Risk Perception and Loan Underwriting in Securitized Commercial Mortgages

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Draft: June 2023

We use mortgage implied volatility to proxy for property risk perceptions in commercial real estate (CRE) lending markets. While loan-to-value ratios (LTVs) unconditionally decreased following the Global Financial Crisis, LTVs conditioned on implied volatility and other theoretically motivated fundamental determinants of optimal leverage show no conclusive trend before or after the crisis. Taking reported property and loan attributes at face value, we find no clear pattern of unwarranted credit being extended to CRE assets. We conclude that systematically higher LTV decisions pre-crisis would have primarily stemmed from risk misperceptions rather than imprudent practices. Our findings suggest that the aggregate LTV level should only be interpreted as a proxy for lending standards after controlling for aggregate risk perceptions, among a host of asset and lending market factors. Our findings also highlight the importance of measuring and tracking aggregate risk perceptions in informing regulators and policymakers.

JEL: C22, D80, G01, G10, G18, G21, R38

Keywords: Loan underwriting, Lending standards, Global Financial Crisis, Mortgages, Real estate finance, Implied volatility

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I. Introduction

Against which of these assets should one extend more credit in 2023: a suburban office or a warehouse facility leased to a large e-commerce company? With the relative uncertainty surrounding work from home trends, the answer seems clear—but it may have been a tossup 20 years ago. This example illustrates the core motivation of our paper. Loan-to-value ratios (LTVs) are often viewed as a primary measure of underwriting standards in commercial real estate (CRE) lending.[1] We argue that variations in observed LTV ratios should be interpreted through the lens of the lender's (or originator's) risk perceptions, which are different not only across types of collateral but also vary with time. What may be interpreted as "aggressively" high LTV in isolation may, in fact, be optimal or justifiable given the riskiness of the collateral and/or state of the business cycle. Theoretically, this idea is not new: Jaffee and Russell (1976) demonstrate that lenders may limit or ration credit (i.e., require more "skin in the game") when it is hard to tell which borrowers are riskier, and Leland and Pyle (1977) provide a framework of firm financing with greater equity accompanying greater risk. A lender's willingness to extend more credit against collateral should reflect the underlying collateral's perceived risk and not only "loose" or "tight" credit conditions (which, to some extent, may reflect lenders' tolerance for bearing risk or their cost of capital).

Despite that, when it comes to measuring aggressiveness of real estate lending, aggregate changes in LTV are frequently interpreted as changes in underwriting standards. For instance, a 2010 Congressional Oversight Panel Report by Elizabeth Warren et al. (2010) after the Global Financial Crisis (GFC) concluded:

> "The development of the commercial real estate bubble [...] resulted in the origination of a significant amount of commercial real estate loans based on dramatically weakened underwriting standards. These loans were based on overly aggressive rental or cash flow projections (or projections that were only sustainable under bubble conditions), had higher levels of allowable leverage, and were not soundly underwritten."

[1]Although income to debt service coverage ratios and income to debt ratios are also prevalent metrics of CRE loan underwriting, the link between property-level risk and LTV appears stronger than the link between property-level risk and income ratios. We report on this in our empirical work.

The identified culprits in the quote are unrealistic cash flow forecasts and overly high LTVs, and this assessment is consistent with that of others (e.g., Levitin and Wachter, 2013). While there is evidence to suggest that property income is, at times, inflated by commercial mortgage-backed security (CMBS) originators (Griffin and Priest, 2023), the diagnosis of aggressive LTV is a narrative that is more difficult to test, though it may often be conjectured. Jacob and Manzi (2005) describe what they believe to be lenders pushing the limit on LTV in a trend towards "weaker lending standards," and Fabozzi, McBride and Clancy (2015) suggest that this was particularly egregious in 2006 and 2007. On the other hand, Wilcox (2012) and Wilcox (2018) argue that aggregate LTVs may not provide a faithful portrayal of underwriting standards.

We provide evidence that, controlling for imputed ex-ante risk perceptions of the collateral as measured by individual properties' implied volatility, the average origination LTV of securitized CRE loans in the period 2000–2004 was only one and a half percentage points more than LTVs during the post risk-retention period of 2016–2020. Average LTVs in 2005–2007 were likewise similar to those in 2008–2015. These differences among origination epochs shrink further when one accounts for property cash yield (cap rate) spreads over the 10-year U.S. Treasury yield. We do find that imputed originator credit rationing thresholds (i.e., maximum underwriting LTV criteria) were most permissive between 2000–2004 even after controlling for risk perceptions. But those thresholds were also most restrictive, as a function of perceived risk, in 2005–2007 (coinciding with the peak of collateralized debt obligation, or "CDO", issuances). Importantly, the imputed lending thresholds explain only a negligible fraction of the variation in LTVs across epochs while perceived risk explains the lion's share.

Our contribution consists of demonstrating that LTVs for securitized CRE loans, throughout various economic episodes between 2000 and 2020, were largely determined by property and market fundamentals as contemporaneously perceived. To proxy for perceived property risk, we use each loan's implied volatility (IV) calculated using a two-factor derivative asset pricing model that allows for standard mortgage contract provisions. The asset implied volatility is the model asset diffusion risk required to rationalize the loan rate given the loan LTV, the loan maturity and amortization schedule, the asset payout ratio, the prevailing term structure of U.S. Treasury yields, and the current mortgage market liquidity premium.[2] Our findings are consistent with trade-off theories of optimal leverage (e.g., Leland, 1994) which imply that observed LTV should decline with perceived collateral risk and payout ratios. On their own, collateral risk perceptions explain two thirds of the cross-sectional and time-series variations in LTV. Including the payout ratio and a host of other fixed effects (including property type and location) helps explain roughly an additional 10% of the variation. While aggregate time-series variation remains after controlling for perceived risk, it appears to be largely random. Our results are in line with Driessen and Van Hemert (2012) and Stanton and Wallace (2018), who find no evidence that underwriting practices among commercial real estate securities deteriorated in the way that underwriting in residential mortgage-backed securities did pre-GFC. They are also consistent with the position in Wilcox (2012) and Wilcox (2018) that LTVs, on their own, may not be informative about aggregate loan underwriting standards.

Given the collapse of the CRE market on the heels of the GFC, it is tempting to say that CRE loan leverage was too aggressive prior to the GFC. We find no evidence that this was expressed through unusually high LTVs, after controlling for perceived collateral risk. On the other hand, we find that risk perceptions, as proxied by implied volatilities, were lowest in the period 2003–2007 than any other five-year period in our sample. Our findings, therefore, point at systematic shifts in perceptions of collateral risk as a compelling explanation for the profound growth in CRE lending between 2003–2007 which, ex-post, fueled the CRE market decline. To the extent that there was a market failure in underwriting CRE loans during the run-up to the GFC, it would be consistent with our findings to point to aggregate risk *misperceptions* as the culprit. Systematic misperceptions of risk would have led to more credit extended because more collateral would have qualified for higher LTVs. The credit extended, however, would have been underpriced (i.e., it would have commanded lower loan spreads).

There is an important difference between viewing a loan as aggressively underwritten versus viewing it as underpriced. In the former case, the lender knowingly undertakes more risk than is warranted by prudent practices. In the

[2]The current mortgage market liquidity premium is proxied by the spread between highly rated (AAA) short-term bonds backed by commercial real estate loans and U.S. Treasury securities with equivalent maturities.

latter case, the lender wrongly believes that they are following best underwriting practices. These are more than nuanced academic distinctions. A regulatory paradigm in which prevailing risk perceptions are used to determine risk-based capital requirements may be chronically vulnerable to systematic biases in those risk perceptions. This points to a need to better understand how macroprudential policy can balance the inefficiency of limited discretion in determining risk-based capital requirements against the risk of systematic biases in risk perceptions.

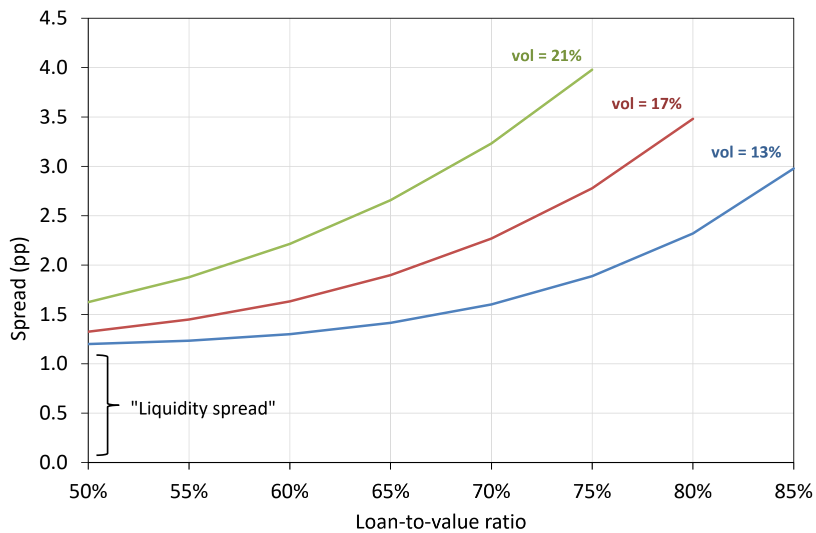
The paper is organized as follows. Section II provides the theoretical background for conceptualizing the aggregate LTV ratio as a dynamic variable that is influenced by risk perceptions and other property and capital market attributes—even in a world where the only frictions arise from taxes and dead-weight costs of default (i.e., in a world where all agents are rational and there is no aggressive lending, risk misperception, or aggregate benefits to financial regulations). Section II also reviews the related literature and provides institutional details on the CMBS market. Section III describes our securitized CRE mortgage data. Section IV reports on attributes of our implied volatility estimates and their link to other measures of property risk. Section V presents our main results, examining CRE loan LTV ratios over time with and without controls for theoretically motivated determinants of optimal leverage, including (and especially) our perceived risk measure proxied by implied volatility. Section VI concludes.

II. Conceptual motivation, methodology roadmap, and institutional background

Figure 1 shows stylized mortgage offer curves for collateral with different perceived risk. Mortgage provision in the primary market depends on the price of liquidity in the secondary mortgage market, competition among lenders, and the availability of capital in the credit market, all of which are held fixed. The offer curves are truncated above a certain LTV as a result of credit rationing due to asymmetric information (Jaffee and Russell, 1976; Leland and Pyle, 1977) and deadweight costs of default (Leland, 1994).

Figure 1. Stylized mortgage offer curves for collateral with different risk

This figure shows stylized five-year zero-coupon mortgage offer curves for collateral with different levels of perceived risk. The risk is proxied by annualized asset volatility ("vol"). The vertical axis shows the mortgage spread over a zero-coupon five-year U.S. Treasury at which lenders would be willing to issue the mortgage loan given the loan-to-value ratio and the annualized asset volatility. The curves are computed using the Merton (1974) model and incorporate a liquidity spread, which represents the price of liquidity in the mortgage-backed security market.



Given the mortgage offer curve appropriate to the collateral, the borrower's choice variable reduces to the LTV.[3] In the Leland (1994) model, for example, the optimal borrower's choice is decreasing in the asset's income yield, increasing in the owner's tax rate, and decreasing in asset volatility. All else equal, one expects that a model with rational borrowers and lenders would yield equilibrium LTV choices that decrease in asset volatility.

There are several important takeaways from the theoretical considerations above. First, time variation in aggregate LTVs may be entirely attributable to changing market fundamentals rather than the tension between regulators and lenders, or agency frictions within lending institutions. Second, in those instances where LTV limits, whether imposed by the lenders or by their regulator, are below the optimal leverage point of a non-negligible number of borrowers, one would

[3]We ignore, in this example, other mortgage contract terms available to the borrower and commensurately priced by the lender. these include maturity, interim coupon payments, and prepayment options. We take such features into account in our empirical methodology.

expect clustering at the LTV maximum threshold. Moreover, the threshold should decline with collateral riskiness. Finally, although lenders and borrowers do not directly observe the asset volatility, their risk assessment is implicit in the loan spread at which the contract is executed. In the figure, it is sufficient to know that a loan with 67% LTV is priced at 2% above the five-year U.S. Treasury yield to conclude that the perceived risk was roughly 17% in asset volatility.

A. *Roadmap to the empirical strategy*

Our null hypothesis is that the cross-section of LTVs results from borrower demand in response to rational offer curves, akin to those depicted in Figure 1. Under this null, it is consistent with prudent lending to provide an infinitely elastic supply of credit at any point on the curve. Risk mis-perception might correspond to systematic lending using offer curves that are below true asset volatilities, and such practices may only become apparent ex-post. On the other hand, aggressive lending in this example is characterized by making a loan that would not normally be made (e.g., in Figure 1, an 80% LTV loan made to an asset with vol of 21%). Thus, to test whether a period like 2005-2007 might have been characterized by aggressive lending, one might check whether the maximum loan threshold, or rationing "frontier", during that period is significantly higher than at other times.[4] Moreover, even if the rationing frontier suggests aggressive lending practices, enough borrowers must take advantage of it in order for there to be meaningful economic impact. In other words, all else equal, changes in the frontier must be significantly related to the distribution of LTVs. This too can be tested.

To examine the null described above, as well as support for its aggressive lending alternative, we first back out the implied volatility of each underlying property in a sample of securitized CRE mortgages. Our mortgage pricing model used to back out implied volatility controls for many realistic features including interest rate risk, default risk throughout the loan term, the property's cash yield (cap rate), the amortization schedule of the loan, and a time-varying mortgage

[4]Note that changes in the frontier could result from fundamental influences, like a systematic change in deadweight costs of default. Thus finding an episode with a higher rationing frontier would be suggestive rather than iron-clad proof of aggressive lending practices.

market liquidity premium.[5] Consistent with the null, the mortgage-implied volatility of an asset can be viewed as a proxy for the lender's risk perception of the collateral. We then identify a lending rationing frontier, as a function of implied collateral volatility, for four separate periods in our sample, to be described in detail shortly. We confirm that, consistent with the null, the frontier weakly declines with implied volatility in each of the four periods.

Next, we test the rationing frontiers to see whether they fit the narrative that lenders were relatively more aggressive in 2005-2007, during the run-up to the Great Financial Crisis. We find no support for this narrative. We subsequently use fundamentals, observable at the time of loan origination, to explain time-series and cross-sectional variations in LTV within the sample. Fundamentals include collateral riskiness (i.e., the estimated property-level implied volatility panel), cash yields, and other fundamental property and market attributes that could influence mortgage offer curves and/or borrower demand for loans. We estimate a censored tobit model with property LTV as a dependent variable, using the identified rationing frontiers as maximum LTV constraints. By estimating the tobit parameters and residual LTV values, we can then reexamine the LTV distribution under "counterfactual" conditions. In particular, by fixing the frontier to counterfactually coincide with that of a benchmark period, we can compare predicted LTVs using the actual frontier with those that use the counterfactual benchmark. This allows us to quantify the impact of changing frontiers on the distribution of LTVs throughout the sample. We find little evidence that changes to the rationing frontier explain LTVs. Thus, even if shifts in the rationing frontiers stem from tightening or loosening of underwriting standards, it seems that these have little impact on the distribution of LTVs in our sample.

What we do find, is that the leading determinant of credit provision in our data, by far, is perceived collateral risk (i.e., property implied volatility). It is important to note that this relationship is not mechanical. For instance, if borrowers randomly select LTVs from the offer curves in Figure 1, then the only link between risk perceptions and LTVs will be through credit rationing (because rationing is more likely to be binding for borrowers with riskier collateral) — this is not consistent with our finding that changes to the rationing frontier explain

[5] We also examine the possibility that prepayment options will meaningfully impact our results. We find that, because of the presence of hefty prepayment penalties in this market, they do not.

little of the LTV distribution. On the other hand, if borrowers optimally choose LTV by trading off costly default against benefits of debt (e.g., lower taxes) then, consistent with our findings, LTV will vary strongly with property risk even when rationing does not materially constrain most borrowers' choices.

B. Literature on commercial real estate risk and mortgage implied volatility

We contribute to a developing understanding of CRE asset volatility. Much prior analysis uses aggregate data to study commercial real estate price dynamics. Ciochetti et al. (2002) create a property value volatility index at the property type-Census district level. Plazzi, Torous and Valkanov (2010) use MSA quarterly average data for broad property types and the Campbell and Shiller (1988) price-dividend decomposition to better understand the stochastic nature of CRE rents, cap rates, and asset returns. They find that CRE returns' predictability at the MSA level is related to the local regulatory environment and population density, and that expected returns are related to factors such as local population, employment, and income growth, as well as construction costs. We find that many of these factors are important for volatility with disaggregated data.

Studies using property-level data have highlighted the magnitude of idiosyncratic volatility, that is, how much larger asset-level volatility is than indices or other area averages. Plazzi, Torous and Valkanov (2010) calculate that, aggregated at the property type-MSA level, the standard deviation of excess returns to property values ranges from 3.7% to 6.1%, depending on property type. By contrast, Downing, Stanton and Wallace (2008) calculate asset-level volatility for CMBS loans using a two-factor Titman and Torous (1989) model. They find implied CRE volatilities in excess of 20%, higher than our estimates for their sample time period but similar to our post-GFC calculations. Sagi (2021) uses asset-level data from the National Council of Real Estate Investment Fiduciaries to measure price appreciation volatility. He finds that, for holding periods of three years or longer, the standard deviation of annual price appreciation volatility is about 13%.[6]

Our mortgage pricing model, from which we back out implied volatility, builds upon an extensive literature using contingent-claims or option-theoretic strategies for pricing mortgage debt. Some models stipulate a partial differential equation for property value that is solved using finite difference methods (Titman and Torous, 1989; Kau et al., 1995). Another popular method, and the one we employ, uses a binomial model for property valuation (Leung and Sirmans, 1990; Giliberto and Ling, 1992; Hilliard, Kau and Slawson, 1998; Ciochetti and Vandell, 1999). Similar to us, many of these models include prepayment as well as default options. Likewise, related model generally assume a single stochastic mean-reverting interest rate process similar to Cox, Ingersoll and Ross (1985), whereas we model interest rates using several competing models. We also include contract characteristics such as interest-only versus amortizing payment schedules, as well as property attributes such as the cash yield. Finally, to better fit the data, we add a tail risk parameter for catastrophic property loss.

Our analysis is most similar to Downing, Stanton and Wallace (2008). They also use a two-factor pricing model which prices the mortgage at par, although their aim is to examine the relationship between implied volatility and CMBS ratings. As in our model, mortgage value is a function of the dynamics in the short rate and property value process. Our model incorporates a richer set of contract and property characteristics, including asset-level income and the length of the interest-only period. Their analysis also ends in 2006, while we examine post-GFC dynamics.

C. Historical evolution of the commercial mortgage-backed security market

The loans in our data set are CMBS loans: loans originated to be pooled within Real Estate Mortgage Investment Conduit trusts that issue mortgage-backed securities. Several changes in CMBS markets took place during our sample

[6]In general, asset pricing models assume that the asset price process is a geometric Brownian motion process, and thus the standard deviation of total price appreciation should be linear in the length of the holding period. As the holding period approaches zero, return volatility should also approach zero. By contrast, Sagi (2021) finds that volatility remains high even for short holding periods. He traces this phenomenon to transaction risk borne of illiquidity and finds that the return data fit well to predictions from a search model.

period which affect both the cost of funding CMBS assets and the market for the riskiest CMBS tranches. CMBS allocate credit risk among different tranches. The tranches least exposed to credit risk typically receive investment-grade ratings, while the tranches which absorb credit losses first are often unrated. During our sample period, the types of investors holding the risky tranches changed several times due to regulatory action and the rise and fall of CDO markets.

Prior to 2005, the unrated tranches were usually held by a specialized set of 'B-piece' investors who were involved in the security design, performing their own due diligence and selecting the special servicer who would be responsible for managing delinquent loans. During the period between 2005 and the GFC, the market for 'B-piece' portions of securitizations changed. It became common for CMBS issuers to repackage the unrated pieces of CMBSs into CRE CDOs. Rating agencies, who made over-optimistic assumptions about the benefits of diversification for CRE CDOs, assigned high ratings to some tranches of CDOs or repackaged CDOs.

Another factor affecting pre-GFC CRE finance was a fall in capital requirements. In addition to the CDO markets' expansion, both commercial bank and investment bank capital requirements for CMBS were reduced in 2004. Duca and Ling (2020) calculates that commercial bank capital requirements fell from 8% to 2%, and the SEC cut requirements from 6% to 3.7%, permitting much greater levels of leverage and reducing the cost of financing these assets. In summary, both commercial and investment banks' cost of funding CMBS fell, and it became much easier for CMBS issuers to sell the riskiest tranches.

After the GFC, the market for CRE CDOs disappeared. CMBS issuance effectively stopped for several years. Capital requirements for CMBS for commercial banks increased at the end of 2010, and by that time the major U.S. investment banks had been absorbed by commercial banks or became bank holding companies subject to these capital requirements (Duca and Ling, 2020). After the GFC, U.S. regulatory agencies proposed Regulation RR, requiring issuers of asset-backed securities, including CMBS, to retain at least five percent of an issue on their balance sheets. This Rule was finalized in October 2014 and came into full effect in

December 2016 with the intention of ensuring that securitization issuers' incentives are aligned with those of investors.[7]

Regulation RR includes exceptions for CMBS. Underwriters may hold a 'vertical' piece of the security, which includes a portion of all tranches, a 'horizontal' piece of the lowest-rated tranche, or a 'L-shaped' hybrid of the two. The underwriter is exempt from risk-retention requirements for relatively conservative 'qualified' CRE loans.[8] In addition, originators of substantial portions of loans in a CMBS pool may retain the unrated piece relevant to their loans. Even with all of the exceptions, Flynn Jr, Ghent and Tchistyi (2020) show that CMBS underwriters often retain risk. These requirements often bind on issuers.

The time variation in CMBS risk and regulations motivates us to subdivide our data sample into four epochs as follows.

1) 2000–2004: Specialized 'B-piece' investors retain the riskiest pieces of CMBSs.

2) 2005–2007: Beginning towards the end of 2004 until the GFC, CMBS underwriters can repackage portions of the riskiest tranches as CRE CDOs, receiving investment-grade ratings for substantial proportions of them. Capital requirements for holding CMBS also fall.

3) 2008–2015: CMBS issuance slowly recovers from almost nothing to pre-2005 levels.

4) 2016–2020: Regulation RR (risk-retention) is introduced, taking effect in December of 2016.

Our epoch boundaries are also motivated by the actual distribution of loans we see in our data (Figure 2). The data show clear cutoffs in the number of originated loans at the end of each of these periods.

[7] See https://www.sec.gov/news/pressrelease/2014-236.html for more information.

[8] According to DiSalvo and Johnston (2018) "The regulation defines a qualifying commercial real estate loan as a fixed-rate loan with a minimum maturity of 10 years and a maximum amortization of 25 years (30 for loans secured by multifamily properties). Lenders must document the income from the property for at least the previous two years. The borrower's debt service ratio must exceed 1.25 for multifamily properties, 1.5 for leased properties, and 1.7 for all other loans. Also, the combined loan-to-value ratio of all loans on the property cannot exceed 70 percent, and the loan-to-value ratio of the first lien loan cannot exceed 65 percent."

III. Data construction and summary statistics

A. Securitized mortgage loan data

Our data consist of 51,365 CRE securitized loan originations (loans included in CMBS deals). The data are provided by Morningstar, who consolidate information from public CMBS disclosures. The Morningstar data include a rich set of loan and property characteristics and provide coverage from 2000 onward.[9] The Morningstar data include loans originated by a variety of institutions, and are not dominated by a single underwriting approach. Many loans were originated by large U.S. banks, such as Bank of America and Citibank. The top ten originators also include large foreign banks such as Deutsche Bank, Credit Suisse, and UBS. Non-depository institutions are a substantial part of the market, but no single such institution has a large market share.

The complete data set includes 171,421 loans. For the purposes of our analysis, we drop CRE loans missing key variables needed for our analysis and loans with problematic observations (see Appendix A for more details). Specifically, we drop loans which have missing or obviously wrong data for key inputs, including the date of origination, loan interest rate, whether the loan is interest-only or amortizing, the amortization schedule if appropriate, and date of maturity.[10] We drop non-fixed-rate and pari-passu loans, which our model currently does not price. Finally, for analytical and expositional simplicity, we restrict our sample to single-property loans, which constitute the overwhelming majority of observations.

The initial culling described above results in a sample of 62,147 CRE loans. We present summary statistics for the characteristics of these loans in Table 1 and their corresponding cross-sectional distributions in Figure F2 of Appendix F. Loans vary widely in origination amount, from less than $2 million to over $2.5 billion. LTV ratios are generally close to 70%. Debt yield, the ratio of NOI to loan amount at origination, varies between 7% and 15%. Debt service coverage

[9] Securitized CMBS loans are a small part of the overall CRE loan market. Black et al. (2017) compare securitized loans in the Morningstar data and data on portfolio CRE loans submitted by large U.S. banks on the FR Y-14 report. They find that these banks are more likely to hold in their portfolio loans that are riskier, including construction loans. Portfolio CRE loans are also more likely to have floating interest rates, shorter loan terms, and lower LTVs than Morningstar loans. Their analysis focuses on loan contract characteristics rather than the collateral. Setting aside construction loans, which are outside the scope of our analysis, it is unclear to what extent Morningstar collateral differs from that of bank portfolio loans.

[10] Amortizing CRE loans generally do not pay down all principal by the maturity date, paying down principal as if the maturity date were much longer, and include a balloon payment at the maturity date.

ratio (DSCR), the ratio of NOI to debt servicing amount at origination, falls roughly between 1.2 and 2.4. The vast majority of loans are 10-year loans, in addition to a few 7- and 5-year loans and other various maturities.

Table 1—Characteristics of sample commercial real estate loans

This table shows summary statistics for key characteristics of commercial real estate mortgage loans in our sample, containing fixed-rate, single-property loans securitized in commercial mortgage-backed securities (CMBSs). From left to right, the columns show the number of observations and the sample mean, standard deviation, as well as the 10^{th} and 90^{th} percentiles of variables in the cross section of sample loans. At the bottom of the table, the respective spread measures represent the percentage point yield spreads of sample loans over the 10-year zero-coupon U.S. Treasury yield and the value-weighted effective yield of the securities constituting the ICE BofA 0-to-3-year AAA U.S. Fixed-Rate CMBS Index.

	Count	Mean	SD	P10	P90
Loan amount ($1,000)	62,147	11,658	17,101	2,000	25,000
Loan term (months)	62,147	113	24	83	120
Amortization period (months)	62,147	312	111	0	360
Interest-only period (months)	62,147	22	35	0	60
Loan-to-value ratio	62,147	0.68	0.12	0.54	0.79
Debt yield	62,147	0.11	0.05	0.07	0.15
Debt service coverage ratio	62,147	1.72	0.81	1.19	2.36
Spread over 10-yr U.S. Treasury (pp)	62,147	1.87	0.74	0.93	2.79
Spread over 0-3-yr AAA CMBS (pp)	62,147	1.86	1.14	0.38	3.30

Table 2—Distribution of sample commercial real estate loans across property types

This table shows the absolute and relative frequencies of commercial real estate mortgage loans in our sample across different collateral property types. The sample contains fixed-rate, single-property loans securitized in commercial mortgage-backed securities.

Property type	Count	Share
Hotel	4,856	7.8%
Industrial	3,047	4.9%
Multi-family	22,673	36.5%
Office	9,354	15.1%
Other	5,948	9.6%
Retail	16,269	26.2%
Total	62,147	100.0%

Figure 2. Annual number of sample commercial real estate loan originations over time

This figure shows the annual number of commercial real estate mortgage loan originations in our sample, color coded by time period (epoch). The sample contains fixed-rate, single-property loans securitized in commercial mortgage-backed securities. Epoch choice is explained in Section II.C.

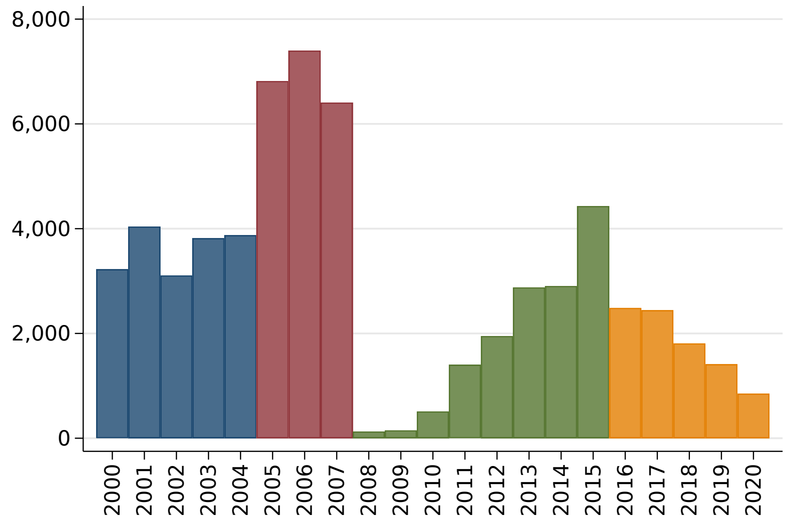


Figure 2 and Table 2 show the distribution of observations over time and across property types. The volume of loans originated steadily increases from 2000 until the GFC, falls to almost nothing until 2011, and slowly recovers to a substantial level continuing until 2020, when we end the analysis. The most common collateral types are retail and multi-family. There are also a substantial number of loans we characterize as "other," which do not cleanly fit into standard broad categories.[11] Industrial properties have the smallest representation in the sample.

We present summary statistics for the properties used as collateral for sample CRE loans in Table 3. The median asset used as collateral is a nine year old nearly fully leased property. Properties range widely in size, from sixteen thousand to over 24 million square feet. Many property-level variables are unevenly populated. This is partly due to heterogeneity among property types. The lead tenant's lease term and share of the property are not relevant for the multi-family properties

[11]The "other" category includes mini-storage and mixed-use properties representing a combination of property types, such as a complex with both multifamily and retail property.

because they have many small units, each leased to a different party. In other cases, variables are simply missing from the data.

Table 3—Characteristics of sample commercial real estate loan properties

This table shows summary statistics for key characteristics of the properties used as collateral for commercial real estate mortgage loans in our sample at the time of loan origination. The sample contains fixed-rate, single-property loans securitized in commercial mortgage-backed securities. From left to right, the columns show the number of observations and the sample mean, standard deviation, as well as the 10^{th} and 90^{th} percentiles of variables in the cross section of sample properties. The area of the property in square feet, as well as the derived variables occupancy rate and lead tenant share, are only available for industrial, office, retail, and most "other" property types. By commercial real estate market convention, the size of hotel and multi-family properties is measured by the number of units.

	Count	Mean	SD	P10	P90
Property value ($1,000)	62,147	17,972	28,667	3,030	37,500
Net operating income ($1,000)	62,147	1,170	1,782	210	2,386
Area (1,000 sqft)	33,834	112.83	151.75	15.97	240.95
Age (years)	56,483	14.84	16.46	1.00	37.00
Occupancy rate	32,168	0.94	0.10	0.83	1.00
Lead tenant area share	29,597	0.42	0.29	0.12	1.00
Lead tenant lease length (years)	29,525	7.87	5.23	2.17	15.16

In our final filtering of the data, we drop loans with debt yield < 0.07 and DSCR < 1.25. These lower bounds correspond to standard underwriting limits (i.e., lenders are reluctant to lend if the debt yield or DSCR are too low) and fall around the 10^{th} percentile in our full sample shown in Table 1. When debt yield and DSCR is very low, it may suggest that the property is not currently stabilized even though it may be anticipated to be so shortly. Because our model uses the property's underwritten cash yield as an input, including non-stabilized properties may distort their true imputed risk. We are left with 51,365 observations which we use in our implied volatility calculations.

B. Adjustment for capital market liquidity dynamics

One limitation of our mortgage valuation model is that it is driven by only two dynamic factors: the short interest rate process and the value process for the collateral. In practice, Christopoulos (2017) shows that mortgage pricing is also affected by a time-varying premium for liquidity. Without adequate controls, our model would attribute an increase in primary CRE mortgage rates due to a higher liquidity premium to increased credit risk, resulting in spuriously higher implied volatility.

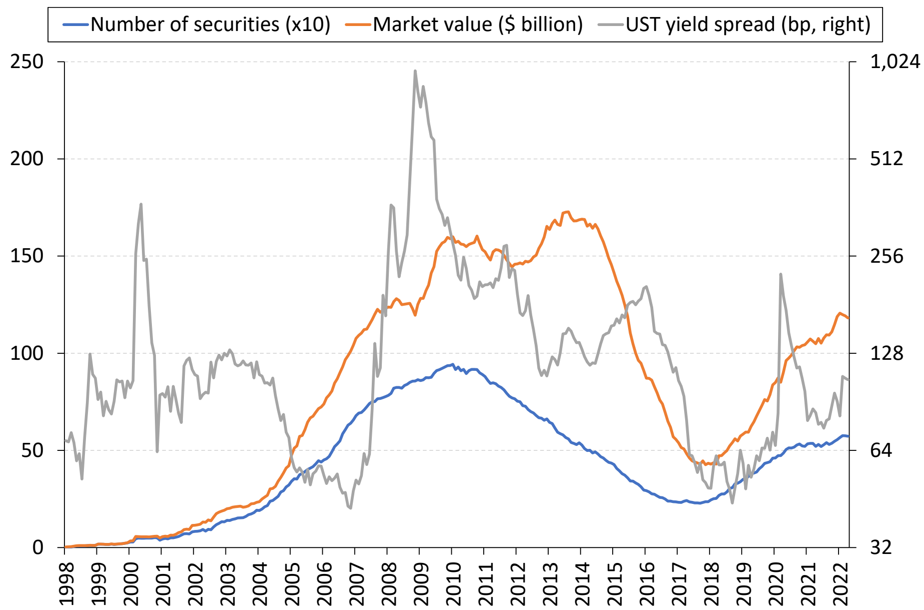
We take the liquidity premium into account by adjusting loan rates in our sample before the model valuation step. Specifically, we calculate a monthly time series of CMBS liquidity spread by taking the value-weighted effective yield of securities in the ICE BofA 0-to-3-Year AAA U.S. Fixed-Rate CMBS Index minus the yield of zero-coupon U.S. Treasury securities with the corresponding effective duration. We then adjust the mortgage rate for each loan by the prevailing liquidity spread as follows:

$$r_{adj} = r_{observed} - (\text{CMBS spread} - 120\text{bp}), \tag{1}$$

where 120bp is the median value of the CMBS yield spread defined above. Figure 3 shows the yield spread and the number and value of the shortest duration CMBS securities. Although this adjustment still leaves a constant baseline level of liquidity premium embedded in mortgage rates, the resulting upward bias should still permit *relative* comparisons of riskiness across time periods based on our model-implied volatilities, which we use as a proxy for property riskiness, rather than as an estimate of actual property diffusion volatility.

Figure 3. Statistics for 0-to-3-year AAA fixed-rate commercial mortgage-backed securities

This figure shows time series aggregates for the commercial-mortgage backed securities (CMBSs) constituting the ICE BofA 0-to-3-Year AAA U.S. Fixed-Rate CMBS Index over time. UST yield spread is defined as the basis point difference between the value-weighted mean effective yield of the index constituents over the yield of the corresponding zero-coupon U.S. Treasury security with maturity equal to the value-weighted mean effective duration of the index constituents.



IV. Implied volatility estimation and diagnostics

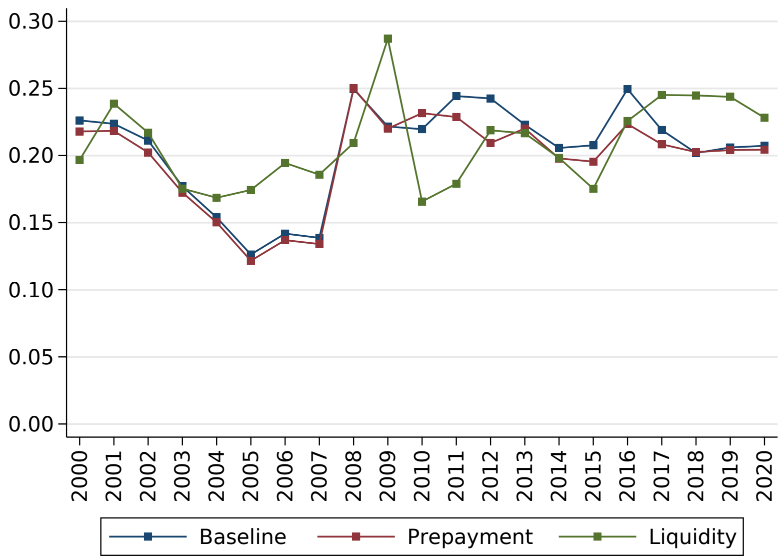
Appendix B describes the the two-factor model (with disaster risk) that we use to estimate property-level implied volatility, which we then use as a measure for the lender-perceived risk of the collateral. The model ignores correlations between U.S. Treasury yields and property values. Although there is no standard way to measure collateral risk in the presence of market frictions and incompleteness, our implied volatility estimate is a sensible proxy measure of risk perception.[12]

Different approaches to backing out implied volatility from our model are plotted in Figure 4. One approach makes no liquidity adjustments to mortgage

[12]Contingent claim models, such as the one we use, assume that a risk-neutral pricing paradigm can be justified when one is able to replicate contingent claims. Clearly, this does not hold in illiquid real estate asset markets. Conceptually, relying on a the risk-neutral valuation methodology is similar to assuming the normality of unobserved shocks in a linear filtering problem. We can acknowledge the limitations of our methodology and attempt to provide validity or robustness tests for the approach, but our estimate is ultimately a proxy for, rather than actual, lender-perceived collateral risk.

Figure 4. Sample means of implied volatility estimates over time

This figure shows the means of the estimated model-implied volatilities of commercial real estate mortgage loans in our sample over time. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The implied volatilites are estimated using the two-factor model described in Appendix B. There are three batches of estimates: a baseline batch without prepayment penalties or market liquidity adjustment, a batch with prepayment penalties, and a batch with market liquidity adjustment. The market liquidity adjustment process is explained in Section III.B.



rates and does not consider the impact of prepayment options. A second approach allows for optimal prepayment in the presence of contract penalties. Save for 2012 and 2016, the presence of prepayment penalties makes little to no perceptible difference, on average, in imputed implied volatility. This is because prepayment penalties, ubiquitous in the commercial property mortgage market, are usually sufficiently punitive so as to render the value of a prepayment option second-order in its impact on mortgage pricing. Because of reliability issues with the Morningstar prepayment data fields (see Appendix A for further details), ignoring prepayment options does confer the advantage of a larger admissible data set. The third plot in Figure 4 corresponds to calculating implied volatilities after making a liquidity adjustment to mortgage rates (but not incorporating prepayment options). It is clear that adjusting rates for mortgage market liquidity profoundly impacts the imputed measure of riskiness. Henceforth, because of concerns raised earlier about mismeasuring relative riskiness over time, we perform the empirical analysis using liquidity-adjusted implied volatilities.

FIGURE 5. SAMPLE QUARTILES OF IMPLIED VOLATILITIES OVER TIME

This figure shows the quartiles of the estimated model-implied volatilities of the commercial real estate mortgage loans in our sample over time. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B.

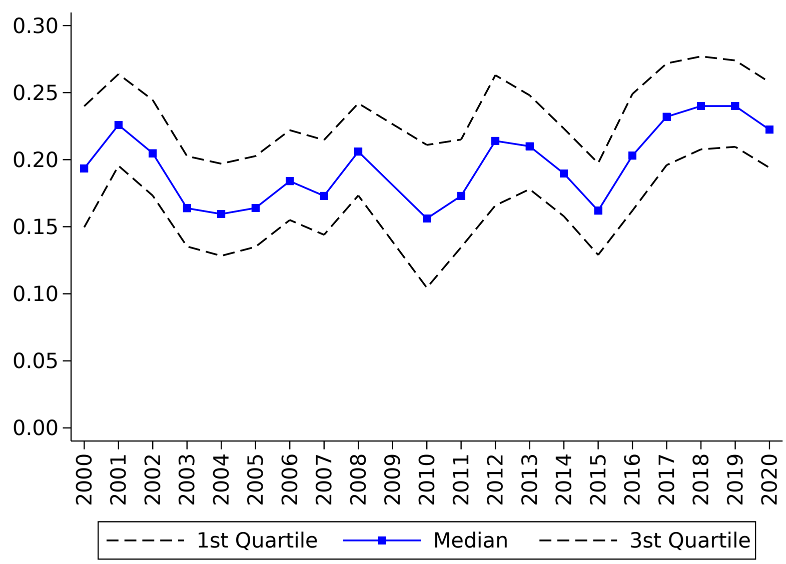


Figure 5 depicts the distribution of implied volatilities calculated using liquidity adjustments (and no prepayment options). The time series mean (median) is 20% (19%) and the standard deviation is 7.5%. The time series variation is pronounced and corresponds to shifts in the entire distribution. This suggests that perceived risk moves systematically through time. It is tempting to expect the variation to coincide with cycles in the property market, but that need not be the case. The reason is that, during times of low perceived risk and high liquidity in credit markets, a greater stock of collateral meets lenders' and borrowers' criteria for financing, while during times of higher perceived risk and low liquidity, only the safest stock of collateral is likely to be financed. In other words, it may be that the aggregated imputed implied volatility is a downward biased measure of the average riskiness of the stock of collateral, with the bias greater during times of credit market dislocation.

Indeed, the five-year period with lowest average implied volatilities is 2003–2007 and coincides with the five-year period of the greatest number of CMBS loan originations (see Figure 2) and relatively liquid mortgage credit markets (Figure 3). There are essentially no data to plot in 2009, accounting for the missing point in Figure 5. Arguably, implied volatilities from 2008 and 2010 originations are biased down because lenders only extended credit to safer properties as credit markets dried (as suggested from Figure 3). The years with highest implied volatility are 2001, and 2017–2019, which were characterized by relative liquidity in mortgage credit markets. This may be a function of higher-than-average perceived risk in the aggregate collateral combined with greater willingness by lenders and borrowers to finance higher-risk assets.

A. *Structural determinants of implied volatility*

One possible objection to our use of implied volatility as a proxy for collateral riskiness is the claim that lenders were risk insensitive when setting CRE loan spreads pre-GFC; in other words, our imputed implied volatilities might reflect something other than property risk in the run-up to the GFC. For instance, pressure to originate for fees during the height of CDO issuances could have spurred frenzied competition for originating CMBS loans, resulting in ultra-low mortgage rates that did not reflect the true risk of the underlying properties.

We address this, and validate the conjecture that implied volatility is related to sources of property risk, by investigating the pre- and post-crisis drivers of implied volatility and verifying whether relevant macroeconomic and property level risk indicators contributed similarly to risk perceptions. Table 4 examines the pre- and post-GFC regression relationships between implied volatility (the dependent variable) and several key variables such as state GDP, real estate sector GDP, unemployment rate, and income per capita, as well as property size and age. Property and interacted state and time fixed effects are included. Property age, state GDP, and state employment rates are positively correlated with risk, potentially consistent with the findings in Fisher et al. (2022) that urban density is associated with greater property market risk. Controlling for these, property size and state income levels are negatively related to risk. Importantly, every coefficient that is significant post-GFC is also significant pre-GFC, and has the same sign. If anything, the marginal effects of these variables on implied volatility are stronger pre-GFC.

Table 4—Marginal effects of structural variables on implied volatility (%)

This table shows the estimated marginal effects of relevant local macroeconomic and property-specific variables on the estimated model-implied volatilities of commercial real estate mortgage loans in our sample. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The marginal effects are estimated on subsamples before and after the Global Financial Crisis (GFC), using a linear regression model with the logarithm of implied volatility as dependent variable. The regression model includes loan originator, property state, and property type-quarter of origination fixed effects. Standard errors are double clustered by state and quarter. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. Local macroeconomic variables are measured at a quarterly frequency and obtained from the Bureau of Economic Analysis. "GDP in sector" stands for the gross domestic product of the real estate industry.

	Pre-GFC	Post-GFC
100 × Log of state real GDP (USD mm)	0.030^{*}	0.014^{**}
100 × Log of state real GDP in sector (USD mm)	0.002	−0.002
100 × Log of state income per capita (USD)	-0.056^{**}	-0.031^{***}
State unemployment rate (%)	-0.243^{**}	-0.161^{***}
Property Age (years)	0.022^{***}	−0.001
100 × Log of property size (sqft)	-0.011^{***}	-0.004^{***}
100 × Log of property size (units)	-0.115^{***}	-0.041^{***}
Number of observations	25,398	17,851

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Additional analysis, not reported here, suggests no significant difference in implied volatility based on whether a loan was issued by systemically important U.S. banks, a systemically unimportant U.S. bank, a foreign bank, a non-bank lender, or an acquired/failed lender. Overall, there is no evidence that mortgage-implied volatility of individual properties was decoupled from fundamentals prior to the GFC.

V. Loan-to-value ratios and risk perceptions

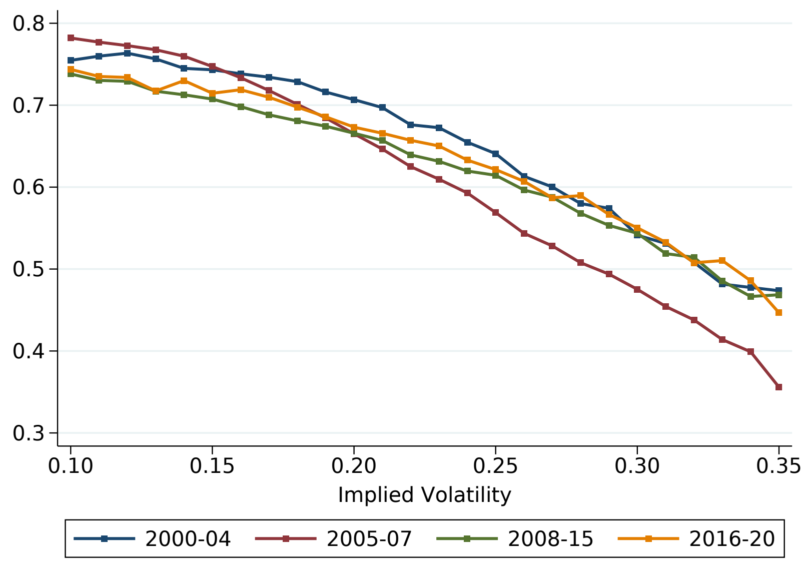
Figure 6 depicts average LTVs for each integer implied volatility "bucket" for loans originated in each of the four epochs described in Section II.C. As might be predicted by a trade-off theory of optimal leverage, across all periods, LTVs exhibit a strong and negative relationship with implied volatility (IV). Indeed a univariate linear regression of IV against LTVs in our sample yields an adjusted R^2 that is an order of magnitude higher than a regression of IV against the other two common CRE mortgage underwriting metrics (debt service coverage ratios and debt yields). Fixing the level of risk, as proxied by implied volatility, it appears that LTVs were generally highest during the first epoch (2000–2004). The only exception comes from loans that were perceived to be relatively low-risk (below 15% implied volatility), for which the most aggressive period was Epoch 2 (2005–2007), when CDO issuance became prevalent. Interestingly, that same much-maligned epoch is associated with seemingly conservative LTVs for loans perceived as higher risk (higher than 20% volatility), and the post risk-retention period (Epoch 4) featured slightly higher LTVs than the prior Epoch spanning the period from the GFC to 2015.

It is important to emphasize that the strong relationship depicted in Figure 6 is not tautological. In a frictionless setting (a so-called Modigliani-Miller world), LTV would be arbitrary and plotting LTV against perceived risk would yield no (or a random) relationship. By contrast, a theory of credit rationing (Jaffee and Russell, 1976; Leland and Pyle, 1977) or tax-bankruptcy trade-offs (Leland, 1994) predicts a downward sloping relationship, which we observe in Figure 6.

Figure 6 suggests differences across time in leverage choice even after controlling for perceived risk. The differences can arise simply from variations in maximum LTV thresholds — the rationing frontier. They can also arise from differences in property cash yield or other, unobserved, differences specific to the collateral. Any variable that can alter optimal leverage can impact observed LTVs. These may include the local credit environment, the marginal tax rate of local investors, capital expenditure expectations that are not reflected in cash yields, etc. To further understand the differences across time in the supply and demand for credit against risky collateral, one ought to attempt to control for such variables and examine how much each explains variation in LTV.

Figure 6. Mean loan-to-value ratios across implied volatility bins and epochs

This figure shows the sample means of the loan-to-value ratios of commercial real estate loans that fall into a given integer model-implied volatility bin and were originated in a given time period (epoch). The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. Epoch choice is explained in Section II.C. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B.



A. *Credit rationing frontier estimation and diagnostics*

In this subsection we investigate whether credit rationing limits (i.e., maximum LTV limits) set by lenders move in time. Our conjecture is that more aggressive lending practices would primarily expressed as increases to such limits. If the distribution of borrowers' demand for optimal leverage is constant, then applying a rationing limit would result in a truncated distribution of observed LTVs, and the observed LTV mean would move monotonically with the truncation point. Thus, once a rationing frontier is identified, a natural question to ask is how much variation in LTV is driven by changes to the frontier.

To begin, we first attempt to identify a lending "frontier" for various IV levels. In other words, what is the maximum credit level that lenders are willing to undertake for a certain risk perception? Given the scarcity of IVs at both extremes, we only do this for implied volatility levels between 5% and 40%. The existence of a frontier can be seen through a quick visual inspection of the data. For instance, Figure F3 shows clustering at around 80% LTV for implied volatility in the rough range of 0.05 to 0.2 across multiple time periods. Using quantile regression, we estimate the frontier as the 95$^{\text{th}}$ LTV percentile within each 1 percent implied volatility interval ("IV bucket") for each of the four time periods.[13]

Figure 7 shows the calculated rationing frontiers by epoch across the implied volatility buckets. The 2005–2007 epoch stands out most in being visually different from, and generally lower than, the other three. Using a quantile regression, a pairwise comparison of marginal linear predictions across periods (Table 5) shows that the LTV frontier for this period is, on average, 4 percentage points lower than 2000–2004, and about 3 percentage points lower than both the 2008–2015 and 2016–2020 epochs. This may be surprising given a common perception that lending standards were looser in the period leading up to the GFC. These results suggest that, on the tail end of maximum loan extension, lending standards in the runup to the GFC were arguably *tighter* after controlling for risk perceptions. The least significant differences in frontiers occur between the two most recent epochs.

Table 5—Testing mean differences between rationing frontier estimates across epochs

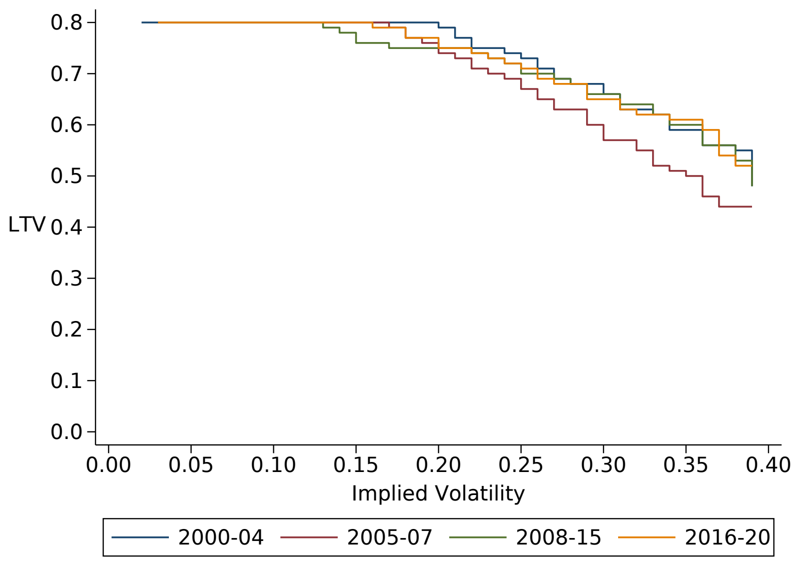
This table shows the the results of statistically testing the mean differences between rationing frontier estimates across different time periods (epochs). The frontiers are estimated by fitting a quantile regression model for the 95$^{\text{th}}$ percentile of the loan-to-value ratios of commercial real estate loans that fall into a given integer model-implied volatility bin and were originated in a given epoch. The estimation sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. Epoch choice is explained in Section II.C. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. From left to right, the columns show the mean differences (Diff.), their standard errors (Std. err.), t-statistics, and corresponding p-values.

Frontier pair	Diff.	Std. err.	t-stat	p-value
2005–2007 vs. 2000–2004	−0.0413	0.0019	−21.62	0.0000
2008–2015 vs. 2000–2004	−0.0127	0.0016	−8.01	0.0000
2016–2020 vs. 2000–2004	−0.0072	0.0018	−4.01	0.0001
2008–2015 vs. 2005–2007	0.0286	0.0020	14.05	0.0000
2016–2020 vs. 2005–2007	0.0341	0.0022	15.40	0.0000
2016–2020 vs. 2008–2015	0.0055	0.0019	2.82	0.0048

[13]The choice of LTV percentile where loans appear to cluster and denoting the frontier is robust to more sophisticated approaches, such as density discontinuity tests.

Figure 7. Credit rationing frontier estimates by epoch

This figure shows our credit rationing frontier estimates across time periods (epochs). The frontiers are estimated by fitting a quantile regression model for the 95^{th} percentile of the loan-to-value ratios (LTVs) of commercial real estate loans that fall into a given integer model-implied volatility bin and were originated in a given epoch. The estimation sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. Epoch choice is explained in Section II.C. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B.



Overall, our analysis of rationing frontiers does not support a narrative that lenders were relatively more aggressive in 2005-2007. It is possible, however, that lenders' perceptions of property risk was systematically lower than what proved realistic, ex-post. Such misperceptions would have unwittingly led to excessive provision of credit (e.g., extending an 80% LTV loan to a property judged to exhibit a property risk of 13% vol when the property was actually characterized by a risk of 21% vol). An alternative explanation is that originators in 2005-2007 did not care about risk when setting mortgage rates because property risk would accumulate in tranches that were subsequently placed in CDOs. While plausible at face value, this alternative is not supported by our analysis of determinants of IV in Section IV.A and leaves unanswered why credit rationing during 2005-2007 was substantially tighter for loans with IV greater than 20%.

B. *Quantification of loan-to-value ratio determinants*

Although there is no support for a looser credit rationing frontier during 2005-2007, it is still useful to understand whether movements in the frontier, which could be driven by imprudent extension of credit, have significant impact on the observed distribution of LTV. After all, based on the frontier analysis, one could argue that the first epoch, 2000-2004, was characterized by lending that was too permissive. Did that matter? In this section, we test this and, more broadly, seek to ask how much of the variation in LTV can be explained using economic fundamentals.

Denote the demand for credit by the borrower for a given observed loan as LTV_i. The amount of credit that is observed to be extended is $cLTV_i = \min[R(c_i, b(IV_i)),\ LTV_i]$, where $R(c,k)$ is the rationing frontier in epoch c and implied volatility bin k, as identified in the previous section. We fit a censored linear regression (tobit) model to $cLTV_i$ of the form:

(2)

$$\begin{aligned} cLTV_i = \max\big[0, \min\big\{ & R(c_i, b(IV_i)), \\ & \mu_{type} + \mu_{s,q} + m_q + o_i + \alpha(c_i)\mathrm{IV}_i + \beta_1 CRS_i + \beta_2 {CRS_i}^2 + \epsilon_i\big\}\big] = \\ = \max\big[0, \min\big\{ & R(c_i, b(IV_i)), x_i\beta + \epsilon_i\big\}\big] \end{aligned}$$

where μ_{type} denotes fixed effects for the property's type, $\mu_{s,q}$ denotes quarterly time fixed and/or state/county fixed effects, m_q is a vector of quarterly macro variables (included if time fixed-effects are absent), o_i is an originator fixed effect, IV_i is the loan's implied volatility (with an epoch-dependent coefficient), CRS stands for the cap rate spread over the 10-year U.S. Treasury yield, and $\epsilon \sim \mathcal{N}(\mu, \sigma_\epsilon^2)$ is a residual term. We use $x_i\beta$ as a shorthand for the linear predictor of the latent part.[14]

Table 6 shows the results of the tobit regression and the strong negative relationship between LTV and IV. Strictly on its own, and with a fixed slope coefficient, IV explains two thirds of the variation in LTV across time. Controlling

[14]The cap rate spread over the 10-year yield is a measure of the cash yield net of the risk-free rate. Leland (1994) shows that optimal LTV should decline with the payout rate. The intuition is that, under the risk-neutral measure, all assets grow at the same rate (the risk-free rate) so an asset that reinvests income will grow more than an asset that distributes income. Correspondingly, a slower-growing asset is more likely to default at loan maturity. We use a net payout rate because interest rates experienced a secular decline between 2000 and 2020, accompanied by a commensurately declining property cash yield.

for quarterly time fixed effects and property type fixed effects, 2005–2007 emerges as the epoch with the greatest sensitivity to IV. This relationship holds even as loan originator fixed effects for the 111 originators in our sample are taken into account, suggesting that individual originators, including those that systematically overstate property financials as documented in Griffin and Priest (2023), are not (on average) driving much of the variation in the LTV-IV relationship. Collateral cash yield is highly significant and appears with the predicted negative sign (Leland, 1994), but only once we control for time, property, and state fixed effects. The CMBS yield spread of short-term AAA CMBS bonds over treasuries of equivalent maturity proxies for illiquidity in the mortgage market and explains a substantial portion of the time-series variation in the data. Viewed as a proportional cost of financing, the presence of illiquidity should negatively impact the choice of optimal leverage and this is consistent with the sign of its coefficient in Table 6.

The estimates in Table 6 do not indicate whether changes in the rationing frontier across epochs materially impacts the distribution of LTVs. To quantify the impact of such changes, we conduct a "counterfactual" LTV analysis. For each loan, the estimated model in (2) can be used to also estimate the residual, ϵ_i, and therefore the amount of credit demanded by the borrower:

$$\hat{\epsilon}_i = \begin{cases} cLTV_i - x_i\hat{\beta} & \text{if } cLTV_i < R(c, b(IV_i)) \\ \mathbb{E}[\epsilon_i \mid \hat{\sigma}_\epsilon, \epsilon_i \geq R(c, b(IV_i)) - x_i\hat{\beta}] & \text{if } cLTV_i = R(c, b(IV_i)) \end{cases} \tag{3}$$

We define a "counterfactual" observed LTV, denoted as $cLTV_i^*$, to be the model estimate for an observation after changing the value of one or more of the independent variables while keeping $\hat{\epsilon}_i$ the same. Specifically,

$$cLTV_i^* = \max\left[0, \min\left\{R(c_i^*, b(IV_i^*)), x_i^*\beta + \hat{\epsilon}_i\right\}\right] \tag{4}$$

where the asterisk denotes an alternative assignment of the independent variables. For this exercise, we use parameter estimates from the 8^{th} censored linear model specification in Table 6. In Figure 8, we plot the mean LTV in each year of the original data set (Panel A) and for various counterfactual data sets (Panels B-F). The original data clearly exhibits a secular decline of LTVs over the sample period. However, this trend disappears in Panel B, which uses a counterfactual data set

Table 6—Estimation results of censored linear regression for the loan-to-value ratio

This table shows the estimation results of the censored linear regression model defined in Equation (2). The estimation sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The dependent variable is the loan-to-value ratio, and IV stands for the model-implied volatility estimate for sample loans. The implied volatilites are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The columns show different model specifications with an expanding set of explanatory variables and fixed effects included. Standard errors are clustered by the quarter of loan origination and reported under the corresponding coefficient estimates in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
IV	-1.29							
	(0.03)							
2000–2004 # IV		-1.22	-1.20	-1.46	-1.39	-1.38	-1.38	-1.17
		(0.04)	(0.05)	(0.07)	(0.06)	(0.06)	(0.06)	(0.04)
2005–2007 # IV		-1.36	-1.31	-1.67	-1.63	-1.60	-1.63	-1.41
		(0.05)	(0.06)	(0.03)	(0.03)	(0.03)	(0.03)	(0.05)
2008–2015 # IV		-1.37	-1.35	-1.24	-1.28	-1.27	-1.27	-1.21
		(0.04)	(0.05)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)
2016–2020 # IV		-1.28	-1.26	-1.25	-1.27	-1.26	-1.25	-1.25
		(0.03)	(0.04)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
Caprate spread			0.18	-0.06	-0.15	-0.36	-0.40	-0.14
			(0.18)	(0.13)	(0.12)	(0.12)	(0.12)	(0.14)
CMBS yield spread								-4.54
								(0.68)
UST 10yr yield								0.23
								(0.33)
Time (quarterly)				x	x	x		
Property type				x	x	x	x	x
Loan originator					x	x	x	x
Property state						x		
Property state × Time							x	
Property county								x
Generalized R^2	0.66	0.67	0.68	0.74	0.76	0.77	0.79	0.75
Number of observations	48,931	48,931	48,931	48,931	48,931	48,892	48,892	48,800

where IV for each loan in equation (4) is fixed at the unconditional sample mean (20%). Further setting the cap rate spread to its unconditional mean of 3.7% (Panel C) does not make much difference (consistent with the estimate in Figure 8, column 8). Fixing the CMBS yield spread does appear to reduce the time-series variation (Panel D) while fixing the US treasury 10-year treasury yield makes little impact (Panel E). Perhaps most importantly, fixing the rationing frontier to correspond to the first epoch (2000-2004) appears to have little impact on distribution of LTV time series means (Panel F). Recalling that the first epoch features the most permissive rationing frontier and the second the least, one would expect to see a large difference in the 2005-2007 data when moving from Panels E to F. The fact that a difference cannot be discerned suggests that changes to the rationing frontier do not have a large impact on the distribution of LTVs.

Table 7 compares the annual counterfactual means throughout the four epochs and shows that they are statistically distinct. This means there is still statistically significant remaining variation between the epochs after controlling for IV, cash yields, CMBS spreads, U.S. Treasury yields, and lender thresholds. That said, the remaining time variation does not clearly fall into a pattern coinciding with macro events and may correspond to unobserved demand factors in the market for CRE loans. Moreover, after controlling for these influences, the differences in mean LTV are economically small: Less than 3% across all three epochs in Regression 6 of Table 7. Even more interesting is that the unexplained average difference between Epochs 1 and 4, as well as that between Epochs 2 and 3, is approximately 1% or less.

It is clear from the adjusted R^2 in Table 6 as well as the counterfactual LTVs in Figure 8 that the first order determinant of LTV in the sample is the perceived risk of the collateral. To provide a sense of the marginal contribution that the explanatory variables provide each sample year, we sequentially decompose the variance of LTV in the sample into components of the form

$$\text{comp}_{n,t} = Cov\Big(cLTV_t, \quad cLTV_t^*|_{(x_1,\ldots,x_{n-1})=(\bar{x}_1,\ldots,\bar{x}_{n-1})} - cLTV_t^*|_{(x_1,\ldots,x_n)=(\bar{x}_1,\ldots,\bar{x}_n)}\Big)\Big/ Var\big(cLTV_t\big), \tag{5}$$

where $cLTV_t^*|_{(x_1,\ldots,x_n)=(\bar{x}_1,\ldots,\bar{x}_n)}$ is the subsample of counterfactual LTVs in year t, generated by fixing variables x_1 to x_n at their sample means.[15] The first component is simply

$$\text{comp}_{1,t} = Cov\Big(cLTV_t, \quad cLTV_t - cLTV_t^*|_{x_1=\bar{x}_1}\Big)\Big/ Var\big(cLTV_t\big),$$

and is essentially the R-squared of regressing $cLTV_t$ against x_1 (because variation in the second term of the covariance only arises from variation in x_1). The last

[15]The variable x_i can be viewed as a vector, or "block", of explanatory variables whose contribution to the variance is sought. This is roughly similar to an ANOVA decomposition, although components can be negative because the covariance is restricted to a subsample.

Table 7—Actual and counterfactual means of loan-to-value ratios across epochs

This table shows the means of the actual (Column 1) and counterfactual (rest of the columns) loan-to-value ratios of commercial lean estate loans in the sample across time periods (epochs). The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The counterfactual loan-to-value ratios are estimated by applying Equation (4) and using the 8^{th} censored linear model specification in Table 6. Each regression, (2)-(6) incrementally fixes the values of various explanatory variables. US10 stands for the 10-year zero-coupon U.S. Treasury yield, while IV and CRS stand for model-implied volatility and capitalization rate spread over the US10, respectively. CMBS stands for the market liquidity spread defined in Section III.B. Robust standard errors are reported in parentheses. At the bottom, F-statistics are reported for the joint mean equality tests across epochs 1 to 4 and epochs 2 to 4, respectively.

Epoch	(1)	(2)	(3)	(4)	(5)	(6)
2000–2004	68.8	69.2	69.7	69.5	69.0	69.0
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
2005–2007	68.8	67.0	68.0	66.7	66.2	66.4
	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
2008–2015	66.1	65.6	66.0	66.9	67.2	67.5
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
2016–2020	62.9	67.4	68.0	67.5	67.8	68.1
	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)
IV = 20%		x	x	x	x	x
CRS = 370bp			x	x	x	x
CMBS = 120bp				x	x	x
US10 = 3.2%					x	x
Frontier set to epoch 1 level						x
Wald F Stat. of 1–4 equality	530.1	516.2	544.1	465.4	420.7	360.0
Wald F Stat. of 2–4 equality	632.4	202.3	339.4	32.4	190.6	188.4
Number of observations	50,734	48,800	48,800	48,800	48,800	48,800

component, $\text{comp}_{n,t}$, is the proportion of variance that cannot be explained with $x_1, \ldots, x_n$.

Panel A of Figure 9 depicts the decomposition when x_1 is IV and x_2 corresponds to all remaining explanatory variables. Implied volatility explains between 45% and 80% of LTV variance in any given year. The incremental contribution of all other independent variables to explaining variance is no more than 20% (in 2000) and averages 10%. Of the non-IV independent variables, cap rate spread, originator fixed effects, and geographic fixed effects are the most important sources of variation. Their contributions are depicted in Panel B of Figure 9 and appear to be most pronounced pre-GFC and especially during the first epoch (2000-2004). One might be troubled by the fact that originator fixed effects capture variation in LTVs in the early part of the sample. A mitigating observation is originator effects substantially decline in 2005-2007 relative to 2000-2004. As suggested by prior analysis, the amount explained by time-varying rationing frontier is third order.

VI. Conclusions

Theory (e.g., Leland, 1994) suggests that optimal leverage choice depends on asset and market-specific factors. One of the most important of these is the risk of the underlying asset. We demonstrate that the single most important determinant of observed loan-to-value ratios (LTVs) in securitized commercial real estate loans is property perceived risk, as measured by a property's mortgage implied volatility. On its own, perceived risk explains roughly two thirds of cross-sectional and time-series variation in LTVs. We find that other theoretically-motivated market-level and asset-specific fundamentals also drive observed choices of LTVs, albeit explaining no more than an additional (roughly) 10% in variation.

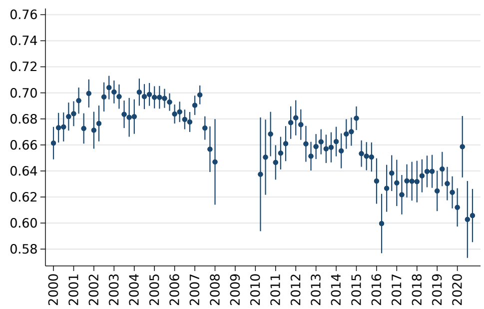
While LTVs have declined throughout our sample period (from 2000 until 2020), this secular decline disappears once one controls for the fundamental factors mentioned above. Remaining time variation does not appear to reflect any market trends. This is significant because LTV is commonly seen as an important metric of lending standards and often referenced in regulations (e.g., DiSalvo and Johnston, 2018). We find some evidence that aggregate LTVs contains information about lending standards through shifting maximum LTV criteria. However, the shifts we identify do not support the narrative that lending standards were least restrictive during the run-up to the Great Financial Crisis. Moreover, there is little evidence changes in maximum LTV criteria materially impacted the distribution of LTVs in our sample. We do, however, find that the key driver of LTV choice in our data set, average perceived collateral risk, was significantly lower in the five years leading to the Great Financial Crisis than at any other five year period in the past 20 years. This raises the possibility that lenders and borrowers systematically underestimated property risk between 2003–2007, leading to more-than-warranted credit being extended against commercial real estate and exacerbating the subsequent market downturn.

To the extent that ex-post poor lending outcomes can be traced to systematic risk misperception, our work can be used to motivate the use of aggregate measures of market-specific risk perceptions, like loan-implied volatility, by regulators and policymakers.

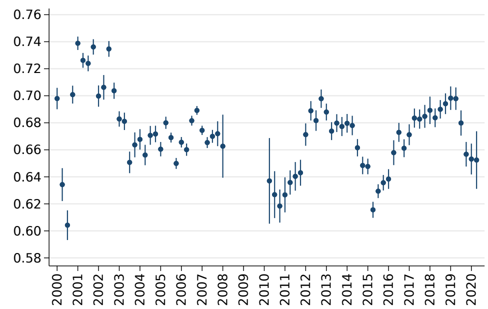
Figure 8. Means of actual and counterfactual loan-to-value ratios over time

This figure shows the means of the actual (Panel A) and counterfactual (rest of the panels) loan-to-value ratios of commercial lean estate loans in the sample over time, with 99% confidence intervals. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The counterfactual loan-to-value ratios are estimated by applying Equation (4) and using the 8$^{\text{th}}$ censored linear model specification in Table 6. Each plot from B-F incrementally fixes the values of various explanatory variables. US10 stands for the 10-year zero-coupon U.S. Treasury yield, while IV and CRS stand for model-implied volatility and capitalization rate spread over the US10, respectively.

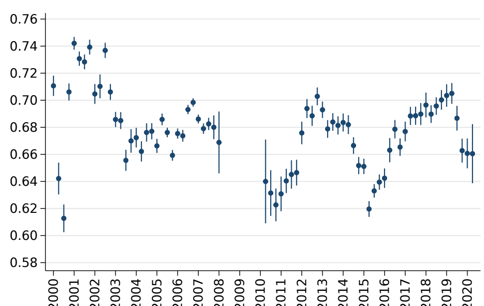
(a) Actual sample means



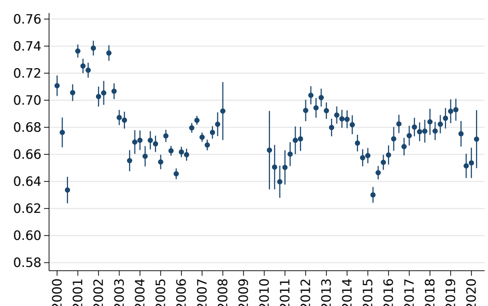
(b) Setting IVs to 20%



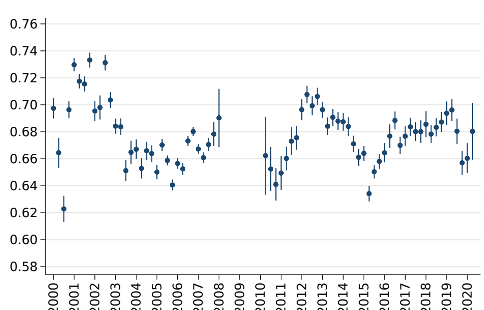
(c) Setting IVs to 20% and CRSs to 370bp



(d) Setting IVs to 20%, CRSs to 370bp, and CMBS spreads to 120bp



(e) Setting IVs to 20%, CRSs to 370bp, CMBS spreads to 120bp, and USTs to 3.2%



(f) Setting IVs to 20%, CRSs to 370bp, CMBS spreads to 120bp, US10 to 3.2%, and fixing rationing frontier at Epoch 1 level

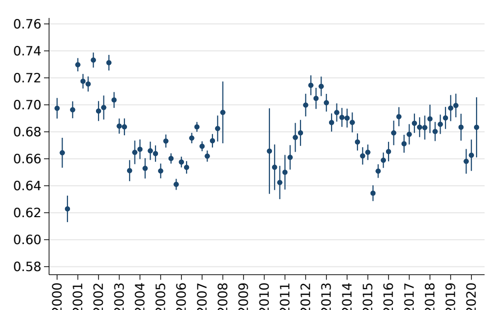
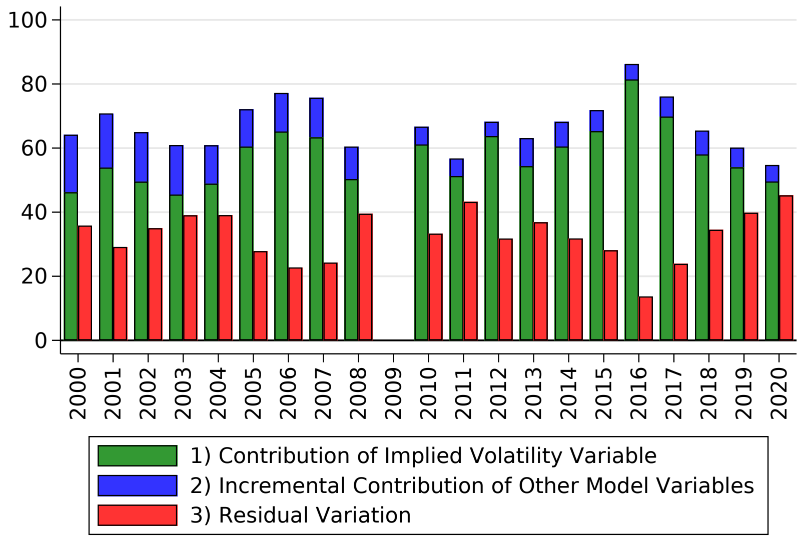


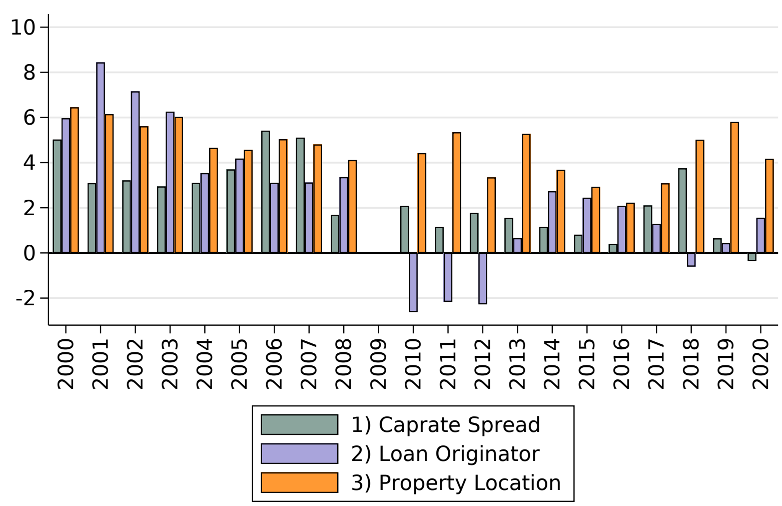
Figure 9. Model-based variance decomposition of loan-to-value ratios over time

This figure shows the model-based variance decomposition of the loan-to-value ratios (LTVs) of commercial lean estate loans in the sample over time. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. The variance decomposition applies the methodology described in the text (see Equation (5)), measuring the contribution of each variable to the variance of LTV in the context of the censored linear model defined in Equation (2). More specifically, LTV is modeled using the 8^{th} censored linear model specification presented in Table 6. Panel A decomposes the variance of LTV into the contribution from implied volatility, other model variables, and residual variation. Panel B presents the variance contribution of the three most relevant model variables other than implied volatility.

(a) Variance attributable to implied volatility and other model variables



(b) Variance attributable to selected other model variables



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Appendix A: Data construction process

The initial data set consists of 171,421 loans.

- We eliminate loans in deals originated by Freddie Mac or Fannie Mae. Both institutions are heavily involved in affordable housing, senior housing, and other subsidized projects. The pricing of such loans may not fully reflect the market perception of risk. Not all of their loans are for subsidized projects, but to our knowledge there is no efficient way to distinguish them from others.

- We eliminate loans with missing key variables such as origination date, maturity date, coupon rate, original loan amount, underwritten NOI, and origination LTV. We also drop loans with unrealistic values for these variables.[16]

- We remove a small number of loans with both defeasance and yield maintenance penalties (our model is not set up to take more than one prepayment penalty), more than three call protection options, and loans with an ambiguous call protection designation such as "prepayment penalty."

- We have a number of loans for which the call protection lengths plus seasoning do not add up to the loan term. We have 326 such loans which undershoot the loan term and 6,908 which overshoot the loan term. When they undershoot, we simply extend the last call protection period. When they overshoot, we start subtracting from the last call protection type, then the second to last, then the first. We end up with 3,506 loans for which the last call protection type ends up getting completely removed. We only calculate IV for these loans in the batches excluding prepayment.

- Our model assumes that dividends, relative to property value, are constant for the life of the loan. In reality, some loans are for renovation purposes or fund other projects that would result in NOI increases. Such loans may include projected cash shortfalls during the beginning of loan life, relative to

[16]We require annual interest rates to be between 1% and 25%, loan amounts to be at least $10,000, LTVs to be less than 100% and greater than 10%, and first year projected NOI at origination to be less than the property value.

the required debt service. We keep only loans whose debt service coverage ratio is greater than 1.25, and whose debt yield is greater than 7%.

- Our model also assumes that the property is collateral for a single loan only. Multiple forms of debt create potentially complicated dynamics between different creditors. We drop 1,850 multi-property loans as well as a number of loans in pari-passu deals.

- We drop a small number of loans with maturities longer than 12 years, as well as those originated before 2000 and are missing zip code data.

- NCF isn't as well populated as NOI, so we multiply NOI by a factor of 0.94 to match the average NCF (this is only done for our implied volatility calculations).

Appendix B: Implied volatility model

B1. Interest rate process

Gupta and Subrahmanyam (2005) run a horse-race among several prevalent pricing models and find that the pricing accuracy of one-factor models is comparable to that of other, more complicated, models. We use two of the models they examine: the Hull and White (1990) (HW) and Black and Karasinski (1991) (BK) models. These are some of the most commonly employed term-structure models for pricing interest rate derivatives in practice. We modify both the HW and BK models so that no more than one tree branch can be above 10% or below zero.[17] This is done to ensure that risk neutral probabilities for the property price model are positive at property diffusion volatilities as low as 3%. We note that, during our sample period, forward rates for a one-year zero coupon U.S. Treasury bond never exceed 7.5% or fall below 0%. Our bounds therefore likely reflect market perceptions for the possible range of interest rates during the life of originated mortgages in our data set.

To calibrate each month's term structure model, yield data are obtained for nominal zero coupon bonds with maturities ranging from one to twelve years.[18] Data for swaptions with exercise maturity of one year, the most liquid contracts, are obtained from Eikon for tenors (underlying swap maturities) of one, five, and ten years. Each month, we fit a HW and a BK model to the data and select the one that best fits the swaptions data.[19] Table B1 summarizes percentage price accuracy across the monthly term structure models that we estimate.

Our term structure models are generally accurate. Periods where the pricing error exceeds 5% are concentrated between 12/2008 to 03/2009, 09/2011 to 12/2012, 02/2016 to 11/2016, and after 03/2020. The BK model seems to be the

[17]This is achieved as follows. If the conventional HW or BK tree is consistent with the bounds, we employ it. Otherwise, we truncate all branches beyond the first that cross the bound by setting their transition probabilities to zero. At any node for which a branch probabilities is set to zero, we solve for the remaining branch probabilities by enforcing the node's expected interest rate to equal the quantity implied by the underlying mean-reverting process. The resulting rate volatility at edge nodes is generally distinct from the constant volatility elsewhere in the tree.

[18]The data are taken from the Federal Reserve: https://www.federalreserve.gov/data/nominal-yield-curve.htm. For the short end of the term structure, we use the 3-month U.S. Treasury constant maturity yield obtained from the St. Louis Fed.

[19]Both HW and BK models can be made to fit any arbitrary term structure of zero coupon bonds.

Table B1—Term Structure Model Precision

This table shows the accuracy of 277 "best-fitting" term structure models, which are estimated each month from 06/1997 to 06/2020 using zero coupon bonds and one-year swaption prices. The two models used for estimation are Hull and White (1990) and Black and Karasinski (1991) models. Each data point corresponds to the root of the weighted mean of squared pricing errors (i.e., percentage accuracy) from a single month's term structure model.

Statistic	TS Model precision
Mean	0.0202
SD	0.0439
P1	0.0001
P5	0.0003
P10	0.0007
P25	0.0015
P50	0.0042
P75	0.0149
P90	0.0534
P95	0.1146
P99	0.2322

better performer in roughly 2/3 of cases, and nearly exclusively so between 2007 and 2015.

B2. Property value process

Property value follows a binomial process similar to Cox, Ross and Rubinstein (1979), but modified to incorporate time-varying short-term interest rates and a possible catastrophic fall in value that triggers immediate default. The latter modification is motivated by the actual distribution of creditor losses. Without the possibility of a sudden (discontinuous) drop in property value, optimal exercise of the default option tends to predict relatively small loan losses relative to what is observed in practice. The "catastrophic" property-level event is Poisson distributed and assumed to arrive with annualized intensity of λ. The event is assumed to permanently reduce the value of the asset to zero, and the rate λ is calibrated to our loan pool to match historical CRE loss given default (LGD) rates of 30-35%.[20]

[20] This range is based on estimates by Esaki, L'Heureux and Snyderman (1999), Ciochetti (1997), and Curry, Blalock and Cole (1991). One could potentially model a distribution of catastrophic losses, but it is unclear to what extent this would make our results more reflective of lender perceptions versus a fixed LGD value. See Appendix D for further methodology.

Let σ be the annual volatility of the value of the property. To modify the binomial model of Cox et al. to accommodate the hazard, we divide the usual "up" and "down" states by $(1 - \lambda\Delta t)$ for each increment of time, Δt:

$$u = \frac{e^{\sigma\sqrt{\Delta t}}}{1 - \lambda\Delta t} \qquad\qquad d = \frac{e^{-\sigma\sqrt{\Delta t}}}{1 - \lambda\Delta t}.$$

The property value changes by a factor of u or d. This keeps the expected price appreciation of the asset, under the risk-neutral measure, independent of the value of λ and thus independent of the idiosyncratic event. It also has the virtue of setting the Arrow-Debreu prices of "up" and "down" non-disaster states equal to $(1 - \lambda\Delta t)$ times their usual values in the Cox model:

$$\pi^{(u)}_{j,t,k} = \frac{e^{r_{k,t}\Delta t} - d(1 - \lambda\Delta t)}{u - d} \qquad\qquad \pi^{(d)}_{j,t,k} = \frac{u(1 - \lambda\Delta t) - e^{r_{k,t}\Delta t}}{u - d},$$

where we denote property state j, time t, and continuously compounded short interest rate state k obtained from the term-structure model. We assume that a one-period binomial "up" or "down" move in the property price process is uncorrelated with the one-period short-term interest rate process.

Commercial properties generate income for their owners, which we incorporate by assuming that the property pays a constant annual "dividend" rate δ_t corresponding to the property's ratio of net cash flow (NCF) to total appraised property value at the time of mortgage origination.[21] We include the dividend in our property value formulation, with the exception of origination, where property value is equal to appraised value.[22] We define "up" and "down" "cum-dividend" property value $V_{j,t+1}$ for all non-origination periods $t \in \{1, ..., T-1\}$ recursively as follows:

$$V_{j_u,t+1} = uV_{j,t}(1 - \delta_t\Delta t) \qquad\qquad V_{j_d,t+1} = dV_{j,t}(1 - \delta_t\Delta t).$$

[21]We use NCF instead of net operating income (NOI), as NCF subtracts CapEx and CapEx reserves and may be a better measure of actual cash flow.

[22]We assume that the property value at mortgage origination is calculated *after* cash flow from operations is distributed.

Mortgage terms comprise the LTV ratio (or, equivalently, the amount borrowed), time to maturity, and the amortization schedule. Together with a complete specification of the property and interest rate model parameters, the mortgage terms imply a fair-market mortgage rate that can be calculated by setting the present value of the mortgage obligation to the amount borrowed. In practice, contract mortgage rates are observed but the underlying property volatility, σ, is unobserved. We therefore solve for the implied property volatility that sets the present value of the mortgage obligation to the amount borrowed given the observed mortgage rate.

We denote property value $V_{j,t,k}$ ($V_{j,t} = V_{j,t,k}$ since property value is independent of interest rate movement), and corresponding equity and debt values $E_{j,t,k}$ and $D_{j,t,k}$. We allow for interest-only or amortizing mortgage payment schedules (or a combination of these). We denote the remaining mortgage balance B_t, and fixed mortgage payment or coupon c_t (B_t remains constant during an interest-only period). As in Cox et al., it is easiest to define our model by working backwards from maturity. Similar to Merton (1974), we define borrower equity and debt at maturity T:

$$E_{j,T,k} = \max(0,\ V_{j,T} - (B_T + c_T))$$
$$D_{j,T,k} = \min(V_{j,T},\ B_T + c_T).$$

These follow from the assumption of "ruthless" default: the borrower will default if the property value falls below the debt value. It is worth emphasizing that the Modigliani-Miller value additivity holds: $D_{j,t,k} + E_{j,t,k} = V_{j,t,k}$.[23] In other words, we assume no deadweight cost of default.[24] Note that there is no prepayment or dividend payment at maturity. For each non-maturity and non-origination period $t \in \{1, ..., T-1\}$, the following equations determine the

[23]See Appendix C for proof that M&M holds at all periods t.

[24]We opt to ignore deadweight cost of default because we do not correspondingly model tax benefits of debt or investors with heterogeneous private values; if mortgage debt only came with costs and no benefits, no rational investor would finance a property with mortgage debt.

borrower's value of equity and debt:

$$E_{j,t,k} = \max\left(0,\ \delta_t \Delta t V_{j,t,k} - c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{E}_{t+1}\right],\ V_{j,t,k} - c_t - B_t - P_{j,t,k}\right)$$
$$D_{j,t,k} = \min\left(V_{j,t,k},\ c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right],\ c_t + B_t + P_{j,t,k}\right),$$

where $\mathbb{E}_{j,t,k}[\tilde{E}_{t+1}]$ and $\mathbb{E}_{j,t,k}[\tilde{D}_{t+1}]$ represent risk-neutral expected values for equity and debt and $P_{j,t,k}$ is the prepayment penalty. The terms in each equation represent values for default, continuation, and prepayment options, respectively. For further clarity, risk neutral expected values for $X \in \{E, D\}$ are defined as follows:

$$\underset{j,t,k}{\mathbb{E}}\left[\tilde{X}_{t+1}\right] = \pi^{(u)}_{j,t,k}\ \left[i^{(u)}_{k,t} X_{j_u,t+1,k_u} + i^{(m)}_{k,t} X_{j_u,t+1,k} + i^{(d)}_{k,t} X_{j_u,t+1,k_d}\right]\ +$$
$$\pi^{(d)}_{j,t,k}\ \left[i^{(u)}_{k,t} X_{j_d,t+1,k_u} + i^{(m)}_{k,t} X_{j_d,t+1,k} + i^{(d)}_{k,t} X_{j_d,t+1,k_d}\right],$$

with $i^{(u)}_{k,t}$, $i^{(m)}_{k,t}$, and $i^{(d)}_{k,t}$ being interest rate up, middle, and down state probabilities. In the initial origination period $t = 0$, we take the values of equity and debt to be their continuation values: $E_0 = (e^{-r_0\Delta t})\, \mathbb{E}_{0,0,0}[\tilde{E}_{t+1}]$ and $D_0 = (e^{-r_0\Delta t})\, \mathbb{E}_{0,0,0}[\tilde{D}_{t+1}]$ (we assume no mortgage coupon payment at origination and no dividend as noted above). After inputting all given mortgage values, we calculate an annual implied volatility figure σ such that D_0 matches the given contract loan amount.

Prepayment rules are specified in the mortgage covenant, and usually vary by time period in the mortgage. For instance, a common feature is a prepayment lockout of several months when prepayment is not allowed, followed by a lengthy period where prepayment is allowed but with penalties (usually defeasance or yield maintenance), followed by another shorter period where prepayment is allowed without penalties (the "open" prepayment period). These periods sum to the length of the mortgage. We model this as follows: we remove the prepayment option during the lockout period, explicitly model defeasance or yield maintenance during penalty periods,[25] and set $P_{j,t,k}$ equal to zero during open periods.

[25]See Appendix E for exact methodology.

Appendix C: Proof of Modigliani & Miller additivity

We would like to show that the Modigliani–Miller additivity $E_{j,t,k} + D_{j,t,k} = V_{j,t,k}$ holds for all $t \in \{0, ..., T\}$.

Part 1 of Proof

We begin by demonstrating that $V_{j,t,k} = \delta_t \Delta t V_{j,t,k} \ + \ (e^{-r_{k,t}\Delta t})\, \mathbb{E}_{j,t,k}[\tilde{V}_{t+1}]$ for $t \in \{1, ..., T-1\}$. We redefine the following (note that $V_{j,t} = V_{j,t,k}$ since property value is independent of interest rate movement):

$$\left.\begin{aligned} V_{j_u,t+1,k} &= uV_{j,t,k}(1-\delta_t\Delta t) \\ V_{j_d,t+1,k} &= dV_{j,t,k}(1-\delta_t\Delta t) \end{aligned}\right\} \text{for } t \in \{1, ..., T-1\}.$$

Now we use these to show that $V_{j,t,k} = \delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right) \mathbb{E}_{j,t,k}\left[\tilde{V}_{t+1}\right]$:

$$\begin{aligned}
&\delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{V}_{t+1}\right] \\
&= \delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right)\left[\pi^{(u)}_{j,t,k} V_{j_u,t+1,k} \ + \ \pi^{(d)}_{j,t,k} V_{j_d,t+1,k}\right] \\
&= \delta_t \Delta t V_{j,t,k} \ + \ \left(e^{-r_{k,t}\Delta t}\right)\left[\pi^{(u)}_{j,t,k} u V_{j,t,k}\left(1-\delta_t\Delta t\right) \ + \ \pi^{(d)}_{j,t,k} d V_{j,t,k}\left(1-\delta_t\Delta t\right)\right] \\
&= V_{j,t,k}\ \left[\delta_t\Delta t \ + \ \left(1-\delta_t\Delta t\right)\left(e^{-r_{k,t}\Delta t}\right) \underbrace{\left(\pi^{(u)}_{j,t,k} u \ + \ \pi^{(d)}_{j,t,k} d\right)}_{= e^{r_{k,t}\Delta t} \text{ (No Arbitrage)}}\right] \\
&= V_{j,t,k}\ \left[\delta_t\Delta t \ + \ \left(1-\delta_t\Delta t\right)\left(e^{-r_{k,t}\Delta t}\right)\left(e^{r_{k,t}\Delta t}\right)\right] \\
&= V_{j,t,k}\ \left[\delta_t\Delta t \ + \ 1-\delta_t\Delta t\right] = V_{j,t,k}.
\end{aligned}$$

Part 2 of Proof

At maturity, T, M&M clearly holds ($E_{j,T,k} + D_{j,t,k} = V_{j,T,k}$):

$$\begin{aligned} E_{j,T,k} &= \max(0,\ V_{j,T} - (B_T + c_T)) \\ D_{j,T,k} &= \min(V_{j,T},\ B_T + c_T). \end{aligned}$$

Now we show that M&M holds at any arbitrary time $t \in \{1, ..., T-1\}$. Assuming $E_{j,t+1,k} + D_{j,t+1,k} = V_{j,t+1,k}$ (true for $t+1 = T$) and using induction:

$$E_{j,t,k} = \max\left(0,\ \delta_t \Delta t V_{j,t,k} - c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{E}_{t+1}\right],\ V_{j,t,k} - c_t - B_t - P_{j,t,k}\right)$$

$$D_{j,t,k} = \min\left(V_{j,t,k},\ c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right],\ c_t + B_t + P_{j,t,k}\right).$$

By hypothesis, $E_{j,t+1,k} + D_{j,t+1,k} = V_{j,t+1,k}$. So the continuation state value of date t equity $=$

$$\begin{aligned}
&\delta_t \Delta t V_{j,t,k} - c_t + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{E}_{t+1}\right] \\
&= \underbrace{\left(\delta_t \Delta t V_{j,t,k} + \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{V}_{t+1}\right]\right)}_{=V_{j,t,k} \text{ by Result 1}} - c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right] \\
&= V_{j,t,k} - c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right].
\end{aligned}$$

So,

$$\begin{aligned}
E_{j,t,k} &= \max\left(0,\ V_{j,t,k} - c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right],\ V_{j,t,k} - c_t - B_t - P_{j,t,k}\right) \\
&= V_{j,t,k}\ +\ \max\left(\underbrace{-V_{j,t,k}}_{-x_{j,t,k}},\ \underbrace{-c_t - \left(e^{-r_{k,t}\Delta t}\right) \underset{j,t,k}{\mathbb{E}}\left[\tilde{D}_{t+1}\right]}_{-y_{j,t,k}},\ \underbrace{-c_t - B_t - P_{j,t,k}}_{-z_{j,t,k}}\right).
\end{aligned}$$

Therefore:

$$\begin{aligned}
E_{j,t,k} + D_{j,t,k} = V_{j,t,k} &+ \max(-x_{j,t,k},\ -y_{j,t,k},\ -z_{j,t,k}) + \\
&+ \min(x_{j,t,k},\ y_{j,t,k},\ z_{j,t,k}) = V_{j,t,k}.
\end{aligned}$$

We can easily show M&M holds at $t = 0$ as well. Taking the appraised property value at origination S_0, we divide by $(1 - \delta_t \Delta t)$ to get V_0.

$$E_0 = (e^{-r_0 \Delta t}) \underset{0,0,0}{\mathbb{E}}[\tilde{E}_{t+1}]$$

$$D_0 = (e^{-r_0 \Delta t}) \underset{0,0,0}{\mathbb{E}}[\tilde{D}_{t+1}]$$

$$V_0 = \frac{S_0}{(1 - \delta_t \Delta t)} \Leftrightarrow S_0 = V_0 - \delta_t \Delta t V_0.$$

Using the continuation values of equity and debt for $t \in \{1, ..., T-1\}$ referenced above and removing the dividend $\delta_t \Delta t V_0$ and coupon c_t, we get:

$$E_0 = S_0 - (e^{-r_0 \Delta t}) \underset{0,0,0}{\mathbb{E}}[\tilde{D}_{t+1}]$$

$$E_0 + D_0 = S_0 - (e^{-r_0 \Delta t}) \underset{0,0,0}{\mathbb{E}}[\tilde{D}_{t+1}] + (e^{-r_0 \Delta t}) \underset{0,0,0}{\mathbb{E}}[\tilde{D}_{t+1}] = S_0.$$

Appendix D: Loss given default

We define LGD at each default node j, t, k as follows:

$$LGD_{j,t,k} = 1 - \frac{V_{j,t,k}}{c_t + B_t}.$$

For each loan in our sample, we obtain an expected LGD figure based on a Monte Carlo simulation run 10,000 times. To do this, we randomly determine property and interest rate movements by weighting these choices by their respective risk neutral probabilities. Upon reaching a default node, the simulation stops and records LGD for simulation number i as $LGD_i = LGD_{j,t,k}$. If no default occurs, $LGD_i = 0$. With probability $\lambda\Delta t$, a catastrophic property loss happens (the risk neutral "up" and "down" property probabilities sum to $1 - \lambda\Delta t$) and default occurs with $LGD_i = 1$. So expected LGD for each loan l is:

$$eLGD_l = \left(\Sigma_{i=1}^{10000} LGD_i\right)/10000.$$

Appendix: Prepayment penalties

We use standard definitions for yield maintenance and defeasance, but modified to fit our term structure models.

The basic principle of defeasance is that the lender is losing a spread when the borrower refinances and requires the risk-free present value of that spread as a penalty. To mimic this spread, we use our term structure calculations to create a portfolio of risk-free assets (in our case, zero coupon bonds) with the same cash flows. The calculation for any interest rate state k, t is as follows:

$$Def_{k,t} = \left(\sum_{i=1}^{T-t} m_{t+i}\ ZCB_{k,t,t+i}\right) - B_t,$$

where m_{t+i} is the mortgage payment at date $t+i$, $ZCB_{k,t,t+i}$ is the value at date t of a zero coupon bond with maturity at date $t+i$, and B_t is the remaining mortgage balance.[26] $ZCB_{k,t,t+i}$ is calculated by creating a sub-tree M of all continuation

[26]Note that, for simplicity, we calculate defeasance and yield maintenance up to maturity T. In practice, there is some heterogeneity, with some lenders calculating the penalty up to the beginning of the open prepayment period instead. We do not observe the exact lender method in our data, though, and the differences between the beginning of the open prepayment period and maturity are usually very minor.

states starting at node k, t of the trinomial interest rate tree. The final column of the tree, which represents time $t+i$, has payoffs of 1 ($M_{k,t+i+1} = 1 \; \forall k$). We then determine the ZCB price by iterating backwards to the original node k, t so that the following recursive formula holds:

$$M_{k,t} = \left(e^{-r_{k,t}\Delta t}\right) \; \left[i_{k,t}^{(u)} M_{k_u,t+1} + i_{k,t}^{(m)} M_{k_m,t+1} + i_{k,t}^{(d)} M_{k_d,t+1}\right].$$

Yield maintenance is slightly different in that it involves replacing the missing spread with a U.S. Treasury security or other risk-free asset of the same remaining term as the mortgage. This is done using the calculated zero coupon bond rates as follows. First we calculate a "risk-free" par bond prevailing rate for the appropriate maturity:

$$rf_{k,t} = (1 - ZCB_{k,t,T}) \; / \; \left(\sum_{i=1}^{T-t} ZCB_{k,t,t+i}\right).$$

Then we calculate an annual "present value factor" f:

$$f_{k,t} = \left((1 - (1 + rf_{k,t}))^{-(T-t)/\Delta t}\right) \; / \; rf_{k,t}.$$

Finally, the yield maintenance penalty is calculated:

$$YM_{k,t} = (rm - rf_{k,t}) \; f_{k,t} \; B_t,$$

where rm is the mortgage rate.

Appendix F: Additional figures and tables

Table F1—Distribution of CRE Loans by Property Type

This table shows the absolute frequencies of commercial real estate mortgage loans in the Morningstar dataset across different collateral property types, with an emphasis on the distinctions between single and multi-property loan frequencies. The sample contains fixed-rate loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities.

	Single-Property Loans	Multi-Property Loans	Total
Hotel	4,849	196	5,045
Industrial	3,043	134	3,177
Mixed	0	175	175
Multi-family	22,642	494	23,136
Office	9,341	216	9,557
Other	5,940	333	6,273
Retail	16,247	297	16,544
Total	62,062	1,845	63,907

Table F2—95^{th} to 99^{th} Percentile of Implied Volatility by Epoch

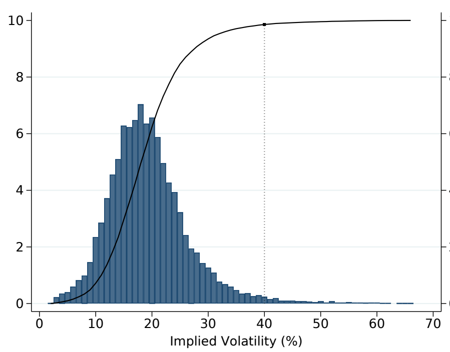
This table shows 95^{th} to 99^{th} percentiles of calculated implied volatility by time period (epoch), as well as overall throughout the sample. Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities.

	P95	P96	P97	P98	P99
2000–2004	0.32	0.34	0.36	0.39	0.44
2005–2007	0.30	0.31	0.32	0.34	0.38
2008–2015	0.31	0.33	0.35	0.40	0.53
2016–2020	0.35	0.38	0.42	0.54	0.61
Total	0.32	0.33	0.35	0.38	0.47

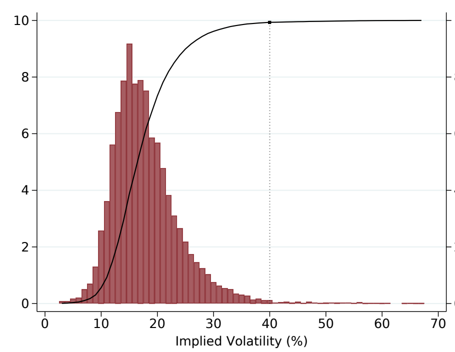
Figure F1. Distribution of Implied Volatility by Epoch

This figure shows the distribution of calculated implied volatility by time period (epoch). Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The sample contains fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities.

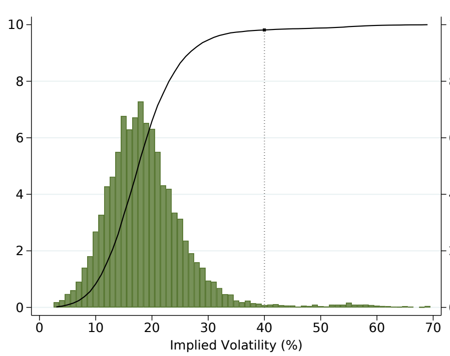
(a) 2000–2004



(b) 2005–2007



(c) 2008–2015



(d) 2016–2020

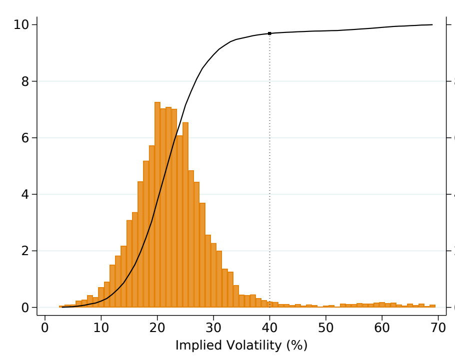


Figure F2. Distribution of Selected CRE Loan Characteristics

This figure shows the distribution of debt service coverage ratios, debt yield, loan-to-value ratios, and loan term lengths (months) in the Morningstar dataset of fixed-rate, single-property loans securitized in commercial mortgage-backed securities. The sample is later cut for further analysis by removing debt yields under 7% and debt service coverage ratios under 1.25 for reasons explained in section III.A.

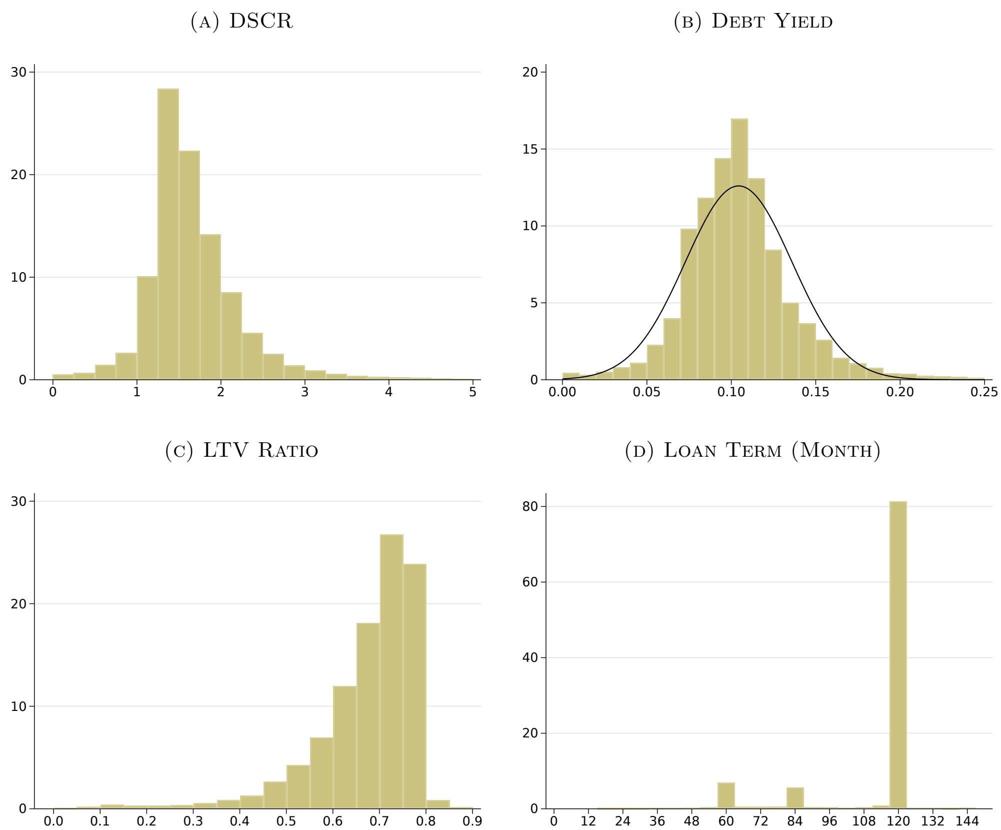


Figure F3. Association Between the LTVs and IVs of CMBS Loans Over Different Epochs

This figure shows the relationship between loan-level loan-to-value ratios and calculated implied volatility by time period (epoch). Epoch choice is explained in Section II.C. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The overlaid frontiers are estimated by fitting a quantile regression model for the 95$^{\text{th}}$ percentile of the loan-to-value ratios (LTVs) of commercial real estate loans that fall into a given integer model-implied volatility bin and were originated in a given epoch. The samples contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities.

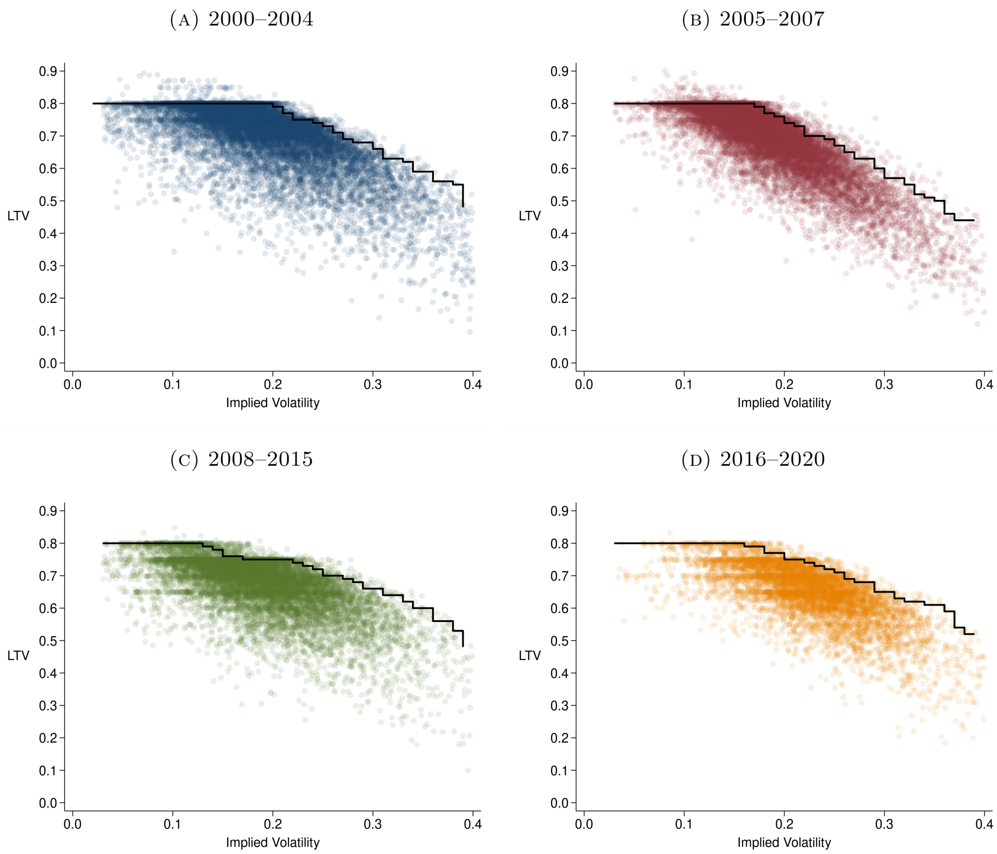


Table F3—Marginal Effects of Explanatory Variables on 100 × Log of Implied Volatility

This table shows the marginal effects of these explanatory variables on 100 × log of implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The samples contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. Property size is measured alternatively in square feet and "units" by Morningstar. Rather than attempt a conversion, we utilize either of these variables when it is available. Certain variables are heterogeneously populated among property types, motivating us to include and exclude hotel, multi-family, and "other" property types in the various columns. GDP, income, and unemployment data are obtained from the Bureau of Economic Analysis (BEA). "Sector" refers to the "real estate industry" as defined by the BEA. The market size and vacancy rate variables are obtained from CBRE.

	(1)	(2)	(3)	(4)	(5)
100 × Log of State Real GDP (USD mm)	0.12*	0.07	0.09		
100 × Log of State Real GDP in Sector (USD mm)	0.01	-0.04	-0.04		
100 × Log of State Income per Capita (USD)	-0.28*	-0.11	-0.10		
State Unemployment Rate (%)	-1.54***	-1.18*	-1.13**		
Property Age (Years)	0.03	0.06***	0.03	0.04*	0.01
100 × Log of Property Size (Sqft)	-0.04***	-0.04***	-0.03***	-0.04***	-0.03***
100 × Log of Property Size (Units)	-0.41***		-0.40***		-0.38***
Property Occupancy (%)		-0.12***		-0.16***	
Lead Tenant Share (%)		0.05***		0.05***	
100 × Log of Market Size (Sqft)			0.02***		0.01
Market Vacancy Rate (%)			-0.03		0.33**
Hotel Included	x		x		x
Multi-Family Included	x		x		x
Other Included	x	x		x	
Loan Originator	x	x	x	x	x
Property State	x	x	x		
Property Type × Time (Quarterly)	x	x	x	x	x
Property County × Time (Quarterly)				x	x
R^2	0.32	0.32	0.32	0.52	0.51
Number of Observations	43,249	19,485	27,860	13,051	22,515

Standard errors are double clustered by the property state and the quarter of origination.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table F4—Marginal Effects of Explanatory Variables on Implied Volatility (%)

This table shows the marginal effects of these explanatory variables on percentage implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. The samples contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. Property size is measured alternatively in square feet and "units" by Morningstar. Rather than attempt a conversion, we utilize either of these variables when it is available. Certain variables are heterogeneously populated among property types, motivating us to include and exclude hotel, multi-family, and "other" property types in the various columns. GDP, income, and unemployment data are obtained from the Bureau of Economic Analysis (BEA). "Sector" refers to the "real estate industry" as defined by the BEA. The market size and vacancy rate variables are obtained from CBRE.

	(1)	(2)	(3)	(4)	(5)
100 × Log of State Real GDP (USD mm)	0.022*	0.013	0.016		
100 × Log of State Real GDP in Sector (USD mm)	0.003	-0.007	-0.007		
100 × Log of State Income per Capita (USD)	-0.052*	-0.021	-0.018		
State Unemployment Rate (%)	-0.290***	-0.220*	-0.211**		
Property Age (Years)	0.006	0.012***	0.006	0.008*	0.003
100 × Log of Property Size (Sqft)	-0.007***	-0.007***	-0.007***	-0.007***	-0.006***
100 × Log of Property Size (Units)	-0.075***		-0.075***		-0.070***
Property Occupancy (%)		-0.022***		-0.031***	
Lead Tenant Share (%)		0.010***		0.009***	
100 × Log of Market Size (Sqft)			0.005***		0.002
Market Vacancy Rate (%)			-0.005		0.061**
Hotel Included	x		x		x
Multi-Family Included	x		x		x
Other Included	x	x		x	
Loan Originator	x	x	x	x	x
Property State	x	x	x		
Property Type × Time (Quarterly)	x	x	x	x	x
Property County × Time (Quarterly)				x	x
Number of Observations	43,249	19,485	27,860	13,051	22,515

Standard errors are double clustered by the property state and the quarter of origination.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table F5—Marginal Effects of Explanatory Variables on IV (%)

This table shows the marginal effects of these explanatory variables on percentage implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. This sample contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. This sample excludes hotel and multi-family properties.

	Pre-GFC	Post-GFC
Property Age (Years)	0.007	0.009
100 × Log of Property Size (Sqft)	-0.008***	-0.004***
Property Occupancy (%)	-0.027***	-0.077***
Lead Tenant Share (%)	0.006**	0.026***
Number of Observations	10,508	2,543

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table F6—Marginal Effects of Explanatory Variables on IV (%)

This table shows the marginal effects of these explanatory variables on percentage implied volatility. The implied volatilities are estimated using the two-factor model described in Appendix B, applying the market liquidity adjustment explained in Section III.B. This sample contain fixed-rate, single-property loans, with debt yields over 7% and debt service coverage ratios over 1.25, securitized in commercial mortgage-backed securities. This sample excludes the "other" property type. The market vacancy rate is obtained from CBRE.

	Pre-GFC	Post-GFC
Property Age (Years)	0.012***	-0.009*
100 × Log of Property Size (Sqft)	-0.006***	-0.006***
100 × Log of Property Size (Units)	-0.059***	-0.076***
100 × Log of Market Size (Sqft)	0.002	0.002
Market Vacancy Rate (%)	0.047*	0.082**
Number of Observations	13,742	8,773

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$