

BANK CAPITAL, BANK CREDIT, AND UNEMPLOYMENT*

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February 23, 2015

Abstract

Since the worst employment slumps follow periods of high household debt and almost all household debt is provided by banks, there is a natural link between bank lending and employment. Building on this, we theoretically investigate whether bank regulation can play a role in stimulating employment. Using a competitive search model, we find that levered households suffer from a *debt overhang* problem that distorts their preferences, making them demand high wages. In general equilibrium, firms internalize these preferences and post high wages but few vacancies. This *vacancy-posting effect* implies that high household debt leads to high unemployment. Unemployed households default on their debt. In equilibrium, the level of household debt is inefficiently high due to a *household-debt externality*—banks fail to internalize the effect that household leverage has on household default probabilities via the vacancy-posting effect. As a result, household debt levels are inefficiently high. Our results suggest that a combination of loan-to-value caps for households and capital requirements for banks can elevate employment and improve efficiency, providing an alternative to monetary policy for labor market intervention.

Keywords: Household debt, employment, banking, screening, financial regulation

JEL Classification Numbers: G21, G28, J63, E24

*We gratefully acknowledge helpful comments from Jonathan Berk, Nittai Bergman, Alex Edmans, Radha Gopalan, Asaf Manela, Ngoc-Khanh Tran and seminar participants at the Bank of England and at Washington University in St. Louis. We alone are responsible for any remaining errors.

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People ... were poor not because they were stupid or lazy ... they were poor because the financial institutions in the country did not help them widen the economic base.

Muhammed Yunus, *Banker to the Poor: Micro-lending and the Battle Against World Poverty*

1 Introduction

An interesting stylized fact that connects labor and credit markets is that the worst employment slumps follow the largest expansions of household debt. For example, in the U.S., household debt as a percentage of GDP climbed from below fifty percent in 1980 to almost one hundred percent by 2006, and the Great Recession that accompanied the 2007–09 subprime crisis saw the U.S. economy shed over eight million jobs. Ng and Wright (2013) note that the increase in leverage prior to the Great Recession was more pronounced for households than for firms. And Mian and Sufi (forthcoming) document that counties in the U.S. that has the most highly levered households had the sharpest drops in employment. The connection between increases in household debt and spikes in unemployment is *not* a unique feature of the last recession—it is rather commonplace in recessions associated with financial crises.¹

Almost all household debt is created by consumer loans made by banks. So the lending decisions of banks influence aggregate household debt, and those lending decisions, in turn, are affected by bank regulation. This raises an interesting question that we address in this paper: can a central bank stimulate employment through bank regulation?

Based on our analysis, the short answer to this question is yes. The central mechanism at work in our model relies on a two-way bridge between the labor and credit markets. Higher household debt induces workers to demand higher wages, and firms respond by posting fewer job vacancies, causing unemployment to go up. Higher unemployment elevates default rates, but households and individual banks fail to internalize this negative labor-market-driven externality of increasing consumer credit, leading to excessive lending to households. We show that a central bank, in its role as a prudential regulator, can diminish this inefficiency and boost employment with a *combination* of capital requirements on banks and caps on household leverage.

This approach to dealing with unemployment generated by the interaction of frictions in the labor and credit markets is in sharp contrast to the approach in the previous

¹See, for example, Reinhart and Rogoff (2009) and Schularick and Taylor (2012).

research, which relies on monetary policy. Specifically, the focus has been on the interest rate set by the central bank as the policy variable through which it influences labor market outcomes. The setting is one in which credit market shocks are transmitted to the labor market via the aggregate demand channel.² When credit tightens for consumers, it reduces demand for goods, leading to falling prices for goods, which causes firms to cut production and hire fewer workers, increasing unemployment. A lowering of bank interest rates by the central bank can ease credit and eliminate the negative employment effect engendered by the initial credit tightening.

We take a different, yet complementary, approach in which we develop a general equilibrium model of household borrowing, bank lending, and the labor market. We focus on the most ubiquitous frictions in these markets: banks face adverse selection frictions when lending to households and households face search frictions when looking for jobs. Households are risk-averse, live for two dates, and come in two types, good and bad. Good types have a labor endowment at the late date, whereas bad types have no labor endowment. At the late date, good households (and only good households) may find employment in a competitive search market. To smooth consumption, households borrow from competitive banks at the early date. Good and bad households are ex ante observationally identical to banks, generating an adverse-selection friction. Banks cope with this friction by screening households, which permits the observation of a noisy signal about borrower types before lending. This information allows banks to avoid lending to bad households, who never repay debt because they have no labor endowment at the late date. The precision of each bank's signal is determined by the bank's private investment in a costly screening technology. Household debt and the level of employment are both endogenously determined in equilibrium.

Analysis of this model produces the following four main results. First, increasing household debt raises the equilibrium unemployment rate. Second, households fail to internalize the effect of their borrowing on unemployment, thus generating an externality on the labor market, which induces inefficiently high household debt. Third, there is a feedback effect between household borrowing and employment that generates a multiplicity of equilibria. There is a high-debt, low-employment equilibrium and a low-debt, high-employment equilibrium. Fourth, a combination of high capital requirements for banks and a cap on household leverage can both eliminate the equilibrium involving low employment and increase employment even above the laissez-faire level in the high-employment equilibrium.

We now explain these results. Consider the first result that increasing household debt raises the equilibrium unemployment rate. Conditional on finding employment,

²See Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2011), Mian and Sufi (forthcoming), Midrigan and Philippon (2011), and Mishkin (1978, 1978).

highly-indebted households must pay a substantial portion of their wages to their creditors and, therefore, attach lower value to finding employment. Hence, highly indebted households are relatively more sensitive to wages than to the probability of finding employment. Competitive firms recognize these household preferences when posting vacancies to attract workers. This results in firms posting fewer vacancies, but at higher wages, as household debt increases. We refer to this as the *vacancy-posting effect* of household debt and it emanates from the fact that leverage distorts household preferences.

The debt-overhang channel that induces the vacancy-posting effect suggests that high levels of household debt lead to higher wages in equilibrium. At first blush, it may appear that this implies that wages should increase during recessions. However, this is not the case. To analyze wages over the business cycle, we need to incorporate macroeconomic variation into the model. We do this in an extension (Subsection 6.3). We demonstrate that wages are in fact higher in booms than in recessions, but that high levels of household debt may be a source of the well-documented rigidity in wages (e.g. Bewley (1999)).

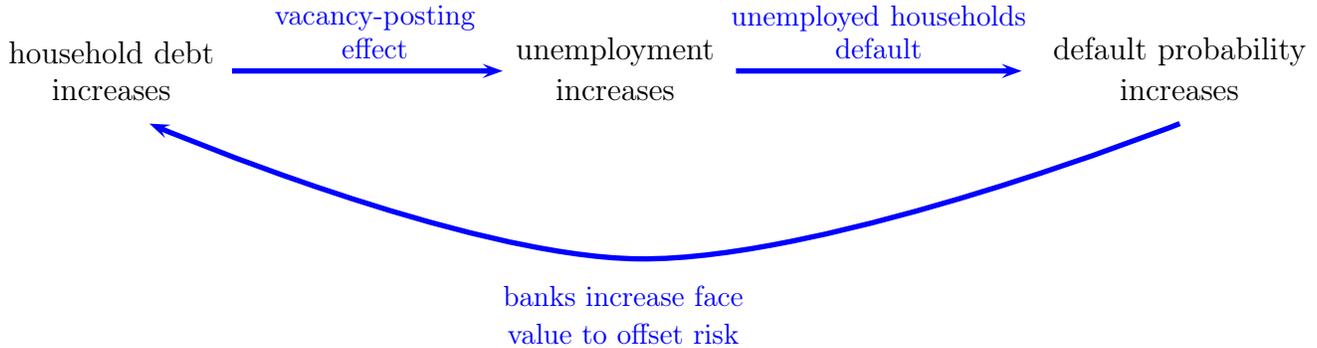
The second result—the level of household debt is *inefficiently high* in equilibrium—arises from the fact that each bank is small compared to the whole economy, so it fails to internalize the vacancy-posting effect that results from the loan that it grants. Thus, the vacancy-posting effect of households generates a negative *household-debt externality* on the labor market.

The third main result—there is a feedback effect between household borrowing and employment—stems from the vacancy-posting effect, which causes an increase in debt to decrease employment. But since unemployed households default on their debt, the lower employment rate leads to a lower probability that households will repay their loans. As a result, banks demand higher face values of debt to compensate for this increase in default probability. This closes the feedback loop by which increases in household debt lead to further increases in household debt. See Figure 1 for an illustration of the feedback loop.

In conjunction with the household debt externality, the feedback effect results in a multiplicity of equilibria. Since banks take the employment rate—and thus households' repayment probability—as given when they determine the face value of debt, beliefs are self-fulfilling. There is one equilibrium in which banks believe that employment will be low and, as a result, they demand high face values of debt. There is another equilibrium in which banks believe that employment will be high and, as a result, they demand low face values of debt.

Output is proportional to employment in our economy because all firms that are matched with workers produce the same output. Therefore, output is high when em-

Figure 1: FEEDBACK LOOP BETWEEN HOUSEHOLD DEBT AND UNEMPLOYMENT



ployment is high, and the high-employment equilibrium is more efficient from the point of view of GDP. However, the inefficiency generated by the household debt externality does not vanish altogether in the high-employment equilibrium. Hence, we ask whether banks' screening of borrowers can reduce this inefficiency by lowering the interest rates charged on household debt. We find that the answer is yes. In the high-employment equilibrium, increasing bank screening increases employment. With more precise screening, banks reduce the likelihood of lending to bad households and, therefore, charge good households a lower interest rate. That is, good households now have to compensate banks less for the failed loans they make to bad households.

This intuition for why increased screening leads to lower face values of household debt does not carry over to the low-employment equilibrium. Increasing screening precision has in fact two countervailing effects on the interest rate of debt—not only the direct effect described above which leads banks to demand lower interest rates, but also a belief-driven indirect effect that leads banks to demand higher interest rates. In the low-employment equilibrium, this indirect effect dominates, leading banks to demand *higher* face values as they increase screening precision.

Finally, let us turn to the fourth result. We ask whether regulating bank capital structure can increase employment and improve welfare. We show that banks raise all new capital via debt and do not issue equity. The reason for this in our model differs from the usual culprits for which banks like high leverage, such as taxes, safety nets and the like. Rather, in our model, high leverage serves as a commitment device for banks to *not* screen loans too intensely. Banks that are somewhat lax in screening are attractive to borrowers who then face a lower risk of being denied credit. Borrowers are impatient and therefore place high value on the probability of receiving credit today.

Thus, high leverage serves as a commitment device for competitive banks to offer easy credit and attract more borrowers. Given that banks in the model have an incentive to lever up and over-lend, we now ask whether a bank regulator can improve welfare by imposing capital requirements to curb this tendency.

Increasing bank equity has a direct positive effect on bank screening. Better-capitalized banks screen loans more intensely. But this higher screening has the effect of lowering interest rates for households *only* in the high-employment equilibrium; in fact, it has a perverse effect in the low-employment equilibrium. Thus, a bank regulator has a delicate task. The regulator should increase capital requirements, but only after having implemented policies that prevent the economy from ending up in the low-employment equilibrium. Fortunately, the regulator can eliminate the low-employment equilibrium with a simple policy that regulates household borrowing: capping household loan interest rates.^{3 4} If the regulator caps the interest rates that banks can charge households, the unique equilibrium is the high-employment equilibrium of the model without regulation. The reason is that if banks are not allowed to charge high interest rates, then the interest rate in the low-employment equilibrium becomes infeasible, and the amount of debt each household can take on will be consequently limited. Thus, our results suggest that capital regulation is valuable, but should not be implemented in isolation. Rather, capital requirements should be implemented in conjunction with limits on household debt. Put a bit differently, bank regulators ought to be concerned with minimum capital requirements *for households as well as for banks*.⁵

We model the labor market within a competitive search framework (Moen (1997), Shimer (1996)). See Rogerson, Shimer, and Wright (2005) for a survey of this literature. Our contribution is to incorporate the provision of household credit by banks into this setting. The only other paper that we know of that embeds a household credit market in a search model of the labor market is Bethune, Rocheteau, and Rupert (2015,), which focuses on self-enforcing contracts for unsecured credit (i.e. credit card loans) in a dynamic environment with limited enforcement. However, in that paper there are no banks and there is no default in equilibrium.

Various other papers have examined the consequences of the interaction between credit and labor markets. Acemoglu (2001) argues that failures of the credit market to channel funds to socially valuable projects can increase unemployment, and that the persistence of high unemployment in Europe, relative to in the US, may be explicable

³In the model, households always borrow the same amount in equilibrium. As a result caps, on interest rates and caps on household leverage will be equivalent in the model.

⁴Such caps have been implemented in several countries in recent years, see, for example, Borio and Shim (2007), Crowe, Dell’Ariccia, Igan, and Rabanal (2011) and Ono, Uchida, Udell, and Uesugi (2014).

⁵Note that we focus only on the effect of bank capital on screening loans and we abstract from other effects that bank capital requirements may have. For discussion and analysis of these other effects, see Opp, Opp, and Harris (2014) and Thakor (forthcoming).

on this basis. Adams (2005) suggests that the ability of households to repay their loans depends on the likelihood of remaining employed, which means that lenders can predict default likelihood by looking at the labor market. Adams, Einav, and Levin (2009) document that automobile demand in the US increases sharply during the rebate season, and that household default rates rise with loan size, indicating the possible desirability of loan caps.⁶ Buera, Fattal-Jaef, and Shin (2014) develop a theoretical model in which a credit crunch leads to a big drop in employment for small, young firms and a lesser drop for large, old firms. Boeri, Garibaldi, and Moen (2012) develop a model and provide evidence that more highly-leveraged sectors in the economy are associated with higher employment-to-output elasticities during banking crises. Kocherlakota (2012) develops an incomplete labor market model with an exogenous interest rate to show that a decline in the price of land can cause a reduction in employment if the real interest rate remains constant. Koskela and Stenbacka (2003) develop a model in which increased credit market competition leads to lower unemployment under certain conditions related to labor force mobility, whereas Gatti, Rault, and Vaubourg (2012) document that reduced banking concentration can lead to lower unemployment, but only under some labor market conditions.

What distinguishes our paper from this earlier research is our focus on the two-way interaction between the level of household debt in the credit market and equilibrium unemployment in the labor market, and the mediating role of bank capital in this interaction. This also enables us to examine how bank regulatory policy can be crafted to deal with the potential inefficiencies arising from the interaction of labor and credit market frictions. Specifically, we study how a regulator can mitigate the adverse effects of the lack of coordination among households that leads to household debt choices that impose a negative externality on the labor market. In other words, our analysis suggests how a central bank can affect labor market outcomes through bank regulation.⁷

Our paper is also related to the literature on bank capital regulation. In the wake of the financial crisis of 2008–09, there has been active debate about the costs and benefits of high bank capital requirements. For example, Holmstrom and Tirole (1997) and Mehran and Thakor (2011) develop theories that highlight the benefits of higher bank capital, and Berger and Bouwman (2013) document that higher capital enhances bank performance during financial crises. See Thakor (2014) for a review. The papers in this literature that are most related to our work study the effects of increasing bank equity in a general equilibrium framework. For example, Opp, Opp, and Harris (2014) show that increasing bank equity has a non-monotonic effect on welfare. The reason is that banks that face competition from outside investors may react to higher capital

⁶The optimality of capping household loan-size is also an implication that arises in our analysis.

⁷The relevance of this is underscored by the fact that controlling unemployment is one of the goals of central banks like the Federal Reserve in the US.

requirements by taking on (socially) inefficient risk. In a structural model that lends itself to calibration, Nguyen (2014) shows that increasing capital requirements from present levels can produce welfare gains.

The rest of the paper is organized in six remaining sections. In Section 2 we develop the model and in Section 3 we solve it. Section 4 discusses the role of key assumptions in the analysis. Section 5 contains a welfare analysis and discusses policy interventions. Section 6 discusses the model’s empirical predictions and discusses the robustness of results by analyzing the following seven extensions: (i) allowing depositors to lend directly to workers; (ii) modeling the labor market with a random matching framework, instead of a directed search framework; (iii) interacting our channel with aggregate shocks (here we comment on how our channel may contribute to wage rigidity); (iv) including a penalty for defaulting households; (v) including explicitly collateral securing household debt; (vi) using a more general specification of household utility than we use to solve the full model; and (vii) considering a simplified model in which we can compute the social optimum explicitly. Section 7 concludes. The Appendix contains all formal proofs as well as a glossary of symbols.

2 Model

This section describes the model, which has two dates, Date 0 and Date 1. There are five types of players: savers, banks, firms, and two types of workers, good workers and bad workers. The good and bad workers are ex ante identical. Firms have capital and good workers have labor; they meet in a directed search market at Date 1. Banks borrow from savers and lend to workers. They use a noisy screening technology to screen out bad workers. Savers and workers consume at both Date 0 and Date 1, whereas banks and firms maximize only expected Date 1 profits.

2.1 Preferences and Action Spaces of Players

In summarizing the preference and action spaces of the players in the mode, we use the terms “savers” and “depositors” interchangeably. Likewise, the words “workers,” “households,” and “borrowers” all refer to the same type of player. Which word we use depends mainly on the context.

2.1.1 *Savers/Depositors*

There is a unit continuum of risk-neutral savers with discount factor one, each with wealth $I - e$. They cannot lend directly to workers, because they lack the technology to distinguish between good workers (who are creditworthy) and bad workers (who are not

creditworthy). They can deposit their capital in a bank for the promised gross return R or consume at Date 0. The deposit market is competitive, so depositors' expected return just satisfies their participation constraint.

2.1.2 Workers/Households

There is a unit continuum of impatient, risk-averse workers. Workers can be either good $\tau = g$ or bad $\tau = b$; g -workers have a unit labor endowment at Date 1, whereas b -workers have no labor endowment. Workers do not know their types initially and they learn them at the end of Date 0. Let $\theta \in (0, 1)$ be the prior probability that the worker is $\tau = g$. Workers consume c_0 at Date 0 and c_1 at Date 1. They have utility $U(c_0, c_1) = u(c_0) + \delta u(c_1)$. Below we will assume that u is piecewise linear (Subsection 2.5), which will enable us to solve the model. Because workers are risk-averse, they want to smooth consumption. Since they have endowments only at Date 1, they can achieve this by borrowing from banks at Date 0. The risk aversion and the impatience of workers are what creates a rationale for the bank to step in and supply credit.

2.1.3 Banks

There is a unit continuum of risk-neutral banks with discount factor one. Each bank has initial equity e and raises an amount D in debt and an amount Δ in equity from a depositor at Date 0; the one-period gross interest rate on debt is R and the equity stake granted to outsiders is $1 - \beta$. Therefore, the Date-0 asset value of the bank is $e + \Delta + D = E + D$, where $E := e + \Delta$ denotes the total value of bank equity. At Date 0, the bank can lend an amount B to a worker in exchange for the worker's promise to repay face value F at Date 1. The credit market is competitive, so each bank earns an expected rate of return equal to the (zero) riskless interest rate.

Banks have a noisy screening technology that enables them to screen out b -type workers. Each bank observes a signal $s \in \{s_g, s_b\}$ about the worker it will potentially lend to. Both the bank's choice of screening precision and the signal realization are public information.⁸ For a signal precision of $\sigma \in [0, 1]$, the type-conditional signal distribution is as follows:

$$\mathbb{P}[s = s_g | \tau = g] = 1 \tag{1}$$

and

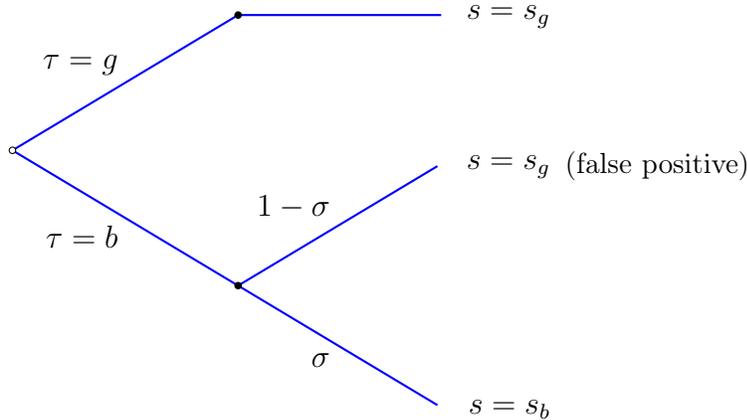
$$\mathbb{P}[s = s_b | \tau = b] =: \sigma. \tag{2}$$

Whenever a worker is g -type, the bank observes the signal s_g . In contrast, when the worker is b -type, the bank observes the signal s_b with probability σ and the signal

⁸Note that this assumption is without loss of generality: since no bad types enter the market at Date 1, this does not affect any inference problem at Date 1.

s_g with probability $1 - \sigma$. This specification implies that $\sigma = 0$ yields a completely uninformative signal—the bank can anticipate observing signal s_g regardless of the borrower’s type. Precision $\sigma = 1$ yields a perfect signal that reveals the borrower’s type with no error. (See Figure 2 for a pictorial representation.) Increasing the screening precision σ allows the bank to reduce the probability of lending to b -type workers, but increasing σ is costly for the bank. Specifically, the bank can pay cost $c(s) = \gamma\sigma^2/2$ to achieve screening precision σ .

Figure 2: A PICTORIAL REPRESENTATION OF THE SIGNAL STRUCTURE



Note that banks never receive negative signals about g -type workers (thereby precluding type I errors), but sometimes receive positive signals about b -type workers (thereby admitting type II errors). This implies that screening workers can help the bank to deny credit to b -type workers, but not to extend more credit to g -type workers—screening reduces type II errors. Not only do we find this assumption realistic, but we also find it useful for technical reasons. This is because this asymmetric signal means that all g -type workers generate the same signal s_g at Date 0. Therefore, banks treat all g -type workers the same way at Date 0, leading them to all have the same amount of debt when they search in the labor market at Date 1. This allows us to abstract from worker heterogeneity in the labor market, which is a major simplification.⁹

⁹Modeling worker heterogeneity in the labor market may be interesting in its own right in future research, especially if employers condition hiring decisions in the labor market on the information they glean from the credit-extension decisions made by banks that specialize in screening loan applicants.

2.1.4 Firms

There is a measure of firms significantly greater than one.¹⁰ Each has a unit of capital, which, in conjunction with a unit of labor, produces output y . Firms pay search cost k to post wages w . Since only g -workers search for work, firms either find a g -worker or remain unmatched. Firms make revenue y if they find a worker in the labor market and zero if they remain unmatched. Firms are competitive and will receive zero expected profit in equilibrium.

2.2 Labor Market

We model the labor market with a one-shot version of a standard competitive search model. In our model, firms post wages w and workers direct their search at a given wage. If the measure of firms posting w is ν_w and the measure of workers directing their search at wage w is μ_w , then the ratio of firms to workers for each w is

$$q_w := \frac{\mu_w}{\nu_w},$$

which is called the *queue length* for wage w (this is the reciprocal of the so-called tightness of submarket w). We assume that for each wage w , workers are matched with firms with intensity $\alpha(q_w)$ and firms are matched with workers with intensity $q_w\alpha(q_w)$. α is decreasing and convex, $q\alpha$ is increasing and concave, and all matches are one-to-one. In general, these properties follow from standard assumptions on a constant-returns-to-scale matching function.

2.2.1 Contracts

There are three types of contracts in the model: (i) the labor contract between workers and firms, (ii) the borrowing contract between workers and banks, and (iii) the bank funding contract between banks and depositors. The bank funding contract is an optimal mix of debt and equity. We discuss these three types of contracts here to introduce notation and to discuss the amount of commitment that contracts provide. The full details of the contractual relationships are formalized by the game form described in Subsection 2.3 below.

The labor contract between workers and firms is defined entirely by a wage w , which is paid at the end of Date 1, after production. Firms post the wage at the beginning of

¹⁰The reason that we assume that this measure is greater than one, which is the measure of workers, is to eliminate the possibility that *all* firms post vacancies. In order to ensure an interior solution in the sense that some firms stay out of the market, we assume that there are many more firms than workers searching for employment.

Date 1 and, if a worker is matched with the firm, he receives w in exchange for devoting his unit of labor toward production.

The borrowing contract between a worker and his bank is defined by the amount B borrowed by the worker and his promised repayment F . We denote this debt contract by (B, F) . We assume that contracts are enforceable but that workers are protected by limited liability, so workers repay whenever they are employed (and have sufficient income), but workers are not punished beyond the loss of their income when they default.¹¹ Since there are only two outcomes (“employed” and “unemployed”) and the cash flow to the worker is zero when he is unemployed, attention can be restricted to debt contracts without loss of generality.

Now turn to the funding contract between the bank and the depositor. The bank raises an amount D via debt and promises to repay RD to bondholders. The bank raises an amount Δ of equity in exchange for an equity stake that promises a proportion $1 - \beta$ of the bank’s cash flows net of debt repayments. We denote this contract by $((D, R), (\Delta, \beta))$.

2.3 Timing and Who Knows What and When

The sequence of moves is as follows. At Date 0, each bank raises capital, is matched with a worker, invests in a screening technology and observes a signal about the worker’s type. We assume, without loss of generality, that each bank’s screening precision and the signal are public information. Each bank funds itself via an optimal mix of debt and equity, which it raises from competitive depositors. Then, each bank proceeds to lend to a worker. This allows the risk-averse worker to smooth his consumption over the two dates. At the end of Date 0 workers learn their types. At Date 1, good workers match with firms in a decentralized labor market. Next, if firms are matched with workers, they produce output and pay wages. Finally, employed workers repay their debts and banks repay their depositors.

Below we specify the timing more formally. The markets for deposits, loans, and workers are competitive. We capture competition via the free entry of banks, depositors, and firms. Note that the only matching frictions are in the labor market.

Date 0 0.1 Each bank competitively posts a contract (B, F) to lend to workers and a contract $((D, R), (\Delta, \beta))$ to raise capital from a random depositor.

- Depositors accept or reject banks’ offers.

0.2 Each worker directs its search at its preferred contract and is matched with a bank.¹²

¹¹In Subsection 6.4 we argue that our results are robust to the inclusion of default penalties as long as the default penalties are bounded from above by a pre-specified maximum.

¹²To focus on adverse selection frictions in the credit market and search frictions only in the labor

0.3 Each bank chooses its screening precision σ and observes the signal s about the type of the worker it is matched with.

- Contingent upon the signal, the bank either agrees to lend B to the worker in exchange for the promise to repay F or does not lend.

0.4 Depositors and workers consume.

0.5 Workers learn their types.

Date 1 1.1 Each firm either pays k to post wage w or stays out.

1.2 Each g -type worker directs his search at a wage w and matches take place (see Subsection 2.2).

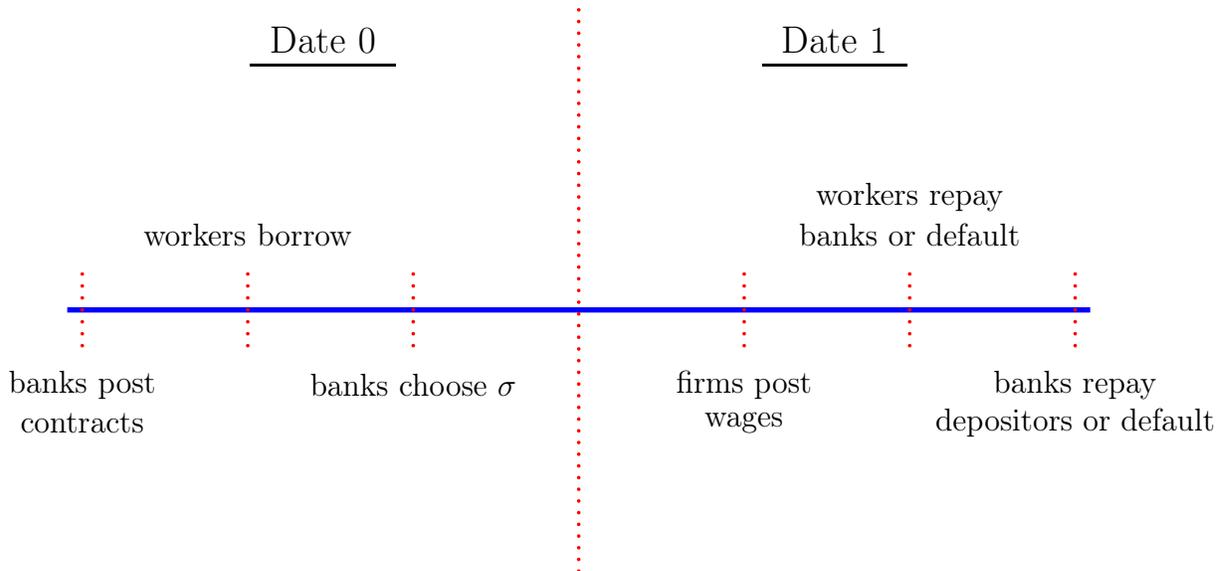
1.3 Each worker who has borrowed either repays F to his bank or defaults.

1.4 Each bank either repays RD to its depositor or defaults. The residual cash flows are split among equity holders.

1.5 Depositors and workers consume. Firms and banks record profits.

See Figure 3 for a timeline representation of the sequence of moves.

Figure 3: A TIMELINE REPRESENTATION OF SEQUENCE OF MOVES



market, we assume that these matches in the credit market are frictionless. Formally, this corresponds to a Leontief matching technology. Note that the contract at which the worker directs its search is $((B, F), ((D, R), (\Delta, \beta)))$ —i.e., the worker takes the bank’s capital structure into account when searching. The reason that this matters is that the bank’s capital structure will affect the bank’s screening precision and therefore the worker’s probability of being granted a loan.

2.4 Solution Concept

The solution concept is symmetric perfect Bayesian equilibrium.

2.5 Assumptions

In this section we impose several restrictions on functional forms and parameters. Specifically, we assume functional forms for the workers' utility and the firm-worker matching function to make the model tractable and we restrict the model's parameters to focus on the cases that we think are economically most important. The implications of relaxing some of these assumptions are discussed in Section 6.

Acemoglu and Shimer (1999) allow workers to be risk-averse in a labor model with directed search, but Rogerson, Shimer, and Wright (2005) note that this "...means it is no longer possible to solve the model explicitly" (p. 976). We avoid this difficulty by assuming that workers' utility functions are piecewise linear. Workers are risk averse globally, giving them the incentive to smooth consumption, but risk-neutral locally, allowing us to solve the Date 1 search model as if they have linear utility. See Chassang (2013) and Dang, Gorton, Holmström, and Ordóñez (2014) for a similar assumption on preferences.

ASSUMPTION 1.

$$u(c) = \begin{cases} c & \text{if } c \leq I, \\ I & \text{otherwise.} \end{cases} \quad (3)$$

We assume a form for the matching probability α that enables us to solve the model in closed form.

ASSUMPTION 2. *The matching function is homogenous and the probability that a worker is employed if he queues at a firm with queue length q is*

$$\alpha(q) = \frac{a}{\sqrt{q}}. \quad (4)$$

This probability satisfies the properties induced by standard matching functions in the literature—the probability α that a worker matches with a firm is decreasing and convex in the queue length, whereas the probability $q\alpha$ that a firm matches with a worker is increasing and concave in the queue length.

We must ensure that these matching probabilities are between zero and one in equilibrium, namely that for the equilibrium queue lengths $\alpha, q\alpha \in [0, 1]$ or

$$a^2 < q < \frac{1}{a^2}. \quad (5)$$

a being sufficiently small suffices for this to hold in equilibrium. Instead we impose the following tighter restriction on primitives.

ASSUMPTION 3.

$$a^2 \left(y + \sqrt{y^2 - 4\Gamma_{\min}} \right) < 4k < y + \sqrt{y^2 - 4\Gamma_{\min}} \quad (6)$$

where

$$\Gamma_{\min} := \frac{2k}{a^2\theta} \left(1 - \frac{(1-\theta)^2 I^2}{2\gamma} \right). \quad (7)$$

Appendix A.12 demonstrates the sufficiency of these bounds for $\alpha, q\alpha \in [0, 1]$.

The worker's utility function is flat whenever consumption is greater than the level I , at which point there is a kink. Therefore, in order to ensure we have an interior solution for workers' wages, it must be that workers' consumption is less than I . Since employed workers repay their debts before consuming, this condition corresponds to $w - F < I$. A sufficient condition for this to hold is given in terms of primitives in the next assumption, that firm output y is not too large relative to the kink parameter I .

ASSUMPTION 4.

$$2I > y. \quad (8)$$

Our last assumption on parameters also ensures that I is not too small. This assumption is useful to ensure the solution of the bank's funding program is well-behaved (we will use it in the proof of Proposition 3).

ASSUMPTION 5.

$$(1 - \theta)^2 I^2 > 4\gamma(I - e). \quad (9)$$

Finally, we state two assumptions on endogenous variables that we maintain throughout the paper. We verify that these assumptions are true in equilibrium in Appendix A.13 and in Appendix A.14. Since these are not assumptions on primitives but conditions on endogenous variables that we later verify, we refer to them as "maintained hypotheses." The first maintained hypothesis simply says that banks' initial book equity $I - RD$ is positive.

MAINTAINED HYPOTHESIS 1.

$$I > RD. \quad (10)$$

The second maintained hypothesis is slightly more technical; it ensures that the workers are on the increasing part of their utility function at Date 1.

MAINTAINED HYPOTHESIS 2.

$$0 < w - F < I. \quad (11)$$

Appendix A.14 demonstrates the sufficiency of Assumption 4 for this maintained hypothesis.

3 Results

The analysis of the model is presented in this section. We look for a symmetric equilibrium in which all workers' loans have the same face value F . We first solve the labor market in terms of F and then proceed by backward induction to find the equilibrium in the credit markets.

3.1 Labor Market

The solution to the directed search model in the labor market is mostly standard,¹³ but there is the twist that in our model workers have debt with face value F . In this section we take the face value F as given; we solve for it in Subsection 3.2 below. Note that we are assuming for now that all g -type workers have the same level of debt. Later, we will verify that this is the case in equilibrium (see Proposition 4).

The key insight for the solution procedure is that each firm posts a wage that makes workers indifferent between directing their search at that wage and directing their search at the most attractive other wage available. If this were not the case, a firm could profitably deviate by posting a slightly lower wage and attracting all the workers. This observation allows us to take as given the worker's utility U from directing his/her search toward the most attractive other wage, and then maximize the firm's profit over the wage it posts. To find the equilibrium queue length, we use the firm's zero-profit condition.

Throughout this section we assume that workers' wages are such that $w - F < I$, and thus workers have linear utility locally, and, further, that $w > F$ so that workers do not default if they are employed. Appendix A.14 demonstrates that these conditions indeed hold in equilibrium given Assumption 4.

We solve the problem for an individual firm; all firms will offer identical wages in equilibrium. Let v denote the workers' Date 1 indirect utility from searching at the most attractive other firm and let w denote the wage that the firm posts. Note that the workers' outside option is zero if he is unemployed, which occurs with probability $1 - \alpha$, so the worker's utility is just the probability of being employed times the wage minus the worker's debt repayment. Thus,

$$v = \alpha(w - F). \tag{12}$$

We can solve this equation to write the wage in terms of v ,

$$w = F + \frac{v}{\alpha}, \tag{13}$$

¹³See Rogerson, Shimer, and Wright (2005).

which in turn will allow us to express the firm's profits in terms of the workers' outside option. The firm pays cost k to post wages w and attract workers with probability $q\alpha$, in which case it generates revenue y . Its expected profit is thus

$$\begin{aligned}\Pi &= q\alpha(y - w) - k \\ &= q\alpha(y - F) - qv - k,\end{aligned}\tag{14}$$

having substituted for w in terms of U from above. The objective is smooth and concave, so the first-order condition defines the global maximum:

$$v = \alpha(y - F) + q\alpha'(y - F),\tag{15}$$

which immediately leads to an expression for the wage w from equation (16):

$$w = y + \frac{q\alpha'(y - F)}{\alpha}.\tag{16}$$

To find the equilibrium queue length and, thus to characterize the equilibrium of the labor market in terms of the face value F of household debt, we substitute this wage into the firm's zero profit condition $\Pi = 0$ or

$$\begin{aligned}k &= q\alpha(y - w) \\ &= -q^2\alpha'(y - F)\end{aligned}\tag{17}$$

or

$$q^2\alpha' = -\frac{k}{y - F}.\tag{18}$$

PROPOSITION 1. *The equilibrium queue length, wage and employment rate are*

$$\sqrt{q} = \frac{2k}{a(y - F)},\tag{19}$$

$$w = \frac{y + F}{2}\tag{20}$$

and

$$\alpha(q) = \frac{a^2(y - F)}{2k}.\tag{21}$$

The equilibrium queue length allows us to find the equilibrium matching probabilities. Unmatched workers are unemployed and, thus, α equals the employment rate.

COROLLARY 1. *The employment rate α is decreasing in the amount of household debt F .*

Corollary 1 above says that household debt has a negative effect on the labor market. The reason is that in equilibrium firms must pay higher wages when they employ more-

highly-indebted workers. Therefore, firms' willingness to post vacancies decreases as household debt increases, and unemployment consequently increases. The reason that wages are increasing in household debt is that, while workers' payoff from unemployment is constant (equal to zero), their payoff from employment (equal to $w - F$) is decreasing in F . To explain this in more detail, we calculate the sensitivity of the worker's utility to the employment rate α and the face value of worker debt F . First, recall that the worker's utility is

$$U = \alpha(w - F).$$

So we have that

$$\frac{\partial}{\partial F} \left(\frac{\partial U}{\partial \alpha} \right) = -1,$$

whereas

$$\frac{\partial}{\partial F} \left(\frac{\partial U}{\partial w} \right) = 0.$$

Observe that the sensitivity of U to w is independent of F , but the sensitivity of U to α is decreasing in F . Thus, the higher F is, the less workers value employment relative to unemployment. Firms recognize these household preferences and post high wages to attract indebted households. But, as a consequence of this, firms can employ fewer workers.

3.2 Credit Markets

We now turn to the two credit markets in the model, namely the market in which workers borrow from banks and the market in which banks borrow from savers. Our main results are about the connection between the labor market and the market in which workers borrow. First, we examine the feedback loop between the face value of worker debt and the unemployment rate, and then we analyze the effect of banks' equity on their screening precision and the face value of worker debt. This section proceeds with solving the game backward. We first solve for screening precision, then for the face value of worker debt, and then for the interest rate that depositors charge banks.

In this section, it will be useful to have notation to refer to the Date-1 asset value of a bank. If the worker repays his debt F , the bank's assets are simply this face value, whereas if the worker defaults, the bank's assets are zero. Furthermore, if the bank does not lend, its asset value is simply its initial value of its assets $E + D$. Formally, let V denote the (random) Date 1 value of a bank's assets:

$$V := \begin{cases} F & \text{if } \tau = g \text{ and the worker is employed,} \\ D + E & \text{if bank does not lend,} \\ 0 & \text{otherwise.} \end{cases}$$

3.2.1 The Level of Worker Debt

Before we turn to our main results of this section, we state a lemma that will simplify the analysis. The lemma pins down the amount workers will borrow at Date 0. Specifically, workers borrow exactly up to the kink in their utility functions, $B = I$. The reason that they do not borrow more is that their utility function is flat above the kink, so their marginal benefit from consuming above the kink at Date 0 is zero. The reason they do not borrow less is that they are impatient (their discount factor δ is relatively small), so they have a strong incentive to move consumption forward. Thus, the amount of credit a worker demands from his bank is exactly I . Since each bank is matched with one worker, and the only alternative to lending is riskless storage, a bank has no incentive to hold assets in excess of I at Date 0. Therefore, a bank borrows $D = I - E$ so that its entire cash holdings at Date 0 are I . The result of this argument is stated formally in the next lemma.

LEMMA 1. *In equilibrium, workers borrow $B = I$ and banks raise capital $D + \Delta = I - e$.*

3.2.2 Bank Screening

Each bank chooses its screening precision σ to maximize its expected Date-1 equity value net of screening costs. The bank will invest in costly screening of the borrower only if the information acquired affects its decision to lend. Since a negative signal s_b indicates a worker is type b with certainty, a bank never extends credit after observing s_b . If a bank lends, it will only be after it observes signal s_g . In other words, there are three possibilities: either (i) a bank does not screen ($\sigma = 0$) and always lends to its loan applicant, (ii) a bank screens ($\sigma > 0$) and lends only when it observes s_g , or (iii) a bank does not screen and does not extend credit. Case (iii) would involve no lending to workers and, therefore, a violation of depositors' participation constraint (equation (35) below) unless the deposits were riskless. Thus, case (iii) is the case of no economic activity. We will see below in Proposition 2 that case (i) will never obtain and there is always strictly positive screening in equilibrium.

When $RD < I$, the bank's shareholders have an incentive to screen in order to avoid lending to b -type borrowers. The benefit of screening in this case is that by not lending the bank preserves the equity value $I - RD > 0$. The bank now chooses its screening precision σ to maximize the expected value of its Date-1 equity net of screening costs. Recall that, conditional on its choosing to screen, the bank lends only when it observes the positive signal s_g . Thus, the terms in the bank's objective function (which is written in full in equation (25) below) are as follows. It lends exactly when it observes a positive signal, which occurs with probability

$$\mathbb{P}[s = s_g] = \theta + (1 - \theta)(1 - \sigma). \tag{22}$$

Conditional on lending, it receives the repayment F exactly when both of two events occur. The first event is that the worker is indeed type g , which occurs with conditional probability

$$\mathbb{P}[\tau = g | s = s_g] = \frac{\theta}{\theta + (1 - \theta)(1 - \sigma)} \quad (23)$$

and the second event is that the worker is employed, which occurs with probability α , where α is the employment rate. When the bank receives repayment F , it must repay its creditors RD , so its equity value, given its borrower's repayment, is $F - RD$. That summarizes the bank's expected payoff when it observes the signal s_g . Alternatively, the bank may observe the negative signal s_b . This occurs with probability

$$\mathbb{P}[s = s_b] = (1 - \theta)\sigma. \quad (24)$$

In this event the bank simply keeps its Date-0 assets in place and has asset value I from which it still repays depositors RD . Its equity value is $I - RD$. Finally, the bank bears the screening cost $c(\sigma) = \gamma\sigma^2/2$. Thus, the objective function of the bank is given by

$$\begin{aligned} \mathbb{P}[s = s_g] \mathbb{P}[\tau = g | s = s_g] \alpha (F - RD) + \mathbb{P}[s = s_b] (I - RD) - c(\sigma) \\ = \theta\alpha (F - RD) + (1 - \theta)\sigma(I - RD) - \frac{\gamma\sigma^2}{2}. \end{aligned} \quad (25)$$

Maximizing this objective gives the bank's equilibrium choice of screening precision, as summarized in Proposition 2 below.

PROPOSITION 2. *Banks screen with precision*

$$\sigma = \begin{cases} \frac{1}{\gamma}(1 - \theta)(I - RD) & \text{if } 0 < I - RD < \frac{\gamma}{1 - \theta}, \\ 1 & \text{if } I - RD > \frac{\gamma}{1 - \theta} \end{cases} \quad (26)$$

The expression for σ in Proposition 2 above allows us to perform comparative statics on the screening precision as a function of bank leverage. In particular, we see that more highly levered banks screen less. The reason is that screening reduces the probability of lending to bad borrowers, and an increasing portion of the cost of these bad loans is borne by the bank's creditors as the bank's leverage increases. Since the screening precision choice is made to maximize the value of the bank's equity, screening precision declines with bank leverage. In the next corollary we look at how changes in D and R affect σ .

COROLLARY 2. *Screening precision is decreasing in bank leverage and deposit rates. In particular,*

$$\frac{\partial \sigma}{\partial D} = -\frac{(1 - \theta)R}{\gamma} < 0$$

and

$$\frac{\partial \sigma}{\partial R} = -\frac{(1-\theta)D}{\gamma} < 0$$

whenever $0 < I - RD < \gamma/(1-\theta)$, and otherwise the derivatives is zero or undefined.

Note that the corollary above takes into account only the direct effects of D and R on σ . In other words, it summarizes the direct effects of bank debt and deposit rates taking the other variable as given. This corresponds to studying what would happen in the abbreviated version of our model that takes bank capital structure as exogenous. In the full model, capital structure is endogenous. In Subsection 3.2.3 below, we will analyze a bank's optimal mix of debt and equity funding. To find the optimum we will have to consider not only the direct effect of an increase in debt D on screening precision σ that we calculate above, but also its indirect effect on screening precision through a change in the deposit interest rate. In particular, we will have to study the total derivative

$$\frac{d\sigma}{dD} = \frac{\partial \sigma}{\partial D} + \frac{\partial \sigma}{\partial R} \frac{\partial R}{\partial D}. \quad (27)$$

Note that Corollary 2 suggests that the direct effects of increasing D and R on σ are negative. In the proof of Proposition 3 below, we show that this intuition carries through to this total derivative, and it is indeed negative.

3.2.3 Bank Capital Structure

In this subsection, we find the optimal capital structure for banks. To do this, we write down the bank's problem to set its borrowing and lending contracts as a constrained maximization program. The constraints are determined by competition. In particular, banks and depositors are competitive, so they break even in expectation. The objective function in the program is the expected utility of the worker. The reason is that in order to be able to make a loan, a bank must appeal to workers who want to borrow. Only banks whose contracts maximize the workers' expected utility receive any loan applications, because these are the only banks at which workers direct their search. Precisely, the bank must maximize the expected utility of a worker subject to four constraints. They are as follows: (1) old shareholders break even; (2) new shareholders break even; (3) depositors break even; and (4) amount of deposits available satisfies the depositor's wealth constraint. The program is thus

$$\text{Maximize } (\theta + (1-\theta)(1-\sigma))I + \delta\theta\alpha(w - F) \quad (28)$$

subject to

$$\beta\left(\theta\alpha(F - RD) + (1 - \theta)\sigma(I - RD) - c(\sigma)\right) = e, \quad (29)$$

$$(1 - \beta)\left(\theta\alpha(F - RD) + (1 - \theta)\sigma(I - RD) - c(\sigma)\right) = \Delta, \quad (30)$$

$$\left(\theta\alpha + (1 - \theta)\sigma\right)RD = D, \quad (31)$$

$$e + \Delta + D = I \quad (32)$$

over $F, \beta, \Delta, R,$ and D , where σ is as in Proposition 2. We now have:

PROPOSITION 3. *In equilibrium, banks raise capital only via debt, i.e. $\Delta = 0$ and $D = I - e$.*

The intuition is as follows. Increasing bank leverage decreases the bank's incentives to screen, i.e. decreases σ , as we discussed earlier (see Corollary 2). This increases the probability that the worker is granted a loan at Date 0 and, therefore, the probability that the worker consumes early. Since workers are impatient, they place a high value on this early consumption and are willing to repay more tomorrow in order to be able to borrow today—they are willing to compensate banks for lending to bad borrowers. Since the only mechanism banks have at their disposal to commit not to screen is leverage on their own balance sheets, a highly levered bank can appeal more to workers in search of loans. This explanation is, of course, only partial. It omits the effect of the program's constraints, and does not fully explicate the effects of bank leverage on the face value F of worker debt in the objective function. The subtleties in the proof come from taking these effects into account. However, the proof shows that the intuition we present here is robust.

3.2.4 The Face Value of Worker Debt

We are now in a position to compute the face value of debt that a bank will post. Banks and depositors are both competitive and therefore break even on average, i.e. the expected value of a bank's Date 1 equity net of screening costs equals its equity E ,

$$\mathbb{E}[\max\{V - RD, 0\} | \sigma] - c(\sigma) = E, \quad (33)$$

and the expected repayment that the bank's depositors receive equals the initial capital D that they provide,

$$\mathbb{E}[\min\{V, RD\} | \sigma] = D. \quad (34)$$

Recall that the bank's total Date 0 capital is I (Lemma 1), so summing the break-even conditions for the bank's equity and the claims of its depositors gives

$$\mathbb{E}[V | \sigma] - c(\sigma) = E + D = I. \quad (35)$$

From now on we will work with this condition to determine the face value F as a function of the interest rate R that depositors charge the bank. After having determined F , we can use the depositors' break-even condition to find R .

The final equation above (equation (35)) is the combined break-even condition for all of the bank's claimants. We now decompose the expectation of the bank's total asset value on the left-hand side. There are two terms in the expression. The first term is the probability that the bank receives a good signal—and hence it lends—multiplied by the expected repayment conditional on a good signal. The second term is the probability that the bank receives a bad signal—and therefore does not lend—multiplied by the Date-0 asset value I . Thus,

$$\mathbb{E}[V | \sigma] = \theta\alpha F + (1 - \theta)\sigma I, \quad (36)$$

where σ is as given in Proposition 2. Thus, the break-even condition expressed in equation (35) gives an expression for the face value F that the bank chooses,

$$F = \frac{(1 - (1 - \theta)\sigma)I + c(\sigma)}{\theta\alpha}. \quad (37)$$

Now observe from the equation (37) above that the face value F depends on the employment rate α . Recall that α depends on the face value of debt that workers have when they enter the labor market (see equation (21)). Thus, the face value F that the bank chooses depends on the face value that the bank believes the other banks are offering, through its dependence on the employment rate. To make the distinction between the face value determined by the bank's zero-profit condition and the face value that the bank believes (or "conjectures") that other banks are offering, denote this conjectured face value by \hat{F} . Therefore, when a bank posts the face value F , it acts as if other banks are offering face value \hat{F} . So the conjectured employment rate—and therefore the conjectured repayment probability—is given by

$$\alpha = \frac{a^2}{2k}(y - \hat{F}). \quad (38)$$

Thus, for a face value F to be an equilibrium face value, it must satisfy two conditions.

The first is the bank's zero-profit condition:

$$F = \frac{2k}{a^2\theta(y - \hat{F})} \left((1 - (1 - \theta)\sigma)I + c(\sigma) \right), \quad (39)$$

where we have substituted in for α in equation (37). The second is the rational expectations condition

$$F = \hat{F}. \quad (40)$$

Combining equations (39) and (40) yields a quadratic equation in F :

$$(y - F)F - \Gamma_D = 0 \quad (41)$$

where the function Γ_D does not depend explicitly on F and is defined as follows:

$$\begin{aligned} \Gamma_D &:= \frac{2k}{a^2\theta} \left((1 - (1 - \theta)\sigma)I + c(\sigma) \right) \\ &= \begin{cases} \frac{2k}{a^2\theta} \left(I - \frac{(1 - \theta)^2(I^2 - R^2D^2)}{2\gamma} \right) & \text{if } 0 < I - RD < \frac{\gamma}{1 - \theta}, \\ \frac{2k}{a^2\theta} \left(\theta I + \frac{\gamma}{2} \right) & \text{otherwise.} \end{cases} \end{aligned} \quad (42)$$

Because of *the self-fulfilling beliefs of banks about future employment*, the quadratic in equation (41) has two solutions. When banks believe that the rate of employment will be high—and therefore that worker default is unlikely—banks demand low face values and employment is indeed high. Likewise, when banks believe that the rate of employment will be low—and therefore that worker default is likely—banks demand high face values and unemployment is indeed high. The self-fulfilling prophecy results from the externality that household debt imposes on the labor market. Because high household indebtedness leads to low employment, banks' beliefs (or conjectures) have a critical impact on employment. The face values are the two solutions of the quadratic equation (41). In Figure 4, we depict the solutions to this quadratic equation for two different values of the constant term in the quadratic equation, Γ_D and Γ'_D . The expressions for these solutions, which we call F_+ and F_- , are summarized in the next proposition.

PROPOSITION 4. *There are two equilibrium face values,*

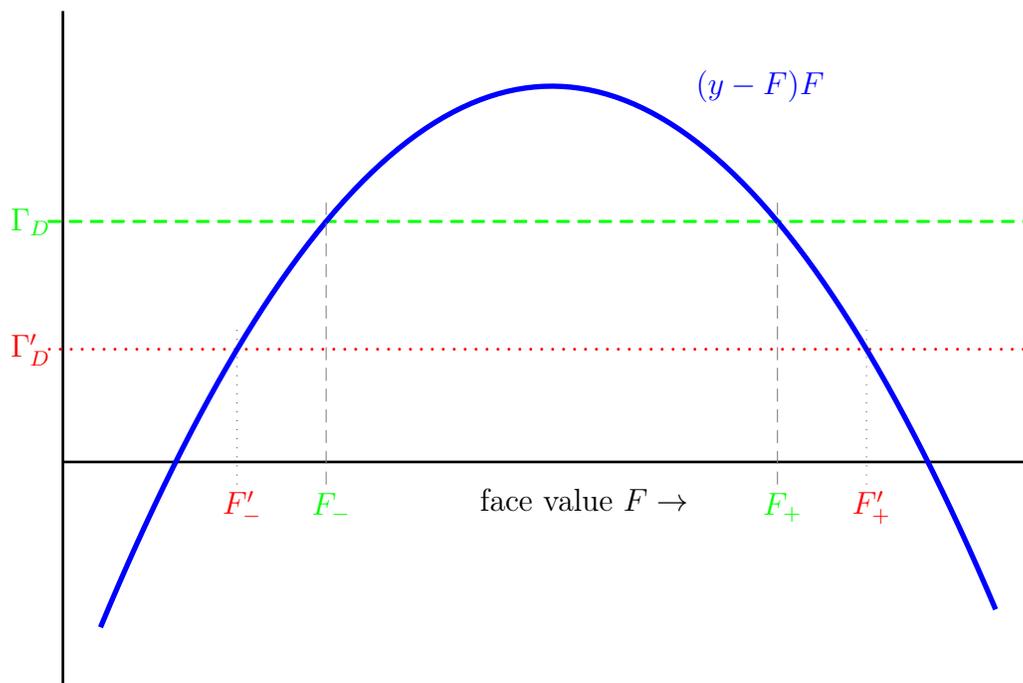
$$F_- = \frac{1}{2} \left(y - \sqrt{y^2 - 4\Gamma_D} \right) \quad (43)$$

and

$$F_+ = \frac{1}{2} \left(y + \sqrt{y^2 - 4\Gamma_D} \right), \quad (44)$$

so long as the discriminant above is positive.

Figure 4: ILLUSTRATION OF COMPARATIVE STATICS OF F



3.2.5 The Effect of Bank Leverage on the Face Value of Worker Debt

In this subsection we analyze the effect of bank leverage on the face value of worker debt. We will show that the two equilibria—the equilibrium associated with face value F_- and the equilibrium associated with face value F_+ —have very different comparative statics properties. In particular, in equilibrium a decrease in the bank debt D leads to a increase in F_- but a decrease in F_+ , as we summarize formally in the proposition below.

PROPOSITION 5. F_- is increasing in bank leverage,

$$\frac{dF_-}{dD} \geq 0,$$

whereas F_+ is decreasing in bank leverage,

$$\frac{dF_+}{dD} \leq 0.$$

The mechanism behind these contrasting comparative statics is rather subtle. We will try to explain the intuition behind the result with the help of the bank's break-even

condition in equation (37), which we can rewrite as

$$\alpha F = \frac{I - (1 - \theta)\sigma I + c(\sigma)}{\theta}. \quad (45)$$

We know that, in equilibrium, σ is decreasing in D (Proposition 2). Further, an increase in D leads to a increase in the right-hand side of equation (45).¹⁴ Looking at the equation, it then appears that to maintain the equality, which is required by competition, a bank must increase F , thereby increasing the left-hand side of the equation to offset the increase in the right-hand side. Economically, a bank must increase the repayment amount F to off set the efficiency lost by screening less. This argument, however, takes into account only the *direct effect* that increasing F has on banks' expected repayment. There is also an *indirect effect* of increasing F that arises in equilibrium: α depends on F and increasing F for all banks leads to a decrease in α via the vacancy-posting effect. It turns out that in the low debt, high employment equilibrium, the direct effect dominates and the argument above is right: increasing D leads to an increase in F . In the high debt, low employment equilibrium, increasing D leads to a decrease in F . The reason is that the indirect effect of a change in F on the expected value of a loan (i.e. on αF) that works through the dependence of α on F is greater than the direct effect of changes in F alone.

We have plotted equation (41)—which re-expresses the bank's break-even condition in equation (45) in terms of the variable Γ_D —in Figure 4. We see how a increase in the right-hand side from in Γ'_D to Γ_D increases the face value of worker debt in the low debt, high employment equilibrium but decreases the face value of worker debt in the low debt, high employment equilibrium. The key to this observation is that the expected value of the worker's loan αF is marginally increasing in F in the low debt, high employment equilibrium; whereas it is marginally decreasing in F in the high debt, low employment equilibrium. In other words, the slope of $(y - F)F$ is positive if $F = F_-$ but negative if $F = F_+$.

3.2.6 The Equilibrium Deposit Rate

In this section we briefly discuss the equilibrium deposit rate R . Our main analysis centers around the connections between household debt, employment, and bank capitalization, but not bank deposit rates. Here we derive the equation that defines R implicitly. We do this more to close the model than to derive further results.

The equilibrium deposit rate is the one that makes the depositors' break-even condition (in equation (34)) bind. By Maintained Hypothesis 1, $I > RD$, the bank defaults either when it lends to a bad worker, which occurs with probability $(1 - \theta)(1 - \sigma)$, or

¹⁴To see this, note that for the equilibrium level of screening, $c'(\sigma) < (1 - \theta)I$.

when it lends to a good worker who remains unemployed, which occurs with probability $\theta(1 - \alpha)$. The equation for R thus reads

$$\theta\alpha RD + (1 - \theta)\sigma RD = D. \quad (46)$$

Recall that α is both the employment rate and repayment rate. Replacing α in the equation above with the expression in terms of the conjectured face value of debt \hat{F} , we have

$$\frac{\theta a^2}{2k}(y - \hat{F}) + (1 - \theta)\sigma = \frac{1}{R}. \quad (47)$$

This reveals immediately that the equilibrium deposit rate also depends on banks' belief about the equilibrium they will be in. The next lemma summarizes how the deposit rate depends on whether the economy is in an equilibrium associated with F_- or an equilibrium associated with F_+ . We find an equation for the deposit rate R in terms of primitives by replacing \hat{F} and σ in equation (47) with their equilibrium values. Note that we express the next lemma in terms of Γ_D , even though when $I - RD < \gamma/(1 - \theta)$, Γ_D depends on R .

LEMMA 2. *If the economy is in an equilibrium associated with F_- , then the deposit rate R_- solves*

$$\frac{\theta a^2}{4k} \left(y + \sqrt{y^2 - 4\Gamma_D} \right) R + \frac{1}{\gamma} (1 - \theta)^2 (I - RD) R = 1 \quad (48)$$

If the economy is in an equilibrium associated with F_+ , then the deposit rate R_+ solves

$$\frac{\theta a^2}{4k} \left(y - \sqrt{y^2 - 4\Gamma_D} \right) R + \frac{1}{\gamma} (1 - \theta)^2 (I - RD) R = 1. \quad (49)$$

4 Discussion of Key Simplifying Assumptions

Because our model has both directed search in the labor market and endogenous information acquisition by banks, it inherits the complexities of both types of models. Nonetheless, it remains tractable. The tractability of our model relies on several useful, but strong, assumptions, which we discuss below.

First, we assume that households' utility is piecewise linear. This allows us to capture households' global risk aversion while maintaining tractability. The local risk neutrality implied by this assumption allows us to solve the directed search model of the labor market (see the discussion in Rogerson, Shimer, and Wright (2005)). But the *global* risk aversion of households implied by this assumption gives them the incentive to borrow. We discuss the robustness of our conclusions to more general forms of utility in Subsection 6.4.

Second, we assume that households have limited liability—indebted households with no wealth consume nothing. Households consume their equity, which is the difference between their income and their debt repayment if it is non-negative (i.e., given limited liability, household consumption is $\max\{w - F, 0\}$). Since the max function is convex, this limited liability generates convexity in households’ preferences near default, and this generates the household debt-overhang problem associated with the vacancy-posting effect. It is now well-established empirically that the implicit insurance provided by limited liability protection distorts household decisions in favor of underinsurance, consistent with our model (see Mahoney (2014)). We discuss the robustness of our results to the inclusion of penalties for household default in Subsection 6.4 below.

Third, we assume that the bank’s screening technology delivers asymmetric signals. Regardless of its precision, the signal is never wrong about good borrowers, but greater signal precision reduces the probability of a “false positive”, i.e., mistakenly identifying a bad borrower as good. This allows us to solve for *symmetric* equilibria in which all households have the same amount of debt when they enter the labor market. If banks’ noisy screening led them to sometimes make the mistake of denying credit to good workers at the early date, then both the good workers with debt and the good workers without debt would search in the labor market at the late date, leading to worker heterogeneity in the labor market, and adding substantial complexity.

5 Welfare and Policy Implications

5.1 Welfare

In this section we analyze welfare in the economy. Banks, depositors, and firms all break even in equilibrium, so the welfare analysis revolves around the utility of workers. Our first result of this section says that if two equilibria of the model are associated with different face values of household debt, then the equilibrium associated with the lower face value corresponds to higher employment and output¹⁵ than the equilibrium associated with the higher face value.

PROPOSITION 6. *Employment and output are higher in the equilibrium associated with F_- than in the equilibrium associated with F_+ .*

This proposition follows from the observation that lower household debt means a diminished externality of debt on unemployment. Thus, the equilibria are efficiency-ranked from the point of view of GDP. Simply the common belief that banks will offer low

¹⁵In Subsection 6.7, we study a simplified version of the model. The simplification allows us to go a step further in our welfare analysis than we do in this section. There we solve for the level that maximizes a measure of utilitarian welfare.

interest rates can arrest a drop in employment and prevent a recession. What is especially noteworthy about this result is that it points to a role for a financial regulator to intervene to restore efficiency in the labor market. We turn to this in the next section.

5.2 Policy Implications

A bank regulator may wish to implement policies that eliminate the low-employment equilibrium. Suppose we have a bank regulator in this economy who wishes to increase employment. To do this, his first task is to implement policies that prevent the economy from ending up in the equilibrium with high debt and low employment. His second task is to increase the rate of employment, given that the economy is in the equilibrium with low debt and high employment.

Since the low-employment equilibrium depends on a high face value of worker debt—i.e., high household interest rates—a bank regulator can step in and eliminate this equilibrium by either prohibiting banks from demanding such high repayments or by directly capping household debt. Given this restriction on the face values of household loans, there will only be one feasible equilibrium face value, the lower face value F_- associated with the high-employment equilibrium. We summarize this in the next proposition.

PROPOSITION 7. If banks are restricted to offer workers debt with face values below a cap $\bar{F} \in (F_-, F_+)$, then the unique equilibrium of the model is the high-employment equilibrium of the model without the cap.

We now turn to the effect of regulating bank equity on the economy. We perform comparative statics on banks' equity E to represent regulatory capital requirements. Our final main result is that increasing bank equity increases employment, provided that the face value of household debt is capped to ensure the economy is in the high-employment equilibrium.¹⁶

PROPOSITION 8. As long as banks's initial equity e is not too small, if banks are restricted to offer workers debt with face values below a cap $\bar{F} \in (F_-, F_+)$, then increasing banks' equity E increases the rate of employment α ,

$$\frac{d\alpha}{dE} > 0. \tag{50}$$

Our analysis thus shows that increasing bank equity increases employment. However, we have also shown that increasing bank equity can be detrimental when the

¹⁶This shows an additional and previously-unexplored beneficial effect of bank capital that goes beyond its role in promoting financial stability, as discussed, for example in Thakor (2014).

economy is in the low-employment equilibrium. Thus, a regulator must use capital requirements *in conjunction* with other regulations that prevent the economy from ending up in the bad equilibrium. In particular, if the regulator caps household debt, he can eliminate the bad equilibrium, so that capital requirements unambiguously increase employment.

6 Extensions and Predictions

This section has two objectives. First, we consider seven extensions of the model and verify that our earlier results are robust in these extensions. Second, we discuss the testable implications of our model.

We analyze seven extensions to verify the model’s robustness: (i) we allow savers to lend directly to workers; (ii) we introduce random matching instead of directed search; we consider the effect of aggregate shocks to output and discuss wage dynamics; (iii) we consider the effects of shocks to aggregate output and analyze their effect on wages; (iv) we relax limited liability to include non-zero default penalties for households; (v) we consider the case in which household debt is secured by collateral; (vi) we permit a more general specification of household utility; and (vii) we consider a simplified version of the model in which we do not consider bank screening in order to conduct a more detailed welfare analysis than in Section 5 above. Additionally, we include a subsection in which we discuss how to test two of the main predictions of our model.

6.1 Depositors Do Not Lend Directly to Workers: Why We Need Banks

The depositors in the model have been largely in the background, funding banks via deposits. We have not explained why banks intermediate between depositors and workers, namely why depositors do not lend to workers directly. In this subsection, we show that if we allow depositors to lend directly to workers, they still prefer to invest in banks. This provides the *raison d’être* of banks.

To do this, we change the model slightly and assume that savers have the same screening technology that banks have, thereby “levelling the playing field” and allowing us to establish a reason for banks to exist even if they are not endowed with any special advantage in screening. We will show that the result that workers always prefer to borrow from banks is effectively a corollary of the result that banks are always maximally levered (Proposition 3).

The reason is as follows. A saver who posts a contract to lend directly to workers is exactly an all-equity funded bank. But Proposition 3 implies that such an all-equity

bank cannot exist in equilibrium. To see this, recall the intuition for the proposition, which says that banks are always maximally levered. The intuition is as follows. Banks' screening precision is decreasing in bank leverage. Thus, the probability that the worker is granted a loan at Date 0—and, therefore, the probability that the worker can consume early—is also decreasing in bank leverage. Since workers are impatient, they place a high value on early consumption. Hence, if a worker is given the choice between borrowing from a levered bank and from an (otherwise identical) unlevered bank, he chooses to borrow from the levered bank. Since a saver who lends directly to workers is effectively an unlevered bank, the worker always prefers to borrow from a bank than from a saver directly. This explains why banks exist in our model: leverage acts as a commitment device by which they can commit not to screen borrowers. We state this result formally as a proposition below.

PROPOSITION 9. In equilibrium, depositors are better off investing via banks than they would be offering loans directly to workers. Further, there cannot be an equilibrium in which depositors lend directly to workers rather than depositing in banks.

6.2 The Effect of Directed Search rather than Random Matching

There are two main search modeling frameworks in labor economics theory: the directed search model that we use in our analysis and the Diamond–Mortensen–Pissarides random-matching model. In this section we briefly demonstrate that our results would not differ if we used the random-matching framework in our analysis. We proceed by outlining a random-matching analog of our labor market search model in which households have debt F . We then explain why higher household debt leads to lower employment via the free-entry condition. Further, we show the results on wages. Finally, we comment on the standard results concerning the efficiency of random matching and emphasize that the inefficiencies in our model are coming from the excessive borrowing of households and not from firms' inefficient wage-setting.

With random search, firms and households divide the surplus generated from their match according to a generalized Nash bargaining rule. This implies that there is a proportion β of the surplus that the household gets and a proportion $1 - \beta$ that the firm gets. Since our model is a one-shot model, all outside options are zero. The total surplus to be shared between the household and the firm is $y - F$, the total output generated by the match minus the amount that must be paid to a third party, namely the household's creditor. Thus, the firm's expected profit from entering the market is

$$\Pi = q\alpha(1 - \beta)(y - F) - k, \tag{51}$$

where, as in the baseline model, $q\alpha$ is the probability a firm is matched with a household

(although here it depends only on the aggregate number of firms entering, not on any submarket), and k is the cost of posting vacancies. Imposing the zero-profit condition $\Pi = 0$ as a result of free entry and rearranging gives

$$q\alpha = \frac{k}{(1-\beta)(y-F)}. \quad (52)$$

This equation says that as F increases, the probability $q\alpha$ must increase. Since $q\alpha$ is increasing in the queue length, this means that queues of job seekers must be longer to induce firms to enter when households are more indebted. Longer queues are tantamount to fewer vacancies. Thus, as household debt increases, there are fewer vacancies. In other words, *the vacancy-posting effect of household debt is robust to the specification of the matching model.*

Note, further, that our predictions about the connection between wages and household debt are also robust to our specification of the matching model. In the directed search model, wages increase as household debt increases. In the random matching model, the wage is just a mechanism to divide the surplus, so the wage is determined according to the equation for the workers' payoff

$$\beta(y-F) = w - F \quad (53)$$

or $w = \beta y + (1-\beta)F$. This coincides exactly with equation (16) for the wage if $\beta = 1/2$. Formally, β is the elasticity of the matching probability $q\alpha$ (see Rogerson, Shimer, and Wright (2005)).¹⁷

Finally, we note that the equivalence of the random matching and directed search models is a result of the efficiency of wage-posting in the directed search model. This is the case in our model, and holds more generally whenever the so-called Hosios condition is satisfied (Hosios (1990)).¹⁸ Thus, the wages firms post in our model are constrained efficient, in the sense that they are the wages a social planner would post, taking into account the search frictions. We think that this lack of a labor-market inefficiency is a desirable feature of our model, since it allows us to focus solely on the inefficiencies generated by household debt.

¹⁷The computation of the elasticity $\varepsilon(q)$ is as follows:

$$\varepsilon \equiv \frac{q(q\alpha)'}{q\alpha} = \frac{(a\sqrt{q})'}{a/\sqrt{q}} = \frac{1}{2}. \quad (54)$$

¹⁸The Hosios condition says that the firms' bargaining power $1-\beta$ must equal the elasticity of the workers' matching probability (equation (54)); it follows immediately from comparing the expressions for the division of surplus in the competitive search and random search models.

6.3 Aggregate Shocks and Wage Dynamics

In this subsection, we discuss the effects of changes to firm output y to employment and wages. We argue that household debt may be a source of sticky wages, and discuss the complementarities between our household debt externality channel of unemployment and the aggregate demand channel that is well-established in the literature.

Here we extend the model to include two possible aggregate states, a boom in which firm output is y_H and a recession in which firm output is $y_L < y_H$. Thus, given (non-contingent) household debt with face value F , the calculations in Subsection 3.1 give the labor market outcomes in the boom and recession. In particular, we have the equations for the wages

$$w_H = \frac{y_H + F}{2} \quad \text{and} \quad w_L = \frac{y_L + F}{2}. \quad (55)$$

Notice that the percentage change of wages across macroeconomic states,

$$\frac{w_H - w_L}{w_H} = \frac{y_H - y_L}{y_H + F} \quad (56)$$

is decreasing in the level of household debt F , suggesting thus that high levels of household debt may be a source of wage rigidity (see, e.g., Bewley (1999)).

Now turn to the employment rates. We see that

$$\alpha_H = \frac{a}{2k}(y_H - F) \quad \text{and} \quad \alpha_L = \frac{a}{2k}(y_L - F), \quad (57)$$

suggesting that high levels of household debt may lower employment in boom times and, more importantly, amplify employment slumps in recessions. Thus, we see our channel of unemployment, based on the externality of household debt on the labor market, as complementary to channels based on varying aggregate output. In particular, when aggregate demand decreases, firm revenues decrease. In our model, this corresponds to a decrease in y . This shock to y has a more severe effect on the labor market when households are more highly levered (F is higher). This is consistent with evidence in studies of the aggregate demand channel, notably Mian and Sufi (2014a).

6.4 Allowing for a Default Penalty

We now examine the effect of the inclusion of a default penalty. In the US, there is large cross-state variation in default penalties. In particular, asset exemption laws, which specify the types and levels of assets that can be seized in bankruptcy, vary across states.¹⁹ Hence, we ask whether our results are robust to the inclusion of a

¹⁹According to Mahoney (2014), “Kansas, for example, allows households to exempt an unlimited amount of home equity and up to \$40,000 in vehicle equity. Neighboring Nebraska allows households to keep no more than \$12,500 in home equity or take a \$5,000 wildcard exemption that can be used for any type of

pre-specified default penalty. In Subsection 6.8, we discuss how one might exploit the cross-state variation in these bankruptcy codes to test the results we find below.

We show that our main results are robust to the inclusion of default penalties, i.e. that as long as the default penalty is capped at a maximum amount, the vacancy-posting effect of household debt will still be at work in the economy. Further, we show that higher default penalties attenuate the vacancy-posting effect. Specifically, for a given level of debt F , higher default penalties lead to higher employment rates.

We now suppose that a household that defaults on its debt suffers a penalty $-d$.²⁰ Thus, if a household has debt F before searching in the labor market, its expected Date 1 utility is

$$v = \alpha(w - F) + (1 - \alpha)(-d) \tag{58}$$

$$= \alpha(w - (F - d)) - d. \tag{59}$$

The last term $-d$ is an additive constant and therefore does not affect household behavior. Comparison with equation (12) reveals that a household with debt F that will suffer a penalty $-d$ in the event of default has equivalent preferences to a household with debt $F' := F - d$ that will suffer no penalty in the event of default. Thus, *the vacancy-posting effect is robust to the inclusion of (bounded) default penalties* and we view the zero default penalty in the model as just a normalization of default penalties to zero.

Higher default penalties also attenuate the vacancy posting effect. The reason is that the preference distortion due the debt overhang problem is mitigated by the default penalty—the effect of $-d$ on preferences exactly offsets the effect of F on preferences. We think that this observation gives a novel cross-sectional prediction of the model: geographical regions with smaller default penalties should have deeper employment slumps following periods of high household leverage, *ceteris paribus*.

6.5 Robustness to the Inclusion of Collateral

In this section we argue that our main results are not dependent on the assumption that household debt is unsecured. This is important because the main example of household debt we have in mind is home mortgage debt, in which a borrower’s residence serves as collateral. We show that our results are robust to the inclusion of collateral as long as the liquidation value of collateral is not too high. However, for high liquidation values

asset.” Chapter 7 is the most popular form of bankruptcy which accounts, according to White (2007), for 70 percent of filings.

²⁰This penalty is in utility terms, which, due to our utility specification, is tantamount to consumption terms as long as consumption is less than I , since the household is risk-neutral before the kink at the point I in its utility function.

of collateral the vacancy posting effect is not present. This suggests that the effects of the vacancy posting effect should be most salient when collateral values are low, i.e. in recessions, especially those associated with large declines in house prices, like the 2008–09 recession.

We now suppose that households have collateral in place. Call the household’s value of the collateral h and call the liquidation value of the collateral λh , where $\lambda \in (0, 1)$ is a parameter that represents the inefficiency of liquidation. This can be interpreted as a loss of value due to the forced sale, because the buyer of the collateral in liquidation is unlikely to be the party that can employ the collateral most efficiently (see, e.g., Shleifer and Vishny (1992)). In our model we should distinguish between the following two cases:

Case 1: $\lambda h \geq F$,

Case 2: $\lambda h < F$.

In Case 1, the liquidation value of the collateral exceeds the face value of debt, or $\lambda h \geq F$. Here the household never defaults, even if $w = 0$. As a result, we can write household’s expected Date 1 utility as

$$v = \alpha(w + h - F) + (1 - \alpha)(\lambda h - F) \tag{60}$$

$$= \alpha(w + h) + (1 - \alpha)\lambda h - F. \tag{61}$$

Here, there is *no interaction between F and α* . Thus, we do not expect the debt overhang resulting from household leverage to distort households’ preferences. As such, the vacancy posting effect is not at work when collateral liquidation values are sufficiently high, or $\lambda h \geq F$.

Now turn to Case 2, in which the liquidation value of collateral is less than the face value of debt, or $\lambda h < F$. In this case, the household defaults when it is unemployed, because without wages it cannot repay its debt even if it liquidates its collateral. As a result, we can write the household’s expected Date 1 utility as

$$v = \alpha(w + h - F). \tag{62}$$

Except for the constant h , this expression coincides with the expression in the baseline model without collateral, as can be seen from comparison with equation (12). Specifically, a household with collateral value h in this model has equivalent preferences to a household with debt $F' := F - h$ in the baseline model. Thus, *whenever liquidation values of collateral are low, the vacancy-posting effect is robust to the inclusion of collateral* and we view the baseline model as just a normalization of collateral values to zero. We think this extension not only confirms the robustness of our model, but also

generates further empirical content, suggesting that the vacancy posting effect should be strongest when collateral values h are low or liquidation discounts $1 - \lambda$ are high.

6.6 Robustness to the Functional Form of Utility

In this section we argue that our main results hold up under more general household preferences than those in our earlier analysis. To do this we demonstrate that the main mechanism behind the vacancy-posting effect would only be amplified if we were to use a more traditional utility function that was strictly increasing and concave everywhere.

Consider the utility of a household with debt F searching in the labor market at Date 1. Its expected utility is given by

$$v = \alpha u(w - F) + (1 - \alpha)u(0), \quad (63)$$

where, as in the main model, the household receives $w - F$ if employed (which occurs with probability α) and zero otherwise (which occurs with probability $1 - \alpha$). In our earlier analysis we used the functional form $u(c) = \min\{I, c\}$ and recovered equation (12). There we solved for the equilibrium of the search model and found the vacancy-posting effect of household debt, as expressed in Corollary 1. We explained that this is due to a household debt-overhang problem: as a household becomes more indebted, its expected utility v becomes more sensitive to wages w than to the probability α of being employed. To see that this holds true even with a more general utility function, differentiate equation (63) with respect to w and α without specifying a specific form for u , but just assuming it is smooth, increasing, and concave. We find that

$$\frac{\partial v}{\partial w} = \alpha u'(w - F), \quad (64)$$

which is increasing in F , i.e.

$$\frac{\partial^2 v}{\partial F \partial w} = -\alpha u''(w - F) > 0. \quad (65)$$

And we find that

$$\frac{\partial v}{\partial \alpha} = u(w - F) - u(0), \quad (66)$$

which is decreasing in F , i.e.

$$\frac{\partial^2 v}{\partial F \partial \alpha} = -u'(w - F) < 0. \quad (67)$$

Thus, increasing F increases the marginal value of higher wages, but decreases the marginal value of a higher probability of employment. This is exactly the mechanism

behind the vacancy-posting effect induced by household debt-overhang. As a result, we expect that *the vacancy-posting effect of household debt is robust to more general utility specifications*.

6.7 The Socially Optimal Household Debt Level

In Section 5 we analyzed welfare by examining which policies can be put in place to maximize GDP and minimize unemployment. In this subsection, we go a step further and analyze utilitarian welfare. However, in order to keep this analysis tractable, we conduct it in a somewhat simplified version of the model. Specifically, only in this subsection, we assume that banks cannot screen borrowers.²¹ In this simplified context, we can ask: what is the socially optimal level of household debt? The social planner’s objective is to maximize utilitarian welfare. The social planner maximizes household welfare over both the face value of debt and the size of the loan, subject to the break-even conditions of the other players in the model.

We find that the socially optimal debt level is always weakly below the equilibrium debt level in the low-debt equilibrium. In other words, as a result of the household-debt externality, households borrow too much even in the low-debt equilibrium. Thus, our main policy prescription in this subsection is that household borrowing should be limited. This is consistent with our prescription to cap household debt in Section 5.

What are the break-even constraints of the other players what the social planner must account for? Without the bank’s screening decision, the break-even constraints for the other players collapse into one equation²² which reads

$$\theta\alpha F = b, \tag{68}$$

where b is the amount borrowed by the household in exchange for the promise to repay F . The household repays F with probability $\theta\alpha$ since it repays only if it is both a g -type—which occurs with probability θ —and finds a job in the labor market—which

²¹This simplified model allows us to abstract from banks’ screening and funding problems and to focus on direct interventions to mitigate the household-debt externality. These interventions are analytically intractable in the full model. One main reason for this is that the bank funding rate is endogenous to the social planner’s intervention—the equation that defines it is non-linear with multiple roots (see Lemma 2). The simplification allows us to perform a more detailed analysis that supports our policy conclusions from the analysis of the full model in Section 5.

²²In particular, we need to consider only the total cash flows accruing to banks and savers together, not how they are divided. The reason is that all these players are risk-neutral, and without the banks’ screening decision, there is no incentive problem. Hence, as long as the borrower’s expected total repayment equals the total amount invested, there is a division of the repayment cash flows that makes savers and banks break even. This is analogous to collapsing the constraints in equations (29)–(32) into a single constraint (cf. the reduction of these constraints into equation (84) in the proof of Proposition 3). In fact, equation (68) is exactly the analog of equation (35) without screening.

occurs with probability α , conditional on being a g -type.

Now, without screening, all borrowers are granted credit, so they all consume b at Date 0. At Date 1, households consume $w - F$ if they are both g -types and employed, which occurs with probability $\theta\alpha$. Thus, the objective function is

$$\mathbb{E}[U] = u(b) + \theta\alpha\delta u(w - F). \quad (69)$$

We have now formulated the social planner's problem, which we state formally in the definition below. Keep in mind that the employment rate α and the wage w are determined in equilibrium and depend on the social planner's choices.

DEFINITION 1. *The social planner's problem is to maximize*

$$u(b) + \theta\alpha\delta u(w - F) \quad (70)$$

over b and F subject to

$$\theta\alpha F = b. \quad (71)$$

We denote the social planner's solution for the optimal amount lent and face value of debt by b^{SP} and F^{SP} .

We are now in a position to present our main results of this section.

LEMMA 3. *The solution to the social planner's problem is given by*

$$F^{\text{SP}} = \frac{1 - \delta}{2 - \delta} y \quad (72)$$

and

$$b^{\text{SP}} = \frac{\theta a^2}{2k} \frac{1 - \delta}{(2 - \delta)^2} y^2. \quad (73)$$

PROPOSITION 10. *The socially optimal amount of household debt is less than the equilibrium amount of household debt (even in the low-debt equilibrium), i.e.*

$$I > b^{\text{SP}} \quad (74)$$

and

$$F_+ > F_- > F^{\text{SP}}. \quad (75)$$

6.8 Testable Implications

In this subsection we discuss how to test two predictions of the model: (i) the vacancy-posting effect of high household debt by which higher household debt causes decreases

in employment (Corollary 1) and (ii) the result that higher default penalties mitigate the vacancy-posting effect (Subsection 6.4).

An empirical test of the vacancy-posting effect would match a sample of counties with relatively low household debt with otherwise identical counties with relatively high household debt. Our model predicts that employment slumps are deeper in the more-indebted counties. This is consistent with existing empirical evidence (see, for example, Mian and Sufi (2014b) for analysis of the 2008–09 recession). However, the level of household indebtedness is endogenous in our model and determined jointly with the unemployment rate in the economy. As a result, this would be a test of association, rather than a causal effect of household debt on the labor market.

To test the causal implication of the vacancy-posting effect, we would require exogenous variation in the level of household indebtedness. In other words, the ideal natural experiment would constitute a subset of identical households undergoing an exogenous shock to their leverage. One way to get closer to an exogenous shock to household leverage is to consider the shocks to household balance sheets induced by changes in house prices, as in Mian, Rao, and Sufi (2013). Since the total amount of debt households have is typically not affected directly by house price changes, households that experience larger declines in asset values are consequently more levered. Thus, house prices can serve as an instrument for household leverage, providing a step toward testing our causal prediction.

To test whether higher default penalties mitigate employment slumps, as our argument in Subsection 6.4 predicts, we would like an exogenous shock to a subset of a group of otherwise identical households. One could test this prediction with a differences-in-differences approach that exploits unexpected changes in state bankruptcy codes. If two states have similar characteristics and one state changes its bankruptcy laws to be more forgiving, effectively lowering default penalties, our model would suggest that unemployment in that state would undergo a relative increase.²³ Absent such a natural experiment, simulated instruments may still be able to identify the causal effect of default penalties on employment. This is how Mahoney (2014) uses cross-sectional variation in state bankruptcy codes to show that uninsured households with larger assets are more likely to pay for medical insurance, conditional on the amount of wealth received. Analogously one could test our prediction that highly-indebted households in states with larger bankruptcy exemptions search for jobs for longer periods of time than highly-indebted households in states with smaller bankruptcy exemptions.

²³This test assumes that labor is not perfectly mobile.

7 Conclusion

This paper examines how the credit and labor markets interact and how this interaction is influenced by bank capital. The analysis shows that household debt is inefficiently high in an unregulated equilibrium, and this contributes to lower employment. The feedback effect between the labor and credit markets generates multiple equilibria, one with higher household indebtedness and lower employment than the other. A combination of high capital requirements for banks and a cap on household leverage can both eliminate the low-employment equilibrium and even increase employment above the laissez-faire level in the high-employment equilibrium.

This role of bank capital requirements, used in conjunction with limits on household debt, in promoting employment is novel. It shows that a well-capitalized banking system not only can contribute to a reduction in financial fragility, as is well recognized, but it can also foster a reduction in unemployment. This role of prudential bank regulation seems significant in light of persistently undercapitalized banks and stubbornly-high unemployment in some parts of the world, e.g., Europe.

Future research may be directed at additional considerations related to the effects of interbank competition and competition between banks and markets on unemployment in the real sector. We know from the theories of relationship banking that these factors affect both the nature and level of relationship lending (e.g., Boot and Thakor (2000)) and there is empirical evidence that banking concentration can affect unemployment (e.g., Gatti, Rault, and Vaubourg (2012)). These insights may be joined to explore a host of additional issues that have potentially rich regulatory implications. One of these is the potential interaction between unemployment insurance, unemployment, consumer indebtedness and bank capital in a general equilibrium setting.

A Appendix

A.1 Proof of Proposition 1

The result follows immediately from substituting $\alpha(q) = a/\sqrt{q}$ into equations (16) and (18). \square

A.2 Proof of Corollary 1

Immediate from the expression for α in equation (21). \square

A.3 Proof of Lemma 1

Before we state the formal proof, we describe the basic idea. Below the kink in workers' utility functions, workers are risk-neutral. This kink is at the point I . Thus, because workers are impatient, there are gains from trade between workers and banks, so long as workers are still below the kink in their utility functions. Workers and banks will always exploit these gains from trade in equilibrium; to do this banks lend to workers I . We show this by conjecturing an equilibrium in which workers borrow less than I and showing that there is a profitable deviation to another contract in which workers borrow I . There is one subtlety in the argument below. It is the following: when the contracts change, banks choose a different level of screening σ . To avoid dealing with these changes in screening, we consider a deviation that keeps banks' incentives to screen unchanged—they screen the same amount under the proposed equilibrium contract and under the (profitable) deviation.

We begin the proof by showing the upper bound on borrowing and then proceed below to show the lower bound. Since savers' wealth is only $I - e$, the bank can raise at most $I - e$ and, as a result, the most that the worker can borrow is I , the maximal Date 0 wealth of the bank.²⁴ This immediately implies that that $B \leq I$ and $E + D \leq I - e$, so to show the equalities $B = I$ and $E + D = I - e$, we need only to show that $B \geq I$ and $E + D \geq I - e$. To do this, we proceed by contradiction.

Suppose (in anticipation of a contradiction) that in equilibrium the bank posts a lending contract (B, F) and funding contract $((D, R), (\Delta, \beta))$ with $B < I$ in equilibrium. Now consider the deviation $(B', F'), ((D', R'), (\Delta', \beta'))$ with $B' \leq I$.

Before writing down the system of incentive constraints for the primed contract to constitute a profitable deviation, we note that so long as $E + D - B < RD$, it must

²⁴Note that this assumption on the wealth of the savers is not driving the result. Workers would not have incentive to borrow more than I even if the funds were available, since the worker does not value consumption above I given his utility function $u(c_0) = \min\{I, c_0\}$. However, the assumption simplifies some analysis (including this proof).

be that $E + D = B$. The reason is that if the bank lends only B and retains cash $E + D - B$, this cash will always accrue to the debt-holders—the bank always defaults if it lends and is not repaid. Thus, the bank will always set $B = E + D$. For this proof we maintain the hypothesis that $E + D - B < RD$, which we verify with Proposition 3 below.

A sufficient condition for the primed contract to be a profitable deviation for the bank is that it make equity holders strictly better off without making workers or debt holders worse off. The corresponding incentive constraints for these conditions are as follows:

$$\begin{aligned} \theta\alpha(F' - R'D') + (1 - \theta)\sigma'(B' - R'D') - c(\sigma') - E' &> \theta\alpha(F - RD) + (1 - \theta)\sigma(B - RD) - c(\sigma) - E \\ (\theta + (1 - \theta)(1 - \sigma'))B' + \delta\theta\alpha(w - F') &\geq (\theta + (1 - \theta)(1 - \sigma'))B + \delta\theta\alpha(w - F), \\ \theta\alpha R'D' + (1 - \theta)\sigma'R'D' - D' &\geq \theta\alpha RD + (1 - \theta)\sigma RD - D, \end{aligned}$$

where σ is the level of screening that the bank chooses optimally given the initial contract and, likewise, σ' is the level of screening that the bank chooses optimally given the primed contract. Note that we have used the notation $E = e + \Delta$ and $E' = e + \Delta'$ and, further, that we have aggregated the incentive constraints of inside and outside equityholders into a single constraint. Since we will show a deviation that strictly increases the total value of equity, we will always be able to find a division of the surplus β' that makes all equity holders are better off. Thus, we write the aggregate constraint down immediately for simplicity.

We will look at a deviation (primed contract) such that $B' - R'D' = B - RD$. This is a sufficient condition for $\sigma' = \sigma$ (we demonstrate this in Proposition 2 below). Substituting for $\sigma' = \sigma$ and reducing these inequalities gives

$$\begin{aligned} \theta\alpha(F' - F) + (1 - \theta)\sigma(B' - B) - (E' - E) &> (\theta\alpha + (1 - \theta)\sigma)(R'D' - RD) && \text{(IC}_E\text{)} \\ (1 - (1 - \theta)\sigma)(B' - B) &\geq \delta\theta\alpha(F' - F) && \text{(IC}_w\text{)} \\ (\theta\alpha + (1 - \theta)\sigma)(R'D' - RD) &\geq D' - D. && \text{(IC}_D\text{)} \end{aligned}$$

Note that the right-hand side of IC_E coincides with the left-hand side of IC_D in the system above. Thus, a sufficient condition for there to exist R' such that the incentive constraints IC_E and IC_D are satisfied is the following condition:

$$\theta\alpha(F' - F) + (1 - \theta)\sigma(B' - B) - (E' - E) > D' - D.$$

Since $E' + D' = B'$ and $E + D = B$, the last inequality can be rewritten as

$$\theta\alpha(F' - F) > (1 - (1 - \theta)\sigma)(B' - B). \quad (76)$$

This constraint and IC_w are the only two constraints that remain to be satisfied. Note that the left-hand side of IC_w coincides with the right-hand side of equation (76). Thus, a sufficient condition for there to exist B' such that the constraints are satisfied is

$$\theta\alpha(F' - F) > \delta\theta\alpha(F' - F).$$

This is always satisfied when $\delta < 1$. Thus, if $B < I$ there exists a profitable deviation for the bank, hence $B \geq I$ in equilibrium. Combined with the arguments above, this says that $B = I$ and it completes the proof. \square

A.4 Proof of Proposition 2

The objective function in equation (25) is a negative quadratic polynomial in σ , so the first-order condition determines the global maximum, i.e., whenever there is an interior solution $\sigma \in (0, 1)$, the optimal screening precision solves

$$(1 - \theta)(I - RD) - \gamma\sigma = 0. \tag{77}$$

When the global maximizer is to the right of the boundary at $\sigma = 1$, $\sigma = 1$ maximizes the quadratic on the domain $[0, 1]$. In contrast, we can see immediately from the first-order condition for $\sigma > 0$ whenever $I > RD$ —there is never a corner solution at zero. \square

A.5 Proof of Proposition 3

To prove that $\Delta = 0$ in equilibrium, we suppose an interior solution $\Delta \in (0, I - e)$ and then show that changing D (equivalent to changing Δ) is a profitable deviation for the bank. As a result, it must be that the program has a corner solution. There are two possible corner solutions, $D = 0$ and $D = I - e$. We compare these directly and show that $D = I - e$ corresponds to a higher value of the objective than $D = 0$, so leverage is maximal in equilibrium.

We divide the proof into five steps. In Step 1, we eliminate the variables that appear linearly in the constraints to simplify the program. In Step 2, we consider a marginal change in leverage. We do this by differentiating the objective along the surface defined by the binding constraints. In Step 3, we show that increasing debt either increases the objective function or first increases the objective and then decreases it. In Step 4, we eliminate the possibility that the objective function is first increasing and then decreasing in debt. In Step 5, we conclude that debt is maximal.

Step 1: Reducing the program. We begin the proof by rewriting the bank's program from equations (28)–(32), having substituted for the equilibrium value of screening

intensity σ , so the program is to maximize

$$\left(1 - \frac{(1-\theta)^2}{\gamma}(I - RD)\right) I + \delta\theta\alpha(w - F) \quad (78)$$

subject to

$$\beta \left(\theta\alpha(F - RD) + \frac{(1-\theta)^2}{2\gamma}(I - RD)^2 \right) = e, \quad (79)$$

$$(1 - \beta) \left(\theta\alpha(F - RD) + \frac{(1-\theta)^2}{2\gamma}(I - RD)^2 \right) = \Delta, \quad (80)$$

$$\theta\alpha RD + \frac{(1-\theta)^2}{\gamma}(I - RD)RD = D, \quad (81)$$

$$e + \Delta + D = I \quad (82)$$

over F, β, Δ, R , and D . Now, observe that the system is linear in the variables β, e , and Δ that appear in the constraints. Thus, we can collapse the three equations (79), (80), and (82) into a single equation, which we then add to equation (81), to eliminate these three variables to rewrite the program again, so it is to maximize

$$\left(1 - \frac{(1-\theta)^2}{\gamma}(I - RD)\right) I + \delta\theta\alpha(w - F) \quad (83)$$

subject to

$$\theta\alpha F + \frac{(1-\theta)^2}{2\gamma}(I^2 - R^2 D^2) = I, \quad (84)$$

$$\theta\alpha R + \frac{(1-\theta)^2}{\gamma}(I - RD)R = 1. \quad (85)$$

Step 2: The effect of an incremental change in D on the objective. Now suppose that $\Delta > 0$ or, equivalently, $D < I - e$ and consider a marginal increase in D . First, we solve for $\partial F/\partial D$ by differentiating the first constraint (as expressed in equation (84)). This gives

$$\theta\alpha \frac{\partial F}{\partial D} - \frac{(1-\theta)^2 RD}{\gamma} \frac{\partial}{\partial D}(RD) = 0. \quad (86)$$

We now differentiate the objective and substitute for $\partial F/\partial D$ from the equation above:

$$\frac{(1-\theta)^2 I}{\gamma} \frac{\partial}{\partial D}(RD) - \delta\theta\alpha \frac{\partial F}{\partial D} = \frac{(1-\theta)^2}{\gamma}(I - \delta RD) \frac{\partial}{\partial D}(RD). \quad (87)$$

Since $\delta < 1$, $I - \delta RD > I - RD$, which is positive by Maintained Hypothesis 1. Thus, a necessary and sufficient condition for the objective to be increasing in D is for RD to be increasing in D . We will now use the second constraint (equation (85)) to show

that this condition is satisfied.

Step 3: Showing the program has a corner solution. Differentiating the second constraint (as expressed in equation (85)) with respect to D gives

$$\left(\theta\alpha + \frac{(1-\theta)^2}{\gamma}(I - RD) \right) \frac{\partial R}{\partial D} - \frac{(1-\theta)^2}{\gamma} R \frac{\partial}{\partial D}(RD) = 0. \quad (88)$$

We now simplify this equation using the fact that

$$\frac{\partial}{\partial D}(RD) = R + D \frac{\partial R}{\partial D}, \quad (89)$$

to recover

$$\frac{\partial R}{\partial D} = \frac{(1-\theta)^2 R^2}{\theta\alpha\gamma + (1-\theta)^2(I - 2RD)}. \quad (90)$$

Now turn to the condition established above—i.e. that RD is increasing in D —and substitute for $\partial R/\partial D$ from the last equation above (equation (90)) to see exactly when it is satisfied:

$$\frac{\partial}{\partial D}(RD) = R + D \frac{\partial R}{\partial D} \quad (91)$$

$$= R + \frac{(1-\theta)^2 R^2 D}{\theta\alpha\gamma + (1-\theta)^2(I - 2RD)} \quad (92)$$

$$= \left(\frac{\theta\alpha\gamma + (1-\theta)^2(I - 2RD) + (1-\theta)^2 RD}{\theta\alpha\gamma + (1-\theta)^2(I - 2RD)} \right) R \quad (93)$$

$$= \left(\frac{\theta\alpha\gamma + (1-\theta)^2(I - RD)}{\theta\alpha\gamma + (1-\theta)^2(I - 2RD)} \right) R. \quad (94)$$

Maintained Hypothesis 1 (that $I > RD$) implies that the numerator is always positive and the derivative above is never zero. Further, at $D = 0$ the denominator is strictly positive, $\theta\alpha\gamma + (1-\theta)^2 I > 0$. Thus, at $D = 0$, RD is increasing in D and therefore so is the objective function. Thus there are two cases:

Case 1: RD is increasing for $D \in [0, I - e]$

Case 2: There is a point $D^* \in (0, I - e)$ at which the denominator in equation (94) is zero; so RD is increasing on $[0, D^*)$ and then decreasing on $(D^*, I - e]$.

Step 4: Showing that RD must be always increasing in equilibrium.

We now show that Case 2 above cannot obtain in equilibrium. We state this result as a lemma, because it will be useful also in subsequent proofs.

LEMMA 4. RD is increasing in D on $[0, I - e]$.

Proof. Suppose (in anticipation of a contradiction) that RD is not increasing in D on $[0, I - e]$. Thus, Case 2 above must obtain. Namely, there is a point D^* after which

RD is decreasing in D . At D^* the denominator of equation (94) must be zero. So,

$$RD^* = \frac{1}{2} \left(I + \frac{\theta\alpha\gamma}{(1-\theta)^2} \right). \quad (95)$$

Now, observe that D^* must maximize RD and, therefore, maximize the objective function. Thus, D^* must be the equilibrium level of debt. Hence, we can solve for R by substituting for RD^* in equation (85):

$$\theta\alpha R + \frac{(1-\theta)^2}{\gamma} \left[I - \frac{1}{2} \left(I + \frac{\theta\alpha\gamma}{(1-\theta)^2} \right) \right] R = 1$$

or

$$R = \frac{2\gamma}{\theta\alpha\gamma + (1-\theta)^2 I}. \quad (96)$$

Solving for D^* from equation (95) now gives

$$D^* = \frac{\left(\theta\alpha\gamma + (1-\theta)^2 I \right)^2}{2\gamma(1-\theta)^2} \quad (97)$$

$$> \frac{(1-\theta)^2 I^2}{4\gamma} \quad (98)$$

$$> I - e, \quad (99)$$

where the last line above follows from Assumption 5. But this conclusion that $D^* > I - e$ violates the hypothesis that $D^* \in (0, I - e)$. This is a contraction. Thus, we conclude that Case 1 must obtain, or that RD is increasing in D on $[0, I - e]$. \square

Step 5: Concluding the in equilibrium $D = I - e$. We have shown that RD is increasing in D on $[0, I - e]$. From equation (86), this implies that the objective is maximized at the corner $D = I - e$. Thus, in equilibrium, banks maximize leverage, setting $D = I - e$ and $\Delta = 0$.

A.6 Proof of Proposition 4

Immediate from applying the quadratic formula to equation (41). \square

A.7 Proof of Proposition 5

We begin the proof by computing the derivative of Γ_D with respect to RD , denoted Γ'_D , whenever it exists:

$$\Gamma'_D = \begin{cases} \frac{2k(1-\theta)^2 RD}{\gamma a^2 \theta} & \text{if } 0 < I - RD < \frac{\gamma}{1-\theta}, \\ 0 & \text{otherwise,} \end{cases} \quad (100)$$

so $\Gamma'_D \geq 0$. Also recall from Lemma 4 in Appendix A.5 that $\partial(RD)/\partial D \geq 0$, so

$$\frac{d\Gamma_D}{dD} = \Gamma'_D \frac{\partial(RD)}{\partial D} \geq 0. \quad (101)$$

Turning to the face value of household debt, we have

$$\frac{dF_-}{dD} = \frac{\partial(RD)}{\partial D} \frac{\Gamma'_D}{\sqrt{y^2 - 4\Gamma_D}} \geq 0 \quad (102)$$

and

$$\frac{dF_+}{dD} = -\frac{\partial(RD)}{\partial D} \frac{\Gamma'_D}{\sqrt{y^2 - 4\Gamma_D}} \leq 0. \quad (103)$$

To complete the argument, note that it is easy to see from the definition of Γ_D that Γ_D is decreasing in D about the kinks, where it is not differentiable. \square

A.8 Proof of Lemma 2

The statement follows immediately from inserting the expression for σ from Proposition 2 and the expressions for F_- and F_+ from Proposition 4 into equation (47). \square

A.9 Proof of Lemma 9

In order for depositors to lend directly to workers, it must be incentive compatible for both workers to borrow directly from depositors and for depositors to lend directly to workers rather than to lend to workers via banks. Denoting by R_s the rate at which the depositor lends to the worker, the incentive compatibility constraint for the worker to borrow from the depositor is

$$R_s I < F.$$

In other words, the depositor must prefer to borrow $R_s I$ from the depositor than F from the bank.

The incentive compatibility constraint for the depositor is

$$\theta\alpha R_s > \theta\alpha R + (1 - \theta)\sigma R.$$

In other words, he must prefer to lend I at rate R_s to his worker (regardless of his type) and get back $R_s I$ if the worker is good and employed, than to lend I at rate R to a bank. If he lends to a bank he gets repaid in two cases: either if the worker is good and employed or if the bank screens, does not lend to a bad worker and does not default.

The incentive compatibility constraints are satisfied if

$$R_s \in \left(\frac{\theta\alpha R + (1 - \theta)\sigma R}{\theta\alpha}, \frac{F}{I} \right).$$

Such a rate exists when the interval above is non-empty. Thus, if

$$\frac{F}{I} < \frac{\theta\alpha R + (1 - \theta)\sigma R}{\theta\alpha},$$

then there is no R_s that satisfies the constraints.

Since $\theta\alpha R + (1 - \theta)\sigma R = 1$, the equation above can be re-written as

$$\theta\alpha F < I.$$

Plugging in for F from equation (37), this can be re-written as

$$\frac{(1 - (1 - \theta)\sigma)I + c(\sigma)}{I} < 1. \tag{104}$$

As a result of the bank's maximization of the objective in equation (25), the left-hand-side in the above equation is minimized in equilibrium. Thus, it is always smaller than its value at $\sigma = 1$:

$$\frac{(1 - (1 - \theta)\sigma)I + c(\sigma)}{I} < \frac{(1 - (1 - \theta)\sigma)I + c(\sigma)}{I} \Big|_{\sigma=1} = 1.$$

This implies that inequality (104) is always satisfied and R_s does not exist. \square

A.10 Proof of Proposition 8

For this proof, we simply compute the total derivative of α with respect to D directly, given that the face value of debt is

$$F_- = \frac{1}{2}(y - \sqrt{y^2 - 4\Gamma_D}).$$

Note that α depends on F_- which, in turn, depends on Γ_D . For $I - RD > \gamma/(1 - \theta)$, Γ_D does not depend on D . For $I - RD < \gamma/(1 - \theta)$, Γ_D depends on D directly and indirectly, via R . Thus,

$$\frac{d\alpha}{dD} = -\frac{a^2}{2k\sqrt{y^2 - 4\Gamma_D}} \left(\frac{\partial\Gamma_D}{\partial D} + \frac{\partial\Gamma_D}{\partial R} \frac{\partial R}{\partial D} \right).$$

We know that $I - RD > \gamma/(1 - \theta)$ we have that $d\alpha/dD = 0$, so we can focus on the case in which $I - RD < \gamma/(1 - \theta)$. First note that

$$\frac{\partial\Gamma_D}{\partial D} = \frac{2kI(1 - \theta)^2 R^2 D}{a^2 \theta \gamma} > 0,$$

and

$$\frac{\partial\Gamma_D}{\partial R} = \frac{2kI(1 - \theta)^2 R D^2}{a^2 \theta \gamma} > 0.$$

As a result, a sufficient condition for $d\alpha/dD$ to be negative is that $\partial R/\partial D$ is positive.

Now, recall that R is defined implicitly by the following equation

$$\frac{\theta a^2}{4k} \left(y + \sqrt{y^2 - 4\Gamma_D} \right) R + \frac{1}{\gamma} (1 - \theta)^2 (I - RD) R = 1. \quad (105)$$

So we use the implicit function theorem to determine the sign of $\partial R/\partial D$. Below, we refer to $\partial R/\partial D$ as R' . Implicitly differentiating the equation above we find

$$\begin{aligned} & \frac{\theta a^2}{4k} \left(y + \sqrt{y^2 - 4\Gamma_D} \right) R' - \frac{\theta a^2 R}{2k\sqrt{y^2 - 4\Gamma_D}} \left(\frac{\partial\Gamma_D}{\partial R} R' + \frac{\partial\Gamma_D}{\partial D} \right) + \\ & + \frac{1}{\gamma} (1 - \theta)^2 [IR' - R^2 - 2RDR'] = 0 \end{aligned}$$

or

$$\begin{aligned} & \left[\frac{\theta a^2}{4k} \left(y + \sqrt{y^2 - 4\Gamma_D} \right) + \frac{1}{\gamma} (1 - \theta)^2 (I - RD) - \frac{(1 - \theta^2)RD}{\gamma} \left(\frac{IRD}{\sqrt{y^2 - 4\Gamma_D}} + 1 \right) \right] R' = \\ & = \frac{(1 - \theta)^2 R^2}{\gamma} \left(\frac{IRD}{\sqrt{y^2 - 4\Gamma_D}} + 1 \right). \end{aligned}$$

Thus, R' is positive as long as

$$\frac{\theta a^2}{4k} \left(y + \sqrt{y^2 - 4\Gamma_D} \right) + \frac{1}{\gamma} (1 - \theta)^2 (I - RD) - \frac{(1 - \theta^2)RD}{\gamma} \left(\frac{IRD}{\sqrt{y^2 - 4\Gamma_D}} + 1 \right) > 0,$$

which holds as long as D is not too large. Since $D = I - e$ in equilibrium, this holds as long as e is not too small. Thus, whenever the bank's initial equity e is not too small, $d\alpha/dD < 0$ or $d\alpha/dE > 0$ as desired. \square

A.11 Proofs of Lemma 3 and Proposition 10

We present the proofs of Lemma 3 and Proposition 10 together. The reason we prove the two of them together is that we prove both results under a maintained hypothesis that we use the second result to verify.

We solve the social planner's problem under the maintained hypothesis that $b^{\text{SP}} < I$ and $w - F^{\text{SP}} < I$, so that households are on the linear increasing part of their utility functions. Under these assumptions, we derive the expressions in Lemma 3. At the end of the proof, we go back and verify that this is indeed the case given the optimal intervention. This verification depends on the findings of Proposition 10.

We begin with the proof of Lemma 3 under the above assumptions. If the solution is indeed on the increasing part of household utility, then the social planner's problem is to maximize

$$b + \theta\alpha\delta(w - F) \tag{106}$$

over b and F subject to

$$\theta\alpha F = b. \tag{107}$$

Plugging the constraint into the objective gives the objective function

$$\theta\alpha F + \theta\alpha\delta(w - F) \tag{108}$$

or, inserting the equilibrium values of α and w from Proposition 1, we have that the social planner maximizes the function

$$\frac{a^2\theta}{2k} \left((y - F)F + \frac{\delta}{2}(y - F)^2 \right) \tag{109}$$

over F . This is a negative quadratic; we solve for the global maximum via the first-order approach, which gives the following equation for the socially optimal level of debt

$$y - 2F^{\text{SP}} - \delta(y - F^{\text{SP}}) = 0 \tag{110}$$

or

$$F^{\text{SP}} = \frac{1 - \delta}{2 - \delta} y. \tag{111}$$

This proves the first part of Lemma 3.

From the equilibrium expression for α , we now find that at the social optimum the

rate of employment is

$$\alpha = \frac{a^2}{2k}(y - F) \quad (112)$$

$$= \frac{a^2}{2k} \left(y - \frac{1 - \delta}{2 - \delta} y \right) \quad (113)$$

$$= \frac{a^2}{2k} \frac{y}{2 - \delta}. \quad (114)$$

From equation (107), we find that the socially optimal level of debt is given by

$$b^{\text{SP}} = \theta \alpha F^{\text{SP}} \quad (115)$$

$$= \theta \cdot \frac{a^2}{2k} \frac{y}{2 - \delta} \cdot \frac{1 - \delta}{2 - \delta} y \quad (116)$$

$$= \frac{\theta a^2}{2k} \frac{1 - \delta}{(2 - \delta)^2} y^2. \quad (117)$$

This proves the second part of Lemma 3.

We now turn to the proof of Proposition 10. First, we show that this socially optimal amount borrowed is less than the equilibrium amount borrowed I . To do this we make use of Assumption 4, which says that $y < 2I$, and Assumption 3, which implies that $y < 4k/a^2$. Together these say that

$$y^2 < \frac{8kI}{a^2}. \quad (118)$$

We use this to express a bound on the socially optimal amount borrowed b^{SP} as follows:

$$b^{\text{SP}} = \frac{\theta a^2}{2k} \frac{1 - \delta}{(2 - \delta)^2} y^2 \quad (119)$$

$$< \frac{\theta a^2}{2k} \frac{1 - \delta}{(2 - \delta)^2} \frac{8kI}{a^2} \quad (120)$$

$$= \frac{4\theta(1 - \delta)I}{(2 - \delta)^2}. \quad (121)$$

Since $\theta < 1$, a sufficient condition for this last expression to be less than I is

$$\frac{4(1 - \delta)}{(2 - \delta)^2} < 1 \quad (122)$$

or

$$4 - 4\delta < 4 - 4\delta + \delta^2, \quad (123)$$

which holds for all δ . Thus, we have proved that the socially optimal amount borrowed is less than the equilibrium amount borrowed, $b^{\text{SP}} < I$.

Now we show further that the socially optimal face value F^{SP} is less than the lower

equilibrium face value F_- . To prove this, we express the condition for $F^{\text{SP}} < F_-$ in this simplified model without screening and then rewrite the condition to show that it always holds as a result of the fact that $b^{\text{SP}} < I$ derived above. First note from the expression for Γ_D in equation (42) that in the model without screening ($\sigma = 0$)

$$\Gamma_D = \frac{2kI}{\theta a^2} \quad (124)$$

and, therefore,

$$F_- = \frac{1}{2} \left(y - \sqrt{y^2 - 4\Gamma_D} \right) \quad (125)$$

$$= \frac{1}{2} \left(y - \sqrt{y^2 - \frac{8kI}{\theta a^2}} \right). \quad (126)$$

Thus, replacing F^{SP} with the expression in equation (72) above, we see that $F^{\text{SP}} < F_-$ if and only if

$$\frac{1-\delta}{2-\delta} y < \frac{1}{2} \left(y - \sqrt{y^2 - \frac{8kI}{\theta a^2}} \right) \quad (127)$$

which can be rewritten as

$$\frac{1-\delta}{(2-\delta)^2} y^2 < \frac{2kI}{\theta a^2}. \quad (128)$$

Observe that this last inequality is equivalent to the inequality $b^{\text{SP}} < I$ derived above (compare with the expression for b^{SP} in equation (119)). Therefore, it is satisfied and we can conclude that $F^{\text{SP}} < F_-$.

It remains to verify the maintained assumptions that $b^{\text{SP}} < I$ and $w - F^{\text{SP}} < I$. That $b^{\text{SP}} < I$ is immediate from the findings above, since the kink is at I and the socially optimal consumption at Date 0 equals the amount borrowed, which is less than or equal to I , i.e. $b^{\text{SP}} < I$. To complete the proof, we show that $w - F^{\text{SP}} < I$ by computation. Recalling that in equilibrium $w = (y + F)/2$, at the social optimum

$$w - F^{\text{SP}} = \frac{y + F^{\text{SP}}}{2} - F^{\text{SP}} \quad (129)$$

$$= \frac{y - F^{\text{SP}}}{2} \quad (130)$$

$$= \frac{y}{2(2-\delta)}. \quad (131)$$

Thus, the household is on the increasing part of its utility function whenever

$$\frac{y}{2(2-\delta)} < I, \quad (132)$$

which always holds by Assumption 4 since $\delta < 1$. □

A.12 Sufficiency of Bounds in Assumption 3

Here we show the sufficiency of the bounds stated in Assumption 3 for the matching probabilities to be well-defined. Substituting the equilibrium q from Proposition 1 and the equilibrium F from Proposition 4, we can rewrite the condition in equation (5) as

$$a^2(y - F) < 2k < (y - F).$$

Plugging in for the smallest F , i.e., F_- , in the left-hand-side of the equation and for the largest F , i.e., F_+ , in the right hand side of the equation we obtain sufficient conditions for the inequality above to hold, namely

$$a^2 \left(y + \sqrt{y^2 - 4\Gamma_D} \right) < 4k < y - \sqrt{y^2 - 4\Gamma_D}.$$

These bounds are tightest when Γ_D is smallest. Thus, we minimize Γ_D over all possible σ . We do this by minimizing the expression²⁵

$$\Gamma_D = \frac{2k}{a^2\theta} \left((1 - (1 - \theta)\sigma)I + c(\sigma) \right) \quad (133)$$

and replacing σ with the minimizer

$$\sigma = \frac{(1 - \theta)I}{\gamma} \quad (134)$$

to find the expression for Γ_{\min} in the statement of the assumption. \square

A.13 Verification of Maintained Hypothesis $I - RD > 0$

We use the equation

$$\theta\alpha RD + \frac{(1 - \theta)^2}{\gamma} (I - RD)RD = D \quad (135)$$

which follows from plugging in for the equilibrium σ in equation (46) to show that the hypothesis $I > RD$ (Maintained Hypothesis 1) holds in equilibrium. This equation says that

$$I - RD = \frac{\gamma(1 - \theta\alpha R)}{(1 - \theta)^2 R}. \quad (136)$$

Therefore $I > RD$ exactly when

$$1 > \theta\alpha R. \quad (137)$$

This holds by equation (135) since $I - RD > 0$ by hypothesis.

Notice that this argument is (appropriately) circular. When $I > RD$, banks anti-

²⁵The expression is a negative quadratic, so the first-order condition suffices to find the global minimizer.

pate that they can recover the value $I - RD$ from avoiding lending to a bad borrower. This induces them to invest in a positive screening precision in equilibrium (Proposition 2). Because the screening precision is positive, depositors get repaid with probability higher than $\theta\alpha$. Thus they demand an interest rate strictly lower than $(\theta\alpha)^{-1}$. \square

A.14 Verification that $0 < w - F < I$

In Subsection 3.1 we solved the model under the hypothesis that $0 < w - F < I$. Here we show that given the equilibrium values of w and F , Assumption 4 suffices for the hypothesis to hold. Substituting in for w from Proposition 1 gives the necessary and sufficient condition

$$F < y < 2I + F. \tag{138}$$

The expressions for F in Proposition 4 show that $0 < F < y$, so Assumption 4 that $y < 2I$ suffices.

A.15 Table of Notations

Labor Market	
q_w	the queue length in the submarket associated with wage w
q	the equilibrium queue length
α	probability that a worker is matched with a firm
$q\alpha$	probability that a firm is matched with a worker
Workers	
U	workers' lifetime utility function
u	workers' single date utility ("felicity") function
I	workers' utility parameter—i.e. the point of the kink in workers' utility
c_t	workers' consumption at Date t
δ	workers' discount factor
$\tau \in \{g, b\}$	worker type
Banks	
$s \in \{s_g, s_b\}$	signal about worker type
σ	banks' screening precision
$c(\sigma)$	banks' cost of screening
γ	cost function parameter
V	banks' total assets
e	banks' initial equity
Γ_D	shorthand notation relating to banks' asset value defined in equation (42)
Firms	
y	firms' output
k	firms' search cost
Π	firms' profit
Contracts	
B	the amount workers' borrow from banks
F	the face value of worker debt
F_-, F_+	the two possible equilibrium face values of worker debt
D	the amount banks borrow from depositors via debt
R	the interest rate on bank deposits
Δ	the amount banks raise from depositors via equity
β	the proportion of equity retained by initial equity holders
w	the wage firms pay workers

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