

# Credit Rating Inflation and Firms' Investments

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## Abstract

Could inflated credit ratings that provide new information have adverse ex-ante real effects? If so, why? In a debt-rollover game featuring a feedback loop, we show that when the risky projects have high upside returns, the credit rating agency's (CRA) ex-ante real effects are negative, despite of its positive informational effects. This arises from the CRAs feedback effects: it strategically assigns high ratings to firms with relatively bad fundamentals, allowing them to gamble for resurrection. Laxer rating standards may not be accompanied by higher rating inflation, and an appropriately designed cost scheme conditional on the firms failure may regulate CRAs.

**Key Words:** Credit rating agency, rating inflation, real effect, feedback effect, global game

**JEL Classification:** D82, D83, G24, G32

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# 1 Introduction

Since the collapse of Enron in 2002, many observers have been questioning the role of credit rating agencies (CRAs) in promoting the financial market's efficiency, criticizing CRAs for issuing overgenerous credit ratings to debt-issuing firms.<sup>1</sup> Later, after the 2007 – 2009 financial crisis, the Financial Crisis Inquiry Report even concludes that “the failures of credit rating agencies were essential cogs in the wheel of financial destruction.” Several empirical studies, such as [Jiang, Stanford, and Xie \(2012\)](#), [Strobl and Xia \(2012\)](#), and [Cornaggia and Cornaggia \(2013\)](#), formally document the credit rating inflation. They further show that credit rating inflation is attributed to the conflicts of interest caused by the “issuer-pays” business model prevailing in the credit rating industry: because CRAs are paid by issuers they are assessing, they will have strong incentives to assign overgenerous ratings, in order to charge higher fees and attract new clients.

It then follows from these criticisms that inflated credit ratings may mislead investors, make some bad firms funded, and thus have adverse real effects. However, to mislead investors, credit ratings must be new informative signals; otherwise, rational and well-informed investors will not even look at them, and thus, CRAs should have no effect. On the other hand, if CRAs are providing investors with informative (though potentially biased) signals, they should be able to promote the market efficiency and thus guide, rather than mislead, investors. Hence, it seems that CRAs who potentially issue inflated credit ratings always have positive ex-ante effects (relative to the world without a CRA), even though they do not help to reach the first-best scenario where they always provide accurate information.

Then, when investors are perfectly rational and well-informed, how do potentially inflated credit ratings affect firms' investments? Will the CRA have positive or negative ex-ante real effects? How do credit ratings acting as new informative signals have negative real effects, if they do have any?

In this paper, we address these questions. We show that potentially inflated ratings can exclude extremely bad firms from creditors' belief supports and thus are positive signals about better-rated firms. Such positive signals then make creditors more optimistic, reduce firms' financial costs, and change certain firms' investment decisions. In particular, with high ratings and thus lower financial costs, some firms take risks instead of default efficiently, while some other firms switch from risky projects to safe projects. The CRA's ex-ante real effects, how-

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<sup>1</sup>The overgenerous credit ratings are usually called *inflated* credit ratings, as they stand for better credit qualities than issuers actually have.

ever, change from positive to negative, as the upside returns of risky projects increase beyond a threshold. Potentially inflated credit ratings that act as informative signals correctly guide creditors and lead some firms to invest in socially optimal projects; hence, such so-called “informational effects” of the CRA are always positive. The CRA’s negative real effects then arise from its “feedback effects,” a component missing in the literature on credit rating inflation and credit ratings’ real effects. Specifically, when issuing ratings, the CRA takes into account the creditors’ and the firm’s best responses to the ratings, hence it strategically assigns high ratings to more firms, allowing them to gamble for resurrection.

The above conclusions rely on two critical features of credit ratings. First, because some of the firm’s decisions are observable and verifiable, credit ratings are subject to a *partial verifiability constraint*. Specifically, we assume that after observing the financial cost, which is determined by the firm’s fundamentals and the aggregation of creditors’ debt-investment decisions, the firm may strategically opt for an early default, or invest either in a viable project (VP) with a high success probability and a positive expected NPV, or a risky project (HR) with a negative expected NPV due to a low probability of success. Importantly, whereas the firm’s choice of VP or HR is hidden and unverifiable, the public can observe its early default. Therefore, the firm’s early default will be the evidence against the CRA in any lawsuit or cause huge reputation costs to the CRA, if the CRA assigns a high rating to the firm. Hence, the CRA would never assign a high rating when it knows that the firm will default early. Consequently, if the CRA assigns a high rating, creditors will learn that the firm’s fundamentals are not extremely bad, although a high rating does not warrant an investment in VP.

Second, a firm’s debt market features coordination among creditors, as emphasized by [Boot, Milbourn, and Schmeits \(2006\)](#). In addition, [Morris and Shin \(2004\)](#) point out that the creditors may have heterogeneous private signals. Therefore, any individual creditor needs to consider the impact of credit ratings not only on his own belief but also on other creditors’ decisions.

Our model, which captures the above two features of credit ratings, has a unique equilibrium. In the equilibrium, the conflicts of interest imply that the CRA always employs a lax rating strategy that assigns all firms, which invest in HR, the rating standing for the VP’s credit quality.<sup>2</sup> However, a high credit rating generated by the lax rating strategy is not a cheap talk

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<sup>2</sup>In the core model, the equilibrium rating can be either the highest possible rating or the lowest possible one. This is so because in the core model, the CRA’s rating behavior is constrained only by the partial verifiability constraint. In Section 7.1, we generalize the partial verifiability constraint to a scheme of rating-dependent reputation costs conditional on the failure of the firm. The analysis there implies that when the firm has several mutually exclusive investment projects with different success probabilities, medium ratings may be assigned in an equilibrium and be potentially inflated.

as in [Crawford and Sobel \(1982\)](#). Due to the partial verifiability constraint, the high rating provides creditors with a public signal about the firm. Such a public signal is endogenous and takes a different form from that in [Morris and Shin \(2002\)](#): it truncates the supports of creditors' interim beliefs from below. Hence, the high credit rating will shift up creditors' beliefs about the firm's fundamentals.

Such a belief shift, then, leads to a non-trivial increase in the amount of creditors who buy the firm's debt.<sup>3</sup> Because the creditors receive heterogeneous private signals, the inflated credit rating, which truncates their interim belief supports from below, will lead some of them to buy the debt. Such a direct effect is reinforced by the creditors' coordination incentives; that is, any individual creditor has stronger incentives to buy the debt when more other creditors are doing so.

Therefore, an inflated credit rating reduces the firm's financial costs and thus changes the firm's investment decisions. The CRA's real effects then vary by the firm's fundamentals. In particular, firms with fundamentals just above the threshold of the high rating will invest in HR, instead of defaulting early when there is no CRA, to gamble for resurrection; hence, for such firms, the CRA's real effects are negative, since HR leads to the lowest ex-ante social welfare. On the other hand, the high rating leads some other firms that invest in HR in the scenario without a CRA to switch to VP, implying positive real effects of the CRA.

Whether the ex-ante real effects of the CRA are positive or negative, however, depends on the exogenous economic environment. Specifically, the CRA's ex-ante real effects are negative, when the upside return of HR is beyond a threshold (but constrained by the assumption that HR leads to the lowest social welfare). To see this, we first notice that there are two direct effects of an increase in the upside return of HR on firm's investments. On the one hand, the CRA is able to assign the high rating to firms with relatively bad fundamentals, because with the high rating, such firms prefer to take risks rather than default early. On the other hand, some firms will choose HR over VP, because HR becomes more profitable. Therefore, as the upside return of HR increases, more firms pick the project with the lowest social welfare, and ultimately, the CRA's ex-ante real effects become negative. This provides a potential explanation for a long-lasting puzzle: the conflicts of interest caused by the issuer-pays business model in the credit rating industry have been recognized for a long time, but they did not attract much

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<sup>3</sup>In Section 7.2, we consider the case where there is only one creditor with a precise public signal; hence, there is no coordination among creditors. Because the high rating can barely shift the creditor's belief about the firm's fundamentals, it cannot change her belief about the firm's investment decision. Consequently, in such a case, the credit rating does not have any real effects on the firm's investment, showing the role of the coordination among creditors in our core model.

attention until the recent subprime crisis. Our argument implies that because of the skyrocketing housing prices right before the crisis, securitizing the huge amount of subprime mortgage loans became a high-return but rather risky project. As most of financial institutions engaged in such high-return high-risk financial products (such as MBSs), the CRAs' ex-ante real effects were negative during the subprime crisis, putting them in criticisms.

The channel through which the CRA affects the firm's investment decisions implies that the CRA's real effects can be decomposed into two parts: its informational effects and its feedback effects. Indeed, the CRA's informational effects are equivalent to the real effects of a CRA who commits to assign the high rating to firms that do not default early in the scenario where there is no CRA. We refer to such a CRA as a "reflecting CRA." We show that though the rating strategy that the reflecting CRA commits to is likely inflated, a high rating assigned by the reflecting CRA will lead more firms to invest in VP than in the case without a CRA, because it provides creditors with a positive informative signal and thus lower the firm's financial costs. Therefore, the CRA's informational effects are always positive; that is, inflated credit ratings acting as new informative signals promote, rather than hurt, social welfare.

Then, the negative real effects of the CRA must arise from its feedback effects. Indeed, the CRA knows that it can assign the high rating to firms with relatively bad fundamentals, because with a high rating, such firms will invest in HR, instead of default early. Such a feedback between credit ratings and firms' investments leads to the CRA's adverse real effects, since it allows more firms to gamble for resurrection, and such adverse real effects are exacerbated when the upside return of HR increases.

Our theory has several empirical implications. Importantly, rating standards and rating inflation are two different endogenous terms, and laxer rating standards are not necessarily accompanied by higher rating inflation. We show this claim in several comparative static analyses, which also provide us with new empirical predictions about CRAs' credit rating standards and credit rating inflation. In particular, a decrease in the firm's transparency,<sup>4</sup> an increase in upside returns of risky projects, and an increase in the market liquidity will all lead to laxer rating standards (assigning high ratings to more firms). However, these changes of economic environments do not necessarily cause higher rating inflation. Specifically, a decrease in the firm's transparency has an ambiguous effect on rating inflation, an increase in upside returns of risky projects will cause higher rating inflation, and an increase in the market liquidity leads to lower rating inflation.

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<sup>4</sup>This shares some similar spirits with [Fong, Hong, Kacperczyk, and Kubik \(2014\)](#), who find that higher analyst coverages provide investors with more information about firms, leading to stricter rating strategies.

Our paper is closely related to the pioneer works about the feedback between credit ratings and the firm's behavior. [Boot, Milbourn, and Schmeits \(2006\)](#) uncover the CRA's coordination role when the firm has moral hazard problems. In their model, a credit rating sets a focal point for equilibrium selection when there exist multiple equilibria in the game between the firm and investors. [Manso \(2013\)](#) analyzes the feedback effect between the firm's endogenous default decision and the CRA's ratings. [Holden, Natvik, and Vigier \(2016\)](#) study how credit ratings reinforce the likelihood of investors' coordination failure and reveals a pro-cyclical impact of credit ratings. In these models, the CRA always wants to provide accurate credit ratings, so there are no conflicts of interest between CRAs and investors, and there is no credit rating inflation. Our paper further develops the research agenda of CRAs' feedback effects. We demonstrate how the CRA takes advantage of the information asymmetry and the feedback between credit ratings and firms' investments to allow more firms to gamble for resurrection, which leads to the CRA's adverse real effects.

Our paper also contributes to the literature on CRAs' rating inflation. Researches in this literature attribute credit rating inflation to investors' imperfect rationality ([Bolton, Freixas, and Shapiro 2012](#); [Skreta and Veldkamp 2009](#)), investors' lack of information ([Frenkel 2015](#)), or regulations tied to ratings ([Opp, Opp, and Harris 2013](#)). Our paper complements to this literature by analyzing CRAs' roles in environments where investors are purely rational and have very precise private information. We also contribute to this literature by focusing on the feedback between credit ratings and firms' credit qualities.

In our model, creditors play a global game ([Carlsson and van Damme 1993](#); [Morris and Shin 2003](#)). Our model differs from the traditional global games mainly in that the creditors' dominant region of not buying the debt is endogenous, which is determined by the CRA's rating strategy. Our model's inclusion of endogenous information provided by the CRA relates it to the growing literature of global games with endogenous information and information manipulation. [Angeletos, Hellwig, and Pavan \(2006\)](#) and [Angeletos and Pavan \(2013\)](#) model the signaling effects of the government's preemptive defending policies, which pools very strong governments and very weak ones together. [Edmond \(2013\)](#) discusses a dictator's costly private information manipulation, but all revolutionaries' interim beliefs have full supports. Hence, the belief updating in these models differs from that in our model. In fact, the belief updating in our model is closer to that in [Angeletos, Hellwig, and Pavan \(2007\)](#) and [Huang \(2017\)](#). Nevertheless, our model has a unique equilibrium, because a high credit rating, the endogenous information, does not completely remove creditors' dominant region of not investing. Finally, our paper is also related to [Goldstein and Huang \(2016\)](#) who show how the government per-

suades depositors not to run a commercial bank by committing to a monitoring policy. In our current model, the CRA cannot commit to a rating strategy, and the firm has severe moral hazard issue; therefore, unlike the government in [Goldstein and Huang \(2016\)](#), the CRA in the current paper may have negative ex-ante real effects.

Our paper also contributes to the literature on experts disclosure or persuasion. [Lizzeri \(1999\)](#) considers the information manipulation of perfectly informed information intermediaries. He shows that a monopoly intermediary's optimal disclosure choice is to reveal that the quality remains above a minimum standard. We model a CRA as a certified expert who discloses information about the firm to creditors. Our conclusion shares some spirit with [Lizzeri \(1999\)](#): a high rating does not mean the firm will invest in VP, but implies that the firm will not default early. However, the lowest type of the firm that receives a high rating is endogenously determined by how the creditors interpret the inflated rating. In addition, our model differs from those in the literature on Bayesian persuasion, such as [Kamenica and Gentzkow \(2011\)](#), in that the CRA cannot commit to a rating rule and thus make no information design.<sup>5</sup>

We organize the rest of the paper as follows. In Section 2, we describe our model. Section 3 establishes a benchmark without a CRA. In Section 4, we analyze credit ratings' informativeness and the CRA's equilibrium rating strategy. Section 5 compares the firm's equilibrium investments in the model with the CRA to those in the benchmark model without a CRA to derive the CRA's real effects. It then further studies the CRA's informational effects and feedback effects. Section 6 presents some empirical implications, and Section 7 discusses two important model assumptions. Section 8 concludes. All proofs appear in the Appendix.

## 2 A Model of Corporate Credit Ratings

We study a model of a CRA that is assigning credit ratings to a firm who needs to roll over its maturing debt. There are three dates,  $t = 0, 1, 2$ . At the beginning of date 0, the firm's existing debt is mature, and so it has to repay \$1 to the existing debt holders. To finance such \$1, the firm can issue new debt (with relatively low costs) or borrow from a predetermined bank credit line (with relatively high costs). At date 0, the CRA assigns credit ratings to the firm.<sup>6</sup> Observing

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<sup>5</sup>In an old version of this paper, we consider the case in which the CRA can commit to a rating strategy before knowing the fundamentals of the firm. Restricting the rating strategy to be monotonic, we show that the optimal committed rating strategy is the same as the equilibrium rating strategy without commitment. Hence, the optimal committed rating strategy is time-consistent.

<sup>6</sup>In the real financial market, CRAs assign credit ratings to both "issuers" and specific "issues." Although these two types of ratings are consistent in our model, we focus mainly on the issuer credit ratings in this paper.

the rating, new creditors in the debt market (with the measure  $1 - \gamma$ ) simultaneously decide whether to buy the newly issued debt or not. At date 1, depending on the financial cost, the firm may choose to default or to continue investing. In the latter case, the cash flow is realized at date 2, and, if possible, creditors are paid in full.

## 2.1 Firm Investment and Social Welfare

If the firm fully repays the existing debt, it can continue investments either in a viable (i.e., low-risk) project VP or a high-risk alternative HR at date 1. VP generates a cash flow  $V > 0$  with probability  $p \in (0, 1)$ ; however, it fails with probability  $1 - p$ . Similarly, HR generates a cash flow  $H > V$  with probability  $q \in (0, p)$  but fails with probability  $1 - q$ . The firm will receive a zero cash flow if the project fails. Since both VP and HR fail with positive probabilities, the firm's investment choice between VP and HR is unobservable and unverifiable.<sup>7</sup>

At date 1, instead of investing in VP or HR, the firm may choose to default. In such a case, the firm will not borrow from the bank credit line, and its liquidation value is  $B \in (0, \gamma]$ . We assume that the liquidation value and the funds from the newly issued debt are used to repay the exiting debt, since the existing debt is senior.<sup>8</sup> If the firm defaults at date 1, the game ends, and thus its early default decision is publicly observable and verifiable.

Social welfare is ranked by the firm's decisions.<sup>9</sup> When the firm invests in VP, the social welfare is  $pV - 1$ ; when the firm invests in HR, the social welfare is  $qH - 1$ ; and if the firm defaults at date 1, the social welfare is  $B - 1$ . We assume that it is very unlikely for HR to generate a positive cash flow; specifically,

$$pV > 1 > B > qH. \quad (1)$$

Therefore, VP has a positive expected NPV and is the social optimal investment project. Assumption 1 also implies that whereas both HR and early default lead to negative social welfare, an investment in HR leads to an even lower social welfare than the early default does.

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<sup>7</sup>In practice, creditors may know the name of the project the firm invests in, but they usually lack the professional knowledge to judge whether the project is HR or VP. Therefore, the choice between VP and HR is unverifiable even ex post.

<sup>8</sup>We assume  $B \leq \gamma$  for simplicity. By this assumption, when the firm defaults at date 1, the amount of funds available is at most \$1. Hence, any creditor who buys the newly issued debt will get nothing.

<sup>9</sup>This notion of social welfare is in fact the sum of all agents' ex-ante payoffs at date 0, including those of the existing debt holders and the bank.



## 2.2 Financing

There is a continuum of creditors with measure  $1 - \gamma$  in the debt market, each having \$1. Here,  $\gamma$  measures the liquidity of the debt market, with a larger  $\gamma$  meaning a lower liquidity level of the debt market. We assume that  $\gamma \in (0, 1)$ , and so it is impossible for the firm to finance all the \$1 by just issuing new debt.

The new debt is a zero-coupon bond with the face value  $F > 1$ . It matures at date 2. So long as the firm does not default either endogenously at date 1 or exogenously at date 2, the creditors who buy the new debt will get full repayment.<sup>10</sup>

We assume  $pF > 1$ , and so if any creditor  $i$  knows that the firm will invest in VP, he will buy the new debt. On the other hand, the probability that HR is successful is so low ( $qF < qH < B < 1$ ) that creditor  $i$  will not buy the new debt, if he knows that the firm will surely invest in HR. Obviously, if creditor  $i$  knows that the firm will default early, he does not buy the new debt either. We denote by  $a_i \in \{0, 1\}$  creditor  $i$ 's debt-investment decision, where  $a_i = 1$  means creditor  $i$  buys the new debt, while  $a_i = 0$  means creditor  $i$  does not buy.

We denote by  $W$  the measure of creditors who buy the new debt, and so the firm needs to finance  $1 - W$  from the bank credit line.<sup>11</sup> The firm can withdraw up to \$1 from the credit line with the constant marginal cost  $f(\theta)$ . Here,  $\theta$  represents the firm's capacity to manage liquidity and is drawn by nature from the real line  $\mathbb{R}$ , according to a common improper uniform distribution. (We also call  $\theta$  the fundamentals of the firm, and call a firm with the fundamentals  $\theta$  the " $\theta$ -firm.") The function  $f(\cdot)$  is differentiable and strictly decreasing. When the firm's fundamentals are extremely good, the marginal cost of the credit line financing approaches the payment of the new short-term debt; that is,  $\lim_{\theta \rightarrow +\infty} f(\theta) = F$ . However, when the firm's

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<sup>10</sup>To focus on the role of the credit rating agency, we take the new debt contract as exogenously given, as in [He and Xiong \(2012\)](#). One might argue that a higher promised payment  $F'$  can attract more creditors, so the firm can reduce its financial cost by offering  $F'$ . However, an endogenously promised payment  $F' > F$  will have signaling effects. Because it immediately rules out all  $\theta$ 's such that  $f(\theta) < F'$ , a higher promised payment  $F'$  may be self-defeated, and so its effect is ambiguous. Furthermore, there exists a pooling equilibrium, in which the firm will always offer the promised payment  $F$ , with the off-equilibrium path belief that any firm choosing  $F' \neq F$  will invest in HR.

<sup>11</sup>Credit lines are important means of corporate liquidity management. The firm has to pay the bank a commitment fee, in order to obtain the right to withdraw urgent money (up to a limit) in the future. So a credit line is like a put option to the firm. Terms of a credit line are usually negotiable between the firm and the bank, so we can also interpret  $\theta$  as the firm's bargaining power in the negotiation. Please refer to the survey by [Almeida, Campello, Cunha, and Weisbach \(2014\)](#) for more details of credit lines. In addition, because the financing terms of the credit line are predetermined, neither the bank nor the firm can make changes. Hence, differing from [Holmström and Tirole \(1997\)](#), the bank is not doing its job in monitoring the firm.

fundamentals are extremely bad, borrowing from the credit line is extremely expensive, so  $\lim_{\theta \rightarrow -\infty} f(\theta) = +\infty$ . Therefore, if the firm decides to invest in either VP or HR, the firm's financial cost is

$$K(\theta) = WF + (1 - W)f(\theta). \quad (2)$$

### 2.3 Firm's Payoff

The firm has limited liability. If it defaults, whether endogenously at date 1 or exogenously at date 2 (when the project fails), its payoff is zero. If the firm generates a positive cash flow at date 2, the firm needs to repay the creditors according to the new debt contract. Therefore, the firm's payoff  $U$  depends on its own investment choice and its financial cost:

$$U = \begin{cases} 0, & \text{if the firm defaults at date 1;} \\ p[V - WF - (1 - W)f(\theta)], & \text{if the firm invests in VP;} \\ q[H - WF - (1 - W)f(\theta)], & \text{if the firm invests in HR.} \end{cases} \quad (3)$$

### 2.4 Information Structure

The firm's liquidity management ability,  $\theta$ , is the firm's private information, which remains unknown to creditors. Before deciding whether to buy the new debt, each creditor  $i$  observes a private signal  $x_i = \theta + \xi_i$ , where  $\xi_i \sim \mathcal{N}(0, \beta^{-1})$  is independent of  $\theta$  and independent across all creditors. In our model, creditors are well-informed if and only if  $\beta$  is sufficiently large, and we aim to analyze credit ratings' effects on rational, well-informed creditors, and so in this paper, we focus on the case when  $\beta$  is sufficiently large. Besides their private signals, creditors will also observe a public credit rating by a credit rating agency (CRA).

### 2.5 Credit Rating Agency

The CRA assigns the firm a credit rating  $R$ .<sup>12</sup> Following [Boot, Milbourn, and Schmeits \(2006\)](#), we restrict the space of ratings to  $R \in \{0, q, p\}$ , because these are the only possible credit qualities of the firm: early default at date 1 means the firm will certainly default, and thus the firm's credit quality is 0; similarly, the firm investing in HR has a credit quality  $q$ , and the firm investing in VP has a credit quality  $p$ .

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<sup>12</sup>This may also be interpreted as a revision of the previous credit rating.

We assume that the CRA knows  $\theta$ , so that we can separate credit rating bias due to the conflicts of interest from that caused by the CRA’s capacity to acquire precise information. Because the CRA knows  $\theta$ , there is no aggregate shock to the CRA. In addition, we consider pure strategies, and so the CRA can perfectly predict the firm’s choice and its corresponding default probability at date 0. Hence, our model captures an important feature of credit ratings — forward-looking.

Due to the “issuer-pays” business model, the CRA always has incentives to assign the firm a high credit rating, in order to please the firm. The CRA’s incentives to assign high credit ratings may also come from issuers’ rating shopping (Bolton, Freixas, and Shapiro 2012), or the CRA’s reputation for being nice to issuers (Frenkel 2015). Therefore, for simplicity, we assume that in the core model, for each  $\theta$ , the CRA wants to maximize the nominal rating. We formally model the CRA’s payoff in section 7.1, where the CRA’s revenue and costs (conditional on the failure of the firm) are both strictly increasing in the nominal rating it assigns.

A partial verifiability condition constrains the CRA’s rating. The CRA must beware of lawsuits resulting from verifiable frauds; that is, the CRA never wants to be caught lying. Consequently, for any given  $\theta$ , the CRA wants to assign the firm the highest possible rating, provided that it cannot be verified as wrong (White 2013).<sup>13</sup>

## 2.6 Timeline and Equilibrium

We summarize the model’s timeline in Figure 1 below.

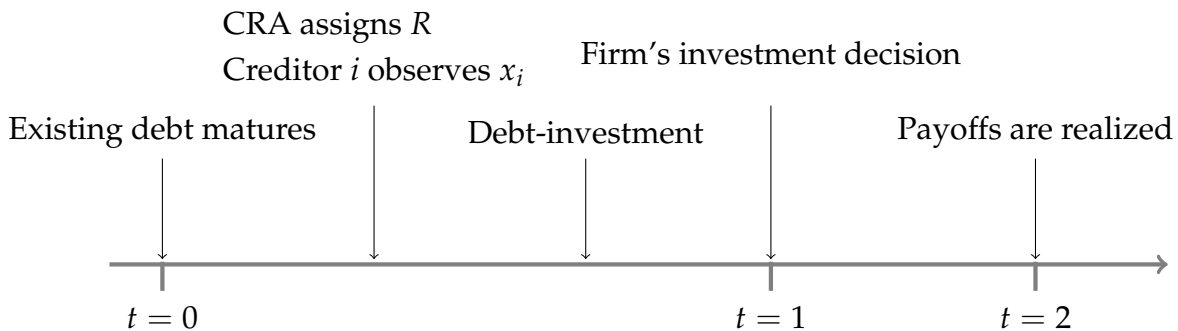


Figure 1: Timing

<sup>13</sup>Credit ratings are viewed as CRAs’ “free speech.” So, protected by the First Amendment, CRAs are not liable for any losses incurred by the inaccuracy of their ratings, unless it is proven that they know the ratings were false.

The CRA's rating strategy, denoted by  $\mathcal{R}$ , maps the firm's fundamentals to the rating space  $\{0, q, p\}$ ; the firm's strategy maps its fundamentals, the CRA's rating, and the measure of creditors investing in the new debt to project choices; and creditors' strategies map their own private signals and the CRA's rating to their debt-investment decisions.

We are interested in monotone equilibria.

**Definition 1** *The CRA's rating strategy, the firm's investment strategy, and creditors' debt-investment strategies constitute a monotone equilibrium, if*

1. *given the firm's investment strategy and creditors' debt-investment strategies, the CRA maximizes the nominal rating  $\mathcal{R}(\theta)$  for all  $\theta \in \mathbb{R}$  subject to the partial verifiability constraint;*
2. *given financial costs in equation (2), the firm's investment strategy will maximize the firm's expected profits;*
3. *given the CRA's rating strategy, the firm's investment strategy, and other creditors' strategies, any creditor  $i$ 's strategy is monotonic in his private signal  $x_i$  and will maximize his expected payoff;*
4. *and, creditors use Bayes' rule to update their beliefs.*

### 3 The Benchmark: No CRA

In order to analyze the CRA's real effects, we set up a benchmark that excludes the CRA. In such a benchmark, creditors decide whether to buy the newly issued debt solely based on their own private information. After observing the measure of creditors who invest in the debt, the firm makes its investment choice. The model is similar to the debt-run model by [Morris and Shin \(2004\)](#), with the key difference being that the firm's choice is not binary.

Let's first analyze the firm's behavior in such a benchmark model. Because of the law of large numbers, given the creditors' strategies, the measure of creditors who buy the debt is a deterministic function  $W(\theta)$ . Hence, any  $\theta$ -firm's financial cost is deterministic:

$$W(\theta)F + (1 - W(\theta))f(\theta).$$

Since  $H > V$ , the  $\theta$ -firm will default early, if and only if,<sup>14</sup>

$$W(\theta)F + (1 - W(\theta))f(\theta) > H. \quad (4)$$

Conditional on that the  $\theta$ -firm decides to continue investing, it invests in VP rather than HR, if and only if,

$$\begin{aligned} p [V - W(\theta)F - (1 - W(\theta))f(\theta)] &\geq q [H - W(\theta)F - (1 - W(\theta))f(\theta)] \\ \Rightarrow W(\theta)F + (1 - W(\theta))f(\theta) &\leq \frac{pV - qH}{p - q}. \end{aligned} \quad (5)$$

The firm's choice between VP and HR is the same as in [Boot, Milbourn, and Schmeits \(2006\)](#). To enhance the model's interest, we assume that if the firm can reach its lowest possible financial cost  $F$ , the firm will choose VP. That is, we maintain the assumption that

$$F < \frac{pV - qH}{p - q}.$$

As a result, given the creditors' strategies, the  $\theta$ -firm's optimal investment strategy is

$$\begin{cases} \text{early default,} & \text{if } W(\theta)F + (1 - W(\theta))f(\theta) > H; \\ \text{HR,} & \text{if } W(\theta)F + (1 - W(\theta))f(\theta) \in \left( \frac{pV - qH}{p - q}, H \right]; \\ \text{VP,} & \text{if } W(\theta)F + (1 - W(\theta))f(\theta) \leq \frac{pV - qH}{p - q}. \end{cases} \quad (6)$$

It then follows from the properties of the function  $f(\cdot)$  that when the firm's fundamentals are extremely good ( $\theta \rightarrow +\infty$ ),  $f(\theta)$  is very close to the par value of the firm's debt,  $F$ ; then  $WF + (1 - W)f(\theta)$  is strictly less than  $(pV - qH)/(p - q)$ , implying that the firm will invest in VP. When the firm's fundamentals are extremely bad ( $\theta \rightarrow -\infty$ ),  $f(\theta)$  is extremely large, so that  $WF + (1 - W)f(\theta) > H$ , implying that the firm will default early.

Hence, as shown in the global game literature, in such a benchmark model, the creditors have both the *dominant regions of not investing* and the *dominant regions of investing*. That is, when creditor  $i$ 's private signal  $x_i$  is extremely negative, he believes that the firm's fundamentals are so weak that even if all other creditors buy the debt, the firm's financial cost of continuing the investment is beyond its highest possible cash flow  $H$ , and thus the firm will default at date

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<sup>14</sup>We assume that the firm will default at date 1, if its financial cost is larger than the highest possible cash flow the firm can generate. Because the firm's investment outcome and debt repayments are both publicly observable at date 2, if the firm's investment is successful, but it still defaults, the firm's manager will receive litigation punishments or incur a reputation loss.

1. Therefore, any creditor  $i$  will not buy the debt, even when all other creditors invest in the debt. This establishes creditors' dominant region of not investing. In contrast, the creditors also have a dominant region of investing. If creditor  $i$ 's private signal  $x_i$  is extremely positive, he believes that the firm's fundamentals are extremely good, and thus the firm will choose VP; as a result, creditor  $i$  will buy the debt, even when all other creditors choose not to. Therefore, as in other global game models, in a monotone equilibrium, any creditor will employ a cutoff strategy with the threshold  $\tilde{x}$ , such that he invests in the debt, if and only if  $x_i \geq \tilde{x}$ .

Given  $\theta$  and the creditors' cutoff strategy, the measure of creditors who hold the debt is

$$W(\theta) = (1 - \gamma) \Pr(x \geq \tilde{x}|\theta) = (1 - \gamma) \left\{ 1 - \Phi \left[ \sqrt{\beta}(\tilde{x} - \theta) \right] \right\},$$

where  $\Phi(\cdot)$  is the CDF of the standard normal distribution. Then, the  $\theta$ -firm's financial cost is

$$\begin{aligned} K(\theta) &= (1 - \gamma) \left\{ 1 - \Phi \left[ \sqrt{\beta}(\tilde{x} - \theta) \right] \right\} F + \left[ \gamma + (1 - \gamma)\Phi \left[ \sqrt{\beta}(\tilde{x} - \theta) \right] \right] f(\theta) \\ &= [(1 - \gamma)F + \gamma f(\theta)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(\tilde{x} - \theta) \right] (f(\theta) - F). \end{aligned} \quad (7)$$

The first term in equation (7) is the financial cost resulting from the insufficient market liquidity, whereas the second term in equation (7) is the endogenous financial cost resulting from the creditors' strategic uncertainties.

As  $\theta$  increases, that is, as the firm's fundamentals improve, the cost of withdrawing funds from the credit line decreases (since  $f(\theta)$  is strictly decreasing), and the measure of the creditors investing in the debt increases. Therefore, given the creditors' cutoff strategies, the firm's financial cost strictly decreases in its fundamentals. In contrast with classical global games, in this benchmark model, the firm has two indifference conditions. First, given the creditors' strategies, the firm will choose to default early if and only if  $\theta < \tilde{\theta}_1$ . This implies

$$K(\tilde{\theta}_1) = [(1 - \gamma)F + \gamma f(\tilde{\theta}_1)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right] (f(\tilde{\theta}_1) - F) = H. \quad (8)$$

Because  $K(\theta)$  is strictly decreasing, for any  $\theta < \tilde{\theta}_1$ , the firm's financial cost will be greater than  $H$ , the upside cash flow of HR; as a result, the firm would default at date 1. But if  $\theta \geq \tilde{\theta}_1$ , the firm can at least choose HR in order to receive a non-negative expected payoff due to its limited liability, and thus the firm will not default early.

When  $\theta \geq \tilde{\theta}_1$ , the firm needs to choose between VP and HR. From equation (6) and the fact that  $K(\theta)$  is strictly decreasing in  $\theta$ , there must be a  $\tilde{\theta}_2 > \tilde{\theta}_1$ , such that the firm will choose VP if and only if  $\theta \geq \tilde{\theta}_2$ . Hence,

$$K(\tilde{\theta}_2) = [(1 - \gamma)F + \gamma f(\tilde{\theta}_2)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(\tilde{x} - \tilde{\theta}_2) \right] (f(\tilde{\theta}_2) - F) = \frac{pV - qH}{p - q}. \quad (9)$$

Following the above arguments, in a monotone equilibrium, the firm will default early if  $\theta < \tilde{\theta}_1$ , invest in HR if  $\theta \in [\tilde{\theta}_1, \tilde{\theta}_2)$ , and invest in VP if  $\theta \geq \tilde{\theta}_2$ .

Any creditor  $i$ , receiving a private signal  $x_i$  about  $\theta$ , first updates his belief about  $\theta$  according to Bayes' rule:

$$\theta|x_i \sim \mathcal{N}(x_i, \frac{1}{\beta}).$$

Then, given the firm's strategy described above, creditor  $i$  calculates his return from holding the debt:

$$\left\{ \Phi \left[ \sqrt{\beta}(\tilde{\theta}_2 - x_i) \right] - \Phi \left[ \sqrt{\beta}(\tilde{\theta}_1 - x_i) \right] \right\} qF + \left\{ 1 - \Phi \left[ \sqrt{\beta}(\tilde{\theta}_2 - x_i) \right] \right\} pF.$$

Because any creditor will receive the payoff 1 if not investing in the debt, the creditor with private signal  $\tilde{x}$  would be the marginal creditor who is indifferent about investing or not investing. As a result, the creditor's indifference condition is

$$\left\{ \Phi \left[ \sqrt{\beta}(\tilde{\theta}_2 - \tilde{x}) \right] - \Phi \left[ \sqrt{\beta}(\tilde{\theta}_1 - \tilde{x}) \right] \right\} qF + \left\{ 1 - \Phi \left[ \sqrt{\beta}(\tilde{\theta}_2 - \tilde{x}) \right] \right\} pF = 1. \quad (10)$$

Proposition 1 below characterizes the equilibrium of the benchmark model.

**Proposition 1 (The Unique Equilibrium in the Benchmark Model)** *There exists a  $\tilde{\beta} > 0$ , such that for all<sup>15</sup>  $\beta > \tilde{\beta}$ , the benchmark model without a CRA has a unique equilibrium described by  $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$ , where  $\tilde{\theta}_1 < \tilde{\theta}_2$ . In particular,*

1. *the firm's investment strategy is*

$$\begin{cases} VP, & \text{if } \theta \geq \tilde{\theta}_2; \\ HR, & \text{if } \theta \in [\tilde{\theta}_1, \tilde{\theta}_2); \\ \text{defaults early,} & \text{if } \theta < \tilde{\theta}_1; \end{cases}$$

2. *and, any creditor  $i$  will invest in the debt if and only if  $x_i \geq \tilde{x}$ .*

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<sup>15</sup>In this paper, we focus on the case that  $\beta$  is sufficiently large. There are two reasons. First, as in the global game literature, sufficiently precise private signals can guarantee equilibrium uniqueness, which is critical for the equilibrium analysis. More importantly, the aim of this paper is to study the credit ratings' effects when creditors have very precise private information, which refers to sufficiently large  $\beta$  in the model.

## 4 Credit Rating Inflation

We now consider our core model where the CRA strategically designs the rating rule. As a step to solve an equilibrium, we first discuss the informativeness of credit ratings. Although the CRA always has incentives to assign overgenerous ratings, its rating strategy is subject to the partial verifiability constraint: the event of the firm's early default is publicly observable and thus verifiable. As a result, in the equilibrium, the CRA will assign a high rating if and only if the firm does not default at date 1. We will show that this partial verifiability constraint plays a critical role in determining the informativeness of the CRA's ratings.

We then solve the CRA's equilibrium rating strategy. Importantly, when assigning credit ratings, the CRA will take into account the effects of the ratings on the creditors' debt-investment decisions and thus the firm's investment choices. As we show later in Section 5, this feature, as well as the informativeness of credit ratings, is the key to understand the credit ratings' real effects.

### 4.1 Informativeness of Ratings

We first argue that an equilibrium rating strategy must be monotonic. Consider a rating strategy  $\mathcal{R}(\theta)$  that assigns the rating  $p$  to  $\theta'$ -firm and the rating  $q$  to  $\theta''$ -firm, where  $\theta'' > \theta'$ . Put differently,  $\mathcal{R}(\theta)$  assigns the lower rating to some firms with relative better fundamentals. Because creditors' strategies are decreasing in their private signals in a monotone equilibrium, among firms that receive the same rating, the ones with better fundamentals have lower financial costs. This is because firms with better fundamentals have both lower bank credit line financing costs and more creditors holding the debt. Hence, if the CRA deviates to assign  $\theta''$ -firm the rating  $p$ ,  $\theta''$ -firm will have a lower financial cost than  $\theta'$ -firm. Because of the partial verifiability constraint, for the rating strategy  $\mathcal{R}(\theta)$  being an equilibrium rating strategy,  $\theta'$ -firm will not default at date 1. Then,  $\theta''$ -firm will not default at date 1 either, if it receives the rating  $p$ . As a result, such a deviation is profitable, and thus, the rating strategy  $\mathcal{R}(\theta)$  under consideration cannot be part of an equilibrium. Similarly, in an equilibrium, the CRA will not assign the rating 0 to  $\theta$ -firm, if it assigns the rating  $p$  to  $\theta'$ -firm with  $\theta' < \theta$ . Therefore, an equilibrium rating strategy is increasing in the firm's fundamentals.

Now, suppose that the rating strategy  $\mathcal{R}(\theta)$  assigns some firms the rating  $q$  in an equilibrium. Because  $\mathcal{R}(\theta)$  is increasing, such firms' fundamentals are worse than those of the firms who receive the rating  $p$ . Then, if the CRA deviates to assign the rating  $p$  to firms that receive the rating  $q$  under  $\mathcal{R}(\theta)$ , these firms' financial costs decrease. Since such firms do not default at



date 1 when assigned the rating  $q$ , with lower financial costs after receiving the rating  $p$ , they will not default at date 1 either. Therefore, the CRA's deviation is also profitable, which implies that the CRA will not assign the rating  $q$  in an equilibrium.<sup>16</sup>

These arguments lead to Lemma 1 below, which characterizes all possible equilibrium rating strategies and simplifies our analysis much.

**Lemma 1 (Cutoff Rating Strategy)** *In an equilibrium (if any exists), the CRA's rating strategy can be described by a threshold  $\theta_1^*$ , such that*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if } \theta < \theta_1^*; \\ p, & \text{if } \theta \geq \theta_1^*. \end{cases} \quad (11)$$

From Lemma 1, when  $\theta_1^*$  decreases, the CRA assigns more firms with the high rating  $p$ . So when the threshold  $\theta_1^*$  of the rating strategy  $\mathcal{R}(\theta)$  decreases, we say that the rating strategy becomes *laxer*. However, a laxer rating strategy may not lead to higher *credit rating inflation*, which refers to the fact that the nominal rating is strictly higher than the real credit quality. Formally:

**Definition 2** *A credit rating assigned to a  $\theta$ -firm is inflated, if in an equilibrium, the  $\theta$ -firm chooses HR and thus has the credit quality  $q$ , but the CRA assigns the rating  $p$ . In addition, a rating strategy is inflated, if credit ratings assigned according to the rating strategy are inflated for a non-negligible subset of fundamentals; and a credit rating strategy is more inflated, if for a larger measure of fundamentals, credit ratings assigned according to the rating strategy are inflated.*

In an equilibrium, the firm receiving the rating  $p$  does not default at date 1, due to the partial verifiability constraint. However, the rating  $p$  cannot guarantee that the firm will invest in VP. Indeed, if all creditors believe that the firm with the rating  $p$  will surely invest in VP, they will all buy the debt, leading to the lowest possible financial cost to any  $\theta$ -firm. Then, the assumption that the  $\theta_1^*$ -firm will invest in VP implies that  $\theta_1^*$ -firm's financial cost is strictly less than  $H$ . Then, the firm with  $\theta$  less than but very close to  $\theta_1^*$  will not default early, if it receives the rating  $p$ , providing the CRA with incentives to deviate to assign the rating  $p$  to such a firm. Therefore, in an equilibrium, some firms with the rating  $p$  will invest in HR, implying credit rating inflation in an equilibrium. Formally:

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<sup>16</sup>On the off-equilibrium path following  $R = q$ , the creditors form the belief that the firm will choose to continue to invest in HR.

**Lemma 2 (No Equilibrium without Rating Inflation)** *There is no equilibrium in which the  $\theta$ -firm invests in VP for all  $\theta \in \{\theta' : \mathcal{R}(\theta') = p\}$ .*

While rating inflation inevitably appears in an equilibrium, credit ratings are still informative to creditors. Lemma 1 implies that if  $R = p$ , all creditors learn that  $\theta \geq \theta_1^*$ . So the rating  $p$  guarantees creditors that the firm's fundamentals are not extremely bad.

**Corollary 1 (Creditors' belief supports following  $R = p$ )** *Following the credit rating  $R = p$ , regardless of his private signal  $x_i$ , the support of any creditor  $i$ 's interim belief about  $\theta$  is truncated from below by  $\theta_1^*$ .*

## 4.2 Firm Investment after the Rating $p$

As shown in Lemma 1, only the rating 0 and the rating  $p$  may appear in an equilibrium. Because the CRA tries to maximize the nominal rating, it will assign the rating 0 only if it knows that the firm will default early even with the rating  $p$ . Therefore, when the creditors observe the rating 0, they all believe that the firm will default early, and so they will not buy the debt. Hence, following  $R = 0$ , there is a unique equilibrium, in which no creditor invests in the debt, and the firm defaults at date 1. Since the rating strategy assigns the rating 0 to the firm if and only if  $\theta < \theta_1^*$ , we must have  $K(\theta) = f(\theta) > H, \forall \theta < \theta_1^*$ . Then, by the continuity of  $f(\cdot)$ , we have the first condition for the equilibrium below:

$$f(\theta_1^*) \geq H. \quad (12)$$

We now focus on the subgame following the rating  $p$ . Given the rating strategy, after observing the rating  $p$ , all creditors believe that the firm's true fundamentals are above  $\theta_1^*$ . However, as shown in Lemma 2, the creditors are not sure whether the firm will invest in HR or VP. In particular, because the firm will invest in HR when  $\theta$  is greater than but very close to  $\theta_1^*$ , the creditors with extremely negative signals believe that the firm will invest in HR and thus choose not to buy the debt. As a result, in a monotone equilibrium, given the belief about the CRA's rating strategy described by  $\theta_1^*$ , after observing the rating  $p$ , any creditor  $i$  will invest in the debt if and only if  $x_i$  lands above some threshold  $x^*$ . Then, as in the benchmark model, because the firm's financial cost strictly decreases in its fundamentals, the firm will invest in HR if and only if  $\theta$  is less than a threshold  $\theta_2^*$ . Hence, given a possible equilibrium rating strategy  $\theta_1^*$ , a monotone equilibrium following the rating  $p$  could be described by  $(x^*, \theta_2^*)$ , such that

1.  $\theta_2^* > \theta_1^*$ ;

2. creditor  $i$  invests in the debt if and only if  $x_i \geq x^*$ ; and
3.  $\theta$ -firm chooses VP if  $\theta \in [\theta_2^*, +\infty)$ , and it chooses HR if  $\theta \in [\theta_1^*, \theta_2^*)$ .

Given the creditors' cutoff strategy with the threshold  $x^*$ ,  $(1 - \gamma) [1 - \Phi(\sqrt{\beta}(x^* - \theta))]$  measure of creditors will invest in the debt, for any  $\theta \geq \theta_1^*$ . Consequently, if the  $\theta$ -firm decides to continue the investment, its financial cost is

$$\begin{aligned} K(\theta) &= (1 - \gamma) [1 - \Phi(\sqrt{\beta}(x^* - \theta))] F + [\gamma + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \theta))] f(\theta) \\ &= [(1 - \gamma)F + \gamma f(\theta)] + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \theta))(f(\theta) - F). \end{aligned}$$

This is precisely the same as equation (7). Because the firm invests in VP if and only if  $K(\theta) \geq (pV - qH)/(p - q)$ , and  $\theta_2^*$ -firm is indifferent between HR and VP, the firm's indifference condition, given the creditors' strategy, is

$$(1 - \gamma) [1 - \Phi(\sqrt{\beta}(x^* - \theta_2^*))] F + [\gamma + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \theta_2^*))] f(\theta_2^*) = \frac{pV - qH}{p - q}. \quad (13)$$

Let's consider a creditor  $i$ 's decision. With his private signal  $x_i$ , creditor  $i$ 's interim belief about  $\theta$  given the CRA's rating strategy  $\theta_1^*$  would be a normal distribution with mean  $x_i$  and precision  $\beta$ , truncated below by  $\theta_1^*$ . This truncation is due to the creditors' belief about the CRA's rating strategy that  $\mathcal{R}(\theta) = p$  if and only if  $\theta \geq \theta_1^*$ . Then, given the firm's strategy, creditor  $i$ 's expected payoff from holding the debt is

$$\frac{\Phi[\sqrt{\beta}(\theta_2^* - x_i)] - \Phi[\sqrt{\beta}(\theta_1^* - x_i)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x_i)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2^* - x_i)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x_i)]} pF.$$

Because refraining from investing in the debt always brings a creditor a payoff 1, a marginal creditor with the private signal  $x^*$  must have

$$\frac{\Phi[\sqrt{\beta}(\theta_2^* - x^*)] - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2^* - x^*)]}{1 - \Phi[\sqrt{\beta}(\theta_1^* - x^*)]} pF = 1. \quad (14)$$

**Lemma 3 (Debt Financing Following  $R = p$ )** *There exists a  $\beta^* > 0$ , such that for any  $\beta > \beta^*$ , if an equilibrium exists, given the CRA's rating strategy  $\theta_1^*$ , following the rating  $p$ , there is a unique solution  $(\theta_2^*, x^*)$  with  $\theta_2^* > \theta_1^*$  to equation (13) and equation (14).*

In the analysis of the interaction between the firm and the creditors above, the CRA's rating strategy  $\theta_1^*$  is given. Lemma 4 below shows how the CRA's rating strategy affects the creditors' debt-investment decisions and the firm's moral hazard.

**Lemma 4 (Laxer Rating Strategy)** For any  $\beta > \beta^*$ , both  $x^*$  and  $\theta_2^*$  are strictly decreasing in  $\theta_1^*$ .

When  $\theta_1^*$  is lower, the CRA's rating strategy is laxer. In this scenario, the creditors discount the good rating by raising their debt-investment threshold. Because more creditors refrain from investing in the debt, the firm's financial cost is higher for any  $\theta$ ; as a result, the threshold that the firm chooses VP is also higher.

### 4.3 Equilibrium Rating Strategy

Lemma 3 shows that the creditors' belief about the CRA's rating strategy  $\theta_1^*$  determines the measure of the creditors holding the debt and thus any  $\theta$ -firm's financial cost. On the other hand, when assigning the rating to the  $\theta$ -firm, the CRA, based on the knowledge of  $\theta$  and the creditors' responses to the ratings, can perfectly predict whether the firm will default early or not. Hence, in an equilibrium, the  $\theta_1^*$ -firm must be indifferent between early default and HR. Because of the firm's limited liability, the firm will choose to default early only if the financial cost is higher than  $H$ , the upside return of HR. Therefore, the  $\theta_1^*$ -firm's indifference condition implies

$$(1 - \gamma) \left[ 1 - \Phi(\sqrt{\beta}(x^* - \theta_1^*)) \right] F + \left[ \gamma + (1 - \gamma)\Phi(\sqrt{\beta}(x^* - \theta_1^*)) \right] f(\theta_1^*) = H. \quad (15)$$

Proposition 2 below shows that the model has a unique equilibrium.

**Proposition 2 (Unique Equilibrium)** There is a  $\beta^* > 0$ , such that when  $\beta > \beta^*$ , the model has a unique equilibrium.<sup>17</sup> The equilibrium is characterized by  $(\theta_1^*, \theta_2^*, x^*)$  with  $\theta_2^* > \theta_1^*$ , such that

1. the CRA will assign a rating  $R = p$ , if the firm's fundamentals  $\theta \in [\theta_1^*, +\infty)$ ; and it will assign a rating  $R = 0$ , if the firm's fundamentals  $\theta < \theta_1^*$ ;
2. if  $R = 0$ , no creditor invests in the debt, and the firm defaults at date 1;
3. if  $R = p$ , a creditor invests in the debt if and only if his private signal lands above  $x^*$ , and the firm will choose HR if  $\theta \in [\theta_1^*, \theta_2^*)$  and VP if  $\theta \in [\theta_2^*, +\infty)$ ; and
4.  $(\theta_1^*, \theta_2^*, x^*)$  solves equations (13), (14), and (15).

<sup>17</sup>In the extreme case where  $\beta = +\infty$ , there is no information asymmetry in our model. The CRA can play a role in coordinating creditors' behavior, but there is no credit inflation. The equilibrium outcomes are similar to those of [Boot, Milbourn, and Schmeits \(2006\)](#), except that when the firm's fundamentals are extremely bad, no creditor invests in the debt, and the firm defaults early.

The equilibrium uniqueness first arises from the creditors' new dominant region of not investing, generated by the credit rating  $p$ . From Lemma 2, because the CRA aims to maximize the nominal rating, it will assign the rating  $p$  to the firm that has the fundamentals just above  $\theta_1^*$  and thus will invest in HR. Consequently, when the creditors receive very negative signals, they believe that the firm invests in HR, and so they will not invest in the debt. Hence, it is impossible for all creditors to hold the debt in an equilibrium, and thus, creditors have a unique best response to the rating  $p$ .

The equilibrium uniqueness also arises from the monotonic effect of the creditors' belief about the CRA's rating strategy  $\theta_1^*$  on their best responses. Lemma 4 shows that if the creditors believe that the CRA employs a rating strategy with a higher  $\theta_1^*$ , they best-respond by decreasing the debt-investment threshold, and any  $\theta$ -firm's financial cost decreases. This is because a tighter rating strategy is more informative and thus is discounted less by the creditors. Since the bank credit line financing cost decreases in the firm's fundamentals,  $\theta_1^*$ -firm's financial cost is strictly decreasing as  $\theta_1^*$  increases. As a result, there will be a unique  $\theta_1^*$ , with which the firm is just indifferent between early default and HR, which, together with creditors' unique best response to the rating  $p$ , implies the equilibrium uniqueness.

Proposition 2 provides us with a clear measure of equilibrium rating inflation. When  $\theta < \theta_1^*$ , the CRA will assign the rating 0 to the firm. Since the firm will default early, the credit rating truly reflects the firm's credit quality. When  $\theta \geq \theta_2^*$ , the firm's fundamentals are sufficiently good, and so it will invest in VP. Hence, the credit rating  $p$  also equals the firm's credit quality in this case. However, when  $\theta \in [\theta_1^*, \theta_2^*)$ , the firm invests in HR and thus has the credit quality  $q$ , but it receives the high rating  $p$ . Then, the credit ratings assigned to such firms are inflated, and the rating inflation is measured by  $\theta_2^* - \theta_1^*$ .

## 5 The CRA's Real Effects

We are now in a position to analyze the CRA's real effects. For a given  $\theta$ -firm, if the assigned credit rating changes its investment (comparing to its investment in the benchmark model without a CRA), we say that the CRA has *real effects* on the  $\theta$ -firm. Such effects are *positive*, if the CRA leads the  $\theta$ -firm to choose an investment with higher social welfare; conversely, if the CRA leads the  $\theta$ -firm to choose an investment with lower social welfare, the CRA's real effects on the  $\theta$ -firm are *negative*. The CRA's ex-ante real effects are then measured by the total change of the ex-ante social welfare. Hence, the ex-ante real effects of the CRA are positive (negative) if the social welfare is higher (lower) with the CRA.

From Proposition 2, we can see that the CRA affects a firm's investment decision through two interacting channels. On the one hand, by assigning the rating  $R = p$ , the CRA separates firms with  $\theta \geq \theta_1^*$  from those with  $\theta < \theta_1^*$ . Hence, the rating  $R = p$  provides creditors with new information about the firm's fundamentals. Such new information affects creditors' debt-investment decisions, and thus the firm's financial cost and investment. We call such effects the CRA's "informational effects."

On the other hand, the CRA strategically chooses  $\theta_1^*$  to pool firms investing in HR with those investing in VP. Hence, the set of types of the firm that invest in either HR or VP may differ in cases with and without a CRA. This also affects firms' investment decisions. We call such effects the CRA's "feedback effects," since the CRA, when choosing  $\theta_1^*$ , takes into account the creditors' and the firm's best responses to the ratings.

In this section, we first compare the firm's equilibrium investment decision in the model with the CRA to those in the benchmark model. Such a comparison shows the CRA's real effects. And then, we decompose the CRA's real effects into its informational effects and its feedback effects to get a full understanding of its real effects.

## 5.1 The CRA's Real Effects

Lemma 5 below shows that, with the CRA, both the firm's early default threshold and VP-investing threshold are lower than those in the benchmark model without a CRA.

**Lemma 5** *Comparing the equilibrium of the model with a CRA (described in Proposition 2) to that of the benchmark model without a CRA (described in Proposition 1), we have  $\theta_1^* < \tilde{\theta}_1$ ,  $\theta_2^* < \tilde{\theta}_2$ , and  $x^* < \tilde{x}$ . However, the sign of  $\theta_2^* - \tilde{\theta}_1$  is undetermined.*

When  $\theta_2^* > \tilde{\theta}_1$ , there are two cases of the CRA's real effects. First, when  $\theta \in [\theta_1^*, \tilde{\theta}_1)$ , without the CRA, the firm's financial cost is so high that it will default early; but when the CRA is present, the firm is assigned the rating  $p$ , which reduces its financial costs. Such a decrease in the financial costs encourages the firm to gamble for resurrection, rather than default early, which shows the CRA's negative real effects. Second, when  $\theta \in [\theta_2^*, \tilde{\theta}_2)$ , because the high rating  $p$  helps the firm reduce financial costs, the firm switches from HR to VP, which implies the CRA's positive real effects.

The CRA's real effects in the case with  $\theta_2^* > \tilde{\theta}_1$  is illustrated in Figure 2 below.

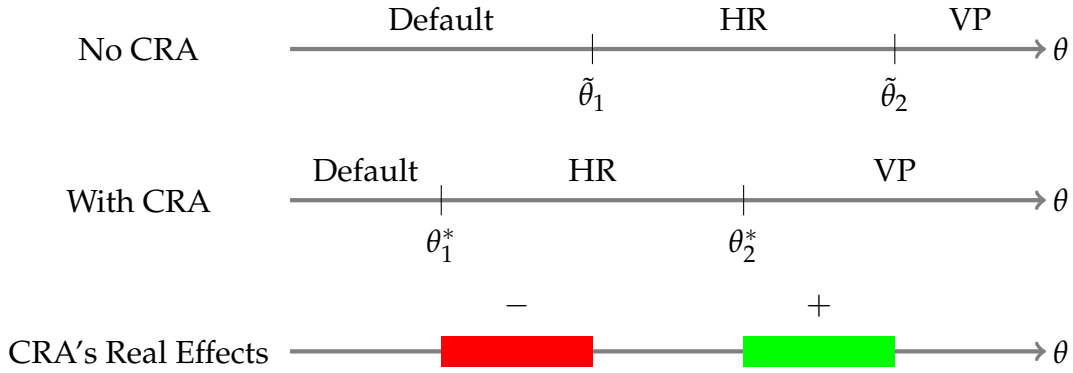


Figure 2: CRA's real effects when  $\theta_2^* > \tilde{\theta}_1$ .

When  $\theta_2^* \leq \tilde{\theta}_1$ , the CRA's real effects are similar. Proposition 3 below formally summarizes the CRA's real effects.

**Proposition 3** *There are two cases of the analysis of the CRA's real effects.*

1. If  $\theta_2^* > \tilde{\theta}_1$ , the CRA has positive real effects when  $\theta \in [\theta_2^*, \tilde{\theta}_2)$  and has negative real effects when  $\theta \in [\theta_1^*, \tilde{\theta}_1)$ ; hence, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\tilde{\theta}_1 - \theta_1^*)(qH - B).$$

2. If  $\theta_2^* \leq \tilde{\theta}_1$ , the CRA has positive real effects when  $\theta \in [\theta_2^*, \tilde{\theta}_2)$  and has negative real effects when  $\theta \in [\theta_1^*, \theta_2^*)$ ; hence, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\theta_2^* - \theta_1^*)(qH - B).$$

Importantly, Proposition 3 shows that the CRA who employs an inflated rating strategy in the equilibrium may have positive or negative real effects, depending on the firm's fundamentals. The CRA's ex-ante real effects, then, depend on the model's parameters. In Figure 3 below, we numerically show that the CRA's ex-ante real effects as a function of the upside return of the risky project,  $H$ .

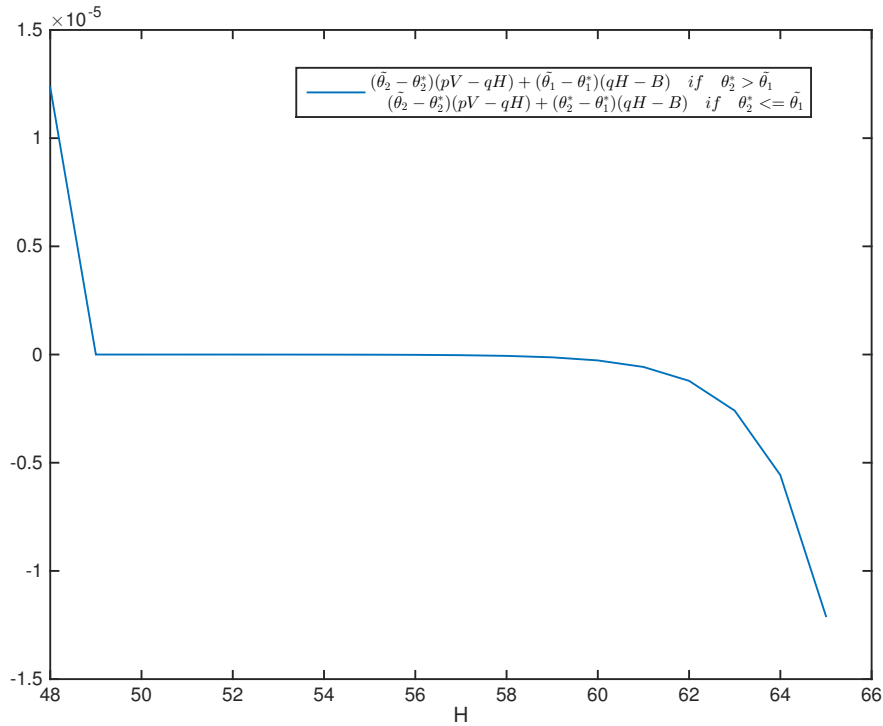


Figure 3: The CRA's Real Effects as a Function of  $H$

Figure 3 shows that when the upside return of the risky project is relatively high, the CRA's real effects are negative. This is because, as shown in Proposition 6 below, when  $H$  is large, the firm has stronger incentives to take risks by investing in HR and thus is less likely to default early. The CRA then will assign more firms the high rating  $R = p$ , which allows those firms to gamble for resurrection, and so have negative real effects. When  $H$  is relatively small, the CRA encourages more firms to switch from HR to VP and thus has positive real effects.

## 5.2 Informational Effects and Feedback Effects

As we have argued, the CRA's real effects can be decomposed into two components: the informational effects, because the CRA provides creditors with new information, and the feedback effects, because the CRA strategically chooses its rating rule, taking advantage of the feedback between credit ratings and firms' investments. We are now analyzing how these two effects interact with each other and determine the CRA's real effects.

We first analyze the CRA's informational effects. Let's consider the case in which the CRA



commits to the following rating strategy:

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if } \theta < \hat{\theta}_1 \equiv \tilde{\theta}_1; \\ p, & \text{if } \theta \geq \hat{\theta}_1. \end{cases} \quad (16)$$

Here,  $\hat{\theta}_1$  equals  $\tilde{\theta}_1$ , which is part of the equilibrium characterized in Proposition 1.

In such a case, the CRA does not behave strategically, though such a rating strategy may still be inflated. Indeed, the committed rating strategy just reflects the firm's investment decision when there is no CRA. So, for ease of exposition, we call such a CRA a "reflecting CRA" and the CRA analyzed in Section 4 a "strategic CRA." Importantly, a reflecting CRA does not have feedback effects, because it does not take into account its effects on the firm's investment decision when committing to its rating strategy. Therefore, the real effects of the reflecting CRA is purely the informational effects. Then, by comparing the strategic CRA's real effects and the reflecting CRA's real effects, we can get the strategic CRA's feedback effects.

Proposition 4 shows the firm's equilibrium investment decision in the case with the reflecting CRA.

**Proposition 4** *Given the committed rating strategy in equation (16), the generated credit ratings lead to two subgames. In particular,*

1. *in the subgame following  $R = 0$ , there is a unique equilibrium in which the firm defaults at date 1; and,*
2. *in the subgame following  $R = p$ , in any equilibrium, the  $\theta$ -firm invests in VP if  $\theta \geq \hat{\theta}_2$  and invests in HR if  $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$ , where  $\hat{\theta}_2 \geq \hat{\theta}_1$ .*

Now, let's analyze the reflecting CRA's real effects, which are also the strategic CRA's informational effects. Proposition 4 shows that with the rating  $p$  assigned by a reflecting CRA, if  $\theta > \tilde{\theta}_2$  (which is strictly greater than  $\hat{\theta}_2$ ), the firm invests in VP, in both the case with a reflecting CRA and the case without a CRA. Therefore, for any  $\theta > \tilde{\theta}_2$ , the reflecting CRA does not have real effects. Same arguments show that the reflecting CRA does not have any real effects when  $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$

However, the reflecting CRA will change the firm's investment decision when  $\theta \in [\hat{\theta}_2, \tilde{\theta}_2)$ . In particular, without a CRA, the firm invests in HR, but with a reflecting CRA, the firm will invest in VP. Therefore, the reflecting CRA has positive real effects, which are measured by

$(\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)$ . That is, the strategic CRA's informational effects are always positive, precisely because its rating  $R = p$ , though potentially inflated, provides creditors with an informative signal and thus correctly guides creditors' debt-investment decisions and the firm's investments.

Figure 4 below illustrates the reflecting CRA's real effects, or equivalently the strategic CRA's informational effects.

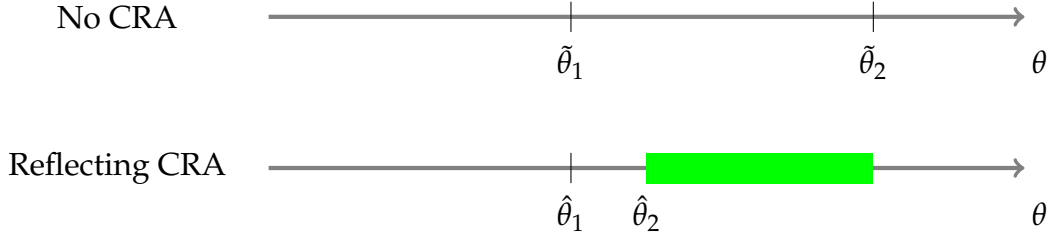


Figure 4: CRA's informational effect.

Finally, let's investigate the strategic CRA's feedback effects. Similarly to Proposition 3, there are two cases:  $\theta_2^* \geq \hat{\theta}_1$  and  $\theta_2^* < \hat{\theta}_1$ . In both cases, the strategic CRA's feedback effects have a negative component. The strategic CRA knows that it can assign more types of the firm the high rating  $R = p$ , because with the rating  $R = p$ , more creditors will invest in the debt, the firm's financial cost decreases, and thus it will not default early. That is, in an equilibrium, the strategic CRA will employ the rating strategy with the threshold  $\theta_1^* < \min\{\hat{\theta}_1, \theta_2^*\}$ . Such a manipulation leads firms with  $\theta \in [\theta_1^*, \min\{\hat{\theta}_1, \theta_2^*\})$  to gamble for resurrection, and thus leads to adverse real effects.

In the case with  $\theta_2^* \geq \hat{\theta}_1$ , the strategic CRA's real effects have another negative component. Because  $\theta_1^* < \hat{\theta}_1$ , the rating  $R = p$  assigned by the strategic CRA is less informative than the rating  $R = p$  assigned by the reflecting CRA. So, with the strategic CRA, after the rating  $R = p$ , less creditors invest in the debt (comparing to the case with the reflecting CRA), the firm's financial cost increases, and thus less types of firms (measured by  $\hat{\theta}_2 - \theta_2^*$ ) switch from HR to VP. That is, in the case with  $\theta_2^* > \hat{\theta}_1$ , the strategic CRA's rating strategy will weaken its informational effects. Figure 5 below illustrates the strategic CRA's feedback effects in this case.

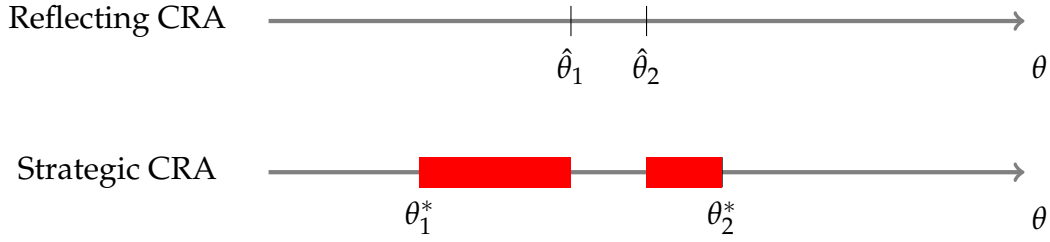


Figure 5: CRA's feedback effect when  $\theta_2^* > \tilde{\theta}_1$ .

In the other case with  $\theta_2^* < \hat{\theta}_1$ , the second component of the strategic CRA's feedback effects is positive. This is because by assigning the rating  $R = p$  to the firm with  $\theta \in [\theta_1^*, \hat{\theta}_1)$ , it is possible for the firm to invest in VP. Indeed, when  $\theta \in [\theta_2^*, \hat{\theta}_2)$ , the firm invests in VP, implying that the strategic CRA has positive feedback effects.

The above arguments are summarized in Proposition 5 below.

**Proposition 5** *The strategic CRA's real effects can be decomposed into its informational effects and its feedback effects. The strategic CRA's informational effects, which are measure by  $(\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)$ , are always positive. When the set of parameters are such that  $\theta_2^* \geq \hat{\theta}_1$ , the strategic CRA's feedback effects, measured by*

$$(\tilde{\theta}_1 - \theta_1^*) (qH - B) + (\theta_2^* - \hat{\theta}_2)(qH - pV),$$

*are negative. When the set of parameters are such that  $\theta_2^* < \hat{\theta}_1$ , the strategic CRA's feedback effects are measured by*

$$(\theta_2^* - \theta_1^*) (qH - B) + (\hat{\theta}_2 - \theta_2^*)(pV - qH),$$

*whose sign is undetermined.*

Proposition 5 implies that credit rating inflation itself does not necessarily lead to negative real effects. In contrast, because inflated ratings are still informative signals, which can increase the market's efficiency and lead to positive real effects. The negative real effects, however, arise from the CRA's feedback effects. Because the CRA knows that the rating will reduce the firm's financial costs and default likelihood, it will assign more firms the high rating, providing them with the opportunities to gamble for resurrection.

## 6 Empirical Predictions

The theory we develop in this paper provides several new empirical predictions about CRAs' rating strategies and credit rating inflation. In this section, we analyze how a CRA's rating strategy and the rating inflation vary when the economic environment changes. That is, we perform comparative static analysis to provide empirical predictions about CRAs' rating strategies and credit rating inflation.

From these comparative static analysis, we show that laxer credit rating strategies are not necessarily accompanied by higher rating inflation. Intuitively, in our model, both the CRA's rating strategy (measured by  $\theta_1^*$ ) and the credit rating inflation (measured by  $\theta_2^* - \theta_1^*$ ) are endogenously determined. Then, an exogenous economic environment changes may lead to a laxer rating strategy and a lower financial cost to the firm at the same time. While the former effect may increase the rating inflation, the latter effect encourages the firm to invest in VP, which reduces the rating inflation. Therefore, whether a laxer rating strategy is accompanied by higher rating inflation depends on which effect dominates. This is related to recent empirical findings. Indeed, [Alp \(2013\)](#) and [Baghai, Servaes, and Tamayo \(2014\)](#) find that CRAs become more conservative by using stricter rating standards (strategies); however, the stricter rating strategies do not reduce credit rating inflation, as shown by [Strobl and Xia \(2012\)](#).

**Proposition 6** *When  $\beta$  is sufficiently large,<sup>18</sup> a decrease in  $\beta$ , an increase in  $H$ , and a decrease in  $\gamma$  will all lead to a decrease in  $\theta_1^*$ . However, a decrease in  $\beta$  has ambiguous effects on  $\theta_2^* - \theta_1^*$ , an increase in  $H$  increases  $\theta_2^* - \theta_1^*$ , and a decrease in  $\gamma$  decreases  $\theta_2^* - \theta_1^*$ .*

First,  $\beta$  is the precision of creditors' private signals, so it measures the firm's transparency. Proposition 6 shows that for more opaque firms, the CRA employs laxer rating strategies. By the properties of the truncated normal random variable's mean, when creditors' private signals become less precise, they believe that the firm is more likely to invest in VP. As a result, more creditors invest in the debt, and the firm's financial cost decreases, which allows the CRA to employ a laxer rating strategy. Equivalently, this comparative static analysis shows that when creditors' private signals are more precise, the CRA will employ a stricter rating standard. This is consistent with a recent empirical finding in [Fong, Hong, Kacperczyk, and Kubik \(2014\)](#) that security analysts can discipline CRAs by providing creditors with more information.

While Proposition 6 implies that CRAs employ laxer rating strategies for more opaque firms, it does not imply that credit ratings assigned to more opaque firms are more inflated.

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<sup>18</sup>This is the condition for the equilibrium uniqueness, which is critical for comparative static analysis.

Since creditors will decrease their debt-investment threshold when the firm is more opaque, the firm's financial cost is lower, causing a smaller  $\theta_2^*$ , which is the firm's VP investment threshold. Consequently, when the creditors' signals are less precise, the CRA is more likely to assign the rating  $p$  to the firm, but the firm with a high rating is more likely to invest in VP. As a result, whether the rating inflation for a more opaque firm is higher or lower depends on which of these two effects dominate. This in turn depends on other parameters of the model.

Second, cross-sectionally, firms differ in the upside returns of their available projects. Equation (15) suggests that the highest upside return among all available projects may determine the credit rating assigned to the firm. Hence, it is interesting to consider how the firm's upside return from HR affects the CRA's rating strategy. An increase in  $H$  does not directly affect the creditors' behavior, because the creditors' payoffs are solely determined by the debt contract, which does not involve the cash flow to the firm, conditional on the success of the investment. Yet,  $H$  has direct effects on both the firm's investment and the CRA's rating strategy. On the one hand, an increase in  $H$  increases the firm's incentives to invest in HR, because the expected return from the HR is higher. On the other hand, an increase in  $H$  decreases the firm's incentives to default early, because the firm has limited liability. As a result, for fixed strategies of the creditors, when  $H$  increases, the CRA's rating strategy will be laxer, and the firm is more likely to invest in HR rather than VP, resulting in higher credit rating inflation.

Finally, suppose that the debt market liquidity increases; that is,  $\gamma$  decreases. A direct effect is that the firm's financial costs will surely decrease, because the firm needs to finance less money from the expensive bank credit line. In addition, a decrease in  $\gamma$  will lead more creditors to hold the debt, due to the strategic complementarities. This further reduces the firm's financial costs. Then, the firm's threshold of investing in VP will decrease, implying that less firms will invest in HR given the CRA's credit rating strategy. In the meanwhile, the lower financial cost of the firm means that less firms may default early. As a result, the CRA would want to employ a laxer rating strategy. Furthermore, as  $\gamma$  decreases, the measure of firms that shift from HR to VP due to the reduced financial cost is greater than the measure of firms that gamble for resurrection because of the high credit rating, leading to lower credit rating inflation.

## 7 Discussions

In our model, the CRA maximizes the nominal rating for each  $\theta$ -firm, and the credit ratings are only regulated by the partial verifiability constraint. That is, unless the firm defaults early, the CRA that assigns a positive rating to the firm will not be punished. However, people may

argue that conditional on the firm's investment's failure, the CRA incurs a higher reputation cost when assigning the rating  $p$  than when assigning the rating  $q$ . In this section, we first analyze how credit ratings' real effects change, when the CRA that assigns the rating  $p$  incurs a higher reputation cost, if the firm's investment fails. This analysis not only generalizes our results in the core model, but also provides an important potential policy to regulate the credit rating industry.

We have also shown that the coordination and heterogeneous information among creditors play critical roles in the inflated credit rating's real effects. In this section, we further study the role of coordination and heterogeneous information among creditors by assuming that there is only one creditor with a precise public signal. We find that when the public signal is sufficiently precise, the CRA will have zero real effect, demonstrating the important role of creditors' coordination in the CRA's real effects.

## 7.1 Reputation Costs when the Firm's Investment Fails

In this subsection, we consider the CRA's reputation cost conditional on the failure of the firm's investment. We assume that the CRA's benefits by providing the rating services are increasing in the ratings, due to the issuer-pays business model. Therefore,  $V^p > V^q > V^0 = 0$ , where  $V^R$  is the CRA's benefit from assigning the rating  $R$ , and we normalize the benefit from assigning the rating  $R = 0$  to be zero. If the firm does not default (either at date 1 or at date 2), the CRA will not incur any cost. However, if the firm defaults early, the CRA will incur a cost greater than  $V^p$ , because the early default is verifiable.<sup>19</sup> Importantly, if the firm invests, and the investment fails at date 2, the CRA incurs a cost  $C^R$ , if it assigns the rating  $R$ . Here,  $C^p > C^q > 0$ .

Given creditors' belief about the CRA's rating strategy and their best responses to credit ratings, the CRA can perfectly predict the firm's investment. Lemma 6 below shows that, the CRA's equilibrium rating strategy depends on the ratio of the benefit increment to the cost increment due to the rating upgrading from  $q$  to  $p$ .

**Lemma 6** *The CRA's equilibrium rating strategy depends on the ratio  $(V^p - V^q) / (C^p - C^q)$ . There are three cases.*

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<sup>19</sup>This is a general form of the partial verifiability constraint assumed in the core model. More importantly, the extension analyzed in this section shows that our analysis of the core model can be extended to the case with several possible projects and multiple rating categories, in which credit rating inflation may occur at some specific rating categories but not at others.

1. if  $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$ , the equilibrium rating strategy is in the form

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta'; \\ 0, & \text{if } \theta < \theta'. \end{cases} \quad (17)$$

2. if  $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$ , the equilibrium rating strategy is in the form

$$\mathcal{R}(\theta) = \begin{cases} q, & \text{if } \theta \geq \theta'; \\ 0, & \text{if } \theta < \theta'. \end{cases} \quad (18)$$

3. if  $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$ , the equilibrium rating strategy is in the form

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta'; \\ q, & \text{if } \theta \in [\theta'', \theta'); \\ 0, & \text{if } \theta < \theta'', \end{cases} \quad (19)$$

where  $\theta'' \leq \theta'$ .

Lemma 6 shows that if the benefit of upgrading the rating from  $q$  to  $p$  is high enough (relative to the change of the cost), the CRA will employ an inflated rating strategy in an equilibrium. This is actually the case in our core model, so the equilibrium will be the same as described in Proposition 2, and credit ratings' real effects are the same as in Proposition 3.

When the benefit of upgrading the rating from  $q$  to  $p$  (relative to the cost) is low enough, the CRA will employ a *deflated* rating strategy in an equilibrium. Since such a rating strategy will not change creditors' belief (following the rating  $q$ ), the equilibrium characterization is very similar to that in Proposition 2, and credit ratings' real effects are the same as in Proposition 3.

When the ratio of the benefit increment to the cost increment due to the rating upgrading from  $q$  to  $p$  is between  $1 - p$  and  $1 - q$ , the CRA will assign a rating, which reflects the firm's true investment choice. In this case, the CRA is "self-disciplined." We now analyze a self-disciplined CRA's equilibrium rating behavior in more details.

Suppose that the CRA's rating is  $R = 0$ . Since the CRA is self-disciplined, all creditors believe that the firm will surely default early. Then, after  $R = 0$ , no creditor will invest in the debt, leading to the firm's financial cost  $K(\theta) = f(\theta)$ . Thus, the firm chooses to default early following  $R = 0$  if and only if

$$f(\theta) \geq H. \quad (20)$$

Let  $z_0$  be the solution to equation (20). Then, following the rating  $R = 0$ , the firm will default early if  $\theta \leq z_0$ .

Suppose  $R = q$ ; then, in the equilibrium, all creditors believe that the firm will invest in HR. Because  $qF < 1$ , no creditor invests in the debt. Thus, we also have the financial cost  $K(\theta) = f(\theta)$ . For the firm to choose HR, we must have

$$\frac{pV - qH}{p - q} \leq f(\theta) < H. \quad (21)$$

Denote the solution to the equation  $\frac{pV - qH}{p - q} = f(\theta)$  by  $z_q$ . Because  $f(\theta)$  is decreasing, and  $H > \frac{pV - qH}{p - q}$ , we have  $z_q > z_0$ . Then, the firm invests in HR following the rating  $R = q$ , if and only if  $\theta \in (z_0, z_q]$ .

Finally, suppose  $R = p$ ; then, in the equilibrium, all creditors invest in the debt, because they believe that the firm will invest in VP. This implies that the firm's financial cost is

$$K(\theta) = (1 - \gamma)F + \gamma f(\theta).$$

Denote the solution to the equation  $(1 - \gamma)F + \gamma f(\theta) = \frac{pV - qH}{p - q}$  by  $z_p$ . Then, the firm invests in VP following the rating  $R = p$ , if and only if  $\theta \geq z_p$ . Note that  $(1 - \gamma)F + \gamma f(\theta) < f(\theta)$  for all  $\theta$ . Therefore,  $z_q > z_p$ .

The arguments above prove Proposition 7 below, which characterizes the equilibrium of the model with a self-disciplined CRA.

**Proposition 7** *When  $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$ , the model has a unique equilibrium. In particular, if  $z_q > z_p > z_0$ , the equilibrium rating strategy is*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if } \theta \leq z_0; \\ q, & \text{if } \theta \in (z_0, z_p]; \\ p, & \text{if } \theta \in (z_p, +\infty). \end{cases} \quad (22)$$

*If  $z_p < z_0 < z_q$ , the equilibrium rating strategy is*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if } \theta \leq z_p; \\ p, & \text{if } \theta > z_p. \end{cases} \quad (23)$$

Proposition 7 has great policy implications. Because both the credit rating and the firm's investment outcome are verifiable, the government can design a cost scheme  $(C^p, C^q)$ , such



that  $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$ , to punish the CRA when the firm's investment fails. Such a cost scheme can make the CRA "self-disciplined" and thus eliminate any inefficiency caused by the "issuer-pays" business model. Specifically, equation (23) shows that when  $z_p < z_0$ , the CRA will assign the rating 0 to weak firms; then, no creditor invests in the debt, the firm defaults early, and thus the CRA has no real effects. In contrast, the CRA assigns the rating  $p$  to other firms, leading them to choose the socially optimal investment project.

However, to design the appropriate cost scheme  $(C^p, C^q)$ , the government has to know the CRA's benefits from ratings,  $(V^p, V^q)$ . While such information may not be available to the government, Lemma 6 at least shows that imposing too high costs is not a correct policy, because this may lead the CRA to employ a deflated rating strategy, which will have the same real effects as those of an inflated rating strategy.

## 7.2 Creditors' Coordination

In this section, we show how creditors' coordination and belief dispersion play roles in determining the CRA's real effects. We analyze an environment in which there is only one creditor in the debt market. We assume that there is a precise public signal about  $\theta$ , and the creditor does not receive any private signal.<sup>20</sup>

Formally, let's assume that the public signal leads to the belief  $\theta \sim \mathcal{N}(\theta_s, \alpha^{-1})$ . We will consider the case when  $\alpha$  is sufficiently large. Assume that the creditor has  $1 - \gamma$  dollars. Then, the firm's financial cost of continue the investment will be

$$K(\theta) = \begin{cases} (1 - \gamma)F + \gamma f(\theta), & \text{if the creditor holds the debt;} \\ f(\theta), & \text{if the creditor does not hold the debt.} \end{cases}$$

When the creditor invests in the debt, the firm's optimal investment choice is

$$\begin{cases} \text{Default early,} & \text{if } (1 - \gamma)F + \gamma f(\theta) > H; \\ \text{HR,} & \text{if } (1 - \gamma)F + \gamma f(\theta) \in \left( \frac{pV - qH}{p - q}, H \right]; \\ \text{VP,} & \text{if } (1 - \gamma)F + \gamma f(\theta) \leq \frac{pV - qH}{p - q}. \end{cases}$$

Denote the solution to the equation  $(1 - \gamma)F + \gamma f(\theta) = H$  by  $y_1$  and that to the equation  $(1 - \gamma)F + \gamma f(\theta) = (pV - qH)/(p - q)$  by  $y_2$ . The firm's optimal investment choice when the

<sup>20</sup>In the core model, we assume for simplicity an improper uniform prior. This assumption is without loss of generality, because we consider the case in which creditors' private signals are sufficiently precise. Adding a normal prior belief to the core model will not change the results, but then the ex-ante economic effects of the CRA depend on the prior mean of the firm's fundamentals.

creditor invests in the debt can be written as

$$\begin{cases} \text{Default early,} & \text{if } \theta < y_1; \\ \text{HR,} & \text{if } \theta \in [y_1, y_2); \\ \text{VP,} & \text{if } \theta \geq y_2. \end{cases}$$

Similarly, we denote the solution to the equation  $f(\theta) = H$  by  $y'_1$  and that to the equation  $f(\theta) = (pV - qH)/(p - q)$  by  $y'_2$ . The firm's optimal investment choice when the creditor holds the debt can be written as

$$\begin{cases} \text{Default early,} & \text{if } \theta < y'_1; \\ \text{HR,} & \text{if } \theta \in [y'_1, y'_2); \\ \text{VP,} & \text{if } \theta \geq y'_2. \end{cases}$$

Because  $f(\theta) > (1 - \gamma)F + \gamma f(\theta)$  for any  $\theta$ , we have  $y_1 < y'_1$  and  $y_2 < y'_2$ . When  $\alpha$  is sufficiently large, the creditor will mainly rely on the public signal to make the debt-investment decision. Because the creditor's behavior determines the firm's investment choice, the public signal and the creditor's behavior determine the CRA's credit rating. Proposition 8 below shows the equilibrium credit rating strategy when there is only one creditor with a precise public signal.

**Proposition 8 (Credit Ratings with One Creditor)** *There exists  $\bar{\alpha} > 0$ , such that for all  $\alpha > \bar{\alpha}$ , the public signal determines the CRA's equilibrium rating strategy. Specifically,*

1. *when  $\theta_s \geq y'_2$ , the CRA will employ the rating strategy  $\theta_1^* = y_1$ ;*
2. *when  $\theta_s < y_2$ , the CRA will employ the rating strategy  $\theta_1^* = y'_1$ ; and*
3. *when  $\theta_s \in [y_2, y'_2)$ , the CRA will set  $\theta_1^* = y_1$  if the creditor invests in the debt after  $R = p$ , while the CRA will set  $\theta_1^* = y'_1$  if the creditor does not invest in the debt after  $R = p$ .*

Significantly, we observe in Proposition 8 that, when the public signal is very positive ( $\theta_s \geq y'_2$ ), the CRA employs a laxer rating strategy, meaning that the good rating is a less positive signal. When the public signal is very negative ( $\theta_s < y_2$ ), the CRA employs a stricter rating strategy, meaning that the good rating is a more positive signal. Such a "substitution" results from the creditor relying more heavily on the public signal when making decisions. When the public signal is in the medium range, there will be multiple equilibria: if the creditor invests in the debt, the CRA will employ a laxer credit rating strategy; and if the creditor refrains from investing in the debt, the CRA will employ a stricter rating strategy.

It follows from Proposition 8 that, when there is only one creditor with a precise public signal, the CRA does not have real effects, because the creditor will ignore the information extracted from the credit ratings. This shows the importance of the creditors' coordination in our core model, where the CRA has significant real effects.

## 8 Conclusion

We study credit rating agencies' effects on firms' investments. We show that high credit ratings, though commonly known to be potentially inflated, exclude extremely bad firms from the creditors' belief supports. Therefore, high ratings make the creditors more optimistic, reduce firms' financial costs, and thus change firms' investments. That is, even in an environment with perfectly rational and well-informed creditors, inflated ratings still have significant real effects.

Such real effects, however, could be positive or negative. With the high ratings, some firms take risky projects instead of default efficiently, implying the CRA's adverse real effects; but some other firms will switch from risky investments to safe investments, showing the CRA's positive real effects. The CRA's ex-ante real effects, then, depend on the economic environment. Specifically, when the upside return of the risky project is high, the CRA's ex-ante real effects are negative. This provides a potential explanation for a long-lasting puzzle: while the conflicts of interest are recognized for a long time, they didn't attract much attention until the 2007-2009 subprime crisis. Our theory implies that because of the high-risk high-return financial products, such as MBSs, the CRAs' ex-ante real effects are negative, putting them in criticisms.

We further decompose the CRA's real effects into its informational effects and its feedback effects. We show that credit ratings that act as new informative signals do positively affect firms' investments. Hence, the CRA's negative real effects arise from its feedback effects. Indeed, the CRA takes advantage of the feedback between credit ratings and firms' investments to assign high ratings to more firms, providing those firms with chances to gamble for resurrection. Such a manipulation leads to negative real effects.

We emphasize that credit rating standards and credit rating inflation are two different concepts, and they are both endogenously determined. Therefore, changes of economic environments that lead to laxer rating strategies do not necessarily cause higher rating inflation.

Our paper offers applied and theoretical contributions. From the applied perspective, we provide a rational framework, enabling us to analyze credit rating inflation and credit rating agencies' real effects. Our model generates several testable empirical predictions and some reasonable policy suggestions. The partial verifiability constraint can be applied to many other

scenarios, such as financial advising, auditing, marketing, and academic recommendation.

From the theoretical perspective, we also analyze an expert information disclosure model with multiple audiences, who have coordination incentives and dispersed beliefs. More importantly, the expert's message will endogenously affect the fundamentals signaled by the message, which may motivate new research on general theoretical disclosure models.

## A Proofs of lemmas and propositions

*Proof of Proposition 1:*

To show there is a unique equilibrium in this benchmark model, we only need to show that there is a unique solution  $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$  to equations (8), (9), and (10).

We first solve  $\tilde{x}$  from equation (9). Define

$$\tilde{\Delta} = \frac{\frac{pV-qH}{p-q} - [(1-\gamma)F + \gamma f(\tilde{\theta}_2)]}{(1-\gamma)(f(\tilde{\theta}_2) - F)}.$$

Because  $f(\theta)$  is strictly decreasing,  $\tilde{\Delta}$  is strictly increasing in  $\tilde{\theta}_2$ . Then we have

$$\tilde{x} = \tilde{\theta}_2 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\tilde{\Delta}), \quad (24)$$

and  $\tilde{x}$  is strictly increasing in  $\tilde{\theta}_2$ .

Plugging  $\tilde{x}$  as a function of  $\tilde{\theta}_2$  into equation (10), we have

$$\tilde{\Delta}(pF - qF) + \Phi \left[ \sqrt{\beta} \left( \tilde{\theta}_2 - \tilde{\theta}_1 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\tilde{\Delta}) \right) \right] qF = 1. \quad (25)$$

The left-hand side of equation (25) strictly increases in  $\tilde{\theta}_2$  and strictly decreases in  $\tilde{\theta}_1$ . So, we have  $\partial\tilde{\theta}_2/\partial\tilde{\theta}_1 > 0$  and  $\partial\tilde{x}/\partial\tilde{\theta}_1 > 0$ .

Let's finally consider equation (8). The derivative of the left-hand side of equation (8) is

$$\frac{\partial K}{\partial\tilde{\theta}_1} + \frac{\partial K}{\partial\tilde{x}} \frac{\partial\tilde{x}}{\partial\tilde{\theta}_1},$$

where

$$\begin{aligned} \frac{\partial K}{\partial\tilde{\theta}_1} &= \left[ \gamma + (1-\gamma)\Phi \left( \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) \right] f'(\tilde{\theta}_1) \\ &\quad - (1-\gamma)\sqrt{\beta}\phi \left( \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) (f(\tilde{\theta}_1) - F) < 0; \\ \frac{\partial K}{\partial\tilde{x}} \frac{\partial\tilde{x}}{\partial\tilde{\theta}_1} &= (1-\gamma)\sqrt{\beta}\phi \left( \sqrt{\beta}(\tilde{x} - \tilde{\theta}_1) \right) (f(\tilde{\theta}_1) - F) \frac{\partial\tilde{x}}{\partial\tilde{\theta}_1} > 0. \end{aligned}$$

Note that  $\tilde{\Delta}$  is between 0 and 1. From equation (25), we have that as  $\beta \rightarrow +\infty$ ,  $\tilde{\Delta}$  is bounded away from both 0 and 1. To see this, let's first suppose  $\tilde{\Delta} \rightarrow 1$ . Then, the left-hand side of equation (25) goes to  $pF$ , which is greater than 1, the right-hand side of equation (25). Similarly, if  $\tilde{\Delta} \rightarrow 0$ , the left-hand side of equation (25) is strictly less than 1.

Hence, from equation (24),  $\tilde{x} \rightarrow \tilde{\theta}_2$ . In addition, as  $\beta \rightarrow +\infty$ ,  $\tilde{\theta}_2$  cannot converge to  $\tilde{\theta}_1$ ; otherwise, equation (8) and equation (9) cannot hold at the same time. Therefore, as  $\beta \rightarrow +\infty$ ,  $\tilde{x} - \tilde{\theta}_1$  is bounded away from 0. This implies that

$$\lim_{\beta \rightarrow +\infty} \sqrt{\beta} \phi \left( \sqrt{\beta} (\tilde{x} - \tilde{\theta}_1) \right) = 0.$$

Therefore, though  $\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0$ , when  $\beta$  is large enough,  $\frac{\partial K}{\partial \tilde{x}} \frac{\partial \tilde{x}}{\partial \tilde{\theta}_1}$  is very close to 0. For the term of  $\frac{\partial K}{\partial \tilde{\theta}_1}$ , it will not go to 0 as  $\beta$  goes to  $\infty$ , because  $f'(\tilde{\theta}_1) < 0$ . Therefore, there is a  $\tilde{\beta} > 0$ , such that for all  $\beta > \tilde{\beta}$ , the left-hand side of equation (8) strictly decreases in  $\tilde{\theta}_1$ . The left-hand side of equation (8) converges to  $F$  when  $\tilde{\theta}_1$  goes to  $+\infty$  and diverges to  $+\infty$  when  $\tilde{\theta}_1$  goes to  $-\infty$ . Then by the continuity of function  $f(\cdot)$ , there is a unique  $\tilde{\theta}_1$ . Then, there is a unique solution to equation (8), (9), and (10).

*Q.E.D.*

*Proof of Lemma 1:*

We first show that the CRA will not assign the rating  $q$  in an equilibrium. Suppose there is an equilibrium, in which the CRA assigns the rating  $q$  to the  $\theta$ -firm when  $\theta \in (\theta_1, \theta_2)$ . There are two cases. In the first case, there is no  $\theta < \theta_1$  such that the  $\theta$ -firm receives the rating  $p$ . Then, observing the rating  $p$ , creditors are more optimistic about the firm's fundamentals. Since  $f$  is strictly decreasing in  $\theta$ , if the CRA deviates to assign the rating  $p$  to the  $\theta$ -firm when  $\theta \in (\theta_1, \theta_2)$ , more creditors will hold the debt, and the  $\theta$ -firm's financial costs decrease. Hence, because the  $\theta$ -firm does not default early following the rating  $q$ , it does not default early following the rating  $p$  either. So, a rating strategy that assigns the rating  $q$  to the  $\theta$ -firm for  $\theta \in (\theta_1, \theta_2)$  but does not assign the rating  $p$  to a positive measure of firms with  $\theta$  below  $\theta_1$  cannot be an equilibrium rating strategy.

Now, consider the second case in which the rating strategy specifies  $\mathcal{R}(\theta) = p$  when  $\theta \in [\theta_3, \theta_4]$  and  $\mathcal{R}(\theta) = q$  when  $\theta \in (\theta_1, \theta_2)$ , where  $\theta_3 < \theta_4 < \theta_1$ . Due to the partial verifiability constraint,  $\theta_3$ -firm does not default early. Then, if the CRA assigns the rating  $p$  to the  $\theta$ -firm for  $\theta \in (\theta_1, \theta_2)$ , the  $\theta$ -firm's financial cost is lower than  $\theta_3$ -firm's. Therefore, the  $\theta$ -firm does not default early, and the deviation is profitable. These arguments show that the CRA will not assign the rating  $q$  in an equilibrium.

Suppose there is an equilibrium, in which  $\theta$ -firm does not default early. So the CRA will assign the rating  $\mathcal{R}(\theta) = p$ , because the CRA wants to maximize the nominal rating  $\mathcal{R}(\theta)$ . Let

$W(p)$  be the measure of creditors who invest in the debt, after observing the credit rating  $\mathcal{R}(\theta)$  and their own private signals. Then the assumption that  $\theta$ -firm does not default early implies

$$K(\theta) = W(p, \theta)F + (1 - W(p, \theta))f(\theta) < H.$$

Now, let's consider any  $\theta'$ -firm with  $\theta' > \theta$ . Again, because the CRA wants to maximize the nominal rating  $R(\theta')$ , if and only if the  $\theta'$ -firm does not default early, the CRA will assign  $R(\theta') = p$ . In a monotone equilibrium, any creditor  $i$ 's strategy is monotonic in his private signal  $x_i$ , and any creditor's private signal conditional on  $\theta'$  First-order Stochastic dominates that conditional on  $\theta$ . So  $W(p, \theta') > W(p, \theta)$ . Recalling that  $f(\theta) > F$  for all  $\theta$ , we have

$$\begin{aligned} K(\theta') &= W(p, \theta')F + (1 - W(p, \theta'))f(\theta') \\ &< W(p, \theta)F + (1 - W(p, \theta))f(\theta') \\ &< W(p, \theta)F + (1 - W(p, \theta))f(\theta) \\ &< H. \end{aligned}$$

Therefore, if  $\theta$ -firm does not default early,  $\theta'$ -firm does not default early either, implying that in the equilibrium,  $R(\theta') = p$ .

Furthermore, independent of creditors' decisions, when  $\theta$  is very negative, the firm will default early, and when  $\theta$  is very positive, the firm will invest in VP. As a result, in any equilibrium (if exists), the CRA's rating strategy must be in the form described by equation (11).

*Q.E.D.*

*Proof of Lemma 2:*

Suppose there is an equilibrium in which the firm invests in VP for all  $\theta$  such that  $\mathcal{R}(\theta) = p$ . All creditors will invest in the debt, leading to the firm's financial cost

$$\gamma f(\theta) + (1 - \gamma)F.$$

For the firm to choose VP if and only if  $\theta \geq \theta_1^*$ , we must have

$$(1 - \gamma)F + \gamma f(\theta_1^*) \leq \frac{pV - qH}{p - q} < H.$$

But because the  $f$  function is continuous and strictly decreasing, there exists  $\hat{\theta}_1^* < \theta_1^*$  such that,

$$\frac{pV - qH}{p - q} < \gamma f(\hat{\theta}_1^*) + (1 - \gamma)F < H.$$

That is, there is a positive measure subset of  $\theta$ 's that are greater than  $\hat{\theta}_1^*$  but very close to  $\hat{\theta}_1^*$ , the firm will invest in HR. Since the firm's investment choice HR is unverifiable, a deviation to the rating strategy with  $\hat{\theta}_1^*$  is profitable to the CRA. Therefore, the rating strategy with  $\theta_1^*$  such that  $(1 - \gamma)F + \gamma f(\theta_1^*) \leq \frac{pV - qH}{p - q}$  cannot be part of an equilibrium. Therefore, if an equilibrium exists, in an equilibrium, the rating strategy must be inflated.

*Q.E.D.*

*Proof of Lemma 3:*

For a given  $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$ , the left-hand side of equation (13) is strictly decreasing in  $\theta_2^*$ . When  $\theta_2^* \rightarrow +\infty$ , the LHS of equation (13) goes to  $F$ , which is assumed strictly less than  $\frac{pV - qH}{p - q}$ . However, if when  $\theta_2^* = \theta_1^*$ , the LHS is still smaller than  $\frac{pV - qH}{p - q}$ , the firm will always choose VP after the rating  $R = p$ . This contradicts Lemma 2. Therefore, for a given  $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$ , there is a unique  $\theta_2^* > \theta_1^*$ , such that equation (13) holds. Then we can solve for  $x^*$  from equation (13)

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}} \Phi^{-1} \left[ \frac{\frac{pV - qH}{p - q} - ((1 - \gamma)F + \gamma f(\theta_2^*))}{(1 - \gamma) [f(\theta_2^*) - F]} \right].$$

Denote

$$\Delta = \frac{\frac{pV - qH}{p - q} - ((1 - \gamma)F + \gamma f(\theta_2^*))}{(1 - \gamma) [f(\theta_2^*) - F]},$$

so  $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta)$ . Because  $f(\cdot)$  is strictly decreasing,  $\Delta$  is strictly increasing in  $\theta_2^*$ , and thus  $x^*$  is strictly increasing in  $\theta_2^*$ .

Then, plugging  $x^*$  as a function of  $\theta_2^*$  into equation (14), we have

$$\frac{\Delta}{\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right]} (pF - qF) = 1 - qF. \quad (26)$$

Differentiating the left-hand side of equation (26), the sign of this derivative would be the same



as the sign of

$$\begin{aligned}
& \frac{\partial \Delta}{\partial \theta_2^*} \frac{\Phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right]}{\Delta} \\
& \quad - \phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \left( \sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \theta_2^*} \right) \\
& = \frac{\partial \Delta}{\partial \theta_2^*} \frac{pF - qF}{1 - qF} - \phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \left( \sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial \Delta}{\partial \theta_2^*} \right).
\end{aligned}$$

The first term is positive for any  $\beta$ . The second, though is negative, will converge to 0 as  $\beta \rightarrow +\infty$ . This is because  $\phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right]$  will converge to 0 higher order faster than  $\sqrt{\beta}$ . We need to consider three cases to prove this argument. First, as  $\beta \rightarrow +\infty$ ,  $\Delta \rightarrow 1$ . In this case, it is trivially that  $\phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \sqrt{\beta}$  converges to 0. Second, as  $\beta \rightarrow +\infty$ ,  $\Delta$  is bounded away from both 0 and 1. Then  $x^* - \theta_2^* \rightarrow 0$ . But because  $\theta_2^* - \theta_1^*$  is positive and bounded away from 0,  $\phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \sqrt{\beta} = \phi \left[ \sqrt{\beta}(x^* - \theta_1^*) \right] \sqrt{\beta}$  must converge to 0. Finally, as  $\beta \rightarrow +\infty$ ,  $\Delta \rightarrow 0$ . Then from equation (26), we must have  $\Phi \left[ \sqrt{\beta}(x^* - \theta_1^*) \right] \rightarrow 0$  and thus  $\sqrt{\beta}(x^* - \theta_1^*) \rightarrow -\infty$  as  $\beta \rightarrow +\infty$ . By L'Hôpital's rule, we have

$$\lim_{\beta \rightarrow +\infty} \frac{1}{\sqrt{\beta}(x^* - \theta_1^*)} = \lim_{\beta \rightarrow +\infty} \frac{\beta^{-\frac{1}{2}}}{(x^* - \theta_1^*)} = \lim_{\beta \rightarrow +\infty} \frac{1}{2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta}} = 0.$$

Therefore,  $\lim_{\beta \rightarrow +\infty} 2\beta^{\frac{3}{2}} \frac{\partial x^*}{\partial \beta} = +\infty$ . Then, simple algebra can lead to the result that

$\phi \left[ \sqrt{\beta} \left( \theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Delta) \right) \right] \sqrt{\beta}$  converges to 0, as  $\beta \rightarrow +\infty$ . Therefore, there is a  $\beta^*$  such that when  $\beta > \beta^*$ , the left-hand side of equation (26) is strictly increasing in  $\theta_2^*$ .

Note by definition,  $\Delta$  must be a number in  $[0, 1]$ . Therefore, there are  $\bar{\theta}$  and  $\underline{\theta}$  such that,  $\theta_1^* < \underline{\theta} < \bar{\theta} < +\infty$ ,  $\Delta(\bar{\theta}) = 1$ , and  $\Delta(\underline{\theta}) = 0$ . Then when  $\theta_2^* \rightarrow \bar{\theta}$ , the left hand side of equation (26) is strictly greater than  $1 - qF$ ; when  $\theta_2^* \rightarrow \underline{\theta}$ , the left hand side of equation (26) is close to 0 and thus strictly smaller than  $1 - qF$ .

Therefore, there is a unique  $\theta_2^*$ , and thus there is a unique  $x^*$ .

*Q.E.D.*

*Proof of Lemma 4:*

The left-hand side of equation (26) is strictly increasing in  $\theta_1^*$ , fixing  $\theta_2^*$ . Combined with the fact that the left-hand side of equation (26) is strictly increasing in  $\theta_2^*$ , the Implicit Function Theorem implies that  $\theta_2^*$  is strictly decreasing in  $\theta_1^*$ . Since  $x^*$  is strictly increasing in  $\theta_2^*$ ,  $x^*$  is strictly decreasing in  $\theta_1^*$  (given  $\theta_2^*$ ,  $x^*$  is determined by equation  $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta]$ ).

*Q.E.D.*

*Proof of Proposition 2:*

When  $\beta$  is sufficiently large, Lemma 3 shows that, for a fixed  $\theta_1^*$ , there is a unique solution,  $(x^*, \theta_2^*)$ , to equation (13) and equation (14) following  $R = p$ . Then, by Lemma 4, we only need to show that there is a unique  $\theta_1^*$  such that equation (15) holds, given  $x^*$  as a function of  $\theta_1^*$ . When  $\beta$  is sufficiently large, by Lemma 4,  $x^*$  is strictly decreasing in  $\theta_1^*$ . Then, the derivative of the left hand side of equation (15) with respect to  $\theta$  is

$$\begin{aligned} & \left[ \gamma + (1 - \gamma)\Phi[\sqrt{\beta}(x^* - \theta)] \right] f'(\theta) \\ & - (1 - \gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}(f(\theta) - F) + (1 - \gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}\frac{\partial x^*}{\partial \theta} < 0. \end{aligned}$$

We know that when  $\theta \rightarrow +\infty$ ,  $f(\theta) \rightarrow F$ , the left hand side of equation (15) converges to  $F$ , which is less than  $H$ ; when  $\theta \rightarrow -\infty$ ,  $f(\theta) \rightarrow +\infty$ , the left hand side of equation (15) diverges to  $+\infty$ , which is greater than  $H$ . Therefore, the solution to equation (15) exists and is unique.

Because  $H > \frac{pV - qH}{p - q}$ , equation (15) and equation (13) imply that  $\theta_2^* > \theta_1^*$ . In addition, equation (15) also implies that  $f(\theta_1^*) > H$ , because  $f(\theta) > F$  for all  $\theta \in \mathbb{R}$ . These complete the proof of the uniqueness of the equilibrium of the model.

*Q.E.D.*

*Proof of Lemma 5:*

Recall that the three equations determining the equilibrium of the benchmark model are

$$[(1 - \gamma)F + \gamma f(\theta_1)] + (1 - \gamma)\Phi[\sqrt{\beta}(x - \theta_1)](f(\theta_1) - F) = H \quad (27)$$

$$[(1 - \gamma)F + \gamma f(\theta_2)] + (1 - \gamma)\Phi[\sqrt{\beta}(x - \theta_2)](f(\theta_2) - F) = \frac{pV - qH}{p - q} \quad (28)$$

$$\left\{ \Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)] \right\} qF + \left\{ 1 - \Phi[\sqrt{\beta}(\theta_2 - x)] \right\} pF = 1; \quad (29)$$

and the three equations determining the equilibrium of the model with the CRA are

$$[(1 - \gamma)F + \gamma f(\theta_1)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x - \theta_1) \right] (f(\theta_1) - F) = H \quad (30)$$

$$[(1 - \gamma)F + \gamma f(\theta_2)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x - \theta_2) \right] (f(\theta_2) - F) = \frac{pV - qH}{p - q} \quad (31)$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} pF = 1 \quad (32)$$

The difference between the equilibrium in the benchmark model and that in the model with the CRA stems from the difference between equation (29) and equation (32). That is, the creditors' indifference conditions differ. If we change equation (29) by dividing both sides by the term  $1 - \Phi[\sqrt{\beta}(\theta_1 - x)]$ , we have

$$\begin{aligned} & \frac{\Phi[\sqrt{\beta}(\theta_2 - x)] - \Phi[\sqrt{\beta}(\theta_1 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} qF + \frac{1 - \Phi[\sqrt{\beta}(\theta_2 - x)]}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} pF \\ &= \frac{1}{1 - \Phi[\sqrt{\beta}(\theta_1 - x)]} \\ &> 1. \end{aligned} \quad (33)$$

Solve  $x$  as a function of  $\theta_2$  from equation (28) and equation (31), and plug it into equation (33) and equation (32), respectively. Then, for a same  $\theta_1$ ,  $\theta_2$  in equation (33) is greater than that in equation (32), because the left-hand sides of these two equations are strictly increasing in  $\theta_2$ . Hence,  $x$  in equation (33) is greater than  $x$  in equation (32). Furthermore, because  $\theta_1$  in the benchmark model is positively correlated to  $\theta_2$ , while  $\theta_1$  in the model with the CRA is negatively correlated to  $\theta_2$ , we know  $\theta_1^* < \tilde{\theta}_1$ . Moreover, we have  $\tilde{\theta}_2 > \theta_2^*$  and  $\tilde{x} > x^*$ .

However, the sign of  $\theta_2^* - \tilde{\theta}_1$  is undetermined. Consider equation (27) and equation (31). Both  $\tilde{\theta}_1$  and  $\theta_2^*$  are strictly increasing functions of  $x$ . While we have shown that  $\tilde{x} > x^*$ , the right-hand side of equation (27) is greater than that of equation (31). Therefore, without specifying parameters' values, we cannot determine the sign of  $\theta_2^* - \tilde{\theta}_1$ .

*Q.E.D.*

*Proof of Proposition 3:*

We denote by  $\Omega$  and  $\tilde{\Omega}$  the social welfare with and without the CRA, respectively. Suppose the CRA assigns the rating  $p$ . When  $\theta \geq \tilde{\theta}_2$ ,  $\Omega = \tilde{\Omega} = pV - 1$ , so the CRA has no effect on the social welfare.

In case 1 where  $\theta_2^* > \tilde{\theta}_1$ , for  $\theta \in [\theta_1^*, \tilde{\theta}_1)$ ,  $\Omega = qH - 1 < B - 1$ , because  $qH$  is assumed to be less than  $B$ . For  $\theta \in [\theta_2^*, \tilde{\theta}_2)$ ,  $\Omega = pV - 1 > qH - 1 = \tilde{\Omega}$ . For all other  $\theta$ 's, the firm's investment choice is the same in both the benchmark model and the model with the CRA; thus, CRA has no effect on the social welfare for such firms. Therefore, in such a case, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\tilde{\theta}_1 - \theta_1^*)(qH - B).$$

In case 2 where  $\theta_2^* \leq \tilde{\theta}_1$ , for  $\theta \in [\theta_1^*, \theta_2^*)$ ,  $\Omega = qH - 1 < B - 1$ . For  $\theta \in [\theta_2^*, \tilde{\theta}_2)$ ,  $\Omega = pV - 1 > qH - 1 = \tilde{\Omega}$ . For all other  $\theta$ 's, the firm's investment choice is same in both the benchmark model and the model with the CRA; thus, CRA has no effect on the social welfare for such firms. Therefore, in this case, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + (\theta_2^* - \theta_1^*)(qH - B).$$

*Q.E.D.*

*Proof of Proposition 4:*

**Part 1:** We first consider the subgame following the rating  $R = 0$ . It then follows from equation (16) that  $\theta < \hat{\theta}_1 = \tilde{\theta}_1$ . Suppose that no creditor invests in the debt, then the  $\theta$ -firm's financial cost is

$$\begin{aligned} K(\theta) &= f(\theta) \\ &> [(1 - \gamma)F + \gamma f(\theta)] + (1 - \gamma)\Phi\left(\sqrt{\beta}(\tilde{x} - \theta)\right)(f(\theta) - F) \\ &> [(1 - \gamma)F + \gamma f(\hat{\theta}_1)] + (1 - \gamma)\Phi\left(\sqrt{\beta}(\tilde{x} - \hat{\theta}_1)\right)(f(\hat{\theta}_1) - F) \\ &= [(1 - \gamma)F + \gamma f(\tilde{\theta}_1)] + (1 - \gamma)\Phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)(f(\tilde{\theta}_1) - F) \\ &= H. \end{aligned}$$

Hence, if no creditor invests in the debt, the  $\theta$ -firm will default early. On the other hand, given that any  $\theta$ -firm will default early, no creditor will invest in the debt, implying that there is an equilibrium in which the firm will default early when receiving the rating  $R = 0$  assigned by the reflecting CRA.

We now show that the subgame following  $R = 0$  has no equilibrium in which the firm will continue to invest in either HR or VP. Suppose there is a (monotone) equilibrium in which a creditor with the private signal  $x_i$  invests in the debt if and only if  $x_i \geq x'$ , when the rating is

$R = 0$ . Here,  $x' \in \mathbb{R}$ . Since some creditors are willing to invest in the debt, they must believe that any  $\theta$ -firm will invest in VP if  $\theta \in [\theta'_2, \hat{\theta}_1)$ , and that by the continuity of the firm's financial cost, any  $\theta$ -firm will invest in HR if  $\theta \in [\theta'_1, \theta'_2)$ , where  $\theta'_1, \theta'_2 \in \mathbb{R}$  and  $\theta'_1 < \theta'_2 < \hat{\theta}_1$ . Therefore, such an equilibrium can be characterized by the following system of equations

$$[(1 - \gamma)F + \gamma f(\theta'_1)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x' - \theta'_1) \right] (f(\theta'_1) - F) = H \quad (34)$$

$$[(1 - \gamma)F + \gamma f(\theta'_2)] + (1 - \gamma)\Phi \left[ \sqrt{\beta}(x' - \theta'_2) \right] (f(\theta'_2) - F) = \frac{pV - qH}{p - q} \quad (35)$$

$$\frac{\Phi[\sqrt{\beta}(\theta'_2 - x)] - \Phi[\sqrt{\beta}(\theta'_1 - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} qF + \frac{\Phi[\sqrt{\beta}(\hat{\theta} - x')] - \Phi[\sqrt{\beta}(\theta'_2 - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} pF = 1 \quad (36)$$

Comparing equation (35) with equation (9), we can see that since  $\theta'_2 < \hat{\theta}_1 = \tilde{\theta}_1 < \tilde{\theta}_2$ ,  $x'$  must be strictly less than  $\hat{x}$ . Solving  $x'$  as a function of  $\theta'_2$  from equation (35) and substituting it into equation (34) and equation (36), we get

$$\begin{aligned} (1 - \gamma)\Phi(\Psi)(f(\theta'_1) - F) + \gamma f(\theta'_1) &= H - (1 - \gamma)F \\ \Delta(pF - qF) + qF\Phi(\Psi) - (pF - 1)\Phi(\Psi') &= 1, \end{aligned}$$

where  $\Delta = \frac{pV - qH - [(1 - \gamma)F + \gamma f(\theta'_2)]}{(1 - \gamma)(f(\theta'_2) - F)}$ ,  $\Psi = \sqrt{\beta}(\theta'_2 - \theta'_1) + \Phi^{-1}(\Delta)$ , and  $\Psi' = \sqrt{\beta}(\theta'_2 - \hat{\theta}_1) + \Phi^{-1}(\Delta)$ .

Totally differentiating the above system of equations, we get

$$A \begin{bmatrix} \frac{\partial \theta'_1}{\partial \hat{\theta}_1} \\ \frac{\partial \theta'_2}{\partial \hat{\theta}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ (pF - 1)\phi(\Psi') \frac{\partial \Psi'}{\partial \hat{\theta}_1} \end{bmatrix}, \quad (37)$$

where

$$A = \begin{bmatrix} [(1 - \gamma)\Phi(\Psi) + \gamma]f'(\theta'_1) + (1 - \gamma)(f(\theta'_1) - F)\phi(\Psi) \frac{\partial \Psi}{\partial \theta'_1} & (1 - \gamma)(f(\theta'_1) - F)\phi(\Psi) \frac{\partial \Psi}{\partial \theta'_2} \\ qF\phi(\Psi) \frac{\partial \Psi}{\partial \theta'_1} & (p - q)F \frac{\partial \Delta}{\partial \theta'_2} + qF\phi(\Psi) \frac{\partial \Psi}{\partial \theta'_2} - (pF - 1)\phi(\Psi') \frac{\partial \Psi'}{\partial \theta'_2} \end{bmatrix}. \quad (38)$$

Note that, when  $\beta$  is sufficiently large,  $|A| < 0$ . We show that  $\partial \theta'_1 / \partial \hat{\theta}_1 < 0$  and  $\partial \theta'_2 / \partial \hat{\theta}_1 < 0$ . Therefore, when  $\hat{\theta}$  goes to  $+\infty$ , the equilibrium converges to an equilibrium with  $x' < \hat{x}$ . However, when  $\hat{\theta}$  goes to  $+\infty$ , equation (36) goes to equation (10). This implies that the benchmark model has another equilibrium with  $x' < \hat{x}$ . This contradicts the conclusion in Proposition 1 that the benchmark model has a unique equilibrium. Therefore, in the case with a reflecting CRA, the subgame following  $R = 0$  has a unique equilibrium in which the firm defaults early and no creditor invests in the debt.

**Part 2:** Similarly to the proof of Part 1, in the subgame following  $R = p$ , there cannot be an equilibrium in which a positive measure of types of the firm defaults early. Otherwise, when  $\hat{\theta}_1$  goes to  $-\infty$ , we show that the benchmark model will have two different equilibria, violating the equilibrium uniqueness result.

Now, suppose that there is  $\hat{\theta}_2 \in [\hat{\theta}_1, +\infty)$ . It follows from Lemma 4 that  $\hat{\theta}_2$  and  $\hat{x}$  are both strictly decreasing in  $\hat{\theta}_1$ . If  $\hat{\theta}_2 > \theta_2^*$  is part of an equilibrium, when the reflecting CRA has the rating strategy  $\hat{\theta}_1 = \theta_1^*$ , there is an equilibrium in which  $\hat{\theta}_2 > \theta_2^*$ , violating the equilibrium uniqueness conclusion in Proposition 2. Therefore, in an equilibrium of the subgame following  $R = p$ ,  $\hat{\theta}_2 < \theta_2^*$ .

*Q.E.D.*

*Proof of Proposition 6:*

We first show the comparative static analysis with respect to  $\beta$ . Recall that

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}[\Delta],$$

where

$$\Delta = \frac{\frac{pV-qH}{p-q} - [(1-\gamma)F + \gamma f(\theta_2^*)]}{(1-\gamma)[f(\theta_2^*) - F]},$$

so  $\partial\Delta/\partial\theta_2^* > 0$ .

Substitute  $x^*$  as a function of  $\theta_2^*$  into equation (14) and equation (15), and denote  $\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) = \Psi$  for simplicity, we have

$$\Delta(pF - qF) - \Phi \left[ \sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) \right] (1 - qF) = 0 \quad (39)$$

$$(1 - \gamma)\Phi \left[ \sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) \right] [f(\theta_1^*) - F] + \gamma f(\theta_1^*) = H - (1 - \gamma)F \quad (40)$$

Total differentiation of the above two equations with respect to  $\theta_2^*$ ,  $\theta_1^*$ , and  $\beta$ , we have

$$A \begin{bmatrix} \frac{\partial\theta_2^*}{\partial\beta} \\ \frac{\partial\theta_1^*}{\partial\beta} \end{bmatrix} = \begin{bmatrix} \phi(\Psi)(1 - qF) \frac{\theta_2^* - \theta_1^*}{2\sqrt{\beta}} \\ -(1 - \gamma)\phi(\Psi) \frac{\theta_2^* - \theta_1^*}{2\sqrt{\beta}} [f(\theta_1^*) - F] \end{bmatrix}, \quad (41)$$

where

$$A = \begin{bmatrix} \frac{\partial\Delta}{\partial\theta_2^*} (pF - qF) - \phi(\Psi) \left( \sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\theta_2^*} \right) (1 - qF) & -\phi(\Psi) \sqrt{\beta} (1 - qF) \\ (1 - \gamma)\phi(\Psi) [f(\theta_1^*) - F] \left( \sqrt{\beta} + \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\theta_2^*} \right) & -(1 - \gamma)\phi(\Psi) [f(\theta_1^*) - F] \sqrt{\beta} + [\gamma + (1 - \gamma)\phi(\Psi)] f'(\theta_1^*) \end{bmatrix} \quad (42)$$

As we have shown in the proofs of Lemma 3, when  $\beta$  is large enough,  $\phi(\Psi)\sqrt{\beta}$  is very close to 0. Therefore, when  $\beta$  is sufficiently large, the determinant of the matrix  $A$  is close to

$$\frac{\partial\Delta}{\partial\theta_2^*}(pF - qF) [\gamma + (1 - \gamma)\Phi(\Psi)] f'(\theta_1^*) < 0,$$

because  $f'(\theta_1^*) < 0$ .

Then further algebra shows that when  $\beta$  is sufficiently large, the sign of  $\partial\theta_1^*/\partial\beta$  is the same as that of

$$\frac{\partial\Delta}{\partial\theta_2^*} \frac{pF - qF}{1 - qF},$$

which is positive. Therefore,  $\theta_1^*$  is strictly increasing in  $\beta$ .

Now, let us consider the comparative static analysis with respect to  $H$ . Total differentiation of equation (39) and equation (40) with respect to  $\theta_2^*$ ,  $\theta_1^*$ , and  $H$ , we have

$$A \begin{bmatrix} \frac{\partial\theta_2^*}{\partial H} \\ \frac{\partial\theta_1^*}{\partial H} \end{bmatrix} = \begin{bmatrix} -(pF - qF) \frac{\partial\Delta}{\partial H} + \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial H} (1 - qF) \\ 1 - (1 - \gamma)\phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial H} [f(\theta_1^*) - F] \end{bmatrix},$$

where  $A$  is defined in equation (42).

Note that  $\phi(\Psi)\sqrt{\beta}$  is very close to 0 when  $\beta$  is sufficiently large, we have

$$\text{sign} \begin{bmatrix} \frac{\partial\theta_2^*}{\partial H} \\ \frac{\partial\theta_1^*}{\partial H} \end{bmatrix} = \text{sign} \begin{bmatrix} [\gamma + (1 - \gamma)\Phi(\Psi)] f'(\theta_1^*) \frac{\partial\Delta}{\partial H} \\ -\frac{\partial\Delta}{\partial\theta_2^*} \end{bmatrix}.$$

Because  $f'(\theta_1^*) < 0$ ,  $\partial\Delta/\partial H < 0$ , and  $\partial\Delta/\partial\theta_2^* > 0$ , we have  $\partial\theta_1^*/\partial H < 0$  and  $\partial\theta_2^*/\partial H > 0$ . Therefore,  $\partial(\theta_2^* - \theta_1^*)/\partial H > 0$ .

We finally show the comparative static analysis with respect to  $\gamma$ . Similar to that about  $H$ , the total differentiation of of equation (39) and equation (40) with respect to  $\theta_2^*$ ,  $\theta_1^*$ , and  $\gamma$ , we have

$$A \begin{bmatrix} \frac{\partial\theta_2^*}{\partial\gamma} \\ \frac{\partial\theta_1^*}{\partial\gamma} \end{bmatrix} = \begin{bmatrix} -(pF - qF) \frac{\partial\Delta}{\partial\gamma} + \phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\gamma} (1 - qF) \\ - \left[ 1 - \Phi(\Psi) - (1 - \gamma)\phi(\Psi) \frac{1}{\phi(\Delta)} \frac{\partial\Delta}{\partial\gamma} \right] (f(\theta_1^*) - F) \end{bmatrix},$$

where  $A$  is defined in equation (42).

Note that  $\phi(\Psi)\sqrt{\beta}$  is very close to 0 when  $\beta$  is sufficiently large. In addition,  $(1 - \Phi(\Psi))/\phi(\Psi) \rightarrow 0$  when  $\beta \rightarrow +\infty$  and  $\partial\Delta/\partial\gamma > 0$ . Then simple algebra will show that  $\partial\theta_1^*/\partial\gamma > 0$  and  $\partial\theta_2^*/\partial\gamma > 0$ . Furthermore, because when  $\beta$  is sufficiently large,  $\partial\theta_1^*/\partial\gamma$  is close to 0, while  $\partial\theta_2^*/\partial\gamma > 0$  is bounded away from 0, we have  $\partial(\theta_2^* - \theta_1^*)/\partial\gamma > 0$ .

*Q.E.D.*

*Proof of Lemma 6:*

We prove each part of this lemma.

1. The case  $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$ .

Because  $p > q$ , this also implies

$$\frac{V^p - V^q}{C^p - C^q} \geq 1 - q > 1 - p. \quad (43)$$

Suppose there is an equilibrium in which the rating strategy  $\mathcal{R}(\theta)$  assigns the rating  $q$  when  $\theta \in (\theta_1, \theta_2)$ . Then, as in the proof of Lemma 1, if the CRA deviates to assign the rating  $p$  to  $\theta$ -firm for all  $\theta \in (\theta_1, \theta_2)$ , the firm will not default early.

Now, consider any  $\theta$ -firm with  $\mathcal{R}(\theta) > 0$ . Equation (43) implies that

$$V^p - (1 - p)C^p > V^q - (1 - p)C^q, \quad (44)$$

$$V^p - (1 - q)C^p > V^q - (1 - q)C^q. \quad (45)$$

Equation (44) implies that if  $\theta$ -firm invests in VP, the CRA wants to assign the rating  $p$ ; equation (45) implies that if  $\theta$ -firm invests in HR, the CRA wants to assign the rating  $p$ . Hence, equation (44) and equation (45) together imply that whether the  $\theta$ -firm invests in VP or HR, it is optimal of the CRA to assign the rating  $p$ . As a result, if the firm does not default early, the CRA will assign the rating  $p$ . Then, because the firm's financial cost is monotonic in  $\theta$ , the equilibrium rating strategy must be in the form of equation (17).

2. The case  $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$ .

This case is same as the previous one, except that we replace the rating  $p$  by the rating  $q$ .

3. The case  $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$ .

In this case, we have

$$V^p - (1 - p)C^p > V^q - (1 - p)C^q > V^q - (1 - q)C^q > V^p - (1 - q)C^p.$$

Therefore, when  $\theta$ -firm chooses VP, the CRA will assign the rating  $p$ ; when  $\theta$ -firm chooses HR, the CRA will assign the rating  $q$ . Furthermore, when  $\theta$ -firm chooses VP following the rating  $p$  and HR following the rating  $q$ , the CRA will assign the rating  $p$ . Because for



any particular  $\theta$ -firm, the financial cost following the rating  $p$  is lower than the financial cost following the rating  $q$ , if the firm invest in HR following the rating  $p$ , it will invest in HR following the rating  $q$ .

Hence, the rating strategy must be in the form as in equation (19). Whether the rating  $q$  can appear in the equilibrium depends on the model's parameters, so  $\theta''$  could be equal to  $\theta'$ , in which case the CRA will not assign the rating  $q$  in an equilibrium.

*Q.E.D.*

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