Oligopolistic Business Cycle Amplification*

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Abstract

We develop a tractable dynamic general equilibrium model with a continuum of different industries, each comprising a finite number of strategic price-setting firms. Each industry coordinates on profit-maximizing markups taking as given the strategic behavior of all other industries. The strategic behavior of all firms jointly determines the level of aggregate consumption in each state, which in turn determines the pricing kernel of the economy. Deviations from efficient consumption are determined state-by-state by the cross-sectional heterogeneity of markups across industries. Heterogeneity of markups rather than the level matters, as it distorts relative goods prices and hence causes deviations from the efficient allocation of labor to industries in the economy. Markups in one particular industry can either be procyclical or countercyclical depending on the risk aversion of the representative agent and the correlation of sector-specific productivity with aggregate consumption. General equilibrium in the model is shown to exist under general conditions. Strategic interaction between firms amplifies business cycles, and the equilibrium outcome can be very sensitive to small changes in long-term growth rates whereas temporary changes have only marginal impact.

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1 Introduction

In 1973 the OPEC cartel raised oil prices, precipitating a global recession. This event was unique but not unprecedented: Several of the largest shocks in financial markets as well as in the real economy in the last century originated from the strategic behavior of firms and other entities. At the industry level, strategic interactions, e.g., through collusion, price wars and new entry, fundamentally determines the profitability and value creation of firms, and shocks are the norm rather than the exception. The general equilibrium asset pricing literature does not speak to such strategic interactions. Typically, in this literature, shocks to the economy are "technological," driven by exogenous processes. However, history suggests that technological shocks, although important, do not capture the whole story. Understanding the general equilibrium effects of such strategic behavior should therefore be an important goal for asset pricing and macro economics more broadly.

We analyze a discrete time, infinite horizon general equilibrium model with many different industries, in which firms within each industry interact strategically in a product market. Specifically, we embed a dynamic oligopoly game in each industry into a general equilibrium model with a continuum of production technologies similar to the seminal paper by Rotemberg and Woodford (1992). Our key departure from their paper is the introduction of sector heterogeneity (in terms of productivity and the number of players) which drives all of our novel implications, in particular endogenous business cycles driven by asynchronous behavior of industries. Within each industry, we model price competition as the subgame perfect outcome in which firms choose their markups. In each period, each firm weighs the value of high short-term profits that can be obtained by aggressive pricing, against the long-term profits that are obtained when firms cooperate. All profits are valued in a standard consumption based asset pricing framework by a representative agent who consumes the goods and values future (risky) cash flows. While each industry takes the macro dynamics, in particular aggregate consumption, as given, industries jointly affect these macro dynamics in a non-trivial way, as long as the different sectors' equilibrium strategies are not perfectly aligned.

Most of the intuition in our paper comes from the interplay between asset pricing and industrial organization. This is because aggregate consumption is endogenous and depends on the output in each industry and also, through the pricing kernel, affects the ability of firms in each industry to sustain collusive outcomes. A small productivity shock in one industry can have an economy-wide effect because firms' profits, values and returns therefore vary with the changing competitive environment. The effects of strategic interaction is important in the sense that small technological shocks can have a drastic impact on the equilibrium outcome and on asset prices, through the mechanism of strategic competition.

Our paper makes two types of contributions. First, given that the setting with multiple industries and strategically interacting firms is fairly complex, we spend substantial effort on developing a rigorous understanding of the model, deriving general equilibrium existence results and exploring the properties of the model. Our main result in this part of the paper is Proposition 5, which shows the existence of equilibrium under minimal assumptions.

Second, we provide several results that show the effect of strategic interaction on the economy's equilibrium. Specifically, strategic interaction can amplify, and even generate, business cycles endogenously. The welfare costs of such interaction are the highest in economies in which competitiveness varies a lot across industries, whereas they are low when the variation is low, even in the case when all industries are monopolistic. Further, business cycle fluctuations are most severe in economies in which there is considerable dispersion across industries' in the ability to sustain high markups when aggregate productivity is low. The equilibrium outcome can be very sensitive to small changes in long term growth-rates whereas temporary changes, on the other hand, have at most a marginal impact. Finally, the economy may have multiple qualitatively very different equilibria. There is therefore a role for policy in our framework.

Our paper is related to the Industrial Organization literature on strategic competition over the business cycle (see, e.g., Bagwell and Staiger, 1997). Our model takes this literature as a starting point, but extends the approach to allow for multiple industries and endogenous pricing of risk in an economy with risk averse agents. This general equilibrium approach connects it to an extensive literature on business cycles (e.g., Kydland and Prescott, 1982; Long and Possner, 1983; Gabaix, 2011; Acemoglu et al., 2011). Although our model is based on a similar framework as real business cycle models (Kydland and Prescott, 1982, Long and Possner, 1983), equilibrium dynamics may be quite different. Specifically, significant business cycle fluctuations may arise even when aggregate "technological" shocks are small. A recent strand of literature has aimed at explaining how technological shocks at the individual firm or industry level do not diversify out, but may affect aggregate productivity. Gabaix (2011) notes that if the distribution of firm size is heavy-tailed, firm-specific shocks may indeed affect aggregate productivity. Acemoglu et al. (2011), suggest that inter-sectoral input-output linkages between industries may lead to "cascades effects" where a shock in one industry spreads through the economy and thereby becomes an aggregate shock.

The mechanism in our model is quite different, more along the lines suggested in Jovanovic (1987), who shows that idiosyncratic shocks may not cancel out in strategic games with a large number of players. We develop examples, in which aggregate productivity is close to constant across states, but because it varies at the sectoral level, the strategic behavior of firms leads to aggregate shocks in equilibrium. We believe that this provides an important mechanism for understanding the sources of aggregate fluctuations in the economy.

Our results highlight how strategic interaction between firms (as an alternative to technological shocks) can generate endogenous business cycle fluctuations that are not just the result of self-fulfilling expectations as in Gali (1994) or Schmitt-Grohe (1997); who build on pioneering work on sunspots by Cass and Shell (1983). Our key contribution is to allow for multiple, *heterogeneous* sectors in which welfare distortions arise from dispersion of markups in the cross-section. The formal expression for the welfare cost is similar to the distortions that arise in sticky-price models in the spirit of Calvo (1983). In contrast to these models, however, prices in our model are fully flexible and are determined *endogenously* as the outcome of a strategic game in each sector. Economically, our results suggest that policy makers as well as empirical studies on markups should not only pay attention to aggregate levels of markups, but also their cross-sectional dispersion.

To get the many different pieces of the model to fit together in a consistent and tractable framework, we have been forced to think carefully about modeling choices. To have a strategic trade-off among firms between competition and cooperation, multiple periods are needed, and an infinite horizon economy turned out to be the most tractable. Given the intricacies of strategic games in continuous time, a discrete approach turned out to be superior to a continuous time approach, especially since the increased tractability of the continuous time setting came at the cost of *ad hoc* assumptions needed about the profitability of firms after off the equilibrium path moves. By choosing a discrete state spaces with time invariant Markov transition processes we were able to use the powerful analytical tools available for such processes. Finally, having a continuum of industries, each of which containing a finite number of firms, allowed us to assume that each firm takes the market price of risk as given, while still generating aggregate general equilibrium effects of firms' strategic behavior.

The rest of the paper is organized as follows. In Section 2 we present the economic framework of the model. The equilibrium analysis of outcomes in each industry is presented in Section 3. Section 4 shows the existence of general equilibrium, and Section 5 analyzes how endogenous business cycles can arise. All proofs are delegated to the Appendix.

2 Model Framework

2.1 Physical Environment

Consider an infinite horizon, discrete time, discrete state economy in which time is indexed by $t \in \mathbb{Z}_+$ and the time t state of the world is denoted by $s_t \in \{1, 2, \dots S\}$.[1] Each period there is a transition between states which is governed by a Markov process with time invariant transition probabilities:

$$\mathbb{P}(s_{t+1} = j | s_t = i) = \Phi_{i,j}. \tag{1}$$

Here, $\Phi_{i,j}$ refers to the element on the ith row and jth column of the matrix $\Phi \in \mathbb{R}_+^{S \times S}$. We assume that Φ is irreducible and aperiodic, so that the process has a unique long-term stationary distribution.

2.1.1 Production

There is a continuum of industries, indexed by $z \in [0, 1]$, each consists of $N(z) \geq 1$ identical strategic firms that produce and sell a unique non-storable consumption good. The nature of the strategic environment is discussed in Section 2.2. The production technology for each good z at time t is linear in labor with stochastic productivity $A(z, t) = A_{s_t}(z)(1 + g)^t$. (That is, one unit of consumption good of z at time t in state s requires $\left[A_s(z)(1 + g)^t\right]^{-1}$ labor units.) Here, with some abuse of notation, $A_{s_t}(z)$ represents a state-dependent and sector-specific productivity component whereas $g \geq 0$ represents a common long-term productivity growth rate across all sectors. For tractability we assume that $A : S \times [0, 1] \to \mathbb{R}_{++}$ is a function that satisfies standard integrability conditions so that aggregation across industries is possible. Labor is supplied inelastically by a representative agent, who in each period divides her one unit of human capital across all the industries and earns a competitive wage, $w(t)$, in return.

[1]Here, $\mathbb{Z}_+ = \{0\} \cup \mathbb{N} = \{0, 1, \dots\}$ is the set of non-negative integers. Also, we follow the standard convention that $\mathbb{R}_+$ is the set of nonnegative real numbers, whereas $\mathbb{R}_{++}$ is the set of strictly positive real numbers.

2.1.2 Preferences / Demand

The representative agent possesses CRRA preferences over aggregate consumption with risk aversion parameter γ and subjective discount factor $\hat{\delta}$, i.e.,:

$$U = \mathbb{E}\left[\sum_{t=0}^{\infty} \hat{\delta}^t \frac{C(t)^{1-\gamma}}{1-\gamma}\right], \tag{2}$$

where $C(t)$ represents the Dixit-Stiglitz *CES* consumption aggregator of goods (see Dixit and Stiglitz, 1977).[2]

$$C(t) = \left(\int_0^1 c(z,t)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}. \tag{3}$$

The parameter $\theta > 1$ is the (constant) elasticity of substitution across goods. We note in passing that preferences with a more general state dependent utility specification are also covered by our specification.[3] The *CES* specification leads to standard *period-by-period* demand functions as a function of prices $p(z,t)$ and total income $y\,(t)$:[4]

$$c(z,t) = \frac{y\,(t)}{p(z,t)^{\theta} P\,(t)^{1-\theta}}, \tag{4}$$

where $P(t) \equiv \left(\int_0^1 p(z,t)^{1-\theta} dz\right)^{\frac{1}{1-\theta}}$ can be interpreted as the ideal price index, total income $y\,(t)$ is derived from wages, and distribution of firm profits, $\pi(z,t)$, across all sectors z:

$$y\,(t) = w\,(t) + \int_0^1 \pi(z,t) dz, \tag{5}$$

$$\pi(z,t) = \left[p(z,t) - \frac{w\,(t)}{A\,(z,t)}\right] c(z,t). \tag{6}$$

Going forward, we will normalize the nominal price index $P(t)$ to 1.[5] Hence, income $y\,(t) = C\,(t)$, wages and profits are measured in units of aggregate consumption.

2.2 Strategic Environment

Within each industry z, $N\,(z)$ identical firms play a dynamic Bertrand pricing game with perfect public information as in Rotemberg and Saloner (1986). The timing of the stage game in each period, t, is as follows. First, the state, s_t is revealed. Then all firms $i \in \{1, 2, \ldots N\,(z)\}$ in industry z simultaneously announce their gross markup, $Q^{(i)}(z,t)$. For tractability, we express each

[2] See van Binsbergen (2007) or Ravn et al. (2006) for using *CES* preferences in a dynamic context.

[3] Consider the more general $\tilde{C}(t) = \left(\int_0^1 v_{s_t}(z) c(z,t)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}$ as in Opp (2010). The state dependent "taste" function $v_s(z)$ can then easily be reduced to the case where $v_s(z) \equiv 1$, by transforming the productivity, $A_s(z) \mapsto v_s(z)^{(\theta-1)/\theta} A_s(z)$. Such a transformation can be interpreted as a numeraire change, where the amount of a unit of goods is redefined in each state. A state dependent taste function could, for example, represent an agent's higher utility of an umbrella in a rainy state than in a sunny state of the world.

[4] The demand functions $c\,(z,t)$ yield maximal $C\,(t)$ given an arbitrary price vector $p\,(z,t)$ and income $y\,(t)$. They are obtained via simple first-order conditions.

[5] This is without loss of generality, since the wage rate $w\,(t)$ is a free variable.

firm's strategy in terms of gross markups instead of prices, satisfying $p^{(i)}(z,t) = Q^{(i)}(z,t)\frac{w(t)}{A(z,t)}$. Consumers demand the product from the producer with the lowest markup. If all firms announce the same Q, total demand in sector z is evenly shared between all $N(z)$ firms. The firms then go out and hire workers to meet demand.

Each industry z coordinates on the *symmetric, subgame* perfect equilibrium outcome that exhibits a maximal degree of collusion, i.e., maximal industry profits. Due to symmetry, the equilibrium gross markup function of each firm i satisfies: $Q^{(i)}(z,t) = Q(z,t)$, with the associated industry price

$$p(z,t) = Q(z,t)\frac{w(t)}{A(z,t)}. \tag{7}$$

While the equilibrium outcome of this game is in general non-trivial (see Section 3.3), the two polar cases of a monopoly, i.e., $N(z) = 1$, and perfect competition provide useful bounds. If the industry is served by a monopolist, he maximizes industry profits (equation 6) subject to consumer demand (equation 4) which leads to an optimal markup of:

$$Q^m(z,t) = Q^m = \frac{\theta}{\theta - 1}. \tag{8}$$

If, on the other hand, $N(z)$ is infinite, then we expect prices to be set competitively. In this case, the markup is 1. If the number of firms is finite but greater than one, we expect equilibrium markups to be somewhere in between the competitive and monopolistic prices, i.e., $Q \in \left[1, \frac{\theta}{\theta-1}\right]$.[6]

3 Analysis

Before proceeding with our formal equilibrium analysis, it is convenient to transform our growing economy into a time-invariant economy in which outcomes only depend on time t through the state at time s_t. The resulting implications and other normalizations are presented in Section 3.1.

Our partial equilibrium analysis consists of two parts. First, for an arbitrary exogenous distribution of markups across industries, we characterize aggregate consumption, and show that it together with a measure of aggregate markups determines the efficiency losses in the economy (Section 3.2). Second, given the aggregate consumption and aggregate markup dynamics, we solve for the partial equilibrium outcome of one sector z in the economy, i.e., the optimal state-contingent markups (Section 3.3).

3.1 Preliminaries

We focus on equilibria which—except for the constant growth rate g—are time invariant in that outcomes are the same at t_1 and t_2 if the states are the same, i.e., if $s_{t_1} = s_{t_2}$. In other words,

[6]Although the condition that $Q \in \left[1, \frac{\theta}{\theta-1}\right]$ arises naturally, for the results in this section it is not crucial, and we need only assume the weaker condition that Q is strictly positive for all z and s.

we study equilibria on the form:

$$C(t) = (1+g)^t C_{s_t}, \tag{9}$$
$$y(t) = (1+g)^t y_{s_t}, \tag{10}$$
$$w(t) = (1+g)^t w_{s_t}, \tag{11}$$
$$\pi(z,t) = (1+g)^t \pi_{s_t}(z), \tag{12}$$
$$c(z,t) = (1+g)^t c_{s_t}(z), \tag{13}$$

where variables on the right hand side are growth-normalized, time invariant, variables which only depend on the state, s_t. In such an economy we immediately obtain that markups are time-invariant

$$Q(z,t) = Q_{s_t}(z). \tag{14}$$

Thus, our following analysis will suppress the time t dependence of all real variables and use the state-dependent form. We note that this does not in any way restrict the off-equilibrium path behavior, it is a restriction on the on-equilibrium path behavior of agents, and the corresponding equilibrium outcomes.

It follows from a standard transformation, using the utility representation (equation 2), that growth-normalized variables can be determined by solving the model for a non-growing economy with a growth-adjusted personal discount rate, i.e., with

$$\delta \stackrel{\text{def}}{=} (1+g)^{1-\gamma}\hat{\delta}. \tag{15}$$

Intuitively, the representative agent's tradeoff between consumption in different times and states is affected in identical ways by changes in the growth rate and the personal discount rate. We will use the formulation with δ going forward.

For ease of exposition, we decompose productivity shocks $A_s(z)$ into the functions $\alpha_s(z)$ and $\bar{A}_s$ where $\alpha : S \times [0,1]$ and the vector $\bar{A} \in \mathbb{R}^S_+$. Specifically,

$$\alpha_s(z) \equiv \frac{A_s(z)^{\theta-1}}{\int_0^1 A_s(z)^{\theta-1}dz} = \left(\frac{A_s(z)}{\bar{A}_s}\right)^{\theta-1}, \qquad \text{where} \tag{16}$$
$$\bar{A}_s \equiv \left[\int_0^1 A_s(z)^{\theta-1}dz\right]^{\frac{1}{\theta-1}}. \tag{17}$$

Here, $\bar{A}$ represents the average productivity shock to the economy and $\alpha_s(z)$ captures the industry productivity shock relative to the economy. In other words, changes in $\alpha(z)$ across states are *idiosyncratic* shocks to individual industries, whereas changes in $\bar{A}$ are *systematic* shocks. We can also view $\alpha(z)$ as an S-vector, $\alpha(z) \in \mathbb{R}^S$.

As a result of the normalization, the average relative industry state is equal to one, i.e., $\int_0^1 \alpha_s(z)dz = 1$. Now instead of specifying A, we can equivalently specify the function of idiosyncratic shocks, α, and the vector of systematic shocks, $\bar{A} \in \mathbb{R}^S_{++}$. Given the previous argument, the exogenous variables in the economy can then be represented by the tuple $\mathcal{E} = (\alpha, \bar{A}, N, \Phi, \theta, \gamma, \delta)$.

3.2 Aggregate Consumption

Aggregate consumption is an important endogenous variable. As outlined above, we will first treat the outcome of the strategic game for each industry and each state as exogenously given, as summarized by the gross markup functions $Q_s(z)$. Together with the functions, $\alpha_s(z)$ and $\bar{A}_s$, the real outcome in the economy or the consumer's consumption bundle is completely determined, state-by-state. We will use aggregate consumption in two ways. First, as a measure of welfare and second to value random future profit streams.

3.2.1 Pareto Efficiency and Aggregate Markups

This section reveals that our economy can exhibit Pareto inefficient outcomes which distinguishes our paper from a standard real business cycle model.[7] Our inefficiency is generated by firms' value-maximizing, strategic price setting behavior and the attendant distortion in labor allocation.

For ease of exposition, we introduce two sufficient statistics of the cross-sectional markup distributions for the macro-economy in each state s:

$$\bar{Q}_s = M_{1-\theta}\left(Q_s\right), \tag{18}$$

$$e_s = \left(\frac{M_{-\theta}\left(Q_s\right)}{M_{1-\theta}\left(Q_s\right)}\right)^{\theta} \leq 1. \tag{19}$$

where $M_p\left(Q_s\right) = \left(\int \alpha_s(z) Q_s\left(z\right)^p dz\right)^{\frac{1}{p}}$ refers to the p-th order cross-sectional power mean of $Q_s\left(z\right)$.[8] Both sufficient statistics capture distinct elements of the cross-sectional markup distribution. The variable $\bar{Q}_s$ captures the notion of aggregate market power, i.e., an appropriate average markup across industries. The variable e_s captures the (inverse of) heterogeneity of markups across industries. By Jensen's inequality, e_s is bounded above by one (obtained when all industries charge the same markup) and is decreasing in the heterogeneity of markups.[9] It can also be interpreted as a measure of relative production efficiency, as the following proposition reveals.

Proposition 1. *Given the functions Q_s, α_s and $\bar{A}_s$, aggregate consumption, C_s, real income y_s, in state s are given by:*

$$C_s = y_s = \bar{A}_s e_s. \tag{20}$$

The fraction of real *income that is derived from labor income is given by:*

$$\omega_s = \frac{1}{e_s \bar{Q}_s}. \tag{21}$$

Real *firm profits in sector z are:*

$$\pi_s\left(z\right) = C_s \alpha_s\left(z\right) \bar{Q}_s^{\theta-1} \frac{Q_s\left(z\right) - 1}{Q_s\left(z\right)^{\theta}}. \tag{22}$$

The outcome in state s is Pareto efficient if $Q_s(z) \equiv k_s$ for all z, so that $e_s = 1$.

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[7] In papers such as Kydland and Prescott (1982) and Long and Possner (1983), the outcome is efficient.

[8] Notice that by construction $\int_0^1 \alpha_s(z) dz = 1$, so we interpret α as a weighting measure where each industry obtains a weight according to its relative productivity.

[9] This follows from the fact that $M_p\left(\tilde{x}\right) > M_q\left(\tilde{x}\right)$ for any non-degenerate random variable $\tilde{x}$ as long as $p > q$.

From equation 20, aggregate consumption only depends on the aggregate shock $\bar{A}_s$ and the heterogeneity of markups embedded in e_s. Since $e_s \leq 1$, the upper bound of aggregate consumption, i.e., potential output, is given by the aggregate shock $\bar{A}_s$. If all industries in the economy are perfectly competitive, so that $Q_s(z) \equiv 1$, then it is easy to see that the outcome is Pareto efficient (i.e., $C_s = \bar{A}_s$). On reflection, it follows that any economy in which markups across industries are the same in each state (i.e., $Q_s(z) \equiv k_s$ for all z and s) will also be Pareto efficient. A special case of this is an economy in which all industries are monopolized as $Q_s^m(z) = \frac{\theta}{\theta-1}$ for all industries and states. We note that Equation 22 provides a bijection, $\pi_s \leftrightarrow Q_s$, where $1 \leq Q_s \leq \frac{\theta}{\theta-1}$, $0 \leq \pi \leq \zeta C_s \alpha_s(z) \bar{Q}_s^{\theta-1}$, and $\zeta = \frac{(\theta-1)^{\theta-1}}{\theta^\theta}$ is a constant.

The intuition for this result is simple. If markups are constant across industries, i.e., $e_s = 1$, relative goods prices are not distorted. Compared to perfect competition, however, the fraction of income derived from labor income will be lower, i.e., $\omega = \frac{\theta-1}{\theta}$ in the monopoly case, since firm profits derived from monopoly rents also accrue back to the representative agent. This feedback of monopoly profits into the budget constraint causes real demand for each good to be unaffected. That is, the labor allocation to industries is not distorted and aggregate consumption is therefore maximal.

We note that the efficiency of the completely monopolized outcome depends on our assumption that labor is the only factor input in the production function, which together with the fact that labor is always fully utilized implies that the relative allocation of labor across industries is what matters. In the fully monopolized case, markups are the same so no relative distortions exist across industries, hence the result. In this respect, the result is model dependent. However, the main points — which will also be valid in a more general setting — are that higher markups in some industries, in general equilibrium, do not necessarily lead to lower efficiency, and that the degree of markup *variation* across industries may be especially important in determining efficiency losses.

It follows from this discussion that the economically interesting case is one in which some industries have high markups and some low. In such economies, the distortions resulting from distorted allocation of labor, or alternatively the welfare costs, are the highest. Figure 1 illustrates the range of feasible welfare losses under the natural restriction that firms charge markups between 1 and $\frac{\theta}{\theta-1}$.

3.2.2 Valuation

To value claims, we assume that there is a complete market of Arrow-Debreu securities in zero net supply traded, in addition to the stocks of the firms. As a result, the unique one period stochastic discount factor of the time-invariant economy, SDF, satisfies:

$$SDF_{t+1} = \delta \left(\frac{C_{s_{t+1}}}{C_{s_t}} \right)^{-\gamma}. \tag{23}$$

This is the textbook stochastic discount factor for power utility, the only exception being that C_{s_t} now represents a properly defined consumption bundle (as opposed to a single consumption good).

For time-invariant economies, since time-t profits of a firm depend only on the state, s, the information about the firm's future profits can be summarized in an S-vector, π, where π_s is

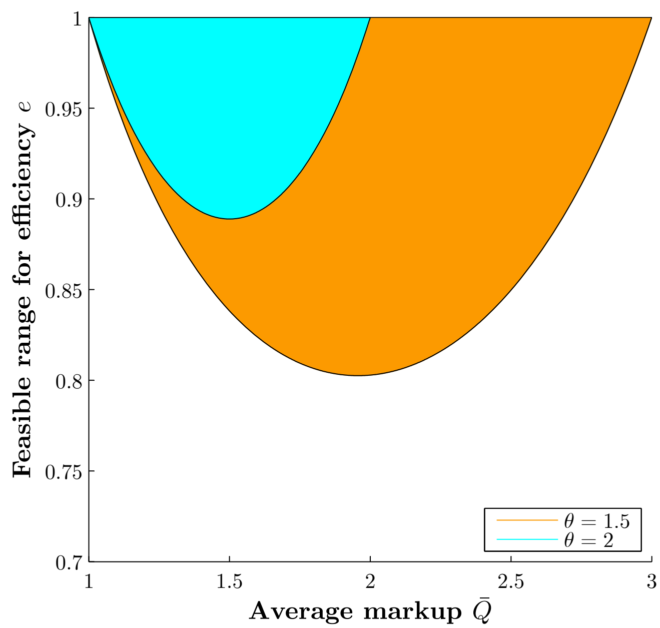

Figure 1. This graph plots the feasible range for efficiency e given an average markup of $\bar{Q}$ in the economy. In the monopolistic economy, i.e., when $\bar{Q} = \frac{\theta}{\theta-1}$, and the competitive economy, i.e., $\bar{Q} = 1$, there are no welfare losses. In the intermediate region, potentially large welfare losses may occur if some industries charge high markups and some low markups. These potential welfare losses are larger for smaller θ, i.e., for higher monopoly markups of $\frac{\theta}{\theta-1}$.

the profit in state s. We can then define the ex-dividend value vector V as an S-vector where the element V_s represents the the value of the firm in state s

$$V = \delta \Lambda_m^{-1} \Phi \Lambda_m (\pi + V). \tag{24}$$

Here, Λ_m is a diagonal matrix, with $m_s = C_s^{-\gamma}$ as its sth diagonal element. This argument leads to the following convenient formula for the value vector of the firm:

$$V = \Theta \pi, \tag{25}$$

where

$$\Theta = \Lambda_m^{-1} (I - \delta \Phi)^{-1} \Lambda_m - I, \tag{26}$$

and I is the $S \times S$ identity matrix. The valuation operator Θ has strictly positive elements. This simply represents the fact that higher profits in some state s increase the present value of future profits, $V_{s'}$, in all states $s' = 1, \ldots, S$.[10]

3.3 Industry equilibrium

Understanding strategic price setting behavior in one industry is the first step towards endogenizing Q. We therefore characterize, as a function of industry and aggregate characteristics,

[10]Recall that Φ is irreducible, so each state will be reached with positive probability, regardless of the initial state.

when firms in a specific industry behave competitively, when a monopolistic outcome can be sustained, and when the outcome is neither of these extremes. Observe that each industry is small compared with the aggregate economy, and that firms in industry z take the dynamics of all other industries as exogenously given, i.e., they take Q as exogenously defined for all $z' \neq z$. This is rational, since, as a result of Proposition 1, aggregate consumption C and average markups $\bar{Q}$ are not affected by an individual industry's behavior, and neither is then the pricing kernel. Thus, the relevant aggregate dynamics of the macro-economy for the outcome in any given industry are characterized by a $S \times 2$ matrix consisting of the vectors C and $\bar{Q}$.

As we have already observed, if an industry is monopolized, the outcome markup is $Q^m = \frac{\theta}{\theta-1}$ and when it is perfectly competitive the markup is $Q^c = 1$. Of course, in the latter case profits are zero, whereas in the former, monopolist profits, π_s^m are

$$\pi_s^m (z) = \zeta \bar{Q}_s^{\theta-1} C_s \alpha_s (z) . \tag{27}$$

Here, since C_s and $\bar{Q}_s$ are macro variables they are systematic and affect profits positively in all industries. Real profits in a particular industry depend positively on the aggregate market power because goods are substitutable. The sector specific component of profits is given by the idiosyncratic productivity shock $\alpha_s (z)$.

Following Abreu (1988), we are interested in industry equilibria that generate the highest industry profits sustainable by credible threats. We restrict attention to *symmetric*, *pure strategy subgame perfect* equilibria of the infinitely repeated stage game described in Section 2.2. Firms condition their action at time t on the entire history of past actions of industry z and states up to time t. The relevant history of each industry z, h_t is defined as the entire sequence of markups, states, and aggregate variables:

$$h_t = \left\{ \left\{ Q^{(i)}(z,\tau) \right\}_{i=1}^{N(z)}, s_\tau, \bar{Q}_{s_\tau}, C_{s_\tau} \right\}_{\tau=0}^{t}, \tag{28}$$

with h_0 representing the empty history. Thus, a time-t, industry-z strategy for firm i is a mapping from $h_{t-1} \times S$ to a chosen markup, Q_τ^i, $f : h_{t-1} \times S \to R_{++} \in R_{++}^{h_{t-1} \times S}$. Here, the second parameter, $s \in S$, represents time t information about the state, which is available for the firm. A strategy for firm i is a sequence of time-τ strategies, $\left\{ f_\tau^i \right\}_{\tau=0}^{\infty}$.

The entire set of subgame perfect equilibria can be enforced with the threat of the worst possible subgame perfect equilibrium. In this case, the most severe punishment is given by the perfectly competitive outcome, i.e., zero profits forever after a deviation.

Any subgame perfect equilibrium must satisfy the following incentive constraints for each state s,

$$\frac{\pi_s (z) + V_s (z)}{N (z)} \geq \pi_s (z) . \tag{29}$$

That is, the share of discounted present value of profits under collusion, $\frac{\pi_s + V_s}{N}$, must be greater or equal to the best-possible one period deviation of capturing the entire industry demand π_s and zero profits thereafter.[11]

[11] We are implicitly assuming that firms can coordinate within an industry to achieve this best outcome with this equilibrium selection mechanism. This trivially rules out any outcomes where markups are higher than $\frac{\theta}{\theta-1}$, and outcomes where markups are lower than necessary. We do *not*, however, assume that firms can coordinate across industries, since in a large economy there are many industries and global coordination therefore typically is not possible.

In the maximum profit equilibria, in each state, s, firms in an industry choose the vector of state contingent markups to maximize the value function, $V_s(z)$, given the value in each of the other states of the world, $V_{-s}(z)$, subject to incentive compatibility (equation 29),

$$V_s(z) = \arg\max_{Q_s} : V_s(z) \,|V_{-s}(z), \tag{30}$$

for all s. Here, Q_s maps to V_s via (22,25). Obviously, the case $N = 1$ is trivial: It leads to profits of π^m. We therefore focus on the case when $N \geq 2$.

In principle (30) is a complex optimization problem. First, strategic firms need to solve a state dependent infinite horizon state problem, second the optimal markup is a highly non-linear function of profits. However, within our model's setting, finding the solution is actually quite straightforward. First, we note that the dynamic equilibrium can be viewed as a linear programming problem in which firms choose profits instead of prices, replacing Q_s in (30) with π_s. Second, the specific form of this corresponding linear programming problem makes it clear that the solution is the same for each state, and the optimization therefore collapses to a static, state independent, linear programming problem.

To see this, we first consider a relaxed optimization problem starting from an arbitrary state j in which we ignore the additional consistency requirements imposed by value-maximizing behavior in other states $s \neq j$, i.e.,

$$\pi^j(z) = \arg\max_{\hat{\pi}(z)} \;\iota_j^T \Theta \hat{\pi}(z), \quad \text{s.t.}, \tag{31}$$

$$\hat{\pi}(z) \leq \pi^m(z), \tag{32}$$

$$0 \leq (\Theta - (N(z) - 1) I)\, \hat{\pi}(z), \tag{33}$$

where ι_j refers to an S-vector with zeros, except for the jth element which is equal to unity, $\iota_j = (\underbrace{0, \ldots, 0, 1}_{j}, \underbrace{0, \ldots, 0}_{S-j})^T$. Feasible profits are bounded above by monopoly profits which is captured by equation 32. The vector of IC constraints following from equation 29 is captured by equation 33.

Lemma 1. *The maximizer of the relaxed program, $\pi^j(z)$, described in equations 31 - 33 is independent of the initial state j, i.e.,*

$$\pi^j(z) = \pi(z), \tag{34}$$

for some $\pi(z) \in \mathbb{R}_+^S$. Further, for each element, s, either constraint 32 or 33 binds in the solution, $\pi_s(z)$.

This technical Lemma implies that the consistency requirements are automatically satisfied leading to the following Proposition.

Proposition 2. *Given aggregate consumption C and the average markup $\bar{Q}$, the industry equilibrium outcome $\pi(z)$ (or equivalently $Q(z)$) is unique. Equilibrium profits in state s are either given by monopoly profits, $\pi_s^m(z)$, or the IC constraint in state s binds, i.e. $\pi_s(z) = \frac{\iota_s^T \Theta \pi(z)}{N(z)-1}$.*

Going forward, it will be important to understand when the incentive constraint binds. This is because, as we have observed, Pareto inefficiencies arise if markups differ across industries. To measure the "tightness" of the monopolistic incentive constraint, we introduce the "tightness" vector, $\Gamma(z)$, with element s denoting the s-state ratio of the present value of industry profits under monopoly markups to monopoly profits:

$$\Gamma_s(z) = \frac{\pi_s^m(z) + V_s^m(z)}{\pi_s^m(z)} = 1 + \frac{V_s^m(z)}{\pi_s^m(z)}. \tag{35}$$

If $\Gamma_{s_1} > \Gamma_{s_2}$ the incentive to deviate in state s_1 is smaller than in state s_2, i.e., the present value of collusion is high relative to current period profits.

Lemma 2. *The tightness vector satisfies:*

$$\Gamma(z) = (\Lambda_{\kappa(z)}^{-1}(I - \delta\Phi)^{-1}\Lambda_{\kappa(z)})\mathbf{1}, \tag{36}$$

where $\Lambda_{\kappa(z)} = diag(\kappa(z))$*, and the vector* $\kappa(z)$ *has elements:*

$$\kappa_s(z) = \pi_s^m(z)\, m_s = \zeta \bar{Q}_s^{\theta-1} C_s^{1-\gamma} \alpha_s(z). \tag{37}$$

The variable κ_s captures an important determinant of the incentive to cheat in a certain state, Γ_s. It consists of the state component of the industry profit, $\pi_s^m(z)$, weighted by marginal utility in state s, $m_s = C_s^{-\gamma}$. We also define the minimum, $\underline{\kappa}(z) = \min_s \kappa_s(z)$.

Using the definition of the tightness vector, we are now able to derive closed-form expressions for the threshold number of players that leads to perfect competition and the monopoly outcome, respectively. Intuitively, for few enough players $N(z) \leq N^m(z)$, the monopoly outcome is sustainable in all states, while too many firms in one industry, $N(z) \leq N^c$, generates the competitive outcome in all states. In between, markups may vary across states. This intuition is formalized in the following proposition.

Proposition 3. *Given aggregate consumption* C *and the average markup* $\bar{Q}$*, equilibrium profits in state* s*,* $\pi_s(z)$*, satisfy:*

$$\begin{array}{lll} \pi_s(z) = \pi_s^m(z) & \textit{for} & N(z) \leq N^m(z), \\ \pi_s(z) \in (\kappa(z)C_s^{\gamma}, \pi_s^m(z)] & \textit{for} & N(z) \in (N^m(z), N^c), \\ \pi_s(z) = C_s^{\gamma}\underline{\kappa}(z) & \textit{for} & N(z) = N^c, \\ \pi_s(z) = 0 & \textit{for} & N(z) > N^c. \end{array}$$

where the respective threshold values satisfy $N^m(z) \stackrel{\text{def}}{=} \min_s(\Gamma_s(z))$ *and* $N^c \stackrel{\text{def}}{=} \frac{1}{1-\delta}$.

The different regions are best shown in example. Assume that aggregate consumption satisfies $C = (1,2,4)^T$, that aggregate markups are competitive in all states, $\bar{Q} = (1,1,1)^T$, and that $\alpha(z) = \left(\frac{1}{2}, \frac{3}{4}, 1\right)^T$. For simplicity, assume that the economy is i.i.d. with all states being equally likely. Finally, assume preference parameters of $\delta = 8/9$, $\gamma = 2$, and $\theta = 2$. Is is easy to show that the tightness vector in this example satisfies:

$$\Gamma(z) = (7, 9, 13)^T \tag{38}$$

Thus, monopoly markups are sustainable for $N(z) \leq N_M(z) = 7$.[12] Given δ, the number of firms necessary to induce the competitive outcome is $N_C = 9$. Figure 2 plots state-contingent profits in the left panel and the corresponding state-contingent markups as a function of the number of firms, confirming the four cases in Proposition 3.

Note that this example features pro-cyclical markups, i.e., $Q_1(z) \leq Q_2(z) \leq Q_3(z)$. While profits are unusually high in the good state of the world (increasing the incentive to cheat), this effect is overwhelmed by the valuation effect, i.e., future profits are discounted at a lower rate in times of low marginal utility. In contrast, in the worst state of the world the incentive to cheat is exacerbated by high marginal utility.

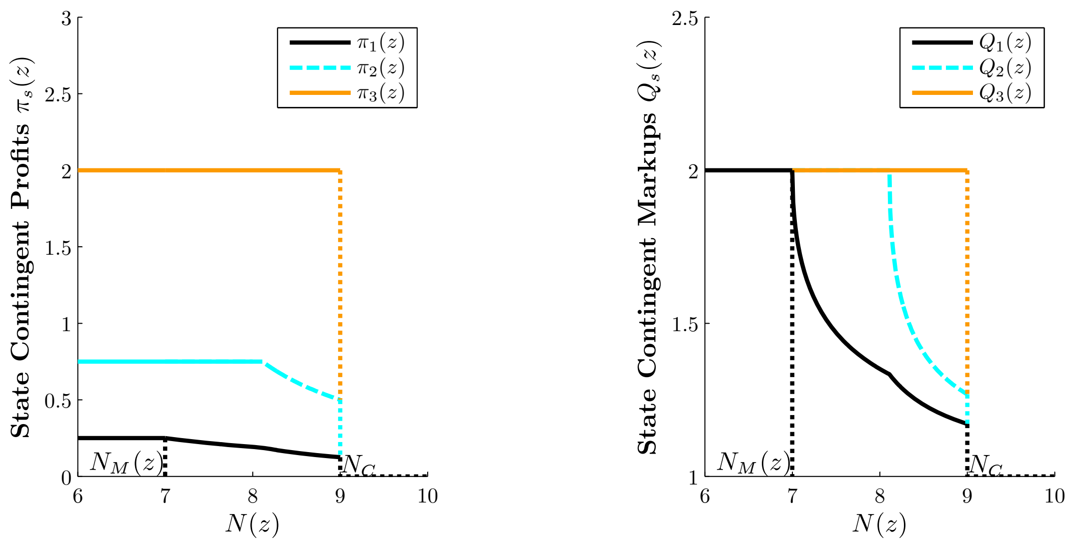

Figure 2. This graph plots the state contingent profits and markups of one particular industry given aggregate consumption of $C = (1, 2, 4)^T$, aggregate markups of $\bar{Q} = (1, 1, 1)^T$, and the relative industry state of $\alpha(z) = (2, 3, 4)^T$. If there are fewer than 7 firms in the industry, monopoly markups are sustainable in all states. Increasing the number of firms further cause the incentive constraint in state 1 to bind first, then in state 2 and finally, at $N_C = 9$, all markups collapse discontinuously to the competitive outcome, i.e., 1.

Given the generality of our setup it is quite surprising how much structure we are able to put on the industry equilibrium outcome. Intuitively, the threshold number of firms that allows the monopoly outcome is directly linked to $\Gamma(z)$. It is determined by the state in which the incentive to deviate is the highest, i.e., the state in which $\Gamma(z)$ attains its minimum. Secondly, the maximum number of players beyond which collusion completely breaks down is simply given by $N^c \stackrel{\text{def}}{=} \frac{1}{1-\delta}$, i.e., it only depends on the growth adjusted discount rate. Quite surprisingly, the threshold value is independent of industry characteristics as captured by $\alpha(z)$ and aggregate properties such as aggregate consumption C or the average markup $\bar{Q}$. At N^c, the incentive constraint is characterized by the indifference condition of a risk-neutral firm that compares the shared perpetuity value under collusion, $\frac{\pi^*(z)}{1-\delta}\frac{1}{N^c}$, and the best possible one-period deviation, $\pi^*(z)$. Moreover, we are able to derive an analytical formula for the profits of any industry z

[12]Recall that aggregate markups are competitive even if a zero measure of industries are non-competitive. Thus, there is no inconsistency in having one non-competitive industry in an economy that in aggregate is competitive.

with N^c firms.

What remains is to characterize the solution for industries with $N^m(z) < N(z) < N^c$ firms. For a special case, this region is empty, i.e., if $N^m(z) = N^c$.

Lemma 3. *If $\kappa_s(z) = k$ for all s and some arbitrary constant k, the threshold value for the monopoly outcome is given by N^c, i.e.: $N^m(z) = N^c$.*

In such industries markups are never state dependent (regardless of the number of firms in the industry), since they are neither state dependent in the monopolistic case, nor in the competitive case. Except for this knife-edge case, the region between $N^m(z)$ and N^c is non-empty, and represents the economically most interesting region, since it gives rise to state-contingent markups.

Proposition 4. *For an industry in which $N^m(z) < N(z) < N^c$,*

1. *There will be at least one state in which monopolistic profits are obtained, $\pi_s(z) = \pi_s^m(z)$ for some s.*
2. *Equilibrium profits, $\pi_s(z)$, are nonincreasing in $N(z)$ for each s, as are markups.*
3. *Equilibrium profits, $\pi_s(z)$, are nondecreasing in $\alpha_{s'}(z)$, for each s, s', as are markups.*
4. *Equilibrium profits and markups depend continuously on all parameters (N, C, $\bar{Q}$, Φ, α, and $\bar{A}$).*

It is straightforward to verify properties 1, 2, and 4 in Figure 2. Thus, given that the aggregate variables of the economy are known, the qualitative behavior of markups in different states of the world in a specific industry is well understood.

4 General Equilibrium

We show the existence of general equilibrium in which firms in each industry choose optimal markups given the (optimal) markups chosen by firms in all other industries. Recall that the economy's environment is characterized by the tuple $\mathcal{E}$, i.e., by the real variables $\alpha : S \times [0,1] \to \mathbb{R}_+$, $N : [0,1] \to \mathbb{N}$, $g \geq 0$, $\bar{A} \in \mathbb{R}^S_{++}$, the irreducible aperiodic stochastic matrix, $\Phi \in \mathbb{R}^{S\times S}_{++}$, and the preference parameters, γ, θ, and $\hat{\delta}$. We note that a given equilibrium is completely characterized by the markup function, $Q : S \times [0,1] \to \left[1, \frac{\theta}{\theta-1}\right]$, together with $\mathcal{E}$, since all other real and financial variables can be calculated from Q using (7) and (18-22). This motivates the following

Definition 1. *General Equilibrium in economy $\mathcal{E}$ is given by a markup function $Q : S \times [0,1] \to \left[1, \frac{\theta}{\theta-1}\right]$ for which,*

1. *$\bar{Q}$ and C are defined by Equations 18 and 20,*

2. *For all z, $Q(z)$ is the solution to the maximization problem given by Equations 31-33, where $\pi^m(z)$ in the optimization problem is given by Equation 27.*

We note that the existence and uniqueness of the second part of the definition is guaranteed by Proposition 2, industry by industry, i.e., given $\bar{Q}$ and C there is a unique optimal markup function. It is a priori unclear, however, whether there exists a general equilibrium, i.e., whether both parts can be solved simultaneously. In other words, both the mappings, $Q \mapsto (\bar{Q}, C)$ (part 1) and $(\bar{Q}, C) \mapsto Q'$ (part 2) are well defined, but it is unclear whether Q can be chosen such that the second step maps to the same markup function that was used in the first step, i.e., such that $Q' = Q$.

It turns out that we are able to prove the existence of equilibrium under very general conditions. Specifically, we assume that the functions N and α are Lebesgue measurable functions, and impose the following technical condition:

Condition 1. *For all s, for almost all z, $c_0 \leq \alpha_s(z) \leq c_1$ for constants, $0 < c_0 \leq c_1 < \infty$.*

Before showing existence, we discuss some invariance results which will be helpful in the proof. We first note that the following result follows immediately from Proposition 2:

Lemma 4. *In any general equilibrium, any two industries with the same N and α have the same markups, Q, and profits, π.*

Also, we observe that it is only the distributional properties of N and α that are important for the aggregate characteristics of an equilibrium. This should come as no surprise given that the aggregate variables important for industry equilibrium only depend on the distributions. To be specific, we define the (cumulative) distribution function $F : \mathbb{N} \times [c, C]^S \to [0, 1]$, where $F(n, s_1, \ldots, s_S) = \lambda(\{(z : N(z) \leq n \wedge \alpha_1(z) \leq s_1 \wedge \cdots \wedge \alpha_S(z) \leq s_S\})$, and λ denotes Lebesgue measure. Thus, $F(n, \alpha_1, \ldots, \alpha_S)$ denotes the fraction of industries with number of firms less than or equal to n, and productivities $\alpha_s(z) \leq \alpha_s$ for all s. We then have

Lemma 5. *Given two economies, 1 and 2, that are identical except for that their functions determining number of firms and productivity, N and α, differ. Assume that the economies have the same distribution function, F. Then they have have the same equilibria in the sense that for each equilibrium in the first economy, there is an equilibrium in the second, such that any two industries, z and z' in the first and second economy, respectively, for which $N^1(z) = N^2(z')$ and $\alpha_s^1(z) = \alpha_s^2(z')$ for all s, have the same industry markups in each state of the world, $Q_s^1(z) = Q_s^2(z')$ for all s.*

We now have the following general result:

Proposition 5. *General equilibrium exists in any economy that satisfies Condition 1.*

Thus, only the technical conditions of integrability and boundedness of productivity functions across industries is needed to ensure the existence of equilibrium.

The proof of Proposition 5 uses Schauder's fixed point theorem. Specifically, the continuity results from the previous section implies that the mapping $Q \mapsto (\bar{Q}, C) \mapsto Q'$ is continuous (technically, in the function space L^1). It is further shown in the proof of the Proposition 5 that the space of markup functions is compact and convex, which via Schauder's theorem then guarantees the existence of a fixed point, i.e., an equilibrium. We note that Proposition 5 makes no claim about equilibrium uniqueness — a subject that will be explored further in the next section.

5 Endogenous Business Cycles

In this section we analyze general equilibrium. Our qualitative analysis is meant to deliver the main intuition for our results, without any attempt toward a real world calibration. Such a calibration would be interesting — and potentially feasible given our flexible set-up — but is outside of the scope of this paper. Our main objective is to show — with a sequence of examples — how strategic competition may endogenously amplify and even generate business cycles.

5.1 Business Cycle Amplification

Previous asset pricing literature has mainly studied the effects of strategic interaction in a partial equilibrium setting. In general equilibrium the decisions of firms in one part of the economy, via the influence they have on aggregate consumption, affect the pricing kernel and thereby the decisions of all other firms in the economy.

We introduce the following measure of business cycle amplification through strategic interaction:

Definition 2. *The* Oligopolistic Business Cycle Amplifier *(OBCA) is defined as*

$$OBCA = \frac{\sigma_C}{\sigma_{\bar{A}}}. \tag{39}$$

Here, σ_C and $\sigma_{\bar{A}}$ are the unconditional standard deviations of aggregate consumption and aggregate productivity, respectively.

Recall that in the first-best competitive outcome, $C \equiv \bar{A}$, leading to an OBCA equal to one. Business cycle fluctuations in this case are purely technology-driven. An OBCA greater than one thus tells us that equilibrium business cycle fluctuations, which are also affected by firms' strategic behavior, are larger than what is motivated by technological shocks. Similarly, an OBCA less than one implies that strategic behavior in equilibrium dampens business cycle fluctuations.[13]

We note that since $C = e\bar{A}$ (see Equation 20), the variation of C will be especially high when e and $\bar{A}$ are positively co-dependent, leading to an all else equal higher OBCA. Further, since e_s is determined by the cross-sectional dispersion of markups across industries in state s

[13] Our OBCA is defined in absolute terms. A "relative" OBCA, a ROBCA, is defined as $ROBCA = \frac{\sigma_C/E[C]}{\sigma_{\bar{A}}/E[\bar{A}]}$. Since $E[C] \leq E[\bar{A}]$, it trivially follows that $OBCA \leq ROBCA$, so business cycle amplifications are at least as severe in relative terms.

(see Equation 19), such that when the dispersion is low then e_s is high (close to one), whereas when the dispersion is high, e_s is low (close to zero), it follows that oligopolistic business cycle amplifications should be especially large in economies in which the cross sectional dispersion of markups is high in low-productivity states.

Consider the economy described in Table 1, with three distinct types of industries, I_1, I_2 and I_3, and $S = 2$ states.

Type, j	I_j	N	A_1	A_2	α_1	α_2
1	$z \in [0, 0.02)$	19	0.25	1	0.8728	1
2	$z \in [0.02, 0.81)$	19	1	1	1.0026	1
3	$z \in [0.81, 1]$	1	1	1	1.0026	1
$\bar{A}$					$\bar{A}_1 = 0.974$	$\bar{A}_2 = 1$

$$\Phi = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\gamma = 6, \qquad \theta = 1.1, \qquad \delta = 0.95.$$

Table 1. Economy with three industries and two states.

Here, all industries with $z \in I_j$ belongs to industry type j. With a slight abuse of terminology, we will call the I_j sets "industries," although each set represents many identical industries. Thus, there is one very small industry (I_1), one large industry (I_2) and one medium-sized industry (I_3). The first two industries have many firms, $N = 19$, but they will still not be perfectly competitive, since $N^c = \frac{1}{1-\delta} = 20$. The third industry is monopolistic, $N = 1$.

Columns 4 and 5 in Table 1 describe the absolute productivity shocks, A, in the two states. We see that only the very small first industry experiences any variation in productivity across the two states. The aggregate variation in productivity will therefore be small. In columns 6 and 7, we show the decomposition of the absolute productivity shocks into relative and aggregate components, α and $\bar{A}$ (see (16) and (17)). The effect on aggregate productivity of the first industry's shock is about 2.5%, since aggregate productivity is 0.974 in the low-productivity state and 1 in the high-productivity state. This would also be the aggregate consumption in the two states in an efficient outcome. Note that the shock to industry 1 also affects the relative productivity in industries 2 and 3, since α is normalized to sum to one across industries, state by state.

Before analyzing the equilibrium in this economy, it is instructive as a reference case to study the economy which is identical to that in Table 1, except for that $A_1 = 1$ in industry 1. This is thus an economy with no productivity shocks, neither idiosyncratic nor aggregate, and it follows that $\bar{A}_1 = \bar{A}_2 = 1$ and $\alpha_s(z) \equiv 1$ in this reference economy. One easily verifies that the monopolistic outcome, in which markups $Q \equiv \frac{\theta}{\theta-1} = 11$ are chosen by all firms in all states, is feasible in this case (this also follows as a consequence from Lemma 3, since $N \leq N^c$ in all industries), leading to the efficient outcome where $C_1 = \bar{A}_1 = 1$, $C_2 = \bar{A}_2 = 1$.

The situation is different for the economy given in Table 1. The fully monopolistic outcome is no longer feasible, because it does not satisfy the IC constraints for firms in industry 2.

Instead, an equilibrium is given by the following markups:

$$
\begin{array}{l|ll}
\text{Markups} & s=1 & s=2 \\
\hline
Q(I_1) & 1.580 & 11 \\
Q(I_2) & 1.465 & 11 \\
Q(I_3) & 11 & 11 \\
 & &
\end{array}
\tag{40}
$$

leading to aggregate consumption

$$C_1 = 0.795, \qquad C_2 = 1.$$

Thus, the small productivity shock ($\approx$ 2.5%) leads to a significant decrease in equilibrium output ($\approx$ 20%) in state 1. The OBCA in this equilibrium is

$$OBCA = \frac{\sigma_C}{\sigma_{\bar{A}}} = \frac{0.103}{0.013} = 7.88,$$

so strategic interaction leads to an almost eight-fold amplification of the business cycle variation. The intuition for why this amplification occurs is exactly in line with our main theme in this paper, that technological shocks that are small in aggregate — in that they only affect a few industries — change the strategic behavior of firms in other industries through the effect they have on the pricing kernel.

This mechanism is explained in Figure 3, focusing on the behaviors of industries 1 and 2.[14] In the upper part of the figure, the reference economy with identical industries is shown, in which case monopolistic profits are feasible for both industries. In the lower part of the figure, the economy in Table 1 is shown. Line A shows the important IC constraint, given the pricing kernel in the monopolistic outcome. Monopolistic profits are indeed feasible in industry 1 (lower left figure), but infeasible in industry 2 (lower right figure). Thus, the lower productivity in industry 1, through its effect on the pricing kernel, affects the outcome in sector 2, which moves to line B. This in turn changes the pricing kernel even further, making monopolistic profits in industry 1 infeasible and further changing the outcome in industry 2, moving to lines C in the two industries, and generating further feedback effect. The ultimate effect of this mechanism is that the equilibrium moves to line D in the two figures, substantially different from monopolistic equilibrium in the reference economy.

5.2 Comparative Statics

The equilibrium outcome may be very sensitive to small changes in some parameter values, whereas it is remarkably stable in other aspects. The results together suggests that cross economy (e.g., cross country) comparisons need to be carefully designed to capture meaningful relationships when studying the determinants of an economy's dynamics.

We show that the equilibrium outcome may be extremely sensitive to small differences in long-term growth growth rates, g, and specifically that small differences can have large welfare

[14] Industry 3 is always monopolistic. The reason that it is still important for the example is that substantial efficiency losses only occur when there is high variability in markups across sectors. If industry 3 was not present then the economy would always be close to efficient, since markups would be the same for the vast majority of industries in each state — almost identical to the markups charged in industry 2. In contrast, when industry 3 is present and industry 2 charges low markups, efficiency will be low.

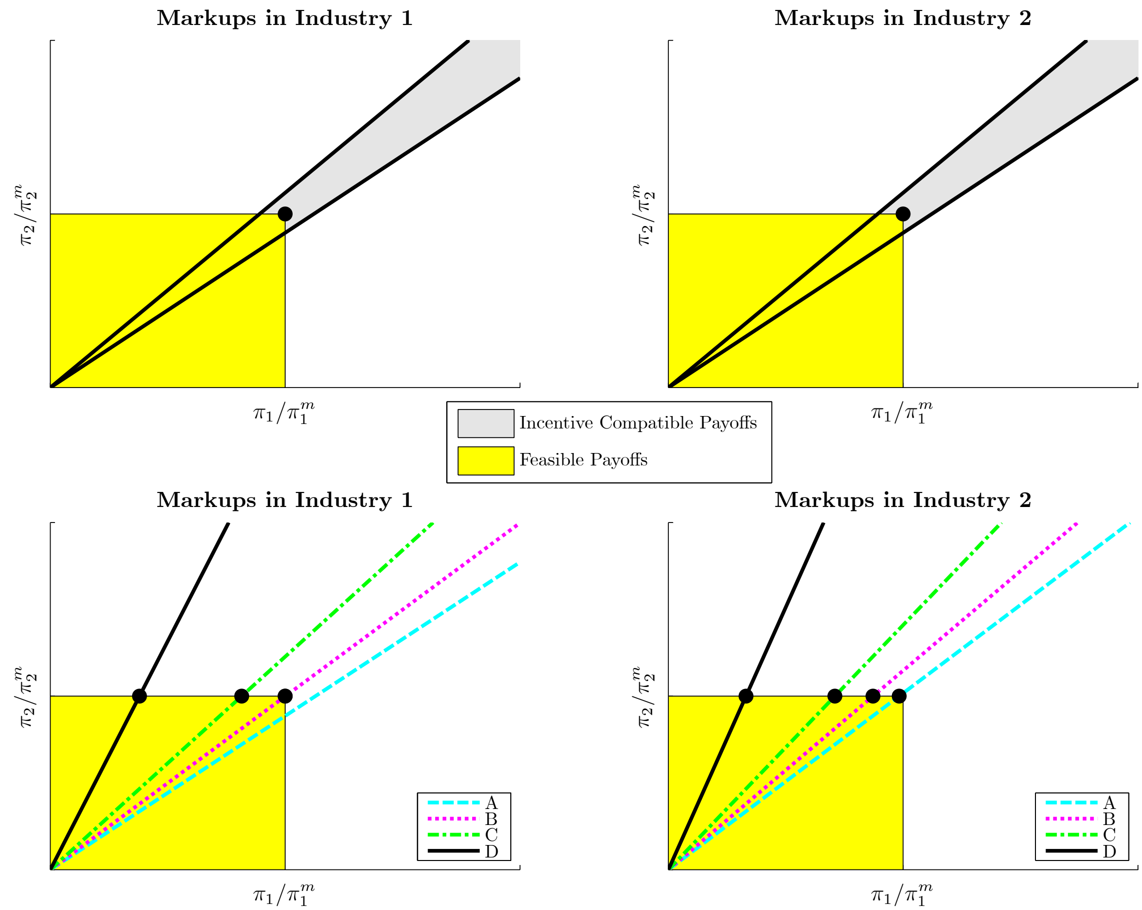

Figure 3. Profits in industry 1 (left) and 2 (right) in economies with identical industries (above) and economy given in Table 1 (below). Above: Monopolistic profits are feasible in all states and both industries. Below: Monopolistic profits violate IC constraint of industry 2 (line A), in turn changing the IC constraints in industry 1 (line B). The resulting equilibrium (line D) is substantially different.

effects by taking the economy from a Pareto efficient, perfectly competitive, outcome to one in which some industries are competitive and others are not. We study a modified version of our workhorse example from the previous section, given in Table 2. The differences are that there are now 20 firms in each industry, that the asymmetry in industry sizes is not as large as in the previous example, and that there are productivity variations across states also in the large industries. It is straightforward to verify that there is an equilibrium with aggregate consumption

$$C_1 = C_2 = 1.19,$$

and markups

Markups	$s = 1$	$s = 2$
$Q(I_1)$	11	11
$Q(I_2)$	11	4.44
$Q(I_3)$	4.44	11

(41)

and that the efficiency therefore is $e_s = \frac{C_s}{A_s} \equiv \frac{1.19}{1.33} = 0.89$, about 11% below the Pareto efficient outcome in both states.

Type, j	I_j	N	A_1	A_2	α_1	α_2
1	$[0, 0.2)$	20	1	1	0.972	0.972
2	$[0.2, 0.6)$	20	1	2	0.972	1.041
3	$[0.6, 1]$	20	2	1	1.041	0.972
$\bar{A}$					$\bar{A}_1 = 1.33$	$\bar{A}_2 = 1.33$

$$\Phi = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\gamma = 6, \qquad \theta = 1.1, \qquad \delta = 0.95,$$

Table 2. Modified economy with three industries and two states.

Now, in an identical economy as the one in Table 2, except for that $\delta = 0.949$ instead of 0.95, only the competitive outcome is an equilibrium, leading to $Q \equiv 1$ and efficient consumption

$$C_1 = C_2 = 1.33.$$

This follows immediately since $N > N^c$ in all industries. Thus, the discontinuity of markups close to N^c, analyzed in Section 3.3, leads to extreme sensitivity of the equilibrium outcome when there is a substantial number of industries in which the number of firms close to N^c. We recall that δ is a function of the long-term growth rate (g), the risk aversion coefficient (γ), and the personal discount rate ($\hat{\delta}$). The equilibrium may therefore be sensitive to these parameters (in other examples, we generate drastic changes in consumption *variation*, not only levels).

Variations in other parameters do typically not have as drastic an effect on the equilibrium. Especially, a property of the economy that turns out to be remarkably insensitive to parameter variations is the number of industries that are competitive. We define

Definition 3. *The* competitiveness, *η, of an equilibrium is defined as the mass of industries that are perfectly competitive,*

$$\eta = \lambda(\{z : Q_s(z) = 1, \forall s\}).$$

We recall from the previous analysis that an industry is either competitive in all states or in no state, so requiring perfect competition in all states is in no way restrictive. We now have

Proposition 6. *The competitiveness of an equilibrium depends on δ and the function N, but not on Φ, α, $\bar{A}$ or θ.*

Thus, somewhat surprisingly, the only real parameter that is important for competitiveness is the long-term growth rate, g. Recall that $\delta = \hat{\delta}(1+g)^{1-\gamma}$. If $\gamma < 1$, a higher long-term growth rate leads to lower competitiveness, i.e., fewer industries in which there is perfect competition. If $\gamma > 1$ on the other hand, a higher long-term growth rate leads to higher competitiveness.

Compared with the risk neutral setting ($\gamma = 0$), competitiveness is higher in the economy with risk averse agents as long as the economy is growing in the long term, $g > 0$ (since $\delta(1+g)^{1-\gamma} < \delta(1+g)$). This is not surprising, since future growth is valued less by an agent with a concave utility function, and it is therefore more tempting for firms to deviate from a cooperative outcome when agents have such preferences.

Interestingly, shorter term fluctuations, represented by Φ, *never* matter for the competitiveness. Neither does the distribution of productivity over industries, α, nor aggregate productivity, $\bar{A}$. Upon reflection, the two results — that competitiveness can be extremely sensitive to the long-term growth rate (g) — although it does not depend at all on temporary fluctuations (Φ) may seem puzzling. Indeed, since our set-up is completely general, we could thus have very different competitiveness in two economies that seem to be identical for an arbitrary long (but finite) time period: one economy with a long-term growth rate $g > 0$, and one that grows at rate g for a long time through state transitions in Φ (recall that Φ can be arbitrary large but finite), but that has a long term growth rate of 0. The technical reason why these economies are indeed very different is because of the infinite horizon nature of dynamic collusion games. In long but finite horizon games, collusion is much harder to sustain, as it is in our model when growth only lasts for a finite time period. The importance of long-term growth rates for asset pricing was recently discussed in Parlour et al. (2011). In that paper, long-term growth rates are important because they determine how much investors care about rare disaster events in the far future. The model in this paper provides another mechanism through which long-term growth rates may be crucial in determining the economy's dynamics and asset prices.

5.3 Uniqueness

Our existence result makes no claims with regards to uniqueness. Given an economy, $\mathcal{E}$, there may be multiple equilibria whenever a nonzero measure of firms fails the condition of perfect competition. The parametrization in Table 1 reveals that there is indeed (exactly) one more equilibrium in the economy supported by the equilibrium markups

Markups	$s=1$	$s=2$
$Q(I_1)$	11	2.104
$Q(I_2)$	11	2.605
$Q(I_3)$	11	11

(42)

and leading to aggregate consumption

$$C_1 = 0.974, \qquad C_2 = 0.884.$$

Again, business cycles are endogenous. However, despite the same technology specification, the second equilibrium is very different from the first one. First, although the state with low productivity is the first, aggregate output is the lowest in the second state in this second equilibrium. There is thus a second way to ensure that firms do not deviate from equilibrium strategies, namely to decrease the attractiveness of state 2. We note that the first equilibrium leads to higher output than the second equilibrium in state 2 (1 versus 0.884), whereas the second equilibrium dominates in state 1 (0.974 versus 0.795). The OBCA in this second equilibrium is

$$OBCA = \frac{\sigma_C}{\sigma_{\bar{A}}} = \frac{0.045}{0.013} = 3.68,$$

lower than the OBCA in the first equilibrium, which suggests that the second equilibrium is better from a welfare perspective. It is indeed easily verified that the second equilibrium Pareto dominates the first in expected utility terms, regardless of the current state.

It turns out that there are multiple equilibria even in the reference economy with no productivity shocks, i.e., in which $A_1 = 1$ also for the first industry. In this economy, one can verify that

Markups	$s = 1$	$s = 2$
$Q(I_1)$	11	1.634
$Q(I_2)$	11	1.634
$Q(I_3)$	11	11

(43)

with aggregate consumption.

$$C_1 = 1, \qquad C_2 = 0.830,$$

is also an equilibrium. Moreover, an third equilibrium (symmetrically) exists in which markups and consumption are low in state 1. Since productivity (idiosyncratic and aggregate) is constant across states in this case, the business cycle in this equilibrium is not just amplified, it is completely endogenous, and the OBCA is infinite. Thus, truly endogenous business cycles arise because of strategic competition in our model.

To summarize, collusion within an industry produces a unique outcome given the behavior of all other industries and the implied stochastic discount factor (see Proposition 2), but general equilibrium effects may lead to multiplicity of equilibria. This suggests that coordination *across* industries becomes relevant and opens up for a role for policy makers. This is noteworthy because our model shares many properties with standard business cycle models, in which outcomes are efficient. For example, there are no standard frictional costs in our model. Of course, the assumption that the number of firms in each industry is exogenously determined could be viewed as equivalent to assuming a friction in terms of high entry costs. However, even if there were no entry costs, the competitive outcome may not prevail. For example, the outcome with $N = 20$ firms in each industry and aggregate consumption in Table 2 is an equilibrium in the economy with zero costs of entry, since profits would immediately drop to zero if another firm entered an industry, so no firm has an incentive to do so. Moreover, there are no bubbles in the model. Instead, our key deviation from the traditional approach is to assume that intra-industry prices are strategically determined, instead of being determined through a Walrasian mechanism.

6 Concluding Remarks

We have developed general equilibrium in a dynamic economy with a continuum of industries each of which comprises a finite number of firms. The framework is quite tractable, and the strategic interaction between firms in each industry is straightforward to characterize. We establish the existence of general equilibrium and establish dynamic properties of the economy including equilibrium markups and attendant asset prices.

The central premise of our model is that firms, maximizing shareholder value, are not always price takers but can be price setters. High prices in an industry can be sustained if firms value the future flow of profits over any immediate increases in market share garnered by undercutting. Of course, the rate at which future profits are discounted depends both on the representative agent's risk aversion and on the behavior of the aggregate economy.

The strategic interaction yields various general equilibrium effects that can be interpreted in light of the macro-economy. Even in an economy with no aggregate uncertainty, if the relative productivity of various industries changes, so does their ability to sustain collusive outcomes. These changes can affect both the level and the volatility of aggregate consumption; in short our model exhibits endogenous volatility. Our welfare results highlight that the cross-sectional dispersion of markups rather than the (average) level of market power across industries, is relevant for welfare assessments.

While conceptually simple, the model is sufficiently rich that it presents many avenues for future research, both on the strategic behavior of firms and on the general equilibrium properties. First, we take the number of firms in each industry as exogenous. However, a natural extension would be to model strategic entry and exit. We speculate that this would lead even more strategic volatility. Second, while we present various stylized examples, it would be natural to structurally calibrate or estimate the model. For example, it is worth noting that the consumption aggregator is highly non-linear. This means that a linear aggregator (such as a price weighted one) will consistently underestimate the volatility of the utility an agents enjoys from a consumption stream. It is an empirical question as to whether this framework provides new empirical insight into the relationship between the volatility of asset prices and aggregate consumption, however it is an interesting avenue for further research. Finally, our model has potentially new implications for policy actions. Indeed, the existence of multiple equilibria suggests that government intervention may be beneficial.

A Proofs

Proof of Proposition 1

As explained in Section 3.1 we focus on time-invariant economies, so that all variables are solely expressed as state-dependent. Using the the expression for prices, $p_s(z)$, (see equation 7) and the definitions of $\alpha_s(z)$, $\bar{A}_s$ and $\bar{Q}_s$ (see equations 16, 17, and 18), we can solve for nominal prices and the nominal wage rate via normalizing the price index $P_s = \left(\int_0^1 p_s(z)^{1-\theta} dz\right)^{\frac{1}{1-\theta}}$ to one.

$$w_s = \frac{\bar{A}_s}{\bar{Q}_s}, \tag{44}$$

$$p_s(z) = \frac{Q_s(z)}{\bar{Q}_s} \alpha_s(z)^{\frac{1}{1-\theta}}. \tag{45}$$

Finally, plugging the demand function of each sector, $c_s(z)$ (see equation 4) into the profit function of each sector $\pi_s(z)$ (see equation 6) yields an expression for y_s via the aggregate budget constraint (see equation 5)

$$y_s = \bar{A}_s e_s, \tag{46}$$

where we have used the expression for nominal wages and prices (see equations 44 and 45) and the definition of e_s (see equation 19). Since the price index is normalized to one, real consumption $C_s = \frac{y_s}{P_s}$ is given by y_s. The fraction of income derived by labor income, $\omega_s = \frac{w_s}{y_s}$, is readily obtained via equations 44 and 46. Real profits following immediately from 4, 6, 44, 45, and 46.

Proof of Lemma 1

The lemma is a special case of the following general lemma (by choosing $b = \Theta^T \iota_j$).

Lemma 6. *Consider a strictly positive vector $\pi^m \in \mathbb{R}^S_{++}$, a strictly positive matrix $\Theta \in \mathbb{R}^{S\times S}_{++}$, and a scalar $n \in \mathbb{R}_{++}$. Then there is a unique $\xi \in \mathbb{R}^S_+$ so that for all strictly positive $b \in \mathbb{R}^S_{++}$,*

$$\begin{aligned} &\xi = \arg\max_x \; b^T x, \;\; s.t., \\ &x \le \pi^m, \\ &0 \le (\Theta - nI)\, x. \end{aligned}$$

For each s, the solution has either the first or the second constraint binding, i.e., for each s, $\xi_s = \pi^m_s$ or $n\xi_s = \Theta\xi_s$.

Proof: Let $x < y$ denote that $x \le y$ and $x \ne y$. Also, define $z = x \vee y \in \mathbb{R}^S$, where $z_s = \max(x_s, y_s)$ for all s. Clearly, $x \le x \vee y$, where the inequality is strict if there is an s such that $y_s > x_s$. Finally, define the set $K = \{x : 0 \le x,\; x \le \pi^*, nx \le \Theta x\}$. Note that K is compact.

Now, there is a unique maximal element of K, that is, there is a unique $\xi \in K$, such that for all $x \in K$ such that $x \ne \xi$, $\xi > x$. This follows by contradiction, because assume that there are two distinct maximal elements, y and x, then clearly $z = x \vee y$ is strictly larger that both x and y. Now, it is straightforward to show that $z \in K$. The only condition that is not immediate is that $\Theta z \ge Nz$. However, this follows from $\Theta(x \vee y) \ge \Theta x \vee \Theta y \ge nx \vee ny = n(x \vee y) = nz$.

Now, since b is strictly positive, it is clear that ξ is indeed the unique solution to the optimization problem regardless of b. That one of the constraint is binding for each s also follows directly, because assume to the contrary that neither constraint is binding in some state s. Then ξ_s can be increased without violating either constraint in state s and, moreover, the constraints in all the other states will actually be relaxed, so such an increase is feasible. Further, since $b_s > 0$, it will also increase the objective function, contradicting the assumption that ξ is optimal.

Proof of Proposition 2

Follows immediately from Lemma 1.

Proof of Lemma 2

By definition: $V = \Lambda_\pi(\Gamma - \mathbf{1})$, so from (27), $\Lambda_\pi(\Gamma - \mathbf{1}) = \delta\Lambda_m^{-1}\Phi\Lambda_m(\pi + \Lambda_\pi(\Gamma - \mathbf{1}))$, leading to $\Gamma - \mathbf{1} = \delta\Lambda_\pi^{-1}\Lambda_m^{-1}\Phi\Lambda_m\Lambda_\pi\Gamma$. Now, observing (from (27)) that $\Lambda_\kappa = \Lambda_\pi\Lambda_m$, the result follows immediately.

Proof of Proposition 3

Let $n = N - 1$ and $K^*(n) \overset{\text{def}}{=} \{x : 0 \leq x, nx \leq \Theta x\}$. Now, $nx \leq (\Lambda_m^{-1}(I - \delta\Phi)^{-1}\Lambda_m - I)$ is equivalent to $Ny \leq (I - \delta\Phi)^{-1}y$, where $y = \Lambda_m x \in \mathbb{R}_+^S$. We first show that $K^*(n) = \{0\}$ when $N > \frac{1}{1-\delta}$, which immediately implies that the only solution to the optimization problem in Lemma 1 is indeed the competitive outcome. Define the matrix norm $\|A\| = \sup_{x\in\mathbb{R}^S\setminus\{0\}} \frac{\|Ax\|}{\|x\|}$, where the l^1 vector norm $\|y\| = \sum_s |y_s|$ is used. Since Φ is a stochastic matrix, $\|\Phi^i\| = 1$ for all i and using standard norm inequalities it therefore follows immediately that

$$\|(I - \delta\Phi)^{-1}\| = \left\|\sum_0^\infty \delta^i\Phi^i\right\| \leq \sum_0^\infty \delta^i\|\Phi^i\| = \frac{1}{1-\delta},$$

and thus $\|(I - \delta\Phi)^{-1}y\| \leq \frac{1}{1-\delta}\|y\|$. Now, $Ny \leq (I - \delta\Phi)^{-1}y$ implies that $N\|y\| \leq \|(I - \delta\Phi)^{-1}y\|$, and therefore it must be the case that $N \leq \frac{1}{1-\delta}$, for the inequality to be satisfied for a non-zero y. Now, consider the case when $N = \frac{1}{1-\delta}$. Since $y = 1$ is an eigenvector to Φ with unit eigenvalue, it is also an eigenvector to $(I - \delta\Phi)^{-1}$ with corresponding eigenvector $\frac{1}{1-\delta}$, leading to $x = \Lambda_m^{-1}1 = m^{-1}$. It is easy to show that this is the unique (up to multiplication) nonzero solution. Given the properties of Φ, the Perron-Frobenius theorem implies that this is indeed the *only* eigenvector with unit eigenvalue, and therefore also the only eigenvector to $(I-\delta\Phi)^{-1}$ with eigenvalue $\frac{1}{1-\delta}$. Now, take an arbitrary $y \in \mathbb{R}_+^S\setminus\{0\}$ as a candidate vector to satisfy the inequality, i.e., such that $z = (I - \delta\Phi)^{-1}y$ satisfies $z_i \geq Ny_i = \frac{1}{1-\delta}y_i$ for all i. Then, since $\|(I - \delta\Phi)^{-1}\| = \frac{1}{1-\delta}$, it follows that $\sum_i z_i \leq \frac{1}{1-\delta}\sum_i y_i$. The two inequalities can only be satisfied jointly if $z_i = \frac{1}{1-\delta}y_i$ for all i, and thus y is the already identified eigenvector. Thus, $K^*\left(\frac{1}{1-\delta}\right) = \{\iota m^{-1}, \iota \geq 0\}$. It follows immediately from the definition of the λ vector that the maximal ι that satisfies $\iota m_s^{-1} \leq \pi_s^* = q_sC_s\alpha_s$ for all s is $\min_s \lambda_s$, leading to the given form of the profit vector.

Proof of Lemma 3

If $\kappa_s = k$, the diagonal matrix Λ_κ becomes $\Lambda_\kappa = kI$ so that we obtain for Γ (see (36)):

$$\Gamma = (I - \delta\Phi)^{-1}\mathbf{1} = \frac{1}{1-\delta}\mathbf{1} = N^c\mathbf{1}. \tag{47}$$

This is because the eigenvalue of $(I - \delta\Phi)^{-1}$ associated with the eigenvector of $\mathbf{1}$ is given by $\frac{1}{1-\delta}$ (see Proof of Proposition 3). So, $N^m = \min_s(\Gamma_s) = N^c$.

Proof of Proposition 4

(1,2) follow from the definition of K in the proof of Lemma 1. It immediately follows that the set K is decreasing in N and increasing in each of α_s, which in turn immediately implies (1,2).

(3) follows from (1), and the fact that $\pi_s > 0$ for all s when the number of firms is N^c.

(4) follows from (1) and that $\pi_s = m_s^{-1}\pi_s^m m_s$ for the s that minimizes μ_s (see Proposition 3).

(5) follows from the fact that the objective function in Lemma 1 is a continuous function of all parameters and that (as long as N is strictly below N^c) the set K is compact, and depends continuously on all parameters, in the sense that if K and K' are defined for two sets of parameter values, then $D(K, K')$ approaches zero when the parameter values that define K' approach those that define K. Here, $D(K, K') = \sup_{x \in K'} \inf_{y \in K} |x - y|$.

Proof of Proposition 5

We wish to prove the proposition with a fixed point argument, and therefore define a fixed point relationship for the markup function, Q, which ensures that it defines an equilibrium. We define $R \stackrel{\text{def}}{=} \bar{N} \times [c, C]^S$, where $\bar{N} = \{1, 2, \lfloor N_c \rfloor + 1\}$, with elements $x = (n, \alpha_1, \ldots, \alpha_S) \in R$. We will then work with functions $Q^0 : R \to [0,1]^S$, and given such a function, the transformation to the standard markup function is given by $Q_s(z) = Q_s^0(\min(N(z), \lfloor N_c \rfloor + 1), \alpha_1(z), \ldots, \alpha_S(z))$. The reason why we work with the canonical domain, R, rather than $S \times [0,1]$, is that compactness properties needed for a fixed point argument are easier obtained in this domain. Given a function, $Q^0 : R \to \left[1, \frac{\theta}{\theta-1}\right]^S$, we define

$$p_s^0 = M_{-\theta}(Q_s) = \left(\int \alpha_s(z) Q_s(z)^{-\theta} dz\right)^{\frac{1}{-\theta}} = \left(\int_{x \in R} x_{s+1} Q^0(x)^{-\theta} dF(x)\right)^{\frac{1}{-\theta}}, \tag{48}$$

$$p_s^1 = M_{1-\theta}(Q_s) = \left(\int \alpha_s(z) Q_s(z)^{1-\theta} dz\right)^{\frac{1}{1-\theta}} = \left(\int_{x \in R} x_{s+1} Q^0(x)^{1-\theta} dF(x)\right)^{\frac{1}{1-\theta}}. \tag{49}$$

It follows immediately that the mapping from Q^0 to p_0 and p_1 is continuous (in L^1 topology) and since $\int \alpha(z) dz = 1$, that p_s^0 and p_s^1 lie in $[1, \theta/(\theta-1)]$. From (20), it follows that

$$C_s = \bar{A}_s \left(\frac{p_1}{p_0}\right)^\theta, \tag{50}$$

and from (27) that

$$\pi_s^m = \frac{1}{p_1^{1-\theta}} \frac{(\theta-1)^{\theta-1}}{\theta^\theta} \alpha_s C_s = \frac{1}{p_1^{1-\theta}} \frac{(\theta-1)^{\theta-1}}{\theta^\theta} x_{s+1} C_s. \tag{51}$$

Now, for each z, given $\pi^m \in \mathbb{R}_+^S$, the program in Lemma 1 provides a continuous mapping from π^m to

$$\pi_s \in \prod_1^S [0, \pi_s^m]. \tag{52}$$

We use (22) to define the operator $\mathcal{F}$, which operates on functions, and which is given by:

$$Q_s^1(x) = (\mathcal{F}(Q^0)(x))_s = 1 + \frac{p_1(s)^{1-\theta}}{C_s x_{s+1}} (Q_s^0(x))^\theta \pi_s.$$

Since each operation in (48-52) is continuous, it follows that $\mathcal{F}$ is a continuous operator (in $L^1(\mathbb{R}^{1+S})$-norm). Further, it also follows that if $Q_s^0(x) \in \left[1, \frac{\theta}{\theta-1}\right]$, then since $0 \leq \pi \leq \pi^m$, $1 \leq Q_s^1(x) \leq 1 + \frac{(\theta-1)^{\theta-1}}{\theta^\theta}(Q_s^0)^\theta \leq \frac{\theta}{\theta-1}$. Define, Z as the set of all functions, $Q : R \to [1, \theta/(\theta-1)]^S$, such that Q is nonincreasing in its first argument and nondecreasing in all other arguments. Then, from what we have just shown, together with Proposition 4, it follows that $\mathcal{F}$ is a continuous operator that maps Z into itself. We also have

Lemma 7. *Z is convex and compact.*

We prove that the set, W, of nondecreasing functions $f:[0,1]\to[0,1]$, is convex and compact. The generalization to functions with arbitrary rectangular domains and ranges, $f:\prod_1^N[a_i,b_i]\to\prod_1^M[c_i,d_i]$, is straightforward, as is the generalization to functions that are nonincreasing in some coordinates and nondecreasing on others (as is Z). Convexity is immediate. For compactness, we show that every sequence of functions $f^n\in W$, $n=1,2,\ldots$, has a subsequence that converges to an element in W. First, note that W is closed, since a converging (Cauchy) sequence of nondecreasing functions necessarily converges to a nondecreasing function. To show compactness, define the corresponding sequence of vectors $g^n\in[0,1]^{2^j}$, for some $j\geq 1$, by $g_k^n=f_n(2^{-j}k)$, $k=0,1,\ldots 2^j-1$. Now, since $[0,1]^{2^j}$ is compact it follows that there is a subsequence of $\{f^n\}$, $\{f^{n_m}\}$ that converges at each point $2^{-j}k$, to some $g^*\in[0,1]^{2^j}$. Define the function $h^j:[0,1]\to[0,1]$ by $h^j(x)=g_k^*$, for $2^{-j}k\leq x<2^{-j}(k+1)$, which is obviously also in W. Next, take the sequence $\{f^{n_m}\}$, and use the same argument to find a subsequence that converges in each point $2^{-(j+1)}k$, $k=0,\ldots,2^{j+1}-1$, and the corresponding function $h^{j+1}(x)$. By repeating this step, we obtain a sequence of functions in W, $h^j,h^{j+1},\ldots$, such that for $m>j$,

$$\int_0^1|h^m(x)-h^j(x)|dx\leq\sum_k(g_{k+1}^j-g_k^j)2^{-j}\leq 2^{-j}.$$

Thus, $h^j,h^{j+1},\ldots$ forms a Cauchy-sequence, which consequently converges to some function $h^*\in W$. Take a subsequence of the original sequence of functions, $\{f^{n_j}\}$, such that $\int|f^{n_j}-h^j|dx\leq 2^{-j}$. Then, for $m>j$, since

$$\begin{aligned}\int_0^1|f^{n_m}(x)-f^{n_j}(x)|dx &= \int_0^1|f^{n_m}(x)+h^m(x)-h^m(x)+h^j(x)-h^j(x)-f^{n_j}(x)|dx\\ &\leq \int_0^1|f^{n_m}(x)-h^m(x)|dx+\int_0^1|f^{n_j}(x)-h^j(x)|dx\\ &+\int_0^1|h^m(x)-h^j(x)|dx\\ &\leq 3\times 2^{-j},\end{aligned}$$

$\{f^{n_j}\}$ is also a Cauchy sequence and converges to $h^*\in W$. Thus, W is compact and the lemma is proved. Given Lemma 7 and the continuity of $\mathcal{F}$, a direct application of Schauder's fixed point theorem

implies that there is a $Q^*\in Z$, such that $\mathcal{F}(Q^*)=Q^*$. Now, given such a Q^*, and its associated π^m defined by (51), and given the functions, $N(z)$ and $\alpha_s(z)$, $0\leq z\leq 1$, Lemma 1 can be used to construct $Q_s(z)$. Since Q and Q^* have the same distributional properties, and C, p_0 and p_1, only depend on distributional properties, it immediately follows that Q constitutes an equilibrium. We are done.

Proof of Proposition 6

Follows immediately from Proposition 3.

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