Can Words Get in the Way?
The Effect of Deliberation in Collective Decision-Making*

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Abstract

We quantify the effect of pre-vote deliberation on the decisions of US appellate courts. We estimate a model of strategic voting with incomplete information in which judges communicate before casting their votes, and then compare the probability of mistakes in the court with deliberation with a counterfactual of no pre-vote communication. The model has multiple equilibria, and judges' preferences and information parameters are only partially identified. We find that deliberation can be useful when judges tend to disagree ex ante and their private information is relatively imprecise; otherwise, it tends to reduce the effectiveness of the court.

Keywords: partial identification, communication, voting, courts.
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1 Introduction

Deliberation is an integral part of collective decision-making. Instances of voting in legislatures, courts, boards of directors, and academic committees are generally preceded by some form of communication among its members, ranging from free to fully structured, and from public to private or segmented. Does deliberation lead to better collective decisions? Or is deliberation among committee members detrimental to effective decision-making?

Consider for instance federal courts of appeal in the US, where a panel of three judges determines by majority vote whether or not errors have been committed at trial. Do the discussions about the case among the three judges in the panel lead to a lower proportion of cases incorrectly overturned and incorrectly upheld, or would mistakes at appeal be fewer on average if judges voted independently? In the first case, we would want to allow judges to freely exchange opinions before voting. But if unbridled communication can lead to worse decisions, citizens might be better served if pre-vote communication were curtailed, or eliminated all together. A similar question can be asked in a variety of contexts. Would trial courts be more effective if jurors were not allowed to deliberate? Would shareholders benefit if boards of directors were to undertake some decisions without deliberation, or with constrained deliberation?

On the face of it, the answer to this question seems straightforward: when committee members have the opportunity to talk with one another they can share their private information and reach a better collective decision. This is for example the message in Coughlan (2000), where jurors can vote in a straw poll before the actual voting takes place. When jurors' preferences are sufficiently similar, there is an equilibrium in which all members communicate their private information in the straw poll, and then vote unanimously in the binding vote in favor of the posterior-preferred alternative. In this context, one round of public communication can lead to an efficient outcome.

The restriction to this particular form of communication, however, is not innocuous. Kamenica and Gentzkow (2011) show that if a "sender" can choose the precision of the information available to a "receiver", the sender can manipulate the beliefs of a fully rational Bayesian receiver to achieve better outcomes for herself. This suggests the possibility that free range communication among committee members might instead be detrimental to decision-making. In fact, as we will see, a model that allows arbitrary communication possibilities among committee members implies that communication can lead to worse outcomes than what would be obtained in equilibria of the voting game without deliberation. Because of the ambiguity of the theoretical results, evaluating the effect of deliberation on outcomes becomes an empirical question, the answer to which depends on committee members' traits, and on the equilibrium strategies they play in the data.

In this paper we quantify the effect of deliberation on collective choices in the context of criminal cases decided in the U.S. courts of appeals. Our empirical strategy is to structurally estimate a model of voting with deliberation. This approach allows us to disentangle committee members' preferences, information, and strategic considerations, and ultimately, to compare equilibrium outcomes under deliberation with a counterfactual scenario in which pre-vote communication is precluded.

We consider a judicial decision-making model, tailored to the application. Three judges decide whether to uphold or overturn the decision of the lower court by simple majority vote. Judges only observe a noisy private signal of whether errors have been committed at trial, and differ in the payoff of incorrectly overturning and upholding a decision of the lower court. We allow arbitrary pre-vote communication possibilities among judges by considering communication equilibria of the game (Forges (1986), Myerson (1986); Gerardi and Yariv (2007)).

Because the incentive for any individual member to convey her information truthfully depends on her expectations about how others will communicate, any natural model of deliberation will have a large multiplicity of equilibria. Since this is also the case in our setting, the equilibrium conditions do not point identify the structural parameters characterizing judges' preferences and quality of information. For this reason, we estimate identified sets for these parameters using a two-step procedure that allows flexibly for characteristics of the alternatives and the individuals, where the identified set is the set of values of judges' preference and information parameters that are consistent with a mixture of equilibria generating the observed vote distribution.

Having recovered the set of court characteristics that are consistent with the data, we turn to outcomes. The fundamental goal of this paper is to evaluate the *effect* of deliberation. To do this we compare outcomes that would emerge with and without deliberation. We focus on the probability of mistakes in the court in equilibrium when judges communicate and vote strategically. Our results show that deliberation can reduce the probability of incorrect decisions when judges' preferences are sufficiently *heterogeneous* or their private information is relatively *imprecise*. In particular, for a prior belief close to the frequency of overturning in the data, we find that communication equilibria consistent with the data have lower probabilities of mistakes than equilibria of the voting game without deliberation. In contrast, we find that pre-vote communication increases the prevalence of mistakes in the court when judges' preferences are not too heterogeneous or when their private information is relatively precise. In other words, deliberation can help precisely when individuals tend to disagree ex ante and cannot do too well if voting independently; otherwise,it tends to reduce the effectiveness of the court.

There are three parts to this result. One, pre-vote communication has the potential to lead to *bad equilibria*, where the court fails to use the private information of its members to its advantage. In these equilibria, judges vote against their own information because they infer during committee deliberation that the information of other judges contradicts their own. Two, in order to be consistent with the data, more heterogeneous courts also have to be "better", in the sense that judges have to have more precise information *and*, for any given level of quality, must shed off the worse equilibria. In other words, heterogeneous courts can be rationalized as generating the observed voting data, but only if they are competent *and* play equilibria in which they use their information effectively.

The third and final component is performance in the counterfactual of no deliberation. This has two parts. First, we find that for preference and information parameters in the estimated identified set (EIS), the set of equilibrium outcomes of the voting game without deliberation are generally close to the best outcomes that can be achieved in any communication equilibrium, including equilibria not consistent with the voting data. Thus the maximum potential gain from deliberation is generally relatively low. Second, voting without deliberation performs worst when quality of private information is low, or when the court is very heterogeneous. It is in these instances when deliberation has a relatively large potential to improve outcomes. As we noted before, it is precisely when the court is heterogeneous that communication equilibria consistent with the data lead to relatively low error rates. The comparison shows that communication can, in fact, improve outcomes in these regions. However it generally leads to higher error rates when judges are like-minded, or when the quality of their private information is not too low, regardless of judges' prior beliefs.

2 Related literature

The structural estimation of voting models with incomplete information is a relatively recent endeavor in empirical economics. This paper extends several recent papers examining voting behavior in committees with incomplete information and common values (Iaryczower and Shum, (2012b, 2012a); Iaryczower, Lewis, and Shum (2013), Hansen, McMahon, and Velasco Rivera (2013)).[1] In those papers committee members are assumed to vote without

[1]Iaryczower, Katz, and Saiegh (2013) uses a similar approach to study information transmission among chambers in the U.S. Congress. For structural estimation of models of voting with private values and complete information see Poole and Rosenthal (1985, 1991), Heckman and Snyder (1997), Londregan (1999),

deliberating prior to the vote. This paper takes the analysis one step further, by allowing explicitly for communication among judges. As we show below, this extension is far from a trivial one, as the deliberation stage introduces multiple equilibria, rendering the conventional estimation approach inapplicable.

In terms of estimation and inference, this paper draws upon recent-developed tools from the econometric literature on partial identification (eg. Chernozhukov, Hong, and Tamer (2007), Beresteanu, Molchanov, and Molinari (2011)). A closely-related paper is Kawai and Watanabe (2013), who study the partial identification of a strategic voting model using aggregate vote share data from Japanese municipalities.

Our basic model of collective decision-making builds on Feddersen and Pesendorfer (1998), allowing for heterogeneous biases and quality of information (all of which are public information). To this we add deliberation as in Gerardi and Yariv (2007), considering communication equilibria.[2] This is an attractive model of voting with deliberation because the set of outcomes induced by communication equilibria coincides with the set of outcomes induced by sequential equilibria of any cheap talk extension of the underlying voting game.

Coughlan (2000), and Austen-Smith and Feddersen, (2005, 2006) introduce an alternative approach in this context, extending the voting game with one round of *public* deliberation. In essence, both papers allow committee members to carry out a straw poll prior to the vote (in the case of Austen-Smith and Feddersen (2005, 2006), this includes a third message, e.g. abstention). Coughlan (2000) shows that if committee members are sufficiently homogeneous, there is an equilibrium in which individuals vote sincerely in the straw poll, making all private information public. Austen-Smith and Feddersen, (2005, 2006) show that a similar result holds for a committee of size three when biases also are private information if committee members are malleable enough, and provide a comparison of equilibria with partial revelation of information under simple majority and unanimity.[3]

While we are not aware of other papers analyzing deliberation with field data in a setting similar to the one considered here, some recent papers have analyzed deliberation in lab-

Clinton, Jackman, and Rivers (2004) – for the US Congress– and Martin and Quinn, (2002, 2007) – for the US Supreme Court. Degan and Merlo (2009), De Paula and Merlo (2009), and Henry and Mourifie (2011) consider nonparametric testing and identification of the ideological voting model.

[2]Our model therefore is a particular case of Gerardi and Yariv (2007). In this paper, Gerardi and Yariv focus on a comparison of the set of communication equilibria across different voting rules. They show that every outcome that can be implemented with a non-unanimous voting rule r can also be implemented in communication equilibria with a non-unanimous rule r'.

[3]The complication in the analysis comes from the fact that players condition on being pivotal both at the voting and the deliberation stage. For other models of deliberation, see Li, Rosen, and Suen (2001), Doraszelski, Gerardi, and Squintani (2003), Meirowitz (2006), and Landa and Meirowitz (2009), Lizzeri and Yariv (2011).

oratory experiments. Guarnaschelli, McKelvey, and Palfrey (2000), using the straw poll setting of Coughlan (2000), show that subjects do typically reveal their signal (above 90% of subjects do so), but that contrary to the theoretical predictions, individuals' private information has a significant effect on their final vote. Goeree and Yariv (2011) show that when individuals can communicate freely, they typically disclose their private information truthfully and use public information effectively (as in Austen-Smith and Feddersen (2005) bias is private information, so individuals are identical ex ante).[4]

3 The Model

We consider a model of voting in a small committee, tailored to cases from the US appellate courts. The appellate court setting is attractive for this analysis for three reasons. First, appellate courts make decisions on issues in which there is an underlying common value component: a correct decision under the law, even if this can be arbitrarily hard to grasp given limited information. This environment allows us to evaluate the effect of deliberation on the efficiency of collective outcomes. Second, courts of appeals are small committees, composed of only three judges. This allows us to capture relevant strategic considerations in a relatively simple environment. Third, within each circuit, judges are assigned to panels and cases on an effectively random basis. The random assignment norm minimizes the impact of "case selection", whereby appellants are more likely to pursue cases in courts composed of more sympathetic judges.

There are three judges, $i = 1, 2, 3$. Judge i votes to uphold ($v_i = 0$) or overturn ($v_i = 1$) the decision of the lower court. The decision of the court, $v \in \{0, 1\}$, is that of the majority of its members; i.e. overturns ($v = \psi(\vec{v}) = 1$) if and only if $\sum_i v_i \geq 2$.

Judge i suffers a cost $\pi_i \in (0, 1)$ when the court incorrectly overturns the lower court ($v = 1$ when $\omega = 0$) and of $(1-\pi_i)$ when it incorrectly upholds the lower court ($v = 0$ when $\omega = 1$). The payoffs of $v = \omega = 0$ and $v = \omega = 1$ are normalized to zero. Thus given information $\mathcal{I}$, judge i votes to overturn if and only if $\Pr^i(\omega = 1|\mathcal{I}) \geq \pi_i$. Note then that π_i can be thought of as the hurdle imposed by judge i on the amount of information that must be available about facts constituting errors in trial for her to be willing to overturn the decision of the lower court. Thus, $\pi_i > 1/2$ reflects a positive hurdle (a bias towards upholding), while $\pi_i < 1/2$ reflects a negative hurdle (a bias towards overturning).[5]

[4] For other experimental results on deliberation, see McCubbins and Rodriguez (2006) and Dickson, Hafer, and Landa (2008).

[5] In the estimation, we will allow the biases π_i of each judge i to vary with case-specific and individual-specific characteristics. The biases that judges can have in any given type of case can reflect a variety of

We assume that whether errors have been committed at trial ($\omega = 1$) or not ($\omega = 0$) in any given case is only imperfectly observed by the appeal panel. Instead, judges $i = 1, 2, 3$ have a common prior $\rho \equiv \Pr(\omega = 1)$, and observe a private signal $t_i \in \{0, 1\}$ that is imperfectly correlated with the truth; i.e., $\Pr(t_i = k|\omega = k) = q_i > 1/2$ for $k = 0, 1$. The parameter q_i captures the informativeness of i's signals.[6] The judges' signals are independent from each other conditional on ω. For convenience, we let $\theta \equiv (\rho, \vec{q})$.

In the absence of deliberation, this setting describes a voting game G, as in Feddersen and Pesendorfer (1998). We extend this model to allow for pre-vote deliberation amongst the judges – that is, for judges to discuss the case with each other, and potentially to reveal their private information to each other. Specifically, following Gerardi and Yariv (2007), we model deliberation by considering equilibria of an extended game in which judges exchange messages *after* observing their signals and *before* voting. In particular, we consider a cheap talk extension of the voting game that relies on a fictional mediator, who helps the judges communicate. In this augmented game, judges report their signals $\vec{t}$ to the mediator, who then selects the vote profile $\vec{v}$ with probability $\mu(\vec{v}|\vec{t})$, and informs each judge of her own vote. The judges then vote. A *communication equilibrium* is a sequential equilibrium of this cheap talk extension in which judges (i) convey their private information truthfully to the mediator, and (ii) follow the mediator's recommendations' in their voting decisions (we describe the equilibrium conditions formally below).[7] A powerful rationale for focusing on the set of communication equilibria, M, is that the set of outcomes induced by communication equilibria coincides with the set of outcomes induced by sequential equilibria of *any* cheap talk extension of G (see Gerardi and Yariv (2007)).

We can now define communication equilibria more formally. As we described above, in a *communication equilibrium* judges (i) convey their private information truthfully to the mediator, and (ii) follow the mediator's recommendations' in their voting decisions. These define two sets of incentive compatibility conditions, which we call the "deliberation stage" and "voting stage" constraints respectively.

Voting Stage. At the voting stage, private information has already been disclosed to the mediator. Still the equilibrium probability distributions $\mu(\cdot|\vec{t})$ over vote profiles $\vec{v}$ must be

factors, inducing a non-neutral approach to this case, such as ingrained theoretical arguments about the law, personal experiences, or ideological considerations.

[6] Assuming $q_i > 1/2$ is without loss of generality, because if $q_i < 1/2$ we can redefine the signal as $1 - t_i$. The assumption that the signal quality does not depend on ω is made only for simplicity.

[7] Note that in equilibrium players do not necessarily infer the information available to the mediator. Thus, the requirement that players report truthfully to the mediator does not imply that players will report truthfully to the other players in a given unmediated communication protocol implementing the same outcomes.

such that each judge i wants to follow the mediator's recommendation v_i. Hence we need that for all $i = 1, 2, 3$, for all $v_i \in \{0, 1\}$, and for all $t_i \in \{0, 1\}$,

$$\sum_{t_{-i}} p(t_{-i}|t_i;\theta) \sum_{v_{-i}} \left[u_i(\psi(v_i, v_{-i}), \vec{t}) - u_i(\psi(1 - v_i, v_{-i}), \vec{t}) \right] \mu(\vec{v}|\vec{t}) \geq 0, \tag{3.1}$$

where as usual $t_{-i} \equiv (t_j, t_k)$ and $v_{-i} \equiv (v_j, v_k)$ for $j, k \neq i$. Here $p(t_{-i}|t_i;\theta)$ denotes the conditional probability mass function of t_{-i} given t_i, and $u_i(\psi(\vec{v}), \vec{t})$ denotes the utility of judge i when the decision is $\psi(\vec{v})$ and the signal profile is $\vec{t}$. Note that $u_i(\psi(v_i, v_{-i}), \vec{t}) - u_i(\psi(1 - v_i, v_{-i}), \vec{t}) = 0$ whenever $v_{-i} \notin Piv^i \equiv \{(v_j, v_k) : v_j \neq v_k\}$. Then (3.1) is equivalent to (3.2) (for $v_i = 1$) and (3.3) (for $v_i = 0$) for $i = 1, 2, 3$ and for all $t_i \in \{0, 1\}$:

$$\sum_{t_{-i}} p(t_{-i}|t_i;\theta) \left[p_\omega(1|\vec{t};\theta) - \pi_i \right] \sum_{v_{-i} \in Piv^i} \mu(1, v_{-i}|\vec{t}) \geq 0, \quad \text{and} \tag{3.2}$$

$$\sum_{t_{-i}} p(t_{-i}|t_i;\theta) \left[\pi_i - p_\omega(1|\vec{t};\theta) \right] \sum_{v_{-i} \in Piv^i} \mu(0, v_{-i}|\vec{t}) \geq 0, \tag{3.3}$$

where $p_\omega(\omega|\vec{t};\theta)$ denotes conditional probability mass function of ω given $\vec{t}$. There are therefore 12 such equilibrium conditions at the voting stage.

Deliberation Stage. At the deliberation stage, communication equilibria require that judges are willing to truthfully disclose their private information to the mediator, anticipating the outcomes induced by the equilibrium probability distributions $\mu(\cdot|\vec{t})$ over vote profiles $\vec{v}$. This includes ruling out deviations at the deliberation stage that are profitable when followed up by further deviations at the voting stage. To consider this possibility we define the four "disobeying" strategies:

$\tau_1(v_i) = v_i$:	always obey
$\tau_2(v_i) = 1 - v_i$:	always disobey
$\tau_3(v_i) = 1$:	always overturn
$\tau_4(v_i) = 0$:	always uphold

We require that for all $i = 1, 2, 3$, all $t_i \in \{0, 1\}$, and $\tau_j(\cdot)$, $j = 1, 2, 3, 4$:

$$\sum_{t_{-i}} p(t_{-i}|t_i;\theta) \sum_{v} \left[u_i(\psi(\vec{v}), \vec{t}) \mu(\vec{v}|t_i, t_{-i}) - u_i(\psi(\tau_j(v_i), v_{-i}), \vec{t}) \mu(\vec{v}|1 - t_i, t_{-i}) \right] \geq 0 \tag{3.4}$$

There are therefore 24 such equilibrium conditions at the deliberation stage.

For any given $(\theta, \vec{\pi})$, the conditions (3.2),(3.3), and (3.4) characterize the set of communication equilibria $M(\theta, \vec{\pi})$; i.e.,

$$M(\theta, \vec{\pi}) = \{\mu \in \mathcal{M} : (\theta, \vec{\pi}, \mu) \text{ satisfies } (3.2), (3.3) \text{ and } (3.4)\}, \tag{3.5}$$

where $\mathcal{M}$ is the set of all possible values that μ can take, and it can be conveniently thought of as the set of 8*8 dimensional matrices whose elements lie in $[0, 1]$ and each row sums to one. Note that $M(\theta, \vec{\pi})$ is convex, as it is defined by linear inequality constraints on μ.

Remark 3.1 (Robust Communication Equilibria). Note that for given v_i, the vote profiles in which the *other* judges vote unanimously to overturn or uphold do not enter the incentive compatibility conditions at the voting stage. Thus, without any additional refinement, the set of communication equilibria includes strategy profiles in which some members of the court vote against their preferred alternative only because their vote cannot influence the decision of the court. These include not only strategy profiles μ that put positive probability only on unanimous votes, but also profiles in which i votes against her preferred alternative only because *conditional* on her signal and her vote recommendation she is sure – believes with probability one – that her vote is not decisive. Consider the example in Table 1.

		Signal Profile							
		(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)
Vote Profile	(1,0,0)	0.000	0.033	0.000	0.015	0.005	0.000	0.077	0.000
	(1,0,1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(1,1,0)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(1,1,1)	0.119	0.282	0.132	0.274	0.216	0.118	0.202	0.132
	(0,0,0)	0.855	0.657	0.859	0.689	0.623	0.850	0.688	0.858
	(0,0,1)	0.000	0.000	0.000	0.000	0.131	0.000	0.000	0.000
	(0,1,0)	0.026	0.027	0.010	0.022	0.025	0.031	0.033	0.009
	(0,1,1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Pr(v=1|t)	0.119	0.282	0.132	0.274	0.216	0.118	0.202	0.132
	Pr(v=0|t)	0.881	0.718	0.868	0.726	0.784	0.882	0.798	0.868
	Pr(w=1|t)	0.069	0.143	0.143	0.273	0.032	0.069	0.069	0.143

Table 1: A Non-Robust Communication Equilibrium for $\rho = 0.1$ and $\pi_i = 0.3, q_i = 0.6$ for $i = 1, 2, 3$. For each row $\vec{v}$ and column $\vec{t}$, the entry gives $\mu(\vec{v}|\vec{t})$.

The strategy profile in Table 1 is a communication equilibrium for $\rho = 0.1$, and $\pi_i = 0.3, q_i = 0.6$ for $i = 1, 2, 3$. However, judge 1 votes to overturn with positive probability even if $\Pr(\omega = 1|\vec{t}) < \pi$ for all $\vec{t}$. This in spite of the fact that non-unanimous vote profiles are played with positive probability. The result is due to the fact that *conditional* on $t_1 = 0$ (columns 5 to 8) and $v_1 = 1$ (rows 1 to 4), judge 1 believes that either $\vec{v} = (1, 0, 0)$ or $\vec{v} = (1, 1, 1)$ are played. As a result, her vote is not decisive in equilibrium, and 1 is willing to vote to overturn. The same is true in this example conditional on $t_1 = 1$. A similar logic holds for judges 2 and 3.

Because these equilibria are not robust to small perturbations in individuals' beliefs about how others will behave, we rule them out. To do this, we require that each individual best responds to beliefs that are consistent with small trembles (occurring with probability η) on equilibrium play, so that all vote profiles have positive probability after any signal profile. Formally, in all equilibrium conditions – at both the voting and deliberation stage – we substitute $\Pr(\vec{v}|\vec{t})$ in place of $\mu(\vec{v}|\vec{t})$, where for any $\vec{t}$ and $\vec{v}$,

$$\Pr(\vec{v}|\vec{t}) = \sum_{\hat{v}:\hat{v}_i=v_i} \mu(\hat{v}|\vec{t}) \prod_{j\neq i} (1-\eta)^{\hat{v}_j=v_j} \eta^{\hat{v}_j\neq v_j}$$

The η we use in the empirical section is 0.01.[8]

Adverse Communication. Having eliminated non-robust equilibria, we know that judges' voting decisions will reflect their posterior beliefs after deliberation. In fact, provided that $\sum_{t_{-i}} \sum_{v_{-i}\in Piv^i} \mu((1,v_{-i})|(t_i,t_{-i})) \Pr(t_{-i}|t_i;(q,\rho)) > 0$, the conditions (3.2) can be written as

$$\Pr(\omega=1|v_i=1,t_i,Piv^i;(\vec{q},\rho,\mu)) \geq \pi_i;$$

i.e., conditional on her vote v_i, signal t_i, and on being pivotal in the court's decision, judge i prefers to overturn the decision of the lower court. Similarly conditions (3.3) boil down to $\Pr(\omega=1|v_i=0,t_i,Piv^i;(\vec{q},\rho,\mu)) \leq \pi_i$. It follows that if communication is to lead to inferior outcomes, it has to be through judges' beliefs after deliberation. The question then is: how much can deliberation influence rational judges' beliefs?

As it turns out, the answer is "quite a lot". We illustrate this with an example. Let $\pi_1 = 0.25, \pi_2 = \pi_3 = 0.6$, $\rho = 1/2$, and suppose that $q_i = 0.90$ for $i = 1,2,3$; i.e., judge 1 is biased towards overturning the lower court, while judges 2 and 3 are biased towards upholding the decisions of the lower court, and judges have uninformative priors and relatively accurate private information. Table 2 describes a particular communication equilibrium $\tilde{\mu}$.[9] This equilibrium is of interest here because it leads to incorrect decisions with high probability, even when $q = 0.9$. Consider for example column 2, corresponding to $\vec{t} = 101$. While the probability that the decision should be overturned given $\vec{t} = 101$ is fairly large – i.e., $\Pr(\omega = 1|\vec{t} = 101)) = 0.9$ – in equilibrium the court overturns when $\vec{t} = 101$ roughly one fourth of the times: $\Pr(v = 1|\vec{t} = 101) = 0.26$.

To understand how this happens, consider the problem of judge 1. Note that judge 1 is

[8] To evaluate the robustness of our results, we replicate the analysis for $\eta = 0.001$ and $\eta = 0.000001$. The results are qualitatively unchanged.

[9] As in Table 1, the cell corresponding to row $\vec{v}$ and column $\vec{t}$ gives the equilibrium probability that $\vec{v}$ is played given a signal realization $\vec{t}$; i.e., $\mu(\vec{v}|\vec{t})$. Thus, for example, $\mu(100|100) = 0.044$.

		Signal Profile								
		(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)	Pr(v)
Vote Profile	(1,0,0)	0.044	0.110	0.321	0.000	0.000	0.066	0.023	0.000	0.025
	(1,0,1)	0.000	0.023	0.141	0.002	0.006	0.006	0.029	0.160	0.019
	(1,1,0)	0.073	0.003	0.044	0.008	0.000	0.003	0.081	0.003	0.012
	(1,1,1)	0.187	0.115	0.080	0.254	0.256	0.252	0.150	0.028	0.223
	(0,0,0)	0.351	0.604	0.403	0.736	0.739	0.537	0.433	0.735	0.676
	(0,0,1)	**0.082**	**0.021**	**0.000**	**0.000**	0.000	0.136	0.170	0.000	0.018
	(0,1,0)	**0.263**	**0.000**	**0.011**	**0.000**	0.000	0.000	0.113	0.000	0.017
	(0,1,1)	0.000	0.124	0.000	0.000	0.000	0.000	0.000	0.074	0.009
	Pr(v=1|t)	0.255	0.259	0.259	0.258	0.256	0.255	0.255	0.259	
	Pr(w=1|t)	0.100	0.900	0.900	0.999	0.001	0.100	0.100	0.900	
	Pr(t)	0.045	0.045	0.045	0.365	0.365	0.045	0.045	0.045	

Table 2: Equilibrium consistent with the data with large error rates, for $q = 0.90$, $\pi_1 = 0.25, \pi_2 = \pi_3 = 0.6$.

predisposed to overturn, as $\pi_1 = 0.25$. Nevertheless, in equilibrium she sometimes votes to uphold, *even after observing a signal that errors have been made in trial.* Now, consider judge 1's equilibrium inference when in equilibrium she votes to uphold ($v_1 = 0$), given that she received a signal in favor of overturning, $t_1 = 1$. Because she is supposed to vote zero, judge 1 can rule out (put probability zero on) the vote profile $\vec{v} = 100$, and thus the entire first row of the matrix. Similarly, she can rule out rows 2, 3 and 4. Because she knows that she received a 1 signal, she can rule out the possibility that $\vec{t} = 000$ (column 5). Similarly, she can rule out columns 6, 7 and 8. Because only events in which she is pivotal to the decision are payoff consequential, she can rule out $\vec{v} = (0, 0, 0)$ and $\vec{v} = (0, 1, 1)$ (rows 5 and 8). We are thus left with the bold cells in the table. But this indicates that the posterior probability that the other two judges received a 0 signal is considerably high. In fact, $\Pr(t_2 = 0, t_3 = 0|v_1 = 0, t_1 = 1, Piv^1)$ is given by

$$\frac{[\mu(001|100) + \mu(010|100)] \Pr(\vec{t} = 100)}{\sum_{(t_2,t_3)}(\mu(001|1, t_2, t_3) + \mu(010|1, t_2, t_3)) \Pr(\vec{t} = (1, t_2, t_3))} = \frac{[0.082 + 0.263]0.045}{0.017} = 0.915$$

Thus, the equilibrium inference about the information of other judges ends up overwhelming her own private information, leading to a posterior probability that errors were not committed at trial (i.e., should uphold) of only 0.17. (The same logic applies to judges 2 and 3, leading to a posterior probability of exactly 0.60 for both judges 2 and 3, consistent with equilibrium.)

The example illustrates that after all incentive constraints are taken into account, deliberation can still have a powerful effect on the beliefs of rational, fully Bayesian judges. The result has the flavor of Kamenica and Gentzkow (2011), albeit in a more elaborate strategic setting. The two games have many differences of course, as here there are three privately informed players, who are both senders and receivers of information, while Kamenica and Gentzkow consider a two player game, where an uninformed sender can choose the information service available to a single decision-maker. Crucially, however, choosing a communication equilibrium μ effectively entails choosing an information service for each judge (as receiver) subject to the equilibrium constraints assuring that each player reports its information truthfully (to the mediator). While a communication equilibrium adds constraints to manipulation of beliefs, the example illustrates that there is still room for players to persuade one another through deliberation.

The fact that deliberation can lead to better or worse outcomes than the corresponding game without deliberation implies that quantifying the effect of deliberation is ultimately an empirical question. In the next sections we develop an empirical framework that allows us to tackle this issue.

4 Data

The data are drawn together from two sources. The main source is the United States Courts of Appeals Data Base (Songer (2008)). This provides detailed information about a substantial sample of cases considered by courts of appeals between 1925 and 1996, including characteristics of the cases, the judges hearing the case, and their votes. Among the roughly 16,000 cases in the full database, we restrict our attention to criminal cases, which make up around 25% of the total. The case and judge-specific variables which we use in our analysis are summarized in Table A.2 in the Appendix. Additional information for judges involved in these decisions was obtained from the Multi-User Data Base on the Attributes of U.S. Appeals Court Judges (Zuk, Barrow, and Gryski (2009)).

Since we are modeling the voting behavior on appellate panels, we distinguish between judges' votes for *upholding* ($v = 0$) versus *overturning* ($v = 1$) the decision of a lower court. Thus, given the majority voting rule, among the eight possible vote profiles, there are four which lead to an outcome of upholding the lower court's decision – $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ – and four leading to overturning – $(1, 1, 1)$, $(1, 1, 0)$, $(1, 0, 1)$ and $(0, 1, 1)$.

For each case, we include a dummy variable ("FedLaw") for whether the case is prosecuted under federal (rather than state) law, as well as dummy variables for the crime in each case. These crime categories are based on the nature of the criminal offense in the case, and do not exhaust the set of possible crimes, but instead constitute "common" issues, bundling a relatively large number of cases within each label. Thus "Aggravated" contains murder, aggravated assault, and rape cases. "White Collar" crimes include tax fraud, and violations of business regulations, etc. “Theft” includes robbery, burglary, auto theft, and larceny. The “Narcotics” category encompasses all drug-related offenses.

In addition to the nature of the crime, we also include information about the major legal issue under consideration in the appeal. In particular, we distinguish issues of Jury Instruction, Sentencing, Admissibility and Sufficiency of evidence from other legal issues.

We also include three variables which describe the makeup of the judicial panel deciding each case: an indicator for whether the panel is a Republican majority (“Rep. Majority”), whether the panel contains at least one woman (“Woman on panel”), and whether there is a majority of Harvard and/or Yale Law School graduates on the panel (“Harvard-Yale Majority”). This latter variable is included to capture possible “club effects” in voting behavior; the previous literature has pointed out how graduates from similar programs may share common judicial views, and vote as a bloc.

Finally, we include four judge-specific covariates. “Republican” indicates a judge’s affiliation to the Republican Party. “Yearsexp” measures the number of years that a judge has served on the court of appeals, at the time that he/she decides a particular case (this variable varies both across judges and across cases). “Judexp” and “Polexp” measure the number of years of judicial and political experience of a judge prior to his/her appointment to the appellate court.

Judges are assigned to cases on an effectively random basis. The particular assignment procedures vary from circuit to circuit, with some circuits using explicitly random assignments (via random number generators) and others incorporating additional factors as dictated by practical considerations (e.g., availability). This semi-random nature of panel assignment means that the parties in each case have little influence over the particular makeup of the panel which hears their case; this minimizes “case selection” problems which may otherwise confound the interpretation of the estimation results.[10]

5 Econometric Model

5.1 Partial identification of model parameters

The immediate goal of the estimation is to recover the signal/state distribution parameters, θ, and the judges’ preference vector $\vec{\pi}$. The information used to recover these parameters is the distribution of the voting profiles, $p_v(\vec{v})$, which can be identified from the data. Here

[10] See Iaryczower and Shum’s (2012b) study of US Supreme Court voting behavior for a more extended discussion and assessment of case selection.

we define the *sharp identified set* for the model parameters.[11] The sharp identified set of $\{\theta, \vec{\pi}\}$ is the set of parameters that can rationalize $p_v(\vec{v})$ under some equilibrium selection mechanism λ – a mixing distribution over $\mu \in M(\theta, \vec{\pi})$. In other words, the sharp identified set $\mathcal{A}_0$ is the set of $(\theta, \vec{\pi}) \in \Theta \times [0,1]^3$ such that there exists a λ that satisfies

$$p_v(\vec{v}) = \int_{\mu \in M(\theta,\vec{\pi})} \lambda(\mu) \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t};\theta) d\mu. \tag{5.1}$$

However, because the set $M(\theta, \vec{\pi})$ of communication equilibria is convex, whenever there exists a mixture λ satisfying (5.1) there exists a single equilibrium $\mu \in M(\theta, \vec{\pi})$ such that $p_v(\vec{v}) = \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t};\theta)$.[12] Thus $\mathcal{A}_0$ boils down to

$$\mathcal{A}_0 = \{(\theta, \vec{\pi}) \in \Theta \times [0,1]^3 : \exists \mu \in M(\theta, \vec{\pi}) \text{ s.t. } p_v(\vec{v}) = \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t};\theta)\}. \tag{5.2}$$

We will also introduce the following set $\mathcal{B}_0$:

$$\mathcal{B}_0 = \{(\theta, \vec{\pi}, \mu) \in \mathcal{B} : \mu \in M(\theta, \vec{\pi}) \text{ and } p_v(\vec{v}) = \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t};\theta)\}, \tag{5.3}$$

where $\mathcal{B} = \Theta \times [0,1]^3 \times \mathcal{M}$ and $\mathcal{M}$ is the set of μ – 64×64 dimensional matrices, the elements of which are positive and each row sums to 1. The set $\mathcal{B}_0$ is the sharp identified set of $\{\theta, \vec{\pi}, \mu\}$, where μ is the true mixture voting assignment probability. The identified set $\mathcal{A}_0$ can be considered as the projection of $\mathcal{B}_0$ onto its first $d_\theta + 3$ dimensions, corresponding to the parameters $(\theta, \vec{\pi})$.

Identification in a Symmetric Model: Intuition. Before proceeding on to the estimation of the identified set, we provide some intuition for the identification of the model parameters by analyzing a stripped-down model in which the three judges are symmetric, in the sense that they have identical preferences and quality of information. That is, the preference parameters are identical across judges ($\pi_1 = \pi_2 = \pi_3 = \pi$) and so are the signal accuracies ($q_1 = q_2 = q_3 = q$). In this simple model, there are only three parameters (ρ, q, π).

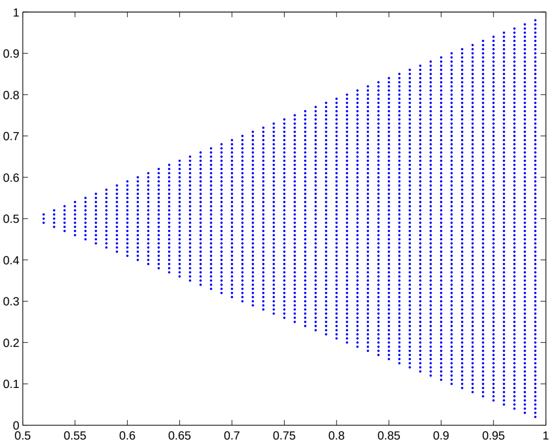
[11] The sharpness of the identified set is in the sense of Berry and Tamer (2006), Galichon and Henry (2011), Beresteanu, Molchanov, and Molinari (2011). However, our estimation approach differs quite substantially from those papers.

[12] This fact implies an observational equivalence between a unique communication equilibrium being played in the data, versus a mixture of such equilibria. Sweeting (2009) and De Paula and Tang (2012) discuss the non-observational equivalence between mixture of equilibria and a unique mixed strategy equilibria in coordination games.

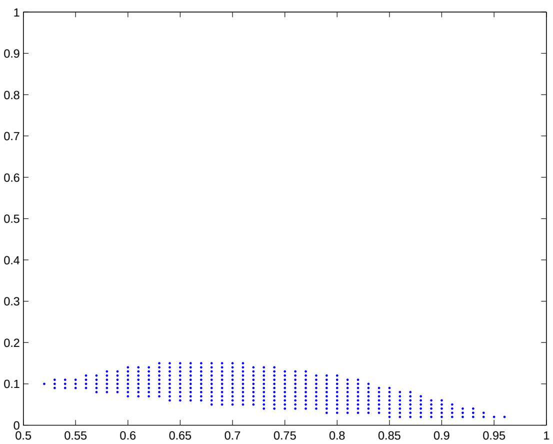
In Figure 1 we show the pairs (π, q) in the identified set for four different hypothetical vote profile vectors and given values of the common prior ρ. The figure on the upper left panel plots the identified set for $\rho = 0.5$ and a uniform distribution of vote profiles, i.e., $p_v(\vec{v}) = 1/8$ for all $\vec{v}$. Because of the symmetry of the vote profile and the characteristics of the individuals, the identified set is also symmetric. Moreover, the set of preference parameters π in the identified set for each value of q is increasing in q. Thus, high ability judges can be very predisposed towards either upholding (requiring considerably more information supporting that mistakes were made at trial to overturn) or overturning (requiring considerably more information supporting that mistakes were not made at trial to uphold) and still play equilibria consistent with the data. However, low ability judges must be highly malleable – willing to uphold or overturn even with little information that errors in trial have been committed or not – if they are to be consistent with the "data".

The figure on the top right plots the pairs (π, q) in the identified set for the uniform distribution over vote profiles and a prior probability of $\rho = 0.1$ that mistakes were made at trial. Because the prior is very favorable towards upholding the decision of the lower court, only judges that are very biased towards *overturning* ($\pi << 1/2$) – who require a high certainty that errors in trial have not been committed in order to uphold – can vote in a way consistent with the data. This is because for these types of judges, the opposite bias and priors compensate each other, effectively making them equivalent to a non-biased judge with uniform priors over the state. Instead, judges that are already predisposed towards upholding only become more extreme once the prior is taken into consideration, and are not inclined to vote to overturn with such large probability.

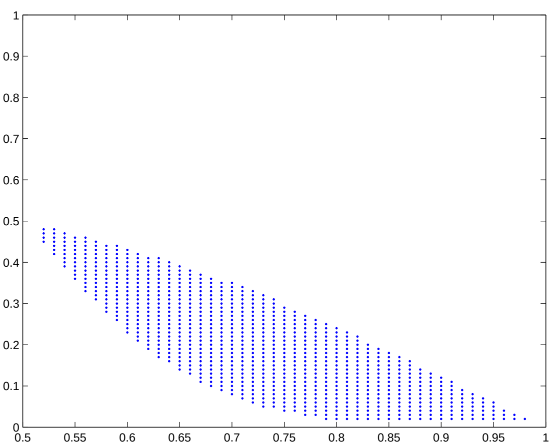
The figures in the lower panel return to $\rho = 0.5$, but consider non-uniform distributions of vote profiles. In the lower-left figure only unanimous votes have positive probability, and the probability of overturning is $p_v(1, 1, 1) = 0.9$, while $p_v(0, 0, 0) = 0.1$. As in the first figure, low ability judges must be willing to uphold or overturn with even little information that errors in trial have been committed or not if they are to be consistent with the "data". However, high ability judges must be biased towards overturning (must demand a high certainty that errors in trial have not been committed in order for them not to overturn), and increasingly so the higher the information precision. The same result holds in the lower right figure, where also overturning is more likely, but only non-unanimous votes have positive probability. In this case, however, more malleable judges are consistent with the data for any given level of q. Note that in all figures, $\pi \to \rho$ as $q \to 1/2$. This is because as signals become less informative, in order to get a judge to vote for both alternatives some of the time, the judge *has* to be increasingly closer to being indifferent between voting one way or the other, after bias and prior beliefs are taken into consideration.



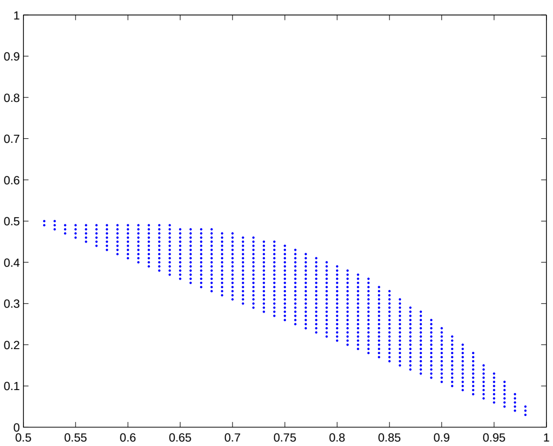
$p_v(\vec{v}) = 1/8$ for all $\vec{v}, \rho = 0.5$



$p_v(\vec{v}) = 1/8$ for all $\vec{v}, \rho = 0.1$



$p_v((1,1,1)) = 0.9, p_v((0,0,0)) = 0.1, \rho = 0.5$



$\vec{v}$: split majority to overturn: $p_v(\vec{v}) = 1/4$
$\vec{v}$: split majority to uphold: $p_v(\vec{v}) = 1/12$
$\rho = 0.5$

Figure 1: Identification of Second-Stage Parameters: A Simplified Model: $q_i = q$ for all i, $\pi_i = \pi$ for all i. X-axis: q (probability of correct signal); Y-axis: π (judges' bias parameter)

5.2 Estimation

To study the estimation of the identified set, we define the criterion function

$$Q(\theta, \vec{\pi}; W) = \min_{\mu \in M(\theta, \vec{\pi})} Q(\theta, \vec{\pi}, \mu; W) \text{ where}$$
$$Q(\theta, \vec{\pi}, \mu, W) = (\vec{p}_v - P_t(\theta)\vec{\mu})' W (\vec{p}_v - P_t(\theta)\vec{\mu})', \tag{5.4}$$

and where $\vec{p}_v = (p_v(111), p_v(110), p_v(101), p_v(100), p_v(010), p_v(001), p_v(000))'$, $\vec{\mu}$ is a 64−vector whose $8k+1$'th to $8k+8$'th coordinates are the $(k+1)$'th row of $\mu(\vec{v}|\vec{t})$ for $k = 0, ..., 7$, $P_t(\theta) = p(\vec{t}, \theta)' \otimes [I_7|0_7]$ and W is a positive definite weighting matrix specified later.

We estimate the vote probabilities by the empirical frequencies of the vote profiles:

$$\hat{p}_v(\vec{v}) = \frac{1}{n}\sum_{l=1}^{n} 1(V_l = \vec{v}), \tag{5.5}$$

where V_l is the observed voting profile for case l and n is the sample size. Assuming that the cases are i.i.d., by the law of large numbers, $\hat{p}_v(\vec{v}) \rightarrow_p p_v(\vec{v})$ for all $\vec{v} \in \mathcal{V}$, where $\mathcal{V} = \{111, 110, 101, 100, 011, 010, 001\}$. One can define a sample analogue estimator for $\mathcal{A}_0$:

$$\hat{\mathcal{A}}_n = \{(\theta, \vec{\pi}) \in \Theta \times [0,1]^3 : Q_n(\theta, \vec{\pi}, W_n) = \min_{(\theta,\vec{\pi}) \in \Theta \times [0,1]^3} Q_n(\theta, \vec{\pi}, W_n)\}, \tag{5.6}$$

where W_n is an estimator of W and Q_n is defined like Q except with $\vec{p}_v$ replaced by its sample analogue $\hat{\vec{p}}_v$. The set $\hat{\mathcal{A}}_n$ is the estimated identified set (EIS) that we compute using data later.

The following theorem establishes the consistency of $\hat{\mathcal{A}}_n$ with respect to the Hausdorff distance:

$$d_H(\hat{\mathcal{A}}_n, \mathcal{A}_0) = \max\left\{ \sup_{(\theta,\vec{\pi}) \in \hat{\mathcal{A}}_n} \inf_{(\theta^*,\vec{\pi}^*) \in \mathcal{A}_0} ||(\theta, \vec{\pi}) - (\theta^*, \vec{\pi}^*)||, \sup_{(\theta^*,\vec{\pi}^*) \in \mathcal{A}_0} \inf_{(\theta,\vec{\pi}) \in \hat{\mathcal{A}}_n} ||(\theta, \vec{\pi}) - (\theta^*, \vec{\pi}^*)|| \right\}. \tag{5.7}$$

In general partially identified models, the sample analogue estimators for the identified sets typically are not consistent with respect to the Hausdorff distance (see e.g. Chernozhukov, Hong, and Tamer (2007)). Our problem has a special structure that guarantees consistency under mild conditions.

Theorem 1. *Suppose that $W_n \rightarrow_p W$ for some finite positive definite matrix W and Θ is* *compact. Also suppose that* $cl(int(\mathcal{B}) \cap \mathcal{B}_0) = \mathcal{B}_0$.[13] *Then,* $d_H(\hat{\mathcal{A}}_n, \mathcal{A}_0) \to_p 0$ *as the sample size* n *goes to infinity.*

Proof. See Appendix 8.1. □

5.3 Confidence Set

Next, we discuss statistical inference in partially identified models based on confidence sets which cover either the true parameter, or the identified set with a pre-specified probability. Following the literature, we construct a confidence set by inverting a test for the null hypothesis $H_0 : (\theta, \vec{\pi}) \in \mathcal{A}_0$ for each fixed $(\theta, \vec{\pi})$. To be specific, we collect all the $(\theta, \vec{\pi})$ such that there is one $\mu \in M(\theta, \vec{\pi})$ at which the H_0 is accepted. The collection of all those $(\theta, \vec{\pi})$ forms a confidence set.[14]

Standard application of the central limit theorem gives us $\sqrt{n}(\hat{\vec{p}}_v - \vec{p}_v) \to_d N(0, \Sigma)$, where Σ is the variance-covariance matrix of the vector of vote probabilities $\vec{p}_v$. Then the law of large number implies $\hat{\Sigma}_n \to_p \Sigma$. Accordingly, we define the following test statistic:

$$T_n(\theta, \vec{\pi}) = nQ_n(\theta, \vec{\pi}; \hat{\Sigma}_n^{-1}). \tag{5.8}$$

By definition, $T_n(\theta, \vec{\pi}) \leq nQ_n(\theta, \vec{\pi}, \mu; \hat{\Sigma}_n^{-1})$ for any $(\theta, \vec{\pi}, \mu) \in \mathcal{B}_0$. Using standard arguments, we can show that for any $(\theta, \vec{\pi}, \mu) \in \mathcal{B}_0$, $nQ_n(\theta, \vec{\pi}, \mu, \hat{\Sigma}_n^{-1}) \to_d \chi^2(7)$. Thus, a test of significance level $\alpha \in (0, 1)$ can use the $1 - \alpha$ quantile of $\chi^2(7)$ as critical value. The confidence set for $(\theta, \vec{\pi})$ is defined as

$$CS_n(1 - \alpha) = \{(\theta, \vec{\pi}) \in \Theta \times [0, 1]^3 : T_n(\theta, \vec{\pi}) \leq \chi^2_{7,\alpha}\}, \tag{5.9}$$

where $\chi^2_{7,\alpha}$ is the $1 - \alpha$ quantile of $\chi^2(7)$.

Theorem 2. *Suppose* Σ *is invertible. Then*
(a) $\liminf_{n\to\infty} \inf_{(\theta,\vec{\pi})\in\mathcal{A}_0} \Pr((\theta, \vec{\pi}) \in CS_n(1 - \alpha)) \geq 1 - \alpha$; *and*
(b) $\liminf_{n\to\infty} \Pr(\mathcal{A}_0 \subseteq CS_n(1 - \alpha)) \geq 1 - \alpha$.

[13] This is a weak assumption that is satisfied if each point in $\mathcal{B}_0$ is either in the interior of $\mathcal{B}$ or is a limit point of a sequence in the interior of $\mathcal{B}$. Unlike seemingly similar assumptions in the literature, it does not require the identified set $\mathcal{B}_0$ to have nonempty interior. In this paper, numerical calculation of the identified sets for different values of $\vec{p}_v$ suggests that this assumption holds.

[14] This inferential method differs from the approach of Pakes, Porter, Ho, and Ishii (2009), which is based on moment inequalities derived from agents' best-response correspondences. While this approach has proved useful in several applications with games of complete information, in the context of our incomplete information environment we have not been able to derive moment inequalities based on best-response behavior.

Proof. See Appendix 8.1. □

Remark 5.1. Part (a) shows that CS_n covers the true value of $(\theta, \vec{\pi})$ with asymptotic probability no smaller than $1 - \alpha$. Interestingly, it is also a confidence set that covers $\mathcal{A}_0$ with asymptotic probability no smaller than $1 - \alpha$, as shown in part (b).[15] The intuition for this phenomenon is that the random components of $T_n(\theta, \vec{\pi}, \mu)$ – which are just the empirical frequencies of the vote probabilities $\hat{\vec{p}}$ – do not depend on the model parameters (θ, π). In contrast, in typicall moment inequality models, the random sample moment functions depend explicitly on the model parameters.

Remark 5.2. Because the confidence set CS_n above is based on the asymptotic critical value for $nQ_n(\theta, \vec{\pi}, \mu; \hat{\Sigma}_n^{-1})$, which is weakly bigger than $T_n(\theta, \vec{\pi})$, it may over-cover asymptotically; that is, it may be larger than necessary. Tighter and nonconservative confidence sets can be constructed by directly approximating the distribution of $T_n(\theta, \vec{\pi})$ using the methods developed in Bugni, Canay, and Shi (2013) and Kitamura and Stoye (2011).[16] The disadvantage of doing this is two-fold: (i) the critical value will need to be simulated and will depend on θ and $\vec{\pi}$ and (ii) a tuning parameter will need to be introduced to reflect the slackness of the inequality constraints. In addition, in our data, we find that the confidence set CS_n is not much larger than the EIS $\hat{\mathcal{A}}_n$, suggesting that not much can be gained by adopting the more complicated methods.

The confidence set can be computed in the following steps:

(1) for each $(\theta, \vec{\pi})$, compute $T_n(\theta, \vec{\pi}) = nQ_n(\theta, \vec{\pi}; \hat{\Sigma}_n^{-1})$ via the quadratic program:

$$\begin{aligned} Q_n(\theta, \vec{\pi}; W_n) = &\min_{\vec{\mu} \in [0,1]^{64}} (\vec{p}_v - P_t(\theta)\vec{\mu})' W (\vec{p}_v - P_t(\theta)\vec{\mu})' \\ &s.t. (3.2), (3.3), (3.4), \text{ and } \sum_{j=k+1}^{k+8} \vec{\mu}_j = 1, \; k = 0, ..., 7. \end{aligned} \tag{5.10}$$

(2) repeat step (1) for many grid points of $(\theta, \vec{\pi}) \in \Theta \times [0, 1]^3$, and

(3) collect the points in step (2) that satisfy $T_n(\theta, \vec{\pi}) \leq \chi^2_{7,\alpha}$ and the points form $CS_n(1-\alpha)$.

For all the results in this paper, we use a value of $\alpha = 0.05$.

[15] Imbens and Manski (2004) initiated a sizable literature regarding these two types of confidence sets.

[16] See Wolak (1989) for the case where the inequality constraints are linear in the structural parameters θ.

5.4 Handling Covariates – Two-step Estimation

Here we describe a two-step estimation approach for this model, which resembles the two-step procedure in Iaryczower and Shum (2012b). This is a simple and effective way to deal with a large number of covariates. Throughout, we let X_t denote the set of covariates associated with case t, including the characteristics of the judges who are hearing case t.

In the first step, we estimate a flexible "reduced-form" model for the vote probabilities $p_v(\vec{v}|X)$.[17] Specifically, we parameterize the probabilities of the eight feasible vote profiles using an 8-choice multinomial logit model. Letting i index the eight vote profiles, we have

$$\begin{aligned} p_v(v_i|X;\beta) &= \frac{\exp(X_i'\beta_i)}{1+\sum_{i'=1}^{7}\exp(X_{i'}\beta_{i'})}, \quad i=1,\dots,7; \\ p_v(v_8|X;\beta) &= \frac{1}{1+\sum_{i'=1}^{7}\exp(X_{i'}\beta_{i'})}, \end{aligned} \tag{5.11}$$

where $v_1,...,v_7$ are the 7 elements in $\mathcal{V}$ and $v_8 = 1-\sum_{i=1}^{7} v_7$.[18] Because the labeling of the three judges is arbitrary, it makes sense to impose an *exchangeability* requirement on our model of vote probabilities. In particular, the conditional probability of a vote profile (v_1, v_2, v_3) given case characteristics X and judge covariates (Z_1, Z_2, Z_3) should be invariant to permutations of the ordering of the three judges; i.e., the vote probability $P(v_1, v_2, v_3|X, Z_1, Z_2, Z_3)$ should be exchangeable in (v_1, Z_1), (v_2, Z_2) and (v_3, Z_3), for all X. These exchangeability conditions imply restrictions on the coefficients on (X, Z_1, Z_2, Z_3) in the logit choice probabilities.[19]

Given the first-stage parameter estimates $\hat{\beta} = \left(\hat{\beta}_1,\dots,\hat{\beta}_7\right)'$, we obtain estimated vote probabilities $\hat{p} = \left(p(v_1|X;\hat{\beta}),\dots,p(v_7|X;\hat{\beta})\right)'$. In the second stage, we use the estimated voting probability vector $\hat{p}$ to estimate the identified set of the model parameters $(\theta, \vec{\pi})$ using arguments from the previous section. This estimation procedure allows the underlying model parameters $(\theta, \vec{\pi})$ to depend quite flexibly on X. The voting assignment μ is allowed to depend on X arbitrarily, $\mu(\vec{v}|\vec{t}, X)$.[20]

[17] This approach is commonplace in recent empirical applications of auction and dynamic game models (see for example Ryan (2012), and Cantillon and Pesendorfer (2006)).

[18] By using a parametrization of the conditional vote probabilities $P(v|X)$ that is continuous in X, we are also implicitly assuming that the equilibrium selection process is also continuous in X. Note that such an assumption is not needed if we estimate $P(v|X)$ nonparametrically and impose no smoothness of these probabilities in X.

[19] In particular, symmetry implies the following constraints: (i) $\beta_{1,111} = \beta_{2,111} = \beta_{3,111}$, (ii) $\beta_{1,011} = \beta_{2,101} = \beta_{3,110}$, (iii) $\beta_{1,100} = \beta_{2,010} = \beta_{3,001}$, (iv) $\beta_{2,011} = \beta_{3,011} = \beta_{1,101} = \beta_{3,101} = \beta_{1,110} = \beta_{2,110}$, (v) $\beta_{2,100} = \beta_{3,100} = \beta_{1,010} = \beta_{3,010} = \beta_{1,001} = \beta_{2,001}$, (vi) $\gamma_{011} = \gamma_{110} = \gamma_{101}$, and (vii) $\gamma_{001} = \gamma_{100} = \gamma_{010}$. See also Menzel (2011) for a related discussion about the importance of exchangeability restrictions in Bayesian inference of partially identified models.

[20] Both the estimation and the inference procedure described in the previous section can be used for each

6 Results

6.1 First-Stage Estimates

The results from the first-stage estimation are given in Table 3. Since these are "reduced-form" vote probabilities, these coefficients should not be interpreted in any causal manner, but rather summarizing the correlation patterns in the data.

Nevertheless, some interesting patterns emerge. First, vote outcomes differ significantly depending on the type of crime considered in each case (cases involving *aggravated assault*, *white collar* crimes and *theft* are significantly less likely to be overturned in a divided decision than other cases) and in response to differences in legal issues (cases involving problems with *jury instruction* or *sentencing* in the lower courts are on average less likely to be overturned in a divided decision, while cases involving issues of *sufficiency* and *admissibility* of evidence are less likely to be overturned in unanimous decisions).

Vote outcomes also change with the partisan composition of the court. A republican judge is less likely to be in the majority of a divided decision to overturn (less so in *assault* and *white collar* cases) and more likely to be in the majority of a divided decision to uphold the decision of the lower court. At the same time, cases considered by courts composed of a majority of republican judges on average have a significantly higher probability of being overturned in both unanimous and divided decisions. The first result indicates that this is due to the voting behavior of the democrat judge when facing a republican majority.

Finally, vote outcomes also differ based on judges' judicial and political experience. Judges with more judicial and political experience, or with more years of experience in the court, are less likely to be in the majority of a divided decision to overturn. Neither having a female judge on the panel, or a majority of graduates from Harvard or Yale Law schools (a possible *club* effect) are significantly related to vote outcomes.

6.2 Second-Stage Estimates: Preferences and Information

In the second stage we use the estimated voting probability vector $\hat{p} = p(\vec{v}|X;\hat{\beta})$ to estimate the identified set of the model parameters $(\theta, \vec{\pi})$. To present the results, we fix benchmark case and judge characteristics, and later on introduce comparative statics from this bench-

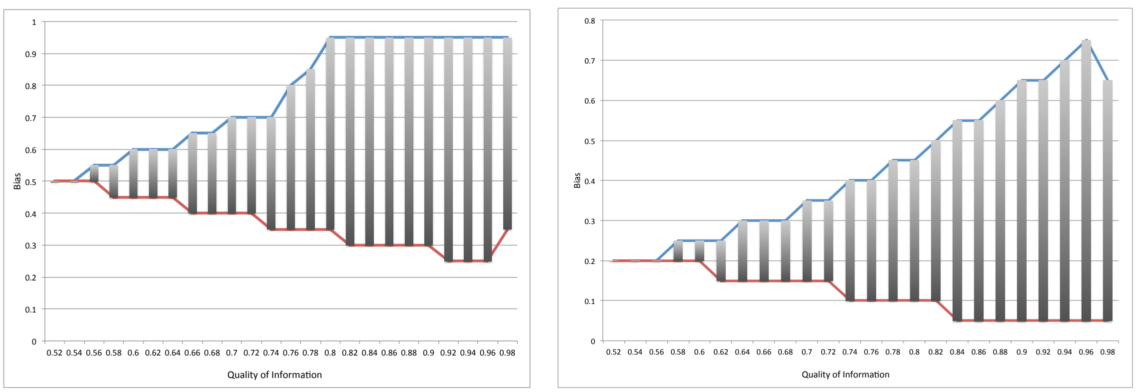
fixed value of $X = x$ in exactly the same way, only with $\hat{p}_v(\vec{v})$, $\hat{\vec{p}}_v$, $p_v(\vec{v})$ and $\vec{p}_v$ replaced by $p_v(\vec{v}|x,\hat{\beta})$, $\vec{p}_v(x,\hat{\beta})$, $p_v(\vec{v}|x,\beta)$ and $\vec{p}_v(x,\beta)$, $(\theta,\vec{\pi},\mu)$ replaced by $(\theta(x),\vec{\pi}(x),\mu(\cdot|\cdot;x))$ and $\hat{\Sigma}_n$ replaced by $\hat{\Sigma}_n(x) = (\partial\vec{p}_v(x,\hat{\beta})/\partial\beta^{'})\hat{\Sigma}_\beta(\partial\vec{p}_v(x,\hat{\beta})/\partial\beta)$, where $\hat{\Sigma}_\beta$ is a consistent estimator of the asymptotic variance of $\sqrt{n}(\hat{\beta}-\beta)$, which can be obtained from the first stage. The consistency and the coverage probability theory go through in the logit case described above as long as Σ_β is invertible.

	$v = (v(i), v(k), v(m))$					
	$v = (1,1,1)$		$v = (1,0,1)$		$v = (0,1,0)$	
	Coef.	Std.Err.	Coef.	Std.Err.	Coef.	Std.Err.
Case Specific:						
FedLaw	-0.124	0.132	-1.001	0.235	-0.304	0.274
Aggravated	-0.216	0.271	-1.261	0.519	0.552	0.560
White Collar	-0.400	0.232	-0.740	0.439	0.449	0.510
Theft	0.024	0.242	-1.371	0.563	1.404	0.510
Narcotics	-0.295	0.244	-0.594	0.474	0.049	0.593
Rep. Majority	0.329	0.179	1.320	0.445	-0.560	0.379
Female	0.043	0.166	0.198	0.346	0.141	0.326
Harvard-Yale Majority	-0.124	0.119	-0.275	0.277	-0.138	0.245
Jury Instruction	-0.120	0.118	-0.906	0.360	0.246	0.216
Sentencing	-0.326	0.130	-0.919	0.384	-0.056	0.267
Admissibility	-0.345	0.100	-0.317	0.229	0.407	0.190
Sufficiency	-0.557	0.116	-0.421	0.277	-0.249	0.226
Judge Specific:						
$J(i)$ Republican	-0.195	0.118	-1.951	0.347	0.641	0.327
$J(i)$ Years of Experience	-0.003	0.004	-0.028	0.010	0.005	0.009
$J(i)$ Prior Judicial Experience	-0.001	0.004	-0.046	0.012	-0.006	0.010
$J(i)$ Prior Political Experience	0.006	0.007	-0.041	0.022	0.034	0.015
$J(i)$ Rep $\times$ Assault	-0.021	0.163	0.920	0.464	0.054	0.399
$J(i)$ Rep $\times$ WhtCol	0.106	0.138	0.844	0.447	-0.217	0.371
$J(i)$ Rep $\times$ Theft	-0.166	0.158	-0.079	0.577	-0.952	0.456
$J(i)$ Rep $\times$ Narctcs	-0.053	0.141	0.419	0.483	-0.088	0.426
$J(k)$ Republican	-0.195	0.118	-0.969	0.363	-0.313	0.431
$J(k)$ Years of Experience	-0.003	0.004	-0.019	0.014	0.014	0.012
$J(k)$ Prior Judicial Experience	-0.001	0.004	-0.004	0.014	-0.027	0.015
$J(k)$ Prior Political Experience	0.006	0.007	-0.047	0.033	-0.036	0.030
$J(k)$ Rep $\times$ Assault	-0.021	0.163	0.738	0.611	-0.108	0.587
$J(k)$ Rep $\times$ WhtCol	0.106	0.138	0.470	0.582	0.539	0.536
$J(k)$ Rep $\times$ Theft	-0.166	0.158	1.752	0.694	-1.451	0.846
$J(k)$ Rep $\times$ Narctcs	-0.053	0.141	0.434	0.618	-0.328	0.630
$J(m)$ Republican	-0.195	0.118	-1.951	0.347	0.641	0.327
$J(m)$ Years of Experience	-0.003	0.004	-0.028	0.010	0.005	0.009
$J(m)$ Prior Judicial Experience	-0.001	0.004	-0.046	0.012	-0.006	0.010
$J(m)$ Prior Political Experience	0.006	0.007	-0.041	0.022	0.034	0.015
$J(m)$ Rep $\times$ Assault	-0.021	0.163	0.920	0.464	0.054	0.399
$J(m)$ Rep $\times$ WhtCol	0.106	0.138	0.844	0.447	-0.217	0.371
$J(m)$ Rep $\times$ Theft	-0.166	0.158	-0.079	0.577	-0.952	0.456
$J(m)$ Rep $\times$ Narctcs	-0.053	0.141	0.419	0.483	-0.088	0.426
Constant	-0.439	0.217	-0.891	0.252	-4.504	0.522

Table 3: First-stage estimates, from a multinomial logit model (baseline vote profile (0,0,0))

mark. For our benchmark case we consider a white collar crime prosecuted under federal law, in which the major legal issue for appeal is admissibility of evidence. Judges 1 and 2 are Republican, and judge 3 is a Democrat (so that the majority of the court is Republican). All three judges are male, and at most one of the judges has a law degree from Harvard or Yale. The three benchmark judges differ in their years of court experience, as well as prior judicial and political experience. (See Table A.3 in the Appendix for the full benchmark specification.)

The left panel of Figure 2 plots points in the estimated identified set (EIS) for an agnostic prior belief, $\rho = 0.5$, which we take as a benchmark to present our results. For simplicity, we begin by presenting results for a symmetric model, in which $\pi_i = \pi$ for all $i \in N$.



$\rho = 0.5$ $\qquad\qquad$ $\rho = 0.2$

Figure 2: Left panel plots points (π, q) in the EIS for $\rho = 0.5$ (left panel) and $\rho = 0.2$ (right panel) in a homogeneous model, where $\pi_i = \pi$ for all $i \in N$.

Two features of the EIS are immediately apparent from the figure. First, as in section 5.1, the range of values of the bias parameter π that are consistent with the data for a given value of competence is increasing in q. Thus, high ability judges can be highly predisposed to uphold or to overturn, but low ability judges must be relatively malleable, willing to overturn (uphold) even when it is slightly more (less) likely than not that the trial court's decision is incorrect ($\pi \approx 1/2$). Second, because the distribution of vote profiles in the data is asymmetric in favor of upholding the decision of the lower court, the EIS for $\rho = 0.5$ is asymmetric towards larger values of π, indicating a higher information hurdle to overturn the decision of the lower court. Thus, with an uninformative prior, malleable judges of all competence levels are consistent with the data, but judges that are highly predisposed to uphold can only be consistent with the data if they are highly competent, and judges that are highly predisposed to overturn are not consistent with the data (irrespective of their competence level).

For comparison purposes, the right panel of Figure 2 plots the EIS for a value of ρ that approximates the empirical frequency of cases in which the court overturned the decision of the lower courts, $\rho = 0.2$. In this case the prior belief that the trial's court decision is flawed is relatively low. Thus, when private signals are not too informative, only judges that are predisposed to *overturn* ($\pi < 1/2$) can vote in a way consistent with the data. To understand this inverse relationship between ρ and π_i among points in the EIS, recall that judge i votes to overturn given information $\mathcal{I}$ if and only if $\Pr^i(\omega = 1|\mathcal{I}) \geq \pi_i$, which can be written in terms of the relative likelihood of the event $\mathcal{I}$ in states $\omega = 1$ and $\omega = 0$ as

$$\frac{\Pr^i(\mathcal{I}|\omega = 1)}{\Pr^i(\mathcal{I}|\omega = 0)} \geq \frac{\pi_i}{1-\pi_i}\frac{1-\rho}{\rho}.$$

The results for the EIS with heterogeneous preferences extend naturally the results of Figure 2 for the symmetric model: while low competence judges must be homogeneous and relatively malleable (willing to uphold or overturn with little supporting information) in order to be consistent with the data, competent judges can have highly *heterogeneous* preferences and still generate a distribution of vote profiles consistent with the data. To illustrate this result in a simple plot, we introduce a measure of preference heterogeneity.

$$H(\vec{\pi}) = \sum_{i \in N} \sum_{j \neq i} (\pi_i - \pi_j)^2.$$

Our index of preference heterogeneity increases as judges' bias parameters are farther apart from one another, reaching a theoretical maximum of two, and decreases as judges' preferences are closer to each other's, reaching a minimum of zero when all judges have the same preferences.

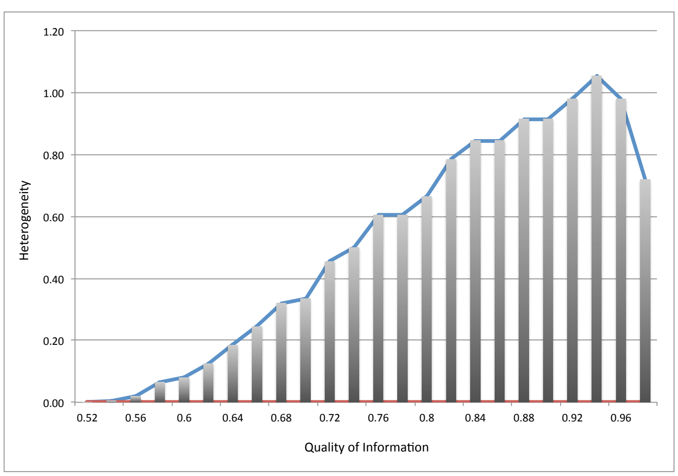
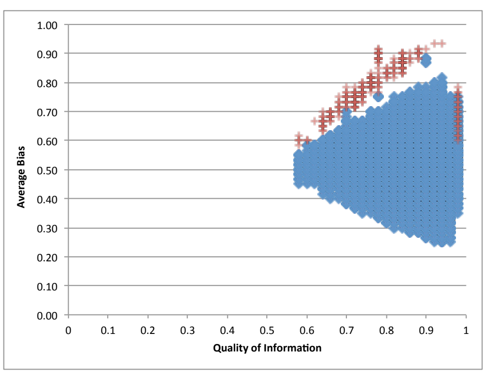


Figure 3: Pairs (H, q) consistent with points $(\vec{\pi}, q)$ in the EIS for $\rho = 0.5$

Figure 3 plots pairs of quality of information and preference heterogeneity that are consistent with points $(\vec{\pi}, q)$ in the EIS for $\rho = 0.5$. For low quality, only very homogeneous courts $(H \to 0)$ are consistent with the data, but as competence increases the allowed heterogeneity in preferences increases as well, reaching values close to 1 for high levels of q.

This result is interesting in its own right, because it clarifies that high unanimity rates (a feature of our voting data) do not *imply* common interests at an ex ante stage. Thus, neither preference homogeneity nor external motives, such as an intrinsic desire to compromise or to put forward a "unified" stance in each case, are required to rationalize the data. While low quality judges would agree as much as they do in the data only if they had very similar preferences, deliberation among *competent* judges can generate the high frequency of unanimous votes observed in the data without requiring these auxiliary motives.

Comparative Statics. In the discussion above, we have focused on the benchmark case and court characteristics. It should be clear, however, that both the confidence set and the set of equilibrium outcomes for each point in the confidence set are functions of the observable characteristics that enter the first stage multinomial logit model. Thus, proceeding as above, we can quantify the changes in types and outcomes associated with alternative configurations of the cases under consideration or the judges integrating the court. To illustrate this, we evaluate the effect of changing the nature of the crime considered in the case from a *White Collar* crime to *Theft* on judges' preferences: are justices more or less predisposed to overturn the lower court in Theft cases?



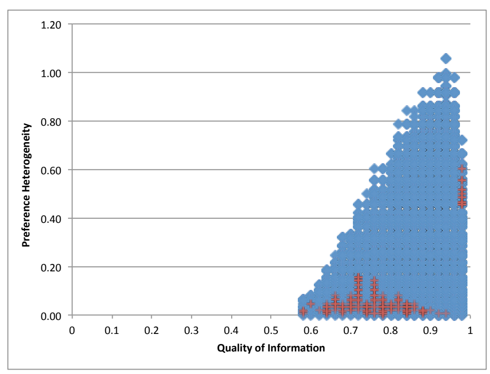


Figure 4: Points in White Collar EIS not in the Theft EIS (blue), and points in the Theft EIS not in the White Collar-EIS (red). Y-axis plots the average bias (left panel) and preference heterogeneity (right panel). X-axis plots quality of information q.

The results are illustrated in Figure 6.2. The top figures show points in "White Collar EIS" not in the "Theft EIS" (blue), and points in the Theft EIS not in the White Collar-

EIS (red). The figures suggest that changing from White-Collar to Theft crimes does not necessarily make the average judge more or less prone to overturning the lower court (top left), but that it does reduce the level of disagreement in the court. As the top right panel shows, the Theft EIS excludes the more heterogeneous courts in the White Collar EIS.

6.3 Equilibrium Outcomes with Deliberation

In the previous section we described the set of characteristics of members of the court that are consistent with the data (points $(\theta, \vec{\pi})$ in the EIS). We now use these results to evaluate the set of *outcomes* that are consistent with the data. We know the voting probabilities, of course. This is in fact the information we used to estimate the EIS in the first place. But knowing the set of parameters consistent with the data allows us to compute more interesting measures of payoff-relevant outcomes. In particular, we focus on the probability that the court reaches an incorrect decision after deliberating and voting strategically.

Note that for any given point $(\theta, \vec{\pi}) \in \mathcal{A}_0$, and any communication equilibrium $\mu \in M(\theta, \vec{\pi})$, we can compute the probability that the court reaches an incorrect decision, $\varepsilon(\mu, (\theta, \vec{\pi}))$. This probability of error $\varepsilon(\mu, (\theta, \vec{\pi}))$ is the weighted average of the type-I error (overturn when should uphold), $\varepsilon_I(\mu, (\theta, \vec{\pi})) = \Pr(v = 1|\omega = 0) = \sum_{\vec{t}} \sum_{\vec{v}:v=1} \mu(\vec{v}|\vec{t})p(\vec{t}|w = 0)$, and the type-II error (uphold when should overturn) $\varepsilon_{II}(\mu, (\theta, \vec{\pi})) = \Pr(v = 0|\omega = 1) = \sum_{\vec{t}} \sum_{\vec{v}:v=0} \mu(\vec{v}|\vec{t})p(\vec{t}|w = 1)$; i.e.,

$$\varepsilon(\mu, (\theta, \vec{\pi})) = (1 - \rho)\varepsilon_I(\mu, (\theta, \vec{\pi})) + \rho\varepsilon_{II}(\mu, (\theta, \vec{\pi})).^{21} \tag{6.1}$$

For each point $(\theta, \vec{\pi}) \in \mathcal{A}_0$ there are in fact multiple equilibria $\mu \in M(\theta, \vec{\pi})$, each being associated with a certain probability of error $\varepsilon(\mu, (\theta, \vec{\pi}))$ computed as in (6.1). Thus, for each point in the EIS, there is a set of error probabilities that can be attained in equilibrium. In order to describe the range of possible equilibrium outcomes for court configurations consistent with the data, we focus on the maximum and minimum equilibrium probability of error for each point in the EIS.

There are two possible sets of such bounds that a researcher might find valuable, depending on the question at hand. First, we can compute the maximum and minimum error probabilities across equilibria that are *consistent with the observed data* $\vec{p}_v$, $\overline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$, and $\underline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$. These bounds rule out error probabilities that are either not attainable in equilibrium given the parameters $(\theta, \vec{\pi})$, or are attainable by mixtures of equilibria that

[21] Note that both the type-I error and the type-II error are functions of the model parameters $\mu, \theta, \vec{\pi}$, and inference on them amounts to projecting the EIS of the model parameters onto the range of these functions.

would lead to a distribution over vote profiles that differs from the one observed in the data. Formally, for each point $(\theta, \vec{\pi})$ in the EIS, and data $\vec{p}_v$, we define

$$\overline{\varepsilon}^*(\theta, \vec{\pi}, p_v) \equiv \max_{\mu \in M(\theta, \vec{\pi})} \left\{ \varepsilon(\mu, (\theta, \vec{\pi})) \quad \text{s.t.} \quad p_v(\vec{v}) = \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t}; \theta) \right\}, \tag{6.2}$$

and similarly for $\underline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$.[22]

A second, more expansive criterion, is to consider the maximum and minimum probability of error across *all equilibria*,

$$\overline{\varepsilon}(\theta, \vec{\pi}) \equiv \max_{\mu \in M(\theta, \vec{\pi})} \varepsilon(\mu, (\theta, \vec{\pi})), \quad \text{and} \quad \underline{\varepsilon}(\theta, \vec{\pi}) \equiv \min_{\mu \in M(\theta, \vec{\pi})} \varepsilon(\mu, (\theta, \vec{\pi})). \tag{6.3}$$

Unlike expression (6.2), expression (6.3) includes error probabilities that are attainable through equilibria that are not consistent with the observed data. The logic behind (6.3) is that equilibrium selection in a given sample is not informative about equilibrium selection in a counterfactual (or in a different sample). Thus, although in the particular data at hand we can rule out that these equilibria were played for these parameter values, it is conceivable that these outcomes can be produced if judges were to play a different selection of equilibria in a counterfactual.

Figure 5 plots the minimum and maximum probability of error in equilibria consistent with the data for pairs of preference heterogeneity and competence (H, q) consistent with points in the EIS for $\rho = 0.5$.[23]

Consider first the minimum error probability, on the left panel. For low competence, q, only very homogeneous courts, composed entirely of malleable judges, are consistent with the data. These courts are highly inaccurate, even after pooling information, and correspondingly make wrong decisions very often (about 45% of the time as $q \to 1/2$). As ability increases, however, more heterogeneous courts can also be consistent with the data. These more able courts are capable of producing decisions with a much lower error rate, even when they are quite heterogeneous.

The right panel focuses on the maximum probability of error in equilibria consistent with the data. The difference between the best and worst equilibria is small for homogeneous

[22]Note that because $M(\theta, \vec{\pi})$ is a convex set and the constraint $p_v(\vec{v}) = \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t}; \theta)$ is linear in μ, μ can be replaced with a linear combination of elements in $M(\theta, \vec{\pi})$ without affecting the value of $\overline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$ or $\underline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$. Therefore, when considering equilibria consistent with the data, we are not assuming that the same equilibrium is played in every case.

[23]When there are multiple points $(\vec{\pi}, q)$ such that $H(\vec{\pi}) = H$, the figure plots the average of extrema across these points.

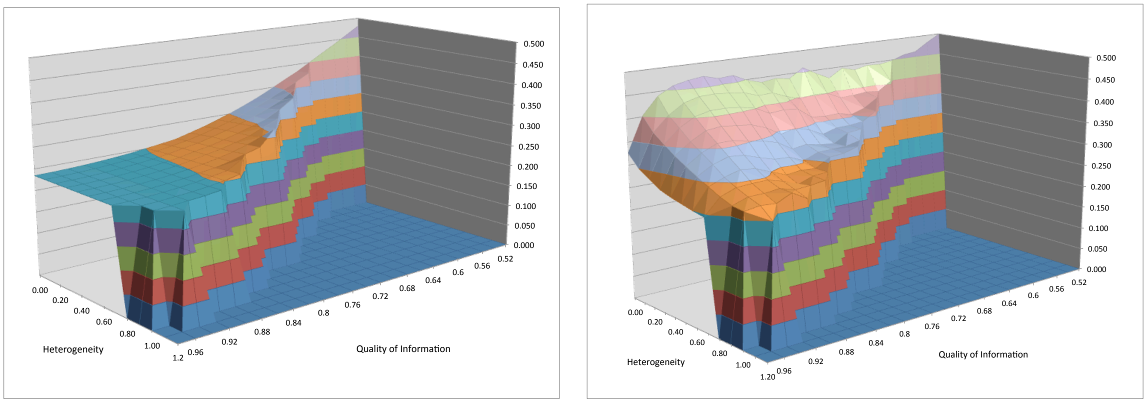


Figure 5: Minimum and maximum probability of error in equilibria consistent with the data, for pairs of preference heterogeneity and competence (H, q) consistent with points in the EIS for $\rho = 0.5$. (Average of extrema across points $(\vec{\pi}, q)$ such that $H(\vec{\pi}) = H$).

courts of low competence and heterogeneous courts with high competence, but is relatively large for courts composed of competent judges with aligned preferences. This is because errors in the worst equilibrium remain high as ability increases precisely when courts are homogeneous. In fact, the last column of Table 2 shows that the example in Section 3 generates a vote distribution equal to the one observed in the data. On the other hand, the maximum equilibrium probability of error decreases sharply with the heterogeneity of the court when courts are competent. Thus, heterogeneous courts can be rationalized as generating the observed voting data, but only if they are competent *and* play equilibria in which they use their information effectively.

Figure 6 reproduces Figure 5 across all equilibria. The right panel of Figure 6 plots the mapping of points in the EIS to the *maximum* probability of error. As the figure illustrates, $\overline{\varepsilon}(\cdot)$ is qualitatively similar to the maximum probability of error across equilibria consistent with the data $\overline{\varepsilon}^*(\cdot, p_v)$.

The left panel plots the minimum equilibrium probability of error across all equilibria. As before, the rate of errors in the best equilibria decreases with competence, but now the minimum error probability approaches zero as $q \to 1$, even when judges are highly heterogeneous. This contrasts with the results for equilibria consistent with the data, in which the minimum equilibrium error probability was bounded above 20%, even as $q \to 1$. The intuition for this result is straightforward. Note that in a large sample, given a prior $\rho = 0.5$, an unbiased high quality court playing the best equilibrium would uphold roughly 50% of the time. Recall however that in the data, the court upholds more than 75% of the time. This means that a court can only match the data by making a relatively large

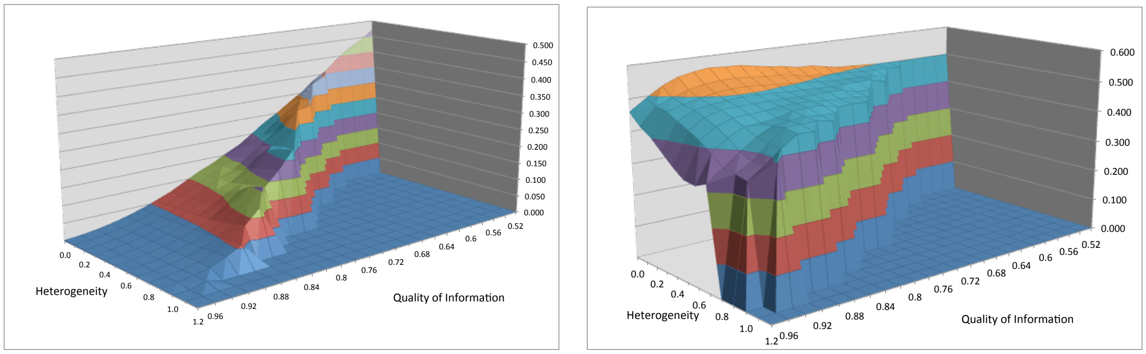


Figure 6: Minimum and maximum probability of error in all equilibria, for pairs of preference heterogeneity and competence (H, q) consistent with points in the EIS for $\rho = 0.5$. (Average of extrema across points $(\vec{\pi}, q)$ such that $H(\vec{\pi}) = H$).

number of errors.

The logic is further emphasized in Figure A.2 in the Appendix, which reproduces Figures 5 and 6 for a prior of $\rho = 0.2$. Since in this case the prior is close to the frequency with which the court overturns the trial courts in the data, the minimum and maximum probability of error in the equilibria consistent with the data is lower overall, and the probability of error in the best equilibrium consistent with the data goes to zero as q goes to one. With this caveat, the mapping of court characteristics to equilibrium outcomes with $\rho = 0.2$ is qualitatively similar to that for $\rho = 0.5$.

6.4 The Impact of Deliberation

Having described the outcomes attained in equilibria with deliberation, our next goal is to quantify the effect of deliberation: how much do outcomes differ *because of* deliberation?

To do this, we compare equilibrium outcomes with deliberation with the outcomes that would have arisen in a counterfactual scenario in which judges are not able to talk with one another before voting. In particular, for each point $(\theta, \vec{\pi})$ in the estimated identified set, we compare the maximum and minimum error probabilities in equilibria with deliberation with the corresponding maximum and minimum error probabilities in *responsive* Bayesian Nash equilibria (BNE) of the voting game *without* communication, $\overline{\varepsilon}^{ND}(\theta, \vec{\pi})$ and $\underline{\varepsilon}^{ND}(\theta, \vec{\pi})$.

To carry out this comparison, we solve for all responsive BNE of the non-deliberation game, for all parameter points in the EIS. In the game without deliberation, the strategy of player i is a mapping $\sigma_i : \{0, 1\} \to [0, 1]$, where $\sigma_i(t_i)$ denotes the probability of voting to overturn given signal t_i. A BNE is a strategy profile σ such that each judge i's strategy is a best response to the voting strategy of the other judges in the court. In particular, it is easy to show that $\sigma_i(t_i) > 0$ (< 1) only if $\Pr(\omega = 1|t_i, Piv^i) \geq \pi_i$ ($\leq \pi_i$), or

$$\frac{\Pr(t_i|\omega = 1)}{\Pr(t_i|\omega = 0)} \frac{\Pr(Piv^i|\omega = 1; \sigma)}{\Pr(Piv^i|\omega = 0; \sigma)} \geq \frac{\pi_i}{1 - \pi_i} \frac{1 - \rho}{\rho} \tag{6.4}$$

Following the convention in the literature, we say that a BNE equilibrium σ is responsive if the probability that the court overturns the decision of the lower court is not invariant to judges' private information. More specifically, let $\Pr(v = 1|\vec{t}; \sigma)$ denote the probability that the court overturns the decision of the lower court when judges receive signals $\vec{t}$, and vote according to σ. Then a BNE σ is responsive if there exist two signal realizations, $\vec{t}$ and $\vec{t'}$ such that $\Pr(v = 1|\vec{t}; \sigma) \neq \Pr(v = 1|\vec{t'}; \sigma)$. Characterizing responsive equilibria in the non-deliberation game is straightforward but somewhat cumbersome, because the set of BNE is not convex and a number of different strategy profiles can form a BNE for different parameter values (e.g., all judges mix after a one signal and uphold after a zero signal, two judges mix after a one signal and uphold after a zero signal while the third overturns, etc). We discuss this further in the Appendix (Section 8.2).

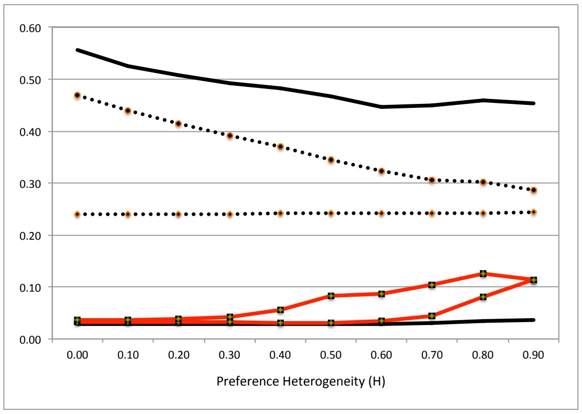
We begin by contrasting the probability of error with and without deliberation for all comparable points in the EIS; i.e., for points in the EIS in which there exists a responsive equilibrium of the game without deliberation. We focus first on how the effect of deliberation changes as a function of initial disagreement among judges in the panel.

Figure 7 plots the maximum and minimum equilibrium probability of error with and without deliberation for various levels of information precision q and prior beliefs ρ. The bounds on equilibrium errors are plotted as a function of the degree of preference heterogeneity in the court, for levels of heterogeneity consistent with points in the EIS (values H such that $H = H(\vec{\pi})$ for some $(\vec{\pi}, \theta) \in \mathcal{A}_0$). Each panel plots the maximum and minimum probability of error in (i) all equilibria with deliberation (black), (ii) in equilibria of the voting game with deliberation that are consistent with the data (grey; round marker), and (iii) in responsive equilibria without deliberation (red, cross marker).[24]

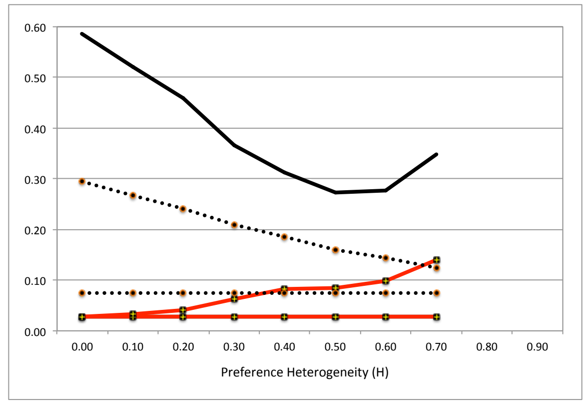
The results show that deliberation can be useful when the court is heterogeneous, but will generally be either ineffectual (all equilibria) or counterproductive (eq. consistent with the data) when the court is relatively homogeneous.

Consider first all equilibria with deliberation. Note that since there is always a "babbling"

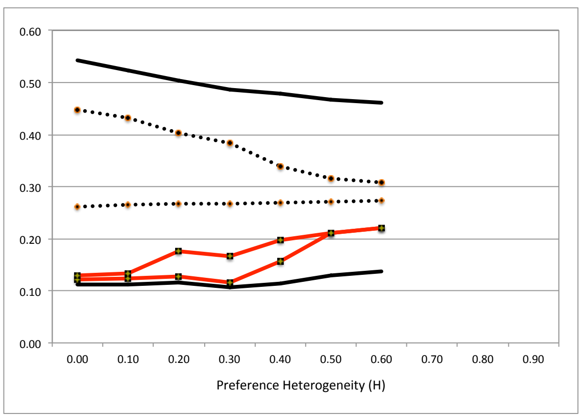
[24]Note that the maximum level of preference heterogeneity consistent with the data is increasing in q (Figure 3). As a result, the solid black lines extend for a larger range of values of H as we move from the figures in the bottom (for $q = 0.70$), to the figures in the top (for $q = 0.90$).



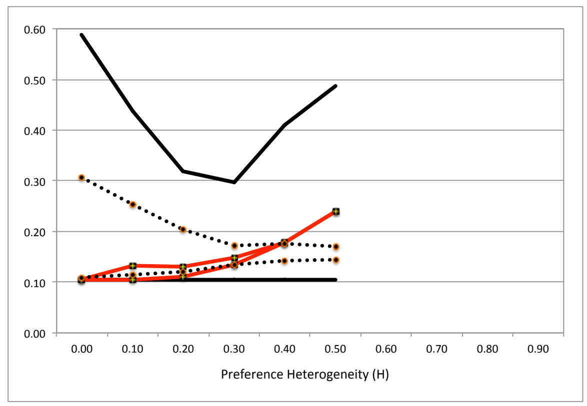
$(\rho = 0.5; q = 0.90)$



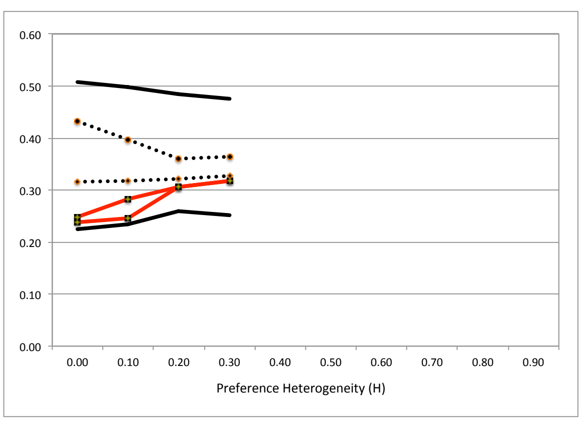
$(\rho = 0.2; q = 0.90)$



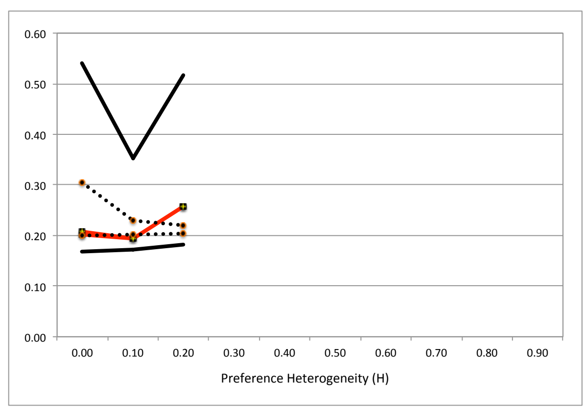
$(\rho = 0.5; q = 0.80)$



$(\rho = 0.2; q = 0.80)$



$(\rho = 0.5; q = 0.70)$



$(\rho = 0.2; q = 0.70)$

Figure 7: Probability of mistakes with and without deliberation for values of preference heterogeneity H consistent with points $(\vec{\pi}, \theta)$ in the EIS for $\rho = 0.5$ (left), and $\rho = 0.2$ (right). Min. and max. eq. probability of error in (i) all equilibria with deliberation (solid black), (ii) in equilibria with deliberation consistent with the data (dotted), and (iii) in responsive equilibria without deliberation (solid red, with marker).

equilibrium, in which all messages are interpreted as uninformative, this set includes the set of equilibria without deliberation. Thus, $\underline{\varepsilon}(\theta, \vec{\pi}) \leq \underline{\varepsilon}^{ND}(\theta, \vec{\pi}) \leq \overline{\varepsilon}^{ND}(\theta, \vec{\pi}) \leq \overline{\varepsilon}(\theta, \vec{\pi})$ for all points $(\theta, \vec{\pi})$ in the EIS. Still, the comparison allows us to put an upper bound on the potential gain or loss that can be attributed to deliberation under any equilibrium selection rule.[25] In fact, Figure 7 shows that when the court is relatively homogeneous (H small), the biggest possible improvement that can be attributed to deliberation is fairly small, under any possible equilibrium selection rule one could use. On the other hand, as the heterogeneity of preferences increases, mistakes in equilibria without deliberation become more frequent (both $\underline{\varepsilon}^{ND}(\theta, \vec{\pi})$ and $\overline{\varepsilon}^{ND}(\theta, \vec{\pi})$ shift up) while the probability of error in the best equilibrium with deliberation remains flat. This shows that at least for some initial prior beliefs, deliberation can have a non-negligible positive effect on outcomes when the level of initial disagreement in the court is relatively high.

Considering equilibria consistent with the data allows a more straightforward assessment of the effect of deliberation. Because the set of outcomes of voting with deliberation in equilibria consistent with the data does not necessarily include the set of outcomes of voting without deliberation, nor is it ranked relatively to it in any way ex ante, the comparison with the counterfactual allows a more conclusive evaluation of the effect of deliberation.

The results reinforce our previous conclusions. As the figures show, in fact, for low levels of conflict *all* responsive equilibria of the game without deliberation lead to a lower probability of mistakes than *all* equilibria consistent with the data of voting with deliberation. Thus, when courts are relatively homogeneous, pre-vote deliberation leads to a larger incidence of errors than responsive equilibria without deliberation, even when we consider the best possible equilibrium with deliberation and the worst responsive equilibrium without deliberation.

As before, this result changes when conflict of interests among judges increases. This is due to two effects. First, as we have seen already, the probability of error in voting without deliberation increases with the heterogeneity of preferences in the court. In addition, the maximum probability of error of voting with deliberation in equilibria consistent with the data decreases with heterogeneity (recall Figure 5). For $\rho = 0.5$, this implies that the negative effect of deliberation on outcomes diminishes with heterogeneity. But for $\rho = 0.2$, where the errors in equilibria consistent with the data are lower to begin with, this

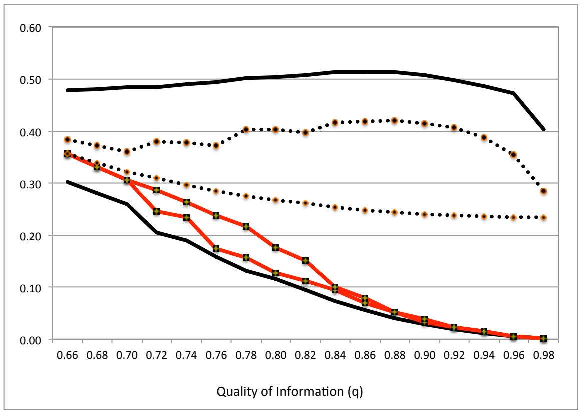
[25] Recall that our equilibrium concept in the game with deliberation is agnostic about the possible communication protocol judges might be using. Thus, a large gain/loss can be due to different equilibrium behavior for a fixed communication protocol, or to our ignorance about which particular communication protocol judges could be using. On the other hand, we know that the equilibrium outcomes of any communication protocol judges could be using is contained in the set of outcomes of communication equilibria. Thus, the maximum gain/loss that can be attributed to deliberation provides an upper bound on the effect of deliberation.

means that deliberation actually improves on no-deliberation when the court is sufficiently heterogeneous. Overall, the results indicate that voting after deliberation can reduce errors when courts are sufficiently heterogeneous, but leads to more erroneous decisions than what we would obtain in responsive equilibria without deliberation when courts are homogeneous.

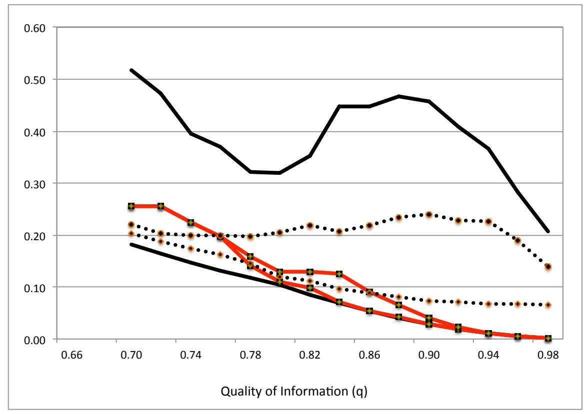
Figure 8 presents the results from a different perspective. The panels in the figure reproduce the structure of Figure 7, but they do so for a fixed level of heterogeneity of preferences and prior beliefs, plotting errors as a function of the level of quality of information in the court. As the figures illustrate, deliberation tends to increase errors in the court when judges' private information is very precise: the probability of mistakes in all responsive equilibria without deliberation is fairly small, and very close to the minimum probability of error across all equilibria with deliberation. Moreover, the probability of errors in all equilibria without deliberation is significantly lower than the probability of errors with pre-vote deliberation in any equilibrium consistent with the data. However, the relative efficacy of voting with deliberation increases as judges' private information gets less precise. In fact, for some values in the EIS for $\rho = 0.2$, deliberation dominates no deliberation when judges' information is sufficiently imprecise. This result is intuitive: exchanging information before the vote can help precisely when it allow judges to overcome deficiencies in their own private information.

Figures 7 and 8 show that pre-vote deliberation can be beneficial when the court is heterogeneous or the quality of justices' private information is low. But just how typical is such a court configuration in comparable points in the EIS? Figure A.3 in the Appendix plots the correspondence between the min/max probability of error in responsive equilibria without deliberation and the min/max probability of error in equilibria with deliberation, for both all equilibria and equilibria consistent with the data. The figure shows that while deliberation typically leads to higher error rates than what can be achieved in responsive equilibria without deliberation, it can also be beneficial in a relatively large number of points in the EIS, in particular when $\rho = 0.2$. This reassures us that the picture presented in Figures 7 and 8 is representative of the results across the EIS.

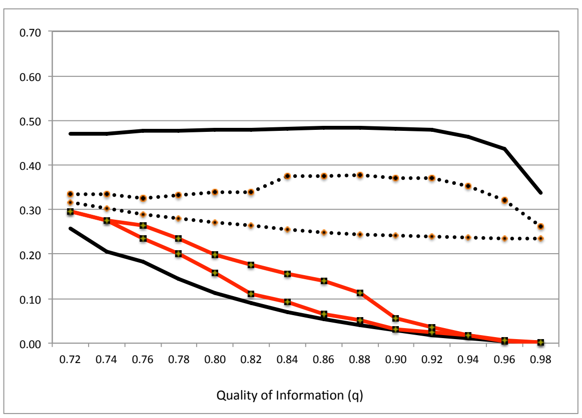
Confidence Set. Up to this point, we have restricted the comparison between communication equilibria and responsive equilibria without deliberation to points in the estimated identified set. These are court types that are consistent with the point estimate of the vote probabilities, $\hat{p}(X)$. When we incorporate the uncertainty in our estimate of the true vote probability $p(X)$, the set of types that are consistent with the data is given by the confidence set CS. Figure A.4 in the Appendix reproduces Figure 7 for all points in the confidence set. Our main conclusion remains unaltered: deliberation can be useful when judges' preferences



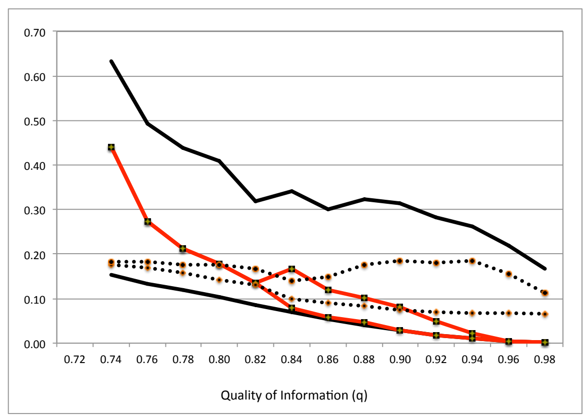
$(\rho = 0.5; H = 0.20)$



$(\rho = 0.2; H = 0.20)$



$(\rho = 0.5; H = 0.40)$



$(\rho = 0.2; H = 0.40)$

Figure 8: Probability of mistakes with and without deliberation as a function of the quality of information q, for points $(\vec{\pi}, q)$ in the EIS for $\rho = 0.5$ (left), and $\rho = 0.2$ (right). Min. and max. eq. probability of error in (i) all equilibria with deliberation (solid black), (ii) in equilibria with deliberation consistent with the data (dotted), and (iii) in responsive equilibria without deliberation (solid red, with marker).

are heterogeneous, but will generally be either ineffectual (if we consider all equilibria with deliberation) or counterproductive (for equilibria consistent with the data) when the court is relatively homogeneous. A similar observation holds for the results in Figure 8.

Non-responsive Equilibria. In the results above, we compared the probability of incorrect decisions in equilibria with deliberation with the corresponding probability of mistakes in responsive Nash equilibria of the voting game without communication. In some points in the EIS, however, the voting game without deliberation only admits unresponsive BNE, where the decision of the court does not depend on judges' private information (i.e., at least two judges vote unconditionally to overturn or uphold). In these unresponsive BNE, the minimum and maximum probabilities of error equal $\min\{\rho, 1-\rho\}$ and $\max\{\rho, 1-\rho\}$ respectively. We should therefore keep in mind that in addition to any positive effect deliberation can have on outcomes relative to responsive equilibria of voting without deliberation, pre-vote communication also expands the set of court configurations for which equilibrium outcomes are responsive to private information. Indeed, in close to 14% of points in the EIS for $\rho = 0.5$ and 8% of points in the EIS for $\rho = 0.2$, private information is too imprecise to overcome differences in preferences, and the only equilibrium of voting without deliberation is completely unresponsive to judges' private signals.

6.4.1 Efficient Deliberation

The results so far are agnostic about equilibrium selection. It could be argued, however, that equilibria that maximize judges' aggregate welfare constitute a focal point, both in the game with deliberation and in the game without deliberation. If this were the case, deliberation could in fact improve welfare, and would certainly do so if we don't restrict to equilibria consistent with the data. In order to quantify this potential gain, we adopt a utilitarian approach, and compare social welfare in the equilibria that maximize the sum of judges' payoffs with and without deliberation, for equilibria consistent with the data, and all equilibria.

For a given point $(\theta, \vec{\pi})$, and given a communication equilibrium μ, judge i's expected utility is given by the expected cost of type I and type II errors,

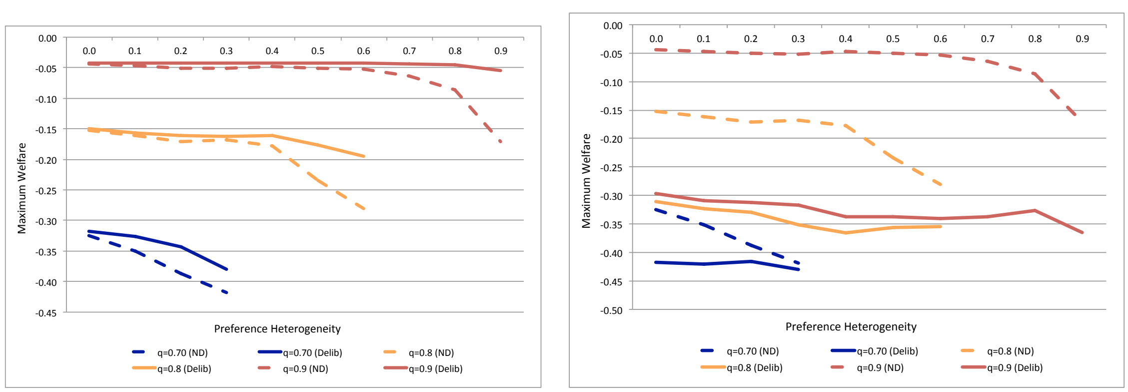
$$U_i(\mu; (\theta, \vec{\pi})) = -\left[\rho\varepsilon_{II}(\mu; (\theta, \vec{\pi}))(1-\pi_i) + (1-\rho)\varepsilon_I(\mu; (\theta, \vec{\pi}))\pi_i\right].$$

Therefore, the equilibrium that maximizes judges' total welfare, $\mu^*(\theta, \vec{\pi})$, is the $\mu \in M(\theta, \vec{\pi})$ that maximizes $\mathcal{U}(\theta, \vec{\pi}, \mu) \equiv \sum_i U_i(\mu; (\theta, \vec{\pi}))$. A similar definition applies for non-deliberation equilibria, giving $\sigma^*(\theta, \vec{\pi})$. For equilibria consistent with the data, the equilibrium that maximizes judges' total welfare, $\tilde{\mu}(\theta, \vec{\pi})$, is

$$\tilde{\mu}(\theta, \vec{\pi}) = \arg \max_{\mu \in M(\theta, \vec{\pi})} \left\{ \mathcal{U}(\theta, \vec{\pi}, \mu) \quad \text{s.t.} \quad p_v(\vec{v}) = \sum_{\vec{t}} \mu(\vec{v}|\vec{t}) p(\vec{t}; \theta) \right\}$$

The left panel of Figure 9 plots the maximum aggregate welfare for points in the EIS for $\rho = 0.5$ across all equilibria of the game with deliberation, $\mathcal{U}^D(\theta, \vec{\pi}) \equiv \mathcal{U}(\mu^*(\theta, \vec{\pi}); (\theta, \vec{\pi}))$, and in the game without deliberation, $\mathcal{U}^N(\theta, \vec{\pi}) \equiv \mathcal{U}(\sigma^*(\theta, \vec{\pi}); (\theta, \vec{\pi}))$. The difference is plotted for various levels of competence q, as a function of the degree of preference heterogeneity in the court.

As can be anticipated from our previous results, the plot shows that the gain from *efficient deliberation* is fairly small, and concentrated at higher levels of competence and preference heterogeneity. Consider for example the highest competence level plotted in the figure ($q = 0.9$). For all levels of heterogeneity $H \leq 0.8$, the change in aggregate welfare attained by introducing efficient deliberation is smaller than the change in welfare that would result from increasing competence from $q = 0.80$ to $q = 0.90$, or from $q = 0.70$ to $q = 0.80$. Only at $H = 0.9$ is the gain from efficient deliberation relatively high, exceeding the change in welfare that would result from increasing competence from $q = 0.80$ to $q = 0.90$ at low levels of preference heterogeneity.



All Equilibria | Equilibria Cons. w/Data

Figure 9: Maximum Equilibrium Welfare under Deliberation and No-Deliberation. Solid lines denote outcomes with deliberation, for $\rho = 0.5$. Dashed lines denote outcomes with no deliberation.

The right panel provides a similar comparison restricting to the maximum aggregate welfare across equilibria consistent with the data, $\tilde{\mathcal{U}}^D(\theta, \vec{\pi}) \equiv \mathcal{U}(\tilde{\mu}(\theta, \vec{\pi}); (\theta, \vec{\pi}))$. The results are dramatically different. For relatively homogeneous courts ($H \leq 0.5$), aggregate welfare at the efficient equilibrium without deliberation for a moderate competence level, $q = 0.80$, actually exceeds aggregate welfare at the most efficient equilibrium with deliberation that is consistent with the data at $q = 0.90$. As the plot shows, the change in welfare is more severe at higher levels of competence. In fact, at $q = 0.9$, the loss of welfare due to deliberation is larger than the change in aggregate welfare that would result from increasing competence from $q = 0.70$ to $q = 0.90$ in equilibria of voting with deliberation consistent with the data.

A similar analysis can be done for $\rho = 0.2$. The previous results show, however, that our previous conclusions do not change when we consider efficient deliberation.

7 Conclusion

Deliberation is ubiquitous in collective decision-making. What is less clear is whether talking can have an effect on what people actually do. In this paper, we quantify the effect of deliberation on collective choices. To do this we structurally estimate a model of voting with deliberation, allowing us to disentangle committee members' preferences, information, and strategic considerations, and ultimately, to compare equilibrium outcomes under deliberation with a counterfactual scenario in which pre-vote communication is precluded.

Because the structural parameters characterizing judges' biases and quality of information are only partially identified, we obtain confidence regions for these parameters using a two-step estimation procedure that allows flexibly for characteristics of the alternatives and the individuals. We find that deliberation can be useful when judges tend to disagree ex ante and their private information is relatively imprecise; otherwise, it tends to reduce the effectiveness of the court. These findings extend the reach of previous theoretical results, and complement findings from laboratory experiments.

Our analysis may be extended in various ways. In this paper, we have been largely agnostic regarding the specific communication protocols used in US appellate courts, and for this reason we focused on communication equilibria because the set of outcomes induced by communication equilibria coincides with the set of outcomes induced by sequential equilibria of any possible communication sequence. In other committee voting settings, however, we may be able to restrict attention to a particular communication protocol; in those cases, equilibrium analysis may yield more precise predictions, which would allow us to further tighten the identified set of parameters and predictions about the effects of deliberation. This we leave for future explorations.

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8 Appendix

8.1 Proofs

Proof of Theorem 1. Because $p(\vec{t},\theta) = p(\vec{t}|w = 1;\theta)\rho + p(\vec{t}|w = 0;\theta)(1-\rho)$ is continuously differentiable in θ, Theorem 2.1 of Shi and Shum (2012) applies and shows that $d_H(\hat{\mathcal{B}}_n, \mathcal{B}_0) \to_p 0$, where

$$\hat{\mathcal{B}}_n = \{(\theta,\vec{\pi},\vec{\mu}) \in \mathcal{B} : Q_n(\theta,\vec{\pi},\mu;W_n) = \min_{(\theta,\vec{\pi})\in\Theta\times[0,1]^3} Q_n(\theta,\vec{\pi},W_n)\}, \tag{8.1}$$

where $Q_n(\theta,\vec{\pi},\mu;W_n)$ is defined like $Q(\theta,\vec{\pi},\mu;W)$ but with $\vec{p}$ and W replaced by $\hat{\vec{p}}$ and W_n. Because $\hat{\mathcal{A}}_n$ and $\mathcal{A}_0$ are the projections of $\hat{\mathcal{B}}_n$ and $\mathcal{B}_0$ onto their first $d_\theta + 3$ dimension, respectively, we have $d_H(\hat{\mathcal{A}}_n, \mathcal{A}_0) \to_p 0$. $\square$

Proof of Theorem 2. (a) For any sequence $\{(\theta_n,\vec{\pi}_n) \in \mathcal{A}_0\}_{n=1}^\infty$, there exists $\{\mu_n \in M(\theta_n,\vec{\pi}_n)\}_{n=1}^\infty$ such that $\vec{p}_v = P_t(\theta_n)\vec{\mu}_n$. Thus, $nQ_n(\theta_n,\vec{\pi}_n,\mu_n;\hat{\Sigma}_n^{-1}) = n(\hat{\vec{p}}_v - \vec{p}_v)'\hat{\Sigma}_n^{-1}(\hat{\vec{p}}_v - \vec{p}_v) \to_d \mathcal{X}^2(7)$. Thus

$$\begin{aligned}\Pr((\theta_n,\vec{\pi}_n) \in CS_n(1-\alpha)) =& \Pr(T_n(\theta_n,\vec{\pi}_n) \le \chi^2_{7,\alpha})\\ &\ge \Pr(nQ_n(\theta_n,\vec{\pi}_n,\mu_n;\hat{\Sigma}_n^{-1}) \le \chi^2_{7,\alpha})\\ &\to \Pr(\chi^2(7) \le \chi^2_{7,\alpha}) = 1-\alpha.\end{aligned} \tag{8.2}$$

This implies part (a).

(b) Part (b) holds because

$$\begin{aligned}\Pr(\mathcal{A}_0 \subseteq CS_n(1-\alpha)) &= \Pr(\sup_{(\theta,\vec{\pi})\in\mathcal{A}_0} T_n(\theta,\vec{\pi}) \le \chi^2_{7,\alpha})\\ &\ge \Pr(\sup_{(\theta,\vec{\pi},\mu)\in\mathcal{B}_0} nQ_n(\theta,\vec{\pi},\mu;\hat{\Sigma}_n^{-1}) \le \chi^2_{7,\alpha})\\ &= \Pr(n(\hat{\vec{p}}_v - \vec{p}_v)'\hat{\Sigma}_n^{-1}(\hat{\vec{p}}_v - \vec{p}_v) \le \chi^2_{7,\alpha})\\ &\to \Pr(\chi^2(7) \le \chi^2_{7,\alpha}) = 1-\alpha,\end{aligned} \tag{8.3}$$

where the second equality holds because for all $(\theta,\vec{\pi},\mu) \in \mathcal{B}_0$, $\vec{p}_v = P_t(\theta)\vec{\mu}$. $\square$

8.2 Appendix: Responsive Equilibria without Deliberation

In Section 6.4 we compare the equilibrium probability of error in voting with deliberation with the corresponding equilibrium probability of error that would have occurred in the absence of deliberation for the same court and case characteristics. Specifically, for each point $(\theta, \vec{\pi})$ in the confidence set we compare the maximum and minimum error probabilities across all equilibria, $\overline{\varepsilon}(\theta, \vec{\pi})$ and $\underline{\varepsilon}(\theta, \vec{\pi})$, and across equilibria consistent with the data, $\overline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$ and $\underline{\varepsilon}^*(\theta, \vec{\pi}, p_v)$, with the corresponding maximum and minimum error probabilities in responsive Bayesian Nash equilibria (BNE) of the voting game *without* communication, $\overline{\varepsilon}^{ND}(\theta, \vec{\pi})$ and $\underline{\varepsilon}^{ND}(\theta, \vec{\pi})$. To carry out this comparison, we solve for all responsive BNE of the non-deliberation game, for all parameter points in the confidence set.

In the game without deliberation, the strategy of player i is a mapping $\sigma_i : \{0,1\} \to [0,1]$, where $\sigma_i(t_i)$ denotes the probability of voting to overturn given signal t_i. It is easy to show that $\sigma_i(t_i) > 0$ (< 1) only if $\Pr(\omega = 1 | t_i, Piv^i) \geq \pi_i$ $(\leq \pi_i)$, or

$$\frac{\Pr(t_i|\omega = 1)}{\Pr(t_i|\omega = 0)} \frac{\Pr(Piv^i|\omega = 1)}{\Pr(Piv^i|\omega = 0)} \geq \frac{\pi_i}{1 - \pi_i} \frac{1 - \rho}{\rho} \tag{8.4}$$

Let $\alpha_{i\omega} \equiv \Pr(v_i = 1|\omega)$ denote the conditional probability that i votes to overturn in state ω, and note that $\alpha_{i1} = q_i\sigma_i(1) + (1 - q_i)\sigma_i(0)$, and $\alpha_{i0} = (1 - q_i)\sigma_i(1) + q_i\sigma_i(0)$. Substituting in (8.4), we have that $\sigma_i(t_i) > 0$ only if (for $j, k \neq i$)

$$\frac{\Pr(t_i|\omega = 1)}{\Pr(t_i|\omega = 0)} \left[\frac{\alpha_{j1}(1 - \alpha_{k1}) + \alpha_{k1}(1 - \alpha_{j1})}{\alpha_{j0}(1 - \alpha_{k0}) + \alpha_{k0}(1 - \alpha_{j0})}\right] \geq \frac{\pi_i}{1 - \pi_i} \frac{1 - \rho}{\rho} \tag{8.5}$$

Under certain conditions (when the court is sufficiently homogeneous) there is an equilibrium in which all judges vote *informatively*; i.e., $\sigma_i(1) = 1, \sigma_i(0) = 0$ for all $i \in N$. Note that with informative voting $\alpha_{i1} = q_i$, and $\alpha_{i0} = (1 - q_i)$. Then informative voting is a best response for each i iff

$$\frac{\rho(1 - q_i)}{\rho(1 - q_i) + (1 - \rho)q_i} \leq \pi_i \leq \frac{\rho q_i}{\rho q_i + (1 - \rho)(1 - q_i)}$$

In general, other responsive equilibria are possible. With binary signals and a symmetric environment ($q_i = q$ and $\pi_i = \pi$ $\forall i \in N$), the literature has focused on symmetric responsive BNE. Here of course the restriction has no bite. Still, there is a relatively "small" class of equilibrium candidates for any given parameter value. The exhaustive list is presented in Table A.1.

	Judge i		Judge j		Judge ℓ		Non-Generic
Eq. Class	$\sigma_1(1)$	$\sigma_1(0)$	$\sigma_2(1)$	$\sigma_2(0)$	$\sigma_3(1)$	$\sigma_3(0)$	
Pure Strategies:							
(EQ1.a)	1	0	1	0	1	0	
(EQ1.b)	1	0	1	0	1	1	
(EQ1.c)	1	0	1	0	0	0	
All judges mix:							
(EQ2)	σ_1	0	σ_2	0	σ_3	0	
(EQ3)	1	σ_1	1	σ_2	1	σ_3	
(EQ4)	σ_1	0	σ_2	0	1	σ_3	
(EQ5)	σ_1	0	1	σ_2	1	σ_3	
Two judges mix:							
(EQ6.a)	σ_1	0	σ_2	0	1	1	
(EQ6.b)	σ_1	0	σ_2	0	0	0	X
(EQ6.c)	σ_1	0	σ_2	0	1	0	
(EQ7.a)	1	σ_1	1	σ_2	1	1	X
(EQ7.b)	1	σ_1	1	σ_2	0	0	
(EQ7.c)	1	σ_1	1	σ_2	1	0	
(EQ8.a)	σ_1	0	1	σ_2	1	1	X
(EQ8.b)	σ_1	0	1	σ_2	0	0	X
(EQ8.c)	σ_1	0	1	σ_2	1	0	

Table A.1: We indicate by σ_j in column $\sigma_j(s)$ that $\sigma_j(s) \in (0,1)$

Characterizing responsive equilibria in the non-deliberation game is a laborious but simple task. We illustrate the main logic in case (8.c) in Table A.1; i.e., $\sigma_i(1) \in (0,1), \sigma_j(0) \in (0,1)$, $\sigma_i(0) = 0, \sigma_j(1) = 1$, and $\sigma_k(1) = 1$, $\sigma_k(0) = 0$. (The analysis of the other cases is similar; full details are available upon request). Note that here $\alpha_{10} = (1-q_1)\sigma_1(1)$, $\alpha_{11} = q_1\sigma_1(1)$, $\alpha_{20} = (1-q_2) + q_2\sigma_2(0)$, $\alpha_{21} = q_2 + (1-q_2)\sigma_2(0)$, $\alpha_{30} = 0$, and $\alpha_{31} = 1$.

In equilibrium, $i = 1$ has to be indifferent between upholding and overturning after $t_1 = 1$. Then if it exists, $\sigma_2^*(0)$ is given by the value of $\sigma_2(0) \in [0,1]$ that solves (8.5) with equality for $i = 1$ and $s_i = 1$, or

$$\sigma_2^*(0) = \frac{[q_1(1-\pi_1)\rho - (1-q_1)\pi_1(1-\rho)][(1-q_2)q_3 + q_2(1-q_3)]}{(2q_3-1)[q_1(1-\pi_1)\rho(1-q_2) + (1-q_1)\pi_1(1-\rho)q_2]},$$

which in turn implies $\alpha_{20}^* = (1-q_2) + q_2\sigma_2^*(0)$ and $\alpha_{21}^* = q_2 + (1-q_2)\sigma_2^*(0)$. Similarly, in equilibrium, $i = 2$ has to be indifferent between upholding and overturning after $t_2 = 0$. Then when it exists, $\sigma_1^*(1)$ is given by the value of $\sigma_1(1) \in [0,1]$ that solves (8.5) with equality for $i = 2$ and $t_2 = 0$, or

$$\sigma_1^*(1) = \frac{(1-q_2)q_3(1-\pi_2)\rho - q_2(1-q_3)\pi_2(1-\rho)}{(2q_3-1)[(1-q_2)q_1(1-\pi_2)\rho + q_2(1-q_1)\pi_2(1-\rho)]},$$

which implies $\alpha_{10}^* = (1-q_1)\sigma_1^*(1)$ and $\alpha_{11}^* = q_1\sigma_1^*(1)$. Finally, in equilibrium $i = 3$ has to have incentives to vote informatively. This means that

$$\underbrace{\frac{1-q_3}{q_3} \leq}_{t_3=1} \quad \frac{\alpha_{21}^*(1-\alpha_{11}^*) + \alpha_{11}^*(1-\alpha_{21}^*)}{\alpha_{20}^*(1-\alpha_{10}^*) + \alpha_{10}^*(1-\alpha_{20}^*)} \cdot \frac{1-\pi_3}{\pi_3} \cdot \frac{\rho}{1-\rho} \quad \underbrace{\leq \frac{q_3}{1-q_3}}_{t_3=0}$$

We can then evaluate numerically, for each point $(\rho, \vec{q}, \vec{\pi})$ in the confidence set, if the conditions for this to be an equilibrium are satisfied. As before, the error associated with this equilibrium σ is $\varepsilon^{ND}(\sigma,\theta) = (1-\rho)\Pr(v = 1|\omega = 0;\sigma,\theta) + \rho\Pr(v = 0|\omega = 1;\sigma,\theta)$, where given majority rule and independent mixing, for $k, \ell \neq j$

$$\Pr(v = 1|\omega,\sigma,\theta) = \sum_{j=1}^{3} \alpha_{k\omega}\alpha_{\ell\omega}(1-\alpha_{j\omega}) + \alpha_{1\omega}\alpha_{2\omega}\alpha_{3\omega}$$

8.3 Appendix: Additional Figures and Tables

Variable:		Mean	Std.Dev.
Case characteristics:			
FedLaw	=1 if case prosecuted under federal law	0.8169	0.3868
Aggravated	=1 if crime is aggravated assault/murder	0.1221	0.3274
White Collar	=1 if white collar crime	0.2038	0.4029
Theft	=1 if crime is theft	0.1418	0.3489
Narcotics	=1 if drug-related crime	0.2062	0.4047
Rep. Majority	=1 if $\geq$ 2 republicans on panel	0.4454	0.4971
Female	=1 if $\geq$ 1 female judge on panel	0.0829	0.2758
Harv-Yale Majority	=1 if $\geq$ 2 Harvard/Yale grads on panel	0.1809	0.3850
Jury instruction	=1 if main legal issue is jury instruction	0.1970	0.3978
Sentencing	=1 if main legal issue is sentencing	0.1628	0.3692
Admissibility	=1 if main legal issue is admissibility of evidence	0.3474	0.4762
Sufficiency	=1 if main legal issue is sufficiency of evidence	0.2543	0.4355
# cases:		3244	
Judge characteristics:			
Republican	=1 if judge is republican	0.5392	0.4989
Yearsexp	Years of appeals court experience	7.1893	7.8409
Judexp	Years of prior judicial experience	1.9197	3.7628
Polexp	Years of prior political experience	6.8547	7.0750
#judges:		523	
Vote Outcomes:			
Unanimous to Overturn		21.4%	
Divided to Overturn		2.6%	
Divided to Uphold		3.8%	
Unanimous to Uphold		72.2%	

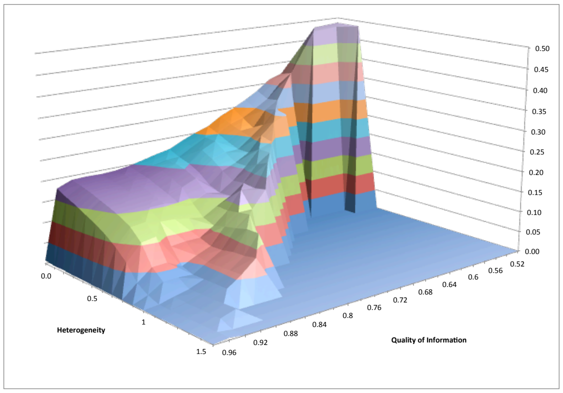
Table A.2: Summary statistics of data variables

Table A.3: Benchmark specification

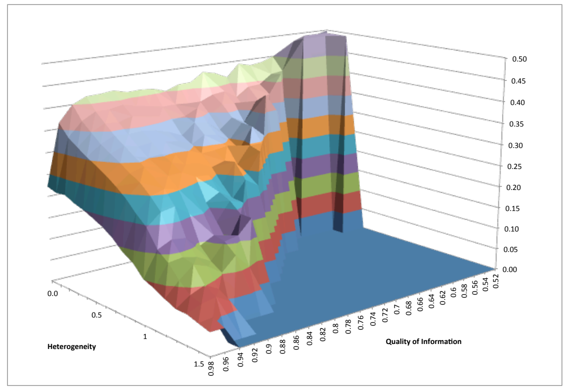
Estimated Vote Probabilities $p_v(\vec{v}\|X)$:			
$\hat{p}_v(111)$	=0.223	$\hat{p}_v(000)$	=0.677
$\hat{p}_v(101)$	=0.020	$\hat{p}_v(010)$	=0.015
$\hat{p}_v(110)$	=0.013	$\hat{p}_v(001)$	=0.018
$\hat{p}_v(100)$	=0.025	$\hat{p}_v(011)$	=0.010

Case characteristics:			
FedLaw	=1	Jury instruction	=0
Narcotics	=0	Sentencing	=0
Aggravated	=0	Admissibility	=1
White Collar	=1	Sufficiency	=0
Theft	=0	Rep. Majority	=1
Female Judge	=0	Harvard-Yale Majority	=0

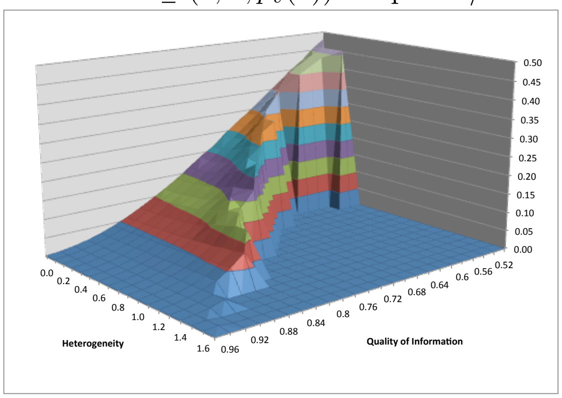
Judge characteristics:	Judge 1	Judge 2	Judge 3
Republican	1	1	0
Yearsexp	7.19	0	7.19
Judexp	1.92	0	1.92
Polexp	0	6.85	6.85



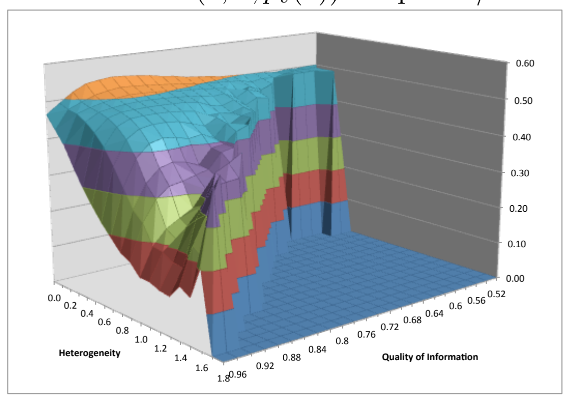
Min. Error $\underline{\varepsilon}^*(\theta, \vec{\pi}, p_v(\vec{v}))$ - Eq. c.w./ data



Max. Error $\overline{\varepsilon}^*(\theta, \vec{\pi}, p_v(\vec{v}))$ - Eq. c.w./ data

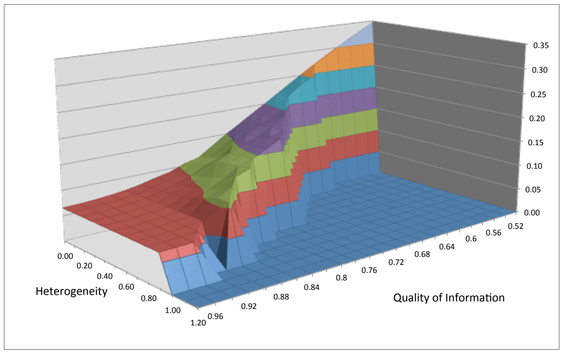


Min. Error $\underline{\varepsilon}(\rho, q, \pi)$ - All Equilibria

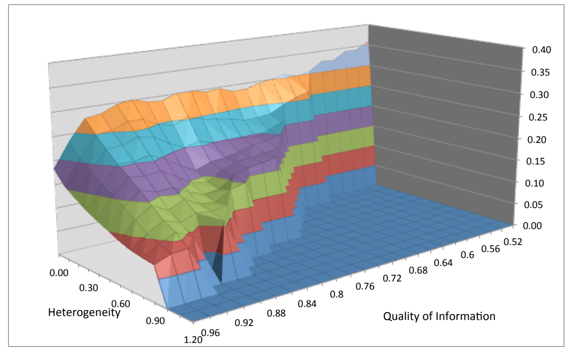


Max. Error $\overline{\varepsilon}(\rho, q, \pi)$ - All Equilibria

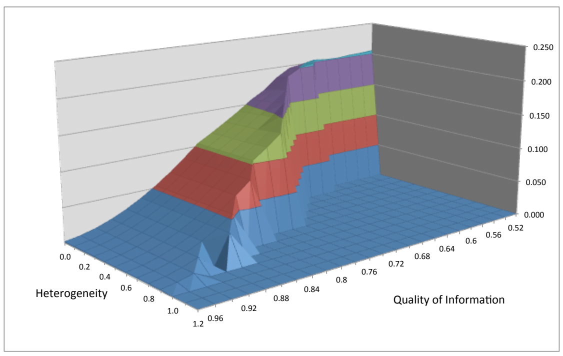
Figure A.1: Minimum and maximum probability of error in equilibria consistent with the data (top) and all equilibria (bottom), for pairs of preference heterogeneity and competence (H, q) consistent with points in the confidence set for $\rho = 0.5$. (Average of extrema across points $(\vec{\pi}, q)$ such that $H(\vec{\pi}) = H$).



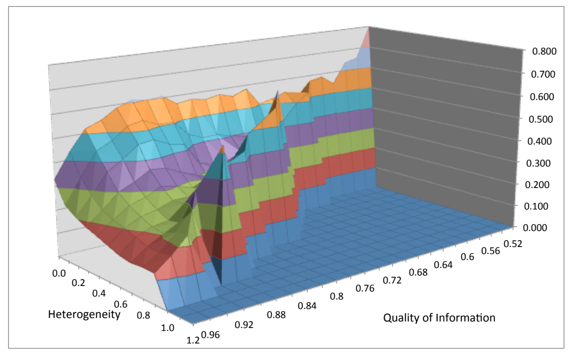
Min. Error $\underline{\varepsilon}^*(\theta, \vec{\pi}, p_v(\vec{v}))$ - Eq. c.w./data



Max. Error $\overline{\varepsilon}^*(\theta, \vec{\pi}, p_v(\vec{v}))$ - Eq. c.w./data

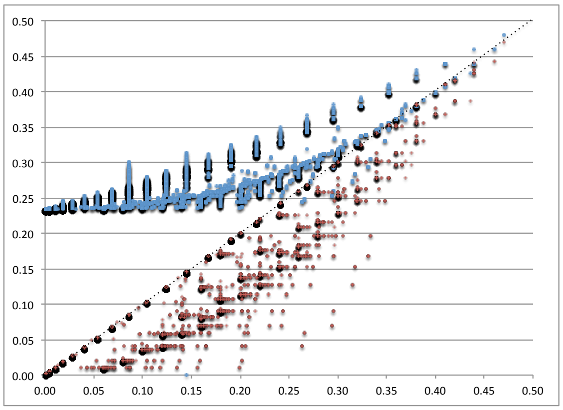


Min. Error $\underline{\varepsilon}(\rho, q, \pi)$ - All Equilibria

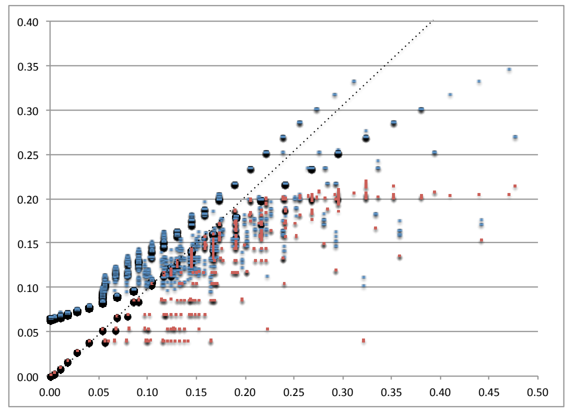


Max. Error $\overline{\varepsilon}(\rho, q, \pi)$ - All Equilibria

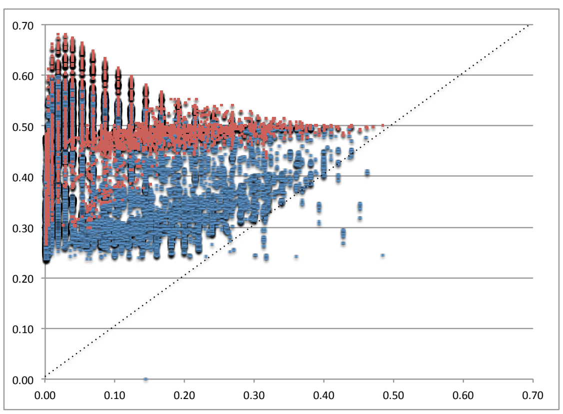
Figure A.2: Minimum and maximum equilibrium probability of error in equilibria consistent with the data (top) and all equilibria (bottom), for pairs of preference heterogeneity and competence (H, q) consistent with points in the EIS for $\rho = 0.2$. (Average of extrema across points $(\vec{\pi}, q)$ such that $H(\vec{\pi}) = H$).



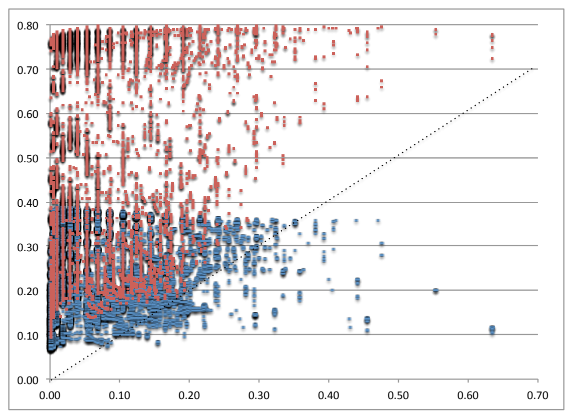
Min. Error ($\rho = 0.5$)



Min. Error ($\rho = 0.2$)

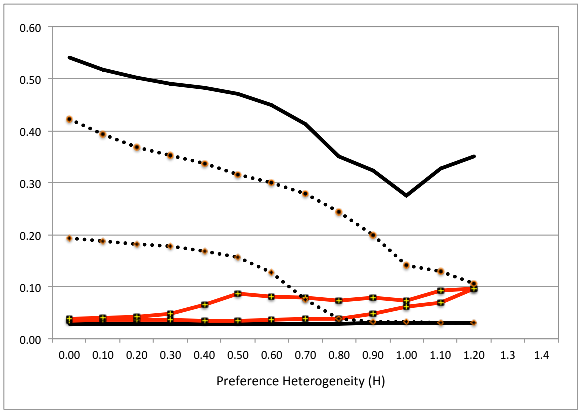


Max. Error ($\rho = 0.5$)

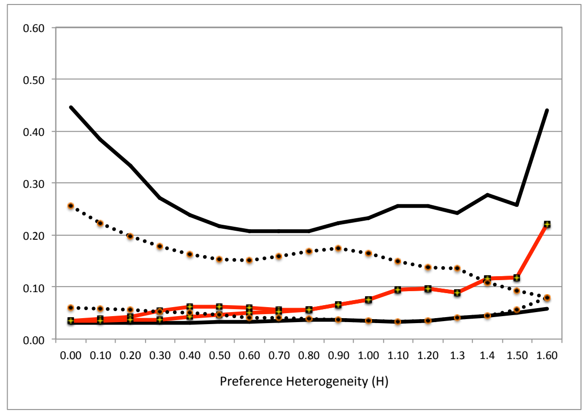


Max. Error ($\rho = 0.2$)

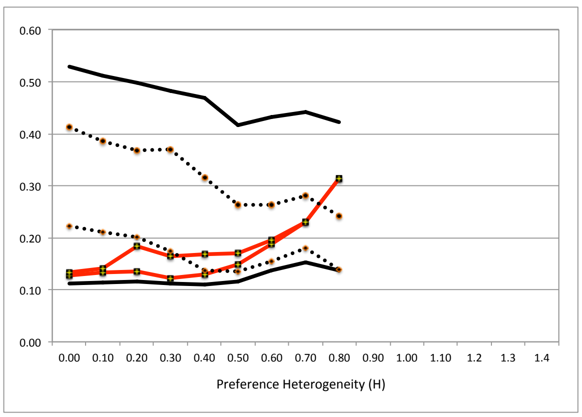
Figure A.3: For each point in the EIS, red dots plot the correspondence between the min/max probability of error in equilibria without deliberation (x-axis) and the min/max probability of error in all equilibria with deliberation (y-axis). Blue dots plot the correspondence between min/max probability of error in equilibria without deliberation (x-axis) and the min/max probability of error in equilibria with deliberation consistent with the data.



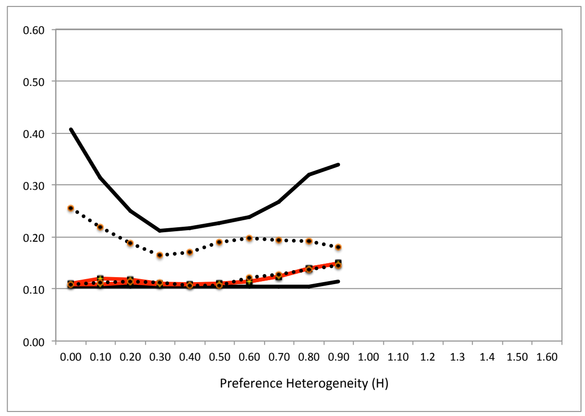
$(\rho = 0.5; q = 0.90)$



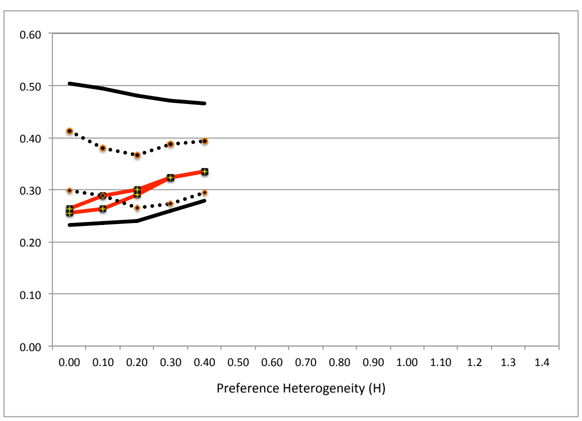
$(\rho = 0.2; q = 0.90)$



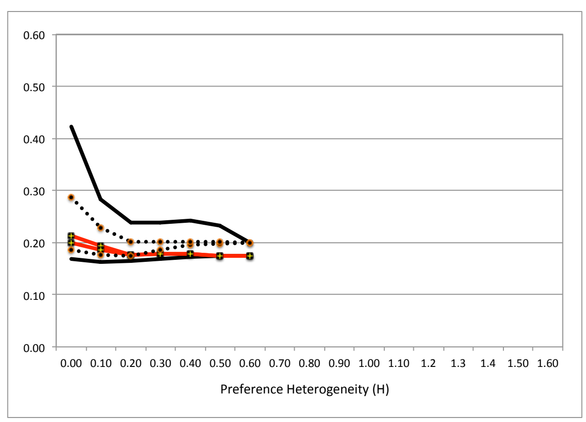
$(\rho = 0.5; q = 0.80)$



$(\rho = 0.2; q = 0.80)$



$(\rho = 0.5; q = 0.70)$



$(\rho = 0.2; q = 0.70)$

Figure A.4: Probability of mistakes with and without deliberation for values of preference heterogeneity H consistent with points $(\vec{\pi}, \theta)$ in the confidence set for $\rho = 0.5$ (left), and $\rho = 0.2$ (right). Min. and max. eq. probability of error in (i) all equilibria with deliberation (solid black), (ii) in equilibria with deliberation consistent with the data (dotted), and (iii) in responsive equilibria without deliberation (solid red, with marker).