of derivatives contracts implicitly assumes that the firm faces no counterparty risk with respect to the payment by the derivative writer, X. We will make this simplifying assumption throughout the analysis, as our focus is primarily on counterparty and credit risk emanating from the firm to its creditors and the derivative writer, i.e., with respect to the firm's repayment of face value of debt R and the derivative premium x.[16].

In what follows, we model the seniority of derivatives by first considering two extreme cases; first the case where derivatives are senior to debt and then the alternative extreme

yet known which of these potential Z-variables will be the relevant one from a risk management perspective. However, once the firm is in operation and learns more about its business environment it can determine the relevant variable Z. This lack of knowledge on the relevant random variable Z ex ante, would effectively prevent the firm from contracting on a particular derivative position, or from making the debt contract contingent on the relevant Z-variable. It is then more plausible that the firm will choose its derivative position only after signing the initial debt contract. Note that this assumption also broadly reflects current market practice. Firms usually choose their derivative exposure for a given amount of debt only ex post. Moreover, in practice few (if any) bonds or loans include restrictions on future derivative positions taken by the debtor.

[16] Note, however, that the basis risk on the derivatives contract could also be interpreted as counterparty risk. For models that explicitly model counterparty risk emanating from the protection seller, see Thompson (2010) and Biais, Heider, and Hoerova (2010).

case in which derivatives are junior. The former situation is one where the premium x is fully collateralized, and where cash collateral in the amount of x can be seized by the derivative counterparty in the event of a default on debt payments.[17] In the other extreme case when derivatives are junior to debt, the premium x is simply not collateralized. In other words, no cash collateral is assigned to the derivative. Moreover, in this case the debt contract then specifies that it is senior to the derivative claim in bankruptcy.

The key question in this polar case is whether the firm can commit not to collateralize its derivative position. Under current U.S. bankruptcy law it is difficult to make such a commitment, for any amount of cash the firm assigns to a derivative counterparty can simply be seized by the derivative writer when the firm files for bankruptcy. It is then extremely difficult to recover any cash collateral that has been assigned to the derivative counterparty, so that the derivative is *de facto* senior. However, under different bankruptcy rules, for example if there was a general stay on all attempts to collect collateral, such a commitment may be contractually feasible.

Following the analysis of the two polar cases, we then also consider the more general, intermediate case in which derivatives can be partially collateralized by only assigning a limited cash collateral $\overline{x} \leq x$ to the derivatives counterparty. In this case, only the amount $\overline{x}$ can be seized by the derivatives writer in the event of default. The remaining amount the firm owes to the derivatives counterparty, $x - \overline{x}$, is then treated as a regular debt claim in bankruptcy. For simplicity we will assume that this remainder is junior to the claims of the debtholder.[18]

[17] The cash the firm assigns as collateral to the derivatives margin account is obtained either from retained earnings or from the initial investment by the creditor. Retained earnings can be modeled by assuming that after the firm sinks the set-up cost F at date 0, the project first yields a sure return C_1^L at date 1^-. At that point it is still unknown whether the full period 1 return will be C_1^H or C_1^L; that is, the firm only knows that it will receive an incremental cash flow at date 1 of $\Delta C_1 = C_1^H - C_1^L$ with probability θ, and 0 with probability $(1 - \theta)$. To hedge the risk with respect to this incremental cash flow, the firm can then take a derivative position by pledging cash collateral $x \leq C_1^L$. Alternatively, the cash collateral x can be obtained from the creditor at date 0 by raising a total amount $F + x$ from the creditor. Either way of modeling cash collateral works in our setup.

[18] In practice, such a claim could be classified in the same priority class as debt. We do not explicitly consider this case, since the *pro-rata* allocation of assets to derivative counterparties and debtholders that arises in this case considerably complicates the formal analysis, without yielding any substantive additional

3 Benchmark: No Derivatives

We first describe the equilibrium in the absence of a derivative market. The results from this section will provide a useful benchmark case against which we can evaluate the effects of introducing derivative markets in Sections 4 and 5.

In the absence of derivatives, the firm always defaults if the low cash flow C_1^L realizes at date 1. We will refer to this outcome as a *liquidity default*. Because $C_1^L < F$, the low cash flow is not sufficient to repay the face value of debt. Moreover, the date 2 cash flow C_2 is not pledgeable, and since the firm has no other cash it can offer to renegotiate with the creditor, the firm has no other option than to default when C_1^L is realized at date 1. The lender then seizes the cash flow C_1^L and shuts down the firm, collecting the liquidation value of the asset L. Early termination of the project leads to a social loss of $C_2 - L$, the additional cash flow that would have been generated had the firm been allowed to continue its operations.

If the high cash flow C_1^H realizes at date 1, the firm has enough cash to service its debt. However, the firm may still choose not to repay its debt. We refer to this choice as a *strategic default*. A strategic default occurs when the firm is better off defaulting on its debt at date 1 than repaying the debt and continuing operations until date 2. In particular, the firm will make the contractual repayment R only if the following incentive constraint is satisfied:

$$C_1^H - R + C_2 \geq C_1^H - C_1^L + S, \tag{2}$$

where S denotes the surplus that the firm can extract in renegotiation after defaulting strategically at date 1. Constraint (2) says that, when deciding whether to repay R, the firm compares the payoff from making the contractual payment and collecting the entire date 2 cash flow C_2 to the payoff from defaulting strategically, pocketing $C_1^H - C_1^L$ and any potential surplus S from renegotiating with the creditor. Repayment of the face value R in the high

economic insights.

cash flow state is thus incentive compatible only as long as the face value is not too high:

$$R \leq C_1^L + C_2 - S. \tag{3}$$

The surplus S that the firm can extract in renegotiation with the creditor after a strategic default depends on the specific assumptions made about the possibility of renegotiation and the relative bargaining powers when renegotiation takes place. To keep things simple, we will assume that the creditor can commit not to renegotiate with the debtor and always liquidates the firm after a strategic default. In this case, $S = 0$.[19]

When the incentive constraint (2) is satisfied, the lender's breakeven constraint (under our simplifying assumption $L = 0$) is given by

$$\theta R + (1 - \theta) C_1^L = F, \tag{4}$$

which, given competitive debt markets, leads to an equilibrium face value of debt of

$$R = \frac{F - (1 - \theta) C_1^L}{\theta}. \tag{5}$$

Inserting this expression for the face value into (3) we find that the project can be financed without strategic default occurring in equilibrium as long as

$$F \leq \overline{F} \equiv C_1^L + \theta C_2. \tag{6}$$

In the absence of derivatives, the project cannot be financed if the IC constraint that governs strategic default is violated, since the creditor cannot break even in that case. We

[19]This assumption is not crucial for our analysis. We could alternatively assume that renegotiation is possible after a strategic default. For example, one could imagine a scenario in which the firm has full bargaining power in renegotiation. In this case, after a strategic default, the firm would offer $C_1^L + L$ to the creditor, making him just indifferent between liquidating the firm and letting the firm continue. The surplus from renegotiation to the firm would then be given by $S = C_2 - L$ and the project can be financed whenever $F < C_1^L + L$. As we show in Appendix B, with slight adjustments, our results on the priority ranking of derivatives relative to debt (Section 5) also carry through in this alternative specification.

summarize the credit market outcome in the absence of derivatives in the following Proposition.

Proposition 1 *In the absence of derivative markets, the firm can finance the project as long as* $F \leq \overline{F} \equiv C_1^L + \theta C_2$*. When the project can attract financing, the face value of debt is given by* $R = \left[F - (1-\theta) C_1^L\right]/\theta$*, and social surplus is equal to expected cash flows minus the setup cost:* $\theta \left(C_1^H + C_2\right) + (1-\theta) C_1^L - F$*.*

Most importantly for the remainder of the paper, Proposition 1 establishes that, in the absence of derivatives, the firm is always shut down after a low cash flow realization at date 1. This early termination results in loss of the date 2 cash flow C_2, which means that the equilibrium is inefficient relative to the first-best (full commitment) outcome. As we will show in the following section, derivatives can reduce this inefficiency by reducing the risk of default at date 1.

4 Financing with Derivatives: No Basis Risk

For simplicity, we first focus on the case in which the derivative has no basis risk. Using the notation introduced above, this corresponds to the situation where $\gamma = 1$, such that the firm can completely eliminate default risk by choosing an appropriate position in the derivative. We will analyze this case in two steps. We first assume that when entering the debt contract the firm can commit to the derivative position it will take ex post. As we will see, in this benchmark case, the firm always takes the socially optimal hedging position and the priority ordering of the derivative relative to debt is irrelevant. We then analyze the case in which the firm cannot commit to the derivative position it takes ex-post. In that case, we will see that the firm's private incentives to hedge are suboptimal. Moreover, making derivatives effectively senior opens the door to ex-post debt dilution in the form of speculative short positions in the derivative (rather than long hedging positions). If the firm cannot commit not to enter such speculative derivative positions, then making derivatives junior to debt is efficient because it discourages such ex-post dilution and leads to optimal hedging decisions by the firm for a strictly larger set of parameters.

For the remainder of this Section and also in Section 5, we will assume that the no-strategic-default constraint (2) is satisfied, which is the case as long as C_2 is sufficiently large. We will return to this issue in Section 5.4, where we examine how the priority ranking of derivatives affects the firm's incentives to default strategically in the high cash flow state.

4.1 No Basis Risk under Full Commitment

Let us first assume that, when entering the debt contract with the creditor, the firm can fully commit to the derivative position it will choose ex post. In this case, the firm's incentives will be to maximize overall surplus: both the creditor and the derivative counterparty will just break even, and all remaining surplus is captured by the firm. The firm will thus choose to hedge whenever it is socially optimal to do so and, since the derivative is costly, when hedging is optimal the firm will always take the minimum position in the derivative that is needed to eliminate default.

We can also immediately see that in this case the priority ranking of debt relative to the derivative is irrelevant from an efficiency standpoint. Whenever the firm chooses to hedge, debt becomes risk free and default will never occur. But when there is never any default, the bankruptcy treatment of debt relative to derivatives is irrelevant.

We see this more formally by comparing the costs and benefits from hedging in either regime. Eliminating default leads to a gain of $(1-\theta)\,C_2$, since now the firm can be kept alive even after the low date 1 cash flow. The net cost of eliminating default is given by the deadweight cost that needs to be incurred in derivative markets. Since the derivative completely eliminates default when there is no basis risk, debt becomes safe, such that $R = F$, irrespective of the priority ranking of debt relative to derivatives. Hence, the deadweight cost of taking the required derivative position $X = F - C_1^L$ is given by $\delta\left(F - C_1^L\right)$. The firm chooses to hedge whenever the presence of derivatives raises surplus, which is the case when

$$(1-\theta)\,C_2 - \delta\left(F - C_1^L\right) > 0. \tag{7}$$

This is satisfied whenever the continuation or going concern value of the firm C_2 is sufficiently large, or when the cost of hedging is sufficiently low.

Proposition 2 *When the derivative has no basis risk* ($\gamma = 1$) *and the firm can commit to a derivative position when entering the debt contract:*

1. *The firm chooses the socially optimal derivative position*
2. *The bankruptcy treatment of derivatives is irrelevant*
3. *Derivatives raise surplus whenever* $(1-\theta)\,C_2 - \delta\left(F - C_1^L\right) > 0$

4.2 No Basis Risk under Limited Commitment

Consider now the case where the firm cannot commit to a derivative position when entering the debt contract with the creditor. As we will see, the priority ranking of debt relative to derivatives may now matter. As before, the bankruptcy treatment of seniority of debt versus derivatives is irrelevant when the firm chooses the minimum derivative position required for hedging, $X = F - C_1^L$. However, if the firm cannot commit to a derivative position, its private ex-post incentives to hedge are lower than the social incentives. Taking the face value of debt $R = F$ as given, it is in the firm's ex post interest to eliminate credit risk by choosing a derivative position of $X = F - C_1^L$ whenever

$$(1-\theta)\,C_2 - (1-\theta+\delta)\left[F - C_1^L\right] > 0. \tag{8}$$

The first term in (8) is the benefit to the firm from being able to continue in the low cash flow state. The second term in (8) is the actuarially fair cost of the derivative plus the deadweight cost of hedging. Comparing this condition to (7) we see that under no commitment the firm's incentives to hedge are strictly lower than is socially optimal. This is simply another illustration of the well-known observation that equityholders have suboptimal hedging incentives once debt is in place.

As long as the firm can only take long positions in the derivative, the hedging incentives are independent of the bankruptcy treatment of derivatives. If, on the other hand, we allow the firm to take short positions in the derivative, an additional effect emerges and the bankruptcy treatment starts to matter. In particular, if the derivative contract is senior, the firm is able to dilute the creditor by taking a short position in the derivative. By doing so, the firm transfers resources that would usually accrue to the creditor in the default state into the high cash flow state, in which they accrue to the equityholder. Hence, under seniority for derivatives, a derivative that could function as a perfect hedge may well be deployed as a vehicle for speculation or risk-shifting.

To see this formally, assume that $(1-\theta)\,C_2 - \delta\left(F - C_1^L\right) > 0$, so that it would be socially optimal for the firm to hedge. Under senior derivatives, we now have to compare the firm's payoff from hedging to the payoff from taking no derivatives position, and also the payoff to taking a short position in the derivative. As it turns out, the firm's incentives are such that it always (weakly) prefers taking a short position in the derivative to taking no position at all. Therefore, the firm will hedge in equilibrium only if the payoffs from hedging exceed the payoffs from speculation by taking a short position. Comparing these payoffs, we see that hedging is now privately optimal if, and only if,

$$(1-\theta)\,C_2 - (1-\theta+\delta)\left[F - C_1^L\right] - \theta\frac{1-\theta}{(\theta+\delta)}C_1^L > 0. \tag{9}$$

The additional term relative to (8) shows that hedging is harder to sustain when short positions in the derivative are possible. In addition, in cases where no position in the derivative is optimal, under senior derivatives the firm now always takes an inefficient short position in the derivative.

Proposition 3 *When the derivative has no basis risk* ($\gamma = 1$) *and the firm cannot commit to a derivative position when entering the debt contract*

1. *The firm's private incentives to hedge are strictly less than the social incentives to hedge.*

2. *When only long positions in the derivative are possible, the bankruptcy treatment of derivatives does not matter for efficiency.*

3. *When the firm can take short 'speculative' positions in the derivative, the bankruptcy treatment of derivatives matters: Under senior derivatives, the firm may choose to take a speculative position in the derivative to dilute its creditors. This is strictly inefficient and restricts the set of parameters for which the efficient hedging position can be sustained.*

Proposition 3 illustrates, in the simplest possible setting, one of the first-order inefficiencies of senior derivatives: Rather than being used as hedging tools, seniority for derivatives may lead firms to channel funds away from creditors, in a form of risk shifting. This is not possible when derivatives are treated as junior to debt.

5 Financing with Derivatives: Basis Risk

We now extend our analysis to the case where the derivative contract has basis risk ($\gamma < 1$) and present the main results of our analysis. We initially continue to assume that the no-strategic-default constraint (2) is satisfied. In Section 5.4, we then examine examine how the priority ranking of derivatives relative to debt affects the firm's incentives to default strategically in the high cash flow state.

We first establish a preliminary Lemma about collateralization of derivatives positions. In particular, Lemma 1 states that once the face value of debt has been set, in the presence of basis risk it is always optimal ex post to maximally collateralize the derivative contract. The reason is that once R is fixed, collateralization of the derivative contract makes hedging cheaper for the firm.

Lemma 1 *Once financing has been secured and the face value of debt R has been set, it is optimal to fully collateralize the derivative position ex post. This is because, the cost of the derivative $x(\overline{x})$ is decreasing in the level of collateralization:*

$$\frac{\partial x(\overline{x})}{\partial \overline{x}} < 0. \tag{10}$$

Lemma 1 illustrates the conventional wisdom supporting the collateralization and effective seniority of derivatives: Collateralization and seniority for derivatives makes hedging cheaper, which benefits the firm. By this rationale, it is often also argued that full collateralization and the concomitant seniority of derivative contracts is optimal, and that reducing collateralization or making derivative contracts junior to debt is undesirable, as it raises the cost of the derivative to the firm and makes hedging more expensive.

However, as we will argue below, changing the level of collateralization of derivatives, while holding the face value of outstanding debt constant is not the correct thought experiment. After all, in the event of default, debtholders and derivative counterparties hold claims on the same pool of assets. Varying the collateralization of derivatives must in equilibrium also have an impact on the pricing of the firm's debt. In fact, we will show below that once we allow the firm's terms in the debt market to adjust in response to the level of collateralization in derivative markets, the argument for full collateralization and effective seniority for derivatives is reversed.

We show this by first considering the two extreme cases: senior derivatives and junior derivatives. These extreme cases contain most of the intuition for why it may be more efficient to make derivatives junior once we take into account the adjustment of the firm's borrowing costs in response to the treatment of derivatives in bankruptcy. We later show that this result generalizes to the intermediate case in which derivatives can be partially collateralized.

As before, we initially assume that the firm can commit to taking the optimal (i.e., surplus-maximizing) derivative position in any given priority structure. This abstracts away from the firm's potential incentive to dilute existing debtholders once debt is in place. We will come back to the issue of dilution through derivative positions when analyzing the non-commitment case in Section 5.5.

5.1 Senior Derivatives under Full Commitment

Senior derivatives (full collateralization of derivatives) is the natural starting point for our analysis because it most accurately reflects the current special bankruptcy status of derivatives discussed in Section 1. The required premium x for a derivative position of a notional size of X, is determined by the counterparty's breakeven constraint. When derivatives are senior, the derivative counterparty is always paid in full as long as $x \leq C_1^L$. The derivative counterparty then receives a payment of x whenever $Z = Z^H$, which happens with probability θ. When $x > C_1^L$, on the other hand, the counterparty cannot be fully repaid when the firm defaults, and then, as the senior claimant, receives the entire cash flow C_1^L. In the interest of brevity, we will focus on the first case, $x \leq C_1^L$, in the main text. The second case is covered in the appendix.

For the counterparty to break even, the expected payment received must equal the expected payments made, $X(1-\theta)$ plus the deadweight cost of hedging $\rho(X)$. The breakeven constraint is thus given by

$$x\theta = X(1-\theta) + \rho(X), \tag{11}$$

which yields a cost of the derivative of

$$x = \frac{(1-\theta)X + \rho(X)}{\theta}. \tag{12}$$

The face value of debt, R, is determined by the creditor's breakeven condition. When derivatives are senior to the creditor and $x \leq C_1^L$, this breakeven condition is given by

$$[\theta + (1-\theta)\gamma] R + (1-\theta)(1-\gamma)\left(C_1^L - x\right) = F. \tag{13}$$

This condition states that the expected payments received by the creditor must equal the initial outlay F. Note that the seniority of the derivative contract becomes relevant in the state when $C_1 = C_1^L$ and $Z = Z^H$, which occurs with probability $(1-\theta)(1-\gamma)$. In that case, the derivative counterparty is paid its contractual obligation x before the creditor can receive any payment. This leads to a face value of debt of

$$R = \frac{F - (1-\theta)(1-\gamma)\left(C_1^L - x\right)}{[\theta + (1-\theta)\gamma]}. \tag{14}$$

The derivative can be a valuable hedging tool for the firm. In particular, when $\gamma = 1$ the derivative is a *perfect hedge* against the cash flow risk at date 1, such that the firm can completely eliminate default by taking a suitable position in the derivative market. When $\gamma < 1$, the derivative is only a partial hedge, as it sometimes does not pay X when $C_1 = C_1^L$ and sometimes pays X when $C_1 = C_1^H$. Nevertheless, hedging can still be valuable for the firm. While the derivative cannot eliminate default, it can still reduce the probability of default at date 1. When $\gamma < 1$, debt remains risky even under hedging. Moreover, since default occurs with positive probability when $\gamma < 1$, the seniority of derivatives relative to debt contracts is then relevant: in states in which the firm defaults and owes payments to both the creditor and protection seller, the protection seller will get paid first.

When hedging in the derivative market, under full commitment the *optimal derivative position* for the firm is the one that just eliminates default when the date 1 cash flow is low and the derivative pays X. This is achieved by setting

$$X = R - C_1^L. \tag{15}$$

Setting $X = R - C_1^L$, the derivative contract just eliminates default in states when $C_1 = C_1^L$ and $Z = Z^L$ (with probability $(1-\theta)\gamma$). Increasing the derivative position beyond this level does not generate any additional surplus; it only increases the deadweight hedging cost ρ and is thus inefficient. As the derivative is an imperfect hedge, the firm still defaults when $C_1 = C_1^L$ and $Z = Z^H$ (with probability $(1-\theta)(1-\gamma)$). Using (12), (14), and (15) we can characterize the equilibrium under senior derivatives as follows.

Proposition 4 ***Senior derivatives.*** *Assume that derivatives are senior and that* $x \leq C_1^L$. *Under full commitment, the optimal derivative position is given by*

$$X = R - C_1^L. \tag{16}$$

This leads to a an equilibrium face value of

$$R = \frac{\theta F - (1+\delta)(1-\gamma)(1-\theta) C_1^L}{\theta - (1+\delta)(1-\gamma)(1-\theta)}, \tag{17}$$

and cost of the derivative of

$$x = \frac{(1-\theta+\delta)\left[F - C_1^L\right]}{\theta - (1+\delta)(1-\gamma)(1-\theta)}. \tag{18}$$

To gain intuition on the above results it is useful to consider the special case in which derivatives provide a perfect hedge against the cash flow risk at date 1 ($\gamma = 1$). In this case, debt becomes risk-free ($R = F$), so that the optimal derivative position is given by $X = F - C_1^L$. When the derivative is not a perfect hedge ($\gamma < 1$), on the other hand, debt remains risky even in the presence of derivatives ($R > F$) and the required derivative position increases to $R - C_1^L > F - C_1^L$.

The social surplus generated in the presence of derivatives depends on how effective derivatives are at hedging the firm's cash flow risks. In particular, when the derivative has more *basis risk* (lower γ), this reduces the effectiveness of the derivative as a hedging tool and thus the probability of continuation of the firm at date 1, $\theta + (1-\theta)\gamma$. In addition, basis risk increases the costs of eliminating default, since the required derivative position, $R - C_1^L$, is strictly larger than the derivative position required in the absence of basis risk.

Corollary 1 ***Social surplus.*** *The social surplus when the firm chooses a derivative position of $X = R - C_1^L$ is given by*

$$\theta C^H + (1-\theta) C_1^L + [\theta + (1-\theta)\gamma] C_2 - F - \rho \left(R - C_1^L\right). \tag{19}$$

Derivatives raise social surplus relative to the outcome without derivatives when the gain from a greater likelihood of continuation of $(1-\theta)\gamma$ outweighs the hedging cost:

$$(1-\theta)\gamma C_2 - \rho \left(R - C_1^L\right) > 0, \tag{20}$$

where R is given by (17). When hedging costs are linear, this is satisfied whenever the hedging cost is not too large:

$$\delta < \delta^* = \frac{(1-\theta)\gamma[\theta - (1-\theta)(1-\gamma)]C_2}{(1-\gamma)\gamma(1-\theta)^2 C_2 + \theta(F - C_1^L)}. \tag{21}$$

Assume for now that (20) is satisfied, so that derivatives can indeed add value. When (20) is satisfied, the socially optimal derivative position is given by $X = R - C_1^L$. When (20) is violated, on the other hand, it is optimal for the firm not to use derivatives at all. Corollary 1 shows that derivatives add value as long as the hedging cost δ is sufficiently low, or equivalently, as long as the setup cost F is not too large. The respective critical values for δ or F depend on the derivative's basis risk. In particular, when basis risk increases (γ decreases), this lowers the benefit from derivatives, $(1-\theta)\gamma C_2$, while raising their cost, $\rho\left(R - C_1^L\right)$. While the reduction in benefits from derivatives is immediate from (20), the increase in the cost of derivatives arises from the higher required face value R for lower γ. This, in turn, implies that a larger derivative position is necessary in order to eliminate default in the states in which $C_1 = C_1^L$ and $Z = Z^L$, thus raising the cost of managing risk through derivatives. Hence, an increase in basis risk implies that derivatives add value for a strictly smaller set of combinations of hedging and setup costs.

5.2 Junior Derivatives under Full Commitment

We now consider the opposite extreme case, junior derivatives. As before, default by the firm occurs in the low cash flow state at date 1 when the derivative bet does not pay off. This happens again with probability $(1-\gamma)(1-\theta)$. Under seniority for derivatives, the protection seller was fully repaid in this state. This changes when derivatives are junior. Now the lender receives the entire cash flow C_1^L in default, whereas the protection seller receives nothing. This changes the protection seller's breakeven constraint, since now the protection seller only receives the premium x with probability $[\theta - (1-\theta)(1-\gamma)]$ rather than with probability θ. The protection seller's breakeven constraint is now given by

$$x^S [\theta - (1-\theta)(1-\gamma)] = (1-\theta) X^S + \rho \left(X^S\right), \tag{22}$$

(where the superscript S refers to the fact that debt is senior), which yields

$$x^S = \frac{(1-\theta) X^S + \rho \left(X^S\right)}{\theta - (1-\theta)(1-\gamma)}. \tag{23}$$

Debt is still risky, but since the creditor is now senior to the derivative counterparty, he receives the entire cash flow in the default state, so that the creditor's breakeven constraint becomes

$$[\theta + (1-\theta)\gamma] R^S + (1-\theta)(1-\gamma) C_1^L = F. \tag{24}$$

As a result, the face value of debt for the senior lender is lower than in the case where derivatives are senior:

$$R^S = \frac{F - (1-\theta)(1-\gamma) C_1^L}{\theta + (1-\theta)\gamma}. \tag{25}$$

By the same argument as before, default can be eliminated in the state where $C_1 = C_1^L$ and $Z = Z^L$ by choosing the size of the derivative contract such that

$$X^S = R^S - C_1^L. \tag{26}$$

As before, default still occurs when $C_1 = C_1^L$ and $Z = Z^H$ when the derivative is an imperfect hedge. We can now use (23), (25) and (26) to characterize the equilibrium under junior derivatives.

Proposition 5 ***Junior derivatives.*** *Assume that derivatives are junior. Under full commitment, the optimal derivative position is given by*

$$X^S = R^S - C_1^L. \tag{27}$$

This leads to an equilibrium face value of

$$R^S = \frac{F - (1-\theta)(1-\gamma) C_1^L}{\theta + \gamma(1-\theta)}, \tag{28}$$

and cost of the derivative of

$$x^S = \frac{(1-\theta+\delta)\left[F - C_1^L\right]}{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}. \tag{29}$$

Analogously to before, we can use the results from Proposition 5 to calculate the surplus when derivatives are junior.

Corollary 2 *With junior derivatives, social surplus is given by*

$$\theta C^H + (1-\theta) C_1^L + \left[\theta + (1-\theta)\gamma\right] C_2 - F - \rho\left(R^S - C_1^L\right). \tag{30}$$

When derivatives are junior, the introduction of derivatives raises social surplus relative to *the outcome without derivatives whenever*

$$(1-\theta)\,\gamma C_2 - \rho\left(R^S - C_1^L\right) > 0. \tag{31}$$

where R^S *is given by (28). When hedging costs are linear, this is satisfied whenever the hedging cost is not too large:*

$$\delta < \delta^{**} = \frac{(1-\theta)\,\gamma\,[\theta + \gamma\,(1-\theta)]\,C_2}{F - C_1^L}. \tag{32}$$

Proposition 5 and Corollary 2 contain the key economic insight of our analysis. *When debt is senior, the required face value on the debt* R^S *is lower* than when derivatives are senior. In other words, when debt is senior to derivatives, the firm's cost of debt is lower despite the fact that the firm's hedging costs are higher. This is a striking result, which is robust to many changes in the model, and which is not entirely obvious a priori. The fact that the cost of debt is lower even though hedging costs are higher is critical, in particular, because according to (27) it implies that the size of the optimal derivative position is lower than when derivatives are senior. Indeed, from Corollaries 1 and 2 it is easy to observe that the optimal derivative position under senior debt, $R^S - C_1^L$, is smaller than the optimal derivative position under senior derivatives. As the deadweight cost of hedging is directly proportional to the size of the hedging position, it follows that the increase in the cost of debt that results when derivatives are senior reduces surplus. This is summarized in the following proposition.

Proposition 6 ***Comparing surplus under junior and senior derivatives.*** *Relative to the case without derivatives, junior derivatives are more likely to raise surplus than senior derivatives. When deadweight costs of hedging costs are linear, in particular, hedging with junior derivatives increases surplus for all* $\delta \leq \delta^{**}$ *while hedging with senior derivatives* *increases surplus for all* $\delta \leq \delta^*$ *where:*

$$\delta^{**} > \delta^*.$$

In addition, when hedging adds value both with senior and junior derivatives, surplus under junior derivatives is unambiguously higher than with senior derivatives. With linear hedging costs the difference in surplus is given by

$$\delta\left(R - R^S\right) = \delta \frac{(1-\gamma)(1-\theta)(1-\theta+\delta)}{[\theta+\gamma(1-\theta)][\theta-(1+\delta)(1-\gamma)(1-\theta)]} \geq 0 \tag{33}$$

Thus, the received wisdom that full collateralization and seniority of derivatives is desirable (Lemma 1) reverses once one takes into account the effects of collateralization of derivatives on the cost of debt. Proposition 6 shows that derivatives are more likely to add value when they are junior as opposed to when they are senior. Moreover, surplus is always higher under junior derivatives than under senior derivatives, except in two special cases. First, when the derivative is a perfect hedge ($\gamma = 1$), the firm never defaults, so that seniority of the derivative contract is irrelevant. Second, when there is no deadweight hedging cost ($\delta = 0$) seniority is irrelevant because of the Modigliani-Miller theorem: in frictionless markets, capital structure does not matter.

Robustness. The superiority of junior derivatives that is established in Proposition 6 is robust to a number of variations in the assumptions of our model. Most importantly, we want to stress that our result is not driven by the fact that there is a deadweight cost of hedging in the derivative markets, but no deadweight cost in the debt markets. The easiest way to see this is to consider the reverse case of what we have assumed up to now: if there was a cost of risk only in the debt market, but not in the derivative market, it would obviously also be optimal to make debt senior, in order to minimize the risk borne by the debtholder.[20]By the same logic, in a model in which there is a cost of risk both in debt and

[20]For example, lenders may have to hold costly regulatory capital when they extend loans. If the regulatory

derivative markets, the cost of risk in the debt market creates an additional reason why debt should be senior: when debt is senior, this minimizes the cost of hedging in the debt market (and thus the required face value of debt), which in turn minimizes the required derivative position, and thus the hedging cost in the derivative market.

To see this more specifically, consider the following example, in which we treat risk in the debt and derivative markets completely symmetrically and still obtain our main result. Assume that parties in each market (the creditor and the counterparty) incur a cost that is proportional to the potential loss they face when their contracts move against them. As before, for the counterparty this cost is proportional to X. For the creditor, this cost is proportional to the loss in case of default, which is given by $F - C_1^L - x$ when derivatives are senior and $F - C_1^L$ when derivatives are junior. It is straightforward to see that making derivatives junior reduces the hedging cost in both markets.

Alternatively, one could also impose a symmetric deadweight cost that is proportional to the volatility of the derivative and the debt contract payoff, respectively. While this is somewhat less tractable than our baseline model, under this specification the ranking of junior and senior derivatives presented in Proposition 6 is also preserved.

The one change that could reverse the superiority of junior derivatives in the context of our model is a potential regime shift in derivative markets following the removal of the special treatment of derivatives in bankruptcy. More specifically, our analysis assumes that the priority treatment of derivatives relative to debt results in a transfer of counterparty risk from one market to the other. While treating counterparty risk symmetrically in this fashion is the natural starting point for our analysis, it is conceivable that following the removal of the privilege there might be a discontinuous increase in the *deadweight* cost of hedging counterparty risk for the derivatives writer: from $\rho(X)$ to, say, $\rho^J(X) >> \rho(X)$. However, while counterparty risk in the derivative market is likely to increase following the removal of the privilege, it is not obvious that the increase risk will lead to significant

capital requirement increases with risk, as it does in Basel II or III, this translates into a cost that is increasing in the face value R.

increases in deadweight costs. For the typical firm the derivatives writer may face only a small increase in the probability of non-payment of the premium x. This small increase in risk would of course result in a higher premium, as is the case in our analysis. However, by far the biggest risk derivatives writers would continue to face is with respect to the payment X which is due when Z^L is realized. This risk and the cost of hedging this risk (say, by holding more capital or purchasing credit default swap protection) is largely unrelated to the risk of non-payment of the premium, and therefore it is unclear why there would be a structural shift in the deadweight cost $\rho(X)$.

Another argument that has been made to justify the super-senior status of derivatives is the resulting reduction in information sensitivity of the derivative contract. Because of full collateralization, derivative counterparties need to know less about firm's default probability. However, note that also in this case there is no free lunch: making the derivative less information sensitive will invariably make the firm's debt contract more information sensitive and, all else equal, raise the firm's cost of financing. In fact, in a number of theories of debt it is precisely their insensitivity to information that makes them optimal financing contracts.

5.3 Partial Collateralization

Having compared the polar cases of senior and junior derivatives, we now characterize the equilibrium for the more general case of partial collateralization of the derivative contract. Under partial collateralization, the firm pledges a maximum amount $\overline{x} \leq x$ of collateral to the derivative counterparty.

The steps required to calculate the equilibrium are analogous to the discussion in the two polar cases above (see the appendix for details). Intuitively, partial collateralization makes the derivative contract senior up to the maximum amount $\overline{x} \leq x$. For the remaining amount $x - \overline{x}$, derivative counterparties are not collateralized and hold a regular debt claim. For simplicity we assume that this remaining claim is junior to the debtholder. As in the two polar cases discussed above, an increase in collateralization reduces the cost of the derivative, but increases the firm's cost of debt. We characterize the equilibrium for a general collateralization amount $\overline{x}$ in Proposition 7 below.

Proposition 7 ***Partial collateralization.*** *When the derivative is partially collateralized up to an amount $\overline{x} \leq x$, the optimal derivative position is given by*

$$X(\overline{x}) = R(\overline{x}) - C_1^L. \tag{34}$$

This leads to an equilibrium face value of

$$R(\overline{x}) = \frac{F - (1-\theta)(1-\gamma)\left(C_1^L - \overline{x}\right)}{\theta + \gamma(1-\theta)}, \tag{35}$$

and a cost of the derivative of

$$x(\overline{x}) = \frac{(1-\theta+\delta)\left[F - C_1^L\right] - (1-\gamma)(1-\theta)\left[\theta - (1-\gamma)(1-\theta) - \delta\right]\overline{x}}{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}. \tag{36}$$

Proposition 7 shows that the case of partial collateralization lies between the two extreme cases above. We see that as collateralization increases, the cost of the derivative, $x(\overline{x})$, decreases. At the same time, however, the required face value of debt increases, as an increase in collateralization of the derivative makes the debt contract riskier. This also means that the required derivative position, $R(\overline{x}) - C_1^L$, is monotonically increasing in the level of collateralization of the derivative.

This proposition shows that the surplus results from the extreme cases of senior and junior derivatives extend to a general setup with partial collateralization. In particular, as the equilibrium face value of debt rises when derivatives are more collateralized, the required derivative position is larger. This reduces total surplus because the firm has to incur a larger hedging cost to eliminate default.

Corollary 3 ***Surplus with partial collateralization.*** *The surplus generated by the introduction of derivatives is decreasing in the level of collateralization of the derivative contract.*

5.4 Default due to Derivative Losses and Inefficient Collateral Calls

Up to now we have assumed that the required debt and derivative payment are such that the firm meets its payment obligations when the firm receives the high cash flow C_1^H, but the derivative moves against the firm. While this helped simplify our analysis, this assumption is not innocuous. The reason is that the firm can only make the required payment $R(\overline{x}) + x(\overline{x})$ if it has sufficient resources to do so. Moreover, even if there are sufficient resources in the firm, the firm may have an incentive to default strategically in states where payment is due both on debt and the derivative (recall that up to now we simply assumed that this incentive constrained is satisfied).

In this Section, we show that default due to derivative losses in the high state is more likely, the higher is the level of collateralization and effective seniority of derivative contracts. Moreover, we show that this problem is exacerbated when the firm has no way of invoking a stay on inefficient collateral calls that may be privately optimal from the perspective of the derivative writer.

5.4.1 Default due to Derivative Losses

The reason that default in the high state is more likely when the level of collateralization of the derivative is higher is that a higher level of collateralization of the derivative contract leads to a larger *overall* required payment $R(\overline{x}) + x(\overline{x})$ in states where the derivative moves against the firm. Intuitively speaking, while more collateralization generally decreases the cost of the derivative $x(\overline{x})$, this is more than outweighed by the concomitant increase in the face value of debt $R(\overline{x})$, such that the overall payment increases. This makes default more likely because it increases the chance of either fundamental or strategic default in the state when the firm receives the high cash flow, but the derivative moves against the firm. This is summarized in the following Proposition.

Proposition 8 ***Default due to losses on the derivative position.*** *The firm meets its payment obligations when it receives the high cash flow but the derivative moves against the firm as long as:*

$$R\left(\overline{x}\right)+x\left(\overline{x}\right)\leq \min\left[C_1^H, C_1^L+C_2\right]. \tag{37}$$

The higher the level of collateralization for derivatives, the less likely it is that this condition holds:

$$\frac{\partial R\left(\overline{x}\right)}{\partial \overline{x}}+\frac{\partial x\left(\overline{x}\right)}{\partial \overline{x}}=\frac{\delta\left(1-\gamma\right)\left(1-\theta\right)}{\left[\theta-\left(1-\gamma\right)\left(1-\theta\right)\right]\left[\theta+\gamma\left(1-\theta\right)\right]}>0 \tag{38}$$

Proposition 8 shows that both fundamental and strategic default are more likely when the derivative is more highly collateralized. This also implies that derivatives can eliminate default in the low cash flow state without causing default in the high cash flow state for a smaller set of parameters when derivatives are more collateralized: The possibility of default due to derivative losses in the high state implies that derivatives can serve as hedging tools only if the ex ante setup cost lies below a cutoff value $F\left(\overline{x}\right)$. This cutoff value is decreasing in the level of collateralization, which means that derivatives can serve as hedging tools for a larger set of ex-ante projects when there is less collateralization.

Corollary 4 *Derivatives can be used to hedge the low cash flow state without causing default in the high cash flow state as long as*

$$F\leq F\left(\overline{x}\right)=\Gamma_0 C_1^L+\Gamma_1\min\left[C_1^H, C_1^L+C_2\right]-\Gamma_2\overline{x}. \tag{39}$$

where Γ_0 and Γ_1 are positive constants and

$$\Gamma_2=\frac{\left(1-\gamma\right)\left(1-\theta\right)\delta}{\theta+\gamma\left(1-\theta\right)+\delta} \tag{40}$$

Since $\Gamma_2\geq 0$, $F\left(\overline{x}\right)$ is decreasing in the level of collateralization.

5.4.2 Inefficient Collateral Calls

We now slightly extend the model to show how the exemption of derivatives from the *automatic stay* under chapter 11 can also lead to inefficient collateral calls by the derivative counterparty. To be able to model collateral calls we introduce into the basic model a working capital demand for the firm, which can also play the role of unassigned cash collateral. Specifically, suppose that the firm requires working capital to generate the date 2 cash flow C_2. Let $y = D - F$ be the amount of working capital in the firm, where D is the amount of funding the firm raises at date 0 and F is the amount is spends on fixed investment. Suppose also for simplicity that the firm can only generate the second period cash flow if there is sufficient working capital in the firm:

$$C_2(y) = \begin{cases} V & \text{if} \quad y \geq \kappa \\ 0 & \text{otherwise} \end{cases}, \tag{41}$$

where $V > 0$ and $\kappa > 0$. Moreover, the working capital used to generate C_2 is spent by the firm before the realization of the date 1 cash flow, so that it is no longer available to make payments to the creditor or derivative counterparty at that point.

Consider briefly the outcome absent derivatives: If V is sufficiently large, it is optimal for the firm to hold sufficient working capital; that is, it is optimal to raise $D = F + \kappa$ at date 0, and to hold working capital $y = \kappa$. The payoff to the firm absent derivatives is then given by

$$\theta \left[C_1^H - R + V \right] \tag{42}$$

where R is given by:

$$\theta R + (1 - \theta) C_1^L = F + \kappa \tag{43}$$

or,

$$R = \frac{F + \kappa - (1 - \theta) C_1^L}{\theta}. \tag{44}$$

Now consider the outcome in the presence of derivatives when the firm does not have the protection of a stay on collateral calls by the derivative counterparty. As we will show, this may then give rise to inefficient collateral calls on the firm. In particular, if the derivative moves against the firm, the counterparty to the derivative transaction may find it privately optimal to make a collateral call on the firm's cash in order to ensure full payment on the derivative, even if this comes at the cost of a lower future value for the firm.

More formally, consider the following time line:

1. Firm writes a debt contract with lender and borrows an amount $D = F + \kappa$.

2. Firm writes derivative contract with counterparty (x, X). This contract involves basis risk γ.

3. Counterparty observes realization of Z before the realization of the first period cash flow; if $Z = Z^H$ counterparty can initiate a procedure to collect x. If there is no stay the counterparty can immediately make a collateral call on the cash available to the firm κ (and subsequently when first-period cash flow is realized on C_1). In that case the firm would be deprived of its working capital, with the consequence that $C_2 = 0$.

4. If the firm has working capital available it spends it *before* the realization of C_1 and then receives $C_2 = V$ at date 2, provided that the firm is not liquidated before then.

5. First-period cash flow is realized; when cash flow is C_1^H and payment is due on the derivative, the firm chooses whether to repay $R + x$ or not; when cash flow is C_1^H and the firm gets a payment X on the derivative, it must decide whether to repay R, and when cash flow is C_1^L and the firm gets a payment X on the derivative, whether to repay R. When cash flow is C_1^L and the firm must make a payment x the firm is liquidated.

6. If the firm continues to the second period and is able to use its working capital it obtains V.

Given this timeline, the firm will be exposed to inefficient collateral calls (effectively a *run* on working capital) if the firm cannot invoke an automatic stay against collateral calls from a derivative counterparty. To see this, suppose that the firm borrows $F + \kappa$ and takes out a derivative promising to pay $X = R - C_1^L$ in the event that $Z = Z^L$, against a payment $x = [(1 - \theta) X + \rho(X)]/\theta$ in the event that $Z = Z^H$ (on the assumption that the derivative is senior to debt and that $x \leq C_1^L + \kappa$).

In this case, it is a best response for the derivative counterparty to make a collateral call immediately on the realization of Z^H. If it makes such a collateral call the firm ends up with insufficient working capital, so that $C_2 = 0$. Because of the collateral call, this also pushes the firm to strategically default when C_1^H is realized, because when $C_2 = 0$ running away with $C_1^H - C_1^L$ is strictly more profitable than making the required payment $R + x$ on the debt and the derivative: $C_1^H - C_1^L > C_1^H - R - x$. The derivative counterparty, however, is still able to fully recover its claim as $x \leq C_1^L + \kappa$. Note that even though in this example it is the collateral call that pushes the firm into default, it is privately optimal for the derivative counterparty to ask for collateral. Should it not make that collateral call the derivative counterparty would lose access to κ and, in the case that C_1^L realizes, can only hope to receive a maximum amount C_1^L at date 1. Thus, making an immediate collateral call is privately optimal for the counterparty if $C_1^L + \kappa \geq x > C_1^L$.

The collateral call by the counterparty is inefficient because it leads to a loss of C_2 (and strategic default) in a state where absent the collateral call the firm would continue. Moreover, the collateral call is not needed for the derivative counterparty to break even. When V is sufficiently large, the firm's incentive to continue can support a high enough payment by the firm such that both the creditor and derivative counterparty break even in expectation.

5.5 Limited Commitment: Hedging or Speculation?

In this Section, we relax the assumption that the firm can commit to an ex-post derivative position and investigate another potential inefficiency that can result from the preferential treatment of derivatives in bankruptcy: if the firm cannot commit to taking an appropriate derivative position, it may choose ex post to take speculative derivative positions at the expense of creditors.

We first illustrate this potential motive for inefficient speculation when derivatives are senior to debt for parameter values such that the firm finds it ex post privately optimal to hedge its cash flow risk in the derivative market. This is the case as long as C_2 is sufficiently large. In this context, we show that the current preferential treatment of derivatives in bankruptcy may incentivize the firm to take excessively large derivative positions or, if it has the choice among multiple derivatives, to choose derivatives that are speculative rather than hedging tools.

First, recall that when hedging is optimal, a social planner would always choose a derivative position that just eliminates default: $X = R - C_1^L$, where R is the face value at which the creditor breaks even given the derivative payoff X. Now consider the firm's ex-post private incentives to take a hedging position X^B when derivatives are senior. If C_2 is large enough that the firm finds it optimal to eliminate default, it would never want to take a derivative position that is smaller than $R - C_1^L$. Under senior derivatives it may, however, have an incentive to take a derivative position that strictly exceeds $R - C_1^L$, which is inefficient given the deadweight cost of hedging. To see this, consider the firm's objective function with respect to hedging after it has already committed to a debt repayment of R, given below. If it is privately optimal for the firm to eliminate the default state, the firm's privately optimal derivative position X^B maximizes the firm's private payoff, subject to the constraint that

$X^B \geq R - C_1^L$:

$$\max_{X^B \geq R - C_1^L} \theta \left[C_1^H - R + \frac{1-\theta}{\theta} (1-\gamma) X^B - \left[1 - \frac{1-\theta}{\theta} (1-\gamma) \right] x \left(X^B \right) \right] + (1-\theta) \gamma \left[C_1^L + X^B - R \right] + \left[\theta + (1-\theta) \gamma \right] C_2. \quad (45)$$

where the premium $x \left(X^B \right)$ the firm pays for the derivative is determined by the protection seller's break-even constraint (11).

To see why the firm may over-speculate in derivative markets, it is instructive to look at the firm's marginal payoff from increasing its derivative position beyond $X = R - C_1^L$:

$$\underbrace{1-\theta}_{\text{marginal derivative payoff}} - \underbrace{\left[1 - \frac{1-\theta}{\theta} (1-\gamma) \right]}_{\leq 1} \underbrace{\left[1 - \theta + \rho'(R - C_1^L) \right]}_{\text{marginal cost of derivative}} \gtrless 0 \quad (46)$$

The first term is the extra derivative payoff to the firm from increasing its derivative position by one unit beyond X. It is equal to $(1-\theta)$ because an increase in the derivative's notional value generates an additional dollar for the firm with probability $(1-\theta)$. The second term is the share of the marginal cost of an additional unit of the derivative that is borne by the firm. The full marginal cost of an additional unit in notional derivative exposure is given by its actuarially fair marginal cost $(1-\theta)$ plus the increase in the hedging cost $\rho'(R - C_1^L)$. However, this cost is only borne by the firm in states in which it is the residual claimant. In the default state, the marginal cost of the derivative is paid by the creditor, since the derivative is senior to debt. Thus, the firm does not internalize the full cost of increasing its derivative position beyond X, and therefore may have an incentive to over-speculate.

To illustrate this more explicitly, suppose that the deadweight hedging costs are linear: $\rho(X) = \delta X$. From (46), we then find that the firm's privately optimal derivative position coincides with the optimal derivative position when the derivative has relatively little basis risk $\gamma \geq \overline{\gamma}$. When the derivative has significant basis risk, $\gamma < \overline{\gamma}$, on the other hand, the firm will enter a derivative position that is too large from a social perspective. This implies that the firm will choose to over-speculate in derivative markets whenever the derivative's basis risk is sufficiently large. Given a linear hedging cost, when the firm chooses to over-speculate it will choose a derivative position that completely expropriates the creditor in the default state (it will choose a position X^B such that $x\left(X^B\right) = C_1^L$). This is summarized in the following Proposition.

Proposition 9 ***Senior derivatives may lead to excessively large derivative positions.*** *Suppose that the deadweight cost of hedging is linear and given by $\rho(X) = \delta X$. Suppose also that it is privately optimal for the firm to hedge default risk via the derivative. Then, when the firm cannot commit to a derivative position ex ante, the firm's privately optimal derivative position coincides with the optimal derivative position only if $\gamma \geq \overline{\gamma}$. When $\gamma < \overline{\gamma}$, the firm enters a derivative position that is too large from a social perspective, where*

$$\overline{\gamma} = 1 - \frac{\delta\theta}{(1-\theta)(1-\theta+\delta)}. \tag{47}$$

When the firm chooses to over-speculate it chooses a position X^B such that $x\left(X^B\right) = C_1^L$, so that

$$X^B_{\gamma<\overline{\gamma}} = \frac{\theta}{1-\theta+\delta} C_1^L. \tag{48}$$

The social loss from the excessively large derivative position is then

$$\delta(X^B - X) = \delta\left[\frac{\theta}{1-\theta+\delta} C_1^L - \frac{\theta}{\theta-(1-\theta)(1-\gamma)(1+\delta)}(F - C_1^L)\right]. \tag{49}$$

The incentive to over-speculate in derivative markets disappears when derivatives are junior to debt. To see this, consider the firm's ex-post objective with respect to hedging with junior derivatives. The firm's surplus is unchanged relative to (45), except that the premium for the derivative $x(X^B)$ is now determined by (22):

$$x\left(X^B\right) = \frac{(1-\theta)\,X^B + \rho\left(X^B\right)}{\theta - (1-\theta)\,(1-\gamma)}. \tag{50}$$

Differentiating (45) and (50) with respect to X^B then reveals that with junior derivatives the firm has no incentive to take an excessively large derivative position. Indeed, the marginal payoff from increasing the derivative position beyond $X^S = R^S - C_1^L$ is now given by $-\rho'\left(R^S - C_1^L\right) < 0$. This is intuitive: Under junior derivatives, the firm bears the full marginal cost of an additional unit of derivative exposure. Since the derivative is priced at actuarially fair terms net of the deadweight hedging cost, the firm cannot gain from increasing its derivative exposure beyond $R^S - C_1^L$.

Proposition 10 ***Under junior derivatives there is no incentive to take excessively large derivative positions.*** *When derivatives are junior, the firm has no incentive to over-speculate in derivative markets. When it is privately optimal for the firm to hedge, it always chooses the efficient derivative position.*

One implication of our analysis is thus that under the current exemption of derivatives from the automatic stay in bankruptcy, firms may take derivative positions that are excessively large from a social perspective. This is true even though derivatives are fundamentally value-enhancing in our model as risk management tools. This incentive to take on excessively large derivative positions are tightly linked to the basis risk of the derivative contract available for hedging. When the derivative has no basis risk, or when basis risk is sufficiently small, the firm has no incentives to take excessively large positions. When, on the other hand, there is a sufficient amount of basis risk, the firm may have an incentive to take on excessive derivative positions, thereby diluting existing creditors. Rather than being a hedging tool, the derivative then becomes a vehicle for speculation.

A natural question to ask is what would happen if the firm had a choice of derivative instruments? Would it choose to hedge as much as possible by choosing little basis risk, or would it choose to speculate at its creditors' expense by choosing a derivative with more basis risk?[21] To answer this question, suppose that after signing the debt contract the firm can

[21] Note that the choice of basis risk is related to the potential incentive to 'short' the derivative discussed in Section 4.2: A short position in a derivative with basis risk γ is the same as a long position in a derivative with basis risk $1 - \gamma$.

choose among a number of derivative contracts that differ in their basis risk: $\gamma \in [\gamma_{\min}, \gamma_{\max}]$. Suppose also, that $\gamma_{\max} > \overline{\gamma} > \gamma_{\min}$ so that under the derivative with minimum basis risk the efficient hedging position could be sustained when derivatives are senior, whereas with maximum basis risk the firm would have an incentive to choose an inefficiently large derivative position, as discussed in Proposition 9.

Observe first that the firm's objective function (45) is linear in γ. This implies that the firm's optimal choice of γ is a *bang-bang* policy: it is either optimal to choose $\gamma = \gamma_{\max}$ or $\gamma = \gamma_{\min}$. In the latter case the firm minimizes the hedging benefit of the derivative and maximizes the dilution of existing creditors. We also see that the firm's incentives to engage in dilution by choosing the highest basis risk depend on the seniority treatment of derivatives in bankruptcy. By differentiating (45) with respect to γ, we can show that the choice of minimum basis risk ($\gamma = \gamma_{\max}$) is easier to sustain when derivatives are junior than when they are senior. Moreover, when minimum basis risk cannot be sustained under senior derivatives the firm then has an incentive to choose an inefficiently large derivative position, as discussed in Proposition 9.

Proposition 11 ***Choice of basis risk.*** *Assume that $\gamma_{\max} > \overline{\gamma} > \gamma_{\min}$. The firm chooses the minimum basis risk derivative and the efficient derivative position when derivatives are junior to debt, if*

$$C_2 - \left[R^S(\gamma = \gamma_{\max}) - C_1^L\right] \geq 0, \tag{51}$$

and when derivatives are senior to debt, if

$$C_2 - \underbrace{\frac{1+\delta}{\theta}}_{\geq 1}\left[R(\gamma = \gamma_{\max}) - C_1^L\right] \geq 0. \tag{52}$$

Condition (52) is strictly harder so satisfy than (51), which means that when derivatives are senior to debt the firm has stronger incentives to choose maximum basis risk. Moreover, when (52) is violated, the firm chooses maximum basis risk $\gamma_{\min}$ and fully dilutes the creditor *by choosing a derivative position that is strictly larger than optimal.*

Proposition 11 establishes, first, that the firm has an incentive to choose the derivative with minimum basis risk when C_2 is sufficiently large. Second, when C_2 is small it is (ex-post) optimal for the firm to choose a derivative instrument with maximum basis risk in order to dilute existing creditors through speculation in the derivative market. Third, choosing minimum basis risk is a more likely outcome when derivatives are junior than when they are senior to debt. The intuition for these results is twofold. First, when derivatives are junior, the required derivative premium increases in basis risk because the derivative counterparty is now less likely to get repaid in full. This decreases the incentive to increase basis risk. Second, the notional derivative position required to hedge cash flow risk is strictly smaller under junior derivatives than under senior derivatives. This reduces the firm's incentives to move this derivative payoff into the high cash flow state at the expense of defaulting more often. All in all, under junior derivatives the firm thus has less to gain from speculating by choosing a position in a derivative with high basis risk.

Let us now extend our discussion to the parameter region in which the firm does not have an incentive to hedge ex post. In this situation seniority of the derivative contract over debt may have a benefit. As is well known (e.g., Smith and Stulz (1985)), once debt is in place, equityholders' benefit from hedging is generally less than the total gain to the firm, as the firm's creditors also stand to gain from the firm's hedge. This means that equityholders incentives to enter hedging contracts ex post could be inefficiently low. Equally, and for the same reason, equityholders may want to unwind efficient existing hedges once debt is in place (due to *risk shifting* gains á la Jensen and Meckling (1976)).

In this situation there could be advantages to having derivatives senior to debt, as then the costs as well as the benefits of hedging will be shared between holders of equity and debt. This is the case for example when C_2 is relatively low. It can be shown that when derivatives are *junior* to debt it is privately optimal for the firm to hedge if $C_2 > \bar{C}_2$, and when derivatives are *senior* to debt it is privately optimal for the firm to hedge when

$C_2 > \tilde{C}_2$. Depending on parameter values, it is possible that $\tilde{C}_2 < \bar{C}_2$, so that a region exists where the firm only chooses to hedge ex post when derivatives are senior to debt.

Note, however, that this situation only arises in a region where hedging is less valuable in the first place (because it occurs for low values of C_2; $C_2 \in (\tilde{C}_2, \bar{C}_2)$). The current privileged treatment of derivatives is thus surely *over-inclusive*, as the benefits of seniority for derivatives arises only for a small subset of parameter constellations, while outside of this region the super-seniority granted by the bankruptcy code induces a number of hedging and excess speculation inefficiencies as pointed out above. Moreover, if the firm has an existing hedge that it entered at the same time that it entered the debt contract, it is precisely the seniority of subsequently entered derivatives that allows the firm to unwind a desirable pre-existing hedge that it would like to commit to ex ante.

6 Conclusion

This paper develops a simple model to analyze in a tractable and transparent way the implications of granting super-seniority protection to derivatives, swaps, and repos. These protections have been put in place with the main objective of providing stability to derivative markets, without any systematic analysis of the likely consequences for firms' overall costs of borrowing and hedging incentives. The presumption has generally been that the effects of super-protection of derivatives on firms' cost of debt are negligible and do not require any in-depth analysis. Our analysis suggests, however, that the strengthening of derivatives' treatment in bankruptcy may have been inefficient. While seen in isolation the super-protection lowers the cost of hedging, this is more than offset by a greater cost of debt and a greater incentive to over-hedge. Based on our analysis, it appears that, at a minimum, further research is required into the consequences for firms' cost of borrowing before one can conclude that the super-priority status of derivatives is warranted.

Our model also points at potential directions for future research. For example, one of the main implications of our model is that, as long as the relocation of credit risk between derivative markets and credit markets are mere transfers, making derivatives junior is more efficient than giving them effective seniority. However, additional effects may arise if moving credit risk to the debt market leads to net surplus gains in the derivative markets, for example by allowing standardization that is not possible in the absence of such seniority. We leave these questions for future research.

7 Appendix

7.1 Appendix A: Proofs

Proof of Proposition 3: The first two statements in the Proposition follow directly from the discussion in the text. To derive equation (9), we need to compare the payoff to the firm from hedging, which is given by the NPV minus the deadweight cost of hedging,

$$\theta C_1^H + (1-\theta)\, C_1^L + C_2 - F - \delta(F - C_1^L), \tag{53}$$

to the payoff from entering a speculative short derivative position. (The deviation to a speculative short position is always more profitable for the firm than a deviation to taking no derivative position at all.) If ex-ante creditors expect the firm to hedge and thus set $R = F$, the payoff to the speculative short derivative position is given by

$$\theta\left(C_1^H + \frac{1-\theta}{\theta+\delta} C_1^L - F\right) + \theta C_2, \tag{54}$$

where $\frac{1-\theta}{\theta+\delta} C_1^L$ is the amount that the firm can transfer to the high state by entering a short derivative that pays C_1^L to the derivative counterparty in the low state (and thus fully expropriates creditors). This follows from the counterparty's breakeven condition $(1-\theta)\, C_1^L = \theta X_{short} + \delta X_{short}$. The firm chooses to hedge when (53) exceeds (54), which leads to equation (9).

Proof of Lemma 1: The steps needed to calculate the cost of the derivative as a function of the level of collateralization $\overline{x}$ are given below in the section characterizing the equilibrium under partial collateralization. Holding R fixed and assuming that $\overline{x} \leq C_1^L$, we know that

$$x(\overline{x}) = \frac{(1-\theta)\left[R - C_1^L\right] + \rho\left(R - C_1^L\right) - (1-\theta)(1-\gamma)\overline{x}}{\theta - (1-\theta)(1-\gamma)}. \tag{55}$$

This implies that, when R is held fixed,

$$\frac{\partial x(\overline{x})}{\partial x} = -\frac{(1-\theta)(1-\gamma)}{\theta - (1-\theta)(1-\gamma)} < 0.$$

This means that when we take face value of debt as given, the cost of the derivative is decreasing in the level of collateralization of the derivative as long as $\overline{x} \leq C_1^L$. When $\overline{x} > C_1^L$, a further increase in collateralization does not change the payoff of the derivative counterparty, such that in this region the cost of the derivative is unchanged.

Senior Derivatives when $x > C_1^L$: In this section we describe the equilibrium under senior derivatives when $x > C_1^L$, which we left out in the main body of the text for space considerations. The main difference to the case discussed in the text is that the equations that the breakeven conditions for the derivative counterparty and the creditor change. When $x > C_1^L$, when the firm defaults, the derivative counterparty receives the entire cash flow, while the creditor receives nothing. Hence, the equilibrium is characterized by

$$X = R - C_1^L \tag{56}$$

$$R = \frac{F}{\theta + \gamma(1-\theta)} \tag{57}$$

$$x = \frac{(1-\theta)X + \rho(X) - (1-\gamma)(1-\theta)C_1^L}{\theta - (1-\theta)(1-\gamma)}. \tag{58}$$

Under linear hedging costs, we can solve for x in terms of the underlying parameters:

$$x = \frac{F(1-\theta+\delta)}{[\theta-(1-\gamma)(1-\theta)][\theta+\gamma(1-\theta)]} - \frac{C_1^L[1-\theta+\delta+(1-\gamma)(1-\theta)]}{[\theta-(1-\gamma)(1-\theta)]} \tag{59}$$

$$= \frac{(1-\theta+\delta)\left[F-C_1^L\right]-(1-\gamma)(1-\theta)[\theta+\gamma(1-\theta)-(1-\theta+\delta)]C_1^L}{[\theta-(1-\gamma)(1-\theta)][\theta+\gamma(1-\theta)]} \tag{60}$$

Characterization of Equilibrium under Partial Collateralization: This section contains the breakeven conditions used to derive the equilibrium under partial collateralization (Proposition 7). Under partial collateralization, the required derivative position is given by

$$X(\overline{x}) = R(\overline{x}) - C_1^L. \tag{61}$$

The creditor's and derivative counterparty's breakeven conditions are given by

$$[\theta+\gamma(1-\theta)]R + (1-\theta)(1-\gamma)\left(C_1^L-\overline{x}\right) = F \tag{62}$$

$$[\theta-(1-\theta)(1-\gamma)]x(\overline{x}) + (1-\theta)(1-\gamma)\overline{x} = (1-\theta)\left[R(\overline{x})-C_1^L\right] + \rho\left(R(\overline{x})-C_1^L\right),$$

which implies that, under linear hedging costs,

$$R(\overline{x}) = \frac{F-(1-\theta)(1-\gamma)\left(C_1^L-\overline{x}\right)}{\theta+\gamma(1-\theta)} \tag{63}$$

$$x(\overline{x}) = \frac{(1-\theta)\left[R(\overline{x})-C_1^L\right]+\delta\left[R(\overline{x})-C_1^L\right]-(1-\theta)(1-\gamma)\overline{x}}{\theta-(1-\theta)(1-\gamma)} \tag{64}$$

Substituting (63) into (64) yields the expression for $x(\overline{x})$ given in the Proposition.

Proof of Proposition 8: Assume that the firm receives the high cash flow C_1^H but has to make a payment $x(\overline{x})$ on its derivative position. The firm will meet its total payment obligation $R(\overline{x})+x(\overline{x})$ under two conditions. First, the cash available to the firm must be sufficient, which is the case whenever

$$C_1^H - [R(\overline{x}) + x(\overline{x})] \geq 0. \tag{65}$$

Second, the firm must have no incentive to default strategically. This is the case whenever

$$C_1^H - \left[R\left(\overline{x}\right) + x\left(\overline{x}\right)\right] + C_2 \geq C_1^H - C_1^L. \tag{66}$$

The left hand side is the payoff from making the contractual payment and continuing, whereas the right hand side is the payoff from declaring default, pocketing $C_1^H - C_1^L$ and letting the creditor and the derivative counterparty split C_1^H. Overall, the firm will thus meet its contractual obligations if

$$R\left(\overline{x}\right) + x\left(\overline{x}\right) \leq \min\left[C_1^H, C_1^L + C_2\right]. \tag{67}$$

Equation (38) follows from taking the derivatives of equations (35) and (36) and simplifying.

Proof of Corollary 4: The result follows from substituting (35) and (36) into (37) and simplifying. The constants not given in the main text are

$$\Gamma_0 = \frac{(1-\theta)(1-\gamma)\left[\theta - (1-\gamma)(1-\theta)\right] + 1 - \theta + \delta}{\theta + \gamma(1-\theta) + \delta}, \tag{68}$$

$$\Gamma_1 = \frac{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}{\theta + \gamma(1-\theta) + \delta}. \tag{69}$$

Proof of Proposition 11: Let us first consider junior derivatives. We know that when the firm has an incentive to hedge, it will choose $X^B = X^S = R^S - C_1^L$. We also know that in this case $x^s = \frac{1-\theta+\delta}{\theta-(1-\theta)(1-\gamma)} X^B$. Inserting this into (45), and taking derivatives with respect to γ (taking the face value of debt as given) we find that the firm has an incentive to choose minimum basis risk as long as

$$C_2 - \left[R^S - C_1^L\right] \geq 0. \tag{70}$$

Minimum basis risk can thus be sustained when, under the expectation that the firm will choose minimum basis risk, the face value of debt $R^S\left(\gamma = \gamma_{\max}\right)$ is such that (70) is satisfied. When (70) cannot be satisfied, the firm chooses maximum basis risk and

$X^B = R^S(\gamma = \gamma_{\min}) - C_1^L$.

Let us now consider senior derivatives. Differentiating (45) with respect to γ, we find that the firm has an incentive to choose minimum basis risk as long as

$$C_2 - \left(R - C_1^L\right) - x\left(X^B\right). \tag{71}$$

Minimum basis risk can thus be sustained when, under the expectation that the firm will choose minimum basis risk, the face value of debt $R^S(\gamma = \gamma_{\max})$ is such that (71) is satisfied, given the firm's optimal derivative position which for $\gamma_{\max} > \overline{\gamma}$ is given by $X^B = R - C_1^L$.. Inserting this into (71) and using $x\left(X^B\right) = \frac{1-\theta+\delta}{\delta} X^B$, we find that minimum basis risk and the optimal derivative position can be sustained as long as

$$C_2 - \frac{1+\delta}{\theta}\left[R(\gamma = \gamma_{\max}) - C_1^L\right] \geq 0 \tag{72}$$

This condition is harder to satisfy than (70) since $\frac{1+\delta}{\theta} > 1$ and $R(\gamma = \gamma_{\max}) > R^S(\gamma = \gamma_{\max})$. When (71) is not satisfied, only $\gamma = \gamma_{\min}$ can be sustained. If $\gamma_{\min} < \overline{\gamma}$, this means that the firm will then also choose a derivative position that fully dilutes creditors in the default state.

7.2 Appendix B: A Model with Renegotiation

This note develops an alternative model that to analyze the bankruptcy status of derivatives in the presence of renegotiation. While in the main text, the firm commits not to renegotiate in the case of default, here we allow for renegotiation following default. For simplicity, we assume that the firm has full bargaining power in renegotiation.

No derivatives: Consider the same setup as in the main text, but assume that renegotiation is possible. As before, in the absence of derivatives, the firm always defaults if the low cash flow C_1^L realizes at date 1. We will refer to this outcome as a *liquidity default.* As $C_1^L < F$, the low cash flow is not sufficient to repay the face value of debt. Moreover, the date 2 cash flow C_2 is not pledgeable, and since the firm has no other cash it can offer to renegotiate with the creditor, the firm has no other option than to default when C_1^L is realized at date 1. The lender then seizes the cash flow C_1^L and shuts down the firm, collecting the liquidation value of the asset L. Early termination of the project leads to a social loss of $C_2 - L$, the additional cash flow that would have been generated had the firm been allowed to continue its operations.

If the high cash flow C_1^H realizes at date 1, the firm has enough cash to service its debt. However, the firm may still choose not to repay its debt. We refer to this choice as a *strategic default*. A strategic default occurs when the firm is better off defaulting on its debt at date 1 than repaying the debt and continuing operations until date 2. In particular, the firm will make the contractual repayment R only if the following incentive constraint is satisfied:

$$C_1^H - R + C_2 \geq C_1^H - C_1^L + S, \tag{73}$$

where S denotes the surplus that the firm can extract in renegotiation after defaulting strategically at date 1. The constraint (73) says that, when deciding whether to repay R, the firm compares the payoff from making the contractual payment and collecting the entire date 2 cash flow C_2 to the payoff from defaulting strategically, pocketing $C_1^H - C_1^L$ and any potential surplus S from renegotiating with the creditor. Repayment of the face value R in the high cash flow state is thus incentive compatible only as long as the face value is not too high:

$$R \leq C_1^L + C_2 - S. \tag{74}$$

In contrast to the analysis in the paper, we now assume that in renegotiation the firm can make a take-it-or-leave-it-offer to the creditor. This means that after strategic default, the firm can always offer L to the creditor (i.e., the creditor receives a total payment of $C_1^L + L$), making him just indifferent between liquidating the firm and letting the firm continue. The surplus from renegotiation to the firm would then be given by $S = C_2 - L$. Hence, the maximum face value that is compatible with repayment is given by $R = C_1^L + L$. This immediately implies that the project can be financed as long as

$$F \leq \overline{F} \equiv C_1^L + L. \tag{75}$$

This is intuitive. Since the firm can always pretend to have received the low cash flow and make a take-it-or-leave-it offer to the creditor, there is no way the creditor can ever extract more than $C_1^L + L$. When the low cash flow realizes, the firm cannot renegotiate and is liquidated.

As in the paper, the *social surplus* generated in the absence of derivatives is equal to the firm's expected cash flows, minus the setup cost F:

$$\theta \left(C_1^H + C_2\right) + (1-\theta)\, C_1^L - F. \tag{76}$$

We summarize the credit market outcome in the absence of derivatives in the following Proposition.

Proposition 12 *In the absence of derivative markets, the firm can finance the project as long as* $F \leq \overline{F} \equiv C_1^L + L$*. When the project can attract financing, the face value of debt is given by* $R = \left[F - (1-\theta)\left(C_1^L + L\right)\right]/\theta$*, and social surplus is equal to* $\theta\left(C_1^H + C_2\right) + (1-\theta)\, C_1^L - F$*.*

Senior derivatives: Now consider senior derivatives. The optimal derivative position is such that it eliminates default in the low state when derivative pays out. This requires setting

$$X = R - C_1^L. \tag{77}$$

The cost x of the derivative position X is determined by the counterparty's breakeven constraint

$$\theta x = (1-\theta)\, X + \delta X. \tag{78}$$

The firm still defaults with probability $(1-\theta)(1-\gamma)$. In that case, the firm is liquidated and the derivative party receives x before the creditor is paid off. Hence, there is $C_1^L + L - x$ left over to pay off the creditor. Hence, as long as $R \leq C_1^L + L - x$ the creditor can be fully paid off even when derivatives are senior, such that $R = F$. Financing with risk-free debt is possible as long as $F \leq C_1^L + L - x$. When $C_1^L + L - x < F \leq C_1^L + L$, the creditor cannot be fully paid off in default. Debt is then risky and the face value determined by the breakeven condition

$$F = [\theta + (1-\theta)\gamma]\, R + (1-\theta)(1-\gamma)\left(C_1^L + L - x\right), \tag{79}$$

which yields

$$R = \frac{F - (1-\theta)(1-\gamma)\left(C_1^L + L - x\right)}{[\theta + (1-\theta)\gamma]}. \tag{80}$$

Junior derivatives: Again, the optimal derivative position eliminates default in the in low state when derivative pays off. This requires setting

$$X^S = R^S - C_1^L. \tag{81}$$

When debt is senior, debt is fully paid off as long as $F \leq C_1^L + L$ (debt is effectively risk-free, but sometimes has to be paid out of liquidation proceeds). Beyond $C_1^L + L$ no financing is possible. This means that whenever financing is possible, debt is safe:

$$R^S = F. \tag{82}$$

The breakeven condition for the junior derivative counterparty is given by

$$[\theta - (1-\theta)(1-\gamma)]\, x + (1-\theta)(1-\gamma)\left(C_1^L + L - F\right) = (1-\theta)\, X + \delta X. \tag{83}$$

Proposition 13 ***Comparing junior and senior derivatives:*** *We are now in a position* *to compare junior and senior derivatives when renegotiation is possible and the firm has all bargaining power in renegotiation. The above analysis implies that there are two cases:*

1. *As long as $F \leq C_1^L + L - x$, debt is safe whether derivatives are junior or senior: $R^S = R = F$. Hence, the seniority of the derivative position does not matter.*

2. *When $C_1^L + L - x < F \leq C_1^L + L$, debt is risk-free when derivatives are junior, but risky when derivatives are senior. Hence, on this interval $R^S < R$ and senior derivatives are more efficient.*

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