

# Technological Innovation: Winners and Losers\*

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## Abstract

We analyze the effect of innovation on asset prices in a tractable, general equilibrium framework with heterogeneous households and firms. Innovation has a heterogeneous impact on households and firms. Technological improvements embodied in new capital benefit workers, while displacing existing firms and their shareholders. This displacement process is uneven: newer generations of shareholders benefit at the expense of existing cohorts; and firms well positioned to take advantage of these opportunities benefit at the expense of firms unable to do so. Under standard preference parameters, the risk premium associated with innovation is negative. Our model delivers several stylized facts about asset returns, consumption and labor income. We derive and test new predictions of our framework using a direct measure of innovation. The model's predictions are supported by the data.

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# Introduction

The history of technological innovation is a story of displacement. New technologies emerge that render old capital and processes obsolete. Further, these new technologies are typically embodied in new vintages of capital, so the process of adoption is not costless. For instance, the invention of the automobile by Karl Benz in 1885 required investment in new types of capital, such as paved highways and an infrastructure for fuel distribution. Resources therefore needed to be diverted into investment in the short run in order for the economy to benefit in the long run. Not all economic agents benefitted from the automobile. Railroad firms, which in the late 19th century accounted for 50% of the market capitalization of all NYSE-listed firms, were displaced as the primary mode of transport.<sup>1</sup>

We analyze the effect of innovation on the stock market using a general equilibrium model. We model innovation as technological change embodied in new vintages of capital goods.<sup>2</sup> A key feature of innovation is that it leads to benefits and losses that are asymmetrically distributed. Hence we consider an economy where both households and firms vary in their exposure to innovation shocks. This heterogeneous impact differentiates innovation from disembodied technical change – in our case a labor augmenting productivity shock – that affects equally all vintages of capital goods.

Innovation results in reallocation of wealth in the cross-section of households through two channels. First, innovation reduces the value of older vintages of capital. In contrast, labor benefits from innovation since their skill is not tied to a particular technology. Motivated by the well-documented empirical facts on limited stock market participation, we assume that the workers do not participate in financial markets. As a result of this break-down of risk sharing, aggregate

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<sup>1</sup>Flink (1990, p. 360) writes: “The triumph of the private passenger car over rail transportation in the United States was meteoric. Passenger miles traveled by automobile were only 25 percent of rail passenger miles in 1922 but were twice as great as rail passenger miles by 1925, four times as great by 1929.”

<sup>2</sup>We study a particular form of technological innovation, specifically innovation that is embodied in new vintages of intermediate goods. Accordingly, our empirical measure of embodied shocks relies on patent data, since innovation that is embodied in new products is more easily patentable (see, for example, Comin, 2008, for a discussion on patentable innovation). The type of innovation that we study could be related to other forms of technical change, such as skill-biased technical change, but the two need not be positively related. For instance, the first industrial revolution, a technological change embodied in new forms of capital – the factory system – led to the displacement of skilled artisans by unskilled workers, who specialized in a limited number of tasks (see e.g. Sokoloff, 1984, 1986; Atack, 1987; Goldin and Katz, 1998). Further, skill-biased technical change need not be related to firms’ growth opportunities in the same manner as the embodied technical change we consider in this paper. Nevertheless, we use the terms innovation and capital-embodied change interchangeably in this paper.

innovation shocks to lead to wealth reallocation between the owners of capital and workers. Second, intergenerational risk sharing is limited in our model. Households have finite lives; each new cohort of households brings with it embodied technological advances in the form of blueprints. Only part of the rents from innovation are appropriated by existing shareholders. Since households cannot share risks with future generations, periods of significant innovation result in wealth transfer from the existing set of households to the newer generations. Both of these effects imply that innovation leads to the displacement of existing owners of capital and therefore to an increase in the marginal utility of consumption of stock market participants.

Embodied technology shocks have a heterogeneous impact on the cross-section of firms. Since innovation shocks carry a negative price of risk in equilibrium, existing shareholders are willing to hold firms that hedge this displacement despite their lower average returns. Improvements in the frontier level of technology benefit firms able to capture a larger share of rent from the new inventions relative to firms that are heavily invested in older vintages of capital. Hence, firms with high growth opportunities are attractive to investors and thus earn lower equilibrium rates of return, consistent with existing evidence. Further, due to their similar exposure to innovation, stock returns of firms with similar access to growth opportunities comove with each other, above and beyond of what is implied by their exposures to the market returns.

We calibrate our model to match several moments of real economic variables and asset returns, including the mean and volatility of the aggregate consumption growth rate, the equity premium, and the risk-free rate. Observable firm characteristics, such as valuation ratios or past investment rates, are correlated with firms' growth opportunities. This endogenous relation allows the model to replicate the empirical patterns of return comovement among firms with similar market-to-book ratios or investment rates, as well as the cross-sectional relations between such characteristics and firms' average stock returns. Further, our model replicates the failure of the CAPM and the consumption CAPM in pricing the cross-section of stock returns, since neither the market portfolio nor aggregate consumption is a sufficient statistic for the marginal utility of market participants.

We test the direct implications of our mechanism using a novel measure of embodied technology shocks constructed in Kogan, Papanikolaou, Seru, and Stoffman (2012), which infers the value

of innovation from stock market reactions to news about patent grants. The Kogan et al. (2012) measure has a natural interpretation in the context of our model; we construct this measure in simulated data and show that it is a close match to the key state variable in our model that captures the current real investment opportunity set in the economy. Armed with a proxy for the unobservable variables in our model, we concentrate our empirical analysis on the properties of the model directly linked to its main economic mechanism – displacement in the cross-section of households and firms generated by embodied innovation shocks.

Our empirical tests support the model’s predictions regarding the cross-section of households and firms. First, innovation shocks generate displacement in the cross-section of households. The level of technological innovation during the year when household heads enter the economy is associated with higher lifetime consumption; by contrast, innovation shocks following the cohort’s entry tend to lower its consumption level relative to the rest of the economy. Moreover, consistent with our model, higher innovation predicts lower consumption growth of stockholders relative to non-stockholders. Next, we relate the measure of innovation to real firm outcomes in the cross-section. Motivated by our model, we proxy for firm growth opportunities with either their market-to-book ratio or their past investment. We find that firms with low growth opportunities exhibit lower output growth following innovation by their competitors relative to firms with high growth opportunities.

We relate our innovation measure to asset returns. Consistent with our model, we find that firms with high growth opportunities have higher return exposure to embodied shocks than firms with low growth opportunities. Further, we confirm empirically that innovation shocks earn a negative price of risk. We approximate the stochastic discount factor of our model using our innovation series and data on total factor productivity or consumption. We find that our specification for the stochastic discount factor prices a cross-section of portfolios sorted on book-to-market and investment-rate with low pricing errors. The point estimates of the market price of innovation risk are negative and statistically significant. Importantly the empirical estimates of the price of innovation risk are close in magnitude to the estimates implied by the calibrated general equilibrium model.

Our work is related to asset pricing models with production (for a recent review of this literature, see Kogan and Papanikolaou, 2012a). Papers in this literature construct structural theoretical

models with heterogeneous firms and analyze the economic source of cross-sectional differences in firms' systematic risk, with a particular focus on understanding the origins of average return differences among value and growth firms. Most of these models are in partial equilibrium (e.g., Berk, Green, and Naik, 1999; Carlson, Fisher, and Giammarino, 2004; Zhang, 2005; Kogan and Papanikolaou, 2011), with an exogenously specified pricing kernel. Some of these papers develop general equilibrium models (e.g. Gomes, Kogan, and Zhang (2003)), yet most of them feature a single aggregate shock, implying that the market portfolio conditionally spans the value factor. In contrast, our model features two aggregate risk factors, one of them being driven by embodied technology shocks. Using a measure of embodied technical change, we provide direct evidence for the model mechanism rather than relying only on indirect model implications

Our work is related to the voluminous literature on embodied technology shocks (e.g., Cooley, Greenwood, and Yorukoglu, 1997; Greenwood, Hercowitz, and Krusell, 1997; Christiano and Fisher, 2003; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010). Technology is typically assumed to be embodied in new capital goods – new projects in our setting. Several empirical studies document substantial vintage effects in the productivity of plants (see Foster, Haltiwanger, and Krizan, 2001, for a survey of the micro productivity literature). For instance, Jensen, McGuckin, and Stiroh (2001) find that the 1992 cohort of new plants was 50% more productive than the 1967 cohort in its entry year, controlling for industry-wide factors and input differences. Further, our paper is related to work that explores the effect of technological innovation on asset returns (e.g., Greenwood and Jovanovic, Greenwood and Jovanovic; Hobijn and Jovanovic, 2001; Laitner and Stolyarov, 2003; Kung and Schmid, 2011; Garleanu, Panageas, and Yu, 2012). The focus of this literature is on exploring the effects of innovation on the aggregate stock market. We contribute to this literature by explicitly considering the effects heterogeneity in both firms and households in terms of their exposure to embodied technology shocks.

The closest related work is Papanikolaou (2011), Garleanu, Kogan, and Panageas (2012) and Kogan and Papanikolaou (2011, 2012b). Papanikolaou (2011) demonstrates that in a general equilibrium model, capital-embodied technology shocks are positively correlated with the stochastic discount factor when the elasticity of intertemporal substitution is less than or equal to the reciprocal

of risk aversion. However, the price of embodied shocks in his model is too small relative to the data. We generalize the model in Papanikolaou (2011), allowing for both firm and household heterogeneity and imperfect risk sharing among households. Our model delivers quantitatively more plausible estimates of the risk premium associated with innovation, as well as additional testable predictions. Our model shares some of the features in Garleanu et al. (2012), namely intergenerational displacement risk and technological improvements embodied in new types of intermediate goods. We embed these features into a model with capital accumulation, limited market participation, and a richer, more realistic cross-section of firms. In addition, we construct an explicit empirical measure of innovation shocks and use it to directly test the empirical implications of our model's mechanism. Last, our work is related to Kogan and Papanikolaou (2011, 2012b), who analyze the effect of capital-embodied technical progress in partial equilibrium. The general equilibrium model in this paper helps understand the economic mechanism for pricing of such innovation shocks, and provides further insights into how these shocks impact the economy.

Last our model replicates several stylized facts documented in the consumption-based asset pricing literature. First, our model is consistent with the findings of Malloy, Moskowitz, and Vissing-Jorgensen (2009) that the return differential between value and growth firms has a relatively high exposure to the consumption growth of stockholders, especially at lower frequencies. Second, our model is consistent with the evidence in Lustig and Van Nieuwerburgh (2008), Lustig, Van Nieuwerburgh, and Verdelhan (2008), who report that human wealth – the present value of wages discounted using the stochastic discount factor implied by no arbitrage – earns lower risk premia than financial wealth. In our model, embodied innovation shocks raise equilibrium wages while reducing dividends on existing firms, resulting in a low correlation between the growth of dividends and labor income and a lower risk premium for human wealth. Last, our model is consistent with the recently reported empirical evidence on the dynamics of income shares of financial and human capital in Lettau and Ludvigson (2011).

# 1 A simple model

To illustrate the main intuition behind our mechanism, we first present a simple two-period model. The economy consists of overlapping generations of capital owners and workers. Capital owners have logarithmic preferences over consumption  $C_0$  and  $C_1$

$$U(C_0, C_1) = \log C_0 + \log C_1. \quad (1)$$

Workers do not participate in the financial markets. There are two technologies available to produce output,  $k \in \{o, n\}$ , each using old or new capital, respectively.

In the first period, only the old technology is available. Existing capital owners are endowed with a unit of capital  $K_o$  that, along with labor  $L_{o,t}$ , can be used to produce output in each period

$$Y_{o,t} = K_o^\alpha L_{o,t}^{1-\alpha}. \quad (2)$$

For simplicity, we normalize the measure of workers and capital owners to unity in the first period. In the second period, a measure  $\mu$  of new workers and new capital owners enter the economy. The new capital owners own a new capital stock  $K_n$ , which produces output according to

$$Y_{n,1} = (\xi K_n)^\alpha L_{n,1}^{1-\alpha}, \quad (3)$$

where  $\xi \sim F(\xi)$  with  $\xi > 0$  and  $E[\xi] = 1$ . The random variable  $\xi$  is the technology shock embodied in the new vintage of capital. A value of  $\xi > 1$  implies that the new capital is more productive than the old. In contrast, the new workers are identical to the old workers; labor can be freely allocated to either the old or to the new technology.

In equilibrium, the allocation of labor between the old and the new technology depends on the embodied shock  $\xi$ ,

$$L_{o,1} = \frac{1 + \mu}{1 + \xi\mu} \quad \text{and} \quad L_{n,1} = \xi\mu \frac{1 + \mu}{1 + \xi\mu}. \quad (4)$$

The consumption of existing capital owners depends on the output of the old technology. Since  $L_{o,1}$  is decreasing in  $\xi$ , so does their consumption growth,

$$\frac{C_1^o}{C_0^o} = \left( \frac{1 + \mu}{1 + \xi\mu} \right)^{1-\alpha}. \quad (5)$$

Equation (5) illustrates the displacive effect of innovation to the owners of existing capital. Unlike workers, who can work in either the new or the old economy, the owners of old capital do not benefit from the embodied shock  $\xi$ . Since these owners compete with owners of new capital in the market for labor, a positive innovation shock leads to lower consumption growth for the owners of existing capital.

Now, suppose that a claim on the output of the new technology were available at time 0. For simplicity, assume that this claim is on an infinitesimal fraction of the output of the new technology, so that (5) still characterizes the consumption growth of the old capital owners. Given the preferences of the existing households (1) and their consumption growth (5), the difference between the realized return to the new and the old technology is

$$R_1^n - R_1^o = \left( \frac{\xi}{E[\xi]} - 1 \right) \left( \frac{1 + \mu}{1 + \xi\mu} \right)^{1-\alpha}. \quad (6)$$

Since the innovation shock  $\xi$  is embodied in new capital, a positive innovation shock  $\xi > 1$  is associated with a higher return of the new technology relative to the old.

**Proposition 1** *In equilibrium, the claim to the new technology has a lower expected return than the claim to the old technology,*

$$E[R_1^n] < E[R_1^o].$$

**Proof.** Let  $f(\xi) = \left( \frac{\xi}{E[\xi]} - 1 \right) \left( \frac{1+\mu}{1+\xi\mu} \right)^{1-\alpha}$ . Since  $f''(\xi) < 0$ , Jensen's inequality implies  $E[f(\xi)] < f(E[\xi]) = 0$  ■

Proposition 1 summarizes the main result of the paper. In contrast to labor, capital is tied to a specific technology. Hence, technological improvements embodied in new vintages of capital lower the value of older vintages. Imperfect inter- and intra-generational risk sharing imply that innovation leads to high marginal utility states for the marginal investor. Given the opportunity,



owners of existing capital are willing to own a claim to the new technology, and accept lower returns on average, to obtain a hedge against displacement. Limited risk sharing across new and old capital owners, as well as shareholders and workers is key for this result. As a result of limited risk sharing, the consumption CAPM fails in the model because the consumption growth of the marginal investor (5) differs from aggregate, per capita, consumption growth

$$\frac{\bar{C}_1}{\bar{C}_0} = \left( \frac{1 + \xi\mu}{1 + \mu} \right)^\alpha. \quad (7)$$

The model in this section illustrates the basic intuition of our paper. However, it is too stylized to allow us to quantify the importance of this mechanism of asset returns and economic quantities. Next, we develop a dynamic general equilibrium model that builds on these basic ideas.

## 2 The Model

In this section we develop a dynamic general equilibrium model that extends the simple model above along several dimensions. First, we endogenize the investment in the capital stock each period; a key part of the mechanism is that workers benefit from the expansion and improvement in the capital stock, but do not share the costs of its acquisition with current capital owners. Second, we include a full cross-section of firms. Existing firms vary in their ability capture rents from new projects. By investing in existing firms, existing capital owners can hedge their displacement from innovation. Differences in the ability of firms to acquire innovation lead to ex-ante differences in risk premia. Third, we consider a richer class of preferences that separate risk aversion from the inverse of the elasticity of intertemporal substitution and allow for relative consumption effects in the utility function. These extensions allow for a better quantitative fit of the model to the data, but do not qualitatively alter the intuition from the simple model above.

### 2.1 Firms and technology

There are three production sectors in the model: a sector producing intermediate consumption goods; a sector that aggregates these intermediate goods into the final consumption good; and a sector

producing investment goods. Firms in the last two sectors make zero profits due to competition and constant returns to scale, hence we explicitly model only the intermediate-good firms.

### Intermediate-good firms

Production in the intermediate sector takes place in the form of projects. Projects are introduced into the economy by the new cohorts of inventors, who lack the ability to implement them on their own and sell the blueprints to the projects to existing intermediate-good firms. There is a continuum of infinitely lived firms; each firm owns a finite number of projects. We index individual firms by  $f \in [0, 1]$  and projects by  $j$ . We denote the set of projects owned by firm  $f$  by  $\mathcal{J}_f$ , and the set of all active projects in the economy by  $\mathcal{J}_t$ .<sup>3</sup>

#### *Active projects*

Projects are differentiated from each other by three characteristics: a) their scale,  $k_j$ , chosen irreversibly at their inception; b) the level of frontier technology at the time of project creation,  $s$ ; and c) the time-varying level of project-specific productivity,  $u_{jt}$ . A project  $j$  created at time  $s$  produces a flow of output at time  $t > s$  equal to

$$y_{jt} = u_{jt} e^{\xi s} k_j^\alpha, \quad (8)$$

where  $\alpha \in (0, 1)$ ,  $\xi$  denotes the level of frontier technology at the time the project is implemented, and  $u$  is a project-specific shock that follows a mean-reverting process. In particular, the random process governing project output evolves according to:

$$du_{jt} = \theta_u(1 - u_{jt}) dt + \sigma_u \sqrt{u_{jt}} dZ_{jt}, \quad (9)$$

All projects created at time  $t$  are affected by the embodied shock  $\xi$ :

$$d\xi_t = \mu_\xi dt + \sigma_\xi dB_{\xi t}. \quad (10)$$

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<sup>3</sup>While we do not explicitly model entry and exit of firms, firms occasionally have zero projects, thus temporarily exiting the market, whereas new entrants can be viewed as a firm that begins operating its first project. Investors can purchase shares of firms with zero active projects.

The embodied shock  $\xi$  captures the level of frontier technology in implementing new projects. In contrast to the disembodied shock  $x$ , an improvement in  $\xi$  affects only the output of *new* projects. In most respects, the embodied shock  $\xi$  is formally equivalent to investment-specific technological change.

All new projects implemented at time  $t$  start at the long-run average level of idiosyncratic productivity,  $u_{jt} = 1$ . Thus, all projects managed by the same firm are ex-ante identical in terms of productivity, but differ ex-post due to the project-specific shocks. Last, active projects expire independently at a Poisson rate  $\delta$ .

*Firm investment opportunities – new projects*

There is a continuum of firms in the intermediate goods sector that own and operate projects. Firms are differentiated by their ability to attract inventors, and hence initiate new projects. We denote by  $N_{ft}$  the Poisson count process that denotes the number of projects the firm has acquired. The probability that the firm acquires a new project,  $dN_t = 1$ , is firm-specific and equal to

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{ft}. \quad (11)$$

The likelihood that the firm acquires a new project  $\lambda_{ft}$  is composed of two parts. The first part  $\lambda_f$  captures the long-run likelihood of firm  $f$  receiving new projects, and is constant over time. The second component,  $\tilde{\lambda}_{ft}$  is time-varying, following a two-state, continuous time Markov process with transition probability matrix  $S$  between time  $t$  and  $t + dt$  given by

$$S = \begin{pmatrix} 1 - \mu_L dt & \mu_L dt \\ \mu_H dt & 1 - \mu_H dt \end{pmatrix}. \quad (12)$$

We label the two states as  $\{\lambda_H, \lambda_L\}$ , with  $\lambda_H > \lambda_L$ . Thus, at any point in time, a firm can be either in the high-growth ( $\lambda_{ft} = \lambda_f \cdot \lambda_H$ ) or in the low-growth state ( $\lambda_{ft} = \lambda_f \cdot \lambda_L$ ). The instantaneous probability of switching to each state is  $\mu_H dt$  and  $\mu_L dt$ , respectively. Without loss of generality, we impose the restriction  $E[\tilde{\lambda}_{f,t}] = 1$ . Our specification implies that the aggregate rate of project creation  $\bar{\lambda} \equiv E[\lambda_{ft}]$  is constant.

### *Implementing new projects*

The implementation of a new project idea requires new capital  $k$  purchased at the equilibrium market price  $q$ . Once a project is acquired, the firm chooses its scale of production  $k_j$  to maximize the value of the project. A firm's choice of project scale is irreversible; firms cannot liquidate existing projects and recover their original costs.

### **Capital-good firms**

Firms in the capital-good sector use labor to produce productive the investment goods needed to implement new projects in the intermediate-good sector

$$I_t = e^{x_t} L_{It}. \quad (13)$$

The labor augmenting productivity shock  $x$  evolves according to

$$dx_t = \mu_x dt + \sigma_x dB_{xt}. \quad (14)$$

### **Final-good firms**

Final consumption good firms using a constant returns to scale technology employing labor  $L_C$  and intermediate goods  $Y_t$

$$C_t = Y_t^\phi (e^{x_t} L_{Ct})^{1-\phi}. \quad (15)$$

Production of the final consumption good is affected by the labor augmenting productivity shock  $x_t$ .

## **2.2 Households**

There are two types of households, each with a unit mass: hand-to-mouth workers who supply labor; and inventors, who supply ideas for new projects. Both types of households have finite lives: they die stochastically at a rate  $\mu$ , and are replaced by a household of the same type. Households have no bequest motive and have access to a market for state-contingent life insurance contracts. Hence, each household is able to perfectly share its mortality risk with other households of the same cohort.

## Inventors

Each new inventor is endowed with a measure  $\bar{\lambda}/\mu$  of ideas for new projects. Inventors are endowed with no other resources, and lack the ability to implement these project ideas on their own. Hence, they sell these projects to existing firms. Inventors and firms bargain over the surplus created by new projects. Each inventor captures a share  $\eta$  of the value of each project. After they sell their project, inventors invest their proceeds in financial markets. Inventors are only endowed with projects upon entry, and cannot subsequently innovate. As a result, each new successive generation of inventors can potentially displace older cohorts. Inventors have access to complete financial markets, including an annuity market.

Inventor's utility takes a recursive form

$$J_t = E_t \int_t^\infty \tilde{f}(C_s, \bar{C}_s, J_s) ds, \quad (16)$$

where the aggregator  $\tilde{f}$  is given by

$$\tilde{f}(C, \bar{C}, J) \equiv \frac{\rho + \mu}{1 - \theta^{-1}} \left( \frac{(C^{1-h} (C/\bar{C})^h)^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)J \right). \quad (17)$$

Household preferences depend on own consumption  $C$ , but also on the consumption of the household relative to the aggregate  $\bar{C}$ . Thus, our preference specification nests 'keeping up with the Joneses' and non-separability across time (see e.g. Abel, 1990; Duffie and Epstein, 1992). The parameter  $h$  captures the strength of the external habit;  $\rho$  is the time-preference parameter;  $\gamma$  is the coefficient of relative risk aversion; and  $\theta$  is the elasticity of intertemporal substitution (EIS). The fact that households face an exponentially distributed time of death leads to an increase in the effective rate of discounting by  $\mu$ .

## Workers

Workers inelastically supply one unit of labor that can be freely allocated between producing consumption or investment goods

$$L_I + L_C = 1. \quad (18)$$

Workers are hand-to-mouth; they do not have access to financial markets and consume their labor income every period.

## 3 Competitive equilibrium

**Definition 1 (Competitive Equilibrium)** *The competitive equilibrium is a sequence of quantities  $\{C_t^S, C_t^W, Y_t, L_{Ct}, L_{It}\}$ ; prices  $\{p_{Yt}, q_t, w_t\}$ ; firm investment decisions  $\{k_t\}$  such that given the sequence of stochastic shocks  $\{x_t, \xi_t, u_{jt}, N_{ft}\}$ : i) shareholders choose consumption and savings plans to maximize their utility (16); ii) intermediate-good firms maximize their value according to (21); iii) Final-good and investment-good firms maximize profits; iv) the labor market (18) clears; v) the market for capital clears (23); vi) the market for consumption clears  $C_t^S + C_t^W = C_t$ ; vii) the resource constraints (13)-(15) are satisfied; and viii) market participants rationally update their beliefs about  $\lambda_{ft}$  using all available information.*

### 3.1 Firm optimization

We begin our description of the competitive equilibrium by characterizing the firms' optimality conditions.

#### Market for intermediate goods

Consumption firms purchase the intermediate good  $Y$  at a price  $p_Y$  and hire labor  $L_C$  at a wage  $w$  to maximize their value. Their first order condition with respect to their demand for intermediate goods yields

$$\phi Y_t^{\phi-1} (e^{x_t} L_{Ct})^{1-\phi} = p_{Yt}. \quad (19)$$

The price of the intermediate good  $p_Y$  is therefore pinned down by the equilibrium allocation of labor to the final good sector  $L_C$  and the supply of intermediate goods,  $Y$ .

The total output of the intermediate good,  $Y_t$ , equals the sum of the output of the individual projects,  $Y_t = \int y_{f,t}$ , and is equal to the effective capital stock

$$Y_t = K_t \equiv \int_{j \in \mathcal{J}_t} e^{\xi_j} k_j^\alpha dj. \quad (20)$$

adjusted for the productivity of each vintage – captured by  $\xi$  at the time the project is created – and for decreasing returns to scale. An increase in the effective capital stock  $K$ , for instance due to a positive embodied shock, leads to a lower price of the intermediate good and to displacement for productive units of older vintages.

### Market for capital

Intermediate good firms choose the scale of investment,  $k_j$ , in each project to maximize its net present value, which equals the market value of a new project, minus its implementation cost. We guess – and subsequently verify – that the equilibrium price of a new project equals  $P_t e^{\xi_t} k^\alpha$ , where  $P$  is a function of only the aggregate state of the economy. Then, the net present value of a project is

$$\max_k NPV = P_t e^{\xi_t} k^\alpha - q_t k. \quad (21)$$

The optimal scale of investment is a function of the ratio of the market value of a new project to its marginal cost of implementation  $q_t$ ,

$$k_t = \left( \frac{\alpha e^{\xi_t} P_t}{q_t} \right)^{\frac{1}{1-\alpha}}. \quad (22)$$

Equation (22) bears similarities to the q-theory of investment (Hayashi, 1982). A key difference here is that the numerator involves the market value of a new project – marginal q – which is distinct from the market value of the firm – average q. Aggregating across firms, the total demand for new

capital equals

$$I_t = \int k_{ft} dN_{ft} = \bar{\lambda} k_t. \quad (23)$$

The equilibrium price of investment goods,  $q_t$ , clears the supply (13) and the total demand for new capital (23)

$$q_t = \alpha e^{\xi t} P_t \left( \frac{\bar{\lambda}}{e^{x_t} L_{It}} \right)^{1-\alpha}. \quad (24)$$

A positive innovation shock leads to an increase in the demand for capital, and thus to an increase in its equilibrium price  $q$ .

### Market for labor

Labor is used to produce both the final consumption good, and the capital needed to implement new projects. The first order condition of the firms producing the final consumption good with respect to labor input links their labor choice  $L_C$  to the competitive wage  $w_t$

$$(1 - \phi) K_t^\phi e^{(1-\phi)x_t} L_{Ct}^{-\phi} = w_t. \quad (25)$$

Similar to the simple model in section 1, an improvement in the effective stock of capital  $K$  benefits laborers due to an increase in the equilibrium wage. In the extended model, a positive innovation shock benefits workers relative to capital owners through an additional channel; labor participates in the production of capital. Hence, not only does labor benefit from the expansion in the effective capital stock, but the costs of capital creation are not shared symmetrically across laborers and shareholders. Specifically, the profit maximization in the investment-goods sector implies that

$$e^{x_t} q_t = w_t, \quad (26)$$

hence a positive innovation shock increases the equilibrium wage on impact.

The equilibrium allocation of labor between producing consumption and investment goods is



determined by the labor market clearing condition (18),

$$(1 - \phi) K_t^\phi e^{(1-\phi)x_t} (1 - L_{It})^{-\phi} = \alpha e^{\alpha x_t + \xi_t} P_t \left( \frac{\bar{\lambda}}{L_{It}} \right)^{1-\alpha}. \quad (27)$$

All else equal, an increase in the embodied shock  $\xi$  increases the demand for new investment goods. As a result, the economy reallocates resources away from producing consumption goods towards producing investment goods.

### 3.2 Household optimization

Here, we describe the household's optimality conditions.

#### Inventors

Upon entry, inventors sell the blueprints to their projects to firms and use the proceeds to invest in financial markets. A new inventor entering at time  $t$  acquires a share of total financial wealth  $W_t$  equal to

$$b_{tt} = \frac{\eta \bar{\lambda} NPV_t}{\mu W_t}, \quad (28)$$

where  $NPV_t$  is the maximand in (21),  $\eta$  is the share of the project value captured by the inventor, and  $W_t$  is total financial wealth in the economy.

As new inventors acquire shares in financial wealth, they displace older cohorts. The share of total financial wealth  $W$  held at time  $t$  by an inventor born at time  $s < t$  equals

$$b_{ts} = b_{ss} \exp \left( \mu(t - s) - \mu \int_s^t b_{uu} du \right). \quad (29)$$

Agents insure the risk of death with other members of the same cohort; hence surviving agents experience an increase in the growth rate of per-capital wealth equal to probability of death  $\mu$ .

We guess – and subsequently verify – that the value function of an inventor born in time  $s$  is given by

$$J_{ts} = \frac{1}{1 - \gamma} b_{ts}^{1-\gamma} F_t, \quad (30)$$

where  $F_t$  is a function of the aggregate state.

Even though the model features heterogenous households, aggregation is simplified due to homotheticity of preferences. Existing inventors vary in their level of financial wealth, captured by  $b_{ts}$ . However, all existing agents at time  $t$  share the same growth rate of consumption going forward, as they share risk in financial markets. Hence, all existing inventors have the same marginal rate of substitution

$$\frac{\pi_s}{\pi_t} = \exp\left(\int_t^s \tilde{f}_J(C_u, \bar{C}_u, J_u) du\right) \frac{\tilde{f}_C(C_s, \bar{C}_s, J_s)}{\tilde{f}_C(C_t, \bar{C}_t, J_t)}, \quad (31)$$

where  $J$  is the utility index defined recursively in equation (16), and  $\tilde{f}$  is the preference aggregator defined in equation (17).

### Workers

Workers inelastically supply one unit of labor and face no investment decisions. Every period, they consume an amount equal to their labor proceeds

$$C_t^W = w_t. \quad (32)$$

### 3.3 Asset prices

The last step in characterizing the competitive equilibrium involves the computation of financial wealth. Since firms producing capital goods and the final consumption good have constant returns to scale technologies and no adjustment costs, they make zero profits in equilibrium. Hence, we only focus on the sector producing intermediate goods. Total financial wealth is equal to the sum of the value of existing assets plus the value of future projects

$$W_t = VAP_t + PVGO_t. \quad (33)$$

The value of financial wealth also corresponds to the total wealth of inventors, which enters the denominator of the displacement effect (28). Next, we solve for the two components of financial

wealth.

### Value of Assets in Place

A single project produces a flow of the intermediate good, whose value in terms of consumption is  $p_{Y,t}$ . The value, in consumption units, of an existing project with productivity level  $u_{j,t}$  equals

$$E_t \left[ \int_t^\infty e^{-\delta s} \frac{\pi_s}{\pi_t} p_{Y,s} u_{j,s} e^{\xi_j} k_j^\alpha ds \right] = e^{\xi_j} k_j^\alpha \left[ P_t + \tilde{P}_t(u_{j,t} - 1) \right], \quad (34)$$

where  $P_t$  and  $\tilde{P}_t$  are functions of the aggregate state of the economy – verifying our conjecture above. The total value of all existing projects is equal to

$$VAP_t \equiv \int_{j \in \mathcal{J}_t} e^{\xi_j} k_j^\alpha \left[ P_t + \tilde{P}_t(u_{j,t} - 1) \right] dj = P_t K_t, \quad (35)$$

where  $K$  is the effective capital stock defined in equation (20).

### Value of Growth Opportunities

The present value of growth opportunities is equal to the present value of rents to existing firms from all future projects

$$PVGO_t \equiv (1 - \eta) E_t \int_t^\infty \left( \int \lambda_{fs} \frac{\pi_s}{\pi_t} NPV_s df \right) ds = \bar{\lambda} (1 - \eta) \left[ \Gamma_t^L + \frac{\mu_H}{\mu_L + \mu_H} (\Gamma_t^H - \Gamma_t^L) \right] \quad (36)$$

where  $NPV_t$  is the equilibrium net present value of new projects in (21),  $1 - \eta$  represents the fraction of this value captured by existing firms;  $\mu_H / (\mu_H + \mu_L)$  is the measure of firms in the high growth state; and  $\Gamma_t^L$  and  $\Gamma_t^H$  determine the value of a firm in the low- and high-growth phase, respectively.

### 3.4 Dynamic evolution of the economy

The current state of the economy is characterized by the vector  $Z_t = [\chi_t, \omega_t]$ , where

$$\chi \equiv (1 - \phi)x + \phi \ln K \quad (37)$$

$$\omega \equiv \alpha x + \xi - \ln K. \quad (38)$$

The dynamic evolution of the aggregate state  $Z$  depends on the law of motion for  $\xi$  and  $x$ , given by equations (10) and (14), respectively, and the evolution of the effective stock of capital,

$$dK_t = \left( i(\omega_t) - \delta \right) K_t dt, \quad \text{where} \quad i(\omega_t) \equiv \bar{\lambda} e^{\xi_t} k_t^\alpha = \bar{\lambda} e^{\omega_t} \left( \frac{L I t}{\bar{\lambda}} \right)^\alpha. \quad (39)$$

At the aggregate level, our model behaves similarly to the neoclassical growth model. The first state variable  $\chi$  is difference-stationary and captures the stochastic trend in the economy. Long-run growth  $\chi$  depends on the disembodied shock  $x$  and the effective capital stock  $K$ . The effective capital  $K$  grows by the average rate of new project creation  $\bar{\lambda}$ , the equilibrium scale of new projects  $k$ , and improvements in the quality of new capital  $\xi$ ; the effective capital depreciates at the rate  $\delta$  of project expiration.

The variable  $\omega$  captures transitory fluctuations along the stochastic trend. Since  $i'(\omega) > 0$ , an increase in  $\omega$  accelerates the growth rate of the effective capital stock, and thus the long-run growth captured by  $\chi$ . We therefore interpret shocks to  $\omega$  as shocks to the investment opportunity set in this economy; the latter are affected both by the embodied innovation shocks  $d\xi_t$  and the disembodied productivity shocks  $dx_t$ . Further, the state variable  $\omega$  is mean-reverting; an increase in  $\omega$  leads to an acceleration of capital accumulation  $K$ , in the future  $\omega$  reverts back to its long-run mean. In addition to  $i(\omega)$ , the following variables in the model are stationary since they depend only on  $\omega$ : the optimal allocation of labor across sectors  $L_I$  and  $L_C$ ; the consumption share of workers  $C^w/\bar{C}$ ; the rate of displacement of existing shareholders  $b$ .

### 3.5 Numerical solution and simulation

The competitive equilibrium involves the computation of six unknown functions  $F$ ,  $P$ ,  $\tilde{P}$ ,  $\Gamma_L$ ,  $\Gamma_H$ , and  $L_I$ , of the aggregate state  $Z$ . These functions are characterized by a system of five nested differential equations and one functional equation, which are relegated to the Appendix. We solve for these equations using finite differences on a grid with 2,000 points.

We simulate the model at a weekly frequency  $dt = 1/52$  and then aggregate the data to form annual observations. We simulate 1,000 model histories of 3,000 firms and 120 years each. We drop the first third of each history to eliminate the impact of initial conditions. When we compare the output of the model to the data, we report the median parameter estimate across simulations.

## 4 Model implications

Here, we calibrate our model and explore its implications for asset returns and aggregate quantities.

### 4.1 Calibration

The model has a total of 18 parameters. We choose these parameters to approximately match a set of moments. Table 1 displays the moments generated by the model, and we mark moments targeted in calibration with a star.

We calibrate the bargaining parameter  $\eta = 0.8$  between innovators and firms to match the volatility of cohort effects. We choose the probability of death  $\mu = 0.025$ , so that the average length of adult life is  $1/\mu = 40$  years. We create returns to equity by levering financial wealth by 2.

Regarding the parameters of the technology shocks, we choose the mean growth rates  $\mu_x = 0.023$  and  $\mu_\xi = 0.005$  to match the growth rate of the economy. We choose the volatilities of the disembodied shock  $\sigma_x = 0.05$  and the embodied shock  $\sigma_\xi = 0.125$  to match the volatility of shareholder consumption growth and investment growth, respectively. We select the parameters of the idiosyncratic shock,  $\sigma_u = 1.15$  and  $\theta_u = 0.05$ , to match the persistence and dispersion in firm output-capital ratios.

We choose the returns to scale parameter at the project level  $\alpha = 0.45$  to approximately match the correlation between investment rate and Tobin's  $Q$ . We choose a depreciation rate of  $\delta$  in

line with typical calibrations of RBC models. We choose the share of capital in the production of final goods  $\phi$  to match the average level of the labor share. We choose the average rate of acquisition of new projects,  $\bar{\lambda}$  to match the average investment-to-capital ratio in the economy. The parameter governing the firm-specific long-run growth rate,  $\lambda_f$  is drawn from a uniform distribution [5, 15]; the parameters characterizing the short-run growth dynamics are  $\lambda_H = 4.25$ ,  $\mu_L = 0.2$  and  $\mu_H = 0.05$ . We choose these parameters to approximately match the persistence, the dispersion and the lumpiness in firm investment rate.

For our preference parameters we choose a low value of time preference  $\rho = 0.005$ , based on typical calibrations. We select the coefficient of risk aversion  $\gamma = 45$  and the elasticity of intertemporal substitution  $\theta = 0.6$  to match the level of the premium of financial wealth and the volatility of the risk free rate. We choose the preference weight on relative consumption  $h = 1/2$  following Garleanu et al. (2012), so that households attach equal weights to own and relative consumption. The presence of the relative consumption concerns in the utility function implies that the effective risk aversion of the marginal investor is shrunk towards one,  $\hat{\gamma} = 1 + (1 - h)\gamma$  for shocks that affect the agents  $C$  and average  $\bar{C}$  consumption symmetrically. Hence, we need a relatively high coefficient of risk aversion to match the equity premium.

## 4.2 Inspecting the mechanism

Here, we detail the model mechanism that leads to cross-sectional dispersion in risk premia. Equilibrium risk premia are determined by the covariance of returns with the equilibrium stochastic discount factor. We first consider the mechanism for how innovation risk is priced – the relation between the innovation shock and the stochastic discount factor. Then, we discuss the determinants of the cross-sectional differences in exposure to innovation risk among firms, and the resulting differences in expected stock returns.

## Equilibrium price of innovation shocks

The stochastic discount factor can be obtained by an application of Ito's lemma on the gradient of the utility function of the stock holders in the model,

$$\frac{d\pi_t}{\pi_t} = [\dots] dt - \theta^{-1} \left( \frac{dC_{ts}}{C_{ts}} - h(1-\theta) \frac{d\bar{C}_{ts}}{\bar{C}_{ts}} \right) - \frac{\gamma - \theta^{-1}}{1-\gamma} \frac{dJ_{ts}}{J_{ts}}, \quad (40)$$

The marginal value of consumption for an existing stockholder of a cohort  $s < t$  depends on her own consumption  $C$ ; aggregate consumption  $\bar{C}$ , due to relative consumption concerns parameterized by  $h$ ; and her growth in continuation utility  $J$ . The price of risk of the innovation shock depends on how it affects each of these three objects.

A positive innovation shock leads to a reallocation of labor from the consumption-good sector to the investment-good sector, as we see in panel *a* of Figure 1.<sup>4</sup> The resulting increased demand for labor services has an additional effect of raising real wages, shifting income from capital to labor as we see in panel *b*. Since workers do not share the costs of creating new capital with shareholders, the consumption of stockholders declines in response to an embodied technological advance by a greater amount than aggregate consumption, as we see in panels *c* and *d* respectively. The decline in shareholder consumption, both in absolute as well as in relative terms, leads a positive relation between the innovation shock and the stochastic discount factor (40).

The embodied shock also affects the value function of stockholders. This continuation value is affected by two additional channels. First, a positive innovation shock accelerates the rate at which new cohorts of inventors enter the economy and reduce profitability of the old capital stock owned by the stockholders. This displacement effect, captured by  $b_{tt}$  in panel *e*, lowers the continuation utility of stockholders; preferences over relative consumption exacerbate this effect. Second, a positive embodied shock leads to an acceleration in capital accumulation and therefore higher future consumption growth in the economy. Hence, a positive embodied shock implies that existing capital owners capture a smaller slice of a larger pie; depending on model parameters, this can lead to a positive or negative relation between innovation and the stochastic discount factor (40). As we

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<sup>4</sup>This part of the mechanism would lead to a negative risk premium for the innovation shock in a representative agent economy, under certain restrictions on preferences, as in Papanikolaou (2011). In our setting, this effect is secondary.

see in panel *f*, in our calibration the value function  $J$  of asset holders is negatively exposed to the innovation shock. Thus, the displacement effect dominates, resulting in a further increase in marginal utility following innovation.

To summarize these effects, we derive the stochastic discount factor as a function of the two technology shocks  $x$  and  $\xi$

$$\frac{d\pi_t}{\pi_t} = -r_{ft} dt - \gamma_x(\omega_t) dB_t^x - \gamma_\xi(\omega_t) dB_t^\xi, \quad (41)$$

where

$$\gamma_x(\omega) = \left[ (\gamma(1-h) + 1)(1-\phi) + \alpha \left( \theta^{-1} \frac{l'(\omega)}{l(\omega)} - \frac{\gamma - \theta^{-1}}{\gamma - 1} \frac{f'(\omega)}{f(\omega)} \right) \right] \sigma_x,$$

$$\gamma_\xi(\omega) = \left( \theta^{-1} \frac{l'(\omega)}{l(\omega)} - \frac{\gamma - \theta^{-1}}{\gamma - 1} \frac{f'(\omega)}{f(\omega)} \right) \sigma_\xi.$$

where  $f(\omega)$  captures the dependence of the value function of stockholders on the embodied shock, and  $l(\omega)$  is a function of the consumption share of stockholders. Both of these functions are defined in the Appendix. In panel *g* of Figure 1 we plot the market price of innovation risk,  $\gamma_\xi(\omega)$ . As we see it is negative, and approximately equal to -0.85 at the mean of the stationary distribution of  $\omega$ .

Last, we plot the market price of the disembodied shock,  $\gamma_x(\omega)$ . The market price of  $x$  depends mainly on the coefficient of risk aversion; however, preferences for relative consumption shrink the effective risk aversion towards one. The fact that the disembodied shock affects the real investment opportunities in the economy  $\omega$  lowers the risk price. As we see in panel *h*, the market price of the disembodied shock is positive and approximately equal to 0.5 at the mean of the stationary distribution of  $\omega$ .

### **Firm exposure to innovation**

The mechanism leading to cross-sectional dispersion in risk premia is that the components of firm value have heterogenous exposure to changes in real investment opportunities  $\omega$ . Specifically, the value of a firm in the intermediate sector consists of the value of assets in place and the value of



growth opportunities

$$\begin{aligned} V_{ft} &= VAP_{ft} + PVGO_{ft} \\ &= \int_{j \in \mathcal{J}_{ft}} e^{\xi_j} k_j^\alpha \left[ P_t + \tilde{P}_t(u_{j,t} - 1) \right] dj + \lambda_f(1 - \eta) \left[ \Gamma_t^L + p_{ft} (\Gamma_t^H - \Gamma_t^L) \right]. \end{aligned} \quad (42)$$

The first term captures the value of assets in place and depends on the firm's current portfolio of projects,  $\mathcal{J}_f$ . The second term captures the value of growth opportunities. This term depends on the current growth state of the firm, captured by the indicator function  $p_{ft}$ , which takes the value one if the firm is in the high-growth state ( $\tilde{\lambda}_{ft} = \lambda_H$ ).

To derive firms' exposures to the fundamental shocks  $x$  and  $\xi$ , we apply Ito's lemma to the value of the firm (42)

$$\frac{dV_{ft}}{V_{ft}} = [\dots] dt + (1 - \phi) \sigma_x dB_t^x + B_{ft} \left( \sigma_\xi dB_t^\xi + \alpha \sigma_x dB_t^x \right). \quad (43)$$

The first stochastic term in (43),  $(1 - \phi) \sigma_x dB_t^x$ , is identical across firms, and is driven solely by the disembodied productivity shocks. The second term,  $B_{ft} \left( \sigma_\xi dB_t^\xi + \alpha \sigma_x dB_t^x \right)$ , represents unanticipated changes in aggregate investment opportunities, and is driven by both by the labor-augmenting and the capital-embodied productivity shocks.

A firm is a portfolio of assets in place and growth opportunities; hence its systematic risk exposure is a weighted average of their corresponding risk exposures,

$$B_{ft} \equiv \left( \zeta'_\nu(\omega) + \zeta'_{\tilde{\nu}}(\omega) \frac{A_{ft}^v}{1 + A_{ft}^v} \right) \frac{VAP_{ft}}{V_{ft}} + \left( \zeta'_g(\omega) + \zeta'_{\tilde{g}}(\omega) \frac{A_{ft}^g}{1 + A_{ft}^g} \right) \frac{PVGO_{ft}}{V_{ft}}. \quad (44)$$

where  $\zeta_\nu, \zeta_g, \zeta_{\tilde{\nu}}$ , and  $\zeta_{\tilde{g}}$  are functions of  $\omega$  alone. The functions  $A^v$  and  $A^g$  depend on the current state of the firm,

$$A_{ft}^v = \frac{\sum_{j \in \mathcal{J}_{ft}} e^{\xi_j} k_j^\alpha (u_{j,t} - 1)}{\sum_{j \in \mathcal{J}_{ft}} e^{\xi_j} k_j^\alpha} e^{\zeta_{\tilde{\nu}}(\omega_t)}, \quad \text{and} \quad A_{ft}^g = \left( p_{ft} - \frac{\mu_H}{\mu_L + \mu_H} \right) (\lambda_H - \lambda_L) e^{\zeta_{\tilde{g}}(\omega_t)}. \quad (45)$$

In Figure 3, we plot the firms' innovation risk exposure, and risk premia, as a function of its current state. The value of assets in place is negatively exposed to innovation shocks,  $\zeta'_\nu(\omega) < 0$ ;

growth opportunities are less subject to displacement, since firms' investment opportunities improve as a result of innovation, thus  $\zeta'_g(\omega) > \zeta'_v(\omega)$ . Hence, the firm's ratio of growth opportunities to firm value  $PVGO/V$  is a primary determinant of the firm's exposure to the embodied shock  $\xi$ . As we see in panel *a*, the firm's return exposure  $B_f$  to the innovation shock is increasing in the share of growth opportunities to firm value  $PVGO/V$ , assuming the firm is in its steady state average ( $A_f^v = 0, A_f^g = 0$ ).

In contrast to the partial equilibrium model of Kogan and Papanikolaou (2011), the firm's ratio of growth opportunities to value,  $PVGO/V$ , is not a sufficient statistic for the firm's systematic risk; the firm's current profitability and current investment opportunities play a role. The timing of cash flows matters for risk exposures, and firms' idiosyncratic productivity shocks and their current growth state,  $\tilde{\lambda}_{ft}$ , are transient in nature. The linearity of our setup implies that these firm-specific risk exposures can be decomposed in the risk exposure of the average firm – captured by  $\zeta'_v$  and  $\zeta'_g$  – and a firm specific exposure that depends on the deviation from the average productivity ( $u = 1$ ) and growth state  $p = \mu_H/(\mu_H + \mu_L)$ , times an aggregate sensitivity  $\zeta'_v$  and  $\zeta'_g$ . In panel *b*, we see that, holding the share of growth opportunities constant, more productive firms have higher exposure to innovation shocks; however, this effect is quantitatively minor. Last, in panel *c*, we see that firms with better current investment opportunities benefit disproportionately more from aggregate innovation, since ceteris paribus,  $B_f$  is increasing in  $\lambda_{ft}$ .

### 4.3 Model properties

Table 1 shows the moments implied by the model. In addition to the moments we target, the model generates realistic moments for aggregate quantities. In line with the data, our model delivers a higher volatility of shareholder consumption growth and a positive correlation between investment and consumption growth. In addition, aggregate payout to capital owners – dividends, interest payments and repurchases minus new issuance – are volatile and weakly correlated with consumption and labor income. Next, we study the implications of our model for asset returns, both at the aggregate level but also about the cross-section of firms.

## Equity premium and the risk-free rate

Our model performs at least as well as most general equilibrium models with production in matching the moments of the market portfolio and risk-free rate (e.g., Jermann, 1998; Boldrin, Christiano, and Fisher, 2001; Kaltenbrunner and Lochstoer, 2010). The equity premium is in line with an empirical estimate, and realized equity returns are sufficiently volatile. The risk-free rate is smooth, despite the relatively low EIS and the presence of consumption externalities. The level of the risk-free rate is somewhat higher than the post-war average, but lower than the average level in the long sample in Campbell and Cochrane (1999). The relatively high level of the risk-free rate is the result of finite lives (see e.g. Blanchard, 1985).

The interaction of the embodied shock with limited risk-sharing is an important source of the high equity premium in our model. Existing asset holders bear most of the displacement risk resulting from innovation. The correlation between the aggregate stock market returns and consumption growth of stockholders is 63%, more than twice the correlation of market returns with consumption growth of non-stockholders (25%). Hence, tests of the standard consumption-CAPM using aggregate equity returns would imply an even larger coefficient of risk aversion than the calibrated value.

## Cross-section of stock returns

Our model features a rich equilibrium cross-section of firms, which differ endogenously in their exposure to the two priced shocks in the economy, the disembodied productivity shock  $x$  and the embodied shock  $\xi$ . To relate the theoretical results in Section 4.2 to the existing body of empirical work, we analyze the cross-sectional relations between expected stock returns and two firm characteristics correlated with cross-sectional differences in growth opportunities among firms: firms' investment rates ( $I/K$ ) and book-to-market ratios ( $B/M$ ). We follow the standard empirical procedure (see e.g. Fama and French, 1993) and sort firms into decile portfolios on their  $I/K$  and  $B/M$  ratios in simulated data.

As we see in Table 2 shows that our model generates a 5.9% spread in average returns between the high- $B/M$  and the low- $B/M$  decile portfolios, compared to 6.4% in the data. Sorting firms on their past investment rates leads to comparable cross-sectional differences in average return. In the

model, the difference in average returns between the high- and low-investment decile portfolios is  $-5.9\%$ , compared to  $-5.3\%$  in the data.

In addition to replicating the relation between average returns and characteristics document by Titman, Wei, and Xie (2004) and Lakonishok, Shleifer, and Vishny (1994), our model also replicates the failure of the CAPM to account for these relations. Table 3 shows that firms' market betas are only weakly correlated with their book-to-market ratios, and returns on the high-minus-low B/M portfolio have a positive alpha with respect to the CAPM ( $3.6\%$  in the model versus  $5.9\%$  in the data). Similarly, CAPM betas are essentially unrelated to the firms' past investment rates in the model, and high-minus-low I/K portfolio has a CAPM alpha of  $-5.01\%$ , compared to  $-7.09\%$  in the data. Importantly, our model also replicates the fact that the high-minus-low B/M and investment rate portfolios are not spanned by the market return, as evidenced by the low  $R^2$  numbers, both in the data and in the model.

### Valuation of human capital

Our model implies that human wealth has lower risk premium than financial wealth. A positive innovation shock leads to an increase in the equilibrium wage and a drop in dividends and the level of financial wealth. Since labor income is positively correlated with the embodied shock, our model implies that human wealth – labor income discount using the equilibrium stochastic discount factor – has a lower risk premium than financial wealth. This prediction is consistent with existing evidence. In recent work, Lustig and Van Nieuwerburgh (2008) and Lustig et al. (2008) document that returns to human wealth are lower than returns to financial wealth. Lustig et al. (2008) calculate the risk premium of financial and human wealth to be  $3.77\%$  and  $2.17\%$  respectively.

To assess the extent to which our model can quantitatively replicate this pattern, we compute the present value of labor income using the stochastic discount factor implied by the model,

$$H_t = E_t \int_t^\infty \frac{\pi_s}{\pi_t} w_s ds. \quad (46)$$

The ratio of human to total wealth  $H/(H + W)$  implied by our model calibration is  $83\%$ , which is close to the  $90\%$  ratio reported in Lustig et al. (2008). Our model implies that the risk premium

associated with the present value of the wage process (46) is equal to 1.98%, compared to 4.11% for an unlevered claim on the stock market.

## 5 Testing new empirical predictions

In this section we analyze the new testable predictions of the model directly tied to its core economic mechanism.

### 5.1 Constructing a proxy for the embodied shock

Our empirical analysis relies on an observable measure of the state variable  $\omega$  that captures the state of real investment opportunities. We exploit the fact that the total net present value of new projects, scaled by the aggregate stock market wealth, is a strictly increasing function of the state variable  $\omega$ ,

$$\frac{1}{W_t} \int NPV_t dN_{ft} \propto b(\omega_t), \quad (47)$$

where  $b(\omega_t) = b_{tt}$  is the share of wealth captured by new inventors (28). As we see in panel A of figure 3,  $\log b(\omega_t)$  is almost a linear function of the state variable  $\omega$  in the model.

We construct our empirical proxy for  $\omega$  using the methodology of Kogan et al. (2012). We consider patents as an empirical equivalent to the projects in our model economy. Kogan et al. (2012) construct an estimate of the dollar value of patents granted to public firms using their stock market reaction around the day that new of the patent issuance becomes public. Given the estimate  $A_{ft}^v$  of the total dollar value of patents granted to firm  $f$  in year  $t$ , we form the empirical equivalent of (47),

$$A_t = \frac{1}{V_t} \sum_{f \in N_t} A_{ft}^v, \quad (48)$$

by aggregating the patent values across the set  $N_t$  of firms in the economy, scaled by their end-of-year total market capitalization of all firms,  $V_t \equiv \sum_{f \in N_t} V_{ft}$ . We plot the series  $\log A_t$  in panel A of figure 3. To study our empirical construction (48) in the context of the model, we replicate the construction of  $A_t$  in simulated data. We follow the same empirical procedure as Kogan et al. (2012), defining the event day  $d$  as the time when a firm acquires a new project.

As we see in panel B of figure 3, the innovation measure  $A_t$  is a good approximation of the variable  $\omega$  in our model. In simulated data, the log innovation measure,  $\ln A_t$ , is highly correlated with the state variable  $\omega$ , both in levels (93.4%) and in first differences (80.1%). In terms of the primitive shocks, changes in  $\ln A_t$  in the model are primarily driven by the innovation shock  $\xi$ ; the median correlation between changes in  $\ln A$  and changes in  $\xi$  and  $x$  is 75.3% and 1.3% respectively.

In Table 4, we compare the moments of our innovation measure in the data and in the model. As we see in Panel A, both in the data and in the model, the cross-sectional distribution of the firm-level innovation measure is highly skewed. Approximately half of the firms do not innovate, and most of the activity is concentrated in the right tail of the distribution. In Panel B, we see that the relation between changes in the aggregate measure and the stock market is negative and comparable in magnitude across the data and the model.

## 5.2 Innovation and consumption displacement

Innovation risk is priced in our model because it affects consumption of existing stockholders. In this section we show that innovation is related to cohort effects in consumption; the consumption share of new cohorts of agents is increasing in the aggregate amount of innovation. Innovation by new generations displaces older cohorts.

### Micro-level evidence

The consumption of shareholders of cohort  $s$ , as a share of aggregate consumption, is equal to

$$\frac{C_{ts}}{C_t} = b(\omega_s) \exp\left(\mu(t-s) - \mu \int_s^t b(\omega_u) du\right) \tilde{l}(\omega_t). \quad (49)$$

We estimate the empirical equivalent of equation (49) using the CEX Family-level extracts by Harris and Sabelhaus (2000), which contain observations of households of different cohorts taken at different points in time. We define the household's cohort as the year in which the head of the household turns 25. Varying this age by plus or minus two years leads to similar results. We exploit the fact that, absent measurement error, our innovation measure  $A$  is linearly related to  $b(\omega)$ , as we see from equations (28) and (47). Taking logs of both sides of (49), we form the econometric

specification,

$$\ln C_{its} - \ln C_t = \beta_0 \ln A_s + \beta_1 \sum_{u=s+1}^{t-1} A_u + \beta_2 \ln A_t + a(t) + c(t-s) + c_2 Z_i + \varepsilon_{its} \quad (50)$$

where,  $i$  indexes households;  $t$  is the observation year;  $s$  is the cohort year;  $C$  denotes log non-durable consumption expenditures;  $a(t)$  is a time trend;  $A$  is our innovation measure;  $c(t-s)$  is a quadratic term parameterizing household age effects; and  $Z_i$  is a vector of household-level controls including years of education and number of earning members. We follow the variable definitions in Harris and Sabelhaus (2000). We cluster standard errors at the cohort level. We estimate (49) separately for stockholders and non-stockholders.<sup>5</sup> We include a deterministic time trend to account for increases in stock market participation and the secular trend in CEX data relative to aggregate consumption.

We focus on the coefficients  $\beta_0$ ,  $\beta_1$  and  $\beta_2$ . The estimate of  $\beta_0$  captures the effect of innovation on the consumption of the entering cohort – corresponding to the term  $b(\omega_s)$ . Our model implies that the coefficient  $\beta_0$  should be positive for stockholders. The estimate of  $\beta_1$  captures the effect of displacement – the integral term inside the exponential. A higher level of innovation results in the displacement of stockholders from earlier cohorts, hence our model predicts that  $\beta_1$  should be negative for stockholders. Last, the coefficient  $\beta_2$  captures both the displacement of the stockholders from cohort  $s$  by the time- $t$  entrants and the contemporaneous consumption distribution between the workers and the owners of capital. In the model, higher recent innovation results in a higher consumption share of the workers. Thus, our model predicts that  $\beta_2$  should be negative for stockholders and positive for non-stockholders.

The results in Panel I of Table 5 largely confirm the prediction of the model regarding displacement. The coefficient  $\beta_0$  is positive and statistically significant across specifications for both stockholders and non-stockholders, suggesting that the level of technological innovation at the time households enters the market has a lasting positive impact on their lifetime consumption. Consistent with our model, the coefficient  $\beta_1$  is negative and statistically significant for stockholders, and positive but not significant for non-stockholders. Hence, our results imply that existing generations

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<sup>5</sup>We define stockholders as households that report owning stocks, bonds or mutual funds. Since many households often do not report their bond and stock holdings in their retirement accounts, restricting the sample in this way is a conservative way of restricting the sample to stockholders.

of stockholders get displaced by subsequent innovation activity, while there is no corresponding effect for non-stockholders. Last, the coefficient  $\beta_2$  is positive and statistically significant for non-stockholders, but not significant for the stockholders.

As a robustness test, we repeat the exercise but we normalize by the mean consumption level of stockholders in the CEX, rather than aggregate consumption. As we see in Panel II of Table 5, relative to the total consumption of stockholders, consumption of the stockholders from cohort  $s$  is positively affected by the innovation at the time of their entry and negatively affected by subsequent innovation activity. This evidence further confirms the model's prediction that innovation leads to displacement of older cohorts of stockholders by the new cohorts of innovators.

### Aggregate evidence

Here, we provide further supporting evidence using time series data. We evaluate the effect of innovation on the consumption growth rate of stockholders  $c^S$  relative to non-stockholders across different horizons  $c^{NS}$ ,

$$(c_{t+k}^S - c_t^S) - (c_{t+k}^{NS} - c_t^{NS}) = a + \beta(T)\Delta \ln A_t + \varepsilon_{tT}. \quad (51)$$

We study horizons from  $k = 1$  to  $k = 4$  years. We use the series constructed in Malloy et al. (2009), which covers the 1982-2004 period.<sup>6</sup> We compute Newey-West adjusted standard errors in (51), setting the maximum number of lags equal to 3 plus the number of overlapping years.

We show the results in Table 6. We use two definitions of stockholders in Malloy et al. (2009). In Panel A, we present results using their baseline definition; in Panel B we present results using their top shareholder definition. Despite the short length of the sample (approximately 20 observations) there is a negative and generally statistically significant relation between our innovation measure and the 1 to 3-year differential growth rate of stockholders relative to non-stockholders. In Panel C, we show that a similar pattern holds in simulated data.

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<sup>6</sup>We follow Jagannathan and Wang (2007) and construct annual consumption growth rates by using end-of-period consumption. In particular, we focus on the sample of households that are interviewed in December of every year, and use the average 4 to 16 quarter consumption growth rate of non-stockholders, stockholders and top-stockholders, defined as in Malloy et al. (2009). Our results remain quantitatively similar when we instead construct annual growth rates by an equal-weighted average of the  $k$ -period consumption growth of all households interviewed in year  $t$ .



### 5.3 Innovation and the cross-section of firms

Firms with few growth opportunities are more vulnerable to displacement than firms with high growth opportunities. Here, we provide two direct tests of this mechanism. First, we show that firm's with high growth opportunities are less subject to displacement by their competitors. Second, we show that differences in firm characteristics related to growth opportunities are related to differences in firms' exposures to the aggregate innovation shock.

#### Innovation and firm displacement

A positive innovation shock  $\xi$  leads to an increase in the total production of the intermediate good  $Y$ , and therefore a reduction in its price  $p_Y$ . In this environment, firms that did not innovate and thus extended their production capacity will experience a reduction in sales. In the medium run, firms with high growth opportunities are less sensitive to this displacement effect because they are likely to acquire projects. Here, we provide a direct test of this model mechanism.

We study the response of firm output – sales plus change in inventories – to the firm's own innovation activity,  $A_f$ , and the innovation activity of its competitors,  $A_{If}$ , where

$$A_{ft} \equiv A_{ft}^v / V_{ft}, \quad \text{and} \quad A_{If t} \equiv \sum_{h \neq f \in N_{It}} A_{ht}^v / \sum_{h \neq f \in N_{If t}} V_{ht}. \quad (52)$$

The set of competing firms  $N_{If t}$  includes all the other firms in the same 3-digit SIC industry as firm  $f$ . Since the model does not have any industries, when constructing the equivalent of (52) in simulated data we use the set of all other firms.

We use the following specification

$$\log y_{ft+k} - \log y_{ft} = a_0 + a_1 A_{ft} + a_2 A_{If t} + a_3 A_{If t} \times D(G_{ft})_H + b Z_{ft} + e_{t+k}, \quad (53)$$

where  $y$  is firm output;  $D(G)_H$  is a dummy variable taking the value 1 if the firm is ranked higher than the industry median in terms of growth opportunities – proxied either by Tobin's  $Q$  or by the investment rate. The vector of controls  $Z$  includes industry effects; time effects; firm size; lagged output growth; and firm and industry stock returns, to control for the possibility that our innovation

measure is inadvertently capturing changes in valuations unrelated to innovation. We cluster the standard errors by firm. We examine horizons of  $k = 1$  to  $k = 7$  years. To facilitate comparison between the data and the model, we scale the variables  $A_f$  and  $A_{If}$  to unit standard deviation. We show the results in Table 7.

Innovation by competitors leads to displacement of firms with low growth opportunities. In panel *A.i* (*A.ii*), we see that firms with below-median Tobin's  $Q$  (investment rate) suffer a reduction in sales following innovation by their competitors, as evidenced by the negative estimate of  $a_2$ . In contrast, the interaction effect  $a_3$  is positive, implying that firms with above-median growth opportunities – measured using  $Q$  or investment rate – are displaced less. This difference in displacement between firms with high versus low growth opportunities is economically meaningful. A one-standard deviation increase in the amount of innovation by firm's competitors is associated with a 3.0-3.3% drop in output over the next five years for the firm's that are below the median industry in terms of growth opportunities. In contrast, the corresponding decrease in firm output of the high-growth firms is only 2.3%. After seven years, the drop in output is 4.0-4.6% and 2.5-2.6% for the low- and high-growth firms respectively. In most cases, these difference in output response is significant at the 1% to 10% level depending on the horizon.

Next, we compare the empirical magnitude of our findings to results in simulated data from the model. In panel B, we see that the magnitude of firm displacement in the model is not far from the data. A one-standard deviation increase in  $A_{If,t}$  is associated with a 2-2.4% drop in firm output for the firms with low growth opportunities after five years, respectively; in contrast, firms with high growth prospects experience only a 0.3-0.7% drop in output after a period of five years.

### **Innovation and return comovement**

In our model, cross-sectional differences in risk premia arise because firms with different levels of growth opportunities have different return exposures to innovation shock. Using our empirical measure of innovation  $A$  we study whether portfolios of stocks, sorted on either their past investment rate, or their book-to-market ratio, have differential innovator risk

$$R_{pt} - r_{ft} = a_p + \beta_p (R_{mt} - r_{ft}) + \gamma_p \Delta \ln A_t + \varepsilon_{pt} \quad (54)$$

As we see in panel A of Table 8, firms with high (low) growth opportunities have positive (negative) stock return exposure to innovation shocks  $\Delta \ln A_t$ , controlling for excess returns to the stock market,  $R_{mt} - r_{ft}$ . The empirical magnitudes are comparable to the magnitudes in simulated data, as we see in panel B.

#### 5.4 Asset pricing tests

Next, we explore whether our innovation measure prices the cross-section of portfolios sorted on the two measures of growth opportunities, book-to-market and investment rate. The stochastic discount factor (41) implied by the model is not available in analytic form. Hence, we estimate a linearized version

$$m = a - \gamma_x \Delta x - \gamma_\xi \Delta \xi. \quad (55)$$

We proxy for the innovation shock  $\Delta \xi$  by changes in our log innovation measure  $\Delta \ln A$ . We proxy for the disembodied technology shock  $x$  by the change in the (log) total factor productivity from Basu, Fernald, and Kimball (2006). In addition, since the disembodied shock  $x$  accounts for most of the short-run variation in aggregate consumption growth, we test an alternative version of the model where, we replace  $\Delta x$  by aggregate consumption growth.

We estimate (55) using the generalized method of moments (GMM). We use the model pricing errors as moment restrictions.<sup>7</sup> As test assets, we use deciles 1,2, 9 and 10 from the book-to-market and investment rate portfolios. We report first-stage GMM estimates using the identity matrix to weigh moment restrictions, and adjust the standard errors using the Newey-West procedure with a maximum of three lags. As a measure of fit, we report the cross-sectional  $R^2$  (one minus the ratio of the sum of squared pricing errors to the cross-sectional dispersion in average returns) and the mean absolute pricing errors. We report the estimation results in Table 9.

The specifications of the stochastic discount factor without the innovation shock result in large pricing errors, both in the data and in the model. As we see in columns A1 and A3, using differences in exposure with total factor productivity or consumption growth are not related to differences in

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<sup>7</sup>We impose that the SDF in equation (55) should price the cross-section of test asset returns in excess of the risk-free rate. Hence, the mean of the stochastic discount factor is not identified. Without loss of generality, we choose the normalization  $E(m) = 1$ , which leads to the moment restrictions  $E[R_i^e] = -cov(m, R_i^e)$ , where  $R_i^e$  denotes the excess return of portfolio  $i$  over the risk-free rate (see Cochrane, 2005, pages 256-258 for details.)

risk premia across portfolios. In Panel B, we report results of the same exercise in simulated data, where we observe similar patterns. In particular, column B3 shows that the consumption CAPM does not hold in the model, as it results in substantial pricing errors, though the magnitude of this failure is not as great as in the data. Adding the innovation measure  $\Delta \ln A$  dramatically improves the ability of the model to price these portfolios, as we see in columns A2 and A4. The price of risk associated with innovation ranges from  $-0.83$  to  $-1.03$  and is statistically significant at the 1% level. Further, as we see in columns B2 and B4, the estimated price of risk in the model are very close to the empirical estimates, ranging from  $-1.01$  to  $-1.15$  across specifications. This point estimate of the price of innovation risk is higher than the theoretical value in the model, likely due to the fact that  $\Delta A$  is a noisy proxy for the innovation shock  $\xi$ , even in simulated data.

## 6 Conclusion

We develop a general equilibrium model to study the effects of innovation on asset returns. Even though the link between technological innovation and aggregate consumption is weak, innovation risk carries a significant risk premium. However, focusing on aggregate moments obscures the effects of innovation in the cross-section of households. Specifically, technological improvements embodied in new capital benefit workers employed in their production, while displacing existing firms and their shareholders. This displacement process is uneven for two reasons. First, newer generations of shareholders benefit at the expense of existing cohorts. Second, firms well-positioned to take advantage of these opportunities benefit at the expense of firms unable to do so. Existing shareholders value firms rich in growth opportunities despite their low average returns, as they provide a hedge against displacement.

Our model delivers rich cross-sectional implications about the effect of innovation on the cross-section of firms and households that are supported by the data. We test the model's predictions using a direct measure of innovation constructed by Kogan et al. (2012) using data on patents and stock returns. Consistent with our model, we find that innovation is associated with a reallocation of wealth from existing shareholders to workers and future generations.

Our work suggests several avenues for future research. Quantifying the role of the wealth

reallocation associated with innovation on the recent increase in inequality is particularly promising, especially given the availability of a direct measure of technology. Further, analyzing the role of government policies in mitigating intergenerational displacement, such as social security or investing labor income in value firms, is another promising path. Last, we only focus on one particular type of innovation, that is technological change embodied in new capital. Analyzing the pricing of more general types of embodied technical change, for instance skill-biased technical change, is potentially fruitful.

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# Tables

**Table 1: Calibration moments**

A. Aggregate Quantities	Model	Data
Consumption growth, aggregate; mean* (%)	1.7	1.8
Consumption growth, aggregate; standard deviation (%)	3.0	2.2
Consumption growth, aggregate; serial correlation (%)	24.5	41.2
Consumption growth, stockholders; standard deviation* (%)	3.7	3.6-5.4
Correlation between dividends (net payout) and consumption (%)	50.6	20.3-30.1
Labor Income; standard deviation (%)	3.2	1.9
Correlation between dividends (net payout) and labor income (%)	29.8	22.9-40.5
Investment growth; standard deviation* (%)	11.6	12.9
Correlation between investment and aggregate consumption (%)	45.0	58.9
Investment to capital; mean* (%)	8.3	8.7
Labor Share; mean* (%)	70.8	68.5
Labor Share; standard deviation (%)	1.8	1.6
First difference of consumption cohort effect, standard deviation* (%)	3.5	3.0
B. Asset returns	Model	Data
Market portfolio, excess returns; mean* (%)	8.2	7.6
Market portfolio, excess returns; standard deviation (%)	12.1	18.5
Risk-free rate; mean (%)	2.6	0.9-2.9
Risk-free rate; standard deviation* (%)	0.6	0.9
Correlation between aggregate consumption and stock market (%)	40.7	18.4
C. Firm-level variables	Model	Data
Investment rate, IQR-to-Median*	1.23	1.21
Investment rate, serial correlation* (%)	46.2	47.8
Investment rate > 1, fraction of firm-year obs* (%)	1.3	1.4
Tobin's $Q$ , IQR-to-Median	0.89	1.27
Tobin's $Q$ , serial correlation (%)	90.8	79.5
Correlation between investment and lagged Tobin's $Q$ * (%)	22.9	23.7
Output to Assets, IQR-to-Median*	0.55	0.80
Output to Assets, serial correlation* (%)	88.0	92.3

Starred moments are targeted in our calibration. Investment, capital and consumption data are from NIPA; investment is non-residential private domestic investment; stock of capital is current-cost from the NIPA Fixed Assets Table; consumption is non-durables plus services; nominal variables are deflated by population and the CPI. Population is from the Census. Moments of shareholder consumption growth are from unpublished version of Malloy et al. (2009); the range depends on the assumptions about measurement error. The moments on net payout are from Larrain and Yogo (2008) using data on net equity and debt payout. Moments of labor income are from Lustig et al. (2008). The volatility of consumption cohorts is from Garleanu et al. (2012). Stock market data are from CRSP. Firm-level accounting data are from Compustat. Labor share is constructed from Flow of Funds data following Sekyu and Rios-Rull (2009). The moments of the real risk-free rate are from Campbell and Cochrane (1999) and Bansal and Yaron (2004); the range refers to the pre- versus post-war sample.

**Table 2: Cross-section of expected returns**

A. Data							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	6.45 (2.32)	6.98 (3.18)	7.62 (3.33)	11.29 (4.00)	11.38 (4.22)	12.83 (3.90)	6.38 (2.46)
$\sigma(\%)$	21.38	17.64	17.74	22.01	21.38	25.74	20.43
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	10.13 (3.42)	8.20 (3.24)	8.32 (4.03)	7.19 (2.68)	7.86 (2.40)	4.87 (1.26)	-5.26 (-2.05)
$\sigma(\%)$	23.52	20.19	17.37	21.01	25.50	28.93	17.11
B. Model							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	4.15 (3.61)	5.53 (4.78)	6.57 (5.48)	9.04 (6.67)	9.31 (6.74)	10.01 (6.96)	5.86 (7.74)
$\sigma(\%)$	10.39	10.56	10.99	12.29	12.53	12.99	6.36
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	8.67 (6.13)	7.28 (5.57)	6.16 (4.96)	5.97 (4.45)	4.86 (3.74)	2.79 (2.21)	-5.92 (-11.49)
$\sigma(\%)$	12.83	11.80	11.26	12.04	11.66	11.12	4.61

Table shows excess returns and standard deviation for portfolios sorted on two measures of growth opportunities: book-to-market and past investment. Data is from CRSP/Compustat. Book to market is book value of common equity divided by CRSP market capitalization in December. Investment rate is growth in property-pant and equipment. Data period is 1950-2008. We form portfolios in June every year. We exclude financial firms (SIC6000-6799), and utilities (SIC4900-4949). When computing investment rates and book to market in simulated data, we measure the book value of capital as the historical cost of firm's capital  $\sum_{\mathcal{J}_{ft}} k_j q_{\tau(j)}$  ( $\tau(j)$  denotes the time of creation of project  $j$ ) divided by its current market value  $V_{ft}$ .

**Table 3: The failure of the CAPM**

A. Data							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\alpha$	-1.51 (-1.38)	0.11 (0.16)	0.87 (1.21)	3.74 (2.44)	4.03 (2.92)	4.35 (2.20)	5.86 (2.07)
$\beta_{mkt}$	1.08 (21.65)	0.92 (22.91)	0.95 (29.54)	1.03 (8.95)	0.99 (10.08)	1.17 (11.14)	0.09 (0.64)
$R^2$	83.89	90.53	93.09	74.97	73.79	70.80	0.66
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$\alpha$	2.64 (1.85)	1.48 (1.48)	2.75 (2.39)	0.10 (0.12)	-0.60 (-0.47)	-4.45 (-2.29)	-7.09 (-2.97)
$\beta_{mkt}$	1.12 (16.38)	1.00 (17.79)	0.83 (13.39)	1.06 (21.26)	1.26 (18.68)	1.39 (16.5)	0.27 (2.79)
$R^2$	77.72	84.86	79.10	87.27	84.41	79.59	8.77
B. Model							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\alpha$	-2.33 (-4.19)	-1.44 (-3.36)	-0.77 (-2.29)	0.60 (3.44)	0.74 (4.00)	1.26 (5.17)	3.55 (5.17)
$\beta_{mkt}$	0.79 (21.25)	0.84 (28.23)	0.89 (38.66)	1.02 (79.78)	1.04 (76.16)	1.07 (61.12)	0.28 (5.74)
$R^2$	84.99	91.43	95.13	95.80	93.65	88.08	29.46
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$\alpha$	2.01 (5.17)	1.05 (3.56)	0.25 (0.93)	-0.43 (-1.66)	-1.30 (-5.47)	-3.03 (-9.62)	-5.01 (-9.34)
$\beta_{mkt}$	1.07 (36.53)	1.00 (44.65)	0.95 (42.45)	1.02 (51.30)	0.99 (53.62)	0.94 (37.58)	-0.14 (-3.40)
$R^2$	92.84	94.72	94.71	92.71	91.95	83.67	12.43

Table shows excess returns and standard deviation for portfolios sorted on two measures of growth opportunities: book-to-market and past investment. See notes to Table 2 for details of the portfolio construction. Moments of market portfolio and risk-free rate are from Kenneth French's website.

**Table 4: Descriptive statistics of innovation measure**

A. Moments of firm-level measure – $A^v/V$		
	Model	Data
Mean	0.044	0.029
Standard deviation	0.129	0.057
50-percentile	0.000	0.000
75-percentile	0.024	0.012
90-percentile	0.129	0.113
95-percentile	0.250	0.145
99-percentile	0.623	0.256
B. Moments of aggregate measure – $\Delta \ln A$		
	Model	Data
Standard deviation	8.57	33.22
Correlation with market excess returns	-60.12	-55.40
Correlation with changes in Tobin's $Q$	-73.21	-57.63

Table compares descriptive statistics for our firm-level and aggregate innovation measure  $A$  in the model and in the data. See text for details of the construction of  $A$ ; moments of the market portfolio and the risk-free rate are from Kenneth French's website. The aggregate Tobin's  $Q$  is computed using NIPA and FRB Flow of Funds Data following Laitner and Stolyarov (2003). Sample period is 1950-2008.

**Table 5: Innovation and consumption displacement**

$c_{its} - \bar{c}_t$	A. Stockholders			
	I. Relative to total consumption		II. Relative to group mean	
$\ln A_s$	0.1600 (3.00)	0.0284 (2.22)	0.1613 (3.25)	0.0207 (1.71)
$\sum_{u=s+1}^{t-1} A_u$	-0.0606 (-3.51)	-0.0588 (-2.36)	-0.0597 (-3.76)	-0.0374 (-2.18)
$\ln A_t$	0.0357 (1.60)	0.0138 (0.91)		
$R^2$	0.128	0.265	0.052	0.185
Observations	13787	12305	13787	12305
$c_{its} - \bar{c}_t$	B. Non Stockholders			
	I. Relative to total consumption		II. Relative to group mean	
$\ln A_s$	0.1640 (2.75)	0.0236 (2.22)	0.1834 (3.32)	0.0261 (2.48)
$\sum_{u=s+1}^{t-1} A_u$	-0.1311 (-7.58)	-0.0023 (-0.13)	-0.1255 (-7.83)	0.0086 (0.60)
$\ln A_t$	0.0769 (3.57)	0.0344 (3.07)		
$R^2$	0.208	0.317	0.132	0.226
Observations	36050	29191	36050	29191
Time Trend	Y	Y	-	-
Household controls	-	Y	-	Y

Table reports results of relating our innovation measure  $A$  to household consumption data (see equation (50) in main text). Household-level consumption data are from the CEX family-level extracts by Harris and Sabelhaus (2000), available through the NBER website. Data covers the period 1980-2003. See main text and Kogan et al. (2012) for details on the construction of  $A$ . Consumption is non-durables, defined as in Harris and Sabelhaus (2000). Stockholders are classified as households reporting ownership of stocks, bonds or mutual funds. Cohort age  $s$  is defined as the age the household turns 25. In panel I we normalize household consumption by per-capital aggregate consumption of non-durables. In Panel II we normalize by group (stockholder versus non-stockholder) means. Depending on the specification, we include a vector of household controls: for linear and quadratic age effects; number of earning members; years of education. All specifications in Panel I include a time trend to control for the historical growth in the rate of stock market participation and the secular trend in the CEX dataset. Standard errors are clustered by cohort.

**Table 6: Innovation and stockholder consumption growth**

A. Data				
i. Stockholders	T=1	T=2	T=3	T=4
$\Delta \ln A_t$	-0.013 (-1.76)	-0.026 (-2.28)	-0.025 (-1.74)	-0.029 (-1.09)
$R^2$	0.075	0.101	0.067	0.070
ii. Top stockholders	T=1	T=2	T=3	T=4
$\Delta \ln A_t$	-0.027 (-0.79)	-0.066 (-1.51)	-0.089 (-2.20)	-0.134 (-1.76)
$R^2$	0.027	0.072	0.103	0.203
B. Model				
i. Stockholders	T=1	T=2	T=3	T=4
$\Delta \ln A_t$	-0.02 (-2.14)	-0.01 (-1.03)	-0.01 (-0.73)	-0.01 (-0.48)
$R^2$	13.36	3.28	2.23	1.97

Table reports results of relating our innovation measure  $A$  to the differential growth rate of stockholders vs non-stockholders  $(c_{i+T}^S - c_t^S) - (c_{i+T}^{NS} - c_t^{NS})$  (see equation (51) in main text) in the data (Panel A) and the model (Panel B). Consumption growth of shareholders and non-shareholders are from Malloy et al. (2009). See Malloy et al. (2009) for definitions of stockholders and top stockholders. We construct annualized growth rates using Dec-Dec growth, following Jagannathan and Wang (2007). See main text and Kogan et al. (2012) for details on the construction of  $A$ . Sample period is 1980-2004. Standard errors are computed using Newey-West with T+1 lags. We standardize right-hand side variables to unit standard deviation.

**Table 7: Innovation and firm displacement**

A. Data							
$y_{t+T} - y_t$	T=1	T=2	T=3	T=4	T=5	T=6	T=7
i. Market-to-book							
$A_{ft}$	0.000 (0.15)	0.005 (2.94)	0.005 (2.50)	0.012 (5.03)	0.018 (6.58)	0.019 (6.37)	0.023 (6.76)
$A_{Ift}$	-0.015 (-7.05)	-0.018 (-6.00)	-0.027 (-6.89)	-0.029 (-6.33)	-0.030 (-5.86)	-0.033 (-5.57)	-0.040 (-6.13)
$A_{Ift} \times D(Q_{ft})_H$	0.009 (4.14)	0.007 (2.37)	0.015 (3.92)	0.014 (3.20)	0.009 (1.80)	0.011 (1.88)	0.015 (2.31)
ii. Investment rate							
$A_{ft}$	0.002 (2.20)	0.008 (5.21)	0.010 (4.97)	0.018 (7.39)	0.024 (8.87)	0.026 (8.67)	0.031 (9.10)
$A_{Ift}$	-0.011 (-5.13)	-0.018 (-6.09)	-0.025 (-6.66)	-0.026 (-5.88)	-0.033 (-6.55)	-0.036 (-6.31)	-0.046 (-7.15)
$A_{Ift} \times D(IK_{ft})_H$	-0.002 (-1.00)	0.004 (1.32)	0.009 (2.25)	0.004 (0.93)	0.010 (1.86)	0.012 (1.99)	0.020 (3.05)
B. Model							
$y_{t+T} - y_t$	T=1	T=2	T=3	T=4	T=5	T=6	T=7
i. Market-to-book							
$A_{ft}$	0.104 (45.76)	0.186 (50.69)	0.239 (53.66)	0.276 (55.86)	0.301 (57.65)	0.319 (59.26)	0.332 (60.64)
$A_{Ift}$	-0.007 (-2.19)	-0.012 (-2.30)	-0.016 (-2.50)	-0.020 (-2.66)	-0.024 (-2.83)	-0.027 (-2.90)	-0.029 (-2.93)
$A_{Ift} \times D(Q_{ft})_H$	0.002 (2.38)	0.005 (4.25)	0.009 (5.58)	0.013 (6.37)	0.017 (6.69)	0.021 (6.77)	0.024 (6.68)
ii. Investment rate							
$A_{ft}$	0.109 (49.12)	0.195 (55.21)	0.250 (58.44)	0.287 (60.45)	0.312 (61.73)	0.329 (62.60)	0.340 (63.17)
$A_{Ift}$	-0.012 (-3.58)	-0.018 (-3.47)	-0.022 (-3.42)	-0.025 (-3.31)	-0.028 (-3.27)	-0.029 (-3.17)	-0.030 (-3.06)
$A_{Ift} \times D(I_{ft})_H$	0.010 (12.14)	0.017 (10.33)	0.021 (9.22)	0.023 (8.40)	0.025 (7.82)	0.026 (7.36)	0.026 (7.05)

Table presents results on the differential rate of firm displacement following innovation by competitors ( $A_{If}$ ) depending on the firm's measure of growth opportunities (Tobin's  $Q$  or past investment rate). We estimate equation (53) in the data (Panel A) and in simulated data from the model (Panel B). Sample period is 1950-2008. Accounting data are from Compustat; investment rate is growth rate in property, plant and equipment (ppeg); Tobin's  $Q$  is CRSP market capitalization, plus book value of debt (dltt), plus book value of preferred shares (pstkvr), minus deferred taxes (txdb) divided by book assets (at); output  $y$  is sales (sale) plus change in inventories (invt). We include a vector of controls  $Z$  containing industry effects; time effects; firm size; lagged output growth; and firm and industry stock returns. We cluster the standard errors by firm. We scale the variables  $A_f$  and  $A_{If}$  to unit 90-50 range and unit standard deviation respectively.

**Table 8: Innovation and return comovement**

A. Data							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	0.17 (4.49)	0.03 (0.84)	-0.03 (-1.35)	-0.08 (-1.07)	-0.12 (-2.14)	-0.22 (-3.91)	-0.39 (-4.74)
$R_{mt} - r_f$	1.21 (23.61)	0.94 (21.98)	0.92 (27.87)	0.97 (7.33)	0.90 (7.86)	0.99 (9.24)	-0.22 (-1.58)
$R^2$	86.87	90.65	93.23	75.61	75.46	74.40	18.69
B. Model							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	-0.13 (-2.35)	-0.05 (-1.44)	-0.04 (-1.01)	0.09 (2.03)	0.17 (3.55)	0.12 (1.75)	0.25 (3.17)
$R_{mt} - r_f$	1.02 (14.07)	0.96 (15.03)	0.80 (13.48)	1.13 (15.54)	1.40 (17.26)	1.49 (13.95)	0.47 (4.33)
$R^2$	79.15	85.22	79.42	88.18	86.66	80.46	19.16
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	0.43 (5.74)	0.31 (5.32)	0.20 (4.18)	-0.07 (-2.54)	-0.11 (-3.57)	-0.17 (-4.75)	-0.61 (-6.27)
$R_{mt} - r_f$	0.96 (22.83)	0.96 (29.18)	0.97 (36.59)	0.99 (62.88)	1.00 (60.24)	1.01 (49.89)	0.05 (0.92)
$R^2$	89.50	93.68	96.07	98.89	98.85	98.49	54.33
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	-0.17 (-5.00)	-0.07 (-2.15)	0.02 (0.62)	-0.06 (-2.07)	0.03 (1.04)	0.21 (4.59)	0.38 (5.75)
$R_{mt} - r_f$	0.99 (51.11)	0.99 (54.84)	0.98 (52.80)	1.00 (60.31)	1.00 (58.34)	0.99 (40.34)	-0.00 (-0.01)
$R^2$	98.52	98.65	98.48	98.87	98.72	97.05	44.98

Table relates our innovation measure  $A$  to stock returns of portfolios sorted on book to market (Part I) and past investment (Part II). We estimate equation (54) in the data (Panel A) and in the model (Panel B). Sample period is 1950-2008. See notes to Table 2 for details on portfolio construction. See main text and Kogan et al. (2012) for details on the construction of  $A$ . Standard errors are computed using Newey-West with 3 lags.



**Table 9: Asset pricing tests**

4 B/M + 4 I/K portfolios								
Factor	A. Data				B. Model			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\Delta \ln X_t$	3.19 [3.99]	-0.54 [-1.08]			0.69 [4.22]	-0.13 [-1.06]		
$\Delta \ln C_t$			1.97 [3.70]	-0.92 [-1.02]			0.88 [4.36]	-0.19 [-0.89]
$\Delta \ln A$		-0.83 [-3.64]		-1.03 [-3.88]		-1.01 [-6.90]		-1.15 [-6.52]
$R^2$	-110.61	64.70	-63.68	74.61	-28.79	82.77	-8.57	81.96
MAPE	3.10	1.06	2.53	1.00	2.86	0.86	2.45	0.87

Table presents results of estimating the stochastic discount factor implied by the model (equation (55) in main text) in the data (Panel A) and in simulated data from the model (Panel B). Sample period is 1950-2008. See notes to Table 2 for details on portfolio construction. See main text and Kogan et al. (2012) for details on the construction of innovation measure  $A$ . Total factor productivity  $X$  is from Basu et al. (2006). Consumption  $C$  is non-durables plus services from NIPA, deflated by CPI and population growth. Standard errors are computed using Newey-West with 3 lags.

Figure 1: Model Solution

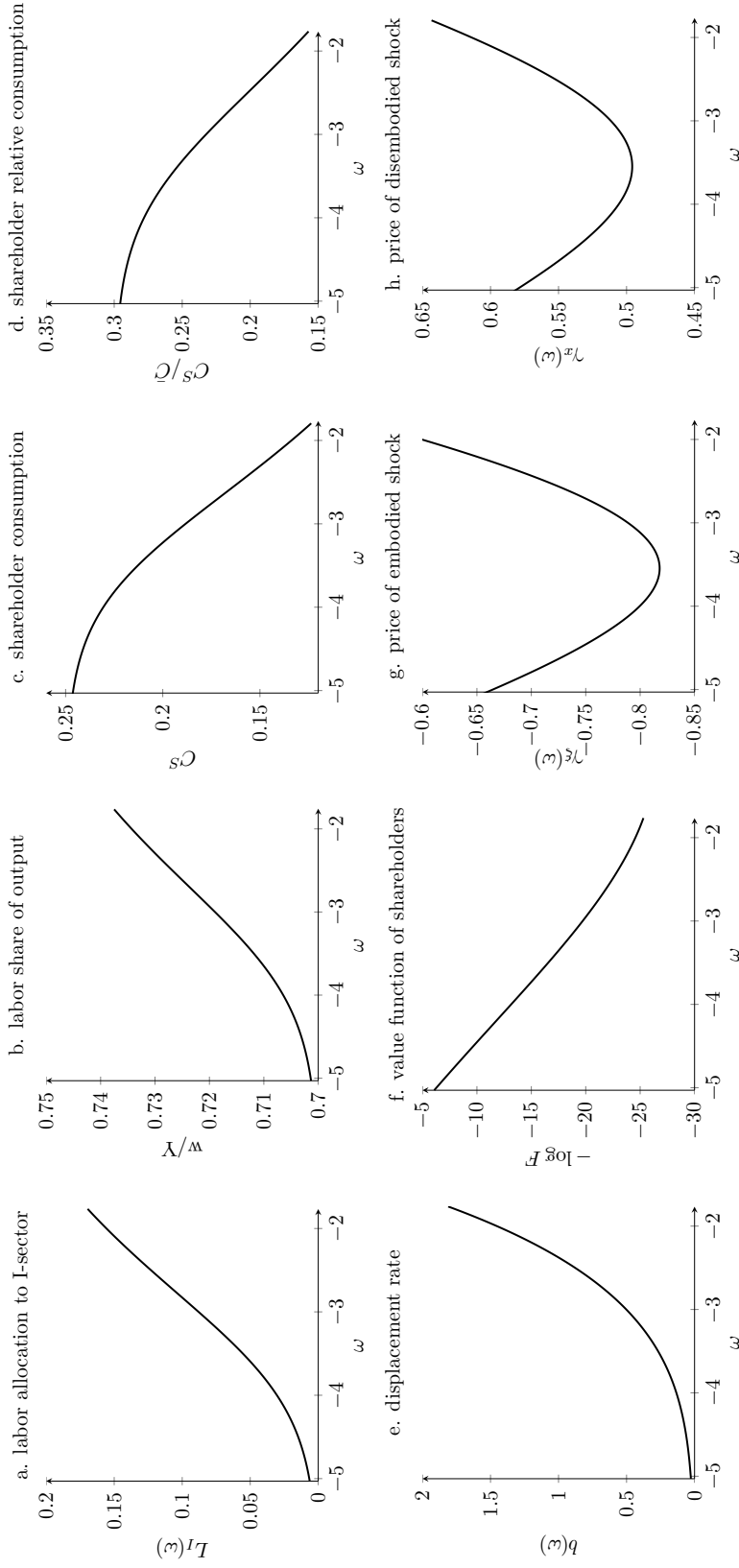


Figure plots the numerical solution to the model as a function of the state variable  $\omega$ . We plot the solution in the relevant range of  $\omega$  based on its stationary distribution.

Figure 2: Firm return sensitivity to innovation shock

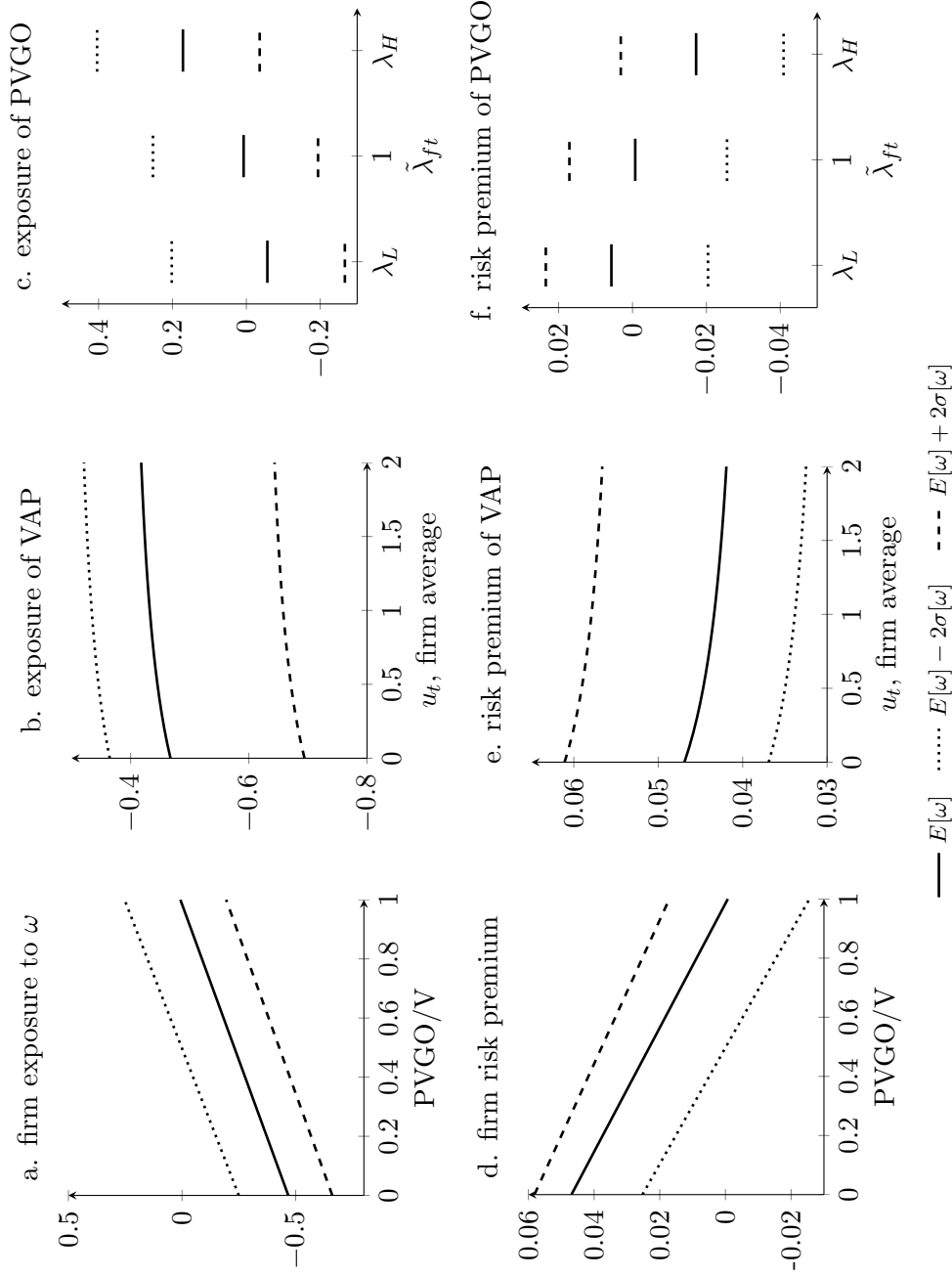
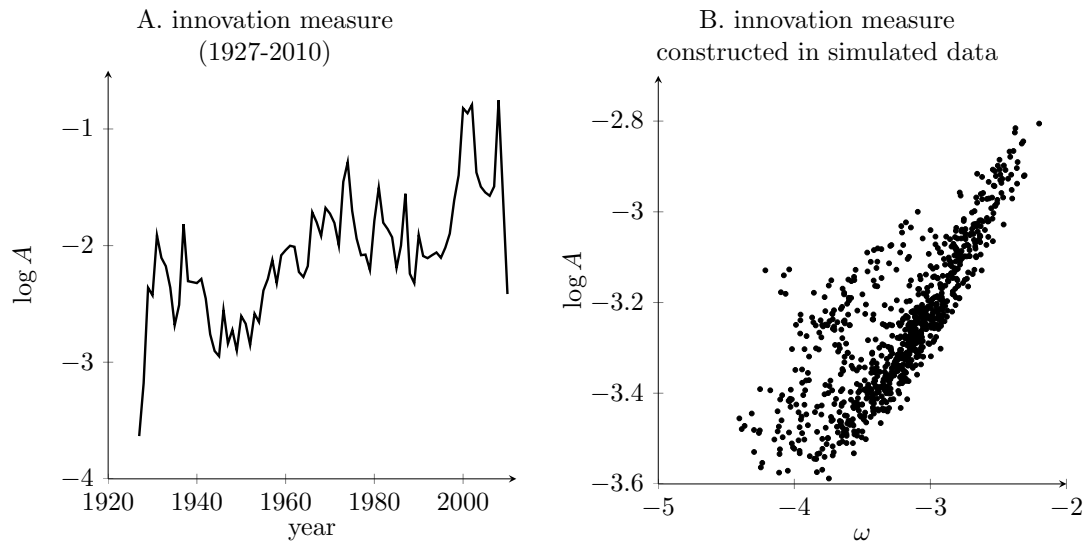


Figure plots the exposure of the firm to changes in the state variable  $\omega$  as a function of its characteristics. We plot the sensitivity of firm value to  $\omega$  as a function of the share of growth opportunities to firm value (panel A); the sensitivity of assets in place as a function of the average project-specific shock at the firm level,  $u$  (panel B); and the sensitivity of growth opportunities as a function of the current growth state of the firm (panel C).

**Figure 3: Aggregate measure of innovation**



In panel A we plot the innovation measure of Kogan et al. (2012) in the data. In Panel B we compare the empirical innovation measure constructed in simulated data to the state variable  $\omega$  over the relevant range of  $\omega$  based on its stationary distribution.

## A Analytical Appendix

In order to solve the fixed point problem, we conjecture that the equilibrium allocation of labor  $L_I$  is only a function of the stationary variable  $\omega$ . We verify that this is indeed the case below.

First, we characterize the consumption allocation. Workers consume their wage (see equation (32)), and shareholders consume the residual. Furthermore, all inventors have the same consumption-to-wealth ratio. As a result, the inventor's share of financial wealth  $b_{ts}$  defined in (29) also determines the fraction of total consumption available to shareholders that he consumes

$$\begin{aligned} C_{ts}^S &= b_{ts} (C_t - C_t^W) \\ &= b_{ts} e^{\chi t} \left( (1 - L_I(\omega))^{1-\phi} - (1 - \phi)(1 - L_I(\omega))^{-\phi} \right), \end{aligned} \quad (\text{A.1})$$

since

$$\bar{C}_t = e^{\chi t} (1 - L_I(\omega_t))^{1-\phi} \quad (\text{A.2})$$

we can write

$$C_{ts}^{1-h} \left( \frac{C_{ts}}{\bar{C}_t} \right)^h = b_{ts} e^{(1-h)\chi t} \hat{l}(\omega_t)$$

where

$$\hat{l}(\omega) \equiv \left( (1 - L_I(\omega))^{1-\phi} - (1 - \phi)(1 - L_I(\omega))^{-\phi} \right) (1 - L_I(\omega_t))^{-h(1-\phi)}.$$

Given the equilibrium consumption process A.1, the value function of an inventor born in time  $s$  is given by

$$J_{ts} = \frac{1}{1-\gamma} b_{ts}^{(1-\gamma)} e^{(1-\gamma)(1-h)\chi t} f(\omega_t), \quad (\text{A.3})$$

where the function  $f$  satisfies the ODE

$$0 = \rho \frac{1-\gamma}{1-\theta^{-1}} \hat{l}(\omega)^{1-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + \rho_f(\omega) f(\omega) + \mathcal{A}f(\omega)$$

where the operator  $\mathcal{A}$  is defined as

$$\mathcal{A}f(\omega) \equiv f'(\omega) \left( \mu_\xi + \delta + \alpha \mu_x + (1-\gamma)(1-\phi)\alpha\sigma_x^2 - \lambda e^\omega \left( \frac{L_I(\omega)}{\lambda} \right)^\alpha \right) + \frac{1}{2} f''(\omega) (\sigma_\xi^2 + \alpha^2 \sigma_x^2), \quad (\text{A.4})$$

and

$$\rho_f(\omega) = -\frac{\rho(1-\gamma)}{1-\theta^{-1}} + (1-\gamma)(\mu - \kappa(\omega)) + (1-h)(1-\gamma) \left( (1-\phi)\mu_x - \phi\delta + \phi\lambda^{1-\alpha} e^\omega L_I(\omega)^\alpha \right) + \frac{1}{2} (1-\phi)^2 \sigma_x^2 (1-\gamma)^2 (1-h)^2. \quad (\text{A.5})$$

Given the consumption allocation (A.1) and the inventor's value function (A.3), we compute the stochastic discount factor,

$$\pi_t = \exp \left( \int_0^t \tilde{f}_J(C_s, \bar{C}_s, J_s) ds \right) \tilde{f}_C(C_t, C_t, J_t),$$

where

$$h_{C,ts} = \rho(e^{\chi t})^{-\hat{\gamma}} b_{ts}^{-\gamma} l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \quad (\text{A.6})$$

$$l(\omega_t) \equiv \left( \left( (1 - L_I(\omega_t))^{1-\phi} - (1 - \phi)(1 - L_I(\omega_t))^{-\phi} \right) (1 - L_I(\omega_t))^{-s(1-\phi)} \right)^{\theta^{-1}} \hat{l}(\omega_t) \quad (\text{A.7})$$

$$\hat{\gamma} \equiv \gamma(1 - h) + 1 \quad (\text{A.8})$$

$$h_J(C, J) = -\frac{\rho}{1 - \theta^{-1}} \left( (\gamma - \theta^{-1}) \left( \hat{l}(\omega_t) \right)^{1-\theta^{-1}} \left( f(\omega_t) \right)^{\frac{1-\theta^{-1}}{\gamma-1}} + (1 - \gamma) \right). \quad (\text{A.9})$$

Next, we determine the value of assets in place and growth opportunities. First, we solve for the two functions  $P$  and  $\tilde{P}$  that determine the value of existing projects (34)

$$P_t = \phi e^{\chi t} K_t^{-1} \left( l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} \nu(\omega_t) \quad (\text{A.10})$$

$$\tilde{P}_t = \phi e^{\chi t} K_t^{-1} \left( l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} \tilde{\nu}(\omega_t), \quad (\text{A.11})$$

where  $\nu(\omega)$  and  $\tilde{\nu}(\omega)$  solve the ODEs

$$0 = (1 - L_I(\omega))^{1-\phi} l(\omega)^{-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + \rho_\nu(\omega) \nu(\omega) + \mathcal{A} \nu(\omega) \quad (\text{A.12})$$

$$0 = (1 - L_I(\omega))^{1-\phi} l(\omega)^{-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} + (\rho_\nu(\omega) - \theta_u) \tilde{\nu}(\omega) + \mathcal{A} \tilde{\nu}(\omega), \quad (\text{A.13})$$

and the function  $\rho_\nu$  is given by

$$\begin{aligned} \rho_\nu(\omega) = & -\frac{\rho}{1 - \theta^{-1}} \left( (\gamma - \theta^{-1}) \hat{l}(\omega_t)^{1-\theta^{-1}} f(\omega_t)^{\frac{1-\theta^{-1}}{\gamma-1}} + (1 - \gamma) \right) + \gamma(\kappa(\omega) - \mu) + \\ & + ((1 - \gamma)\phi - 1) \lambda^{1-\alpha} e^{\omega} L_I(\omega)^\alpha + (1 - h)(1 - \gamma)((1 - \phi)\mu_x - \phi\delta) + \frac{1}{2}(1 - \gamma)^2(1 - h)^2(1 - \phi)^2 \sigma_x^2. \end{aligned} \quad (\text{A.14})$$

Using (A.10) and (A.11), the value of a firm's existing assets can be written as

$$\begin{aligned} VAP_{ft} = & \phi e^{\chi t} \left( l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} \\ & \times \left( \nu(\omega_t) \sum_{j \in \mathcal{J}_{ft}} \varepsilon^{\xi_j} k_j^\alpha / K_t + \tilde{\nu}(\omega_t) \sum_{j \in \mathcal{J}_{ft}} \varepsilon^{\xi_j} k_j^\alpha (u_{j,t} - 1) / K_t \right). \end{aligned} \quad (\text{A.15})$$

The relative contribution of the functions  $\nu$  and  $\tilde{\nu}$  in the value of assets in place depends on the size and profitability of existing projects, as we can see from the last term in (A.15).

Second, we solve for the two functions  $\Gamma^H$  and  $\Gamma^L$  that determine the value of growth opportunities

$$\Gamma_t^H = (1 - \alpha) e^{\chi t} \left( l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} \left( g(\omega_t) + (\lambda_H - \lambda_L) \frac{\mu_L}{\mu_L + \mu_H} \tilde{g}(\omega_t) \right) \quad (\text{A.16})$$

$$\Gamma_t^L = (1 - \alpha) e^{\chi t} \left( l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} \left( g(\omega_t) - (\lambda_H - \lambda_L) \frac{\mu_H}{\mu_L + \mu_H} \tilde{g}(\omega_t) \right) \quad (\text{A.17})$$

where  $g(\omega)$  and  $\tilde{g}(\omega)$  solve the ODEs

$$0 = \nu(\omega)e^\omega \left( \frac{L_I(\omega)}{\lambda} \right)^\alpha + \rho_g(\omega)g(\omega) + \mathcal{A}g(\omega) \quad (\text{A.18})$$

$$0 = \nu(\omega)e^\omega \left( \frac{L_I(\omega)}{\lambda} \right)^\alpha + (\rho_g(\omega) - \mu_L - \mu_H)\tilde{g}(\omega) + \mathcal{A}\tilde{g}(\omega), \quad (\text{A.19})$$

and the function  $\rho_g$  is given by

$$\rho_g(\omega) \equiv \rho_\nu(\omega) + \lambda^{1-\alpha}e^\omega L_I(\omega)^\alpha. \quad (\text{A.20})$$

Using (A.16)-(A.17) the value of the firm's growth opportunities (??) equals

$$\begin{aligned} PVGO_{ft} &= \lambda_f(1-\eta)(1-\alpha)e^{\lambda t} \left( l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \\ &\quad \times \left[ g(\omega_t) + \left( p_{ft} - \frac{\mu_H}{\mu_L + \mu_H} \right) (\lambda_H - \lambda_L) \tilde{g}(\omega_t) \right], \end{aligned} \quad (\text{A.21})$$

so the contribution of the functions  $g$  and  $\tilde{g}$  to the value of growth opportunities depends on current growth state of the firm  $p_{ft}$ .

Aggregating (A.15) and (A.21) across firms, the aggregate value of assets in place and growth opportunities is

$$VAP_t = \phi e^{\lambda t} \left( l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \nu(\omega_t) \quad (\text{A.22})$$

$$PVGO_t = \bar{\lambda}(1-\eta)(1-\alpha)e^{\lambda t} \left( l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} g(\omega_t). \quad (\text{A.23})$$

Given (A.22) and (A.23), we next determine the amount of inter-generational displacement

$$b_{tt} = b(\omega) \equiv \frac{\bar{\lambda}\eta(1-\alpha)\phi\nu(\omega)e^\omega \left( \frac{L_I(\omega)}{\lambda} \right)^\alpha}{\phi\mu\nu(\omega) + \bar{\lambda}\mu(1-\eta)(1-\alpha)g(\omega)}. \quad (\text{A.24})$$

The last step is to determine the equilibrium allocation between the two sectors  $L_I$  and verify that it depends only on  $\omega$ . The first order condition (27) simplifies to

$$(1-\phi)(1-L_I)^{-\phi} = \alpha\phi e^{\omega t} \left( l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \nu(\omega_t) \left( \frac{\bar{\lambda}}{L_I} \right)^{1-\alpha}. \quad (\text{A.25})$$

Last, the functions characterizing firm's exposure to changes in aggregate growth opportunities are

$$\begin{aligned} \zeta_\nu(\omega) &= \ln \left( \left( l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \nu(\omega) \right), & \zeta_{\tilde{\nu}}(\omega) &= \ln \left( \frac{\tilde{\nu}(\omega)}{\nu(\omega)} \right), \\ \zeta_g(\omega) &= \ln \left( \left( l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} g(\omega) \right), & \zeta_{\tilde{g}}(\omega) &= \ln \left( \frac{\tilde{g}(\omega)}{g(\omega)} \right), \end{aligned} \quad (\text{A.26})$$