Collective Self-Control

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Abstract

Behavioral economics presents a "paternalistic" rationale for intervention by a benevolent government. This paper studies the desirability of various forms of collective action when government decisions are determined via the political process in response to preferences of time inconsistent voters. We consider an economy where the only "distortion" is the agents' time inconsistency. We first examine a fully decentralized economy where agents can make private "investments" in a commitment technology. We show that the demand for commitment is non monotone in the degree of time inconsistency, with agents with intermediate intensity of present bias exhibiting the highest value of commitment. We then study several forms of collective action. If only commitment decisions are centralized, commitment investment is more moderate than if all decisions are centralized. Welfare consequences of full centralization (of both commitment and consumption decisions) are ambiguous and depend on the distribution of time inconstistency in the population.

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1 Introduction

Traditional public economics provides efficiency rationales for government intervention that are commonly founded in payoff or information externalities. In particular, none of these rationales justify government policy in areas in which agents make private decisions that have limited impact on other agents. The behavioral economics literature has introduced a novel justification for government intervention arising from "paternalistic attitudes."[1] Roughly, a paternalistic government is a benevolent planner that designs policy to help agents make better decisions. This form of paternalism is controversial, partly because it drastically departs from standard normative economics.[2] Nevertheless, it has proven influential. For instance, the current U.K. government coalition program states that: "The Government believes that action is needed to protect consumers, particularly the most vulnerable, and to promote greater competition across the economy. We need to promote more responsible corporate and consumer behaviour through greater transparency and by harnessing the insights from behavioural economics and social psychology." In the U.S., some discussion of social security takes an explicitly paternalistic approach by viewing it as a neccessary program to correct for many individuals' inability to properly save for retirement.[3] In fact, one of the leading proponents of a form of paternalism (Cass Sunstein) has been "regulation czar" in the Obama administration for three years.

The goal of this paper is to develop a tractable model of the potential political economy constraints to a specific form of such paternalistic attitudes. Just as for textbook public policy analysis, it is useful to consider what happens when we abandon the idea of a benevolent planner and instead explicitly model the fact that the political process determines the design of policy. Will politicians seeking election exploit/indulge the voters' behavioral distortions? Are behavioral distortions amenable to aggregation into collective action? What are the implications for the constitutional scope of government activity?

There are of course many types of behavioral distortions, and each of those may lead to its own collective action environment. We focus here on time inconsistency: agents have preferences that display present-bias or quasi-hyperbolic discounting a-la Phelps and Polak (1968) and Laibson (1997).[4] Because of this time inconsistency, agents may display self-control

[1] Camerer et al (2004) contains a number of "second generation" contributions to behavioral economics. Frederick et al. (2002) surveys the experimental evidence on time discounting. Della Vigna (2009) surveys evidence from the field. See also Thaler and Sunstein (2009).

[2] See, for instance, the essays in Caplin and Schotter (2008).

[3] See for instance Diamond (1977), Akerlof (1998), Feldstein (1985), and Imrohoroglu et al. (2003).

[4] Some of the issues related to time inconsistent preferences that we highlight can be represented as problems of self-control, and can also be studied in models of temptation and self-control a-la Gul and

problems that lead to phenomena such as procrastination (doing things too late), preproperation (doing things too early) (see O'Donoghue and Rabin 1999), insufficient savings for retirement (Laibson et al. 1998), harmful obesity and addictions (Gul and Pesendorfer 2007, O'Donoghue and Rabin 2000), etc. These self-control problems also identify a demand for commitment (rehab clinics, illiquid assets with costly withdrawals, and so on) that cannot arise with exponential discounting. In particular, a benevolent government could, in principle, offer commitment instruments that would help the electorate overcome some of the harmful symptoms of time inconsistency.

Time inconsistency and commitment problems have been the focus of a large literature in political economy and macroeconomics, especially in models of government debt and monetary policy. There is evidence that such problems have affected the design and the history of pension systems.[5] In this literature time inconsistency of political choices emerges from the interaction among *time consistent* agents who act at different points in time. Our analysis complements this literature by studying the consequences of having agents with heterogeneous amounts of time inconsistency participate in the political process. For instance, as mentioned above, a public pension system is sometimes defended as a desirable solution to a potential problem of under-saving due to self-control problems. However, the design of such a system should then take into account the political constraints generated by these same self-control problems. For example, this may affect the choice between a pay as you go system and a funded system, the kind of safeguards that are designed into the system, as well as the timing and evolution of the system. It turns out that some of the issues that arise from consideration of time inconsistent agents are quite different from those that have been considered in the literature on time inconsistent policy driven by sequences of time consistent agents.

Once we depart from a world in which policy is determined by hypothetical benevolent social planners, the set of feasible outcomes is constrained by the political incentives faced by politicians or by the bargaining protocols that govern collective action. Time inconsistency offers a simple case study to illustrate how political forces driven by 'behavioral' voters may induce outcomes that differ from those offered by a benevolent social planner. It is easy to construct scenarios in which government intervention leads to worse outcomes than laissez faire. In order to understand the effects of the political process we contrast the outcomes of several political institutions. In the setting of time inconsistency, an important dimension

Pesendorfer (2001, 2004, 2007).

[5] See for instance Persson and Tabellini (1990) for a review of the early literature. Jacobs (2011) provides a comparative history of pension systems, where problems of commitment are emphasized to explain why some countries chose, or eventually turned to, a pay as you go system.

of this comparison of political institutions relates quite naturally to the classic problem regarding the optimal degree of (de-)centralization of resource allocation.

Specifically, we study a simple Wicksellian tree-cutting problem, under the standard specification that the tree is growing in value over time. In our baseline setting, agents have the option of cutting a tree at period 2, which generates a value of v_2, or at period 3, which generates a value of v_3, where $v_3 > v_2$. A tension arises since agents exhibit present bias. At any period, *all future periods* are discounted with a factor of $\beta \leq 1$. Thus, from the perspective of period 1, agents prefer to wait until period 3 to cut the tree. But when period 2 arrives, agents compare an immediate value of v_2 with a discounted value of βv_3 and could potentially prefer to cut the tree early. This problem has been studied by O'Donoghue and Rabin (1999), who show that time inconsistent agents tend to consume (cut the tree) inefficiently early, and that these agents would find it valuable to be able to commit to cut the tree later than they would absent commitment.

We modify the O'Donoghue-Rabin model to allow for continuous choices and costly commitment: by investing resources in period 1, agents can make it costly for their future selves to depart from some pre-specified plan of action. The more investment there is early on in commitment, the costlier it is for future selves to cut the tree too early.

We first consider a fully decentralized environment, a benchmark case in which government plays no role. We show that preferences for investing in commitment are non monotonic in the strength of present bias, and that these preferences may not be single-peaked. Agents with severe present bias (very low β parameters) require large investments in commitment in order to alter the timing of future consumption: commitment may be too expensive for these individuals. On the other hand, agents who exhibit mild present bias (very high β parameters) are able to postpone consumption even absent commitment instruments: commitment is unnecessary for these individuals. Consequently, when all decisions are decentralized, extreme agents on both sides of the spectrum choose little or no investment in commitment, while moderate agents choose more substantial investments.

We then introduce collective action. We assume collective decisions are determined by the outcome of competition between two office-seeking candidates. We outline three different scenarios that vary in terms of which choices (investment in commitment and/or the timing of consumption) are subject to the political process, and which ones are left to individuals. We believe these scenarios offer a simple taxonomy for an array of plausible environments. They also help highlight the sensitivity of generated welfare levels to the aspects, or timing, of choices in which collective action comes into play.[6]

[6] We abuse terminology by referring to "welfare" as the utilitarian social surplus measured for the period

Suppose first that commitment decisions are decentralized, while allocation decisions, regarding when to cut the tree, are taken by a centralized government through a voting mechanism. More concretely, in the second period each of the two candidates offers a platform specifying the fraction of the tree that they would cut in that period and majority vote determines which platform gets implemented. The median β is decisive and determines the equilibrium amount of tree-cutting in the second period. However, no individual makes any investment in commitment: government intervention completely undermines incentives to invest in commitment because of free riding in commitment investments. Agents know that their individual commitment decisions have no effect on the allocation decision that results from the voting mechanism, and therefore have no incentive to invest in commitment. Nonetheless, if the median agent is not prone to a strong present-bias, i.e., the decisive agent is virtuous, the political process would lead to delayed consumption and high welfare levels. If, in contrast, the median agent is prone to a strong present bias, consumption would occur early and the process would be particularly inefficient.

Consider next the case in which allocation decisions are taken privately in a decentralized manner, but commitment decisions are centralized. I.e., the two candidates compete in period 1 (via majority rule) over platforms specifying the levels of commitment. This scenario is a natural way to think of many applications as commitment decisions might involve, for instance, setting up fines for consuming savings (say, retirement savings) too early, prohibition legislation, etc. Analysis of this case is more subtle because of the non monotonic amount of commitment desired in the population, and since it is possible that preferences are not single-peaked. Equilibrium outcomes depend crucially on the characteristics of the commitment technology. A pure strategy equilibrium may not exist, and even when it does, because of the non monotonicity of ideal levels of commitment, the decisive voter is typically not the agent with the median present-bias parameter. When a pure strategy equilibrium does not exist, we show that there is a continuum of mixed strategy equilibria with a two-point support. The equilibrium with minimal commitment level places a 50% probability of zero investment in commitment. The maximal one places 50% probability on the level of commitment that provides zero value of commitment for a marginal voter. The generated welfare levels in this setting are always dominated by those generated in the fully decentralized setting. Indeed, in the fully decentralized environment nothing prevents agents from privately choosing the level of commitment that emerges in the centralized commitment scenario.

The last system we consider is one that is fully centralized, where both commitment

1 selves of the voters. We acknowledge that other criteria are relevant and we discuss this more explicitly in Section 6.

levels and the timing of consumption are decided upon collectively. In this case, the median present-bias voter determines both decisions. When collectively deciding on commitment investments, voters aim at providing commitment for the median voter in the subsequent period. In contrast with the decentralized consumption scenarios, where the desired commitment levels are non monotone in β, in the fully centralized environment these desired commitment levels are increasing in β. In equilibrium, positive commitment may take place if the median voter is sufficiently moderate, and the population inherits the virtues or biases of its median voter. As a consequence, when the median voter exhibits a weak but substantial present bias, this system generates the highest welfare levels. Indeed, when commitment decisions are decentralized, the median voter would still opt for early consumption. Thus, in this case full centralization allows the electorate to tailor commitment levels to the median voter, who does not require very costly commitment investments in order to delay consumption.

The comparison of these scenarios that differ in degrees and timing of centralization shows that the welfare consequences of government intervention are fairly nuanced when we take into account the fact that behavioral agents are also political actors, electing the government that is charged with "solving" their behavioral biases. Thus, for instance, outcomes can be worse under centralization than those generated by a laissez faire economy in which all decisions are decentralized. However, particular forms of intervention can be useful. Welfare consequences are sensitive both to the distribution of preferences in the electorate, and to the precise timing in which government intervenes. In particular, when the median voter is not prone to strong present biases, interventions under which the timing of consumption is decided upon collectively are welfare enhancing. They allow the electorate to effectively delegate decisions to a virtuous median voter.

2 Related Literature

Some authors (Benjamin and Laibson 2003, Caplan 2007, Glaeser 2006, Rizzo and Whitman 2009 a, b) have informally made the point that when government is not run by a benevolent social planner but by politicians influenced by voting decisions, it is not clear that government intervention is beneficial. In fact, Glaeser and Caplan explicitly make the case that, if voters are boundedly rational, then the case for limited government may be even stronger than in standard models.[7] Krusell et al. (2002, 2010) examine government policy for agents who

[7] Bendor et al. (2011) presents models of boundedly rational voters that are successful in matching some features of elections that are hard to explain with rational voter models.

suffer self-control problems. Krusell et al. (2002) consider a neoclassical growth model with quasi-hyperbolic consumers. They show that, when government is benevolent but cannot commit, decentralized allocations are Pareto superior. This is due to a general equilibrium effect of savings that exacerbates an under-saving problem. Benabou and Tirole (2006) discuss how endogenously biased beliefs that are chosen by individuals for self-motivation can generate a belief in a just (unjust) world and ultimately affect redistributive politics.

As mentioned earlier, the current paper is also related to the literature on dynamic inconsistency of political-economic decision making (e.g., Persson and Tabellini 1990, Alesina and Tabellini 1990). In those models, voters are time consistent, but the identity of the decision maker (or decisive voter) changes over time, generating time inconsistent policies. This in turn creates an incentive for early decision makers to manipulate state variables, such as debt, in order to influence subsequent decisions.

Bisin, Lizzeri, and Yariv (2011) studies a model of fiscal irresponsibility and public debt in the presence of time inconsistent voters. The model they consider captures environments where it is either impossible for government to help agents to achieve commitments or it is positively harmful for the government to do so. Their model does not quite fit into any of the scenarios that we discuss in this paper, but does highlight the potential harmful effects government intervention may have in the realm of fiscal policy when voters exhibit time inconsistencies. In fact, the paper offers a new rationale for balanced budget rules in constitutions as they restrain governments' responses to voters' desires.[8] Hwang and Mollerstrom (2012) study political reform with time inconsistent voters and show that gradualism emerges in equilibrium as a consequence of time inconsistency. They also show that election of a patient agenda setter can arise in equilibrium.

3 A Tree Cutting Model

2.1 Preferences and Consumption Possibilities

A continuum of agents decides collectively on the timing of consumption. There are three periods. In period 1 agents make "commitment" decisions (that we specify below). In periods 2 and 3 agents consume fractions of a "tree" of growing value. The tree is worth

[8]Gottlieb (2008) studies the optimal design of nonexclusive contracts when firms compete over time inconsistent consumers. The paper studies the asymmetry between immediate-cost goods and immediate-reward goods that are generated by nonexclusivity. To the extent that firms are akin to political competitors, some of the underlying forces in that paper are relevant for the study of political processes with a time inconsistent electorate.

v_2 in period 2, and v_3 in period 3. We assume that $v_2 < v_3$.[9] In period 2 agents choose a fraction x of the tree to consume in period 2, with $1-x$ remaining to be consumed in period 3. We interpret period 3 as the natural moment of maturity of the tree so that there is an extra cost in cutting part of the tree in period 2. This cost is given by the function $k(x,c)$, where c is a parameter that is determined in the first period. We assume that $\frac{\partial k(x,c)}{\partial x} \geq 0$, $\frac{\partial^2 k(x,c)}{\partial x^2} > 0$, $\frac{\partial k(x,c)}{\partial c} > 0$, $\frac{\partial^2 k(x,c)}{\partial x \partial c} > 0$. That is, cutting costs are weakly increasing and convex in the amount of the tree that is cut x and in the extent of commitment in place, as given by the commitment parameter c. The marginal cost of early consumption is also increasing in the commitment parameter c.

Agents have $\beta - \delta$ preferences. That is, for any payoffs u_2 and u_3 in periods 2 and 3, respectively, the assessed utility at time t, denoted by U_t, is given by:

$$\begin{aligned} U_1 &= \beta\delta u_2 + \beta\delta^2 u_3, \\ U_2 &= u_2 + \beta\delta u_3, \\ U_3 &= u_3. \end{aligned}$$

We assume that the parameter β is distributed according to a continuous distribution $G[0,1]$ in the population with a median parameter of β_M.

An agent with parameter β has a utility at $t=2$ given by:

$$U_2(x,c,\beta) = v_2 x - k(x,c) + \beta\delta v_3 (1-x)$$

In period 1 a parameter c is chosen (by a collective action process that we soon specify). This parameter raises the cost of cutting the tree early: we assume that $k(x,c)$ is increasing in c for all x. Thus, c serves as a commitment mechanism to delay consumption to period 3. This commitment is costly in period 1: choosing c costs $I(c)$, where we assume $I(0) = 0, I'(0) = 0, I'(c) \geq 0$, and $I''(c) > 0$ for all c.[10] Utility in period 1 is given by

$$U_1(x,c,\beta) = \beta\delta\left(v_2 x - k(x,c)\right) + \beta\delta^2 v_3 (1-x) - I(c).$$

Agents are assumed to be *sophisticated*, in the sense that they are aware that they are time inconsistent. O'Donoughue and Rabin (1999) analyzed the single person decision problem in this environment, by using the notion of perception perfect equilibrium. When agents are sophisticated, this boils down to preferences that are specified a-la Strotz (1955) who perform backwards induction. We assume sophistication because we want to study how the demand for commitment is mediated by the political system.[11]

[9] Our qualitative results remain in the presence of uncertainty over future tree values.

[10] The assumption that $I(0) = 0$ is not restrictive. Indeed, assuming $I(0) > 0$ is tantamount to assuming there is a fixed cost to entering our economy.

[11] We return to a discussion of the impact of naievete in our setting in the Conclusions.

For notational simplicity, we assume $\delta = 1$ for the remainder of the analysis. This assumption is effectively without loss of generality (as discounting can be encoded in the v sequence of tree values).

2.2 The Political Process

There are two candidates running for office. Candidates are office motivated, receiving some positive payoffs from each electoral victory. It will be clear that candidates' time preferences play no role.[12] We assume that the electorate has no ideological attachment to the candidates. In Appendix B, we allow agents to have idiosyncratic ideological preferences (as in Lindbeck and Weibull 1987). We distinguish between three types of environments. These are meant to capture different collective action settings and highlight the effects of the timing of collective decisions on commitment choices.

Centralized commitment, Centralized choice. Elections occur in periods $t = 1, 2$. At $t = 1$, each candidate offers a platform consisting of a cost c that determines the cost of consumption in period 2 later on. Majority voting determines which outcome, and corresponding platform, is elected (we assume that ties are broken with a toss of a coin). All agents experience an immediate commitment cost of $I(c)$ at $t = 1$. At $t = 2$, the candidates each offer a fraction x of the tree to be consumed in period 2 and majority rule (with random breaking of ties) determines which policy is implemented. If an amount x of the tree is consumed at $t = 2$, an agent with taste parameter β receives the value of $v_2 x + \beta v_3(1 - x)$. All agents experience an immediate cost of $k(x, c)$.

Dentralized commitment, Centralized choice. At $t = 1$, agents choose individually the parameter c that will induce their commitment-breaking costs at time $t = 2$, the cost of which is immediate and given by $I(c)$. At $t = 2$, fraction x of the tree to be consumed in period 2 and majority rule (with random breaking of ties) determines which policy is implemented for the entire population. An individual with taste parameter β who chose a commitment parameter of c at $t = 1$ receives a net value of $v_2 x + \beta v_3(1 - x)$ and experiences an immediate cost of $k(x, c)$.

Centralized commitment, Decentralized choice. Elections occur only in period $t = 1$, when each candidate offers a platform consisting of a commitment parameter c

[12] This is not to say that time inconsistencies cannot take place directly at the political level. As mentioned in the discussion of the related literature, there is a body of work that focuses on time consistency of government policy. There is also work (e.g., Lagunoff, 2008) that shows that, if one considers governments that have policy preferences and that know that they may be kicked out of office with positive probability, endogenous present bias may emerge.

involving an immediate commitment cost of $I(c)$. Majority voting determines which outcome, and corresponding platform is elected (again, ties are broken randomly). At $t = 2$, each of the individual agents decides what fraction x of the tree to consume. An individual with taste parameter β who chooses to consume a fraction x of the tree $t = 2$ receives a net value of $v_2 x + \beta v_3 (1 - x)$ and experiences an immediate cost of $k(x, c)$.

Notice that had commitment been free, all agents that would naturally consume early in period 2 would commit themselves to later consumption. Consequently, decentralized decisions would lead to first-best outcomes in which all agents would efficiently and fully delay their consumption. The introduction of commitment costs introduces a non-trivial cost-benefit trade-off in decentralized economies, that we soon analyze, which creates room for potentially beneficial government intervention.

4 Decentralized Outcomes

Before inspecting the impacts of collective action on commitment decisions, we describe each agent's individual decisions. This analysis corresponds to the case in which all decisions are made in a decentralized fashion.

Given the value of c determined in the first period, in the second period the agent's problem is given by:

$$\max_x U_2(x, c, \beta) \Longleftrightarrow \max_x v_2 x - k\left(x, c\right) + \beta v_3 \left(1 - x\right).$$

Let $x\left(c, \beta\right)$ denote the solution to this problem.

$$\frac{\partial U_2}{\partial x} = 0 \Longrightarrow x(c, \beta) = \begin{cases} 1 & \beta < \left(v_2 - \frac{\partial k(1,c)}{\partial x}\right)/v_3 \\ (v_2 - \beta v_3) = \frac{\partial k(x(c,\beta),c)}{\partial x} & \left(v_2 - \frac{\partial k(1,c)}{\partial x}\right)/v_3 \leq \beta < \left(v_2 - \frac{\partial k(0,c)}{\partial x}\right)/v_3 \\ 0 & \beta \geq \left(v_2 - \frac{\partial k(0,c)}{\partial x}\right)/v_3 \end{cases} . \tag{1}$$

Intuitively, whenever the agent either experiences less present bias or higher marginal costs of immediate consumption, delay is more likely. At the extremes, if marginal costs of cutting the whole tree are not too high (namely, $\frac{\partial k(1,c)}{\partial x} < v_2$), very impatient agents will not delay any consumption. Virtuous agents, for whom the marginal costs of very little early consumption outweigh the benefits, will cut the entire tree in period 3. The monotonicity of the consumption function $x(c, \beta)$ is captured by the following lemma, which will be useful for our analysis of the collective choice settings.

Lemma 1 (Consumption Monotonicity) *The fraction of the tree consumed in period* 2, $x(c, \beta)$, *is decreasing in both* c *and* β.

Assume that the fundamentals are such that $x(c, \beta)$ is differentiable with respect to both c and β whenever positive. The first period problem can then be written as:

$$\max_c U_1(x, c, \beta) \Longleftrightarrow \max_c \beta v_3 + x(c, \beta)(\beta v_2 - \beta v_3) - \beta k(x(c, \beta), c) - I(c)$$

Let $c(\beta)$ be the solution of this problem. We want to understand the dependence of the commitment parameter $c(\beta)$ on β, will be an essential input into the collective action problem.

In order to glean some intuition on the dependence of c on β, suppose that fundamentals are such that $x(c, \beta)$ is differentiable with respect to c. Notice that:

$$\frac{\partial U_1}{\partial c} = \frac{\partial x(c, \beta)}{\partial c} \beta \left((v_2 - v_3) - \frac{\partial k(x(c, \beta), c)}{\partial x} \right) - \beta \frac{\partial k(x(c, \beta), c)}{\partial c} - I'(c).$$

In contrast to the standard dynamic optimization problem with geometric discounters, the envelope condition fails and the indirect effect on period 2 consumption does not disappear. Indeed, substituting the second period first-order conditions we obtain:

$$\frac{\partial U_1}{\partial c} = -\beta \left(\frac{\partial x(c, \beta)}{\partial c} ((1 - \beta) v_3) + \frac{\partial k(x(c, \beta), c)}{\partial c} \right) - I'(c).$$

There are several effects of changes in β on the optimal choice of c. As β increases more weight is put on the future, pushing for more investment now. Furthermore, the fraction of the tree consumed in period 2, $x(c, \beta)$, is smaller, leading to a smaller marginal cost $\frac{\partial k(x(c,\beta),c)}{\partial c}$ tomorrow. Nonetheless, time inconsistency is less relevant, so the benefit of $(1 - \beta) v_3$ is smaller. When β is close to zero or close to $\frac{v_2}{v_3}$ period 1 investment will be zero, so investment is not monotone. Intuitively, agents for whom time inconsistency is very severe foresee that reasonably priced commitments will not save them from excessive consumption in period 2 and therefore acquire limited commitment. On the other side of the spectrum, agents for whom time inconsistency is very weak, do not suffer from great temptation in period 2 and therefore do not require extreme commitment to enable them to postpone consumption.

In general, $c(\beta)$ can achieve several local maxima between 0 and $\frac{v_2}{v_3}$. The following example illustrates a case in which $c(\beta)$ is concave in this region and only one maximum exists.

Example (Quadratic Commitment Costs) Consider the case of $k(x, c) = (c + v_2) \frac{x^2}{2}$ and $I(c) = \frac{c^2}{2}$. The second period utility is then given by:

$$U_2(x, c, \beta) = \beta v_3 + x(v_2 - \beta v_3) - (c + v_2) \frac{x^2}{2}$$

and the corresponding first-order condition requires that:

$$x\left(c,\beta\right)=\begin{cases}\frac{(v_2-\beta v_3)}{(c+v_2)} & \beta\leq\frac{v_2}{v_3}\\ 0 & \beta>\frac{v_2}{v_3}\end{cases}.$$

Notice that $x(c,\beta)$ is decreasing in β,achieving the maximal value of 1 when $\beta=0$.[13] This generates a second period utility of:

$$U_2(x(c,\beta),c,\beta)=\beta v_3+\frac{\left(v_2-\beta v_3\right)^2}{2\left(c+v_2\right)}.$$

Plugging these values into period 1's objective function yields:

$$U_1=\beta v_3+\beta\frac{\left(v_2-\beta v_3\right)}{\left(c+v_2\right)}\left(v_2-v_3\right)-\beta\frac{\left(v_2-\beta v_3\right)^2}{2\left(c+v_2\right)}-\frac{c^2}{2}.$$

The optimum is given by:

$$c(\beta)=\begin{cases}\frac{\alpha_1}{\alpha_2\sqrt[3]{P_3(\beta)+\sqrt{P_6(\beta)}}}+\sqrt[3]{P_3(\beta)+\sqrt{P_6(\beta)}}-\alpha_3 & 0<\beta\leq\frac{v_2}{v_3}\\ 0 & \frac{v_2}{v_3}<\beta\leq 1\end{cases},$$

where $\alpha_1,\alpha_2,\alpha_3$ are positive constants depending on v_2 and v_3, while $P_k(\beta)$ is a polynomial of degree k in β (with coefficients determined by v_2 and v_3).

Figure 1 illustrates the emerging result of $c(\beta)$. As highlighted by the figure, the greatest commitment constraints are chosen by individuals with moderate levels of time inconsistency.

5 Electoral Outcomes

We now turn to inspect the effects of collective action on agents' choices. We start by analyzing the case in which only the choice of commitment levels is done through an electoral process. We then proceed to a case in which both commitment and the timing of consumption are decided upon collectively.

5.1 Collective Commitment with Decentralized Choice

In this setting, the commitment parameter c is determined collectively. From the point of view of an agent of type β, the voting problem is determined as follows. From the analysis of

[13] In particular, our specification of the cost function $k(x,c)$ assures that consumption is interior for $\beta\in(0,\frac{v_2}{v_3})$.

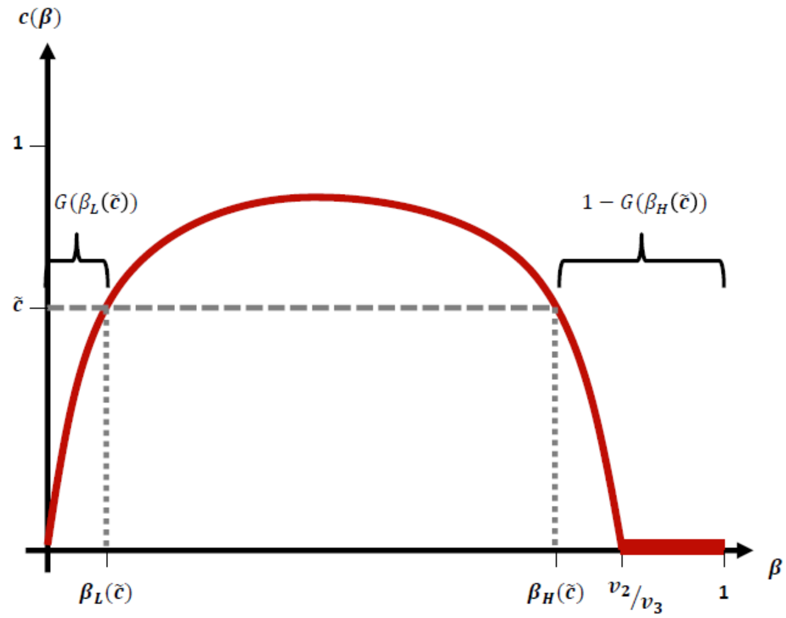

Figure 1: Commitment for the Quadratic Case

the private decision problem of an agent of type β, if a commitment parameter c is chosen, and subsequent choices are made optimally by the agent, period 1 utility is given by

$$U_1\left(x\left(c,\beta\right),c,\beta\right) = \beta v_3 + x\left(c,\beta\right)\left(\beta v_2 - \beta v_3\right) - \beta k\left(x\left(c,\beta\right),c\right) - I\left(c\right).$$

Thus, the agent votes for candidate 1 offering commitment c_1 over candidate 2, who offers commitment c_2, whenever

$$U_1\left(x\left(c_1,\beta\right),c_1,\beta\right) > U_1\left(x\left(c_2,\beta\right),c_2,\beta\right).$$

Proposition 1 *Assume that* $\frac{\partial k(1,c)}{\partial x} \geq v_2$ *for all* c. *There is a unique pure strategy equilibrium of the collective commitment game in which both candidates offer a platform* c^{CD}. *Furthermore, when* $c(\beta)$ *has a unique local maximum in* $\left(0, \frac{v_2}{v_3}\right)$, *the platform* c^{CD} *corresponds to the ideal policy for a voter of type* β^{CD}, *where* β^{CD} *is higher than the median* β, $\beta^{CD} \geq \beta_M$.

The quadratic case in the example above is useful in illustrating the intuition underlying Proposition 1. Consider Figure 1. If $1 - G(\frac{v_2}{v_3}) \geq 1/2$, there is a majority of agents who prefer no commitment and the equilibrium commitment parameter is naturally $c^{CD} = 0$, which coincides with that preferred by the median. Otherwise, for every $\tilde{c} > 0$, define

$\beta_L(\tilde{c})$ and $\beta_H(\tilde{c})$ such that $\tilde{c}$ is their ideal point, i.e. $c(\beta_L(\tilde{c})) = c(\beta_H(\tilde{c})) = \tilde{c}$. All agents with preference parameters below $\beta_L(\tilde{c})$ and above $\beta_H(\tilde{c})$ prefer commitment parameters lower than $\tilde{c}$, while agents with preference parameters between $\beta_L(\tilde{c})$ and $\beta_H(\tilde{c})$ prefer preference parameters above $\tilde{c}$. In particular, the equilibrium commitment parameter c^{CD} is chosen so that these two classes of agents are of equal proportions. That is, $G(\beta_L(c^{CD})) + (1 - G(\beta_H(c^{CD}))) = 1/2$. By construction, $\beta_M \in (\beta_L(c^{CD}), \beta_H(c^{CD}))$ and the result follows. In fact, note that in this case the equilibrium commitment level corresponds to a voter of type β^{CD} that is strictly higher than the median, $\beta^{CD} > \beta_M$.[14]

This construction of the equilibrium level of commitment can be adapted to environments in which $c(\beta)$ entails several local maxima, it is only the relation to the median agent's preferred level of commitment that hinges on $c(\beta)$ having a unique maximum. However, the construction does rely on all agents having single-peaked preferences with respect to the commitment parameter c. Indeed, in this case, agents with high taste parameter β prefer no investment in commitment, while all others prefer a positive amount of commitment. The condition on $\frac{\partial k(1,c)}{\partial x}$ assure that even individuals with very low parameters β benefit from some level of commitment.

When preferences are not single-peaked, this analysis breaks down. This case arises when $\frac{\partial k(1,c)}{\partial x} < v_2$ for some c, and a policy of no commitment is a local optimum for most β's. We will now outline what happens when preferences are not single peaked by considering the special case of linear costs. This case is useful since its structure is rather simple.

When consumption costs are linear, we can normalize the problems' fundamentals so that $k(x, c) = cx$. Furthermore, the optimal choice in the second period is generically either $x = 0$ or $x = 1$. In case of indifference, we will assume that an agent breaks the indifference to favor her "commitment self," i.e., she chooses $x = 0$.[15]

Suppose that in period 1 a cost c was chosen, and consider the period 2 choice problem of a voter of type β. She will wait until time 3 if and only if

$$U_2 = v_2 - c \leq \beta v_3. \tag{2}$$

[14]The construction suggests that median preserving spreads of the distribution G would lead to lower equilibrium commitment levels.

[15]This setting can be fit a special case of Gul and Pesendorfer (2001, 2004, 2007) type of preferences. Namely, suppose that two functions govern an individual's utility from consumption: $u(x)$ is the direct utility of x, while $v(y)$ is the temptation cost of not having consumed y available at the time of choice. In such a setting, in order to delay consumption in period 2, $u(v_3) - v(v_2) \geq u(v_2)$. Suppose $u(x) = x$ and $u(y) = \alpha y$, where $\alpha > 0$. Then delayed consumption in period 3 occurs when $v_3 \geq v_2(1+\alpha)$, which is analogous to our linear costs case when taking $\beta = \frac{1}{1+\alpha}$.

Thus, as before, agents with $\beta > \frac{v_2}{v_3}$ are not willing to pay for commitment: they do not find it necessary.

Commitment is perceived beneficial in period 1 if the delay in consumption due to commitment is worth its costs $I(c)$. That is, whenever there is a commitment parameter c such that:

$$\beta v_3 - I(c) \geq \beta v_2 \iff \beta(v_3 - v_2) \geq I(c). \tag{3}$$

How do investment incentives vary with β? It is very difficult (and costly) to make low β agents wait until period three to consume. On the other side of the spectrum, high β agents are virtuous and will wait till period 3 even with no commitment instruments. Therefore, investment only pays for intermediate β's.

Thus, as in the case studied previously, incentives to invest are not monotonic in β since both low and high β's dislike investment (for different reasons). However, unlike the previous case, utilities are *not single-peaked* with respect to the commitment c: for intermediate β's payoffs are first decreasing in c because we violate condition (2) and so commitment initially affects utility only through its costs, but carries no benefits in terms of the timing of consumption, until we reach a level of commitment c^* such that condition (2) is satisfied, so that $c = 0$ and $c = c^*$ are both local optima.

Recall that for all agents of preference parameter $\beta \geq \frac{v_2}{v_3}$, there is no willingness to pay for commitment no matter what the commitment technology is. We denote by $\beta_H \equiv \frac{v_2}{v_3}$. If $1 - G(\beta_H) \geq 1/2$, there is a majority supporting no commitment and, as before, there is a unique equilibrium in which both candidates offer commitment $c^{CD} = 0$. Suppose there is a substantial fraction of the population that is moderate, $1 - G(\beta_H) < 1/2$. Now note that by raising c we obtain an increasing mass of β's for which $\beta v_3 \geq v_2 - c$. Let $\beta(c) \equiv \frac{v_2 - c}{v_3}$. The mass is given by $G(\beta(c))$. Define c_L such that

$$G(\beta_H) - G(\beta(c_L)) = \frac{1}{2}$$

and let $\beta_L \equiv \beta(c_L)$.

Let $\tilde{c}$ be the unique commitment level such that[16]

$$\beta(\tilde{c})(v_3 - v_2) = I(\tilde{c}).$$

The next result characterizes the equilibria in this environment.

Proposition 2 *Assume that $k(x, c) = cx$.*

[16]Note that $\beta(\tilde{c})(v_3 - v_2)$ is decreasing in $\tilde{c}$. Since $\beta(0)(v_3 - v_2) > I(0) = 0$ and $0 = \beta(v_2)(v_3 - v_2) < I(v_2)$, the existence of a unique $\tilde{c} \in (0, v_2)$ satisfying the equality is guaranteed.

1. *If* $\beta_L (v_3 - v_2) \leq I(c_L)$*, there exists a unique equilibrium with investment of zero in commitment instruments.*

2. *If* $\beta_L (v_3 - v_2) > I(c_L)$*, there is no pure strategy equilibrium. In this case, there is a continuum of equilibria in mixed strategies. All symmetric profiles having a two-point support* $c_1 < c_2$ *such that there is a mass of 50% of* β*'s between* c_1 *and* c_2*, where* $c_2 \in [c_L, \tilde{c}]$*, constitute part of an equilibrium.*

The intuition for the non existence of positive commitment pure strategy equilibria is the following. Assume $c > 0$ is part of an equilibrium. A deviation to a slightly lower commitment level attracts votes from two groups of voters: all (low) $\beta' s$ for whom c is not sufficient to generate delay and so a lower c is preferable, and all (high) β-agents for whom c is more than enough. Thus, support for the deviating candidate is overwhelming, with the extremes 'squeezing' the middle. Zero commitment is an equilibrium if the commitment technology is not 'too efficient.' If, however, investment is very cheap ($I(c)$ is very low), then zero commitment cannot be an equilibrium because a 'global' deviation to a large commitment would attract a majority of support. The proposition describes the mixed strategy equilibria in this case.

5.2 Collective Commitment with Centralized Choice

We now discuss the case in which the second period choice is also taken via collective action. Two office-motivated candidates, 1 and 2, offer platforms x_1 and x_2 in the second period.

From the analysis of individual choices, recall that (1) provides the second period optimal choice $x(c, \beta)$ for any given commitment parameter c selected in period 1. From Lemma 1, $x(c, \beta)$ is decreasing in β. It is then clear that for any given choice of c in the first period, both candidates will choose to offer the ideal policy of the median voter β_M. Thus, the second period choice will be $x(c, \beta_M)$.

We can now step back and consider a generic voter's first period utility in this scenario.

$$U_1 (x(c, \beta_M), c, \beta) = \beta v_3 + x(c, \beta_M)(\beta v_2 - \beta v_3) - \beta k(x(c, \beta_M), c) - I(c). \qquad (4)$$

Since $x(c, \beta_M)$ is fixed for all β, the choice of commitment in the first period is driven by the desire to commit of an agent of median taste parameter β_M. Denote by $c(\beta, \beta_M)$ the (constrained) optimal commitment parameter for an agent of taste β, foreseeing the second period choice being determined according to the taste of the median parameter β_M. As it turns out, $c(\beta, \beta_M)$ is monotonic in β, with individuals who care more about future consumption preferring greater investment in commitment, as illustrated in the following lemma.

Lemma 2 (Constrained Commitment Monotonicity) *The optimal constrained commitment $c(\beta, \beta_M)$ is increasing in β.*

Not that the monotonicity in β of desired commitment is in contrast with the analysis of both the fully decentralized scenario as well as of the centralized commitment with decentralized choice scenario. The logic for this is the following. The value of investment in commitment is now in reducing incentives for the median agent to cut the tree early. This is particularly valuable for the high-β agents. The force behind the desired commitment level being decreasing in β_M is that marginal benefit of commitment $(1 - \beta_M) v_3$ is higher when β_M is lower.

Lemma 2 implies that it is median preferences that determine first period choices as well. This is captured in the following proposition.

Proposition 3 *When both commitment and consumption are chosen collectively, equilibrium outcomes coincide with those chosen optimally by agents with the median taste parameter β_M.*

It also interesting to highlight how optimal constrained commitment $c(\beta, \beta_M)$ changes as β_M changes. For the case of interior solutions we can show the following.

Remark *Assume that $\frac{\partial k(1,c)}{\partial x} \geq v_2$. Then the optimal constrained commitment $c(\beta, \beta_M)$ is decreasing in β_M.*

The following example illustrates how the optimal constrained commitment and the equilibrium outcome work for the case of quadratic consumption costs.

Example 2 (Quadratic Costs – Fully Centralized Solutions) Consider the setting of Example 1. Assume first $\beta_M < \frac{v_2}{v_3}$. Plug in $x(c, \beta_M)$ into U_2 to get

$$U_2(x(c, \beta_M), c, \beta) = \beta v_3 + \frac{v_2 - \beta_M v_3}{c + v_2}(v_2 - \beta v_3) - \frac{(v_2 - \beta_M v_3)^2}{2(c + v_2)}.$$

Moving back to period 1 we obtain:

$$U_1(x(c, \beta_M), c, \beta) = \beta v_3 + \frac{v_2 - \beta_M v_3}{c + v_2}(\beta v_2 - \beta v_3) - \beta \frac{(v_2 - \beta_M v_3)^2}{2(c + v_2)} - \frac{c^2}{2}.$$

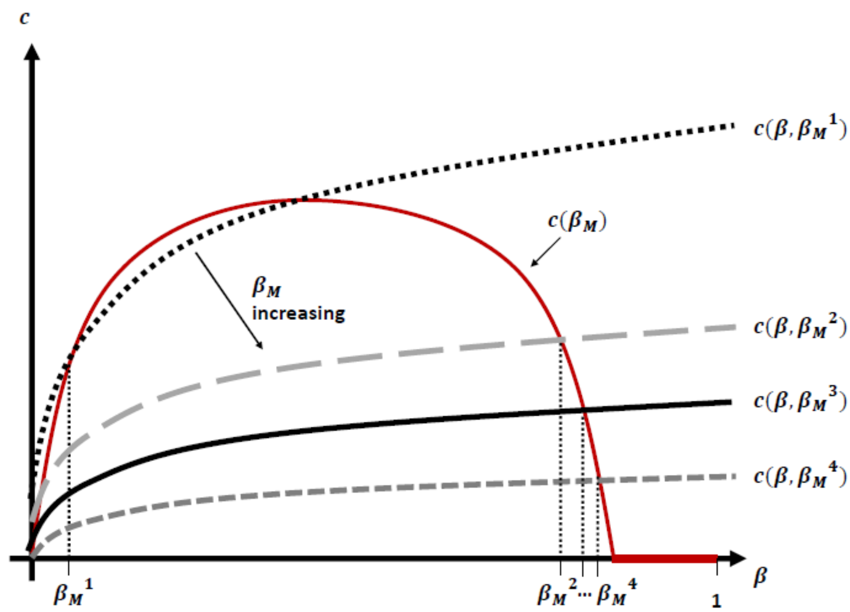

Figure 2: Constrained Commitment for Different Median Preferences

The optimal choice of commitment is given by:

$$c(\beta,\beta_M) = \frac{\tilde{\alpha}_1}{\sqrt[3]{P_1(\beta)+\sqrt{P_2(\beta)}}} + \sqrt[3]{P_1(\beta)+\sqrt{P_2(\beta)}} - \tilde{\alpha}_2,$$

where the positive constants $\tilde{\alpha}_1, \tilde{\alpha}_2$, as well as the coefficients of the polynomials $P_1(\beta)$ and $P_2(\beta)$ (of degrees 1 and 2,respectively) are functions of v_1, v_2, and β_M.

Figure 2 depicts $c\left(\beta,\beta_M\right)$ for different values of $\beta_M \in \left(0, \frac{v_2}{v_3}\right)$.

As Figure 2 illustrates, the optimal desired amount of commitment $c(\beta,\beta_M)$ is increasing in β and decreasing in β_M. Notice, however, that the equilibrium level of commitment is given by $c(\beta_M,\beta_M) \equiv c(\beta_M)$, which is not monotonic.

We can now compare the level of commitment in the two collective action scenarios. When preferences are single-peaked and $c(\beta)$ has a unique maximum, Proposition 1 assures that the platform c^{CD} chosen in equilibrium corresponds to the ideal policy for a voter of type β^{CD}, where β^{CD} is higher than the median β, $\beta^{CD} \geq \beta_M$. Furthermore, the construction of the proof of Proposition 1 (extending that appearing for the quadratic case in Example 1) illustrates that $c(\beta^{CD}) \leq c(\beta_M)$ when $c(\beta)$ has a unique local maximum. In particular, we have the following proposition.

Proposition 4 *Assume that* $\frac{\partial k(1,c)}{\partial x} \geq v_2$ *and suppose that* $c(\beta)$ *has a unique local maximum in* $\left(0, \frac{v_2}{v_3}\right)$. *The equilibrium choice of commitment is higher under full centralization than in the decentralized choice scenario.*

When $c(\beta)$ has multiple local maxima, the comparison between the equilibrium commitment levels generated by full centralization and decentralized choice is inconclusive and, in principle, can go either way.

We now discuss environments in which the commitment technology does not lead agents to have single-peaked preferences. As before, for presentation simplicity, we discuss the case of linear consumption costs, $k(x, c) = cx$.

In this case, incentives to vote for investment in the first period may be high for high β individuals. The optimal commitment parameter c is either 0 or the c^* that is just sufficient to make the median β individual choose consumption at period 3,i.e., the minimal level of cost that solves

$$v_2 - c^* \geq \beta_M v_3 \text{ or } c^* = \max\{v_2 - \beta_M v_3, 0\}.$$

In period 1, all voters such that $\beta(v_3 - v_2) \geq I(c^*)$ or β's such that $\beta \geq \frac{I(c^*)}{(v_3 - v_2)}$ prefer c^* to 0; all agents with lower β's prefer 0. Thus, there can be a broad consensus in favor of investing.

Proposition 5 *Suppose* $k(x, c) = cx$. *There exist* $\check{\beta}, \hat{\beta}$ *such that if* $\beta_M \leq \check{\beta}$ *or* $\beta_M \geq \hat{\beta}$, *there is a unique equilibrium with* $c = 0$, *and if* $\beta_M \in \left(\check{\beta}, \hat{\beta}\right)$, *there is an equilibrium with positive commitment.*

In terms of welfare, there are two effects that play an important role. First, some individuals (high β's) would not have needed investment left to their own devices, but are pushed to support non-trivial commitment so as to tie the hands of the median voter in period 2. On the other hand, other individuals (low β's) get to experience consumption at period 3 due to the effective delegation of the timing decision to the median voter, when they themselves would have consumed at period 2. We return to this point below when we discuss welfare.

5.3 Decentralized commitment with Centralized Choice

We now consider the case in which individuals privately invest in commitment, but at time 2 there is an election that determines the time for consumption for all individuals.

Proposition 6 *There is a unique equilibrium of the decentralized commitment, centralized choice case in which all voters choose* $c = 0$.

The intuition for this result is that there is free riding in commitment investment. Investment in commitment is only useful if it affects the choice in period 2. But, this choice is made collectively, and the probability that an agent is pivotal in period 2 is vanishingly small when there are many agents. This result does rely on the continuum assumption. In a world with a finite number of voters more care would be needed. Zero investment in commitment would still be an equilibrium in the limit.

This result leads us to make the following observation. Suppose that in the decentralized setting we observe a median individual making responsible choices in period 2. One may naively conclude that centralizing consumption would be beneficial because it would lead to responsible choices for the entire population, including those who were choosing irresponsibly. However, this would undermine commitment that allowed the median person to choose responsibly in period 3. For instance, if $\beta_M < \frac{v_2}{v_3}$, and $k(x, 0) = 0$, then in this scenario the median choice would be to consume the entire tree in period 2.

6 Welfare Consequences

We now turn to the welfare consequences of each of the political processes analyzed above. In the case of time inconsistent agents, the appropriate welfare criterion is debatable. We measure welfare as the utility of first period agents.[17]

Let β^* be the threshold preference parameter corresponding to agents who are just indifferent between postponing consumption or consuming immediately in period $2 : v_2 = \beta^* v_3$.

We denote by $\Pi^{DD}(G), \Pi^{DC}(G), \Pi^{CD}(G)$, and $\Pi^{CC}(G)$ the expected utilitarian welfare corresponding to the fully decentralized, decentralized-centralized, centralized-decentralized, and centralized-centralized systems, respectively, when the underlying preference distribution is given by G. We will at times abuse notation and drop the argument of the welfare function when clarity is not compromised. The following proposition summarizes the welfare comparison of the political institutions we consider.

Proposition 7 *Assume that* $k(0, c) = k(x, 0) = 0$ *for all* x *and* c. *For all preference distributions,* $\max\left\{\Pi^{DD}, \Pi^{CC}\right\} \geq \max\left\{\Pi^{CD}, \Pi^{DC}\right\}$. *Furthermore, suppose* $\frac{\partial k(1,c)}{\partial x} \geq v_2$ *and consider any sequence of distributions* $\{G_k\}_{k=1}^{\infty}$ *with corresponding medians* $\left\{\beta_M^k\right\}_{k=1}^{\infty}$.

1. *If there exists* $\tilde{\beta} > 0$ *such that* $\left\{G_k(\tilde{\beta})\right\}$ *is uniformly bounded below* 1 *and* $\lim_{k\to\infty} \beta_M^k = 0$, *then there exists* k^* *such that for all* $k > k^*$, $\Pi^{DD}(G_k) > \Pi^{CC}(G_k)$; *and*

[17] We could also consider welfare from the point of view of a period zero agent, who does not have a present bias in terms of the commitment investment decision. Some of the comparisons across the institutions that we consider would be similar.

2. *If there exists* $\tilde{\beta} < \beta^*$ *such that* $\left\{G_k(\tilde{\beta})\right\}$ *is uniformly bounded above* 0 *and* $\lim_{k\to\infty} \beta_M^k \geq \beta^*$*, then there exists* k^* *such that for all* $k > k^*$, $\Pi^{DD}(G_k) < \Pi^{CC}(G_k)$.

Intuitively, when the median β is high, a centralized political process allows all agents to delegate choice to a virtuous voter who commits to efficient actions at low costs. On the other hand, when the median voter is prone to a strong present-bias, collective decisions lead to investment in commitment and early consumption. In these cases, decentralized decisions do well. Government intervention does not help and laissez faire policies are best from a welfare perspective.

The quadratic costs example is useful in illustrating how the different political processes fare in terms of welfare as a function of the underlying preference distribution in the electorate.

Example 3 (Quadratic Costs – Welfare Comparisons) Consider the settings of Examples 1 and 2 above and suppose that G is a triangular distribution with a peak at $d \in (0, 1)$. Figure 3 depicts the welfare levels generated by the different processes as a function of the median agent's preferences when $v_2 = 1$ and $v_2 = 3/2$, and $I(c) = 0.0005c^2$.[18] We use the fully decentralized setting as a baseline for comparison.The figure illustrates the way that the four scenarios compare in terms of first-period welfare. Full centralization reaches the highest welfare when the median β is high. Full centralization and decentralized commitment-decentralized choice have the same level of welfare when the median is above $2/3$ because $2/3 = v_2/v_3$ for our parameters and in these cases no commitment is necessary to induce zero tree-cutting in period two. However, the decentralized commitment-centralized choice scenario is a lot worse for lower values of the median β. When the median β is lower full decentralization leads to the best outcome, with centralized commitment-decentralized choice a close second. The reason why the comparison between these scenarios becomes much more favorable to full decentralization for high values of median β is that when the median β is high there is no commitment in equilibrium, and, under decentralized choice, this harms the individuals with lower β. One interesting aspect of the comparison among welfare levels is that commitment and consumption, are 'complementary':

[18] For a triangular distribution with a peak at d, the corresponding median is given by:

$$\beta_M = \begin{cases} \sqrt{d}/\sqrt{2} & d \geq 1/2 \\ 1 - \sqrt{1-d}/\sqrt{2} & d < 1/2 \end{cases}.$$

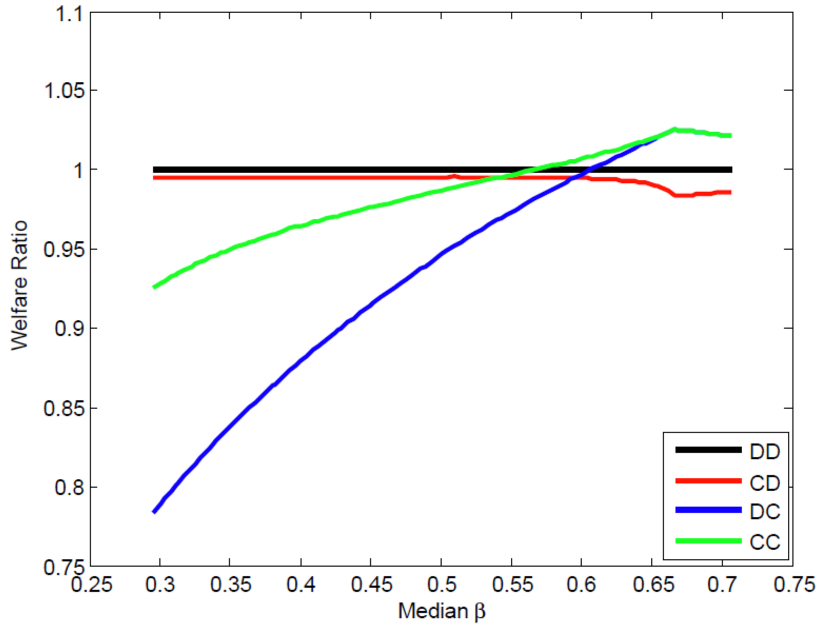

Figure 3: Welfare Comparison for Different Median Preferences

either full centralization or full decentralization generate the greatest levels of welfare, whereas partial centralization yields inferior welfare results.

It is interesting to compare the welfare resulting from our political processes and that generated by an economy that does not allow for commitment. Denote by Π^S the expected first period utilitarian surplus absent any commitment instruments:

$$\Pi^S = \int_0^{\beta^*} \beta v_2 dG(\beta) + \int_{\beta^*}^1 \beta v_3 dG(\beta). \tag{5}$$

In the fully decentralized setting agents can always emulate the no commitment environment by choosing a commitment level of 0. Thus, whenever a positive commitment level is chosen by an individual, the induced first-period utility is higher than that absent commitment. Hence, the fully decentralized process dominates a decentralized one absent commitment, $\Pi^S \leq \Pi^{DD}$. With respect to the fully centralized process, whenever $\beta_M \geq \beta^*$, the centralized commitment-decentralized choice process yields zero investment in commitment, followed by individual consumption decisions and therefore equivalent to an environment without commitment instruments. In particular, $\Pi^S = \Pi^{CD}$. From Proposition 7 above, $\Pi^S \leq \max\left\{\Pi^{DD}, \Pi^{CC}\right\}$ (in fact, as shown in the proof, $\Pi^S \leq \min\left\{\Pi^{DD}, \Pi^{CC}\right\}$ as well). When $\beta_M < \beta^*$, the relative performance of a commitment-less economy depends on the underlying preference distribution. If there is a substantial mass of agents with low preference parameters β and the median β_M is sufficiently high, much in the spirit of point 2 of Proposition 7, $\Pi^S < \Pi^{CC}$. On the other hand, if there is a substantial mass of virtuous agents and the median β_M is very low, $\Pi^S > \Pi^{CC}$.

Let us now consider the potential surplus from imposing a commitment level of c (at a collective cost $I(c)$). In order to get a closed-form solution for the induced welfare, we now focus on the case of linear early consumption costs, $k(x,c) = cx$. We consider a situation in which, a social planner sets the consumption parameter cost c for the entire electorate: all agents experience the cost $I(c)$ at the outset, but only agents who are sufficiently patient, with $\beta \geq \beta(c) = \frac{v_2 - c}{v_3}$ resist the temptation to consume early come period 2. The resulting expected welfare as a function of the cost c, $\Pi(c)$, is therefore:

$$\Pi(c) = \int_0^{\beta(c)} \beta(v_2 - c)\, dG(\beta) + \int_{\beta(c)}^1 \beta v_3 dG(\beta) - I(c)$$

As before, since agents in our environment are sophisticated, if they had access to private commitment instruments they would choose ones that maximize their period 1 utility. Therefore, centralized commitment decisions can only harm the quality of their individual utilities. In other words, $\Pi^* \equiv \max \Pi(c) \leq \Pi^{DD}$.

We now inspect when a fully centralized process leads to superior outcomes relative to an environment with no commitment capacities. When there is a substantial mass of agents who can easily be tipped over to delaying consumption at costs that are fairly low, a positive level of commitment would be chosen centrally and generate superior outcomes relative to the world in which no commitment were available, as the following proposition illustrates.

Proposition 8 *Suppose that* $\frac{v_3 - v_2}{v_3} g\left(\frac{v_2}{v_3}\right) > G\left(\frac{v_2}{v_3}\right)$ *and* $I'(0) = 0$ *then, for sufficiently low* β_M,

$$\Pi^{DD} \geq \Pi^* \geq \Pi^S = \Pi^{CD} \geq \Pi^{DC} = \Pi^{CC}.$$

Note that zero investment may be optimal even if $I'(0) = 0$: there is a first order loss in raising costs from zero because of costs that are born all the (low β) agents who will choose in period 2. This loss does not exist in the fully decentralized case. The proposition suggests that while the centralized political processes we consider can generate inferior collective outcomes, in principle, intervention (in the form of imposing exogenous commitment) could generate positive welfare surplus.

7 Extensions

7.1 Naive Agents

In the literature, when modeling time inconsistent agents, an assumption of naivete is sometimes made in contrast to the assumption of sophistication we have assumed so far.[19] Naive agents have $\beta - \delta$ preferences, but they believe that they will have standard geometric preferences starting tomorrow. Sometimes agents are assumed to be partially naive. This is modeled as agents having beliefs about their future selves that are intermediate between full sophistication and full naivete.

Most of our analysis would go through, with some modifications, if agents are partially sophisticated. However, it is useful to comment on what happens if some of the agents are fully naive.

In our model naive agents behave like time consistent (high β) individuals in period 1: they do not have any demand for commitment because they are unaware of their time inconsistency problem. Therefore, the higher the mass of naive agents in the economy, the lower the investment in commitment in equilibrium. However, once period 2 arrives, these agents are tempted by immediate consumption, lowering the effective pivotal β in the centralized consumption scenario. Overall, the presence of these naive agents reduces welfare for the sophisticated agents.

7.2 Alternative Voting Rules

In the centralized choice scenario, supermajorities to cut the tree early unambiguously raise welfare of all period 1 agents because they increase the β of the second period pivotal agent. However, raising the β of the pivotal agent in the centralized commitment scenario may be harmful because it can lower commitment.

7.3 Commitment Subsidies

Instead of considering a centralized commitment scenario where the elected government chooses the amount of commitment in period 1, one could consider a scenario where candidates propose subsidies to commitment. If a voter receives a subisidy s, the choice of commitment in period 1 can be obtained by maximizing

$$U_1 \left(x \left(c, \beta \right), c, \beta, s \right) = \beta v_3 + x \left(c, \beta \right) \left(\beta v_2 - \beta v_3 \right) - \beta k \left(x \left(c, \beta \right), c \right) - I \left(c, s \right)$$

[19] See for instance O'Donoughue and Rabin (1999).

where $\frac{\partial I(c,s)}{\partial s}$ is decreasing in s. Thus, the amount of commitment chosen by each individual is increasing in s. However, the voting decision between two candidates who offer different levels of subsidies needs to take into account the budgetary impact of the subsidies and how this is distributed in the population. The total amount of subsidies depends on the aggregate amount of commitment. If the burden is shared equally, then it can be shown that the pivotal agent remains β^{CD} (the pivotal agent in the centralized commitment decentralized choice scenario). If this agent has a relatively low investment in commitment, then the value of subidies for him is lower than his contribution to paying for the aggregate cost of the subsidies. In this case the outcome of the election would be zero subsidies. On the other hand, if this agent has a relatively high investment in commitment, so that he is a net beneficiary of the subsidies, he will want to vote for high subsidies. In this case, the outcome would lead to higher investment in commitment by all agents relative to the decentralized scenario.

7.4 Supplementing Commitment

One possible extension is to consider what happens if agents can supplement commitment investment that are chosen by the government. The outcome in this scenario depends on the relative efficiency of the private technology to supplement commitment and the government technology for centralized commitment. As long as the government technology is not inferior to the private technology, the amount of commitment chosen by the government is given by our characterization in Proposition 1. Those who want additional commitment will then supplement by using the private technology. If instead the private technology is superior, then in equilibrium there is no centralized provision of commitment.

7.5 Bargaining

We now discuss a case where collective actions are taken via bargaining. Assume first a two-person bargaining problem. Agent i has present bias β_i and discount factor δ_i, $i = 1, 2$.

The agents bargain over shares of a pie of changing value: if they stop at period t, they get to share value v_t.

Proposition 9 *Consider a bargaining procedure and a subgame perfect equilibrium such that agreement takes place immediately in every subgame. Then, the equilibrium of the game with agents with time preferences characterized by* $\{\beta_i, \delta_i\}$ *coincides with that of the game in which time preferences are given by present bias* $\beta_i = 1$ *and discount factor* $\hat{\delta}_i = \beta_i \delta_i$*,* $i = 1, 2$.

The meaning of this proposition is that agents behave as if they were geometric discounters with lower discount factor. This extends to general changing time preferences: only the first discount factor matters.

An immediate consequence of this Proposition is that in many circumstances bargaining can lead to inefficiently early agreements.

In these environments it is possible to construct examples in which an agent prefers a partner with high discount factors. There are two effects: 1. The standard negative effect of raising the discount factor of the bargaining partner is that this leads to a lower share of the pie. 2. Having a partner with high discount factor leads to later stopping. This is potentially valuable for a time inconsistent agent. High discount factor partners serve as commitment devices.

Analogously, it can be shown that bargaining frictions can be useful since they permit efficient delays. In other words, there are circumstances in which agents have little incentive to set up effective/smoothly functioning bargaining protocols since these would lead to inefficiently early agreements.

8 Conclusions

The paper considers a simple setting in which behavioral agents, who in our case suffer from present bias, are also political actors, electing the government that is charged with "solving" their behavioral biases. The main message that comes out of our analysis is that collective outcomes can be worse than those generated by a laissez faire economy in which all decisions are decentralized. While commitment instruments can be beneficial to individuals left to their own devices, we show the sensitivity of collective outcomes to the precise timing in which political processes take place and the underlying distribution of biases in the population.

When decisions are fully decentralized, very patient individuals do not require commitment to delay consumption, while individuals suffering from moderate present bias acquire varying levels of commitment that allow them to consume greater amounts at later dates. The starkest case in which government intervention can be harmful occurs when the timing of consumption itself is decided upon collectively. In this case, no commitment can be sustained and the political process destroys private commitment incentives. On the other hand, if the median voter is not prone to a strong present bias, a fully centralized process in which both commitment decisions and the timing of consumption are decided upon collectively can be beneficial. Such a process allows the electorate to delegate decisions to a virtuous median voter. However, it is important to delegate all decision power to the median voter. Indeed, a fully decentralized economy is superior in terms of welfare to one in which only commitment choices (but not the timing of consumption) are decided upon collectively.

Our analysis also highlights the sensitivity of political outcomes to the commitment technology in place. Indeed, technologies that entail a fixed marginal cost of early consumption generate qualitatively different outcomes than those that do not. From a technical perspective, commitments that do not involve a fixed marginal cost of early consumption generate an electorate that has single-peaked preference with respect to the volume of commitment, while other technologies generate a greater wedge between voters' preferences (i.e., preferences are not single-peaked).

9 Appendix A – Proofs

Proof of Lemma 1. Consider the intermediate region of β parameters:

$$\beta \in \left[\left(v_2 - \frac{\partial k(1,c)}{\partial x} \right) / v_3, \left(v_2 - \frac{\partial k(0,c)}{\partial x} \right) / v_3 \right).$$

Since $\frac{\partial^2 k(x(c,\beta),c)}{\partial x \partial c} > 0$, increasing c leads to an increase in the right hand side of the first order condition specified in (1). Since $\frac{\partial^2 k(x(c,\beta),c)}{\partial x^2} > 0$ it follows that $x(c,\beta)$ is decreasing in c. Similarly, notice that $\frac{d(v_2 - \beta v_3)}{d\beta} < 0$ so that the left hand side of the first order condition is decreasing in β. The assumption that $\frac{\partial^2 k(x(c,\beta),c)}{\partial x^2} > 0$ then assures that $x(c,\beta)$ is decreasing in β. ■

Proof of Proposition 1. The assumption that $\frac{\partial k(1,c)}{\partial x} \geq v_2$ together with our assumptions on k, which assure that $\frac{\partial k(0,0)}{\partial x} = 0$, imply through the first order conditions specified in (1) that for any $\beta < \frac{v_2}{v_3}$, there exists $\bar{c} > 0$, such that $x(c,\beta) \in (0,1)$ for all $c < \bar{c}$. Recall that the first period's utility changes with c in regions where $x(c,\beta) \in (0,1)$ according to:

$$\frac{\partial U_1}{\partial c} = -\beta \left(\frac{\partial x(c,\beta)}{\partial c} ((1-\beta) v_3) + \frac{\partial k(x(c,\beta),c)}{\partial c} \right) - I'(c).$$

Our assumptions guaranty that $x(c,\beta)$ is decreasing in c, while $\frac{\partial k(x,0)}{\partial c} = I'(0) = 0$. Therefore, optimization requires that $c(\beta) > 0$ for all $\beta \in \left(0, \frac{v_2}{v_3}\right)$. Furthermore, since we assume that fundamentals are such that $x(c,\beta)$ is well-behaved, $c(\beta)$ is continuous.

Now, if $1 - G(\frac{v_2}{v_3}) \geq 1/2$, there is a majority of agents who prefer no commitment and the equilibrium commitment parameter is $c^{CD} = 0$, which coincides with that preferred by the median.

Suppose that $1 - G(\frac{v_2}{v_3}) < 1/2$. Let $B(\tilde{c}) = \{\beta \mid c(\beta) \leq \tilde{c}\}$. For any $0 < c_1 < c_2 \leq \max_\beta c(\beta)$, $B(c_1) \subsetneq B(c_2)$. Therefore, there exists a unique c^{CD} such that $G(B(c^{CD})) = 1/2$. For any parameter $c > c^{CD}$, there is a strict majority preferring lower commitment, while for any $c < c^{CD}$, there is a strict majority preferring greater commitment. It follows that c^{CD} defines the unique equilibrium commitment level.

If $c(\beta)$ has a unique maximum, then there exist $\beta_L, \beta_H \in \left(0, \frac{v_2}{v_3}\right)$ such that $B(c^{CD}) = [0, \beta_L] \cup [\beta_H, 1]$. Since G is continuous, by construction $G([0,\beta_L]), G([\beta_H,1]) < 1/2$, implying that $\beta_M \in (\beta_L, \beta_H)$. The result follows. ■

Proof of Proposition 2. We first show that with linear costs there is no pure strategy equilibrium with positive commitment. Assume by way of contradiction that candidate 1 chooses $c > 0$ with probability 1. Then candidate 2 can win with probability 1 by choosing $c - \epsilon$ for ϵ sufficiently small. All voters with preference parameter β such that $\beta v_3 \geq v_2 - (c - \epsilon)$ prefer candidate 2 because they still get to consume in period 3 but the lower investment in commitment is sufficient to do so. Furthermore, all voters with β such $\beta v_3 < v_2 - c$ prefer candidate 2 because they consume in period 2 with both levels of commitment, so prefer the candidate who offers the lower level. The only voters who may prefer c over $c - \epsilon$ are those whose preference parameter β is such that $\beta v_3 \geq v_2 - c$ and $\beta v_3 < v_2 - (c - \epsilon)$. However, because the distribution G is continuous, the mass of these voters can be made arbitrarily small by choosing ϵ small enough.

If $\beta_L (v_3 - v_2) \leq I(c_L)$, then all agents with preference parameter β such that $\beta \leq \beta_L$ prefer $c = 0$ to c_L. Since $I(c)$ is convex, they prefer $c = 0$ to all $c > c_L$. Furthermore, any $0 < c < c_L$ is also worse than $c = 0$ for these agents because $\beta v_3 < v_2 - c$ by the definition of c_L and β_L. Since $(1 - G(\beta_H)) + G(\beta_L) = \frac{1}{2}$, there is a majority in favor of $c = 0$ against all other c's.

If $\beta_L (v_3 - v_2) > I(c_L)$, then all β's between β_H and β_L strictly prefer $c_L + \epsilon$ to $c = 0$. Furthermore, some β's slightly higher than β_L also prefer $c_L + \epsilon$ to $c = 0$. Since there half the mass of voters is concentrated between β_L and β_H, $c_L + \epsilon$ defeats $c = 0$. As shown above, there is no pure strategy equilibrium with positive commitment. This establishes that when $\beta_L (v_3 - v_2) > I(c_L)$, there is no pure strategy equilibrium.

We now show that when $\beta_L (v_3 - v_2) > I(c_L)$ the mixed-strategy profiles in the statement of the proposition constitute equilibria. Note first that c_1 and c_2 as defined in the proposition tie. Consider now a policy $\hat{c} > c_2$. This policy may win against c_1. However, $\hat{c}$ loses against c_2 because all agents of preference parameter $\beta > \beta(c_2) - \delta$ (for some δ) would vote for c_2 over $\hat{c}$. Since $G(\beta(c_1)) - G(\beta(c_2)) = \frac{1}{2}$, there is more than 50% of the voters supporting c_2. Thus, $\hat{c}$ wins with probability 1/2. Consider now a policy $c_1 < \hat{c} < c_2$. Such a policy may win against c_2. However, against c_1, the only potential supporters are agents with preference parameters within $[\beta(\hat{c}), \beta(c_1))$, which by construction entails less than 50% of the population. In particular, c_L is a policy that would lose against c_1. Last, consider a policy $\hat{c} < c_1$. This policy may win against c_1. Against c_2, its only potential supporters are agents with preference parameters $\beta \leq \beta(c_2)$ or $\beta \geq \beta(\hat{c})$, which from the definition of the pair (c_1, c_2) account for less than 50% of the voters. Thus, the candidate equilibrium strategy profile wins with probability at least 1/2 against all possible deviations and no deviation is strictly beneficial. ■

Proof of Lemma 2. Let us first consider the case in which $x(c, \beta_M)$ is interior.

From the first order condition of the median voter in period 2 we know that whenever

$x(c, \beta_M) > 0$,

$$v_2 - \beta_M v_3 = \frac{\partial k(x(c, \beta_M), c)}{\partial x}.$$

Consider the effect of c on an agent of taste parameter β who foresees that period 2 decisions will be made by the median voter.

$$\frac{\partial U_1}{\partial c} = \frac{\partial x(c, \beta_M)}{\partial c}(\beta v_2 - \beta v_3) - \beta \frac{\partial k(x(c, \beta_M), c)}{\partial c} - \beta \frac{\partial k(x(c, \beta_M), c)}{\partial x}\frac{\partial x(c, \beta_M)}{\partial c} - I'(c).$$

The first order condition for β_M implies that

$$v_2 - v_3 = -(1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial x}$$

and so

$$\frac{\partial U_1}{\partial c} = -\beta \left(\frac{\partial x(c, \beta_M)}{\partial c}(1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} \right) - I'(c).$$

Now, if $\frac{\partial x(c,\beta_M)}{\partial c}(1 - \beta_M) v_3 + \frac{\partial k(x(c,\beta_M),c)}{\partial c} \geq 0$, all agents prefer $c = 0$ and the claim follows. Otherwise, $\frac{\partial x(c,\beta_M)}{\partial c}(1 - \beta_M) v_3 + \frac{\partial k(x(c,\beta_M),c)}{\partial c} < 0$ and $\frac{\partial U_1}{\partial c}$ is increasing in β. At a maximum, U_1 is (weakly) concave and the claim follows.

We now consider the cases in which $x(c, \beta_M)$ may be at a corner solution. Note first that when $\beta_M \geq \frac{v_2}{v_3}$, then $x(c, \beta_M) = 0$ for all c. In this case, all agents prefer $c = 0$ in period 1 and the claim follows. More generally, we have

$$x(c, \beta_M) = \begin{cases} 0 & c \geq c_H(\beta_M) \\ (v_2 - \beta_M v_3) = \frac{\partial k(x(c,\beta_M),c)}{\partial x} & c_L(\beta_M) < c < c_H(\beta_M) \\ 1 & c \leq c_L(\beta_M) \end{cases}.$$

Clearly, there is no value in choosing $c > c_H(\beta_M)$. thus, $c \geq c_H(\beta_M)$ and

$$U_1(\beta, c_H(\beta_M)) = \beta v_3 - I(c_H(\beta_M)).$$

Comparing this to interior cases:

$$\begin{aligned} U_1(\beta, c_H(\beta_M)) - U_1(\beta, c) &= \beta v_3 - I(c_H(\beta_M)) - \beta v_3 + x(c, \beta_M)(\beta v_2 - \beta v_3) - \\ &- \beta k(x(c, \beta_M), c) - I(c) = \\ &= \beta(x(c, \beta_M)(v_3 - v_2) - k(x(c, \beta_M), c)) - (I(c_H(\beta_M)) - I(c)). \end{aligned}$$

If $c_H(\beta_M)$ is optimal for some $\hat{\beta}$, it has to be the case that

$$\hat{\beta}(x(c, \beta_M)(v_3 - v_2) - k(x(c, \beta_M), c)) > (I(c_H(\beta_M)) - I(c))$$

for all $c < c_H(\beta_M)$. But then, this also holds for all $\beta > \hat{\beta}$.

It is easy to see that it must be the case that when $c \leq c_L(\beta_M)$, then the optimal c is zero: there is no point in investing anything in commitment if it does not help. In this case, the payoff in the first period is $U_1(\beta, 0) = v_2$. Comparing this to interior cases:

$$\begin{aligned} U_1(\beta, c_H(\beta_M)) - U_1(\beta, c) &= \beta v_2 - (\beta v_3 + x(c, \beta_M)(\beta v_2 - \beta v_3) - \beta k(x(c, \beta_M), c) - I(c)) \\ &= -\beta((v_3 - v_2)(1 - x(c, \beta_M)) - k(x(c, \beta_M), c)) + I(c). \end{aligned}$$

If a choice of zero commitment is optimal for some $\hat{\beta}$ it has to be the case that

$$\hat{\beta}((v_3 - v_2)(1 - x(c, \beta_M)) - k(x(c, \beta_M), c)) < I(c)$$

for all $c > c_L(\beta_M)$. But then this also holds for all $\beta < \hat{\beta}$. ■

Proof of Proposition 6. In period 1, all agents but the foreseen pivotal voter of period 2 best respond by choosing $c = 0$, as their choice of commitment parameter affects only the commitment and consumption costs they experience, but not the levels of future consumption. If any agent of taste parameter $\beta \neq \beta_M$ invests in commitment in period 1, the median preferences in period 2 would correspond to those of the median agent with preferences β_M and so investment by the agent of taste parameter β are strictly sub-optimal. Suppose the median agent invests in period 1. In that case, in period 2 her preferences no longer coincide with the median preferences and so her commitment investment does not affect ultimate choice and is thus strictly sub-optimal. The claim then follows. ■

Proof of Proposition 7. Notice first that when decisions are fully decentralized, agents can emulate the equilibrium commitment choices performed when commitment is decided upon collectively. In particular, $\Pi^{DD} \geq \Pi^{CD}$. Whenever $\beta_M \geq \beta^*$, both systems in which choices are made collectively entail delayed consumption at no commitment cost and $\Pi^{CC} = \Pi^{DC}$. Suppose $\beta_M < \beta^*$. When only consumption choices are centralized, since $x(0, \beta_M) = 1$, Proposition 6 suggests that consumption will occur only at period 2 and thus generate a period 1 utility of v_2 to the entire electorate. When decisions are fully decentralized, agents can emulate this outcome by choosing themselves to invest nothing in commitment. It follows that whenever individuals choose a positive level of commitment, their period 1 utility must exceed v_2 and $\Pi^{DD} \geq \Pi^{DC}$. In particular, for all preference distributions, $\max\left\{\Pi^{DD}, \Pi^{CC}\right\} \geq \max\left\{\Pi^{CD}, \Pi^{DC}\right\}$.

Consider now any sequence of distributions $\{G_k\}_{k=1}^{\infty}$ with corresponding medians $\left\{\beta_M^k\right\}_{k=1}^{\infty}$.

1. Suppose there exists $\gamma > 0$ such that $1 - G_k(\tilde{\beta}) \geq \gamma$ for some $\tilde{\beta} > 0$ for all k and assume that $\lim_{k\to\infty} \beta_M^k = 0$. It follows that:

$$\lim_{k\to\infty} c(\beta_M^k) = 0 \quad \text{and} \quad \lim_{k\to\infty} x(c\left(\beta_M^k\right), \beta_M^k) = 1.$$

Therefore, for any $\varepsilon > 0$, there exists $\bar{k}(\varepsilon)$ such that for all $k > \bar{k}(\varepsilon)$, $\Pi^{CC}(G_k) < v_2 \mathbb{E}_{G_k}\beta + \varepsilon$.

The assumption $\frac{\partial k(1,c)}{\partial x} \geq v_2$ implies that $c(\beta) > 0$ for all $\beta \in (0, \beta^*)$ (see proof of Proposition 1). When decisions are decentralized, each individual with preference parameter $\beta \in (0, \beta^*)$ can assure a value of v_2 by choosing a commitment level of 0. Furthermore, individuals of preference parameter $\beta \geq \beta^*$ are assured a utility of v_3 when decisions are decentralized. Therefore, for $\beta \in (0, 1]$, $U_1(x(c(\beta)), c, \beta) > v_2$. Define

$$\tilde{U} \equiv \min_{\beta \geq \tilde{\beta}} U_1(x(c(\beta)), c, \beta) > v_2.$$

It follows that for all k,

$$\begin{aligned}
\Pi^{DD}(G_k) &\geq G_k(\tilde{\beta}) v_2 \mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right) + (1 - G_k(\tilde{\beta}))\tilde{U}\mathbb{E}_{G_k}\left(\beta \mid \beta > \tilde{\beta}\right) = \\
&= v_2 \mathbb{E}_{G_k}\beta + (1 - G_k(\tilde{\beta}))\left(\tilde{U} - v_2\right)\mathbb{E}_{G_k}\left(\beta \mid \beta > \tilde{\beta}\right) \geq \\
&\geq v_2 \mathbb{E}_{G_k}\beta + \gamma\left(\tilde{U} - v_2\right)\mathbb{E}_{G_k}\left(\beta \mid \beta > \tilde{\beta}\right).
\end{aligned}$$

In particular, choosing $\varepsilon = \gamma\left(\tilde{U} - v_2\right)\mathbb{E}_{G_k}\left(\beta \mid \beta > \tilde{\beta}\right) > 0$, we get that for any $k > k^* = \bar{k}\left(\gamma\left(\tilde{U} - v_2\right)\mathbb{E}_{G_k}\left(\beta \mid \beta > \tilde{\beta}\right)\right)$, $\Pi^{DD}(G_k) > \Pi^{CC}(G_k)$, as needed.

2. Suppose there exists $\gamma > 0$ such that $G_k(\tilde{\beta}) \geq \gamma$ for some $\tilde{\beta} < \beta^*$ for all k and assume that $\lim_{k\to\infty} \beta_M^k \geq \beta^*$. It follows that:

$$\lim_{k\to\infty} c(\beta_M^k) = 0 \quad \text{and} \quad \lim_{k\to\infty} x(c\left(\beta_M^k\right), \beta_M^k) = 0.$$

Therefore, for any $\varepsilon > 0$, there exists $\bar{k}(\varepsilon)$ such that for all $k > \bar{k}(\varepsilon)$, $\Pi^{CC}(G_k) > v_3 \mathbb{E}_{G_k}\beta - \varepsilon$. From the proof of Proposition 1, for individuals with preference parameter $\beta \leq \tilde{\beta}$, $c(\beta) > 0$, and so $U_1(x(c(\beta)), c, \beta) < v_3$. Define

$$\tilde{U} \equiv \max_{\beta \leq \tilde{\beta}} U_1(x(c(\beta)), c, \beta) < v_3.$$

Then, for all k,

$$\begin{aligned}
\Pi^{DD}(G_k) &\leq G_k(\tilde{\beta})\tilde{U}\mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right) + (1 - G_k(\tilde{\beta})) v_3 \mathbb{E}_{G_k}\left(\beta \mid \beta > \tilde{\beta}\right) = \\
&= v_3 \mathbb{E}_{G_k}\beta - (G_k(\tilde{\beta}))\left(v_3 - \tilde{U}\right)\mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right) \leq \\
&\leq v_3 \mathbb{E}_{G_k}\beta - \gamma\left(v_3 - \tilde{U}\right)\mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right).
\end{aligned}$$

Furthermore, since we consider continuous distributions, $\mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right) > 0$. Therefore, choosing $\varepsilon = \gamma\left(v_3 - \tilde{U}\right)\mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right)$, for any $k > k^* = \bar{k}\left(\gamma\left(v_3 - \tilde{U}\right)\mathbb{E}_{G_k}\left(\beta \mid \beta \leq \tilde{\beta}\right)\right)$, $\Pi^{DD}(G_k) < \Pi^{CC}(G_k)$,and the claim follows. ■

Proof of Proposition 8. Using (5), and noting that $\beta(c) < \beta^*$ for all $c > 0$, the relative surplus from full centralization can be written as:

$$W(c) \equiv \Pi(c) - \Pi^0 = -\int_0^{\beta(c)} \beta c dG(\beta) + \int_{\beta(c)}^{\beta^*} \beta(v_3 - v_2)\, dG(\beta) - I(c)$$

Using Leibnitz' rule,

$$\begin{aligned}
&\tfrac{d}{dc}\textstyle\int_0^{\beta(c)} \beta c dG(\beta) = \int_0^{\beta(c)} \beta dG(\beta) + \beta'(c)\beta(c)cg(\beta(c)); \text{ and}\\
&\tfrac{d}{dc}\textstyle\int_{\beta(c)}^{\beta^*} \beta(v_3 - v_2)\, dG(\beta) = -\beta'(c)\beta(c)(v_3 - v_2)\, g(\beta(c)),
\end{aligned}$$

so that:

$$W'(c) = -\beta'(c)\beta(c)g(\beta(c))\left[c + v_3 - v_2\right] - \int_0^{\beta(c)} \beta dG(\beta) - I'(c).$$

Recall that $\beta(c) = \frac{v_2 - c}{v_3}$. Therefore, $\beta'(c) = -\frac{1}{v_3}$, so that:

$$W'(c) = \frac{v_2 - c}{v_3^2} g(\beta(c))\left[c + v_3 - v_2\right] - \int_0^{\beta(c)} \beta dG(\beta) - I'(c).$$

Notice that:

$$\int_0^{\beta(c)} \beta dG(\beta) \leq \beta(c)G(\beta(c)) = \frac{v_2 - c}{v_3} G\left(\frac{v_2 - c}{v_3}\right),$$

so whenever

$$\frac{c + v_3 - v_2}{v_3} g\left(\frac{v_2 - c}{v_3}\right) > G\left(\frac{v_2 - c}{v_3}\right),$$

if $I(c)$ is sufficiently flat around 0, $W'(0) > 0$. In particular, whenever $\frac{v_3 - v_2}{v_3} g\left(\frac{v_2}{v_3}\right) > G\left(\frac{v_2}{v_3}\right)$ and $I'(0) = 0$, there is a positive level of common commitment. ■

10 Appendix B – Probabilistic Voting with Time Inconsistent Voters

We now modify our model by following Lindbeck and Weibull (1987) and allowing each individual voter to have an idiosyncratic 'ideology' preference of Candidate L over Candidate R. We assume this takes the form of a random utility advantage $x \sim F$, with density f, determined independently for each voter (and independently of their time preference β).

For presentation simplicity, we focus on the case in which early consumption costs are linear, or commitment investment is binary.

We consider equilibria in pure strategies. Suppose Candidate J offers commitment level c_J, J = L, R. Recall that for any cost c of breaking the commitment, $\beta(c) \equiv \frac{v_2 - c}{v_3}$ defines the threshold parameter such that individuals with $\beta \geq \beta(c)$ will wait till the third period to consume.

Therefore, the expected utility of an agent of type β from commitment level c is given by:

$$U(\beta; c) = \begin{cases} \beta v_3 - I(c) & \beta \geq \beta(c) \\ \beta(v_2 - c) - I(c) & \beta < \beta(c) \end{cases},$$

where weak inequalities are specified arbitrarily throughout as they occur with zero probability since G is continuous.

Assume now that $c_L \geq c_R$, and notice that $\beta(c_L) \leq \beta(c_R)$. It follow that:

$$U(\beta; c_R) - U(\beta; c_L) = \begin{cases} I(c_L) - I(c_R) & \beta \geq \beta(c_R) \\ \beta(v_2 - v_3 - c_R) + I(c_L) - I(c_R) & \beta(c_L) \leq \beta < \beta(c_R) \\ \beta(c_L - c_R) + I(c_L) - I(c_R) & \beta < \beta(c_L) \end{cases}.$$

For any realization of the ideology advantage x, Candidate R is chosen if and only if

$$U(\beta; c_R) - U(\beta; c_L) \geq x.$$

It follows that the probability that Candidate R is selected is given by:

$$\begin{aligned} \Pr(\text{Candidate R wins}) &= F\left(I(c_L) - I(c_R)\right)\left(1 - G\left(\beta(c_R)\right)\right) + \\ &+ \int_{\beta(c_L)}^{\beta(c_R)} F\left(\beta(v_2 - v_3 - c_R) + I(c_L) - I(c_R)\right) dG(\beta) + \\ &+ \int_{0}^{\beta(c_L)} F\left(\beta(c_L - c_R) + I(c_L) - I(c_R)\right) dG(\beta), \end{aligned} \tag{6}$$

and clearly $\Pr(\text{Candidate L wins}) = 1 - \Pr(\text{Candidate R wins})$.

We now look for a symmetric equilibrium $c \equiv c_R = c_L$. Consider then the first order condition corresponding to maximizing (6) with respect to c_R.

$$\begin{aligned} & f(0)I'(c)\left(1-G\left(\beta(c)\right)\right)+F(0)g\left(\beta(c)\right)\beta'(c)= \\ & =\beta'(c)F\left(\beta(c)(v_2-v_3-c)\right)g(\beta(c))-\int_0^{\beta(c_L)}\left(\beta-I'(c)\right)f(0)dG(\beta)= \\ & =\beta'(c)F\left(\beta(c)(v_2-v_3-c)\right)g(\beta(c))-I'(c)f(0)G(\beta(c))-f(0)\mathbb{E}_G\left(\beta\mid\beta<\beta(c)\right). \end{aligned}$$

This is equivalent to:

$$f(0)\left[I'(c)+\mathbb{E}_G\left(\beta\mid\beta<\beta(c)\right)\right]=-\beta'(c)g\left(\beta(c)\right)\left[F(0)-F\left(\beta(c)(v_2-v_3-c)\right)\right].$$

Plugging in $\beta(c)=\frac{v_2-c}{v_3}$, so that $\beta'(c)=-1/v_3$, we get:

$$f(0)\left[I'(c)+\mathbb{E}_G\left(\beta\mid\beta<\beta(c)\right)\right]=\frac{1}{v_3}g\left(\beta(c)\right)\left[F(0)-F\left(\beta(c)(v_2-v_3-c)\right)\right].$$

Recall that marginal welfare is given by

$$W'(c)=\frac{v_2-c}{v_3^2}g(\beta(c))\left[c+v_3-v_2\right]-\mathbb{E}_G\left(\beta\mid\beta<\beta(c)\right)-I'(c)$$

Notice that when F is uniform, the first order condition coincides, up to a constant, with the marginal welfare (see proof of Proposition 8 above). Conceptually, whenever marginal welfare at 0 is positive, so that a certain level of commitment is efficient, $c=0$ is not part of an equilibrium and political competition would generate a positive level of commitment.

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