Teaching a Quantitative Core Course

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Background

- Finance core course for EWMBA students, since 2006
  - Covers discounted cash flow analysis, CAPM, company valuation, intro to derivatives, etc.
  - ~120 students
  - Relatively quantitative course
  - A lot of material to digest in 7-9 weeks
  - Heterogeneity in backgrounds and levels of interest

- Stochastic calculus course for MFE students, since 2011
  - Covers Ito's lemma, differential equations, martingales, etc.
  - ~65 students
  - Very quantitative course
  - A lot of material to digest in 8 weeks
  - Homogeneous backgrounds and high levels of interest

Deriving the CAPM formula* (2/3)

2: Use portfolio formulas to calculate expected returns and standard deviation of portfolio of: market, stock i, and risk-free asset

- Specifically, invest w_m=100% in market portfolio, $w_i=\theta$ in stock & $w_f=-\theta$ (borrow from bank).
- $w_m+w_i+w_f$=100%
- Expected return of portfolio: $r_p = r_m + \theta(r_i - r_f)$ (see page 34)
- Variance of portfolio: $\sigma_p^2 = \sigma_m^2 + 2\theta\sigma_{im} + \theta^2\sigma_i^2$ (see page 35)
- From previous page: $\left(\frac{r_p - r_f}{r_m - r_f}\right)^2 \le \frac{\sigma_p^2}{\sigma_m^2}$
- Plugging in: $\left(\frac{r_m - r_f + \theta(r_i - r_f)}{r_m - r_f}\right)^2 \le \frac{\sigma_m^2 + 2\theta\sigma_{im} + \theta^2\sigma_i^2}{\sigma_m^2}$ ⇨
- A few manipulations: $\left(1 + \theta\frac{r_i - r_f}{r_m - r_f}\right)^2 \le 1 + 2\theta\frac{\sigma_{im}}{\sigma_m^2} + \theta^2\frac{\sigma_i^2}{\sigma_m^2}$ ⇨

$$\cancel{1} + 2\theta\frac{r_i - r_f}{r_m - r_f} + \theta^2\left(\frac{r_i - r_f}{r_m - r_f}\right)^2 \le \cancel{1} + 2\theta\frac{\sigma_{im}}{\sigma_m^2} + \theta^2\frac{\sigma_i^2}{\sigma_m^2}$$

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The stochastic ODE- Existence and uniqueness

Consider the linear first order SDE:

$$d\mathbf{X} = \mu(t, \mathbf{X})dt + \sigma(t, \mathbf{X})d\mathbf{W}_t, \qquad \mathbf{X}_0 = \mathbf{x}$$

where $\mathbf{W}_t$ is a k-dimensional Wiener process, $\mathbf{X}_t \in \mathbb{R}^n$, $\mu : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^{n \times k}$. Assume that there is constant, K, such that $\forall \mathbf{x}, \forall \mathbf{y}, \forall t$

- $\|\mu(t, \mathbf{x}) - \mu(t, \mathbf{y})\| + \|\sigma(t, \mathbf{x}) - \sigma(t, \mathbf{y})\| \leq K\|\mathbf{x} - \mathbf{y}\|$,
- $\|\mu(t, \mathbf{x})\| + \|\sigma(t, \mathbf{x})\| \leq K(1 + \|\mathbf{x}\|)$.

Then the SDE has a unique solution, $\mathbf{X}$. This solution:

- is $\mathcal{F}_t^W$-adapted,
- has continuous trajectories,
- is a Markov process,
- satisfies the following growth bound, for some constant, K',

$$E[\|\mathbf{X}\|^2] \leq K' e^{K't}(1 + \|\mathbf{x}\|^2).$$

Agenda

- Justification of scope
- Efficiency
- One size does not fit all

Making choices is crucial for any business

How much should we spend on marketing?

Should we replace the machine?

Should we buy company X?

In which country should we initially launch our product?

Is there a business case for the new product?

Should we close down the factory?

Should we issue new stock?

Should we expand in the supply chain?

The need to compare "apples" and "oranges"

Which "project" is best?

You give me...

$10,000 today

... & I give you

1. $11,000 today
2. $14,000 today
3. $15,000 in a year
4. $30,000 in a year if DJIA is above 14,000
5. $30,000 in a year if last digit of DJIA is odd

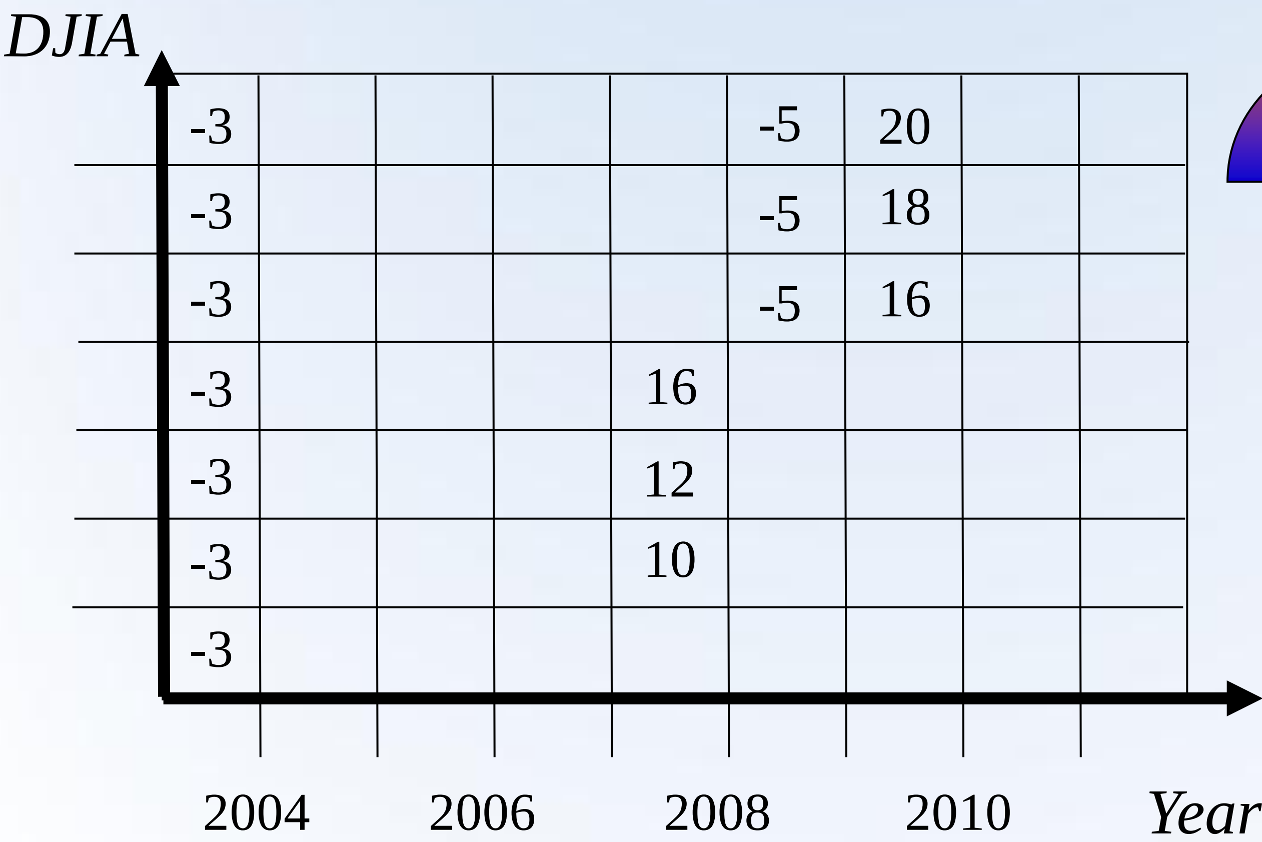
Theory of Finance

- Provides a rational way to approach business decisions
- Provides a common language of "value"
- Provides a structured & consistent approach to comparing cash flows across different situations

Map projects into one single number: NPV!

Collapsing “projects” into NPVs

Cash flows



NPV = 10

Easy to compare: Higher NPV is better!

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Current status of our knowledge

How to make investment decisions:

?

1. Estimate (expected) cash flows in each time period
2. Choose an appropriate discount rate
3. Use discounted cash flow analysis to calculate NPV
4. Make decision that maximizes NPV

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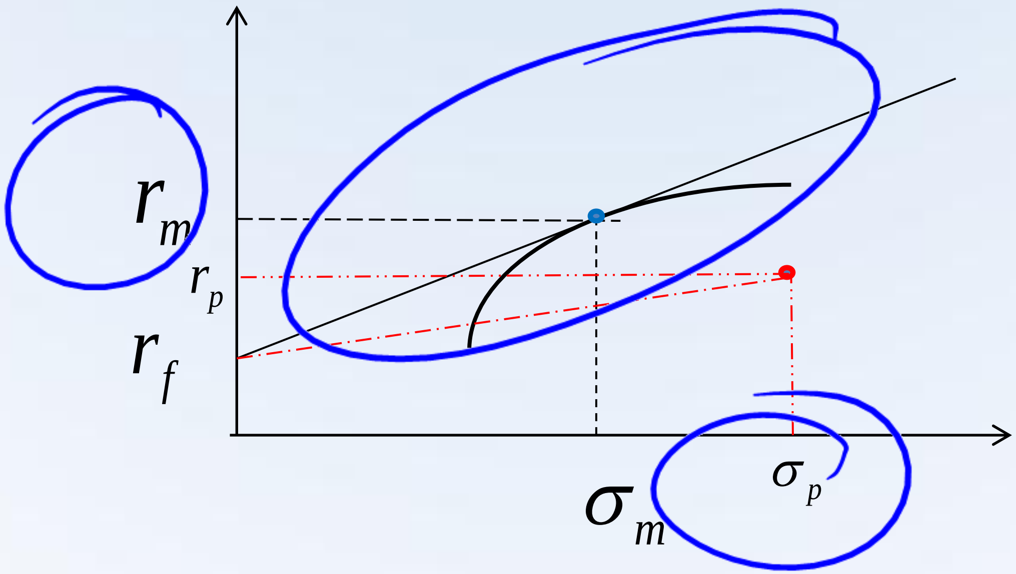
Deriving the CAPM formula* (1/3)

1: Since portfolio S has the highest slope, adding any stock to portfolio can not increase slope:



– So, $\frac{r_p - r_f}{\sigma_p} \le \frac{r_m - r_f}{\sigma_m}$ or equivalently*, $\frac{(r_p - r_f)^2}{\sigma_p^2} \le \frac{(r_m - r_f)^2}{\sigma_m^2}$, i.e., $\left(\frac{r_p - r_f}{r_m - r_f}\right)^2 \le \frac{\sigma_p^2}{\sigma_m^2}$

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*Assume $r_p > r_f$

Deriving the CAPM formula* (2/3)

2: Use portfolio formulas to calculate expected returns and standard deviation of portfolio of: market, stock i, and risk-free asset

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- A few manipulations: $\left(1 + \theta\frac{r_i - r_f}{r_m - r_f}\right)^2 \le 1 + 2\theta\frac{\sigma_{im}}{\sigma_m^2} + \theta^2\frac{\sigma_i^2}{\sigma_m^2}$ ⇒

$$1 + 2\theta\frac{r_i - r_f}{r_m - r_f} + \theta^2\left(\frac{r_i - r_f}{r_m - r_f}\right)^2 \le 1 + 2\theta\frac{\sigma_{im}}{\sigma_m^2} + \theta^2\frac{\sigma_i^2}{\sigma_m^2}$$

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Deriving the CAPM formula* (3/3)

3: Use that the inequality must hold for arbitrary θ to derive CAPM

- From previous page: $2\theta \frac{r_i - r_f}{r_m - r_f} + \theta^2 \left(\frac{r_i - r_f}{r_m - r_f} \right)^2 \leq 2\theta \frac{\sigma_{im}}{\sigma_m^2} + \theta^2 \frac{\sigma_i^2}{\sigma_m^2}$
- Holds *for all* θ*,* e.g., for *very* small θ*:*
  - Small positive θ*:* $\frac{r_i - r_f}{r_m - r_f} \leq \frac{\sigma_{im}}{\sigma_m^2}$
  - Small negative θ*:* $\frac{r_i - r_f}{r_m - r_f} \geq \frac{\sigma_{im}}{\sigma_m^2}$
- Therefore $\frac{r_i - r_f}{r_m - r_f} = \frac{\sigma_{im}}{\sigma_m^2}$
- So,

$$r_i = r_f + \frac{\sigma_{im}}{\sigma_m^2}(r_m - r_f)$$

CAPM!!

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Example 6

Q: One-year risk-free spot rate is 5%, market risk premium is 7%. current DJIA is 12,000. Please value & rank the following five projects

I give you... | **... & you give me**

$10,000 today

1. $11,000 today
2. $14,000 today
3. $15,000 in a year
4. $30,000 in a year if DJIA is above 13,000
5. $30,000 in a year if last digit of DJIA is odd

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Agenda

- Justification of scope
- Efficiency
- One size does not fit all

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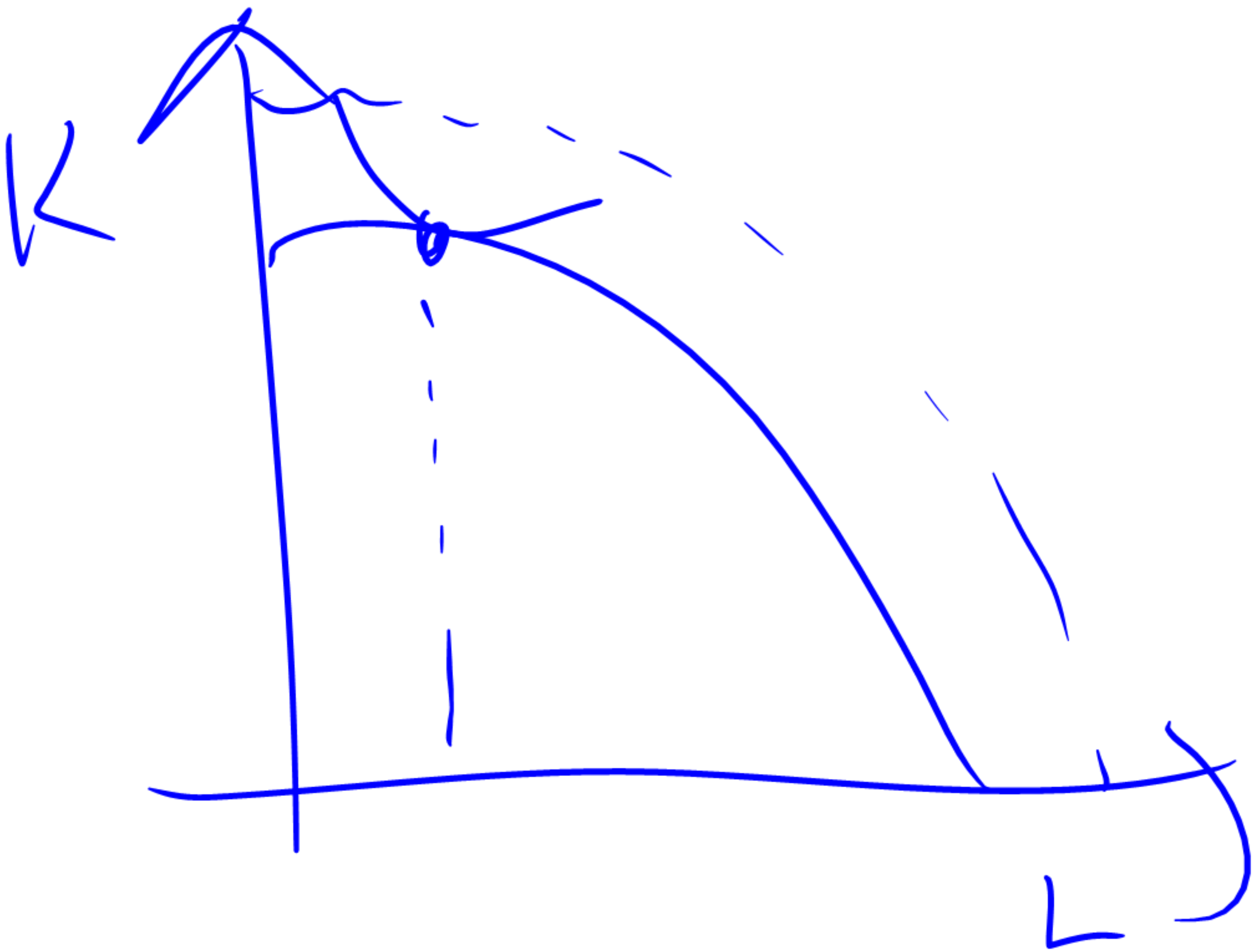

K
L

Material

1. *Time value of money – NPV*

- Calculating NPV/PV of cash-flows
    - General
    - Specific (perpetuity, annuity, growth, deferral)
    - Calculating PV for combinations of specific cash flow patterns (e.g., problem 2b on practice midterms)
    - How to go between different cash flow patterns (see e.g., retirement problem in lecture 2)
    - Other measures: IRR and payback period
- Time varying discount rates
    - Concepts of spot rates and forward rates
    - How to go between spot rates and forward rates
    - How to create a synthetic forward contract (e.g., Problem 4b on problem set 2)
- Bonds
    - Coupon, principal, STRIPS
    - How to calculate price and yield to maturity (YTM), (see, e.g., problem 2d on practice midterm I).
- Real versus nominal rates
    - How to take into account in calculations (see retirement problem in lecture 2 and saving for college question on problem set 2)
- Compounding
    - General: Going between, and using, interest rates for different time intervals
    - APR versus EAR
- Simple equity & project valuation using:
    - Free Cash Flow/Discounted Cash Flow analysis - Backing out cash flows from Balance/Income sheet (starting with EBIAT, backing out depreciation, capital expenditures and changes in net working capital). Calculating NPV.
    - Dividend growth model - How to use & interpret (see, e.g., problem 4 on practice midterm II)
    - Present value of growth opportunities, PVGO - Being able to separate cash flows into a perpetuity and a growth opportunity part.
- Taxes
    - Calculating NPV when taxes are present (taking into account effects both on CF and discount rate)

2. *Risk – CAPM*

- Portfolio choice
    - Calculating expected return, variance and standard deviation of portfolios of stocks
    - Risk aversion – tradeoff of mean versus variance/standard deviation
    - Concepts of: diversification, systematic risk and unique risk
    - Perfectly diversified portfolios
- CAPM
    - Concepts of minimum variance frontier, efficient frontier, tangency portfolio, market portfolio, capital market line (CML) and security market line (SML).
    - Calculating risk adjusted discount rate, using CAPM
    - Understanding risk/return trade-off (SML)
    - Betas and expected returns of portfolios (Part II, slide 57)
    - Estimating CAPM using a regression
    - Different ways of benchmarking a stock or portfolio – Sharpe ratio, Treynor ratio, alpha

3. *Valuation*

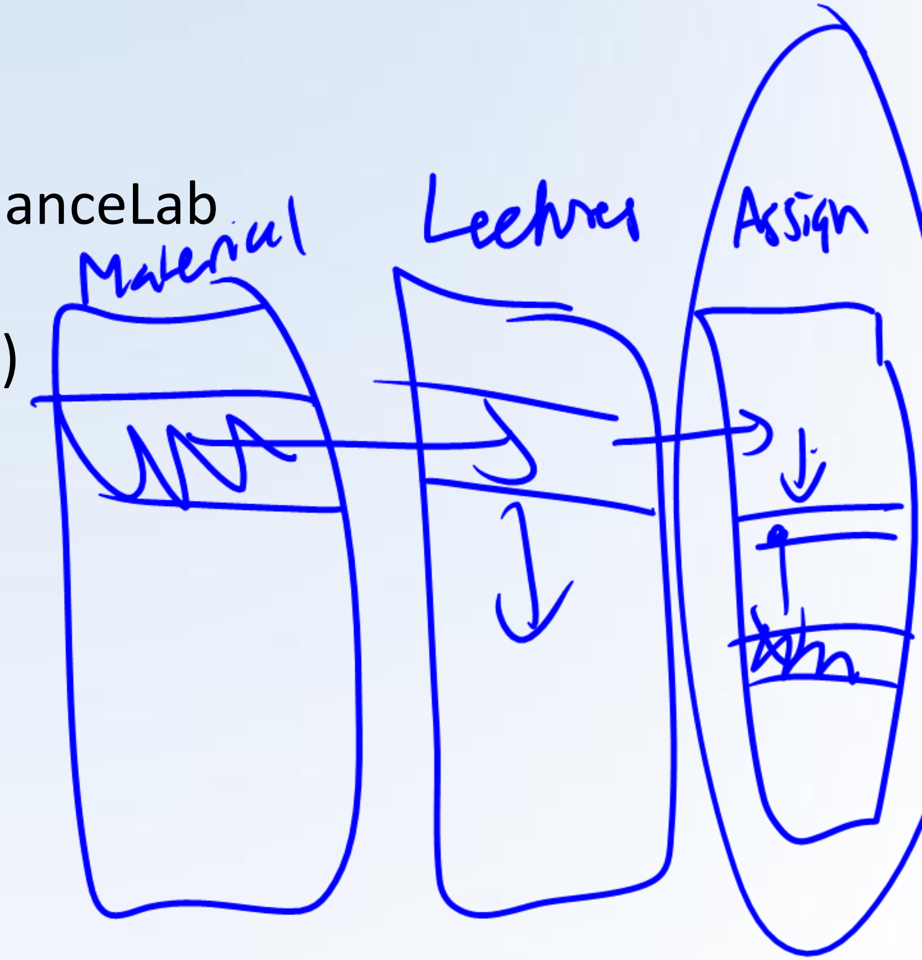
- No taxes - effect of leverage on risk and expected returns (Part III, slide 7-15)
    - Irrelevance of capital structure (Miller-Modigliani)
    - Beta of a leveraged company
- Taxes – Value of tax shields
- WACC
    - Assumptions (proportional debt)
        - How to calculate WACC (Part III, slides 17-31)
            - r_A
            - r_E by unlevering and relevering beta of comparable company
                - Simplifying assumption: beta of debt = 0
                - General: Using CAPM to get beta of debt
    - Calculating NPV using WACC
        - Understanding which cash flows to use and who are the stakeholders
- APV
    - Assumptions (constant debt)
    - Calculating NPV using APV method
        - Understanding which cash flow to use and who are the stakeholders

4. *Derivatives*

- The concept of option value, e.g., option to wait and option to exercise.
- Different types of derivatives (forwards, futures, call options, put options, swaps, American versus European).
- Put-call parity (Part IV, slide 64-67).
- Using Black-Scholes' formula to value call & put* options (Part IV, slides 70-72)

Channels

- Text book & slides
- Problems on MyFinanceLab
- Assignments
    - Problem sets (4)
    - Cases (4)
- Exams
    - Mid-term
    - Final
- Class participation
    - Cases
    - Other



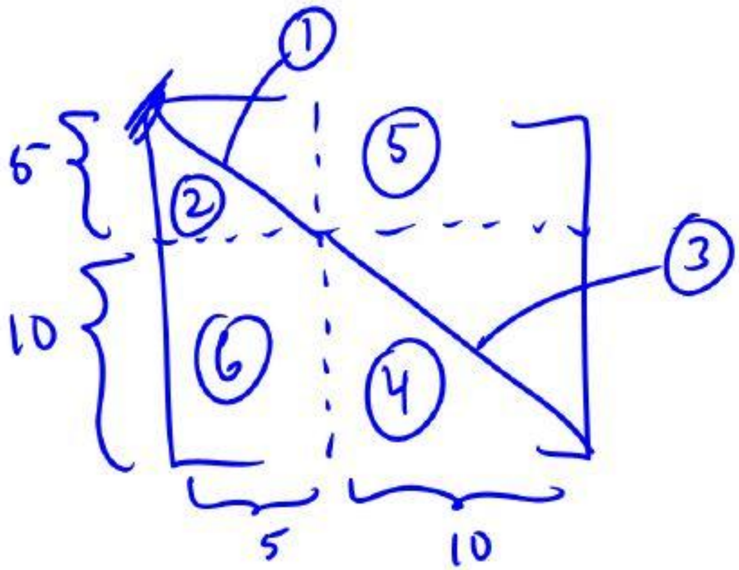
In-class exercise

Assumptions: You have a portfolio of 15 stocks:

- The first 5 stocks have portfolio weights 0.1;
- The remaining stocks have portfolio weight .05;
- Each stock has a variance of 10,
- All stocks have a covariance of 2 with each other.

Question: What is the variance of your portfolio?

Solution



3.1
26
2.427
10
6.77
2

(1) + (2): $5 \times (10\%)^2 \times 10 + 20 \times (10\%)^2 \times 2 = 0.9$

w_i

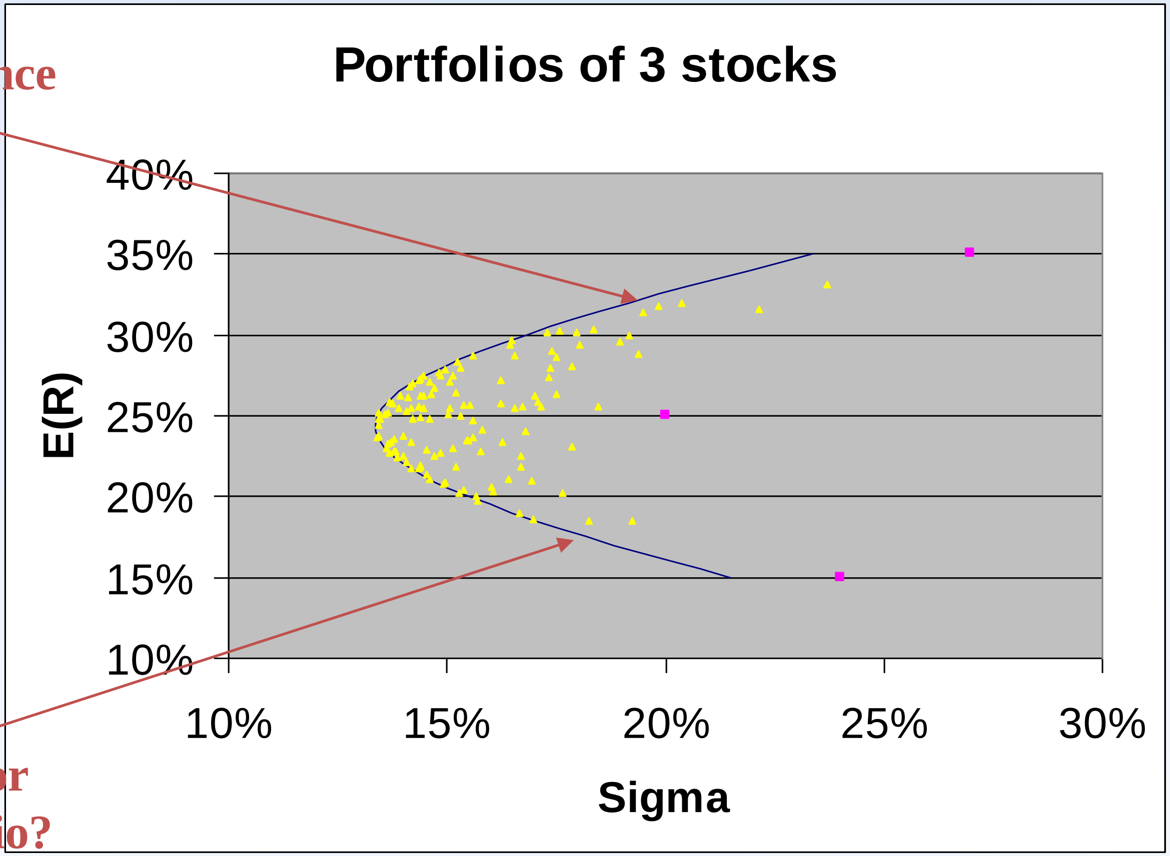
(3) (4): $10 (5\%)^2 \times 10 + 90 \times (5\%)^2 \times 2 = 0.7$

(5) (6): $100 \times 10\% \times 5\% \times 2 = 1$

Σ 2.6

Application

Minimum variance frontier



Might an investor pick this portfolio?

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Assignment

Company X needs to know its cost of capital, to be able to evaluate investment opportunities. A complication is that company X is not publicly traded, so it cannot use market data to estimate its risk. Fortunately, there are several other companies in the same industry as X, which are all publicly traded, and which can be used to estimate the riskiness of the business X operates in.

Information: Company X operates within a computer service consulting niche. There are two major business lines within this industry: private and business customers, and these two lines have different riskiness: the business customer segment is known to be heavily pro-cyclical, whereas the private customer segment is known to be only weakly related to the business cycle. There are three publicly traded companies that compete with X: companies A, B, and C. Company A mainly targets the business segment, whereas B provides 80% of its services to private customers, and C splits its services roughly equally between the two segments. All four companies have very similar operations for each business segment. The companies' total operations can therefore be viewed as "portfolios" of business and private segments, with different weights on each group.

Using regressions, it has been estimated that company A, B and C's market betas are 1.2, 0.4, and 0.7, respectively.

Company X mainly targets private customers. The risk-free rate is 4%, and the market risk premium is 6%. The interpretation here is that 80% of B's market value is generated from its private customer business, whereas the number is 50% for C.

…

Agenda

- Justification of scope
- Efficiency
- One size does not fit all

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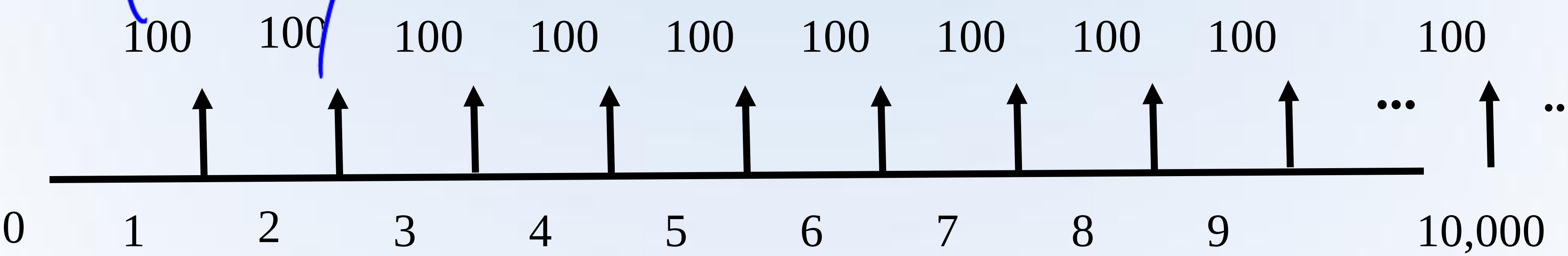
Value of perpetuity

- Payments are made 'in perpetuity'
  - Payment C, constant interest rate r:



- General formula for **perpetuity**:

$$PV = \frac{C}{r}$$

- **Question**: Show formula using no-arbitrage argument. Assume r=10%

Remember:
***r* is constant**

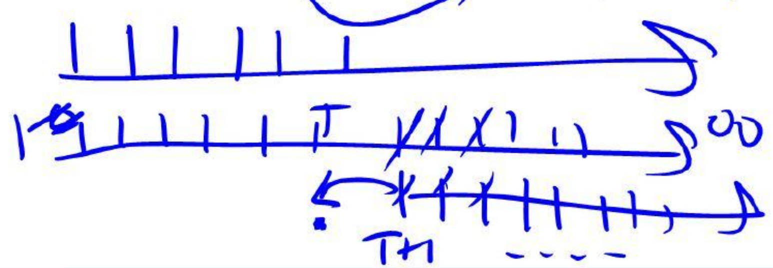
Value of perpetuity- again

$|a| < 1$

$$\sum_{t=0}^{\infty} a^t = \frac{1}{1-a} \quad (*)$$

$$\sum_{t=1}^{\infty} \frac{C}{(1+r)^t} = \frac{C}{1+r} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \quad (*)$$

$$= \frac{C}{(1+r)\left(1-\frac{1}{1+r}\right)} = \frac{C}{1+r-1} = \frac{C}{r}$$



$$\frac{C}{r} - \frac{C}{r}\frac{1}{(1+r)^T} \quad (**)$$

Q7: Can you derive the formula for the growing annuity mathematically?*

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^{T}\right]$$

The discounted dividend model (1)

$$P = DIV / (r - g)$$

$$r = DIV / P + g$$

Plowback ratio = 1 – DIV/EPS

ROE = E/BE = EPS/BVPS

g = Plowback ratio x ROE

- Identical to bond perpetuity formula, but
  - Dividends uncertain implies $r > r_{riskfree}$
  - (Expected) Dividends and growth may change as new investment opportunities occur!

- Extra assumptions about
  - Earnings = Cash flows
  - Plowed back earnings give same returns (ROE constant)

EXAMPLE

The discounted dividend model (2)

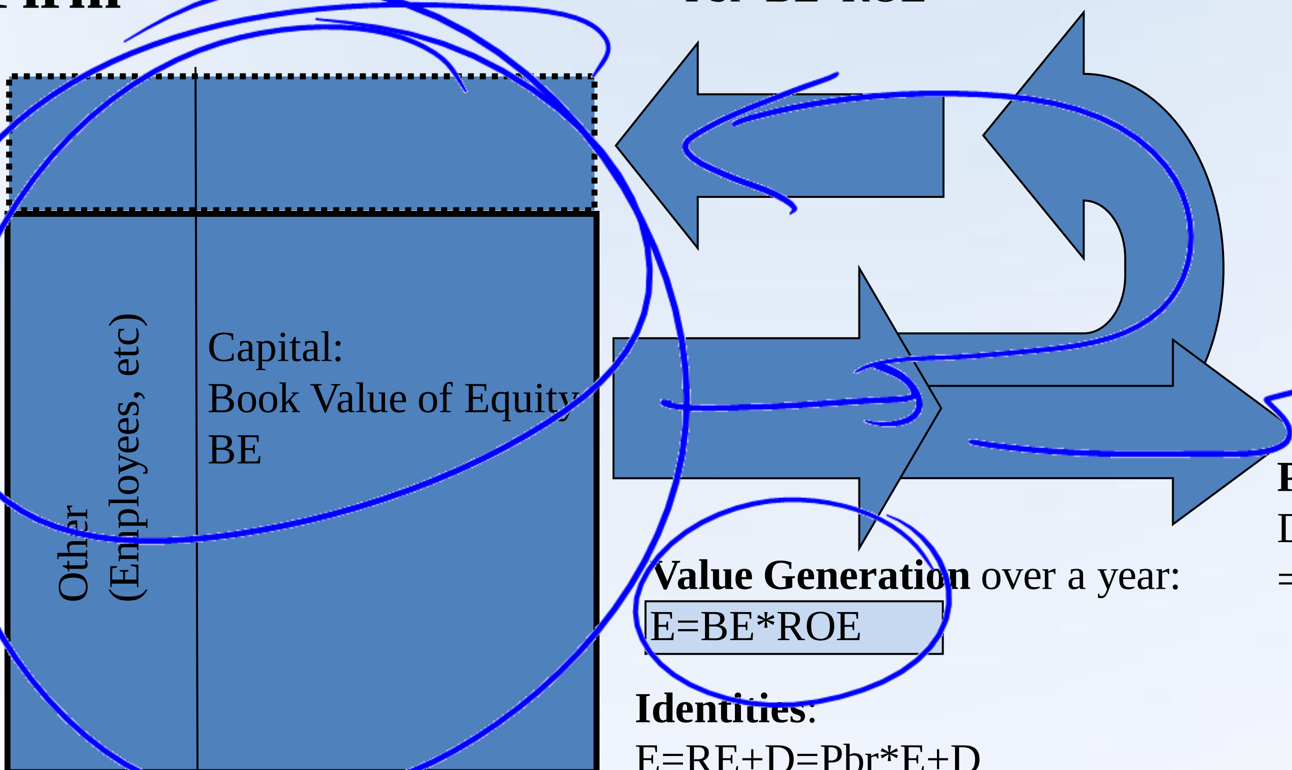
Growth:
g=Pbr*E/BE=Pbr*ROE

Reinvested:
RE=Pbr*E=
Pbr*BE*ROE

Cost of capital, r

Firm

Shareholder



Paid out:
D=(1-Pbr)*E
=(1-Pbr)*BE*ROE

Value Generation over a year:
E=BE*ROE

Identities:
E=RE+D=Pbr*E+D
1=Pbr+D/E
Pbr=1-D/E

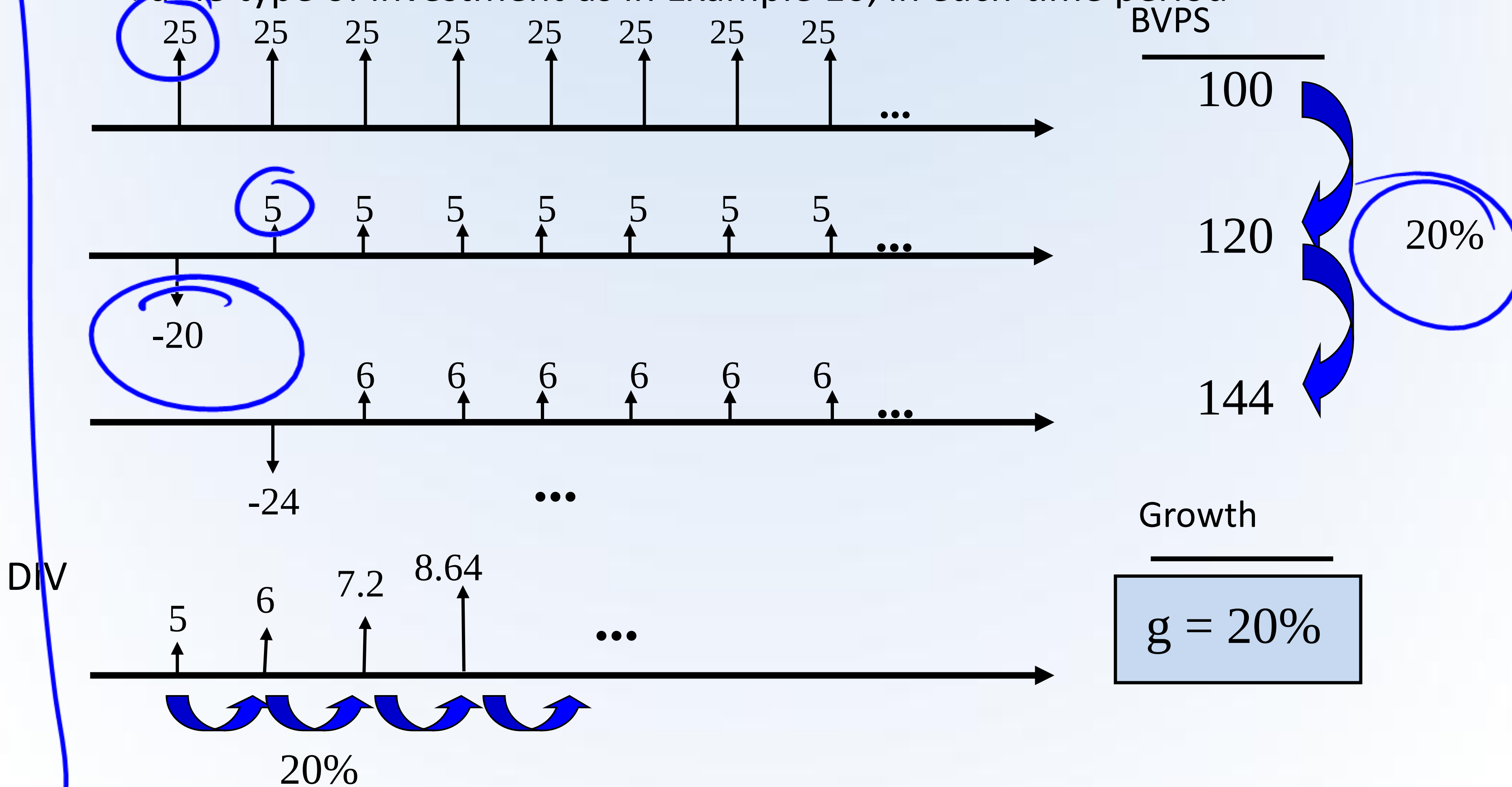
N shares:
•BE=N*BVPS
•E=N*EPS
•D=N*DIV

Value (V) & Stock price (P)
•V=D/(r-g)
•P=DPS/(r-g)

The discounted dividend model (3)

The cell-phone example: New opportunities in each time period

- Same type of investment as in Example 10, in *each* time period



EXAMPLE

Assignment

Mark Chimney, the famous Industrialist, has his eyes set on BB Dolls (full company name “Boring Babies, Dolls and Accessories, Inc.) a less successful toy manufacturer. Through a careful analysis, Mr. Chimney’s executive team has realized that by drastically changing its product line, BB Dolls can become more profitable, and thereby immediately increase its Return on Equity from 10% to 25%. The team now wishes to understand the NPV of implementing the proposed change.

Additional information: The opportunity cost of capital for BB Dolls is 15%. Without the change, BB Dolls is expected to pay dividends of 1 million dollars a year from now, and have earnings of 2 million dollars. The company will keep the same plowback ratio after the change as it had before.

a) (5 points) What is the PVGO for the company *before* the change?

b) (5 points) What is the NPV of the proposed change?

The executive team also believes that the company will have the opportunity to grow more aggressively, beginning in the third year, by increasing its plowback ratio. This “Phase 2” of the retooling of BB Dolls would occur *in addition* to the previous product line change, and would not affect the company’s Return on Equity of 25%. Under the assumptions of “Phase 2”, the company would pay the same dividends as before at the end of year 1, but then at the end of year 2 decrease its dividends and increase its plowback ratio so that a growth rate of 14% is obtained in future years.

c) (5 points) Under these assumptions, what is the NPV of “Phase 2”?