Auctions with Endogenous Initiation*

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Abstract

We study initiation of auctions by potential buyers and the seller. A bidder's decision to approach the seller reveals that she is optimistic about the asset's value. If bidders' values have a substantial common component, as in takeover battles between financial bidders, this revelation effect disincentivizes bidders from approaching the seller, and auctions are seller-initiated. Conversely, if bidders' values have a substantial private component, as in takeover battles between strategic bidders, the revelation effect encourages bidders to initiate, and equilibria can feature both seller- and bidder-initiated auctions. We highlight implications linking the initiating party to bids and auction outcomes.

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1 Introduction

Over the last several decades, auction theory has developed into a highly influential field with many important practical results, including those related to applications in finance.[1] To focus on the insights about the auction stage, with rare exceptions, the literature has examined situations in which the asset is already up for sale. In some cases, exogeneity of a sale is an innocuous assumption. For example, the U.S. Treasury auctions off bonds at a known frequency. In many cases, however, the decision to put the asset up for an auction is a strategic one. For example, the board of directors of a firm (or an art collector) has a right but not an obligation to sell a division (or an art piece). In practice, an auction can be either bidder-initiated, when a potential bidder approaches the seller expressing an interest, in which case the seller then decides to auction the asset off, or seller-initiated, when the seller decides to auction the asset off without being approached.

To give a flavor of this heterogeneity, consider the following two examples of auctions of companies. The acquisition of Taleo, a provider of cloud-based talent management solutions, by Oracle on February 9, 2012 for \$1.9 billion was conducted via a bidder-initiated auction. In January 2011, the CEO of a publicly traded technology company contacted Taleo expressing an interest in acquiring it. Following this contact, Taleo hired a financial adviser that conducted an auction, engaging four more bidders. Oracle was the winning bidder, ending up acquiring Taleo. By contrast, the acquisition of Blue Coat Systems, a provider of Web security, by a private equity firm Thoma Bravo on December 9, 2011 for \$1.1 billion was conducted via a seller-initiated takeover auction. In early 2011, Elliot Associates, an activist hedge fund, amassed 9% ownership stake in Blue Coat and forced its board to auction off the company. Twelve bidders participated in the auction, and Thoma Bravo was the winner. Overall, in an extensive study of takeover initiations, Eckbo, Norli, and Thorburn (2020) find that 42% of all deals done over 1996–2016 were initiated by the seller's board, 29% by the buyer, and 15% by another bidder.[2]

[1]The formal analysis of auctions goes back to Vickrey (1961). The overview of results on auction theory can be found, for example, in Krishna (2010). Dasgupta and Hansen (2007) review applications of auction theory to corporate finance problems.

[2]The remaining groups (joint effort, seller shareholder, and merger of equals) account for 14%. The working paper of Eckbo, Norli, and Thorburn (2020) is not publicly available at the time of writing this article. See Eckbo, Malenko, and Thorburn (2020) for a summary of this evidence.

Which characteristics of auctions and the economic environment determine whether auctions are bidder- or seller- initiated? How do bidding strategies and auction outcomes differ depending on how the auction was initiated? What are the implied inefficiencies and what are the potential remedies, if any? To study these questions, we develop a theory of the seller's choice to put his asset up for sale and the potential buyers' choice to indicate their interest to the seller in the presence of private information about their values.

We consider a two-period framework, in which a seller owns an asset and faces two potential buyers. We later extend the framework to multiple periods. Each buyer has a signal about her value of the asset. The signals are independently distributed, but the value of a buyer can depend on both signals. Thus, the model nests the cases of pure private values and pure common values, but also allows for mixes of private and common components. The signals of buyers can simultaneously change over time as a result of an exogenous shock. In any period each buyer decides whether to send a cheap talk message indicating her interest in buying the asset. After observing the messages, the seller decides whether to put the asset up for an auction or not. In addition, with some probability the seller may also be hit by a liquidity event, in which case she has no choice but to sell the asset. Thus, the auction can be initiated by a buyer when the seller auctions the asset off after receiving an indication of interest, or by the seller when the seller auctions the asset off without receiving any interest.

In this framework, the seller's benefit of waiting is that with some likelihood signals of potential buyers will increase in the next period, resulting in a higher transaction price. Conversely, the benefits of selling without waiting for an indication of interest is the lack of delay and the possibility that values of potential buyers will decrease. We analyze cut-off equilibria, in which each potential buyer communicates an indication of interest to the seller if her signal is above a certain period-specific cut-off. Our base model assumes that the auction format is first-price, but in an extension we consider a rich class of sale formats, which nests first-price and, in the limit, ascending auction, but also allows for intermediate formats.

The terminal period has a simple equilibrium: neither buyer indicates her interest, and all auctions are seller-initiated. From now on, we will focus on earlier periods, which are more interesting and empirically relevant. Our main insight is that the degree of commonality of buyers' values is a key determinant of whether the equilibrium features seller- or bidder-initiated auctions. Specifically, we establish the following results:

1. If the common component of bidders' values is sufficiently high, no buyer indicates her interest to the seller, no matter how high her signal is, and all auctions, if they occur, are seller-initiated.

2. If the private component of bidders' values is sufficiently high, in equilibrium both bidder-initiated and seller-initiated auctions can occur. A buyer approaches the seller if her value is sufficiently high. The seller auctions the asset off either upon receiving an indication of interest from a potential buyer or upon being hit by a liquidity shock. In addition to this equilibrium, there is also a non-responsive equilibrium in which no buyer indicates her interest to the seller and all auctions are seller-initiated.

3. If the seller is highly likely to be hit by a liquidity shock that forces him to put the asset up for sale, then, regardless of the commonality of values, no potential buyer indicates her interest, and the auction is initiated by the seller.

One of the applications of our theory is to auctions of companies. Here, it is natural to relate the commonality of values to whether bidders are strategic (e.g., companies in an industry related to the target) or financial (e.g., private equity firms). Strategic bidders acquire targets for synergies and integrate them into their existing businesses. Because synergies are often acquirer-specific, strategic bidders likely have high private components of values. In contrast, financial bidders' post-acquisition value-creating strategies for targets include more aggressive use of leverage and higher-powered managerial compensation. Because these strategies are similar across financial bidders, they likely have high common components of values. In auctions of companies, our first two results then imply that we expect battles among financial bidders to be more frequently seller-initiated, while battles among strategic bidders to be more frequently bidder-initiated. This implication is consistent with empirical evidence: approximately 60% (35%) of strategic (private-equity) deals are initiated by the bidders (Fidrmuc et al, 2012).[3] Further, multiplicity of equilibria when the private component of values is high implies that sales of otherwise similar assets in different markets (e.g., otherwise similar companies in different countries) may have very different patterns of initiation. Our third result implies that sales of distressed companies, which are expected to go bankrupt and be put up for sale with a high probability, would be mostly seller-initiated, regardless of whether values are private or common. In contrast, sales of companies that are far away from distress would be seller-initiated if values are common, but would be more frequently bidder-initiated if values are private.

The key driving force behind our results is that the initiating bidder approaching the seller reveals that her value is sufficiently high. In a bidder-initiated auction, ex-ante identical bidders can become endogenously asymmetric at the auction stage: the signal of the initiating bidder is drawn from a more optimistic distribution than that of the non-initiating rival. The rival uses this information to choose her bidding strategy and potentially re-value the asset. Similarly, the lack of an initiating bidder reveals information about values of all bidders: in a seller-initiated auction, each bidder knows that the value of the rival is sufficiently low, as she would have initiated the auction otherwise.[4]

This force creates two effects that impact bidder initiation in opposite directions. The first effect is the *asset revaluation effect*: if values have a common component, upon learning that the auction is bidder-initiated, the rival updates her value of the asset upwards. As a result, she bids aggressively not only because she competes against a strong bidder but also because of her own higher value. In a pure common-value setting, a bidder earns

[3]These deals include both negotiations and competitive processes (controlled sales and auctions). Approximately 79% (59%) of strategic (private-equity) deals completed through a negotiation and approximately 47% (29%) of strategic (private-equity) deals completed through a competitive process are initiated by bidders. While our main model speaks to competitive processes only, its results extend, in section 5.2, to the case of negotiations, which occur when one of the buyers is unwilling to pay a cost of information acquisition to learn her signal about the asset. In the presence of potentially uninformed rivals, informed buyers are more willing to initiate deals, because the fact of initiation can preempt their rivals. However, the value of preemption is much lower for bidders with high common components of values. Modeling negotiations as having multiple potential buyers instead of a single such buyer is consistent with the practice: Takeover deal backgrounds often detail multiple contacts of target companies with potential buyers before either a negotiation with one such buyer or a competitive process with multiple buyers is initialized. On a separate note, initiation is also related to characteristics of the seller and auction outcomes (Masulis and Simsir, 2018; Eckbo, Norli, and Thorburn, 2020).

[4]When both bidders attempt to initiate the auction in the same period, this also reveals information about values of all bidders: each bidder knows that the rival's value is sufficiently high.

rents only through informational advantage. The initiating bidder with the lowest signal among those that lead to initiation (the cut-off signal) has no informational advantage, as the rival knows that the bidder's signal is at least equal to the cut-off. Thus, the cut-off type of the initiating bidder earns zero rents, and consequently prefers not to approach the seller but instead enter either the rival- or seller-initiated auction, as she would be able to get information rents then. Because this argument holds for any hypothetical equilibrium cut-off signal that leads to initiation, bidder-initiated auctions do not occur in equilibrium. Importantly, this is not a knife-edge result: if values are not purely common but the degree of commonality is sufficiently high, the cut-off type of the initiating bidder still obtains a sufficiently low profit from the auction to make bidder initiation suboptimal.

The second effect is the *rival selection effect*. If a bidder does not indicate her interest and the auction is initiated by the rival, she ensures that she will compete against a strong rival who assigns a sufficiently high value to the asset. Conversely, if the bidder indicates her interest but the rival does not, she will compete against a weaker rival, as the lack of rival interest implies that her value is not very high. As a consequence, the bidder with a sufficiently high signal finds it profitable to bet on the chance that the rival's value is low by approaching the seller. While the asset revaluation effect lowers the payoff of the initiating bidder, the rival selection effect increases it. The rival selection effect is absent when values are purely common. The asset revaluation effect is absent when values are purely private. In turn, when the private component of values is sufficiently high, the equilibrium can potentially feature bidder-initiated auctions.

The reason for equilibrium multiplicity is interdependence of initiation decisions by bidders and the seller. To see the intuition, consider the case of pure private values. If bidders perceive a seller-initiated auction to be a very unlikely event in this period, they will have strong incentives to initiate the auction, because, as described above, a rival-initiated auction makes them worse off and is likely to occur before the seller-initiated auction. In contrast, if bidders expect the seller to auction the asset off in this period, they will have weak incentives to initiate the auction. This is because the seller-initiated auction makes bidders better off by allowing a bidder with a high value to hide it. This interdependence has two implications. First, if the seller is expected to be hit by a liquidity shock that forces her to sell the asset in the near future, no bidder approaches the seller, regardless of the commonality of values, and the only equilibrium dictates immediate initiation of the auction by the seller. Second, if the probability of such a shock and the commonality of values is low, there is a potential for multiple equilibria.

In addition to initiation patterns, the model generates a set of implications relating bidders' values to their bidding strategies in the first-price auction and the identity of initiating bidders. For example, auctions are likely to be initiated by stronger bidders, who will submit, on average, higher bids and will be more likely to win than non-initiating bidders. However, conditional on the same valuation, non-initiating bidders will bid more aggressively than initiating bidders. The model also implies that bidders will bid less aggressively in seller-initiated auctions, even conditionally on having the same values.

Beyond the set of main results, the model points to a value-enhancing role of shareholder activists in intermediating intercorporate asset sales. Consider an inefficiently-run firm followed by potential bidders, each of whom can restore its efficiency. Then, these bidders' values have a high common component, and hence our results imply that each bidder would be reluctant to approach the seller. If the firm's management and board are entrenched, the seller would not initiate the auction either. The result would be the failure of the market for corporate control as a corporate governance mechanism precisely in situations when it is most needed. While the market alone may be insufficient to resolve such inefficiencies, an activist investor can use it to acquire a block of shares and force the firm to auction itself off. In this respect, shareholder activism and the market for corporate control are complements, rather than two different mechanisms for turning around poorly managed companies. Alternatively, an investment bank, which intermediates a transaction and can commit (e.g., due to reputational concerns by being a long-run player) to hide the presence of buyers' indications of interest from their rivals, would provide strong buyers with incentives to communicate interest. Lastly, the model points to a potentially value-enhancing role of toeholds, implying that in a dynamic environment the welfare effect of toeholds trades off allocative inefficiency of the auction against greater incentives of strong bidders to acquire toeholds and initiate auctions.

Our paper belongs to the large literature on auction theory. Virtually all of it only considers a stage when the auction takes place. Three exceptions are papers by Board (2007), Cong (2019), and Gorbenko and Malenko (2018), which also feature strategic timing of the auction. Board (2007) and Cong (2019) study the problem of a seller auctioning an option, such as the right to drill oil, where the timing of the sale and option exercise are decision variables. Gorbenko and Malenko (2018) assume that M&A contests are bidder-initiated and study how their financial constraints affect the timing of initiation. These papers do not study joint initiation by bidders and the seller and restrict attention to independent private values, so the issues examined in our paper do not arise.

Second, the paper is related to the literature that studies takeover contests as auctions. They have been modeled using both the common-value (e.g., Bulow, Huang, and Klemperer, 1999) and private-value framework (e.g., Fishman, 1988; Burkart, 1995; Povel and Singh, 2006).[5] Like us, Bulow, Huang, and Klemperer (1999) interpret competition between strategic (or financial) bidders as a private-value (or common-value) auction. However, these papers do not study endogenous initiation of takeover contests. Initiation has an interesting relation to preemptive bidding, analyzed by Fishman (1988).[6] Focusing on private values, Fishman (1988) shows that the possibility of preemption gives a first-mover advantage to the initial bidder over an uninformed rival. In a model extension, we allow for information acquisition and preemption by bidders, and show that when values are common, bidders still prefer not to approach the seller, because the aforementioned first-mover advantage shrinks as values become more common.

The implications of our results for shareholder activism relate our paper to recent literature that studies interactions between activism and the market for corporate control focusing on other aspects of the interaction – Burkart and Lee (2020) focus on the free-rider problem in tender offers, while Corum and Levit (2019) focus on the commitment problem of the bidder in a proxy fight. Greenwood and Schor (2009), Jiang, Li, and Mei (2016), and Boyson, Gantchev, and Shivdasani (2017) provide empirical evidence on interactions between activism and the market for corporate control, which is broadly consistent with shareholder activists creating value in the M&A market.

[5]Bulow and Klemperer (1996, 2009) provide motivations for why running a simple auction is often a good way for the seller to sell the asset.

[6]See also Che and Lewis (2007) who study lockups in a model based on Fishman (1988).

Finally, the paper is related to models of auctions with asymmetric bidders. Most of the literature on auction theory assumes that bidders are symmetric in the sense that their signals are drawn from the same distribution. Some recent literature examines issues that arise when bidders are asymmetric.[7] The novelty of our paper is that asymmetries at the auction stage are not assumed: they arise endogenously and are driven by incentives to approach the seller, which differ with the bidder's information. While bidders are ex-ante symmetric, at the auction stage they may not be: the decision of one bidder to approach the seller makes it commonly known that bidders' signals come from different partitions of the same ex-ante distribution of signals.[8]

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. Section 3 solves for the equilibrium bidding in bidder- and seller-initiated auctions and analyzes how the resulting payoffs depend on the commonality of values. Section 4 characterizes equilibrium initiation strategies. Section 5 extends the baseline model to a richer class of selling mechanisms and to costly information acquisition by bidders. Section 6 discusses the role of additional features of the market for corporate control that are absent from the base model. Section 7 concludes.

2 The model setup

The economy consists of one risk-neutral seller (male) and two risk-neutral potential buyers (female), referred to as bidders, indexed by $i = 1, 2$.[9]. There are two periods, $t = 0$ and $t = 1$. All agents value period-1 flows at multiple $\beta \in (0, 1)$. The seller has an asset for sale. For example, the asset can be the whole company or a business unit. The seller's value of the asset is normalized to zero.

At $t = 0$, each bidder randomly draws a private signal. Signals are independent draws from the uniform distribution over $[0, 1]$. Conditional on all signals, the value of the asset

[7] For example, Maskin and Riley (2000, 2003), Campbell and Levin (2000), Lebrun (2006), Kim (2008), and Liu (2016).

[8] While on a different topic, the learning effect in common-value auctions is related to Ely and Siegel (2013), who develop a static model of firms interviewing and hiring workers and show that in equilibrium only the highest-ranked firm interviews the applicant.

[9] The model can be extended to $N \geq 2$ potential bidders with the main qualitative effects intact.

to bidder i is $\alpha v(s_i) + (1-\alpha) v(s_{-i})$, where s_{-i} is the signal of the rival.[10]

Assumption 1. *Function $v(\cdot)$ is continuous, strictly increasing, and satisfies $v(0) = 0$. Parameter α takes values between $\frac{1}{2}$ and 1.*

Assumption 1 is standard. Continuity means that there are no gaps in possible values of the asset. Strict monotonicity means that a higher private signal is always good news about the bidder's value. Finally, $\alpha \in \left[\frac{1}{2}, 1\right]$ means that the model can capture pure private values, pure common values, as well as intermediate cases. At the extremes, it covers two valuation structures commonly used in the literature:

- **Pure private values:** $\alpha = 1$. A bidder's signal provides information only about her own value $v(s_i)$, but not about the value of her competitor $v(s_{-i})$.
- **Pure common values:** $\alpha = \frac{1}{2}$. Conditional on both signals, bidders have the same value of the asset, $\frac{1}{2}(v(s_1) + v(s_2))$. However, bidders differ in their assessments of the value, because of different private signals.

If $\alpha \in \left(\frac{1}{2}, 1\right)$, the valuation structure includes elements of both private and common values. The importance of common values is strictly decreasing in α.

There are three natural interpretations of parameter α in the context of auctions of companies and business units. The first interpretation deals with different types of bidders that compete for an asset: We can interpret battles between two financial (strategic) bidders as situations in which α is relatively low (high). Financial bidders use similar strategies after they acquire the target, but may have different estimates of potential gains. In contrast, because synergies that strategic bidders expect to achieve from acquiring the target are often bidder-specific, they are likely to provide little information about value of the target to the other bidder. Thus, it is natural to expect that values of financial bidders have a greater common component than values of strategic bidders. The second interpretation deals with different types of targets rather than bidders. Broadly, value in an acquisition

[10]Because $v(\cdot)$ is a general function, the assumption of uniform distribution is, to a large extent, a normalization.

can be created either because the incumbent target management is inefficient or because the target and the acquirer have synergies that cannot be realized by the stand-alone target. To the extent that inefficiency can be resolved by many bidders, acquisitions of the first type are expected to have a higher common component of bidders' values. Finally, commonality of values can be generated by resale opportunities among bidders with otherwise private values (e.g., Hafalir and Krishna, 2008). In this interpretation, parameter α captures the degree of frictions in the resale market.

In practice, the environment changes over time, as either external economic shocks arrive or the business nature of the asset for sale changes. In the model, with probability $\lambda > 0$ a shock arrives at $t = 1$, in which case both bidders' values of the asset change. Specifically, each bidder i draws a new independent signal from the uniform distribution over $[0, 1]$, denoted s'_i, $i \in \{1, 2\}$, and her value becomes $\alpha v(s'_i) + (1 - \alpha) v\left(s'_{-i}\right)$. In addition, each period with probability $\nu \geq 0$, the seller experiences a liquidity shock and has no choice but to sell the asset immediately. Examples of such liquidity shocks are an arrival of an attractive investment opportunity that is mutually exclusive with the asset under consideration, a change in the strategy of the firm, or a bankruptcy in which the judge liquidates the target by auctioning its assets among potential bidders.[11]

In addition to being forced to sell due to a liquidity shock, the seller has the right to auction the asset off to the bidders at any time. Prior to the auction, each bidder communicates with the seller by sending a private message signaling her interest in acquiring the asset. Communication takes the form of cheap talk (Crawford and Sobel, 1982). Messages are costless, and the message space is binary, 0 and 1. Without loss of generality, we interpret message $m_{i,t} = 1$ from bidder i in period t as an indication of interest in acquiring the asset, and we interpret message $m_{i,t} = 0$ as the lack of such interest. Having observed messages $\mathbf{m}_t = (m_{1,t}, m_{2,t})$, the seller decides whether to place the asset up for sale ($d_t = 1$) or not ($d_t = 0$).

The auction format is sealed-bid first-price with no reserve price. Each bidder simulta-

[11] As an example of an investment opportunity triggering the sale of an existing asset, consider the case of the seller acquiring another firm in a horizontal merger. As a condition of approval, it is common for antitrust authorities to require a spin-off of some of the existing assets to ensure that the product market does not become too concentrated.

neously submits a bid to the seller in a concealed fashion. The two bids are compared, and the bidder with the highest bid acquires the asset and pays her bid. Once the asset is sold, the game ends. The winning bidder obtains the payoff that equals her value less the price she pays. The losing bidder obtains zero payoff. The seller obtains the winning bid.[12] Our focus on the first-price auction is motivated by two factors, besides its popularity. First, it has a unique equilibrium, even if bidders are asymmetric, which makes the analysis unambiguous. Second, it is the simplest auction format that features strategic bidding, because the strategy of each bidder depends on her expectations of bids of the rival. Because of this, the model delivers interesting implications regarding strategic bidding, which should be also relevant in practice.[13] In Section 5, we generalize the analysis to a large class of auctions, which includes the first-price auction and (in the limit) the second-price auction. The main result that bidders do not indicate their interest for an asset with a sufficiently high common value component holds for any auction in that class.

We assume that the indications of interest $\mathbf{m}_t$ are publicly observable. The conceptually important assumption is that they become observable to the bidders prior to bidding.[14] This assumption can be justified as follows. The seller may voluntarily disclose whether the auction is bidder- or seller-initiated. In many contexts, whether the auction was bidder- or seller-initiated can be verified ex post – for example, any publicly-traded U.S. target is required to report background of the transaction as part of its SEC filings, and lying in it has legal consequences. By the standard reasoning (Grossman, 1981; Milgrom, 1981), because it is common knowledge that the seller knows whether the auction is bidder- or

[12] The seller's private value of the asset can also be important for his decision to offer it for sale. Lauermann and Wolinsky (2017) study common-value first-price auctions in which the seller obtains a private signal about his value and solicits a different number of bidders at a cost depending on the signal. Being solicited is thus a signal of the seller's information to bidders. While modeling the two-sided private information is beyond the scope of this paper, it is potentially interesting to examine interactions between seller and bidder initiation in the presence of the solicitation effect.

[13] A typical selling mechanism in auctions of companies, described in Hansen (2001), has several elements of the first-price auction despite having multiple rounds of bidding. First, there is often a final round in which, in contrast to informal preliminary bids, bidders submit formal sealed bids. Second, jump bidding is prevalent. As Avery (1998) and Daniel and Hirshleifer (2018) show, equilibria and outcomes of the ascending auction with jump bids have features of the first-price auction.

[14] In other words, we assume that messages are publicly observed, whether the seller decides to hold an auction or not. Alternatively, we could assume that the messages become observable only if the seller decides to hold the auction. This change affects some off-equilibrium-path payoffs, but the analysis largely remains the same.

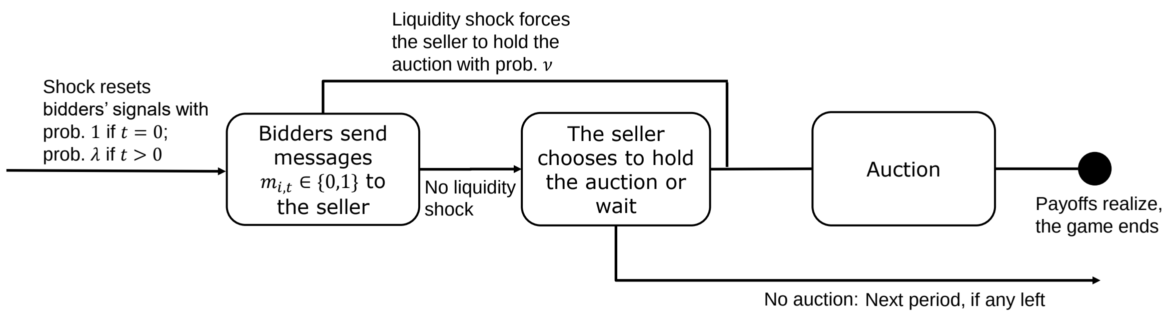

Figure 1: **Timing of the model.**

seller-initiated and this information is verifiable, he will always disclose it.

As we shall see, we will examine equilibria in which upon receiving an indication of interest, $m_{i,t} = 1$, the seller auctions the asset off immediately. We refer to such an event as a *"bidder-initiated"* auction. We refer to an event of the seller auctioning the asset off without receiving any indications of interest as a *"seller-initiated"* auction.

Figure 1 summarizes the timing of actions in each period.

2.1 The equilibrium concept

The equilibrium concept is Perfect Bayesian Equilibrium in pure strategies.[15] In the auction, the bidding strategy of each bidder is a non-decreasing mapping from her own signal s_i and the history of the game into a non-negative bid. Prior to the auction, the communication strategy of each bidder is a mapping from her own signal s_i and the history of the game into message $m_{i,t} \in \{0, 1\}$, i.e., to send an indication of interest to the seller or not. Because bidders are ex-ante symmetric, we look for equilibria in which they follow symmetric communication strategies.

Furthermore, we look for equilibria in which bidders follow cut-off communication strategies, such that a bidder sends message $m_{i,t} = 1\{s_i \geq \hat{s}_t\}$ for some time- and history-contingent cut-off $\hat{s}_t \in [0, 1]$.[16] If in equilibrium all bidder types send the same message

[15] In the appendix, we also consider mixed strategies by the seller. Their analysis is more complicated but does not alter the main findings.

[16] We do not know whether the restriction to equilibria in cut-off strategies is with or without loss of generality, for example, if there can exist equilibria with multiple thresholds. This is because the analysis

at some time t (e.g., if $\hat{s}_t \leq 0$ or $\hat{s}_t \geq 1$), we denote this message by 0, without loss of generality.

An equilibrium is *responsive* if there exist message profiles $\mathbf{m}$ and $\mathbf{m}' \neq \mathbf{m}$ that are sent on equilibrium path such that $d_t(\mathbf{m}) \neq d_t(\mathbf{m}')$ for some t. In other words, an equilibrium is responsive if a bidder's indication of interest has an effect on the decision of the seller to put the asset up for sale. As we shall see below, any responsive equilibrium has a property that a single indication of interest is sufficient to induce an auction. We will refer to an auction triggered by an indication of interest as *a bidder-initiated auction.* Otherwise, we refer to an auction as *a seller-initiated auction.*

An equilibrium is *non-responsive* if $d_t(\mathbf{m}) = d_t(\mathbf{m}')$ for all t and all message profiles $\mathbf{m}$ and $\mathbf{m}'$ that are sent on equilibrium path. In other words, an equilibrium is non-responsive if the decision of the seller to put the asset up for sale is not affected by bidders' indications of interest. As we will see, in a non-responsive equilibrium, all types of bidders will send the same message, which will thus convey no information, and thus all auctions will be seller-initiated.

We solve the model by backward induction. First, we solve for the equilibrium at the auction stage for all possible initiation scenarios. Second, we analyze equilibria at the initiation game, first at $t = 1$, and then at $t = 0$.

3 Auction stage

We consider the following three cases in a sequence. First, we consider an auction following an indication of interest from a single bidder. Second, we consider a seller-initiated auction, when the seller puts the asset up for sale without receiving any indications of interest. Finally, we consider an auction following indications of interest from both bidders. With slight abuse of terminology, we refer to the first and third cases as a bidder-initiated auction and an auction initiated by both bidders, respectively. Throughout this section, we assume that signals that are distributed uniformly over $[0, \bar{s}]$ and that the indication of interest is sent by bidders with signals $[\hat{s}, \bar{s}]$, where $\hat{s} \leq \bar{s}$ and $\bar{s}$ are arbitrary values in $(0, 1]$ to be

of first-price auctions when distributions of signals have an arbitrary number of gaps is, to our knowledge, an open problem.

determined in equilibrium of the initiation game.

3.1 A bidder-initiated auction

In this case, the auction is triggered by one of the two bidders indicating her interest to the seller, i.e., $m_{1,t} = 1 - m_{2,t}$. Denote the initiating and non-initiating bidders by I and N, respectively. From the point of view of bidder N and the seller, the signal of bidder I is distributed uniformly over $[\underline{s}_I, \bar{s}_I] = [\hat{s}, \bar{s}]$. Similarly, from the point of view of bidder I and the seller, the signal of bidder N is distributed uniformly over $[\underline{s}_N, \bar{s}_N] = [0, \hat{s}]$. Thus, even though bidders are ex-ante symmetric, initiation endogenously creates an asymmetry at the auction stage. For brevity, we suppress $\hat{s}$ and $\bar{s}$ from sets of arguments of all functions in this subsection.[17]

Let $a_I(s)$ and $a_N(s)$ be the equilibrium bids of bidder I with signal s and bidder N with signal s, respectively, in a candidate equilibrium. Conjecture that each bid is strictly increasing in s in the relevant range.[18] Denote the corresponding inverses in s by $\phi_I(b)$ and $\phi_N(b)$. Intuitively, $\phi_j(b)$ is the signal of bidder $j \in \{I, N\}$ that submits bid b. Then, the expected payoff of bidder of type $j \in \{I, N\}$ with signal s from bidding b is

$$\Pi_j(b, s) = \int_{\underline{s}_k}^{\phi_k(b)} (\alpha v(s) + (1-\alpha) v(x) - b) \frac{1}{\bar{s}_k - \underline{s}_k} dx, \tag{1}$$

where $j \neq k \in \{I, N\}$. The intuition behind (1) is as follows. For example, consider the initiating bidder that bids b. She wins the auction if and only if the bid of the non-initiating bidder is below b, which happens if such bidder's signal is below $\phi_N(b)$. Conditional on winning when the rival's signal is $x \in [0, \phi_N(b)]$, the value of the asset to the initiating bidder is $\alpha v(s) + (1-\alpha) v(x)$. Integrating over $x \in [0, \phi_N(b)]$ yields (1) for $j = I$.

Taking the first-order conditions of (1), we obtain

$$\frac{\partial \phi_k(b)}{\partial b} (\alpha v(s) + (1-\alpha) v(\phi_k(b)) - b) - (\phi_k(b) - \underline{s}_k) = 0 \tag{2}$$

[17] In particular, equilibrium bids $a_I(\cdot)$ and $a_N(\cdot)$, their inverses $\phi_I(\cdot)$ and $\phi_N(\cdot)$, range of bids $[\underline{a}, \bar{a}]$, and bidders' payoffs depend on $\hat{s}$ and $\bar{s}$.

[18] This conjecture is proven in Lemma 1.

for $k \in \{I, N\}$. The first and second terms of equations (2) represent the trade-off between the marginal benefit and the marginal cost of increasing a bid by a small amount. The marginal benefit is that bidder j wins a marginal event in which the signal of the rival bidder k is exactly $\phi_k(b)$. Her payoff from winning this event is therefore $\alpha v(s) + (1-\alpha) v(\phi_k(b)) - b$. The marginal cost is that bidder j must pay more in case she wins. In equilibrium, $b = a_j(s)$ must satisfy (2) for $j \neq k$, implying $s = \phi_j(b)$. Plugging in and rearranging the terms, for $j \in \{I, N\}$ we obtain:

$$\frac{\partial \phi_j(b)}{\partial b} = \frac{\phi_j(b) - \underline{s}_j}{\alpha v(\phi_k(b)) + (1-\alpha) v(\phi_j(b)) - b}. \tag{3}$$

The system of two differential equations (3) is solved subject to the appropriate boundary conditions. The first condition is that in equilibrium, the highest bid submitted by both bidders, $a_j(\bar{s}_j) \equiv \bar{a}$ for $j \in \{I, N\}$, must be the same. The condition implies $\phi_I(\bar{a}) = \bar{s}$ and $\phi_N(\bar{a}) = \hat{s}$. By contradiction, if, say, $a_I(\bar{s}) > a_N(\hat{s})$, then bidder I with a signal that is sufficiently close to $\bar{s}$ would be better off reducing her bid: she would still win the auction with certainty but her payment would be lower.[19]

Let $a_I(\hat{s}) \equiv \underline{a}$ be the lowest bid submitted by bidder I, which implies $\phi_I(\underline{a}) = \hat{s}$. The second boundary condition is that in equilibrium, bidder N that submits bid $\underline{a}$ must be indifferent between winning and losing: $\alpha v(\phi_N(\underline{a})) + (1-\alpha) v(\hat{s}) = \underline{a}$. Intuitively, if bidder N with signal $\phi_N(\underline{a})$ strictly preferred to win, then this bidder would be better off deviating to a higher bid that wins with a strictly positive probability.

The final boundary condition arises from solving for bid $\underline{a}$, i.e., the lowest bid that can potentially win. In principle, this bid can depend on bids of bidder N with very low signals. Because such a bidder wins with probability zero, one can rationalize a variety of her bids. In the following analysis, we impose the following natural restriction on equilibrium bids, which will also allow us to pin down the unique equilibrium in the auction:[20]

Assumption 2. *A bidder's belief about the signal of the rival is weakly monotone in the*

[19] The argument for why $a_N(\hat{s})$ cannot exceed $a_I(\bar{s})$ is identical.

[20] As Kaplan and Zamir (2015) show in the special case of pure private values, without this restriction, multiple equilibria in the first-price auction with asymmetric bidders arise, in which some bidders submit counterintuitive "non-serious" bids (i.e., bids that win with probability zero) above their values.

rival's bid. No bidder bids above her expected (conditional on winning) value in equilibrium.

Assumption 2 holds trivially for bids in $(\underline{a}, \bar{a})$, i.e., bids that win with positive probability. First, because a bid is monotone in the bidder's signal, a higher bid is indicative of a higher signal. Second, because a bidder can always bid zero and lose the auction with certainty, she would never bid above her expected (conditional on winning) value. Assumption 2 disciplines the bids of bidder N with a very low signal. Consider such a bidder contemplating some bid $b \leq \underline{a}$. The first part of Assumption 2 means that the fact that her bid is winning (which never happens in equilibrium) means that the signal of bidder I must be $\hat{s}$. The second part of Assumption 2 means that bidder N with a very low signal s cannot bid above $\alpha v(s) + (1-\alpha) v(\hat{s})$: such a bid is irrational, because bidder N achieves a negative payoff if she wins. As we show in the proof of Lemma 1 in the appendix, Assumption 2 uniquely pins down the minimum "serious" bid $\underline{a}$ in the following way:

$$\underline{a} = \arg\max\nolimits_b v^{-1}\left(\tfrac{b-(1-\alpha)v(\hat{s})}{\alpha}\right) \times \left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x) | x \leq v^{-1}\left(\tfrac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - b\right). \quad (4)$$

The next lemma summarizes the equilibrium in the bidder-initiated auction:[21]

Lemma 1 (equilibrium in the bidder-initiated auction). *The bidder-initiated auction has a unique (up to the non-serious bids of the non-initiating bidder with a low signal) equilibrium. The equilibrium bidding strategies of the initiating and non-initiating bidders, $a_j(s)$, $j \in \{I, N\}$, are increasing functions with the lowest serious bid $\underline{a}$ given by (4), such that their inverses satisfy (3) with boundary conditions*

$$\bar{s} = \phi_I(\bar{a}), \quad \hat{s} = \phi_N(\bar{a}), \quad \hat{s} = \phi_I(\underline{a}), \quad \phi_N(\underline{a}) = v^{-1}\left(\frac{\underline{a} - (1-\alpha) v(\hat{s})}{\alpha}\right). \quad (5)$$

Examples 1 and 2 in the appendix solve for bidding strategies in a bidder-initiated auction

[21] Note that our analysis can be extended to a model with any number $N \geq 2$ of bidders. The lemma then covers the auction with one initiating bidder and $N-1$ non-initiating bidders. Its solution is still a system of two similar differential equations with similar boundary conditions. When there is more than one initiating bidder, they compete against each other and erode each others' profits as illustrated in Section 3.3., irrespective of the number of non-initiating bidders.

for $\alpha = 1$ (pure private values) and $\alpha = \frac{1}{2}$ (pure common values), when $v(s) = s$, $\bar{s} = 1$, and $\hat{s} = \frac{1}{2}$. These bidding strategies are illustrated in Figure 1, Panels A and B.

We denote the equilibrium payoff of bidder $j \in \{I, N\}$ from the bidder-initiated auction by $\Pi_j^* (s, \hat{s}, \bar{s})$, where we are now explicit about its dependence on $\hat{s}$ and $\bar{s}$. Equilibrium payoffs of bidders $j \in \{I, N\}$ with cut-off signal $\hat{s}$ (useful in the analysis of the initiation stage) can be computed without knowing equilibrium bids of all types:

$$\Pi_I^* (\hat{s}, \hat{s}, \bar{s}) = \max_y \frac{v^{-1}(y)}{\hat{s}} \left((2\alpha - 1) v(\hat{s}) + (1-\alpha) \mathbb{E}\left[v(x) | x \leq v^{-1}(y) \right] - \alpha y \right), \quad (6)$$

$$\Pi_N^* (\hat{s}, \hat{s}, \bar{s}) = \alpha v(\hat{s}) + (1-\alpha) \mathbb{E}\left[v(x) | x \in [\hat{s}, \bar{s}] \right] - \bar{a}(\hat{s}, \bar{s}). \quad (7)$$

We denote the expected revenue of the seller from the bidder-initiated auction by

$$R_B(\hat{s}, \bar{s}) = \mathbb{E}\left[\max\left(a_I(s_1), a_N(s_2) \right) | s_1 \in [\hat{s}, \bar{s}], s_2 \in [0, \hat{s}] \right]. \quad (8)$$

3.2 A seller-initiated auction

Next, consider a seller-initiated auction. In this case, the seller puts the asset up for sale without receiving any indications of interest, i.e., $m_{1,t} = m_{2,t} = 0$. Thus, from the point of all agents, the signal of each bidder is distributed uniformly over $[0, \hat{s}]$ for some $\hat{s}$ to be solved for later. Indeed, if any of the bidders had a signal above $\hat{s}$, she would have indicated her interest to the seller. Because the two bidders are symmetric, the equilibrium is also symmetric. Denote the equilibrium bid by a bidder with signal s by $a_S(s)$ and its inverse in s by $\phi_S(b)$, where we again suppress the possible dependence on $\hat{s}$ for brevity. The expected payoff of a bidder with signal s from bidding b is:

$$\Pi_S(b, s) = \int_0^{\phi_S(b)} \left(\alpha v(s) + (1-\alpha) v(x) - b \right) \frac{1}{\hat{s}} dx. \quad (9)$$

Intuitively, the bidder wins if and only if the signal of the rival bidder x is below $\phi_S(b)$, in which case the asset is worth $\alpha v(s) + (1-\alpha) v(x)$. Taking the first-order condition of (9), we obtain

$$\frac{\partial \phi_S(b)}{\partial b} = \frac{\phi_S(b)}{v(\phi_S(b)) - b}, \quad (10)$$

which is solved with the initial value condition that the bidder with the lowest signal bids zero: $\phi_S(0) = 0$. This differential equation leads to the following equilibrium bid:

$$a_S(s) = \mathbb{E}\left[v(x) \,|x \le s\right]. \tag{11}$$

The next lemma summarizes the equilibrium in the seller-initiated auction:

Lemma 2 (equilibrium in the seller-initiated auction). *The seller-initiated auction has a unique equilibrium. A bidder with signal $s \in [0, \hat{s}]$ bids $a_S(s) = \mathbb{E}\left[v(x) \,|x \le s\right]$.*

Panels A and B of Figure 1 illustrate bidding strategies in the seller-initiated auction for $\alpha = 1$ (pure private values) and $\alpha = \frac{1}{2}$ (pure common values) when $v(s) = s$, $\bar{s} = 1$, and $\hat{s} = \frac{1}{2}$. Note that the equilibrium in the seller-initiated auction depends neither on whether the valuation structure is closer to common or private values nor on the cut-off $\hat{s}$. Intuitively, $\hat{s}$ is irrelevant, because when choosing a bid, a bidder conditions on the rival's signal below the bidder's signal. The degree of commonality of values α is irrelevant, because the marginal event that a bidder with signal s wins has the opponent with exactly the same signal, and therefore the bidder infers that the value of the asset in this marginal event is $v(s)$. In contrast, both the valuation structure and the initiation cut-off matter for the equilibrium in the bidder-initiated auction.

We denote the equilibrium payoff of a bidder from the seller-initiated auction by $\Pi_S^*(s, \hat{s})$, where we are now explicit about the payoff's dependence on $\hat{s}$. The equilibrium payoff of a bidder with cut-off signal $\hat{s}$ is:

$$\Pi_S^*(\hat{s}, \hat{s}) = \alpha v(\hat{s}) - \alpha \mathbb{E}\left[v(x)|x \le \hat{s}\right]. \tag{12}$$

We denote the expected revenues of the seller from the seller-initiated auction by

$$R_S(\hat{s}) = \mathbb{E}\left[v\left(\min\left(s_1, s_2\right)\right)|s_i \le \hat{s}, i \in \{1, 2\}\right]. \tag{13}$$

Note that $R_S(\hat{s})$ does not depend on α, because bidders' equilibrium bidding strategies in the seller-initiated auction do not depend on α, as shown above.

3.3 An auction initiated by both bidders

The remaining possibility is that the seller puts the asset up for sale after both bidders indicate their interest, i.e., $m_{1,t} = m_{2,t} = 1$. We will call this outcome the dual bidder-initiated auction. In this case, from the point of view of all agents, the signal of each bidder is distributed uniformly over $[\hat{s}, \bar{s}]$ for some $\hat{s}$ and $\bar{s}$ to be determined later. Denoting the equilibrium bid by a bidder with signal s by $a_D(s)$ and following the derivation analogous to the case of a seller-initiated auction, we obtain the following equilibrium bid:

$$a_D(s) = \mathbb{E}\left[v(x)\,|x \in [\hat{s}, s]\right]. \tag{14}$$

As in the seller-initiated auction, the equilibrium bid is independent of the degree of commonality of values α and the highest signal $\bar{s}$. However, it depends on the cut-off signal $\hat{s}$: it becomes common knowledge that the value of the asset is at least $v(\hat{s})$. In turn, a bidder with signal $\hat{s}$ bids $a_D(\hat{s}) = v(\hat{s})$, wins only if the rival bidder's signal is also $\hat{s}$, and obtains zero surplus from the auction. Formally, letting $\Pi_D^*(s, \hat{s}, \bar{s})$ denote the equilibrium payoff of a bidder with signal s from such an auction, $\Pi_D^*(\hat{s}, \hat{s}, \bar{s}) = 0$.

3.4 Ranking of bidders' payoffs

Given the equilibria in the bidder- and seller-initiated auctions, we next compare the payoffs of a bidder with the cut-off signal $\hat{s}$ in three cases: (1) in a bidder-initiated auction that she initiates; (2) in a bidder-initiated auction initiated by the rival bidder; (3) in a seller-initiated auction. The case of a dual bidder-initiated auction is trivial, because in it the payoff of a bidder with signal $\hat{s}$ is zero. The next proposition establishes this comparison:

Proposition 1 (ranking of payoffs for the cut-off type). *The expected payoffs of a bidder with signal $\hat{s}$ from the seller-initiated auction, $\Pi_S^*(\hat{s}, \hat{s})$, the auction initiated by her, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$, and the auction initiated by the rival bidder, $\Pi_N^*(\hat{s}, \hat{s}, \bar{s})$, compare as follows:*

1. $\Pi_S^*(\hat{s}, \hat{s}) > \Pi_I^*(\hat{s}, \hat{s}, \bar{s})$ *for any α.*

2. *There exists* $\alpha' \in \left(\frac{1}{2}, 1\right)$ *such that for any* $\alpha > \alpha'$, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s}) > \Pi_N^*(\hat{s}, \hat{s}, \bar{s})$.

3. *There exists* $\alpha'' \in \left(\frac{1}{2}, 1\right)$ *such that for any* $\alpha < \alpha''$, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s}) < \Pi_N^*(\hat{s}, \hat{s}, \bar{s})$. *Furthermore,* $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$ *approaches zero as* $\alpha \to \frac{1}{2}$.

Panels C and D of Figure 1 illustrate these payoffs for $\alpha = 1$ (pure private values) and $\alpha = \frac{1}{2}$ (pure common values) when $v(s) = s$, $\bar{s} = 1$, and $\hat{s} = \frac{1}{2}$. Proposition 1 is key for the results about initiation derived in the next section, so it is worth explaining the intuition in detail. The first result shows that a bidder with signal $\hat{s}$ is always better off in the auction initiated by the seller than in the auction initiated by her. This result may seem surprising, because the distribution of the signal of the other, non-initiating bidder is the same in both kinds of auctions. However, the important difference is in the non-initiating bidder's perception of the strength of her rival. In a bidder-initiated auction, the non-initiating bidder believes that she competes against a strong rival whose signal is distributed over $[\hat{s}, \bar{s}]$. In contrast, in a seller-initiated auction, each bidder believes that she competes against a weak rival whose signal is distributed over $[0, \hat{s}]$. In response, the non-initiating bidder bids more aggressively in the bidder-initiated auction. As a result, a bidder with signal $\hat{s}$ obtains a higher payoff in the seller-initiated auction. Note that this argument and the result do not depend on the degree of commonality of bidders' values.

The second result of Proposition 1 shows that a bidder with signal $\hat{s}$ is better off in the auction initiated by her than in the auction initiated by the rival bidder if the private value component is sufficiently high. In contrast, if the common value component is sufficiently high, then the opposite is true, as shown in the third result of Proposition 1. Intuitively, there are two opposite effects. It is easiest to see them in the two extreme cases, pure common values ($\alpha = \frac{1}{2}$) and pure private values ($\alpha = 1$).

The first effect, which we call the *asset revaluation* effect, discourages initiation. If bidders' values have a common component, the non-initiating bidder updates the belief about her value from the fact that the auction is initiated by the rival bidder: she learns that the signal of the rival bidder is at least $\hat{s}$. In the case of pure common values, the non-initiating bidder learns that the value of the asset is at least $\frac{1}{2}(v(s) + v(\hat{s}))$, where s is her signal. In particular, it becomes common knowledge that the value of the asset is at least

$\frac{1}{2}v(\hat{s})$, because the signal of the non-initiating bidder cannot be below zero and the signal of the initiating bidder cannot be below $\hat{s}$. Thus, no bidder bids below $\frac{1}{2}v(\hat{s})$. Consequently, the initiating bidder with signal $\hat{s}$ wins the auction only when the non-initiating bidder's signal is zero and at price $\frac{1}{2}v(\hat{s})$, leaving the initiating bidder with zero surplus. In contrast, the non-initiating bidder with signal $\hat{s}$ preserves her rents, as the initiating bidder does not know that her signal is that high. If the common component of values becomes smaller (α increases), rents of the initiating bidder of the cut-off type grow, as they decline less due to the re-valuation of the rival bidder. However, as long as values are sufficiently close to common, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s}) < \Pi_N(\hat{s}, \hat{s}, \bar{s})$.

The second effect, which we call the *rival selection* effect, encourages initiation. If bidders' values have a private component, the fact that the rival bidder has not indicated an interest implies that the initiating bidder competes against a weak rival whose signal is distributed over $[0, \hat{s}]$. In contrast, if a bidder with signal $\hat{s}$ waits until the auction is initiated by the rival bidder, she will compete against a strong rival whose signal is distributed over $[\hat{s}, \bar{s}]$, resulting in lower rents. In the case of pure private values, only this effect is present as there is no re-valuation, implying a higher surplus for the initiating bidder with signal $\hat{s}$. If the private component of values becomes smaller (α decreases), this effect becomes weaker. However, as long as values are sufficiently close to private, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s}) > \Pi_N(\hat{s}, \hat{s}, \bar{s})$.

Related to the argument of Proposition 1, the next proposition generates a set of testable implications about the bidding behavior in bidder-initiated and seller-initiated auctions:[22]

Proposition 2 (empirical implications about bidding and values). *Suppose that $\alpha > \frac{1}{2}$, i.e., that values are not pure common. Then:*

1. *In a bidder-initiated auction, conditional on the same signal, the non-initiating bidder bids more aggressively than the initiating bidder: $a_N(\hat{s}) > a_I(\hat{s})$.*

2. *In a bidder-initiated auction, unconditional on the signal, the initiating bidder bids more aggressively and wins more often: $\mathbb{E}[a_I(s) | \hat{s} \leq s \leq \bar{s}] > \mathbb{E}[a_N(s) | s \leq \hat{s}]$.*

[22]See Landsberger et al. (2001) and Section 4.3 of Krishna (2010) for related results on the comparison of bidding strategies of strong and weak bidders in the first-price auction under independent private values when asymmetry is assumed. Proposition 2 relates asymmetry to initiation and goes beyond private values.

3. *Any non-initiating bidder that makes a serious bid in the bidder-initiated auction (i.e.,* $a_N(s) \geq \underline{a}$*) bids less aggressively in a seller-initiated auction:* $a_S(s) < a_N(s)$.

4. *The value of the winning bidder is, on average, higher in a bidder-initiated auction than in a seller-initiated auction, whether she is an initiating bidder or not.*

These implications are testable for auctions of companies, as the data on initiation and bidding behavior are available from SEC deal backgrounds.[23]

4 Initiation game

Having solved for the equilibria in the auction for all combinations of bidders' messages, we proceed by analyzing the initiation game. We solve the model by backward induction. First, we consider bidders' and seller's initiation decisions in the terminal period, $t = 1$. It turns out that this subgame has a unique and very simple equilibrium. Using this equilibrium, we analyze initiation decisions in period $t = 0$, which is our main object of interest. Finally, we derive the key properties of initiation equilibria.

4.1 Initiation in the terminal period

Consider the initiation game in period $t = 1$, conditional on the seller choosing to not hold the auction at $t = 0$. At $t = 1$, each bidder's signal is distributed uniformly over $[0, \bar{s}]$, where either $\bar{s} = 1$ if a shock arrives earlier in the period leading to a reset of bidders' signals, or $\bar{s} = \hat{s}_0$ if no such shock arrives. Here, $\hat{s}_0$ denotes the cut-off signal at $t = 0$, such that bidders with all signals above $\hat{s}_0$ indicate their interest to the seller at $t = 0$. This cut-off will be determined below.

The next proposition shows that for all α and $\bar{s}$ the terminal period has a very simple equilibrium: no bidder sends an indication of interest to the seller regardless of how high her signal is, and the seller always initiates the auction:

[23]To test some of the implications in Proposition 2, one can estimate values either from announcement returns (if the acquirer and the target are publicly traded firms) or structurally using data on bids.

Proposition 3 (equilibrium in the initiation game at $t = 1$). *There is a unique non-responsive equilibrium, in which both bidders do not send indications of interest ($m_{i,1} = 0\ \forall s_i, i \in \{1,2\}$), and the seller holds the auction at $t = 1$ regardless of the messages: $d_1(\mathbf{m}_1) = 1\ \forall \mathbf{m}_1$.*

The intuition is as follows. Because the seller values the asset less than bidders and the game ends at $t = 1$, there is no benefit for him to not hold an auction at $t = 1$ regardless of the indications of interest. Expecting this reaction from the seller, no bidder finds it optimal to indicate her interest, as by Proposition 1 the payoff of the bidder with any hypothetical cut-off signal is higher when the rival perceives her to be weak. Thus, the only equilibrium at $t = 1$ is that no bidder indicates her interest, and the auction is seller-initiated.

Given Proposition 3, we can compute the continuation payoffs in period $t = 1$ for the seller and each bidder. Suppose that at $t = 0$ each bidder sends message $m_{i,0} = 1$ if and only if $s_i \geq \hat{s}_0$ for some $\hat{s}_0 \in [0,1]$, and the seller holds the auction unless $m_{1,0} = m_{2,0} = 0$, which will be shown to be the case in the next subsection. From Lemma 2, whether bidders' signals reset at $t = 1$ or not, they follow the same bidding strategy $a_S(s) = \mathbb{E}[v(x)\,|x \leq s]$ at $t = 1$. Thus, before the signal-resetting shock, the expected revenues of the seller are

$$R_1(\hat{s}_0) = \lambda \mathbb{E}[v(\min(s_1, s_2))] + (1-\lambda)\,\mathbb{E}[v(\min(s_1, s_2))\,|s_i \leq \hat{s}_0, i \in \{1,2\}]\,. \tag{15}$$

Likewise, before the signal-resetting shock, the expected payoff of a bidder with signal $\hat{s}_0$ is

$$\Pi_1(\hat{s}_0) = (1-\lambda)\,\Pi_S^*(\hat{s}_0, \hat{s}_0) + \lambda \Pi_R = (1-\lambda)\,\alpha\,(v(\hat{s}_0) - \mathbb{E}[v(x)\,|x \leq \hat{s}_0]) + \lambda \Pi_R, \tag{16}$$

where Π_R is the expected payoff of a bidder conditional on a reset of signals at $t = 1$:

$$\Pi_R = \alpha \mathbb{E}[v(\max(s_1, s_2)) - v(\min(s_1, s_2))]\,. \tag{17}$$

Intuitively, after the signal-resetting shock but before learning a new signal, a bidder expects to win with a 50% chance; her expected value conditional on winning is $\mathbb{E}[\alpha v(\max(s_1, s_2)) + (1-\alpha)\,v(\min(s_1, s_2))]$; and her expected payment is $\mathbb{E}[v(\min(s_1, s_2))]$.

4.2 Initiation in the initial period

Given the equilibrium in the terminal period and the continuation payoffs (15)–(16), consider the initiation game in period $t = 0$. Denoting the decision of the seller whether or not to hold the auction in response to bidders' messages by $d_0(\mathbf{m}_0) \in \{0, 1\}$, the next proposition characterizes all possible equilibria of the model:

Proposition 4 (equilibria in the initiation game at $t = 0$). *The set of possible equilibria is as follows:*

1. *A non-responsive equilibrium, in which both bidders do not send indications of interest ($m_{i,0} = 0 \; \forall s_i, i \in \{1, 2\}$), and the seller holds the auction at $t = 0$ regardless of the messages: $d_0(\mathbf{m}_0) = 1 \; \forall \mathbf{m}_0$.*

2. *A responsive equilibrium, in which each bidder sends an indication of interest if and only if $s_i \geq \hat{s}_0$ for some $\hat{s}_0 \in (0, 1)$, and the seller that is not hit by the liquidity shock holds the auction unless $\mathbf{m}_0 = (0, 0)$:*

$$d_0(\mathbf{m}_0) = \begin{cases} 1, & \text{if } \mathbf{m}_0 \neq (0,0), \\ 0, & \text{if } \mathbf{m}_0 = (0,0). \end{cases} \tag{18}$$

The cut-off signal $\hat{s}_0$ satisfies:

$$\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) = \left(\frac{1}{\hat{s}_0} - 1\right) \Pi_N^*(\hat{s}_0, \hat{s}_0, 1) + \nu \Pi_S^*(\hat{s}_0, \hat{s}_0) + (1 - \nu) \beta \Pi_1(\hat{s}_0). \tag{19}$$

The non-responsive equilibrium always exists. The responsive equilibrium exists if and only if the solution to (19) satisfies

$$R_S(\hat{s}_0) \leq \beta ((1 - \lambda) R_S(\hat{s}_0) + \lambda R_S(1)) \leq R_B(\hat{s}_0, 1), \tag{20}$$

and inequalities (57) and (58) in the appendix.

Proposition 4 show that the equilibrium is either non-responsive, so that all auctions are seller-initiated, or responsive, so that a single indication of interest triggers a bidder-initiated auction. Interestingly, there cannot be a responsive equilibrium in which the seller not hit by the liquidity shock holds an auction only if both bidders indicate their interest. Intuitively, if such an equilibrium existed, then an initiating bidder with the cut-off signal $\hat{s}_0$ would obtain a zero payoff in such an auction, as she would have the lowest value and thus would win with zero probability. Furthermore, if the rival's signal is below $\hat{s}_0$, the bidder with signal $\hat{s}_0$ would be worse off indicating her interest: When the seller finally holds the auction at $t = 1$, her interest reveals her high signal and leads to a more aggressive bidding by the rival. Across these two possibilities, the bidder with signal $\hat{s}_0$ is strictly better off not indicating her interest. Hence, any responsive equilibrium is such that a single indication of interest triggers an auction.[24]

The non-responsive equilibrium always exists from the following argument. If bidders expect the seller to put the asset up for sale in the absence of interest, they do not benefit from indicating their interest. And if the lack of interest does not reveal any negative information about bidders' signals, then the seller does not benefit from delaying the auction until $t = 1$, because he does not expect an improvement in the bidders' willingness to pay.

To see the intuition for responsive equilibria, consider equation (19) that determines the cut-off signal $\hat{s}_0$ and is also key for analyzing the existence of a responsive equilibrium. In the responsive equilibrium, a bidder with signal $\hat{s}_0$ is indifferent between indicating her interest ($m_{i,0} = 1$), which leads to the auction with certainty, and not indicating it ($m_{i,0} = 0$), in which case the auction occurs only if the rival indicates her interest or if the seller is hit by the liquidity shock with probability ν. Consider the payoff of bidder i with signal $\hat{s}_0$ from indicating her interest. With probability $1 - \hat{s}_0$, the rival also sends $m_{-i,0} = 1$. Because in this case the rival's signal is distributed over $[\hat{s}_0, 1]$, bidder i loses the auction with certainty and gets the payoff of zero. With probability $\hat{s}_0$, the rival sends message $m_{-i,0} = 0$. In this case, we have an asymmetric auction, in which bidder i's expected payoff is $\Pi_I^* (\hat{s}_0, \hat{s}_0, 1)$. Thus, sending $m_{i,0} = 1$ yields the expected payoff of $\hat{s}_0 \Pi_I^* (\hat{s}_0, \hat{s}_0, 1)$.

Next, suppose that bidder i with signal $\hat{s}_0$ does not indicate her interest. With probability $1-\hat{s}_0$, the rival indicates her interest, and the auction occurs with certainty. In this case,

[24] This argument does not rely on the presence of two bidders only. In a model with any number $N \geq 2$ of bidders, a responsive equilibrium would be such that a single indication of interest triggers an auction.

bidder i obtains the expected payoff of $\Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$. With probability $\hat{s}_0$, the rival does not indicate her interest, and the auction occurs only if the seller is hit by the liquidity shock. In this case, bidder i obtains the expected payoff of $\Pi_S^*(\hat{s}_0, \hat{s}_0) = \alpha(v(\hat{s}_0) - \mathbb{E}[v(x)|x \leq \hat{s}_0])$. With probability $\hat{s}_0(1-\nu)$, the auction does not occur at $t=0$, and the game proceeds to $t=1$. In this case, bidder i obtains the expected payoff of $\beta\Pi_1(\hat{s}_0)$, where $\Pi_1(\hat{s}_0)$ is given by (16). To be indifferent between indicating and not indicating her interest, cut-off $\hat{s}_0$ must satisfy the indifference condition (19).

Condition (99) ensures that the seller prefers to hold the auction upon receiving a single indication of interest (the right inequality) and prefers to wait until $t=1$ if she receives no interest at all (the left inequality). If the condition does not hold, the responsive equilibrium does not exist because the seller's action is not responsive to bidders' messages. For example, if the seller discounts the future very strongly ($\beta \to 0$), he prefers to hold the auction immediately even in the absence of any interest, so only the non-responsive equilibrium exists.[25]

4.3 Equilibrium Properties

Equation (19) highlights several equilibrium properties. The first property relates to the role of common versus private values. Proposition 1 shows that a high common component of values implies $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) < \Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$: The asset revaluation effect discussed in Section 3.4 makes a bidder reluctant to indicate her interest. Conversely, a low common component of values implies $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) > \Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$: The rival selection effect discussed in Section 3.4 makes a bidder eager to indicate her interest. The following result links the commonality of values to the existence of the responsive equilibrium:

Proposition 5 (equilibrium when values are close to common). *There exists $\hat{\alpha} \in \left(\frac{1}{2}, 1\right)$, such that for any $\alpha < \hat{\alpha}$, only the non-responsive equilibrium exists at $t=0$.*

For a stark example, consider (19) and the case of almost pure common values, $\alpha \to \frac{1}{2}$.

[25] Additional conditions (57) and (58) ensure that if the bidder with signal $\hat{s}_0$ is indifferent between indicating her interest and not, then bidders with signals below $\hat{s}_0$ prefer to not indicate their interest, while those with signals above $\hat{s}_0$ prefer to indicate it.

By Proposition 1, $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) \to 0$, as re-valuation by the rival erodes almost all payoff of the initiating bidder with signal $\hat{s}_0$. In contrast, waiting results in a strictly positive payoff. Not expecting to receive any indications of interest, the seller does not benefit from delaying the auction until $t = 1$, so he sells the asset immediately.

The second property highlighted by equation (19) is the role of liquidity shocks for the seller. Proposition 1 shows that $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) < \Pi_S^*(\hat{s}_0, \hat{s}_0)$. In turn, if the seller is likely to initiate the auction in the absence of any interest, each bidder's incentives to indicate interest and face a more aggressive bidding by the rival are low. The following result shows that irrespective of the commonality of values, only seller-initiated auctions occur in equilibrium if the seller is sufficiently likely to receive liquidity shocks:

Proposition 6 (equilibrium when liquidity shocks are likely). *For any $\alpha \in \left[\frac{1}{2}, 1\right]$, there exists $\hat{\nu} \in (0, 1)$, such that for any $\nu > \hat{\nu}$, only the non-responsive equilibrium exists at $t = 0$.*

For a stark example, consider (19) and $\nu \to 1$. For any $\hat{s}_0$, the benefits of waiting are above the benefits of initiation, so the bidders do not indicate their interest, and the seller sells the asset immediately. As a practical application, Proposition 6 suggests that in the market for distressed assets sales would tend to be seller-initiated. Bidders are reluctant to approach the seller who is close to liquidation, because they expect him to put the asset up for sale soon regardless of the expressed demand for it. This result holds regardless of whether the asset is commonly or privately valued by market participants.

It is natural to extend the model by assuming that the seller can have some disutility $C > 0$ from selling the asset. In the application to auctions of companies, C can be interpreted as the level of entrenchment of the target's board. Suppose that $C > R_S(1)$, so that the seller would not sell the asset unless he is either hit by the liquidity shock or receives some indications of interest. The following result, a corollary to Proposition 5, shows that if values are close to common and bidders are reluctant to show their interest, then in equilibrium the asset will only be sold if the seller is hit by the liquidity shock:

Corollary 1 (no strategic initiation). *If $C > R_S(1)$, there exists $\hat{\alpha} \in \left(\frac{1}{2}, 1\right)$, such that for any $\alpha < \hat{\alpha}$, only the non-responsive equilibrium exists in each period, in which the seller does not put the asset up for sale unless he is hit by the liquidity shock.*

In contrast, when values are close to private, responsive equilibria can exist even if $C > R_S(1)$. Intuitively, in this case, the bidder with signal $\hat{s}_0$ is eager to indicate her interest, provided that this leads to the auction. If C does not exceed $R_S(1)$ by too much and $\hat{s}_0$ is relatively high, the seller finds it optimal to put the asset up for sale upon receiving interest and inferring that one of the bidder's signals is above $\hat{s}_0$.

4.4 Special case: linear values

For the special case of $v(s) = s$, all expressions of bidders' payoffs $\Pi_I^*(\hat{s}, \hat{s}, 1)$, $\Pi_N^*(\hat{s}, \hat{s}, 1)$, $\Pi_S^*(\hat{s}, \hat{s})$, and Π_0 admit closed-form solutions. Thus, we can solve for equilibria in closed-form, illustrate Propositions 5 and 6, and obtain general comparative statics in α and ν.

For the details of the solution, we refer the reader to the appendix. In it, we show that

$$\Pi_I^*(\hat{s}, \hat{s}, 1) = \tfrac{(2\alpha-1)^2}{2(3\alpha-1)}\hat{s},\ \Pi_N^*(\hat{s}, \hat{s}, 1) = \tfrac{1-\alpha}{2}\left(\hat{s} - \hat{s}^2\right) + \tfrac{\alpha^2}{2(3\alpha-1)}\hat{s}^2,\ \Pi_S^*(\hat{s}, \hat{s}) = \tfrac{\alpha}{2}\hat{s},\ \Pi_R = \tfrac{\alpha}{6}. \quad (21)$$

Plugging (21) into (19) yields a quadratic equation, whose positive solution is

$$\hat{s}_0 = \frac{(3\alpha-1)\left((1+\nu+(1-\nu)\beta(1-\lambda))\alpha - 1 + \sqrt{(1-(1+\nu+(1-\nu)\beta(1-\lambda))\alpha)^2 + 4\frac{(2\alpha-1)^2}{3\alpha-1}\left(1-\alpha+(1-\nu)\beta\lambda\frac{\alpha}{3}\right)}\right)}{2(2\alpha-1)^2}. \quad (22)$$

If the parameters are such that $\hat{s}_0 \geq 1$, then only the non-responsive equilibrium exists.

The next proposition studies the effects of the commonality of values α, and the probability of the seller's liquidity shock ν:

Proposition 7. *Consider parameters for which the responsive equilibrium exists. Then:*

1. *A marginal increase in α strictly decreases $\hat{s}_0$;*

2. *A marginal increase in ν strictly increases $\hat{s}_0$.*

The probability of a bidder-initiated auction is increasing in α and decreasing in ν.

Proposition 7 generalizes Propositions 5 and 6 beyond existence results: The likelihood of a bidder-initiated sale monotonically decreases in the degree of commonality of potential buyers' values and in the likelihood of the seller's liquidity shock. Figure 2 illustrates how the initiation cut-off $\hat{s}$ changes with respect to α and ν when $v(s) = s$, $\beta = 0.9$, and $\lambda = 0.9$.

5 Extensions

5.1 Generalization to other auction formats

We assume that the sale proceeds as a first-price auction for two reasons. First, under regularity conditions, the first-price auction has a unique equilibrium even when signals of bidders are distributed asymmetrically. In contrast, the difficulty with other auction formats, such as the second-price auction, is that in the common value setting there exist multiple equilibria (e.g., see Milgrom, 1981b). When bidders are symmetric, it is natural to focus on the equilibrium in which both bidders play identical strategies, which is unique. However, when bidders are asymmetric, as is the case in this paper, there is no clear way to select one equilibrium from many. Second, a first-price auction is the simplest format that highlights how bidders respond to the perceived aggressiveness of the rival. Such strategic considerations are common to a variety of auction formats but are often absent from the second-price auction. For example, if the ascending (English) auction has jump bids, which are common in takeover bidding, then its outcome will have properties of both the first-price and the second-price auctions both in the common-value setting (Avery, 1998) and in the private-value setting (Daniel and Hirshleifer, 2018).

Nevertheless, it is important to explore whether the results of our analysis generalize beyond the first-price auction. In this section, we show that the main implication of the baseline model generalizes to a large class of auction formats, which covers first-price and (in the limit) second-price auctions, as well as intermediate formats.

Specifically, consider the base model but assume that the auction format is a combination of first- and second-price auctions. Bidders simultaneously submit bids in a concealed fashion. The bidder with the highest bid wins the auction and pays the weighted average of her bid and the rival's bid with weights f and $1-f$, respectively, where $f \in (0,1]$ captures the proximity of the auction format to first-price. If $f=1$, the format is identical to the first-price auction, and the model reduces to the base model. If $f \to 0$, the format is arbitrarily close to the second-price and ascending (English) auctions.[26] This model's application is to auctions of companies, as they have features of both first-price and ascending auctions (e.g., Hansen, 2001). We maintain Assumption 2 of the base model.

Consider a seller-initiated auction.[27] Because the derivation of the equilibrium is similar to that in Section 3.2, we leave the details of the proof of Proposition 8 in the appendix. There, we show that the equilibrium inverse bidding function, $\phi_S(b)$, solves

$$\frac{\partial \phi_S(b)}{\partial b} = \frac{f\phi_S(b)}{v(\phi_S(b)) - b}, \tag{23}$$

subject to the initial value condition $\phi_S(0) = 0$. Note that (23) is equivalent to (10) in the base model but has multiple f in the numerator on the right-hand side. Intuitively, bidders bid more aggressively if f is lower, as they expect the payment upon winning to weigh less on their own bid and more on the (lower) bid of the rival. In the appendix, we show that the expected payment of a bidder with any signal s, conditional on winning, is $\mathbb{E}[v(x)|x \leq s]$, independent of the auction format. Thus, there is revenue equivalence of seller-initiated auction formats. Consequently, the expected payoff of a bidder with signal $\hat{s}$ in the seller-initiated auction is the same as in the base model: $\Pi_S^*(\hat{s},\hat{s}) = \alpha(v(\hat{s}) - \mathbb{E}[v(s)|s \leq \hat{s}])$. The analysis of the auction initiated by both bidders is very similar.

Next, consider a bidder-initiated auction, similar to Section 3.1. Now the expected payoff of bidder of type $j \in \{I, N\}$ with signal s from bidding b is

$$\Pi_j(b,s) = \int_{\underline{s}_k}^{\phi_k(b)} (\alpha v(s) + (1-\alpha) v(x) - fb - (1-f) a_k(x)) \frac{1}{\bar{s}_k - \underline{s}_k} dx. \tag{24}$$

[26] When there are two bidders, the English auction with a continuously increasing price and the second-price auction are strategically equivalent.

[27] As in Sections 3.1 and 3.2, we omit the dependence of bidding functions on $\hat{s}$ and $\bar{s}$ to keep notations simpler.

Taking the first-order condition of (24), we obtain

$$\frac{\partial \phi_j(b)}{\partial b} = \frac{f\left(\phi_j(b) - \underline{s}_j\right)}{\alpha v\left(\phi_k(b)\right) + (1-\alpha) v\left(\phi_j(b)\right) - b}. \tag{25}$$

for $j \neq k \in \{I, N\}$. This system of differential equations reduces to (3) if $f = 1$. It must be solved subject to boundary conditions $\phi_I(\bar{a}) = \bar{s}$, $\phi_N(\bar{a}) = \hat{s}$, $\phi_I(\underline{a}) = \hat{s}$, and $\phi_N(\underline{a}) = v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)$. These boundary conditions are the same as boundary conditions in the base model and follow from the same arguments, so we omit them here. In the base model, we had the additional condition (4), which pinned down $\underline{a}$, leading to the unique equilibrium. When the auction format is not first-price ($f < 1$), the argument from the base model does not apply. Consequently, there is a possibility of multiple equilibria.[28]

Because multiple equilibria can exist, a detailed analysis of equilibrium properties would require additional equilibrium selection. In the example below, we offer one selection criterion. However, the main implication of our base model holds regardless of the equilibrium selection: The next proposition shows that if the commonality of values is sufficiently high, then the equilibrium features no bidder-initiated auctions, regardless of the equilibrium that is expected to be played in a bidder-initiated auction:

Proposition 8 (equilibrium with seller-initiated auctions when values are close to common: robustness to auction format). *Fix any $f \in (0,1)$. There exists $\tilde{\alpha} \in \left(\frac{1}{2}, 1\right)$, such that for any $\alpha < \tilde{\alpha}$, only the non-responsive equilibrium exists at $t = 0$, in which the seller holds the auction immediately.*

The specific equilibrium in a bidder-initiated auction affects the payoff of the initiating bidder with signal $\hat{s}$, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$. However, Proposition 8 shows that this payoff converges to zero as the common component of bidders' values becomes large ($\tilde{\alpha} \to \frac{1}{2}$) in any equilibrium. As a result, the initiating bidder with signal $\hat{s}$ prefers not to approach the seller for any cut-off $\hat{s}$, leading to the seller's rational response to initiate the auction himself.

[28] A loose intuition for equilibrium uniqueness in the first-price auction and multiplicity in the auction with $f < 1$ is as follows. In the first-price auction, bids of very low types of the non-initiating bidder play a limited role, as they never win and the initiating bidder pays her own bid. In contrast, here such bids play an important role, as they can affect the payment of the initiating bidder.

Once multiple auction formats are allowed, it is natural to ask which format should optimally be chosen by a seller to maximize his revenues. The analysis of revenues, first, depends on the specific equilibrium in a bidder-initiated auction and, second, has to be done numerically, as (25) does not have a closed-form solution for intermediate auction formats. The equilibrium selection criterion we propose is that a non-serious bid by a non-initiating bidder with sufficiently low signal x is simply equal to her value, taking into account that the initiating bidder's signal is at least $\hat{s}$: $a_N(x) = \alpha v(x) + (1-\alpha)v(\hat{s}) \leq \underline{a}$.[29] Our numerical analysis shows that for reasonable model parameters, the seller prefers the first-price auction to all other formats. In fact, we were unable to find parameters, for which the second-price format is preferred, and only located a small subset of parameters, for which an intermediate format is preferred. As an example, when $\alpha = 1$, $v(s) = s^{0.35}$, $\nu = 0$, $\beta = 0.936$, and $\lambda = 0.965$, the first-price format is preferred to the second-price format in both the responsive and non-responsive equilibrium, and the optimal format in the responsive equilibrium has $f = 0.64$ (the seller is indifferent among formats in the non-responsive equilibrium). The implication is that the optimal auction heavily weighs on features that make its format closer to the first-price format.

5.2 Costly information acquisition and preemption

We assume that potential bidders always know their signals. As Fishman (1988) shows, if potential bidders need to acquire this information at a cost, in a sequential sale process the first bidder may preempt the second potential bidder from acquiring information and participating in the sale. Such a sale process is classified as a negotiation in practice. Although Fishman's model has independent private values, it is natural to ask if this first-mover advantage is also relevant when values have a significant common component and if it can overturn the result of the base model that sales are seller-initiated when values are close to common.

To explore this question, we alter the base model to capture the key ingredients of Fishman (1988). We assume that initially at $t = 0$, one bidder is informed about her signal,

[29] This criterion is reasonable when bidders, on the margin, prefer winning the auction to losing and when bidders may make mistakes in their bids with a small probability. These two conditions allow non-initiating bidders with very low signals to win at their value with a vanishingly small probability.

while the other is not. At $t = 0$, the informed bidder sends message $m_{i,0} \in \{0, 1\}$, where $m_{i,0} = 1$ is interpreted as an indication of interest and $m_{i,0} = 0$ is interpreted as lack of such indication. After observing the message, the seller decides whether to put the asset up for sale or not; with probability ν, she is hit by the liquidity shock and has no choice but to sell the asset. In a responsive equilibrium, the informed bidder sends $m_{i,0} = 1$ if and only if her signal exceeds some cut-off $\hat{s}_0$.

If the sale is bidder-initiated (i.e., if it is triggered by the informed bidder sending $m_{i,0} = 1$), it proceeds via a sequential format.[30] At the first stage, the informed bidder chooses bid b to submit. Having observed this bid, the uninformed bidder decides whether to pay cost Ψ to observe her signal (in case she does not already know it). If she chooses not to pay the cost, the informed bidder acquires the asset at price b in a single-bidder sale. In this case, the bid of the initiating bidder preempts the rival from participating. If the uninformed bidder chooses to pay the cost to learn her signal, the two bidders compete in the first-price auction with the initiating bidder's first-round bid b serving as the reservation price. This timing is thus similar to Fishman (1988), with the difference in that we assume that the second-round format is first-price, while Fishman assumes the English auction.[31] If the sale is seller-initiated (i.e., it is not triggered by the informed bidder sending $m_{i,0} = 1$), the uninformed bidder decides whether to pay cost Ψ to learn her signal, after which the two bidders also compete in the first-price auction. We assume that Ψ is not too high, so that the uninformed bidder prefers to learn her signal if she knows nothing about the signal of the her rival beyond her prior. The other assumptions are as in the base model.

The next proposition shows that if values are close to common, then the equilibrium can only be non-responsive, and thus all sales are seller-initiated.

Proposition 9 (equilibrium with costly information acquisition and sequential bidding). *Suppose that $\Psi \leq \bar{\Psi}$, defined in the proof in the appendix. There exists $\tilde{\alpha} \in \left(\frac{1}{2}, 1\right)$, such that for any $\alpha < \tilde{\alpha}$, only the non-responsive equilibrium exists at $t = 0$, in*

[30] Alternatively, one could assume that the informed bidder makes an initial bid and submits an indication of interest simultaneously.

[31] As in the baseline model, the English auction format would result in equilibrium multiplicity with equilibrium selection being non-obvious due to asymmetric bidders. In Fishman (1988) multiplicity does not arise because he assumes independent private values.

which the seller holds the auction immediately.

The intuition is as follows. Consider a hypothetical responsive equilibrium in which the informed bidder sends $m_{i,0} = 1$ if and only if her signal exceeds some cut-off $\hat{s}_0$, and the seller puts the asset up for sale at $t = 0$ if either the informed bidder indicates her interest or the liquidity shock occurs. When values are close to common, the initiating bidder with cut-off signal $\hat{s}_0$ cannot successfully preempt the rival: The rival already knows that the signal of the initiating bidder is at least $\hat{s}_0$ and updates her value accordingly. Therefore, the argument of the base model applies: The initiating bidder with signal $\hat{s}_0$ obtains very small rents from indicating her interest and is better off not approaching the seller. Because the argument holds for any $\hat{s}_0$, the responsive equilibrium cannot exist, and the sale is initiated by the seller at $t = 0$. Thus, while the ability to preempt associated with costly information acquisition gives the first-mover advantage to the initiating bidder when values are close to private, there is no such advantage to the initiating bidder with the cut-off signal when values are close to common.

Although not analyzed in the model formally, Proposition 9 points to a potential valuable role of lockup clauses as means of transferring rents to the initiating bidder and thereby encouraging bidders to approach the target. Che and Lewis (2007) analyze lockups in a static model based on Fishman (1988). In the future work, it can be interesting to incorporate them into our dynamic model of endogenous initiation.

5.3 A model with a general number of periods

The base model is two-period, but its results extend to the multi-period setting. Consider a $T+1$-period model. The model setup is the same as in the base model. The only difference is that a signal-resetting shock arrives with probability λ in each period $t \geq 1$, in which case both bidders' values of the asset for sale change as described in the base model.[32]

In any period t, the state of the world can be described by two variables: t and the highest signal the bidders can possibly possess in this period, denoted $\bar{s}_t$ ($\bar{s}_t$ is determined

[32] We have also considered a continuous-time model without a terminal period. For it, we derived the differential equation for a decreasing cut-off $\hat{s}_t$. Its solution is numerical and very computationally involved, and is therefore much less transparent than the solution of the discrete-time model with a terminal period.

by the previous period's initiation cut-off, if signals do not reset, and is equal to 1 otherwise). The current-period initiation cut-off that summarizes the bidders' strategy is then denoted $\hat{s}_t(\bar{s}_t)$ (so that bidders with signals $s \geq \hat{s}_t(\bar{s}_t)$ initiate and those with signals $s \leq \hat{s}_t(\bar{s}_t)$ do not), and the seller's probability of initiation is denoted $\mu_t(\bar{s}_t) \in \{\nu, 1\}$. To simplify notation, we will suppress state-of-the world variables and use $\bar{s} \equiv \bar{s}_t$, $\hat{s} \equiv \hat{s}_t(\bar{s}_t)$, and $\mu \equiv \mu_t(\bar{s}_t)$ unless the additional notation is required for clarity.

As in the two-period model, there is only a non-responsive equilibrium in period $t = T$: $\hat{s}_T = \bar{s}_T$ and $\mu_T = 1$, so that seller always initiates the auction. Take any period $t < T$. First, consider the bidders' problem. The indifference condition of a bidder with signal $\hat{s}$ is

$$\tfrac{\hat{s}}{\bar{s}}\Pi_I^*(\hat{s},\hat{s},\bar{s}) = \left(1-\tfrac{\hat{s}}{\bar{s}}\right)\Pi_N^*(\hat{s},\hat{s},\bar{s}) + \tfrac{\hat{s}}{\bar{s}}\mu\Pi_S^*(\hat{s},\hat{s}) + \tfrac{\hat{s}}{\bar{s}}(1-\mu)\beta\left((1-\lambda)U_B(\hat{s},t+1) + \lambda V_B(1,t+1)\right), \quad (26)$$

where $U_B(\hat{s}, t+1)$ denotes the continuation value of a bidder with signal $\hat{s}$ when (with probability $1-\lambda$) signals do not reset in period $t+1$, such that the rival signal remains in $[0, \hat{s}]$; and $V_B(\hat{s}, t+1)$ denotes the continuation value of a bidder when (with probability λ) both her and her rival's signals reset to $[0, \hat{s}]$ in period $t+1$ (this is a general notation for $V_B(\cdot)$, but in the model both signals reset to $[0, 1]$).

Let us characterize $U_B(\bar{s}, t)$ and $V_B(\bar{s}, t)$ for any t (recall that $\bar{s}_t = \hat{s}_{t-1}(\bar{s}_{t-1})$ in period t, making this notation consistent with the above definitions). If the cut-off in period t is $\hat{s} \leq \bar{s}$ (the case of $\hat{s} > \bar{s}$ is identical to the case when the two are equal), then:

$$U_B(\bar{s}, t) = \frac{\hat{s}}{\bar{s}}\Pi_I^*(\bar{s},\hat{s},\bar{s}) + \left(1-\frac{\hat{s}}{\bar{s}}\right)\Pi_D^*(\bar{s},\hat{s},\bar{s}), \quad (27)$$

where the first summand captures the outcome in case the bidder with signal $\bar{s}$ is the only one sending message $m_{i,t} = 1$ to the seller and the second summand captures the outcome when both bidders send message $m_{i,t} = 1$. Note that the profit in the dual bidder-initiated auction $\Pi_D^*(\bar{s},\hat{s},\bar{s}) = \Pi_S^*(\bar{s}-\hat{s},\hat{s})$: only the fraction of the signal above the lowest common signal $\hat{s}$ is important for the bidders' profits.

Next, consider $V_B(\bar{s},t)$. Because the future reset signal is unknown,

$$\begin{aligned} V_B\left(\bar{s},t\right) &= \int_{\hat{s}}^{\bar{s}}\left(\frac{\hat{s}}{\bar{s}}\Pi_I^*\left(s,\hat{s},\bar{s}\right)+\left(1-\frac{\hat{s}}{\bar{s}}\right)\Pi_D^*\left(s,\hat{s},\bar{s}\right)\right)\frac{1}{\bar{s}}ds \\ &+ \int_0^{\hat{s}}\left(\left(1-\frac{\hat{s}}{\bar{s}}\right)\Pi_N^*\left(s,\hat{s},\bar{s}\right)+\frac{\hat{s}}{\bar{s}}\mu\Pi_S^*\left(s,\hat{s}\right)\right)\frac{1}{\bar{s}}ds \\ &\quad+\left(\frac{\hat{s}}{\bar{s}}\right)^2\left(1-\mu\right)\beta\left(\left(1-\lambda\right)U_B\left(\hat{s},t+1\right)+\lambda V_B\left(1,t+1\right)\right), \end{aligned} \tag{28}$$

where the first summand captures the bidder's payoff when her signal is reset to the value above $\hat{s}$, so that she is either the only one sending message $m_{i,t}=1$ to the seller or both bidders send message $m_{i,t}=1$; the second summand captures the bidder's payoff when her signal is reset to the value below $\hat{s}$, so that either no bidder sends message $m_{i,t}=1$ to the seller (and the auction is initiated by the seller with probability μ) or the rival is the only one sending message $m_{i,t}=1$; and the third summand captures the continuation value of the bidder when no bidder sends message $m_{i,t}=1$ to the seller and the auction is not initiated by the seller, so that the outcome is delayed until the next period.

The boundary conditions in period T take into account that this period only has a non-responsive equilibrium with seller-initiated auctions:

$$U_B\left(\bar{s},T\right)=\Pi_S^*(\bar{s},\bar{s}),\quad V_B\left(\bar{s},T\right)=\Pi_R \text{ defined in (17).} \tag{29}$$

Second, consider the seller's problem. Let $V_S\left(\bar{s},t\right)$ denote the seller's value function given the state variables:

$$V_S\left(\bar{s},t\right)=\left(1-\tfrac{\hat{s}}{\bar{s}}\right)^2R_D^*\left(\hat{s},\bar{s}\right)+2\tfrac{\hat{s}}{\bar{s}}\left(1-\tfrac{\hat{s}}{\bar{s}}\right)R_B^*\left(\hat{s},\bar{s}\right)+\left(\tfrac{\hat{s}}{\bar{s}}\right)^2\max\left\{R_S^*\left(\hat{s}\right),\beta\left(\left(1-\lambda\right)V_S\left(\hat{s},t+1\right)+\lambda V_S\left(1,t+1\right)\right)\right\}, \tag{30}$$

where $R_S^*\left(\hat{s}\right)$ are the seller's expected revenues from the seller-initiated auction; $R_D^*\left(\hat{s},\bar{s}\right)$ are the seller's expected revenues from the dual bidder-initiated auction; and $R_B^*\left(\hat{s},\bar{s}\right)$ are the seller's expected revenues from the single bidder-initiated auction. The boundary condition in period T takes into account that this period only has seller-initiated auctions:

$$V_S\left(\bar{s},T\right)=R_S^*\left(\bar{s}\right). \tag{31}$$

The solution to the seller's problem is:

$$\mu = \begin{cases} \nu, & \text{if } R_S^*(\hat{s}) \leq \beta\left((1-\lambda)V_S(\hat{s},t+1) + \lambda V_S(1,t+1)\right), \\ 1, & \text{if } R_S^*(\hat{s}) \geq \beta\left((1-\lambda)V_S(\hat{s},t+1) + \lambda V_S(1,t+1)\right). \end{cases} \tag{32}$$

It is straightforward to establish our main result in the $T+1$-period setting:

Proposition 10 (equilibrium with seller-initiated auctions when values are close to common: the $T+1$-period model). *There exists $\hat{\alpha} \in \left(\frac{1}{2}, 1\right)$, such that for any $\alpha < \hat{\alpha}$, only the non-responsive equilibrium exists at $t = 0$, in which the seller holds the auction immediately.*

The proof is by induction. At $t = T-1$, there are two periods left, and Proposition 3 establishes that if $\bar{s}_{T-1} = 1$, there exists $\hat{\alpha}(\bar{s}_{T-1}) = \alpha(1) \in \left(\frac{1}{2}, 1\right)$ such that $\hat{s}_{T-1}(1) = 1$ and $\mu_{T-1}(1) = 1$. Directly following proof of Proposition 3, we can show that for any $\bar{s}_{T-1} < 1$, there exists $\hat{\alpha}(\hat{s}_{T-1}) \in \left(\frac{1}{2}, 1\right)$ such that $\hat{s}_{T-1}(\bar{s}_{T-1}) = \bar{s}_{T-1}$ and $\mu_{T-1}(\hat{s}_{T-1}) = 1$. Let $\hat{\alpha} \equiv \inf_{\hat{s}_{T-1}} \alpha(\hat{s}_{T-1})$. Because for any $\alpha < \hat{\alpha}$ the game ends in a non-responsive equilibrium with seller initiation at $t = T-1$, the agents at $t = T-2$ effectively face the same two-period problem as the agents at $t = T-1$, so $\hat{s}_{T-2}(\bar{s}_{T-2}) = \bar{s}_{T-2}$ and $\mu_{T-2}(\bar{s}_{T-2}) = 1$ for any $\bar{s}_{T-2}$. Continuing the logic, $\hat{s}_1(\bar{s}_1) = 1$ and $\mu_1(\bar{s}_1) = 1$ for any $\bar{s}_1$, including $\bar{s}_1 = 1$.

If values are sufficiently far from common, the model requires numerical analysis. As a simple illustration, in the online appendix we provide a semi-closed form solution of the $T+1$-period model for the case of private values and $v(s) = s$. Additionally, for this case and parameters $\nu = 0$, $\beta = 0.9$, and $\lambda = 0.9$, Figure 4 illustrates the evolution of responsive equilibria in the three-period model. As the terminal period nears, in which the seller always puts the auction up for sale, bidders indicate their interest more aggressively compared to the period before (whether their signals are reset at the beginning of the period or not). As always, there is also the non-responsive equilibrium in which the seller initiates at $t = 0$ and ends the game immediately.

6 Additional discussion

The base model provides a general theory of endogenous initiation of auctions, with auctions of companies being our lead application. In this section, we discuss additional features of this application and their potential role in alleviating the problem that bidders are reluctant to communicate information about their values through indications of interest.

6.1 Shareholder activists as facilitators of auctions of companies

Corollary 1 shows that if a company is underperforming (inducing common value of restructuring in potential bidders), its management is entrenched, and the likelihood of a liquidity shock is low, such a company can remain without a change in ownership for a long time, even when the value added from the change is large. Thus, the role of takeovers as a corporate governance mechanism can be limited. The lack of an incentive for bidders to initiate auctions when the common component of their values is large gives rise to alternative ways of promoting takeovers, such as shareholder activism. If an activist finds it beneficial to buy a fraction of the company's shares and undertake an activism campaign, which results in putting the company up for sale, seller-initiated takeovers can occur in equilibrium even in the presence of an entrenched management.[33] The model can be extended to capture this feature (see the online appendix for a simple extension).

The implication is that shareholder activism and the market for corporate control are not simply two alternative mechanisms for disciplining the management but rather complement each other: Activists use the market for corporate control to facilitate transactions of targets, inefficiencies in which would not be corrected otherwise.

6.2 Toeholds

When the common component of bidders' values of a company is large, the initiating bidder with the cut-off signal can obtain a positive expected profit if she secretly acquires a toehold (a block of shares) in the company prior to the auction (see the online appendix for a simple

[33]This value-enhancing role of shareholder activism also applies to settings with significant private values if managerial entrenchment is so high that the management of the target prefers not to sell it even conditional on learning about a high value of the initiating bidder.

extension). Such a bidder can thus find it optimal to indicate her interest to the company in the first place. Thus, while toeholds are often considered to be a source of inefficiency, they help bidders initiate positive-value deals that would not occur otherwise.

6.3 Investment banks

The lack of indications of interest from bidders when values are close to common relies on the assumption that whether the auction is bidder- or seller-initiated is publicly known. While it is in the ex-post interest of the seller to disclose the interest of the initiating bidder to rivals, such a disclosure can be ex-ante suboptimal for all parties combined as it impedes initiation. This inconsistency between ex-ante and ex-post objectives can create a role for an intermediary, such as an investment bank, to alleviate the lack-of-commitment problem. The investment bank can centralize communication among all participating parties, and, because it is a long-run player that interacts with buyers and sellers over time across a range of different services, can incentivize the seller to not disclose the ex-post beneficial but ex-ante harmful, for all parties combined, information in the context of a single transaction.[34]

7 Conclusion

In this paper, we theoretically examine endogenous initiation of auctions by potential buyers and the seller. Our model aims to capture many real-world environments in which initiation of an auction is a strategic choice. Our lead application is to auctions of companies and intercorporate asset sales. We show that approaching the seller signals that the initiating bidder is sufficiently optimistic about her value of the asset. This leads to two opposite effects, one of which reduces bidders' incentives to approach the seller, while the other one improves them. The asset revaluation effect implies that the non-initiating bidder updates her value of the asset upwards, which reduces the surplus of the initiating bidder and discourages her from approaching the seller. The rival selection effect implies that not initiating the auction in hopes that the rival initiates instead is costly for a bidder because

[34]Hiding whether the auction is bidder- or seller-initiated gives positive rents to the initiating bidder with the cut-off signal, because the rival believes that, with some probability, the auction is seller-initiated and the initiating bidder's signal is below the cut-off.

it results in competition against a stronger rival. When bidders' values are common, only the first effect is present. When values are private, only the second effect is present.

The model establishes three main results about the equilibrium patterns of initiation. First, if the common component of bidders' values is sufficiently important, the asset revaluation effect dominates, so that that no potential buyer indicates her interest to the seller, and all auctions are seller-initiated. Second, if the private component of values is important, both bidder- and seller-initiated auctions can occur in equilibrium. At the same time, there is also an equilibrium in which all auctions are seller-initiated. Finally, if potential buyers expect the seller to be forced to sell his asset in the near future, no potential buyer indicates her interest, and all auctions are seller-initiated, regardless of the commonality of values. We derive a set of testable implications relating the identity of the initiating party to bids and auction outcomes.

Two extensions of the paper could be interesting. First, it can be useful to consider bids in securities, such as a bidder's stock, and the interaction of securities used in a bid and initiation decisions. Second, the asset for sale can be made divisible: for example, bankrupt companies are often sold piecemeal in a liquidation auction. A more general extension is to consider multiple sellers and allow bidders to choose which asset to pursue and sellers to choose which bidders to invite to an auction in a dynamic model of matching.

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Appendix

A Proofs

Proof of Lemma 1. First, we prove that an equilibrium must satisfy a system of two equations (3) for $\phi_I(b)$ and $\phi_N(b)$ with four boundary conditions (5) and $\underline{a}$ given by (4). To do this, first, we prove that each bidder's strategy $a_i(s)$, $i \in \{I, N\}$ cannot have a jump in the range of bids that win with non-zero probability. By contradiction, suppose that there is a jump in $a_i(s)$ at some s' from a' to a''. Then, types s of the other bidder that bid between a' and a'' benefit from a deviation to bidding infinitesimally close to a': This deviation has no effect on the probability of winning and the value conditional on winning but discontinuously reduces the payment. However, in this case type $s = a'' + \varepsilon$ of bidder i for an infinitesimal positive ε benefits from a deviation to bidding just above a': this deviation has no effect on the probability of winning and the value conditional on winning but discontinuously reduces the payment. Second, we prove that each bidder's strategy $a_i(s)$, $i \in \{I, N\}$ must be strictly increasing in the range of bids that win with non-zero probability. By contradiction, suppose not, i.e., $\Pr(a_i(s) = b) > 0$ for some b. Then, no type of bidder j wants to bid $b - \varepsilon$ for an infinitesimal ε: bidding b rather than $b - \varepsilon$ leads to a discontinuous increase in the probability of winning, in the value conditional on winning, and only an infinitesimal increase in the price paid. However, this contradicts the result above that there is no jump in $a_i(s)$, $i \in \{I, N\}$ in the range of bids that win with non-zero probability. Therefore, $\phi_I(b)$ and $\phi_N(b)$ are continuous and strictly increasing. Then, optimality implies that equations (3) are satisfied. The proof of the boundary conditions is in the main text of the section with the exception of (4).

Let us prove (4). Because bid $\underline{a}$ must be optimal for the initiating bidder with signal $\hat{s}$,

$$\phi_N(\underline{a})(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x)|x \leq \phi_N(\underline{a})] - \underline{a}) \geq \phi_N(b)(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x)|x \leq \phi_N(b)] - b) \quad \forall b. \tag{33}$$

Because no non-initiating bidder with signal s bids above $\alpha v(s) + (1-\alpha) v(\hat{s})$, it must be the case that $\phi_N(b) \geq v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)$. Using the fact that the right-hand side of the above inequality is a strictly increasing function of $\phi_N(b)$ and the fact that $\phi_N(\underline{a})$ is described by (5), the inequality implies

$$\begin{aligned} & v^{-1}\left(\tfrac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \leq v^{-1}\left(\tfrac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a}\right) \\ \geq\ & v^{-1}\left(\tfrac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \leq v^{-1}\left(\tfrac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - b\right) \ \forall b, \end{aligned} \tag{34}$$

which implies (4).

Second, we prove existence of equilibrium. For this we use the results of Athey (2001). Without loss of generality, limit the action (bid) space to $[0, v(\bar{s})]$. Because no bidder bids above her expected value by Assumption 2, this restriction is indeed without loss of generality. The problem also satisfies Assumption A1 in Athey (2001) (bounded and atomless type distribution). Next, we verify the single-crossing condition (Definition 3 in Athey, 2001): Whenever an opponent uses an increasing strategy ($a_j(s_j)$ increasing in s_j), player i's objective function $\Pi_i(b_i, s_i)$ satisfies SCP-IR in (b_i, s_i). The expected payoffs of bidders are

$$\Pi_I(b,s) = \int_0^{\phi_N(b)} (\alpha v(s) + (1-\alpha) v(x) - b) \frac{1}{\hat{s}} dx; \tag{35}$$

$$\Pi_N(b,s) = \int_{\hat{s}}^{\phi_I(b)} (\alpha v(s) + (1-\alpha) v(x) - b) \frac{1}{\bar{s} - \hat{s}} dx. \tag{36}$$

Their cross-partial derivatives in b and s are:

$$\frac{\partial \Pi_I}{\partial b \partial s}(b,s) = \frac{\alpha v'(s)}{\hat{s}} \frac{\partial \phi_N(b)}{\partial b} \geq 0; \quad \frac{\partial \Pi_N}{\partial b \partial s}(b,s) = \frac{\alpha v'(s)}{\bar{s} - \hat{s}} \frac{\partial \phi_I(b)}{\partial b} \geq 0. \tag{37}$$

Therefore, the problem satisfies the single-crossing condition. The payoffs also satisfy Assumptions A2 and A3 (A3 follows from Theorem 7, Part 1). By Theorem 6 in Athey (2001), we conclude that there exists an equilibrium in nondecreasing strategies. Because equilibrium bidding strategies cannot have flat regions, as shown above, there exists an equilibrium in increasing strategies.

Finally, we prove uniqueness ot the solution to (3). Let $v(\phi_j(b)) \equiv v_j(b)$ for $j \in \{I, N\}$. Then, system (3) can be re-written as

$$\frac{d}{db} \ln \left(v_j^{-1}(v_k(b)) - \underline{s}_j \right) = \frac{1}{\alpha v_k(b) + (1-\alpha) v_j(b) - b}, \tag{38}$$

and boundary conditions (5) can be re-written as $v_I(\bar{a}) = v(\bar{s})$, $v_N(\bar{a}) = v(\hat{s})$, $v_I(\underline{a}) = v(\hat{s})$, and $v_N(\underline{a}) = \frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}$. This system of equations satisfies the standard assumptions of the theory of differential equations and thus admits a unique solution. The specific argument is the same as in Lebrun (2006), page 147.

Proof of Lemma 2. The derivation of the equilibrium is in the main text. Uniqueness is established in, e.g., Lebrun (2006).

Proof of Proposition 1. First, compare $\Pi_S^*(\hat{s}, \hat{s})$ and $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$. Consider two cases. First, consider case $a_S(\hat{s}) \le \underline{a}$, where we omit the dependence of $\underline{a}$ on $\hat{s}$ and $\bar{s}$ for brevity. Then,

$$\begin{aligned} \Pi_S^*(\hat{s}, \hat{s}) &\ge \alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x) | x \le \hat{s}] - \underline{a} \\ &> \alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x) | x \le v^{-1}\left(\frac{\underline{a} - (1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a} \\ &\ge \frac{\phi_N(\underline{a})}{\hat{s}} \left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x) | x \le v^{-1}\left(\frac{\underline{a} - (1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a} \right) = \Pi_I^*(\hat{s}, \hat{s}, \bar{s}), \end{aligned} \tag{39}$$

where the second inequality follows from $v(\hat{s}) \ge \underline{a}$, which in turn follows from the optimality of type $\hat{s}$ of the initiating bidder bidding $\underline{a}$. The inequality is strict if $\hat{s} > 0$. The last inequality follows from $\phi_N(\underline{a}) \le \hat{s}$. Second, consider case $a_S(\hat{s}) > \underline{a}$. Then,

$$\begin{aligned} \Pi_S^*(\hat{s}, \hat{s}) &> \frac{\phi_S(\underline{a})}{\hat{s}} \left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x) | x \le \phi_S(\underline{a})] - \underline{a} \right) \\ &> \frac{1}{\hat{s}} v^{-1}\left(\frac{\underline{a} - (1-\alpha)v(\hat{s})}{\alpha}\right) \left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x) | x \le v^{-1}\left(\frac{\underline{a} - (1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a} \right) = \Pi_I^*(\hat{s}, \hat{s}, \bar{s}). \end{aligned} \tag{40}$$

The first inequality follows from the fact that a bidder with signal $\hat{s}$ must prefer bidding $a_S(\hat{s})$ over bidding $\underline{a}$. The second inequality is implied by $\phi_S(\underline{a}) > v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)$. Therefore, $\Pi_S^*(\hat{s}, \hat{s}) > \Pi_I^*(\hat{s}, \hat{s}, \bar{s})$.

Second, compare $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$ and $\Pi_N^*(\hat{s}, \hat{s}, \bar{s})$. The payoff of type $\hat{s}$ of the initiating bidder solves

$$\begin{aligned} \Pi_I^*(\hat{s}, \hat{s}, \bar{s}) &= \max_b \frac{1}{\hat{s}} v^{-1}\left(\frac{b - (1-\alpha)v(\hat{s})}{\alpha}\right) \left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x) | x \le v^{-1}\left(\frac{b - (1-\alpha)v(\hat{s})}{\alpha}\right)\right] - b \right) \\ &= \max_y \frac{v^{-1}(y)}{\hat{s}} \left((2\alpha - 1) v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x) | x \le v^{-1}(y)\right] - \alpha y \right), \end{aligned} \tag{41}$$

where $y \equiv \frac{b-(1-\alpha)v(\hat{s})}{\alpha}$. Therefore,

$$\Pi_I^*(\hat{s},\hat{s},\bar{s}) \leq (2\alpha-1)\max_y \frac{v^{-1}(y)}{\hat{s}}(v(\hat{s})-y) = (2\alpha-1)\max_x \frac{x}{\hat{s}}(v(\hat{s})-v(x)). \tag{42}$$

It immediately follows that as $\alpha \to \frac{1}{2}$, $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ converges to zero. Next, the payoff of type $\hat{s}$ of the non-initiating bidder solves:

$$\begin{aligned} \Pi_N^*(\hat{s},\hat{s},\bar{s}) &= \alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x)|x \leq \hat{s}] - \bar{a} \\ &= \max_b \frac{\phi_N(b,\hat{s})-\hat{s}}{\bar{s}-\hat{s}}(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x)|x \in [\hat{s},\phi_N(b,\hat{s})]] - b). \end{aligned} \tag{43}$$

Because $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ converges to zero as $\alpha \to \frac{1}{2}$, but $\Pi_N^*(\hat{s},\hat{s},\bar{s})$ does not, $\Pi_N^*(\hat{s},\hat{s},\bar{s}) > \Pi_I^*(\hat{s},\hat{s},\bar{s})$ for any α sufficiently close to $\frac{1}{2}$. This proves the last statement of the proposition.

Finally, compare $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ and $\Pi_N^*(\hat{s},\hat{s},\bar{s})$ for $\alpha = 1$:

$$\begin{aligned} \Pi_I^*(\hat{s},\hat{s},\bar{s}) &= \frac{\phi_N(\underline{a})}{\hat{s}}(v(\hat{s})-\underline{a}) = \max_b \frac{\phi_N(b)}{\hat{s}}(v(\hat{s})-b) \\ &> \frac{\phi_N(\bar{a})}{\hat{s}}(v(\hat{s})-\bar{a}) = v(\hat{s})-\bar{a} = \Pi_N^*(\hat{s},\hat{s},\bar{s}). \end{aligned}$$

Therefore, $\Pi_I^*(\hat{s},\hat{s},\bar{s}) > \Pi_N^*(\hat{s},\hat{s},\bar{s})$ for $\alpha = 1$. By continuity of $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ and $\Pi_N^*(\hat{s},\hat{s},\bar{s})$ in α, $\Pi_I^*(\hat{s},\hat{s},\bar{s}) > \Pi_N^*(\hat{s},\hat{s},\bar{s})$ for any α sufficiently close to 1. This proves the second statement of the proposition.

Proof of Proposition 2. As before, we omit the dependence of all functions on $\hat{s}$ for brevity. Type $\hat{s}$ of the non-initiating bidder bids $\bar{a}$. Type $\hat{s}$ of the initiating bidder bids $\underline{a}$. Because $\bar{a} > \underline{a}$, $a_N(\hat{s}) > a_I(\hat{s})$, proving the first statement of the proposition.

To prove the second statement of the proposition, let $H_k(b)$ and $h_k(b)$ denote the c.d.f. and p.d.f., respectively, of bids of type $k \in \{I,N\}$. Note that $H_k(b) = \frac{\phi_k(b)-\underline{s}_k}{\bar{s}_k-\underline{s}_k}$, and therefore $h_k(b) = \frac{1}{\bar{s}_k-\underline{s}_k}\frac{\partial\phi_k(b)}{\partial b}$. Using the equilibrium condition for optimality of bids, (3),

$$\frac{H_I(b)}{h_I(b)} = \alpha v(\phi_N(b)) + (1-\alpha)v(\phi_I(b)) - b < \alpha v(\phi_I(b)) + (1-\alpha)v(\phi_N(b)) - b = \frac{H_N(b)}{h_N(b)}, \tag{44}$$

where the inequality follows from $\phi_I(b) > \phi_N(b)$ and $\alpha > 1-\alpha$. Inequality $\phi_I(b) > \phi_N(b)$ holds because $\phi_I(b) \geq \hat{s} > \phi_N(b)$ for any $b < \bar{a}$ and $\bar{s} = \phi_I(b) > \hat{s} = \phi_N(b)$ for $b = \bar{a}$. Therefore, the distribution of bids of the initiating bidder dominates the distribution of bids of the non-initiating bidder in terms of the reverse hazard rate. In turn, dominance in terms of the reverse hazard rate implies first-order stochastic dominance (e.g., Krishna, 2010, Appendix B). Therefore, $\mathbb{E}[a_I(s)|s \in [\hat{s},\bar{s}]] > \mathbb{E}[a_N(s)|s \leq \hat{s}]$.

To prove the third statement, let $\tilde{s}$ denote the type of the non-initiating bidder that submits the lowest serious bid: $a_N(\tilde{s}) = \underline{a}$. In the seller-initiated auction, this type bids

$$a_S(\tilde{s}) = \mathbb{E}[v(x)|x \leq \tilde{s}] < v(\tilde{s}) < \alpha v(\tilde{s}) + (1-\alpha)v(\hat{s}) = \underline{a}. \tag{45}$$

Therefore, type $\tilde{s}$ bids less aggressively in the seller-initiated auction: $a_S(\tilde{s}) < a_N(\tilde{s})$. We next prove that $a_S(s) < a_N(s)\ \forall s \in (\tilde{s}, \hat{s}]$ by contradiction. Suppose that there exists type z for which $a_S(z) \geq a_N(z)$. Because $a_S(\tilde{s}) < a_N(\tilde{s})$ and both $a_S(\tilde{s})$ and $a_N(\tilde{s})$ are continuous, there exists type $z' \in (\tilde{s}, z)$ for which $a_S(z') = a_N(z')$. Pick the lowest such z'. Then, because $a_S(\tilde{s}) < a_N(\tilde{s})$, $a_N(s)$ crosses $a_S(s)$ from above at $s = z'$. Using (3) and (9) and $\phi_I(b) > \hat{s} > z'$ for any $b \in [\underline{a}, \bar{a}]$,

$$\frac{\partial a_N(s)}{\partial s}\Big|_{s=z'} = \frac{\alpha v(\phi_I(a_N(s)))+(1-\alpha)v(z')-a_N(z')}{z'} > \frac{v(z')-a_S(z')}{z'} = \frac{\partial a_S(s)}{\partial s}\Big|_{s=z'}. \quad (46)$$

Therefore, at any intersection point, $a_N(s)$ must cross $a_S(s)$ from below, which is a contradiction. Therefore, $a_S(s) < a_N(s)\ \forall s \in [\tilde{s}, \hat{s}]$.

Finally, the last statement trivially follows from (1) the fact that the distribution of the initiating bidder's signal is uniform over $[\hat{s}, \bar{s}]$, while the distribution of the non-initiating bidder's signal and each bidder's signal in the seller-initiated auction is uniform over $[0, \hat{s}]$; and (2) the fact that, conditional on winning, the expected non-initiating bidder's signal exceeds the expected winning bidder's signal in the seller-initiated auction.

Proof of Proposition 3. The proposition follows from the following two observations. The first observation is that the strategy profile described in the proposition constitutes an equilibrium. Indeed, because the seller is expected to auction the asset off for any set of messages, each bidder finds it optimal to send $m_{i,1} = 0$. And because the game ends at $t = 1$, it is optimal for the seller to auction the asset off for any set of messages he receives. The second observation is that there is no other equilibrium. By contradiction, suppose there exists $\hat{s}_1 \in (0, \bar{s})$, such that bidder i sends message $m_{i.1} = 1\{s_i \geq \hat{s}_1\}$. Because the game ends at $t = 1$, it is optimal for the seller to auction the asset off for any pair of messages she receives, including $(0, 0)$. By Proposition 1, $\Pi^*_S(\hat{s}_1, \hat{s}_1) > \Pi^*_I(\hat{s}_1, \hat{s}_1, \bar{s})$ and $\Pi^*_N(\hat{s}_1, \hat{s}_1, \bar{s}) > \Pi^*_D(\hat{s}, \hat{s}, \bar{s}) = 0$. Consider a bidder with signal $\hat{s}_1$: because the seller's decision to auction the asset off is the same but the bidder's auction payoff is strictly higher if she sends message $m_{i,1} = 0$, the bidder (and any bidder with the signal just above $\hat{s}_1$) is better off deviating to sending message $m_{i,1} = 0$. Hence, the equilibrium described in the proposition is unique.

Proof of Proposition 4. First, we prove that the non-responsive equilibrium always exists by showing that neither bidders nor the seller benefit from any deviation from the equilibrium strategies. A bidder does not benefit from sending message $m_{i,0} = 0$, because the seller's reaction does not change: The seller will hold the auction. The seller does not benefit from delaying the auction until $t = 1$, because the distribution of values of bidders will not change (bidders will have the same signals, if there is no shock at $t = 1$, or will draw new signals from the same distribution if there is a shock at $t = 1$) and the payoff will be discounted.

Second, we solve for the responsive equilibria. We proceed in the following three steps.

Step 1: Any responsive equilibrium must have the seller's reaction described by equation (18).

Proof of Step 1. By symmetry, the seller's reaction must be the same for $\mathbf{m}_0 = (1, 0)$ and $\mathbf{m}_0 = (0, 1)$. Thus, step 1 means that there cannot be a responsive equilibrium in which the seller's reaction is

$$d_0(\mathbf{m}_0) = \begin{cases} 1, & \text{if } \mathbf{m}_0 = (1,1), \\ 0, & \text{if } \mathbf{m}_0 \neq (1,1). \end{cases}$$

By contradiction, suppose that such an equilibrium exists with some cut-off $\hat{s}_0 \in (0,1)$. Consider bidder i with signal $\hat{s}_0$. Suppose she sends message $m_{i,0} = 1$. With probability $1 - \hat{s}_0$, the rival also sends $m_{-i,0} = 1$. Because conditional on this event the rival's signal s_{-i} is distributed uniformly over $[\hat{s}_0, 1]$, bidder i loses the auction with certainty and obtains the payoff of zero. With probability $\hat{s}_0$, the rival sends $m_{-i,0} = 0$. In this case, the seller waits until the next period, so the bidder's payoff becomes $\beta\left((1-\lambda)\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) + \lambda\Pi_R\right)$. Hence, the bidder's expected payoff from message $m_{i,0} = 1$ is

$$\hat{s}_0\beta\left((1-\lambda)\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) + \lambda\Pi_0\right). \tag{47}$$

Suppose instead that bidder i with signal $\hat{s}_0$sends message $m_{i,0} = 0$. In this case, the auction will take place at $t = 1$ with certainty. The bidder's payoff will be

$$\beta\left((1-\hat{s}_0)(1-\lambda)\Pi_N^*(\hat{s}_0, \hat{s}_0, 1) + \hat{s}_0(1-\lambda)\Pi_S^*(\hat{s}_0, \hat{s}_0) + \lambda\Pi_R\right) \tag{48}$$

By Proposition 1, $\Pi_S^*(\hat{s}_0, \hat{s}_0) > \Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$. Therefore, (48) is strictly higher than (47). Thus, bidders with signals at or just above $\hat{s}_0$are better off deviating to sending message $m_{i,0} = 0$. We have reached a contradiction.

Step 2: Optimality conditions for the bidder.

The indifference condition (19) for the but-off signal $\hat{s}_0$ was derived in the main text. It remains to show that if a bidder with signal $\hat{s}_0$ is indifferent between sending messages $m_{i,0} = 0$ and $m_{i,0} = 1$, then any bidder with signal $s < \hat{s}_0$ prefers to send message $m_{i,0} = 0$ and any bidder with signal $s > \hat{s}_0$ prefers to send message $m_{i,0} = 1$:

$$\Pi_I^*(s, \hat{s}_0, 1) \geq \left(\tfrac{1}{\hat{s}_0} - 1\right)\Pi_N^*(s, \hat{s}_0, 1) + \nu\Pi_S^*(s, \hat{s}_0) + (1-\nu)\beta\left((1-\lambda)\Pi_S^*(s, \hat{s}_0) + \lambda\Pi_R\right) \text{ for any } s > \hat{s}_0; \tag{49}$$

$$\Pi_I^*(s, \hat{s}_0, 1) \leq \left(\tfrac{1}{\hat{s}_0} - 1\right)\Pi_N^*(s, \hat{s}_0, 1) + \nu\Pi_S^*(s, \hat{s}_0) + (1-\nu)\beta\left((1-\lambda)\Pi_S^*(s, \hat{s}_0) + \lambda\Pi_R\right) \text{ for any } s < \hat{s}_0. \tag{50}$$

Because equality is achieved at $s = \hat{s}_0$, it is sufficient to prove

$$\frac{\partial}{\partial s}\Pi_I^*(s, \hat{s}_0, 1) \geq \left(\frac{1}{\hat{s}_0} - 1\right)\frac{\partial}{\partial s}\Pi_N^*(s, \hat{s}_0, 1) + (\nu + (1-\nu)\beta(1-\lambda))\frac{\partial}{\partial s}\Pi_S^*(s, \hat{s}) \text{ for all } s. \tag{51}$$

First, consider $s > \hat{s}_0$. In this range,

$$\Pi_N^*(s, \hat{s}_0, 1) = \alpha v(s) + (1-\alpha)\mathbb{E}\left[v(x) \mid x \in [\hat{s}_0, 1]\right] - \bar{a}(\hat{s}_0) \Rightarrow \frac{\partial}{\partial s}\Pi_N^*(s, \hat{s}_0, 1) = \alpha v'(s); \tag{52}$$

$$\Pi_S^*(s, \hat{s}) = \alpha v(s) + (1-\alpha)\mathbb{E}\left[v(x) \mid x \leq \hat{s}_0\right] - a_S(\hat{s}_0, \hat{s}_0) \Rightarrow \frac{\partial}{\partial s}\Pi_S^*(s, \hat{s}_0) = \alpha v'(s). \tag{53}$$

Thus, the derivative of the right-hand side of (49) in s is

$$\left(\frac{1}{\hat{s}_0}-1+\nu+(1-\nu)\beta(1-\lambda)\right)\alpha v'(s). \tag{54}$$

To calculate the derivative of the left-hand side of (49), apply the envelope theorem to

$$\begin{aligned}\Pi_I^*(s,\hat{s}_0,1) &= \max_b\left\{\frac{\phi_N(b,\hat{s}_0,1)}{\hat{s}_0}\left(\alpha v(s)+(1-\alpha)\mathbb{E}\left[v(x)\,|x\le\phi_N(b,\hat{s}_0,1)\right]-b\right)\right\}\\ &\Rightarrow \frac{\partial}{\partial s}\Pi_I^*(s,\hat{s})=\alpha v'(s)\frac{\phi_N(a_I(s,\hat{s}_0,1),\hat{s}_0,1)}{\hat{s}_0}. \end{aligned}\tag{55}$$

Thus, (51) is equivalent to

$$\frac{\phi_N(a_I(s,\hat{s}_0,1),\hat{s}_0,1)}{\hat{s}_0}\ge\frac{1}{\hat{s}_0}-1+\nu+(1-\nu)\beta(1-\lambda). \tag{56}$$

Because the left-hand side is strictly increasing in $s\in[\hat{s}_0,1]$ and the right-hand side is independent of s, it is sufficient to verify the inequality for $s\downarrow\hat{s}_0$, in which case it reduces to

$$\nu<1-\frac{1-v^{-1}\left(\frac{a(\hat{s}_0,1)-(1-\alpha)v(\hat{s}_0)}{\alpha}\right)}{\hat{s}_0(1-\beta(1-\lambda))}, \tag{57}$$

where we used the final initial value condition in (5) and re-arranged (56) for $s\downarrow\hat{s}_0$. Next, consider $s<\hat{s}_0$. In this case, condition (50) must be satisfied in equilibrium. Because non-serious bids of low types of the non-initiating bidder are not uniquely pinned down in equilibrium, there can be multiple equilibrium values of $\Pi_I^*(s,\hat{s}_0,1)$ for $s<\hat{s}_0$. However, it is necessary and sufficient to verify (50) for a specific value of $\Pi_I^*(s,\hat{s}_0,1)$:

$$\tilde{\Pi}_I^*(s,\hat{s}_0,1)\le\left(\tfrac{1}{\hat{s}_0}-1\right)\Pi_N^*(s,\hat{s}_0,1)+\nu\Pi_S^*(s,\hat{s}_0)+(1-\nu)\beta\left((1-\lambda)\Pi_S^*(s,\hat{s})+\lambda\Pi_R\right)\ \text{ for all } s<\hat{s}_0, \tag{58}$$

where

$$\tilde{\Pi}_I^*(s,\hat{s}_0,1)\equiv\max\left\{0,\max_{b\ge(1-\alpha)v(\hat{s}_0)}\left\{v^{-1}\left(\tfrac{b-(1-\alpha)v(\hat{s}_0)}{\alpha}\right)\times\left(\alpha v(s)+(1-\alpha)\mathbb{E}\left[v(x)\,|x\le v^{-1}\left(\tfrac{b-(1-\alpha)v(\hat{s}_0)}{\alpha}\right)\right]-b\right)\right\}\right\}. \tag{59}$$

Intuitively, this expression is the payoff of the bidder with signal $s<\hat{s}_0$ from deviating and indicating the interest under the assumption that the non-initiating bidder with a low signal s bids $\alpha v(s)+(1-\alpha)v(\hat{s}_0)$. Thus, this expression is the lowest possible payoff of the deviating bidder. If (58) holds, then there exist equilibrium bids of the non-initiating bidder with a low signal that satisfy (50): For example, the equilibrium bids of the non-initiating bidder with a low signal that win with probability zero could be $\alpha v(s)+(1-\alpha)v(\hat{s}_0)$. Of course, there can be other non-serious bids of the non-initiating bidder (which imply higher values of $\Pi_I^*(s,\hat{s}_0)$ for $s<\hat{s}_0$) that also satisfy (50). In contrast, if (58) does not hold for some $s<\hat{s}_0$, then it cannot hold for any equilibrium $\Pi_I^*(s,\hat{s}_0,1)$ for the same s. Note that (58) trivially holds for sufficiently small values of s (as $\tilde{\Pi}_I^*(s,\hat{s}_0,1)=0$ in this case) and holds for $s\uparrow\hat{s}_0$ (it is implied by (57) in this case). However, we are unable to prove that these results imply that (58) necessarily also holds for intermediate values of s, so it needs to be numerically verified.

Step 3: Optimality condition for the seller.

Finally, in the responsive equilibrium the seller must have incentives to hold an auction if he gets at least one indication of interest, and wait until $t = 1$ if he gets no indications of interest. We need to consider three cases: (1) the seller deviates to not holding the auction upon receiving $\mathbf{m}_0 = (1, 0)$; (2) the seller deviates to not holding the auction upon receiving $\mathbf{m}_0 = (1, 1)$; (3) the seller deviates to holding the auction upon receiving $\mathbf{m}_0 = (0, 0)$. By symmetry, the case of $\mathbf{m}_0 = (0, 1)$ is identical to the case of $\mathbf{m}_0 = (1, 0)$.

Consider the case of $\mathbf{m}_0 = (1, 0)$. If the seller holds an auction, he obtains an expected payoff of $R_B(\hat{s}_0, 1)$. If the seller does not hold an auction, the game proceeds to $t = 1$, when the seller-initiated auction occurs with certainty by the argument identical to Proposition 3. The seller's discounted payoff from not holding the auction at $t = 0$ is therefore $\beta((1 - \lambda) R_S(\hat{s}_0) + \lambda R_S(1))$, where $R_S(\hat{s})$ are the expected revenues of the seller from the seller-initiated auction given by (13). Thus, the seller finds it optimal to hold an auction upon receiving $\mathbf{m}_0 = (1, 0)$ if and only if

$$R_B(\hat{s}_0, 1) \geq \beta((1 - \lambda) R_S(\hat{s}_0) + \lambda R_S(1)). \tag{60}$$

Because $R_B(\hat{s}_0, 1) > R_S(\hat{s}_0)$ and $\beta < 1$, (60) holds for $\hat{s}_0$ sufficiently close to 1, but may be violated for low values of $\hat{s}_0$. As can be seen, it always holds for λ sufficiently close to zero.

Next, consider the case of $\mathbf{m}_0 = (1, 1)$. The expected revenues of the seller from immediate initiation of the auction are $R_D(\hat{s}_0) = \mathbb{E}[v(\min(s_1, s_2)) | s_i \in [\hat{s}_0, 1], i \in \{1, 2\}]$. If the seller does not hold an auction, the game proceeds to $t = 1$, when the seller-initiated auction occurs with certainty by the argument identical to Proposition 3. The seller's discounted payoff from this option is $\beta((1 - \lambda) R_D(\hat{s}_0) + \lambda R_S(1))$. Because $R_D(\hat{s}_0) > R_S(1)$ and $\beta < 1$, $R_D(\hat{s})$ exceeds $\beta((1 - \lambda) R_D(\hat{s}_0) + \lambda R_S(1))$. Thus, the seller always finds it optimal to hold the auction upon receiving two indications of interest.

Finally, consider the case of $\mathbf{m}_0 = (0, 0)$. If the seller waits until $t = 1$, her expected payoff would be $\beta((1 - \lambda) R_S(\hat{s}_0) + \lambda R_S(1))$. If the seller holds the auction immediately, her expected revenues would be $R_S(\hat{s}_0)$. Thus, the seller finds it optimal to wait if and only if

$$R_S(\hat{s}_0) \leq \beta((1 - \lambda) R_S(\hat{s}_0) + \lambda R_S(1)). \tag{61}$$

Combining with (60), we obtain (99).

Proof of Proposition 5. Consider a single bidder-initiated auction. The payoff of the initiating bidder with signal $\hat{s}_0$ is $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) \leq (2\alpha - 1) \max_x \frac{x}{\hat{s}_0}(v(\hat{s}_0) - v(x))$, as shown by (42) in the proof of Proposition 1. Note that $\max_x \frac{x}{\hat{s}_0}(v(\hat{s}_0) - v(x))$ is the profit of the bidder with signal $\hat{s}_0$ in the first-price auction with pure private values, in which the rival bids according to schedule $v(x)$ for x distributed uniformly over interval $[0, \hat{s}_0]$. Suppose instead that the rival with signal x bids $\mathbb{E}[v(s) | s \leq x]$. Because the bid of the rival under this schedule is lower than

$v(x)$ for any realization of x, the payoff of the bidder under consideration goes up. Therefore,

$$\max_x \frac{x}{\hat{s}_0}\left(v\left(\hat{s}_0\right)-v\left(x\right)\right) \leq \max_x \frac{x}{\hat{s}_0}\left(v\left(\hat{s}_0\right)-\mathbb{E}\left[v(s)|s\leq x\right]\right) = v\left(\hat{s}_0\right)-\mathbb{E}\left[v\left(s\right)|s\leq \hat{s}_0\right], \tag{62}$$

where the equality comes from the fact that $\mathbb{E}\left[v\left(s\right)|s\leq \hat{s}_0\right]$ is the optimal bid for the bidder with signal $\hat{s}_0$ in the symmetric first-price auction with pure private values (see Section 3.2).

Consider equation (19) for the cut-off signal $\hat{s}_0$. For the left-hand side and the right-hand side of it we have:

$$\begin{aligned} LHS &\leq (2\alpha-1)\left(v\left(\hat{s}_0\right)-\mathbb{E}\left[v\left(s\right)|s\leq \hat{s}_0\right]\right); \\ RHS &> \alpha\left(\nu+(1-\nu)\beta\left(1-\lambda\right)\right)\left(v\left(\hat{s}_0\right)-\mathbb{E}\left[v\left(s\right)|s\leq \hat{s}_0\right]\right) \end{aligned} \tag{63}$$

Therefore, a sufficient condition for there to be no responsive equilibrium is:

$$2\alpha-1\leq\left(\nu+(1-\nu)\beta(1-\lambda)\right)\alpha \quad \Rightarrow \quad \alpha\leq\hat{\alpha}\equiv\frac{1}{2-\nu-(1-\nu)\beta(1-\lambda)}.$$

Note that $\hat{\alpha}>\frac{1}{2}$.

Proof of Proposition 6. From the proof of Proposition 5, a sufficient condition for there to be no responsive equilibrium is $\alpha\leq\frac{1}{2-\nu-(1-\nu)\beta(1-\lambda)}$. We can alternatively re-write a sufficient condition as:

$$\nu\geq\hat{\nu}\equiv 1-\frac{1-\alpha}{\alpha\left(1-\beta(1-\lambda)\right)}, \tag{64}$$

which proves the proposition.

Proof of Corollary 1. If $C>R_S\left(1\right)$, the seller's revenues from the seller-initiated auction are below C for any $\hat{s}_0$. Hence, the seller never initiates the auction voluntarily. Because $\Pi_I^*\left(\hat{s}_0,\hat{s}_0,1\right)$ approaches zero as $\alpha\to\frac{1}{2}$, the payoff of the bidder with cut-off signal $\hat{s}_0$ from sending an indication of interest to the seller is close to zero, while the payoff from not sending it is strictly positive as long as $\nu>0$. Thus, no responsive equilibrium exists.

Proof of Proposition 7. If the responsive equilibrium exists, then the bidder with the cut-off signal $\hat{s}_0$ is given by (22). First, consider the comparative statics with respect to α. Differentiating the quadratic equation (88) for $\hat{s}_0$ in α and re-erranging the terms yields:

$$\frac{d\hat{s}_0}{d\alpha}=\frac{\left(1+\nu+(1-\nu)\,\beta\left(1-\lambda\right)\right)\hat{s}_0-\left(1-(1-\nu)\,\beta\lambda\frac{1}{3}\right)-\left[\frac{(2\alpha-1)^2}{3\alpha-1}\right]'\hat{s}_0^2}{\frac{(2\alpha-1)^2}{3\alpha-1}2\hat{s}_0+\left(1-\left(1+\nu+(1-\nu)\,\beta\left(1-\lambda\right)\right)\alpha\right)}. \tag{65}$$

The denominator is positive, as it can be seen from (88) that it is equal to $\frac{(1-\alpha)+(1-\nu)\beta\lambda\frac{\alpha}{3}}{\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0}$ +

$\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0 > 0$. Hence, it is sufficient to analyze the numerator. From (88):

$$\alpha\left[(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\left(1-(1-\nu)\beta\lambda\frac{1}{3}\right)\right]=\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0^2-1. \tag{66}$$

Because $\left[\frac{(2\alpha-1)^2}{3\alpha-1}\right]'=\frac{2\alpha-1}{(3\alpha-1)^2}\left[24\alpha-7-12\alpha^2\right]>0$ for $\alpha\in\left[\frac{1}{2},1\right]$, $\frac{(2\alpha-1)^2}{3\alpha-1}$ is strictly increasing in α. Using that $\hat{s}_0\in(0,1)$, as otherwise the responsive equilibrium would not exist,

$$\alpha\left[(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\left(1-(1-\nu)\beta\lambda\frac{1}{3}\right)\right]<\frac{1}{2}\hat{s}_0^2-1<0. \tag{67}$$

We can now sign the numerator:

$$\left[(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\left(1-(1-\nu)\beta\lambda\frac{1}{3}\right)\right]-\left[\frac{(2\alpha-1)^2}{3\alpha-1}\right]'\hat{s}_0^2<0, \tag{68}$$

because both terms are negative. Therefore, $\hat{s}_0$ is strictly decreasing in α.

Second, consider the comparative statics with respect to ν. Differentiating the quadratic equation (88) for $\hat{s}_0$ in ν and re-erranging the terms yields:

$$\frac{d\hat{s}_0}{d\nu}=\frac{(1-\beta(1-\lambda))\alpha\hat{s}_0-\beta\lambda\frac{\alpha}{3}}{\frac{(2\alpha-1)^2}{3\alpha-1}2\hat{s}_0+(1-(1+\nu+(1-\nu)\beta(1-\lambda))\alpha)}. \tag{69}$$

The denominator is the same as above, so it is positive. Hence, it is sufficient to analyze the numerator. Re-arranging the terms in (88) yields

$$(1-\nu)\left[(1-\beta(1-\lambda))\alpha\hat{s}_0-\beta\lambda\tfrac{\alpha}{3}\right]=(1-\alpha)+(2\alpha-1)\hat{s}_0-\tfrac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0^2>\min\left\{1-\alpha,\alpha-\tfrac{(2\alpha-1)^2}{3\alpha-1}\right\}, \tag{70}$$

where the inequality is from the fact that the right-hand side of the equation is inverted U-shaped in $\hat{s}_0\in[0,1]$. Finally,

$$\min\left\{1-\alpha,\alpha-\tfrac{(2\alpha-1)^2}{3\alpha-1}\right\}=\min\left\{1-\alpha,\tfrac{3\alpha-\alpha^2-1}{3\alpha-1}\right\}>\min\left\{1-\alpha,\tfrac{\frac{3}{2}-\frac{1}{4}-1}{3\alpha-1}\right\}>0, \tag{71}$$

where the first inequality is from the fact that $\left[3\alpha-\alpha^2-1\right]'=3-2\alpha>0$ for all $\alpha\in\left[\frac{1}{2},1\right]$. Hence, the numerator is positive. Therefore, $\hat{s}_0$ is strictly increasing in ν.

Proof of Proposition 8. First, we prove that the equilibrium in the seller-initiated auction is as described in the text. Denote the equilibrium bidding strategy by $a_S(s)$ and its inverse by $\phi_S(b)$. The expected payoff of a bidder with signal s and bid b is

$$\Pi_S(b,s)=\int_0^{\phi_S(b)}(\alpha v(s)+(1-\alpha)v(x)-fb-(1-f)a_S(x))\frac{1}{\hat{s}}dx. \tag{72}$$

Differentiating in b and using the equilibrium condition that the maximum is reached at $b=a_S(s)$, or, equivalently, that $s = \phi_S(b)$ yields (23). Because a bidder with signal $s = 0$ does not obtain any value from the asset, the initial value condition is $0 = \phi_S(0)$. Equation (23) is equivalent to

$$\frac{ds}{da_S(s)} = \frac{fs}{v(s) - a_S(s)}. \tag{73}$$

It can be further re-written as

$$\frac{d}{ds}\left[s^{\frac{1}{f}} a_S(s)\right] = s^{\frac{1}{f}-1}\frac{v(s)}{f}, \tag{74}$$

which is solved by

$$a_S(s) = \int_0^s v(x)\, d\left[\left(\frac{x}{s}\right)^{\frac{1}{f}}\right] = \tilde{\mathbb{E}}\left[v(x)\,|x \le s\right], \tag{75}$$

where $\tilde{\mathbb{E}}[v(x)]$ denotes the expectation of $v(x)$ when random variable x is distributed with c.d.f. $x^{\frac{1}{f}}$. Now, we can calculate the expected payment of a bidder with signal s, conditional on winning:

$$f\tilde{\mathbb{E}}\left[v(x)\,|x \le s\right] + (1-f)\int_0^s \tilde{\mathbb{E}}\left[v(y)\,|y \le x\right]\frac{1}{s}dx \tag{76}$$

$$= \frac{1}{s^{\frac{1}{f}}}\int_0^s x^{\frac{1}{f}-1} v(x)\, dx - \frac{1}{s^{\frac{1}{f}}}\int_0^s v(x)\left(x^{\frac{1}{f}-1} - s^{\frac{1}{f}-1}\right)dx = \frac{1}{\hat{s}}\int_0^{\hat{s}} v(x)\,dx = \mathbb{E}\left[v(x)\,|s \le \hat{s}\right],$$

where we use integration by parts to calculate the second term. Therefore, the payment is as described in the main text.

Consider the payoff of the initiating bidder with signal $\hat{s}$:

$$\begin{aligned}
\Pi_I^*(\hat{s},\hat{s},\bar{s}) &\le \frac{v^{-1}(y)}{\hat{s}}\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)\,|x \le v^{-1}(y)\right] - f(\alpha y + (1-\alpha)v(\hat{s})) - (1-f)(1-\alpha)v(\hat{s})\right) \\
&= \frac{v^{-1}(y)}{\hat{s}}\left((2\alpha-1)v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)\,|x \le v^{-1}(y)\right] - f\alpha y\right) \qquad (77) \\
&\le (2\alpha-1)\max_y \frac{v^{-1}(y)}{\hat{s}}\left(v(\hat{s}) - \frac{(f+1)\alpha-1}{2\alpha-1}y\right) = (2\alpha-1)\max_x \frac{x}{\hat{s}}\left(v(\hat{s}) - \frac{(f+1)\alpha-1}{2\alpha-1}v(x)\right).
\end{aligned}$$

As in Proposition 1, $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ converges to zero as $\alpha \to \frac{1}{2}$ for any f. Because the payoff of the bidder with signal $\hat{s}$ in the seller-initiated auction is the same as in the first-price auction ($\Pi_S^*(\hat{s},\hat{s}) = \alpha(v(\hat{s}) - \mathbb{E}[v(s)\,|s \le \hat{s}])$), the final part of the proof of Proposition 5 applies, leading to the statement of the proposition.

Proof of Proposition 9. As in the base model, the seller initiates the auction in the terminal period. Consider a single bidder-initiated auction at $t = 0$. With a slight abuse of notation, we use the same notation to denote the payoffs of bidders as in the base model. We will show that the payoff of the initiating bidder with the cut-off signal $\hat{s}$, $\Pi_I^*(\hat{s}_0,\hat{s}_0,1)$, approaches zero in the sequential format as $\alpha \to \frac{1}{2}$. Once this result is established, the final part of the proof of Proposition 5 applies, leading to the statement of the proposition.

Let $\hat{b}$ denote the lowest bid that preempts the uninformed non-initiating bidder from learning her signal. Let $\tilde{s} \ge \hat{s}_0$ be the lowest signal of the initiating bidder that leads her to preempt the uninformed bidder by submitting bid $b \ge \hat{b}$. There are two possible cases, $\tilde{s} > \hat{s}_0$ and $\tilde{s} = \hat{s}_0$. If $\tilde{s} > \hat{s}_0$, then the initiating bidder with signal $\hat{s}_0$ does not preempt the uninformed bidder from learning her own signal. In this case, we have the first-price auction in which the signal of one bidder is drawn from the uniform distribution over $[\hat{s}_0, \tilde{s}]$, while the signal of the other bidder is drawn from the uniform distribution over $[0,1]$. Aside from a different distribution of signals of the rival bidder ($[0,1]$ instead of $[0,\hat{s}_0]$), the argument of the baseline model (Proposition 5) applies in the same way to show that the payoff of the initiating bidder with the cut-off signal $\hat{s}_0$ converges to zero as $\alpha \to \frac{1}{2}$.

If $\tilde{s} = \hat{s}_0$, then the initiating bidder with signal $\hat{s}_0$ preempts the uninformed non-initiating bidder from learning her own signal. The highest bid that the initiating bidder with signal $\hat{s}_0$ is willing to make cannot exceed the expected payoff of the asset:

$$\hat{b} \leq \alpha v(\hat{s}_0) + (1-\alpha)\mathbb{E}[v(s)] \equiv \bar{b}(\hat{s}_0). \tag{78}$$

If the uninformed bidder competes against bid $\hat{b}$, her expected payoff from participating in the auction, denoted $E\Pi_N\left(\hat{s}_0, \hat{b}\right)$, satisfies $E\Pi_N\left(\hat{s}_0, \hat{b}\right) \geq E\Pi_N\left(\hat{s}_0, \bar{b}(\hat{s})\right) > 0$ (this expected payoff is positive because there exist realizations of the signal close to one such that the non-initiating bidder with this signal obtains positive information rents). Therefore, if $\Psi \leq \bar{\Psi} \equiv \min_{\hat{s}_0 \in [0,1]} E\Pi_N\left(\hat{s}_0, \bar{b}(\hat{s})\right)$, the initiating bidder with signal $\hat{s}_0$ cannot preempt the non-initiating bidder in equilibrium. In this case, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$ approaches zero as $\alpha \to \frac{1}{2}$ according to the same argument as in the base model.

B Special case of linear valuations

We first derive the closed-form solutions of (21) for the special case $v(s) = s$. Using $\mathbb{E}[v(x)|x \leq s] = \frac{s}{2}$, expression (12) specializes to $\Pi_S^*(\hat{s}, \hat{s})$ in (21). Consider optimization (4). Re-write it as

$$\Pi_I^*(\hat{s}, \hat{s}, 1) = \max_x \frac{x}{\hat{s}}\left((2\alpha - 1)\hat{s} - \left(\frac{3\alpha}{2} - \frac{1}{2}\right)x\right). \tag{79}$$

The first-order condition yields $x = \frac{2\alpha-1}{3\alpha-1}\hat{s}$. This gives $\Pi_I^*(\hat{s}, \hat{s}, 1)$ in (21). It follows that the minimum bid is

$$\underline{a}(\hat{s}) = \alpha\frac{2\alpha - 1}{3\alpha - 1}\hat{s} + (1-\alpha)\hat{s} = \frac{3\alpha - 1 - \alpha^2}{3\alpha - 1}\hat{s}. \tag{80}$$

To obtain $\Pi_N^*(\hat{s}, \hat{s}, 1)$, rewrite differential equations (3) that determine functions $\phi_j(b, \hat{s})$, $j \in \{I, N\}$ for $v(s) = s$ as:

$$\frac{\partial \phi_j(b, \hat{s})}{\partial b}\left(\alpha\phi_k(b, \hat{s}) + (1-\alpha)\phi_j(b, \hat{s}) - b\right) = \phi_j(b, \hat{s}) - \underline{s}_j. \tag{81}$$

Here, $\underline{s}_I = \hat{s}$ and $\underline{s}_N = 0$. Adding up both equations,

$$[\alpha\phi_I(b, \hat{s})\phi_N(b, \hat{s})]' + \left[\frac{1-\alpha}{2}\phi_I(b, \hat{s})^2\right]' + \left[\frac{1-\alpha}{2}\phi_N(b, \hat{s})^2\right]' = [(\phi_I(b, \hat{s}) + \phi_N(b, \hat{s}) - \hat{s})b]' \tag{82}$$

Integrating,

$$\alpha\phi_I(b, \hat{s})\phi_N(b, \hat{s}) + \frac{1-\alpha}{2}\phi_I(b, \hat{s})^2 + \frac{1-\alpha}{2}\phi_N(b, \hat{s})^2 = (\phi_I(b, \hat{s}) + \phi_N(b, \hat{s}) - \hat{s})b + c, \tag{83}$$

where c is the constant of integration. Evaluating it at $b = \underline{a}(\hat{s})$ and using $\underline{a}(\hat{s}) = \frac{3\alpha-1-\alpha^2}{3\alpha-1}\hat{s}$, $\phi_N(\underline{a}(\hat{s}), \hat{s}) = \frac{2\alpha-1}{3\alpha-1}\hat{s}$, and $\phi_I(\underline{a}(\hat{s}), \hat{s}) = \hat{s}$ yields the constant of integration in closed form:

$$c = \frac{\alpha^2}{2(3\alpha-1)}\hat{s}^2. \tag{84}$$

Evaluating (83) at $b = \bar{a}(\hat{s})$ and using $\phi_N(\bar{a}(\hat{s}), \hat{s}) = \hat{s}$, $\phi_I(\bar{a}(\hat{s}), \hat{s}) = 1$, and c from (84) yields $\bar{a}(\hat{s})$:

$$\bar{a}(\hat{s}) = \alpha\hat{s} + \frac{1-\alpha}{2} + \frac{1-\alpha}{2}\hat{s}^2 - \frac{\alpha^2}{2(3\alpha-1)}\hat{s}^2. \tag{85}$$

Now, we can obtain the payoff of the non-initiating bidder with signal $\hat{s}$. This bidder submits bid $\bar{a}(\hat{s})$, wins with certainty, and values the asset at $\alpha\hat{s} + (1-\alpha)\frac{1+\hat{s}}{2}$. This gives $\Pi_N^*(\hat{s}, \hat{s}, 1)$ in (21):

$$\Pi_N^*(\hat{s}, \hat{s}) = \alpha\hat{s} + (1-\alpha)\frac{1+\hat{s}}{2} - \bar{a}(\hat{s}) = \frac{1-\alpha}{2}\left(\hat{s} - \hat{s}^2\right) + \frac{\alpha^2}{2(3\alpha-1)}\hat{s}^2. \tag{86}$$

Finally,

$$\Pi_R = \int_0^1 \Pi_S^*(s, 1)\, ds = \alpha\int_0^1 \frac{s^2}{2} ds = \frac{\alpha}{6}. \tag{87}$$

Plugging expressions (21) into equation (19) that determines threshold $\hat{s}_0$ in the responsive equilibrium, we obtain the following quadratic equation:

$$\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0^2 + (1 - (1+\nu+(1-\nu)\beta(1-\lambda))\alpha)\hat{s}_0 - (1-\alpha) - (1-\nu)\beta\lambda\frac{\alpha}{3} = 0. \tag{88}$$

It has two roots, but one of them is negative. The positive root is given by (22).

Next, we consider three examples of the model with linear values.

Example 1: auction stage, pure private values. Suppose that $\alpha = 1$ and $v(s) = s$. This type of asymmetric auction with private values was analyzed by Kaplan and Zamir (2012). The solution in inverse bidding strategies is

$$\phi_I(b) = \hat{s} + \frac{\hat{s}^2}{(\hat{s}-2b)\, c_I e^{-\frac{\hat{s}}{\hat{s}-2b}} - 4b}, \phi_N(b) = \frac{\hat{s}^2}{(\hat{s}-2b)\, c_N e^{\frac{\hat{s}}{\hat{s}-2b}} + 4(\hat{s}-b)},$$

where constants c_N and c_I are determined from the first two boundary conditions in (5), $\phi_I\left(\hat{s} - \frac{\hat{s}^2}{4}\right) = 1$ and $\phi_N\left(\hat{s} - \frac{\hat{s}^2}{4}\right) = \hat{s}$. The range of bids is $b \in \left[\frac{\hat{s}}{2}, \hat{s} - \frac{\hat{s}^2}{4}\right]$ for both bidders. For $\hat{s} = \frac{1}{2}$, Figure 1, Panel A illustrates these bidding strategies as well as bidding strategies $a_S(s) = \frac{s}{2}$ in the seller-initiated auction.

Example 2: auction stage, pure common values. Suppose that $\alpha = \frac{1}{2}$ and $v(s) = s$. To find the equilibrium, let $\phi_j(b) = \gamma_j + \beta_j b$, $j \in \{I, N\}$. Then, the first two boundary conditions in (5) become

$$1 = \gamma_I + \beta_I\bar{a}, \quad \hat{s} = \gamma_N + \beta_N\bar{a} \quad \Rightarrow \quad 1 + \hat{s} = \gamma_I + \gamma_N + (\beta_I + \beta_N)\bar{a}. \tag{89}$$

Using (4), $\underline{a} = \frac{\hat{s}}{2}$. Then, the final two conditions in (5) become:

$$\hat{s} = \gamma_I + \beta_I \frac{\hat{s}}{2}, \quad 0 = \gamma_N + \beta_N \frac{\hat{s}}{2} \quad \Rightarrow \quad \hat{s} = \gamma_I + \gamma_N + (\beta_I + \beta_N) \frac{\hat{s}}{2}. \tag{90}$$

Note that, particular to pure common values, the lowest serious non-initiating bidder has signal 0, so that there are no non-serious bids. The difference between (89) and (90) yields $\beta_I + \beta_N = \frac{1}{\bar{a} - \hat{s}/2}$. Next, plugging $\phi_j(b)$ into differential equations (3) yields:

$$\beta_j = \frac{\alpha_j + \beta_j b - \underline{s}_j}{\frac{1}{2}(\gamma_I + \gamma_N + (\beta_I + \beta_N) b) - b}. \tag{91}$$

Adding the two equations up at $b = \bar{a}$ results in $\beta_I + \beta_N = \frac{1}{(1+\hat{s})/2 - \bar{a}}$. Combining the two equations for $\beta_I + \beta_N$ yields $\bar{a} = \frac{1+2\hat{s}}{4}$ and the range of bids $b \in \left[\frac{\hat{s}}{2}, \frac{1+2\hat{s}}{4}\right]$ for both bidders. With $\bar{a}$ known, the coefficients in $\phi_j(b, \hat{s})$ can be found from boundary conditions. The resulting inverses of bidding strategies are:

$$\phi_I(b) = \hat{s}(2\hat{s} - 1) + 4(1 - \hat{s})b, \quad \phi_N(b) = -2\hat{s}^2 + 4\hat{s}b. \tag{92}$$

The bidding strategies given signals are, in turn, inverses of $\phi_j(b)$. They are linear in own signals:

$$a_I(s) = \frac{s + \hat{s}(1 - 2\hat{s})}{4(1 - \hat{s})}, \quad a_N(s) = \frac{s + 2\hat{s}^2}{4\hat{s}}.$$

For $\hat{s} = \frac{1}{2}$, Figure 1, Panel B illustrates these bidding strategies as well as bidding strategies $a_S(s) = \frac{s}{2}$ in the seller-initiated auction.

Example 3: initiation stage, pure private values. Suppose that $\alpha = 1.$ and $v(s) = s$. Then, (22) simplifies to

$$\hat{s}_0 = \nu + (1 - \nu)\beta(1 - \lambda) - 1 + \sqrt{(\nu + (1 - \nu)\beta(1 - \lambda))^2 + (1 - \nu)\beta\lambda\frac{2}{3}}. \tag{93}$$

If this expression exceeds one, then the responsive equilibrium does not exist.

C Mixed equilibria

In this section we look at responsive equilibria in mixed seller strategies. An equilibrium is responsive if there exist bidder message profiles $\mathbf{m}$ and $\mathbf{m}' \neq \mathbf{m}$ on equilibrium path such that $h(\tilde{d}_t(\mathbf{m})) \neq h(\tilde{d}_t(\mathbf{m}'))$ for some t, where $\tilde{d}_t \in \{0, 1\}$ is the possibly randomized strategy of the seller to put the auction up for sale or not, and $h(\cdot)$ is its p.d.f.

While the presence of mixed seller strategies does not affect pure-strategy equilibria at $t = 0$ and $t = 1$ and does not add new equilibria at $t = 1$ (the only equilibrium remains non-responsive), it can add new responsive equilibria at $t = 1$. The following modification of Proposition 4 characterizes all possible mixed equilibria of the extended model:

Proposition 4a (mixed equilibria in the initiation game at $t = 0$). *In addition to the* *pure equilibria described in Proposition 4, the set of possible mixed equilibria is as follows:*

1. *A responsive equilibrium, in which each bidder sends an indication of interest if and only if* $s_i \geq \hat{s}_0$ *for some* $\hat{s}_0 \in (0,1)$*, and the seller holds the auction with certainty if* $\max_i \mathbf{m}_{i,0} = 1$ *and with probability* $\mu_0 \geq \nu$ *if* $\mathbf{m}_0 = (0,0)$*:*

$$h\left(\tilde{d}_0\left(\mathbf{m}_0\right) = 1\right) = \begin{cases} 1, & \text{if } \mathbf{m}_0 \neq (0,0)\,, \\ \mu_0, & \text{if } \mathbf{m}_0 = (0,0)\,. \end{cases} \tag{94}$$

   The cut-off signal $\hat{s}_0$ *satisfies:*

$$\Pi_I^*\left(\hat{s}_0, \hat{s}_0, 1\right) = \left(\frac{1}{\hat{s}_0} - 1\right)\Pi_N^*\left(\hat{s}_0, \hat{s}_0, 1\right) + \mu_0 \Pi_S^*\left(\hat{s}_0, \hat{s}_0\right) + \left(1 - \mu_0\right)\beta\Pi_1\left(\hat{s}_0\right). \tag{95}$$

   This equilibrium exists if and only if the solution to (95) satisfies

$$R_S\left(\hat{s}_0\right) = \beta\left(\left(1-\lambda\right)R_S\left(\hat{s}_0\right) + \lambda R_S\left(1\right)\right) \leq R_B\left(\hat{s}_0, 1\right), \tag{96}$$

   and inequalities similar to (57) and (58) in the appendix, in which exogenous ν *is substituted with strategic* μ*.*

2. *A responsive equilibrium, in which each bidder sends an indication of interest if and only if* $s_i \geq \hat{s}_0$ *for some* $\hat{s}_0 \in (0,1)$*, and the seller holds the auction with certainty if* $\mathbf{m}_0 = (1,1)$*, with probability* $\mu_0 \geq \nu$ *if* $\mathbf{m}_0 = (0,1)$ *or* $(1,0)$*, and with exogenous probability* ν *if* $\mathbf{m}_0 = (0,0)$*:*

$$h\left(\tilde{d}_0\left(\mathbf{m}_0\right) = 1\right) = \begin{cases} 1, & \text{if } \mathbf{m}_0 = (1,1)\,, \\ \mu_0, & \text{if } \mathbf{m}_0 = (1,0) \text{ or } (0,1), \\ 0, & \text{if } \mathbf{m}_0 = (0,0)\,. \end{cases} \tag{97}$$

   The cut-off signal $\hat{s}_0$ *satisfies:*

$$\mu_0\Pi_I^*\left(\hat{s}_0, \hat{s}_0, 1\right) + (1-\mu_0)\beta\Pi_{I,1}(\hat{s}_0) = \mu_0\left(\tfrac{1}{\hat{s}_0} - 1\right)\Pi_N^*\left(\hat{s}_0, \hat{s}_0, 1\right) + (1-\mu_0)\beta\left(\tfrac{1}{\hat{s}_0} - 1\right)\Pi_{N,1}(\hat{s}) + \nu\Pi_S^*\left(\hat{s}_0, \hat{s}_0\right) + \left(1-\nu\right)\beta\Pi_{S,1}\left(\hat{s}_0\right), \tag{98}$$

   where $\Pi_{j,1}(\hat{s}_0)$*,* $j \in \{I, N, S\}$ *are continuation values for bidder* i *if* $(m_{i,0}, m_{-i,0}) = (1,0)$*,* $(0,1)$*, and* $(0,0)$ *correspondingly, when the initiation cut-off at* $t=0$ *is* $\hat{s}_0$*. This equilibrium exists if and only if the solution to (952) satisfies*

$$R_S\left(\hat{s}_0\right) < \beta\left(\left(1-\lambda\right)R_B\left(\hat{s}_0\right) + \lambda R_S\left(1\right)\right) \leq R_B\left(\hat{s}_0, 1\right), \tag{99}$$

   and modified inequalities (57) and (58) in the appendix.

The proof and intuition for the existence of the first of the two mixed responsive equilibria is effectively the same as the proof of Proposition 4. Interestingly, this mixed equilibrium exists only if the pure responsive equilibrium of Proposition 4 exists. The proof for the existence of the second of the two mixed responsive equilibria is different in details but fundamentally still follows the lines of the proof of Proposition 4. For reasonable parameters, this mixed equilibrium typically does not exist: In order to delay the auction with some likelihood despite interest from one of the bidders, the benefit from a signal reset at $t = 1$ to the seller must be large. This only occurs when the initiation cut-off $\hat{s}_0$ is low. However, at a low hypothetical equilibrium cut-off, the bidder with the cut-off signal finds it not worthwhile to indicate her interest for most parameters.

For the case of $\alpha = 1$, $v(s) = s$, $\nu = 0$, $\beta = 0.9$, and $\lambda = 0.9$, Figure 3 illustrates the bidders' and seller's best responses, and two responsive equilibria, in pure and mixed seller strategies at $t = 0$: $(\hat{s}_0, \mu_0) = (0.83, 0)$ and $(0.89, 0.09)$ correspondingly. Additionally, there is a non-responsive equilibrium $(\hat{s}_0, \mu_0) = (1, 1)$, in which the seller puts the asset up for sale immediately. The responsive equilibrium, in which the seller delays the auction with some likelihood despite interest from one of the bidders, does not exist.

Internet appendix

D For online publication: Extension for costly participation

The base model assumes that upon initiation of the auction, both bidders enter it. If participating in the auction is costly, bidders with sufficiently low signals may prefer to avoid entering the auction. In general, analyzing the effects of participation costs properly is a difficult problem, because the implications can be specific to the assumptions of the setting, in particular to whether the seller can reimburse the participation costs of the bidders[35] and whether the entry of other bidders is observable. In this section, we analyze one specific extension of the model: pure common values $\alpha = \frac{1}{2}$, no reimbursement of participation costs, and observable entry.

Suppose that a bidder has to pay a cost $\Psi > 0$ to participate in the auction. A positive participation cost implies possibility of a single-bidder auction. This leads to the question of how the transaction price is determined in this case, and whether it is different upon the seller receiving a liquidity shock, at which point it has to involuntarily sell the asset.[36] It is natural to assume that the transaction price equals to the seller's reservation value and that this reservation value declines when he is hit by the shock. Let the seller's reservation value in these two cases be $v_s > 0$ and 0. We keep the same assumptions for the bidder's signals and values but shift the value by v_s, so that $v(0,0) = v_s$. The next proposition shows the main result of the extension:

Proposition 11 (equilibrium with costly participation). *Suppose that* $\Psi < \mathbb{E}\left[\frac{1}{2}v(1) + \frac{1}{2}v(x)\right] - v_s$. *Then, the only equilibrium has no bidder initiation and immediate initiation by the seller:* $\hat{s} = 1$ *and* $\mu = \infty$. *Each bidder enters this seller-initiated auction if and only if her signal exceeds cut-off* $\hat{s}_S$, *defined as:*

$$\int_0^{\hat{s}_S} \left(\frac{1}{2}v(\hat{s}_S) + \frac{1}{2}v(x) - v_s\right) dx = \Psi. \tag{100}$$

The restriction on Ψ simply means that the participation cost does not exceed the surplus from the auction when one bidder's signal is the highest possible. The intuition behind this result is as follows. Because of positive participation costs, a bidder-initiated auction results in the rival bidder not entering the auction by the unraveling argument similar to that in the base model. Because of low entry, a bidder-initiated auction results in low seller's revenues, and thus from the seller's point of view there is no benefit from waiting until he gets approached by the bidder. Thus, in equilibrium the auction is initiated immediately by the seller.

E For online publication: Extension for shareholder activism

We extend our base model by adding an activist investor, and show how it can be beneficial in facilitating auctions. Consider the case of pure common values, $\alpha = \frac{1}{2}$, for which the lack of bidder (and seller, for an entrenched target) initiation is most stark. Suppose that there is a different

[35]Because participation has a positive externality on the seller, the seller benefits from subsidizing participation. Compensation of due diligence costs is quite common in M&A transactions - see Wang (2018) for examples.

[36]These questions were irrelevant in the base model, because both bidders always participated and the seller's reservation value was $0 \leq \alpha v(s_1) + (1-\alpha)v(s_2)\ \forall s_1, s_2$.

activist each period that arrives to the market (e.g., by discovering the company) with probability $\lambda_A > 0$. After it arrives, the activist can submit an order to buy fraction γ of the target asset. In addition, there is a different liquidity trader each period that arrives with probability $\lambda_L > 0$ and submits an order to buy fraction γ of the target asset.[37] If an activist buys fraction γ, it can undertake an activism campaign at cost $A > 0$, which results in putting the target asset up for sale. A competitive market maker prices the purchase at its expected value conditional on receiving the orders. In this setup, the activist cannot create value outside of the campaign, so it can only be optimal to buy fraction γ to subsequently undertake the campaign. To save on notation, suppose that upon discovering the target, an activist buys fraction γ with probability 1.

In each period, there are three scenarios: first, there are no orders to to buy the target's shares; second, there is one order (from either a liquidity trader or an activist); third, there are two orders (from the liquidity trader and activist). From the market maker's perspective, these scenarios happen with probabilities

$$(1-\lambda_A)(1-\lambda_L); \qquad \lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L; \qquad \lambda_A\lambda_L, \tag{101}$$

and market prices of the target's shares are

$$\begin{aligned} P(0) &= V_S(1); \\ P(1) &= \frac{\lambda_A(1-\lambda_L)}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}R_S(1)+\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}V_S(1); \\ P(2) &= R_S(1), \end{aligned} \tag{102}$$

where the continuation value of the seller's shareholders (which does not consider managerial entrenchment) is $V_S(\hat{s}) \in \{0, \beta\lambda_A R_S(1)\}$ in the initial period (the value is zero if the final-period activist is not expected to buy fraction γ of the target asset, and positive otherwise) and $V_S(\hat{s}) = 0$ in the final period.

Unlike the market maker, the activist does not know if there are any rival orders, and therefore has to assess their likelihood. In particular, it knows that if it purchases fraction γ, $P(0)$ cannot realize, but $P(1)$ and $P(2)$ are likely. The price the activist expects from its purchase is

$$EP = (1-\lambda_L)P(1)+\lambda_L R_S(1). \tag{103}$$

In the initial period, the expected price is
$EP \in \left\{R_S(1)\left(1-(1-\lambda_L)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}\right), R_S(1)\left(1-(1-\lambda_L)(1-\beta\lambda_A)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}\right)\right\}$
(the price is lower if the final-period activist is not expected to buy fraction γ of the target asset). In the final period, the price is $EP = R_S(1)\left(1-(1-\lambda_L)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}\right)$. The activist's payoff from undertaking a campaign, net of cost of acquiring fraction γ of the target asset, is $\Pi_A = \gamma R_S(1) - \gamma EP - A$. In the initial period, the payoff is
$\Pi_A \in \left\{\gamma R_S(1)(1-\lambda_L)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L} - A, \gamma R_S(1)(1-\lambda_L)(1-\beta\lambda_A)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L} - A\right\}$
(the payoff is higher if the final-period activist is not expected to buy fraction γ of the target asset). In the final period, the payoff is $\Pi_A = \gamma R_S(1)(1-\lambda_L)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L} - A$.

[37] Here, γ is an exogenous parameter but it can be endogenized in a richer model.

By comparing activist's payoffs across periods, note that (1) if the market maker expects the arrived initial-period activist to buy fraction γ of the asset, it must expect the arrived final-period activist to buy fraction γ; (2) in turn, the willing initial-period activist's expected price to buy fraction γ embeds a positive continuation value of the target's shareholders and therefore is always higher than the willing final-period activist's expected price; (3) finally, the threshold on the cost of a valuable campaign for the initial-period activist is always lower than that for the final-period activist. The next proposition immediately follows:

Proposition 12. *In equilibrium, upon discovering the target in the final period, the activist buys γ of the target asset's shares if $A \leq A_T \equiv \gamma R_S(1)(1-\lambda_L)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}$; upon discovering the target in the initial period, the activist buys γ of the target asset's shares if $A \leq A_1 \equiv \gamma R_S(1)(1-\lambda_L)(1-\beta\lambda_A)\frac{(1-\lambda_A)\lambda_L}{\lambda_A(1-\lambda_L)+(1-\lambda_A)\lambda_L}$. For $A \in [0, A_1]$, the seller-initiated auction in the presence of the activist occurs in the initial period. For $A \in (A_1, A_T]$, the seller-initiated auction in the presence of the activist occurs in the final period. For $A \geq A_T$, the company is never sold.*

F For online publication: Extension for toeholds

Consider the case of pure common values, $\alpha = \frac{1}{2}$, for which the lack of bidder initiation is most stark. The analysis can be extended to other values of α. Suppose that a bidder who considers approaching the seller can submit an order to acquire fraction γ of the target asset (e.g., the company's outstanding shares) without disclosing her intent. In addition, there is a different liquidity trader each period that arrives with probability λ_L and submits an order to buy fraction γ of the target asset. A competitive market maker prices the purchase at its expected value conditional on receiving the orders. This setup captures, in reduced form, the U.S. practice that a bidder may acquire up to 5% of a company's outstanding shares secretely, beyond which she is required to publicly file Schedule 13(D). For simplicity, assume that the holding cost of the target's share is sufficiently high, such that a bidder never acquires a toehold unless she approaches the target immediately after. To further save on notation, suppose that a bidder that considers approaching the target acquires fraction γ with probability 1.

For reasonably small γ encountered in practice, the ability to purchase a toehold does not change the outcome of the final period, as it remains beneficial for bidders to let the target initiate the auction.[38] Consider the initial period and four scenarios: first, there are no orders to buy the target's shares; second, there is one order (from either a liquidity trader or one of the bidders); third, there are two orders (from either one of the bidders and the liquidity trader or both bidders); fourth, there are three orders (from the liquidity trader and both bidders). From the market maker's perspective, these scenarios happen with probabilities

$$\begin{aligned} &\hat{s}^2(1-\lambda_L); && \hat{s}^2\lambda_L + 2(1-\hat{s})\hat{s}(1-\lambda_L); \\ &2(1-\hat{s})\hat{s}\lambda_L + (1-\hat{s})^2(1-\lambda_L); && (1-\hat{s})^2\lambda_L, \end{aligned} \tag{104}$$

[38]The case of large γ and a theoretical possibility of bidder-initiated auctions in both periods introduces additional notational complexity and is omitted for brevity. Additionally, in our numerical analysis of the case $v(s) = s$, we were unable to find parameters, which result in bidder-initiated auctions in the final period even when γ is close to 1, as the bidders still prefer to let the target initiate the auction.

and market prices of the target's shares are

$$\begin{aligned}
P(0) &= \mu R_S(\hat{s}) + (1-\mu)V_S(\hat{s}); \\
P(1) &= \frac{\hat{s}^2\lambda_L}{\hat{s}^2\lambda_L + 2(1-\hat{s})\hat{s}(1-\lambda_L)}\left(\mu R_S(\hat{s}) + (1-\mu)V_S(\hat{s})\right) + \frac{2(1-\hat{s})\hat{s}(1-\lambda_L)}{\hat{s}^2\lambda_L + 2(1-\hat{s})\hat{s}(1-\lambda_L)} R_B(\hat{s}); \\
P(2) &= \frac{2(1-\hat{s})\hat{s}\lambda_L}{2(1-\hat{s})\hat{s}\lambda_L + (1-\hat{s})^2(1-\lambda_L)} R_B(\hat{s}) + \frac{(1-\hat{s})^2(1-\lambda_L)}{2(1-\hat{s})\hat{s}\lambda_L + (1-\hat{s})^2(1-\lambda_L)} R_D(\hat{s}); \\
P(3) &= R_D(\hat{s}), \qquad (105)
\end{aligned}$$

where the continuation value of the seller is $V_S(\hat{s}) = \beta\left((1-\lambda)R_{S,0}(\hat{s}) + \lambda R_{S,0}(1)\right)$.

Unlike the market maker, the initiating bidder knows her signal. However, she does not know if there are any rival orders, and therefore has to assess their likelihood. In particular, she knows that if she purchases a toehold, $P(0)$ cannot realize, but $P(1)$, $P(2)$, and $P(3)$ are likely. The price the initiating bidder expects upon her purchase is:

$$EP = \hat{s}(1-\lambda_L)P(1) + \left(\hat{s}\lambda_L + (1-\hat{s})(1-\lambda_L)\right)P(2) + (1-\hat{s})\lambda_L P(3). \qquad (106)$$

The indifference equation for the bidder with signal $\hat{s}$ in the initial period is similar to (19):

$$\hat{s}\left(\Pi_I^*(\hat{s},\hat{s}) - \gamma EP\right) = (1-\hat{s})\Pi_N^*(\hat{s},\hat{s}) + \hat{s}\mu\Pi_S^*(\hat{s},\hat{s}) + \hat{s}(1-\mu)\left((1-\lambda)\Pi_S^*(\hat{s},\hat{s}) + \lambda\Pi_R\right). \qquad (107)$$

Note that, as before, this indifference condition does not include the payoff of the bidder with signal $\hat{s}$ in case both bidders acquire symmetric toeholds and initiate: because the dual-initiated auction is symmetric, this payoff is zero. The indifference condition for the seller also remains unchanged, because seller-initiated auctions are unaffected by the toehold:

$$R_S(\hat{s}) = \beta\left((1-\lambda)R_S(\hat{s}) + \lambda R_S(1)\right). \qquad (108)$$

Together, the two indifference conditions determine equilibria. We are left with deriving the equilibrium payoffs in the bidder-initiated auction in the presence of toeholds.

Consider a bidder-initiated auction when initiating bidder(s) have toehold γ. The expected payoffs of bidders with signal s and bid b are

$$\Pi_j(b,s,\hat{s}) = \int_{\underline{s}_k}^{\phi_k(b,\hat{s})} \left(v(s,x) - (1-\gamma_j)b\right)\frac{1}{\bar{s}_k - \underline{s}_k}dx + \int_{\phi_k(b,\hat{s})}^{\bar{s}_k} \gamma_j a_k(x,\hat{s})\frac{1}{\bar{s}_k - \underline{s}_k}dx, \qquad (109)$$

where $j,k \in \{I,N\}$ (three combinations of initiating and non-initiating bidders are possible), $(\gamma_I,\gamma_N) = (\gamma,0)$, and the common value is denoted by $v(s,x) = \alpha v(s) + (1-\alpha)v(x)$. For intuition, consider bidder I. The first term in (109) captures the payoff of bidder I upon winning. In this case, bidder I acquires the remaining $1-\gamma$ shares at bid b, and obtains asset value $v(s,x)$. The second term in (109) captures the payoff of bidder I upon losing. In this case, bidder I sells fraction γ to the rival at her bid $a_k(s,\hat{s})$. In contrast, bidder N only obtains the payoff if she wins. Taking the first-order conditions of (109), we obtain

$$\frac{\partial\phi_j(b,\hat{s})}{\partial b}\left(v(s,\phi_j(b,\hat{s})) - (1-\gamma_k)b - \gamma_k a_j(\phi_j(b,\hat{s}),\hat{s})\right) - (1-\gamma_k)\left(\phi_j(b,\hat{s}) - \underline{s}_j\right) = 0 \qquad (110)$$

for $j,k \in \{I,N\}$. Compared to (2) in the model without toeholds, (110) includes an additional term. In equilibrium, $b = a_j(s,\hat{s})$ must satisfy (110) for $j \in \{I,N\}$, implying $s = \phi_j(b,\hat{s})$. Also, note that $a_j(\phi_j(b,\hat{s}),\hat{s}) = b$. Plugging in and rearranging the terms, we obtain

$$\frac{\partial \phi_j(b,\hat{s})}{\partial b} = \frac{(1-\gamma_k)(\phi_j(b,\hat{s}) - \underline{s}_j)}{v(\phi_k(b,\hat{s}),\phi_j(b,\hat{s})) - b}. \tag{111}$$

When $j = k = I$ (both bidders are initiating), the auction is symmetric, because both bidders' signals are above $\hat{s}$ and their toeholds are γ. The system of equations (111) is then equivalent to the system (25) for intermediate auction formats with $1-\gamma = m$. This, in turn, implies revenue equivalence of the dual-initiated auction with toeholds to the dual-initiated auction without toeholds. When $j \neq k \in \{I,N\}$, the system of equations (111) is solved subject to boundary conditions $a_j(\bar{s}_j,\hat{s}) \equiv \bar{a}(\hat{s})$ and $a_j(\underline{s}_j,\hat{s}) \equiv \underline{a}(\hat{s})$, so that the support of possible equilibrium bids for both bidders is $[\underline{a}(\hat{s}),\bar{a}(\hat{s})]$. In turn, the equilibrium inverse bidding functions solve (111) subject to boundary conditions $1 = \phi_I(\bar{a}(\hat{s}),\hat{s})$, $\hat{s} = \phi_N(\bar{a}(\hat{s}),\hat{s})$, $\hat{s} = \phi_I(v(\hat{s},0),\hat{s})$, $0 = \phi_N(v(\hat{s},0),\hat{s})$.

The payoff of bidder I with signal $\hat{s}$ is $\Pi_I^*(\hat{s},\hat{s}) = \Pi_I(\underline{a}(\hat{s}),\hat{s},\hat{s}) = \gamma\int_0^{\hat{s}} a_N(x,\hat{s})\frac{1}{\hat{s}}dx$. Payoffs of bidder N and bidders in the seller-initiated auction, $\Pi_N^*(\hat{s},\hat{s})$ and $\Pi_S^*(\hat{s},\hat{s})$, are the same as in the main model. The necessary condition to acquire a toehold and initiate the auction is that $\Pi_I^*(\hat{s},\hat{s}) - \gamma EP$ is positive. This condition is never satisfied if $\hat{s}$ is low enough and always satisfied if $\hat{s}$ is high enough. Intuitively, if too many types are expected to initiate, the price of a toehold is high, so it is too expensive for the bidder with signal $\hat{s}$. Thus, factors that increase the price of a toehold also limit bidder initiation.

The following example provides a quasi-closed form solution for the case $v(s) = s$. Even though the presence of a toehold improves bidders' incentives to initiate the auction, for realistically small γ we were unable to find a set of parameters with equilibrium bidder initiation (for this to occur, γ needs to be well in excess of 0.5).

Example 4: the common-value framework with toeholds. Suppose that $\alpha = \frac{1}{2}$ and $v(s) = s$. When $j \neq k \in \{I,N\}$, the general solution to the system of differential equations (111) is given by

$$\phi_I(b,\hat{s}) = \hat{s} + c_1\left((1-\gamma)\phi_N(b,\hat{s})\right)^{\frac{1}{1-\gamma}}, \tag{112}$$

$$b = \frac{\phi_N(b,\hat{s})}{2(2-\gamma)} + c_2\phi_N(b,\hat{s})^{-\frac{1}{1-\gamma}} + \frac{\hat{s}}{2} + \frac{c_1}{4}\left((1-\gamma)\phi_N(b,\hat{s})\right)^{\frac{1}{1-\gamma}}. \tag{113}$$

First, use boundary conditions $\hat{s} = \phi_I(v(\hat{s},0),\hat{s})$ and $0 = \phi_N(v(\hat{s},0),\hat{s})$ to show that (113) can only be satisfied with equality if $c_2 = 0$. Second, plug in $1 = \phi_I(\bar{a}(\hat{s}),\hat{s})$ and $\hat{s} = \phi_N(\bar{a}(\hat{s}),\hat{s})$ in (112) to obtain $c_1 = \frac{1-\hat{s}}{((1-\gamma)\hat{s})^{\frac{1}{1-\gamma}}}$.

Plugging $\hat{s} = \phi_N(\bar{a}(\hat{s}),\hat{s})$ and expressions for c_1 and c_2 into (113), we obtain $\bar{a}(\hat{s})$:

$$\bar{a}(\hat{s}) = \frac{1+\hat{s}}{4} + \frac{\hat{s}}{2(2-\gamma)}. \tag{114}$$

The upper boundary on bids is increasing in γ, consistent with the intuition that toeholds result in a more aggressive bidding. Additionally, (113) becomes

$$b = \frac{\phi_N(b,\hat{s})}{2(2-\gamma)} + \frac{\hat{s}}{2} + \frac{1-\hat{s}}{4}\left(\frac{\phi_N(b,\hat{s})}{\hat{s}}\right)^{\frac{1}{1-\gamma}}. \tag{115}$$

Equivalently, the equilibrium bid of bidder N with signal s is

$$a_N(s,\hat{s}) = \frac{s}{2(2-\gamma)} + \frac{\hat{s}}{2} + \frac{1-\hat{s}}{4}\left(\frac{s}{\hat{s}}\right)^{\frac{1}{1-\gamma}}, \tag{116}$$

so that $\underline{a}(\hat{s}) = \frac{\hat{s}}{2}$. Plugging the expression for c_1 into (112), we obtain

$$\phi_I(b,\hat{s}) = \hat{s} + (1-\hat{s})\left(\frac{\phi_N(b,\hat{s})}{\hat{s}}\right)^{\frac{1}{1-\gamma}}. \tag{117}$$

Substituting the non-linear term in (117) using (115), we obtain

$$\phi_I(b,\hat{s}) = 4b - \hat{s} - \frac{2\phi_N(b,\hat{s})}{2-\gamma}, \tag{118}$$

Equivalently, the equilibrium bid of bidder I with signal s is

$$a_I(s,\hat{s}) = \frac{s+\hat{s}}{4} + \frac{\phi_N(a_I(s,\hat{s}),\hat{s})}{2(2-\gamma)}, \tag{119}$$

where $\phi_N(a_I(s,\hat{s}),\hat{s})$ is the signal of bidder N corresponding to bid $a_I(s,\hat{s})$ made by bidder I with signal s. For the case $\gamma = 0$ (no toeholds), $\phi_N(a_I(s,\hat{s}),\hat{s}) = 4\hat{s}a_I(s,\hat{s}) - 2\hat{s}^2$, and we have the same solution as in Example 2. For the case $\gamma > 0$, a numerical inversion is needed.

To solve for $\hat{s}$ in the indifference condition (107), we compute the equilibrium payoff from the auction of bidder I with signal $\hat{s}$:

$$\Pi_I^*(\hat{s},\hat{s}) = \gamma\int_0^{\hat{s}} a_N(s,\hat{s})\,\frac{1}{\hat{s}}ds = \gamma\left(\frac{1-\gamma(1-\hat{s})}{4(2-\gamma)} + \frac{\hat{s}}{2}\right). \tag{120}$$

Equilibrium payoffs of bidder N and both bidders in a seller-initiated auction with signals $\hat{s}$ are

$$\Pi_N^*(\hat{s},\hat{s}) \;=\; \int_{\hat{s}}^1\left(\frac{\hat{s}+x}{2} - \overline{a}(\hat{s})\right)\frac{1}{1-\hat{s}}dx = \frac{\hat{s}}{2}\frac{1-\gamma}{2-\gamma}; \tag{121}$$

$$\Pi_S^*(\hat{s},\hat{s}) = \int_0^{\hat{s}}\left(\frac{\hat{s}+x}{2} - a_S(\hat{s})\right)\frac{1}{\hat{s}}dx = \frac{\hat{s}}{4}. \tag{122}$$

Finally, the final-period expected payoffs of both bidders in a seller-initiated auction when their signals reset are

$$\Pi_R = \frac{1}{12}. \tag{123}$$

Next, we need to calculate the expected price (106) at which the initiating bidder can buy fraction

γ of the asset for sale. This price depends on seller revenues $R_S(\hat{s})$, $R_B(\hat{s})$, $R_D(\hat{s})$, and $R_{S,0}(\hat{s})$ (the revenue in the final period). The first, third, and fourth values are

$$R_S(\hat{s}) = \frac{\hat{s}}{3}, \qquad R_D(\hat{s}) = \hat{s} + \frac{1-\hat{s}}{3}; \qquad R_{S,0}(\hat{s}) = \frac{\hat{s}}{3}. \tag{124}$$

For $R_B(\hat{s})$, let $H_B(b, \hat{s})$ denote the c.d.f. of the winning bid b in the bidder-initiated auction:

$$\begin{aligned} H_B(b,\hat{s}) &= \Pr\left(a_I(s_1,\hat{s}) \le b, a_N(s_2,\hat{s}) \le b\right) = \Pr\left(s_1 \le \phi_I(b,\hat{s}), s_2 \le \phi_N(b,\hat{s})\right) \\ &= \frac{\phi_I(b,\hat{s}) - \hat{s}}{1-\hat{s}} \frac{\phi_N(b,\hat{s})}{\hat{s}} = \frac{4b - 2\hat{s} - \frac{2\phi_N(b,\hat{s})}{2-\gamma}}{1-\hat{s}} \frac{\phi_N(b,\hat{s})}{\hat{s}}, \end{aligned} \tag{125}$$

so that the corresponding p.d.f. is

$$\begin{aligned} h_B(b,\hat{s}) &= \frac{4 - \frac{2}{2-\gamma}\phi'_{N,1}(b,\hat{s})}{1-\hat{s}} \frac{\phi_N(b,\hat{s})}{\hat{s}} + \frac{4b - 2\hat{s} - \frac{2}{2-\gamma}\phi_N(b,\hat{s})}{1-\hat{s}} \frac{\phi'_{N,1}(b,\hat{s})}{\hat{s}} \\ &= \frac{4\phi_N(b,\hat{s}) + 4b\phi'_{N,1}(b,\hat{s}) - 2\hat{s}\phi'_{N,1}(b,\hat{s}) - \frac{4}{2-\gamma}\phi_N(b,\hat{s})\phi'_{N,1}(b,\hat{s})}{\hat{s}(1-\hat{s})}. \end{aligned} \tag{126}$$

Here, $\phi_N(b, \hat{s})$ is the numerical solution of (115). Applying the implicit function theorem to (115),

$$\phi'_{N,1}(b,\hat{s}) = \frac{1}{\frac{1}{2(2-\gamma)} + \frac{1-\hat{s}}{4}\frac{1}{1-\gamma}\phi_N(b,\hat{s})^{\frac{1}{1-\gamma}-1}\hat{s}^{-\frac{1}{1-\gamma}}}. \tag{127}$$

P.d.f. $h_B(b, \hat{s})$ can be calculated numerically. $R_B(\hat{s})$ is then

$$R_B(\hat{s}) = \int_{\underline{a}(\hat{s})}^{\bar{a}(\hat{s})} b h_b(b,\hat{s}) db = \int_{\frac{\hat{s}}{2}}^{\frac{1+\hat{s}}{4} + \frac{\hat{s}}{2(2-\gamma)}} b h_b(b,\hat{s}) db. \tag{128}$$

We now have all the ingredients to compute the expected price (106) of the toehold and the indifference condition for the seller (108).

The standard parameters used throughout the paper ($\nu = 0$, $\beta = 0.9$, and $\lambda = 0.9$) does not produce toeholds or responsive equilibria with bidder initiation in the pure common value case for any γ. In fact, the set of parameters, for which these outcomes appear in equilibrium, is rather small. For example, when $\nu = 0$, $\beta = 0.975$, $\lambda = 0.95$, $\lambda_L = 0.25$, and $\gamma = 0.75$, the responsive equilibrium with toeholds and bidder initiation in the initial period has $\hat{s}_0 = 0.952$. As always, there is also the non-responsive equilibrium with seller initiation only. The final period does not feature bidder initiation.

G For online publication: Example for the model with a general number of periods

Example 5: initiation stage, the pure private value environment, $T+1$-period model. Suppose that $\alpha = 1$ and $v(s) = s$. Further, suppose that the highest signal the bidders can possibly possess in a given period is $\bar{s}$, and that $\hat{s}$ is the period's cut-off signal. Similar to Example 1 in Appendix B, the solution to the system of differential equations (3) subject to boundary conditions

(5) for general $\bar{s}$ is

$$\begin{aligned}
\phi_I(b) &= \hat{s} + \frac{\hat{s}^2}{(\hat{s}-2b)c_I e^{-\frac{\hat{s}}{\hat{s}-2b}} - 4b}, \quad \phi_N(b) = \frac{\hat{s}^2}{(\hat{s}-2b)c_N e^{\frac{\hat{s}}{\hat{s}-2b}} + 4(\hat{s}-b)}; \qquad (129)\\
c_I &= \frac{\frac{\hat{s}^2}{\bar{s}-\hat{s}} + 4\bar{a}}{-2(\bar{a}-\underline{a})} e^{-\frac{\hat{s}}{2(\bar{a}-\underline{a})}}, \quad c_N = \frac{\hat{s}+4(\bar{a}-\hat{s})}{-2(\bar{a}-\underline{a})} e^{\frac{\hat{s}}{2(\bar{a}-\underline{a})}}; \qquad (130)\\
\underline{a} &= \frac{\hat{s}}{2}, \quad \bar{a} = \hat{s} - \frac{\hat{s}^2}{4\bar{s}}. \qquad (131)
\end{aligned}$$

Further, the bidding strategy in the dual bidder-initiated auction is:

$$a_D(s) = \hat{s} + \frac{s-\hat{s}}{2}. \qquad (132)$$

Then, the expected value of a bidder with signal $\bar{s}$ in period t when signals do not reset, given by (27), is

$$U_B(\bar{s},t) = \frac{\hat{s}}{\bar{s}}\left(\bar{s} - \left(\hat{s} - \frac{\hat{s}^2}{4\bar{s}}\right)\right) + \left(1-\frac{\hat{s}}{\bar{s}}\right)\left(\bar{s} - \left(\hat{s} + \frac{\bar{s}-\hat{s}}{2}\right)\right). \qquad (133)$$

Note that the computation of $\Pi_I^*(\bar{s},\hat{s},\bar{s})$ and $\Pi_D^*(\bar{s},\hat{s},\bar{s})$ in (27) incorporates that the initiating bidder with signal $\bar{s}$ always wins. Next, the expected value of a bidder when both her and her rival's signals reset to $[0,\bar{s}]$ in period t, given by (28), can be obtained in semi-closed form:

$$\begin{aligned}
V_B(\bar{s},t) &= \int_{\hat{s}}^{\bar{s}} \left(\frac{\hat{s}}{\bar{s}} \frac{\phi_N \phi_I^{-1}(s)}{\hat{s}}(s - \phi_I^{-1}(s)) + \left(1-\frac{\hat{s}}{\bar{s}}\right)\frac{s-\hat{s}}{\bar{s}-\hat{s}}\left(s - \left(\hat{s} + \frac{s-\hat{s}}{2}\right)\right)\right)\frac{1}{\bar{s}}ds\\
&+ \int_0^{\hat{s}} \left(\left(1-\frac{\hat{s}}{\bar{s}}\right)\frac{\phi_I\phi_N^{-1}(s)-\hat{s}}{\bar{s}-\hat{s}}(s-\phi_N^{-1}(s)) + \frac{\hat{s}}{\bar{s}}\mu\frac{s}{\hat{s}}\left(s-\frac{s}{2}\right)\right)\frac{1}{\bar{s}}ds\\
&+ \left(\frac{\hat{s}}{\bar{s}}\right)^2 (1-\mu)\,\beta\left((1-\lambda)\,U_B(\hat{s},t+1) + \lambda V_B(1,t+1)\right). \qquad (134)
\end{aligned}$$

The bidder's boundary conditions (29) are

$$U_B(\bar{s},T) = \frac{\bar{s}}{2}, \quad V_B(\bar{s},T) = \frac{1}{6}. \qquad (135)$$

Finally, consider the seller's value function (30). When $v(s)=s$, $R_S^*(\hat{s})$ is equal to $\frac{\hat{s}}{3}$; $R_D^*(\hat{s},\bar{s})$ is equal to $\hat{s} + \frac{\bar{s}-\hat{s}}{3}$; and $R_B^*(\hat{s},\bar{s})$ can only be expressed in a semi-closed form. Lastly, the seller's boundary condition (31) is

$$V_S(\bar{s},T) = \frac{\bar{s}}{3}. \qquad (136)$$

For the case of $\beta = 0.9$ and $\lambda = 0.9$, Figure 4 illustrates the evolution of responsive equilibria in the three-period model. As the terminal period nears, in which the seller always puts the asset up for sale, bidders indicate their interest more aggressively compared to the period before (whether their signals are reset at the beginning of the period or not). As always, there is also the non-responsive equilibrium in which the seller initiates at $t = 0$ and ends the game immediately.

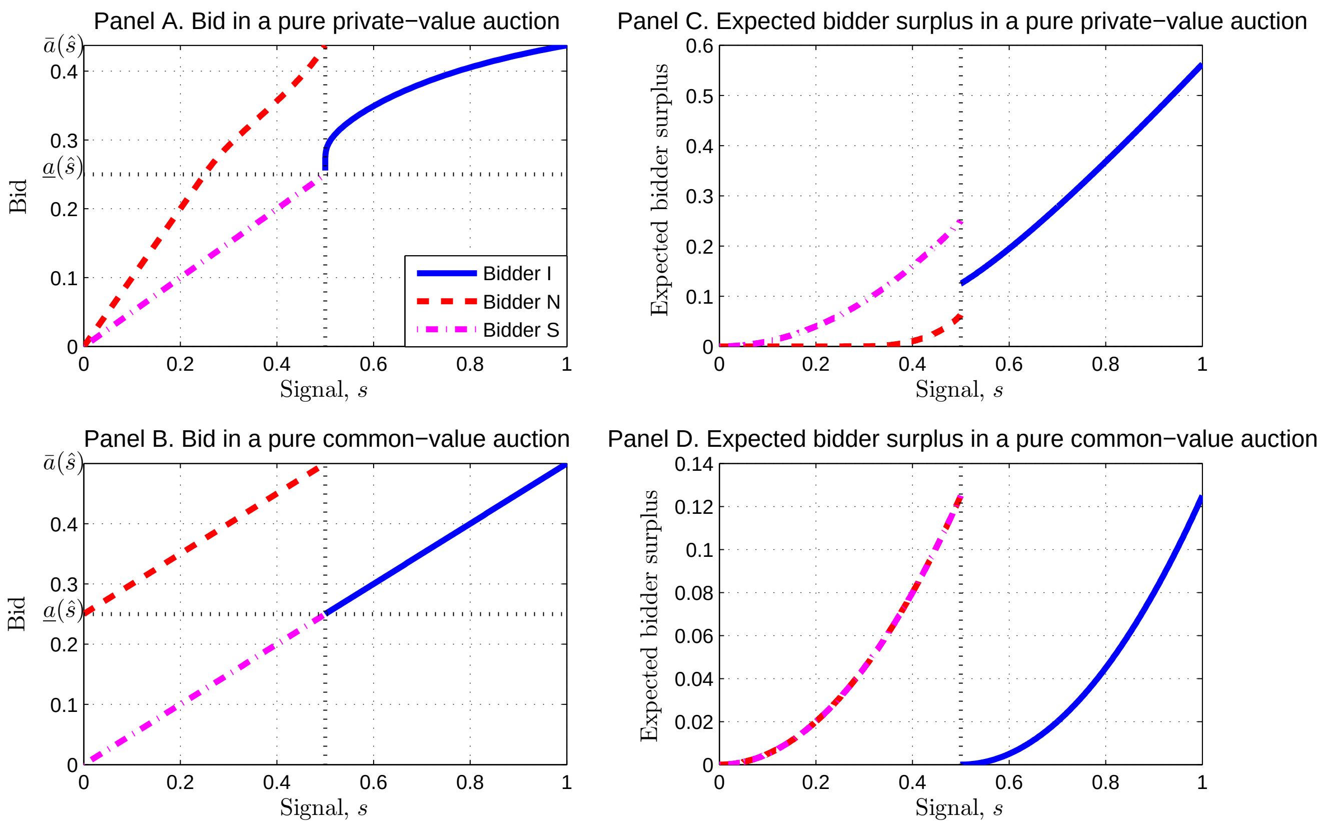

Figure 1: **Equilibrium bids and expected payoffs of bidders.** For pure private values $\alpha = 1$ and pure common values $\alpha = 1/2$, and values $v(s) = s$, Panels A and B plot equilibrium bids as functions of signal s for the initiating bidder (the normal line) and the non-initiating bidder (the dashed line) in a bidder-initiated auction, and either of the two bidders (the dash-dotted line) in a seller-initiated auction. Panels C and D plot the corresponding expected surpluses. The cut-off signal for the initiation, $\hat{s}$, is 0.5. The lowest and highest serious bids are marked as $\underline{a}(\hat{s})$ and $\bar{a}(\hat{s})$.

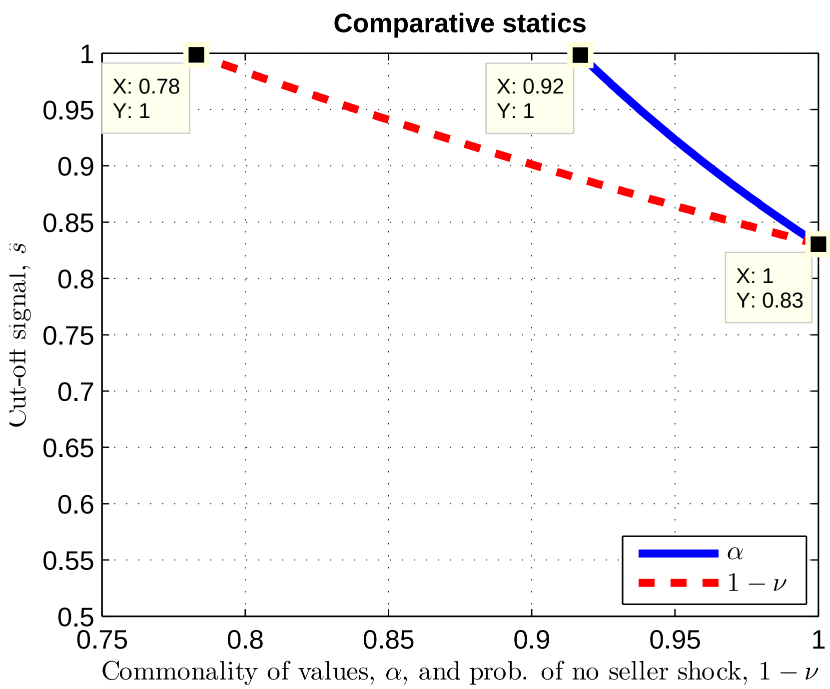

Figure 2: **Comparative statics of the initiation cut-off signal.** For values $v(s) = s$ and parameters $\beta = 0.9$ and $\lambda = 0.9$, the figure plots comparative statics of the period-0 responsive-equilibrium initiation cut-off signal $\hat{s}_0$ with respect to commonality of values α and the probability of no exogenous liquidity shocks, $1 - \nu$. The responsive equilibrium when $\alpha = 1$ and $\nu = 0$ is $\hat{s}_0 = 0.83$. The responsive equilibrium ceases to exist when $\alpha < 0.92$ and $\nu > 0.22$.

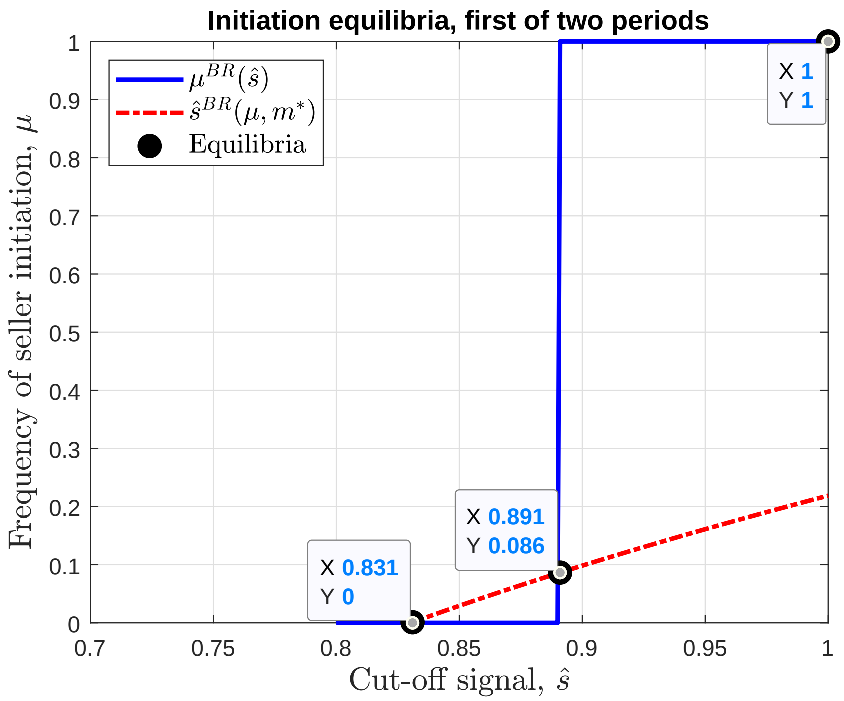

Figure 3: **Equilibria with seller- and bidder-initiated auctions, pure private values.** For pure private values $\alpha = 1$, values $v(s) = s$, no exogenous liquidity shocks $\nu = 0$, and parameters $\beta = 0.9$ and $\lambda = 0.9$, the figure plots period-0 best responses (BR) of the seller, who chooses the probability of putting the asset up for sale without receiving any indications of interest μ (the normal line), and each bidder, who chooses the cut-off signal for initiation $\hat{s}$ (the dashed line), as well as multiple pure and mixed equilibria (circle markers).

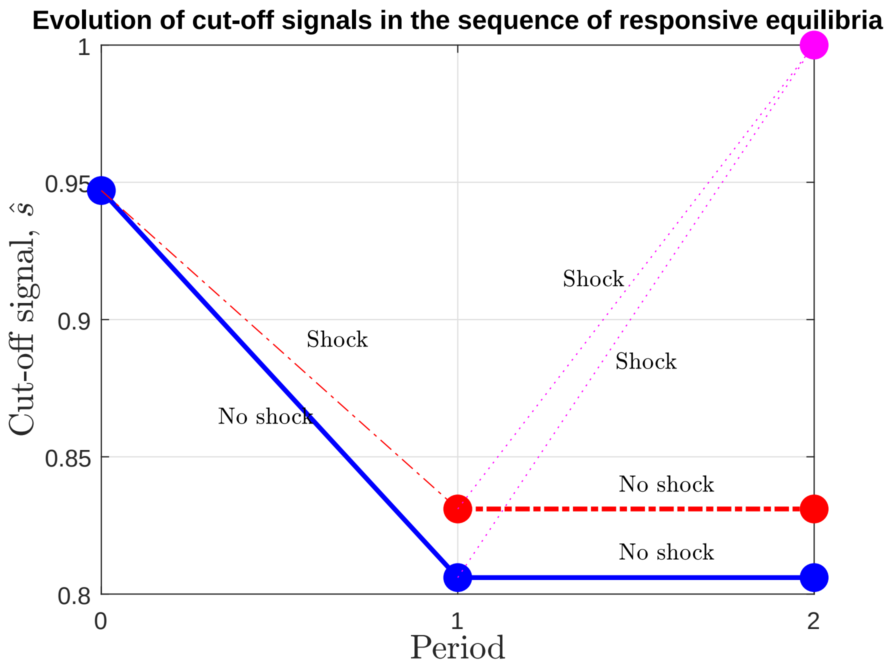

Figure 4: **The evolution of responsive equilibria in the three-period game, pure private values.** For pure private values $\alpha = 1$, values $v(s) = s$, no exogenous liquidity shocks $\nu = 0$, and parameters $\beta = 0.9$ and $\lambda = 0.9$, the figure plots the evolution of initiation cut-off signals $\hat{s}_t$ in the sequence of responsive equilibria of the three-period game. In any period except $t = 0$, bidders' signals can either reset to $[0, 1]$ (with probability λ) or remain bounded by the last-period's cut-off signal $\hat{s}_{t-1}$ (with probability $1 - \lambda$), implying different period-t cut-off signals. These outcomes are marked with "Shock" and "No shock" markers in the figure.