The Social Cost of Near-Rational Investment*

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Abstract

We show that the stock market may fail to aggregate information even if it appears to be efficient and that the resulting decrease in the information content of stock prices may drastically reduce welfare. We solve a macroeconomic model in which information about fundamentals is dispersed and households make small, correlated errors around their optimal investment policies. As information aggregates in the market, these errors amplify and crowd out the information content of stock prices. When stock prices reflect less information, the volatility of stock returns rises. The increase in volatility makes holding stocks unattractive, distorts the long-run level of capital accumulation, and causes costly (first-order) distortions in the long-run level of consumption.

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1 Introduction

An important function of financial markets is to aggregate information that is dispersed across market participants. Market prices should reflect the information held by countless investors and direct resources to their most efficient use. If stock prices reflect information, investors have an incentive to learn from equilibrium prices and to update their expectations accordingly. But if investors learn from equilibrium prices, *anything* that moves prices has an impact on the expectations held by *all* market participants. We explore the implications of this basic dynamic in a world in which people are less than perfect – a world in which they make small, correlated errors when investing their wealth.

We find that relaxing the rational paradigm in such a minimal way results in a drastically different equilibrium with important consequences for the functioning of financial markets, capital accumulation, and welfare: If information is dispersed across investors, the private return to making diligent investment decisions is orders of magnitude lower than the social return. If we allow for individuals to have an economically small propensity to make correlated errors in their investment decisions, information aggregation endogenously breaks down precisely when it is most socially valuable (i.e. when information is highly dispersed). This endogenous informational inefficiency results in higher equilibrium volatility of asset returns and socially costly first-order distortions in the level of capital accumulation and in the level of consumption.

Our model builds on the standard real business cycle model in which a consumption good is produced from capital and labor. Households supply labor to a representative firm and invest their wealth by trading claims to capital ('stocks') and bonds. The consumption good can be transformed into capital, and vice versa, by incurring a convex adjustment cost. The accumulation of capital is thus governed by its price relative to the consumption good (Tobin's Q). The only source of real risk in the economy are shocks to total factor productivity. We extend this standard setup by assuming that each household receives a private signal about productivity in the next period and solve for equilibrium expectations.

As a useful benchmark, we first examine two extreme cases in which the stock market has no role in aggregating information. In the first case, the private signal is perfectly accurate such that all households know next period's productivity without having to extract any information from the equilibrium price. In this case, our model is very close to the "News Shocks" model of Jaimovich and Rebelo (2009), in which all information about the future is common (everybody knows everything there is to know from the outset). The opposite extreme is the case in which the private signal is perfectly inaccurate (it contains no information at all and consists only of noise). In this case our model resembles the standard real business cycle model in which no one in the economy has any information about the future and there is consequently nothing to learn from the equilibrium stock price. We show that households face less financial risk in the former case than in the latter: The more households know about the future, the more information is reflected in the equilibrium price, and the lower is the volatility of equilibrium stock returns.

The paper centers on the more interesting case in which households' private signals are neither perfectly accurate nor perfectly inaccurate: private signals contain both information about future productivity and some idiosyncratic noise (information is dispersed). In this case, households' optimal behavior is to look at the equilibrium stock price and to use it to learn about the future. When information is dispersed, the stock market thus serves to aggregate information.

We call the equilibrium in which all households behave perfectly rationally the "rational expectations equilibrium". If households are perfectly rational in making their investment decisions, the stock market is a very effective aggregator of information: As long as the noise in the private signal is purely idiosyncratic, the equilibrium stock price becomes perfectly revealing about productivity in the next period. (This is the well-known result in Grossman (1976).) Since the equilibrium stock price in the rational expectations equilibrium reflects all information about tomorrow's productivity, the equilibrium volatility of stock returns is just as low as it is in the case in which the private signal is perfectly accurate. Loosely speaking, the level of financial risk depends on how much information is in the equilibrium stock price and not on how it got there.

We then show that the rational expectations equilibrium is unstable in the sense that the economy behaves very differently if we allow households to make small, correlated errors around their optimal investment policy. We refer to this case as the "near-rational expectations equilibrium" to emphasize that households have no or very little economic incentive to avoid such small correlated errors in their investment decisions as, by the envelope theorem, the expected utility cost accruing to an individual household due to small deviations from its optimal policy is economically small.

In the near-rational expectations equilibrium the average household is slightly too optimistic in some states of the world and slightly too pessimistic in others. Consider a state of the world in which the average household is slightly too optimistic about future productivity: If the average investor is slightly too optimistic, the stock price must rise. Households who observe this higher stock price may interpret it in one of two ways: It may either be due to errors made by their peers or, with some probability, it may reflect more positive information about future productivity received by other market participants. Rational households should thus revise their expectations of future productivity upwards whenever they see a rise in the stock price. As households revise their expectations upwards, the stock price must rise further, triggering yet another revision in expectations, and so on. Small errors in the investment decision of the average household may thus lead to much larger deviations in equilibrium stock prices. The more dispersed information is across households the stronger is this feedback effect, because households rely more heavily on the stock price when their private signal is relatively uninformative. In fact, we show that small near-rational errors in investor behavior may completely destroy the stock market's capacity to aggregate information if information is sufficiently dispersed. In other words, the stock market's ability to aggregate information is most likely to break down precisely when it is most valuable. As near-rational errors may drastically reduce the amount of information reflected in the equilibrium stock price, they result in an increase in the volatility of equilibrium stock returns, and thus in a rise in the amount of financial risk faced by households.

A large literature in behavioral finance has developed a wide range of psychologically founded mechanisms that prompt households to make correlated mistakes in their investment decisions.[1] We thus deliberately remain open to many possible interpretations of the small, correlated errors that households' make in our model. In Hassan and Mertens (2011) we give one such interpretation where households who want to insulate their investment decision from the errors made by their peers ("market sentiment") have to pay a small mental cost. In equilibrium, households then choose to make small, correlated errors of the type we assume in this paper. Regardless of the exact mechanism, the point is that marginal deviations from the optimal investment policy are (by the envelope theorem) costless in terms of utility, such that small correlated deviations from the optimal investment behavior are consistent with standard economic theory.

While households have little incentive to avoid such near-rational errors, they entail a first-order cost to society. This is easiest to see for the example of a small open economy in which households can borrow and lend at an exogenous international interest rate. Risk-averse investors demand a higher risk premium for holding stocks when stock returns are more volatile. This risk premium determines the marginal product of capital in the long run (at the stochastic steady state). In the near-rational expectations equilibrium the marginal unit of capital installed must therefore yield a higher expected return than in the rational expectations equilibrium, in order to compensate investors for the additional risk they bear. It follows that an increase in the volatility of stock returns depresses the equilibrium level of capital installed at the stochastic steady state and consequently lowers the level of output and consumption in the long run.[2] Moreover, returns to capital rise while wages fall.[3] A decrease in the informational efficiency of stock prices thus has a level effect on output and consumption at the stochastic steady state.

We calibrate our model match key macroeconomic and financial data. Our results suggest that stock prices aggregate a significant amount of dispersed information, but that much of this

[1]Some examples are Odean (1998); Odean (1999); Daniel, Hirshleifer, and Subrahmanyam (2001); Barberis, Shleifer, and Vishny (1998); Bikhchandani, Hirshleifer, and Welch (1998); Hong and Stein (1999) and Allen and Gale (2001).

[2]The stochastic steady-state is the vector of capital, bonds, and prices at which those quantities do not change in unconditional expectation.

[3]In a closed economy the fact remains that any distortion in the level of output and consumption is associated with first-order welfare losses. However, the effects are slightly more complicated (due to the precautionary savings motive), such that rises in the volatility of stock returns may drive consumption at the stochastic steady state up or down.

information is crowded out in equilibrium. In our preferred calibration the conditional variance of stock returns in the near-rational expectations equilibrium is 18% higher than in the rational expectations equilibrium.

We quantify the aggregate welfare losses attributable to near-rational behavior as the percentage rise in consumption that would make households indifferent between remaining in an equilibrium in which the volatility of stock returns is high (the near-rational expectations equilibrium) and transitioning to the stochastic steady state of an economy in which all households behave fully rationally until the end of time (the rational expectations equilibrium). We find that aggregate welfare losses attributable to near-rational behavior amount to 2.36% of lifetime consumption, while the incentive to the individual household to avoid these near-rational errors is economically small (0.01% of lifetime consumption). Almost all of the aggregate welfare losses are attributable to distortions in capital accumulation. The results for a closed economy are quantitatively and qualitatively similar.[4]

Related Literature This paper is to our knowledge the first to address the welfare costs of pathologies in information aggregation within a full-fledged dynamic stochastic general equilibrium model.

In our model, near-rational errors on the part of investors are endogenously amplified and result in a deterioration of the information content of asset prices. The equilibrium of the economy thus behaves as if irrational noise traders are de-stabilizing asset prices, although all individuals are almost perfectly rational. In this sense, our paper relates to the large literature on noisy rational expectations equilibria following Hellwig (1980) and Diamond and Verrecchia (1981), in which the aggregation of information is impeded by exogenous noise trading (or equivalently by a stochastic supply of the traded asset).[5] Relative to this literature we make progress on two dimensions. First, we are able to make statements about social welfare as the introduction of near-rational behavior puts discipline on the amount of noise in equilibrium asset prices which is consistent with the notion that the losses accruing to individual households who cause this noise must be economically small. Second, we show that a given amount of near-rational errors has a more detrimental effect on the aggregation of information when information is more dispersed.[6]

[4] An important caveat with respect to our quantitative results is that we use the standard real business cycle model as our model of the stock market. This model cannot simultaneously match the volatility of output and the volatility of stock returns. We address this issue by calibrating the model to match the volatility of stock returns observed in the data and make appropriate adjustments to our welfare calculations to ensure that they are not driven by a counterfactually high standard deviation of consumption.

[5] Most closely related are Wang (1994), where noise in asset prices arises endogenously from time-varying private investment opportunities, and Albagli (2009) where noise trader risk is endogenously amplified due to liquidity constraints on traders.

[6] The notion of near-rationality is due to Akerlof and Yellen (1985) and Mankiw (1985). Our application is closest to Cochrane (1989) and Chetty (2009) who use the utility cost of small deviations around an optimal policy to derive "economic standard errors". Other recent applications include Woodford (2005) and Dupor (2005).

The recent literature on pathologies in information aggregation in financial markets has focused on information externalities arising either from strategic complementarities or from higher order uncertainty: Amador and Weill (2007) and Goldstein et al. (2009) study models in which individuals have an incentive to over-weight public information relative to private information due to a strategic complementarity. In their models errors in public signals are endogenously amplified due to this over weighting. In Allen et al. (2006), Bacchetta and van Wincoop (2008), and Qiu and Wang (2010), agents have differing information sets about multi-period returns and therefore must form expectations about the expectations of others. The dynamics of these higher order expectations drive a wedge between asset prices and their fundamentals. This paper highlights a third, even more basic type of information externality which arises even when there are no strategic complementarities and asset prices are fully determined by first-order expectations:[7] Individuals do not internalize how errors in their investment decisions affect the equilibrium expectations of others. Pathologies similar to those outlined in this paper are thus likely to arise in any setting in which households observe asset prices which aggregate dispersed information.

We also contribute to a large literature that studies the welfare cost of pathologies in stock markets, including Stein (1987) and Lansing (2008). Most closely related are DeLong, Shleifer, Summers, and Waldmann (1989) who analyze the general equilibrium effects of noise-trader risk in an overlapping generations model with endogenous capital accumulation. A large literature in macroeconomics and in corporate finance focuses on the sensitivity of firms' investment to a given mispricing in the stock market. Some representative papers in this area are Blanchard, Rhee, and Summers (1993); Baker, Stein, and Wurgler (2003); Gilchrist, Himmelberg, and Huberman (2005); and Farhi and Panageas (2006).[8] One conclusion from this literature is that investment responds only moderately to mispricings in the stock market or that the stock market is a "sideshow" with respect to the real economy (Morck, Shleifer, and Vishny (1990)). We provide a new perspective on this finding: In our model welfare losses are driven mainly by a distortion in the stochastic steady state rather than by an intertemporal misallocation of capital. The observed sensitivity of the capital stock with respect to stock prices may therefore be uninformative about the welfare consequences of highly volatile stock returns. Pathologies in the stock market may thus have large welfare consequences even if the stock market appears to be a "sideshow".

This finding also relates to a large literature on the costs of business cycles:[9] First, we emphasize that macroeconomic fluctuations affect the *level* of consumption if they create financial risk. This level effect is not captured in standard cost-of-business cycles calculations in the spirit

[7] The provision of public information thus always raises welfare in our framework (see Appendix C).

[8] Also see Galeotti and Schiantarelli (1994); Polk and Sapienza (2003); Panageas (2005); and Chirinko and Schaller (2006)

[9] See Barlevy (2004) for an excellent survey.

of Lucas (1987).[10] Second, our model suggests that this level effect may cause economically large welfare losses if macroeconomic fluctuations are indeed responsible for the large amounts of financial risk which we observe in the data.

At a methodological level, an important difference to existing work is that our model requires solving for equilibrium expectations under dispersed information in a non-linear general equilibrium framework. While there is a large body of general equilibrium models with dispersed information, existing models feature policy functions which are (log) linear in the expectation of the shocks that agents learn about (e.g. Hellwig (2005), Lorenzoni (2009), Angeletos et al. (2010), and Angeletos and La'O (2010)). However, in the standard real business cycles model with capital accumulation and decreasing returns to scale, households' policies are non-linear functions of the average expectation of future productivity. We are able solve our model due to recent advances in computational economics. We follow the solution method in Mertens (2009) which builds on Judd (1998) and Judd and Guu (2000) in using an asymptotically valid higher-order expansion in all state variables around the deterministic steady state of the model in combination with a nonlinear change of variables (Judd (2002)).[11] This paper is thus one of the first to model dispersed information within a full-fledged dynamic stochastic general equilibrium model.

In a closely related paper, Mertens (2009) derives welfare improving policies for economies in which distorted beliefs create too much volatility in asset markets. He shows that the stabilization of asset markets enhances welfare and that history-dependent policies may improve the information content of asset prices.

In the main part of the paper we concentrate on the slightly more tractable small open economy version of the model (alternatively we may think of it as a closed economy in which households have access to a certain type of storage technology). After setting up the model in section 2 we discuss equilibrium expectations and how near-rational behavior may lead to a collapse of information aggregation and to a rise in financial risk in section 3. In section 4 we build intuition for the macroeconomic implications of a rise in financial risk by presenting a simplified version of the model which allows us to show all the main results with pen and paper. In this simplified version of the model households consist of two specialized agents: a "capitalist" who has access to the stock and bond markets and a "worker" who provides labor services but is excluded from trading in the stock market. We then solve the full model computationally in section 5. Section 6 gives the results of our calibrations and also gives results for a closed economy version of the model.

[10] This finding is similar to the level effect of uninsured idiosyncratic investment risk on capital accumulation in Angeletos (2007).

[11] Closely related from a methodological persepective are Tille and van Wincoop (2008) who solve for portfolio holdings of international investors using an alternative approximation method developed in Tille and van Wincoop (2007) and Devereux and Sutherland (2008).

2 Setup of the Model

The model is a de-centralization of the standard Mendoza (1991) framework: A continuum of households work and trade in stocks and bonds. A representative firm produces a homogeneous consumption good by renting capital and labor services from households. Total factor productivity is random in every period and the firm adjusts factor demand accordingly. An investment goods sector has the ability to transform units of the consumption good into units of capital, while incurring convex adjustment costs.[12] All households and the representative firm are price takers and plan for infinite horizons.

At the beginning of each period, households receive a private signal about productivity in the next period. Given this signal and their knowledge of prices and the state of the economy, they form expectations of future returns. Households make correlated near-rational errors when forming expectations about future productivity.

2.1 Economic Environment

Technology is characterized by a linear homogeneous production function that uses capital, K_t, and labor, L as inputs

$$Y_t = e^{\eta_t} F\left(K_t, L\right), \tag{1}$$

where Y_t stands for output of the consumption good. Total factor productivity, η_t, is normally distributed with a mean of $-\frac{1}{2}\sigma_\eta^2$ and a variance of σ_η^2. The equation of motion of the capital stock is

$$K_{t+1} = K_t\left(1-\delta\right) + I_t, \tag{2}$$

where I_t denotes aggregate investment and δ is the rate of depreciation. Furthermore, there are convex adjustment costs to capital,

$$AC = \frac{1}{2}\chi\frac{I_t^2}{K_t}, \tag{3}$$

where χ is a positive constant. There is costless trade in the consumption good at the world price, which we normalize to one. All households can borrow and lend abroad at rate r. Foreign direct investment and international contracts contingent on η are not permitted.

[12] The alternative to introducing an investment goods sector is to incorporate the investment decision into the firm's problem. The two modeling devices are equivalent as long as there are no frictions in contracting between management and shareholders.

2.2 Households

There is a continuum of identical households indexed by $i \in [0,1]$. At the beginning of every period each household receives a private signal about tomorrow's productivity:

$$s_t(i) = \eta_{t+1} + \nu_t(i). \tag{4}$$

where $\nu_t(i)$, represents *i.i.d.* draws from a normal distribution with zero mean and variance σ_ν^2.[13]

Given $s_t(i)$ and their knowledge about the economy, households maximize lifetime utility by choosing an intertemporal allocation of consumption, $\{C_t(i)\}_{t=0}^{\infty}$, and by weighting their portfolios between stocks and bonds at every point in time, $\{\omega_t(i)\}_{t=0}^{\infty}$, where ω represents the share of equity in their portfolio. Formally, an individual household's problem is

$$\max_{\{C_t(i)\}_{t=0}^{\infty}, \{\omega_t(i)\}_{t=0}^{\infty}} U_t(i) = \mathcal{E}_{it} \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \log(C_s(i)) \right\} \tag{5}$$

subject to

$$W_{t+1}(i) = [(1-\omega_t(i))(1+r) + \omega_t(i)(1+\tilde{r}_{t+1})](W_t(i) + w_t L - C_t(i)) \quad \forall t, \tag{6}$$

where $\mathcal{E}_{it}$ stands for household i's conditional expectations operator, $W_t(i)$ stands for financial wealth of household i at time t and $\tilde{r}_{t+1}$ is the equilibrium return on stocks. We denote the market price of capital with Q_t and dividends with D_t:[14]

$$1 + \tilde{r}_{t+1} = \frac{Q_{t+1}(1-\delta) + D_{t+1}}{Q_t}. \tag{7}$$

Households have rational expectations, but they have a small propensity to make correlated mistakes when forming their expectation of η_{t+1}. The expectations operator $\mathcal{E}$ is thus the rational expectations operator with the only exception that the conditional probability density

[13] In appendix C we show that the conclusions of our model hold for more general information structures in which the noise in the private signal is correlated across households and households observe a public signal in addition to their private signal.

[14] We implicitly assume here that stocks split proportionally to the percentage change in aggregate capital stock at the end of each period. The stock price is then always equal to the price of a claim to one unit of capital.

function of η_{t+1} is shifted by $\tilde{\epsilon}_t$:[15]

$$\mathcal{E}_{it}\left(\eta_{t+1}\right) \equiv E_{it}\left(\eta_{t+1}\right) + \tilde{\epsilon}_t, \tag{8}$$

where, E_{it} denotes the rational expectations operator, conditional on all information available to household i at time t,

$$E_{it}\left(\cdot\right) = E\left(\cdot | Q_t, s_t(i), K_t, B_{t-1}, \eta_t\right). \tag{9}$$

For simplicity we assume that all households make the same small mistake. Alternatively, we may think of $\tilde{\epsilon}_t$ as the average mistake made by households trading in the stock market. The deviation caused by $\tilde{\epsilon}_t$ is zero in expectation and its variance, $\sigma^2_{\tilde{\epsilon}}$, is small enough such that the expected utility loss from making this mistake is below some threshold level.[16]

This formulation of near-rational errors is a reduced form of a number of microfounded mechanisms that have been suggested in the literature. In Hassan and Mertens (2011) we give our favorite interpretation in which investors decide whether or to what degree they want to allow their behavior to be influenced by "market sentiment". Investors who choose to insulate their decision from market sentiment earn higher expected returns, but incur a small mental cost. We show that if information is moderately dispersed across investors, even a very small mental cost (on the order of 0.001% of consumption) may generate a significant amount of sentiment in equilibrium. Alternatively, we may think of "animal spirits", small menu costs in the portfolio decision (Mankiw (1985)), or of some form of expectational bias as in Dumas, Kurshev, and Uppal (2006), where households falsely believe that an uninformative public signal contains a tiny amount of information about future productivity.

In order to avoid having to keep track of the wealth distribution across households, we assume that households can insure against idiosyncratic risk which is due to their private signal: At the beginning of each period (and before receiving their private signal), households can buy claims that are contingent on the state of the economy and on their individual idiosyncratic shock $\nu_t\left(i\right)$. These claims are in zero net supply and pay off at the beginning of the next period. Contingent claims trading thus completes markets between periods. In equilibrium all households thus enter each period with the same amount of wealth. In order to keep the exposition simple, we suppress the notation relating to contingent claims except for when we define the equilibrium and relegate details to Appendix A.

[15] This implies that households have the correct perception of all higher moments of the conditional distribution of η_{t+1}:

$$\mathcal{E}\left[\left(\eta_{t+1} - \mathcal{E}\left(\eta_{t+1}|s_t\left(i\right), Q_t\right)\right)^k | s_t\left(i\right), Q_t\right] = E\left[\left(\eta_{t+1} - E\left(\eta_{t+1}|s_t\left(i\right), Q_t\right)\right)^k | s_t\left(i\right), Q_t\right] \text{ for all } k \neq 1.$$

[16] More precisely, $\tilde{\epsilon}_t(i)$ has a mean of $-\frac{1}{2}\sigma^2_{\tilde{\epsilon}}$ such that agents hold the correct expectation of log returns in expectation.

2.3 Firms

A representative firm purchases capital and labor services from households. As it rents services from an existing capital stock, its maximization collapses to a period-by-period problem.[17] The firm's problem is to maximize profits

$$\max_{K_t^d, L_t^d} e^{\eta_t} F\left(K_t^d, L_t^d\right) - w_t L_t^d - D_t K_t^d, \tag{10}$$

where K_t^d and L_t^d denote factor demands for capital and labor respectively. First order conditions with respect to capital and labor pin down the fair wage and the dividend. Both factors receive their marginal product; and the fair wage is

$$e^{\eta_t} F_L\left(K_t^d, L_t^d\right) = w_t. \tag{11}$$

As the production function is linear homogeneous, the representative firm makes zero economic profits.

2.4 Investment Goods Sector

The representative firm owns an investment goods sector which converts the consumption good into units of capital, while incurring adjustment costs. It takes the price of capital as given and then performs instant arbitrage:

$$\max_{I_t} Q_t I_t - I_t - \frac{1}{2}\chi \frac{I_t^2}{K_t}, \tag{12}$$

where the first term is the revenue from selling I_t units or capital and the second and third terms are the cost of acquiring the necessary units of consumption goods (recall the price of the consumption good is normalized to one) and the adjustment costs respectively. Since there are decreasing returns to scale in converting consumption goods to capital, the investment goods sector makes positive profits in each period. Profits are paid to shareholders as a part of dividends:[18]

$$e^{\eta_t} F_K\left(K_t^d, L_t^d\right) + \frac{1}{2}\chi \frac{I_{t+1}^2}{K_{t+1}} = D_t. \tag{13}$$

[17] Note that by choosing a structure in which firms rent capital services from households, we abstract from all principal agent problems between managers and stockholders. Managers therefore cannot prevent errors in stock prices from impacting investment decisions, as in Blanchard, Rhee, and Summers (1993). On the other hand, they do not amplify shocks or overinvest as in Albuquerque and Wang (2005).

[18] Alternatively, profits may be paid to households as a lump-sum transfer; this assumption matters little for the results of the model.

Taking the first order condition of (12), gives us equilibrium investment as a function of the market price of capital:

$$I_t = \frac{K_t}{\chi}(Q_t - 1) \tag{14}$$

Whenever the market price of capital is above one, investment is positive, raising the capital stock in the following period. When it is below one the investment goods sector buys units of capital and transforms them back into the consumption good. Note that the parameter χ scales the adjustment costs and can be used to calibrate the sensitivity of capital investment with respect to the stock price.

2.5 Definition of Equilibrium

Definition 2.1

Given a time path of shocks $\{\eta_t, \tilde{\epsilon}_t, \{\nu_t(i) : i \in [0,1]\}\}_{t=0}^{\infty}$ *an equilibrium in this economy is a time path of quantities* $\{\{C_t(i), B_t(i), W_t(i), \omega_t(i), \varphi(i;n) \;:\; i \in [0,1]\}, C_t, B_t, W_t, \omega_t, K_t^d, L_t^d, Y_t, K_t, I_t\}_{t=0}^{\infty}$*,signals* $\{s_t(i) : i \in [0,1]\}_{t=0}^{\infty}$ *and prices* $\{Q_t, r, D_t, w_t, \rho_t(n)\}_{t=0}^{\infty}$ *with the following properties:*

1. $\{\{C_t(i)\}, \{\omega_t(i), \varphi(i;n)\}\}_{t=0}^{\infty}$ *solve the households' maximization problem (5) given the vector of prices, initial wealth, and the random sequences* $\{\tilde{\epsilon}_t, \{\tilde{\nu}_t(i)\}\}_{t=0}^{\infty}$*;*

2. $\{K_t^d, L_t^d\}_{t=0}^{\infty}$ *solve the representative firm's maximization problem (10) given the vector of prices;*

3. $\{I_t\}_{t=0}^{\infty}$ *is the investment goods sector's optimal policy (14) given the vector of prices;*

4. $\{w_t\}_{t=0}^{\infty}$ *clears the labor market,* $\{Q_t\}_{t=0}^{\infty}$ *clears the stock market,* $\rho_t(n)$ *clears the contingent claims market, and* $\{D_t\}_{t=0}^{\infty}$ *clears the market for capital services;*

5. *There is a perfectly elastic supply of the consumption good and of bonds in world markets. Bonds pay the rate* r *and the price of the consumption good is normalized to one;*

6. $\{Y_t\}_{t=0}^{\infty}$ *is determined by the production function (1),* $\{K_t\}_{t=0}^{\infty}$ *evolves according to (2),* $\{\{W_t(i)\}\}_{t=0}^{\infty}$ *evolve according to the budget constraints (6), and* $\{s_t(i)\}_{t=0}^{\infty}$ *is determined by (4);*

7. $\{\{B_t(i)\}, C_t, B_t, W_t, \omega_t\}_{t=0}^{\infty}$ *are given by the identities*

$$B_t(i) = (1 - \omega_t(i))\,(W_t(i) - C_t(i))\,, \tag{15}$$

$$X_t = \int_0^1 X_t(i)di \;,\;\; X = C, B, W \tag{16}$$

and

$$\omega_t = \frac{Q_t K_{t+1}}{W_t - C_t}, \tag{17}$$

where n is a realization of the vector $\left[\eta_{t+1}, \tilde{\varepsilon}_t, \nu_t\right]$, $\varphi_t\left(i;n\right)$ is the quantity of contingent the claims bought by household i that pay off one unit of consumption at time $t+1$ if state n occurs and ν_t coincides with $\nu_t(i)$, and $\rho_t\left(n\right)$ is the time t price of these claims.

The *rational expectations equilibrium* is the equilibrium in which $\sigma_{\tilde{\epsilon}} = 0$, such that the expectations operator $\mathcal{E}$ in equation (5) coincides with the rational expectation in (9). The *near-rational expectations equilibrium* posits that $\sigma_{\tilde{\epsilon}} > 0$; households make small errors around their optimal policy, as given in (8). The idea behind the near-rational expectations equilibrium is that small errors in households' policies result in minor welfare losses for the individual household. The following definition formalizes what it means for near-rational households to suffer only "economically small" losses:

Definition 2.2
A near-rational expectations equilibrium is k-percent stable if the welfare gain to an individual household of obtaining rational expectations is less than k% of consumption.

3 Equilibrium Expectations

In this section we show that if we allow for individuals to have an economically small propensity to make correlated errors in their investment decisions, information aggregation endogenously breaks down precisely when it is most socially valuable (i.e. when information is highly dispersed).

To fix ideas, let us define the error in market expectations of η_{t+1} as the difference between the average expectation held by households in the near-rational expectations equilibrium and the average expectation they would hold if $\tilde{\epsilon}_t$ happened to be zero in this period. We call the error in the market expectations

$$\epsilon_t = \gamma \tilde{\epsilon}_t$$

and solve for γ below.[19] The main insight is that the multiplier γ may be large. This amplification of errors is a result of households learning from equilibrium prices: a rise in prices causes households to revise their expectations upwards; and when households act on their revised expectations, the price rises further. Trades that are correlated with the average error made by investors thus represent an externality on other households' expectations.

[19] More formally, $\epsilon_t = \int \left(\mathcal{E}_{it}\left(\eta_{t+1}\right) + \tilde{\epsilon}_t\right) di|_{\tilde{\varepsilon}_t > 0, \sigma_{\tilde{\epsilon}} > 0} - \left[\int \mathcal{E}_{it}\left(\eta_{t+1}\right) di|_{\tilde{\epsilon}_t = 0, \sigma_{\tilde{\epsilon}} > 0}\right]$.

3.1 Solving for Expectations in General Equilibrium

In order to say more about the relationship between $\tilde{\epsilon}_t$ and ϵ_t we need to solve for equilibrium expectations. This is a challenge because our model is non-linear, and in particular because the market price of capital (Q_t) is a non-linear function of η_{t+1}. Households' optimal behavior is characterized by two Euler equations which take the form

$$\begin{gathered}\mathcal{E}_{it}\left(C_t\left[\kappa_t\left(i\right)\right]^{-1}-\beta\left[C_{t+1}\left[\kappa_{t+1}\left(i\right)\right]^{-1}\left(1+\tilde{r}_{t+1}\left[\kappa_{t+1}\left(i\right)\right]\right)\right]\right)=0\\ \mathcal{E}_{it}\left(C_t^{-1}\left[\kappa_t\left(i\right)\right]-\beta\left[C_{t+1}^{-1}\left[\kappa_t\left(i\right)\right]\right]\left(1+r\right)\right)=0,\end{gathered} \tag{18}$$

where equilibrium consumption is a function of $\kappa_t\left(i\right)=\left(K_t,B_{t-1},\eta_t,\eta_{t+1},\tilde{\varepsilon}_t,\nu_t(i)\right)$; a vector containing the state variables, the productivity shocks at time t and $t+1$, the near-rational error, and the idiosyncratic noise in the private signal.

Solving for equilibrium behavior thus poses two difficulties: First, households care about the payoff they receive from stocks and about their future consumption, but they receive information about η_{t+1}, and there is a complicated non-linear relationship between these variables. Second, households learn from Q_t about η_{t+1}, but Q_t is again a non-linear function of η_{t+1}.

We use two tricks developed in Mertens (2009) to transform (18) into a form which we can solve with standard techniques: First, we use perturbation methods to show that given the households' information sets, their conditional expectation of η_{t+1} is a sufficient statistic for their expectation of both future consumption and of future stock returns; i.e. there is a deterministic relationship between households' expectations of tomorrow's productivity and what they expect to happen in the future more generally. Moreover, K_t, B_{t-1} and η_t have no predictive power over and above the information contained in Q_t and $s_t(i)$. This reduces the problem to solving for $\int\mathcal{E}\left(\eta_{t+1}|Q_t,s_t(i)\right)di$. Second, we use a nonlinear change of variables to obtain a transformation of the equilibrium stock price which is linear in households' average expectation of tomorrow's productivity. This linear transformation, we call it $\hat{q}_t$, is a linear function of η_{t+1}, but has the same information content as Q_t (i.e. both variables span the same σ-*algebra*). The basic intuition is that Q_t is a monotonic function of η_{t+1}, such that learning from Q_t is just as good as learning from its linear transformation. Framed in terms of this $\hat{q}_t$, the equilibrium boils down to computing prices and expectations such that the following equation is satisfied:

$$\hat{q}_t=\int E\left(\eta_{t+1}|\hat{q}_t,s_t(i)\right)di+\tilde{\epsilon}_t, \tag{19}$$

where $\hat{q}_t$ is a function of the state variables and shocks known at time t. Equation (19) is the familiar linear equilibrium condition of a standard noisy rational expectations model. We can now apply standard methods to solve for equilibrium expectations in terms of $\hat{q}_t$ (Hellwig (1980)) and then transform the system back to recover the equilibrium Q_t. Technical details are given in Appendix B.1.

3.2 Amplification of Small Errors

We now obtain equilibrium expectations by solving for $\hat{q}$. As it turns out we are able to show all the main qualitative results on the aggregation of information in this linear form. In section 6, we map the solution back into its non-linear form to show the quantitative implications for the equilibrium stock price and for stock returns.

Since $\hat{q}_t$ equals the market expectation of η_{t+1} in (19), we may guess that the solution for $\hat{q}_t$ is some linear function of η_{t+1} and $\tilde{\epsilon}_t$:

$$\hat{q}_t = \pi_0 + \pi_1 \eta_{t+1} + \gamma \tilde{\epsilon}_t. \tag{20}$$

This guess formally defines the multiplier γ. Our task is to solve for the coefficients in this equation. Assuming that our guess for $\hat{q}_t$ is correct, the rational expectation of η_{t+1} given the private signal and $\hat{q}_t$ is

$$E_{it}\left(\eta_{t+1}\right) = A_0 + A_1 s_t(i) + A_2 \hat{q}_t, \tag{21}$$

where the constants A_0, A_1 and A_2 are the weights that households give to the prior, the private signal and the market price of capital respectively. We get market expectations by adding the near-rational error and summing up across households. Combining this expression with our guess (20) yields

$$\int E\left(\eta_{t+1} | \hat{q}_t, s_t(i)\right) di + \tilde{\epsilon}_t = \left(A_0 + A_2 \pi_0\right) + \left(A_1 + A_2 \pi_1\right) \eta_{t+1} + A_2 \gamma \tilde{\epsilon}_t + \tilde{\epsilon}_t, \tag{22}$$

where we have used the fact that $\int s_t(i) di = \eta_{t+1}$. This expression reflects all the different ways in which $\tilde{\epsilon}_t$ affects market expectations: The last term on the right hand side is the direct effect of the near-rational error on individual expectations. If we introduced a fully rational household into the economy and gave it the same private signal as one of the near-rational households, the two households' expectations of η_{t+1} would differ exactly by $\tilde{\epsilon}_t$. The third term on the right hand side represents the deviation in market expectations that results from the fact that the market price transmits the average error as well as information about future fundamentals. The extent of this amplification depends on how much weight the market price has in the rational expectation (21) and on how sensitive $\hat{q}_t$ is to $\tilde{\varepsilon}_t$ in (20). Finally, the second term on the right hand side tells us that the mere fact that households make near-rational errors may reduce the extent to which the market can predict η_{t+1} by changing the coefficients A_1 and A_2.

Plugging (22) into (19) and matching coefficients with (19) allows us to show solve for the amplification of $\tilde{\epsilon}_t$:

Proposition 3.1
Through its effect on the market price of capital, the near-rational error, $\tilde{\epsilon}_t$, feeds back into the rational expectation of η_{t+1}. The more weight households place on the market price of capital when forming their expectations about η_{t+1}, the larger is the error in market expectations relative to $\tilde{\epsilon}_t$. We have that

$$\gamma = \frac{1}{1 - A_2}. \tag{23}$$

Proof. See appendix B. ■

It follows that the larger the weight on the market price of capital in the rational expectation, A_2, the larger is the variance in ϵ_t relative to the variance in $\tilde{\epsilon}$. Small, near-rational errors may thus generate large deviations in the equilibrium price, if households rely heavily on the market price of capital when forming their expectations about the future.

The same matching coefficients algorithm also gives us the coefficient determining the amount of information reflected in the market price of capital: $\pi_1 = \frac{A_1}{1-A_2}$. We can solve for the weights A_1, A_2 by applying the projection theorem. With explicit solutions in hand, we can show that:

Proposition 3.2
The absolute amount of information aggregated in the stock price decreases with $\sigma_{\tilde{\epsilon}}$,

$$\frac{\partial \pi_1}{\partial \sigma_{\tilde{\epsilon}}} < 0$$

Proof. See appendix B. ■

While near-rational errors amplify and lead to potentially large deviations in the stock price, they simultaneously hamper the capacity of the stock market to transmit and aggregate information. The conditional variance of η_{t+1} in the near-rational expectations equilibrium therefore exceeds the conditional variance in the rational expectations equilibrium for two reasons: First, because the stock price becomes noisy and second because it contains less information about the future.[20]

When information is highly dispersed in the economy, households rely relatively more on the stock price when forming their expectations. But when households pay a lot of attention to the stock price (A_2 is large), near-rational errors are amplified most, and the information content of prices is most vulnerable to near-rational behavior. The following proposition takes this insight to its logical conclusion:

Proposition 3.3
For any given level of $\sigma_{\tilde{\epsilon}}$, the noise to signal ratio in the market price of capital becomes arbitrarily

[20] See Appendix B.4 for an analytical solution for the conditional variance of η_{t+1}.

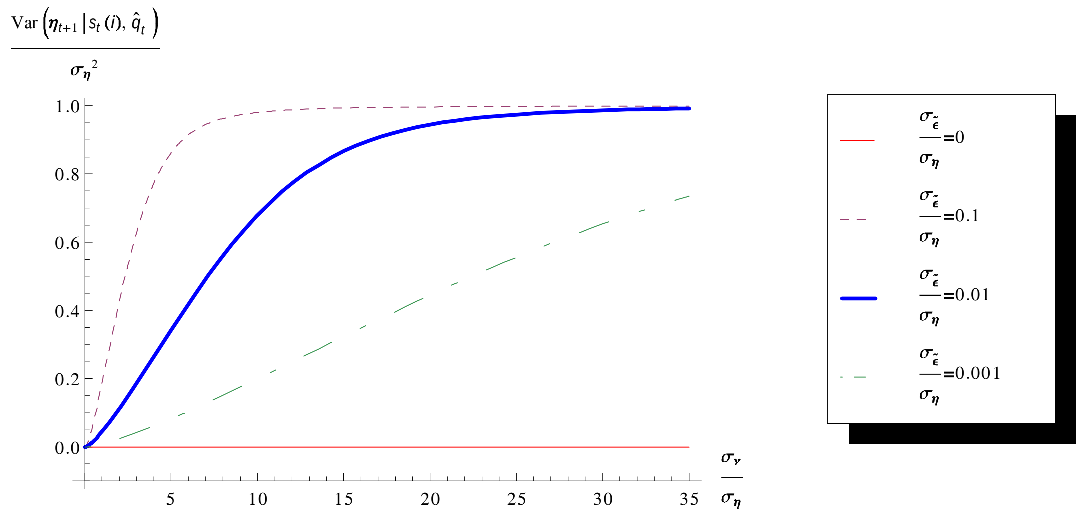

Figure 1: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information, σ_ν/σ_η.

large as the precision of the private signal goes to zero,

$$\lim_{\sigma_\nu \to \infty} \frac{\sqrt{var\left(\gamma \tilde{\epsilon}_t\right)}}{\sqrt{var\left(\pi_1 \eta_{t+1}\right)}} = \infty.$$

Proof. See appendix B. ■

As information becomes more dispersed across households, the private signal becomes less informative relative to the stock price. Households adjust by paying relatively more attention to the public signal. If households put less weight on their private signal, less information enters the equilibrium price; and the more attention they pay to the market price, the larger is the amplification of $\tilde{\epsilon}_t$. Both effects result in a rising noise to signal ratio in equilibrium stock prices. The implication of this finding is that information aggregation in financial markets is most likely to break down precisely when it is most socially valuable – when information is highly dispersed. If the private signal received by households is sufficiently noisy, *any given amount of near-rational errors* in investor behavior may completely destroy the market's capacity to aggregate information.

Figure 1 illustrates this point. It plots the ratio of the conditional variance of the productivity shock to its unconditional variance over the level of dispersion of information. To facilitate the interpretation of the results, we scale all standard deviations with the amount of real risk in the economy, σ_η. With this scaling all standard deviations have a natural interpretation. In particular, the ratio $(\frac{\sigma_\nu}{\sigma_\eta})^2$ measures the level of dispersion of information in the economy as the number of individuals who, in the absence of a market price, would need to pool their private information to reduce the conditional variance of η_{t+1} by one half. A value of zero on the vertical axis indicates that households can perfectly predict tomorrow's realization of η_{t+1}, whereas a value of 1 indicates that η_{t+1} is completely unpredictable from the perspective of a household in the economy. The thick blue line shows that in the rational expectations equilibrium ($\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_\eta} = 0$), productivity is perfectly predictable, regardless of how dispersed information is in the economy. If all households are perfectly rational, the conditional variance of η_{t+1} is always zero, because the market price of capital perfectly aggregates the information in the economy. This situation changes drastically when $\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_\eta} > 0$: The solid line plots the results for the case in which the standard deviation of the near-rational error is 1% of the standard deviation of the productivity shock. The curve rises steeply and quickly converges to one. When information is highly dispersed and we allow for near-rational behavior, the aggregation of information collapses.

The implication of Proposition 3.3 is that this qualitative result does not depend on *how* near-rational households are. Figure 1 plots the results for near-rational errors that are an order of magnitude larger ($\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_\eta} = 0.1$) and an order of magnitude smaller ($\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_\eta} = 0.001$) for comparison. In each case, the productivity shock becomes completely unpredictable if information is sufficiently dispersed.

One important feature of such a breakdown in the aggregation of information is that it affects everyone in the economy: If we placed a fully rational household into our economy, this fully rational household would do only a marginally better job at predicting η_{t+1} than a near rational household: Conditional on receiving the same private signal, the difference in the expectation of the a rational and a near-rational household is $\tilde{\varepsilon}$. In fact, the conditional variance we plotted in Figure 1, is the conditional variance of η_{t+1} from the perspective of such a fully rational household. We can write it as

$$\frac{Var\left(\eta_{t+1}|s_t\left(i\right),\hat{q}_t\right)}{\sigma_\eta^2} = \frac{1}{\sigma_\eta^2}\left(A_1^2\sigma_\nu^2 + \left(1-\pi_t\right)^2\sigma_\eta^2 + \left(\gamma-1\right)^2\sigma_{\tilde{\varepsilon}}^2\right), \tag{24}$$

where the same conditional variance from the perspective of a near-rational households is identical, except that the third term in brackets is then $\gamma^2\sigma_{\tilde{\varepsilon}}^2$.

Figure 2 decomposes the conditional variance into its three components. The solid line in Figure 2 is the same as the solid line in Figure 1, it plots the ratio of the conditional variance of the productivity shock to its unconditional variance over the level of dispersion of information for the case in which $\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_\eta} = 0.01$. The dotted line plots the first term on the right hand side of (24), which is the error that households make in their forecast of η_{t+1} due to the error in their private signal. It is close to zero throughout, reflecting the fact that households downweight their private signal when it contains more noise, such that differences of opinion remain small in equilibrium. The broken green line plots the second term, which is the error that households

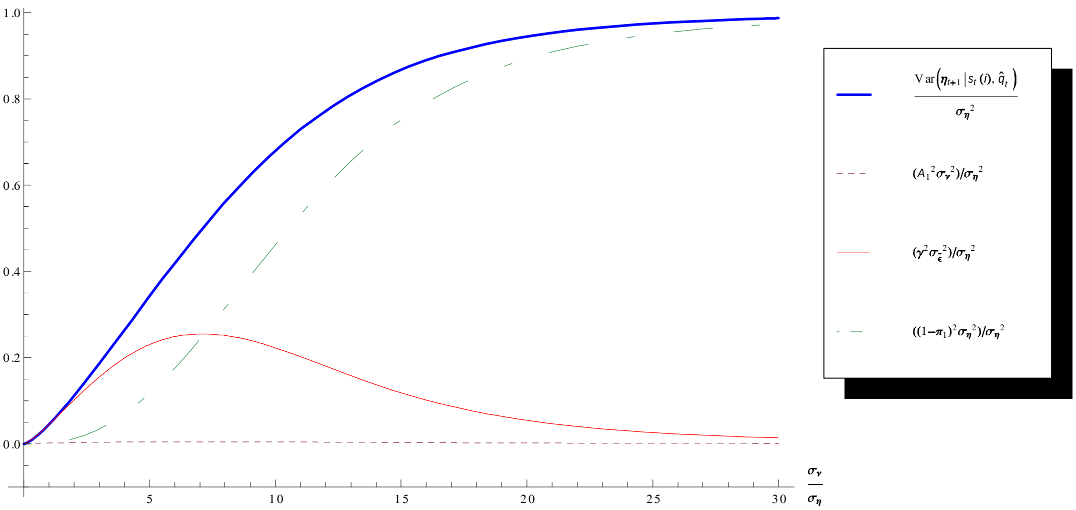

Figure 2: Decomposition of the ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information, σ_ν/σ_η.

make in their forecast because the stock price does not reflect all information about η_{t+1}, and the third component is the error that they make due to amplified near-rational errors in the stock price.

At low levels of σ_ν, amplified near-rational errors are the main source of households' forecast errors. As information becomes more dispersed, the amplification rises and eventually peaks as households, confronted with noisy private signals and a noisy stock price begin to rely more on their priors. At the same time, the information content of the stock price begins to fall rapidly. In the region in which the broken line approaches one, small near-rational errors result in a complete collapse of information aggregation.

Finally, note the basic logic of these results is not particular to the exact information structure we choose. For example, we may think of a situation in which the noise in the private signal is correlated across agents, such that $\int s_t(i)di \neq \eta_{t+1}$, in which case the stock price would not be fully revealing in the rational expectations equilibrium (the solid red line in Figure 1 and the intercept of the other lines would shift upwards as shown Appendix Figure 8); or we may think of a situation in which households receive a public as well as a private signal about η_{t+1}, in which case the information contained in the public signal would survive in the near-rational expectations equilibrium (the solid and broken lines in Figure 1 would converge a value less than one as shown Appendix Figure 7). In each case, near-rational errors impede the aggregation of the part of the information which is dispersed across households. The information externality we highlight here is thus relevant whenever financial markets play an important role in aggregating dispersed information, regardless of the exact information structure.

Moreover, note that aggregate noise in the private signal as well as noise in a public signal are conceptually very different from the near-rational errors which we introduce in this paper. An increase in the noise of a public signal increases the *amount* of information that is available to be aggregated; an increase in aggregate noise in the private signal decreases the *amount* of information that is available to be aggregated; whereas an increase in near-rational behavior reduces the *quality* of information aggregation given the amount of information that is available to be aggregated. In this sense near-rational behavior has implications which are similar to the implications of noise trader risk, with the important difference of course that near-rational behavior endogenously determines the quality of information aggregation whereas noise trader risk represents an exogenous assumption about the quality of information aggregation. (See Appendix C for details.)

Now that we understand the aggregation of information in our model we can ask how near-rational behavior impacts the economy as a whole. Intuitively, the less information is reflected in the stock price, the higher is the conditional variance of stock returns and the more financial risk households face in equilibrium. It follows that the conditional variance of stock returns must be strictly higher in the near-rational expectations equilibrium than in the rational expectations equilibrium. For the purposes of our discussion below, we define this difference in financial risk as “excess volatility”:

Definition 3.4
Excess volatility in stock returns is the percentage amount by which the conditional standard deviation of stock returns in the near-rational expectations equilibrium, σ, exceeds the conditional standard deviation of stock returns in the rational expectations equilibrium, σ^*,

$$\frac{\sigma - \sigma^*}{\sigma} 100.$$

The amount of excess volatility in stock returns that may arise due to near-rational errors depends on the non-linearities of the model. Before we turn to quantifying these effects we first build some intuition for the impact that this particular pathology in financial markets may have on the macroeconomy.

4 Intuition: The Macroeconomic Effects of Financial Risk

In this section we turn to the effect that near-rational behavior has on the macroeconomic equilibrium. To provide a maximum of intuition for the mechanisms at work, this section focuses on a simplified version of the model for which we are able to derive the main results analytically. In section 6 we show computationally that the relevant implications of the simplified model carry over to the full model.

Assume that households consist of two specialized agents, a "capitalist" who trades in the stock and bond markets and a "worker" who provides labor services, receives wages and the profits from the investment goods sector, but is excluded from trading in financial markets. This division eliminates labor income from the capitalist's portfolio choice problem such that we can solve it with pen and paper. A capitalist's budget constraint is

$$W_{t+1}(i) = ((1-\omega_t(i))(1+r) + \omega_t(i)(1+\tilde{r}_{t+1}))(W_t(i) - C_t(i)) \quad \forall t. \tag{25}$$

Taking as given that the distribution of equilibrium asset returns is approximately log-normal (this is true to a first-order approximation), we can solve for the capitalist's optimal consumption and portfolio allocation:[21]

Lemma 4.1
Capitalists' optimal consumption is a constant fraction of financial wealth

$$C_t(i) = (1-\beta)W_t(i) \tag{26}$$

and the optimal portfolio share of stocks is the expected excess return divided by the conditional variance of stock returns, σ^2

$$\omega_t(i) = \frac{\mathcal{E}_{it}\left(1+\tilde{r}_{t+1}\right) - (1+r)}{\sigma^2}. \tag{27}$$

Proof. Appendix D gives a detailed derivation which proceeds analogous to Samuelson (1969). ■

In this version of the model, only the capitalist, rather than the entire household, makes small mistakes as defined in (8) when investing in the stock market. The stock market clears when the value of shares demanded equals the value of shares in circulation:

$$\int_0^1 \beta \frac{\mathcal{E}_{it}(1+\tilde{r}_{t+1}) - (1+r)}{\sigma^2} W_t(i) di = Q_t K_{t+1}. \tag{28}$$

It is this condition that links the stock market to the real economy. We can apply the definition (7), as well as (26) and use the fact that all capitalists hold the same beginning of period wealth in equilibrium to get

$$\int_0^1 \mathcal{E}_{it}\left(\frac{Q_{t+1}(1-\delta) + D_{t+1}}{Q_t}\right) di = 1 + r + \omega_t \sigma^2, \tag{29}$$

where ω_t is defined in equation (17) and represents the aggregate degree of leverage required in

[21] We require approximate log-normality for the analytical solution below but not for the computational results.

order to finance the domestic capital stock. In equilibrium, the average capitalist holds a share ω_t of her wealth in stocks. The left hand side of (29) is the market expectation of stock returns; the right hand side is the required return that investors demand given the risk that they are exposed to. The equity premium, $\omega_t \sigma^2$, rises with the conditional variance of stock returns and with the amount of leverage required to hold the domestic capital stock.

Any error in aggregate expectations has two important channels through which it affects the real side of the model. First, it causes a temporary misallocation of capital by distorting Q_t and aggregate investment (14). Second, a rise in the conditional variance of stock returns raises the equity premium and with it the expected dividend demanded by capitalists in general equilibrium. While the former channel mainly influences the dynamics of the model, the latter channel has a direct effect on the stochastic steady state. We discuss each in turn.

4.1 Distortion of Capital Accumulation

Definition 4.2
The stochastic steady-state is the level of capital, bonds, and prices at which those quantities do not change in unconditional expectation.

In the simplified version of the model we are able to obtain a closed form solution for the stochastic steady state and thus analytically show the following result:

Proposition 4.3
The equilibrium has a unique stochastic steady state iff $\beta \leq \frac{1}{1+r}$. At the stochastic steady state the aggregate degree of leverage is

$$\omega_o = \sqrt{\frac{1}{\sigma^2}\left(\frac{1-\beta}{\beta} - r\right)}; \tag{30}$$

and the stochastic steady state capital stock is characterized by

$$(1 + \delta\chi)\left(r + \omega_o \sigma^2 + \delta\right) = F_K\left(K_o, L\right). \tag{31}$$

Proof. See Appendix E. ■

The intuition for the first result is simple: If the time discount factor is larger than $\frac{1}{1+r}$, investors are so patient that even those holding a perfectly riskless portfolio containing only bonds would accumulate wealth indefinitely. In that case, no stochastic steady state can exist. However, if $\beta \leq \frac{1}{1+r}$, there exists a unique value ω_o at which the average capitalist has an expected portfolio return that exactly matches her time discount factor: $\beta = \left(1 + r + \omega_o^2 \sigma^2\right)^{-1}$. At this value, there is no expected growth in consumption and the economy is at its stochastic steady state.[22]

The second result, (31), follows directly from applying the steady state to equation (29). On the left hand side, $1+\delta\chi$ is the market price of a unit of capital at the stochastic steady state. This is multiplied with the required return to capital: the risk free rate plus the equity premium and the rate of depreciation. At the stochastic steady state, the required return on one unit of capital must equal the expected dividend, which is precisely the expected marginal product of capital (on the right hand side of the equation). This brings us to one of the main results of this paper:

Proposition 4.4
A rise in the conditional variance of stock returns unambiguously depresses the stochastic steady state level of capital stock and output.

$$\frac{\partial K_o}{\partial \sigma} < 0$$

Proof. We use (30) to eliminate ω_o in (31) and take the total differential, see Appendix E for details. ■

The higher the risk of investing in stocks, the higher is the risk premium demanded by capitalists. A higher risk premium implies higher dividends at the stochastic steady state and, with a neoclassical production function, a lower level of capital stock. The conditional variance of stock returns thus has a *level* effect on the amount of capital accumulated at the stochastic steady state; and less installed capital in turns implies lower production.

Interestingly, this level effect may operate even if the stock market seems to have little influence on the allocation of capital in the economy:

Corollary 4.5
A rise in the conditional variance of stock returns depresses the stochastic steady state level of output even if the sensitivity of the capital stock with respect to stock prices is low.

Proof. From (14) we have that $\frac{\partial(I_t/K_t)}{\partial Q_t} = \frac{1}{\chi}$. The sensitivity of physical investment as a share of the existing capital stock with respect to the stock price is fully determined by the adjustment cost parameter χ. From (30) and (31) we have that $\frac{\partial^2 F_K(K_o,L)}{\partial\sigma^2\partial\chi} = \delta\sqrt{\frac{1}{\sigma^2}\left(\frac{1-\beta}{\beta}-r\right)} > 0$. ■

If the adjustment cost parameter χ is sufficiently large, the stock market in this economy may appear as a "sideshow" (Morck, Shleifer, and Vishny (1990)) in the sense that a given change in the stock price has little influence on investment. To the casual observer it may therefore seem as though pathologies in the stock market should not have much influence on the real economy. However, a low responsiveness of physical investment to the stock price is uninformative about

[22] Conversely we can determine the wealth of our economy relative to the value of its capital stock at the stochastic steady state by choosing an appropriate time discount factor. We shall make use of this feature when we calibrate the model in section 5.

the impact that excess volatility has on the stochastic steady state. Excess volatility in stock returns may cause a large depression of output at the stochastic steady state while leaving virtually no evidence to the econometrician. Since our model does not exempt replacement investments from capital adjustment costs, the impact of an incremental rise in stock market volatility on the stochastic steady state level of capital actually *rises* with χ, implying that excess volatility may actually have a larger effect on the stochastic steady state in economies in which the stock market appears to be a "sideshow".

Finally, the volatility of stock returns has an important implication for the distribution of income in the economy:

Corollary 4.6
A rise in the conditional variance of stock returns unambiguously lowers wages and raises dividends at the stochastic steady state.

Proof. The result follows directly from (13), (11) and proposition 4.4. ■

Excess volatility may paradoxically raise the incomes of stock market investors: At lower levels of K, dividends rise relative to wages, increasing the return to each unit of capital. Over some range, such a rationing raises the total payments to capital. As the conditional variance of stock returns rises, it pushes the economy towards higher dividends, compensating capital for the loss of aggregate output at the expense of payments to labor.

4.2 Dynamics of the Model

Solving the dynamics of the model requires a computational algorithm that we discuss in section 5. However, we can gain some intuition from the simplified version of our model. Equations (2), (13), (14), (29), and the standard transversality condition jointly determine the market price of capital. Every vector of state variables and shocks is therefore associated with a unique stock price.

Regardless of initial conditions, the economy transitions to a unique stochastic steady state in expectation. To understand this, imagine an economy that is at its stochastic steady state and receives a positive productivity shock. Capitalists will save a fraction of the currently high dividends and are now on average richer than they were before. This implies that the aggregate portfolio share required to finance the domestic capital stock in the following period falls, $\omega_{o+1} < \omega_o$. As capitalists are now less leveraged, they require a lower risk premium for the next period. Expected returns therefore tend to be lower following a positive shock and higher following a negative shock. Equilibrium returns thus exhibit negative autocorrelation and thereby generate stationary dynamics and a unique ergodic distribution.[23]

[23] There is a large body of literature discussing the non-stationarity of small open economy models (see for

In the rational expectations equilibrium, the market price of capital reflects households' knowledge about productivity in the next period. In the near-rational expectations equilibrium, the market price contains amplified noise and less information about the future, resulting in potentially large errors in market expectations about future productivity. These errors in market expectations increase the conditional variance of stock returns in the near-rational expectations equilibrium relative to the rational expectations equilibrium. Near-rational behavior thus results in an increase in financial risk and a depression in the stochastic steady state level of capital accumulation and output. Moreover, each error in market expectations passes into physical investment through the arbitrage performed by the investment goods sector, causing a temporary misallocation of capital.

To summarize, the near-rational expectations equilibrium of the simplified version of our model exhibits a higher volatility of returns around a lower stochastic steady state level of capital and output. Expected returns to capital are higher and expected wages are lower than in the rational expectations equilibrium. As we show below, these conclusions carry over to the full version of the model.

5 Quantifying Welfare Cost

In this section we return to the full version of our model and quantify the welfare cost of near-rational behavior. To this end, we first derive a standard welfare metric, based on a simple experiment in which near-rational behavior is purged from financial markets and the economy transitions to the stochastic steady state of the rational expectations equilibrium. We then briefly describe the computational algorithm used to solve this problem and calibrate the model to the data.

5.1 Welfare Calculations

Consider an economy that is at the stochastic steady state of the near-rational expectations equilibrium and suppose that at time 0, there is a credible announcement that all households henceforth commit to fully rational behavior until the end of time. Immediately after the announcement, the conditional variance of stock returns falls and households require a lower risk-premium for holding stocks. The stochastic steady state levels of capital and output rise. Although the economy does not jump to the new stochastic steady state immediately, it accumulates capital over time and converges to it in expectation. Over the adjustment process,

example Schmitt-Grohé and Uribe (2003)). The issue of non-stationarity is, however, a consequence of the linearization techniques typically employed to solve these models and *not* an inherent feature of the small open economy setup. Since we solve our model using higher order expansions we obtain stationary dynamics. See also Coerdacier et al. (2011).

output rises, wages rise and returns to capital fall. The level of consumption increases; and due to the reduction in uncertainty about future productivity the variance of consumption may fall as well.

Formally, we ask by what fraction λ we would have to raise the average household's consumption in order to make it indifferent between remaining in the near-rational expectations equilibrium and transitioning to the stochastic steady state of the rational expectations equilibrium. λ then indicates the magnitude of the welfare loss attributable to near-rational behavior as a fraction of lifetime consumption. It is defined as

$$E \int_0^1 \sum_{t=0}^{\infty} \beta^t \log\left((1+\lambda) C_t(i)\right) di \equiv E \int_0^1 \sum_{t=0}^{\infty} \beta^t \log\left(C_t^*(i)\right) di, \tag{32}$$

where we denote variables pertaining to the rational expectations equilibrium with an asterisk. From (32) we can see that welfare losses may result either from a lower *level* of consumption or from a higher *volatility* of consumption. In appendix F we decompose λ into two components, λ^{Δ}, measures the change in welfare due to a change in the level of consumption and λ^{σ} measures the change in welfare due to a change in the volatility of consumption, where $1+\lambda = \left(1+\lambda^{\Delta}\right)\left(1+\lambda^{\sigma}\right)$.

5.2 Numerical Solution

The numerical solution of our model employs perturbation methods in combination with a non-linear change of variables. It proceeds in three stages. First, we expand the conditions of optimality around the deterministic steady state. Second, we employ the non-linear change of variables described in section 3.1 in order to bring the equilibrium conditions of the model into a form which allows us to solve for conditional expectations in closed form. Finally, we make a natural guess for the equilibrium price function, solve for conditional expectations taking equilibrium prices as given, and verify the validity of the guess as described in section 3.2.

For the first step, we obtain the two conditions of optimality (18) and plug in for the households' budget constraint, stock returns, optimal investment, wages and dividends. Ultimately, we obtain two functions of known and unknown state variables and shocks K_t, B_{t-1}, K_t, $\int \mathcal{E}_{it}\left(\eta_{t+1}\right) di$, and $\nu_t(i)$ which characterize the optimal behavior of the individual.

We solve the Euler equations (18) for the optimal policies, impose market clearing, and solve for the deterministic steady state of the model. We then begin with a higher-order expansion in state variables and shocks around this point.

The crucial step which gets us back to a stochastic economy is to build at least a second-order expansion in the standard deviation of η and in the standard deviation of the conditional expectation of η. Financial risk thus affects the economy through the second moments of shocks. We use a fourth order expansion to generate the results below. All variances and covariances reported are calculated at the deterministic steady state of the system by analytically integrating over the second order expansion. For details on perturbation methods see Judd (2002).

5.3 Calibration

An important caveat with respect to our quantitative results is that we use the standard real business cycle model as our model of the stock market. A well known issue with this model is that it cannot simultaneously match the volatility of output and the volatility of asset prices. Rather than complicating the analysis by incorporating one of the standard remedies for this issue (such as habit formation, long-run risk, or rare disasters) into our model, we side-step the issue by choosing σ_η to match the standard deviation of stock returns in the data and then adjust our welfare calculations to ensure that they are not driven by a counterfactually high standard deviation of consumption.[24]

In our preferred calibration we set the standard deviation of $\tilde{\epsilon}$ to a very low level as to ensure that the losses of individual households due to their near-rational errors remain economically small; we set $\frac{\sigma_{\tilde{\epsilon}}}{\sigma_\eta} = 0.01$. We choose an adjustment cost parameter of $\chi = 2$, a risk free rate of $r = 0.04$, and a rate of depreciation of $\delta = 0.15$. We pick the time discount factor β such that the entire capital stock is owned by domestic households at the stochastic steady state of the near-rational expectations equilibrium, $\omega_o = 1$. Finally, we choose a Cobb-Douglas production technology with a capital share of $\frac{1}{3}$. Since our economy is scale independent, we normalize labor supply to one without loss of generality. Finally, we choose σ_η to match the conditional standard deviation of stock returns in the data for our preferred calibration at which $\frac{\sigma_\nu}{\sigma_\eta} = 7.6$. We begin by presenting comparative statics with respect to $\frac{\sigma_\nu}{\sigma_\eta}$ and then calibrate it to match various moments in the data that reflect the information content of stock prices.

6 Results

Figure 3 relates the equilibrium level of financial risk to the level of dispersion of information in the economy. The solid red line plots the conditional standard deviation of stock returns in the rational expectations equilibrium (σ^*). The line is perfectly horizontal, as the stock market perfectly aggregates the information held by all households regardless of how dispersed information is in the economy. However, the fact that the stock price is perfectly informative about η_{t+1} does not mean that households do not face financial risk. In the next period, they

[24] We suspect that our results would look very similar if we introduced habit formation (Campbell and Cochrane (1999)) or if instead of learning about productivity shocks, households learned about growth rates (Bansal and Yaron (2004)) or about disaster probabilities (Barro (2009), Gabaix (2010), and Gourio (2010))

learn about η_{t+2}, and the stock price adjusts to this information such that stock returns remain uncertain. This is why the solid line intercepts the vertical axis at a positive value. Learning about tomorrow's productivity can reduce the variance of stock returns to 0.14, but not to zero. The solid upward sloping line gives the conditional variance of stock returns in the near-rational expectations equilibrium (σ). When information is dispersed and we allow for near-rational errors, the aggregation of information in the economy deteriorates and eventually collapses. The result is a higher volatility of stock returns and thus more financial risk. If information is highly dispersed the stock price becomes completely uninformative about the future and σ converges to 0.20.

The conditional standard deviation of stock returns is the standard deviation of stock returns from the perspective of a household who knows the state variables of the economy K_t, B_{t-1}, η_t, and extracts information about future productivity from Q_t and $s_t(i)$. Figure 3 also plots the unconditional standard deviation of stock returns which is not conditional on any information about future productivity (i.e. from the perspective of a household that knows the K_t, B_{t-1}, η_t, but does not receive a private signal and does not know the equilibrium stock price). In the rational expectations equilibrium the conditional and unconditional standard deviation are identical, because all information is common in equilibrium and there remain no differences of opinion about tomorrow's productivity. The equilibrium stock price thus always adjusts such that the expected stock return equals the return required by investors. The dashed line in Figure 3 is the unconditional standard deviation of stock returns in the near-rational expectations equilibrium. It is almost identical with the conditional standard deviation as even in the near-rational expectations equilibrium differences of opinion remain small regardless of the level of dispersion of information in the economy (compare this to Figure 2 where the term $A_1^2\sigma_\nu^2$ remains small throughout).

The vertical distance between the two solid lines in Figure 3 reflects the amount of excess volatility in stock returns which is attributable to near-rational behavior. Figure 4 plots this excess volatility as a percentage amount. It shows that if information is highly dispersed, the conditional standard deviation of stock returns is up to 28.1% lower in the rational expectations equilibrium than in the near-rational expectations equilibrium. Moreover, it is striking that even relatively moderate levels of dispersion of information make the stock market vulnerable to near-rational behavior. At $\frac{\sigma_\nu}{\sigma_\eta} = 5$ near-rational errors result in excess volatility of 13% although a mere $\left(\frac{\sigma_\nu}{\sigma_\eta}\right)^2 = 25$ private signals contain enough information to reduce the conditional variance of η_{t+1} by half.

An even more striking result of our calibration is that the welfare cost of near-rational behavior is very large, even for moderate levels of excess volatility in stock returns. The solid line in Figure 5 plots the compensating variation for households over a range of σ_ν/σ_η. For our preferred calibration in which $\sigma_\nu/\sigma_\eta = 7.6$ excess volatility in stock returns is 18.38% and

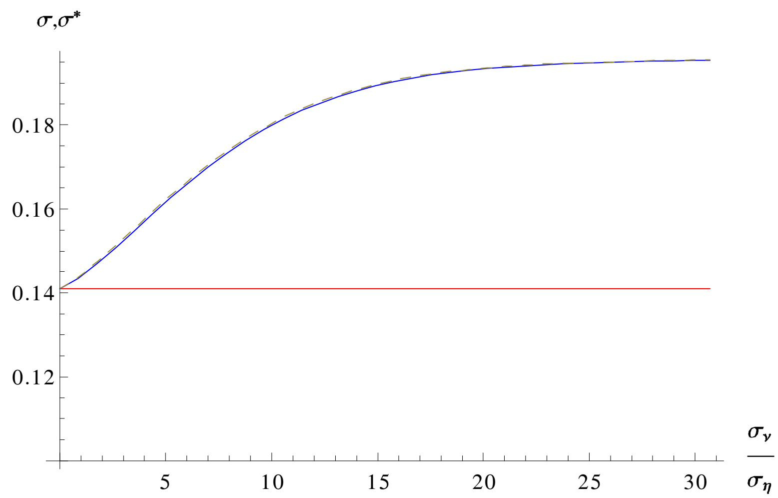

Figure 3: Solid line: Conditional standard deviation of stock returns in the near-rational expectations equilibrium. Dashed line: Unconditional standard deviation of stock returns in the near-rational expectations equilibrium. Thick line: Conditional and unconditional standard deviation of stock returns in the rational expectations equilibrium.

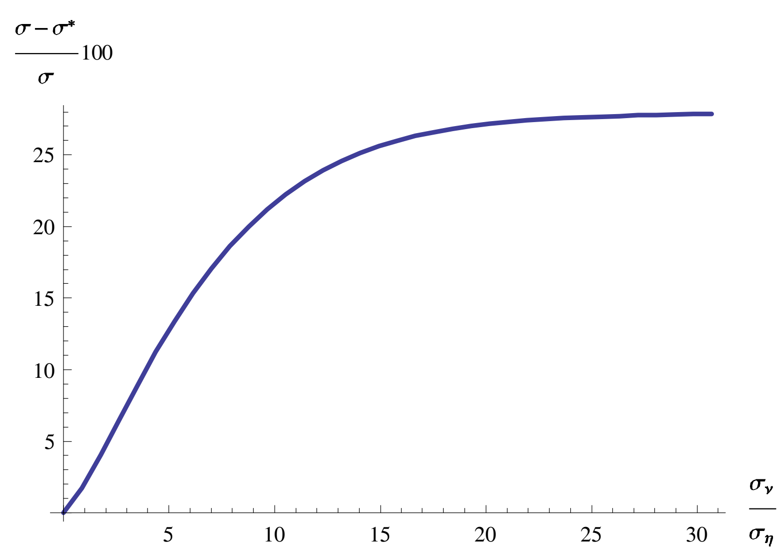

Figure 4: Excess volatility in stock returns for $\sigma_{\tilde{\varepsilon}}/\sigma_\eta = 0.01$ plotted over a range of σ_ν/σ_η.

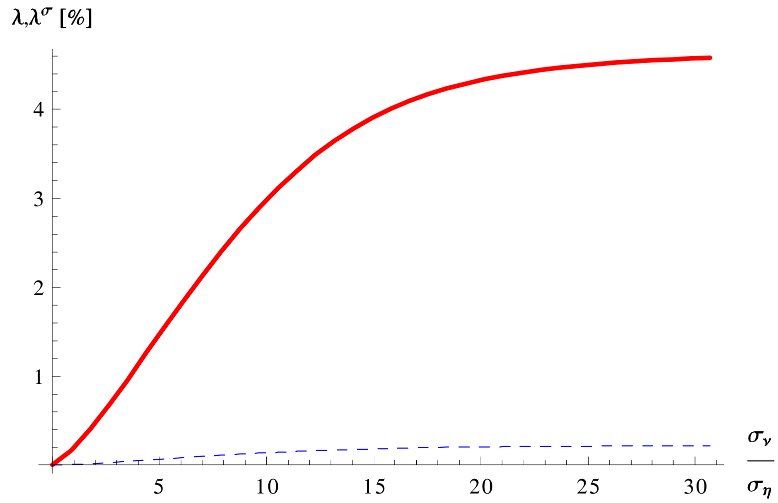

Figure 5: Solid line: Compensating variation for eliminating all present and future near-rational errors from the behavior of all households and transitioning to the stochastic steady state of the rational expectations equilibrium. Dashed line: Upper bound for the amount of the total compensating variation that could be attributable to a higher volatility of consumption in the near-rational expectations equilibrium versus the rational expectations equilibrium

the compensating variation amounts to 2.32% of consumption. At higher levels of dispersion of information welfare losses are even higher and reach up to 4.65% of consumption in the extreme case in which information is infinitely dispersed.

Households would thus be willing to give up a significant fraction of their consumption if they could get all other households in the economy to behave fully rationally. In contrast, individual households stand almost nothing to gain by eliminating near-rational errors from their own investment behavior. Figure 6 plots the compensating variation for eliminating near-rational errors from an individual household's behavior. The potential gain from behaving fully rather than near-rationally is uniformly less than 0.015% of consumption and thus at least two orders of magnitude lower than the social gain.

Another interesting result is that while the social gain of eliminating near-rational errors in Figure 5 rises monotonically, the individual gain peaks around the levels of dispersion of information at which the amplification of the near-rational error is largest (compare to the solid red line in Figure 2), and then falls off towards zero. When information is highly dispersed the social gain from eliminating near-rational behavior is largest, while the private gain becomes economically infinitessimal (in the limit it reaches 0.001% of consumption). The reason is that as the information content of stock prices falls and investors learn less and less about future productivity a hypothetical rational household would also be less and less able to hedge against suspected near-rational errors. If information is highly dispersed even an infinitesimally small

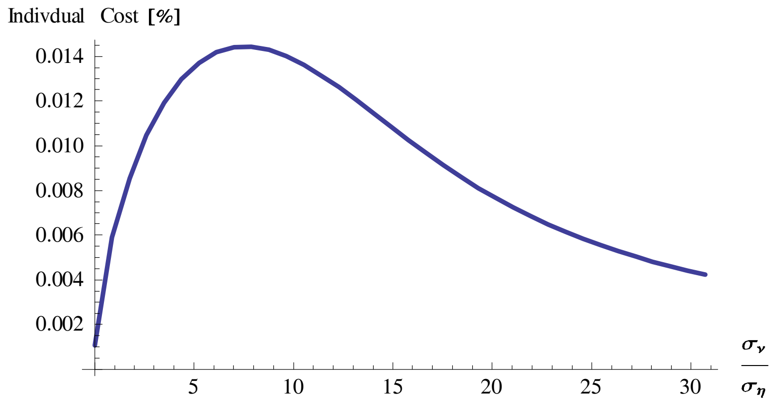

Figure 6: Compensating variation for eliminating near-rational errors from an indvidual household's behavior.

propensity on the part of households to make correlated mistakes in their investment decisions will thus cause large aggregate welfare losses.

Our estimates for the welfare losses attributable to near-rational behavior exceed even relatively high estimates of the costs of business cycles (see for example Alvarez and Jermann (2005)). The reason is that standard costs of business cycles calculations consider the welfare cost of fluctuations around a given level of consumption, and these are typically small (Lucas (1987)). In our model, near-rational behavior affects both the volatility and the level of consumption. The dashed line in Figure 5 shows an upper bound for the share of the overall welfare costs that could be attributable to a higher volatility of consumption in the near-rational expectations equilibrium versus the rational expectations equilibrium, λ^{σ}. (It is the willingness to pay of the average household for eliminating all of the variability in consumption which is due to productivity shocks and near-rational errors, while keeping the path and the level of capital accumulation the same as in the near-rational expectations equilibrium.) Throughout, this upper bound is less than 0.2% of consumption, indicating that the vast majority of the welfare loss caused by near-rational behavior is attributable to the distortions it causes in the level of consumption.

We now impose discipline on the two remaining free parameter in our calibration, σ_η and σ_ν, by using them to match key financial and macroeconomic data. Column 1 gives the four moments of the data which we attempt to match. In our preferred calibration we choose σ_η to match the conditional standard deviation of stock returns, and choose σ_ν to match the correlation of stock price growth with GDP growth one year ahead, the standard deviation of the average forecast of GDP growth, the standard deviation of the average forecast of GDP growth, and the average forecast error of GDP growth (the latter two moments are normalized with the standard deviation of (1600 HP filtered) GDP). All moments are taken from US data as they are not readily available for other countries. Details are in Appendix G.

Column 2 gives our preferred calibration in which the standard deviation of the error in the private signal is 7.6 times the standard deviation of the productivity shock (this implies that in the 57.76 private signals contain enough information to reduce the conditional variance of the aggregate shocks driving US stock returns by half). Columns 3 and 4 contrast it with two limiting cases in which the stock market has no role in the aggregation of information in the economy. The calibration in column 3 is the case in which the private signal is perfectly accurate ($\frac{\sigma_\nu}{\sigma_\eta} = 0$) such that all households know next period's productivity without having to extract any information from the equilibrium stock price (we call this the "News Shocks" calibration). The calibration in column 4 gives the other extreme in which the private signal is perfectly inaccurate ($\frac{\sigma_\nu}{\sigma_\eta} = \infty$) such that no one in the economy has any information about the future and there is consequently nothing to learn from the equilibrium stock price (we call this the "RBC" calibration). Column 5 gives the results for the rational expectations equilibrium. It uses our preferred calibration and imposes perfectly rational behavior ($\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_\eta} = 0$).

In our preferred calibration we match the correlation of stock price growth with GDP growth perfectly, and come close to matching the two other moments: The standard deviation of the average forecast comes in slightly too low (0.69 rather than 0.73 in the data), and the standard deviation of the average forecast error comes in slightly too high (0.72 rather than 0.59 in the data). The last three lines show that our preferred calibration implies an excess volatility of 18.38% of the conditional standard deviation of stock returns, an aggregate welfare loss attributable to near-rational errors of 2.32% of consumption, and an individual welfare loss from near-rational behavior of 0.014% of lifetime consumption.

By construction, the standard deviation of stock returns is lower in the News Shocks calibration, and all three variables reflecting the information content of stock prices reflect (almost) perfect aggregation of information, as households are perfectly informed about future productivity from the outset. In this case, near-rational behavior is of almost no consequence: Excess volatility is minimal (0.01) as near-rational errors are not amplified when the stock market has no role in aggregating information.

In the RBC calibration the conditional standard deviation of stock returns is higher than in the near-rational expectations equilibrium and all three variables reflecting the information content of stock prices reflect no aggregation of information, as households have no information about the future in the first place. For example, stock price growth and future GDP growth are almost completely uncorrelated.

Table 1

	(1)	(2)	(3)	(4)	(5)
	Data	N-REE	"News Shocks"	"RBC"	REE
Near-rational error, $\frac{\sigma_{\tilde{\varepsilon}}}{\sigma_{\eta}}$		0.01	0.01	0.01	0
Dispersion of Information, $\frac{\sigma_{\nu}}{\sigma_{\eta}}$		7.6	0	∞	7.6
Conditional standard deviation of stock returns, σ	≈ 0.17	0.17	0.14	0.2	0.14
Corr. (growth in stock prices, future GDP growth)	≈ 0.67	0.67	0.99	0.01	0.99
Std.(average forecast of GDP)/Std.(GDP)	0.73	0.69	1.00	0.01	1.00
Std.(average forecast error of GDP)/Std.(GDP)	0.59	0.72	0.00	1.00	0.00
Excess Volatility, $\frac{\sigma-\sigma^*}{\sigma}$		18.38	0.01	28.1	
Welfare loss attributable to near-rational errors, λ		2.32	0.00	4.65	
Individual cost of near-rational behavior		0.014	0.001	0.001	

In summary, our calibration suggests that stock markets do aggregate some information about the future, but that near-rational behavior may crowd out much of this information and result in an economically significant rise in financial risk as well as large aggregate welfare losses.

6.1 Closed Economy

In the closed economy version of our model the interest rate r becomes an endogenous variable and bonds are in zero net supply, $B_t = 0$. The dynamics of the model are slightly more involved in the closed economy case as the capital stock at the stochastic steady state of the rational expectations equilibrium may be either higher or lower. This is due to the precautionary savings motive which may or may not dominate the effect of a higher risk premium. Nevertheless the basic economic intuition holds: Any distortion in capital accumulation causes a distortion in the level of consumption; and any distortion in the level of consumption causes first-order welfare losses.

We calibrate the closed economy version to the parameters of our preferred calibration above. The compensating variation for eliminating near-rational behavior in this specification is 2.44% of consumption, which is very close to the result we get for the small open economy.

7 Conclusion

This paper showed that financial markets may fail to aggregate information even if they appear to be efficient (in the sense that all economic actors are arbitrarily close to their optimal behavior) and that a decrease in the information content of asset prices may drastically reduce welfare. In our model, each household has some information about future productivity. If all households behave perfectly rationally, the equilibrium stock price reflects the information held by all market participants and directs resources to their most efficient use. We show that this core function of financial markets may break down if we allow for the possibility that households do not respond to incentives which are economically infinitessimal (i.e. on the order of 0.01% of consumption). In particular, if households make small correlated errors in their investment decisions, these errors give rise to an information externality: households do not internalize how errors in their investment decisions affect the equilibrium expectations of others. This information externality leads information aggregation to break down precisely when it is most socially valuable, i.e. when information is highly dispersed. This information externality operates whenever information is dispersed and households observe an endogenous price. It relies neither on strategic complementarities nor on higher order uncertainty.

When households make small correlated errors in their investment decisions these errors collectively move the equilibrium price and generate potentially large errors in market expectations of future productivity. If information is sufficiently dispersed, small correlated errors in households' investment decisions may cause a complete collapse of the information content of stock prices. Such a collapse of the information content of stock prices increases the amount of financial risk faced by households and thus induces them to demand higher risk premia for holding stocks. Higher risk premia in turn distort the level of capital accumulation, output and consumption in the long run. The social return to diligent investor behavior is thus orders of magnitude larger than the private return.

Our model is one of the first non-linear general equilibrium (DSGE) models with dispersed information. We calibrate it to the data and estimate that small correlated errors in investor behavior result in an 18.38% rise in the conditional standard deviation of stock returns. The social cost of near-rational errors is on the order of 2.32% of lifetime consumption, while the incentive to individual households to avoid near-rational errors in their investment decisions is economically small – on the order of 0.01% of lifetime consumption.

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Appendix

A Details on Contingent Claims

This appendix gives formal details on the contingent claims which ensure that all households hold the same amount of wealth in equilibrium. Each period is divided in two subperiods. In the first subperiod the productivity shock realizes, contingent claims pay off and households buy state contingent claims for next period. In the second subperiod they consume, receive their private signal of next period's productivity shock, choose their consumption and a their portfolio. We can write equations (5) and (6) as

$$\max_{\{\varphi_t(i;n)\}} \mathcal{E}\left[\max_{C_t(i),\omega_t(i),} U_t(i) = \mathcal{E}_{it}\left\{\sum_{s=t}^{\infty}\beta^{s-t}\log(C_s(i))\right\}|K_t, B_{t-1}, \eta_t\right] \tag{33}$$

and

$$W_{t+1}(i) = [(1-\omega_t(i))(1+r)+\omega_t(i)(1+\tilde{r}_{t+1})]\left(W_t(i) + w_t L - C_t(i) + \int \varphi_t\left(i;n\right)\left(\Phi_{t+1}\left(i;n\right) - \rho_t\left(n\right)\right)dn\right) \quad \forall t, \tag{34}$$

where n is a realization of the vector $\left[\eta_{t+1}, \tilde{\varepsilon}_t, \nu_t\right]$, $\varphi_t\left(i;n\right)$ is the quantity of contingent claims bought by household i that pay off in state n, $\Phi_{t+1}\left(i;n\right)$ is an indicator function that is one if state n occurs at time $t+1$ and ν coincides with $\nu_t(i)$ and zero otherwise, and $\rho_t\left(n\right)$ is the time t price of a claim that pays off one in state n at time $t+1$ and zero otherwise.

The crucial assumption is that households trade state contingent claims in the first subperiod, during which they have homogeneous information. Contingent claims are in are in zero net supply ($\int \varphi_t\left(i;n\right)di = 0\ \forall t$), such that, in equilibrium households use these claims to insure against the idiosyncratic risk arising from the heterogeneity in the private signal they receive.

Recall that the errors in private signals, $\nu_t\left(i\right)$, are by definition uncorrelated and independent of the realization of aggregate shocks in the next period. Since households do not know their own realization of $\nu_t\left(i\right)$, households cannot predict the payoff they receive from the state-contingent securities and this payoff is uncorrelated and independent of any of the other variables influencing their decisions. It follows that trading in state contingent claims does not distort the portfolio and consumption decisions of the household, while ensuring that the wealth distribution collapses to its average at the beginning of each period.

B Equilibrium Expectations

B.1 Non-linear Change in Variables

We start with the optimality conditions in (18) which we rewrite in the form

$$Q_t = \int \mathcal{E}_{it} \left(\beta \frac{C_t[\kappa_t(i)]}{C_{t+1}[\kappa_{t+1}(i)]} \left((1-\delta) Q_{t+1} + D_{t+1} \right) \right) di \tag{35}$$

$$C_t(i) = \mathcal{E}_{it} \left(\beta C_{t+1}[\kappa_{t+1}(i)]^{-1} (1+r) \right)^{-1} \tag{36}$$

For a more concise exposition, we only demonstrate how to solve for the stock price. However, the method applies to the consumption function analogously.

Lemma B.1

The average expectation of the rewritten optimality condition (35) can be written in the following form[25]

$$Q_t = \int \mathcal{E}_{it} \left(\beta \frac{C_t[\kappa_t(i)]}{C_{t+1}[\kappa_{t+1}(i)]} \left((1-\delta) Q_{t+1} + D_{t+1} \right) \right) di = h^1_{S^k_t} \left(\int \left(E_{it}[\eta_{t+1} | s_t(i), Q_t] di + \tilde{\epsilon}_t \right) di \right)$$

where $h_{S^k_t}(\cdot)$ depends solely on a vector of known state variables, $S^k_t = [K_t, B_{t-1}, \eta_t]$, and moments, as well as the market expectation of next period's productivity conditional on the information set at time t.

For computational purposes, it turns out that we can reduce the state space by replacing η_{t+1} and $\tilde{\varepsilon}_t$ with the average expectation of productivity in the next period. The reason is that households cannot distinguish between productivity and near-rational errors and hence the coefficient on either shock in the perturbation is identical.

To see the result in the lemma, take an infinite order Taylor series expansion of the product of next periods marginal utility with the asset return in K_{t+1}, B_t, η_{t+1}, $E[\eta_{t+2} | s_t(i), \log(Q_t)]$, σ_η, and take the expectation conditional on $s_t(i)$ and Q_t. This gives us a series of terms depending on K_{t+1}, B_t, and σ_η, which are known at time t. Moreover, we get a series of terms depending on the conditional expectation of η_{t+2}. Since η_{t+2} is unpredictable for an investor at time t, the first-order term is 0, and all the higher-order terms depending on $E[\eta_{t+2} | s_t(i), Q_t]$ are just

[25] In the simplified version of our model in which households consist of specialized capitalists and workers we can solve for the consumption policy in closed form. The optimal behavior of households (18) and market clearing in the stock market : The equation to be inverted is then

$$\left((1+r) Q_t + \frac{Q_t^2 K_t \left(1 - \delta + \frac{1}{\chi} (Q_t - 1) \right)}{\beta (B_{t-1}(1+r) + Q_t K_t (1-\delta) + e^{\eta_t} F_K(K_t, L) K_t)} \sigma^2 \right) = \int \mathcal{E}_{it} \left(Q_{t+1}(1-\delta) + D_{t+1} \right) di. \tag{37}$$

cumulants of the unconditional distributions of η and $\tilde{\epsilon}$. The only interesting terms are then those depending on η_{t+1}. We can write

$$\mathcal{E}_{it}\left[\beta\left[C_{t+1}\left[\kappa_{t+1}\left(i\right)\right]^{-1}\left(1+\tilde{r}_{t+1}\left[\kappa_{t+1}\left(i\right)\right]\right)\right]\right]=\sum_{j=0}^{\infty}c_j(K_{t+1},B_t)\mathcal{E}[(\eta_{t+1}-E[\eta_{t+1}])^j|s_t(i),Q_t],$$

where the coefficients $c_j(K_{t+1},B_t)$ involve all the terms depending on the K_{t+1}, B_t, σ_η, and the higher cumulants of η and $\tilde{\epsilon}$.

Next, take the term in the expectations operator on the right hand side and expand it to get

$$\begin{aligned}
&\mathcal{E}[(\eta_{t+1}-E[\eta_{t+1}])^j|s_t(i),Q_t]\\
&\quad=\mathcal{E}[\left((\eta_{t+1}-\mathcal{E}[\eta_{t+1}|s_t(i),Q_t])+(\mathcal{E}[\eta_{t+1}|s_t(i),Q_t]-E[\eta_{t+1}])\right)^j|s_t(i),Q_t]\\
&\quad=\sum_{k=0}^{j}\left(\begin{array}{c}j\\k\end{array}\right)\mathcal{E}\left[(\eta_{t+1}-\mathcal{E}[\eta_{t+1}|s_t(i),Q_t])^k(\mathcal{E}[\eta_{t+1}|s_t(i),Q_t]-E[\eta_{t+1}])^{j-k}|s_t(i),Q_t\right]\\
&\quad=\sum_{k=0}^{j}\left(\begin{array}{c}j\\k\end{array}\right)\mathcal{E}\left[(\eta_{t+1}-\mathcal{E}[\eta_{t+1}|s_t(i),Q_t])^k|s_t\left(i\right),Q_t\right](\mathcal{E}[\eta_{t+1}|s_t(i),Q_t]-E[\eta_{t+1}])^{j-k}\\
&\quad=\sum_{k=0}^{j}\left(\begin{array}{c}j\\k\end{array}\right)m(k)(\mathcal{E}[\eta_{t+1}|s_t(i),Q_t]-E[\eta_{t+1}])^{j-k},
\end{aligned}$$

where $m(k)=\mathcal{E}\left[(\eta_{t+1}-\mathcal{E}[\eta_{t+1}|s_t(i),Q_t])^k|s_t\left(i\right),Q_t\right]$. Now we can use the fact that the operator $\mathcal{E}$ is a rational expectations operator in which the probability density function of η has been shifted by $\tilde{\epsilon}$. This means that we can replace

$$\mathcal{E}\left[(\eta_{t+1}-\mathcal{E}[\eta_{t+1}|s_t(i),Q_t])^k|s_t\left(i\right),Q_t\right]=E\left[(\eta_{t+1}-E[\eta_{t+1}|s_t(i),Q_t])^k|s_t\left(i\right),Q_t\right]\text{ for all }k,$$

where for $k=1$, the expression collapses to zero. $m\left(k\right)$ is then just the k-th moment of the conditional distribution of η.

The conditional expectation that households hold of all higher moments of η_{t+1} is thus a non-linear function of their conditional expectation (the first moment) of η_{t+1} and all higher conditional moments, $m(k)$. However, since η_{t+1} is normally distributed, we know that its conditional distribution must also be normal. Therefore all the higher conditional moments depend only on the conditional variance and on known parameters. Moreover, the conditional variance is constant.

We can now collect terms in the expression above and integrate to get

$$\int \mathcal{E}_{it}\left(\beta\left[C_{t+1}\left[\kappa_{t+1}\left(i\right)\right]^{-1}\left(1+\tilde{r}_{t+1}\left[\kappa_{t+1}\left(i\right)\right]\right)\right]\right)di \tag{38}$$

$$= \int \sum_{j=0}^{\infty} c_j\left(K_{t+1},B_t\right)\left(\sum_{k=0}^{j}\left(\begin{array}{c} j \\ k \end{array}\right) m\left(k\right)\left(\mathcal{E}\left[\eta_{t+1}|s_t\left(i\right),Q_t\right]-E\left[\eta_{t+1}\right]\right)^{j-k}\right)di \tag{39}$$

The last step is to use (8) and (21) in combination with (4) and integrate over households to write

$$\mathcal{E}[\eta_{t+1}|s_t(i),Q_t] = A_1\nu_t(i) + \int \mathcal{E}[\eta_{t+1}|s_t(i),Q_t]di,$$

where $A_1\nu_t(i)$ is the weight households put on their private signal multiplied with the error they receive in their private signal. This term represents the only source of idiosyncratic variation in household expectations. We then substitute this expression into (38) and expand the sum in its polynomial terms. We then integrate over households. In the resulting expression, all terms containing $\nu_t(i)$ give us the unconditional moments of the distribution of ν, which is known. Finally, we can define the resulting expression on the right hand side as $h^1_{S^k_t}\left(\int \mathcal{E}_{it}[\eta_{t+1}]di\right)$.

The only remaining piece of the puzzle is then to obtain the conditional expectation and the conditional variance of η_{t+1}, as well as the coefficient A_1. See section 3.2 for a derivation of the conditional expectation and of A_1. Appendix B.4 gives the conditional variance. ■

Moreover, we can show computationally that $h^i_{S^k_t}(\cdot)$, $i=1,2$, is invertible with

$$h^i_{S^k_t}(0)=0 \qquad (h^i_{S^k_t})'(\cdot)>0 \qquad h^i_{S^k_t}(\infty)=\infty. \tag{40}$$

Using B.1, we can re-write equation (35) in the linear form

$$\hat{q} = \int E\left(\eta_{t+1}|\hat{q}_t, s_t(i), K_t, B_t, \eta_t\right)di + \tilde{\epsilon}_t,$$

where $\hat{q} \equiv (h^1_{S^k_t})^{-1}(Q_t)$. Analogously, we solve for the consumption function. See Mertens (2009) for a more detailed derivation of these results.

B.2 Proof of Proposition 3.1

Matching coefficients between (22) and (20) yields three equations: $A_0 + A_2\pi_0 = \pi_0$, $A_1 + A_2\pi_1 = \pi_1$, and $1 + A_2\gamma = \gamma$. Solving the three equations and three unknowns yields

$$\pi_0 = \frac{A_0}{1 - A_2}, \tag{41}$$

$$\pi_1 = \frac{A_1}{1 - A_2}, \tag{42}$$

and

$$\gamma = \frac{1}{1 - A_2}. \tag{43}$$

B.3 Proof of Proposition 3.3

The vector $(\eta_{t+1}, s_t(i), \hat{q}_t)$ has the following variance covariance matrix:

$$\Sigma = \begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_{\tilde{\epsilon}}^2 \end{pmatrix}$$

Applying the projection theorem yields the coefficients A_1 and A_2 that correspond to the rational expectation of η_{t+1} given $s_t(i)$ and $\hat{q}_t$ in (21):

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} \sigma_\eta^2 & \pi_1\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_{\tilde{\epsilon}}^2 \end{pmatrix}^{-1},$$

yielding

$$A_1 = \frac{\gamma^2\sigma_\eta^2\sigma_{\tilde{\epsilon}}^2}{\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2 + \sigma_\eta^2\left(\pi_1^2\sigma_\nu^2 + \gamma^2\sigma_{\tilde{\epsilon}}^2\right)}, \quad A_2 = \frac{\pi_1\sigma_\eta^2\sigma_\nu^2}{\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2 + \sigma_\eta^2\left(\pi_1^2\sigma_\nu^2 + \gamma^2\sigma_{\tilde{\epsilon}}^2\right)}. \tag{44}$$

These coefficients are still functions of endogenous variables π_1 and γ. Combining them with equations (42) and (43) yields the following closed-form solutions:

$$\gamma = \frac{1}{6\sigma_\eta^4}\left[\begin{array}{c} 2\sigma_\eta^2\left(\sigma_\eta^2 - 2\sigma_\nu^2\right) + \frac{22^{1/3}\sigma_\eta^4\left(\sigma_\eta^2+\sigma_\nu^2\right)^2\sigma_{\tilde{\epsilon}}^2}{\left(27\sigma_\eta^{12}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^4+2\sigma_\eta^6\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^6+3\sqrt{3}\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^8\left(27\sigma_\eta^6\sigma_\nu^2+4\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^2\right)}\right)^{1/3}} \\ +\frac{2^{2/3}\left(27\sigma_\eta^{12}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^4+2\sigma_\eta^6\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^6+3\sqrt{3}\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^8\left(27\sigma_\eta^6\sigma_\nu^2+4\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^2\right)}\right)^{1/3}}{\sigma_{\tilde{\epsilon}}^2} \end{array}\right]$$

and

$$\begin{aligned}\pi_1 \quad = \quad & (92^{2/3}\sigma_\eta^{16}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^6 + 92^{2/3}\sigma_\eta^{14}\sigma_\nu^4\sigma_{\tilde{\epsilon}}^6 + 2^{2/3}\sqrt{3}\sigma_\eta^4\sigma_{\tilde{\epsilon}}^2\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^8\left(27\sigma_\eta^6\sigma_\nu^2 + 4\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^2\right)} \\ & +2^{2/3}\sqrt{3}\sigma_\eta^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^8\left(27\sigma_\eta^6\sigma_\nu^2 + 4\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^2\right)} - 92^{1/3}\sigma_\eta^{12}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^4\left(\Psi\right)^{1/3} \\ & -2^{1/3}\sqrt{3}\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^8\left(27\sigma_\eta^6\sigma_\nu^2 + 4\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^2\right)}\left(\Psi\right)^{1/3} + 6\sigma_\eta^{10}\sigma_{\tilde{\epsilon}}^2\left(\Psi\right)^{2/3})/\left(6\sigma_\eta^{10}\sigma_{\tilde{\epsilon}}^2\left(\Psi\right)^{2/3}\right),\end{aligned}$$

where $\Psi = 27\sigma_\eta^{12}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^4 + 2\sigma_\eta^6\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^6 + 3\sqrt{3}\sqrt{\sigma_\eta^{18}\sigma_\nu^2\sigma_{\tilde{\epsilon}}^8\left(27\sigma_\eta^6\sigma_\nu^2 + 4\left(\sigma_\eta^2+\sigma_\nu^2\right)^3\sigma_{\tilde{\epsilon}}^2\right)}$. Given these results

$$\lim_{\sigma_\nu\to\infty}\frac{var\left(\gamma\tilde{\epsilon}_t\right)}{var\left(\pi_1\eta_{t+1}\right)} = \infty$$

can easily be calculated using a mathematical software package.

B.4 Conditional Variance

The projection theorem also gives us the conditional variance of η_{t+1} as

$$\begin{aligned} var\left(\eta_{t+1}|\hat{q}_t, s_t(i)\right) &= \sigma_\eta^2 - \begin{pmatrix}\sigma_\eta^2 & \pi_1\sigma_\eta^2\end{pmatrix}\begin{pmatrix}\sigma_\eta^2+\sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2+\gamma^2\sigma_{\tilde{\epsilon}}^2\end{pmatrix}^{-1}\begin{pmatrix}\sigma_\eta^2 \\ \pi_1\sigma_\eta^2\end{pmatrix} \quad (45) \\ &= \frac{\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2}{\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2 + \sigma_\eta^2\left(\pi_1^2\sigma_\nu^2+\gamma^2\sigma_{\tilde{\epsilon}}^2\right)}. \quad (46)\end{aligned}$$

A closed form solution follows from combining this expression with equations (42) and (43).

B.5 Proof of Proposition 3.2

The derivative $\frac{\partial\pi_1}{\partial\sigma_{\tilde{\epsilon}}}$ can easily be calculated from (42). However, the resulting expression is too complex to be reproduced here. The fact that $\frac{\partial\pi_1}{\partial\sigma_{\tilde{\epsilon}}} < 0$ can be verified using a mathematical software package.

C Alternative Information Structures

This appendix discusses the case of more complex information environments.

C.1 Public Signal

Assume that households observe a public signal about future productivity in addition to the private signal they receive,

$$g_t = \eta_{t+1} + \zeta_t,$$

where ζ_t represents i.i.d. draws from a normal distribution with zero mean and variance σ_ζ^2. We may then guess that the solution for $\hat{q}_t$ is some linear function of η_{t+1}, ζ_t, and $\tilde{\epsilon}_t$:

$$\hat{q}_t = \pi_0 + \pi_1 \eta_{t+1} + \pi_2 \zeta_t + \gamma \tilde{\epsilon}_t,$$

where the rational expectation of η_{t+1} given $\hat{q}_t$ and the private and public signals is

$$E_{it}\left(\eta_{t+1}\right) = A_0 + A_1 s_t(i) + A_2 \hat{q}_t + A_3.$$

A matching coefficients algorithm parallel to that in Appendix B.2 gives

$$\pi_1 = \frac{A_1 + A_3}{1 - A_2},\ \pi_2 = \frac{A_3}{1 - A_2},\ \gamma = \frac{1}{1 - A_2}$$

The amplification of near-rational errors is thus influenced only in so far as that the presence of public information may induce households to put less weight on the market price of capital when forming their expectations.

The vector $(\eta_{t+1}, s_t(i), \hat{q}_t, g_t)$ has the following variance covariance matrix:

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_2^2\sigma_\zeta^2 + \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_{\tilde{\epsilon}}^2 & \pi_2\sigma_\zeta^2 + \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 & \pi_2\sigma_\zeta^2 + \pi_1\sigma_\eta^2 & \sigma_\zeta^2 + \sigma_\eta^2 \end{pmatrix}$$

Solving the signal extraction problem returns

$$\begin{aligned} A_1 &= \frac{\gamma^2\sigma_\zeta^2\sigma_\eta^2\sigma_{\tilde{\epsilon}}^2}{\sigma_\zeta^2\left(\sigma_\eta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+(\pi_1-\pi_2)^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2} \\ A_2 &= \frac{(\pi_1-\pi_2)\sigma_\zeta^2\sigma_\eta^2\sigma_\nu^2}{\sigma_\zeta^2\left(\sigma_\eta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+(\pi_1-\pi_2)^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2} \\ A_3 &= \frac{\sigma_\eta^2\sigma_\nu^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+\pi_2(\pi_2-\pi_1)\sigma_\zeta^2\right)}{\sigma_\zeta^2\left(\sigma_\eta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+(\pi_1-\pi_2)^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2} \end{aligned}$$

Based on these results Figure 7 plots the conditional variance of η_{t+1} for the rational and near-rational expectations equilibrium and for varying levels of precision of the public signal .

In the rational expectations equilibrium the provision of public information makes no difference, as households are already fully informed from the outset. In the near-rational expectations equilibrium the presence of the public signal is relevant only insofar as a collapse of information aggregation affects only the subset of information that is dispersed across households and not the information that is publicly available. If the public information provided is relatively precise, the conditional variance of stock returns now converges to lower values as σ_ν increases.

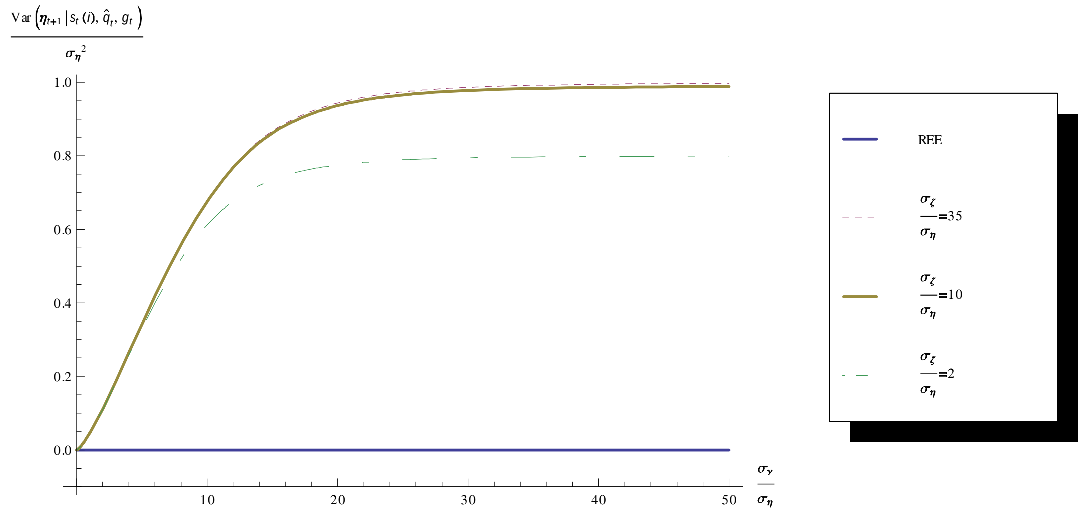

Figure 7: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information, σ_ν/σ_η, and for varying precisions of the public signal. In each case, $\sigma_{\tilde{\varepsilon}}/\sigma_\eta$ is set to 0.01.

C.2 Aggregate Noise in Private Signals

Alternatively, we may consider a situation in which that the private signal received by households contains some aggregate noise:

$$s_t\left(i\right) = \eta_{t+1} + \nu_t\left(i\right) + \zeta_t$$

In this case we may guess that

$$\hat{q}_t = \pi_0 + \pi_1\left(\eta_{t+1} + \zeta_t\right) + \gamma\tilde{\epsilon}_t,$$

where both the rational expectation (21), and the coefficients π_1 and γ are the ones given in the main text. However, the variance covariance matrix of the vector $(\eta_{t+1}, s_t(i), \hat{q}_t)$ changes to

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\zeta^2 + \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\left(\sigma_\zeta^2 + \sigma_\eta^2\right) \\ \pi_1\sigma_\eta^2 & \pi_1\left(\sigma_\zeta^2 + \sigma_\eta^2\right) & \left(\sigma_\zeta^2 + \sigma_\eta^2\right)\pi_1^2 + \gamma^2\sigma_{\tilde{\epsilon}}^2 \end{pmatrix},$$

and we get

$$\begin{aligned} A_1 &= \frac{\gamma^2\sigma_\eta^2\sigma_{\tilde{\epsilon}}^2}{\sigma_\zeta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2} \\ A_2 &= \frac{\pi_1\sigma_\eta^2\sigma_\nu^2}{\sigma_\zeta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_{\tilde{\epsilon}}^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_{\tilde{\epsilon}}^2} \end{aligned}.$$

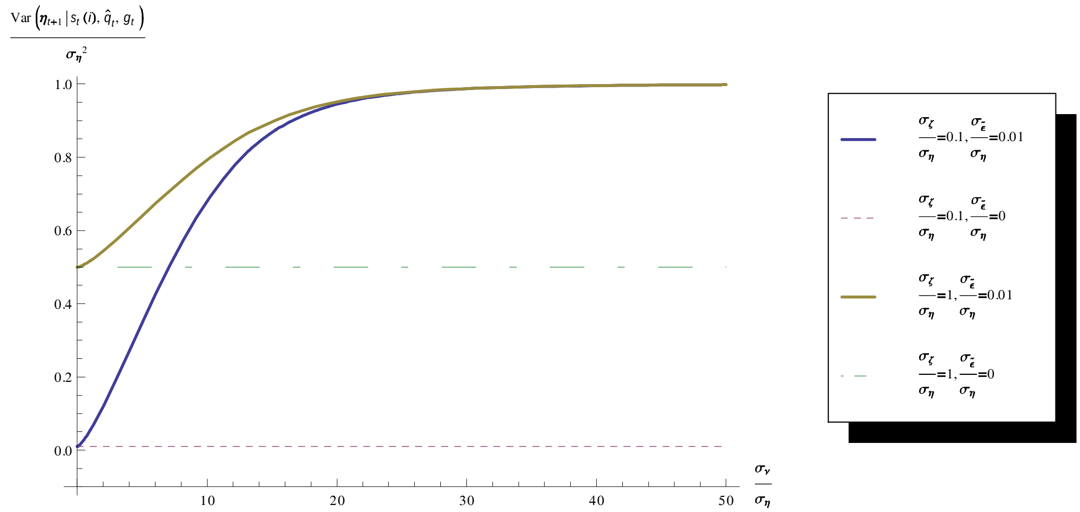

Figure 8: Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information, σ_ν/σ_η, and for varying amounts of aggregate noise in the private signal.

Based on these calculations Figure 8 plots the conditional variance of η_{t+1} for the rational and near-rational expectations equilibrium and for varying levels of aggregate noise in the private signal.The more aggregate noise there is in the private signal the less information is there to aggregate, and the intercept of the curves in Figure 8 shift upwards.

D Proof of Lemma 4.1

We can re-write (5) in Bellman form:

$$V(W_t(i), \pi_t(i)) = \max_{C_t(i), \omega_t(i)} \log(C_t(i)) + \beta \mathcal{E}_{it} \left[V(W_{t+1}(i), \pi_{t+1}(i)) \right],$$

where we abbreviate $\pi_t(i) = E_{it}(1 + \tilde{r}_{t+1}) - (1 + r)$. The conditions of optimality are:

$$\frac{1}{C_t(i)} = \beta \mathcal{E}_{it} \left[R^p_{i,t+1} V'(W_{t+1}(i), \pi_{t+1}(i)) \right], \tag{47}$$

$$\mathcal{E}_{it} \left((\tilde{r}_{t+1} - r)(W_t(i) - C_t(i)) V'(R^p_{i,t+1}(W_t(i) - C_t(i)), \pi_{t+1}(i)) \right) = 0, \tag{48}$$

and

$$V'(W_t(i), \pi_t(i)) = \beta \mathcal{E}_{it} \left(R^p_{i,t+1} V'(W_{t+1}(i), \pi_{t+1}(i)) \right),$$

where $R^p_{i,t+1} \equiv ((1-\omega_t(i))(1+r)+\omega_t(i)(1+\tilde{r}_{t+1}))$ and V' denotes $\frac{\partial V}{\partial W}$. It follows immediately that

$$\frac{1}{C_t(i)} = V'(W_t(i)). \tag{49}$$

Guess the value function:

$$V_t(W_t(i)) = \kappa_1 \log\left(W_t(i)\right) + \kappa_2(\pi_t(i)) + \kappa_3 \tag{50}$$

Verification yields:

$$\begin{aligned}
\kappa_1 &= \frac{1}{1-\beta} \\
\kappa_2 &= \frac{1}{1-\beta}\mathcal{E}_{it}\left\{\sum_{s=1}^{\infty}\beta^s \log(R^{p*}_{t+s}(i))\right\} \\
\kappa_3 &= \frac{1}{1-\beta}\log(1-\beta) + \frac{\beta}{(1-\beta)^2}\log(\beta),
\end{aligned}$$

where R^{p*}_t is the optimized portfolio return. Furthermore, the transversality condition has to hold:

$$\lim_{s\to\infty}\beta^s\kappa_2(R^{p*}_{t+s}(i)) = 0$$

The first result in Proposition 4.1 follows directly from taking the derivative with respect to $W_t\,(i)$ in (50) and combining it with (49). For the second result, combine (48) with (50) to obtain

$$(1+r)\mathcal{E}_{it}\left(R^p_{t+1}(i)\right)^{-1} = \mathcal{E}_{it}\left(\left(1+\tilde{r}_{t+1}\right)\left(R^p_{t+1}(i)\right)^{-1}\right),$$

take logs on both sides, use the fact that

$$\log\mathcal{E}_{it}\left(\cdot\right) = \mathcal{E}_{it}\log\left(\cdot\right) + \frac{1}{2}var\left(\log\left(\cdot\right)\right),$$

and re-arrange the resulting expression to recover (27).

E Solving for the stochastic steady state

E.1 Proof of Proposition 4.3

If at any time o the economy is at its stochastic steady state, we can write $E_oB_{o+1} = B_o$, $E_oK_{o+1} = K_o$ and $I_o = \delta K_o$, where E_o is the unconditional expectations operator, which conditions only on public information available at time o, $E_o\left(\cdot\right) = E\left(\cdot|Q_o, K_o, B_o, \eta_o\right)$. From equation (14) it immediately follows that $Q_o = E_oQ_{o+1} = 1+\delta\chi$. We first calculate the steady state dividend, from which we then back out the steady state capital stock. Finally we derive the steady state value of ω.

From equation (13),

$$D_{t+1} = e^{\eta_{t+1}} F_K\left(K_{t+1}, L\right),$$

At the steady state:

$$E_o D_{o+1} = F_K\left(K_o, L\right)$$

Taking the unconditional expectation of (29) and plugging in yields

$$r + \omega_o \sigma^2 = -\delta + \frac{1}{1+\delta\chi}\left(F_K\left(K_o, L\right)\right)$$

and

$$\left(1+\delta\chi\right)\left(r + \omega_o\sigma^2 + \delta\right) = F_K\left(K_o, L\right).$$

This proves the second statement in Proposition 4.3.[26]

We now turn to solving for ω_o. The first step is to derive the equilibrium resource constraint for capitalists from (2), (13), (14), (25) and (17): From (17) we get that $W_t - C_t = Q_t K_{t+1} + B_t$ plugging this into (25) yields

$$Q_t K_{t+1} + B_t + C_t = (1+r)B_{t-1} + \left(Q_t\left(1-\delta\right) + D_t\right)K_t.$$

Now we can use (2) to eliminate K_{t+1}:

$$Q_t\left(1-\delta\right)K_t + Q_t I_t + B_t + C_t = (1+r)B_{t-1} + \left(Q_t\left(1-\delta\right) + D_t\right)K_t.$$

This simplifies to

$$Q_t I_t + B_t + C_t = (1+r)B_{t-1} + D_t K_t. \tag{52}$$

The next step is to re-write (52) in terms of K_o and ω_o. For this purpose note that

$$C_o = (1-\beta)W_o,$$

$$\beta W_o = K_o\left(1+\delta\chi\right) + B_o,$$

$$B_o = \beta W_o(1-\omega_o),$$

and

$$(1+\delta\chi)K_o = \beta W_o \omega_o$$

[26] With a Cobb-Douglas specification and a capital share of α we can further write

$$\left(\frac{\left(1+\delta\chi\right)\left(r+\omega_o\sigma^2+\delta\right)}{\alpha L^{1-\alpha}}\right)^{\frac{1}{\alpha-1}} = K_o. \tag{51}$$

$$\rightarrow B_o = \frac{1-\omega_o}{\omega_o}(1+\delta\chi)K_o$$

Plugging these conditions into (52) and simplifying yields

$$(1+\delta\chi)\left(\delta + \frac{1-\beta}{\beta} + \frac{1-\omega_o}{\omega_o}\left(\frac{1-\beta}{\beta} - r\right)\right) = F_K\left(K_o, L\right) \tag{53}$$

We can eliminate K_o from this equation by substituting in (31). Some manipulations yield

$$\omega_o = \sqrt{\frac{1}{\sigma^2}\left(\frac{1-\beta}{\beta} - r\right)},$$

proving the first statement in Proposition 4.3.

E.2 Proof of Proposition 4.4

Combining (30) and (31) and taking the total differential gives

$$\frac{dK_o}{d\sigma} = \frac{1+\delta\chi}{F_{KK}\left(K_o, L\right)}\left(\frac{1-\beta}{\beta} - r\right)^{.5}.$$

Proposition 4.3 states that a stochastic steady state exists iff $\beta \leq \frac{1}{1+r}$. Proposition 4.4 then follows directly from the fact that $F_{KK}\left(K_t, L\right) < 0$.

F Decomposition of welfare losses

This section decomposes households' total welfare loss into components attributable to additional variability of consumption and a distortion in the capital accumulation. Given the parameters of the model and initial conditions K_o, ω_o, B_o (see Appendix E), define the expected utility level of the average household in the near-rational expectations equilibrium U as

$$U = E_o \int_0^1 \sum_{t=0}^{\infty} \beta^t \log\left(C_t(i)\right) di,$$

where E_o is the unconditional expectations operator, which conditions only on public information available at time o, $E_o\left(\cdot\right) = E\left(\cdot | K_o, B_o, \eta_o, Q_o\right)$. Similarly, given the same parameters and initial conditions define the expected utility level U^* of transitioning to the stochastic steady state of the rational expectations equilibrium as

$$U^* = E_o \int_0^1 \sum_{t=0}^{\infty} \beta^t \log\left(C_t^*(i)\right) di.$$

We can solve (32) for λ to obtain

$$1+\lambda=\exp\left[\left(E_oU^*-E_oU\right)(1-\beta)\right]. \tag{54}$$

We now define a reference level of utility, C^σ. In this scenario, the path and the level of capital accumulation remain the same as in the near-rational expectations equilibrium, but households exogenously receive compensation for the variability in consumption which is due to productivity shocks and near-rational errors. In technical terms, we calculate the lifetime utility of households which consume $C_t^\sigma=C\left(K_t,B_{t-1},1,1,1,0\right)$, rather than $C\left(K_t,B_{t-1},\eta_t,\eta_{t+1},\tilde{\varepsilon}_t,\nu_t\left(i\right)\right)$: Households are exogenously given the level of consumption they would have received had productivity shocks and near rational errors been at their mean.

$$U^\sigma=E_o\int_0^1\sum_{t=0}^{\infty}\beta^t\log\left(C_t^\sigma(i)\right)di.$$

This reference level of utility allows us to calculate an upper bound for the welfare costs that could be attributable to a higher volatility of consumption in the near-rational expectations equilibrium versus the rational expectations equilibrium. The remainder of the difference between the reference utility and welfare in the rational expectations equilibrium must thus be due to a distortion in the accumulation of capital.[27] We can write

$$U^\Delta=U^*-U^\sigma$$

We can now apply these definitions in (32):

$$1+\lambda=\exp\left[\left(U^*-U^\sigma+U^\sigma-U\right)(1-\beta)\right]$$

and

$$\begin{aligned}1+\lambda=&\exp\left[\left(U^*-U^\sigma\right)(1-\beta)\right]\\&\cdot\exp\left[\left(U^\sigma-U\right)(1-\beta)\right].\end{aligned}$$

This implies that

$$1+\lambda=\left(1+\lambda^\Delta\right)\left(1+\lambda^\sigma\right).$$

[27] We subsume the second order effect due to the variability of the capital stock in this category.

G Construction of Moments

The unconditional standard deviation of stock returns is around 0.18 in international data (Campbell (2003)). Regressing stock returns on lagged price dividend and price earnings ratios (both variables available to the households in our model) yields an R^2 of about 4% (Cochrane, 2005, p.393), suggesting that the conditional variance of stock returns in the data is about 0.17. The correlation of stock price growth with one year ahead GDP growth is from Backus et al. (2007), and the remaining moments are from the author's calculations based on data provided by the Philadelphia Federal Reserve.[FIXME to be completed]