MBA 203 Executive Summary

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Class 1. Present and future value

- C: Cash flow at a future date
- r: Interest rate
- g: Growth rate of cash flows
- n: Number of cash flows (or date of a single amount)
- PV: Present value
- FV: Future value
- C_0: Cash flow at date 0 (today). C_1: Cash flow at date 1 (one period from today). Etc.

1. **Future value:** Future value of a single cash flow invested for n periods:
$$FV = C \times (1+r)^n$$
2. **Present value:** Present value of a single cash flow received n periods from now:
$$PV = C \times \frac{1}{(1+r)^n}$$
3. **Present value of a stream of cash flows:**
$$PV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{C_n}{(1+r)^n}$$

4. **Perpetuity:** Present value of a perpetuity
$$PV = \frac{C}{r}$$
5. **Growing perpetuity:** Present value of a constant growth perpetuity, when $r - g > 0$,
$$PV = \frac{C}{r-g}$$
6. **Annuity:** Present value of an annuity
$$PV = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$
7. **Growing annuity:** Present value of a growing annuity
$$PV = \frac{C}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^n\right) \qquad \text{Works for } r < g \text{ or } r > g.$$
   If $r = g$, then $PV = C \times \frac{n}{1+r}$.

8. **Excel functions for annuities:** PV, PMT, FV, NPER, RATE. These functions solve the following equation:
$$0 = PV + \frac{PMT}{RATE}\left(1 - \frac{1}{(1+RATE)^{NPER}}\right) + \frac{FV}{(1+RATE)^{NPER}}$$
   or, equivalently,
$$0 = PV + \frac{PMT}{1+RATE} + \frac{PMT}{(1+RATE)^2} + ... + \frac{PMT+FV}{(1+RATE)^{NPER}}$$

Class 2. Putting Present Value to Work

- k: Number of compounding periods per year
- APR: Annual percentage rate
- EAR: Effective annual rate
- C_t^{real}: Real cash flow at date t
- C_t^{nominal}: Nominal cash flow at date t
- g^{real}: Growth rate of real cash flows
- g^{nominal}: Growth rate of nominal cash flows
- r^{real}: Real interest rate
- r^{nominal}: Nominal interest rate
- i: The rate of inflation
- τ: Tax rate

1. **Interest rate per compounding period:**

$$r_{period} = \frac{APR}{k}$$

Effective annual rate (EAR): If you invest \$1 at APR with compounding k times per year, then after one year you have

$$(1 + EAR) = \left(1 + \frac{APR}{k}\right)^k$$

and after t years you have $(1 + EAR)^t$.

2. **Dealing with cases with multiple interest rates:**

- Calculate loan payments using the loan interest rate
- Calculate the principal still owed by discounting the remaining payments using the loan interest rate
- Calculate present value (market value) using the going market rate.

3. **Nominal vs. real cash flows:** (i is the expected inflation rate)

$$C_t^{\text{real}} = \frac{C_t^{\text{nominal}}}{(1+i)^t}$$

Growth rates of real and nominal cash flows:

$$1 + g^{\text{real}} = \frac{1 + g^{\text{nominal}}}{1 + i}$$

Nominal vs. real interest rates:

$$1 + r^{\text{real}} = \frac{1 + r^{\text{nominal}}}{1 + i}.$$

Discounting real cash flows at the real interest rate or nominal cash flows at the nominal interest rate gives the **same present value**:

$$PV = \frac{C_t^{\text{nominal}}}{(1 + r^{\text{nominal}})^t} = \frac{C_t^{\text{nominal}}}{((1 + r^{\text{real}}) \times (1 + i))^t} = \frac{C_t^{\text{nominal}} / (1+i)^t}{(1 + r^{\text{real}})^t} = \frac{C_t^{\text{real}}}{(1 + r^{\text{real}})^t}.$$

4. **After tax cash flows and interest rates at tax rate τ:**

$$\begin{aligned} C_t^{\text{After-tax}} &= (1 - \tau) \times C_t^{\text{Before-tax}} \\ r^{\text{After-tax}} &= (1 - \tau) \times r^{\text{Before-tax}}. \end{aligned}$$

Class 3. Decision Rules

1. The **net present value (NPV)** of a project's cash flows is:

$$\text{NPV} = C_0 + \frac{C_1}{1+r} + \cdots + \frac{C_n}{(1+r)^n}$$

The NPV decision rule:

(a) In the case of a **single project**, or several **independent projects**, accept the project if and only if NPV > 0.

(b) In the case of **mutually exclusive projects**, accept the project with the highest NPV, if that NPV > 0.

2. A project's **internal rate of return (IRR)** is the interest rate that sets the NPV of the project cash flows equal to zero:

$$0 = C_0 + \frac{C_1}{(1+\text{IRR})} + \frac{C_2}{(1+\text{IRR})^2} + \cdots + \frac{C_n}{(1+\text{IRR})^n}.$$

The IRR decision rule:

- **Independent projects**: Accept a project if its IRR is greater than the cost of capital r.
- **Mutually exclusive projects**: Among the projects having IRR greater than r, accept the one with the highest IRR.

The IRR is a useful number to know even if you use the NPV decision rule: It tells you how high the cost of capital can be before the NPV goes negative.

IRR as a decision rule is less useful – at best it agrees with NPV.
But, sometimes you need to **communicate** with people who don't do NPV.
In that case, remember:

- If you look at mutually exclusive projects with different scale or length, use the **incremental cash flows IRR approach**.
- Use **modified IRR**:
  (a) To calculate the actual return on the project.
  (b) If cash flows flip sign more than once.
- If first cash in, then cash out, look for low IRR, not high.

$$\text{Modified IRR} = \left[\frac{\text{FV of cash inflows (as of date n, using assumed reinvestment rate)}}{-\text{PV of cash outflows (as of date 0, using the cost of capital)}}\right]^{1/n} - 1$$

(Can be calculated using the MIRR function in Excel.)

3. A project's **profitability index** is the ratio of the net present value to the constrained resource consumed:

$$\text{PI} = \frac{\text{NPV}}{\text{Constrained resource consumed}}$$

Useful tool to maximize overall NPV from a set of projects. This will be better than just picking projects in order of decreasing NPV.

4. Other criteria are used, but should not be:
   - Payback period: The number of years it takes the project to pay back the initial investment.
   - "Best-case" cash flows and discount rate above the cost of capital.

Class 4. Capital budgeting

1. **Cash flow formulas:**

$$
\begin{aligned}
\text{Free cash flow, } C_t = \quad & \underbrace{(1-\tau_c) \times (\text{Revenues}_t - \text{Costs}_t - \text{Depreciation}_t)}_{\text{Unlevered net income}_t} \\
+ \quad & \text{Depreciation}_t \\
- \quad & \text{Net capital expenditure}_t \\
- \quad & \text{Increase in operating working capital from t-1 to t}
\end{aligned}
$$

   - Operating working capital = Inventory + Accounts Receivable − Accounts Payable
   - Increase in operating working capital from t-1 to t
     = Operating working capital t − Operating working capital t-1
   - Sale of capital enters as negative capital expenditure, and remember taxes:

     $$\text{After-tax cash flow from asset sale} = \text{Sale price} - \tau \times (\text{Sale price} - \text{Book value})$$

     where the book value is the amount of the purchase price that has not yet been depreciated.
   - ($\text{Revenues}_t - \text{Costs}_t - \text{Depreciation}_t$) is called EBIT. If EBIT is negative in a particular year, you should still multiply by $(1-\tau)$ if the firm has sufficient profits from other projects. If not, you'll need to deal with tax carrybacks/carryforwards.

Class 6. Stock Valuation

1. **Dividend discount model:** The price of a stock is the PV of its expected future dividends

$$P_0 = \Sigma_{t=1}^{\infty} \frac{DIV_t}{(1+r)^t}.$$

2. **Dividend discount model with constant growth (forever):**

$$P_0 = \frac{DIV_1}{r-g}$$

Under the simplifying assumptions that (a) the number of shares is constant (no issues or repurchases), (b) the firm has no debt, and (c) absent any new investments, earnings would be constant over time:

- $DIV_1 =$ Dividend payout rate $\times$ EPS_1
- $g =$ Retention rate $\times$ Return on new investment
- Dividend payout rate $+$ Retention rate $= 1$

3. **Total payout model:**

$$P_0 = \frac{\text{PV(Future total dividends and repurchases)}}{\text{Shares outstanding at time 0}}$$

where:

- PV(Future total dividends and repurchases) $= \frac{\text{Total dividends and repurchases at time 1}}{r-g}$
- $g =$ Retention rate $\times$ Return on new investment
- Dividend payout rate $+$ Retention rate $+$ Repurchase rate $= 1$

4. **Free cash flow model:**

$$P_0 = \frac{\text{Enterprise value}_0 + \text{Cash}_0 - \text{Debt}_0}{\text{Shares outstanding}_0}$$

Enterprise value$_0$ is the PV of the free cash flows. To calculate enterprise value make sure to use the firm's overall cost of capital (the **weighted-average cost of capital**), not just it's equity cost of capital (more on this later in the class).
Note that from the above equation:

$$\text{Enterprise value}_0 = (P_0 \times \text{Shares outstanding}_0) + \text{Debt}_0 - \text{Cash}_0$$

5. **Multiples valuation:** Using the multiple Enterprise value/Free cash flows as an example, the general approach to estimating the enterprise value of a particular firm (Teuer in our case) using a multiples approach is:

$$\text{Enterprise value}_{2012}^{\text{Teuer}} = \text{Free cash flows}_{2012}^{\text{Teuer}} \times \left(\frac{\text{Enterprise value}}{\text{Free cash flows}}\right)_{2012}^{\text{Comparable firms}}$$

Class 7. Bonds, Term Structure, and Interest Rate Risk

- F: Face value of bond, paid at maturity, in \$
- B_n: Price today of an n-year zero coupon bond
- y_n: Yield today of an n-year zero coupon bond
- C: Coupon payment, in \$
- c: Coupon rate, in percent
- P_n: Price today of an n-year coupon bond
- y_n^{coupon}: Yield today of an n-year coupon bond.

1. **Price of a n-year zero coupon bond** with face value F:

$$B_n = \frac{F}{(1+y_n)^n}.$$

This formula also defines the yield to maturity on the n-year zero coupon bond, y_n, as the (constant) discount rate that equates the discounted cash flow to the price. Reorganizing:

$$y_n = \left(\frac{F}{B_n}\right)^{1/n} - 1.$$

2. **n-year coupon bond with annual coupon payments**:
   - The annual coupon payment is: $C = F \times c$
   - Price and yield are related by:

$$P_n = \frac{C}{(1+y_n^{coupon})} + \frac{C}{(1+y_n^{coupon})^2} + ... + \frac{C}{(1+y_n^{coupon})^{n-1}} + \frac{C+F}{(1+y_n^{coupon})^n}.$$

3. **n-year coupon bond with semi-annual coupon payments**:
   - The coupon payment every 6 months is: $C = F \times \frac{c}{2}$
   - Price and yield are related by:

$$P_n = \frac{C}{\left(1+\frac{y_n^{coupon}}{2}\right)} + \frac{C}{\left(1+\frac{y_n^{coupon}}{2}\right)^2} + ... + \frac{C+F}{\left(1+\frac{y_n^{coupon}}{2}\right)^{2\times n}}.$$

4. Relationship between **coupon bond prices and zero coupon bond prices** (with annual coupon payments):

$$P_n = \frac{C}{(1+y_1)} + \frac{C}{(1+y_2)^2} + ... + \frac{C}{(1+y_{n-1})^{n-1}} + \frac{C+F}{(1+y_n)^n}$$

or expressed directly in terms of the zero-coupon bond prices

$$P_n = C \times \frac{B_1}{100} + C \times \frac{B_2}{100} + ... + C \times \frac{B_{n-1}}{100} + (C+F) \times \frac{B_n}{100}.$$

5. **PVs of riskless cash flow streams (when the term structure is not necessarily flat):**

$$NPV = C_0 + C_1 \times \frac{B_1}{100} + ... + C_n \times \frac{B_n}{100} \quad \text{Method 1: Using prices of zero coupon bonds}$$

$$NPV = C_0 + \frac{C_1}{1+y_1} + ... + \frac{C_n}{(1+y_n)^n} \quad \text{Method 2: Using yields of zero coupon bonds}$$

6. Relationship between **prices of zero coupon bonds and forward interest rates**:

$$f_n = \frac{B_{n-1}}{B_n} - 1.$$

- f_n: Forward interest rate for year n (which goes from time $n-1$ to time n)
- B_n: Today's price of a n-year zero coupon bond
- B_{n-1}: Today's price of a $n-1$-year zero coupon bond

Relationships between **yields on zero coupon bonds and forward interest rates**:

$$f_n = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}} - 1$$

- y_n: Today's yield of a n-year zero coupon bond
- y_{n-1}: Today's yield of a $n-1$-year zero coupon bond.

7. For bonds with **default risk**:

$$Price = \frac{E(C_1)}{(1+E(r))} + \frac{E(C_2)}{(1+E(r))^2} + ... + \frac{E(C_n)+E(F)}{(1+E(r))^n}$$

$$Price = \frac{C_1^{promised}}{(1+y)} + \frac{C_2^{promised}}{(1+y)^2} + ... + \frac{C_n^{promised} + F^{promised}}{(1+y)^n}$$

Class 8. Diversification

1. **A single asset**

   Expected return: $E(r) = \Sigma_{j=1}^{n} p_j \times r_j$

   Variance of return: $\sigma^2 = E([r - E(r)]^2) = \Sigma_{j=1}^{n} p_j \times [r_j - E(r)]^2$

   Standard deviation of return: $\sigma = \sqrt{\sigma^2(r)}$.

2. **Portfolios of two risky assets, A and B**

   Covariance of returns:

$$\text{cov}(r_A, r_B) = \sigma_{A,B} = E[(r_A - E(r_A))(r_B - E(r_B))] = \sum_{j=1}^{n} p_j \times (r_{A,j} - E(r_A))(r_{B,j} - E(r_B))$$

   Return (realized return): $r_p = x_A r_A + x_B r_B$, where $x_B = 1 - x_A$.

   Expected return: $E(r_p) = x_A E(r_A) + x_B E(r_B)$

   Variance of return:

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_{A,B} = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \rho_{A,B} \sigma_A \sigma_B.$$

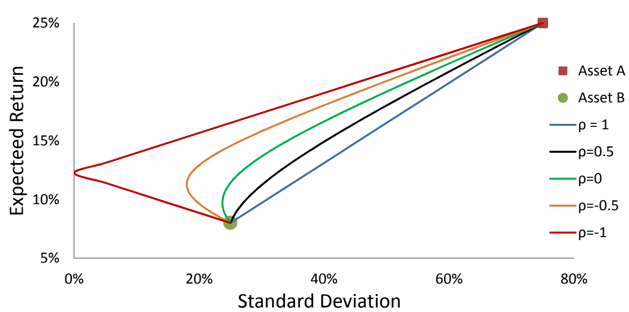
   Standard deviation of return: $\sigma_p = \sqrt{\sigma_p^2}$.

- For $\rho_{A,B} = 1$ there is is **no diversification benefit**. Then

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_A \sigma_B = (x_A \sigma_A + x_B \sigma_B)^2, \text{ and } \sigma_p = x_A \sigma_A + x_B \sigma_B$$

  so the portfolio standard deviation is simply the weighted average of the standard deviations of the two assets.

- For $\rho_{A,B} < 1$ there is a **diversification benefit**. The portfolio standard deviation is now less than the weighted average of the standard deviations of the two assets. Thus by diversifying you can get less risk for the same expected return or higher expected return for the same risk.



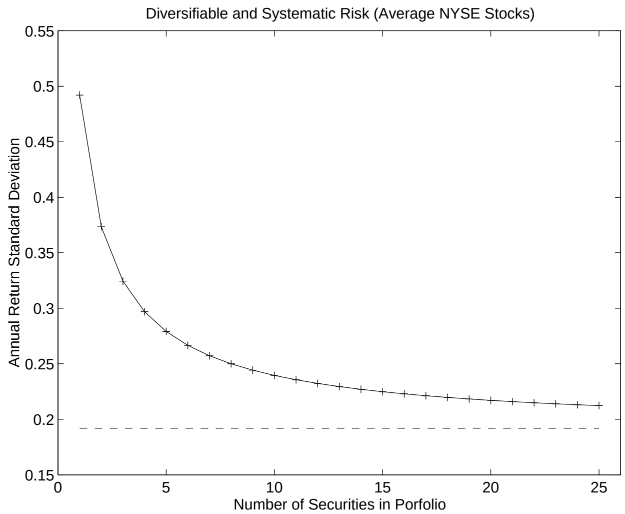
3. **Equal-weighted portfolios of many risky assets:**

   Equal weighted portfolio of n risky assets. Each portfolio share is $1/n$.

   - If all assets have the same σ and all pairs of assets are equally correlated with correlation ρ, then

$$\sigma_p^2 \;=\; \left(\frac{1}{n}\right)\sigma^2 + \left(\frac{n-1}{n}\right)\rho\sigma^2 \rightarrow \rho\sigma^2 \qquad\qquad \text{as n gets large.}$$

Graphing $\sigma_p = \sqrt{\sigma_p^2}$ against n illustrates the role of ρ:



- If all assets don't have the same σ and all pairs of assets are not equally correlated, then:

$$\sigma_p^2 = \left(\frac{1}{n}\right)\overline{\sigma^2} + \left(\frac{n-1}{n}\right)\overline{cov}$$

where $\overline{\sigma^2}$ is the average variance of the n assets and $\overline{cov}$ is the average covariance of the n assets. As n becomes large

$$\left(\frac{1}{n}\right) \rightarrow 0,\ \left(\frac{n-1}{n}\right) \rightarrow 1,\ \text{so } \sigma_p^2 \rightarrow \overline{cov}.$$

For very well diversified portfolios of risky assets, portfolio risk is determined only by the average covariance of the assets in the portfolio, not by the variances of the assets.

Class 9. Optimal portfolio choice

1. **Portfolios of a risky asset (A) and a riskless asset (f).**

   Return (realized return): $r_p = xr_A + (1-x)\, r_f$

   Expected return: $E(r_p) = xE(r_A) + (1-x)\, r_f = r_f + x\,(E(r_A) - r_f)$

   Variance of return: $\sigma_p^2 = x^2\sigma_A^2$

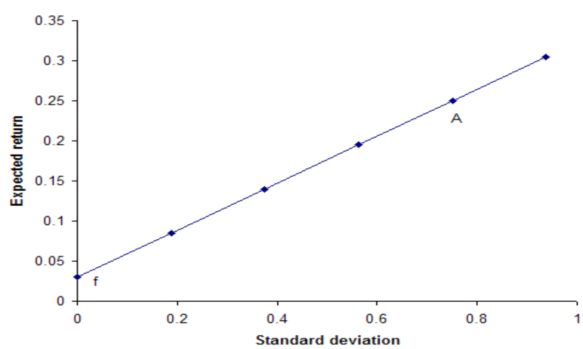
   Standard deviation of return: $\sigma_p = x\sigma_A$.

   If you are willing to tolerate portfolio risk of σ_p, choose a portfolio share for the risky asset of $x_A = \frac{\sigma_p}{\sigma_A}$. If you do this, your portfolio's expected return will be

$$\begin{aligned} E(r_p) &= r_f + \frac{\sigma_p}{\sigma_A}\,(E(r_A) - r_f) \\ &= r_f + \frac{E(r_A) - r_f}{\sigma_A}\sigma_p : \ \textbf{Capital Allocation Line (CAL)}. \end{aligned}$$

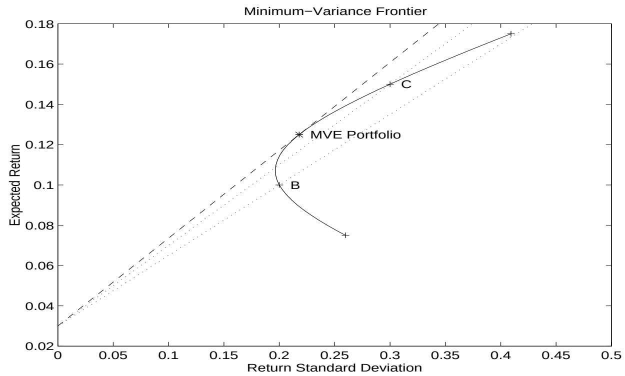
   Intercept: The riskless interest rate. Slope: Sharpe ratio of the risky asset.

   The CAL gives the expected return on a portfolio invested in a combination of a riskless asset and a risky asset, for each possible chosen value of the portfolio standard deviation.



2. **Optimal portfolio choice with a riskless asset and two or more risky assets**

   - For any choice of risky asset portfolio we can draw the CAL resulting from combining this risky asset portfolio with holdings of the riskless asset.
   - The best portfolio of risky assets with which to combine the riskless asset is the one which leads to the steepest CAL (less risk for same expected return/higher expected return for same risk).
   - The best portfolio of risky assets is called the **mean-variance efficient (MVE)** portfolio of risky assets, or the tangency portfolio.



- The resulting CAL is the "**optimal CAL**". The expression for the optimal CAL is

$$E\left(r_p\right) = r_f + \frac{E\left(r_{MVE}\right) - r_f}{\sigma_{MVE}} \sigma_p.$$

- **Efficient portfolios** lie on the optimal CAL and combine f and MVE.
- **Punch line: Two funds are enough** - All (smart) investors should choose an efficient portfolio, i.e. a combination of the riskless asset and the MVE portfolio of risky assets. The exact point chosen depends on how **risk averse** the investor is. Investors should control the risk of their portfolio not by reallocating among risky assets, but through the split between risky and riskless assets.
- **Formulas for efficient portfolios:**

Return (realized return): $r_p = x_{MVE} r_{MVE} + x_f r_f$
Expected return: $E(r_p) = x_{MVE} \cdot E(r_{MVE}) + x_f \cdot r_f = r_f + x_{MVE} \cdot (E(r_{MVE}) - r_f)$
Variance: $\sigma_p^2 = x_{MVE}^2 \sigma_{MVE}^2$
Standard deviation: $\sigma_p = x_{MVE} \sigma_{MVE}$.

- In the case of two risky assets, you can find the MVE portfolio of risky assets using Solver in Excel (see spreadsheet posted for example).
- Adding more risky assets improves the minimum variance frontier of risky assets. We do not solve for the MVE in cases with more than two risky assets, but once you have the MVE portfolio of risky assets, everything works as in the case with two risky assets (and a riskless asset).

Class 10. The CAPM

- **Equilibrium:** Asset prices must adjust to ensure that

MVE portfolio of risky assets=Market portfolio of risky assets.

- **Capital market line (CML):**

When investors invest in a combination of the riskless asset and the market portfolio, their portfolio will be on the **capital market line (CML)**.

The CML is the capital allocation line CAL for which the risky asset portfolio is the market portfolio. Therefore, on the CML:

$$\begin{aligned} r_p &= r_f + x_m \left(r_m - r_f\right) \ , \\ E\left(r_p\right) &= r_f + x_m \left(E\left(r_m\right) - r_f\right), \\ \sigma_p &= x_m \sigma_m \end{aligned}$$

so, combining these

$$E\left(r_p\right) = r_f + \frac{E\left(r_m\right) - r_f}{\sigma_m} \sigma_p.$$

- **Capital asset pricing model (CAPM):**

A model of what the expected return on a security i will be in equilibrium. According to the CAPM

$$E\left(r_i\right) = r_f + \beta_i \left[E\left(r_m\right) - r_f\right]$$

β **is a regression coefficient** in this regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t}$$

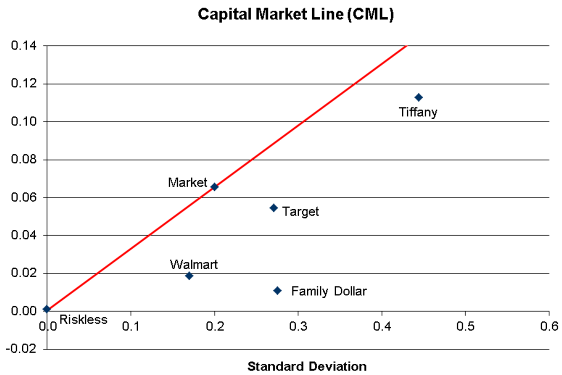
$$\beta_i = \frac{cov(r_{i,t} - r_{f,t}, r_{m,t} - r_{f,t})}{V(r_{m,t} - r_{f,t})}$$

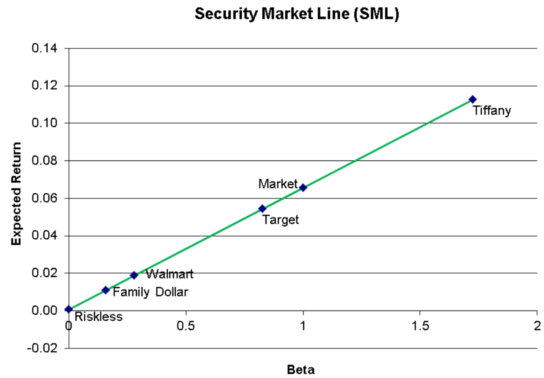
If the expected return on any asset i differed from the above, the market portfolio would not be mean-variance efficient and the supply of risky assets would not equal the demand for risky assets.

- **Security market line (SML):**

Illustrates the CAPM by plotting the formula $E\left(r_i\right) = r_f + \beta_i \left[E\left(r_m\right) - r_f\right]$ to show that a security's expected return depends linearly on its β.

- **Comparing the CML and SML:**





- **Decomposing realized (excess) returns for security i:**

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

  - $\beta_i(r_{m,t} - r_{f,t})$: The undiversifiable (=systematic=market) component. How well you'd expect security i to do this period (in terms of it's excess return) given the market's excess return in this period.
  - $\epsilon_{i,t}$: The part of $r_{i,t} - r_{f,t}$ that is not explained by $r_{m,t} - r_{f,t}$. The security-specific component. Across stocks, $\epsilon_{i,t}$ averages to zero for each t.

- **Decomposing (excess) return variance:**

$$\underbrace{\mathrm{Var}(r_{i,t} - r_{f,t})}_{\text{Total Risk, } \sigma_i^2} = \underbrace{\mathrm{Var}(\beta_i(r_{m,t} - r_{f,t}))}_{\text{Systematic Risk, } \beta_i^2\sigma_m^2} + \underbrace{\mathrm{Var}(\varepsilon_{i,t})}_{\text{Firm-specific Risk, } \sigma_{i,\varepsilon}^2}$$

  - The R^2 of the regression: The fraction of the total variance of in a security's excess return can be explained by movements in the market excess return.

$$R_i^2 = \frac{\text{Explained Variance}}{\text{Total Variance}} = \frac{\text{Systematic Risk}}{\text{Total Risk}} = \frac{\beta_i^2\sigma_m^2}{\beta_i^2\sigma_m^2 + \sigma_{i,\varepsilon}^2}$$

Class 11. The cost of capital for equity financed projects

The capital budgeting process involves the following steps:

1. Find a set of **comparable firms** whose business consists of projects similar to yours, and which are publicly traded so you can get data for them.

   - Trade-off: Balance between using a lot of comparables (to get more precise statistical estimates) and using a few but very good comparable (whose business is very close to the project's).

   - Include your own firm if the project is similar to your existing business and your firm is publicly traded.

   - If you cannot find other good comparables, and the project is similar to your existing business, you may decide to only use your own firm.

2. Estimate the **equity betas** of these comparable firms using regressions. Also estimate **debt betas** if the firms have enough debt that it is risky.

3. For each firm, i, calculate the **asset beta** or firm beta, using the equity betas, debt betas, and capital structure:

$$\beta_A^i = \frac{E^i}{V^i}\beta_E^i + \frac{D^i}{V^i}\beta_D^i.$$

   where

$$V^i = E^i + D^i, \qquad E^i/V^i = \frac{1}{1 + D^i/E^i}, \qquad D^i/V^i = \frac{D^i/E^i}{1 + D^i/E^i}.$$

   E^i is the market value of equity for firm i: E =Price per share$\times$Number of shares.
   D^i is the market value of debt for firm i: Often approximated by the book value of debt (the principal).

4. Use an **average** of the comparable firms' asset betas as an estimate of the project beta.

$$\beta_A = \frac{1}{n}\Sigma_{i=1}^N \beta_A^i$$

5. **Plug it into the CAPM** to get the cost of capital for the project

$$\mathbb{E}(r_A) = r_f + \beta_A\left(\mathbb{E}(r_m) - r_f\right)$$

6. From this estimated project beta, calculate a discount rate using the **CAPM**, and discount the expected cash flows to get the **NPV**.

The beta of a portfolio of assets can come in handy when working with companies that have multiple divisions (or cash holdings):

$$\beta_A = \frac{V_{Div.1}}{V_{Div.1} + V_{Div.2}} \times \beta_{A,Div.1} + \frac{V_{Div.2}}{V_{Div.1} + V_{Div.2}} \times \beta_{A,Div.2}.$$

Class 12-13. Capital Structure, WACC, and APV

Modigliani and Miller laid out exactly when **capital structure is irrelevant** for the value of a project or firm. This is the case when:

- Capital structure doesn't affect the free cash flows
- Capital structure doesn't change the risk of the firm (or project)
- The firm's financing decision doesn't reveal new information about the free cash flows
- Investors and firms can trade the same set of securities at the same prices
- There are no taxes, transactions costs, or issuance costs associated with security trading.

When this set of conditions are satisfied we say that **capital markets are perfect**.

But even in perfect capital markets, capital structure does affect something: The risk of equity is increasing in the amount of debt (so is the risk of debt for sufficiently high debt).

$$\beta_A = \left(\frac{E}{V}\right)\beta_E + \left(\frac{D}{V}\right)\beta_D \quad \Longleftrightarrow \quad \beta_E = \beta_A + \frac{D}{E}(\beta_A - \beta_D)$$

$$\mathbb{E}(r_A) = \frac{E}{V}E(r_E) + \frac{D}{V}\mathbb{E}(r_D) \quad \Longleftrightarrow \quad \mathbb{E}(r_E) = \mathbb{E}(r_A) + \frac{D}{E}\left[\mathbb{E}(r_A) - \mathbb{E}(r_D)\right].$$

There are two approaches to deal with the tax shield benefit of interest on debt.

- WACC: Accounted for via discount rate. APV: Accounted for via cash flows.

WACC, the steps:

1. **Determine the free cash flows** of the investment (a firm or project) using our usual formula for free cash flow.
2. **Compute the WACC as:**

$$\begin{aligned}\mathbb{E}(r_{wacc}) &= \frac{E}{V}\mathbb{E}(r_E) + \frac{D}{V}\mathbb{E}(r_D)(1-\tau_C) \\ &= \mathbb{E}(r_A) - \frac{D}{V}\mathbb{E}(r_D)\tau_C\end{aligned}$$

   where $V = D + E$ and $E(r_A)$ is the cost of capital for an unlevered project (or firm), also called the **"unlevered cost of capital"**.

   - For a project: $\mathbb{E}(r_A)$ is calculated as last class based on comparables and the CAPM:

   $$\mathbb{E}(r_A) = r_f + \beta_{Project}\left(E(r_m) - r_f\right).$$

   - For a firm: $\mathbb{E}(r_A)$ is calculated from the firm's $\beta_A = \frac{E}{V}\beta_E + \frac{D}{V}\beta_D$ and the CAPM. Equivalently, calculate the firm's $\mathbb{E}(r_A)$ using $\mathbb{E}(r_A) = \frac{E}{V}\mathbb{E}(r_E) + \frac{D}{V}\mathbb{E}(r_D)$ where $\mathbb{E}(r_E)$ and $\mathbb{E}(r_D)$ are themselves based on the CAPM.

3. **Calculate the NPV of the investment**, including the tax benefit of debt, by discounting the free cash flows using the WACC.

$$NPV = FCF_0 + \frac{FCF_1}{1 + \mathbb{E}(r_{wacc})} + \frac{FCF_2}{(1 + \mathbb{E}(r_{wacc}))^2} + ...$$

Class 14. Market Efficiency

Market efficiency: Markets are (informationally) efficient **if prices reflect information** about cash flows and discount rates. To consider how efficient markets are we look at how much information seems to be reflected in prices.

1. **Weak Form Efficient:** Prices reflect all past price information.
   - You cannot make abnormal returns by trading on information contained in historical prices (e.g., technical analysis).

2. **Semi-Strong Form Efficient:** Prices reflect all publicly available information.
   - You cannot make abnormal returns trading on Reuters, the WSJ or fundamental analysis based on public information.

3. **Strong Form Efficient:** Prices reflect **all information** (public and private).
   - You cannot make abnormal returns trading on inside information.

Joint Hypothesis Problem: Any test of market efficiency is a joint test of both the asset pricing model (e.g., CAPM) and the efficient market hypothesis.

Testing the CAPM

1. **What does the CAPM claim?**
   - Expected returns are increasing in β_i: $E(r_{i,t}) = r_{f,t} + \beta_i [E(r_{m,t}) - r_{f,t}]$.
   - Unsystematic risk does not drive expected returns.
   - The alpha estimate should be insignificantly different from zero for any traded security.

2. **Alpha (α):** The difference between a security or portfolio's actual expected return and what the expected return should be given the risk, according to our asset pricing model. Using the CAPM as our asset pricing model:
$$\alpha_i = E(r_i) - [r_f + \beta_i (E(r_m) - r_f)]$$
So for an asset with α_i, the expected return is
$$E(r_i) = \alpha_i + r_f + \beta_i (E(r_m) - r_f).$$
You estimate the α as:
$$\alpha_i = \overline{r}_i - (\overline{r_f} + \beta_i[\overline{r}_m - \overline{r_f}])$$
Graphically: α_i is the vertical distance to the SML and the intercept in the beta regression.

   Evidence on alpha's of individual investors, mutual fund managers, and hedge fund managers suggests that markets are close to semi-strong form efficient. Only the top hedge fund managers seem to have significantly positive alpha.

3. **Multi factor models:** The Fama French Carhart 4-factor model is the leading multi-factor model. Estimate betas in this regression:
$$\begin{aligned} r_{i,t} - r_{f,t} = \alpha_i &+ \beta_{i,1} (r_{m,t} - r_{f,t}) + \beta_{i,2} (r_{small,t} - r_{big,t}) \\ &+ \beta_{i,3} (r_{high,t} - r_{low,t}) + \beta_{i,4} (r_{winners,t} - r_{losers,t}) + \varepsilon_{i,t}. \end{aligned}$$
Then calculate expected returns from:
$$\begin{aligned} \mathbb{E}(r_i) - r_f = \beta_{i,1} [\mathbb{E}(r_m - r_f)] &+ \beta_{i,2} [\mathbb{E}(r_{small} - r_{big})] \\ &+ \beta_{i,3} [\mathbb{E}(r_{high} - r_{low})] + \beta_{i,4} [\mathbb{E}(r_{winners} - r_{losers})] \end{aligned}$$