Inefficient provision of liquidity

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Abstract

We study an economy where the lack of a simultaneous double coincidence of wants creates the need for liquidity. We show that the private provision of liquidity is inefficient: the private sector invests too much in collateralizeable assets and too much in relatively safe assets. The reason is that liquidity affects prices and the welfare of others, and creators do not internalize this. The government can eliminate the inefficiency by restricting the creation of liquidity by the private sector.

Key Words: liquidity, money, collateral.
JEL Codes:E41, G21, E51, D5.

1. Introduction

Between January 1994 and March 2008, while the stock of M2 grew by a factor of 2.4, the stock of overnight repos grew seven fold (Shin, 2010). This growth has been so remarkable that Adrian and Shin (2008) claim that "repos are the rightful successors of `money'." In response to the 2008 crisis several authors (e.g., Acharya and Naqvi (2010) and Brunnermeier and Pedersen (2009)) have focused on the instability effects that this repo growth generates. There is, however, another important welfare question that deserves attention. Even ignoring the instability consequences, is the growth in this private money socially efficient?

In this paper, we analyze this issue. To do so, we develop a simple general equilibrium model where there is a need for money and this money can be endogenously created by the private sector. Our model, which is a finite horizon one, contains two groups of agents, whom we call doctors and builders. Doctors buy building services from builders and then builders buy doctors' services from doctors, or the other way round. Each builder requires a doctor at a different date and typically one with different skills from the doctor she is building for, and vice versa for doctors. In other words there is no simultaneous double coincidence of wants (see Jevons (1876), and for a modern treatment Kiyotaki and Wright (1989)). Since we assume that future labor income is not pledgeable, this generates a need for means of payment. Agents are endowed with wheat. Wheat can be invested in various projects. These projects differ in their degree of collateralizeability, their yield, and their risk, where we suppose that riskier or less collateralizeable projects yield more. Given this assumption, we show that the decentralized equilibrium exhibits an excessive investment in collateralizeable assets and in relatively safe assets--the assets that are good as a means of payment.

We begin our analysis with a model that has no uncertainty. In this case the choice is between lower yield collateralizeable assets (i.e., assets whose future income can be fully pledged) and higher yield non-collateralizeable ones (whose future income cannot be pledged at all). We study the effect of the supply of liquidity, or notes, on equilibrium prices and on social welfare. Notifying the wheat of a doctor who buys building services before he sells doctor services imposes a negative externality on other doctors since it increases the price of building services. Moreover, this externality has first order welfare consequences given that doctors are liquidity constrained. Because of this externality too much wheat is invested in the collateralizeable asset to create liquidity instead of being invested in socially productive projects.

The distortion in liquidity that we identify is present even when the notification process is not done by individuals but by banks, as long as banks are competitive. A monopolistic bank, in contrast, ends up under-producing money; this is the standard result that a profit maximizing monopolist restricts production. As a result, there is one intermediate level of competition that generates the efficient amount of liquidity but it would be fortuitous for the economy to end up there.

The unappealing feature of this model is that whether an asset is collateralizeable or not is taken as given. In our second model we relax this assumption. We assume that all asset returns can be pledged but that the returns from some activities are risky. Moreover, risky returns are positively correlated, i.e., there are aggregate shocks. We also suppose that uncertainty about returns is resolved before trading in doctors' and builders' services takes place. Then, a high return realization of risky assets provides a large amount of liquidity for the economy, while a low return realization provides a low amount of liquidity. Since there are diminishing returns to liquidity—the marginal value of liquidity falls to zero when the gains from trade have been exhausted—this induces the equivalent of risk aversion in agents: the yield on the high return assets is discounted in the good state, and safe assets are favored for liquidity purposes. We find the same externality as before: doctors' investment in safe assets drives up the price of doctor services, hurting other doctors. In this version of the model, our main result is that an unregulated market economy allocates an excessive amount of resources to assets whose returns are relatively stable (there is little volatility in their value before trading takes place).

The overinvestment in collateralizeable assets can provide some explanation for the large investment in housing before the financial crisis. Also the huge expansion of the financial sector can be understood as an overinvestment in the production of safe assets, e.g., AAA securities.

Others, such as Sargent and Wallace (1983), have shown that commodity money is inefficient. Our findings are different. Sargent and Wallace's result is concerned with the inefficiency of commodity money relative to fiat money. As Sargent and Wallace acknowledge (p. 171): "It is well-known that the no-last-period feature of our model is necessary for producing such outcomes [i.e., the inefficiency of commodity money]. If there were a last period, a last generation, then under our assumptions any competitive equilibrium is both efficient and Pareto optimal." In contrast, ours is a finite horizon model with no fiat money (although fiat money can be introduced). We show that a planner can increase welfare by restricting commodity money

even if the planner is unable to introduce fiat money. More important, we show that this result is a general feature of a model where there is an opportunity cost of creating liquidity (whether this is because higher return assets are less collateralizable or because they are riskier).

Our result that the creation of inside money is excessive is similar to Stein (2011). In his model, however, it is assumed that agents have a discontinuous demand for a riskless claim (money). Similarly, his inefficiency arises from an assumed friction in the financial markets (that patient investors cannot raise additional money), while ours arises endogenously. Stein's model, however, is richer in terms of implications for monetary policy. Our results are also related to Chapter 7 of Holmstrom and Tirole (2011). They show that investors tend to hold an excessive amount of liquidity to take advantage of potential firesales. Our model shows that this excess liquidity is not specific to the storage role of money; it applies also to its transactional role.

There are parallels between our work and the literature on incomplete markets. In that literature a competitive equilibrium is typically inefficient and a planner operating under the same constraints as the market can do better (see, e.g., Geanakoplos and Polemarchakis (1986)). One feature of the incomplete markets literature is that the market structure is taken as given. This raises the question: why can't the private sector create new securities to complete the market? Our work differs in that we endogenize the market structure: markets are complete with respect to verifiable events such as aggregate shocks, but the inability to borrow against human capital and some types of physical capital creates liquidity problems[1]. Related to this, we focus on whether a market economy overinvests in non-collateralizeable or safe assets, something that, as far as we know, the incomplete markets literature has not considered.

Finally, our paper is connected to a vast and growing literature on the welfare effects of pecuniary externalities in the presence of financial constraints. This literature, from Holmstrom and Tirole (1998) to Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009), typically finds that a competitive equilibrium delivers an inefficiently low quantity of liquid assets. We find the opposite: a competitive equilibrium delivers an inefficiently high quantity of liquidity.

With respect to Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009), the reason for this difference is twofold. First, there is a difference in the meaning of liquidity. In these papers liquidity is the ability to provide insurance against a sudden consumption need:

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[1] In this respect our paper has parallels with Kiyotaki and Moore (1997), which also endogenizes the market structure, although in a different context.

short term assets are liquid and long term assets are not. By contrast, in our model liquidity is the ability to use an asset as a means of payment and, in the second version of our model, this is an endogenous characteristic of an asset. Second, in Allen and Gale (2004) the inefficiency arises from an exogenous incompleteness of markets; while in Farhi, Golosov, and Tsyvinski (2009) it arises from the existence of asymmetric information (in combination with the inability of parties to commit not to carry out side deals). In our model there is no asymmetric information, there are complete markets (except for the inability to pledge human capital), and there is no scope for insurance.

Holmstrom and Tirole (1998) rely on a definition of liquidity similar to ours, but in their model the government has a power—the power to tax — that the private sector does not have. As a result, the government can increase the amount of pledgeable income. Holmstrom and Tirole do not consider the effect of liquidity on relative prices, which is the focus of our paper.

The paper is structured as follows. In Section 2 we lay out the framework and describe the Walrasian equilibrium for the case where there are no pledgeability problems. In Section 3 we analyze the effect of the nonpledgeability of labor income for the case where high return projects cannot be collateralized. In Section 4 we allow both high return and low return projects to be collateralized but introduce aggregate uncertainty. Section 5 considers extensions and Section 6 concludes.

2. The Framework

We consider an economy that lasts four periods:

1 -------------------------2----------------------3-------------------4
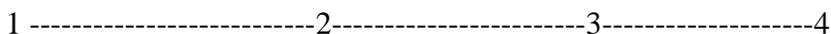
Figure 1

There are two types of agents in equal numbers: doctors and builders. The doctors want to consume building services and the builders want to consume doctor services. Consumption of these services occurs in periods 2 and 3. Doctors and builders can also consume wheat in period 4 and there is no discounting. Each doctor and builder has an endowment of wheat in period 1 equal to e. We will assume that $e > 1$.

We write agents' utilities as:

Doctors: $U_d = w_d + b_d - \dfrac{1}{2}l_d{}^2$

Builders: $U_b = w_b + d_b - \dfrac{1}{2}l_b{}^2$

where w_i is the quantity of wheat consumed by individual $i = d, b$; b_d is the quantity of building services consumed by the doctors; l_d is the labor supplied by the doctors; d_b is the quantity of doctor services consumed by the builders; and l_b is the labor supplied by the builders. We assume constant returns to scale: one unit of builder labor yields one unit of building services and one unit of doctor labor yields one unit of doctor services. We normalize the price of wheat in period 4 to be 1. Let p_b and p_d be the price respectively of building and doctor services. In words, doctors and builders have a constant marginal utility of wheat, a constant marginal utility of the service provided by the other group of agents, and a quadratic disutility of labor.

In period 1 each agent learns whether he will first buy or sell. Ex ante both events are equally likely. For convenience, we assume that in the east side of town doctors buy builders' services in period 2, while builders buy doctors' services in period 3. In the west side of town, the order is reversed. After each agent learns whether he will first buy or sell (equivalently, whether he is in the east or west side of town), he can invest part of his wheat endowment in a project that pays off in period 4 (in the form of wheat), and whose return is $\bar{R} > 1$. For the moment we take the return from the project to be perfectly certain. Wheat can also be stored, in which case it earns a return of 1.

In periods 2 and 3 the markets for doctor and building services meet and the doctors and builders trade in the order determined in period 1.

We have deliberately set up the model to be very symmetric; this helps with the welfare comparisons later. Throughout the paper we will analyze the east part of town, where doctors buy in period 2 and builders in period 3; the reverse case is completely symmetric.

We assume there are many doctors and many builders, and so the prices for both services are determined competitively. It is crucial for our analysis that there is *no simultaneous* double coincidence of wants: builders and doctors are in either the market for buying or the market for selling: they cannot do both at the same time. Hence, even if the builder a doctor buys from wants the doctor services from her customer, she cannot buy them at the same time as she is selling building services.

2.1 A benchmark: the Walrasian equilibrium

In an ideal world the doctors could pledge to pay the builders out of income from supplying doctor services that they will earn in period 3 *and* from the return from their investment project. This is the assumption made in classic Walrasian or Arrow-Debreu theory and it is easy to compute the Walrasian equilibrium, which is characterized by the following proposition:

Proposition 1: In the unique Walrasian equilibrium all wheat is invested in the project and $p_b = p_d = b_d = l_b = d_b = l_d = 1$. The utilities of the doctors and builders are $U_d = e\bar{R} + \dfrac{1}{2}$, $U_b = e\bar{R} + \dfrac{1}{2}$, respectively, and total welfare (social surplus) equals $W \equiv U_d + U_b = 2e\bar{R} + 1$.

The proof is straightforward and we leave it to the reader. The intuition is the following. Markets cannot clear if either $p_b > 1$ or $p_d > 1$, since demand will be less than supply for building/doctor services, respectively (consumers will prefer wheat). On the other hand, we cannot have both $p_b < 1$ and $p_d < 1$ because then the demand for wheat would be zero, while the supply is $2e\bar{R}$. Hence, either $p_b < 1$ and $p_d = 1$, or $p_b = 1$, $p_d < 1$, or $p_b = p_d = 1$. By symmetry we would expect $p_b = p_d$ (this needs to be checked, of course). We are left with $p_b = p_d = 1$. Utility maximization then implies $b_d = l_b = d_b = l_d = 1$. Each consumer receives a net surplus of ½ from buying one unit of a service and supplying one unit of labor.

Note that the Walrasian allocation and prices are independent of the initial endowment e and $\bar{R}$ (except for consumption of wheat, which varies one to one with $e\bar{R}$). Also the Walrasian equilibrium achieves maximal social surplus; this follows from the first theorem of welfare economics and the symmetry of the parties. In what follows we will refer to the Walrasian equilibrium allocation as the first best.

3. Introducing Non-Pledgeability

We now suppose that parties cannot pledge their future labor income.[2] We also assume that the pledegability of investment income depends upon the type of investment. The project, which yields $\bar{R} > 1$, is supposed not to be collateralizeable—its investment income cannot be pledged—while storage, which yields a return of 1, can be collateralized. Given that storage can be pledged we suppose that an individual who stores wheat receives notes that can be used as a means of payment. We provide an interpretation of this below.

3.1 Individual optimization

In what follows we conjecture that p_d and p_b are strictly less than one. We confirm this below.

Let's consider the doctors first. Suppose that, in period 1, a doctor invests $e - f_d$ units of wheat in the investment project and deposits f_d units of wheat in storage, receiving f_d in notes. In period 2 he uses these notes to purchase $\dfrac{f_d}{p_b}$ units of building services. In period 3, he will choose his labor supply l_d to maximize $p_d l_d - \dfrac{1}{2} l_d^2$, i.e., set $l_d = p_d$. Note that it is too late for the doctor to buy more building services and so his marginal return from work is p_d. A doctor's labor yields revenue p_d^2 in the form of notes, which he redeems for wheat in period 4; in addition he incurs an effort cost of $\dfrac{1}{2} p_d^2$, and so his net utility is $\dfrac{1}{2} p_d^2$. Finally, he will also receive and consume the payoff from the project in period 4 in the form of wheat.

Therefore, a doctor's utility when he buys first is

$$(3.1) \qquad U_d = \frac{f_d}{p_b} + \frac{1}{2} p_d^2 + (e - f_d) \bar{R} \ .$$

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[2] We have in mind that borrowers can breach any promise to pay out of future labor income by "disappearing". In practice some specialist agents may be able to keep track of borrowers and force them to repay their debts. We discuss this in Section 5.

Each doctor chooses f_d to maximize (3.1), taking prices as given. The first order condition for an interior solution is

$$(3.2) \qquad \frac{1}{p_b} = \bar{R}.$$

Similarly, a builder's utility is

$$(3.3) \qquad U_b = \frac{f_b}{p_d} + \frac{1}{2}\frac{p_b^2}{p_d^2} + (e - f_b)\bar{R}.$$

The second term arises from the fact that the builder chooses her labor supply l_b to maximize $\frac{p_b}{p_d}l_b - \frac{1}{2}l_b^2$. Note that a builder's marginal return from work is $\frac{p_b}{p_d}$, since she will use her income to buy doctor services. We shall see shortly that builders will be a corner, so their first order condition is

$$(3.4) \qquad \frac{1}{p_d} \leq \bar{R}.$$

3.2 Market Equilibrium

We solve for the equilibrium under the conjecture that $f_b = 0$. In due course we will confirm this. Since only $f_d > 0$, we will drop the subscript and set $f_d \equiv f$.

We work backwards, starting with the market for doctors in period 3. After the doctors have bought building services, the builders find themselves with a quantity f of notes. Hence, their demand for doctor services will be given by $\frac{f}{p_d}$. The supply of doctor services is $l_d = p_d$. Hence, market clearing requires

$$(3.5) \qquad \frac{f}{p_d} = p_d.$$

Similarly, the market clearing condition in the building services market in period 2 is

$$(3.6) \qquad \frac{f}{p_b} = \frac{p_b}{p_d}$$

since the labor supply of builders is $\dfrac{p_b}{p_d}$. Combining this with (3.5) yields

(3.7) $$p_d = f^{\frac{1}{2}}, \ p_b = f^{\frac{3}{4}}.$$

Substituting the equilibrium price p_b into (3.2) we have:

(3.8) $$f^{-\frac{3}{4}} = \bar{R}$$

or

$$f = \hat{f} = \frac{1}{\bar{R}^{\frac{4}{3}}} \quad .$$

Since $\bar{R} > 1$, $\hat{f} < 1$, which implies (given $e > 1$) that doctors are not at a corner solution. Also $1 > p_d > p_b$, as initially conjectured. Hence (3.4) holds as a strict inequality and the builders will be at a corner solution with $f_b = 0$, again as initially conjectured.

Note that the level of trade of doctors' services is $\dfrac{f}{p_d} = f^{\frac{1}{2}} < 1$ and of builders' services is

$\dfrac{f}{p_b} = f^{\frac{1}{4}} < 1$. In other words trade levels are lower than in the Walrasian equilibrium. (Recall that

in the Walrasian equilibrium $p_d = p_b = 1$ and one unit of each service is traded.)

Proposition 2: If $\bar{R} > 1$, there is less trade and prices are lower in the market equilibrium when agents cannot pledge future income than in the Walrasian equilibrium.

3.3 Social Optimum

Obviously the market equilibrium is not first best optimal given that it operates below the Walrasian equilibrium level of trade. We now show that the market equilibrium is also not second best optimal: a planner operating under the same constraints as the market can do better. Recall that ex ante it is not known who will buy first: doctors or builders. Thus the expected

utility of each group is $\dfrac{1}{2}U_d + \dfrac{1}{2}U_b$ .The social optimum is obtained by maximizing $U_d + U_b$

taking into account the effect of f on prices (i.e., substituting (3.7)). We suppose that the planner sets $f_b = 0$.

That is, the planner maximizes

(3.9) $$U_d + U_b = f^{\frac{1}{4}} + \frac{1}{2}f + \frac{1}{2}f^{\frac{1}{2}} + (e-f)\bar{R} + e\bar{R}.$$

The first order condition is

(3.10) $$\frac{1}{4}f^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}f^{-\frac{1}{2}} = \bar{R}.$$

Comparing the left-hand side of (3.8) and (3.10) we have

Proposition 3: If $\bar{R} > 1$, the market equilibrium leads to an excessive amount of private money f, and an excessive trade level, relative to the second best social optimum.

Proof: Since the l.h.s. of both (3.8) and (3.10) are decreasing in f, to prove that the solution of (3.8) is large than the solution of (3.10) it is enough to show that $f^{-\frac{3}{4}} > \frac{1}{4}f^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4}f^{-\frac{1}{2}}$. This can be rewritten as $\frac{1}{2}(f^{-\frac{3}{4}} - 1) + \frac{1}{4}(f^{-\frac{3}{4}} - f^{-\frac{1}{2}}) > 0$, which is true as long as $f < 1$. When $f=1$, the inequality becomes an equality. Trade levels are higher in the market equilibrium since equilibrium trade levels, given by $f^{\frac{1}{2}}$ for doctor services and $f^{\frac{1}{4}}$ for building services, are monotonically increasing in f. 　　　　　　　　　　　　　　　　　　Q.E.D.

The social and private returns from varying f are illustrated in Figure 2.

[Figure 2 here]

There are two types of inefficiency. With respect to the Walrasian equilibrium, there is too little trade but at the same time too much wheat is invested in liquidity-creating unproductive storage instead of productive projects. This inefficiency is due to the lack of pledgeabilty of future income. In the Walrasian equilibrium there is no liquidity problem since future income can be pledged and so no wheat is invested in unproductive storage.

In addition, there is an inefficiency with respect to the second best. The creation of more means of payment imposes a positive externality on the builders (who see the price of their building services go up) and a negative externality on the other doctors (who see the price of what they are buying go up). In standard models the effect of these "pecuniary" externalities is second order and thus does not create a divergence between social and private optimality. Here, however, while the positive externality on other builders is second order, the negative externality on other doctors is first order, since the doctors are liquidity constrained. (The builders, who sell before they buy, are not liquidity constrained.) Thus the market equilibrium yields too much liquidity and too much trade relative to the second best, even though too little trade relative to the first best.

Notice that if $\bar{R} = 1$, the social and private solutions do not differ: they are both $f = 1$. In this case the first best is achieved. There is still a divergence between private and social incentives, but this divergence is infra-marginal.

We discuss optimal intervention/regulation by the planner in Section 5.

Note that if some investment projects become collateralizeable, their prices will jump from $\frac{1}{\bar{R}}$ to $\left(\frac{1}{\bar{R}}\right)^2$, since they can be used both to produce $\bar{R}$ in period 4 and to buy goods in period 1, which has a return of $\frac{1}{p_b} = \bar{R}$.

This conclusion can be generalized: assets that can be collateralized to back credit will trade at a higher price than otherwise identical assets that cannot be collateralized. Furthermore, if resources are used to produce various assets, there will be an overproduction of assets that can be collateralized to produce liquidity; this is similar to Madrigan and Philippon (2011)'s result that there is overproduction of housing. To derive the overproduction result in the above model assume $\bar{R} = 1$ but that there is a cost of credibly storing and notifying wheat: it requires $\frac{1}{1-\mu}$ units of raw wheat to produce 1 unit of notified wheat. The results of the model are unchanged with $\frac{1}{1-\mu}$ replacing $\bar{R}$.

3.4 Robustness

One legitimate question to ask is how sensitive are our results to the assumption that agents know their type before they make their investment. The results are unchanged if agents can write insurance contracts contingent on the type of agent they will turn out to be (assumed to be verifiable). Let f be the storage chosen under the veil of ignorance and let Δ be the insurance supplement that doctors will receive and that builders will pay ex post. (Since being a doctor and a builder are equally likely events this is actuarially fair.)[3] From (3.1) and (3.3), the marginal return of an extra unit of wheat for a doctor is $\dfrac{1}{p_b}$, which is bigger than the marginal return of an extra unit of wheat for a builder, $\dfrac{1}{p_d}$. Thus, an optimal insurance contract will maximize Δ.

Hence, it will be exactly at the limit in which the builders have zero wheat and all the stored wheat is in the hands of the doctors, as in our basic model where individuals know their type before making their storage decision:

$$f + \Delta = f_d$$
$$f - \Delta = 0.$$

A further case is worth considering. Return to the situation where individuals know their type before making their storage decisions. Suppose that they can obtain insurance against their type. Will they want to do so? The answer is no. To see why, recall that $f_d < e$, $f_b = 0$. Hence in (3.1) and (3.3) the marginal utility of income of doctors and builders in period 1 equals $\bar{R}$ —the same for both groups.

3.5 Banks as Third Party Storage

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[3] A natural question to ask is how this insurance contract is enforced, i.e., what ensures that the builders pay up. One possibility is that agents store their wheat with an institution, such as an insurance company or a bank, which redistributes the wheat from those who turn out to be builders (i.e., those who will sell before they buy) to those who turn out to be doctors (i.e., those who will buy before they sell). See also 3.5 below.

So far we have glossed over the question of how wheat is transferred from doctors to builders (and vice versa). One possibility is that doctors carry around wheat in their pockets and transfer it to the builders in period 2. If it is too cumbersome to carry around or wheat can rot or be stolen, then an alternative is to deposit the wheat and use a proof of deposit as a means of payment. But if each doctor stores his wheat in his own facility, why should a builder believe that she will receive the promised wheat? This is where a third-party storage system can provide a solution, as Mattesini et al. (2009) show. Such storage facilities resemble early banks.

3.6 Non-Competitive Banking

Once we admit that storage is not done individually, but by some storage facilities that we call banks, the competitive structure of the storage facility/bank market becomes relevant. The other relevant dimension is in whose interest these storage facilities/banks operate.

These questions are studied in a previous version of the paper (Hart and Zingales, 2011). There we show that in a competitive market mutual (non-profit) banks choose too high a level of deposits with respect to social efficiency. In a monopolistic market mutual banks choose too low a level of deposits with respect to social efficiency. The intuition for the competition result is as above. The intuition for the monopoly one is simple. Large mutual banks restrict liquidity, i.e., issue too few notes, to lower the price of building services; this helps their members who are doctors (the builders do not deposit) since their members consume these services. In doing this, however, large banks ignore the positive externality they impose on builders, who gain from high prices since this allows them to buy more doctor services. Small banks choose a high quantity of liquidity because their impact on prices is limited.

Similar results are obtained in the case of for-profit banks.

Since a competitive banking sector generates too much liquidity and a monopolistic one too little, by continuity there exists a degree of oligopoly that delivers the efficient level of private money. Note, however, that this level is contingent upon $\bar{R}$; thus if the level of $\bar{R}$ changes with the business cycle, so does the level of competition that delivers the first best. In other words, unless the government can somehow fine tune the level of competition over time, this does not seem a very reliable method to achieve the first best level of money.

4. All Investments Are Pledgeable

A weakness of our analysis so far is that we have assumed that the higher yield investment is not collateralizeable. It is easy to show that if this investment is pledgeable the results of Section 3 cease to hold: shares in the project can be used as a means of payment. We now show that, if the higher yield project's return is stochastic and there is sufficiently high aggregate uncertainty, which is resolved before trading takes place, then our results are restored, albeit in a slightly different form.

Consider an economy where all investment income (but not labor income) is pledgeable, but there is aggregate uncertainty. The time-line is as in Figure 1. In period 1 wheat can be invested in two possible technologies. There is a riskless technology (storage) where one unit of wheat is transformed into one unit of wheat in period 4; and there is a risky technology where one unit of wheat is transformed into $R^H > 1$ units of wheat in period 4 with probability π and $R^L < 1$ units with probability $1 - \pi$, where $0 < \pi < 1$ and $\bar{R} = \pi R^H + (1-\pi)R^L > 1$. The returns of the various risky projects are perfectly correlated. Agents learn about the aggregate state of the world, i.e., whether $R = R^H$ or $R = R^L$, between periods 1 and 2. We refer to the two states as high (H) or low (L). All agents are risk neutral.

It is convenient to suppose now that investments are carried out by firms rather than individuals. Thus we suppose that there is free entry of firms possessing the two technologies described above and that these firms face constant returns to scale.

4.1 Arrow-Debreu Equilibrium with Non-Pledgeable Labor Income

We assume that the state of the world H or L is verifiable. It is therefore natural to suppose that markets for two Arrow securities exist in period 1, one paying a unit of wheat in period 4 in state H and the other paying a unit of wheat in period 4 in state L. These securities will be supplied by firms investing in projects. The securities will be collateralized by the project returns in each state and so there will be no default in equilibrium (asset returns cannot be stolen by firms' managers).

The Arrow securities will be used as a means of exchange in the period 2 and 3 doctor and builder markets. Let x_d^H and x_d^L be the quantities of the two Arrow securities bought by doctors and x_b^H and x_b^L the quantities bought by builders in period 1. Write q^H and q^L as the respective prices of the Arrow securities, where we normalize the price of wheat to be 1 in period

1 (as well as in period 4). Markets for builder and doctor services open in periods 2 and 3, just as in the model of Section 3. Let p_b^H, p_b^L, p_d^H, and p_d^L be the prices of builder and doctor services in the high and low state, respectively; the price of the H (resp., L) Arrow security is 1 in periods 2 and 3 if H (resp., L) occurs since the security guarantees a unit of wheat in period 4. As in Section 3, builder and doctor labor supply decisions are determined competitively: each will be on his Walrasian supply curve and the net utility from labor will be $\frac{1}{2}(p_d)^2$ for doctors and

$\frac{1}{2}(\frac{p_b}{p_d})^2$ for builders.

Thus a doctor chooses x_d^H and x_d^L to solve:

(*) $\qquad$ Max $\quad \pi\left[\dfrac{x_d^H}{p_b^H} + \dfrac{1}{2}\left(p_d^H\right)^2\right] + (1-\pi)\left[\dfrac{x_d^L}{p_b^L} + \dfrac{1}{2}\left(p_d^L\right)^2\right]$

subject to

$$q^H x_d^H + q^L x_d^L \leq e.$$

A builder chooses x_b^H and x_b^L to solve:

(**) $\qquad$ Max $\quad \pi\left[\dfrac{x_b^H}{p_d^H} + \dfrac{1}{2}\left(\dfrac{p_b^H}{p_d^H}\right)^2\right] + (1-\pi)\left[\dfrac{x_b^L}{p_d^L} + \dfrac{1}{2}\left(\dfrac{p_b^L}{p_d^L}\right)^2\right]$

subject to

$$q^H x_b^H + q^L x_b^L \leq e.$$

Note that in equilibrium the prices of building and doctor services cannot exceed 1 since otherwise consumers would strictly prefer wheat to them. Thus for the purpose of calculating utility in the above formulae we can assume that doctors (resp., builders) use all the proceeds from Arrow securities to buy builder (resp., doctor) services. Note also that firm profits are zero in equilibrium given constant returns to scale, and so we do not need to keep track of any dividends received by consumers.

Let y^s and y^r be the quantity of period 1 wheat invested respectively in the safe and risky technology. As noted, profit maximization and constant returns to scale imply zero profit: the value of the return stream of each technology cannot exceed the cost of investing in that technology (i.e., 1); and if the inequality is strict the technology will not be used. In other words,

(4.1) $\qquad\qquad q^H + q^L \leq 1 \qquad$ where $y^s = 0$ if the inequality is strict;

(4.2) $\qquad\qquad q^H R^H + q^L R^L \leq 1 \quad$ where $y^r = 0$ if the inequality is strict.

The market clearing conditions in the securities and wheat market in period 1 are given respectively by

(4.3) $\qquad x_d^H + x_b^H = y^s + y^r R^H$

(4.4) $\qquad\qquad x_d^L + x_b^L = y^s + y^r R^L$

(4.5) $\qquad\qquad y^s + y^r = 2e$

Finally, market clearing conditions in the builder and doctor markets in periods 2 and 3 in each state are:

(4.6) $\qquad\qquad p_b^H \leq 1$. If $p_b^H < 1$, then $\dfrac{x_d^H}{p_b^H} = \dfrac{p_b^H}{p_d^H}$. If $p_b^H = 1$, then $x_d^H \geq \dfrac{1}{p_d^H}$

(4.7) $\qquad\qquad p_b^L \leq 1$. If $p_b^L < 1$, then $\dfrac{x_d^L}{p_b^L} = \dfrac{p_b^L}{p_d^L}$. If $p_b^L = 1$, then $x_d^L \geq \dfrac{1}{p_d^L}$

(4.8) $\qquad\qquad p_d^H \leq 1$. If $p_d^H < 1$, then $\dfrac{x_b^H + x_d^H}{p_d^H} = p_d^H$. If $p_d^H = 1$ then $x_b^H + x_d^H \geq 1$.

(4.9) $\qquad\qquad p_d^L \leq 1$. If $p_d^L < 1$, then $\dfrac{x_b^L + x_d^L}{p_d^L} = p_d^L$. If $p_d^L = 1$ then $x_b^L + x_d^L \geq 1$.

(4.6) - (4.9) reflect the fact that, if the price of building or doctor services equals 1, consumers are indifferent between buying the service and wheat and so the market clears as long as liquidity is at least equal to supply.

In summary, the above describes a standard Arrow-Debreu equilibrium with one wrinkle: consumers cannot borrow against future labor income.

As in Section 3 it is useful to compare the competitive equilibrium with what a planner could achieve. In the first best the planner maximizes the sum of utilities subject to the aggregate feasibility constraints. That is, the planner solves:

$$Max\left\{\pi[w_d^H + b_d^H - \frac{1}{2}(l_d^H)^2 + w_b^H + d_b^H - \frac{1}{2}(l_b^H)^2] + (1-\pi)[w_d^L + b_d^L - \frac{1}{2}(l_d^L)^2 + w_b^L + d_b^L - \frac{1}{2}(l_b^L)^2]\right\}$$

subject to:
$$b_d^H = l_b^H$$

$$d_b^H = l_d^H$$

$$b_d^L = l_b^L$$

$$d_b^L = l_d^L$$

$$w_d^H + w_b^H = y^s + y^r R^H$$

$$w_d^L + w_b^L = y^s + y^r R^L$$

$$y^s + y^r = 2e$$

where w_d^i, w_b^i stands for wheat consumption of doctors and builders in state $i=H,L$, l_d^i, l_b^i, stands for labor services of doctors and builders in state i, etc.

The solution is easily seen to be
$$b_d^H = l_b^H = d_b^H = l_d^H = b_d^L = l_b^L = d_b^L = l_d^L = 1$$

$$y^s = 0 \text{ and } y^r = 2e.$$

Note that, as in Section 3, it is optimal to trade one unit of each service (now in each state), and to invest everything in the high yield, risky technology. Also the net surplus of each agent from trade is ½. Thus first best welfare is given by W= 2e $\bar{R}$ +1.

The next two lemma and proposition characterize the competitive equilibrium and provide a comparison to the first best.

Lemma 1: In a competitive equilibrium, the prices of doctor and builder services equal one in the high state (i.e., $p_d^H = p_b^H = 1$).

Proof: See Appendix.

The (rough) intuition is that, given $R^H > 1$ there is enough liquidity in the high state to support efficient trade at prices of 1.

Lemma 2: A necessary and sufficient condition for a competitive equilibrium to be first best optimal is $2eR^L \geq 1$.

Proof: See Appendix.

The (rough) intuition is that if $2eR^L \geq 1$ there is enough liquidity to support efficient trade when all resources are invested in the risky technology, even in the low state.

Proposition 4: If $2eR^L \geq 1$, then a competitive equilibrium delivers the first best. If

$$1 > 2eR^L \geq \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} \right)^{\frac{4}{3}} \text{ then a competitive equilibrium is such that investment is efficient}$$

(only the risky technology is used), but trading in doctor and building services is inefficiently

low. If $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} \right)^{\frac{4}{3}}$ a competitive equilibrium is such that investments and trading in

labor services are both inefficient: the riskless technology is operated at a positive scale and trade is inefficiently low.[4]

Proof: See Appendix.

Proposition 4 says that the competitive equilibrium is in one of three regions. If *2e R^L* exceeds 1 then we achieve the first best (which we already know from Lemma 2). If *2e R^L* is smaller than but close to 1 then there is not enough liquidity in the low state and so trade is inefficient. However, all resources are invested in the risky technology –this follows from the fact that the safe technology is strictly unprofitable at the prices supporting the first best and so will continue to be unprofitable in a neighborhood of the first best (see the Appendix). Finally, if *2e R^L* is much below 1 then the demand for liquidity is sufficiently great relative to the supply

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[4] In each region the competitive equilibrium is unique. We leave the proof to the reader.

that some resources are switched to the safe technology , and so both trade and investment are inefficient.

Note that if $R^L = 0$, the economy will never be at the first best.

Another way to understand Proposition 4 is as follows. Given that $2eR^L < 1$, what determines the type of distortion present is the comparison between the total amount of pledgeable wealth in the bad state and the ratio of the expected capital loss in the bad state $((1-\pi)(1-R^L))$ to the expected capital gain in the good one ($\pi(R^H - 1)$). If the expected capital loss is small relative to the expected capital gain, then it is still optimal to invest all resources in the risky technology (the first best outcome) and the inefficiency is limited to trading in doctor and builder services. If the expected capital loss is relatively large, then the optimal allocation will require some investment in the storage technology; this reduces the overall losses in the bad state and also the inefficiency in trading.

A natural question to ask is one we raised in Section 3. Since individuals know their type before they trade they could in principle obtain insurance against their type. Will they want to do so? As in Section 3 the answer is no. It is easy to show that, in every region in Proposition 4, in equilibrium both doctors and builders buy some high state Arrow securities. Thus, in period 1 the builders' marginal utility of income will equal $\dfrac{\pi}{p_b^H q^H}$ and the doctors' marginal utility of income will equal $\dfrac{\pi}{p_d^H q^H}$. However, we have shown in Lemma 1 that $p_b^H = p_d^H = 1$. Hence the marginal utilities of the two groups are equal, and there is no role for individual insurance.

4.2 Overinvestment in safe assets

We now consider whether a planner operating under the same constraints as the market can do better. We assume that the planner can constrain the production decisions of firms, i.e., choose y^r (or equivalently y^s), but cannot interfere in markets in other ways.

We will focus on the case $2eR^L < \left(\dfrac{1-\pi}{\pi}\dfrac{1-R^L}{R^H - 1}\right)^{\frac{4}{3}}$. It is easy to see from the proof of

Proposition 4 that in a neighborhood of the competitive equilibrium both technologies are used and $x_d^L > 0, x_b^L = 0$. Therefore, from (4.4)-(4.5) there is a one-to-one relationship between y^r and

x_d^L: decreases in the former correspond to decreases in the latter. In what follows we therefore assume that the planner picks $x_d^L = x^{CP}$ rather than y^r (or equivalently y^s). We will show that the planner can increase surplus by reducing x^{CP} below the competitive level.

Suppose that the planner picks $x_d^L = x^{CP}$ in a neighborhood of the competitive equilibrium. Then the market clearing conditions (4.7) and (4.9) yield

$$p_d^L = \left(x^{CP}\right)^{\frac{1}{2}}$$

$$p_b^L = \left(x^{CP}\right)^{\frac{3}{4}}.$$

Also q_H, q_L q_H, q_L will satisfy (4.1)-(4.2) with equality.

The doctors' utility, which is given by $\pi\left[x_d^H + \dfrac{1}{2}\right] + (1-\pi)\left[\dfrac{x_d^L}{p_b^L} + \dfrac{1}{2}\left(p_d^L\right)^2\right]$, becomes

$\pi\left[\dfrac{e - q^L x^{CP}}{q^H} + \dfrac{1}{2}\right] + (1-\pi)\left[\left(x^{CP}\right)^{\frac{1}{4}} + \dfrac{1}{2}x^{CP}\right]$. Similarly, the builders' utility, which is given by

$\pi\left[x_b^H + \dfrac{1}{2}\right] + (1-\pi)\left[\dfrac{1}{2}\left(\dfrac{p_b^L}{p_d^L}\right)^2\right]$, becomes $\pi\left[\dfrac{e}{q^H} + \dfrac{1}{2}\right] + (1-\pi)\left[\dfrac{1}{2}\left(x^{CP}\right)^{\frac{1}{2}}\right]$.

The planner maximizes $U^d + U^b$. Differentiating the welfare function with respect to x^{CP} yields

(4.10) $\qquad\qquad -\pi\dfrac{q^L}{q^H} + (1-\pi)[\dfrac{1}{4}\left(x^{CP}\right)^{-\frac{3}{4}} + \dfrac{1}{2}] + (1-\pi)\dfrac{1}{4}\left(x^{CP}\right)^{-\frac{1}{2}}$

We want to prove that (4.10) is negative when we evaluate it at the market equilibrium, $x^{CP} = \left(\dfrac{1-\pi}{\pi}\dfrac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$. From (4.1) and (4.2) we know that $\dfrac{1-R^L}{R^H-1} = \dfrac{q^H}{q^L}$. Hence, we can rewrite

(4.10) calculated at $x^{CP} = \left(\dfrac{1-\pi}{\pi}\dfrac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ as

$$-\pi\dfrac{q^L}{q^H} + (1-\pi)[\dfrac{1}{4}\left(\dfrac{1-\pi}{\pi}\dfrac{q^H}{q^L}\right)^{-1} + \dfrac{1}{2}] + (1-\pi)\dfrac{1}{4}\left(\dfrac{1-\pi}{\pi}\dfrac{q^H}{q^L}\right)^{-\frac{2}{3}}.$$

Simplifying, this becomes

$$-\frac{3}{4}\pi\frac{q^L}{q^H}+\frac{1}{2}(1-\pi)+\frac{1}{4}(1-\pi)\left(\frac{\pi}{1-\pi}\frac{q^L}{q^H}\right)^{\frac{2}{3}}.$$

Since $\dfrac{\pi}{1-\pi}\dfrac{q^L}{q^H}>1$, we can simplify further to obtain

$$-\frac{3}{4}\pi\frac{q^L}{q^H}+\frac{1}{2}(1-\pi)+\frac{1}{4}(1-\pi)\frac{\pi}{1-\pi}\frac{q^L}{q^H}\quad=-\frac{1}{2}\pi\frac{q^L}{q^H}+\frac{1}{2}(1-\pi)<0.$$

It follows that the planner can increase surplus by reducing x^{CP} below the competitive level, or equivalently by reducing y^s.

Proposition 5:   When $2eR^L<\left(\dfrac{1-\pi}{\pi}\dfrac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$ *,the economy overinvests in safe assets.*

In other words, even if the all investments are pledgeable, the competitive equilibrium will be inefficient as long as there is sufficiently high aggregate uncertainty before trading takes place ($2eR^L<\left(\dfrac{1-\pi}{\pi}\dfrac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$). The nature of the distortion is somewhat different from that in

Section 3. It is an overinvestment not in collateralizeable assets (since all assets are collateralizeable), but in safe assets. The intuition, though, is the same: the non-pledgeability of future labor income creates an additional demand for relatively safe assets. The reason is that transactional needs generate a form of risk aversion even in risk neutral people. When an agent has the opportunity/desire to buy, having a great deal of pledgeable wealth in some states does not compensate her for the risk of having very little pledgeable wealth in other states, because there are diminishing returns to liquidity: in the former states the gains from trade have been exhausted and the marginal value of liquidity is zero, whereas in the latter states the agents are wealth-constrained and the marginal value of liquidity is high. As a result, agents are willing to hold relatively safe assets even if they have a lower yield. However, in doing so they ignore the negative externality they impose on other agents (doctors).

We have shown that too many resources are invested in manufacturing these relatively safe assets. An example of this overproduction is the huge expansion of the finance sector in the

first decade of the new millennium, an expansion that cannot be explained by any of the traditional roles performed by finance (Philippon (2008)). While the production of AAA mortgage-backed securities might have been privately optimal, our model suggests that it was not necessarily socially optimal.

5. Extensions and Further Thoughts

5.1 Uncollateralized lending

So far, we have considered borrowers who can breach any promise to pay out of future labor income by "disappearing". In practice some specialist agents may be able to keep track of borrowers and force them to repay their debts. This is especially plausible when we assume that storage (or investment) takes place through banks and so all payments are endorsements of some storage/investment held at a bank. Specifically, suppose that all payments for building and doctor services take place through check transfers and that the bank is able to seize these before they are cashed for consumption. In this way labor income becomes contractible. However, a bank cannot force anyone to work. That is, all a bank can do is to ensure that someone who defaults has zero consumption (apart from any non-pledgeable investment income). As a result, uncollateralized lending against future labor income is possible, but there is a repayment constraint. Each worker can borrow up to the point at which ex post he is indifferent between exerting effort and repaying the loan and doing nothing and defaulting.

In a previous version of the paper (Hart and Zingales, 2011) we show that in this context lending does not resolve the tension between private and social objectives. Nevertheless, lending does improve welfare since it increases the volume of trade without sacrificing the higher return of the alternative investment. Interestingly, lending is not a perfect substitute for the notification of wheat in the model of Section 3. The reason is that with no notes in the system, the amount borrowed by doctors equals the purchasing power in the hands of builders, which in turn equals the revenue received by doctors for their services. But if the revenue equals the debt, it is not in the interest of the doctors to work, given that they have to exert costly effort. Hence, the doctors will default. To have a functioning lending market, we need a minimum amount of deposits.

5.2 Regulation

Since the amount of money created by the private sector is inefficient, it is natural to ask whether the government can fix this inefficiency. Let's start with the model described in section 3, where investments other than storage are not contractible. Then, the government can achieve the second best by restricting the level of deposits f at the second best level. Interestingly, however, the same result cannot be achieved with a simple (linear) Pigovian tax on deposits. The reason is that while an appropriate linear tax can restore the right incentive *at the margin*, it will be suboptimal, since it will raise revenues, worsening the liquidity constraints of doctors.

The optimal level of deposits can be implemented with a nonlinear tax $t = 0$ if $f \leq f^*$ and $t = f$ if $f > f^*$, since in equilibrium this tax will raise no revenues. Yet, this tax is very informational intensive. It requires the planner to know the optimal level of deposits for each individual. This is trivial in the model, since everybody is the same, but not in reality.

In practice, there are ways to deal with this problem that do not require so much information. Since what matters is the aggregate level of deposits, the planner can limit herself to controlling that instead. Even with multiple banks, she can do this by issuing a fixed number of permits and allowing banks to accept deposits only if they own the right amount of permits.

This is exactly what central banks do with reserve requirements. The central bank requires each bank to deposit a fraction of its own deposits at the central bank. In this way a bank can accept a deposit only if it has enough deposits at the central bank. Thus, by controlling the level of its own deposits, a central bank can control the level of deposits in all the banks and (in this simple economy) the liquidity of the overall system.[5]

The same argument applies to the second version of our model, where investment income is contractible, but uncertain. In this case, a central planner can achieve the second best by constraining the amount of investment in the safe asset y^s or the amount of Arrow securities x^L sold. Interestingly, our Arrow securities are equivalent to debt and equity in a bank that invests in the two technologies.[6] Thus, one is tempted to think that the government can achieve the second

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[5] This example applies to a situation of fiat money not of specie money, as in our model. However, even in a specie money economy central banks control the total amount of deposits with some fractional reserve.

[6] Let the debt D be

$$D = y^s + y^r R^L.$$

Then the equity E is

$$E = 0 \qquad \text{in the bad state}$$
$$= y^r (R^H - R^L) \qquad \text{in the good state}$$

best by imposing a minimum capital requirement on banks. However, this is the case only if the government can at the same time prohibit any investment in the safe technology outside the banking sector. If it cannot, then the bank capital requirement can easily be bypassed by independent firms that set up to produce Arrow securities. In a sense, this is the problem of the Shadow Banking sector, which bypasses the regulated banking sector, offering products that are functionally equivalent.

5.3 Government Money

Another, more direct, way in which the government can intervene is by introducing fiat money. We have dealt with this issue in a previous version of the paper (Hart and Zingales, 2011). In this finite horizon economy, to introduce government money we need to specify why it is accepted. Following a long tradition (e.g., Cochrane (1998)), we assume that government money is valuable because one can pay taxes with it. The moment we introduce taxes in a world with limited contractability, however, we need to consider also their deadweight costs. In fact, even a lump sum tax has some potential distortions. What happens if an individual refuses to pay the lump sum tax? Presumably, he would be thrown in jail. To make this a credible threat, however, the government must build prisons in advance, which is itself distortionary. In Hart and Zingales (2011) we show that the introduction of government money does not fully crowd out the need for private money, to the extent that taxes impose deadweight losses.

More generally, the distortions identified in this paper survive the introduction of government money, as long as government money has some cost. If, in an infinite horizon economy, government money can be introduced without cost in sufficiently large quantities, then the distortions identified in this paper disappear.

Doctors will buy a fraction θ of the equity and all the debt. Builders will buy a fraction $1-\theta$ of the equity and no debt. Thus,

$$x_d^L = y^s + y^r R^L$$
$$x_b^L = 0$$
$$x_d^H = y^s + y^r R^L + \theta y^r (R^H - R^L)$$
$$x_b^H = (1-\theta) y^r (R^H - R^L).$$

If we sum x_d^H and x_b^H, we obtain $y^s + y^r R^H$; hence the market clears.

If government money has a cost, then in the presence of uncertainty it should be used only in bad states. This can be thought of as a monetary ease during a recession. As in many models there is an issue of how this money will be pumped into the economy. If it is pumped in through banks, this will generate a standard moral hazard problem, where banks take too much risk because they know that they will be bailed out in bad states of the world. If this risk can be contained, it might provide a natural offset for the excessive investment in safe assets in the absence of any government intervention. This is an interesting topic for future research.

6. Conclusions

We have built a simple framework to analyze the general equilibrium implications of the private creation of liquidity. We have shown that this creation tends to be inefficient: the private incentive to generate liquidity exceeds the social incentive. This distortion manifests itself in different ways. When some investments are not collateralizeable, there is excessive investment in collateralizeable asstes. When some investments are risky, there is excessive investment in safe assets.

If we introduce the possibility of manufacturing safe assets, e.g., by combining various returns, perhaps at a cost, then the inefficiency would manifest itself as too many resources spent in manufacturing relatively safe assets. An example of this overproduction might be the huge expansion of the finance sector in the first decade of the new millennium, an expansion that cannot be explained by any of the traditional roles performed by finance (Philippon (2008)). While the production of AAA mortgage-backed securities might have been privately optimal, our model suggests that it was not necessarily socially optimal.

The existing literature (e.g., Allen and Gale (2004)) has emphasized that competitive markets under-provide liquidity, interpreted as insurance against future consumption risk. Our model is based on a different notion of liquidity --the ability to use an asset as a means of exchange--and we find that competitive markets over-provide liquidity. We regard both results as relevant.

How these two effects trade off in practice is an interesting topic for future research. It is important to notice, however, that the two effects do not necessarily offset each other. For example, in our model a risky asset whose value does not get revealed for a long time provides liquidity (in our sense) but not insurance. Thus, there can easily be an over-production of liquid

assets that does not alleviate the under-provision of systemic insurance. We regard the creation of AAA mortgage backed securities as an example of the over-production of liquidity that does not reduce systemic risk.

In this paper we deliberately ignore any potential financial contagion effects. However, we think that our simple framework lends itself naturally to an analysis of these effects, which we plan to consider in future work.

It is also important to emphasize that our result that a competitive economy overinvests in safe assets is dependent upon the absence of government bail-outs. If in bad states of the world the government intervenes to inject liquidity, then we have a standard moral hazard problem, where financial intermediaries want to take too much risk because they anticipate being bailed out. In fact, our model provides a natural trade-off between the benefit of government intervention (to reduce the overinvestment in safe assets) and its cost (excessive investment in risky assets due to moral hazard). We hope to analyze this in future work.

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Appendix

Lemma 1: In a competitive equilibrium, the prices of doctor and builder services equal one in the high state (i.e., $p_d^H = p_b^H = 1$).

Proof: Suppose $p_d^H < 1$. Then, by (4.8), $x_b^H + x_d^H = (p_d^H)^2 < 1$, which contradicts (4.3), given that $R^H > 1$ and $y^s + y^r > 1$ by (4.5). Hence $p_d^H = 1$.

To prove that $p_b^H = 1$, assume the contrary: $p_b^H < 1$. We first show that $x_d^H \geq x_d^L$. Suppose not: $x_d^H < x_d^L$. Then $x_d^L > 0$. From the first order conditions for (*),

(A1) $$\frac{\pi}{p_b^H q^H} \leq \frac{1-\pi}{p_b^L q^L}$$

That is, the utility rate of return on the low state Arrow security for doctors must be at least as high as that on the high state Arrow security. We also know that there is more output in the high state, so, if $x_d^H < x_d^L$, builders must be buying the high state security, which means that it must give them an attractive return, or , from their first order condition,

(A2) $$\frac{\pi}{q^H} \geq \frac{1-\pi}{p_d^L q^L} ,$$

where we are using the fact that $p_d^H = 1$.

Putting (A1)-(A2) together yields

(A3) $$\frac{p_b^L}{p_b^H} \leq p_d^L .$$

If $p_b^L = 1$, (A3) implies $p_b^H = 1$, which we have supposed not to be the case. Hence $p_b^L < 1$. Then we have, from (4.6) and (4.7), $\left(p_b^H\right)^2 = x_d^H$ and $\left(p_b^L\right)^2 = x_d^L p_d^L$. Therefore (A3) becomes

$$\frac{\left(x_d^L\right)^{\frac{1}{2}}}{\left(x_d^H\right)^{\frac{1}{2}}} \le \left(p_d^L\right)^{\frac{1}{2}} \le 1$$

or $x_d^H \ge x_d^L$, which is a contradiction.

Hence, $x_d^H \ge x_d^L$. Since a doctor's utility is increasing in x_d^L and x_d^H, a doctor's budget constraint will hold with equality. Thus $q^H x_d^H + q^L x_d^L = e$, which implies $(q^H + q^L)x_d^H \ge e$. Hence, by (4.1), $x_d^H \ge e > 1$, implying $p_b^H = 1$ by (4.6).

<div align="right">Q.E.D.</div>

Lemma 2: A necessary and sufficient condition for a competitive equilibrium to be first best optimal is $2eR^L \ge 1$.

Proof:

To achieve the first best the supply of building and doctor services must be 1 in each state. Since the supply of doctor services is given by p_d^H, p_d^L in (4.8), (4.9), it follows that $p_d^H = p_d^L = 1$. The supply of building services is given by $\frac{p_b^H}{p_d^H}, \frac{p_b^L}{p_d^L}$ in (4.6)-(4.7), and, substituting $p_d^H = p_d^L = 1$, we obtain $p_b^H = p_b^L = 1$. Hence, again from (4.6)-(4.7), $x_d^H \ge 1$ and $x_d^L \ge 1$.

In the first best all wheat is invested in the high yield project: $y^s = 0$ and $y^r = 2e$. Therefore, from (4.4), $2eR^L = x_d^L + x_b^L \ge x_d^L \ge 1$. Hence, $2eR^L \ge 1$ is a necessary condition.

To prove sufficiency consider a candidate equilibrium where the prices of doctor and builder services equal 1 in both states, $q^H = \frac{\pi}{R}$, $q^L = \frac{(1-\pi)}{R}$, all wheat is invested in the high yield project, and the doctors buy at least one unit of each Arrow security. Since $q^H + q^L < 1 < e$, they can afford to do so. Doctors and builders satisfy their first order conditions and firms maximize profit. Hence this is indeed a competitive equilibrium.

<div align="right">Q.E.D.</div>

Proposition 4: If $2eR^L \geq 1$, *then a competitive equilibrium delivers the first best. If*

$$1 > 2eR^L \geq \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} \right)^{\frac{4}{3}}$$ *then a competitive equilibrium is such that investment is efficient*

(only the risky technology is used), but trading in doctor and building services is inefficiently

low. If $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} \right)^{\frac{4}{3}}$ *a competitive equilibrium is such that investments and trading in*

labor services are both inefficient: the riskless technology is operated at a positive scale and trade is inefficiently low.

Proof:

Lemma 2 shows that we achieve the first-best if $2eR^L \geq 1$. Consider the case $2eR^L < 1$. We know from Lemma 1 that $p_d^H = p_b^H = 1$. We show first that doctors will hold both securities. Given $p_b^H = 1$, (4.6) implies $x_d^H > 0$. Suppose $x_d^L = 0$. This is inconsistent with $p_b^L = 1$ in (4.7). But if $p_b^L < 1$, then, from (4.7), $x_d^L = 0$ implies $p_b^L = 0$. This in turn implies that the marginal return on the low state Arrow security for a doctor is infinite, which means that the first order condition in (*) cannot hold. Therefore, $x_d^L > 0$.

Since doctors hold both securities, we have

(A4) $$\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L}$$

Let's assume first that both technologies are used. Then (4.1) and (4.2) hold as an equality and

$q^L = \dfrac{R^H - 1}{R^H - R^L}$ and $q^H = \dfrac{1 - R^L}{R^H - R^L}$. Therefore

(A5) $$p_b^L = \frac{1-\pi}{\pi} \frac{1-R^L}{R^H - 1} < 1$$

since $\bar{R} > 1$.

It is easy to see that $p_b^L < p_d^L$. This is clear from (A5) if $p_d^L = 1$. Suppose $p_d^L < 1$. Then

(4.9) implies $\left(p_d^L \right)^2 = x_b^L + x_d^L \geq x_d^L = \dfrac{\left(p_b^L \right)^2}{p_d^L}$ by (4.7). Hence, $p_d^L \geq (p_b^L)^{\frac{2}{3}} > p_b^L$ since $p_b^L < 1$. This

proves $p_b^L < p_d^L$. It follows that the rate of return on the low security is strictly less than that on

32

the high security for builders. So builders will not hold the low security (from the first order condition for (**)): $x_b^L = 0$.

From (4.7),

(A6) $$p_d^L x_d^L = \left(p_b^L \right)^2,$$

from which it follows, since $p_b^L < 1$ and $p_b^L < p_d^L$, that $x_d^L < 1$. But then (4.9) in combination with $x_b^L = 0$ implies $p_d^L < 1$. Hence, again from (4.9),

(A7) $$x_d^L = \left(p_d^L \right)^2.$$

Combining (A6) and (A7) we have

(A8) $$x_d^L = \left(p_b^L \right)^{\frac{4}{3}}.$$

If the solution $x_d^L = \left(\dfrac{1-\pi}{\pi} \dfrac{1-R^L}{R^H-1} \right)^{\frac{4}{3}} > 2eR^L$, then this candidate equilibrium where both technologies are used is feasible. Also trade of doctor and builder services is inefficient since p_b^L, p_d^L are both less than 1.

If $x_d^L = \left(\dfrac{1-\pi}{\pi} \dfrac{1-R^L}{R^H-1} \right)^{\frac{4}{3}} \leq 2eR^L$, then we solve instead for an equilibrium with $x_d^L = 2eR^L$, $x_b^L = 0$. In this case, there will be no investment in the storage technology (thus $y^s = 0$) and the market clearing condition for securities, (4.3)-(4.4), simplifies to

$$x_d^H + x_b^H = 2eR^H,$$
$$x_d^L + x_b^L = 2eR^L.$$

(4.7) and (4.9) become

$$p_d^L = \left(2eR^L \right)^{\frac{1}{2}}$$
$$p_b^L = \left(2eR^L \right)^{\frac{3}{4}},$$

which are below one since $2eR^L < 1$. Doctors hold both Arrow securities since $p_b^H, p_b^L > 0$ and so we must have

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L} = \frac{1-\pi}{\left(2eR^L\right)^{\frac{3}{4}} q^L}.$$

This, together with (4.2) with equality, determines q^H and q^L. Thus, in this equilibrium investment is efficient but the level of trading is not.

Q.E.D.
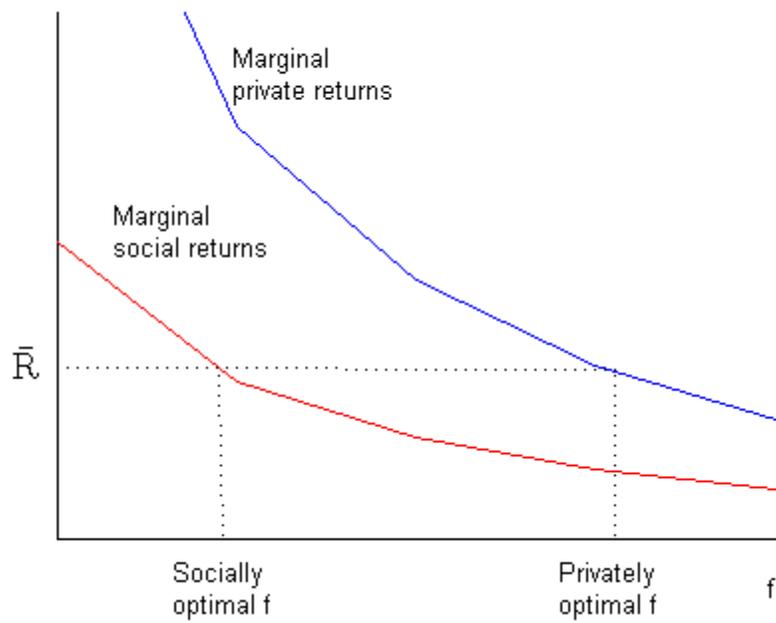
Figure 2 - Difference between private and social optimality