

# Liquidity, Business Cycles, and Monetary Policy

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## Abstract

This paper presents a model of monetary economy with differences in liquidity across assets. Our purpose is to study how aggregate production and asset prices fluctuate with shocks to productivity and liquidity. In so doing, we examine what role government policy might have through open market operations that change the mix of assets held by the private sector. We also show that certain apparent anomalies of asset markets are in fact normal features of a monetary economy in which the circulation of money is essential for a better allocation of resources.

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# 1 Introduction

In this paper, we provide a model of monetary economy with differences in liquidity across assets. Our purpose is to understand how aggregate production and asset prices fluctuate with recurrent shocks to productivity and liquidity. In so doing, we want to find out what role government policy might have through open market operations that change the mix of assets held by the private sector. The present paper takes fiat money to be one of the assets under consideration. We investigate under what circumstances money is essential to a better allocation of resources. We show that certain apparent anomalies of the non-monetary economy are in fact normal features of an economy where money is essential. Among the well-known puzzles we have in mind are: the low risk-free rate puzzle; the excess volatility of asset prices; the anomalous savings behaviour of certain households, and their low participation in asset markets. Before describing our monetary economy, we should start with some remarks about modeling strategy.

In broad terms, there are two ways of getting fiat money into a competitive macroeconomic model. One approach is to endow money with some special function – for example, cash-in-advance or sticky nominal prices.<sup>1</sup> The other approach is to starve agents of alternatives to money – as in an overlapping generations framework where money is the sole means of saving.<sup>2</sup> Although the first approach, in particular the cash-in-advance model and the dynamic sticky price model, has proved important to monetary economics and policy analysis, it is not well-suited to answering larger questions to do with liquidity. By endowing money with a special function in the otherwise frictionless economy with complete Arrow-Debreu security market, one is imposing rather than explaining the use of money, which precludes the possibility that other assets or media of denomination may substitute for money. And the second approach rules out any general discussion of liquidity if there are no alternative assets to money.

There are many noncompetitive models of money, leading with the random matching framework. In principle, such models are suited to analyzing liquidity. But they are necessarily special, and it is difficult to incorporate them into the rest of macroeconomics.<sup>3</sup> We believe there is a need for a work-

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<sup>1</sup>See Clower (1967), Lucas and Stokey (1987) and Woodford (2003).

<sup>2</sup>P.A.Samuelson (1958). Bewley (1980) also models money in a context where there are no alternative assets with which to save.

<sup>3</sup>See Kiyotaki and Wright (1989) and Duffie, Garleanu and Pedersen (2005) for exam-

horse model of money and liquidity, with competitive markets, which does not stray too far from the other workhorse, the real business cycle model.

In our framework, markets are competitive, money is not endowed with any special function, and there are other assets traded besides money. The basic model presented in Section 2 has two kinds of agents, entrepreneurs and workers, homogeneous general output, and three assets: fiat money, physical capital (or equity of physical capital), and human capital. The supply of fiat money is fixed. The supply of capital changes through investment and depreciation. A worker's human capital is inalienable, which means that he or she cannot borrow against future labour income: in any period, the only labour market is a spot market for that period's labour services. There is a commonly available technology for combining labour with capital to produce general output.

In each period a fraction of the entrepreneurs (but none of the workers) can invest in producing new capital from general output. The arrival of such an investment opportunity is randomly distributed across entrepreneurs through time. Because not all entrepreneurs can invest in each period, there is a need to transfer resources from those who don't have an investment opportunity (that period's savers) to those who do (that period's investors). To acquire general output as input for the production of new capital, investing entrepreneurs sell equity claims to the future returns from the newly produced capital. The crucial feature of the model is that, because the investing entrepreneur is still needed to run the project to produce output and he cannot precommit to work throughout its life, he is able to pledge only a fraction (say  $\theta$ ) of future returns from the new capital. As the investing entrepreneur can only issue new equity up to  $\theta$  fraction of his investment, he faces a borrowing constraint.

Because of the borrowing constraint, the investing entrepreneur needs to finance the investment cost partly by selling his holding of money and equity of the other agents (which he acquired in the past). Another important feature of our model is that the existing equity (the claim to the return of the existing capital stock) cannot be sold as quickly as money. Specifically, we assume that, in any given period  $t$ , an agent can sell only a fraction  $\phi_t$  of his equity holding. In contrast to the upper bound of new equity issue

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ples. For recent attempts to make matching models applicable for policy analysis, see Lagos and Wright (2005), Huang and Wright (2005, 2008) and Aruoba, Waller and Wright (2007).

$\theta$ , the value of  $\phi_t$  is the limit that the investing entrepreneur can resell his equity holding before he misses the investment opportunity. Thus, we call  $\theta$  as borrowing constraint, call  $\phi_t$  as "resaleability constraint", and call both constraints together as "liquidity constraints". Here, we take both  $\theta$  and  $\phi_t$  as exogenous parameters and consider a stochastic shock to  $\phi_t$  as "liquidity shock".<sup>4,5</sup>

The question is to what extent does these liquidity constraints inhibit the efficient transfer of resources from savers to investors. There may be a role for money to lubricate the transfer of additional resources. Whether or not agents use money is determined endogenously. We show that for high enough values of  $\theta$  and average  $\phi_t$  money is not used and has no value in the neighbourhood of the steady state. But for lower values of  $\theta$  and average  $\phi_t$ , money plays an essential role. In the latter case, we call the economy a monetary economy.

We find that a necessary feature of a monetary economy is that the investment of entrepreneurs is limited by liquidity constraints. He cannot raise the entire cost of investment externally, given that the borrowing constraint binds for the sale of new equity. That is, he has to make a downpayment for each unit of investment from his own internal funds. But in trying to raise funds to make this downpayment, he is constrained by how much of his equity holding can be sold in time: the resaleability constraint binds here. In this sense, an investing entrepreneur finds money more valuable than equity, because he can use all of his money to finance new investment whereas he can use only a fraction  $\phi_t$  of his equity: money is more liquid than equity.<sup>6</sup>

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<sup>4</sup>In Kiyotaki and Moore (2003, 2005b), we develop a framework in which the resaleability constraint arises endogenously due to adverse selection in resale market. Each new capital comprises a large number of parts, some of which will eventually fail (depreciate completely), although nobody knows which when the new capital is produced. Overtime, the insiders (producing entrepreneurs and those who bought the new equity) learn privately which parts will fail. If the fraction of failing parts is large enough, no outsider will buy second-hand equity for fear of being sold lemons.

In order to overcome the adverse selection problem, the investing entrepreneur can spend extra resource to bundle all the parts of the new capital together in such a way that they cannot later unbundled. Then the equity issued against bundled new capital will be resold freely.

<sup>5</sup>In the analysis of financial market, Brunnermeier and Perdersen (2007) use notion of "funding liquidity" to refer borrowing constraint and "market liquidity" to refer resaleability constraint.

<sup>6</sup>In practice, there are clearly differences between kinds of equities – e.g. between the share of a large publicly-traded company and stock of a small privately-held business.

We find that in a monetary economy, the rate of return on money is very low, less than the return on equity. Nevertheless, a saving entrepreneur chooses to hold some money in his portfolio, because, in the event that he has an opportunity to invest in the future, he will be liquidity constrained, and money is more liquid than equity. The gap between the return on money and the return on equity is a liquidity premium. This may help explain the low risk-free rate puzzle.

We also find that both the returns on equity and money are lower than the rate of time preference. This means that agents such as workers, who don't anticipate having investment opportunities, will choose to hold neither equity nor money. They will simply consume their labour income, period by period. This may help explain why certain households do not save nor participate in asset markets. It is not that they don't have free access to asset markets nor that they are particularly impatient, but rather that the return on assets isn't enough to attract them.<sup>7</sup>

In most real business cycle models, there is no feedback from asset market to output. That is not true in our monetary economy. Consider a shock which reduces resaleability of equity  $\phi_t$  persistently. (This liquidity shock is meant to capture an aspect of the recent financial turmoil in which many assets - such as auction rate bonds - that used to be liquid suddenly have become only partially resaleable). Then, the amount the investing entrepreneurs can use as downpayment for investment shrinks. Moreover, anticipating a lower resaleability, the equity price falls, which can be thought as "a flight to liquidity". This raises the size of the required downpayment per unit of new investment. Altogether, investment suffers from the negative shock to the resaleability of equity. This feedback mechanism causes asset prices and investment to be vulnerable to liquidity shocks unlike standard general equilibrium asset pricing model without liquidity constraints.

In a later section of the paper we introduce government. Our interest is in seeing the effect of policy on the behaviour of the private economy. We consider that the government holds equity and can costlessly change the supply of fiat money. In our framework with flexible prices, a change of money supply through lump-sum transfer to the entrepreneurs (helicopter drop) does not have any effect on aggregate real variables. The open market operation to purchase equity by issuing money, however, will increase the

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<sup>7</sup>If workers face their own investment opportunity shocks, then workers would save but only in money. See discussion of the later section.

ratio of the value of liquid money to illiquid equity of the private sector, and will expand investment through a larger liquidity of investing entrepreneurs.<sup>8</sup> Using this framework, we analyze how government (or central bank) can use the open market operation to accommodate the effects of the shock to the productivity and how it can offset the effects of shock to the liquidity.

## 2 The basic model without government

Consider an infinite-horizon, discrete-time economy with four objects traded: a nondurable general output, labour, equity and fiat money. Fiat money is intrinsically useless, and is in fixed supply  $M$  in the basic model of this section.

There are two populations of agents, entrepreneurs and workers, each with unit measure. Let us start with the entrepreneurs, who are the central actors in the drama. At date  $t$ , a typical entrepreneur has expected discounted utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \quad (1)$$

of consumption path  $\{c_t, c_{t+1}, c_{t+2}, \dots\}$ , where  $u(c) = \log c$  and  $0 < \beta < 1$ . He has no labour endowment. All entrepreneurs have access to a constant-returns-to-scale technology for producing general output from capital and labour. An entrepreneur holding  $k_t$  capital at the start of period  $t$  can employ  $l_t$  labour to produce

$$y_t = A_t (k_t)^\gamma (l_t)^{1-\gamma} \quad (2)$$

general output, where  $0 < \gamma < 1$ . Production is completed within the period  $t$ , during which time capital depreciates to  $\lambda k_t$ ,  $0 < \lambda < 1$ . We assume that the productivity parameter,  $A_t > 0$  which is common to all entrepreneurs, follows a stationary stochastic process. Given that each entrepreneur employs labour at competitive market with the real wage rate  $w_t$ , the gross profit is proportional to the capital stock as:

$$y_t - w_t l_t = r_t k_t, \quad (3)$$

where the gross profit per unit of capital  $r_t$  depends upon productivity, aggregate capital stock and labour supply condition as will be seen shortly.

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<sup>8</sup>This idea can be traced back to Metzler (1951).

The entrepreneur may also have an opportunity to produce new capital stock. Specifically, at each date  $t$ , with probability  $\pi$  he has access to a constant-returns technology that produces  $i_t$  units of capital from  $i_t$  units of general output good. The arrival of such an investment opportunity is independently distributed across entrepreneurs and through time, and is independent of aggregate shocks. Again, investment is completed within the period  $t$  – although newly-produced capital does not become available as an input to the production of general output until the following period  $t+1$ :

$$k_{t+1} = \lambda k_t + i_t. \tag{4}$$

We assume there is no insurance market against having an investment opportunity.<sup>9</sup> We also make a regularity assumption that the subjective discount factor is larger than the fraction of capital left after production (one minus the depreciation rate):

$$\textit{Assumption 1} : \beta > \lambda.$$

This mild restriction is not essential, but will make the distribution of capital and asset holdings across of individual entrepreneurs well-behaved.

In order to finance the cost of investment, the entrepreneur who has an investment opportunity can issue equity claim to the future returns from the newly produced capital. Normalize one unit of equity at date  $t$  to be claim to the future returns from the one unit of investment of date  $t$ : it pays  $r_{t+1}$  output at date  $t+1$ ,  $\lambda r_{t+2}$  at date  $t+2$ ,  $\lambda^2 r_{t+3}$  at date  $t+3$ , and so on.

We make two critical assumptions. First, we assume that the entrepreneur who produces new capital cannot precommit to work through the lifetime even though he is needed to produce full output described by the production function; thus an investing entrepreneur can pledge at most  $\theta$  fraction of future returns from his new capital. As he can issue new equity only up to  $\theta$  fraction of new capital, the parameter  $\theta$  represents the tightness

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<sup>9</sup>This assumption can be justified in a variety of ways. For example, it may not be possible to verify that someone has an investment opportunity; or verification may take so long that the opportunity has gone by the time the claim is paid out. A long-term insurance contract based on self-reporting does not work here because the people are able to trade assets covertly. Each of these justifications warrants formal modelling. But we are reasonably confident that even if partial insurance were possible our broad conclusions would still hold. So rather than clutter up the model, we simply assume that no insurance scheme is feasible.

of the borrowing constraint an investing entrepreneur faces.<sup>10</sup> Because an entrepreneur who finds an investment opportunity faces this borrowing constraint, he must finance the cost of investment partly from selling his holding of equity and money.

The second critical assumption is that entrepreneurs cannot sell their equity holding as quickly as money. More specifically, before the investment opportunity disappears, the investing entrepreneur can sell only  $\phi_t$  fraction of his equity holding within a period even though he can use all his money holding. It is tantamount to a peculiar transaction cost per period: zero for the first fraction  $\phi_t$  of equity sold, and then infinite. We take  $\phi_t$  as an exogenous parameter of liquidity of the equity, and call  $\phi_t$  as "resaleability constraint". We consider that the aggregate productivity  $A_t$  and the liquidity of equity  $\phi_t$  jointly follow a stationary Markov process in the neighbourhood of the constant unconditional mean  $(A, \phi)$ . A shock to  $A_t$  is productivity shock, and shock to  $\phi_t$  is considered as "liquidity shock".

In general, an entrepreneur has three kinds of asset in his portfolio: money, equity of the other entrepreneur, and unmortgaged capital stock (a fraction of own capital stock against which the entrepreneur has not issued the equity: own capital stock minus own equity issued).

Balance sheet	
money	own equity issued
equity of others	
own capital stock	net worth

It turns out to be difficult to analyze the aggregate fluctuations of the economy with these three assets, because there is a rich dynamic interaction between the distribution of assets and aggregate production. Thus, we make a simplifying assumption: at every period, an entrepreneur can remortgage up to a fraction  $\phi_t$  fraction of his unmortgaged capital stock. Then, equity of the other entrepreneurs and unmortgaged capital stock become perfect substitute as means of saving: both pays the same returns stream of  $r_{t+1}$  at date t+1,  $\lambda r_{t+2}$  at date t+2,  $\lambda^2 r_{t+3}$  at date t+3, and so on per unit; and the holder can sell up to  $\phi_t$  fraction of his holding of both. Because equity held by the agents other than the producer is sometimes called "outside equity" and unmortgaged capital stock is called "inside equity", we call both together as

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<sup>10</sup>See Hart and Moore (1994) which explains how the borrowing constraint arises from inalienability of human capital of the entrepreneur.



"equity".

Let  $n_t$  be the quantity of equity and let  $m_t$  be money held by an individual entrepreneurs at the start of period  $t$ . The liquidity constraints (borrowing constraint and resaleability constraint) are expressed as

$$n_{t+1} \geq (1 - \theta)i_t + (1 - \phi_t)\lambda n_t, \text{ and} \quad (5)$$

$$m_{t+1} \geq 0. \quad (6)$$

The entrepreneur who invests  $i_t$  can issue at most  $\theta i_t$  equity and can resell at most  $\phi_t$  fraction of outside and inside equity holding after depreciation during this period. Taken together, (5) implies the minimum holding of equity at the beginning of period  $t+1$  is equal to  $1 - \theta$  fraction of investment plus  $1 - \phi_t$  fraction of depreciated equity holding of this period. (6) implies his fiat money holding cannot be negative - the only agent who can have a negative position of fiat money is government (or central bank) in the next section.

Let  $q_t$  be the price of equity in terms of general output. It is also equal to Tobin's  $q$ : the ratio of stock market value to the replacement cost of capital, (noting that production cost of capital is unity per unit). Let  $p_t$  be the price of money in terms of general output. (Warning!  $p_t$  is customarily defined as the inverse: the price of general output in terms of money. But, a priori, money may not have value, so we prefer not to make it the numeraire.) The entrepreneur's flow of funds constraint at date  $t$  is then given by

$$c_t + i_t + q_t(n_{t+1} - i_t - \lambda n_t) + p_t(m_{t+1} - m_t) = r_t n_t. \quad (7)$$

The left-hand side (LHS) is his expenditure on consumption, investment and net purchases of equity and money. The right-hand side (RHS) is his dividend income, which is proportional to the holding of equity at the start of this period.

Turn now to the workers. At date  $t$ , a typical worker has expected discounted utility

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U \left[ c'_s - \frac{\omega}{1 + \nu} (l'_s)^{1+\nu} \right], \quad (8)$$

of consumption path  $\{c_t, c_{t+1}, c_{t+2}, \dots\}$  given his labour supply path  $\{l_t, l_{t+1}, l_{t+2}, \dots\}$ , where  $\omega > 0, \nu > 0$  and  $U[\cdot]$  is increasing and strictly concave. The flow-of-funds constraint of the worker is

$$c'_t + q_t(n'_{t+1} - \lambda n'_t) + p_t(m'_{t+1} - m'_t) = w_t l'_t + r_t n'_t. \quad (9)$$

The consumption expenditure and net purchase of equity and money in the LHS is financed by wage and dividend income. Workers do not have investment opportunities, and cannot borrow against their future labour income.

$$n'_{t+1} \geq 0, \text{ and } m'_{t+1} \geq 0. \quad (10)$$

An equilibrium process of prices  $\{p_t, q_t, w_t\}$  is such that: entrepreneurs choose labour demand  $l_t$  to maximize the gross profit (3) subject to production function (2) for a given start-of-period capital stock, and choose consumption, investment, capital stock and start-of-next-period equity and money holdings  $\{c_t, i_t, k_{t+1}, n_{t+1}, m_{t+1}\}$ , to maximize (1) subject to (4) - (7); workers choose consumption, labour supply, equity and money holding  $\{c'_t, l'_t, n'_{t+1}, m'_{t+1}\}$  to maximize (8) subject to (9) and (10); and the markets for general output, labour, equity and money all clear.

Before we characterize equilibrium, it helps to clear the decks a little by suppressing reference to the workers. Given that their population has unit measure, it follows from (8) and (9) that their aggregate labour supply equals  $(w_t/\omega)^{1/\nu}$ . Maximizing the gross profit of a typical entrepreneur controlling capital  $k_t$ , we find his labour demand,  $k_t [(1 - \gamma)A_t/w_t]^{1/\gamma}$  which is proportional to  $k_t$ . So if the aggregate stock of capital controlled by entrepreneurs at the start of date  $t$  is  $K_t$ , labour-market clearing requires that

$$(w_t/\omega)^{1/\nu} = K_t [(1 - \gamma)A_t/w_t]^{1/\gamma}.$$

Substituting back the equilibrium wage  $w_t$  into the LHS of (3), we find that the individual entrepreneur's maximized gross profit equals  $r_t k_t$  where

$$r_t = a_t (K_t)^{\alpha-1}, \quad (11)$$

and the parameters  $a_t$  and  $\alpha$  are derived from  $A_t, \gamma, \omega$  and  $\nu$ :

$$\begin{aligned} a_t &= \gamma \left( \frac{1 - \gamma}{\omega} \right)^{\frac{1-\gamma}{\gamma+\nu}} (A_t)^{\frac{1+\nu}{\gamma+\nu}} \\ \alpha &= \frac{\gamma(1 + \nu)}{\gamma + \nu}. \end{aligned} \quad (12)$$

Note from (12) that  $\alpha$  lies between 0 and 1, so that  $r_t$  – which is parametric for the individual entrepreneur – declines with the aggregate stock of capital  $K_t$ , because the wage increases with  $K_t$ . But for the entrepreneurial sector

as a whole, gross profit  $r_t K_t$  increases with  $K_t$ . Also note from (12) that  $r_t$  is increasing in the productivity parameter  $A_t$  through  $a_t$ . Later we will show that in the neighbourhood of the steady state monetary equilibrium, a worker will choose to hold neither equity nor money. That is, the worker simply consumes his labour income at each date:

$$c'_t = w_t l'_t. \quad (13)$$

We are now in a position to characterize the equilibrium behaviour of the entrepreneurs. Consider an entrepreneur holding equity  $n_t$  and money  $m_t$  at the start of date  $t$ . First, suppose he has an investment opportunity: let this be denoted by a superscript  $i$  on his choice of consumption, and start-of-next-period equity and money holdings,  $(c_t^i, n_{t+1}^i, m_{t+1}^i)$ . He has two ways of acquiring equity  $n_{t+1}^i$ : either produce it at unit cost 1, or buy it in the market at price  $q_t$ . (See the LHS of the flow-of-funds constraint (7), where, recall, investment  $i_t$  corresponds to investment.) If  $q_t$  is less than 1, the agent will not invest. If  $q_t$  equals 1, he will be indifferent. If  $q_t$  is greater than 1, he will invest by selling as much equity as he can subject to the constraint (5). The entrepreneur's production choice is similar to Tobin's  $q$  theory of investment.

As the aggregate productivity and liquidity of equity  $(A_t, \phi_t)$  follow a stochastic process in the neighbourhood of constant  $(A, \phi)$ , we have the following claim in the neighbourhood of the steady state equilibrium (All the proofs are in Appendix):

**Claim 1** *Suppose that  $\theta$  and  $\phi$  satisfy*

$$\text{Condition 1 : } (1 - \lambda)\theta + \pi\lambda\phi > (1 - \lambda)(1 - \pi).$$

*Then in the neighbourhood of the steady state:*

- (i) the allocation of resource is the first best*
- (ii) Tobin's  $q$  is equal to unity:  $q_t = 1$ ;*
- (iii) money has no value:  $p_t = 0$ ;*
- (iv) the gross dividend is roughly equal to the time preference rate plus the depreciation rate:  $r_t \simeq \frac{1}{\beta} - \lambda$ .*

If the investing entrepreneurs can issue new equity relatively freely and existing equity is relatively liquid to satisfy Condition 1, then the equity market transfers enough resources from the savers to the investing entrepreneurs

to achieve the first best allocation.<sup>11</sup> There would be no extra advantage of having investment opportunity; Tobin's  $q$  is equal to 1 (or the market value of capital is equal to the replacement cost) and both investing entrepreneurs and savers earn the same net rate of return on equity which is approximately equal to the time preference rate. (Note that the usual risk premium would be fairly negligible in the first best with our logarithmic utility function). Because the economy achieves the first best allocation without money, money has no value in the equilibrium.

In the following we want to restrict attention to an equilibrium in which  $q_t$  is greater than 1. We also want money to have value in equilibrium. Let us assume that  $\theta$  and  $\phi$  satisfy:

*Assumption 2* :  $0 < \Phi(\theta, \phi)$ , where

$$\begin{aligned} \Phi(\theta, \phi) \equiv & \pi\lambda\beta^2(1-\pi)(1-\phi)[(1-\lambda)(1-\pi) - (1-\lambda)\theta - \pi\lambda\phi] \\ & + [(\beta-\lambda)(1-\pi) - (1-\lambda)\theta - \pi\lambda\phi] [1-\lambda + \pi\lambda - (1-\lambda)\theta - \pi\lambda\phi] \\ & \cdot [\lambda(1-\beta)(1-\pi) + (1-\lambda)\theta + \lambda(\beta + \pi - \pi\beta)\phi]. \end{aligned}$$

Because both  $\theta$  and  $\phi$  are between 0 and 1, we observe all the terms in the RHS are positive, except for the terms  $(1-\lambda)(1-\pi) - (1-\lambda)\theta - \pi\lambda\phi$  and  $(\beta-\lambda)(1-\pi) - (1-\lambda)\theta - \pi\lambda\phi$ . Thus a sufficient condition for Assumption 2 is

$$(1-\lambda)\theta + \pi\lambda\phi < (\beta-\lambda)(1-\pi),$$

and a necessary condition is

$$(1-\lambda)\theta + \pi\lambda\phi < (1-\lambda)(1-\pi).$$

We observe that if the Condition 1 for Claim 1 is satisfied, then the necessary condition is not satisfied and there would be no equilibrium with valued fiat money. Under Assumption 2, however, upper bound on  $\theta$  and  $\phi$  is tight enough to ensure that the following claim holds.

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<sup>11</sup>In the steady state, the aggregate saving (which is equal to aggregate investment) is equal to the depreciation of capital. The RHS of Condition 1 is equal to the ratio of aggregate saving of non-investing entrepreneurs (who are  $1-\pi$  fraction of total entrepreneurs) to aggregate capital stock in the first best allocation. The LHS is the ratio of maximum equity sold by the investing entrepreneurs to aggregate capital stock:  $\theta(1-\lambda)$  corresponds to the new equity issued and  $\pi\lambda\phi$  corresponds to the existing equity sold by the investing entrepreneurs (which is  $\pi$  fraction of total entrepreneurs). Thus Condition 1 says that the maximum equity sold by the investing entrepreneurs is enough to shift the aggregate saving of the non-investing entrepreneurs.

**Claim 2** *Under Assumption 2, in the neighbourhood of the steady state:*

- (i) *the price of money,  $p_t$ , is strictly positive;*
- (ii) *the price of capital,  $q_t$ , is strictly greater than 1;*
- (iii) *an entrepreneur with an investment opportunity faces the binding liquidity constraints and will not choose to hold money:  $m_{t+1}^i = 0$ .*

We will be in a position to prove the claim once we have laid out the equilibrium conditions - we use a method of guess-and-verify in the following. For values of  $\theta$  and  $\phi$  which do not satisfy Assumption 2 nor the condition for Claim 1, we can show that money has no value even though the liquidity constraint (5) still binds. To streamline the paper, we have chosen not to give an exhaustive account of the equilibria throughout the parameter space.

There is a caveat to Claim 2(i). Fiat money can only be valuable to someone if other people find it valuable, hence there is always a non-monetary equilibrium in which the price of fiat money is zero. Thus when there is a monetary equilibrium in addition to the non-monetary equilibrium, we restrict attention to the monetary equilibrium:  $p_t > 0$ . Claim 2(iii) says that the entrepreneur prefers investment with the maximum leverage to holding money, even though the return is in the form of equity which at date  $t+1$  is less liquid than money. (Incidentally, even though the investing entrepreneurs don't want to hold money for liquidity purposes, the non-investing entrepreneurs do – see below. This is why Claim 2(i) holds.)

Thus, for an investing entrepreneur, the liquidity constraints (5) and (6) are both binding. His flow of funds constraint (7) can be rewritten

$$c_t^i + (1 - \theta q_t) i_t = (r_t + \lambda \phi_t q_t) n_t + p_t m_t. \quad (14)$$

In order to finance investment cost  $i_t$ , the entrepreneur issues equity as much as  $\theta i_t$  at price  $q_t$ . Thus the second term in the LHS is the investment cost that has to be financed internally - the downpayment for investment. The LHS is the total liquidity needs of the investing entrepreneur. The RHS corresponds to the maximum liquidity supplied from dividend, sales of resaleable fraction of equity after depreciation and the value of money. When we solve this flow-of-funds constraint with respect to the equity of the next period, we have

$$c_t^i + q_t^R n_{t+1}^i = r_t n_t + [\phi_t q_t + (1 - \phi_t) q_t^R] \lambda n_t + p_t m_t, \quad (15)$$

$$\text{where } q_t^R \equiv \frac{1 - \theta q_t}{1 - \theta} < 1, \text{ as } q_t > 1. \quad (16)$$

The value of  $q_t^R$  is the effective replacement cost of equity to the investing entrepreneur: because he needs downpayment  $1 - \theta q_t$  for every unit of investment in order to retain  $1 - \theta$  inside equity, he needs  $(1 - \theta q_t)/(1 - \theta)$  in order to acquire one unit of inside equity. The RHS of (15) is his net worth: gross dividend, the value of his depreciated equity  $\lambda n_t$  - of which resaleable  $\phi_t$  fraction is valued by the market price and the non-resaleable  $1 - \phi_t$  fraction is valued by the effective replacement cost -, and the value of money.

Given the discounted logarithmic preferences (1), the entrepreneur saves a fraction  $\beta$  of his net worth, and consumes a fraction  $1 - \beta$ :<sup>12</sup>

$$c_t^i = (1 - \beta) \{ r_t n_t + [\phi_t q_t + (1 - \phi_t) q_t^R] \lambda n_t + p_t m_t \}. \quad (17)$$

And so, from (14), we obtain an expression for his investment in period t:

$$i_t = \frac{(r_t + \lambda \phi_t q_t) n_t + p_t m_t - c_t^i}{1 - \theta q_t}. \quad (18)$$

The investment is equal to the ratio of liquidity available after consumption to the downpayment per unit of investment.

Next, suppose the entrepreneur does not have an investment opportunity: denote this by a superscript  $s$  to stand for a pure saver. The flow-of-funds constraint (7) reduces to

$$c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s = r_t n_t + q_t \lambda n_t + p_t m_t. \quad (19)$$

For the moment, let us assume that constraints (5) and (6) do not bind. Then the RHS of (19) corresponds to the entrepreneur's net worth. It is the same as the RHS of (15), except that now his depreciated equity is valued at the market price,  $q_t$ . From this net worth he consumes a fraction  $1 - \beta$ :

$$c_t^s = (1 - \beta)(r_t n_t + q_t \lambda n_t + p_t m_t). \quad (20)$$

Note that consumption of entrepreneur who does not have investment opportunity is larger than consumption of the investing entrepreneur if both hold the same equity and money at the start of period. The remainder is split across a savings portfolio of  $m_{t+1}$  and  $n_{t+1}$ .

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<sup>12</sup>Compare (1) to a Cobb-Douglas utility function, where the expenditure share of present consumption out of total wealth is constant and equal to  $1/(1 + \beta + \beta^2 + \dots) = 1 - \beta$ .

To determine the optimal portfolio, consider the choice of sacrificing one unit of consumption  $c_t$  to purchase either  $1/p_t$  units of money or  $1/q_t$  units of equity, which are then used to augment consumption at date  $t+1$ . The first-order condition is

$$\begin{aligned}
u'(c_t) &= E_t \left\{ \frac{p_{t+1}}{p_t} [(1 - \pi) u'(c_{t+1}^s) + \pi u'(c_{t+1}^i)] \right\} \\
&= (1 - \pi) E_t \left\{ \frac{r_{t+1} + \lambda q_{t+1}}{q_t} u'(c_{t+1}^s) \right\} \\
&\quad + \pi E_t \left\{ \frac{r_{t+1} + \lambda \phi_{t+1} q_{t+1} + \lambda (1 - \phi_{t+1}) q_{t+1}^R}{q_t} u'(c_{t+1}^i) \right\}.
\end{aligned} \tag{21}$$

The RHS of the first line of (21) is the expected gains from holding  $1/p_t$  additional units of money at date  $t+1$ : money will always yields  $p_{t+1}$  which will increase utility by  $u'(c_{t+1}^s)$  times as much when he will not have an investment opportunity with probability  $1 - \pi$  and will increase utility by  $u'(c_{t+1}^i)$  times as much when he will have an investment opportunity with probability  $\pi$  at date  $t+1$ . The second line is the gain in expected discounted utility from holding  $1/q_t$  additional units of equity at date  $t+1$ . Per unit, this additional equity yields  $r_{t+1}$  dividend, plus its depreciated value. With probability  $1 - \pi$  the entrepreneur will not have an investment opportunity and the depreciated equity will be valued at the market price,  $q_{t+1}$ , and these yields will increase utility by  $u'(c_{t+1}^s)$  times as much. With probability  $\pi$  the entrepreneur will have an investment opportunity at date  $t+1$ , in which case he will value depreciated equity by the market price  $q_{t+1}$  for resaleable fraction and by the effective replacement cost  $q_{t+1}^R$  for non-resaleable fraction, and these yield will increase utility by the marginal utility conditional on investment  $u'(c_{t+1}^i)$ . Because the effective replacement cost is lower than the market price in monetary economy and the equity is only partially resaleable, the equity will have lower contingent return in the state in which the entrepreneur needs fund most with an arrival of investment opportunity and the marginal utility of consumption is high (as  $c_{t+1}^i < c_{t+1}^s$ ).

Equity is "risky" to the saving entrepreneur not only because the rate of return is correlated with aggregate consumption (aggregate risk) but also because the contingent rate of return is low due to limited resaleability when the entrepreneur's marginal utility is high (idiosyncratic risk). Money is "free" from idiosyncratic risk of having investment opportunity, because its rate of return is independent of whether or not the entrepreneur has an

investment opportunity in the next period.

We are now in a position to consider the aggregate economy. The great merit of the expressions for an investing entrepreneur's consumption and investment choices,  $c_t^i$  and  $i_t$ , and a non-investing entrepreneurs' consumption and savings choices,  $c_t^s$ ,  $n_{t+1}$  and  $m_{t+1}$ , is that they are all linear in start-of-period equity and money holdings  $n_t$  and  $m_t$ .<sup>13</sup> Hence aggregation is easy: we do not need to keep track of the distributions. Notice that, because workers do not choose to save, the aggregate holdings of equity and money of the entrepreneurs are equal to aggregate capital stock  $K_t$  and money supply  $M$ . Since investment opportunities are independently distributed, we can work with the total capital and money holdings in the economy,  $K_t$  and  $M$ . At the start of date  $t$ , a fraction  $\pi$  of  $K_t$  and  $M$  is held by entrepreneurs who have an investment opportunity. From (18), total investment,  $I_t$ , in new capital therefore satisfies

$$(1 - \theta q_t) I_t = \pi \left\{ \beta [(r_t + \lambda \phi_t q_t) K_t + p_t M] - (1 - \beta)(1 - \phi_t) \lambda q_t^R K_t \right\}. \quad (22)$$

Goods market clearing requires that total output (net of labour costs, which equals the consumption of workers),  $r_t K_t$ , equals investment plus consumption of entrepreneurs— which, using (17) and (20), yields

$$\begin{aligned} r_t K_t &= a_t K_t^\alpha = I_t + (1 - \beta) \cdot \\ &\quad \left\{ [r_t + (1 - \pi + \pi \phi_t) \lambda q_t + \pi (1 - \phi_t) \lambda q_t^R] K_t + p_t M \right\}. \end{aligned} \quad (23)$$

It remains to find the aggregate counterpart to the portfolio equation (21). During period  $t$ , the investing entrepreneurs sell a fraction  $\theta$  of their investment  $I_t$ , together with a fraction  $\phi_t$  of their depreciated equity holdings  $\pi \lambda K_t$ , to the non-investing entrepreneurs. So the stock of equity held by the group of non-investing entrepreneurs at the end of the period is given by  $\theta I_t + \phi_t \pi \lambda K_t + (1 - \pi) \lambda K_t \equiv N_{t+1}^s$ . And, by claim 2(iii), we know that this group also hold all the money stock,  $M$ . We also know that the marginal utility of consumption is proportional to the inverse of consumption (that is proportional to the net worth) due to logarithmic utility function. The

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<sup>13</sup>From (19) and (20), the value of savings,  $q_t n_{t+1}^s + p_t m_{t+1}^s$  is linear in  $n_t$  and  $m_t$ , and (the reciprocal of) the portfolio equation (21) is homogeneous in  $(n_{t+1}^n, m_{t+1}^n)$ , noting that  $u'(c) = 1/c$  with logarithmic utility function.



group's savings portfolio  $(N_{t+1}^s, M)$  satisfies (21), which can be simplified to:

$$\begin{aligned} & (1 - \pi) E_t \left[ \frac{(r_{t+1} + \lambda q_{t+1})/q_t - p_{t+1}/p_t}{(r_{t+1} + q_{t+1}\lambda)N_{t+1}^s + p_{t+1}M} \right] \\ &= \pi E_t \left[ \frac{p_{t+1}/p_t - [r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R]/q_t}{[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R]N_{t+1}^s + p_{t+1}M} \right]. \end{aligned} \quad (24)$$

Equation (24) lies at the heart of the model. When there is no investment opportunity at date  $t+1$ , so that the partial liquidity of equity doesn't matter, the return on capital,  $(r_{t+1} + \lambda q_{t+1})/q_t$  exceeds the return on money,  $p_{t+1}/p_t$ : the LHS of (24) is positive. However, when there is an investment opportunity, the contingent rate of return on equity,  $[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R]/q_t$  is less than the return on money: the RHS of (24) is positive. These return differentials have to be weighted by the respective probabilities and marginal utilities. The liquidity premium of equity over money in the LHS can be substantial and time-varying, because the equity holder faces the idiosyncratic cost of limited resaleability in the RHS in case of having an investment opportunity (in addition to usual aggregate risk).<sup>14</sup>

Aside from the liquidity shock  $\phi_t$  and the technology parameter  $A_t$  which follow an exogenous stationary Markov process, the only state variable in this system is  $K_t$ , which evolves according to

$$K_{t+1} = \lambda K_t + I_t. \quad (25)$$

Restricting attention to stationary price process, the competitive equilibrium can be defined recursively as function  $(I_t, p_t, q_t, K_{t+1})$  of aggregate state  $(K_t, A_t, \phi_t)$  that satisfy (11), (22) – (25), together with the law of motion of  $A_t$  and  $\phi_t$ . From these four equations to characterize the equilibrium, we observe that there are rich interaction between quantities  $(I_t, K_{t+1})$  and

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<sup>14</sup>Holmstrom and Tirole (1998, 2001) develop models of three-period production economy with financial intermediaries in which pledgeable future returns are limited. One of the main differences from ours is that the liquidity needs of each individual entrepreneur is contractible and that there is no constraint on resaleability. In Holmstrom and Tirole (2001), the liquidity premium of each asset depends upon the covariance between its rate of return and the aggregate liquidity needs. Because of limited insurance, our approach is perhaps closer to Luttmer (1996, 1999) which examine the implications of transaction costs and short-sales constraints for consumption and asset prices.

Atkeson and Kehoe (2008) argue for the need for the time-varying risk premium to analyzing monetary policy.

asset prices  $(p_t, q_t)$ . In this sense, our economy is similar to Keynes (1936). In fact, perhaps the closest ancestor of our model is Tobin (1969) where he considers Tobin's  $q$  as the key variable to analyze the interaction between goods market and asset market. From methodological point of view, our model is similar to more modern macroeconomics because we derive all the behavioral relationship from individual optimization under the constraints of technology and liquidity.

In steady state, when  $a_t = a$  (the RHS of (12) with  $A_t = A$ ) and  $\phi_t = \phi$ , capital stock  $K$ , investment  $I$ , and prices  $p$  and  $q$ , satisfy  $I = (1 - \lambda)K$  and

$$\pi\beta aK^{\alpha-1} + \pi\beta\frac{pM}{K} = \left[1 - \lambda + \pi\lambda(1 - \beta)\frac{1 - \phi}{1 - \theta}\right](1 - \theta q) - \pi\beta\lambda\phi q \quad (26)$$

$$\begin{aligned} \beta aK^{\alpha-1} - (1 - \beta)\frac{pM}{K} &= \left[1 - \lambda + \pi\lambda(1 - \beta)\frac{1 - \phi}{1 - \theta}\right] \\ &+ (1 - \beta)\left(1 - \pi\frac{1 - \phi}{1 - \theta}\right)\lambda q \end{aligned} \quad (27)$$

$$aK^{\alpha-1} - (1 - \lambda)q = \pi\lambda\frac{1 - \phi}{1 - \theta}(q - 1)\frac{q + (pM/\chi K)}{aK^{\alpha-1} + \lambda\frac{1 - \phi}{1 - \theta} + \lambda\frac{\phi - \theta}{1 - \theta}q + (pM/\chi K)}, \quad (28)$$

where  $\chi \equiv \theta(1 - \lambda) + (1 - \pi + \pi\phi)\lambda$ , the steady-state fraction of equity held by non-investing entrepreneurs at the end of a period.

Equations (26), (27) and (28) can be viewed as a simultaneous system in three unknowns: the price of capital,  $q$ ; the gross profit rate on capital,  $r = aK^{\alpha-1}$ ; and the value of the money stock as a fraction of total capital,  $pM/K$ . (26) and (27) can be solved for  $aK^{\alpha-1}$  and  $pM/K$ , each as affine functions of  $q$ , which when substituted into (28) yield a quadratic equation in  $q$  with a unique positive solution. Assumption 2 is sufficient to ensure that this solution lies strictly above 1 (but below  $1/\theta$ ). We can also show that Assumption 2 is the necessary and sufficient condition for money to have value:  $p > 0$ .

As a prelude to the dynamic analysis that we undertake later on, notice that the technology parameter  $A$  only affects the steady-state system through the gross profit term  $aK^{\alpha-1}$ . That is, a rise in the steady state value of  $A$  increases the capital stock,  $K$ , but does not affect  $q$ , the price of capital. The price of money,  $p$ , increases to leave  $pM/K$  unchanged.

It is interesting to compare our economy, in which the liquidity constraints (5) and (6) bind for investing entrepreneurs, to a "first-best" economy without such constraints. Consider steady states. In the first-best economy, the price of capital would equal its cost, 1; and the capital stock,  $K^*$  say, would equate the return on capital,  $aK^{\alpha-1} + \lambda$ , to the agents' common subjective return,  $1/\beta$ . (See Claim 1). We can show that, in our constrained economy, the level of activity – measured by the capital stock  $K$  – is strictly below  $K^*$ . Hence, by continuity, the same is true in the neighbourhood of the steady state. Because of the partial liquidity of equity, the economy fails to transfer enough resources to the investing entrepreneurs to achieve the first-best level of investment.

The liquidity constraint creates the wedges between the marginal product of capital and the expected rate of returns on equity, and turns out to keep both the expected rates of return on equity and that of money below the time preference in the neighborhood of the steady state. Intuitively, the rate of returns on assets to savers are below their time preference rate so that the savers will not save enough to escape the liquidity constraint when they find an opportunity to invest in future.

**Claim 3** *In the neighbourhood of the steady state monetary economy,*

(i) *the stock of capital,  $K_{t+1}$  is less than in the first-best (unconstrained) economy:*

$$K_{t+1} < K^* \Leftrightarrow E_t (a_{t+1}K_{t+1}^{\alpha-1} + \lambda) > \frac{1}{\beta},$$

(ii) *the expected rate of return on equity (if the saver does not have investment opportunity at date  $t+1$ ) is lower the time preference rate:*

$$E_t \frac{a_{t+1}K_{t+1}^{\alpha-1} + \lambda q_{t+1}}{q_t} < \frac{1}{\beta},$$

(iii) *the expected rate of return on money is lower than the expected rate of return on equity:*

$$E_t \frac{p_{t+1}}{p_t} < E_t \frac{a_{t+1}K_{t+1}^{\alpha-1} + \lambda q_{t+1}}{q_t},$$

(iv) *the expected rate of return on equity contingent on having an investment opportunity in the next period is lower than the expected rate of return*

on money:

$$E_t \frac{a_{t+1} K_{t+1}^{\alpha-1} + \phi_{t+1} \lambda q_{t+1} + (1 - \phi_{t+1}) \lambda q_{t+1}^R}{q_t} < \frac{p_{t+1}}{p_t}.$$

Claim 3(iii) follows directly from (28), given that in steady state  $q > 1$ . This difference between the expected return on equity and money reflects a liquidity premium. It equals the nominal interest rate on equity.<sup>15</sup> Because entrepreneurs are constrained when they have an investment opportunity, they have to be compensated for holding less liquid equity in their savings portfolio. If there were no binding liquidity constraints, money would have no value.

In our monetary economy, there are a spectrum of interest rates: from the highest, we have the expected marginal product of capital, the time preference rate, the expected rate of return on equity, the expected rate of return on money, and the expected rate of return on equity contingent on the saver having an investment opportunity in the next period. These spreads across different interest rates come from the very feature of monetary economy in which the circulation of money is essential for resource allocation. Thus in our economy the impact of the asset markets on aggregate production cannot be summarized by the expectations of a single real interest rate from present to future as in some popular models such as Woodford (2003). It is equally misleading to use the average real rates of returns on money and equity for the time preference rate to calibrate our economy.

The fact that the expected rates of return on equity and money are both lower than the time preference rate justifies our earlier assertion that workers will not choose to save by holding capital or money.<sup>16</sup> (Of course, if workers could borrow against their future labour income they would do so. But we

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<sup>15</sup>By the Fisher equation, the nominal interest rate on equity equals the net real return on equity plus the inflation rate. But minus the inflation rate equals the net real return on money. Hence the nominal interest rate on equity equals the real return on equity minus the return on money, i.e. the liquidity premium. Because our money is broad money - asset readily resaleable, our nominal interest rate is similar to the interest rate in Keynes (1936); the gap between the rate of return on partially resaleable assets and broad money.

<sup>16</sup>Workers would save if workers were to face their own investment opportunity shocks. Suppose, instead, that each worker randomly meets a "health shock" with which the worker has to spend immediately some fixed amount  $\zeta$  of co-payment in order to maintain his human capital. (The health insurance covers some costs, but the patient has to cover the co-payment from his own pocket here). Then, if the resaleability of equity is low, we can show that worker save only in money up to the value enough to cover the co-payment  $\zeta$ .

have ruled this out.) In steady state, workers enjoy a constant consumption equal to their wages.

The reason why an entrepreneur saves, and workers do not, is because the entrepreneur is preparing for his next investment opportunity. And the entrepreneur saves using money as well as equity, despite money's particularly low return, because he anticipates that he will be liquidity constrained at the time of investment. Along a typical time path, he experiences episodes without investment, during which he consumes part of his saving. As the return on saving – on both capital and money – is less than his time preference rate, the value of his net worth gradually shrinks, as does his consumption. He only expands again at the time of investment. In the aggregate picture, we do not see all this fine grain. But it is important to realize that, even in steady state, the economy is made up of a myriad of such individual histories.

### 3 Dynamics

In order to examine the dynamics of our economy, we can use the recursive equilibrium - function  $(I_t, p_t, q_t, K_{t+1})$  of aggregate state  $(K_t, A_t, \phi_t)$  that satisfy (11) and (22) – (25), together with the law of motion of  $(A_t, \phi_t)$ .

To start with, let's look at how the economy fluctuates with deterministic shifts of aggregate productivity, like seasonal cycle.<sup>17</sup> Suppose that the aggregate productivity alternate between high level and low level for every  $T > 1$  periods deterministically. We take the length of one period very short in order to examine the dynamics. (In Appendix, we layout the continuous time limit of our economy). *Figure 1* shows how asset prices and quantities fluctuate with the deterministic shifts of aggregate productivity.

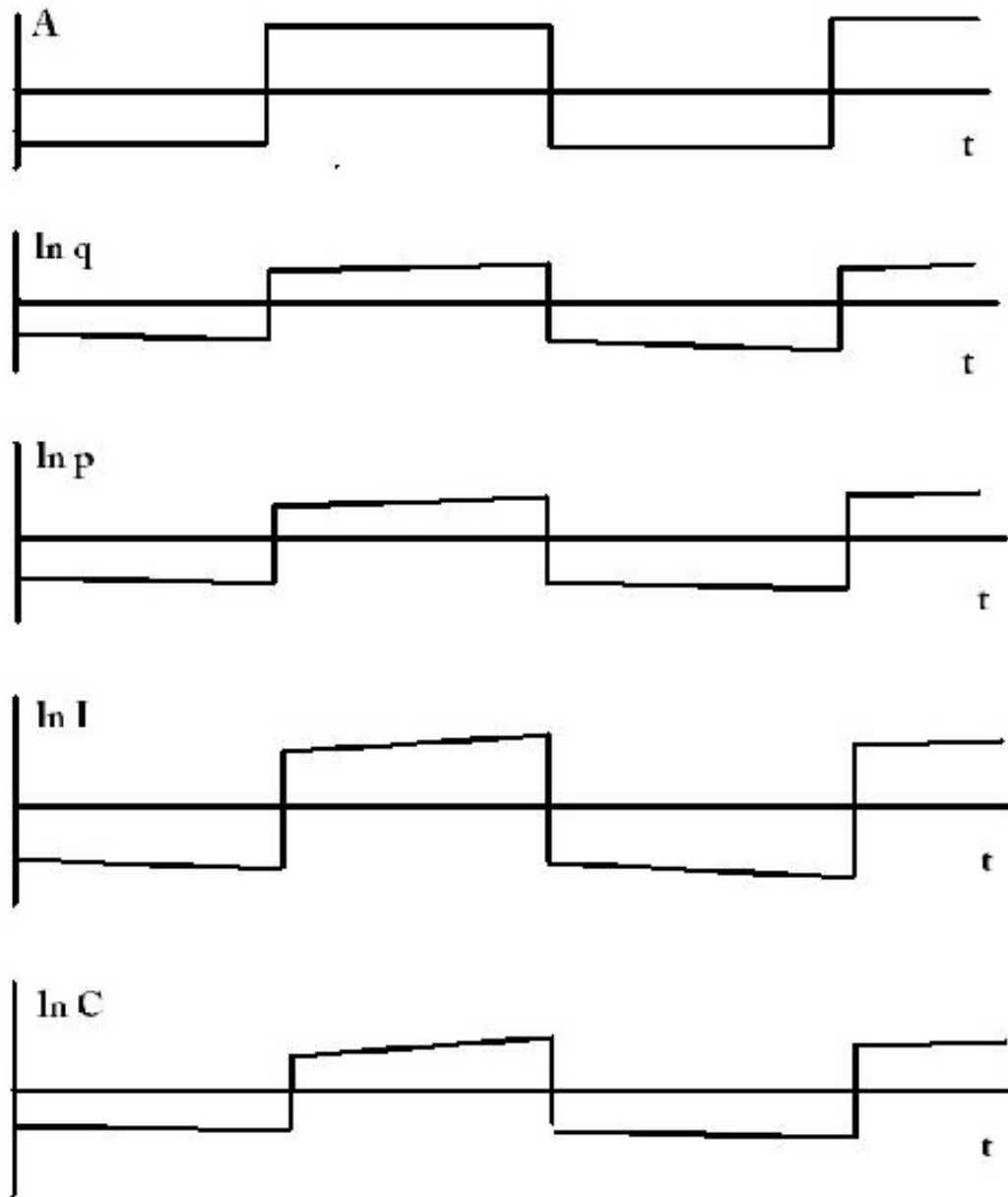
Because the timing of the productivity shifts is known in advance, there is no discontinuous jump in the nominal price of equity  $q_t/p_t$  at the time of productivity shift. (If the nominal price of equity jumped, say jumped up, then saving entrepreneurs would have held only equity immediately before the

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Even though the rate of return on equity is higher than money, the worker would require to save more in equity than money in order to compensate the resaleability constraint, which would be more costly, given that the rate of return on equity is lower than the time preference rate. See Kiyotaki and Moore (2005a) for the detail.

<sup>17</sup>Barsky and Miron (1989) and Miron (1996) document that there are many similarities between seasonal cycle and business cycle in terms of relative volatility and co-movement of aggregate prices and quantities. Thus, we may learn about the property of our economy by examining how the economy fluctuates with the deterministic cycle of the productivity.

Figure 1: Deterministic Productivity Shifts



jump). From the goods market equilibrium condition, (23), we observe the real asset prices  $(p_t, q_t)$  in terms of general output have to increase together at the time of productivity increase in order to increase the consumption and investment in line with a larger output supply. Although investment is more sensitive to the asset prices and thus increases more than consumption in proportion, consumption also jumps up as the net worth increases. This is different from the first best allocation, in which consumption of the entrepreneurs does not jump against the deterministic shift of the productivity.<sup>18</sup> After the productivity improvement, the real asset prices, investment and consumption continue to increase with capital accumulation, until productivity switch from a high to a low level.

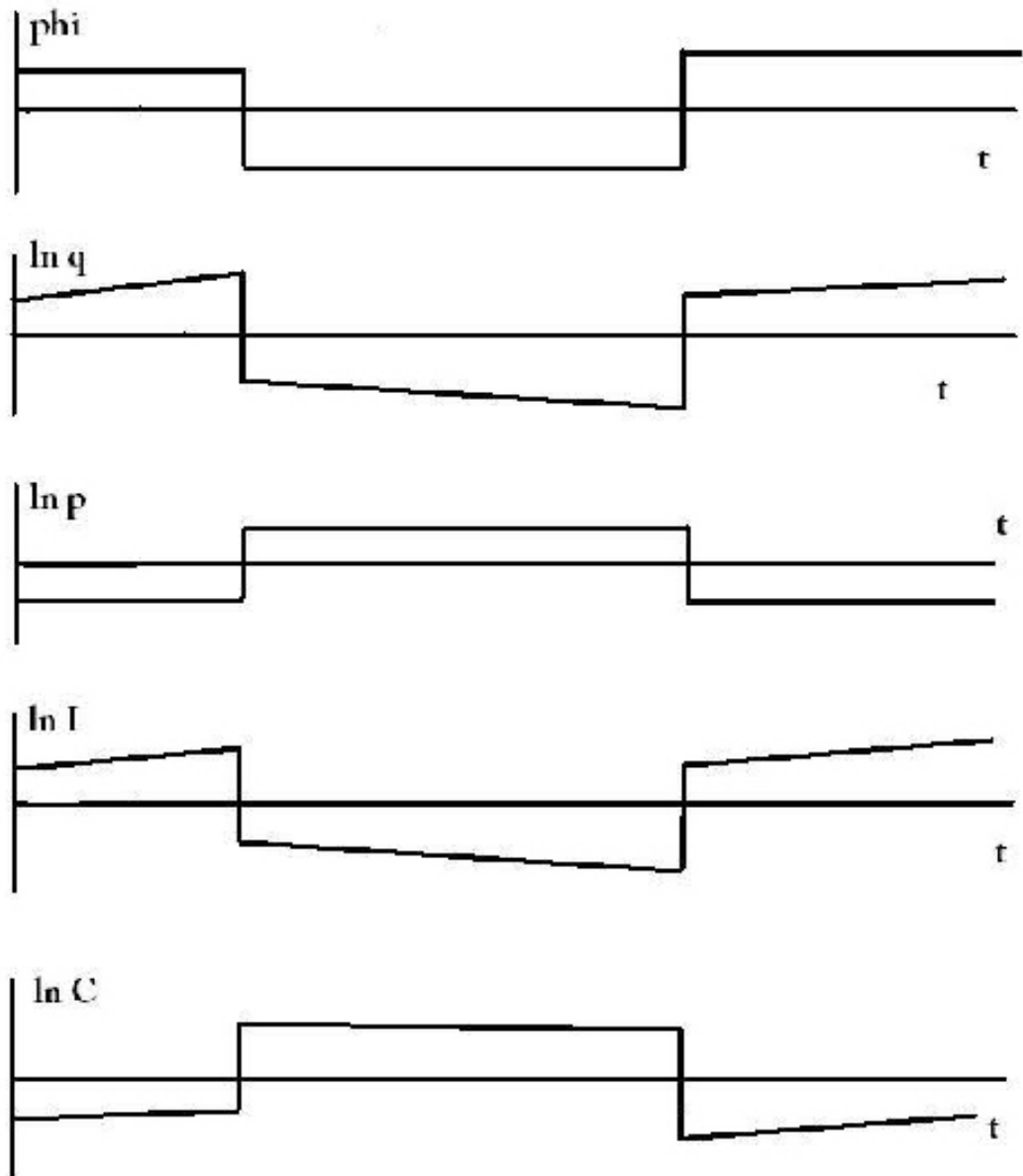
Now, let's consider liquidity shocks: a stochastic shock to the resaleability of equity,  $\phi_t$ . Suppose that  $\phi_t$  follows a Markov process with two states,  $\phi(1 + \Delta_\phi)$  and  $\phi(1 - \Delta_\phi)$  and constant arrival rate  $\eta_\phi$  of switch every period, where  $\Delta_\phi$  and  $\eta_\phi$  are small positive parameters. *Figure 2* shows the process of the asset prices and quantities with a particular realization of the stochastic process of the liquidity  $\phi_t$ .

When the resaleability of equity falls to a low level persistently with a stochastic arrival of the switch, then the investing entrepreneurs can finance only a smaller downpayment from selling his equity holding. Also the entrepreneurs without investment opportunity find equity less attractive than money as means of saving (if the expected rate of returns were unchanged), because he can resale only a smaller fraction of equity holding if he has an investment opportunity in the following period and he has to revalue the non-resaleable fraction of equity by the effective replacement costs (which is lower than the market price). (See (24)). Thus, the equity price falls and the value of money increases in order to restore the asset market equilibrium. This can be thought of "a flight to liquidity". Because of the lower equity price, the investing entrepreneur needs more downpayment per unit of investment. Taken together, aggregate investment decreases substantially, despite that the increase of value of money partially offsets the decrease. (See (22)). Because output is not affected with full employment, consumption increases to restore goods market equilibrium. Overtime, capital decumulates with lower investment, and the real asset prices, investment and consumption all

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<sup>18</sup>Consumption of workers jumps with the deterministic productivity shift, because the marginal utility of consumption is affected by labour supply (and labour supply depends upon productivity).

## Liquidity Shock under Laissez-Faire





decrease, until the resaleability of equity switches from a low to a high level.

## 4 Introducing government

Our goal here is simply to explore the effects on equilibrium of an exogenous government policy. We make no attempt to explain government behaviour.

At the start of date  $t$ , suppose the government holds  $N_t^g$  equity. Unlike entrepreneurs, the government cannot produce new capital. However, it can engage in open market operations, to buy (sell) equity by issuing (taking in) money – it has sole access to a costless money-printing technology.<sup>19</sup> Finally, the government can purchase goods, or transfer to the workers (which can be negative if it is lump-sum tax). Let  $G_t$  denote the total government purchase of general output and real lump-sum net transfer to workers. We assume  $G_t$  does not affect utility of entrepreneurs. This leaves intact our analysis of entrepreneurs' behaviour. We assume that  $N_t^g$  and  $G_t$  are not so large that the private economy switches regimes. That is, we are still in an equilibrium in which the liquidity constraints bind for investing entrepreneurs, and money is valuable.

If  $M_t$  is the stock of money privately held by entrepreneurs at the start of date  $t$ , then the government's flow-of-funds constraint is given by

$$G_t + q_t (N_{t+1}^g - \lambda N_t^g) = r_t N_t^g + p_t (M_{t+1} - M_t). \quad (29)$$

That is, government net purchase of general output and equity must equal dividend of equity plus seigniorage revenues. Since the government is a large agent, at least relative to each of the atomless private citizens, open market operations will affect the prices  $p_t$  and  $q_t$ .

All of our earlier analysis goes through, but with obvious adjustments. Total supply of equity (which is equal to aggregate capital stock by the way of defining the equity) is equal to the sum of the government holding and aggregate holding of the entrepreneurs (denoted as  $N_t$ )

$$K_t = N_t^g + N_t. \quad (30)$$

The tax changes workers' consumption (as they consume all the disposable income), but, given the form of their preferences in (8), does not affect their

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<sup>19</sup> Any sales of equity are subject to the same constraint as (5):  $N_{t+1}^g \geq (1 - \phi_t)\lambda N_t^g$ .

labour supply. Equations (22), (23) and (24) are modified to:

$$(1 - \theta q_t) I_t = \pi \left\{ \beta [(r_t + \lambda \phi_t q_t) N_t + p_t M_t] - (1 - \beta)(1 - \phi_t) \lambda q_t^R N_t \right\} \quad (31)$$

$$r_t K_t = a_t K_t^\alpha = I_t + G_t + (1 - \beta) \cdot \left\{ [r_t + (1 - \pi + \pi \phi_t) \lambda q_t + \pi (1 - \phi_t) \lambda q_t^R] N_t + p_t M_t \right\} \quad (32)$$

$$(1 - \pi) E_t \left[ \frac{(r_{t+1} + \lambda q_{t+1}) / q_t - p_{t+1} / p_t}{(r_{t+1} + q_{t+1} \lambda) N_{t+1}^s + p_{t+1} M_{t+1}} \right] \quad (33)$$

$$= \pi E_t \left[ \frac{p_{t+1} / p_t - [r_{t+1} + \phi_{t+1} \lambda q_{t+1} + (1 - \phi_{t+1}) \lambda q_{t+1}^R] / q_t}{[r_{t+1} + \phi_{t+1} \lambda q_{t+1} + (1 - \phi_{t+1}) \lambda q_{t+1}^R] N_{t+1}^s + p_{t+1} M_{t+1}} \right],$$

where  $N_{t+1}^s = \theta I_t + \phi_t \pi \lambda N_t + (1 - \pi) \lambda N_t + \lambda N_t^g - N_{t+1}^g$ . Suppose we take both the policy variables  $(G_t, N_{t+1}^g, M_{t+1})$  and the parameters of technology and liquidity  $(A_t, \phi_t)$  to follow exogenous stationary Markov processes. Then, restricting attention to stable price process, the competitive equilibrium can be defined recursively as functions  $(I_t, p_t, q_t, N_{t+1}, K_{t+1})$  of the aggregate state  $(N_t, K_t, M_t, A_t, \phi_t, N_t^g)$  that satisfy (11), (25), (29) – (33) together with the laws of motion of  $(A_t, \phi_t, G_t, N_{t+1}^g, M_{t+1})$ .<sup>20</sup>

How should the central bank use the open market operation against the deterministic shifts of productivity (as in *Figure 1*)? The problem of laissez-faire monetary economy is that consumption is not smooth even if the productivity shift is deterministic. Thus, the central bank can conduct the open market purchase operation of equity immediately before the upward productivity shift. As the central bank purchase equity with money, the private entrepreneurs hold a larger amount of liquid money and hold a smaller amount of partially resaleable equity, and thus the entrepreneurs can invest more. From the RHS of (31), precisely because the equity is less liquid than money  $\phi_t < 1$ , we see investment will increase if the value of money increase as much as the decrease of the value of private holdings of equity by the open market operation:

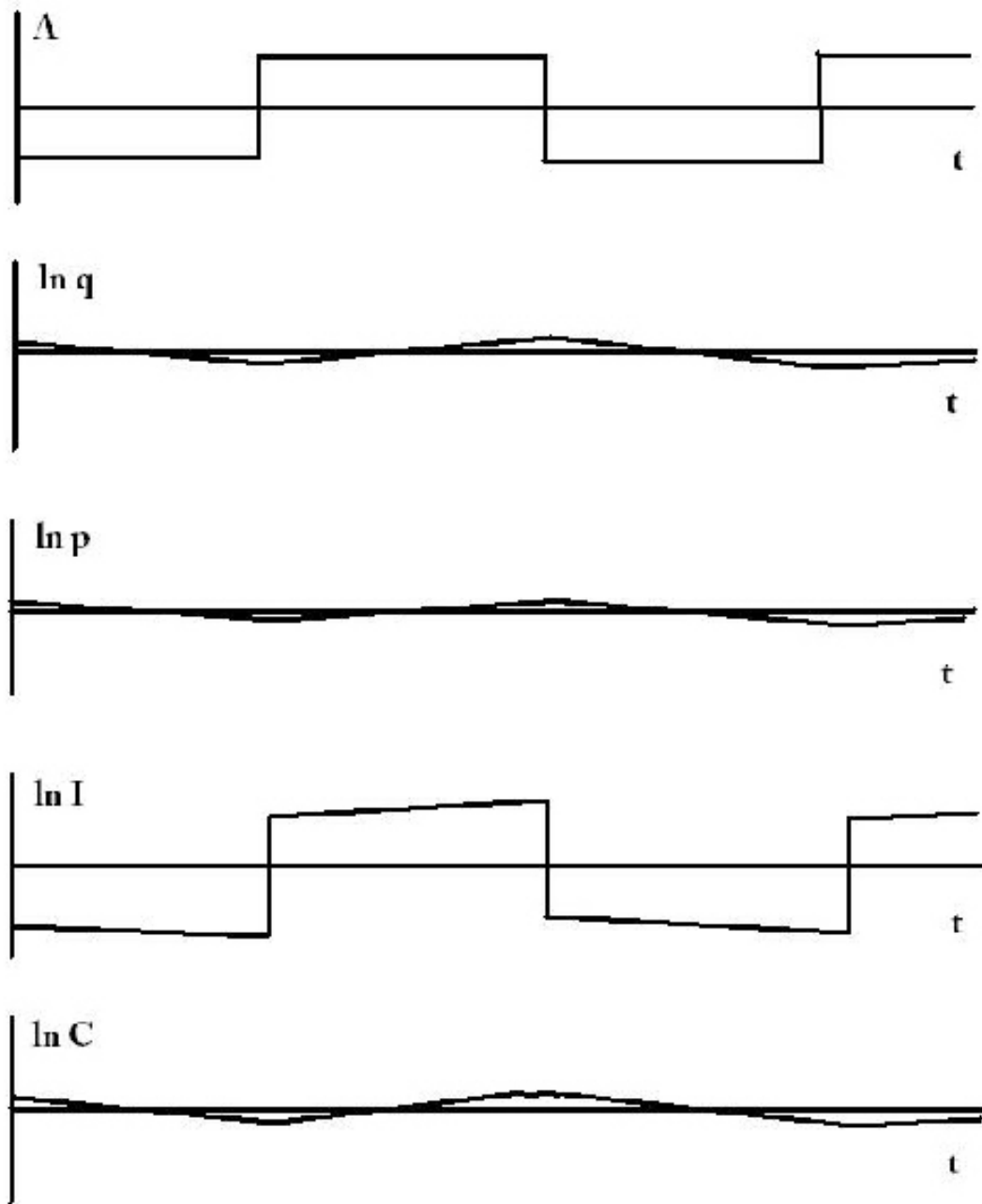
$$p_t \Delta M_t = - q_t \Delta N_t > 0. \quad (34)$$

If the size of the open market operation is such that investment and out-

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<sup>20</sup>If there were lump-sum transfer of money to the entrepreneurs (helicopter drop), then the aggregate quantities do not change in our economy because the prices are flexible. The consumption and investment of the individual entrepreneur, however, is affected by the helicopter drop, because there is redistribution from rich to poor entrepreneurs through lump-sum money transfer and inflation.

**Figure 3: Open Market Operation against Productivity Shifts**



put increase by equal amount with the productivity improvement, then the consumption is perfectly smooth at the time of deterministic productivity shift. In other words, the central bank can use the open market operation to accommodate the productivity shifts in order to smooth consumption. See *Figure 3*.

When the resaleability of equity fluctuates stochastically and the productivity is unchanged as in *Figure 2*, then the central bank can use the open market operation in order to offset the effect of the liquidity shock. When the resaleability of equity falls with an arrival of the liquidity shock, the central bank can do the open market purchase operation to increase the liquidity of the investing entrepreneurs as in (34). Then the quantities and asset prices will be insulated from the liquidity shock.<sup>21</sup> See *Figure 4*.

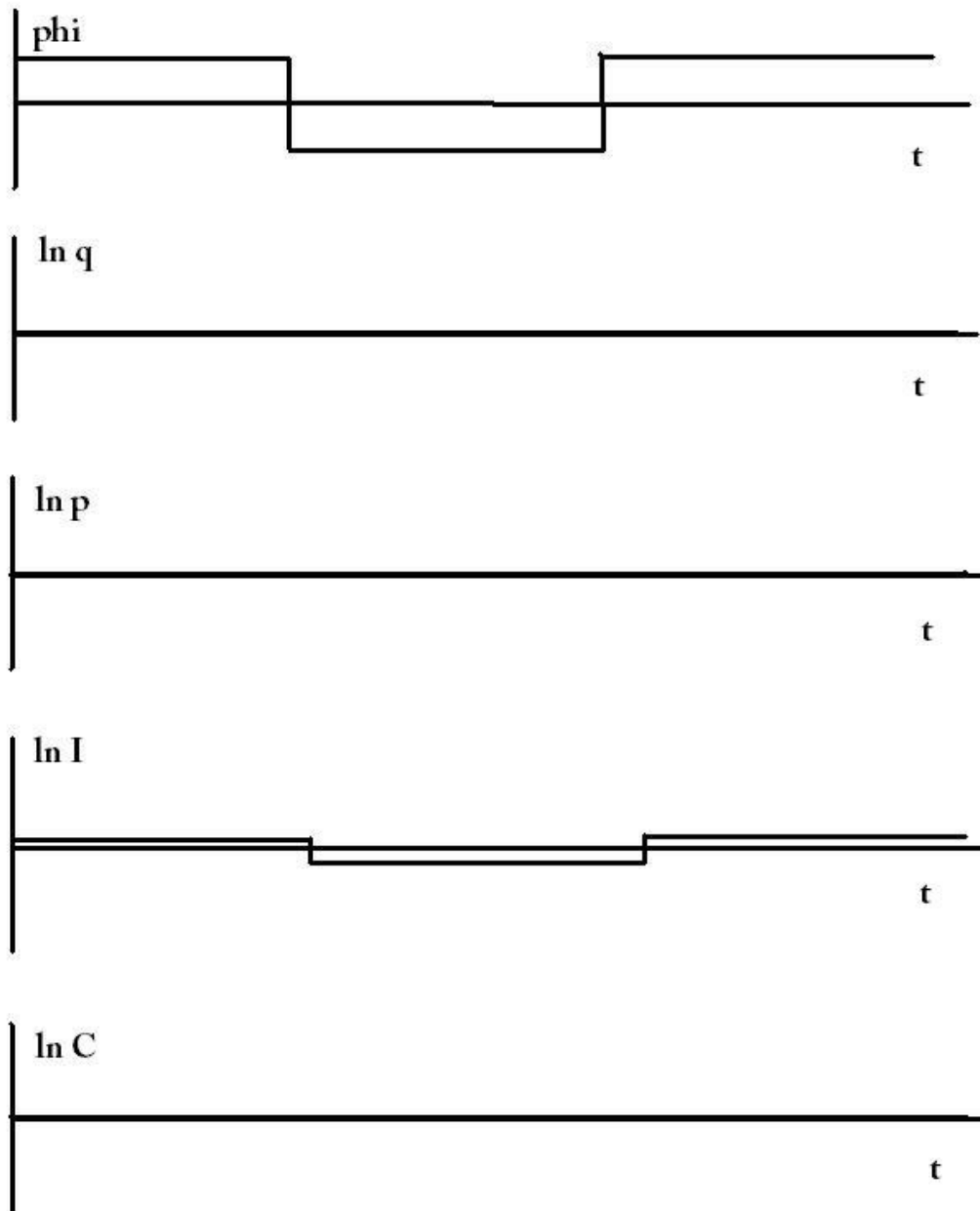
Here the open market operation must be to purchase the asset which has partial resaleability and a substantial liquidity premium. If the liquidity premium of the short-term government bond is very low (as in Japan from the late 1990s to the 2000s), then the traditional open market operation only changes the composition of broad money and has limited effects. The recent unorthodox policy of the Federal Reserve Bank and the Bank of England, such as Term Security Lending Facility, is an attempt of increasing the liquidity by supplying more treasury bills against partially resaleable securities, such as mortgage backed securities. Perhaps one of the main reasons that the Fed lends treasury bills instead of selling treasury bills in exchange of mortgage backed securities is that the Fed concerns about the adverse selection problem in the mortgage backed securities that the current holders may have better information about the quality. (See footnote 4 and the reference within).

In order to analyze the effect of the open market operation over the business cycles, we assume that the government and central bank can pre-commit to conduct a particular policy. If we were to extend our analysis to explain monetary and fiscal policy in the long-run, including policy to increase the return to money holders (such as Friedman's rule or paying a higher interest on broad money), then perhaps we have to explain why government may be able to commit more than private agents. Also we have to take into account how government enforces people to pay tax and how the

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<sup>21</sup>This is true only approximately. After open market purchase of equity, the government earns extra dividend income, which is spent on government purchase of general output in (29). Thus investment decreases a little to clear the goods market .

Figure 4: Open Market Operation against Liquidity Shocks



enforcement of taxation may crowd out people's pledgeable future returns to the other people.<sup>22</sup>

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<sup>22</sup>A related question would be: If government has a superior power to enforce people to pay, why not government directly finances people's liquidity needs?

## 5 Reference

Aruoba, Boragan, Christopher Waller, and Randall Wright. 2007. "Money and Capital: A Quantitative Analysis." working paper. University of Pennsylvania.

Atkeson, Andrew, and Patrick Kehoe. 2008. "On the Need for New Approach to Analyzing Monetary Policy." Forthcoming in *2008 NBER Macroeconomic Annual*. Cambridge, MA: MIT Press.

Barksy, Robert, and Jeffrey Miron. 1989. "The Seasonal Cycle and the Business Cycle." *Journal of Political Economy*. 97: 503-535.

Bewley, Truman. 1980. "The Optimum Quantity of Money." In John H. Kareken and Neil Wallace (eds.) *Models of Monetary Economics*, Minneapolis, Federal Reserve Bank of Minneapolis: 169-210.

Brunnermeier, Markus. and Lasse Pedersen. 2007. "Market Liquidity and Funding Liquidity." Forthcoming in *Review of Financial Studies*.

Clower, Robert W. 1967. "A Reconsideration of the Microfoundations of Monetary Theory." *Western Economic Journal*. 6: 1-8.

Duffie, Darrel, Nicolae Garleanu, and Lasse Heje Pedersen. 2005. "Over-the-Counter Markets." *Econometrica*, 73: 1815-1847

Hart, Oliver, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital." *Quarterly Journal of Economics*, 109: 841-879.

He, Ping, Lixin Huang, and Randall Wright. 2005. "Money and Banking in Search Equilibrium." *International Economic Review*. 46-2: 637-670.

\_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_. 2008. "Money, Banking and Monetary Policy." Forthcoming in *Journal of Monetary Economics*.

Holmstrom, Bengt. and Jean Tirole. 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy*. 106: 1-40.

\_\_\_\_\_, and \_\_\_\_\_. 2001. "LAPM: A Liquidity-Based Asset Pricing Model." *Journal of Finance*. 56: 1837-1867.

Keynes, John Maynard. 1936. *The General Theory of Employment, Interest, and Money*. London, Macmillan.

Kiyotaki, Nobuhiro, and John Moore. 2000. "Inside Money and Liquidity." mimeo. London School of Economics. Revised (2003).

\_\_\_\_\_ and \_\_\_\_\_. 2005a. "Liquidity and Asset Prices." *International Economic Review*. 46: 317-349.

\_\_\_\_\_ and \_\_\_\_\_. 2005b. "Financial Deepening." *Journal of European Economic Association: Papers and Proceedings*. 3: 701-713.

- \_\_\_\_\_ and Randall Wright. 1989. "On Money as a Medium of Exchange." *Journal of Political Economy*, 97, 927-954.
- Lagos, Ricardo and \_\_\_\_\_. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*. 113:463-484.
- Lucas, Robert E. and Nancy L. Stokey. 1987. "Money and Interest in a Cash-in-Advance Economy." *Econometrica*, 55: 491-513.
- Luttmer, Erzo. 1996. "Asset Pricing in Economies with Frictions." *Econometrica*. 64: 1439-1467.
- \_\_\_\_\_. 1999. "What Level of Fixed Costs Can Reconcile Consumption and Stock Returns?" *Journal of Political Economy*. 7: 969-1033.
- Metzler, Loyd A. 1951. "Wealth, Saving, and the Rate of Interest." *Journal of Political Economy*. 59: 93-116.
- Miron, Jeffrey. 1996. *The Economics of Seasonal Cycles*. Cambridge, MA: MIT Press.
- Samuelson, Paul A. 1958. "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money", *Journal of Political Economy*, 66, 467-482.
- Tobin, James. 1969. "A General Equilibrium Approach to Monetary Theory." *Journal of Money, Credit, and Banking*, 1: 15-29.
- Townsend, Robert. 1987. "Asset Return Anomalies in a Monetary Economy." *Journal of Economic Theory*, 41(5): 219-247.
- Woodford, Michael. (2003). *Interest and Prices*. Princeton, NJ: Princeton University Press.