The Consumption Risk of Bonds and Stocks

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November 18, 2015

Abstract

Aggregate consumption growth reacts slowly, but significantly, to bond and stock return innovations. As a consequence, slow consumption adjustment (SCA) risk, measured by the reaction of consumption growth cumulated over many quarters following a return, can explain most of the cross-sectional variation of expected bond and stock returns. Moreover, SCA shocks explain about a quarter of the time series variation of consumption growth, a large part of the time series variation of stock returns, and a significant (but small) fraction of the time series variation of bond returns, and have substantial predictive power for future consumption growth.

Keywords: Pricing Kernel, Stochastic Discount Factor, Consumption Based Asset Pricing, Bond Returns, Stock Returns, Slow Consumption Adjustment.
JEL Classification Codes: G11, G12, G13, C52.

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I Introduction

The central insight of consumption based macro-finance models is that equilibrium prices of financial assets should be determined by their equilibrium risk to households' marginal utilities and, in particular, current and future marginal utilities of consumption: agents are expected to demand a premium for holding assets that are more likely to yield low returns when the marginal utility of consumption is high i.e. when consumption (current and expected) is low. Nevertheless, in the data the contemporaneous covariance of asset returns and consumption growth is small and not disperse cross-sectionally, making it challenging to rationalised both average risk premia (e.g., Mehra and Prescott (1985), Weil (1989)) and their wide cross-sectional dispersion (e.g., Hansen and Singleton (1983), Mankiw and Shapiro (1986), Breeden, Gibbons, and Litzenberger (1989), Campbell (1996)).[1]

In this paper, we document that consumption growth reacts slowly, but significantly, to bond and stock returns common innovations. These slow consumption adjustment shocks account for about a quarter of the time series variation of aggregate consumption growth, and its innovations explain most of the time series variation of stock returns (on average about 79%), and a significant, but small, share of the time series variation of bond returns, and generate substantial predictability for future consumption growth.

Since consumption responds with a lag to changes in wealth, the contemporaneous covariance of consumption and wealth understates and mismeasures the true risk of an asset, rendering empirically measured risk premia hard to rationalise. On the contrary, slow consumption adjustment (SCA) risk, measured by the cumulated response of consumption growth to asset return innovations, can *jointly* explain the average term structure of interest rates and the cross-section of a broad set of stock returns (including industry portfolios and Fama-French size and book to market portfolio).

To assess the role of SCA risk in a robust manner, and using post-war data on a large cross section of both stock and US treasury returns, we perform our empirical analysis following two very different approaches and identification strategies.

First, we consider a flexible parametric setting in which consumption growth is modelled as being the sum of two independent processes: a (potentially, since parameters are estimated) long memory moving average component that (potentially) co-moves with asset returns and a transitory component orthogonal to financial assets. Innovations to asset return are in turn modelled as depending (potentially) on the long memory component of consumption plus an orthogonal component.

[1] Recently, Julliard and Ghosh (2012) show that pricing kernels based on consumption growth alone cannot explain either the equity premium puzzle, or the cross-section of asset returns, even after taking into account the possibility of rare disasters.

Empirically, we find that: *a*) consumption reacts very slowly (i.e. over a period of two to four years), but significantly, to asset returns innovations, and these innovations account for about 27% of the time series variation of consumption growth; *b*) returns on portfolios of stocks load significantly on the SCA component, with a pattern that closely mimics the value and size pricing anomalies, and this component tends to explain between 36% and 95% of their time series variation; *c*) returns on US treasury bonds load significantly on the SCA component, with loadings increasing with the time to maturity, but this component explains no more than 3.5% of their time series variations (an additional latent variable, independent from both consumption and stock returns, seems to drive most of the time series variation of bonds); *e*) SCA risk, measured as the loading of asset returns on the SCA component, can explain between 57% and 90% of the joint cross-section of stocks and bond returns.[2]

Second, not to take an ex-ante stand on a parametric model of consumption dynamics, we consider a broad class of consumption-based equilibrium models (see, e.g., Ghosh, Julliard, and Taylor (2013)) in which the stochastic discount factor can be factorized into a component that depends on consumption growth and an additional, model specific, component. In this setting, following Parker and Julliard (2005), we show that a pricing kernel can be constructed by measuring asset risk via the covariance between an asset return and the change in marginal utility over several quarters following the return. Using this measure, we demonstrate that the SCA risk is priced in the cross-section of bond holding returns, as well as the joint cross-section of stocks and bonds. Moreover, we show that the slow consumption adjustment risk creates a 'fanning out' pattern in consumption betas, leading to both more *pronounced* and *dispersed* covariance with the stochastic discount factor. As a result, the model captures 85% of the cross-sectional variation in bonds returns, and 37-94% of the joint cross-sectional variation in stocks and bonds.

Interestingly, our findings are consistent (both qualitatively and quantitatively) with the consumption dynamics postulated by the Long Run Risk (LRR) literature (see e.g. Bansal and Yaron (2004), Hansen, Heaton, Lee, and Roussanov (2007), Bansal, Kiku, and Yaron (2012)), but are also supportive of a broader class of consumption based asset pricing models.

Our analysis builds upon the finding of Parker and Julliard (2005) that consumption risk measured by the covariance of an assets return and consumption growth cumulated over many quarters following the return – that is, measured as slow consumption adjustment risk – can explain a large fraction of the variation in average returns across the 25 Fama-French portfolios and, more broadly, on the empirical evidence linking slow movements in consumption and asset returns (see, e.g., Daniel and Marshall (1997), Bansal, Dittmar, and

[2] In our baseline specification we consider a cross section of 46 asset given by 12 industry portfolios, 25 size and book-to-market portfolios, and 9 bond portfolio, but the results appear robust to alternative specifications.

Lundblad (2005), Jagannathan and Wang (2007), Hansen, Heaton, and Li (2008), Malloy, Moskowitz, and Vissing-Jorgensen (2009)). We expand upon this framework by both *i*) identifying the SCA risk component from, and quantifying its relevance for, the time series properties of consumption and asset returns, and *ii*) by showing that this component can price jointly different classes of assets and tends to act as a driving factor of the term structure of interest rates. We also show that an additional, non-spanned (i.e. that does not seem to require a risk premium), factor is also required to rationalise the time series behaviour of bonds, and that this factor tends to behave as a slope type component.[3]

More broadly, our work is connected to the large literature on the co-pricing of stocks and bonds.[4] In particular, our focus on the role of macroeconomic risk is related to a series of works that combine the affine asset pricing framework with a parsimonious mix of macro variables and bond factors for the joint pricing of bonds and stocks. In particular: Bekaert and Grenadier (1999) and Bekaert, Engstrom, and Grenadier (2010), that presents a linear model for the simultaneous pricing of stock and bond returns that jointly accommodate the mean and volatility of equity and long term bond risk premia; Brennan, Wang, and Xia (2004), that assumes that the investment opportunity set is completely described by two state variables given by the real interest rate and the maximum Sharpe ratio, and the state variables (estimated using US Treasury bond yields and inflation data) are shown to be related to the equity premium, the dividend yield, and the Fama-French size and book-to-market portfolios; Lettau and Wachter (2011), that focus on matching an upward sloping bond yield term structure and a downward sloping equity yield curve via an affine model that incorporates persistent shocks to the aggregate dividend, inflation, risk-free rate, and price of risk processes; Koijen, Lustig, and Nieuwerburgh (2010), that develops an affine model in which three factors –the level of interest rates, the Cochrane and Piazzesi (2005) factor,[5] and the dividend-price ratio– have explanatory power for the cross-section of bonds and stock returns, while the latter two factors have explanatory power for the time series of these assets; Ang and Ulrich (2012), that considers an affine model in which returns to bonds (real and nominal) and stocks, are decomposed into five components meant to capture the real short rate dynamics as well as term premium, inflation related components (a nominal

[3]This last finding is consistent with Chernov and Mueller (2012) that identify an unspanned latent factor driving in bond yields.

[4]E.g.: Fama and French (1993) expands the original set of Fama and French (1992) stock market factors (meant to capture the overall market return, as well as the value and the size premia), with two bond factors (the excess return on a long bond and a default spread), meant to capture term and default premia; Mamaysky (2002) built upon the affine term structure framework canonically used in term structure modelling (see, e.g., Duffie and Kan (1996)) by adding affine dividend yields to help pricing jointly bonds and stocks.

[5]Cochrane and Piazzesi (2005) find that a single factor (a single tent-shaped linear combination of forward rates), predicts excess returns on one- to five-year maturity bonds. This factor tends to be high in recessions, but forecasts future expansion, i.e. this factor seems to incorporate good news about future consumption.

premium, an expected inflation as well as an inflation risk component) as well as a real cash flow risk element.

The reminder of the paper is organized as follows. Section II formally defines the concept of slow consumption adjustment risk in a broad class of consumption based asset pricing models. Sections III presents the econometric methodology, while a description of the data is reported in Section IV. Our empirical findings are reported in Section V while Section VI concludes. Additional methodological details, as well as robustness checks and additional empirical evidence, are reported in the Appendix.

II The Slow Consumption Adjustment Risk of Asset Returns

Representative agent based consumption asset pricing models with either CRRA, Epstein and Zin (1989), or habit based preferences, as well as several models of complementary in the utility function, and models with either departures from rational expectations, or robust control, or ambiguity aversion, and even some models with solvency constraints,[6] all imply a consumption Euler equation of the form

$$C_t^{-\phi} = \mathbb{E}_t \left[C_{t+1}^{-\phi} \tilde{\psi}_{t+1} R_{j,t+1} \right] \tag{1}$$

for any gross asset return j including the risk free rate R_{t+1}^f, and where $\mathbb{E}_t$ is the rational expectation operator conditional on information up to time t, C_t denotes flow consumption, $\tilde{\psi}_{t+1}$ depends on the particular form of preferences (and expectation formation mechanism) considered, and the ϕ parameter is a function of the underlying preference parameters.[7] Rearranging terms, moving to unconditional expectations, and using the definition of covariance, we can rewrite the above equation as a model of expected returns

$$\mathbb{E}\left[\mathbf{R}_{t+1}^e\right] = -\frac{Cov\left(M_{t+1}; \mathbf{R}_{t+1}^e\right)}{\mathbb{E}\left[M_{t+1}\right]}. \tag{2}$$

where $M_{t+1} := \left(C_{t+1}/C_t\right)^{-\phi} \tilde{\psi}_{t+1}$ represents the stochastic discount factor between time t and $t+1$ and $\mathbf{R}^e \in \mathbb{R}^N$ denotes a vector of excess returns. Log-linearizing the above relationship,

[6] See, e.g.: Bansal and Yaron (2004); Abel (1990), Campbell and Cochrane (1999), Constantinides (1990), Menzly, Santos, and Veronesi (2004); Piazzesi, Schneider, and Tuzel (2007), Yogo (2006); Basak and Yan (2010), Hansen and Sargent (2010); Chetty and Szeidl (2015); Ulrich (2010); Lustig and Nieuwerburgh (2005).

[7] E.g., ϕ would denote relative risk aversion in the CRRA framework, while it would be a function of both risk aversion and elasticity of intertemporal substitution with Epstein and Zin (1989) recursive utility.

expected returns can be expressed as

$$\mathbb{E}\left[\mathbf{R}^e_{t+1}\right] = \left[\phi Cov\left(\Delta c_{t,t+1}; \mathbf{r}^e_{t+1}\right) - Cov\left(\log\tilde{\psi}_{t+1}; \mathbf{r}^e_{t+1}\right)\right]\lambda \tag{3}$$

where $\Delta c_{t,t+1} := \ln\left(C_{t+1}/C_t\right)$, $\mathbf{r}^e \in \mathbb{R}^N$ denotes log excess returns, and λ is a positive scalar. Since, in the data, the covariance between one period consumption growth and asset returns is small and has a much smaller cross-sectional dispersion than average excess returns, the first term of the above equation is not sufficient for pricing a cross-section of asset returns, and most of the modelling effort in the literature has been devoted to identifying a $\tilde{\psi}$ component that can help rationalise observed returns.

Note that Equation (1) above implies that

$$C_t^{-\phi} = \mathbb{E}_t\left[C^{-\phi}_{t+1+S}\psi_{t+1+S}\right]$$

where $\psi_{t+1+S} := R^f_{t,t+1+S}\prod_{j=0}^{S}\tilde{\psi}_{t+1+j}$. Hence, the Euler equation

$$\mathbf{0}_N = \mathbb{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\phi}\tilde{\psi}_{t+1}\mathbf{R}^e_{t+1}\right] \tag{4}$$

where $\mathbf{0}_N$ denotes and N-dimensional vector of zeros, can be equivalently rewritten as

$$\mathbf{0}_N = \mathbb{E}\left[\left(\frac{C_{t+1+S}}{C_t}\right)^{-\phi}\psi_{t+1+S}\mathbf{R}^e_{t+1}\right]. \tag{5}$$

Once again, using the definition of covariance, we can rewrite the above equation as a model of expected returns

$$\mathbb{E}\left[\mathbf{R}^e_{t+1}\right] = -\frac{Cov\left(M^S_{t+1}; \mathbf{R}^e_{t+1}\right)}{\mathbb{E}\left[M^S_{t+1}\right]}. \tag{6}$$

where $M^S_{t+1} := \left(C_{t+1+S}/C_t\right)^{-\phi}\psi_{t+1+S}$. That is, under the null of the model being correctly specified, there is an entire family of SDFs that can be equivalently used for asset pricing: M^S_{t+1} for every $S \geq 0$. Log-linearizing the above expression, we have the linear factor model

$$\mathbb{E}\left[\mathbf{R}^e_{t+1}\right] = \left[\phi Cov\left(\Delta c_{t,t+1+S}; \mathbf{r}^e_{t+1}\right) - Cov\left(\log\psi_{t+1+S}; \mathbf{r}^e_{t+1}\right)\right]\lambda_S \tag{7}$$

where $\Delta c_{t,t+1+S} := \ln\left(C_{t+1+S}/C_t\right)$ and λ_S is a positive scalar.

But why measure risk, and price returns, using the slow consumption adjustment framework as in equations (5)-(7) instead of the contemporaneous risk as in equations (2)-(4)? First, it is a well-known fact that consumption displays excessive smoothness in response to the wealth shocks (Flavin (1981), Hall and Mishkin (1982)), which can be caused by various adjustment costs (Gabaix and Laibson (2001)) and asynchronous consumption/investment decisions (Lynch (1996)). Moreover, the problem could be further exacerbated if the agent has a nonseparable utility function, potentially including labour or other state variables that are also costly to adjust, and hence leading to further staggering in the consumption adjustment in response to wealth innovations. Second, if there is measurement error in consumption, then using a one-period growth rate does not reflect the true pricing impact of the SDF. Indeed, in a recent paper Kroencke (2013) demonstrates that one of the reasons for the failure of the standard consumption-based model to solve equity premium and risk-free rate puzzles, is that NIPA consumption data is filtered to eliminate the impact of the measurement error. The unfiltered data, in turn, produces substantially better results. The fourth quarter to fourth quarter consumption growth of Jagannathan and Wang (2007), as well as the ultimate consumption risk of Parker and Julliard (2005), are related to the reconstructed unfiltered time series of consumption growth, and therefore provide a better measure for the overall consumption risk.

To model parametrically the–potential–slow reaction of consumption to financial market shocks, we postulate that the consumption growth process can be decomposed in two terms: a white noise disturbance, w_c with variance σ_c^2, that is independent from financial market shocks, plus a (covariance stationary) autocorrelated process–the slow consumption adjustment component–that depends on current and past stocks to asset returns. In order not to have to take an ex ante stand on the particular time series structure of the slow adjustment component, we work with its (potentially infinite) moving average representation. That is we model the consumption growth process as:

$$\Delta c_{t-1,t} = \mu_c + \sum_{j=0}^{\bar{S}} \rho_j f_{t-j} + w_t^c; \tag{8}$$

where $\bar{S}$ is a positive integer (potentially equal to $+\infty$), the ρ_j coefficients are square summable, and most importantly f_t, a white noise process normalised to have unit variance, is the fundamental innovation upon which all asset returns loads contemporaneously i.e. given a vector of log excess returns, $\mathbf{r}^e$, we have

$$\underset{N\times 1}{\mathbf{r}_t^e} = \underset{N\times 1}{\boldsymbol{\mu}_r} + \underset{N\times 1}{\boldsymbol{\rho}^r} f_t + \underset{N\times 1}{\mathbf{w}_t^r} \tag{9}$$

where $\boldsymbol{\mu}_r$ is a vector of expected values, $\boldsymbol{\rho}^r$ contains the asset specific loadings on the common risk factor, $\mathbf{w}_t^r$ is a vector of white noise shocks with diagonal covariance matrix Σ_r (the diagonality assumption can be relaxed as explained below and in Appendix A.1), that are meant to capture asset specific idiosyncratic shocks.

The dynamic system in equations (8)-(9) can be reformulated as a state-space model, and Bayesian posterior inference can be conducted to estimate both the unknown parameters (μ_c, $\boldsymbol{\mu}_r$, $\{\rho_j\}_{j=0}^{\bar{S}}$, $\boldsymbol{\rho}^r$, σ_c^2, Σ_r) and the time series of the unobservable common factor of consumption and asset returns ($\{f_t\}_{t=1}^{T}$). This estimation procedure is described in detail in the next section and Appendix A.1.

Note that, since $\Delta c_{t-1,t+S} \equiv \sum_{j=0}^{S} \Delta c_{t-1+j,t+j} \equiv \ln\left(C_{t+S}/C_{t-1}\right)$, from the one period consumption growth process in equation (8) we can recover the dynamic of cumulated consumption growth with a simple rotation since

$$[\Delta c_{t-1,t}, \Delta c_{t-1,t+1}, ..., \Delta c_{t-1,t+S}]' \equiv \Gamma\, [\Delta c_{t-1,t}, \Delta c_{t,t+1}, ..., \Delta c_{t-1+S,t+S}]'$$

where Γ is a lower triangular square matrix of ones (of dimension S). From this last expression it is easy to see that the ρ_j coefficients identify the impulse response function of slow consumption adjustment to the fundamental asset market shock f_t as

$$\frac{\partial \mathbb{E}\left[\Delta c_{t-1,t+S}\right]}{\partial f_t} = \sum_{j=0}^{S} \rho_j \tag{10}$$

where $\rho_{j>\bar{S}} := 0$. Moreover, the consumption betas of the factor model of asset returns in equation (7) are fully characterised by the loadings of the dynamic system on the factor f_t since

$$Cov\left(\Delta c_{t-1,t+S}; \mathbf{r}_t^e\right) \equiv \sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r. \tag{11}$$

That is, the time series estimates of the latent factor loadings ($\hat{\rho}_j$ and $\hat{\boldsymbol{\rho}}^r$) can be used to assess whether the slow consumption adjustment component has explanatory power for the cross-section of risk premia (via, for instance, simple cross-sectional regressions of returns on these estimated covariances).

The formulation in Equations (8)-(9) can be generalize to allow for a bonds specific latent factor (g_t) to which consumption, potentially, reacts slowly over time. This is an appealing extension since the factor f_t, as shown in the empirical section, explains most of the time series variability of stocks, a quarter of the one of consumption growth, but a small share of the time series variation of bonds. The dynamic system in this case becomes:

$$\Delta c_{t-1,t} = \mu_c + \sum_{j=0}^{\bar{S}} \rho_j f_{t-j} + \sum_{j=0}^{\bar{S}} \theta_j g_{t-j} + w_t^c; \tag{12}$$

$$\underset{N\times 1}{\mathbf{r}_t^e} = \underset{N\times 1}{\boldsymbol{\mu}_r} + \underset{N\times 1}{\boldsymbol{\rho}^r} f_t + \left[\underset{N_b\times 1}{\boldsymbol{\theta}'^b}, \quad \mathbf{0}'_{N-N_b} \right]' g_t + \underset{N\times 1}{\mathbf{w}_t^r}; \tag{13}$$

where N_b is the number of bonds and they are ordered first in the vector $\mathbf{r}_t^e$, $\boldsymbol{\theta}^b \in \mathbb{R}^{N_b}$ contains the bond loadings on the factor g_t–a white noise process with variance normalized to one. Note that in this case the implied covariance of consumption and returns becomes:

$$Cov\left(\Delta c_{t-1,t+S}; \mathbf{r}_t^e\right) \equiv \sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r + \left[\boldsymbol{\theta}'^b, \quad \mathbf{0}'_{N-N_b} \right]' \sum_{j=0}^{S} \theta_j. \tag{14}$$

III Econometric Methodology

Our empirical analysis is based on both parametric and nonparametric inference, ensuring the results are robust to the methodology employed. The main approach (Section III.1) consists in rewriting the model in Equations (8)-(9) in state-space form and employ standard Bayesian filtering techniques to recover the unobservable latent consumption factor (f_t) and other model parameters. Since the model is tightly parametrised, with the factor loadings driving not only the time series, but also the cross-sectional relationships between asset returns, this in turn allows us to assess model performance in both time series and cross-sectional dimensions, using variance decomposition and Fama-MacBeth (1973) cross-sectional regressions.

In addition, we also use the standard semi-parametric techniques (e.g. GMM and Empirical Likelihood estimation) to assess whether ultimate consumption risk of Parker and Julliard (2005) can successfully capture the cross-section of stock and bond returns. Section III.2 provides further details on the moment construction, parameter estimation and tests used for inference.

III.1 Parametric Inference

We can rewrite the dynamic model in Equations (8)-(9) in state-space form, assuming Gaussian innovations, as

$$\mathbf{z}_t = \mathbf{F}\mathbf{z}_{t-1} + \mathbf{v}_t, \quad \mathbf{v}_t \sim \mathcal{N}\left(\mathbf{0}_{\bar{S}+1}; \Psi\right); \tag{15}$$

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{H}\mathbf{z}_t + \mathbf{w}_t, \quad \mathbf{w}_t \sim \mathcal{N}\left(\mathbf{0}_{N+1}; \Sigma\right). \tag{16}$$

where $\mathbf{y}_t := [\Delta c_t, \mathbf{r}_t^{e\prime}]$, $\mathbf{z}_t := [f_t, ..., f_{t-\bar{S}}]'$, $\boldsymbol{\mu} := [\mu_c, \boldsymbol{\mu}_r']'$, $\mathbf{v}_t := \left[f_t, \mathbf{0}_{\bar{S}}'\right]'$, $\mathbf{w}_t := [w_t^c, \mathbf{w}_t'^r]'$,

$$\Psi := \underbrace{\begin{bmatrix} 1 & \mathbf{0}_{\bar{S}}' \\ \mathbf{0}_{\bar{S}} & \mathbf{0}_{\bar{S}\times\bar{S}} \end{bmatrix}}_{(\bar{S}+1)\times(\bar{S}+1)}, \quad \mathbf{F} := \underbrace{\begin{bmatrix} \mathbf{0}_{\bar{S}}' & 0 \\ I_{\bar{S}} & \mathbf{0}_{\bar{S}} \end{bmatrix}}_{(\bar{S}+1)\times(\bar{S}+1)}, \tag{17}$$

$$\Sigma := \underbrace{\begin{bmatrix} \sigma_c^2 & \mathbf{0}_N' \\ \mathbf{0}_N & \Sigma_r \end{bmatrix}}_{(N+1)\times(N+1)}, \quad \mathbf{H} := \underbrace{\begin{bmatrix} \rho_0 & \rho_1 & ... & \rho_{\bar{S}} \\ \boldsymbol{\rho}^r & \mathbf{0}_N & ... & \mathbf{0}_N \end{bmatrix}}_{(N+1)\times(\bar{S}+1)}. \tag{18}$$

and $I_{\bar{S}}$ and $\mathbf{0}_{\bar{S}\times\bar{S}}$ denote, respectively, an identity matrix and a matrix of zeros of dimension $\bar{S}$.

Similarly, the dynamic system in Equations (12)-(13) can be represented in the state-space form (15)-(16) with: $\mathbf{z}_t := [f_t, ..., f_{t-\bar{S}}, g_t, ..., g_{t-\bar{S}}]'$; $\mathbf{v}_t := \left[f_t, \mathbf{0}_{\bar{S}}', g_t, \mathbf{0}_{\bar{S}}',\right]' \sim \mathcal{N}\left(\mathbf{0}_{\bar{S}+1}; \Psi\right)$; Ψ and F block diagonal with blocks repeated twice and given, respectively, by the two matrices in equation (17); and with space equation coefficients given by

$$\mathbf{H} := \underbrace{\begin{bmatrix} \rho_0 & ... & ... & \rho_{\bar{S}} & \theta_0 & ... & ... & \theta_{\bar{S}} \\ \rho_1^r & 0 & ... & 0 & \theta_1^b & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ \rho_{N_b}^r & 0 & ... & 0 & \theta_{N_b}^b & 0 & ... & 0 \\ ... & 0 & ... & 0 & 0 & 0 & ... & 0 \\ \rho_N^r & 0 & ... & 0 & ... & 0 & ... & 0 \end{bmatrix}}_{(N+1)\times 2(\bar{S}+1)}. \tag{19}$$

The above state-space system implies the following conditional likelihood for the data:

$$\mathbf{y}_t | \mathcal{I}_{t-1}, \boldsymbol{\mu}, \mathbf{H}, \Psi, \Sigma, \mathbf{z}_t \sim \mathcal{N}\left(\boldsymbol{\mu} + \mathbf{H}\mathbf{z}_t; \Sigma\right) \tag{20}$$

where $\mathcal{I}_{t-1}$ denotes the history of the state and space variables until time $t-1$. Hence, under a diffuse (Jeffreys') prior and conditional on the history of $\mathbf{z}_t$ and $\mathbf{y}_t$, and given the diagonal structure of Σ, we have the standard Normal-inverse-Gamma posterior distribution for the parameters of the model (see e.g. Bauwens, Lubrano, and Richard (1999)). Moreover, the posterior distribution of the unobservable factors $\mathbf{z}_t$ conditional on the data and the parameters, can be constructed using a standard Kalman filter and smoother approach (see, e.g., Primiceri (2005)).

Using equation (7), the above specification for the dynamics of consumption and asset returns implies, in the presence of only one latent factor (f_t) common to both assets and consumption

$$\mathbb{E}\left[\mathbf{R}_t^e\right] = \alpha + \left(\sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r\right) \lambda_f \tag{21}$$

where λ_f is a positive scalar variable that captures the price of risk associated with the slow consumption adjustment risk, and $\alpha \in \mathbb{R}^N$. If consumption fully captures the risk of asset returns, the above expression should hold with $\alpha = \mathbf{0}_N$, otherwise α should capture the covariance between the omitted risk factors and asset returns.

Similarly, if we also allow for a bond specific latent factor (g_t), the implied cross-sectional model of returns is

$$\mathbb{E}\left[\mathbf{R}_t^e\right] = \alpha + \left(\sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r\right) \lambda_f + \left[\begin{array}{cc} \boldsymbol{\theta}'^b, & \mathbf{0}'_{N-N_b} \end{array}\right]' \sum_{j=0}^{S} \theta_j \lambda_g \tag{22}$$

with the additional testable restriction $\lambda_f = \lambda_g$.

Equation (21) (and similarly Equation (21)), conditional on the data and the parameters of the state-space model, defines a standard cross-sectional regression, hence the parameters α, λ_f and λ_g can be estimated via standard Fama and MacBeth (1973) cross-sectional regressions. This implies that, not only we can compute posterior means and confidence bands for both the coefficients of the state space model and for the unobservable factor's time series, but we can also compute means and confidence bands for the Fama and MacBeth (1973) estimates of the cross sectional regressions defined in Equations (21) and (22). That is, we can jointly test the ability of the slow consumption adjustment risk of explaining both the time series and the cross-section of asset returns with a simple Gibbs sampling approach described in detail in Appendix A.1.

III.2 Semi-parametric Inference

We start with the pricing restriction in Euler Equation (5):

$$\mathbf{0} = \mathbb{E}\left[M_{t+1}^S \mathbf{R}_{t+1}^e\right]$$

where $M_{t+1}^S := \left(C_{t+1+S}/C_t\right)^{-\phi} \psi_{t+1+S}$ and $S \geq 0$.

The fact that the stochastic discount factor can be decomposed into the product of the consumption growth over several consecutive periods (C_{t+1+S}/C_t) and an additional, potentially unobservable, component, makes the above setting particularly appealing for the application of Empirical Likelihood -based techniques (for an excellent overview, see Kitamura (2006)) as discussed in Ghosh, Julliard, and Taylor (2013).

Consider the following transformation of the Euler equation:

$$
\begin{aligned}
\mathbf{0} = \mathbb{E}\left[M_t^S \mathbf{R}_t^e\right] &\equiv \int \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \psi_{t+S} \mathbf{R}_t^e dP = \int \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \frac{\psi_{t+S}}{\bar{\psi}} \mathbf{R}_t^e d\mathbf{P} \\
&= \int \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e d\Psi = \mathbb{E}^{\Psi}\left[\left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right] \qquad (23)
\end{aligned}
$$

where P is the unconditional physical probability measure, $\bar{\psi} = \mathbb{E}[\psi_{t+S}]$, Ψ is another probability measure, related to the physical one through the Radon-Nikodym derivative[8] $\frac{d\Psi}{dP} = \frac{\psi_{t+S}}{\bar{\psi}}$.

Empirical Likelihood provides a natural framework for recovering parameter estimates and probability measure Ψ defined by Equation (23), by minimising Kullback-Leibler Information Criterion (KLIC):

$$
(\hat{\Psi}, \hat{\phi}) = \arg\min_{\Psi,\phi} D(P||\Psi) \equiv \arg\min_{\Psi} \int \ln \frac{dP}{d\Psi} dP \quad \text{s.t.} \quad \mathbf{0} = \mathbb{E}^{\Psi}\left[\left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right] \qquad (24)
$$

Equation (24) provides a nonparametric maximum likelihood estimation of the probability measure, induced by the unobservable components of the SDF, and has been used in various applications, including the recovery of the risk-neutral probability density (Stutzer (1995)). For more information on the rationale behind this change of measure, see Ghosh, Julliard, and Taylor (2013).

Following Csiszar (1975) duality approach, one can easily show that:

$$
\hat{\Psi}_t = \frac{1}{T\left(1 + \hat{\boldsymbol{\lambda}}(\theta)' \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\hat{\phi}} \mathbf{R}_t^e\right)} \quad \forall t = 1..T, \qquad (25)
$$

where $\hat{\phi}$ and $\hat{\boldsymbol{\lambda}} \equiv \hat{\boldsymbol{\lambda}}(\hat{\phi}) \in \mathbb{R}^n$ are the solution to the dual optimisation problem:

$$
\hat{\phi} = \arg\min_{\phi \in \mathbb{R}} \; -\sum_{t=1}^{T} \ln\left(1 + \hat{\boldsymbol{\lambda}}(\phi)' \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right) \qquad (26)
$$

$$
\hat{\boldsymbol{\lambda}}(\phi) = \arg\min_{\boldsymbol{\lambda} \in \mathbb{R}^n} \; -\sum_{t=1}^{T} \ln\left(1 + \boldsymbol{\lambda}(\phi)' \left(\frac{C_{t+S}}{C_{t-1}}\right)^{-\phi} \mathbf{R}_t^e\right) \qquad (27)
$$

The dual problem is usually solved via the combination of internal and external loops (Kitamura (2001)): first, for each ϕ find the optimal values of the Langrange multipliers λ, as in Equation (27); then minimize the value of the dual objective function w.r.t. $\phi(\hat{\lambda})$, following

[8] We assume absolute continuity of both P and Ψ.

Equation (26).

Empirical likelihood estimator is known not only for its nonparametric likelihood interpretation, but also for its convenient asymptotic representation and properties. It belongs to the family of Generalised Empirical Likelihood estimators (Newey and Smith (2004)), with other notable members including the Exponentially Tilted Estimator (ET, Kitamura and Stutzer (1997)) and the Continuously Updated GMM (CU-GMM, Hansen, Heaton, and Yaron (1996)). While the whole family enjoys the same asymptotic distribution of the parameter estimates, achieves the semiparametric efficiency bound of Chamberlain (1987), and shares the standard battery of tests for moment equalities (e.g. J-test), the empirical likelihood estimator is also higher-order efficient (Newey and Smith (2004), Anatolyev (2005)).

We can also capture the average pricing error of the model implied by Equation (5) simply by introducing additional parameters in the following way:

$$\mathbf{0} = \mathbb{E}\left[M_{t+1}^{S}\left(\mathbf{R}_{t+1}^{e} - \alpha\right)\right], \tag{28}$$

where α stands for the average rate of return that is not cross-sectionally captured through the covariance between M_{t+1}^{S} and $\mathbf{R}_{t+1}^{e}$, since Equation (28) implies

$$\mathbb{E}\left[\mathbf{R}_{t+1}^{e}\right] = \alpha - \frac{Cov\left(M_{t+1}^{S}, \mathbf{R}_{t+1}^{e}\right)}{\mathbb{E}[M_{t+1}^{S}]}. \tag{29}$$

Parameter estimation proceeds in exactly the same way, following the procedure outlined in Equations (24)-(27). We consider several versions of Equation (28): $\alpha = 0$ (correct model specification); average pricing errors; error specific to a particular asset class ($\alpha_b \neq \alpha_s$); and a common level of mispricing for both stocks and bonds ($\alpha_b = \alpha_s$).

For each model we also report the cross-sectional adjusted R-squared

$$R_{adj}^{2} = 1 - \frac{n-2}{n-1}\widehat{Var}_c\left(\frac{1}{T}R_{i,t+1} - \hat{\alpha} - \frac{\widehat{Cov}\left(\left(C_{t+1+S}/C_t\right)^{-\hat{\phi}}, \mathbf{R}_{t+1}^{e}\right)}{\mathbb{E}[(C_{t+1+S}/C_t)^{-\hat{\phi}}]}\right) \Big/ \widehat{Var}_c\left(\frac{1}{T}R_{i,t+1}\right) \tag{30}$$

where $\widehat{Var}_c$ is the sample cross-sectional variance and $\widehat{Cov}$ is the sample time series covariance.

Finally, for the sake of completeness we also use two-stage Generalised Method of Moments (GMM, Hansen (1982)) to estimate consumption-based asset pricing models on the cross-section of stock and bond returns, and report its results alongside those for EL. While the estimator-implied probabilities no longer have the convenient nonparametric maximum likelihood interpretation (unlike those in Equation (25)), if one restricts the class of admissible SDF to the external habit models, asset pricing implications and inference based on the *ultimate consumption risk* only, remain valid. Under fairly general conditions, this result is a direct consequence of Proposition 1 in Parker and Julliard (2003), implying that GMM estimates of risk aversion retain consistency and asymptotical normality, and do not require an explicit knowledge of the habit function, if one relies on the ultimate consumption risk in the estimation.

IV Data Desciption

Bond holding returns are calculated on a quarterly basis using the zero coupon yield data constructed by Gurkaynak and Wright (2007)[9] from fitting the Nelson-Siegel-Svensson curves daily since June 1961, and excess returns are computed subtracting the return on a three-month Treasury bill. We consider the set of the following maturities: 6 months, 1, 2, 3, 4, 5, 6, 7, and 10 years, which gives us a set of 9 bond portfolios.

We consider several portfolios of stock returns. The baseline specification relies, in addition to the bond portfolios, on the 25 size and book-to-market Fama-French portolios (Fama and French (1992)), and 12 industry portfolios, available from Kenneth French data library. We consider monthly returns from July, 1961 to December, 2013, and accumulate them to form quarterly returns, matching the frequency of consumption data. Excess returns are then formed by subtracting the correponding return on the three-month Treasury bill.

Consumption flow is measured as real (chain-weighted) consumption expenditure on non-durable goods per capita available from the National Income and Product Accounts (NIPA). We use the end-of-period timing convention and assume that all of the expenditure occurs at the end of the period between t and $t+1$. We make this (common) choice because under this convention the entire period covered by time t consumption is part of the information set of the representative agent before time $t+1$ returns are realised. All the returns are made real using the corresponding consumption deflator.

Overall, this gives us consumption growth and matching real excess quarterly holding returns on a number of portfolios, from the forth quarter of 1961 to the end of 2013.

V Empirical Evidence

While our model allows for a potentially infinite number of lags for the consumption process, in order to proceed with the actual estimation one has to choose a particular value of $\bar{S}$. For

[9]The data is regularly updated and available at:
http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html

the rest of the section we use $S = 15$ for a number of reasons.

First, we rely on the previous results of Parker and Julliard (2003), who demonstrate that most of the pricing ability of the ultimate consumption risk is contained within the time span of 15 quarters. They define a proxy for the signal-to-noise ratio, taking into account both the time-series and cross-sectional variation of the data, and find that the maximum (as well as the best overall fit) is obtained around $S = 11$.

Second, Equation (8) implies a certain autocorrelation structure of the nondurable consumption growth through the combination of the common factor lags and its loadings. Hence, the value of S should be high enough to capture most of the time series autocorrelation in the consumption growth. Figure A1 in the Appendix presents the sample autocorrelation coefficients and the results of Ljung-Box (1978) and Box-Pierce (1970) tests. Since most of the dependence occurs within the first 15 lags, this value also becomes a natural choice for the lag truncation.

Further, intuitively most of the pricing impact from the consumption adjustment is probably taking place within the business cycle frequency, consistent with a number of recent empirical studies (e.g. Bandi and Tamoni (2015)). Therefore, $S = 15$ is a rather conservative choice, since it provides a 4 year window to capture most of the interaction between the ultimate consumption and returns.

Finally, our results remain robust to including additional lags.

V.1 Parametric approach

We start our analysis by examining the time-series properties of a one (common) factor model implied by Equations (8)–(9). We then turn to the 2-factor specification described by Equations (12)–(13). Finally, we present the cross-sectional properties of the model and demonstrate that the slow consumption adjustment risk is a priced factor, explaining a significant proportion of the cross-sectional variation in returns.

V.1.1 Time series properties of stocks and bonds

Our baseline cross-section consists of 9 bond portfolios, 25 Fama-French portfolios sorted by size and book-to-market, and 12 industry portfolios. We estimate the model in Equations (8)–(9) using the inference procedure outlined in Section III.1. Figures 1 and 2 present stock and bond loadings on the common factor, along with the 68% and 90% confidence bounds.

All the portfolios in Figure 1 display significant and positive exposures to the common factor. However, even more interesting is a widely recognisable pattern in the factor loadings: decreasing from the smallest to the largest decile on size, with a similar effect for book-to-market sorting. This is in line with the size and value anomalies and, in addition, provides

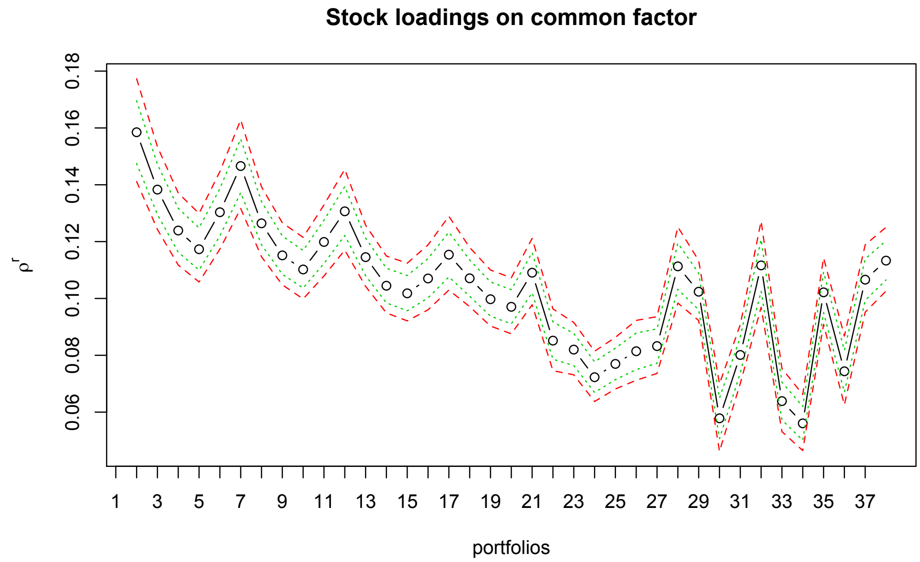


Figure 1: Common factor loadings ($\boldsymbol{\rho}^r$) of the stock portfolios in the one-factor model.

Note. The graph presents posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions. Ordering of portfolios: 25 Fama and French (1992) size and book-to-market sorted portfolios (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio), and 12 industry portfolios.

some preliminary evidence that the SCA risk plays an important role in explaining the cross-sectional dispersion of stocks returns. These findings also remain unchanged, when a second, bond-specific factor is added into the model (see Figure 3, lower panel).

In a single factor model, bond loadings, however, are not as prominent (Figure 2). While there is some evidence in favour of their increase with the bond maturity, the magnitude is still considerably smaller than that of the stocks.

Figures 2 (upper panel) and 4 highlight the importance of adding a bond-specific factor into the model. While the cross-section of bonds reveals a very pronounced maturity-driven pattern of loadings on the bond-specific factor, g_t, its presence also allows to highlight the effect of the consumption-related component. Compared to a one factor specification, these loadings are still not as high as those of the stocks, however, they are contained within very tight confidence bounds, are significantly different from zero (except for the 6 months return), and generally increase with maturity.

To summarise, not only (nearly) all the assets in the mixed cross-section of stocks and bonds have a significant positive exposure to the innovations in the ultimate consumption growth, the pattern of those loadings reflects well-known features of the data: size and value anomalies for stocks, and positive slope of the yield curve for bonds.

One of the possible concerns could be that we inadvertently capture a factor that heavily

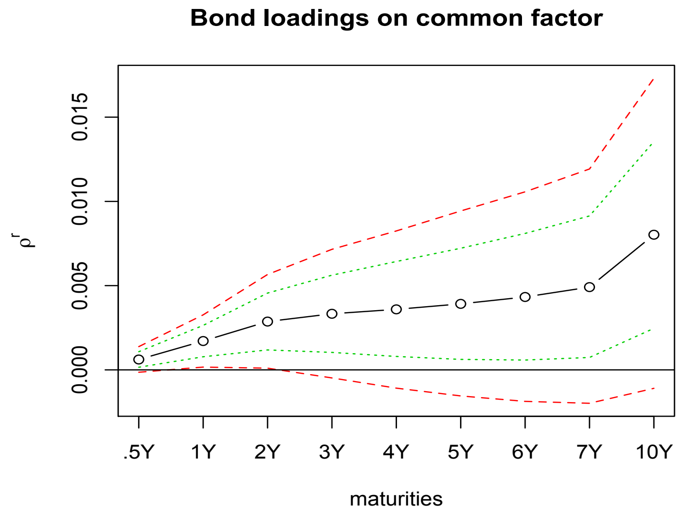


Figure 2: Bond loadings ($\boldsymbol{\rho}^r$) on the common factor (f_t).

Note. The graph presents posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions.

loads on one of the principal components of the cross-section of asset returns and thus mechanically has rather high factor loadings (Lewellen, Nagel, and Shanken (2010)). However, this is not the case. While there is indeed some correlation with the principal components of the cross-sections, composed of different assets (see Table 1), the common factor does not heavily correlate with any of them in particular, but rather displays a certain degree of spread in loadings. For example, it is related to the first, third and fourth principal components of the joint cross-section of stocks and bonds. Therefore, we conclude that our results are not driven by a particular implied factor structure of a given cross-section, but rather reflect a more general feature of the data.

Table 1: Correlation of Slow Consumption Adjustment with Principal Components

	Correlation of:									
	$\sum_{j=0}^{S}\hat{\rho}_j\hat{f}_{t-j}$					$\sum_{j=0}^{S}\hat{\theta}_j\hat{g}_{t-j}$				
PCA of:	I	II	III	IV	V	I	II	III	IV	V
$\mathbf{r}^e$	-.37	.01	-.13	-.17	.03	-.03	-.32	-.01	-.54	.04
$\mathbf{r}^e_{\text{bonds}}$	.11	-.12	.10	.15	-.03	.64	-.10	.01	.06	-.08
$\mathbf{r}^e_{\text{stocks}}$	.38	.08	-.11	.01	-.01					

The economic magnitude of asset exposure to the SCA risk can in turn be assessed by the standard variance decomposition techniques. Figure 5 summarises our results. The common factor explains on average 79% of the time series variation in the stock returns, ranging from 36% to nearly 95% for individual portfolios. Moreover, this level of fit in our model is produced by a single consumption-based factor, as opposed to some of the alternative successful specifications, relying on 3 and sometimes 4 explanatory variables.

The same common factor accounts for a small (about 1.5%), but significant proportion of the time series variation in bond returns as well. The bond-specific factor, in turn, manages to capture most of the residual time series in variation in returns. While the model captures just about 55% of the variation in the 6-month bond returns, its performance rapidly improves with maturity and results in a nearly perfect fit for the time horizon of 2 years and more.

V.1.2 Consumption process and its properties

Slow consumption adjustment explains a significant proportion of the time series variation in consumption growth. As Figure 5 demonstrates, the common factor is responsible for roughly 27% of the variation in the one-period nondurable consumption growth, 33% of the two-period consumption growth, and so on, followed by a slow decline towards just above 5% for the 15-period growth. The bond-specific factor amounts for an additional 5% of the explanatory power. While these numbers may not seem as impressive as those for the cross-section of stocks, the pattern is highly persistent and significant, confirming a common factor structure between nondurable consumption growth and asset returns. Further, it also allows to use the model in Equations (12)–(13) for predictive purposes.

The upper right panel in Figure 5 displays the outcome of the predictive regression for the one-period consumption growth, should one rely on the factor loadings from Equation 12. Ultimate consumption risk predicts about 27% of the time series variation in the next period consumption and 18% of the consumption growth 2 quarters from now. Interestingly, the model retains significant predictive power (albeit, much lower) even for the one-period consumption that will occur nearly 4 years from now. A bond-specific factor increases the quality of predictive regressions by roughly another 5%.

The consumption growth process in Equation (12) is similar to the moving average decomposition, which allows us to model the dynamics of the slow consumption adjustment ($\Delta c_{t,t+1+S}$) in response to a common and/or a bond-specific shock. Figure 6 depicts SCA loadings on the factors as a function of the horizon S. If $S = 0$, the case of a standard consumption-based asset pricing model, SCA virtually does not load on the common factor. This is expected, since the factor manifests itself at a lower frequency. Indeed, as S increases, the impact of the common factor becomes more and more pronounced, levelling off at around $S = 11$. Interestingly, the pattern of the loadings observed in our two-factor model, is very similar to the one implied by the moving average representation of the consumption process

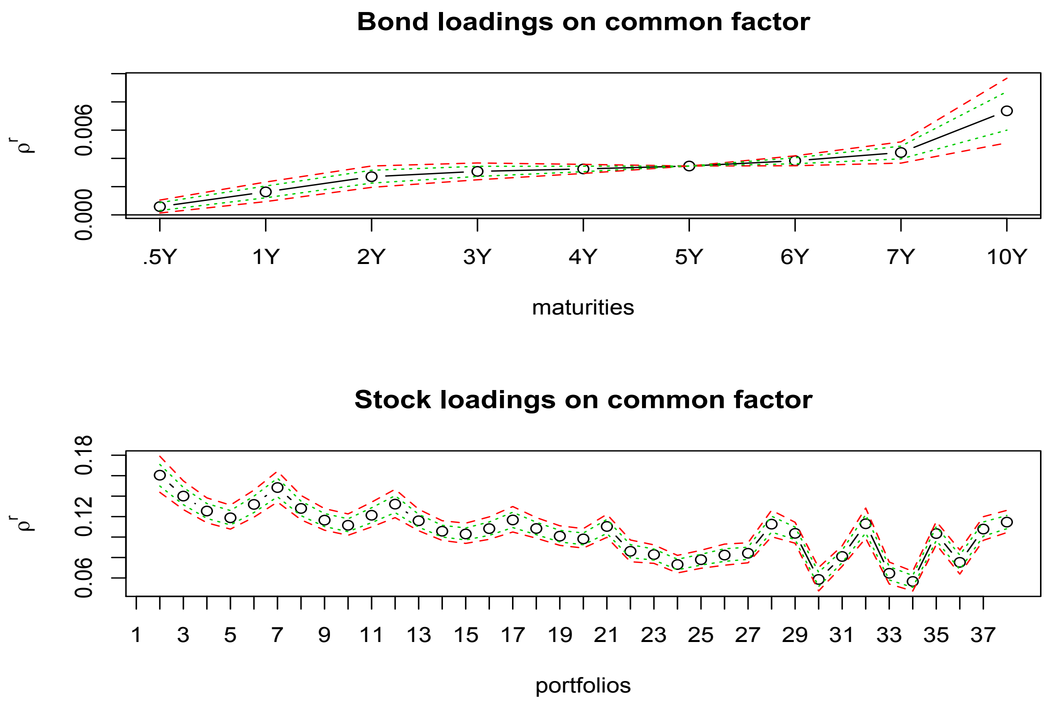


Figure 3: Bond and stock loadings on the common factor (f_t).

Note. Upper panel: loadings of bonds ($\boldsymbol{\rho}^r$) on common factor (f_t). Lower panel: loadings of stock portfolios ($\boldsymbol{\rho}^r$) on common factor (f_t). The graph presents posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions. Ordering of the portfolios: 25 Fama and French (1992) size and book-to-market sorted portfolios (e.g. portfolio 2 is the smallest decile of size and the second smaller decile of book-to-market ratio), and 12 industry portfolios.

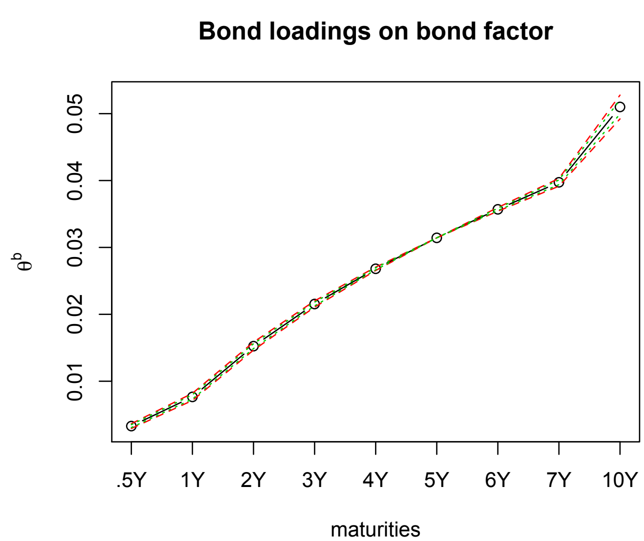
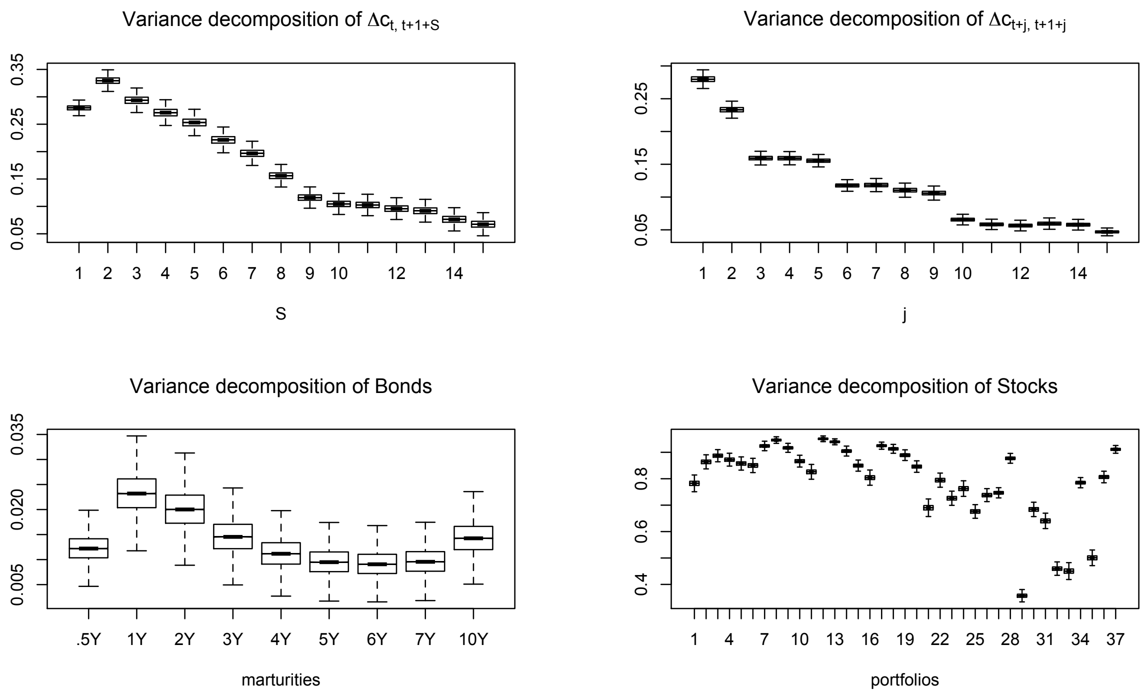
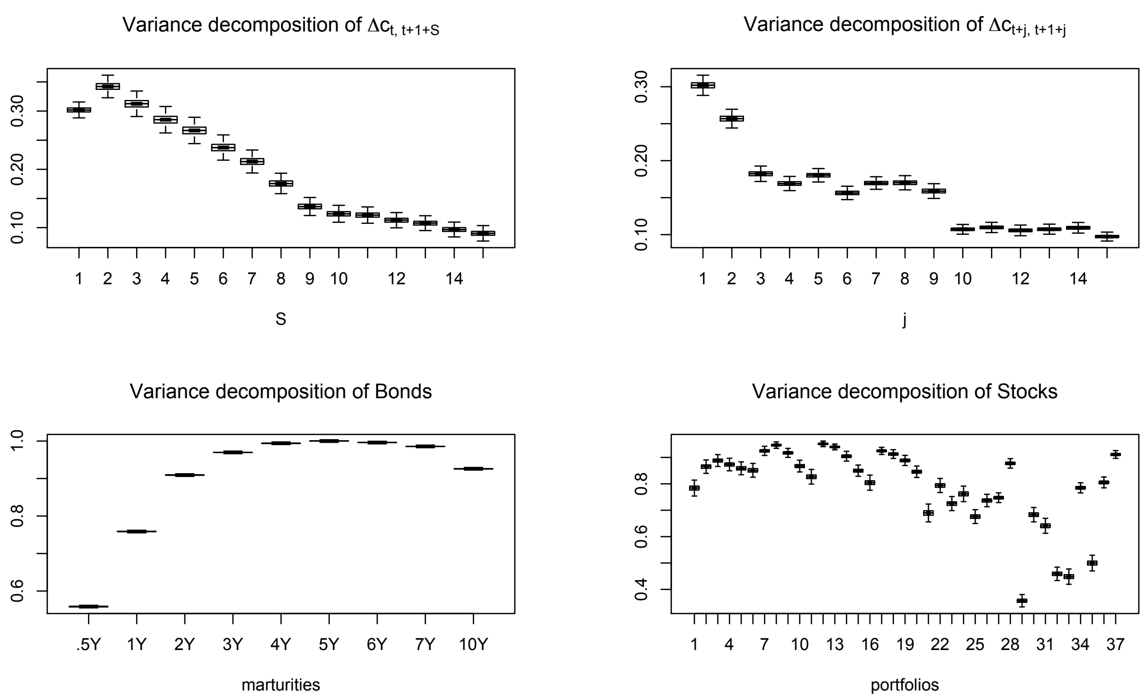


Figure 4: Bond loadings ($\boldsymbol{\theta}^b$) on the bond-specific factor (g_t).

Note. The graph presents posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions.



(a) Box-plots of percentage of time series variances explained by the common component f_t.



(b) Box-plots of percentage of time series variance explained jointly by the common component f_t and the bond component g_t.

Figure 5: Variance decomposition box-plots of asset returns and consumption growth

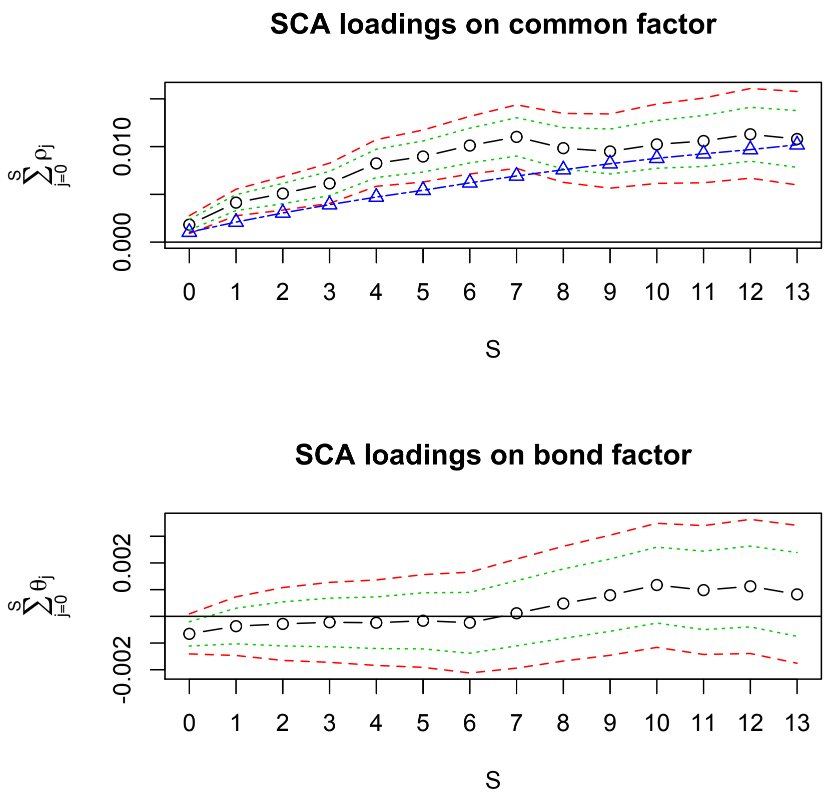


Figure 6: Slow consumption adjustment ($\Delta c_{t,t+1+S}$) response to shocks.

Note. Upper panel: SCA response to common factor (f_t) shocks, lower panel: SCA response to bond only factor (g_t) shocks. The graph presents posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions. Triangles denote Bansal and Yaron (2004) implied values.

in Bansal and Yaron (2004)[10]. In short, our setting reveals a similar degree of persistency and response rates, as their consumption process. The pricing of stocks and bonds, however, differs, because we consider a more flexible, reduced form model that nevertheless uncovers a very similar consumption-related pattern in the data as the one implied by the long-run risk model.

As a robustness check, we recover the long-run impact of common innovations to financial market returns and nondurable consumption using a simple bivariate SVAR model for the market excess return and consumption growth. We achieve identification via long-run restrictions on the impulse response functions á la Blanchard and Quah (1989). In particular, we distinguish a fundamental long-run shock, that can have a long-run impact on both market return and consumption, and a transitory shock that is restricted not to have a long-run impact on asset prices.

[10]For more details on the construction of the MA representation, see Appendix A.2

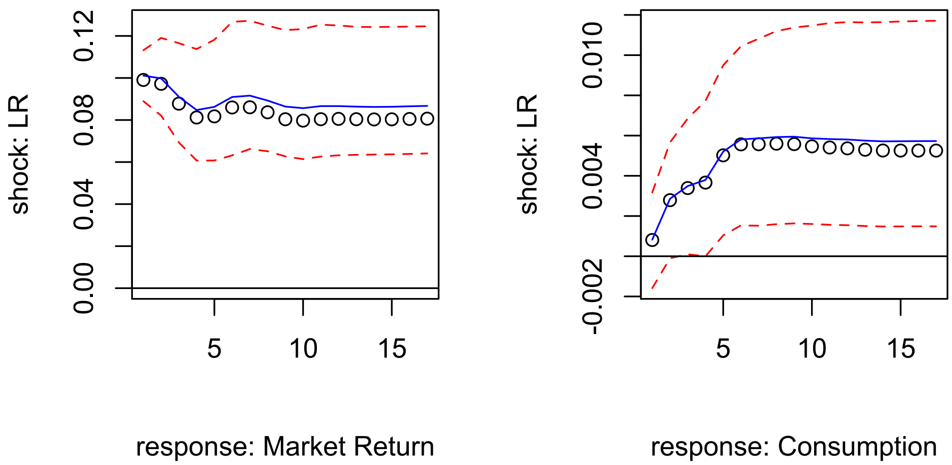


Figure 7: Cumulated response functions to a long-run shock

Note. The shock identified via a VAR and imposed long-run restrictions. Left panel depicts the cumulated response function for the market return, while the right one - for consumption growth. The graphs include posterior median (continuous line), mean (circles), and centred 95% coverage region (dotted lines).

Figure 7 displays the cumulated impulse response functions to a long-run fundamental shock, that is allowed to have a potemtially permanent impact on both the market excess return and nondurable consumption. In line with our previous reasoning, the latter response to a shock (right panel) is very similar to the one we observed from the SCA loadings on the common factor (Figure 6), while the response of the market returns (left panel), is consistent with an immediate and complete reaction of asset returns to the long-run shock as in our state-space model in Equations (8)-(9).

All these observations confirm that within the stream of nondurable consumption flow there is a rather persistent slow-moving component, accounting for 28% of the one-period time series variation in consumption growth, with innovations of that factor driving most of the contemporaneous changes in stocks returns and a small, but significant proportion in bonds. Next, we investigate whether this risk is actually *priced* in the cross-section of assets.

V.1.3 The price of consumption risk

Recovering factor loadings in Equations (12)–(13) also produces a cross-section of average returns on the set of portfolios. Figure 8 displays the scatterplot of the average vs. fitted excess returns for the baseline mixed cross-section of 46 assets. While the subset of bond returns demonstrates an almost perfect fit (lower left corner of the plot), the variation in the cross-section of stocks is also well-captured.

Further, as Equation (22) demonstrates, model-implied factor loadings of the asset returns determine their full exposure of the SCA risk and thus allow not only to assess the

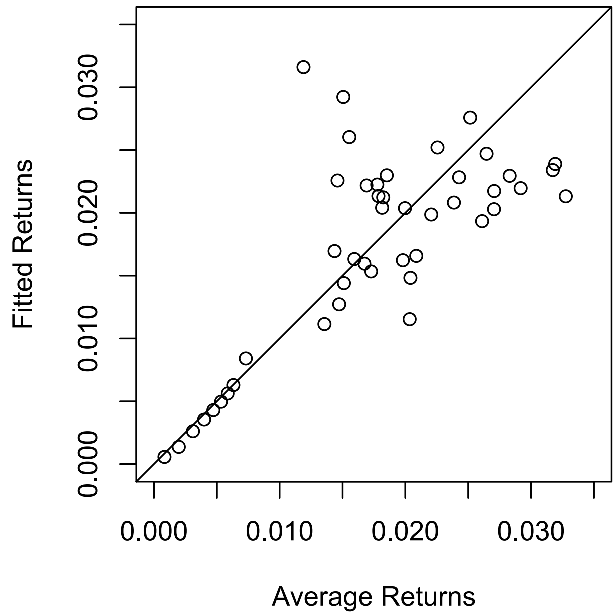


Figure 8: SCA risk: Average and Fitted Excess returns.

Note. Fitted versus average returns using the consumption betas implied by the latent factor specification in Equations (12)–(13).

cross-sectional fit of the model, but also to test whether the slow consumption adjustment is indeed a *priced* risk factor, and whether the common and bond factors share the same value of the risk premium.

Following the critique of Lewellen, Nagel, and Shanken (2010), we are using a mixed cross-section of assets to ensure that there is no dominating implied factor structure of the returns. Indeed, if that was the case, it could lead to spuriously high significance levels, quality of fit, and significantly complicate the overall model assessment. However, as Table 1 indicated, the slow consumption adjustment factor does not heavily load on any of the main principal components of the returns. Further, we provide confidence bounds for the cross-sectional measure of fit to ensure the point estimates reflect the actual pricing ability of the model. Finally, since both stocks and bonds have significant loadings on the common factor (and in the case of bonds, also on the bond-specific one), we do not face the problem of *irrelevant*, or *spurious* factors (Kan and Zhang (1999)), that could also lead to the unjustifiably high significance levels.

Table 2 summarizes the cross-sectional pricing performance of our parametric model of consumption on a mixed cross-section of 9 bond portfolios, 25 Fama-French portfolios sorted

Table 2: Cross-Sectional Regressions with State-Space Loadings

Row:	α	λ_f	λ_g	$\lambda_f = \lambda_g$	$\bar{R}^2$
One latent factor specification					
(1)	.0056 [0.0051, .0062]	14.77 [8.89, 26.01]			.57 [.54, .60]
(2)		20.00 [12.05, 35.16]			.90 [.89, .91]
Two latent factor specification					
(3)	.0057 [.0052, .0061]	14.97 [8.72, 27.45]			.57 [.54, .60]
(4)		20.30 [11.85, 37.18]			0.90 [0.89, 0.91]
(5)	.0069 [−539.5, 497.7]	13.79 [7.96, 25.49]	−1.44 [−539.5, 497.7]		.56 [0.53, 0.59]
(6)		20.27 [11.83, 37.12]	19.57 [−1140, 1218]		.91 [.90, .92]
(7)	.0053 [.0042, 0.0064]			15.24 [8.80, 28.40]	.57 [.53, .60]
(8)				20.29 [11.85, 37.19]	.90 [.89, .91]

Note. The table presents posterior means and centred 95% posterior coverage (in square brackets) of the Fama and MacBeth (1973) cross-sectional regression of excess returns on $\sum_{j=0}^{S} \rho_j \boldsymbol{\rho}^r$ (with associated coefficient λ_f) and $\left[\ \boldsymbol{\theta}'^b,\ \ \mathbf{0}'_{N-N_b}\ \right]' \sum_{j=0}^{S} \theta_j$ (with associated coefficient λ_g). The column labeled $\lambda_f = \lambda_g$ reports restricted estimates. Cross-section of assets: 25 Fama and French (1992) size and book-to-market portfolio; 12 industry sorted portfolios; 9 bond portfolios.

by size and book-to-market, and 12 industry portfolios. For each of the specifications, we recover the full posterior distribution of the factor loadings, and estimate the associated risk premia using Fama-MacBeth (1973) cross-sectional regressions. Regardless of the specification, there is strong support in favour of the slow consumption adjustment being a priced risk in the composite cross-section of assets with the risk premia of about 14-20% per quarter.

The average pricing error is about 0.005% per quarter, and the cross-sectional R^2 varies from 57% to 91%, depending on whether the intercept is included in the model. While allowing for a common intercept in the estimation substantially lowers cross-sectional fit, 95% posterior coverage bounds remain very tight, providing a reliable indicator of the model performance.

While the risk premium on the common factor is strongly identified and seems to play an important role in explaining the cross-section of both stock and bond returns, the bond factor loadings do not provide an equally large spread for recovering its pricing impact with the same degree of accuracy. As a result, the risk premium appears to be insignificant, unless its value is restricted to that of the common factor. To summarise, the bond-specific factor is *unspanned*, in the sense that while it is essential for explaining most of the time series variation in bond returns and producing a correct slope of the yield curve, it does not have any cross-sectional impact on bond returns.

V.2 Semi-parametric approach

Since the relevance of slow consumption adjustment risk for the cross-section of stocks has already been highlighted by Parker and Julliard (2005), we first focus on the cross-section of bonds only, and provide empirical evidence that the SCA risk is important for explaining their cross-section of returns. We then turn to analysing the model performance for pricing a composite set of bonds and stocks.

Table 3 summarizes the performance of the consumption-based asset pricing model on the cross-section of bond returns for various values of S of the ultimate consumption measure of Parker and Julliard (2005). While EL estimation remains valid in the presence of the multiplicative unobservable part of the stochastic discount factor, evaluating GMM output requires a certain degree of caution, since in this case, to the best of our knowledge, the same robustness is achieved only within the class of external habit models (see Proposition 1 of Parker and Julliard (2003)). Nevertheless, for the sake of completeness we report both sets of results.

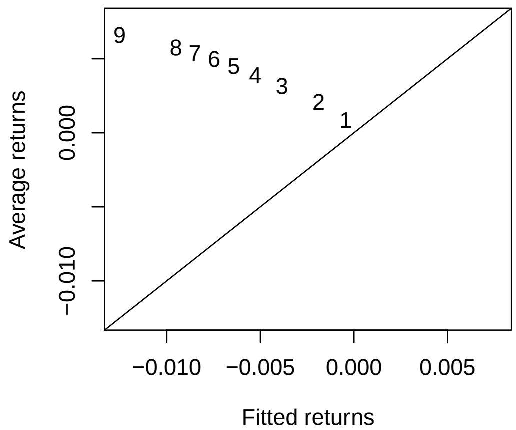
The $S = 0$ case corresponds to the standard consumption-based asset pricing model, where the spread of the returns is driven only by their contemporaneous correlation with the consumption growth. Both EL and GMM output reflect the well-known failure of the classical model to capture the cross-section of bond returns: according to the J-test, the model is rejected in the data, and the cross-sectional adjusted R-squared is negative. Increasing the span of consumption growth to 2 or more quarters drastically changes the picture: J-test no longer rejects the model, and the level of cross-sectional fit increases up to 85% for $S = 12$, for example.

Further, the estimates of the power coefficient ϕ (which in the case of additively separable CRRA utility corresponds to the Arrow-Pratt relative risk-aversion coefficient) not only appear to be much smaller (hence more in line with the economic theory), but also more precisely estimated. The large standard error associated with this parameter for the standard consumption-based model ($S = 0$) is due to the fact that the level and spread of the contemporaneous correlation between asset returns and consumption growth is rather low. This in turn leads to substantial uncertainty in parameter estimation. As the number of quarters used to measure consumption risk increase, the link between bond returns and the slow moving component of the consumption becomes more pronounced, resulting in lower standard errors, better quality of fit, and the overall ability of the model to match the cross-section of bond returns. In fact, model-implied average excess returns are very close to the actual ones, in drastic contrast to the standard consumption-based asset pricing

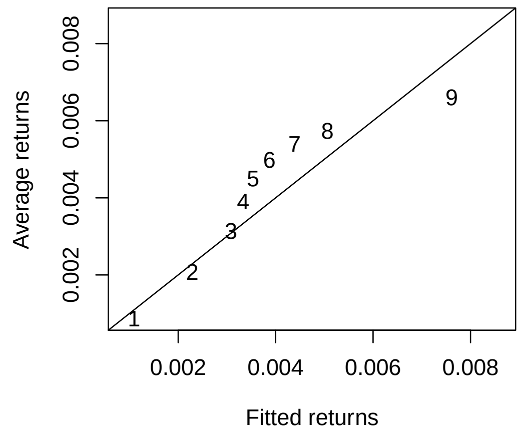
Table 3: Cross-Section of Bond Returns and Ultimate Consumption Risk

	Empirical Likelihood				Generalised Method of Moments			
Horizon S (Quarters)	$R^2_{adj}(\%)$ (1)	α (2)	ϕ (3)	J-test (4)	$R^2_{adj}(\%)$ (5)	α (6)	ϕ (7)	J-test (8)
0	-837	0.0007	100	13.0888	-10	0.0000	4	19.5597
		(0.0003)	(28.5)	[0.0700]		(0.0002)	(73.5)	[0.0066]
1	-167	0.0009	88	7.6457	-35	0.0005	42	11.5448
		(0.0004)	(24.8)	[0.3649]		(0.0005)	(47.3)	[0.1166]
2	70	0.0030	120	2.8778	43	0.0009	50	4.6351
		(0.0005)	(21.8)	[0.8961]		(0.0011)	(52.6)	[0.7044]
3	39	0.0010	70	4.5187	61	0.0006	35	5.1968
		(0.0004)	(16.2)	[0.7185]		(0.0005)	(20.5)	[0.6360]
4	69	0.0008	55	3.4531	48	0.0004	33	3.2207
		(0.0003)	(13.4)	[0.8402]		(0.0004)	(16.1)	[0.8639]
5	5	0.0008	45	6.8134	38	0.0004	27	6.0294
		(0.0003)	(10.5)	[0.4486]		(0.0003)	(13.0)	[0.5363]
6	3	0.0008	42	8.9256	42	0.0002	23	6.8397
		(0.0003)	(10.0)	[0.2580]		(0.0003)	(11.5)	[0.4458]
7	64	0.0004	33	9.8236	64	0.0001	22	6.4740
		(0.0003)	(9.9)	[0.1988]		(0.0003)	(10.7)	[0.4856]
8	70	0.0006	35	9.6027	69	0.0003	24	6.5862
		(0.0003)	(10.1)	[0.2122]		(0.0003)	(12.3)	[0.4732]
9	53	0.0008	55	8.2778	67	0.0004	26	6.8314
		(0.0003)	(10.5)	[0.3087]		(0.0003)	(14.7)	[0.4466]
10	77	0.0008	38	10.2472	73	0.0004	25	6.8649
		(0.0002)	(12.3)	[0.1750]		(0.0003)	(18.4)	[0.4431]
11	77	0.0008	44	8.2683	72	0.0006	26	7.7110
		(0.0002)	(14.3)	[0.3095]		(0.0003)	(23.7)	[0.3588]
12	85	0.0008	78	6.1561	88	0.0008	34	6.8054
		(0.0002)	(16.3)	[0.5216]		(0.0003)	(26.5)	[0.4494]
13	69	0.0007	85	5.8494	89	0.0007	37	6.0817
		(0.0002)	(17.5)	[0.5574]		(0.0003)	(28.7)	[0.5302]
14	88	0.0006	72	8.0283	90	0.0007	41	6.7445
		(0.0002)	(19.6)	[0.3301]		(0.0004)	(30.3)	[0.4560]
15	77	0.0006	70	7.3656	69	0.0008	46	7.2723
		(0.0002)	(22.1)	[0.3918]		(0.0005)	(36.4)	[0.4011]

Note. The table reports the pricing of 9 excess bond holding returns for various values of the horizon S, and allowing for an intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and two-stage GMM.



(a) S=0



(b) S=12

Figure 9: Slow consumption adjustment factor and the cross-section of bond returns

Note. The figures show average and fitted returns on the cross-section of 9 bond portfolios (1961Q1-2013Q4), sorted by maturity. The model is estimated by Empirical Likelihood for various values of consumption horizon S. $S = 0$ corresponds to the standard consumption-based asset pricing model; $S = 12$ corresponds to the use of ultimate consumption risk, where the cross-section of returns is driven by the their correlation with the consumption growth over 13 quarters, starting from the contemporaneous one.

model. This is shown in Figure 9 which presents fitted and actual average excess returns on the cross-section of 9 bond portfolios for several values of the consumption horizon S. The contemporaneous correlation between bond returns and consumption growth (Panel A, $S = 0$) is so low that not only it results is rather poor fit, but actually reverses the order of the portfolios: i.e. the fitted average return from holding long-term bonds is smaller than that of the short term ones. And again, once the horizon used to measure consumption risk is increased, the quality of fit substantially improves, leading to an R-squared of 85% for $S = 12$ (see Panel on the right).

The ability of slow consumption adjustment risk to capture a large proportion of the cross-sectional variation in returns is not a feature of the bond market alone: it works equally well on the joint cross-section of stocks and bonds, providing a simple and parsimonious one factor model for co-pricing securities in both asset classes.

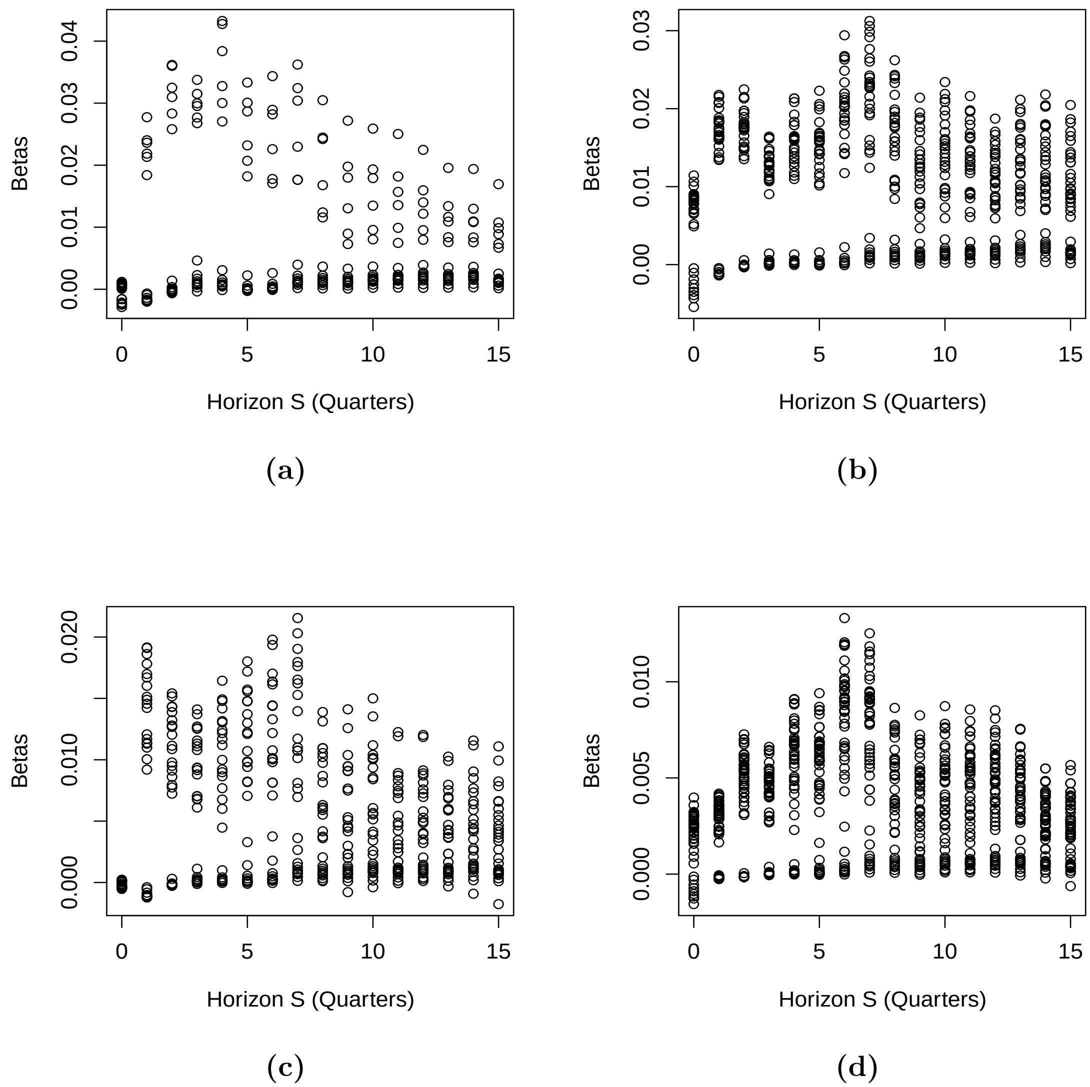
Table 4 summarises the model performance with various joint cross-sections of stocks and bonds for different consumption horizons S. Compared to the standard case of $S = 0$, slow consumption adjustment substantially improves model performance in a number of ways. While a simple consumption-based asset pricing model is rejected by the J-test on all the cross-section of stocks, the test values are dramatically improved over the range of $S = 10 - 12$: in fact, based on Empirical Likelihood Estimation, the model is no longer

Table 4: Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

	Empirical Likelihood			Generalised Method of Moments		
Horizon S	$R^2_{adj}(\%)$	ϕ	*J*-test	$R^2_{adj}(\%)$	ϕ	*J*-test
(Quarters)	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 9 Bonds and Fama-French 6 portfolios						
0	-13	-7	36.8568	70	60	36.3730
		(26.3)	[0.0013]		(27.7)	[0.0016]
10	95	23	7.275274	89	30	28.3589
		(6.0)	[0.9495]		(6.8)	[0.0194]
11	94	23	6.389318	94	32	29.3379
		(6.5)	[0.9724]		(8.5)	[0.0145]
12	91	22	5.864083	96	35	30.5354
		(6.3)	[0.9819]		(9.4)	[0.0101]
Panel B: 9 Bonds and Fama-French 25 portfolios						
0	46	41	56.7788	64	73	157.2452
		(17.8)	[0.0084]		(15.0)	[0.0000]
10	75	20	24.3141	24	41	31.8799
		(3.8)	[0.8899]		(6.1)	[0.5719]
11	76	20	21.2727	45	21	26.3571
		(3.7)	[0.9563]		(6.3)	[0.8224]
12	70	18	20.9430	49	22	22.3989
		(3.4)	[0.9612]		(7.7)	[0.9364]
Panel C: 9 Bonds, Fama-French 6, and Industry 12 portfolios						
0	-6	-3	59.7497	59	68	156.2215
		(21.2)	[0.0003]		(15.0)	[0.0000]
10	54	13	24.2148	-68	40	22.9235
		(3.9)	[0.6184]		(7.0)	[0.6891]
11	51	12	24.2189	-38	42	22.0777
		(3.7)	[0.6181]		(7.0)	[0.7334]
12	52	12	22.1532	-3	45	22.0186
		(3.5)	[0.7295]		(6.4)	[0.7364]
Panel D: 9 Bonds, Fama-French 25, and Industry 12 portfolios						
0	22	19	82.6606	50	86	213.7053
		(15.1)	[0.0007]		(14.2)	[0.0000]
10	37	8	52.2543	-48	42	48.9612
		(2.5)	[0.2440]		(5.3)	[0.3551]
11	37	8	49.6145	-7	44	47.8821
		(2.3)	[0.3312]		(5.6)	[0.3963]
12	36	8	47.4384	16	48	41.7552
		(2.2)	[0.4138]		(6.3)	[0.6506]

Note. The table reports the pricing of excess returns of stocks and bonds, allowing for no intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.

Figure 10: Cross-sectional spread of exposure to slow consumption adjustment risk



Note. Panels present the spread of normalised betas for the various sets of assets and horizon S (0-15): (a) 9 bonds and 6 Fama-French portfolios, (b) 9 bonds and 25 Fama-French portfolios, (c) 9 bonds, 12 Industry and 6 Fama-French portfolios, (d) 9 bonds, 12 Industry and 25 Fama-French portfolios. All the parameters were estimated by Empirical Likelihood.

rejected in any of the cross-sections. Combined with the improved values of the power parameter (ϕ), the accuracy of its estimation (lower standard errors), and a substantial increase in the cross-sectional quality of fit, measured by the R^2, Table 4 presents compelling evidence in favour of the slow consumption adjustment risk being an important driver for the cross-sections of both stocks and bonds. Appendix A.3 provides similar empirical evidence for the alternative model specifications that also include a common or asset class-specific intercept as a proxy for model misspecification.

But why does the slow consumption adjustment risk provide a better fit for the cross-sectional spread in expected returns? The empirical evidence, presented in the previous section, suggests that both stocks and bonds tend to co-vary more with the consumption growth over the next few periods (captured by the common unobservable factor and the loadings on it). However, not only the SCA risk measure increases the average asset exposure to consumption growth, it also improves the *spread* of the latter. While the standard one-period consumption growth does not perform well in either dimensions, leading to the equity premium puzzle and a relatively poor cross-sectional fit, the SCA factor seems to achieve both objectives: it increases the amount of measured risk as well as its cross-sectional dispersion.

Figure 10 displays the dispersion of the model-implied scaled betas,[11] associated with the consumption growth over different horizon values and for different cross-sections of assets. As we move away from the standard case of $S = 0$, two observations immediately arise. First, there is a substantial improvement in the average asset exposure to consumption growth, which leads to lower and more accurate estimates of the risk aversion. However, it is the increase in the spread of betas, with a particular contribution from the stocks, which is most striking. The 'fanning out' effect, observed for the higher values of the consumption horizon S, further supports the hypothesis that the fundamental source of risk in the asset returns is related to the aggregate consumption growth, and should take into account its slow speed of adjustments to the common shocks.

Finally, the fact that there is a significant correlation between asset returns and consumption growth over the several periods (both in terms of its level and spread), also serves as an additional robustness check against a potential problem of *spurious factors* type (Kan and Zhang (1999)), i.e. factors that are only weakly related to the asset returns and thus only *appear* to be driving the cross-section of asset returns.

VI Conclusions

This paper provides empirical evidence that the slow consumption adjustment risk is an important driver for both stock and bond returns. A flexible parametric model with common factors driving asset dynamics and consumption identifies a slow varying component of consumption that responds to financial shocks. Both stocks and bonds load significantly on SCA risk factor, generating a sizeable risk premium and a dispersion in returns, consistent with the size and value anomalies, as well as the positive slope of the yield curve. As a

[11] We define betas as the ratio between the asset covariance with the model-implied scaled SDF and its variance.

result, our model explains between 36% and 95% of the time series variation in returns and between 57% and 90% of the joint cross-sectional variation in stocks and bonds.

Moreover, we find that slow consumption adjustment innovations drive more than a quarter of the time series variation of consumption growth, indicating that financial market related shocks are first order drivers of consumption risk.

While generally consistent with the consumption dynamics postulated in the long run risk framework, these empirical findings nevertheless pose several important questions. Can the results be applied to other asset classes, such as currencies or commodities? What is the nature of the unspanned factor, driving most of the time series variation in bonds?

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A Appendix

A.1 State Space Estimation and Generalisations

Let $\Pi' := [\boldsymbol{\mu}, \mathbf{H}]$, $x_t' := [1, \mathbf{z}_t']$. Under a (diffuse) Jeffreys' prior the likelihood of the data in equation (20) implies the posterior distribution

$$\Pi' | \, \Sigma, \{\mathbf{z}_t\}_{t=1}^T, \{\mathbf{y}_t\}_{t=1}^T \sim \mathcal{N}\left(\hat{\Pi}'_{OLS}; \Sigma \otimes (x'x)^{-1}\right)$$

where x contains the stacked regressors, and the posterior distribution of each element on the main diagonal of Σ is given by

$$\sigma_j^2 \big| \, \{\mathbf{z}_t\}_{t=1} \sim \text{Inv-}\Gamma\left((T - m_j - 1)/2, T\hat{\sigma}^2_{j,OLS}/2\right)$$

where m_j is the number of estimated coefficients in the j-th equation. Moreover, $\mathbf{F}$ and Ψ have a Dirac posterior distribution at the points defined in equation (17). Therefore, the missing part necessary for taking draws via MCMC using a Gibbs sampler, is the conditional distributions of $\mathbf{z}_t$. Since

$$\begin{array}{c} \mathbf{y}_t \\ \mathbf{z}_t \end{array} \Bigg| \mathcal{I}_{t-1}, \mathbf{H}, \Psi, \Sigma \sim \mathcal{N}\left(\begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{F}\mathbf{z}_{t-1} \end{bmatrix}; \begin{bmatrix} \Omega & \mathbf{H} \\ \mathbf{H}' & \Psi \end{bmatrix}\right),$$

where $\Omega := Var_{t-1}(\mathbf{y}_t) = \mathbf{H}\Psi\mathbf{H}' + \boldsymbol{\Sigma}$, this can be constructed, and values can be drawn, using a standard Kalman filter and smoother approach. Let

$$\mathbf{z}_{t|\tau} := E\left[\mathbf{z}_t | \mathbf{y}^\tau, \mathbf{H}, \Psi, \Sigma\right]; \quad \mathbf{V}_{t|\tau} := Var\left(\mathbf{z}_t | \mathbf{H}, \Psi, \Sigma\right).$$

where $\mathbf{y}^\tau$ denotes the history of $\mathbf{y}_t$ until τ. Then, given $\mathbf{z}_{0|0}$ and $\mathbf{V}_{0|0}$, the Kalman filer delivers:

$$\mathbf{z}_{t|t-1} = \mathbf{F}z'_{t-1|t-1}; \;\; \mathbf{V}_{t|t-1} = \mathbf{F}\mathbf{V}_{t-1|t-1}\mathbf{F}' + \Psi; \;\; \mathbf{K}_t = \mathbf{V}_{t|t-1}\mathbf{H}'\left(\mathbf{H}\mathbf{V}_{\mathbf{t|t-1}}\mathbf{H}' + \Sigma\right)^{-1}$$
$$\mathbf{z}_{t|t} = \mathbf{z}_{t|t-1} + \mathbf{K}_t\left(\mathbf{y}_t - \boldsymbol{\mu} - \mathbf{H}\mathbf{z}_{t|t-1}\right); \;\; \mathbf{V}_{t|t} = \mathbf{V}_{t|t-1} - \mathbf{K}_t\mathbf{H}\mathbf{V}_{t|t-1}.$$

The last elements of the recursion, $\mathbf{z}_{T|T}$ and $\mathbf{V}_{T|T}$, are the mean and variance of the normal distribution used to draw $\mathbf{z}_T$. The draw of $\mathbf{z}_T$ and the output of the filter can then be used for the first step of the backward recursion, which delivers the $\mathbf{z}_{T-1|T}$ and $\mathbf{V}_{T-1|T}$ values necessary to make a draw for $\mathbf{z}_{T-1}$ from a gaussian distribution. The backward recursion can be continued until time zero, drawing each value of $\mathbf{z}_t$ in the process, with the following updating formulae for a generic time t recursion:

$$\mathbf{z}_{t|t+1} = \mathbf{z}_{t|t} + \mathbf{V}_{t|t}\mathbf{F}'\mathbf{V}_{t+1|t}^{-1}\left(\mathbf{z}_{t+1} - \mathbf{F}\mathbf{z}_{t|t}\right); \;\; \mathbf{V}_{t|t+1} = \mathbf{V}_{t|t} - \mathbf{V}_{t|t}\mathbf{F}'\mathbf{V}_{t+1|t}^{-1}\mathbf{F}\mathbf{V}_{t|t}.$$

Hence parameters and states can be drawn via Gibbs sampler using the following algorithm:

1. Take a guess $\tilde{\Pi}'$ and $\tilde{\Sigma}^{-1}$ (e.g. freq. estimate), and use it to construct initial draws for $\boldsymbol{\mu}$ and $\mathbf{H}$. Using also $\mathbf{F}$ and Ψ, draw the $\mathbf{z}_t$ history using the Kalman recursion above with (Kalman step)
$$\mathbf{z}_t \sim \mathcal{N}\left(\mathbf{z}_{t|t+1}; \mathbf{z}_{t|t+1}\right).$$
2. Conditioning on $\{\mathbf{z}_t\}_{t=1}^{T}$ (drawn at the previous step) and $\{\mathbf{y}_t\}_{t=1}^{T}$ run *OLS* imposing the zero restrictions and get $\hat{\Pi}'_{OLS}$ and $\hat{\Sigma}_{OLS}$, and draw $\tilde{\Pi}'$ and $\tilde{\Sigma}^{-1}$ from the N-i-Γ. Use the draws as the initial guess for the previous point of the algorithm (N-i-Γ step), and repeat.

Computing posterior confidence intervals for the cross-sectional performance of the model, conditional on the data, is relatively simple since, conditional on a draw of the time series parameters, estimates of the risk premia (λ's in equations (21) and (22)) are just a mapping obtainable via the linear projection of average returns on the asset loadings in $\mathbf{H}$. Hence, to compute posterior confidence intervals for the cross-sectional analysis, we repeat the cross-sectional estimation for each posterior draw of the time series parameters, and report the posterior distribution of the cross-sectional statistics across these draws..

A.2 The Moving Average Representation of The Long Run Risk Process

We we assume the same data generating process as in Bansal and Yaron (2004), with the only exception that we introduce a square-root process for the variance, as in Hansen, Heaton, Lee, and Roussanov (2007), that is:

$$\Delta c_{t,t+1} = \mu + x_t + \sigma_t\eta_{t+1}; \;\; x_{t+1} = \rho x_t + \phi_e\sigma_t e_{t+1}; \;\; \sigma_{t+1}^2 = \sigma^2(1-\nu_1) + \nu_1\sigma_t^2 + \sigma_w\sigma_t w_{t+1},$$

where η_t, e_t, w_t, $\sim$ iid$\mathcal{N}(0,1)$. The calibrated *monthly* parameter values are: $\mu = 0.0015$, $\rho = 0.979$, $\phi_e = 0.044$, $\sigma = 0.0078$, $\nu_1 = 0.987$, $\sigma_w = 0.00029487$. To extract the quarterly frequency moving average representation of the process, we proceed in two steps. First, we simulate a long sample (five million observations) from the above system treating the given parameter values as the truth. Second, we aggregate the simulated data into quarterly observation and we use them to estimate, via MLE, the moving average representation of consumption growth in equation (8).

A.3 Additional Empirical Results

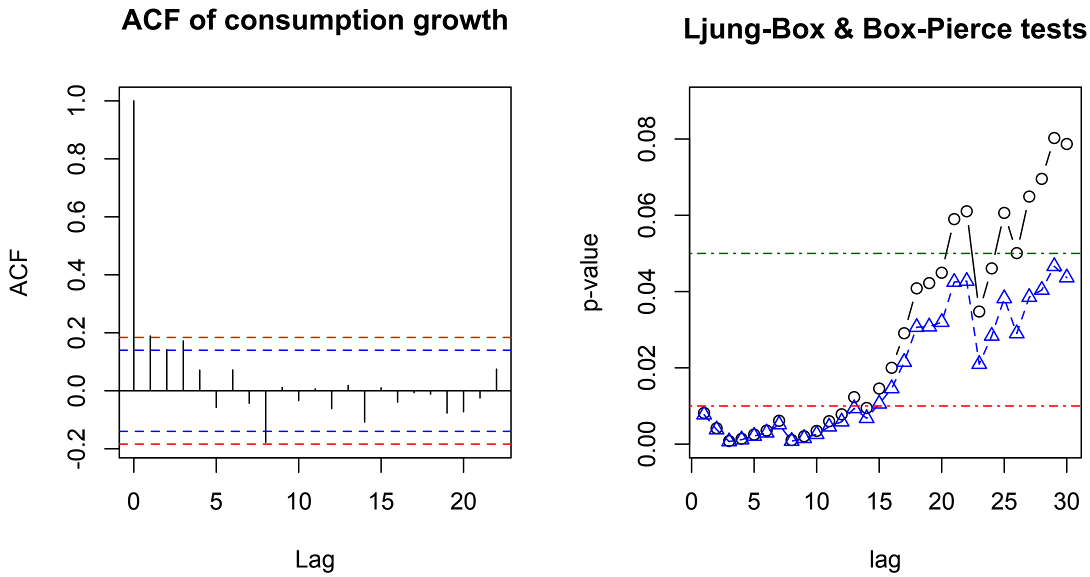

Figure A1: Autocorrelation structure of consumption growth.

Note. Left panel: autocorrelation function of consumption growth ($\Delta c_{t,t+1+S}$) with 95% and 99% confidence bands. Right panel: $p-$values of Ljung and Box (1978) (triangles) and Box and Pierce (1970) (circles) tests.

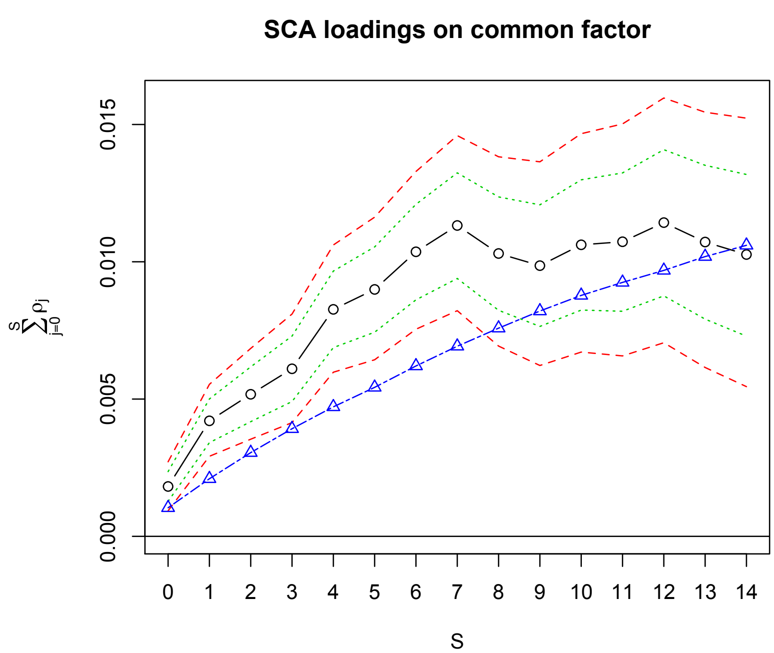

Figure A2: Slow Consumption Adjustment response to the common factor (f_t) shock.

Note. Posterior means (continuous line with circles) and centred posterior 90% (dashed line) and 68% (dotted line) coverage regions. Triangles denote Bansal and Yaron (2004) implied values.

Table A1: Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

	Empirical Likelihood					Generalised Method of Moments				
Horizon S (Quarters)	R^2_{adj}(%) (1)	α_b (2)	α_s (3)	ϕ (4)	J-test (5)	R^2_{adj}(%) (6)	α_b (7)	α_s (8)	ϕ (9)	J-test (10)
Panel A: 9 Bonds and Fama-French 6 portfolios										
0	-6	0.0001	0.0162	-74	59.7497	71	0.0003	0.0137	33	38.3181
		(0.0002)	(0.0045)	(21.2)	[0.0003]		(0.0003)	(0.0069)	(42.1)	[0.0001]
10	54	0.0004	0.0105	22	24.2148	29	0.0006	0.0132	28	17.3421
		(0.0003)	(0.0046)	(3.9)	[0.6184]		(0.0004)	(0.0046)	(6.9)	[0.1372]
11	51	0.0005	0.0099	24	24.2189	44	0.0008	0.0131	30	17.6300
		(0.0003)	(0.0047)	(3.7)	[0.6181]		(0.0003)	(0.0048)	(8.3)	[0.1274]
12	52	0.0005	0.0093	22	22.1532	53	0.0009	0.0136	32	18.6997
		(0.0003)	(0.0049)	(3.5)	[0.7295]		(0.0003)	(0.0050)	(9.0)	[0.0960]
Panel B: 9 Bonds and Fama-French 25 portfolios										
0	50	-0.0006	0.0130	60	62.3266	61	0.0011	0.0125	50	226.2077
		(0.0002)	(0.0045)	(21.2)	[0.0007]		(0.0001)	(0.0038)	(15.3)	[0.0000]
10	72	-0.0002	0.0104	19	23.1802	26	0.0019	0.0063	37	44.4437
		(0.0003)	(0.0038)	(3.9)	[0.8425]		(0.0003)	(0.0018)	(5.9)	[0.0558]
11	79	-0.0002	0.0096	18	20.8589	56	0.0020	0.0052	39	33.7601
		(0.0002)	(0.0039)	(3.9)	[0.9156]		(0.0003)	(0.0020)	(6.5)	[0.3355]
12	78	-0.0001	0.0096	17	20.4496	64	0.0018	0.0065	42	28.8556
		(0.0002)	(0.0040)	(3.7)	[0.9257]		(0.0002)	(0.0015)	(7.2)	[0.5768]
Panel C: 9 Bonds, Fama-French 6, and Industry 12 portfolios										
0	64	0.0000	0.0119	-14	59.4323	-33	0.0006	0.0239	31	124.6547
		(0.0002)	(0.0041)	(22.3)	[0.0001]		(0.0001)	(0.0027)	(18.4)	[0.0000]
10	72	0.0003	0.0131	14	21.9269	-77	0.0016	0.0140	32	44.9201
		(0.0003)	(0.0039)	(4.3)	[0.5836]		(0.0003)	(0.0024)	(6.3)	[0.0060]
11	70	0.0004	0.0119	11	24.8752	-53	0.0018	0.0140	34	37.2377
		(0.0002)	(0.0040)	(3.9)	[0.4126]		(0.0003)	(0.0023)	(7.0)	[0.0414]
12	72	0.0004	0.0115	10	24.4976	2	0.0019	0.0107	38	29.5539
		(0.0002)	(0.0041)	(3.7)	[0.4335]		(0.0003)	(0.0024)	(7.7)	[0.2000]
Panel D: 9 Bonds, Fama-French 25, and Industry 12 portfolios										
0	54	0.0005	0.0124	23	78.2258	36	0.0007	0.0146	58	269.4971
		(0.0002)	(0.0036)	(16.7)	[0.0008]		(0.0002)	(0.0027)	(13.4)	[0.0000]
10	61	-0.0002	0.0114	6	55.7091	-29	0.0018	0.0093	36	71.4739
		(0.0003)	(0.0034)	(2.5)	[0.0926]		(0.0003)	(0.0013)	(4.7)	[0.0041]
11	62	-0.0002	0.0112	6	53.6016	8	0.0020	0.0090	37	60.1299
		(0.0002)	(0.0034)	(2.4)	[0.1289]		(0.0003)	(0.0013)	(4.8)	[0.0430]
12	62	-0.0002	0.0111	6	51.8898	25	0.0019	0.0082	42	47.2360
		(0.0002)	(0.0034)	(2.2)	[0.1659]		(0.0003)	(0.0012)	(5.4)	[0.3036]

Note. The table reports the pricing of excess returns of stocks and bonds, allowing for separate asset class-specific intercepts. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.

Table A2: Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

	Empirical Likelihood				Generalised Method of Moments			
Horizon S (Quarters)	$R^2_{adj}(\%)$ (1)	α (2)	ϕ (3)	ELR-test (4)	$R^2_{adj}(\%)$ (5)	α (6)	ϕ (7)	J-test (8)
Panel A: B Bonds and Fama-French 6 portfolios								
0	-30	0.0002	-16	30.1955	73	0.0007	73	35.0646
		(0.0002)	(25.0)	[0.0044]		(0.0003)	(27.1)	[0.0008]
10	94	0.0008	23	11.5946	91	0.0009	29	24.9738
		(0.0003)	(6.0)	[0.5611]		(0.0004)	(6.8)	[0.0233]
11	95	0.0006	24	10.4758	94	0.0011	32	24.4029
		(0.0003)	(6.7)	[0.6546]		(0.0003)	(8.5)	[0.0276]
12	92	0.0005	23	11.1154	96	0.0012	34	25.2110
		(0.0003)	(6.5)	[0.6011]		(0.0003)	(9.3)	[0.0217]
Panel A: B Bonds and Fama-French 25 portfolios								
0	54	-0.0004	52	78.6597	60	0.0018	61	321.3738
		(0.0002)	(18.2)	[0.0000]		(0.0001)	(15.3)	[0.0000]
10	74	-0.0001	19	68.5008	38	0.0025	38	48.0606
		(0.0002)	(3.7)	[0.0002]		(0.0003)	(5.7)	[0.0340]
11	76	0.0000	20	67.9188	62	0.0024	40	35.1659
		(0.0002)	(3.7)	[0.0002]		(0.0003)	(6.0)	[0.3205]
12	70	0.0000	18	71.0791	67	0.0029	44	30.5687
		(0.0002)	(3.4)	[0.0001]		(0.0002)	(7.4)	[0.5390]
Panel C: 9 Bonds, Fama-French 6, and Industry 12 portfolios								
0	-6	0.0001	-6	63.2328	61	0.0017	55	273.0204
		(0.0002)	(21.9)	[0.0002]		(0.0002)	(15.2)	[0.0000]
10	56	0.0009	14	56.6896	-24	0.0037	35	51.9830
		(0.0003)	(4.0)	[0.0003]		(0.0003)	(6.4)	[0.0012]
11	51	0.0009	12	58.4329	-9	0.0042	37	38.4378
		(0.0002)	(3.7)	[0.0002]		(0.0003)	(6.9)	[0.0419]
12	52	0.0009	12	58.0225	10	0.0039	41	29.1776
		(0.0002)	(3.6)	[0.0002]		(0.0003)	(6.5)	[0.2566]
Panel D: 9 Bonds, Fama-French 25, and Industry 12 portfolios								
0	26	-0.0003	22	146.685	54	0.0016	69	356.9325
		(0.0002)	(15.2)	[0.0000]		(0.0002)	(13.5)	[0.0000]
10	38	-0.0002	8	141.4802	-25	0.0039	39	77.4115
		(0.0002)	(2.5)	[0.0000]		(0.0003)	(4.8)	[0.0014]
11	38	-0.0002	8	140.6384	16	0.0041	39	66.0979
		(0.0002)	(2.3)	[0.0000]		(0.0003)	(4.9)	[0.0172]
12	37	-0.0002	8	140.8904	29	0.0041	43	51.9741
		(0.0002)	(2.2)	[0.0000]		(0.0003)	(5.6)	[0.1912]

Note. The table reports the pricing of excess returns of stocks and bonds, allowing for a common intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.

Table A3: Expected Excess Returns and Consumption Risk, 1967:Q3-2013:Q4

	Empirical Likelihood				Generalised Method of Moments			
Horizon S (Quarters)	R^2_{adj}(%) (1)	α (2)	ϕ (3)	ELR-test (4)	R^2_{adj}(%) (5)	α (6)	ϕ (7)	J-test (8)
0	-30	0.0002	-16	30.1955	73	0.0007	73	35.0646
		(0.0002)	(25.0059)	[0.0044]		(0.0003)	(27.1)	[0.0008]
1	50	0.0005	55	19.8352	64	0.0006	50	26.7987
		(0.0003)	(16.5936)	[0.0994]		(0.0003)	(16.4)	[0.0133]
2	3	0.0008	50	15.9515	39	0.0008	45	20.5230
		(0.0004)	(11.4430)	[0.2517]		(0.0004)	(11.3)	[0.0829]
3	27	0.0007	45	14.3198	55	0.0007	40	20.5313
		(0.0004)	(9.3960)	[0.3517]		(0.0004)	(9.4)	[0.0827]
4	-33	0.0004	40	12.6842	16	0.0004	36	18.8278
		(0.0003)	(7.8412)	[0.4725]		(0.0003)	(7.7)	[0.1285]
5	58	0.0004	29	11.8102	42	0.0005	31	19.6120
		(0.0003)	(6.5887)	[0.5433]		(0.0003)	(6.7)	[0.1053]
6	67	0.0004	27	12.0794	53	0.0005	29	20.0162
		(0.0003)	(6.0256)	[0.5211]		(0.0003)	(6.2)	[0.0948]
7	61	0.0002	26	11.9012	43	0.0004	28	22.5791
		(0.0003)	(5.8619)	[0.5358]		(0.0003)	(6.0)	[0.0470]
8	89	0.0003	25	12.3113	74	0.0006	29	23.9049
		(0.0003)	(5.8866)	[0.5023]		(0.0003)	(6.3)	[0.0320]
9	95	0.0003	25	13.0312	92	0.0009	29	24.9160
		(0.0003)	(5.9862)	[0.4454]		(0.0003)	(6.4)	[0.0237]
10	94	0.0008	23	11.5946	91	0.0009	29	24.9738
		(0.0003)	(5.9595)	[0.5611]		(0.0004)	(6.8)	[0.0233]
11	95	0.0006	24	10.4758	94	0.0011	32	24.4029
		(0.0003)	(6.7275)	[0.6546]		(0.0003)	(8.5)	[0.0276]
12	92	0.0005	23	11.1154	96	0.0012	34	25.2110
		(0.0003)	(6.5436)	[0.6011]		(0.0003)	(9.3)	[0.0217]
13	86	0.0004	22	11.8978	96	0.0012	35	26.5862
		(0.0003)	(6.3313)	[0.5360]		(0.0003)	(9.6)	[0.0142]
14	85	0.0004	23	11.7044	97	0.0013	42	18.5716
		(0.0003)	(6.5983)	[0.5520]		(0.0005)	(13.2)	[0.1370]
15	79	0.0005	21	13.4734	96	0.0021	43	32.4073
		(0.0003)	(6.1575)	[0.4120]		(0.0004)	(12.7)	[0.0021]

Note. The table reports the pricing of 9 excess bond holding returns and 6 Fama-French portfolios, sorted on size and book-to-market. We report the results for various values of the horizon parameters S and allow for a common intercept. Standard errors are reported in parentheses and p-values in brackets. Estimation is done using EL and GMM.