Credit Cycles with Market-Based Household Leverage*

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Abstract

We develop a model in which mortgage leverage available to households depends on the risk bearing capacity of financial intermediaries. Our model features a novel transmission mechanism from Wall Street to Main Street, as borrower households choose lower leverage and consumption when intermediaries are distressed. The model has financially-constrained young and unconstrained middle-aged households in overlapping generations. Young households choose higher leverage and riskier mortgages than the middle-aged, and their consumption is particularly sensitive to credit supply. Relative to a standard model with exogenous credit constraints, the macroeconomic importance of intermediary net worth is magnified through its effects on household leverage, house prices, and consumption demand. The model quantitatively demonstrates how recessions with housing crises differ from those driven only by productivity, and how a growing demand for safe assets replicates many features of the 2000s credit boom and increases the severity of future financial crises.

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1 Introduction

The financial sector grew during the boom of the early 2000s and crashed in the financial crisis of 2008, contributing to a boom and bust in asset prices. Low spreads between mortgage rates and risk free interest rates during the boom meant that highly levered households could borrow cheaply, while after 2008 spreads on risky mortgages reached historical highs, making household borrowing extremely expensive. Household leverage surged during the boom, fueling growth in consumption and output, while a reduction in household leverage and consumption following the financial crisis contributed to a large recession. To what extent was household leverage and consumption in this episode driven by the cost of loans offered by the financial sector?

This paper presents a quantitative general equilibrium model in which the price of credit offered by the financial sector impacts the quantity of leverage chosen by households. Most quantitative work on the 2000s credit cycle assumes that household leverage is exogenously constrained by a fraction of their home value or their income, making households' demand for leverage inelastic to its price. Booms and busts in such models are generated by exogenous loosening or tightening of these constraints. The modeling approach in this paper, where household leverage is determined by supply and demand forces like any other good, allows us to study how asset price fluctuations impact households' leverage choices. As a result, our model features a key mechanism by which events on Wall Street that impact asset prices can have macroeconomic consequences for Main Street.

To understand the general equilibrium implications of this supply-and-demand determination of household leverage, we jointly model the frictions faced by households and financial intermediaries in a unified framework. Our model's financial sector follows recent work in the intermediary asset pricing literature, in which intermediaries issue riskless deposits, invest in risky assets such as mortgages, and face frictions in raising the equity capital needed to bear the risk in their asset portfolios. The capital of intermediaries is a key state variable that drives asset price fluctuations. In addition, our model features heterogeneous households in overlapping generations, where households must pledge housing as collateral in order to borrow. Young households expect rapid growth in their non-pledgeable future income and would want to borrow against this non-pledgeable income to consume today. The consumption of the young is therefore highly sensitive to the supply of credit, and they choose risky, highly levered mortgages in order to increase their present consumption.

Relative to existing quantitative models of credit cycles, our approach to modelling leverage is new and builds on recent theoretical work (Geanakoplos (2010), Simsek (2013), Diamond (2018)). Financial intermediaries offer households a menu of competitively priced mortgage contracts, where the severity of the intermediaries' financing constraints endogenously determine the risk premium they charge for default risk. This menu yields a "credit surface" which determines how the interest rate on a mortgage depends on the leverage of the mortgage and the creditworthiness of the borrower. Households choose their optimal mortgage from the credit surface, with no exogenous constraints placed on the amount of leverage they choose. Different generations of households face different credit surfaces, reflecting their different degrees of creditworthiness. Because older households are less financially constrained, they choose safer and less levered mortgages than younger households. When credit spreads on risky mortgages increase, households endogenously choose to reduce their leverage.

Although our model features rich heterogeneity and imperfect risk sharing between households, the total wealth of households in each generation is the only state variable required to describe their aggregate behavior. This aggregation result follows because households have homogenous utility functions and choice sets that scale linearly in their wealth. In a standard model of non-pledgeable endowment income, households become increasingly financially constrained as their liquid wealth decreases, so their choice set does not scale linearly in their wealth. Our innovation in this paper is to allow households to trade their endowment income with other households of the same age, but not with other agents. This allows us to model financial distortions due to the lifecycle dynamics of endowment income while maintaining enough tractability to solve the model. As in Constantinides and Duffie (1996), households face only multiplicative shocks to their income, so they choose portfolios that scale linearly in their wealth even with incomplete financial markets.

We use our model for three quantitative counterfactuals: comparing non-financial recessions driven by productivity with housing crises, understanding the equilibrium effects of a drop in the equity capital of intermediaries, and analyzing the effects of a growing demand for safe assets on the financial system and real economy. Housing recessions in our model are caused by shocks to the cross-sectional dispersion of house values, which we refer to as "housing risk shocks". High house price dispersion pushes more borrowers underwater and causes more mortgage defaults. In our first results, comparing the two types of recessions, we find several key differences. First, a productivity driven recession leads to an increase in the risk free rate, reflecting high growth expectations as the economy returns to trend. In a housing crisis, the losses of intermediaries' equity capital due to mortgage defaults impairs their ability to create safe assets. The resulting scarcity of safe assets leads to a drop in the risk free rate, consistent with the low rates during and after the 2008 financial crisis. Second, all households have similarly sized drops in consumption in the productivity driven recession, while in a housing crisis borrowing constrained young households face a disproportionate drop in consumption. This is because young consumers must borrow in order to consume, and intermediaries are less willing to bear the risk of lending to these risky borrowers after their loss of equity capital in the housing crisis. Third, the impaired ability of intermediaries to bear risk also leads to a large increase in mortgage spreads and to a drop in the leverage of all households in a household crisis, while the mortgage market is nearly unaffected by a productivity driven recession.

Next, we analyze the effects of a 40% drop in the equity capital of financial intermediaries on the economy, without any other exogenous. This loss impairs the ability of the intermediary to bear risk, leading to a shrinking of the intermediary's balance sheet and an increase in the spread charged on risky mortgages. This induces households to reduce their leverage. Because households cannot borrow as much against their houses, house prices experience a moderate drop that reflects their reduced value as a form of collateral. In addition, because households now have safer mortgages, default rates drop sharply. The intermediary gradually rebuilds its equity both through retained earnings, which are high due to the low risk and high risk premium of mortgages. As intermediary capital converges back to its original level, other variables gradually revert as well. This simulation isolates the quantitatively large effect of intermediary net worth fluctuations on mortgage spreads and household leverage.

Finally, we consider the general equilibrium effects of a growing demand for safe assets, considering both the effects during the credit and housing boom of the 2000s and the following bust and crisis. As discussed by Bernanke (2005), Caballero and Farhi (2018), and others, an increase in the demand for safe assets was a key macroeconomic feature of the economy before the financial crisis. Relative to existing literature, our model allows us to study the indirect effects of this growing demand on intermediary leverage and risk taking, household leverage, consumption, and house prices. Along a range of dimensions, we find that this increase in the demand for safe assets replicates features of the pre-crisis lending boom. In particular, we find that the size of the financial sector, the amount of mortage debt outstanding, the leverage of households, and the price of houses increase. We can also generate flat mortgage spreads and declining mortgage risk premia despite a rise in credit risk.

We then study how the economy with inflated house prices and mortgage debt, following the elevated demand for safe assets, responds to a housing risk shock, which causes more homeowners to default. Relative to an average economy drawn from the ergodic distribution, our high-safe-asset-demand economy is more vulnerable to this shock. The economy faces a sharp increase in mortgage defaults, which depletes the majority of the financial system's equity capital. This in turn induces banks to charge substantially higher spreads on mortgage debt for any given leverage, and as a result households cut back drastically on their mortgage leverage. Because our model connects household leverage choices with intermediary risk taking capacity, this counterfactual provides a rich illustration of how a growing demand for safe assets increases the size and riskiness of the financial sector and therefore the severity of financial crises.

Related Literature. A key feature of research in macroeconomics and finance since the 2008 financial crisis is a new understanding of how financial frictions impact the overall economy. A large and growing body of empirical research documents the macroeconomic roles of house prices, credit supply, and their impact on households' leverage and consumption. [1] Another recent body of empirical work documents how distressed financial institutions reduce the supply of credit to households and firms, contributing to a drop in output and employment.[2] Our goal is to develop a model that is consistent with and unifies findings in both empirical literatures as a framework for counterfactual analysis.

The quantitative macroeconomics literature after the housing boom has focused on models with exogenous housing collateral constraints following Iacoviello (2005), such as in Kiyotaki, Michaelides, and Nikolov (2011), Landvoigt, Piazzesi, and Schneider (2015), Favilukis, Ludvigson, and Van Nieuwerburgh (2017), and Guerrieri and Lorenzoni (2017). More recent work emphasized the importance of high household indebtedness and credit frictions for the severity of the bust, for example Guren, Krishnamurthy, and McQuade (2018) and Hedlund and Garriga (2018), or the relevance of household-level credit frictions for the transmission of monetary policy and other aggregate shocks, e.g. Elenev (2018), Wong (2018) and Greenwald (2018). Papers in this literature tighten or loosen exogenous collateral, leverage, or payment to income constraints to simulate a boom or bust. As pointed out by Justiniano, Primiceri, and Tambalotti (forthcoming), among others, it is necessary in such a framework to shock both households' borrowing constraints

[1] for example Mian and Sufi (2011), Mian, Rao, and Sufi (2013), Favara and Imbs (2015), Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017), Di Maggio and Kermani (2017), Adelino, Schoar, and Severino (2016), and Foote, Lowenstein, and Willen (2016)

[2] for example Chodorow-Reich (2014), Chodorow-Reich and Falato (2018), Benmelech, Meisenzahl, and Ramcharan (2014), and Ramcharan, Verani, and Van Den Heuvel (2016)

and constraints on the supply of mortgages to explain movements in both the prices and quantity of high leverage mortgages during the 2000-2006 housing boom. Our framework, in which a reduction in intermediaries' funding cost both lowers the price and raises the leverage of household debt, provides a single explanation for these facts.

Corbae and Quintin (2015) is the only paper we know in this literature where households choose the leverage of their mortgage. They study a framework where households face a menu of mortgage contracts offered by a risk-neutral lender subject to exogenous constraints and select endogenously into high and low leverage mortgages. To our knowledge, ours is the first quantitative paper to model heterogeneous borrowers facing a menu of leverage choices priced by constrained intermediaries in general equilibrium.

A separate research agenda in finance on intermediary asset pricing (He and Krishnamurthy (2013), Adrian, Etula, and Muir (2014), He, Kelly, and Manela (2017)) has shown empirically and quantitatively that the risk taking capacity of financial intermediaries is a key driver of asset prices. This approach to asset pricing successfully explains prices in a range of asset classes and is particularly important for pricing highly intermediated assets such as derivatives, bonds, and commodities (Haddad and Muir (2018)). Of particular relevance to our model is the empirical evidence (Gabaix, Krishnamurthy, and Vigeron (2007), Hanson (2014)) that the pricing of mortgage risk is sensitive to the risk taking capacity of specialized intermediaries. Relative to this literature, our contribution is to connect the pricing kernel of a constrained financial intermediary to the leverage choices of the agents that borrow from it.

Finally, our paper provides a potential resolution to the question whether the boom-bust episode was caused by loose credit constraints, or high expected future house prices (Kaplan, Mitman, and Violante (2017)). In our framework, a large positive shock to the demand for safe assets leads to a relaxation of credit constraints, and simultaneously puts the economy on a path of rising house prices. This integrates the three narratives about the origins of the financial crisis mentioned above, in a manner that depends crucially on the role of financial intermediaries in our model as both mortgage lenders and creators of safe assets. While existing work connects the demand for safe assets to financial fragility (Caballero and Krishnamurthy (2009)), the indirect effects of a safe asset shortage on househould leverage and consumption is new to our paper.

2 Model

2.1 Income and housing endowment

Aggregate output is $Y_t = \bar{Y}_t \tilde{Y}_t$, where $\bar{Y}_t$ is the income trend and grows at the deterministic rate g, i.e. $\bar{Y}_t = \bar{Y}_{t-1}\exp(g)$. $\tilde{Y}_t$ is the cyclical component and follows an AR(1) in logs

$$\log(\tilde{Y}_t) = (1 - \rho_y)\mu_y + \rho_y \log(\tilde{Y}_{t-1}) + \varepsilon_t,$$

where ε_t is i.i.d. and normally distributed with mean zero.

The economy is endowed with a constant stock of housing capital $\bar{H}$. Housing capital produces housing services according to the linear technology

$$s_t = n(h_t, \bar{Y}_t) = h_t \bar{Y}_t,$$

and owners of housing capital need to spend fraction δ_H of the capital value on maintenance each period.

2.2 Demographics

The economy is populated a continuum of households in three generations: old, middle-aged, and young. Young and middle-aged households respectively have probabilities π^Y and π^M of becoming middled-aged and old, drawn i.i.d across households. Old households live for one period. Each period, a measure one of new young households are born and the same measure of old households die. The population of each generation is constant, with measure $\frac{1}{\pi^Y}$ young, $\frac{1}{\pi^M}$ middle aged, and 1 old households.

2.3 Markets

Households have access to competitive markets for housing, mortgages, bank deposits, and bank equity. In addition, households within each generation are able to trade shares of their endowment of labor income but not with agents from other generations. This endowment pays in aggregate Y_t^*, equal to Y_t plus an additional term that reflects rebates of bankruptcy costs to households, to be defined below in equation (18).[3] Agents are also

[3]Mortgage default creates losses for banks. We view these losses as stemming from fire sales or factor payments to actors involved in the foreclosure process. Hence we rebate mortgage losses to households in proportion to endowment income.

able to frictionlessly rent housing from other members of their own generation. Because the rental markets for housing are segmented by generation, the equilibrium rental rate for housing is different for agents of different generations. Bank equity is available only to middle-aged agents, while all agents face the same risk-free interest rate on bank deposits.

In addition, agents of each generation face a menu of mortgage contracts offered by a financial intermediary from which they can choose. Mortgages can only be held by the financial intermediary, so households cannot buy them directly. These mortgages are priced by the intermediary's stochastic discount factor, which is influenced both by the preferences of middle-aged households who own its equity, as well as the severity of financial frictions that affect intermediaries.

2.4 Individual household problem

Preferences and timing. All households maximize expected utility with discount factor β and constant relative risk aversion γ.

Old households only live for one period. They decide how to split their wealth between consumption c_t^O and bequests b_t^O in order to maximize their utility function

$$u^O(c_t^O, b_t^O) = \frac{1}{1-\gamma}((c_t^O)^{1-\gamma} + \phi(b_t^O)^{1-\gamma}). \tag{1}$$

Middle-aged and young households obtain utility from consuming non-durables and housing as well as from their holdings of bank deposits. They can spend their wealth by consuming non-durables, renting housing, buying housing, and investing in a generation-dependent set of other assets. In addition they can take out a mortgage from the bank collateralized by their house and can choose the loan-to-value ratio of the mortgage, taking as given the menu of contracts offered by the bank.

Denote by $a \in \{Y, M\}$ the generation of an individual young or middle-aged household, respectively. The utility function depends on nondurable consumption c_t^a, housing consumption s_t^a and real deposit holdings d_t^a as follows:

$$u^a(c_t^a, s_t^a, d_t^a) = \frac{1}{1-\gamma}((c_t^a)^{1-\theta}(s_t^a)^{\theta})^{1-\gamma} + \psi^a \frac{(d_t^a)^{1-\gamma}}{1-\gamma}.$$

Households face an idiosyncratic shock to the value of their housing. If they finished period t owning h units of housing, they have $\epsilon_{t+1}^a h$ units of housing at the beginning of period $t+1$, where ϵ_{t+1}^a is a mean one lognormal random variable, drawn i.i.d. across households and across time. The variance of ϵ^a_{t+1} evolves as a binary Markov chain, which is the second source of aggregate risk in the economy (the first source being endowment shocks). We refer to high realizations of $\mathrm{Var}_t(\epsilon^a_t)$ as *housing risk shocks*. Households choose whether to default on their mortgage after these shocks are realized. If they default, a fraction λ^a of their wealth is lost. Each household chooses whether or not to default in order to maximize their continuation utility.

The precise timing of events within each period is:

1. Aggregate shocks, idiosyncratic housing and aging shocks are realized.
2. Households decide whether to default on their mortgage.[4]
3. All households make consumption and portfolio choices (including bequest choice for the old) given their post-default-decision wealth.

The post-default-decision wealth w^a_t is the only individual state variable of a household in generation a. Denote all other state variables exogenous to the household by $\mathcal{Z}_t$.

Old generation. The old begin the period with post-default-decision wealth w^O_t. Given their utility function in (1), they optimally choose to consume $c^O_t = \frac{1}{1+\phi^{\frac{1}{\gamma}}} w^O_t$ and bequeath $b^O_t = \frac{\phi^{\frac{1}{\gamma}}}{1+\phi^{\frac{1}{\gamma}}} w^O_t$ yielding a total amount of utility

$$V^O(w^O_t) = \frac{(w^O_t)^{1-\gamma}}{1-\gamma} \left[\left(\frac{1}{1+\phi^{\frac{1}{\gamma}}} \right)^{1-\gamma} + \phi \left(\frac{\phi^{\frac{1}{\gamma}}}{1+\phi^{\frac{1}{\gamma}}} \right)^{1-\gamma} \right]. \tag{2}$$

Recursive optimization problem for the young and middle-aged. Denote the vector of portfolio choices of an individual generation-a household, $a \in \{Y, M\}$, by $\alpha^a_t = [h^a_t, b^a_t, m^a_t, d^a_t]$. Here b^a_t is a vector of portfolio holdings over a set of assets $\mathcal{B}^a$ available to generation a for trade, with the corresponding price vector P^a_t. Let $q^a(\alpha^a_t, \mathcal{Z}_t)m^a_t$ be the amount the intermediary is willing to lend if the household chooses a portfolio α^a_t and mortgage face value of m^a_t. Note that the mortgage pricing function $q^a(\alpha^a_t, \mathcal{Z}_t)$ generally depends on the generation of the borrowing household.

[4] Old households also optimally default on their mortgage. Since the old only live for one period, they have the same beginning-of-period portfolio as the middle-aged, and their default decision is identical to that of middle-aged households.

The full problem of a household of age a is

$$
\begin{aligned}
V^a(w_t^a, \mathcal{Z}_t) = \max_{c_t^a, s_t^a, \alpha_t^a} & \frac{\left((c_t^a)^{1-\theta}(s_t^a)^{\theta}\right)^{1-\gamma}}{1-\gamma} + \psi_a \frac{(d_t^a)^{1-\gamma}}{1-\gamma} \\
& + \beta(1-\pi^a)\mathrm{E}_t\left[\max\left\{V^a(w_{t+1}^{a,nd}, \mathcal{Z}_{t+1}), V^a(w_{t+1}^{a,d}, \mathcal{Z}_{t+1})\right\}\right] \\
& + \beta\pi^a\mathrm{E}_t\left[\max\left\{V^{a+}(w_{t+1}^{a+,nd}, \mathcal{Z}_{t+1}), V^{a+}(w_{t+1}^{a+,d}, \mathcal{Z}_{t+1})\right\}\right],
\end{aligned} \tag{3}
$$

subject to the budget constraint

$$
w_t^a = c_t^a + \rho_t^a s_t^a + h_t^a P_t - \rho_t^a h_t^a \bar{Y}_t + b_t^a \cdot P_t^a + \frac{d_t^a}{1+r_t} - q^a(\alpha_t^a, \mathcal{Z}_t) m_t^a, \tag{4}
$$

and the definition of next-period wealth for non-defaulters

$$
w_{t+1}^{a,nd} = (1-\delta_H) P_{t+1} \epsilon_{t+1}^a h_t^a + d_t^a + b_t^a \cdot (P_{t+1}^a + x_{t+1}^a) - m_t^a, \tag{5}
$$

and for defaulters

$$
w_{t+1}^{a,d} = (1-\lambda^a)(d_t^a + b_t^a \cdot (P_{t+1}^a + x_{t+1}^a)), \tag{6}
$$

where x_t^a is the vector of payoffs paid by the assets available to agents in the generation if the agent *does not age*. If the agent does age, its wealth is

$$
w_{t+1}^{a+,nd} = (1-\delta_H) P_{t+1} \epsilon_{t+1}^a h_t^a + d_t^a + b_t^a \cdot (P_{t+1}^{a+} + x_{t+1}^{a+}) - m_t^a, \tag{7}
$$

and for defaulters

$$
w_{t+1}^{a+,d} = (1-\lambda^a)(d_t^a + b_t^a \cdot (P_{t+1}^{a+} + x_{t+1}^{a+})), \tag{8}
$$

where $x_t^{a^+}$ is the vector of payoffs paid by the assets available to agents in generation a if the agent *ages*.

To apply this general structure of the optimization problem to both generations, we need to specify the precise timing of the aging shocks, and the continuation value functions $V^{a+}(w_t, \mathcal{Z}_t)$. For middle-aged households, we simply have that $V^{M+}(w_t, \mathcal{Z}_t) = V^O(w_t)$ as defined in (2). Further, the asset portfolio available to the middle-aged, $\mathcal{B}^M$, does not pay off differently depending on their age transition, which implies that $x_{t+1}^M = x_{t+1}^{M+}$ and the distinction between equations (5) – (6) and (7) – (8) is not necessary.

We assume that young households can "age twice" within one period, meaning that they can transition from young to middle-aged, then immediately to old. This implies that their continuation value function is

$$V^{Y+}(w_t, \mathcal{Z}_{t+1}) = \pi^M V^O(w_t) + (1 - \pi^M) V^M(w_t, \mathcal{Z}_t). \tag{9}$$

We further allow for different payoffs to the young asset portfolio based on the outcome of the age transition, such that generally $x^Y_{t+1} \neq x^{Y+}_{t+1}$.

2.4.1 Characterization of the Household Problem

The following proposition provides the key result for characterizing the optimization problem in (3).

Proposition 1. 1. *The household value function of has the form*

$$V^a(w^a_t, \mathcal{Z}_t) = v^a(\mathcal{Z}_t) \frac{(w^a_t)^{1-\gamma}}{1-\gamma}, \tag{10}$$

where $v^a(\mathcal{Z}_t)$ only depends on aggregate state variables.

2. *The choice vector $[c^a_t, s^a_t, \alpha^a_t]$ is linear in individual wealth w^a_t, conditional on the aggregate state. As a result, the decisions made at time t by generation a households are independent of the time t wealth distribution within the generation.*

Proof. See appendix. □

Proposition 1 has two key implications. First, although agents within each generation behave like a representative agent in their consumption and portfolio choice ex-ante, they are not insured ex-post against the idiosyncratic shocks to the value of the houses they own. As a result, only households who face a sufficiently bad idiosyncratic housing shock will choose to default. The key to this combination of aggregating to a representative agent ex-ante (necessary for numerical tractability) and heterogeneity ex-post (necessary to meaningfully calibrate the model) is that agents face shocks which are multiplicative in the amount of housing they own. As explained in Constantinides and Duffie (1996), this sort of multiplicative shock cannot be avoided by trading financial assets within the generation when agents have CRRA utility, so choices naturally aggregate as if each generation was a representative agent in autarky.

Second, the mortgages given to all households in the same generation are equally risky. Richer households borrow more and buy more housing than poorer ones, but the endogenously optimal degree of leverage is the same for all households in the same generation.

This result (which is key for proving the aggregation proposition above) comes from the fact that the mortgage pricing function $q^a(\alpha_t^a, \mathcal{Z}_t)$ is homogeneous of degree zero in the portfolio vector α_t^a. That is, an agent with twice as much housing, twice as much mortgage face value, and twice the financial portfolio of another will be provided with twice as much of a loan by the intermediary. The property of the mortgage pricing function is derived from the optimal behavior of the financial intermediary below.

Households default if and only if their wealth (inclusive of the costs of default) will be higher than if they did not default. Define the non-housing wealth of a household, conditional on the outcome of the age transition at $t+1$, as

$$w_{t+1}^{a_{t+1},nh} = d_t^a + b_t^a \cdot (P_{t+1}^{a_{t+1}} + x_{t+1}^{a_{t+1}})$$

where

$$a_{t+1} = \begin{cases} a & \text{if the household does not age,} \\ a^+ & \text{if the household ages.} \end{cases}$$

Then households default for any realization of ϵ_{t+1}^a such that

$$(1-\lambda^a) w_{t+1}^{a_{t+1},nh} > \underbrace{\epsilon_{t+1}^a (1-\delta_H) P_{t+1} h_t^a - m_t^a}_{\text{home equity}} + w_{t+1}^{a_{t+1},nh}, \tag{11}$$

This relation defines a cutoff value $\bar{\epsilon}_{t+1}^{a_{t+1}}$, conditional on the age transition, such that the household defaults if and only if their realized ϵ_{t+1}^a is lower.

Corollary 1. *There exists default thresholds $\bar{\epsilon}_{t+1}^{a_{t+1}}$ such that generation-a households with $\epsilon_{t+1}^a < \bar{\epsilon}_{t+1}^{a_{t+1}}$ default. The aggregate default rate of generation a is $\pi^a F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^{a+}) + (1-\pi^a) F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^a)$.*

It is obvious from the definition of the default threshold in (11) that absent any default costs (i.e. if $\lambda^a = 0$), households default if and only if they have negative home equity at the beginning of the period. A greater cost of default ($\lambda^a > 0$) requires more negative home equity to trigger default.

2.5 Financial Intermediary

The financial intermediary is a publicly traded firm in a competitive financial market that maximizes the market value of its equity, which is owned in equilirium by the middle-aged generation. It makes mortgages and issues deposits and equities backed by these mortgages. The intermediary starts period t with inside equity e_t. It must pay a dividend τe_t each period, so that it does not save its way out of financial constraints over time. It can raise outside equity I_t at a cost $C(I_t, \bar{Y}_t) = \frac{\chi}{\bar{Y}_t} I_t^2$. Further, the intermediary faces a regulatory capital constraint that its inside equity can never be less than some fraction $\bar{e}$ of the value of its assets, ensuring its deposits are riskless. Its mortgage lending is financed by the inside equity it does not pay out, newly issued equity, and deposits.

Mortgages provided by the intermediary are priced competitively, so that the intermediary makes zero economic profits from each loan. The present value of cash flows paid by a borrower (valued with the intermediary's pricing kernel) therefore determines how much the intermediary is willing to lend. This is true for mortgages at any degree of leverage and made to any household, so borrowers naturally face a menu of mortgage contracts from which to choose. If $\mathcal{M}^I_{t,t+1}$ is the intermediary's pricing kernel (derived in the appendix) at time t for valuing cash flows paid at time $t+1$, a mortgage that pays δ_m will induce the intermediary to lend l_m at time t equal to

$$l_m = E_t(\mathcal{M}^I_{t,t+1}\delta_m).$$

Note that the cash flows an intermediary receives from a mortgage depend both on the mortgage's face value as well as the portfolio choices of the borrower, since these endogenously affect the borrower's incentives to default. A mortgage made to a borrower with portfolio α^a_t and face value m^a_t has present value $q^a(\alpha^a_t, \mathcal{Z}_t)m^a_t$, for $a = Y, M$, which intermediaries determine based on the expected payoff of the mortgage.

Although the intermediary could in principle make loans with arbitrary amounts of leverage, the aggregation results derived above drastically simplify the intermediary's problem. We can assume that the intermediary only makes mortgages optimally selected by households from the menu of mortgage contracts offered by the intermediary. The intermediary then only optimizes over the total face value N^Y_t and N^M_t of mortgages lent to each generation. Further, the law of large numbers implies that all idiosyncratic risk in the payoffs of mortgages diversifies away.

Based on corollary 1, the payoff to one dollar of face value of this portfolio for the middle generation is

$$\mathcal{P}^M_{t+1} = 1 - F^M_{\epsilon,t+1}(\bar{\epsilon}^M_{t+1}) + F^M_{\epsilon,t+1}(\bar{\epsilon}^M_{t+1})(1-\xi)(1-\delta_H)\frac{E_t(\epsilon^M|\epsilon^M < \bar{\epsilon}^M_{t+1})P_{t+1}h^M_t}{m^M_t}, \quad (12)$$

where a fraction ξ of the house value is lost when a house of a defaulting borrower is repossessed.

Similarly, for the young generation, the payoff per dollar of face value is

$$\mathcal{P}^Y_{t+1} = (1-\pi^Y)(1-F^Y_{\epsilon,t}(\bar{\epsilon}^Y_{t+1})) + \pi^Y(1-F^Y_{\epsilon,t+1}(\bar{\epsilon}^{Y+}_{t+1})) + (1-\xi)(1-\delta_H)\frac{P_{t+1}h^Y_t}{m^Y_t}\times \Big[(1-\pi^Y)E_t(\epsilon^Y|\epsilon^Y < \bar{\epsilon}^Y_{t+1})F^Y_{\epsilon,t+1}(\bar{\epsilon}^Y_{t+1}) + \pi^Y E_t(\epsilon^Y|\epsilon^Y < \bar{\epsilon}^{Y+}_{t+1})F^Y_{\epsilon,t+1}(\bar{\epsilon}^{Y+}_{t+1})\Big]. \quad (13)$$

The intermediary's inside equity at the start of a period is the value of its mortgage portfolio minus its payments to depositors.

$$e_{t+1} = N^Y_t \mathcal{P}^Y_{t+1} + N^M_t \mathcal{P}^M_{t+1} - D_t. \quad (14)$$

The budget constraint of the intermediary is

$$(1-\tau)e_t + I_t - C(I_t, \bar{Y}_t) + \frac{D_t}{1+r_t} = N^Y_t q^Y(\alpha^Y_t, \mathcal{Z}_t) + N^M_t q^M(\alpha^M_t, \mathcal{Z}_t). \quad (15)$$

Let $\mathcal{M}^M_{t,t+1}$ be the stochastic discount factor of the middle aged, who own the intermediary's equity. The full optimization problem of the intermediary is

$$V^I(e_t, \mathcal{Z}_t) = \max_{I_t, D_t, N^M_t, N^Y_t} \tau e_t - I_t + \mathrm{E}_t\left[\mathcal{M}^M_{t,t+1} V^I(e_{t+1}, \mathcal{Z}_{t+1})\right], \quad (16)$$

subject to the budget constraint (15). The regulatory capital constraint requires limits intermediary leverage for all possible states at time $t+1$. Denote the current state by $\mathcal{Z}_t$ and all possible future states conditional on the current state as $z_{t+1}|\mathcal{Z}_t$. Then the capital constraint leads to the following set of constraints in t

$$e_{t+1} \geq \bar{e}^Y N^Y_t \mathcal{P}^Y(z_{t+1}) + \bar{e}^M N^M_t \mathcal{P}^M(z_{t+1}) \ \forall z_{t+1}|\mathcal{Z}_t$$

or equivalently[5]

$$(1-\bar{e}^Y)N^Y_t \mathcal{P}^Y(z_{t+1}) + (1-\bar{e}^M)N^M_t \mathcal{P}^M(z_{t+1}) \geq D_t \ \forall z_{t+1}|\mathcal{Z}_t.$$

At time t, it suffices to impose the constraint for the worst possible aggregate state in $t+1$: if the solvency condition is binding for the worst possible payoff of the mortgage

[5]Since mortgages to young and middle-aged households may have different default risk in equilibrium, we allow for different capital requirements by borrower generation, $\bar{e}^Y$ and $\bar{e}^M$.

portfolio, it will be slack for all higher payoff realizations. This implies that we can define

$$\underline{z}_t = \underset{z_{t+1}|\mathcal{Z}_t}{\operatorname{argmin}}\ N_t^Y \mathcal{P}^Y(z_{t+1}) + N_t^M \mathcal{P}^M(z_{t+1}),$$

and impose a single constraint at time t

$$(1-\bar{\epsilon}^Y) N_t^Y \mathcal{P}^Y(\underline{z}_t) + (1-\bar{\epsilon}^M) N_t^M \mathcal{P}^M(\underline{z}_t) \geq D_t. \tag{17}$$

This regulatory capital constraint is only occasionally binding. If the intermediary is sufficiently well capitalized, it has a precautionary incentive to save because of the risk of the constraint binding in the future.

The following proposition verifies what was assumed above about mortgage pricing in order to show that households in each generation make decisions equivalent to those of a representative agent. It follows simply from the fact that there is some pricing kernel that prices mortgages and is therefore general and robust.

Proposition 2. *The mortgage pricing functions $q^a(\alpha_t^a, \mathcal{Z}_t)$ for $a = Y, M$ are homogeneous of degree zero in the wealth of borrowing households, w_t^a.*

Proof. Suppose a household of generation a chooses a portfolio α_t^a and mortgage face value m_t^a. Let δ_{t+1}^a be the cash flows paid by this borrower to the intermediary. If the borrower multiplied each element of its portfolio and its mortgage face value by a constant $k > 0$, the intermediary would get cash flows repaid of $k\delta_{t+1}^a$. The intermediary is therefore willing to lend $q^a(k\alpha_t^a, \mathcal{Z}_t)(km_t^a) = E_t(M_{t,t+1}^I k\delta_{t+1}^a) = kE_t(M_{t,t+1}^I \delta_{t+1}^a) = q^a(\alpha_t^a, \mathcal{Z}_t)(km_t^a)$. This implies $q^a(k\alpha_t^a, \mathcal{Z}_t) = q^a(\alpha_t^a, \mathcal{Z}_t)$. □

Inspection of the expression in (12) and (13) shows that mortgage payoffs only depend on the default thresholds $(\bar{\epsilon}_{t+1}^Y, \bar{\epsilon}_{t+1}^{Y+}, \bar{\epsilon}_{t+1}^M)$ and the inverse mortgage leverage ratios $P_{t+1}h_t^a/m_t^a$. From the aggregation result in proposition 1, these objects are independent of borrower wealth. Intuitively, mortgage pricing depends on mortgage leverage, and the ratio of housing to total wealth. Since all households choose to invest equal shares of wealth in housing and mortgage debt, these ratios are the same for all borrowers within a generation.

2.6 Aggregation and Equilibrium

Tradable endowment income. Both young and middle-aged households can trade shares of an asset whose payoff depends on aggregate endowment income Y_t. Specifically, we first define aggregate income

$$Y_t^* = Y_t + B_t^O + \Lambda_t, \tag{18}$$

where Λ_t is the sum of all default penalties (of households) and losses (of intermediaries) in period t, to be defined below in equation (24). These penalties and losses are rebated to households as part of the endowment income. Further, B_t^O is the aggregate bequest from the current old households.

To describe the payoff structure of the endowment assets, we first define the generation-dependent payoffs

$$y_t^Y = \nu Y_t^*, \tag{19}$$

and

$$y_t^M = (1-\nu)Y_t^*. \tag{20}$$

The parameter $\nu \in [0,1]$ determines the share of aggregate endowment income paid to young households. Young households can purchase shares in unit net supply of their generation-specific endowment asset at price p_t^Y. While households remain young, this asset is a perpetuity with payoff y_t^Y per share as defined in (19). Newly born households are endowed with one share of the young-generation-specific endowment asset.

Upon turning middle-aged, the shares of this asset expire, after yielding the final payoff per share[6]

$$\frac{\delta^M}{\pi^Y} y_t^M,$$

where y_t^M is defined in (20) and and $\delta^M \in [0,1]$ parameterizes the degree to which the endowment asset available to the middle-aged has front-loaded payoffs.

Similarly, middle-aged households can purchase shares in unit net supply of their generation-specific endowment asset at price p_t^M. This asset is a perpetuity with payoff per share[7]

$$(1-\delta^M)y_t^M.$$

[6]Recall that each period, there is a total mass of $1/\pi^Y$ young households, of which π^Y turn into newly middle-aged. Thus at the beginning of the period, the newly middle-aged hold fraction π^Y of the shares of the young-generation-specific asset.

[7]The shares of this asset never expire, i.e. the old sell their shares to the middle aged in the terminal period of their lives.

Parameters ν and δ^M allow us to parsimoniously specify the life-cycle income profile of households. If $\nu < 1/2$, young households receive a smaller fraction of the aggregate endowment each period in total, and thus face an upward-sloping lifetime income path. We can then use δ^M to generate a hump-shape in the income profile. To see this, consider the extreme case of $\delta^M = 1$. In that case, each household receives the total middle-aged portion of its lifetime income as a lump-sum when turning middle-aged, corresponding to a strongly hump-shaped income profile that peaks in early middle age. In contrast, when $\delta^M = 0$, middle-aged income is paid out as constant stream until old age. In our model, δ^M regulates the savings demand of the middle-aged and thus equilibrium interest rates and mortgage leverage. We will discuss the effect of δ^M further in the calibration section.

Generation-specific assets. Given the endowment assets described above, we can specify the generation-specific sets of assets $\mathcal{B}^Y$ and $\mathcal{B}^M$.

For the middle-aged, the set consists of (1) their generation-specific endowment asset, and (2) intermediary equity that in aggregate pays the intermediary's dividend $\tau e_t - I_t$. Thus we have the portfolio vector $b_t^M = [b_t^{M,M}, b_t^{M,I}]$, with $b_t^{M,M}$ denoting holdings of the endowment assets, and $b_t^{M,I}$ denoting holdings of intermediary equity, respectively. The corresponding price vector is $P_t^M = P_t^{M+} = [p_t^m, p_t^I]'$, and the payoff vector is

$$x_t^M = x_t^{M+} = [(1 - \delta^M) y_t^M, \tau e_t - I_t]'.$$

For the young, the set only consists of their generation-specific endowment asset.[8] Thus the portfolio vector is simply $b_t^Y = b_t^{Y,Y}$, where $b_t^{Y,Y}$ are holdings of the young endowment asset. The prices and payoffs depend on the age transition, with

$$P_t^Y = p_t^Y, \; x_t^Y = y_t^Y,$$

and

$$P_t^{Y+} = 0, \; x_t^{Y+} = \frac{\delta^M}{\pi^Y} y_t^M.$$

Equilibrium. Uppercase letters denote aggregate choice variables for young, middle-aged and old generations throughout. Market clearing for mortgage debt requires that

[8] We could allow the young to hold bank equity, and impose a short-sale constraint on bank equity positions. For our calibration, this would yield the same equilibrium allocation, since the young are natural borrowers and do not want to hold a long position in bank equity.

intermediaries purchase the full portfolio of mortgages of both borrowing generations:

$$
\begin{aligned}
N_t^Y &= M_t^Y, \\
N_t^M &= M_t^M.
\end{aligned}
$$

Market clearing for housing capital requires that

$$
H_t^Y + H_t^M = \bar{H},
$$

and the rental market needs to clear within each generation

$$
\begin{aligned}
S_t^Y &= H_t^Y \bar{Y}_t, \\
S_t^M &= H_t^M \bar{Y}_t.
\end{aligned}
$$

Market clearing for intermediary liabilities requires

$$
\begin{aligned}
D_t &= D_t^Y + D_t^M, \\
B_t^{M,I} &= 1.
\end{aligned}
$$

Shares to the endowment assets of the young and middle aged are in unit supply such that

$$
\begin{aligned}
B_t^{Y,Y} &= 1, \\
B_t^{M,M} &= 1.
\end{aligned}
$$

Finally, market clearing for non-durables requires that

$$
Y_t = C_t^Y + C_t^M + C_t^O + C(I_t, \bar{Y}_t).
$$

An equilibrium is a set of prices and allocations such that all 3 generations and the intermediary solve their optimization problems above (equations 3 and 16) and all markets clear.

Aggregation. In every period, there is a total measure of $1/\pi^Y$ young, a fraction $1 - \pi^Y$ of whom are "incumbent" young and the remaining fraction π^Y are newly born households (with a fraction π^Y having moved on to middle-age). Denote the total beginning-of-period liquid wealth of the incumbent young that do not turn middle-aged as

$$\begin{aligned}\overrightarrow{W}_t^{Y,Y} =& (1-\pi^Y)\underbrace{\left(1-F^Y_{\epsilon,t}(\bar{\epsilon}^Y_t)\right)((1-\delta_H)\mathrm{E}_t[\epsilon^Y|\epsilon^Y>\bar{\epsilon}^Y_t]P_tH^Y_{t-1}-M^Y_{t-1})}_{\text{home equity of non-defaulters}}\\ &+(1-\pi^Y)\underbrace{\left(1-\lambda^Y F^Y_{\epsilon,t}(\bar{\epsilon}^Y_t)\right)(D^Y_{t-1}+\mathbf{1}\cdot(P^Y_t+x^Y_t))}_{\text{other wealth}-\text{def. penalty}}.\end{aligned}$$

The home equity of the non-defaulters depends on the conditional expectation $\mathrm{E}_t[\epsilon^Y|\epsilon^Y > \bar{\epsilon}^Y_t]$, which is the average realization of the idiosyncratic house price shock conditional on not defaulting. Similarly, we define the aggregate wealth of the young that turn middle-aged as

$$\begin{aligned}\overrightarrow{W}_t^{Y,M} =& \pi^Y\underbrace{\left(1-F^Y_{\epsilon,t}(\bar{\epsilon}^{Y+}_t)\right)((1-\delta_H)\mathrm{E}_t[\epsilon^Y|\epsilon^Y>\bar{\epsilon}^{Y+}_t]P_tH^Y_{t-1}-M^Y_{t-1})}_{\text{home equity of non-def.}}\\ &+\pi^Y\underbrace{\left(1-\lambda^Y F^Y_{\epsilon,t}(\bar{\epsilon}^{Y+}_t)\right)\left(D^Y_{t-1}+\mathbf{1}\cdot(P^{Y+}_t+x^{Y+}_t)\right)}_{\text{other wealth}-\text{def. penalty}}.\end{aligned}$$

Since newly born households are endowed with one share of the young-generation-specific endowment asset, the aggregate wealth of the young generation is

$$W^Y_t = \pi^Y(y^Y_t+p^Y_t)+\overrightarrow{W}_t^{Y,Y}. \tag{21}$$

We further define the wealth of incumbent middle-aged as

$$\begin{aligned}\overrightarrow{W}_t^{M} =& \left(1-F^M_{\epsilon,t}(\bar{\epsilon}^M_t)\right)((1-\delta_H)\mathrm{E}_t[\epsilon^M|\epsilon^M>\bar{\epsilon}^M_t]P_tH^M_{t-1}-M^M_{t-1})\\ &+\left(1-\lambda^M F^M_{\epsilon,t}(\bar{\epsilon}^M_t)\right)\left(D^M_{t-1}+\mathbf{1}\cdot(P^M_t+x^M_t)\right).\end{aligned}$$

Then the aggregate wealth of the middle generation is

$$W^M_t = (1-\pi^M)\left(\overrightarrow{W}_t^{Y,M}+\overrightarrow{W}_t^{M}\right), \tag{22}$$

and the aggregate beginning-of-period wealth of the old is

$$W^O_t = \pi^M\left(\overrightarrow{W}_t^{Y,M}+\overrightarrow{W}_t^{M}\right). \tag{23}$$

The rebate term Λ_t in (18) consists of bankruptcy costs of intermediaries Λ^ξ_t and monetary default penalties Λ_t^λ for households. The total bankruptcy costs are

$$\Lambda_t^\xi = \xi P_t \Big[F_{\epsilon,t}^M(\bar{\epsilon}_t^M) \mathrm{E}_t[\epsilon^M | \epsilon^M < \bar{\epsilon}_t^M] H_{t-1}^M \\ + \Big((1-\pi^Y) F_\epsilon^M(\bar{\epsilon}_t^Y) \mathrm{E}_t[\epsilon^Y | \epsilon^Y < \bar{\epsilon}_t^Y] + \pi^Y F_\epsilon^Y(\bar{\epsilon}_t^{Y+}) \mathrm{E}_t[\epsilon^Y | \epsilon^Y < \bar{\epsilon}_t^{Y+}] \Big) H_{t-1}^Y \Big].$$

Monetary default penalties are

$$\Lambda_t^\lambda = \lambda^Y (1-\pi^Y) F_{\epsilon,t}^Y(\bar{\epsilon}_t^Y)(D_{t-1}^Y + \mathbf{1}\cdot(P_t^Y + x_t^Y)) + \lambda^Y F_{\epsilon,t}^Y(\bar{\epsilon}_t^{Y+})(D_{t-1}^Y + \mathbf{1}\cdot(P_t^{Y+} + x_t^{Y+})) \\ + \lambda^M F_{\epsilon,t}^M(\bar{\epsilon}_t^M) \Big(D_{t-1}^M + \mathbf{1}\cdot(P_t^M + x_t^M) \Big).$$

The sum of both rebate terms,

$$\Lambda_t = \Lambda_t^\xi + \Lambda_t^\lambda, \tag{24}$$

is included in the payoff of the endowment along with the output of the economy.

3 Calibration and Solution Method

3.1 Parameterization

We calibrate the model to annual U.S. data. Table 1 lists the parameter values. As indicated in the table, several parameters are directly set to external estimates. The remaining parameters are chosen jointly so that simulated moments from the model's stationary distribution match a set of corresponding moments in the data. We choose 1998 as base year for the calibration, since in the boom-bust simulation below, this will be the starting point of the trend of declining real interest rates.

Growth Rate and Productivity shocks. We calibrate the trend growth rate and productivity shocks based on real disposable income per capita from 1929-2017. The annual growth rate is exactly 2%. The standard deviation and autocorrelation of the cyclical HP-filtered series are 2.7% and 45%, respectively. We convert the continuous AR(1) productivity process to a 3-state Markov chain using the Rouwenhorst (1995) method. The aggregate endowment income per year is normalized to 1, as is the fixed housing stock $\bar{H}$.

Table 1: Parameter choices

Parameter		Value	Target	Data	Model
			TFP		
Growth rate	g	0.02	Average growth rate income p.c.		
Income shocks std.dev.	σ^Y	0.027	Std. dev. HP-filtered income p.c.		
Income shocks AC	ρ^Y	0.45	Autocorrelation HP-filtered income p.c.		
			Preferences		
Risk aversion (1/IES)	γ	1	standard		
Patience	β	0.85	1998 real yield (1-year T-bill %)	3.21	3.3
Weight on housing	θ	0.19	Housing wealth/income (1998 SCF)	2.05	2.04
Liquidity preference M	ψ^M	0.0125	Liquidity premium KVJ	70bp	100bp
Liquidity preference Y	ψ^Y	0.02	Money-like assets of young/income (1998 SCF)	0.15	0.15
			Life-cycle		
Transition prob Y	π^Y	0.10	Young lifespan 25-34		
Transition prob M	π^M	0.033	Middle-aged lifespan 35-64		
Income share of young	ν	0.34	Consumption share young (Wong)	0.32	0.3
Middle-aged income profile	δ^M	0.93	Middle-aged wealth/agg. income (1998 SDF)	3.51	3.46
Bequest parameter	ϕ	1	Consumption share old (Wong)	0.11	0.08
			Housing and mortgages		
Forced maintenance	δ_H	0.025	Housing depreciation (BLS)		
Idiosyncratic shock std.dev Y,M	σ_ϵ^0	0.3	Mortgage delinquency rate (%)	4.2	3.1
Idiosyncratic shock std.dev Y,M	σ_ϵ^1	0.48	Delinquency rate, crisis (%)	9.1	7.6
Trans. prob. $\sigma_\epsilon^0 \to \sigma_\epsilon^1$	$\Gamma_{1,2}$	0.05	% periods in housing recession		0.2
Trans. prob. $\sigma_\epsilon^1 \to \sigma_\epsilon^0$	$\Gamma_{2,1}$	0.2	Duration (years) housing recession		4.5
Default penalty Y	λ^Y	0.02	LTV of young (1998 SCF)	0.64	0.64
Default penalty M	λ^M	0.01	LTV of middle-aged (1998 SCF)	0.43	0.39
Foreclosure loss to bank	ξ	0.3	Charge-off rate mortgages (%)	0.5	0.47
			Intermediary		
Capital requirement	$\bar{e}^M$	0.01	Basel requirement Agency MBS		
Capital requirement	$\bar{e}^Y$	0.08	Basel requirement Mortgage Loans		
Target payout ratio	τ	0.051	Dividend payout rate (Baron)		
Equity issuance cost	χ	600	Effective payout rate (Baron)	3.2	3.8

Parameters without corresponding data and model moments in the two rightmost columns are set directly based on external data. All other parameters are jointly chosen to match the target moments listed in the table as closely as possible, in the ergodic distribution of the simulated stochastic model.

Preferences and Life-Cycle. Risk aversion is set to a standard value of 1, implying log utility. We choose the discount factor β to match the deposit rate in the model to the annualized real yield of 1-year treasury bills in 1998, which is 3.2%. Several preference and life-cycle related parameters are chosen to match moments from the Survey of Consumer Finances (SCF). We compute means for the target moments from the 1998 SCF wave, using SCF sampling weights. Following Wong (2018), we categorize households by age of the household head, with the young being 25-34 years of age, the middle-aged between 35-64, and the old 65 and older. Accordingly, we set $\pi^Y = 0.1$ to achieve an average duration of 10 years in young age, and $\pi^M = 0.033$ to achieve average duration of 30 years in middle age. We set the weight on housing in the Cobb-Douglas consumption aggregator to 0.19 to match the aggregate housing wealth-to-income ratio in the SCF, which comes out to 2.05. We pick the aging parameter ψ^M to match the liquidity premium estimated by Krishnamurthy and Vissing-Jorgensen (2012) of 70 bp for the middle-aged.[9] For young households, we choose ψ^Y to match the ratio of liquid wealth-to-income for young households from the SCF of 15%.

The share of the aggregate endowment received by the young, ν, is set to 34% to match the consumption share of the young, which Wong (2018) estimates to be 32% of aggregate consumption. Similarly, we choose the bequest utility parameter to match the consumption share of the old at 11%, also as reported by Wong (2018). Naturally, the middle-aged consume the remaining 57%.

Housing and Mortgages. We set the forced maintenance of housing δ_H to match depreciation of residential fixed assets based on the BEA fixed asset tables. Idiosyncratic house price dispersion follows a two-state Markov Chain with transition matrix Γ, with state 0 indicating normal times, and state 1 indicating elevated housing risk. The probability of staying in the normal state in the next year is 95% and the probability of staying in the crisis state in the next quarter is 80%. Under these parameters, the economy spends 80% of the time in the normal state and 20% in the high housing risk state, and the average duration of the high risk state is 4.5 years. These transition probabilities are independent of the aggregate endowment state. The low uncertainty state has $\bar{\sigma}_{\epsilon,0} = 0.30$ and the high uncertainty state has $\bar{\sigma}_{\omega,1} = 0.48$. We use the same values for idiosyncratic housing risk of young and middle-aged households. These numbers allow the model to achieve an average mortgage delinquency rate of 3.1% per year unconditionally and of 7.6% per year in housing recessions, which are periods defined by low endowment growth and high

[9] We calculate the liquidity premium in the model as the difference of a counterfactual risk free rate that does not provide any liquidity services and the deposit rate.

housing risk. In the data, the average mortgage delinquency rate is 4.2% unconditionally, and peaks at 11.5% in 2011.[10] Combined with a foreclosure loss for banks of 30%, the model generates a loss-given-default rate of 47%, and thus an overall loss rate on banks' mortgage portfolio 1.6% unconditionally and 3.5% in housing recessions, in line with data on loss rates on residential loans for the 1991-2017 period.

Given the housing risk parameters, we choose the pecuniary default penalties λ^Y and λ^M to match mortgage leverage of young and middle-aged households in the SCF. Holding fixed other parameters, households choose lower leverage at a higher level of λ. A value of $\lambda^Y = 1.5\%$ delivers young leverage of 64%, and $\lambda^M = 0.5\%$ gives middle-aged leverage of 39%. Even though we make it more costly for the young to default, they still choose higher leverage than the middle-aged because of the severity of their liquidity constraints.

Intermediary. We set the equity capital requirements for the intermediary sector based on Basel risk-weighted regulatory requirements for mortgage assets. Since mortgages of young households are far riskier than those of middle-aged borrowers in equilibrium, we assign 100% risk weight to these assets, which combined with a simple equity ratio requirement of 8% yields $\bar{e}^Y = 0.08$. We calibrate the capital requirement for middle-age mortgages, which are close to risk-free, to the risk weight of GSE-issued mortgage backed securities of 20%, yielding $\bar{e}^M = 0.016$.[11] We calibrate the remaining two parameters of the intermediary objective based on evidence in Baron (2018). We set τ to match the dividend payout rate of banks, and χ to target the effective payout rate net of equity issuances, both measured as a fraction of book equity.

3.2 Solution Method

We solve the model numerically using a global projection method. The two aggregate exogenous state variables of the economy are the cyclical component of the endowment, and the time-varying cross-sectional dispersion of idiosyncratic housing shocks. Both shocks are jointly approximated by a discrete-time Markov chain. The model features four endogenous aggregate state variables, which span the wealth distribution across the different optimizing agents. They are aggregate wealth of the young, middle-aged, and

[10] Data are from Mortgage Bankers Association for 1979-2017. The model produces lower delinquency rates than the data, but in the model delinquencies always turn into full mortgage defaults.

[11] These capital requirements for portfolio mortgages and agency MBS have not changed since Basel I regulations.

old, and intermediary equity. Since the wealth of all agents has to add up to aggregate tradable wealth, we only need to keep track of any three of these four endogenous state variables when computing the model.

The solution technique involves approximating the unknown functions that characterize the equilibrium of the economy over the domain of the state variables. The Appendix summarizes the set of equations and unknowns that fully characterize the equilibrium. For details on the solution method, see Elenev, Landvoigt, and Van Nieuwerburgh (2018).

4 Results

We perform a long simulation of the calibrated model and report both the average behavior of the economy as well as conditional on an expansion, recession, and housing recession. In addition, we use the model for three quantitative exercises. First, we compare the behavior of the economy in regular productivity-driven recessions and in "housing recessions" that feature both low endowment realizations and high house price shock dispersion. Second, we show how the economy responds to an unanticipated 30% drop in intermediary capital. Finally, we consider the effects of an unanticipated shock: in the model calibrated to the 1998 base year, the supply of assets available for middle-aged savings declines. The decline is modeled as a perfect-foresight trend over 10 years. As result, demand for savings increases relative to supply and the deposit rate gradually declines to 1.5%. We show how that this trend shock leads to a credit boom and increases the severity of future housing crisis.

4.1 Properties of Ergodic Distribution

Borrowers. Table 2 report means from the ergodic distribution of the model. Young households own 24% of the total housing stock, while the middle-aged generation own the remaining 76% (lines 1 and 2). Even though the middle-aged own more housing, they have significantly less leverage, with a LTV ratio of 37% (line 4), as compared to 64% for young households (line 4). Since the middle-aged also hold the majority of deposits issued by banks (lines 5-6), as well as bank equity, they are substantially wealthier than the young. Both types of households face the same idiosyncratic housing risk. Therefore, the young have much higher default risk as result of higher mortgage leverage: the average mortgage default rate of young borrowers is 7.77%, whereas it is only 0.46% for the middle-aged (lines 7-8). Why do young households take on so much higher leverage than the old, despite facing the same amount of mortgage risk? This is because young households are financially constrained, in the sense that they expect greater income and wealth in future life-cycle phases. Higher mortgage leverage for a given house size allows them to trade the possibility of greater future mortgage payments in exchange for more consumption in the present. Although there are costs of mortgage default, they are willing to bear some of these costs in exchange for moving consumption forward in time.

The average default rate for lenders holding a diversified portfolio of young and middle-aged mortgages is 3.12%, which results in an effective loss rate on mortgage assets of 1.50% (lines 9-10). How do leverage and defaults vary of the business and housing cycle? Leverage of both types of borrowers is *highest* in regular recessions, as default risk in those periods is not much higher than during expansions, and households use their home equity for consumption smoothing. In particular, young household leverage rises to 65% in regular recessions. During housing recessions, leverage of all borrowers declines as default rates spike, and lenders raise interest rates. The default rate of middle-aged households rises to 2.05% on average, while that of young borrowers jumps to 17%. As a result, the average default rate rises to 7.67%, and intermediaries make credit losses of 3.89% of their portfolio per year in housing recessions.

Banks. Total mortgages assets are roughly equal in size to endowment income at 0.942 (line 12), in line with the data for the base year of 1998. Banks fund on average 8.17% of their assets with equity (line 13), and the remaining fraction with deposits, implying a total average deposit base of 0.85 (line 11). About 46% of bank equity represents a buffer that banks must hold to satisfy their solvency constraint (line 14). This buffer is largest during expansions at 55%, when the difference between the current state (expansion) and the worst possible future state (housing recession) is largest. The banking sector shrinks substantially during housing recessions: mortgage assets and deposits decline by almost 14% and 13%, relative to their unconditional average. The equity ratio drops down to 4.47%, as banks suffer large losses. Banks' equity buffer during housing recessions is depleted to 6.73% of total equity, which is sufficient to satisfy the solvency constraint: once the economy transitioned into a housing recession, there is little downside risk going forward.

These equity dynamics are reflected in the fraction of periods during which the intermediary's solvency constraint is binding (line 15). It is binding 26.86% of expansions years. During these periods banks expand lending, but also earn high realized returns on their mortgages. Thus expansions are also periods of high payouts to shareholders (line

Table 2: Means of ergodic distribution

	Uncond.	**Exp.**	**Rec.**	**Housing Rec.**
		Households		
1. Housing Y	0.242	0.244	0.239	0.232
2. Housing M	0.758	0.756	0.761	0.768
3. Leverage Y	63.80%	64.83%	65.35%	59.26%
4. Leverage M	37.38%	38.90%	39.13%	31.19%
5. Deposits Y	0.040	0.042	0.039	0.035
6. Deposits M	0.813	0.850	0.837	0.670
7. Default rate Y	7.77%	5.44%	7.36%	17.05%
8. Default rate M	0.46%	0.12%	0.19%	2.05%
9. Default rate (dollar weighted)	3.12%	2.02%	2.73%	7.67%
10. Loss rate (dollar weighted)	1.50%	0.93%	1.26%	3.89%
		Intermediary		
11. Total deposits	0.853	0.891	0.876	0.705
12. Total mortgage assets	0.942	0.989	0.972	0.757
13. Equity ratio (% of assets)	8.17%	8.99%	8.97%	4.47%
14. Equity buffer (% of equity)	46.50%	55.82%	50.31%	6.73%
15. Fraction solvency constr binds	32.57%	26.86%	23.11%	70.45%
16. Net payout rate	3.89%	4.39%	4.36%	1.35%
17. External equity excess ret	1.78%	1.72%	1.80%	2.03%
18. Internal equity excess ret	4.31%	3.33%	2.96%	11.76%
		Prices		
19. House Price	2.044	2.089	1.995	1.885
20. Deposit rate	3.43%	3.21%	5.83%	3.75%
21. Convenience yield	1.09%	1.05%	1.02%	1.27%
22. Mortgage spread Y	4.37%	3.76%	3.95%	7.01%
23. Mortgage spread M	0.22%	0.20%	0.19%	0.33%
24. Mortgage Y excess ret	0.65%	0.59%	0.55%	1.19%
25. Mortgage M excess ret	0.05%	0.06%	0.04%	0.04%
		Welfare		
26. Consumption Y	0.307	0.312	0.293	0.294
27. Consumption M	0.570	0.575	0.551	0.552
28. Consumption O	0.072	0.073	0.070	0.070
29. Consumption gr vol, Y	3.31%	3.05%	2.83%	3.03%
30. Consumption gr vol, M	2.46%	2.28%	2.10%	2.27%
31. Default cost rebates	0.009	0.007	0.009	0.020
32. Housing maintenance	0.051	0.052	0.050	0.047

The table reports averages from a long simulation (10,000 periods) of the benchmark model. First column: unconditional average, second column: expansions (high or average endowment), third column: regular recessions (low endowment), and fourth column: housing recessions (low endowment, high housing risk σ_ϵ^1).

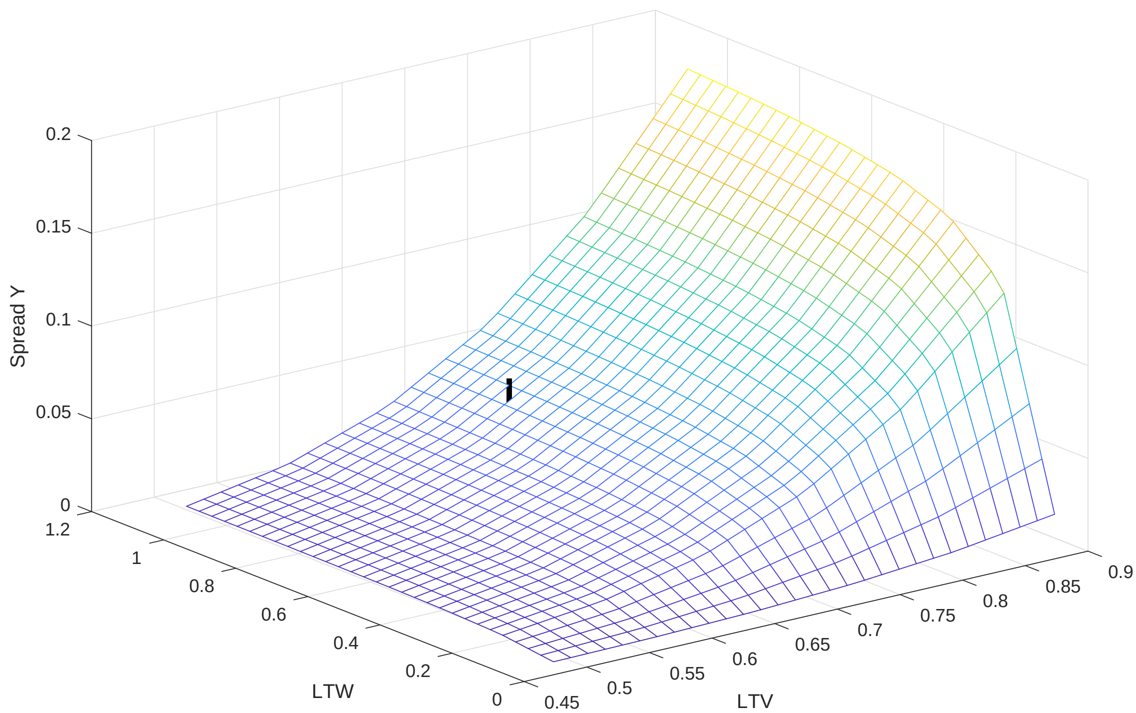
16), with a payout rate of 4.39%. The constraint is slack 77% of periods during regular recessions, as lending contracts and banks only suffer moderate losses. During these periods, both realized and *expected* returns are low. The constraint binds in 70% of housing recession periods. In these periods, banks raise new equity, reducing the payout rate to 1.35% to recover from mortgage losses by retaining earnings.

While the external expected excess return (EER) to bank equity is relatively low at 1.78% (line 17), the internal EER is much higher at 4.31%. The difference shows that making mortgage loans to young households, while borrowing "cheap" deposits from middle-aged is a very profitable business for banks. However, due to the equity-related frictions, there is a large wedge between internal and external equity. Forward-looking profit opportunities for banks become largest during housing recessions, with the EER on internal equity rising to 11.8%. This reflects countercyclical risk compensation: once the economy has arrived in a housing recession, recovery of house prices and mortgage debt to normal levels involves a large expansion of banking. During the recovery from housing recession, young households that were forced to deleverage have high demand for mortgage debt, and banks can earn large excess returns going forward (line 24).

Prices. The model generates substantial house price fluctuations, close to the volatility of aggregate house prices in the data. House prices are 13% lower during housing recessions than during expansions (line 19). The deposit rate is countercyclical (line 20), a feature our model has in common with other endowment economies that have mean-reverting shocks and agents with a low intertemporal elasticity of substitution: during recessions, agents expect higher income in the future and would like to move this future income to the present, reducing savings demand. To clear the deposit market, the interest rate needs to rise. Interestingly, the interest rate rises by much less during housing recessions than during regular recessions. This is because in housing recessions deposit *supply* by banks also shrinks. The convenience yield contained in deposit rates is 1.09% on average (line 21), and rises in housing recessions when deposits become scarce. Mortgage spreads for young households are much higher at 4.37% than those of middle-aged households at 0.22%, primarily due to large differences in risk (lines 22-23). At 0.65%, almost one quarter of the mortgage spread for the young is a risk premium (line 24). The risk premium on young mortgages is highest during housing recessions.

Consumption. The consumption distribution reflects the income and wealth distribution between generations (lines 26-28). Consumption fluctuations are mainly driven by endowment shocks, with aggregate consumption in regular and housing recessions being roughly equal. Larger default costs from bankruptcies in housing recessions are offset by less expenditure on housing maintenance due to lower house prices (lines 31, 33). While consumption growth of the young is more volatile than endowment income (3.31% vs. 2.7%), middle-aged consumption growth vol is lower (2.46%). The young prioritize asset accumulation though levered exposure to housing over consumption smoothing.

Figure 1: Average Credit Surface for Young

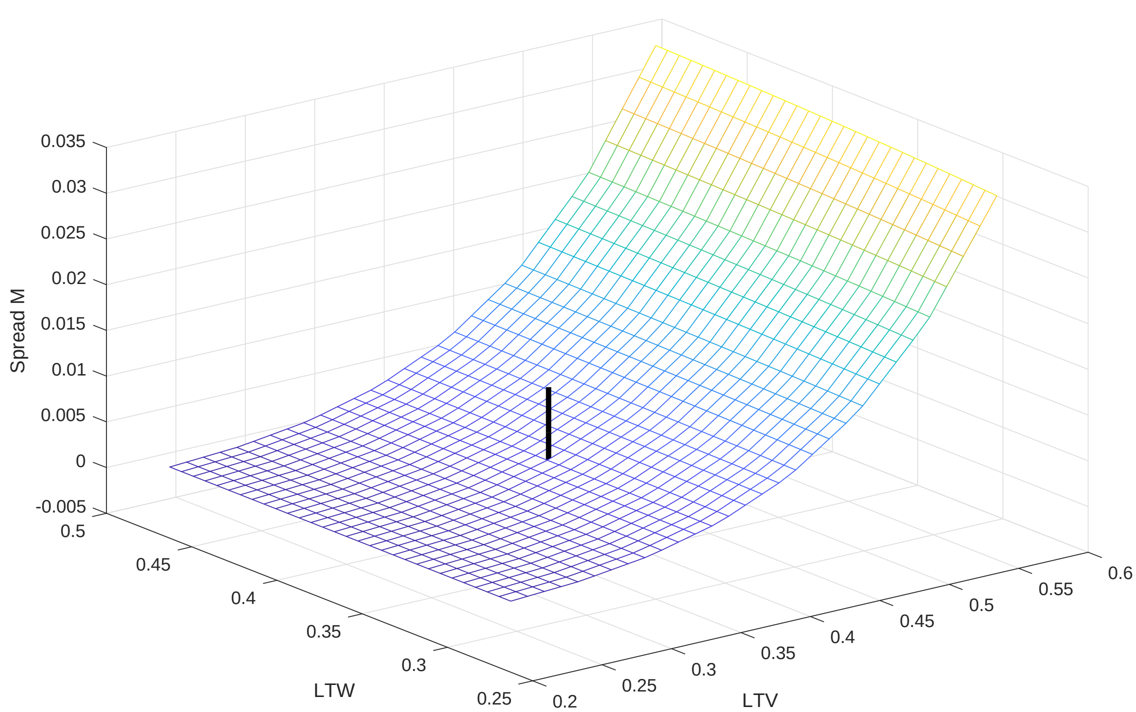


Credit Surfaces. Figures 1 and 2 plot the average "credit surfaces" facing young and middle aged borrowers. The credit surface for a borrower reports the interest rate it would be charged on a loan if it had a given loan to value (LTV) ratio and loan to wealth (LTW) ratio. The credit surface defines the menu of mortgage contracts available to a borrower, who then selects an optimal contract. The plots below are created by computing the credit surface available to young and middle aged borrowers at each point in the economy's state space and then reporting an average over the ergodic distribution.

As is intuitive, young borrowers face higher interest rates than middle aged borrowers at the same LTV and LTW, since the non-pledgeability of their large future endowment income makes them more likely to default. In addition, the interest rate charged for a given young borrower is increasing and convex in LTW and LTV, consistent with empirical properties of credit surfaces estimated by Geanakoplos and Rappoport (2019). Middle aged borrowers face a credit surface that is increasing and convex in LTV but roughly flat in LTW, which suggests that they default primarily when their house is severely underwater in a manner that is not quantitatively very connected to their overall degree of wealth.

The black line on each credit surface plot reports the average LTV and LTW chosen by households of each generation of households. Young households choose considerably higher leverage than middle aged households (roughly 63 % for the young and 39 % for the middle aged), and they are charged a much larger spread on their mortgage interest rate. Because young households are constrained in their ability to consume today against their non-pledgeable wealth, they choose to increase their consumption by having a highly levered mortgage. This high leverage exposes the lender to default risk, who therefore charges a large credit spread. Middle aged households instead do not need to use mortgages for the purpose of financing consumption because they have liquid wealth that can be consumed. As a result, they choose a lower degree of leverage, and their mortgages are almost riskless.

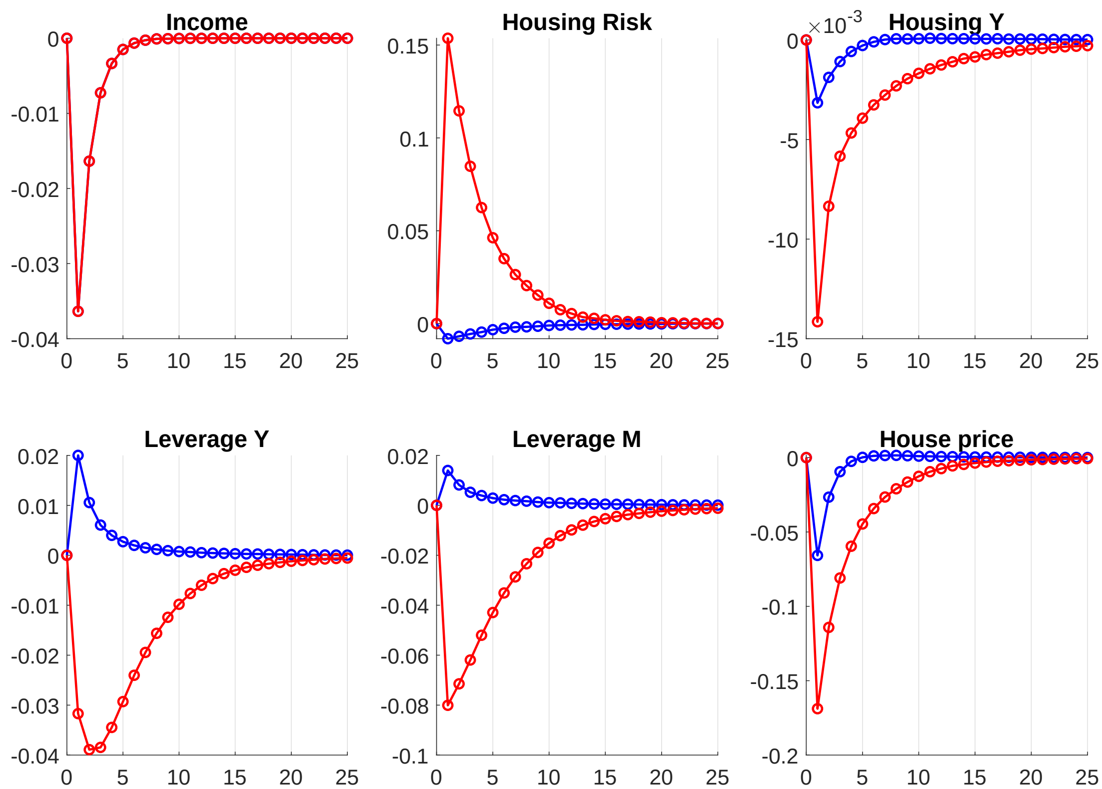
Figure 2: Average Credit Surface for Middle Aged



4.2 Response to productivity and housing risk shocks

Next, we will examine the how the economy reacts to endowment (productivity) and housing risk shocks. Figure 3 shows impulse response functions to a pure negative endowment shock (blue) and the combination of a negative endowment shock and a housing risk shock (red). By construction, the blue and red lines coincide in the top left panel of Figure 3. However, as can be seen from the second panel in the top row, housing risk spikes during housing recessions, and reverts to normal levels over ten years on average.[12] Aggregate consumption of agents is primarily driven by endowment shocks, but the differences in consumption responses across agents is substantial.

Figure 3: Regular vs. housing Recession (part 1)



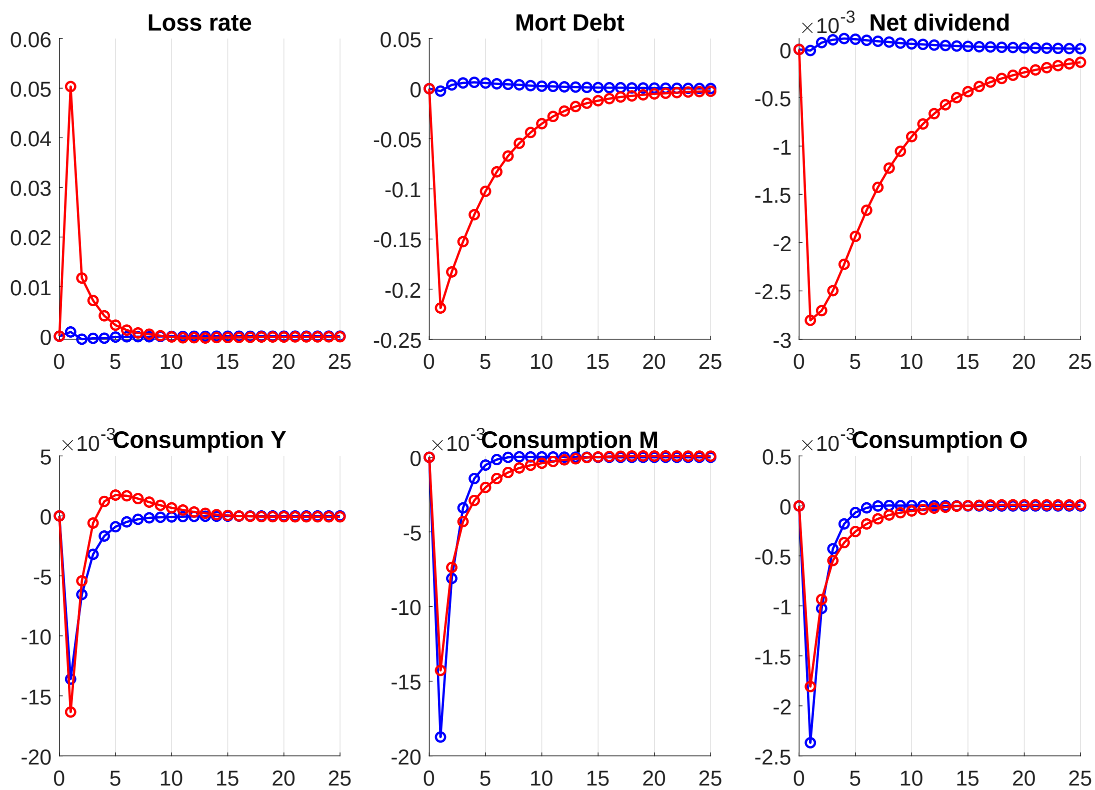
Blue: regular recession, **Red:** housing recession. The generalized IRF plots are created by simulating the economy 10,000 times for 25 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing-risk state $\bar{\sigma}_{\epsilon,0}$. The plots indicate deviations from the unconditional path in levels.

However, as can be seen in the top right panel of Figure 3, housing recessions cause reallocation of housing capital from young to middle-aged borrowers. This is mainly

[12] Recall that σ_ϵ is a two-state Markov chain with the average duration of a high-housing-risk episode being 4.5 years.

due to a strong wealth effect: young households have much greater exposure to house price risk than middle-aged households due to their high leverage. As house prices drop in housing recessions, young households lose more wealth than middle-aged, and sell housing capital.

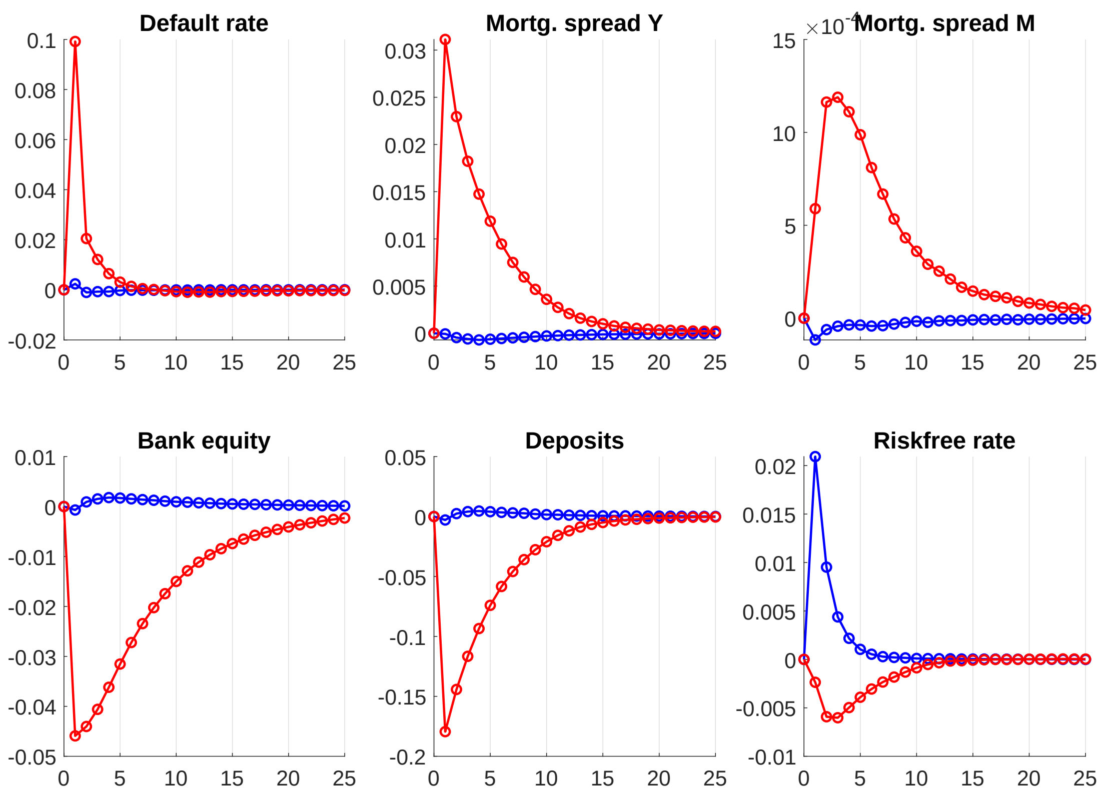
Figure 4: Regular vs. housing Recession (part 2)



Blue: regular recession, **Red:** housing recession. The generalized IRF plots are created by simulating the economy 10,000 times for 25 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing-risk state $\bar{\sigma}_{\epsilon,0}$. The plots indicate deviations from the unconditional path in levels.

We can see the large drop in house prices during housing recession in the bottom right panel of figure 3. While households increase leverage in regular recessions, they reduce leverage in housing recessions. Despite the sharp deleveraging of young borrowers, the mortgage spread they face spikes during housing recessions, while it remains flat in regular recessions. Middle-aged households reduce leverage roughly twice as much as the young, since they can fund their consumption out of financial wealth rather than borrowing. The difference in responses of young and middle-aged households highlights the differing degrees of financial constraints the two generations face. Young households have large exposure to housing risk and low wealth. When mortgage borrowing becomes more expensive, they are not only forced to delever, but also shed housing capital and cut consumption. The more wealthy middle-aged are less affected by reduced mortgage borrowing. They simply re-optimize their portfolio to be less levered, and buy housing capital cheaply from the young. As house prices recover, the earn large returns. During regular recessions, the deposit rate exhibits the typical properties of the riskfree rate in an endowment economy: it rises sharply to increase savings demand. However, the rate drops in housing recessions, reflecting the scarcity of safe assets that can be issued by distressed financial intermediaries.

Figure 5: Regular vs. housing Recession (part 3)



Blue: regular recession, **Red:** housing recession. The generalized IRF plots are created by simulating the economy 10,000 times for 25 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing-risk state $\bar{\sigma}_{\epsilon,0}$. The plots indicate deviations from the unconditional path in levels.

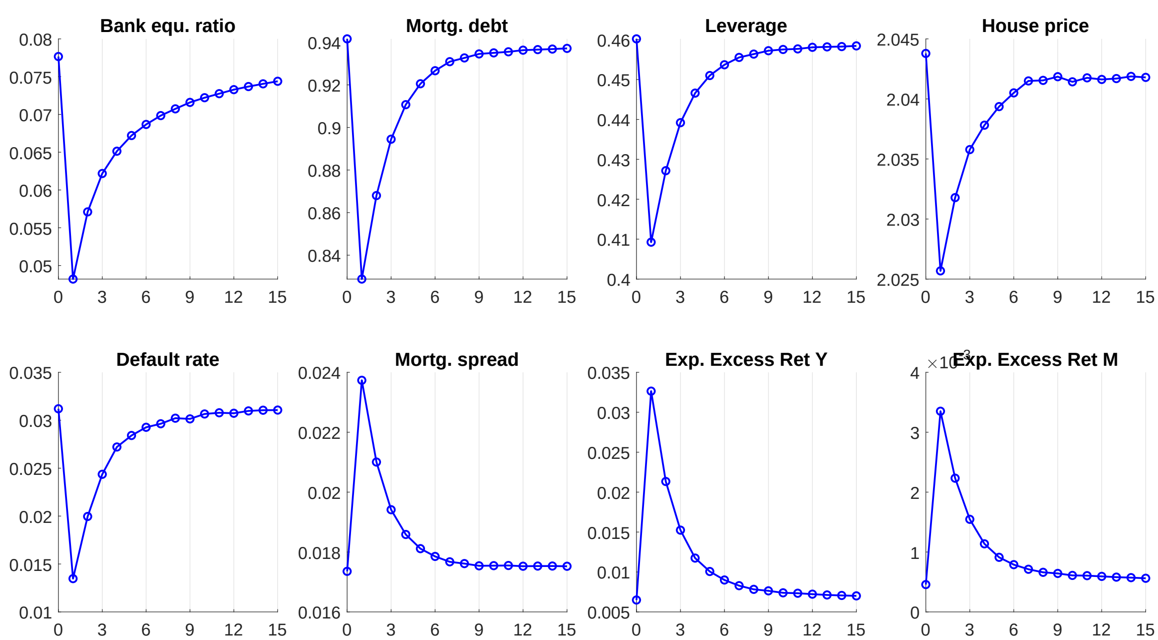
The reason for this difference can be seen in figure 5. In housing recessions, the mortgage default rate (top left) spikes sharply, and then drop back quickly as consequence of households deleveraging. Bank equity is depleted by more than 50% relative to its baseline level following these losses (bottom left), dropping by rougly 3.5 % of output. The banking sector shrinks, reducing both assets (mortgage debt) and liabilities (deposits) sharply as a result. The recovery of bank equity takes substantially longer than the mean-reversion of the housing risk shock due to equity issuance costs. The net dividend paid by banks (top right panel of figure 4) remains low for more than 15 years following the initial shock during a housing recession, reflecting the slow build-up in equity. The dynamics of the banking sector are in stark contrast to regular recessions, during which bank equity and dividends increase, and the banking sector expands slightly.

4.3 Bank equity, leverage, and housing booms

The results of the previous section suggest that the interaction between household leverage and constrained credit supply from intermediaries is a powerful amplification mechanism in housing recessions.

Role of bank equity. To further isolate the effect of bank equity capital as state variable on equilibrium dynamics, we analyze the effect of an unanticipated reduction in internal bank equity, similar in magnitude to the losses banks suffer in a housing recession.

Figure 6: Effect of reduction in bank equity



The generalized IRF plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states. The plots report the evolution of variables in levels.

Figure 6 illustrates the pure effect of a loss in bank equity. In the initial period, there is an unanticipated drop in bank equity by approximately 40% (top left), roughly the same magnitude of drop that banks experience in housing recessions. Having less equity to back deposits, banks need to raise mortgage rates, and households need to cut back on borrowing. Total mortgage debt declines by roughly 1/2 of the decline in housing recessions. House prices also fall, albeit by a much smaller magnitude than during housing recessions. As house prices drop by less, there is only a smaller wealth effect for young households, and less reallocation of housing capital from young to middle-aged, which in turn is consistent with the smaller drop in house prices.

Overall, figure 6 shows that the bank balance sheet effect has quantitatively large effects on credit supply and household leverage. It also clarifies that the deep fall in house prices during housing recessions (figure 4) is mainly caused by the combination of low income and high housing risk shocks. These shocks jointly diminish the viability of mortgage borrowing for young households, forcing them to shed housing capital to the middle-aged, who in turn have a lower marginal valuation of housing.[13]

The cause of these large effects on quantities are large shifts in mortgage prices. Even though aggregate leverage declines by enough to cut default risk in half (bottom left), the mortgage spread spikes (bottom left-middle), reflecting greater risk premia in mortgage spreads. As bank equity is depleted, the bank's solvency constraint is tightened, causing a large rise in mortgage spreads despite endogenously smaller credit risk.

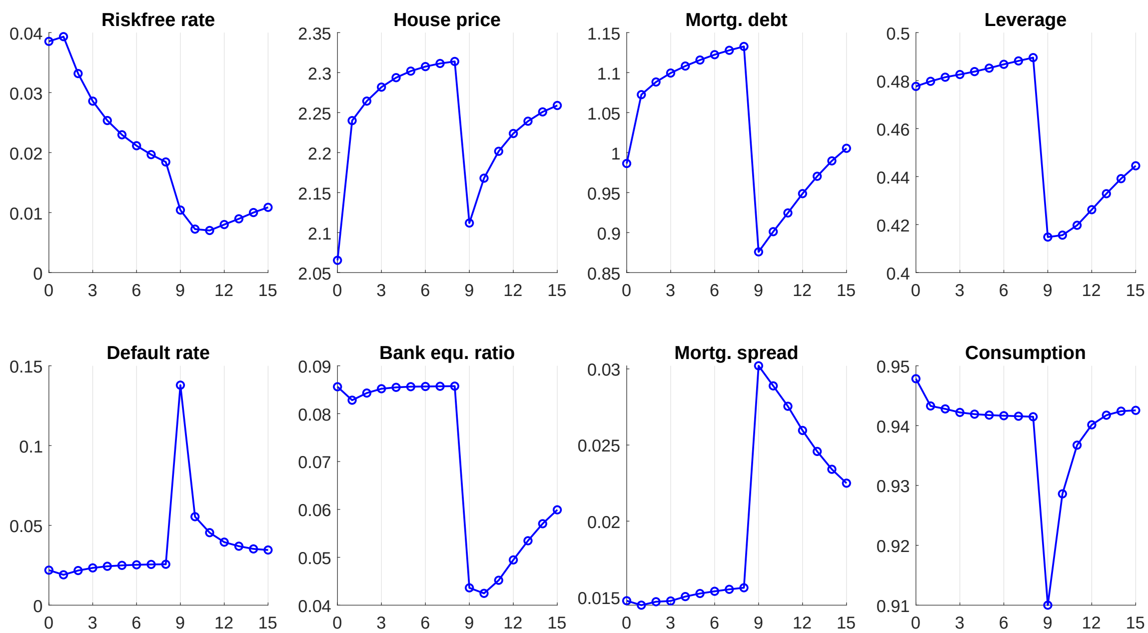
Increased demand for assets. To which extent can low interest rates, driven by a greater demand for assets, explain a credit cycle? To answer this question, we simulate the economy for a specific sequence of shocks. The simulation starts at the ergodic distribution of all state variables. Then the economy experiences two unanticipated shocks: (i) households learn that over the next 9 years, the fraction of middle-aged income y_t^M that goes to the newly middle-aged households linearly increases from .93 to .98. Over the same 9 years, the taste parameter ψ_M of the middle-aged will linearly increase from its current value of .0125 to a new steady state value of .02. Our first shock, allocates the income of middle-aged households earlier in their life, effectively requiring them to hold a larger portfolio of financial assets in order to save for retirement. This greater imbalance between asset demand and supply exerts downward pressure on the riskfree interest rate. The shock can be interpreted broadly as one which increases the relative demand for sav-

[13]Interestingly, it is the combination of both shocks that causes large housing recessions in the model. A pure housing risk (i.e. second moment) shock, has effects similar to the unanticipated reduction in bank equity in figure 6. As discussed above, a pure endowment shock does not lead to a contraction of the banking sector.

ings compared to the supply of existing assets and therefore bids up asset prices (the "global savings glut"). Our second shock, increasing the preference for safe assets, makes it cheaper for intermediaries to issue riskless deposits and thus induces intermediaries to borrow more. The combination of both shocks causes a fall in the riskfree rate but also in the yields on risky assets, without leading to a counterfactually large rise in the convenience yield on safe assets.

Over the 9 years of these parameter adjustments, the productivity shock is realized to be its median level in all periods, while the housing risk shock is realized to be low. In year 9, a high housing risk shock is realized, triggering an event similar to the wave of mortgage defaults in 2007 that contributed to the financial crisis. After that, the simulation progresses stochastically for 6 more years as the economy recovers.

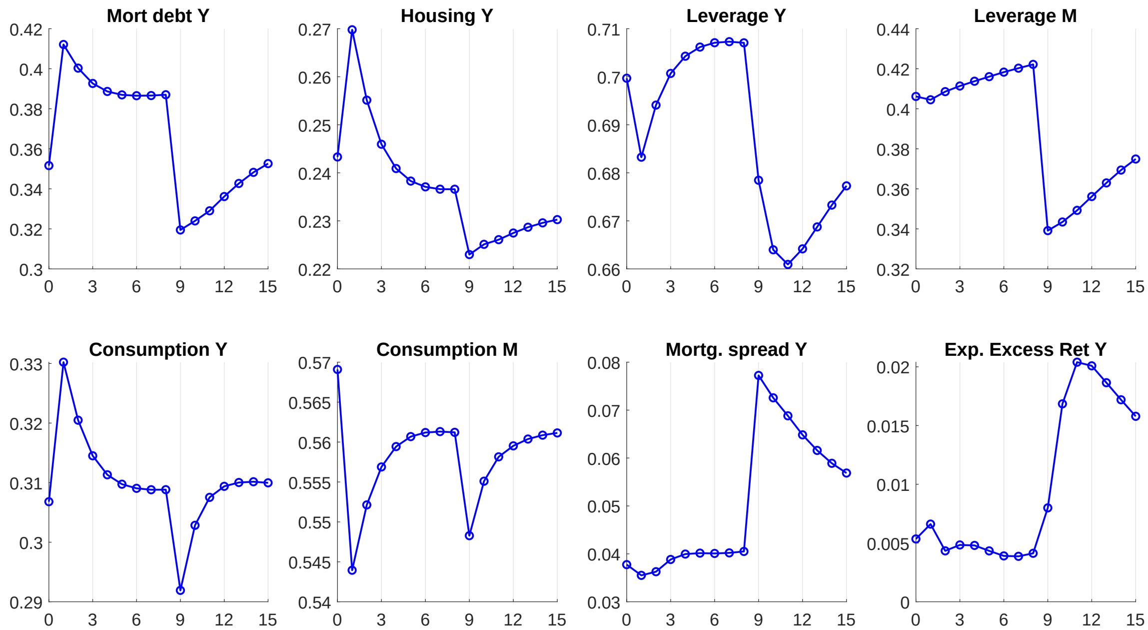
Figure 7: Housing boom and bust



The generalized IRF plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states. All simulations have the same sequence of shocks for the first seven years, which are six years of average endowment and low housing risk realizations, followed by a housing recession (low endowment, high housing risk) in year seven. The plots report the evolution of variables in levels.

Figures 7 and 8 show the response of the economy to this sequence of shocks. The increased demand for assets drives a reduction in the risk free rate throughout the first 9 years. Because of the resulting supply of cheap credit, mortgage debt surges to provide a larger supply of assets to meet the growing demand. Since house prices rise sharply, a large rise in mortgage debt only requires a moderate rise in leverage as a whole, consis-

Figure 8: Housing boom and bust



The generalized IRF plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states. All simulations have the same sequence of shocks for the first seven years, which are six years of average endowment and low housing risk realizations, followed by a housing recession (low endowment, high housing risk) in year seven. The plots report the evolution of variables in levels.

tent with the aggregate data for the boom period. Young households, whose consumption is highly sensitive to credit supply, use their increase in mortgage debt to consume more non-durables and more housing. Because of the exogenous aggregate endowment, market clearing requires that the middle-aged reduce their consumption as well. Similar to empirical findings by Justiniano, Primiceri, and Tambalotti (2017), mortgage spreads are very low at the beginning of this credit boom and widen slightly as the boom continues. Even as mortgage debt, leverage, and aggregate credit risk build up over the course of the boom, mortgage risk premia are weakly declining (bottom right of 8). House prices and mortgage debt each grow by roughly 20% during the boom.

After the realization of the high housing risk shock, a wave of mortgage defaults occur. Because the housing risk shock is persistent, mortgages as a whole remain risky after this shock and default rates remain elevated. The high housing risk shock also leads to large losses for financial intermediaries who own mortgages, depleting their equity capital and constraining their ability to lend. As a result, the spread on mortgages to risky young households spikes from under 4% to over 8%, and excess return on these mortgages goes from 0.5% to 2%. This shows that fluctuations in the price of risk driven by frictions in financial intermediation contribute significantly to the tightening of credit conditions after a housing crisis. The young consumers who must borrow to finance their consumption cut back sharply, and all households reduce the leverage they choose on their mortgages. This illustrates how shocks to the financial system propagate in a manner that looks similar to a tightening of a borrowing constraint for households as occurs in more conventional models of the housing boom and bust. The effects of a housing risk shock is considerably stronger in this simulation than in a “standard" housing recession analyzed above, showing that a shortage of assets makes the financial system particularly vulnerable to shocks.

A key feature of this boom-bust episode is the impatience of young households. After the initial shock that pushes interest rates down over time, they use the new supply of cheap credit to fund a sharp increase in their consumption. As the boom continues, even though interest rates continue to fall, the young households have exhausted their new ability to consume and must gradually cut back while maintaining a constant level of mortgage debt. In this sense, they seem to behave akin to a “hand-to-mouth” consumer. This is true even though they own housing wealth, and their choice of mortgage leverage is a meaningful channel by which they can intertemporally substitute consumption between the presence and the future. Their behavior is similar to the “wealthy hand-to-mouth" (of Kaplan and Violante (2014), Kaplan, Moll, and Violante (2018)), though with the added feature that their consumption is highly dependent on the supply of credit from financial intermediaries.

5 Conclusion

The main innovation of our model is that the supply of credit to borrowing-constrained households depends on the risk-taking capacity of financial intermediaries. As a result, when financial intermediaries are distressed, constrained households endogenously choose to reduce their mortgage leverage and must cut back on their consumption. This connection between the health of the financial sector and real economy gives us a novel propagation mechanism for shocks to the financial system.

In our first quantitative counterfactual, we find that an increase in mortgage defaults inhibits the ability of intermediaries to create safe assets and thereby leads to a low interest rate. In addition, the depletion of intermediary equity due to these losses lead to deleveraging and a reduction in consumption by the most constrained households, as well as a drop in house prices due to their reduced collateral value. We also find in a second similar exercise that an exogenous reduction in the equity capital of intermediary leads to an increase in mortgage spreads, a decrease in mortgage leverage, a decrease in the consumption of the most constrained households, and a reduction in house prices. Finally, we use our model to show that a growing demand for safe assets leads to many features of the housing and credit boom of the 2000s and increases the severity of future financial crises after a shock to mortgage default. In particular we find that an increase in the demand for safe assets induces intermediaries to expand their balance sheet and make riskier loans, which induces households to borrow more and boosts the consumption of constrained households. However, after an increase in mortgage defaults, this increased size and riskiness of the financial sector leads to a more severe drop in household leverage, consumption of constrained households, and house prices.

Broadly speaking, our model implies that shocks to intermediary capital emphasized by the intermediary asset pricing literature building on He and Krishnamurthy (2013) cause a negative credit supply shock that induces households to delever and consume less as emphasized by the literature following Mian and Sufi (2011). Our model therefore has a novel transmission mechanism of distress from Wall Street to Main Street, because leverage is endogenously determined. Going forward, we hope to enrich the general equilibrium effects of this transmission mechanism by making several features of our model endogenous. First, the fact that we have an endowment economy does not let us consider effects on output. Second, adding nominal rigidities would allow us to study additional mechanisms by which aggregate demand can affect output, which may feed back onto the health of the financial sector. Third, our current model of mortgage borrowing misses the fact that mortgages are long-term and can only be refinanced when homeowners have sufficient home equity.

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A Model

A.1 Proofs

A.1.1 Proposition 1

In order to use variables that are stationary if the economy grows at a trend rate g, we renormalize the choices $(c_t^M, s_t^M, \alpha_t^M)$ as well as prices (P_t, p_t^I), and the bank dividend x_t^I. The rents ρ_t^a, interest rate r_t, and housing and asset holdings (h_t^a, b_t^a) are stationary variables along such a balanced growth path.

Thus, defining $G = exp(g)$, the detrended problem along the BGP of a household in generation a, $a \in \{Y, M\}$ is

$$\begin{aligned} V^a(w_t^a, \mathcal{Z}_t) = \max_{c_t^a, s_t^a, \alpha_t^a} & \frac{\left((c_t^a)^{1-\theta}(s_t^a)^\theta\right)^{1-\gamma}}{1-\gamma} + \psi \frac{(d_t^a)^{1-\gamma}}{1-\gamma} \\ & + \beta(1-\pi^a)\mathrm{E}_t\left[G^{1-\gamma}\max\left\{V^a(w_{t+1}^{a,d}, \mathcal{Z}_{t+1}), V^Y(w_{t+1}^{a,nd}, \mathcal{Z}_{t+1})\right\}\right] \\ & + \beta\pi^a\mathrm{E}_t\left[G^{1-\gamma}\max\left\{V^M(w_{t+1}^{a+,d}, \mathcal{Z}_{t+1}), V^M(w_{t+1}^{a+,nd}, \mathcal{Z}_{t+1})\right\}\right], \end{aligned}$$

subject to

$$w_t^a = c_t^a + \rho_t^a s_t^a + (P_t - \rho_t^a)h_t^a - q^a(\alpha_t^a, \mathcal{Z}_t)m_t^a + \frac{d_t^a}{1+r_t} + b_t^a \cdot P_t^a.$$

Next-period non-housing wealth, conditional on the age transition is $k \in \{a, a+\}$,

$$w_{t+1}^{k,nh} = \frac{d_t^a}{G} + b_t^a \cdot (P_{t+1}^k + x_{t+1}^k).$$

Thus next-period wealth conditional on defaulting is

$$w_{t+1}^{k,d} = (1-\lambda^a)w_{t+1}^{k,nh},$$

and next-period wealth conditional on not defaulting is

$$w_{t+1}^{k,nd} = \epsilon_{t+1}^a(1-\delta_H)P_{t+1}h_t^a - \frac{m_t^a}{G} + w_{t+1}^{k,nh}.$$

Denote the savings of an individual household by

$$\Sigma_t^a = h_t^a(P_t - \rho_t^a) + \frac{d_t^a}{1+r_t} - q^a(\alpha_t^a, \mathcal{Z}_t)m_t^a + b_t^a \cdot P_t^a.$$

Further define the portfolio return conditional on defaulting and not defaulting, respectively, as

$$R_{t+1}^{k,j} = G\frac{w_{t+1}^{k,j}}{\Sigma_t^a},$$

for $j \in \{d, nd\}$ and $k \in \{a, a+\}$, where

$$\begin{aligned}
R_{t+1}^{k,nd} &= G(1-\delta_H)P_{t+1}\epsilon_{t+1}^a \hat{h}_t^a - \hat{m}_t^a + \hat{d}_t^a + \hat{b}_t^a \cdot (P_{t+1}^k + x_{t+1}^k) \\
R_{t+1}^{k,d} &= (1-\lambda^a)(\hat{d}_t^a + \hat{b}_t^a \cdot (P_{t+1}^k + x_{t+1}^k))
\end{aligned}$$

and we have defined quantity portfolio shares $\hat{h}_t^a = h_t^a/\Sigma_t^a$, $\hat{d}_t^a = d_t^a/\Sigma_t^a$, $\hat{m}_t^a = m_t^a/\Sigma_t^a$, and $\hat{b}_t^a/\Sigma_t^a$.

The usual results for Cobb-Douglas utility functions imply that the optimal expenditure on non-durable and housing services consumption are

$$c_t^a = (1-\theta)(w_t^a - \Sigma_t^a), \tag{25}$$

$$s_t^a = \frac{\theta}{\rho_t^a}(w_t^a - \Sigma_t^a). \tag{26}$$

We conjecture and the verify that the value function has the form

$$V^a(w_t^a, \mathcal{Z}_t) = v^a(\mathcal{Z}_t)\frac{(w_t^a)^{1-\gamma}}{1-\gamma},$$

as in (3), where $v^a(\mathcal{Z}_t)$ only depends on aggregate states exogenous to the individual household.

This allows us to rewrite the value function as

$$V^a(w_t^a, \mathcal{Z}_t) = \max_{\Sigma_t^a} \frac{\Theta^a(\mathcal{Z}_t)}{1-\gamma}(w_t^a - \Sigma_t^a)^{1-\gamma} + (\Sigma_t^a)^{1-\gamma}A^a(\mathcal{Z}_t),$$

where we defined the portfolio choice problem per dollar of savings

$$
\begin{aligned}
A^a(\mathcal{Z}_t) = \max_{\hat{\alpha}_t^a} \ & \psi \frac{(\hat{d}_t^a)^{1-\gamma}}{1-\gamma} \\
& + \beta(1-\pi^a)\mathrm{E}_t \left[\max \left\{ \frac{(R_{t+1}^{a,nd})^{1-\gamma}}{1-\gamma}, \frac{(R_{t+1}^{a,d})^{1-\gamma}}{1-\gamma} \right\} v^a(\mathcal{Z}_{t+1}) \right] \\
& + \beta\pi^a \mathrm{E}_t \left[\max \left\{ \frac{(R_{t+1}^{a+,nd})^{1-\gamma}}{1-\gamma}, \frac{(R_{t+1}^{a+,d})^{1-\gamma}}{1-\gamma} \right\} v^{a+}(\mathcal{Z}_{t+1}) \right]
\end{aligned} \tag{27}
$$

subject to the budget constraint

$$
1 = \hat{h}_t^a(P_t - \rho_t^a) + \frac{\hat{d}_t^a}{1+r_t} + \hat{b}_t^a \cdot P_t^a - q^a(\hat{\alpha}_t^a, \mathcal{Z}_t)\hat{m}_t^a \tag{28}
$$

and where $\Theta^a(\mathcal{Z}_t) = \left((1-\theta)^{1-\theta} \left(\frac{\theta}{\rho_t^a} \right)^{\theta} \right)^{1-\gamma}$. The last term of the portfolio budget constraint (28) uses the property that the mortgage price $q^a(\alpha_t^a, \mathcal{Z}_t)$ is homogeneous of degree zero in household wealth and savings, conditional on the conjectured value function, such that $q^a(\hat{\alpha}_t^a, \mathcal{Z}_t) = q^a(\alpha_t^a, \mathcal{Z}_t)$, see also proposition 2.

Taking the first-order condition with respect to Σ_t^a and solving, we get

$$
\Sigma_t^a = \underbrace{\frac{((1-\gamma)A^a(\mathcal{Z}_t))^{1/\gamma}}{\Theta^a(\mathcal{Z}_t)^{1/\gamma} + ((1-\gamma)A^a(\mathcal{Z}_t))^{1/\gamma}}}_{=B^a(\mathcal{Z}_t)} w_t^a. \tag{29}
$$

Equation (29) implies that all households in generation a save the same fraction of their wealth, with this fraction given by $B^a(\mathcal{Z}_t)$.

Reinserting this solution for Σ_t^a into the value function gives

$$
V^a(w_t^a, \mathcal{Z}_t) = \frac{(w_t^a)^{1-\gamma}}{1-\gamma} \left[\Theta^a(\mathcal{Z}_t) \left(1 - B^a(\mathcal{Z}_t)\right)^{1-\gamma} + (1-\gamma)A^a(\mathcal{Z}_t)B^a(\mathcal{Z}_t)^{1-\gamma} \right].
$$

This confirms the conjecture from (3) with

$$
v^a(\mathcal{Z}_t) = \Theta^a(\mathcal{Z}_t) \left(1 - B^a(\mathcal{Z}_t)\right)^{1-\gamma} + (1-\gamma)A^a(\mathcal{Z}_t)B^a(\mathcal{Z}_t)^{1-\gamma}. \tag{30}
$$

Equation (30) is a recursion in $v^a(\mathcal{Z}_t)$, since $A^a(\mathcal{Z}_t)$ depends on the expectation of $v^a(\mathcal{Z}_{t+1})$ and $v^{a+}(\mathcal{Z}_t)$. In order for the proposition to hold, $V^{a+}(\mathcal{Z}_t)$ must also be homogeneous in wealth of degree $1-\gamma$. This is the case for both generations: for middle-aged households

$V^{M+} = V^O$, which satisfies this property from (2). This implies that V^M is homogeneous of degree $1-\gamma$. Since $V^{Y+} = \pi^M V^O + (1-\pi^M)V^M$, it follows that V^Y inherits the same homogeneity.

Since the optimization problem in (27) is independent of individual wealth, all households in the same generation choose the same portfolio and savings shares, irrespective of their level of wealth.

A.1.2 Proposition 2

Proposition 2 was proven in the main text taking as given the stochastic discount factor of the intermediary. Here we derive the intermediary's stochastic discount factor from its optimization problem. As above, we normalize all variables to grow at a rate $G = exp(g)$ each period so that the first order conditions we derive are consistent with a balanced growth path. The proof does not assume this balanced growth path exists, but this provides expressions useful for the numerical solution of the model. The detrended value function is

$$V^I(e_t, \mathcal{Z}_t) = \max_{I_t, D_t, N_t^Y, N_t^M} \tau e_t - I_t - \frac{\chi}{2} I_t^2 + \mathrm{E}_t \left[G\mathcal{M}_{t,t+1}^M V^I(e_{t+1}, \mathcal{Z}_{t+1}) \right],$$

subject to the budget constraint

$$(1-\tau)e_t + I_t - C(I_t, \bar{Y}_t) + \frac{D_t}{1+r_t} = N_t^Y q^Y(\alpha_t^Y) + N_t^M q^M(\alpha_t^M),$$

the transition law for equity

$$e_{t+1} = \left(N_t^Y \mathcal{P}_{t+1}^Y + N_t^M \mathcal{P}_{t+1}^M - D_t \right) / G,$$

and the regulatory capital constraint for the worst-payoff state next period

$$D_t \le (1-\bar{e}^Y) N_t^Y \mathcal{P}^Y(\underline{z}_t) + (1-\bar{e}^M) N_t^M \mathcal{P}^M(\underline{z}_t). \tag{31}$$

The regulatory capital constraint is effectively an endogenous leverage constraint.

The Lagrangian form of the problem, with Lagrange multiplier $\mu_t^{I^*}$ on the (occasionally binding) regulatory capital constraint and multiplier κ_t^I on the intratemporal budget constraint, is

$$\max_{I_t, D_t, N_t^Y, N_t^M} \tau e_t - I_t + \mathrm{E}_t \left[G \mathcal{M}^M_{t,t+1} V^I \left(\left(N_t^Y \mathcal{P}^Y_{t+1} + N_t^M \mathcal{P}^M_{t+1} - D_t \right) / G, \mathcal{Z}_{t+1} \right) \right]$$
$$+ \mu_t^{I^*} \left[D_t - (1 - \bar{e}^Y) N_t^Y \mathcal{P}^Y(\underline{z}_t) - (1 - \bar{e}^M) N_t^M \mathcal{P}^M(\underline{z}_t) \right]$$
$$+ \kappa_t^I \left[(1-\tau) e_t + I_t - \frac{\chi}{2} I_t^2 + \frac{D_t}{1 + r_t} - \left(N_t^Y q^Y(\alpha_t^Y) + N_t^M q^M(\alpha_t^M) \right) \right].$$

Assets held by the intermediary have value for two reasons. First, their payoff in the worst aggregate state loosens the regulatory capital constraint of it is binding. Second, assets provide wealth in the future, which is valued by a stochastic discount factor determined by the intermediary's shadow value of equity. Taking the FOC for issuance I_t, the shadow value of internal funds is

$$\kappa_t^I = \frac{1}{1 - \chi I_t}.$$

Hence, the marginal value of equity is

$$\frac{\partial V^I(e_t, \mathcal{Z}_t)}{\partial e_t} = \tau + (1-\tau)\kappa_t^I = \tau + \frac{1-\tau}{1 - \chi I_t}.$$

We define the intermediary's shadow value SDF (which captures only this second source of value) as

$$\mathcal{M}^I_{t,t+1} = \mathcal{M}^M_{t,t+1} (1 - \chi I_t) \left(\tau + \frac{1 - \tau}{1 - \chi I_{t+1}} \right).$$

Letting $\mu_t^I = \mu_t^{I^*}(1 - \chi I_t)$ be a renormalization of the Lagrange multiplier on the constraint (31), the FOCs for deposits and loans are

$$\frac{1}{1 + r_t} = \mu_t^I + \mathrm{E}_t \left[\mathcal{M}^I_{t,t+1} \right] \tag{32}$$
$$q^M(\alpha_t^M) = \mu_t^I \mathcal{P}^M(\underline{z}_t) + \mathrm{E}_t \left[\mathcal{M}^I_{t,t+1} \mathcal{P}^M_{t+1} \right], \tag{33}$$
$$q^Y(\alpha_t^Y) = \mu_t^I \mathcal{P}^Y(\underline{z}_t) + \mathrm{E}_t \left[\mathcal{M}^I_{t,t+1} \mathcal{P}^Y_{t+1} \right]. \tag{34}$$

The first-order conditions (33) and (34) define the mortgage pricing functions faced by borrowers, $q^j(\alpha_t^j)$, which depend on mortgage payoffs $\mathcal{P}^j_{t+1}$. From the definitions of these payoffs in (12) and (13), it is clear that they depend on borrower choices through the inverse mortgage leverage ratio $P_{t+1} h_t^j / m_t^j$, and default thresholds, which depend on choices through ratios of non-default to default wealth $w^{a,nd}_{t+1} / w^{a,d}_{t+1}$. Then by propositions

1, these payoffs are homogeneous of degree zero in borrower wealth. Individual borrowers choose identical portfolio *shares* of wealth, thus keeping these ratios independent of wealth levels.

A.2 Characterization of Portfolio Problems

SDF. Since the solution to the optimization problem of households scales in individual wealth (Proposition 1), we can construct the stochastic discount factor of a representative household for generation a. To do so, first note that the growth of wealth of any generation-a household, conditional on the default decision and age transition, is given by

$$\frac{w_{t+1}^{k,j}}{w_t^a} = \frac{\Sigma_t^a R_{t+1}^{k,j}}{w_t^a} = \frac{B^a(\mathcal{Z}_t) R_{t+1}^{k,j} w_t^a}{w_t^a} = B^a(\mathcal{Z}_t) R_{t+1}^{k,j},$$

for $j \in \{nd, d\}$ and $k \in \{a, a+\}$.

Thus we can construct the SDF of generation a as

$$\mathcal{M}_{t+1}^{k,j} = \beta \frac{(B^a(\mathcal{Z}_t) R_{t+1}^{k,j})^{-\gamma} v^k(\mathcal{Z}_{t+1})}{v^a(\mathcal{Z}_t)}.$$

We can assemble the SDF for assets that only pay off in the non-default state, namely housing and mortgages, as

$$M_{t+1}^{a,nd} = \pi^a \int_{\bar{\epsilon}_{t+1}^{a+}}^{\infty} \mathcal{M}_{t+1}^{a+,nd}(\epsilon) dF_{\epsilon,t+1}^a(\epsilon) + (1-\pi^a) \int_{\bar{\epsilon}_{t+1}^{a}}^{\infty} \mathcal{M}_{t+1}^{a,nd}(\epsilon) F_{\epsilon,t+1}^a(\epsilon), \tag{35}$$

and the SDF of defaulters is

$$M_{t+1}^{a,d} = \pi^a (1-\lambda^a) F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^{a+}) \mathcal{M}_{t+1}^{a+,d} + (1-\pi^a)(1-\lambda^a) F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^{a}) \mathcal{M}_{t+1}^{a,d}.$$

The SDF for discounting payoffs that do not depend on the default decision or the age transition (deposit) is

$$\mathcal{M}_{t+1}^{a} = \mathcal{M}_{t+1}^{a,nd} + \mathcal{M}_{t+1}^{a,d}. \tag{36}$$

We can also construct SDFs for discounting the age-specific assets that condition on the age transition status, but not on the default decision

$$\mathcal{M}_{t+1}^{ak} = (1-\pi^a)^{\mathbb{1}[k=a]}(\pi^a)^{\mathbb{1}[k=a+]}\left[\int_{\bar{\epsilon}_{t+1}^k}^{\infty} \mathcal{M}_{t+1}^{k,nd}(\epsilon)dF_{\epsilon,t+1}^a(\epsilon) + (1-\lambda^a)F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^k)\mathcal{M}_{t+1}^{k,d}\right], \tag{37}$$

for $k \in \{a, a+\}$.

First-order conditions. The portfolio problem of the young is analogous to that of the middle-aged. Using the SDF definitions in (35), (36), and (37), the first-order conditions are

$$\frac{1}{1+r_t} - \hat{m}_t^a q_d^a(\hat{\alpha}_t^a) = \frac{\psi}{v^a(\mathcal{Z}_t)}(\hat{d}_t^a B^a(\mathcal{Z}_t))^{-\gamma} + \beta \mathrm{E}_t\left[\mathcal{M}_{t+1}^a\right] \tag{38}$$

$$q^a(\hat{\alpha}_t^a) + \hat{m}_t^a q_m^a(\hat{\alpha}_t^a) = \beta \mathrm{E}_t\left[\mathcal{M}_{t+1}^{a,nd}\right] \tag{39}$$

$$P_t - \rho_t^a - \hat{m}_t^a q_h^a(\hat{\alpha}_t^a) = \beta \mathrm{E}_t\left[\mathcal{M}_{t+1}^{a,nd}(1-\delta_H)P_{t+1}\right] \tag{40}$$

$$P_t^a - \hat{m}_t^a q_b^a(\hat{\alpha}_t^a) = \beta \mathrm{E}_t\left[\mathcal{M}_{t+1}^{aa}(P_{t+1}^a + x_{t+1}^a) + M_{t+1}^{aa+}(P_{t+1}^{a+} + x_{t+1}^{a+})\right], \tag{41}$$

where we use the shorthand notation

$$q_\ell^a(\hat{\alpha}_t^a) = \frac{\partial q^a(\hat{\alpha}_t^a, \mathcal{Z}_t)}{\partial \ell}.$$

Note that equations (38) – (41) only characterize the relative portfolio shares of assets that households invest in. To fully characterize the complete savings and portfolio choice problem of the middle generation, we can reduce these equation to three excess return equations by first defining the effective returns to mortgage borrowing and housing as

$$R_{t+1,m}^a = \frac{1}{q^a(\hat{\alpha}_t^a) + \hat{m}_t^a q_m^a(\hat{\alpha}_t^a)},$$

and

$$R_{t+1,h}^a = G\frac{(1-\delta_H)P_{t+1}}{P_t - \rho_t^a - \hat{m}_t^a q_h^a(\hat{\alpha}_t^a)}.$$

Further, the effective return to deposits is

$$R_{t+1,d}^a = \frac{1+r_t}{1-(1+r_t)\hat{m}_t^a q_d^a(\hat{\alpha}_t^a)},$$

and to the generation-specific assets for $k = \in \{a, a+\}$

$$R^k_{t+1,b} = [P^k_{t+1} + x^k_{t+1}] \oslash [P^a_t - \hat{m}^a_t q^a_b(\hat{\alpha}^a_t)].$$

The resulting excess return restrictions are

$$0 = \frac{\psi}{v^a(\mathcal{Z}_t)}(\hat{d}^a_t B^a(\mathcal{Z}_t))^{-\gamma} R^a_{t+1,d} + \beta \mathrm{E}_t \left[\mathcal{M}^a_{t+1} R^a_{t+1,d} - \mathcal{M}^{a,nd}_{t+1} R^a_{t+1,m}\right] \tag{42}$$

$$0 = \mathrm{E}_t \left[\mathcal{M}^{a,nd}_{t+1}(R^a_{t+1,m} - R^a_{t+1,h})\right], \tag{43}$$

$$0 = \mathrm{E}_t \left[\mathcal{M}^{a,nd}_{t+1} R^a_{t+1,h} - \left(\mathcal{M}^{aa}_{t+1} R^a_{t+1,b} + \mathcal{M}^{aa+}_{t+1} R^{a+}_{t+1,b}\right)\right]. \tag{44}$$

Jointly with the optimal savings choice (29) and the recursive definition of the value function (30), these equations fully characterize the dynamic problem of the middle-generation.

Mortgage pricing function derivatives. To compute the effective returns on all assets, we need to calculate the derivative of the mortgage pricing function $q^Y(\hat{\alpha}^Y_t, \mathcal{Z}_t)$ with respect to the elements of $\hat{\alpha}^Y_t$. The first step is to differentiate the payoff functions (12) and (13) with respect to these portfolio choices. We first define the home equity per dollar of mortgage debt of the marginal defaulter after bankruptcy losses, conditional on $k \in \{a, a+\}$,

$$\hat{\epsilon}^k_{t+1} = \frac{(1-\xi)(1-\delta_H) P_{t+1} h^a_t \bar{\epsilon}^k_{t+1}}{m^a_t} - 1.$$

Then we get

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial m^a_t} = \frac{\hat{f}^a_{\epsilon,t+1}}{(1-\delta_H)P_{t+1}h^a_t} - \hat{F}^a_{\epsilon,t+1}(1-\xi)(1-\delta_H)\frac{P_{t+1}h^a_t}{(m^a_t)^2},$$

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial h^a_t} = (1-\delta_H)P_{t+1}\left[\hat{F}^a_{\epsilon,t+1}\frac{1-\xi}{m^a_t} - \frac{\hat{\hat{f}}^a_{\epsilon,t+1}}{(1-\delta_H)P_{t+1}h^a_t}\right],$$

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial d^a_t} = -\hat{f}^a_{\epsilon,t+1}\frac{\lambda^a}{(1-\delta_H)P_{t+1}h^a_t},$$

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial b^a_t} = -\hat{\hat{f}}^a_{\epsilon,t+1}\frac{\lambda^a}{(1-\delta_H)P_{t+1}h^a_t},$$

where we use the auxiliary functions

$$\hat{f}^{a}_{\epsilon,t+1} = \pi^{a} f^{a}_{\epsilon,t+1}(\bar{\epsilon}^{a+}_{t+1})\hat{\epsilon}^{a+}_{t+1} + (1-\pi^{a}) f^{a}_{\epsilon,t+1}(\bar{\epsilon}^{a}_{t+1})\hat{\epsilon}^{a}_{t+1},$$
$$\hat{F}^{a}_{\epsilon,t+1} = \pi^{a} \int_{0}^{\bar{\epsilon}^{a+}_{t+1}} \epsilon F^{a}_{\epsilon,t+1}(\epsilon) + (1-\pi^{a}) \int_{0}^{\bar{\epsilon}^{a}_{t+1}} \epsilon F^{a}_{\epsilon,t+1}(\epsilon),$$
$$\hat{\hat{f}}^{a}_{\epsilon,t+1} = \pi^{a} f^{a}_{\epsilon,t+1}(\bar{\epsilon}^{a+}_{t+1})\hat{\epsilon}^{a+}_{t+1}(\bar{\epsilon}^{a+}_{t+1})^{2} + (1-\pi^{a}) f^{a}_{\epsilon,t+1}(\bar{\epsilon}^{a}_{t+1})\hat{\epsilon}^{a}_{t+1}(\bar{\epsilon}^{a}_{t+1})^{2},$$
$$\hat{\hat{\hat{f}}}^{a}_{\epsilon,t+1} = \pi^{a} f^{a}_{\epsilon,t+1}(\bar{\epsilon}^{a+}_{t+1})\hat{\epsilon}^{a+}_{t+1}(P^{a+}_{t+1} + x^{a+}_{t+1}) + (1-\pi^{a}) f^{a}_{\epsilon,t+1}(\bar{\epsilon}^{a}_{t+1})\hat{\epsilon}^{a}_{t+1}(P^{a}_{t+1} + x^{a}_{t+1}).$$

For any argument ℓ of the mortgage pricing function q^a we have that

$$\frac{\partial q^{a}(\alpha^{a}_{t}, \mathcal{Z}_{t})}{\partial \ell} = \mu^{I}_{t} \frac{\partial \mathcal{P}^{a}(\underline{z}_{t})}{\partial \ell} + \mathrm{E}_{t}\left[\mathcal{M}^{I}_{t,t+1} \frac{\partial \mathcal{P}^{a}_{t+1}}{\partial \ell}\right].$$

With the first order conditions above, this characterizes the portfolio choice problem.