Pricing the Upside Potential to Downside Risk*

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Abstract

Shopping centers represent a rare example wherein prices reflect the internalization of externalities. The relatively lower rent anchors pay which other tenants subsidize proxies for positive externalities anchors create. A related proxy we theoretically model and empirically analyze are co-tenancy lease provisions which capture the cost of negative externalities triggered when an anchor leaves. This real option provides temporary rent relief and early lease termination. We show this option price increases (decreases) with base rent (rent abatement, lease term, bond price, and default time). We also show this option enhances property value if favorable market conditions prevail.

Keywords: Real Estate, Credit Risk, Arbitrage Pricing, Financial Stability

JEL Codes: G12, G13, G28, R32, R33

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Prominent stores who draw customers to a shopping center and who attract other stores to locate in that center are known as anchor tenants. Prior research shows that consistent with the Coase Theorem (Coase, 1960), center owners internalize the positive externalities[1] (spillovers) that anchor stores generate offering these stores lower rent at the expense (higher rent premiums) of non-anchor stores who subsidize this rent discount. When an anchor tenant departs from the center, negative externalities arise. The departure of an anchor not only diminishes economies of agglomeration (e.g., decreases a center's drawing power) but also reduces the sales productivity of the remaining center tenants which in turn could lead to further store closures.[2] When negative externalities exist, the allocation of rent according to the Coase theorem is negotiated based on the externality cost that non-anchors incur. The co-tenancy lease provision that is triggered when an anchor leaves the center captures this externality cost that non-anchor stores incur. This co-tenancy real option provides insurance to non-anchor stores which reduces their rent temporarily until a suitable replacement anchor is found and provides early lease termination options if it takes too long to find a replacement anchor. Tenants insured by this provision are called co-tenants. While academics and practitioners recognize the importance of this real option especially with respect to bankruptcy (for example, see Benmelech et al. (2018) and Bernstein et al. (2019)), no study has evaluated and priced this co-tenancy option either theoretically or empirically. This is the purpose of our paper.

Ex-ante, it is unclear whether the inclusion of this embedded real option as a lease provision is detrimental to the value of the shopping center. On the one hand, the presence of co-tenants encumbers the center's cash flows since the landlord must reduce rent for those with co-tenancy options if an anchor departs (i.e., downside risk). On the other hand, the landlord can set rents higher when the anchor remains in the center (e.g., option is

[1] Benjamin et al. (1992); Gatzlaff et al. (1994); Wheaton (2000); Konishi and Sandfort (2003); Liu and Liu (2013); Zhou and Clapp (2015); Benmelech et al. (2018); Kuiper et al. (2021) are some of the studies which examine spillover effects.

[2] Benmelech et al. (2018) show the stores in the same shopping center are more likely to close after a shutdown of an anchor tenant. Consistent with this evidence, Shoag and Veuger (2018) show shoppers reduce their visits to nearby stores after an anchor store closes.

out-of-the-money) to more than compensate for any rent shortfalls if the co-tenancy option is triggered such that the owner receives higher cash flows overall. Intuitively, the weighted average rent from giving a tenant this lease provision exceeds the rent a tenant pays in the absence of a co-tenancy clause. Stated differently, the landlord treats a co-tenancy lease provision as a profit center (e.g., upside potential) wherein the owner profits from the difference between the insurance premium and payout.

To analyze the co-tenancy real option, we first construct a theoretical pricing model that initially consists of one anchor tenant and one co-tenant based on the arbitrage pricing theory (Jarrow, 2021). Within this framework, we view a co-tenancy option as insurance on a coupon-bearing bond. We develop three variations of the co-tenancy option starting from the most restrictive case (e.g., only rent abatement) to the most general case (e.g., rent relief and exit option). Ex-ante, we show that the option price increases with base rent and decreases with rent abatement, lease term, bond price, and default time. To test whether our theoretical price drivers are valid, we perform 40 Monte Carlo simulations each with 100,000 iterations. We next extend our model to investigate the case of multiple anchor tenants and co-tenants and apply different default structures (e.g., Poisson conditional independence, counterparty risk). Finally, we develop a measure of systemic risk in shopping centers called *locally systemically important merchant* (L-SIM), which is defined as a reference tenant to two or more co-tenants.[3]

To empirically investigate the impact of the co-tenancy real option on a center's expected sales price, we use hedonic and logistic regressions. Our cross-sectional data consists of hand collected information from offering memorandums of 236 U.S. shopping centers including but not limited to sold and list prices, presence of co-tenants, grocery and shadow anchor tenants, total gross leasable area landlord-owned and anchor-owned, parking spaces owned, year built, occupancy rate, walkscore, and building quality. Each shopping center's

[3]As an extension of our model, we develop a static shopping center trading game of incomplete information based on the Akerlof (1970) lemon problem. We analyze a Bayes Nash equilibrium in which a shopping center with co-tenancy is traded at a premium and provide a theoretical basis for potential market inefficiency. See Appendix.

memorandum is a snapshot of these variables at the time the memorandum was written. Of the 236 centers, 115 centers have sales and list price information wherein 31 centers have co-tenancy, and 84 centers are without co-tenancy. We use the data from these 115 centers to estimate our empirical model. The remaining 121 centers (out-of-sample) are used to test the performance of our estimated equation.

We first perform a hedonic regression to investigate the impact of co-tenancy options on its *expected* sales price. We partition the presence of co-tenants into two categories: (i) a center that goes from having zero to a positive number of co-tenants and (ii) a center that keeps adding co-tenants. In the former case, we construct a dummy variable that indicates whether a shopping center has a co-tenant or not. In the latter case, we count the number of co-tenants in each center as an ordinal variable. The economic intuition for having two measures of co-tenancy is that a shopping center that has no (i.e., zero) co-tenant is intrinsically different from a center that has multiple co-tenants in the way their respective landlords view co-tenancy as insurance (e.g., risk versus a profit-center). We find that a center's expected sales price *decreases* by $1.3 million when a center transitions from having no (zero) to some co-tenants. In contrast, when a center that already has a co-tenant adds an additional co-tenant, its expected sales price *increases* by $5.4 million. We argue that these contrasting results in part explain the co-tenancy puzzle; a center's expected sales price correctly adjusts downward when it shifts from having zero to some co-tenants. However, for the centers that already have co-tenants, they might use co-tenancy options as profit centers to magnify their cash flows and bid the sales prices up.

We next estimate a logistic regression on the sold-list price ratio to examine how the odds of selling a shopping center for more than it was listed changes when co-tenancy options are added. We find that the odds of the expected sales price exceeding its list price increases 6.33 times higher when switching from having no co-tenants to having some co-tenants in a shopping center. The odds increase 15 times higher for each co-tenant added to a shopping center. In addition to the presence of co-tenancy the odds of selling a shopping center for more than its listed offering price are enhanced the higher the walkscore (i.e., better location) and the higher the building quality. In contrast, the presence of a grocery anchor and building age work against the odds. Finally, we analyze the predictive margin for each additional co-tenant and its impact on the odds. On average, we find a 33% higher likelihood exists that a center sells for a higher price than its list price for the first co-tenant added. However, the incremental contribution to the odds becomes smaller each additional co-tenant added to a shopping center.

As an enhancement to our hedonic and logistic regressions, we construct a simulation model for an unlevered acquisition of a 225,000 square feet shopping center. The center consists of one anchor tenant, one inline tenant, and one co-tenant. In our base case, the co-tenancy provision permits a 50% reduction in rent and allows for an immediate exit option after a 12-month grace period. The reference anchor tenant in this scenario carries some risk since it has a below investment grade (BBB) credit rating. On average, when the co-tenant leaves, it takes six months to find a new occupant for their space. Under our base scenario, the co-tenancy premium is calculated at $3.17 per square foot per year, representing a 10.56% increase in rent compared to the absence of co-tenancy provisions. We examine how this premium varies based on two factors: the probability of the anchor tenant experiencing a credit event (indicating their riskiness) and if default occurs, the parameters affecting the loss. Following an anchor tenant's failure, both endogenous factors influence losses, such as the co-tenant's ability to exit the center, the extent of rent reduction (rent abatement), and the length of the grace period during which the co-tenant receives the reduced rent. Exogenous market conditions, specifically the duration of co-tenant vacancy after their departure, also contribute to losses. As the cumulative default probability of an anchor tenant decreases from 1% to 0.1% (indicating a transition from risky to safe), the co-tenancy premium declines from 10.56% to 0.37%. Similarly, removing the option for the co-tenant to exit the center results in a 3.23% decrease in the co-tenancy premium (from 10.56% to 7.33%). Compared to a scenario with no rent reduction, a co-tenancy provision that provides for a 75% rent-reduction increases the premium from 2.13% to 16.97%. The length of the grace period significantly influences the pricing of co-tenancy provisions, although its impact is contingent on market conditions. In a weak market, longer grace periods provide a safety net for landlords, ensuring some rental income instead of extended vacancy. In contrast, in a strong market, shorter grace periods enable landlords to quickly re-lease the co-tenant space at full rent. In a weak (strong) market, co-tenancy rent premiums range from 27.86% (0%) to 10.21% (8.42%) when the grace period is set at one and twelve months, respectively.

These simulation results suggest that co-tenancy provisions can potentially enhance a property's value in certain situations, particularly if market conditions improve during the holding period. A landlord with an optimistic outlook on the market should offer co-tenancy provisions with short grace periods to capitalize on significant rent premiums in a weak market, while benefiting from stronger cash flows as market conditions improve without a substantial increase in risk. Conversely, landlords anticipating a decline in market conditions should favor longer grace periods, even in a strong market, as the rent premiums can offset losses during worsening market conditions. In an efficient market, grace periods can serve as a signaling mechanism, providing insight into the landlord and co-tenant's expectations for future market conditions.

Our study makes three contributions to the literature. First, our study contributes to bridging a larger literature concerning derivative-based pricing and valuation of contingencies embedded in real estate lease contracts. While the extant literature has examined various applications of real options in the context of real estate (Grenadier, 1996; Ott, 2002; ., 2009; Clapp et al., 2009, 2013), the highly non-standard nature of these options makes their valuations challenging. To the best of our knowledge, our paper is the first study to apply the derivative-pricing approach to non-standard real estate options wherein it provides a closed-form solution to price the co-tenancy option. Our model has two major strengths. First, under an arbitrage pricing framework, our pricing is a fair valuation (Melnikov, 2012; Feng, 2018; Jarrow, 2021). Second, it is readily generalizable to a set of contingency clauses in a commercial real estate lease. Although a large economics literature studied agglomeration economies and externalities in retail space (Wolinsky, 1983; Pashigian and Gould, 1998; Agrawal and Cockburn, 2003; Gould et al., 2005; Liu et al., 2018; Rosenthal and Strange, 2020; Liu et al., 2020; Kuiper et al., 2021), the valuation of these externalities and associated contingent claims has not been extensively examined. For example, a shopping center lease typically includes contingencies such as escalation, exclusive use, going dark, and going dim to which our fair pricing technology can be applied. Therefore, our model can price not only the co-tenancy option but also more complex and highly non-standard real estate contingent claims. This paper's approach provides a starting point for future research on developing novel pricing techniques at the intersection of insurance economics, real estate, finance, and option pricing.

Second, our model empirically measures the externality cost that non-anchors incur as reflected in the co-tenancy lease provision when negative externalities arise due to the departure of anchor tenant and shows what impact this externality cost exerts on a shopping center's sales price. A large body of literature focused on the differential rental rates between anchor tenants and smaller stores based on spatial proximity and sales incentives (Benjamin et al., 1992; Gatzlaff et al., 1994; Pashigian and Gould, 1998; Wheaton, 2000; Konishi and Sandfort, 2003; Gould et al., 2005). However, little research exists on the impact of various contingent claims on a shopping center's price. Stated differently, existing lease valuation models[4] exclude most embedded options except for overage rent (e.g., additional rent based on a percentage of sales beyond a threshold level of sales and/or renewal options). To the best of our knowledge, our paper is the first to empirically measure not only the monetary impact of a co-tenancy real option on a center's sales price but also provide a probabilistic assessment on the odds of selling a center for more than its offering price. Our empirical methodology provides a starting point to statistically measure and

[4]See, for example, Glascock et al. (1990); Benjamin et al. (1992); Wheaton and Torto (1994); Webb and Fisher (1996); Mooradian and Yang (2000)

test the financial impact of different contingent claims in leases on the expected sales price and profitability.

Finally, our model calls into question whether real estate markets are efficient with respect to shopping center transactions (Harsanyi, 1967; Akerlof, 1970; Rothschild and Stiglitz, 1976; Stiglitz and Weiss, 1981). A rational shopping center buyer should perceive the presence of co-tenancy options as a negative factor to a center's future cash flows. An anchor-driven credit event can trigger multiple co-tenancy options, and this credit risk should adjust a center's expected sales price and profitability downward. However, it is conceivable that a prospective buyer lacks information or technology to value a co-tenancy option correctly. Our empirical and simulation results are consistent with the view that market inefficiency is present in shopping center transactions. We show that adding a co-tenant positively influences a shopping center's expected sales price. Furthermore, we find that adding a co-tenant contributes to increasing the odds of selling the center for more than an asking price. However, our co-tenancy pricing algorithm in our simulation model correctly risk-adjusts the expected returns with and without co-tenancy shopping centers. These contrasting results further provide further evidence of the potential market inefficiency present in the retail real estate market where co-tenancy real options are the norm for larger centers.

An outline of our paper is as follows. Section 1 discusses why co-tenancy matters. Section 2 provides a theoretical framework for pricing the co-tenancy option based on the the arbitrage-free valuation framework. Section 3 presents the paper's empirical strategy and results. Section 4 discusses the input and output from our co-tenancy pricing algorithm using a simulation model. Section 5 concludes.

1 Why Co-Tenancy Matters

Co-tenancy clauses represents additional risk to a landlord since they could cause a domino (multiplier) effect (e.g., a mass exodus of tenants, and/or significantly lower rent income). According to Tom Mullaney of JLL, "in a typical anchor co-tenancy clause, if one or more identified anchors goes out of business, rents immediately reset, often down to 50 percent of what they currently are, typically for about a year".[5] The severity of the domino effect that arises from co-tenancy clauses is echoed in a Forbes[6] article that quotes CREModels[7], a real estate consultant in St. Petersburg, Florida

> "One of the best tools to have at your fingertips is an abbreviated co-tenancy and kick-out performance matrix, which allows you to quickly see how these effects can compound in a worst-case scenario. We recently looked at a property with three anchor tenants, which was originally over 95% occupied. If two of those anchors were to leave, the property could lose half of its inline tenants due to the effect of co-tenancy and occupancy requirements."

Given the potential ripple effect, some landlords have taken drastic measures to prevent retailers from triggering their co-tenancy clause. For example, shopping center landlords Simon Property Group and Brookfield Property Partners partnered to purchase J.C. Penney in 2020 which is a major anchor for their centers. The reason given for the purchase is that "if J.C. Penney continues to close stores, these landlords could have to deal with other retailers at their properties invoking their co-tenancy clauses."[8] The deal postpones the challenge of finding new anchors for their centers. More recently in April 2022, these landlords pooled their resources to bid $8.6 billion for Kohl's, a J.C. Penney rival.[9] Previously, they purchased

[5] Wolf, Liz. "As Retail Anchors Go under, Landlords Offer Rent Concessions in Exchange for Other Tenants' Co-Tenancy Clauses." Wealth Management, 14 July 2021.

[6] Harris, Mike. "Store Closings Put Co-Tenancy Clauses in the Hot Seat." Forbes Magazine, 4 Apr. 2018.

[7] https://www.cremodels.com/

[8] Wolf, Liz. "As Retail Anchors Go under, Landlords Offer Rent Concessions in Exchange for Other Tenants' Co-Tenancy Clauses." Wealth Management, 14 July 2021.

[9] Howland, Daphne. "J.C. Penney Owners Simon and Brookfield Prepared to Buy Kohl's for $8.6B, NY Post Reports." Retail Dive, 26 Apr. 2022.

Aeropostale in 2016 and Forever 21 in 2020 out of bankruptcy.

Lenders are also attuned to co-tenancy provisions and include this as part of their risk assessment since these provisions substantially reduce the landlord's rental revenue on the departure or bankruptcy of an anchor tenant. For example, in BB-UBS Trust 2012-TFT, Commercial Mortgage Pass-Through Series 2012-TFT certificates a $567,752,000 transaction secured by three fixed-rate, first-lien mortgage loans on three super-regional shopping malls - Tucson Mall in Tucson, Arizona, Fashion Place in Murray, Utah and Town East Mall in Mesquite, Texas. Morningstar rating agency pre-sale report notes that "the majority of the tenants at each property have co-tenancy provisions that are based on a minimum number of anchor tenants located at the related property or a minimum level of occupancy at the related property. Several of the mall shop tenants also have termination options that are based on gross sales performance, and several of the in-line tenants have some form of co-tenancy clause that is tied to occupancy thresholds and/or the closing of one of more of the anchor tenants at the related property. Co-tenancy provisions are a common feature of many retail leases at larger shopping malls."

2 Theoretical Framework: Pricing Co-tenancy

In this section, we provide the rationale for using the arbitrage-free valuation method in pricing the co-tenancy provisions (Jarrow, 2021). This is also referred to as the derivative-based pricing in insurance contracts, and we apply this framework to our real option pricing model. We construct a model of co-tenancy pricing in a simple environment that consists of a landlord, an anchor, and a co-tenant. We then extend this model to incorporate multiple anchors and inline tenants.

2.1 Set-up

2.1.1 Introduction to Co-tenancy

There are two types of premia associated with the co-tenancy option: (i) a proximity premium and (ii) a co-tenancy insurance premium. The proximity premium is the additional fee an inline tenant pays to a landlord for being close to an anchor store and exposed to positive spillover effects. The insurance premium is the amount an inline tenant pays to a landlord to protect itself against an anchor-driven credit event. We assume the proximity and insurance premia are unobserved and are included in the contract rent:

$$\text{Contract Rent} = \text{Base Rent} + \text{Proximity Premium} + \text{Insurance Premium} \tag{1}$$

This study's objective is to price the co-tenancy insurance premium, however neither the proximity nor insurance premium are directly observable. First, a finalized rent undergoes several individualized and closed-door negotiations between a shopping center owner and tenant. Second, even when a shopping center owner lists its property for sale, an offering memorandum typically does not decompose a contract rent into the proximity and insurance premia; a typical rent roll section simply displays a set of inline tenants with associated cash flows. These ambiguities call for a critical examination of the co-tenancy clause and its fair pricing.

Table 1: Sample Co-tenancy Clause for Inline Tenant in a Shopping center. Once an anchor tenant triggers a credit event (e.g., exiting the center, ceasing operation), then an inline tenant with a co-tenancy insurance has several options such as terminating a lease or remaining in the center until the grace period ends.

Co-tenancy Clause (Anchor: X, Inline: Y)
If X or any similar substitute is not open for business, Y can terminate the current rental lease or pay 50 percent of base rent for 12 months. If an acceptable tenant has not opened, Y can either terminate the current lease or return to paying the full base rent.

Table 1 provides a sample co-tenancy clause. Since inline tenants pay both base rent and proximity premium, these tenants want to hedge against an anchor-driven credit event such as cease-of-operation and bankruptcy. For example, if an anchor tenant suddenly ceases its operation, then neighboring inline tenants can suffer from a significant loss in foot traffic and sales. Once the anchor tenant ceases operation or defaults, the provision grants the landlord and inline tenant a grace period to respond to the credit event. The co-tenancy clause gives the inline tenant the option to abate the base rent for the foreseeable future or terminate the lease if the landlord cannot find a suitable replacement (e.g., a tenant that has equivalent drawing power to the departing anchor tenant) during the adjustment period. If the inline tenant decides to ultimately remain at the property, the contract stipulates that it resumes paying the full base rent after the grace period. In effect, the co-tenancy clause *insures* the inline tenant against the adverse consequences (e.g., a decline in foot traffic and sales) of the anchor-driven credit events.

2.1.2 Shopping Center Rental Lease: A Coupon-bearing Bond

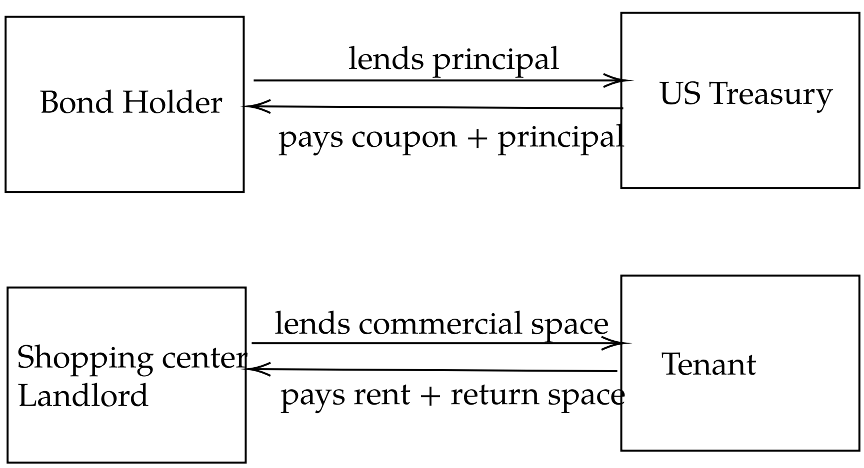

Figure 1: A Shopping Center Rental Lease is a Coupon-bearing Bond. It illustrates that a shopping center lease is a type of coupon-bearing bond.

In a frictionless and competitive market setting, an operating lease contract is equivalent to a coupon-bearing bond. Indeed, the U.S. Treasury is a debtor owing the principal and interest to its bondholders. Similarly, a shopping center tenant is a borrower of the value of the space (i.e., the bond's principal); the tenant borrows the space to open her store and make sales returning the store space to the center owner at the end of her lease term. The landlord is a creditor and receives rent (i.e., the bond's coupon).

2.1.3 Co-tenancy: Insurance on the Coupon-bearing Bond

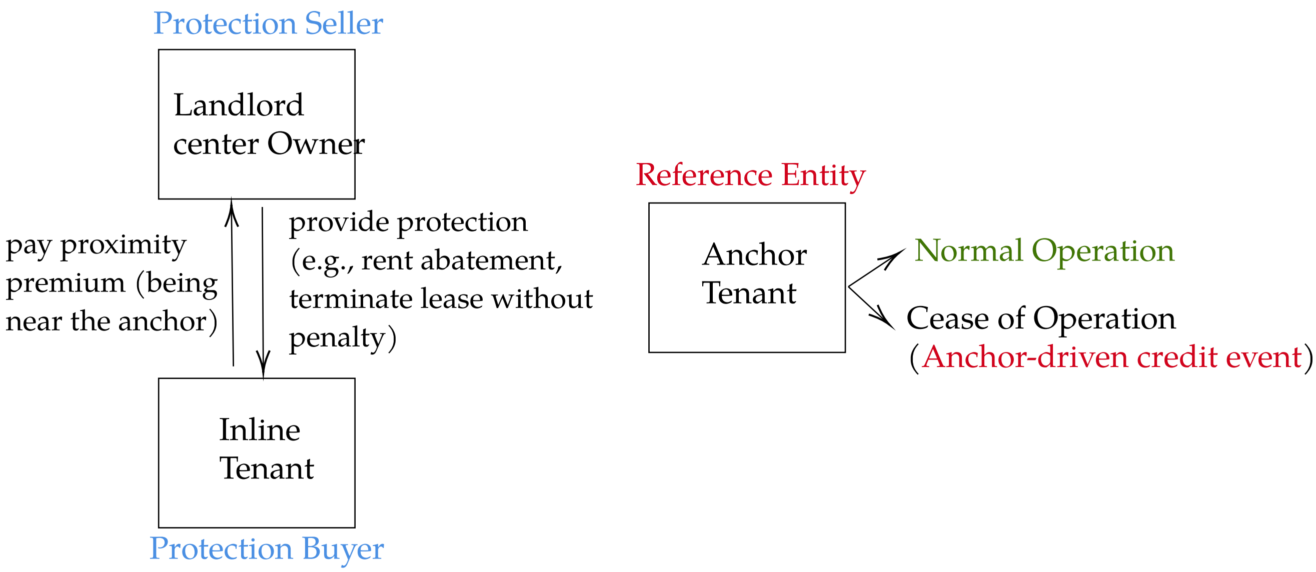

Figure 2: Co-tenancy Provision: An Insurance on the Rental Lease. It shows the co-tenancy provision is an insurance on a rental lease. The reference entity is the anchor tenant, and the protection seller and buyer are landlord and inline tenant respectively.

A tenant might be interested in buying insurance on her operating lease, because it is locationally close to a major anchor tenant that generates significant foot traffic. If the anchor tenant abruptly leaves the shopping center, then it can adversely impact the neighboring tenant's sales. Therefore, the tenants can resort to adding an insurance clause in their lease contracts to minimize their exposure to the anchor-driven credit risk. These neighboring smaller tenants are called inline tenants or co-tenants.[10] This insurance is known as the co-tenancy provision. In this study, the terms co-tenancy insurance, provision, clause, and option are used interchangeably. In the provision, there are three parties: protection seller (landlord), protection buyer (inline tenant), and reference entity (anchor tenant).

[10]Note buying an insurance is an option not an obligation. Although a tenant might be extremely close to a major anchor tenant, it might choose to not buy the insurance.

2.1.4 The Arbitrage Free Valuation

Since lease terms are finite, we assume a continuous time model on a finite horizon $[0, T^*]$. We characterize the uncertainty in the model with a complete filtered probability space $(\Omega, \mathscr{F}, \mathcal{F} = (\mathscr{F}_t)_{t\in[0,T^*]}, \mathbb{P})$ satisfying the usual hypotheses where Ω is the state space, $\mathscr{F}$ is a σ-algebra of events, $\mathcal{F} = (\mathscr{F}_t)_{t\in[0,T^*]}$ is an information filtration, and $\mathbb{P}$ is the statistical probability measure.

Consider two continuously traded objects, a default-free money market account (mma) and default-free zero-coupon bonds.[11] They are traded in a frictionless (e.g., no transaction costs or trading constraints) and competitive market (e.g., traders are price takers). We denote the time t price of a zero-coupon bond maturing at T paying a sure dollar as $B(t, T)$, and r_t is the default-free spot rate of interest.

The market is arbitrage-free. By the First and Third Fundamental Theorem of Asset Pricing, there exists an equivalent probability measure $\mathcal{Q}$ with respect to $\mathbb{P}$ such that

$$B(t, \mathbb{T})e^{-\int_0^t r_u du} \tag{2}$$

is a martingale for all $\mathbb{T} \in [0, T^*]$.[12] The equivalent probability measure $\mathcal{Q}$ is referred to as a risk-neutral probability measure, and it is equivalent to $\mathbb{P}$ since they agree on zero probability events in $\mathcal{F}$. In order to characterize the anchor-driven credit event, we assume $\tau \in [0, T]$ is a stopping time with respect to the information filtration where $T < T^*$ and T is the final date of an inline tenant's lease term. The stopping time τ represents the first instance when the anchor tenant triggers a credit event such as default or ceasing-of-operation (e.g., store-wide closure).

Finally, the strength of the reduced-form approach for valuing the co-tenancy option pertains to its ability to characterize a probability distribution via an intensity process; it

[11]These assumptions can be relaxed and the following results still apply in an extended fashion; see Jarrow (2023).

[12]For the formal characterizations of arbitrage-free market (i.e., No Free Lunch with Vanishing Risk) and the fundamental theorems, see Jarrow (2018).

encompasses a behavioral finance notion. Contrary to the structure approach, the reduced-form does not require the knowledge of the intensity of the co-tenancy insurance process.[13]

2.2 One Anchor & One Co-tenant

In this section, we begin our analysis with the simple case of one landlord, one anchor tenant, and a co-tenancy provision with an inline tenant. First, we introduce the key elements and notations for an inline tenant's lease contract and co-tenancy option. We then investigate three scenarios based on the different types of the grace period stipulated in the co-tenancy clause. For each case, we provide the appropriate analytic expression for the co-tenancy premium.

2.2.1 Environment

Consider a shopping center with one anchor tenant and one inline tenant. The inline tenant's lease is over a finite horizon $[0, T]$.[14] It pays the base rent B to the landlord starting at t_1 and until the final lease period t_m.

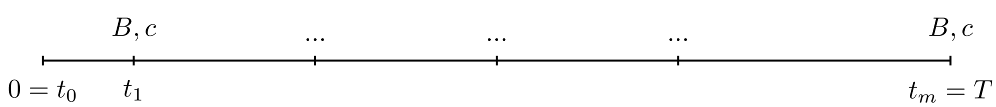

Figure 3: A Rent and Co-tenancy Insurance Payment Schedule. A typical rent and co-tenancy insurance schedule comprises paying both the base rent B and insurance premium c over the entire lease term.

Suppose the inline tenant buys a co-tenancy insurance on her lease. In the co-tenancy option, the protection buyer is the inline tenant, the protection seller is the shopping center owner, and the reference entity is the anchor tenant that is located near the inline tenant. Under the co-tenancy provision, the inline tenant is obligated to pay an insurance premium

[13] See Jarrow et al. (2010) for a more explicit comparison between the structural and reduced-form approach to valuing callable corporate bonds. The structural approach assumes that a firm calls the bond to minimize the market value of the particular bond. In the context of co-tenancy option, it is analogous to a shopping center owner choosing some form of a stopping time to minimize the payment or market value associated with the option, which is a simplifying assumption.

[14] Using a time subscript $m \in \mathbb{N}$, $[0, T] = [0, t_m]$.

(c) regularly along with her rent.[15] Figure 3 illustrates the payment schedule of an inline tenant's rent and co-tenancy premium. If the anchor tenant triggers a credit event (e.g., ceasing operation or vacating the premise), then the landlord reduces the inline tenant's base rent to δB where $\delta \in (0, 1)$. In effect, the landlord subsidizes the rent in the amount of $(1 - \delta)B$.

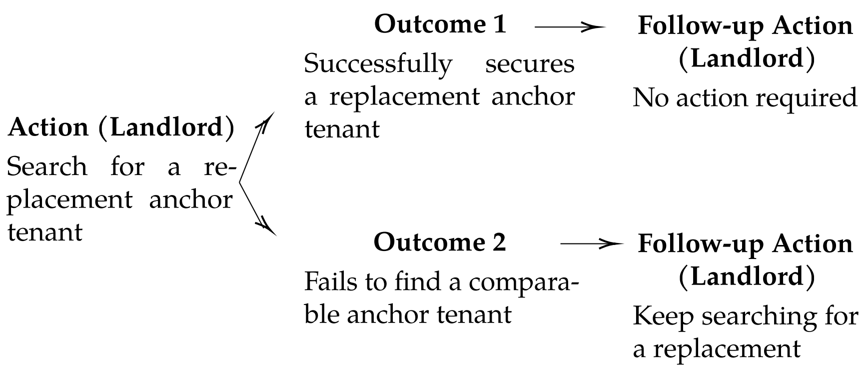

Figure 4: A Landlord's Action, Potential Outcomes, and Follow-up Actions. Once an anchor-driven credit event occurs, a landlord attempts to find a replacement anchor tenant during a grace period. If the landlord successfully finds a comparable anchor, then the rent schedule is restored and no follow-up action is required. Otherwise, the landlord continues searching.

Although a landlord subsidizes rent of a co-tenant during an anchor-driven credit event, it typically makes a profit overall given the probability of such event is relatively small during normal times. An important element of the co-tenancy option is the grace period. When an anchor-driven credit event occurs, the landlord is given a grace period to find a comparable replacement. The typical grace period ranges from three to six months. If the landlord successfully secures a replacement anchor tenant during a grace period, then the inline tenant resumes paying the full base rent B. In effect, a successful search for the replacement anchor tenant *deactivates*[16] a co-tenancy provision. If the landlord fails to find

[15] The timing of the base rent (B) and insurance premium (c) payments can be adjusted on a case-by-case basis. For example, if a shopping center owner requires both payments due before each month's new leasing start date, then the time index subtracts one period from the current set-up. A rent roll consists of a list of the property's current tenants and information about their rents and addendum clauses (e.g., co-tenancy, go-dark). In a typical offering memorandum of a shopping center that is listed for a sale, a rent roll reveals only the base rent. Here, we explicitly separate the base rent from the co-tenancy premium.

[16] An anchor-driven credit event *triggers* or *activates* a co-tenancy provision. A grace period provides the last chance for a landlord to successfully secure a replacement anchor tenant, and it can *deactivate* the provision.

a comparable key tenant by the end of a grace period, the inline tenant typically has two options: (i) remain at the shopping center and resume paying the full base rent or (ii) terminate the rental lease and exit the center.[17]

In general, there are three variations in how the grace period and subsequent payments are stipulated in a co-tenancy option. The first option is most restrictive, and the latter options give more choices to an inline tenant. In the first option, if landlord fails to find a comparable anchor tenant after the grace period, then an inline tenant resumes to paying her full base rent. In the second option, the landlord's failure to find a replacement anchor tenant provides the inline tenant with an option of not only remaining in the center but also exiting the shopping center without a penalty. Finally, the most comprehensive option gives an inline tenant a choice to enter a grace period or not. If the inline tenant enters a grace period, the second option ensues. Otherwise, the inline tenant can simply exit the premise without any penalty.

As we apply the standard insurance economics to tenant leasing, a landlord offers a co-tenancy option to all tenants at a fair price. The tenants either buy the insurance or not. Since the anchor's positive spillover effects are internalized into rent, those tenant who are closer to an anchor store would be most interested in purchasing the insurance. If a tenant is distant from an anchor store, its proximity premium internalized into rent will be lower; there is less incentive to purchase the insurance.

2.2.2 Option 1: Reverting to Paying Full Base Rent

The anchor-driven credit event is denoted as τ, and it can occur in an interval such that $\tau \in (t_{h-1}, t_h]$ for $h \in \{1, ..., m\}$. The default of an anchor tenant activates a grace period (e.g., 180 days) denoted as $g \in \mathbb{N}$. The landlord (insurer) pays the inline tenant (insured) $(1-\delta)B$ during a specified grace period.

[17] In general, $c \leq (1-\delta)B$ implying the co-tenancy insurance premium is weakly less than the payout. A shopping center owner (i.e., insurer) holds more cash than it has to payout per co-tenancy contract it signs. If two or more anchor tenants depart, it can trigger a cascade of co-tenancy provisions straining a landlord's ability to stay afloat.

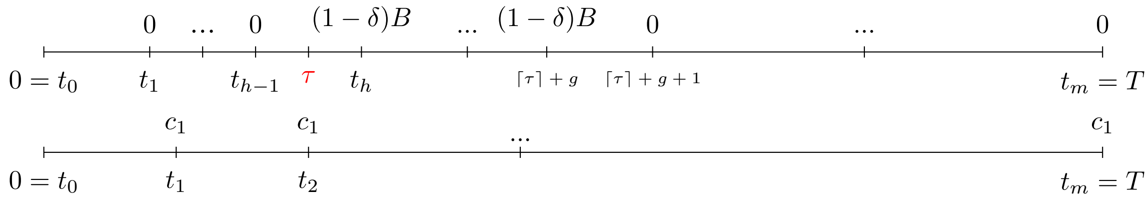

Figure 5: Insurance Payout (top) & Premium (bottom) Schedule for Option 1

We denote the first time the landlord successfully finds a replacement anchor tenant as $\xi \in (t_{j-1}, t_j]$ for $j \in \{\lceil \tau \rceil, ..., m\}$.[18] Hence, Option 1 incorporates two stopping times: τ and ξ. If the landlord finds a replacement anchor tenant, then the inline tenant pays the full base rent. If the landlord fails to find another anchor tenant and once the grace period is over, the inline tenant is required to resume paying the full rent to the landlord.[19]

The Valuation Formula for Option 1

$$V(0) = 0 = \mathbb{E}_{\mathcal{Q}}\left[\sum_{k=1}^{m} c_1 e^{-\int_0^{t_k} r_u du} - \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} (1-\delta)B\mathbb{1}_{\{t_0<\tau\le t_m=T\}} e^{-\int_0^{t_h} r_u du}\right]$$

$$c_1 = \frac{(1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum\limits_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\le t_m=T\}} e^{-\int_0^{t_h} r_u du}\right]}{\sum\limits_{k=1}^{m} B(0,t_k)} \tag{3}$$

The time 0 value of the insurance contract is zero under the arbitrage-free condition (i.e., $V(0) = 0$).[20] (3) is the analytic expression for the option 1 co-tenancy insurance premium c_1.

[18] Note the landlord might find a replacement anchor tenant immediately following a credit event which implies $\xi \in (t_{\lceil\tau\rceil-1}, t_{\lceil\tau\rceil}]$. It might be unable to find one until the inline tenant's original rental lease ends meaning $\xi \in (t_{m-1}, t_m]$.

[19] Note that $\lceil\tau\rceil, \lfloor\tau\rfloor \in \mathbb{N}$ denote rounding up and down of the stopping time τ respectively to its nearest integer. Since $\tau \in (t_{h-1}, t_h]$ for $h \in \{1, ..., m\}$, rounding to its nearest integer accurately accounts for the insurance payments over the discrete time interval.

[20] Note $\lceil\xi\rceil \wedge (\lceil\tau\rceil + g) \equiv \min\{\lceil\xi\rceil, \lceil\tau\rceil + g\}$. Under Option 1, the co-tenancy insurance payout stops either after the landlord finds a comparable anchor tenant (i.e., $\lceil\xi\rceil$) or after the grace period ends (i.e., $\lceil\tau\rceil + g$).

2.2.3 Option 2: Exiting the Shopping Center

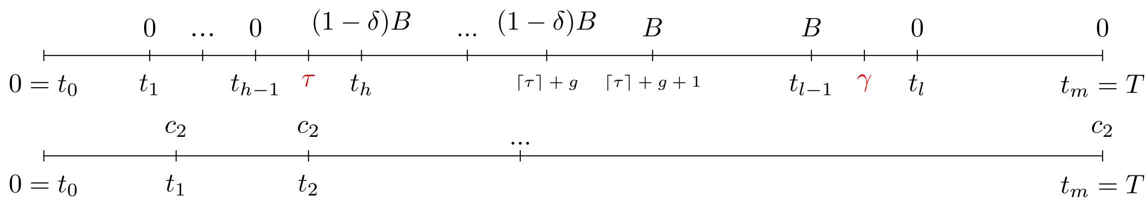

Figure 6: Insurance Payout (top) & Premium (bottom) Schedule for Option 2. If the inline tenant decides to leave, the landlord faces another search-and-replace risk once a grace period terminates.

In Option 1, regardless of the landlord's success in securing a replacement anchor tenant, the inline tenant has no choice but to resume paying its full base rent (B) once a grace period ends. Option 2 enhances the protection buyer's choice set by enabling the inline tenant to exit the shopping center if the landlord fails to find a comparable anchor tenant. In other words, Option 2 is Option 1 with the added choice of a premature lease termination. For this reason, the price of co-tenancy Option 2 should be at least as valuable as that of Option 1.

To evaluate the price of co-tenancy Option 2, we formalize three objects: (i) an indicator function representing the success or failure of the landlord's search for a replacement anchor tenant, (ii) an indicator function representing the inline tenant's stay-or-leave decision, and (iii) a distinct stopping time (γ) representing the moment at which the landlord secures a replacement inline tenant once the previous one leaves. First, let $S_L = \lceil\xi\rceil \wedge (\lceil\tau\rceil + g) \equiv \min\{\lceil\xi\rceil, \lceil\tau\rceil + g\}$.[21] Then, an indicator function representing the landlord's search for a replacement anchor can be defined as:

$$\mathbb{1}(S_L) := \begin{cases} 1 & \text{if } S_L = \lceil\tau\rceil + g, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose $\mathbb{1}(S_L) = 1$. It implies the grace period ended without the landlord finding

[21] If $S_L = \lceil\tau\rceil + g$, this implies the landlord has failed to fine a replacement anchor and the associated grace period has ended.

a replacement anchor tenant. The inline tenant then decides either to stay or leave the shopping center. Define S_I as the inline tenant's choice set to leave or remain in the shopping center.[22] The indicator function representing the inline tenant's stay-or-leave decision can be defined as:

$$\mathbb{1}(S_I) := \begin{cases} 1 & \text{if } S_I = \text{"leave after a grace period"}, \\ 0 & \text{otherwise.} \end{cases}$$

To describe the possible outcomes following a grace period, the two indicator functions are multiplied: $\mathbb{1}(S_L)\mathbb{1}(S_I)$.

Table 2: The Payout Matrix for the Landlord.

Table 3: The Option 2 Co-tenancy Insurance Payout Matrix for the Landlord. If the landlord fails to find a replacement anchor by the end of a grace period (i.e., $\mathbb{1}(S_L) = 1$) and the inline tenant exits (i.e., $\mathbb{1}(S_I) = 1$), the landlord must pay the base rent B until it finds a replacement inline tenant.

	$\mathbb{1}(S_L) = 1$	$\mathbb{1}(S_L) = 0$
$\mathbb{1}(S_I) = 1$	$-B$	0
$\mathbb{1}(S_I) = 0$	0	0

Finally, γ denotes a distinct and independent stopping time (from τ) representing the successful replacement lease singing of a new inline tenant where $\gamma \in (t_{l-1}, t_l]$ for $l \in \{\lceil\tau\rceil + g + 1, ..., m\}$.

The Valuation Formula for Option 2 The time 0 value of the insurance contract is zero under the arbitrage-free condition (i.e., $V(0) = 0$):

[22] In other words, $S_I :=$ {"leave after a grace period", "remain in the center"}

$$V(0) = 0 = \mathbb{E}_{\mathcal{Q}}\left[\sum_{k=1}^{m} c_2 e^{-\int_0^{t_k} r_u du} - \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} (1-\delta)B\mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du} - \sum_{l=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor} B\mathbb{1}_{S_L}\mathbb{1}_{S_I} e^{-\int_0^{t_l} r_u du}\right]$$

$$c_2 = \frac{(1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right] + \mathbb{E}_{\mathcal{Q}}B\left[\sum_{l=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor} \mathbb{1}_{S_L}\mathbb{1}_{S_I} e^{-\int_0^{t_l} r_u du}\right]}{\sum_{k=1}^{m} B(0,t_k)}$$

The valuation formula of c_2 for the Option 2 co-tenancy insurance premium. Since Option 2 expands the choice set available in Option 1, it is at least as valuable as Option 1.

Lemma 1 The Option 2 co-tenancy insurance is at least as valuable as the Option 1.

2.2.4 Option 3: Immediate Exit Option

Consider the most comprehensive and general co-tenancy clause form, Option 3 in our simple set-up. An example of the Option 3 co-tenancy clause states:

Table 4: An Example of the Most General Co-tenancy Clause in the Simple Set-up.

Co-tenancy Clause (Anchor: X, Inline: Y)
If X or any similar substitute is not open for business, Y can terminate her lease or pay 50 percent of base rent for 12 months. If an acceptable tenant has not opened even after the grace period ends, Y can either terminate her lease or return to paying the full base rent.

The distinct feature of Option 3 is that *the inline tenant has a choice to not enter a grace period* when the credit event occurs; it can simply terminate the lease obligation and exit the shopping center. If the inline tenant chooses to enter the grace period, then the co-tenancy follows the trajectory of the previously studied Option 2. Hence, Option 3 has the first exit option with the embedded Option 2. This establishes the most general co-tenancy in the simple model of one landlord, one anchor tenant, and one inline tenant; it provides the most protection to the insured inline tenant who opts to have a co-tenancy clause.

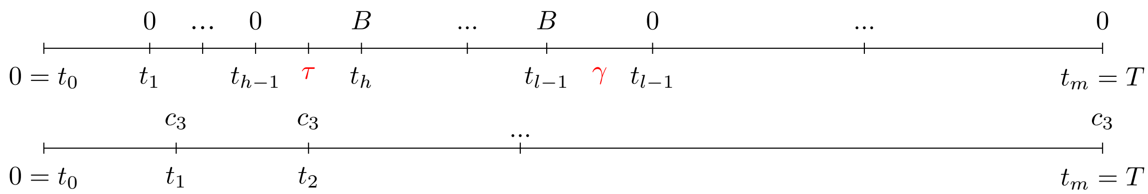

Figure 7: Insurance Payout (top) & Premium (bottom) Schedule for Option 3. If an inline tenant decides to leave immediately an anchor tenant ceases operation, then a grace period become irrelevant. The landlord must try to find a replacement inline tenant, because the base rent costs B in each period. If the inline tenant decides to stay, then the scenario follows Option 2.

In Option 2, we defined an indicator function $\mathbb{1}(S_I)$ to represent an inline tenant's decision to leave the shopping center after a grace period ends. Since Option 3 adds an exit choice before a grace period starts, we define a new choice function S_P.[23] Then, the associated indicator function $\mathbb{1}(S_P)$ equals 1 when the inline tenant decides to exit the center prior to a grace period. The landlord is responsible for paying the departed inline tenant's base rent B until it secures a replacement inline tenant.

$$\mathbb{1}(S_P) := \begin{cases} 1 & \text{if } S_P = \text{"leave before a grace period"}, \\ 0 & \text{otherwise.} \end{cases}$$

The Valuation Formula for Option 3

The time 0 value of the insurance contract is zero under the arbitrage-free condition (i.e., $V(0) = 0$):

$$V(0) = 0 = \mathbb{E}_{\mathbb{Q}} \left\{ \sum_{k=1}^{m} c_3 e^{-\int_0^{t_k} r_u du} - \mathbb{1}_{S_P} \underbrace{\sum_{j=\lceil\tau\rceil}^{\lfloor\gamma\rfloor} B e^{-\int_0^{t_j} r_u du}}_{(1)} - (1 - \mathbb{1}_{S_P}) \underbrace{\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} (1-\delta) B \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du} + \sum_{l=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor} B \mathbb{1}_{S_L} \mathbb{1}_{S_I} e^{-\int_0^{t_l} r_u du} \right]}_{(2)} \right\}$$

$$c_3 = \frac{B \mathbb{1}_{S_P} \mathbb{E}_{\mathbb{Q}} \left[\sum_{j=\lceil\tau\rceil}^{\lfloor\gamma\rfloor} e^{-\int_0^{t_j} r_u du} \right] + (1 - \mathbb{1}_{S_P}) \mathbb{E}_{\mathbb{Q}} \left[(1-\delta) B \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du} + B \sum_{l=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor} \mathbb{1}_{S_L} \mathbb{1}_{S_I} e^{-\int_0^{t_l} r_u du} \right]}{\sum_{k=1}^{m} B(0, t_k)}$$

The expression in (1) represents the sum of the base rent the landlord must pay out of its pocket once the inline tenant exits the center before a grace period begins. If the inline

[23]Similar to S_I, $S_P \coloneqq$ {"leave before a grace period", "remain in the center"}

tenant decides to remain in the shopping center, then $\mathbb{1}(S_P) = 0$ and Option 3 follows the cash flow structure of Option 2, which leads to the following lemma.

Lemma 2 The Option 3 co-tenancy insurance is at least as valuable as the Option 2.

Combining Lemma 1 and Lemma 2 yields the following proposition.

Proposition 1 Suppose there exist a landlord, an anchor tenant, and an inline tenant in a shopping center. The Option 3 co-tenancy is at least as valuable as the Option 2 co-tenancy. Similarly, the Option 2 is at least as valuable as the Option 1 co-tenancy.

2.2.5 Interpretation

The value of a co-tenancy option can be interpreted as the present value of an annuity or the net present value of a risky project. We provide the economic intuition using Option 1; the interpretation easily extends to Option 2 and Option 3 as their prices increase with respect to the number of exit choices they embed.

Present Value of Annuity

By rearranging (3), we obtain:

$$(1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}}e^{-\int_0^{t_h} r_u du}\right] = c_1\sum_{k=1}^{m} B(0,t_k) \tag{4}$$

The left-hand side of (4) is the sum of the stochastic payments (i.e., insurance payout); it is the expected protection cost incurred by the landlord. The right-hand side is the present value of an annuity paying a constant coupon c per period over the lease term discounted by the spot rate of interest, $r(t)$, under the risk-neutral measure $\mathcal{Q}$. Hence, it conforms to the economic intuition that under the risk neutral measure, the present value of the expected protection cost equals the discounted sum of the co-tenancy premium.

Net Present Value of Risky Project

Using the time 0 price of a zero-coupon bond paying a sure dollar at t_k and $y(0,t_k)$ as one plus its internal rate of return, the zero-coupon bond's price at time 0 is

$$B(0,t_k) = \frac{1}{y(0,t_k)^{t_k}} \tag{5}$$

Summing over m periods yields

$$\sum_{k=1}^{m} B(0,t_k) = \sum_{k=1}^{m} \frac{1}{y(0,t_k)^{t_k}} \tag{6}$$

Applying this convention to Option 1, we re-write (3) as:

$$\sum_{k=1}^{m} \frac{c_1}{y(0,t_k)^{t_k}} = (1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right] \tag{7}$$

This equation is an alternate expression of the annuity representation in (4) using the internal rate of return. Re-arranging (7) yields the Net Present Value (NPV) interpretation of the co-tenancy option:

$$0 = -\sum_{k=1}^{m} \frac{c_1}{y(0,t_k)^{t_k}} + (1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right] \tag{8}$$

One can interpret the co-tenancy provision as an insurance on a risky project.[24] For an inline tenant, the act of purchasing a co-tenancy option is an insurance, because it does not know whether an anchor-driven credit event will occur or not. In order to buy this insurance, an inline tenant pays the initial outlay, which is the present value of the stream of the insurance premium c over the lease term. The first-term in (8) is a lump-sum initial investment required (e.g., sweat equity) for the risky project to take off. The second term represents the stochastic cash flows generated by the risky project contingent on an

[24]The set-up is similar to a farmer considering to buy a crop insurance. Buying a crop insurance is an act of investing into a risky project.

anchor-driven credit event. The underlying intuition is that an insured benefits from the purchased co-tenancy option when it partially subsidizes the base rent, which depends on the random time at which an anchor-driven credit event occurs.

2.2.6 Sensitivity Analysis

We are interested in how the price of a co-tenancy provision changes with respect to the key parameters based on the analytic expression in (3): rent abatement degree (δ), base rent (B), rental lease term (T), a set of zero-coupon bond prices $(B(0,t_k))$, and the stopping times (τ, ξ, γ). We find the following proposition.[25]

Proposition 2 Consider the simple model of one anchor and one co-tenant. Suppose the co-tenancy premium is c and it is continuously differentiable with respect to the degree of rent abatement (δ), base rent (B), rental lease term (T), and a set of zero-coupon bond prices $(B(0,t_k))$. The co-tenancy premium increases with base rent and with the support of the distribution for the first time a landlord finds a replacement anchor shifting to the right by one time interval. The co-tenancy premium decreases with rent abatement, lease term, bond price, and with the support of the distribution for the first time an anchor-driven credit event occurs shifting to the right by one time interval.

Degree of Rent Abatement

δ represents the degree of rent abatement when an anchor tenant defaults or ceases operation. The closer δ is to 1, the less rent abatement the in-line tenant receives. In this case, the co-tenancy option does not offer as much protection against an anchor-driven credit event; the intuition suggests the value should decrease accordingly. Suppose δ increases by a small amount. Since the sum of the random payouts provided by the landlord decreases, this takes some protection power away from the co-tenancy provision. To match the lower

[25]Note for an increase in lease term, the first time an anchor-driven event occurs, and the first time a landlord successfully secures a replacement anchor tenant imply these events are, in the context of time, *delayed* by one period.

level of protection, the co-tenancy premium also decreases.

Base Rent

B is the base rent the in-line tenants pay during a normal rental lease term. As the base rent increases, the landlord has to provide a stronger degree of protection. In effect, the landlord is selling a more expensive protection. The co-tenancy premium incorporates this additional cost, hence it increases.

Lease Term

A lease ends at time T. Note $T = t_m$ since our set-up is based on premiums and rents being paid over the equi-spaced time intervals: $\{0 = t_0, \ldots, t_m = T\}$. Consider a unit increase in the lease term, t_{m+1}. There exist two channels in which a small increase in the term impacts the co-tenancy price: (i) the stochastic payout in the numerator and (ii) the sum of the zero-coupon prices in the denominator. The extension of the lease term elongates the period subject to the discounting under the risk-neutral measure; the sum of the zero-coupon prices in the denominator increases. However, the numerator remains unchanged since the first time an anchor-driven credit event occurs is either in $(t_0, t_m]$ or $(t_m, t_{m+1}]$; the numerator is not impacted by a longer lease term.

The stochastic cash flows are generated once an anchor-driven credit event occurs, and it continues until a given grace period ends (i.e., $[\lceil\tau\rceil, \lceil\tau\rceil + g])$. Extending the lease term does not impact this random payment window as the terminal date of the lease is strictly larger than the grace period window (i.e., $\lceil\tau\rceil + g < T$). Hence, extending the lease horizon forces the insurance premium to be discounted by one more period under the risk-neutral measure while the payment remains unchanged. Therefore, the price of the co-tenancy insurance premium decreases with respect to a marginal increase in the lease term.

Zero-coupon Bond Price

The underlying intuition is that an increase in a given zero-coupon bond price provides steeper discounting under the risk-neutral measure. The co-tenancy premium decreases with respect to the zero-coupon bond price.

First Anchor-driven Credit Event

Consider a case when there is a one-period delay in an anchor-driven credit event. Since an anchor-driven credit event occurs within a fixed lease term, this delay effectively shortens the potential horizon for a co-tenancy clause to produce stochastic cash flows. It weakens the protection of the co-tenancy provision. Hence, the co-tenancy insurance premium decreases with respect to a delay in an anchor-driven credit event.

Successful Replacement for A Departing Anchor

Suppose a shopping center owner's search for a replacement anchor tenant is postponed one period. Since a co-tenancy premium is paid until either a replacement anchor is successfully found or a grace period terminates, the postponement of the anchor replacement extends the potential window for receiving insurance money. The underlying intuition is that a delay in the search for a replacement anchor provides more time for an insured to benefit from a rent abatement. This in turn enhances the protection of a co-tenancy clause at the margin. Consequently, the co-tenancy insurance premium increases with respect to a small delay in finding a replacement anchor.

2.2.7 Cox Process with Intensity

In the reduced form credit risk models, one can specify the credit event indicator variables to follow a stochastic process with default intensity. The purpose of which is to give an analytic expression whose parameters can be estimated. Using the analytic expression of one anchor and co-tenant case, we specify that an anchor-driven credit event follows the Cox process with intensity λ_t. The intensity is a function of common state variables of the market, Γ_t, such as short-term interest rates.[26]

Proposition 3 Consider the simple model of one landlord, one anchor, and one co-tenant. Suppose the co-tenancy premium is c. Assume an anchor-driven credit event follows a Cox process $N_t \in \{0, 1, 2, ...\}$ with $N_0 = 0$ an intensity process $\lambda_t(\Gamma_t)$ where $\Gamma_t = (\Gamma_1(t), ..., \Gamma_m(t))' \in \mathbb{R}^m$ is a set of stochastic processes characterizing the state of the retail market at time t with the Γ_t-generated filtration $\mathbb{F}_t^\Gamma$ where $N_t \perp\!\!\!\perp r_t$. The stopping time τ represents the first time an anchor-driven credit event occurs. Then, the co-tenancy premium is

$$c = \frac{(1-\delta)B}{\sum_{k=1}^{m} B(0, t_k)} \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil \wedge (\lceil\tau\rceil + g)} \int_0^T \hat{\lambda}_s e^{-\int_0^s \hat{\lambda}(u)du} B(0, s+j)ds.$$

3 Data & Variables

We hand collect our data from 236 offering memorandums of U.S. shopping centers. The variables include the sold and list prices (if any), presence of co-tenants, whether the anchor is a grocery store or a shadow anchor, total gross leasable area that is landlord-owned and also anchor owned, parking spaces owned, year built, occupancy rate, walkscore, and building quality of a given center. One hundred fifteen (115) shopping centers report their sales and list price. There are 31 centers that have at least one or more co-tenancy and 115 centers that have zero co-tenancy. Our empirical model considers the 115 centers that have price information as in-sample data to construct our estimation equation. The remaining 121 centers are used to test the out-of-sample performance of our estimation equation.

Our regressands are `SOLD_PRICE` and `SLPR`, which represent a shopping center's sold price and its sold-list price ratio respectively. The former regressand is used in our hedonic regression. The latter is used in our logistic regression. To measure whether co-tenancy exists in a shopping center, we construct two variables. First, we generate `DUM_COT`, which

[26]One interprets $\lambda_t(\Gamma_t)$ as the probability of credit event over a small time (e.g., $\varepsilon > 0$) interval $[t, t+\varepsilon]$ conditional on the fact that no credit event has occurred prior to t. See Lando (1998), Jarrow and Yu (2001), and Jarrow (2009, 2018) for a class of reduced form credit risk models using the Cox process.

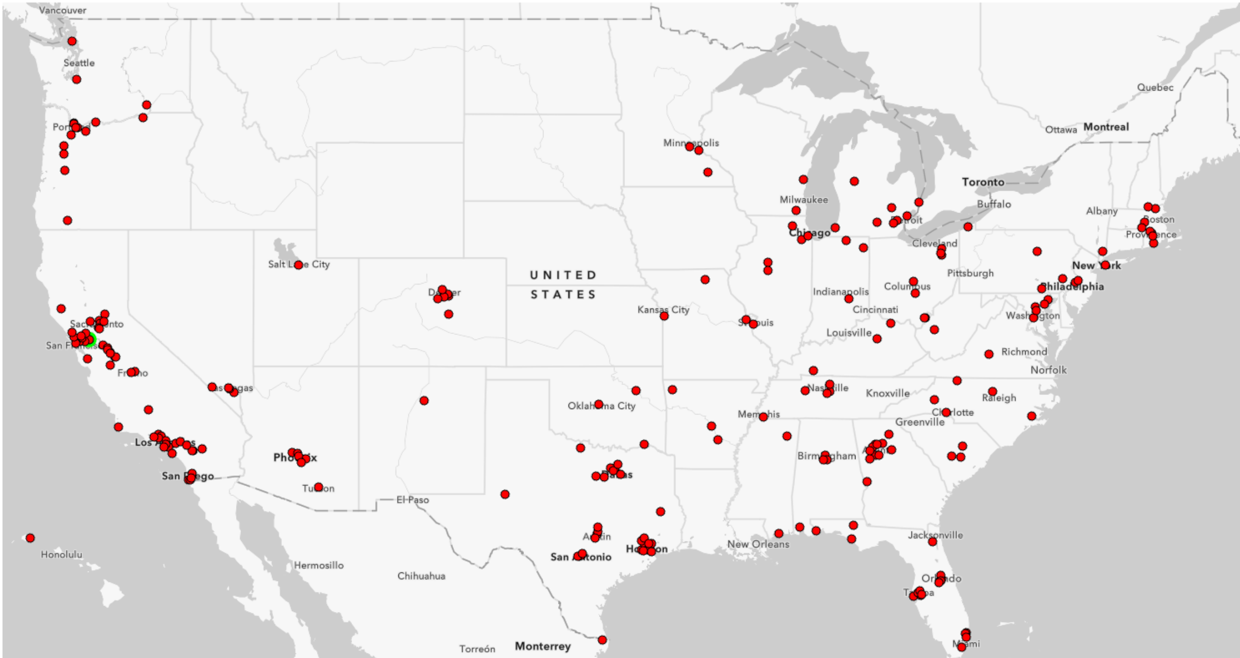

Figure 8: The Map of 238 Shopping Centers in the Data Set. *Source*: getLatLong.net & ©2022 ArcGIS Pro, Esri. All rights reserved.

equals to 1 if the center has has a positive number of co-tenants and 0 otherwise. Our sample is thus divided into two groups: zero co-centers that have no (zero) co-tenants and centers that have at least one co-tenant. This categorization allows us to measure the marginal effect of transitioning from a no co-tenancy center to a center that has one or more co-tenancy provisions. Our second constructed variable COT_NO, enumerates the number of co-tenants present in a shopping center. This enriches our information set available for the hedonic regression and allows us to measure the marginal contribution of adding a co-tenant to a center's lease portfolio.

REF_NO provides the information on the number of reference entities (e.g., an anchor tenant) in a shopping center. LSIM represents the number of times a reference entity is mentioned; if this number is greater than 2, then it indicates that an anchor tenant is referenced more than twice in distinct co-tenancy clauses and this reference tenant qualifies as a locally systemically important merchant. STORE_INBTW counts the number of stores in between the closest co-tenant and its reference entity. The in-between stores can be

Table 5: The List of Variables Used in the Hedonic and Logistic Regressions

No.	Variable	Description
Y-1	SOLD_PRICE	Sold Price
Y-2	SLPR	Sold-List Price Ratio
1-A	DUM_COT	At Least 1 Co-tenant or More
1-B	COT_NO	Number of Co-tenants
1-B	REF_NO	Number of Reference Entity
1-B	LSIM	Number of Times Reference Entity Mentioned
1-B	STORE_INBTW	Number of Stores between Reference and Co-tenant
2	DUM_GROCERY	Grocery Anchor
3	DUM_SHADOW	Shadow Anchor
4	TOT_GLA	Total Gross Leasable Area (in thousands)
5	TOT_ANC_GLA	Total Anchor Gross Leasable Area (in thousands)
6	TOT_LL_GLA	Lanlodrd Owned Gross Leasable Areas (in thousands)
7	PARKING	Parking Spaces Owned
8	YEAR	Year Built
9	OCC	Occupancy Rate
10	WALKSCORE	Walk Score
11	BLDG	Building Quality

co-tenants or not; the intuition is to capture the impact of spatial distance with respect to co-tenancy on the expected sales price. This recognizes that the impact of positive externalities that an anchor generates attenuates (decays) with distance. DUM_GROCERY and DUM_SHADOW equal to 1 if there is a grocery and shadow anchor respectively. A shadow anchor is defined as an anchor that is physicallypart of a shopping center, but the anchor owns its own space. As such, the shopping center owner has no control over its operations. When a shadow anchor departs, the center owners has no say in who will replace the anchor tenant nor how comparable the replacement tenant is in terms of drawing power which increases the risk and decreases the sal price. TOT_GLA, TOT_ANC_GLA, and TOT_LL_GLA measure the total gross leasable area of a center, the anchor-owned portion, and the portion that the landlord owns respectively. PARKING is the number of parking spaces owned by a center. YEAR and OCC provide the year built and occupancy information of a center. The WALKSCORE obtained from https://www.walkscore.com/ is a location metric that measures the ability to walk to amenities within 30 minutes or less while BLDG measures the walkability and building quality.

4 Empirical Strategy & Models

Our empirical strategy consists of three analyses to investigate the impact of co-tenancy on a shopping center's expected sales price. We first perform a hedonic regression on a shopping center's expected sales price. We then validate our estimated equation with out of sample data. Next, we run a logistic regression on the sold-list price ratio dummy and the number of co-tenants to isolate the impact of co-tenancy on the odds of selling a shopping center for more than its offering price. We extend the logistic regression results to the associated predictive margin analyses to capture the diminishing marginal odds with respect to each co-tenant added to a shopping center. Finally, we complement these results with a Monte Carlo simulation model examining an acquisition of a 225,000 square feet shopping center and price the co-tenancy premium per square foot.

4.1 Empirical Model

4.1.1 Hedonic Regression

We perform two hedonic regressions based on our measure of co-tenancy presence. Out of 236 shopping centers, we utilize 115 shopping center offering memorandums given the available information on their sold and list prices. Our first hedonic model uses the co-tenancy dummy variable, `DUM_COT`, to measure the impact of transitioning from a center that hs no co-tenancy (zero co-tenancy) in their lease provisions to a center that has one or more leases that have a co-tenancy real option on its expected price. Equation (9) provides the regression specification.

$$\begin{aligned} \texttt{SOLD\_PRICE}_i = \beta_0 &+ \beta_1 \texttt{DUM\_COT}_i + \beta_2 \texttt{DUM\_GROCERY}_i + \beta_3 \texttt{DUM\_SHADOW}_i + \beta_4 \texttt{TOT\_GLA}_i \\ &+ \beta_5 \texttt{TOT\_ANC\_GLA}_i + \beta_6 \texttt{TOT\_LL\_GLA}_i + \beta_7 \texttt{PARKING}_i \\ &+ \beta_8 \texttt{YEAR}_i + \beta_9 \texttt{OCC}_i + \beta_{10} \texttt{WALKSCORE}_i + \beta_{11} \texttt{BLDG}_i + \varepsilon_i \end{aligned} \tag{9}$$

In our second hedonic model, we replace our co-tenancy dummay variable with four related co-tenancy variables designed to capture a more granular perspective of the spatial relationship between the co-tenant and the anchor tenant. These four variables are the exact number of co-tenants (COT_NO), reference a.k.a. anchor tenants (REF_NO), the number of stores between a co-tenant and an anchor store (STORE_INBTW), and the number of times that a particular reference entity or anchor store is mentioned (LSIM). Ex-ante, we should expect any positive externalities that an anchor generates to decay with distance from the reference entity. Our second hedonic regression specification is as follows:

$$
\begin{aligned}
\texttt{SOLD\_PRICE}_i = \beta_0 + \beta_1\texttt{COT\_NO} + \beta_2\texttt{REF\_NO} + \beta_3\texttt{LSIM} + \beta_4\texttt{STORE\_INBTW} \\
\beta_5\texttt{DUM\_GROCERY}_i + \beta_6\texttt{DUM\_SHADOW}_i + \beta_7\texttt{TOT\_GLA}_i \\
+ \beta_8\texttt{TOT\_ANC\_GLA}_i + \beta_9\texttt{TOT\_LL\_GLA}_i + \beta_{10}\texttt{PARKING}_i \\
+ \beta_{11}\texttt{YEAR}_i + \beta_{12}\texttt{OCC}_i + \beta_{13}\texttt{WALKSCORE}_i + \beta_{14}\texttt{BLDG}_i + \varepsilon_i \qquad (10)
\end{aligned}
$$

4.1.2 Logistic Regression

In the second part of our empirical strategy, we perform a logistic regression on the sold-list price ratio.

$$
Y_i = \begin{cases} 1, & \text{if } \frac{P_i^{sold}}{P_i^{list}} > 1 \\ 0, & \text{otherwise} \end{cases}
$$

The binary dependent variable Y is a measure of a shopping center's sale profitability. If i^{th} shopping center's sold price is higher than its listed offering price, then Y equals 1 otherwise equals zero. Using a similar line of reasoning in our logistic regression that we used in our hedonic model, we use our first measure of co-tenancy presence, DUM_COT, in the following logistic regression specification:

$$
\begin{aligned}
log\left[\frac{p}{1-p}\right] &= \beta_1\texttt{DUM\_COT}_i + \beta_2\texttt{DUM\_GROCERY}_i + \beta_3\texttt{DUM\_SHADOW}_i + \beta_4\texttt{TOT\_GLA}_i \\
&+ \beta_5\texttt{TOT\_ANC\_GLA}_i + \beta_6\texttt{TOT\_LL\_GLA}_i + \beta_7\texttt{PARKING}_i \\
&+ \beta_8\texttt{YEAR}_i + \beta_9\texttt{OCC}_i + \beta_{10}\texttt{WALKSCORE}_i + \beta_{11}\texttt{BLDG}_i + \varepsilon_i
\end{aligned} \tag{11}
$$

The left-hand side of (11) provides the probabilistic interpretation of log odds. Each $\hat{\beta}$ represents how much the odds of selling a shopping center for more than it is listed changes when a one unit change occurs in the given co-variate. Our second logistic specification in (12) is based on (10). This specification recognizes that externalities can decay with distance from the anchor store. We additionally extend our logistic model specification to predictive margin analysis on the number of co-tenants and how changes the odds at margin.

$$
\begin{aligned}
log\left[\frac{p}{1-p}\right] &= \beta_1\texttt{COT\_NO} + \beta_2\texttt{REF\_NO} + \beta_3\texttt{LSIM} + \beta_4\texttt{STORE\_INBTW} \\
&\beta_5\texttt{DUM\_GROCERY}_i + \beta_6\texttt{DUM\_SHADOW}_i + \beta_7\texttt{TOT\_GLA}_i \\
&+ \beta_8\texttt{TOT\_ANC\_GLA}_i + \beta_9\texttt{TOT\_LL\_GLA}_i + \beta_{10}\texttt{PARKING}_i \\
&+ \beta_{11}\texttt{YEAR}_i + \beta_{12}\texttt{OCC}_i + \beta_{13}\texttt{WALKSCORE}_i + \beta_{14}\texttt{BLDG}_i + \varepsilon_i
\end{aligned} \tag{12}
$$

4.2 Results

In this section, we provide the regression results based on our hedonic pricing model as well as our logistic specification inclusive of predictive margin analyses. When a shopping center transitions from having no co-tenants to having at least one or more co-tenants, the expected sales price decreases by $1.3 million. However, for each co-tenant added to a center that already has a co-tenant, the expected sales price increases by $5.4 million.

Moreover, the center that sells for more than its asking price experiences a 1.8 times (15 times) higher increase in its sold price with the addition of each co-tenant. Finally, each co-tenant added contributes less to the odds of selling a center for more than asked (i.e., diminishing marginal returns of co-tenants on the odds).

4.2.1 Hedonic Pricing: Expected Sales Price

Table 6: Hedonic Regression Result: Expected Sold Price & List Price (in millions)

	OLS (Prices in millions)			
	(1)	(2)	(3)	(4)
	List Price	Sold Price	Sold Price	List Price
At Least 1 Co-tenant or More	-1.292	-1.334		
	(-1.01)	(-0.97)		
Number of Co-tenants			5.439	0.492
			(0.57)	(0.07)
Number of Reference Entity			-2.208	1.489
			(-0.27)	(0.26)
Number of Times Reference Entity Mentioned			-4.957	-1.390
			(-0.61)	(-0.24)
Number of Stores between Reference and Co-tenant			0.595	0.526
			(0.59)	(0.76)
Grocery Anchor	-0.345	-1.868	1.640	0.198
	(-0.24)	(-1.39)	(0.40)	(0.05)
Shadow Anchor	0.770	1.818	-0.482	-2.330
	(0.62)	(1.48)	(-0.17)	(-0.59)
Total Gross Leasable Area (in thousands)	-0.0153	-0.0262*	-0.0377	0.0114
	(-1.03)	(-1.66)	(-0.47)	(0.14)
Total Anchor Gross Leasable Area (in thousands)	0.0171	0.0382	0.0330	-0.0114
	(0.74)	(1.60)	(0.47)	(-0.16)
Lanlodrd Owned Gross Leasable Areas (in thousands)	0.152***	0.142***	0.0866	0.118*
	(6.32)	(4.33)	(1.11)	(2.09)
Parking Spaces Owned	-0.00403	-0.00537*	0.00429	0.00238
	(-1.48)	(-1.77)	(0.95)	(0.65)
Year Built	0.196***	0.144**	0.104	0.306**
	(3.62)	(2.50)	(0.64)	(2.51)
Occupancy Rate	0.157***	0.149***	0.233	0.249**
	(3.32)	(3.51)	(1.74)	(2.40)
Walk Score	0.113***	0.101***	0.0670	0.218
	(3.37)	(3.37)	(0.65)	(1.56)
Building Quality	-0.424	0.844	1.241	-1.711
	(-0.44)	(0.98)	(0.38)	(-0.64)
Observation	109	109	25	25
R^2	0.652	0.640	0.528	0.733

The columns (2) and (3) provide the main hedonic regression results on the expected sales price on 115 in-sample shopping centers using the specifications in (10) and (10) respectively. The columns (1) and (4) are the hedonic regression results on the expected list price as a part of the robustness check.

Table 6 reports our hedonic regression results. When a shopping center transforms from having no co-tenancy to having one tenant with a co-tenancy lease provisions, the center's expected sales price decreases by $1.3 million. In contrast, when a shopping center already has one co-tenant with a co-tenancy real option, adding another co-tenant increases the expected sales price by $5.4 million. What is notable is that when we focus on the offering (list) price of a shopping center and perform the same hedonic regression similar results obtain. In particular, when the center has a single co-tenant, its expected list price decreases by $1.3 million. However, each additional co-tenant added increases the expected list price by $492,000. For every thousand square feet increase in the landlord's owned gross leasable area, the expected list and sold prices increase by $152,000 and $142,000 for centers with at least one co-tenant or more. We also find that a higher occupancy rate and a higher walkscore (i.e., residents can walk to a center within half an hour or less) exert a positive influence on the expected sales price. Neither having a grocery anchor nor shadow anchor has an economically meaningful impact on the expected sales price.

4.2.2 Hedonic Pricing: Out-of-Sample Performance

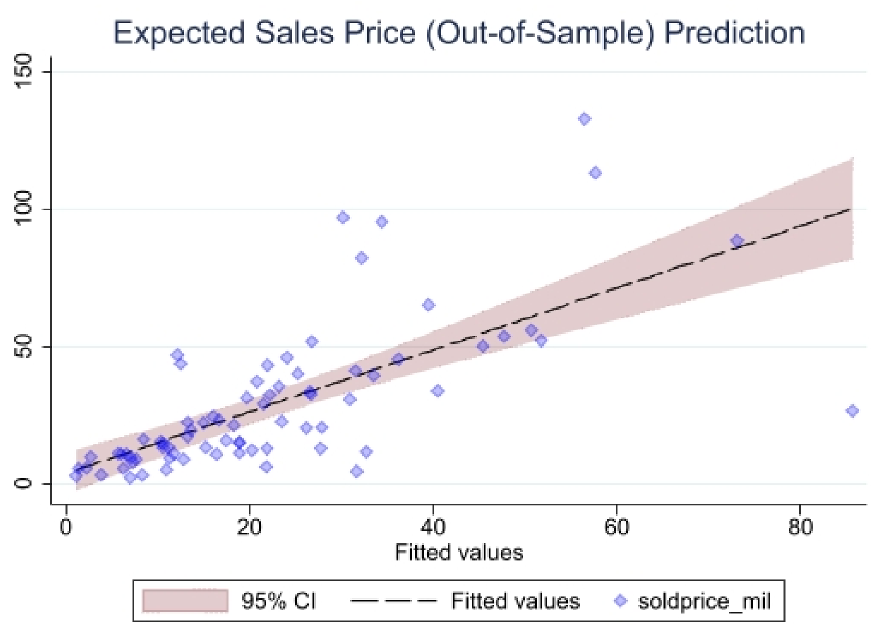

Figure 9: The Goodness of Fit of the Out-of-Sample for the Hedonic Estimate on the Expected Sales Price

We test the performance of our first hedonic regression model (i.e., no co-tenancy to at least one co-tenancy center) data for the 121 withheld out-of-sample shopping centers applied to our estimation equation in (9). The goodness of fit is strongest for the shopping centers whose sold prices are less than or equal to $40 million. Given that only few centers exist that sold for a price that exceeds $40 million, not surprisingly our model suffers a loss of predictive power.

4.2.3 Logistic Regression: Sold-List Price Ratio (SLPR)

Table 7 provides the logistic regression results. Our dependent variable is a dummy that equals to 1 if the sold-list price ratio is greater than 1 and equals to 0 otherwise. The columns (1-A) and (2-A) report the main logistic regression results on the sold-list price ratio dummy on 115 in-sample shopping centers using the specifications in (11) and (12) respectively. The corresponding columns (1-B) and (2-B) provide the odds-ratio interpretation of the corresponding log-odds coefficient estimates.

When a shopping center transitions from a center that has no co-tenants to a center that has at least one co-tenant, the odds of selling the center for more than the offering price increases 6.3 times. While adding a shadow anchor increases the odds of selling a center for a higher price than its offering price by 38.7 times, the odds decreases 95 times when a grocery anchor is added. These contrasting results support our ex-ante hypotheses that shadow anchors generate positive externalities (i.e., increasing foot traffic and potential revenue for neighborhood stores) without forcing a landlord to incur operational costs. However, grocery anchors tend to be large (e.g., less redeployable and replaceable in vacancy) and difficult to pair with other tenants (e.g., the Whole Foods and Safeway are often standalone competing parking spaces of other tenants' customers). We also find that an increase in the building quality increases the odds of selling a center for a higher price than its offering price by approximately 3 times.

Table 7: Logistic Regression: Sold-List Price Ratio (SLPR)

Logistic Regression				
	Sold-List Price Ratio			
	(1-A)	(1-B)	(2-A)	(2-B)
	Log-odds	Odds Ratio	Log-odds	Odds Ratio
At least 1 Co-tenancy	1.846**	6.34		
	(1.97)			
Number of Co-tenancy			2.74	15.48
			(0.96)	
Number of Reference Entity			-0.739	0.48
			(-0.32)	
Same Reference Entity Mentions			-1.378	0.25
			(-0.58)	
Number of Stores between Reference and Co-tenant			0.0136	1.01
			(0.12)	
Grocery Anchor	-3.096***	0.05	-3.099**	0.05
	(-2.80)		(-2.56)	
Shadow Anchor	3.655**	38.66	5.600***	270.45
	(2.45)		(2.86)	
Total Gross Leasable Area	-0.0000138	1.00	-0.0000299	1.00
	(-1.14)		(-1.45)	
Total Anchor Gross Leasable Area	0.0000138	1.00	0.0000118	1.00
	(0.90)		(0.58)	
Center Owned Gross Leasable Area	0.0000222*	1.00	0.0000426**	1.00
	(1.84)		(2.37)	
Center Owned Parking Spot	-0.00353	1.00	-0.00345	1.00
	(-1.48)		(-1.32)	
Center Built Year	-0.0614**	0.94	-0.0463	0.95
	(-2.02)		(-1.41)	
Occupancy Rate	-0.0174	0.98	0.00396	1.00
	(-0.50)		(0.09)	
Walkscore	0.0645**	1.07	0.0632**	1.07
	(2.31)		(2.09)	
Building Quality	1.211*	3.36	1.351	3.86
	(1.67)		(1.47)	
Observation	109		109	
R^2	0.3063		0.3581	

4.2.4 Logistic Regression: Predictive Margin Analysis

Co-tenancy

As shown in column (2) Table 8, the predicted probabilities are 9%, 34.8%, and 65.6% that a shopping center with one, two, and three co-tenants will sell more than listed respectively. The non-linearity is captured in column (3) where the difference in the predictive margins increases up to the third tenant added then subsequently diminishes. Ex ante, the economic intuition underlying the diminishing odds is that adding the first few co-tenants represents a good source of profit for a landlord. However, having too many co-tenants represents will erode the future cash flows given an anchor-driven credit event can trigger multiple co-tenancy options. The diminishing nature of the predictive margins evidence this.

Table 8: Predictive Margin Analysis: # of Co-tenants

(1) # of Co-tenants	(2) Margin	(3) Δ_{Margin}	(4) SE	(5) Z-score	(6) $P > z$	(7) 95% CI LB	(8) 95% CI UB
1	0.096		.023	4.13	0.000	0.050	0.141
2	0.348	.252	.308	1.13	0.259	-0.256	0.952
3	0.656	.308	.439	1.49	0.135	-0.205	1.517
4	0.839	.183	.251	3.34	0.001	0.347	1.331
5	0.927	.088	.124	7.45	0.000	0.683	1.171
6	0.966	.040	.063	15.29	0.000	0.843	1.090
7	0.990	.024	.053	18.62	0.000	0.886	1.094
8	0.999	.009	.011	90.11	0.000	0.977	1.021
9	1.000	.001	.000	1003.22	0.000	0.998	1.002
10	1.000	.000	.000	1.3e+04	0.000	1.000	1.000

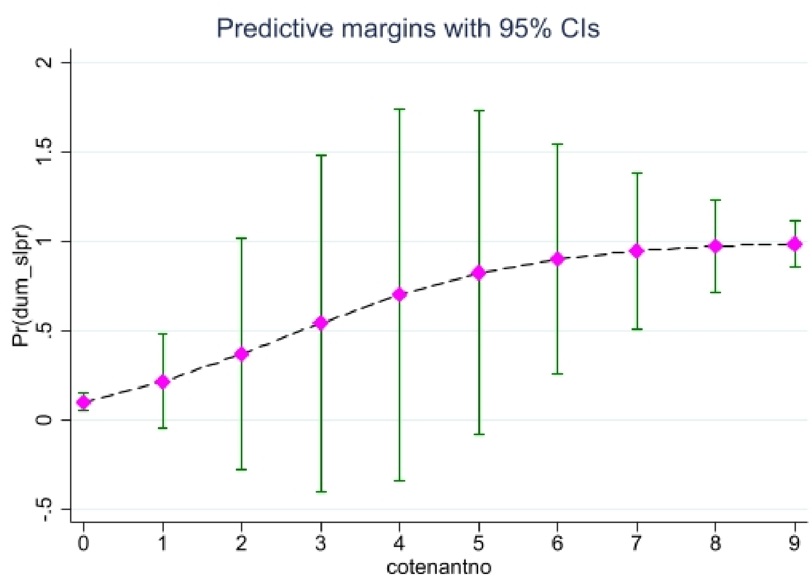

Figure 10: The Mean Predicted Probability of Selling a Center for More Than Asked

Co-tenancy by Reference Tenants (RTs) & Reference Tenant Mentions (RTMs)

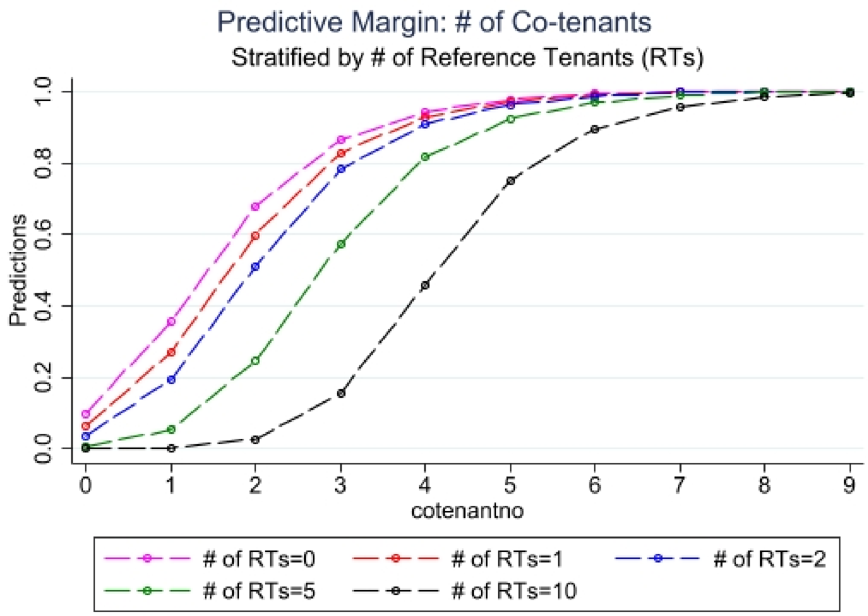

Figure 11: The Mean Predicted Probability of Selling a Center for More Than Asked

We have shown that diminishing marginal odds exist when we analyze the impact of the number of co-tenants present in a shopping center on its odds of selling more the offering price. We how that the diminishing marginal odds is present. We are interested in how this diminishing nature varies with respect to the number of reference (anchor) tenants.

We stratify the predictive margin analysis of co-tenants on the odds of selling a shopping center for more than its list by the number of reference tenants. *Ex ante*, adding more reference tenants to a shopping center is similar to adding more anchor stores to the tenant portfolio; this implies that these store are likely to be referenced in a co-tenancy lease provision. Hence, we expect the predictive margin to shift downward as we add more reference tenants. We observe this pattern in Figure 11.

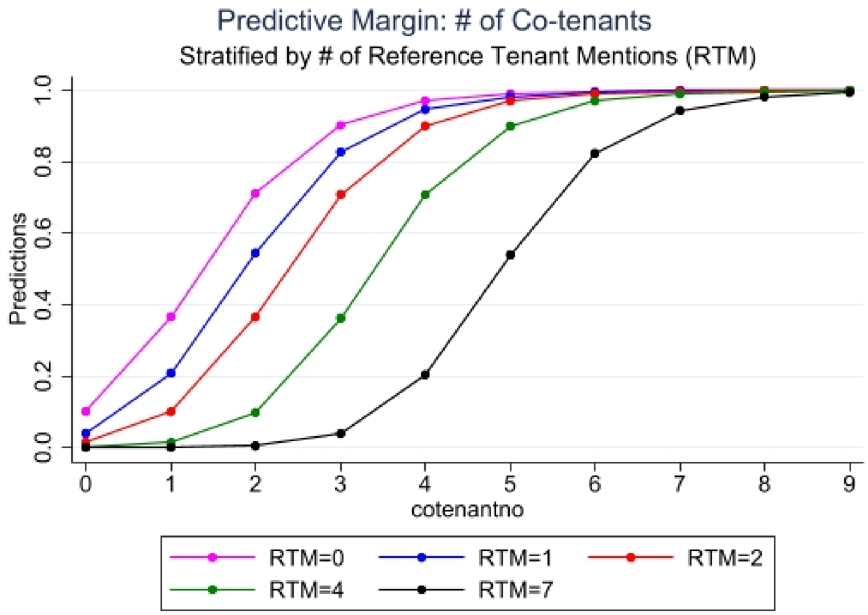

Figure 12: The Mean Predicted Probability of Selling a Center for More Than Asked

A similar logic applies to the reference tenant mentioned. When a reference tenant is mentioned in two or more distinct co-tenancy options (e.g., two or more tenants in their co-tenancy lease provisions specifically mentioned the same anchor), we defined this reference tenant to be locally systemically important merchant (L-SIM). The departure of this reference tenant from a center can trigger two or more co-tenancy provisions resulting in an exacerbated disruption. Therefore, *ex ante*, we expect the predictive margin for adding co-tenants on the odds of selling a center's sold price exceeding its asking price should shift downward ex-ante as the frequency that a reference tenant is mentioned in distinct co-tenancy clauses increases. Figure 12 validates this conjecture.

Co-tenancy by Grocery & Shadow Anchors

A grocery store can undermine the sales prospects of a shopping center. Not only is the space that grocery stores occupy harder to repurpose to another tenants use but also grocery stores occupy a large portion of the available leasable area.

The presence of a shadow anchor is often advertised as a significant positive factor in

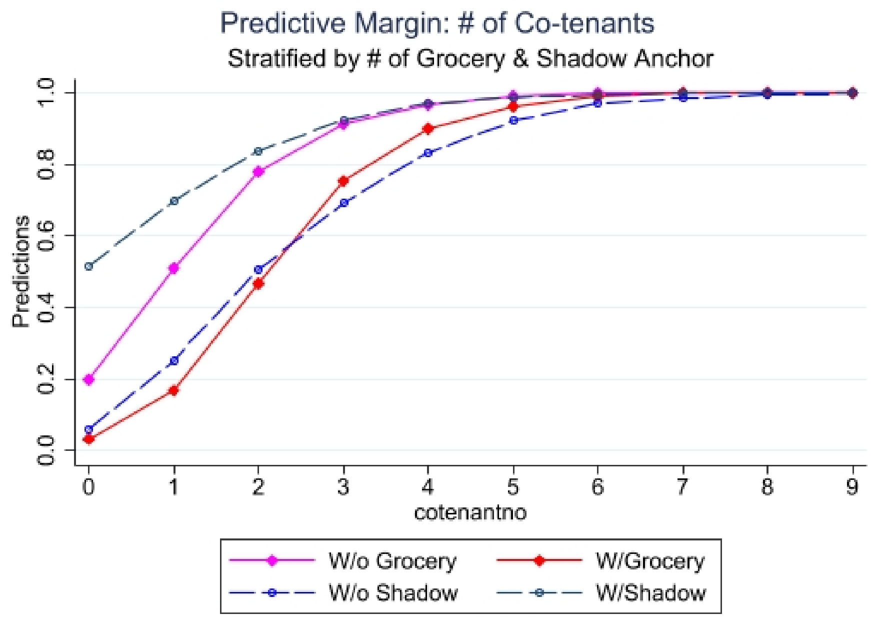

Figure 13: The Mean Predicted Probability of Selling a Center for More Than Asked

a shopping center sales deal, because a prospective owner would not be responsible for its operations but can still exploit the positive spillover effects such as extra foot traffic. Therefore, we anticipate that the predictive margin line to shift upward if a center adds a shadow anchor. Figure 13 shows that the predictive margin shifts downward (upward) when the anchor is a grocery store (shadow anchor).

Co-tenancy by Distance Between a Reference Tenant and Co-tenant

We measure the number of stores between a reference tenant and co-tenant to examine whether the positive externalities that a co-tenant receives from the reference tenant which results in higher sales productivity and rent paid attenuates with spatial distance. Ex ante, we would expect the closer a co-tenant is to its reference tenant, the higher the expected sales price and also sold-list price ratio. Our logistic regression results are inconclusive in determining the direction of the marginal spatial effect of having an additional store in between a reference tenant and co-tenant. Figure 14 stratifies the predictive margin analysis by the number of stores between a reference tenant and co-tenant. We observe that

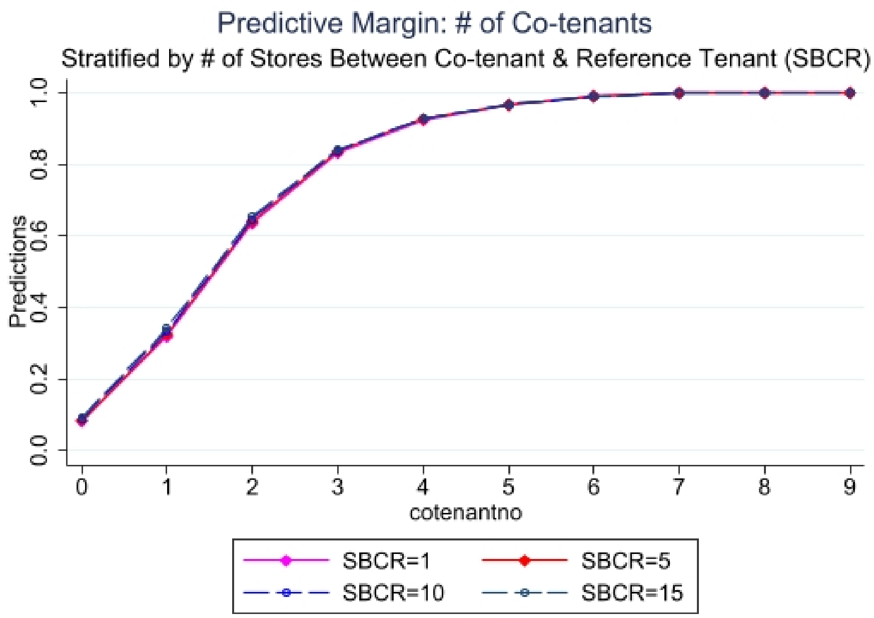

Figure 14: The Mean Predicted Probability of Selling a Center for More Than Asked

a the spatial distance between the reference tenant and co-tenant has a negligible impact if any. This suggests that externalities don't attenuate rapidly; it doesn't appear to matter whether a co-tenant is one or five stores away from the anchor tenant.

4.3 Numerical Simulations

In this section, we model an unlevered acquisition of a 225,000 square feet shopping center wherein there is one anchor tenant, one inline tenant, and one co-tenant. A co-tenancy option consists of four elements: anchor quality, exit power, rent abatement, and grace period. Our base case co-tenancy lease provision assumes a 50% rent abatement and an immediate exit option following a 12-month grace period in an average market where the space a co-tenant occupies remains vacant for six months on average when the co-tenant leaves. We also assume that its corresponding reference anchor tenant is risky (i.e., cumulative probability of default is 1%). We set the co-tenancy premium (i.e., the difference between a tenant with and without co-tenancy *ceteris paribus*) at $3.17 per square foot per year for our base case scenario. We investigate how the co-tenancy premium varies with the anchor quality (default probability), a co-tenant's ability to exit the center, rent abatement (degree of rent reduction), grace period (length of time the co-tenant receives the rent reduction) and market conditions (length of co-tenant vacancy following exit).

4.3.1 Environment

Consider an unlevered acquisition of a 225,000 square feet shopping center with three tenants: an anchor tenant, an inline tenant, and co-tenant with their lease terms being 7, 4, and 2 years respectively. The landlord holds the shopping center for 5 years.

Tenant Profiles

The anchor tenant occupies 120,000 square feet and pays $15 per square foot in base rent per year, with a lease maturity set to seven years.[27] The in-line tenant occupies 50,000 square feet and pays $30 per square foot in base rent per year. This tenant serves as the control group and does not have a co-tenancy option. Their lease term is set for four years. The co-tenant occupies 55,000 square feet and has a co-tenancy option with a lease term of two years.

Key Parameters

Table 9: The Key Elements of A Co-tenancy Option

Parameters	Measure					
Anchor Quality (%)	Cumulative Probability of Default	0.1	0.5	1		
Exit Power (%)	Probability of Lease Termination	0	25	50	75	100
Rent Abatement (%)	Degree of Rent Reduced	0	25	50	75	
Grace Period (month)	Rent Relief Duration	0	3	6	9	12

The key parameters of a co-tenancy option that we consider in our analysis are listed in Table 9. We define anchor quality as the probability the anchor will default at any

[27]There is no lease turnover during the holding period

time during its lease term. The cumulative probabilities .1%, .5%, and 1% correspond to a safe, moderately risky, and risky anchor respectively. We define the exit power of the co-tenant as the probability the co-tenant terminaties its lease following a grace period. If the co-tenant exits with certainty, this corresponds to having the most exit power. If the lease termination probability equals zero, then the co-tenant has no option but to resume paying its full base rent. Rent abatement corresponds to the percentage of rent reduction a co-tenancy option provides. The grace period is the length of time that the co-tenant receives the rent reduction.[28] We also consider in our analysis how the interaction between the grace period and different market conditions impacts the co-tenancy premia. In a strong, average and weak market, the space the co-tenant occupies remains vacant for one, six and eleven months on average respectively when the co-tenant leaves the center.

4.3.2 Methodology

As our initial point of departure, we set the rent of the base case co-tenant and inline tenant at $30 per square foot per year and perform 40 Monte Carlo simulations. Each simulation consists of 100,000 iterations. The economic intuition is that the expected returns of a shopping center with the co-tenant would be lower than that of a center without the co-tenant. Consequently, we increase successively the rent the co-tenant's pays in 50 cents increments until the expected returns of both centers converge. In each scenario, we compare expected return and its excess expected return per unit of standard deviation of the two shopping centers. We normalize by the standard deviation to adjust for the different total risks of the alternatives, due to the existence of the co-tenancy insurance. To construct 95% confidence intervals for the ratio, we exclude the simulations with the lowest and highest ratios. The upper and lower bounds of the confidence interval are the second highest and second lowest ratios calculated for each co-tenancy scenario respectively. We stop the pricing algorithm when the upper bound of the 95% confidence interval falls

[28] Rent abatement of 0% provides the co-tenant only with the option to exit following an anchor-driven credit event without benefits during the grace period.

within the 95% confidence interval of the co-tenant. Finally, we use linear interpolation to approximate the convergence point and to calculate the co-tenancy premium as the difference in the dollar per square foot. We report co-tenancy premia as a percentage of rent without co-tenancy.

4.3.3 Results

The Base Case

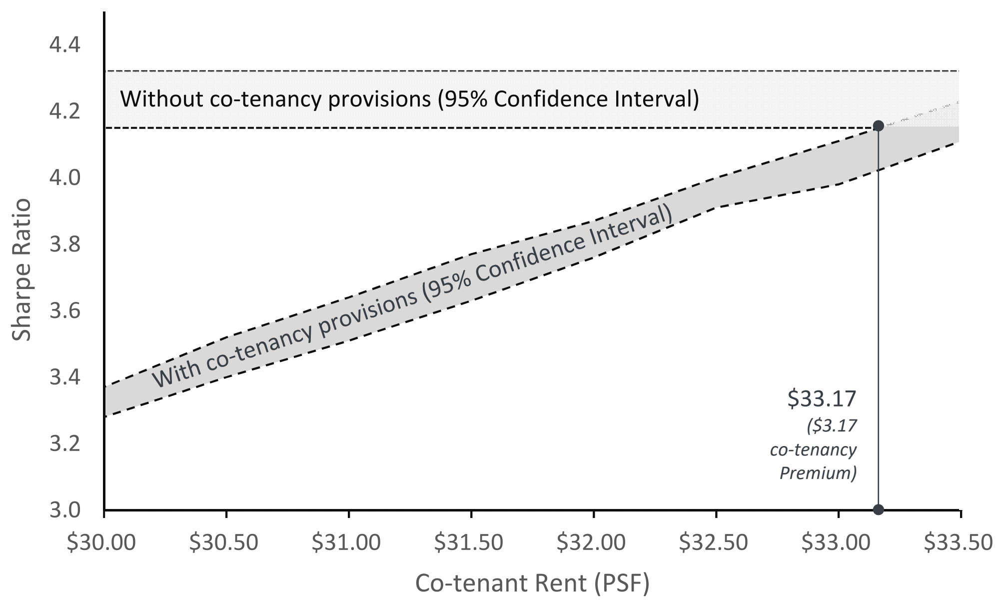

Figure 15: The Base Case Simulation

The base case co-tenancy option consists of the following parameters: a risky anchor,[29] immediate exit option following the grace period, 50% rent abatement, and 12-month grace period in an average market. Figure 15 reports the ratios 95% confidence intervals for varying base rent with and without co-tenancy provisions. We can observe the convergence of the ratios as the rent increases in 50 cents increments in the base case scenario. Without co-tenancy, the 95% confidence interval of the ratio ranges from 4.15 to 4.32. At the base rent of $30 per square foot, the ratio ranges from 3.28 to 3.37 for the shopping center that

[29]The risky anchor is based on 1% cumulative default probability.

has co-tenancy provisions. At $33.17 per square foot, the upper bound of the confidence interval of the co-tenancy shopping center reaches the ratio of 4.15. Therefore, the base case co-tenancy premium is $3.17 per square foot. This corresponds to a 10.56% rent premium over the base rent without co-tenancy provisions. [30]

Anchor Quality

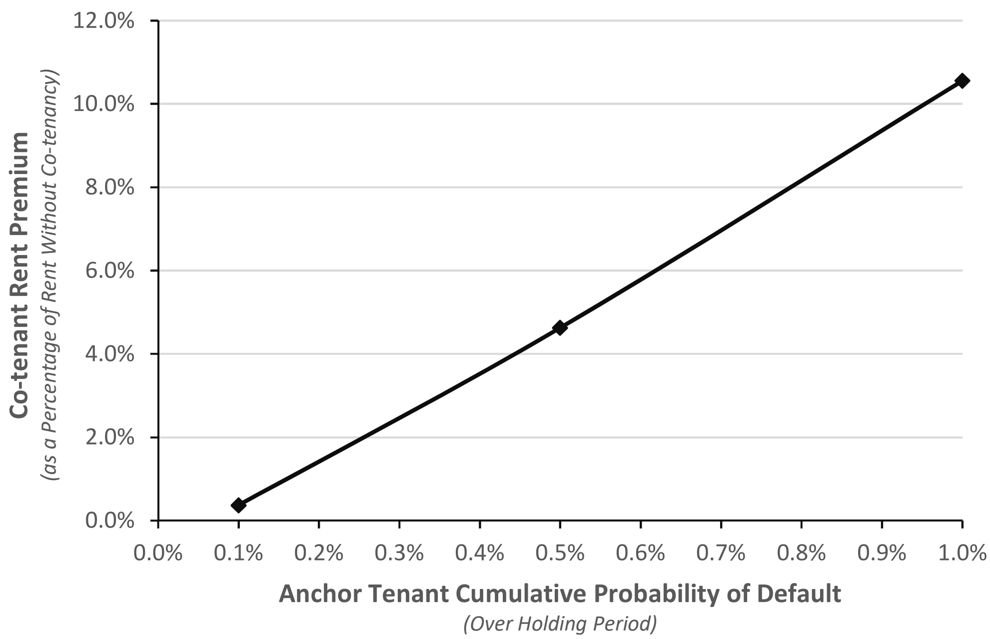

Figure 16: Anchor Quality

To test how the co-tenancy premium changes, we use a 1% cumulative default probability as the benchmark for a risky anchor then lower the cumulative default probability to .5% and .1% respectively. Ceteris paribus, one should expect a lower co-tenancy premium the higher the credit quality of an anchor. The simulation results in Figure 16 are consistent with this economic intuition. The co-tenancy premia are 10.56%, 4.63% and 0.37% for the 1%, .5% and .1% cumulative default probabilities respectively.[31]

[30] $\frac{3.17}{30} = 10.56\%$

[31] $3.17, $1.39 and $.11 per square foot rent premia respectively.

Exit Option Power

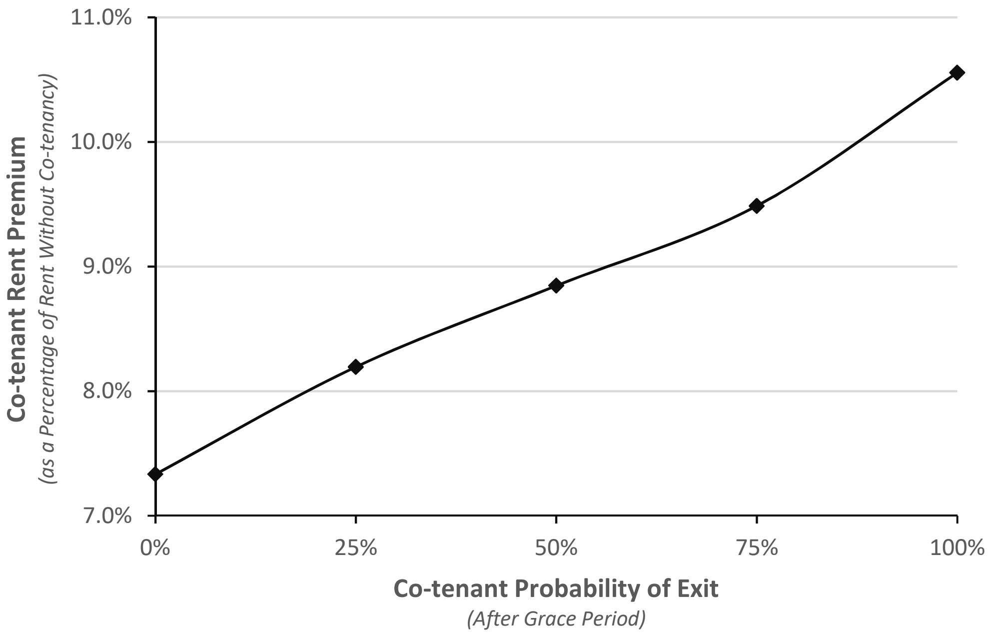

Figure 17: Co-tenant Exit Power

Figure 17 reports the risk premia for varying levels of exit power. We categorize the exit option as no exit, weak exit, moderate exit, strong exit, and absolute exit power. Their corresponding probabilities of lease termination after a grace period (co-tenancy premia) are 0% (7.33%), 25% (8.19%), 50% (8.85%), 75% (9.49%) and 100% (10.56%) respectively. A co-tenancy option with stronger exit power in the lease provision should enhance the degree of protection it provides to the co-tenancy. Our numerical results support this intuition.

Rent Abatement

Since our base case considers a 50% reduction in rent, we investigate the impact of a 0%, 25% and 75% rent reduction on the co-tenancy premium.[32] Results in Figure 18 are consistent with our economic intuition. A larger rent reduction enhances the co-tenant's real option.

[32] We do not consider the 100% case that implies complete rent waiver. The central element of a co-tenancy option is rent reduction, the ability for a co-tenant to get some degree of rent relief. In practice, the extreme case of a rent waiver is rare to find.

As rent abatement increases from 0% to 25% and further to 75% the co-tenant option premium rises from 2.13% to 5.83% and 16.97% respectively. [33]

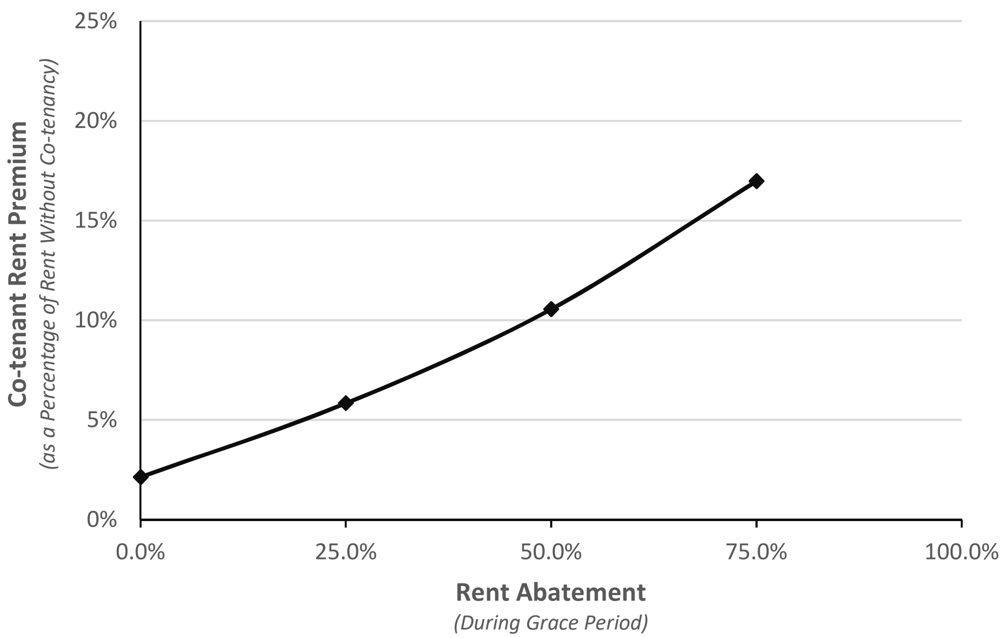

Figure 18: Rent Abatement Analysis

Grace Period

The duration of the grace period, which involves rent abatement, has a significant impact on the pricing of co-tenancy provisions. However, the influence of this impact depends greatly on the length of time that the space the co-tenant occupies remains vacant after an exit following the grace period. This finding helps explain some of our preliminary empirical results, where we observed a price increase in properties with co-tenancy provisions.

In a weak market, the space the co-tenant occupies remains vacant for 11 months on average and long grace periods act as a safety net. The landlord receives a fraction of the rent as opposed to no rent at all had the space become vacant immediately. This is particularly significant if the anchor tenant's credit event occurs in the last year of the holding period. A short grace period results in a prolonged vacancy period from the

[33]$0.64, $1.65 and $5.09 respectively.

co-tenant, precisely when the property is up for sale. This situation negatively impacts the resale value and returns the most. This represents the worst-case scenario where co-tenancy provisions lead to cash flow contagion and property value deterioration. In a weak market, the standard deviation of returns is highest with short grace periods and decreases as the grace period increases. As a result, expected returns per unit of total risk increase with longer grace periods. This implies that the co-tenancy premium is the largest when short grace periods exists as can be observed in Figure 19 where rent premia range from 27.86% ($8.36) to 10.21% ($3.06) with a grace period of one and twelve months respectively.

Conversely, in a strong market, the space the co-tenant occupies remains vacant for an average of only one-month and long grace periods are an opportunity cost. During this period, the co-tenant pays only a fraction of the rent. The owner would prefer the co-tenant to vacate the space so it can be quickly re-leased at full rent. In such a scenario, a long grace period implies a significant drop in revenue compared to the expected revenue after an immediate exit by the co-tenant (with a short grace period). If the anchor tenant experiences a credit event during the last year of the holding period, the impact of a long grace period (and the subsequent revenue loss) amplifies the losses and increases the volatility of returns. Consequently, expected returns decrease as the grace period increases. This indicates that the co-tenancy premium are the largest with long grace periods as reported in Figure 19 where rent premia range from 0% ($0) to 8.42% ($2.53) with a grace period of one and twelve months respectively.

In an average market, the space the co-tenant occupies remains vacant for an average of six months, and short (long) grace periods act as an opportunity cost (safety net). If a credit event occurs near the end of the holding period, a short grace period is beneficial for the resale value and helps maintain a low standard deviation of returns. A grace period shorter than the average vacancy period of the co-tenant, implies a brief period of reduced rent and the potential to quickly re-lease the co-tenant space at full rent. The worst-case scenario arises with a grace period of six months. In this situation, the property experiences six months of reduced rent followed by six months of vacancy, significantly impacting the resale value and resulting in the lowest expected returns. Consequently, co-tenancy provisions command the largest premium when the length of the grace period is equal to the length of the expected vacancy from the co-tenant. As the grace period extends past six months, the safety net effect takes hold with longer periods of reduced rent preferable to no rent from a vacant space. This pattern in Figure 19 corresponds to a relatively small 10% co-tenancy premium with a grace period of zero months. This co-tenancy premium increases to a maximum of 16.94% for a grace period of six months, then decreases to 10.56% premium for a grace period of twelve months, forming a frown (an inverted smile-shaped curve).

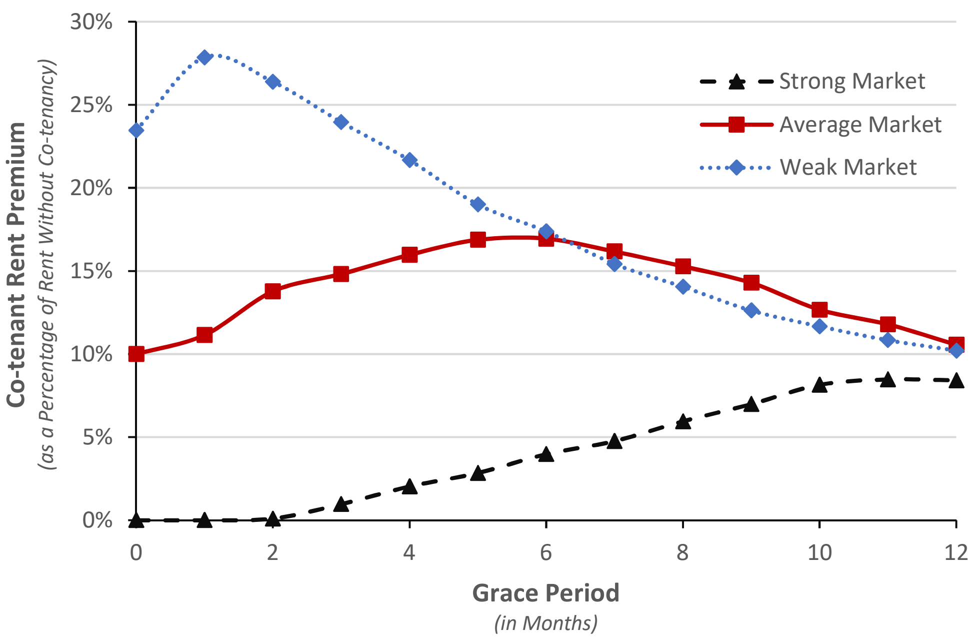

Figure 19: Grace Period and Market Conditions

These findings indicate that co-tenancy provisions can potentially enhance the value of a property in certain situations, particularly if market conditions improve during the holding period. This novel insight offers an explanation for our empirical findings, where properties with co-tenancy provisions have resulted in a price increase.

Landlords who are optimistic about the market should consider offering co-tenancy provisions with short grace periods. This approach allows them to take advantage of significant rent premiums when the market is weak, while also benefiting from stronger cash flows as market conditions improve, without significantly increasing their risk. It is worth noting that this situation would arise from co-tenants paying an excessive amount for an insurance policy that may not longer be necessary. Additionally, in a robust market, we would anticipate that co-tenants would not exercise the fish-or-cut-bait option and would instead continue leasing the space. Regardless of the co-tenant's choice, the property would swiftly resume collecting full rent reaping the benefits of the rent premium from the co-tenancy provision without an increase in risk. On the other hand, landlords who anticipate a downturn in the market should opt for longer grace periods, even in a strong market. This strategy leverages rent premium to offset potential losses during periods of worsening market conditions. In an efficient market, the length of grace period can serve as a signaling mechanism, providing valuable insights into the expectations of both landlords and co-tenants regarding future market conditions.

5 Conclusion

Shopping centers are a practical representation of the Coase theorem wherein in the presence of externalities, an economic efficient allocation that is optimal arises from bargaining. To internalize positive externalities, center owners offer anchor tenants lower rent that corresponds to these externalities e.g., increased customer traffic these stores generate. To subsidize this lower rent, non-anchor tenants pay a higher rent premium. If negative externalities exist, the allocation of rent ala the Coase theorem is negotiated based on the externality cost that non-anchors incur.

The 2008 financial crisis and more recent 2020 pandemic, both exogenous events, have forced major retailers who anchor shopping centers to close some or all their stores. The store closures of major tenants have triggered co-tenancy provisions in shopping centers that address the externality cost incurred from negative externalities. In particular, when non-anchor stores face the loss of traffic that anchor stores generate and consequently a loss of sales, these stores are allowed to pay reduced rent and/or exit the shopping center after a grace period. To date, while the previous literature has recognized the importance of this co-tenancy provision especially in the case of bankruptcy, this lease provision has not been priced, and the sensitivity of the price drivers of this real option has yet to be examined. This is the purpose of our study.

We first construct a theoretical closed form pricing model that consists of one anchor tenant and one co-tenant using arbitrage-pricing and a reduced-form credit risk framework. However, our model is easily extendable so that one can value a broader set of real options embedded in commercial real estate leases such as going dark, going dim, escalation, and exclusive use. Our model identifies both positive and negative spillover effects of an anchor tenant. While an in-line tenant's base rent internalizes the anchor's drawing power, our study prices the downside risk of the corresponding credit event in the form of an insurance premium. Quantifying these spatial risks and enabling tenants to hedge against them have important implications for commercial spatial structure and risk allocation.

In our empirical analysis we address a current phenomenon where shopping centers are sold at a premium relative to its offering (list) price when co-tenancy options exist. Our hedonic pricing model and logistic regression results are consistent with our theoretical conjecture. We find that co-tenancy options can have a positive impact on a center's expected sales price and its profitability. In particular, a center's sold price can exceed its offering price (e.g., sold-list price ratio is greater than 1). In our numerical simulation analyses, we examine an unlevered acquisition of a 225,000 square feet shopping center. Our base case assumes an immediate exit option, a 50% rent reduction, a 12-month grace period and a co-tenancy premium of 10.56% ($3.17 per square foot) per year. Our numerical results are consistent with our economic intuition, namely that any enhancement that increases externality cost such as having a risky anchor, higher rent reduction, and/or a more immediate exiting option increases the co-tenancy premium. Furthermore, in an efficient market, the duration of the grace period can serve as a signaling mechanism, offering valuable insights into the expectations of both landlords and co-tenants regarding future market conditions. When fairly priced, the co-tenancy option can enhance efficiency in risk-sharing and reduce information asymmetry. Our valuation technology provides a novel starting point for future research around insurance economics, real estate finance, and credit risk.

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Internet Appendix

A The Extensions

A 1 Anchor & N Co-tenants for Option 1

In this subsection, we examine an extended case where there exist one anchor and N co-tenants. Since there are N co-tenants, there are N distinct co-tenancy (i.e., N potentially different lease terms) provisions whose reference entity is the sole anchor tenant. For demonstration, the relevant figures show the case with $N = 2$.

A.1 The Setup

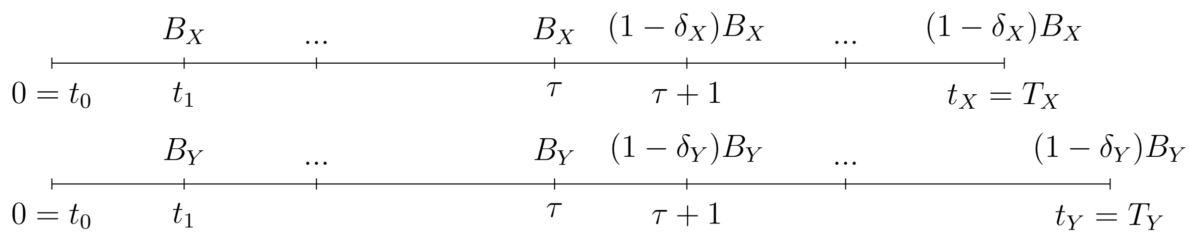


Figure 20: Rent Schedule with One Anchor and Two Co-tenants

The setup is similar to that of the simple one anchor and one co-tenant model. With multiple co-tenants, we allow N distinct lease terms among the N inline tenants such that the sole anchor's credit event activates N co-tenancy clauses. Denote the index set representing the number of co-tenants to be $I = \{1, ..., N\}$. Without loss of generality, suppose they are ordered in an ascending order such that for any distinct co-tenants $X, Y \in I$ with $X < Y$, we assume the lease term for Y is weakly longer than that of X (i.e., $t_X = T_X \leq T_Y = t_Y$). Figure 20 demonstrates the stochastic rent schedule for the case of two co-tenants X and Y where B_i, δ_i, and t_i represent co-tenant i's base rent, rent abatement, and lease expiry respectively and $i \in \{X, Y\}$. Note τ represents the first time an anchor-driven credit event occurs. Since there is only one anchor and it is the only reference entity of the co-tenancy option, τ is the stopping time for all N inline tenants. Figure 21 illustrates the insurance payment schedule for the two co-tenant case.

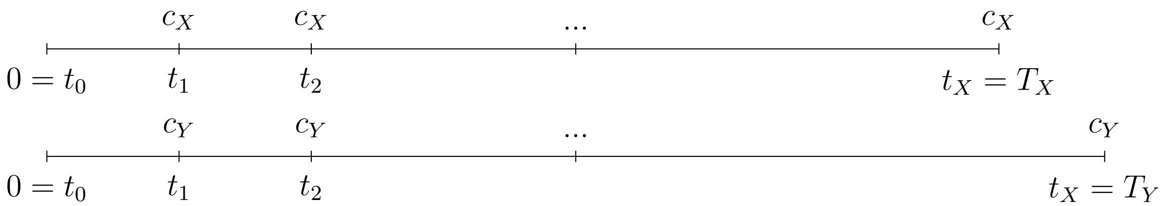


Figure 21: Premium Payment Schedule with One Anchor and Two Co-tenants

A.2 The Valuation Formula for Option 1 (1 Anchor, N Co-tenants)

$$V_u(0) = 0 = \mathbb{E}_{\mathcal{Q}} \left[\sum_{k=1}^{i} c_i e^{-\int_0^{t_k} r_u du} - \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil \wedge (\lceil\tau\rceil + g_i)} (1-\delta_i) B_i \mathbb{1}_{\{t_0 < \tau \le t_i = T_i\}} e^{-\int_0^{t_h} r_u du} \right]$$

$$c_i = \frac{(1-\delta_i) B_i \mathbb{E}_{\mathcal{Q}} \left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil \wedge (\lceil\tau\rceil + g_i)} \mathbb{1}_{\{t_0 < \tau \le t_i = T_i\}} e^{-\int_0^{t_h} r_u du} \right]}{\sum_{k=1}^{i} B(0, t_k)} \tag{13}$$

Similar to the simple model's set-up, we begin with the time 0 values of the insurance contracts being zero under the arbitrage-free condition. In the set-up, there are N distinct co-tenancy provisions such that $V_i(0) = 0$ for $i \in I = \{1, ..., N\}$. (13) is the analytic expression for the i^{th} co-tenant insurance premium c.

B M Anchors & N Co-tenants

In this subsection, we generalize the simple model to include M anchors and N co-tenants in a shopping center. Since anchor-driven exits lead to an activation of various co-tenancy options, it is important to investigate the systematic risk structure present in a shopping center. In the context of a co-tenancy option in shopping centers, we define a locally systemically important merchant (L-SIM) to be an anchor tenant who is a reference entity in at least two or more inline tenants' co-tenancy clauses.[34] Next, using the measure of systemic risk developed by Jarrow et al. (2022) we define a measure of systemic risk in a shopping center that could lead to a cascade of defaults within the shopping center. The number of anchors in a center is generally smaller than that of co-tenants, because a few

[34]The definition of locally systemically important merchants (L-SIMs) is motivated in the same spirit the Financial Stability Board (FSB) annually defines its list of globally systemically important banks (G-SIBS). One can think of L-SIMs as "too big to fail" version for shopping center.

anchors tend to take up a large space creating a strip of suites on which multiple in-line tenants occupy. Hence, we assume $M \ll N$.

B.1 Conditional Independence

First, we explore the case where the conditional independence assumption is maintained. We characterize the default contagion among the anchors by examining the anchors' joint default probability before the expiry of the lease term. With M anchors with possibly M different credit events, the joint default distribution function under the risk-neutral measure is:

$$P\big[(\tau_1 \le T_1), ..., (\tau_M \le T_M)\big] = \mathbb{E}_{\mathbb{Q}}\big[(1 - e^{-\int_0^{T_1} \lambda_1(u)du})...(1 - e^{-\int_0^{T_M} \lambda_M(u)du})\big] \tag{14}$$

The multiplicative nature in (14) conforms to the conditional independence assumption that the credit events occurs independently and each anchor's default intensity is influenced only through the common state variables.

Square-Root Process

One can specify the evolution of the default intensity to derive an analytic solution when the model assumes the conditional independence. Consider the following evolution of the default intensity process of anchor j with the constants $\kappa_j, \mu_j, \sigma_j > 0$, $2\kappa_j\mu_j > \sigma_j^2$ for all $j \in \{1, ..., M\}$, and $W_{j,t}$ is an independent standard Brownian motion adapted to $(\mathscr{F}_t)_{t\in[0,\tau_j]}$:[35]

$$d\lambda_{j,t} = \kappa_j(\mu_j - \lambda_{j,t})dt + \sigma_j\sqrt{\lambda_{j,t}}dW_{j,t} \tag{15}$$

Then, using (15) the join default distribution function is:[36]

$$P\big[(\tau_1 \le T_1), ..., (\tau_M \le T_M)\big] = \prod_{j=1}^{M}\Big[1 - e^{\alpha_j(T_j)+\beta_j(T_j)\lambda_j(0)}\Big] \tag{16}$$

We derive the conditions under which a small increase tenant j's lease term increases the joint default probability.

Proposition 4 Consider M tenants where tenant j's default intensity follows the square-root process: $d\lambda_{j,t} = \kappa_j(\mu_j - \lambda_{j,t})dt + \sigma_j\sqrt{\lambda_{j,t}}dW_{j,t}$ with $\kappa_j, \mu_j, \sigma_j > 0$, $2\kappa_j\mu_j > \sigma_j^2$ for all $j \in \{1, ..., M\}$, and $W_{j,t}$ is an independent standard Brownian motion adapted to $(\mathscr{F}_t)_{t\in[0,\tau_j]}$.

[35] Cox et al. (1985) proposed the square-root diffusion process as an extension of the model by Vasicek (1977). The proof of this version is in Lando (2004) (see p.293).

[36] The computation follows the convention in Jarrow (2009) (see p. 49).

The join default distribution of M tenants is a weakly-increasing function of any tenant j's lease term, T_j if and only if $\kappa_j > 3\gamma_j$ and $\frac{\gamma_j(\gamma_j-2)}{\gamma_j+\kappa_j}e^{\gamma_j T_j} < \beta_j$.

Proposition 4 states that when a tenant's lease term extends, the longer horizon admits more opportunity for credit events; it weakly increases the joint default probability. It is intuitive that if a tenant elongates her contract to be at a shopping center, then it creates more room for potential defaults.

The bound conditions also confrom to economic intuition. The first condition ($\kappa_j > 3\gamma_j$) bounds the rate at which the default intensity reverts to its long term mean from below. Tenant j's intensity's convergence rate has to be somewhat large, and this ensures j's elongated lease term positively contributes to the joint default probability. The second condition bounds β_j from being a significantly negative number, which ensures that j's individual default probability before its expiry to not become explosive.

By maintaining the conditional independence assumption in a standard doubly stochastic Possion model of default, the above square root process demonstrates that one can explicilty solve for a condition under which the joint default probability is influenced by a small increase in a tenant's lease term.

B.2 Discussion: Conditional Dependence

It is not unusual to witness a cascade of defaults in shopping centers where it initially starts with one empty store space (e.g., anchor exiting the center) followed by an exodus of neighboring shops. In this section, we provide a discussion on how one can relax the conditional independence assumption of the doubly stochastic Possion process of defaults. In particular, we introduce the three important spatial counterparty risks (i.e., geographical, industrial, and contractual) that are most germane to the shopping center co-tenancy analysis and default contagion.

Spatial Counterparty Risk

In the class of reduced-form credit risk models, the environment assumes that the default intensity's randomness is driven by a set of common state variables such as inflation, GDP, and short-term interest rates.[37] Under this conditional independence assumption, the default events in a shopping center are independent across anchor tenants. By relaxing the assumption, Jarrow and Yu (2001) expands the extant literature by introducing the concept

[37]This provides the basis for the conditional independence assumption in a standard doubly stochastic Poisson model of default. It implies the default correlations across tenants stem from their default intensities dependent on the common state variables. In this paper, we allow its relaxation. See Jarrow (2009, 2018) and Lando (2004) for the reduced-form credit risk modeling.

of counterparty risk. This framework captures the aspects of inter-firm and industrial organization interdependence that influence firms' default intensities.

In the context of investigating anchors' credit events in a shopping center, this paper argues that relaxing the conditional independence assumption is appropriate. Tenants' default intensities are influenced not only by the common state variables but also by their unique geographical locations, industry relations, and contractual interdependence such as co-tenancy provisions. The nature of these interdependencies is *spatial*.

For example, regardless of the presence of co-tenancy provisions, a tenant can incur sales loss if a neighboring anchor suddenly ceases operation. Similarly, the co-tenancy protections are geographical in nature, and this insurance might decelerate an inline store's own default time. Finally, being in a closely-related industry (e.g., apparels and jewelry) can promote default contagion; one can measure out how distant one industry is from another, for example, by looking at the Standard Industrial Classification (SIC) codes.[38] Hence, the *spatial counterparty risk* (SCR) is defined as the risk that the default of a tenant's counterparty might affect her own default probability where the major categories of the counterparty are geographical (i.e., physical distance from each other), industrial (i.e., industrial organization structure), and contractual (i.e., co-tenancy).

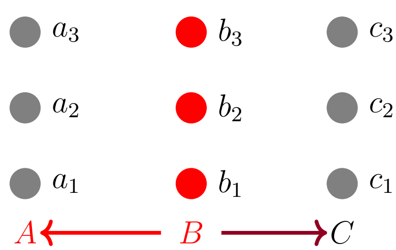


Figure 22: Co-tenancy Activation & Default Acceleration

Figure 22 illustrates a case where anchor B's credit event activates anchor A's co-tenancy since it is A's reference entity (red arrow). Anchor C's credit event might become accelerated as the landlord is unable to find a suitable replacement for B (brown arrow).

For example, consider Figure 22 where anchor A and B have a co-tenancy contract and C is simply close to B. Anchor B's credit event activates A's co-tenancy provision and can ac(de)celerate C's credit event. If A and C are major anchors in a class A shopping center, finding a suitable replacement for B might be relatively easy. Their presence can help reduce the turnaround time in finding a substitute anchor for B. Finally, being in a

[38]For example, in the Standard Industrial Classification (SIC) system, unique four-digit numeric codes are assigned to classify various industries. A nice feature of the SIC system is that it numercailly codes similar industries tightly with each other's number. For example, various types of farming and its industries (e.g., wheat, rice, corn, soybean) are recorded from 0111 to 0279.

similar industry or not can amplify or subdue the anchor-driven credit event's influence on other tenants survivability.

Primary-Secondary Framework

In this section, we utilize the primary-secondary framework developed by Jarrow and Yu (2001). In their original work, the authors divides the set of firms into two mutually exclusive groups where the primary firms' default processes depend only on macroeconomic variables while those of the secondary firms depend on both the macroeconomic variables and default processes of primary firms. For example, a secondary firm can hold a significant amount of liabilities of a primary firm in its portfolio where the dichotomy in the primary-secondary framework is not uncommon in the universe of defaultable corporate bonds.

In the context of shopping center leases and co-tenancy, the well-being and profitability of an inline tenant depends not only the macroeconomic variables but also the default processes of its neighboring tenants and its respective geographical, industrial, and contractual relations. For example, consider the following default intensity of an inline tenant B:

$$\begin{aligned}\lambda_t^B =& \alpha_{0,t}^B + \alpha_{A,t}^B \mathbb{1}_{\{t\geq\tau^A\}} + \alpha_{C,t}^B \mathbb{1}_{\{t\geq\tau^C\}} + \alpha_{D,t}^B \mathbb{1}_{\{t\geq\tau^D\}} + \alpha_{A,t}^B\alpha_{C,t}^B \mathbb{1}_{\{t\geq\tau^A\}}\mathbb{1}_{\{t\geq\tau^C\}} + \\ & \alpha_{A,t}^B\alpha_{D,t}^B \mathbb{1}_{\{t\geq\tau^A\}}\mathbb{1}_{\{t\geq\tau^D\}} + \alpha_{C,t}^B\alpha_{D,t}^B \mathbb{1}_{\{t\geq\tau^C\}}\mathbb{1}_{\{t\geq\tau^D\}}\end{aligned} \tag{17}$$

In this simple specification, the sign of $\alpha_{i,t}^B$ of primary tenant i where $i \in \{A, C, D\}$ influences the default probability of tenant B. For example, tenant B might be closely related to tenant A in terms of industry-relation but if B and C are geographically far from each other, then the corresponding coefficients could be such that $\alpha_{A,t}^B > 0$ and $\alpha_{C,t}^B < 0$.[39]

The idea behind utilizing the primary-secondary framework is that one can compartmentalize any spatially pertinent tenants around tenant B into geographical, industrial, and contractual counterparty risk categories and model and solve for the joint default probability distribution to take the default contagion into account. With a specific evolution of each tenant's default intensity and a set of mild assumptions (e.g., similar to the square-root process in the conditional independence case), we will be able to solve for the joint default probability distribution in the primary-secondary framework using the spatial counterparty risk.

[39]See p1775 of Jarrow and Yu (2001) for a general form of the default intensity in the primary-secondary framework where the default is independent of the default-free term structure.

B.3 Locally Systemically Important Merchants (L-SIMs)

We develop a measure of systemic risk called locally systemically important merchants (L-SIMs) in the same spirit the Financially Stability Board (FSB) defines and updates its list of globally systemically important banks (G-SIBs). One can think of L-SIMs as the "too-big-to-fail" version in the shopping center universe.

Definition 1 (A Locally Systemically Important Merchant (L-SIM)) In a shopping center, a locally systemically important merchant (L-SIM) is an anchor tenant that is a reference entity to two or more distinct co-tenancy clauses.

The underlying intuition is that if an anchor tenant is a reference entity to only one inline tenant, then an anchor-driven credit event is less likely to strain a shopping center owner's ability to payout the co-tenancy compensation in a form of rent abatement or its ability to quickly find a replacement anchor tenant; a landlord can manage and contain the risk without impacting the entire shopping center community. However, if an anchor is a reference entity of two or more co-tenancy options, a credit event activates two or more distinct co-tenancy clauses significantly undermining its ability to deliver insurance payouts and simultaneously and potentially manage a set of one large and two small vacant retail spaces. Hence, an anchor tenant that is a reference entity of two or more co-tenancy clauses is a systemically important merchant within a local shopping center ecosystem.

B Market Inefficiency

In this section, we incorporate the presence of asymmetric information induced by co-tenancy clause into shopping center sales. We model how an informed shopping center owner who better understands the default risks associated with its co-tenancy clauses can downplay these risks and increase the co-tenancy premium cash flows against a less informed buyer as a static trading game of incomplete information.

A The Source of Asymmetric Information

In a typical shopping center sales transaction, a retail brokerage firm compiles an offering memorandum highlighting the list property's key features, site plans, demographics analysis, tenant profile, and rent roll. Co-tenancy clauses are often embedded in a base rent, which makes it difficult for a prospective buyer to fully understand how much the proximity premium is and to what extent a landlord collects the co-tenancy insurance premium. Even if prospective buyers carefully examine the offering memorandum with a third party property appraiser, the valuation and associated default risks are sufficiently complex making them ultimately less informed than a shopping center seller, and this is the source of information asymmetry.

B Shopping Center Trading Game

Consider a static game of incomplete information with two players, S and B, a shopping center seller and buyer respectively. Player S owns a shopping center and wants to sell it. The value of the shopping center depends on the quality of location, tenants, and valuation of the co-tenancy clause. Since player S has negotiated with its individual tenants over lease terms, base rent, and co-tenancy provisions, it is more informed relative to player B about the shopping center quality (e.g., tenant quality and their credit-riskiness with respect to co-tenancy clauses).

Suppose the quality of the shopping center is inversely correlated with the number of the Locally Systemically Important Merchants (L-SIMs); the more the L-SIMs there are in the center, the higher the default chance exists undermining the quality of the shopping center. For simplicity, consider a case in which there is either one L-SIM or none. Let the quality of the shopping center be $\{L, H\}$ that corresponds to having one or zero L-SIM respectively, and we represent it as player S's type $\Theta_S = \{L, H\}$ with the corresponding probabilities p and $1-p$ respectively for $p \in (0,1)$.(Harsanyi, 1967; Akerlof, 1970).

For each quality type of the shopping center, it yields different values for the seller and buyer. Given the shopping center sales are completed in a fiat currency, we assume the following personal values with (i) $s_l < b_l, s_h < b_h$ and (ii) $s_l < s_h, b_l < b_h$.[40]

$$v_S(\theta_S) = \begin{cases} s_l & \text{if } \theta_S = L, \\ s_h & \text{if } \theta_S = H \end{cases} \qquad v_S(\theta_S) = \begin{cases} b_l & \text{if } \theta_S = L, \\ b_h & \text{if } \theta_S = H \end{cases} \tag{18}$$

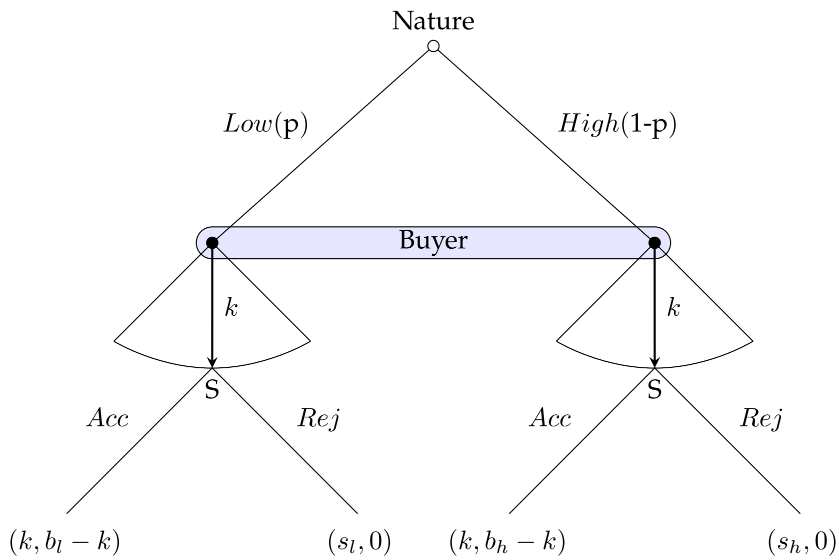
Suppose nature picks the quality of shopping center; the seller is born as type L with probability p and as type H with probability $1-p$. Equivalently, the seller lists its shopping center for a sale, which can be a low or high quality center dependent on the existence of the L-SIM. The buyer does not know which quality of the shopping center it receives in the offering memorandum.[41] The buyer makes an offer price to the seller to which it can respond by either accepting (A) or rejecting (R). If the seller accepts the offer, the shopping center sale is complete, otherwise the deal fails. Figure 23 provides an extensive-form of

[40]The condition (i) implies that the list shopping center is more valuable to the buyer than to the seller regardless of the center quality. The condition (ii) implies regardless of the player, the high quality center yields more monetary value than the low quality one.

[41]The buyer understands the probability distribution over the two quality types, because it can read the rent roll, but the exact information about the number of the L-SIMs present in this particular shopping center is unknown.

the shopping center sales game.

Figure 23: An extensive-form of the shopping center sales game with incomplete information.



The seller's strategy is a mapping from the offered price and its type space to a accept-or-reject response such that $s_s : [0, \infty) \times \Theta_s \to \{A, R\}$, and the buyer's strategy is one-time price offer $k \in \mathbb{R}_+$. Knowing the probability distribution over the quality of the shopping center (i.e., the seller type), the buyer's expected monetary payoff is $pb_l + (1-p)b_h$. Since each player's payoff is common knowledge, the buyer understands that the shopping center is worth $ps_l + (1-p)s_h$ for the seller, which is a natural point of departure for an initial offer price. Denote this offer price as k_0.

Since the seller is the shopping center owner, it knows the shopping center quality. This implies, the seller would accept k_0 if and only if

$$s_l \leq ps_l + (1-p)s_h \quad \text{if } \theta_S = L \tag{19}$$

$$s_h \leq ps_l + (1-p)s_h \quad \text{if } \theta_S = H \tag{20}$$

Simplifying the equations yields:

$$s_l \leq s_h \quad \text{if } \theta_S = L \tag{21}$$

$$s_h \leq s_l \quad \text{if } \theta_S = H \tag{22}$$

Regardless of the shopping center quality, the seller yields a higher monetary value for the high quality center, hence it must be that $s_l \leq s_h$, which implies the seller accepts k_0 if and only if the listed shopping center is of low quality with one L-SIM. The buyer gets only a low quality shopping center from the seller imposing her expect value from the trade to be b_l. If the buyer proceeds with this trade, then its expected payoff is $b_l - k_0$, which can be written as:

$$pb_l - ps_l + (1-p)s_h \tag{23}$$

Since the seller values a high quality shopping center, it follows that the seller would accept k_0 only if the shopping center is of low quality. Knowing the seller's response, the buyer expects to receive ps_l. Hence, the seller's expected payoff is $ps_l - b_l$. If $ps_l - b_l < 0$, the seller is better off simply walking away from the shopping center deal. In theory, a co-tenancy provision is provided to all tenants with a price. Depending on a tenant's risk profile and preference, it might choose to purchase or not. In reality, a small and family-owned (e.g., mom and pop stores) tenant tend to not have a co-tenancy provision in their contracts. It might be the case that smaller but still reputable and financially healthy stores are more likely to buy a co-tenancy insurance.

C Proof & Lemma

A Proofs

A.1 Lemmas

Proof of Lemma 1

Proof. I want to show that $c_2 \geq c_1$ from the set-up in Section 2.2.

Recall the analytic expression for c_2 is:

$$c_2 = \underbrace{\frac{(1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge\lceil(\rceil\tau+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right]}{\sum_{k=1}^{m} B(0,t_k)}}_{=c_1} + \underbrace{\frac{\mathbb{E}_{\mathcal{Q}}\left[\sum_{l=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor} B\mathbb{1}_{S_L}\mathbb{1}_{S_I} e^{-\int_0^{t_l} r_u du}\right]}{\sum_{k=1}^{m} B(0,t_k)}}_{\geq 0}$$

Hence, $c_2 \geq c_1$. □

Proof of Lemma 2

Proof. I want to show that $c_3 \geq c_2$ from the set-up in Section 2.2.

Recall the valuation formula for c_3 is:

$$c_3 = \frac{\underbrace{B\mathbb{1}_{S_P}\mathbb{E}_{\mathcal{Q}}\left[\sum_{j=\lceil\tau\rceil}^{\lfloor\gamma\rfloor} e^{-\int_0^{t_j} r_u du}\right]}_{(*)} + (1-\mathbb{1}_{S_P})\underbrace{\mathbb{E}_{\mathcal{Q}}\left[(1-\delta)B\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}}e^{-\int_0^{t_h} r_u du} + B\sum_{l=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor}\mathbb{1}_{S_L}\mathbb{1}_{S_I}e^{-\int_0^{t_l} r_u du}\right]}_{c_2}}{\sum_{k=1}^{m} B(0,t_k)}$$

Suppose $S_P = 0$. Then, $c_3 = c_2$, hence $c_3 \geq c_2$.
Suppose $S_P = 1$. Then, $(*)$ can be re-written as:

$$\mathbb{E}_{\mathcal{Q}}\left[B\sum_{j=\lceil\tau\rceil}^{\lceil\tau\rceil+g} e^{-\int_0^{t_j} r_u du} + B\sum_{j=\lceil\tau\rceil+g+1}^{\lfloor\gamma\rfloor} e^{-\int_0^{t_j} r_u du}\right] \tag{24}$$

Since $\max\{\xi, \lceil\tau\rceil + g\} = \lceil\tau\rceil + g$, $S_P = 1$, and $\delta \in (0,1)$, it follows that $c_3 > c_2$.
$\therefore c_3 \geq c_2$. □

A.2 Propositions

Proof of Proposition 1

Proof. In the simple set-up of a landlord, an anchor tenant, and an inline tenant:
By Lemma 1, $c_2 \geq c_1$. By Lemma 2, $c_3 \geq c_2$.
Hence, $c_3 \geq c_2 \geq c_1$. □

Proof of Proposition 2

Proof. Consider the simple model of one landlord, one anchor tenant, and one co-tenant. Assume the co-tenancy premium, c, is continuously differentiable. We want to show that:

1. $\frac{\partial c}{\partial \delta} < 0$.
2. $\frac{\partial c}{\partial B} > 0$.
3. $\frac{\partial c}{\partial T} < 0$.
4. $\frac{\partial c}{\partial B(0,t_k)} < 0$ for $k \in [1, m]$.
5. $\frac{\partial c}{\partial \tau} < 0$ for $\tau \in [0, T]$.

Consider the numerator term of the analytic expression of the co-tenancy premium c for the one landlord, anchor, and tenant model in (3):

$$c = \frac{(1-\delta)B\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right]}{\sum_{k=1}^{m} B(0,t_k)} \tag{25}$$

Degree of Rent Abatement

$$\delta \uparrow \implies (1-\delta)B \downarrow \implies c \downarrow \tag{26}$$

Base Rent

$$B \uparrow \implies (1-\delta)B \uparrow \implies c \uparrow \tag{27}$$

Lease Term

Suppose the final date $T = t_m$ increases by one time unit, so that the lease now ends at $T+1 = t_{m+1}$. We show both numerator and denominator of (3) increase.

First, the numerator increases as the lease term extends.

Recall (3)'s numerator is:

$$(1-\delta)B\mathbb{E}_{\mathcal{Q}}\underbrace{\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}}e^{-\int_0^{t_h} r_u du}\right]}_{(N_0)} \tag{28}$$

When the lease term increases by a time unit, the numerator becomes:

$$(1-\delta)B\mathbb{E}_{\mathcal{Q}}\underbrace{\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \left(\mathbb{1}_{\{t_0<\tau\leq t_m\}}+\mathbb{1}_{\{t_m<\tau\leq t_{m+1}\}}\right)e^{-\int_0^{t_h} r_u du}\right]}_{(N_1)} \tag{29}$$

Since $\mathbb{E}_{\mathcal{Q}}[\cdot] \geq 0$ and $(t_0, t_m] \cap (t_m, t_{m+1}] = \emptyset$, it follows that $N_0 = N_1$. However, in the denominator, the zero-coupon bond prices are assumed to be strictly positive:

$$\sum_{k=1}^{m} B(0,t_k) < \sum_{k=1}^{m+1} B(0,t_k) \tag{30}$$

Therefore, $\dfrac{\partial c}{\partial T} < 0$.

Zero-coupon Bond Price

$B(0, t_k)$ is the zero-coupon bond price at time 0 with maturity t_k. Let $\mathscr{T} = \{1, ...m\}$, and fix $i \in \mathscr{T}$. If $B(0, t_i)$ increases by $\varepsilon > 0$, it follows:

$$\sum_{k=1}^{m} B(0,t_k) < \sum_{i\notin\mathscr{T}} B(0,t_k) + \left[B(0,t_i)+\varepsilon\right] \implies c\downarrow \tag{31}$$

First Anchor-driven Credit Event

Consider the expression inside the expectation under the risk-neutral probability:

$$\mathbb{E}_{\mathcal{Q}}\underbrace{\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}}e^{-\int_0^{t_h} r_u du}\right]}_{(K_1)} \tag{32}$$

Suppose τ increases by a small amount such that $\tilde{\tau} = \tau + 1$. Since $\tau \in (t_{h-1}, t_h]$ for

$h \in H = \{1, ..., m\}$, one can alternatively characterize an increase in τ as $\tilde{\tau} \in (t_j, t_{j+1}]$ for $j \in J = \{1, ..., m-1\}$. Then, it follows:

$$\bigcup_{j \in J}(t_j, t_{j+1}] \subset \bigcup_{h \in H}(t_h, t_{h+1}] \tag{33}$$

Under the risk-neutral probability $\mathcal{Q}$, it follows that:

$$\mathcal{Q}\left\{\bigcup_{j \in J}(t_j, t_{j+1}]\right\} \subset \mathcal{Q}\left\{\bigcup_{h \in H}(t_h, t_{h+1}]\right\} \tag{34}$$

$$\sum_{j \in J} \mathcal{Q}\left\{(t_j, t_{j+1}]\right\} \leq \sum_{h \in H} \mathcal{Q}\left\{(t_h, t_{h+1}]\right\} \tag{35}$$

Therefore,

$$\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil \tilde{\tau} \rceil}^{\lceil \xi \rceil \wedge (\lceil \tilde{\tau} \rceil + g)} \mathbb{1}_{\{t_0 < \tau \leq t_m = T\}} e^{-\int_0^{t_h} r_u du}\right] < (K1) \implies c \downarrow \tag{36}$$

Successful Replacement for A Departing Anchor

Recall ξ represents the first time a shopping center owner successfully finds a replacement anchor such that $\xi \in (t_{j-1}, t_j]$ for $j \in \{\lceil \tau \rceil, ..., m\}$. Suppose the first successful replacement time is delayed by one period such that $\hat{\xi} \in (t_{j-1}, t_j]$ for $j \in \{\lceil \tau \rceil + 1, ..., m\}$.

$$\min\{\xi, \lceil \tau \rceil + g\} \leq \min\{\hat{\xi}, \lceil \tau \rceil + g\} \tag{37}$$

It implies:

$$\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil \tau \rceil}^{\lceil \xi \rceil \wedge (\lceil \tau \rceil + g)} \mathbb{1}_{\{t_0 < \tau \leq t_m = T\}} e^{-\int_0^{t_h} r_u du}\right] \leq \mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil \tau \rceil}^{\lceil \hat{\xi} \rceil \wedge (\lceil \tau \rceil + g)} \mathbb{1}_{\{t_0 < \tau \leq t_m = T\}} e^{-\int_0^{t_h} r_u du}\right] \tag{38}$$

Hence, c weakly increases.

□

Proof of Proposition 3

Proof. Consider the expectation term of the analytic expression of the co-tenancy premium c for the one landlord, anchor, and tenant model in (3):

$$\underbrace{\mathbb{E}_{\mathcal{Q}}\left[\sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right]}_{(*)} \tag{39}$$

Simplifying the expression in $(*)$,

$$(*) = \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \mathbb{E}_{\mathcal{Q}}\left[\mathbb{1}_{\{t_0<\tau\leq t_m=T\}} e^{-\int_0^{t_h} r_u du}\right] \tag{40}$$

$$= \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \int_0^T \mathbb{E}_{\mathcal{Q}}\left[\mathbb{1}_{\{\tau=s\}} e^{-\int_0^{s+j} r_u du}\right] ds \tag{41}$$

$$= \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \int_0^T \mathbb{E}_{\mathcal{Q}}\left[\mathbb{1}_{\tau=s}\right] \mathbb{E}_{\mathcal{Q}}\left[e^{-\int_0^{s+j} r_u du}\right] ds \tag{42}$$

$$= \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \int_0^T \hat{\lambda}_s e^{-\int_0^s \hat{\lambda}_u du} B(0, s+h) ds \tag{43}$$

Hence, the co-tenancy premium is:

$$c = \frac{(1-\delta)B}{\sum_{k=1}^{m} B(0,t_k)} \sum_{h=\lceil\tau\rceil}^{\lceil\xi\rceil\wedge(\lceil\tau\rceil+g)} \int_0^T \hat{\lambda}_s e^{-\int_0^s \hat{\lambda}(u) du} B(0, s+j) ds.$$

□

Proof of Proposition 4

$$d\lambda_{j,t} = \kappa_j(\mu_j - \lambda_{j,t})dt + \sigma_j\sqrt{\lambda_{j,t}}dW_{j,t} \tag{44}$$

$$P_{JDP} = P\big[(\tau_1 \leq T_1), ..., (\tau_M \leq T_M)\big] = \prod_{j=1}^{M}\Big[1 - e^{\alpha_j(T_j)+\beta_j(T_j)\lambda_j(0)}\Big] \tag{45}$$

Proof. Assume $\alpha_j(T_j)$ and $\beta_j(T_j)$ are continuously differentiable with respect to T_j such that

$$\alpha_j(T_j) = \frac{2\kappa_j\mu_j}{\sigma_j^2}\ln\Big(\frac{2\gamma_j e^{(\gamma_j+\kappa_j)\frac{T_j}{2}}}{2\gamma_j + (\gamma_j+\kappa_j)(e^{\gamma_j T_j}-1)}\Big) \tag{46}$$

$$\beta_j(T_j) = \frac{-2(e^{\gamma_j T_j}-1) + e^{\gamma_j T_j}(\gamma_j - \kappa_j)}{2\gamma_j + (\gamma_j+\kappa_j)(e^{\gamma_j T_j}-1)} \tag{47}$$

$$\gamma_j = \sqrt{\kappa_j^2 + 2\sigma_j^2} \tag{48}$$

Differentiate the joint default probability P_{JDP} in (45) with respect to T_j:

$$\frac{\partial P_{JDP}}{\partial T_j} = \prod_{j=1}^{M}\Big[1 - e^{\alpha_j+\beta_j\lambda_j(0)}\frac{\partial\alpha_j}{\partial T_j} - e^{\alpha_j+\beta_j\lambda_j(0)}\frac{\partial\beta_j}{\partial T_j}\lambda_j(0)\Big] \tag{49}$$

Given $-e^{\alpha_j+\beta_j\lambda_j(0)} < 0$, we observe the following four cases:

Table 10: The Joint Default Probability Sensitivity with respect to Lease Term

Case	$\frac{\partial\alpha_j}{\partial T_j}$	$\frac{\partial\beta_j}{\partial T_j}$	$\frac{\partial P_{JDP}}{\partial T_j}$
1	+	+	$\downarrow$
2	+	-	indet
3	-	+	indet
4	-	-	$\uparrow$

We explore Case 4. Differentiate $\alpha_j(T_j)$ and $\beta_j(T_j)$ with respect to T_j:

$$\frac{\partial\alpha_j T_j}{\partial T_j} = \frac{2\kappa_j\mu_j}{\sigma_j^2}\Big[\frac{\gamma_j+\kappa_j}{2} - \frac{\gamma_j(\gamma_j+\kappa_j)e^{\gamma_j T_j}}{2\gamma_j + (\gamma_j+\kappa_j)(e^{\gamma_j T_j}-1)}\Big] \tag{50}$$

$$\frac{\partial\beta_j T_j}{\partial T_j} = \frac{(-2\gamma_j e^{\gamma_j T_j} + \gamma_j^2 e^{\gamma_j T_j})(2\gamma_j + (\gamma_j+\kappa_j)(e^{\gamma_j T_j}-1)) - (\gamma_j+\kappa_j)[-2(e^{\gamma_j T_j}-1) + e^{\gamma_j T_j}(\gamma_j-\kappa_j)]}{(2\gamma_j + (\gamma_j+\kappa_j)(e^{\gamma_j T_j}-1))^2} \tag{51}$$

Hence, we observe that the join default probability increases with respect to the tenant j's lease term *if and only if*:

$$\frac{\partial \alpha_j}{\partial T_j} < 0 \iff \kappa_j > 3\gamma_j \tag{52}$$

$$\frac{\partial \beta_j}{\partial T_j} < 0 \iff \frac{\gamma_j(\gamma_j - 2)}{\gamma_j + \kappa_j} e^{\gamma_j T_j} < \beta_j \tag{53}$$

□

D Simulation

A Market Leasing Profiles

We use the following market leasing profiles when any of the three types of leases end due to vacancy turnover or default.

A.1 Market Rent

1. All new leases are underwritten to avoid rental income jumps when a new lease starts. New leases are also triple net to keep the expense reimbursement unchanged and avoid discrepancies between in-place and future miscellaneous income.

2. The base rent for the anchor, inline tenant, and co-tenant are set to $15, $30, and $30 per square foot at t=0, with a 3% annual rent adjustment.

3. Co-tenancy provisions (and higher base rent) for the co-tenant are continued to future co-tenants should the co-tenant vacate the property following their lease maturity, a default or an early exit due to the anchor default.

A.2 Leasing Costs

When a new tenant lease starts due to vacancy turnover or lease default, the property incurs the following expenses on the first day of the new lease for all types of tenants:

1. Tenant improvements of $9 per square foot, grown at 3% annually.

2. Leasing commissions of 3% of the total new rental income generated from the new lease.

When a tenant renews at the end of their lease, the property incurs the following expenses on the first day of the new lease for all types of tenants:

1. Tenant improvements of $5 per square foot, grown at 3% annually.

2. Leasing commissions of 1% of the total new rental income generated from the new lease.

B Co-tenancy Provisions

The co-tenancy provisions included provide the co-tenant with the following options:

1. Rent Remedy: following the anchor tenant vacancy from default, rent remedy is available for the co-tenant after a cure period. The size of the rent remedy (between 0% and 100%), the length of the cure period (between 0 and 12 months) and the length of the rent remedy period (between 0 and 24 months) can be changed.

2. Early Exit: Once the rent remedy period is over, the co-tenant must decide whether they want to stay and pay full rent or break the lease at no cost ("fish or cut-bait"). The model allocates a probability that the co-tenant will terminate ("cut bait") that can be set to anything between 0% and 100%. When the co-tenant decides to exit following an anchor tenant prolonged vacancy following default, the space remains vacant for a random period of time between 0 and 12 months (varies with market conditions) and incurs tenant improvements and leasing commissions on the first day of the new lease. The new tenant rent and lease type will be based on the market leasing profile.

C Anchor Tenant Scenarios

We underwrite three types of anchor tenant scenarios to evaluate the impact of the likelihood of the credit event on the price of the co-tenancy provisions. In each case, the rent per square foot from the several tenants remains unchanged.

1. High Credit Worthiness Anchor Tenant: This corresponds to a Walmart type of anchor tenant. In this case the cumulative probability of default is set to 0.1% (over 6 years). The property acquisition price is set to $62 million (7.95% going-in cap). The model uses an exit cap rate of 8.5% to avoid magnifying returns from the resale (no cap rate compression).

2. Moderate Credit Worthiness Anchor Tenant: This corresponds to an anchor tenant 5 times as likely to default. In this case the cumulative probability of default is set to 0.5% (over 6 years). The property acquisition price is set to $57.5 million (8.57% going-in cap). The model uses an exit cap rate of 9.0% to avoid magnifying returns from the resale (no cap rate compression).

3. Low Credit Worthiness Anchor Tenant: This corresponds to an anchor tenant 10 times as likely to default. In this case, the cumulative probability of default is set to 1% (over 6 years). The property acquisition price is equal to $53 million (which corresponds to a 9.29% going-in cap rate). The model uses an exit cap rate of 9.5% to avoid magnifying returns from the resale (no cap rate compression).

D Distributional Assumption

Table 11: Distributional Assumption of the Random Events

Name	Function	Minimum	Maximum	Mean	Std. Deviation
MonthsVacantAnchor	Triangular Distribution (2,14,20)	2.00	20.00	12.00	3.74
MonthsVacantinlineTenant	Triangular Distribution (2,9,12)	2.00	12.00	7.67	2.09
MonthsVacantCoTenant	Triangular Distribution (0,6,12)	0.00	12.00	6	2.00
Random Draw / Renew	Uniform(0,1)	0.00	1.00	0.50	0.29
Random Draw / Renew	Uniform(0,1)	0.00	1.00	0.50	0.29
Random Draw / Vacate	Uniform(0,1)	0.00	1.00	0.50	0.29
Default / 6	Poisson	0.00	infty	0.01	0.09
Default after Fish or Cut Bait / 1	Uniform(0,1)	0.00	1.00	0.50	0.29

E Renewal, Replacement & Default Probabilities

1. Vacancy uncertainty: The space will remain vacant for a random period between two and twenty months.

2. If the anchor tenant defaults, the space will remain vacant for a random period between 2 and 20 months. The new anchor tenant's rent and lease type will be based on market leasing profile.

3. Inline Tenant (without co-tenancy): The renewal probability is 60%. The tenant can also default during the holding period with a cumulative probability of default set to 4%. If the tenant vacates the property at the end of their lease or after a default, the space will remain vacant for a random period of time between 2 and 12 months, and the new in-line tenant's rent and lease type will be based on the market leasing profile (but without co-tenancy provisions).

4. Co-tenant (with co-tenancy): The rent varies depending on the type of co-tenancy provisions. The tenant can also default during the holding period with a cumulative probability of default set to 4%, or after the "fish and cut bait" period following the anchor default. If the tenant vacates the property at the end of their lease or after a default, the space will remain vacant for a random period of time between 0 and 12 months, and the new in-line tenant's rent and lease type will be based on the market leasing profile (but with co-tenancy provisions).