Targeted Advertising: How Do Consumers Make Inferences?

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Abstract

Using increasingly granular customer data, firms have improved their targeting capabilities to proactively reach customers who are not even aware of their needs or wants for the product. The mere fact that consumers get targeted by firm's advertising can influence their inference about unknown utility from a product. We build a micro-model in which multiple firms compete through targeted advertising, and consumers make inferences from targeted advertising about their unknown match value for the product category, as well as the advertising firm's unobserved quality. We show that in equilibrium, upon being targeted by a firm, consumers make optimistic inferences about the product category, as well as the quality of the firm. So, With the improved beliefs, a targeted consumer may be more likely to engage in costly search throughout the category. We find that this increase in consumer search creates advertising spillover and firms' equilibrium amount of targeted advertising can be non-monotonic in the targeting accuracy. Therefore, consumer search can mitigate competition in targeted advertising. We show that, without consumer search, advertising competition intensifies significantly that it can be optimal for firms to relinquish the customer data, and instead engage in non-targeted advertising. The results provide insights into trade-offs between advertising reach and targeting precision.

Keywords: targeted advertising, targeting accuracy, consumer inference, consumer search, reach and precision, value of information, prominence, free-riding

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1 Introduction

With the rise of big data and artificial intelligence, firms have been able to collect and process an unprecedented amount of consumer-level data, which helps firms to gain an even more in-depth understanding of their customers. Using increasingly granular customer data, they can identify customers who are more likely to need their products or services and benefit from the product category (Davenport et al., 2001; Braun and Moe, 2013; Summers et al., 2016). For example, advertisers on Facebook can use customers' demographic information (e.g., age, gender, and location), their social activities on the platform (e.g., wall postings, clicked ads, "likes", and "sharing"), and social networks (e.g., who are their friends, and what they do and like)[1] to *target* their desired group of customers. In 2016 Lexus launched highly personalized ads for individual Facebook users by matching video clips based on data including social media usage profile and other behavioral data. A woman who purchased traveling luggage or likes many traveling sites sees one video, while a man in mid-30s who likes music and fashion would see another.[2] Leveraging an increasing ability to harvest and interpret consumer-level data, companies identify those target customers and reach out to them with highly targeted advertising even before customers are aware of their needs and wants. As a result, consumers are often exposed to targeted ads about the products which they were not even aware of the existence.

In the past few decades, the targeting technology has become more valuable. The consumers who are likely to have a strong preference will receive the targeted message instead of those who have no interest and whose preferences do not match a product's benefit. Research has shown that digital targeting meaningfully improves the response to advertisements and that ad performance declines when marketers' access to consumer data is reduced (John et al., 2018). Armed with customer data,[3] targeted advertising becomes more effective in increasing both click-through and conversion rates (e.g., Ansari and Mela, 2003; Joshi et al., 2011; Lambrecht and Tucker, 2013; Summers et al., 2016; Yan et al., 2009), and more and more firms expand their spending on targeted advertising mainly through

[1]See a New York Times article, "Facebook and Cambridge Analytica: What You Need to Know as Fallout Widens" and CNBC news article, "How Facebook ads target you" at https://www.cnbc.com/2018/04/14/how-facebook-ads-target-you.html.

[2]"Beyond Utility: 1000 to 1 – The Shorty Awards" at https://shortyawards.com/8th/beyond-utility-1000-to-1.

[3]Given the Facebooks recent scandal involving Cambridge Analytica, privacy issues raise significant concerns for both marketers and consumers. As a result, many firms like Facebook and Google try to avoid using sensitive information such as race, sexual condition, and health conditions. Privacy issue and its effects on information sharing (especially, a third-party data sharing see Goldfarb and Tucker, 2011a, Goldfarb and Tucker, 2011b, Goldfarb and Tucker, 2012, Tucker, 2012) is an important topic, but this is not the focus of the current research and we will leave this important issue for future research.

digital channels.[4] Such targeted advertising is crucial in a product category where consumers' default engagement level is low because it is an infrequently purchased product, or a new product category which consumers are not familiar with or even aware of. In these circumstances, firms can use targeted advertising to induce customer engagement and create demand by identifying those prospects who are more likely to become interested in the product category.

However, targeting those potential customers in an early stage of consumers decision can be risky. Sometimes, less than 50% of qualified initial leads initiated by a brand's targeted advertising are moving toward to the final purchase stage of the same brand (Court et al. 2009 *McKinsey Quarterly*). In this early phase of consumers' decision journey,[5] firms need to convince and encourage them to deliberate their potential needs, and thus increasing product acceptance (Lu and Shin, 2018). The initial efforts to identify and attract new prospects who do not understand the products uses and benefits (sometimes, they are even unclear as to whether they need the product) can be substantial. Furthermore, those initial targeted advertising spending can be wasted if consumers eventually make a purchase from another firm (Shin, 2007). In other words, firms can *free-ride* on competitor's' advertising efforts to enhance customer awareness or interests in the product category. This can reduce incentives for investing in advertising.

Consider the following incident. While on Facebook, one of the authors was shown an advertisement featuring a new scanning app for iPhone iOS on Facebook. He clicked the ad and downloaded the free version of the app. Although he did not like this particular app (especially, he did not even know the existence of such a product and, after a few trials, he could not appreciate the value of a mobile scanning function over simple camera), he is aware of the fact that Facebook ads are often highly relevant. Thus, instead of simply ignoring the mobile scanning function entirely, he further searched for other scanning apps in Google. Then, he realized that it could be extremely useful in scanning documents instantaneously and export them as multi-page PDF files. As a result, he purchased a different scanning app with such useful functions. Clearly, the targeted advertising by one seller

[4] In 2017, Google garnered $35 billion in the US market which is up 18.9% over the previous year, and Facebook captures $17.37 billion (https://www.emarketer.com/Article/Google-Facebook-Tighten-Grip-on-US-Digital-Ad-Market/1016494).

[5] The customer journey (Court et al. 2009, Lemon and Verhoef 2016, and Richardson 2010) is an idea that conceptualize customer experience as a "journey" with a firm over time during the purchase cycle across multiple touch points. The literature in customer journey emphasizes the purchase funnel such as AIDA (Awareness-Interest-Desire-Action) model. In particular, Shin (2005) conceptualizes the costs associated with the early stage of purchase funnel as selling costs and its importance in the sales process.

has motivated his interest in such product category, but this advertising spillover effect benefits its competitor who free-rides off of the firm's costly efforts on targeted advertising. Without this targeted ad, consumers would not have been prompted to further search a product.

Advertising in this case is a key to engage consumers and create the category demand. On the one hand, Facebook's targeting ability helps firms to reach those people who might be interested in the product feature or benefit based on customer information. Facebook might have first filtered customers who are in education (students or academics) or running business for whom document-scanning can be a useful option for several reasons such as reducing the amount of paperwork and unnecessary filing cabinets, improved data security and protection, etc. In this example, the mere fact that the ad is targeted made one of us more interested in the product category and eventually purchase a product. Consumers' distinct response to targeted advertising implies that consumers acknowledge the *relevance* of targeted advertising, consciously and unconsciously, and make an inference based on the fact that they are targeted. We focus on the mechanism that triggers this additional effect of targeting beyond the simple advertising effect of increasing awareness through consumers' inferences and their subsequent search behavior.

On the other hand, enhancing consumers' belief about the match value from a product, or the product category in general, by a firm's targeted advertising can not only help the firm, but also benefit all other firms in the category. This advertising spillover effect can be a serious issue and dissuade firms from investing in targeted advertising because all of their advertising efforts can be wasted when consumers eventually switch to competing firms.

This paper focuses on investigating the effect of targeting accuracy, which is the key determinant of consumer inference, and market outcomes. There are several forces that affect firms decision whether or not to invest in targeted advertising. First, the more a firm advertises, the more likely it is that it becomes a *prominent* firm that consumers consider first (e.g., Armstrong et al., 2009, Armstrong and Zhou, 2011), which helps to preempt demand under competition. As the accuracy of targeting improves, prominence can be valuable as most consumers will be satisfied with their first search and buy immediately. Second, targeting accuracy can increase the advertising efficiency by reducing the wasted advertising to consumers who may not be interested in the product category (Goldfarb, 2014). These two benefits are the direct effects of improved accuracy (i.e., *demand preemption through prominence* and *cost efficiency*).

There is another indirect effect of improved accuracy, which works as an opposing force; namely, *free-riding of competitors* due to increased consumer search (Shin, 2007). As targeting becomes more accurate, their beliefs about category becomes more optimistic, which may encourage consumers search for other alternatives beyond the prominent firm. This effect reduces each firm's incentives to invest in costly advertising. Therefore, the improved accuracy makes the prominence through targeted ads more desirable, but it also induces more consumer search and free-riding.

In this paper, we build a game-theoretic model to formally study how the use of customer data for targeted advertising affects consumers' search behavior and purchasing decisions when there are multiple firms in the market. We first begin by providing a micro-model about consumer inference process when they encounter a targeted advertising. With this understanding, we identify firms' equilibrium targeted advertising strategy, which accounts for the effect of the advertising strategy on consumers' search and purchasing decisions. We show that, in equilibrium, firms focus their advertising efforts on consumers who are likely to benefit from the product category, and higher quality firm invests more on targeted advertising. Therefore, upon being targeted by a firm's advertising, consumers rationally make more optimistic beliefs about both their unknown match value for the product category and unobserved quality of the firm.

We also find several interesting implications of targeting accuracy on equilibrium outcomes. First, we find that the targeting accuracy has non-monotonic effects on the extent of *consumer search*. On the one hand, more accurate targeting improves match between consumers and the product category, which reduces the need for further search because it increases the chance that consumers will be satisfied with the first firm they visit. This implies that more accurate targeting may eliminate the need for search beyond the first firm in the category. On the other hand, conditional on being dissatisfied with the first firm, consumers who are targeted still hold optimistic beliefs about the product category, and therefore, they may search for a better alternative. Based on these two opposing forces, we show that the amount of search is increasing in targeting accuracy when the targeting accuracy is high enough.

Second, consumers' extensive search in the product category induces free-riding, which reduces each firm's incentives to invest in targeted advertising. So, we show that the equilibrium amount of advertising can be non-monotonic in targeted advertising. In particular, it can decrease when the targeting accuracy is sufficiently high.

Third, despite this non-monotonic effect of targeting accuracy on the amount of targeted advertising, we find that targeted advertising is highly appealing for firms, and therefore its amount can exceed that of optimal level of non-targeted advertising. This is surprising, given that by definition targeted advertising can be sent to a smaller market of consumers who might have a good match with the product category. And yet, targeted advertising can be highly effective, and therefore firms invest in it aggressively. And, this aggressive investments in targeted advertising can drive up the total cost of advertising beyond the level spent under non-targeting case.

Finally, we find that firms can benefit from this free-riding effect induced by consumer search because it mitigates competition in targeted advertising. In particular, when advertising cost is sufficiently high, firms can be better off relinquishing all customer data and instead engage in non-targeted advertising. This result provides an insight into a recent debate on how companies need to cope with trade-offs between advertising reach and targeting precision.[6] Under targeted advertising, firms concentrate their advertising on a smaller group of the entire customers. Moreover, it is highly effective in identifying attractive customers. Therefore, competition in targeted advertising can be fierce, which will amplify the total cost of advertising. On the other hand, the competition can be mitigated in non-targeted advertising, which allows for greater advertising reach. This describes the tradeoffs between precision attainable under targeting and reach under un-targeting. Our analysis implies that the extent of consumer search plays an important role in this trade-off.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 and 4 present the model and analyze consumers' inference processes and search behaviors when they receive targeted advertising. Section 5 identifies firms' optimal targeted advertising decisions incorporating these consumer behaviors. Section 6 analyzes profits, and Section 7 concludes.

2 Literature Review

This paper is closely related and contributes to the literature in targeted advertising and consumer search. First, the literature in online advertising has emphasized the importance of targeting using various customer data, such as demographic information (Joshi et al., 2011), cognitive styles (Hauser et al., 2009), browsing behaviors such as ad clicks (e.g., Agarwal et al., 2009; Chen et al., 2009), or past purchases (e.g., Rossi et al., 1996; Fader et al., 2005; Malthouse and Elsner, 2006). Research in

[6]https://www.wsj.com/articles/p-g-to-scale-back-targeted-facebook-ads-1470760949.

this area consistently finds that tailoring the message based on the target segment characteristics or specific content of online website improves the performance of communications and consumer response (Goldfarb and Tucker, 2011a, Zhang and Katona, 2012). Our paper contributes to the literature by providing a micro-foundation for such effectiveness based on consumer's rational inference process from the mere fact of getting targeted.

There also exists a stream of research in targeted advertising which investigates the effects of targeting precision on the equilibrium outcomes. An early contribution in this area is Chen et al. (2001). They show that imperfect targeting can soften price competition among firms. This is because firms can misconceive price-sensitive consumers (switchers) as price-insensitive consumers (loyal customers) and therefore would charge greater prices than under the case of perfect targeting. Iyer et al. (2005) find that, with targeted advertising, firms benefit from targeting, compared to no-targeting, because it differentiates firms. For similar reasons, Bergemann and Bonatti (2011) find that the equilibrium price of advertising can decrease in targeting accuracy, even though its marginal product is increasing in targeting accuracy. From the perspective of an ad platform, Levin and Milgrom (2010) argue that platforms have incentives to limit advertisers' access to detailed customer data in order to make them less differentiated. Zhong (2016) studies a similar issue when the platform can control the accuracy of consumer search technology. He investigates how it affects the consumer search and its implications on the firms prices and platform revenue. Rafieian and Yoganarasimhan (2017) document empirical evidence to support an ad platform's incentives to withhold information from advertisers. Similar to these researches, we also study the effects of targeting accuracy on the equilibrium outcomes. In contrast, our focus is on the on the micro-process of consumer inference and we investigate trade-offs between targeting accuracy and the advertising intensity.

Also, there is a stream of research which extends the domain of targeting beyond advertising and focuses on the customized pricing using customer information (Rossi et al., 1996; Besanko et al., 2003). In particular, the literature on behavior-based pricing investigates the optimal price discrimination strategy based on customers' past purchase history (e.g., Fudenberg and Tirole, 1998, 2000; Villas-Boas, 1999; Shaffer and Zhang, 2000; Villas-Boas, 2004; Shin and Sudhir, 2010).

Our model also builds on the consumer search theory in differentiated product market. Wolinsky (1986) was the first to propose a sequential search in a horizontally differentiated market in the context of random search order. Subsequently, Anderson and Renault (1999) analyzed the effects of search costs on the relationship between the degree of product differentiation and equilibrium prices. A more recent line of research has examined more realistic market situations where search is non-random and consumers search in a deliberate order (see Armstrong, 2017 for an extensive literature on this ordered search literature). Armstrong et al. (2009) demonstrates that, when consumers engage in costly search across firms, *prominence*, or being the first shopping destination, can be valuable as it can preempt demand. Armstrong and Zhou (2011), Chen and He (2011), and Zhou (2011) endogenize the prominence by allowing firms to obtain prominence, for example, by charging a lower price than other firms which consumers visit later in their search process. An extensive consumer search implies that a consumer can eventually buy from a firm different from the first firm they visit. Therefore, consumer search endogenously creates advertising spillover effect. Bronnenberg et al. (2016) empirically document that consumers engage in an extensive search on-line beyond the first prominent firm and often times choose a product that they see toward the end of their search. Also, Honka et al. (2017) finds that advertising makes consumers search more and eventually find better alternatives.

Several papers attend to the effects of consumer search on equilibrium profits.[7] On the one hand, if consumers engage in extensive search, the market would become more competitive. To prevent competition, firms can strategically deter consumer search by making it more costly to acquire product information (Ellison and Wolitzky, 2012) or making exploding offers (Armstrong and Zhou, 2015). On the other hand, sometimes firms strategically encourage consumer search which can help to create demand for all firms as consumers become better informed about product category. In markets where adverse selection in unobserved quality of firms can discourage consumers' engagement, non-attribute focused advertising (Mayzlin and Shin, 2011) or multi-product retailing (Rhodes, 2014) can credibly convey product information and encourage consumer search. Lu and Shin (2018) show that in an innovative product category sharing innovation with its competitors can also serve as an invitation to search, which increases the category demand.

This paper is different from other works in targeted advertising and consumer search in that we focus on the micro mechanism of consumer inference from simply receiving a targeted ad, which in

[7]Another stream of research focuses on the strategic implications of search costs on equilibrium outcomes such as product design and prices (Lynch and Ariely, 2000; Kuksov, 2004; Bar-Isaac et al., 2012; Branco et al., 2012). In particular, Branco et al. (2012) develop a tractable gradual learning model in which an agent incurs a search cost to learn product value in the context of a single agent's decision making under sequential information search, which is extended to a multiple-product cases for a single agent (Ke et al. (2016)) or two-sided learning in the context of sales process where a buyer and a seller learn information gradually over time (Ning, 2018).

turn influences their extent of search in the product category. In that sense, this paper is related to a recent work by Summers et al. (2016) who demonstrate this phenomenon through a series of behavioral experiments. In their work, the mechanism that explains consumers' inferences upon seeing a targeted advertising is a psychological one of social labeling. Consumers receive information about what others think about them (by targeted advertising), resulting in adjustments to self-perceptions and behavior consistent with the label. On the other hand, the mechanism in this paper is based on Bayesian updating by rational consumers who recognize the relevance of targeted advertising which is based on customer data. Importantly, consumers observe *one signal*, which is the realized advertising state, and update inferences about *two unknowns* – their own match type for the product category and the advertisers' quality types.

3 Model

There are two firms, $j \in \{A, B\}$ that compete with each other in the same product category. Both firms sell a product to a unit mass of consumers. A consumer (she) may have a bad match with the product category, and therefore cannot benefit from buying any product in this category. On the other hand, the consumer may have a good match with the product category, and therefore may enjoy a product in this category, if the product satisfies the consumer's needs. The following utility function captures this idea:

$$u_{ij} = m_i \cdot v_j, \tag{1}$$

where $u_{ij} \in \{0, 1\}$ is the consumer i's utility from buying firm j's product in the product category. $m_i \in \{0, 1\}$ is consumer i's category *match* for this particular product category, and $v_j \in \{0, 1\}$ is *value* of the product j to the consumer. $m_i = 1$ if the consumer is of a *good* match-type for the product category, whereas $m_i = 0$ if the match-type is *bad*. This category match is drawn from a common distribution such that $\Pr(m_i = 1) = \mu \in (0, 1)$, but the realization is unknown to the consumer. Second, $v_j \in \{0, 1\}$, the firm j's product value to a consumer, takes value $v_j = 1$ if it can address the customer's needs, and otherwise $v_j = 0$. The realization of v_j depends on quality type of firm j, denoted by q_j, which is drawn independently from a distribution $F(\cdot)$ on $[0, 1]$, where $q_0 = E[q]$. More precisely, the product value $v_j = 1$ with a probability q_j and $v_j = 0$ with a probability $1 - q_j$. Therefore, a higher quality firm's product is able to meet consumers' need or want with a greater probability. Each firm's quality type is private information of the firm.

Once consumer i visits firm j, she learns $u_{ij} \in \{0, 1\}$, i.e., whether or not she likes a product. If she likes the product, her utility is $u_{ij} = 1$ and she knows that this product category is a good match ($m_i = 1$) *and* the product is sufficiently good such that it satisfies her specific need ($v_j = 1$). However, if she does not like the product, i.e., $u_{ij} = 0$, then she is unable to identify the source of displeasure. That is, she does not observe the exact realization for the consumer's own match-type (m_i) and product value (v_j) separately.

This is a critical assumption for our model which implies that if consumers have a bad experience with a product, the consumer makes inferences about her own match-type and the firm's unobserved quality type. Based on these two inferences, the consumer will make subsequent decisions.[8]

Information and Targeting Technology

Firms have an access to customer data which provides a noisy signal $s_i \in \{g, b\}$ for m_i, or the consumer i's true match type for the product category. We assume that both firms have an access to the same data from a platform, such as Facebook or an on-line web publisher such as the New York Times. Therefore, they receive a *common* signal about each consumer.[9] Based on the signal that firms receive about each customer, firms can classify customers into two segments; *perceived good-type* customers whose signal is good, $s_i = g$, and thus underlying match-type is likely to be good, and *perceived bad-type* customers whose signal is bad, $s_i = b$, and therefore match with the category is likely to good. This is the perceived market segmentation from the firms' perspectives. For example, if a person had previously purchased a energy-saving light bulb, the platform may perceive her as being interested in an environmentally sustainable product category in general (Summers et al., 2016). How informative the noisy signals are depends on the type and amount of customer data.[10]

We measure the informativeness of the signal by $\alpha \in (0, 1)$, which allows a possibility of imperfect

[8]Sometimes consumers can identify the source of their dissatisfaction with a product. Our analysis can accommodate this situation as a limit case of our model where one of prior beliefs goes to 1 or 0. However, our focus is on many other situations such as new product category or infrequently purchased product category where consumers have little experience with.

[9]We consider typical advertising situations where advertisers use an accessible advertising networks such as Google, Facebook and Amazon, who provide the same customer information to all the advertisers. However, in some cases, it is possible that different firms may have access to different data using their own first party data.

[10]It is reported while the precision of data in most platforms can be anywhere between 10% and 20% (even gender is usually only 75% accurate), targeting accuracy in Facebook can be an order of magnitude better than anywhere else, except for a few exceptions like Google Search (*Forbes*,"How Accurate is Marketing Data?" 2017 July 5).

targeting, for example, due to the lack of sufficient customer information (i.e., the platform has no history information for a new customer) or imperfect information processing technology (Chen et al. 2001). If the true match of the customer i is $m_i = 1$, firms receive a correct signal $s_i = g$, indicating the consumer to be a "perceived" good type with probability $\alpha \in (0, 1)$. Otherwise, with probability $1-\alpha$, the platform provides a signal which is randomly drawn from the prior beliefs about the customer types so that $s_i = g$ with probability μ and $s_i = b$ with probability $1 - \mu$. Likewise, if the true match of the consumer i is $m_i = 0$, firms receive a correct signal, $s_i = b$, with probability α, and otherwise they receive a random signal according to the prior beliefs. So, the signal structure can be summarized formally as following:

$$\begin{aligned} &\Pr(s_i = g|m_i = 1) = \alpha + (1-\alpha)\cdot\mu, \quad \Pr(s_i = b|m_i = 1) = (1-\alpha)\cdot(1-\mu) \\ &\Pr(s_i = b|m_i = 0) = \alpha + (1-\alpha)\cdot(1-\mu), \quad \Pr(s_i = g|m_i = 0) = (1-\alpha)\cdot\mu \end{aligned} \tag{2}$$

Here, one can think of $\alpha = 0$ as the case of non-targeting where firms do not have any information for a new customer, and thus, they can only rely on the prior distribution of the consumer types in the market. On the other hand, the case of $\alpha = 1$ would imply the perfect targeting where firms can perfectly identify each customer's type.[11] So, α captures the additional informativeness of signals beyond the prior distribution over consumers' types, i.e., $\Pr(m_i)$ for $m_i \in \{0, 1\}$. Accordingly, we refer to α as the "targeting accuracy."

Targeted advertising

Given the customer data which allows firms to execute targeted advertising with accuracy $\alpha \in (0, 1)$, firms choose how much to advertise. Firm j of private quality type q_j decides its *advertising intensity* for two segments of consumers: the perceived good-type with $s_i = g$, and the perceived bad-type with $s_i = b$. More formally, the firm's advertising strategy is defined as a mapping $\sigma_j(q) = (\sigma_j^g(q), \sigma_j^b(q))$, where $\sigma_j^s(q) \in [0, 1]$ denotes the fraction of consumers with signal $s \in \{g, b\}$ to be reached by the firm's advertising.

For example, in the extreme case, if $\sigma_j^g(q) = 1$ and $\sigma_j^b(q) = 0$, the firm sends an advertising to all

[11]The distribution of signals in Equation (2) is a special case of copula formula which captures a dependence between two distributions; one for prior and the other for a noisy signal. Similar specifications have been adopted by Klemperer (1995), Shin and Sudhir (2010), and Shen and Villas-Boas (2017).

consumers perceived as good-type and none of those perceived as bad-type. We focus on a symmetric equilibrium in which firm A and B adopt the same strategy, i.e., $\sigma_j(q) \equiv \sigma(q)$ for all $j \in \{A, B\}$ and q.

Each firm's actual advertising level is not observed by the other firm, or by consumers. However, consumers have a rational expectation about each firm's advertising strategy, $\sigma_j(q) = (\sigma_j^g(q), \sigma_j^b(q))$, and therefore it is useful to distinguish notations between each firm's actual advertising choice, $\tilde{\sigma}_j = (\tilde{\sigma}_j^g, \tilde{\sigma}_j^b)$, and the expected advertising strategy, $\sigma_j(q) = (\sigma_j^g(q), \sigma_j^b(q))$.[12] It is costly to send each unit of advertising, and we assume that the total cost of advertising is quadratic in the total amount of advertising, $\mu \cdot \tilde{\sigma}_j^g + (1-\mu) \cdot \tilde{\sigma}_j^b$, so that it is an increasing and convex function:

$$c(\tilde{\sigma}) = k \cdot \frac{(\mu \cdot \tilde{\sigma}^g + (1-\mu) \cdot \tilde{\sigma}^b)^2}{2}, \tag{3}$$

where $k > 0$ captures the unit cost of advertising.

Consumers may receive an advertising from both firms, just one firm, or no firms. So, there are four distinct segments of consumers who belong to different advertising states. A consumer i's advertising state is defined by $\theta_i = (a_A, a_B)$, where $a_j \in \{0, 1\}$ indicates whether the consumer received an advertising from firm j (denoted by $a_j = 1$) or not (denoted by $a_j = 0$). For simplicity, we denote this state by θ^{a_A, a_B}. For example, $\theta^{1,1}$ represents the state in which the consumer received both firms' advertising; $\theta^{1,0}$ and $\theta^{0,1}$ if she received firm A's and B's advertising only, respectively; $\theta^{0,0}$ if no advertising was received.

Then, the realized distribution over the set of advertising state is:

$$\begin{aligned}
\Pr(\theta_i^{1,0}) &= \mu \cdot \tilde{\sigma}_A^g(1-\tilde{\sigma}_B^g) + (1-\mu) \cdot \tilde{\sigma}_A^b(1-\tilde{\sigma}_B^b) \\
\Pr(\theta_i^{0,1}) &= \mu \cdot (1-\tilde{\sigma}_A^g)\tilde{\sigma}_B^g + (1-\mu) \cdot (1-\tilde{\sigma}_A^b)\tilde{\sigma}_B^b \\
\Pr(\theta_i^{1,1}) &= \mu \cdot \tilde{\sigma}_A^g \cdot \tilde{\sigma}_B^g + (1-\mu)\tilde{\sigma}_A^b\tilde{\sigma}_B^b \\
\Pr(\theta_i^{0,0}) &= \mu \cdot (1-\tilde{\sigma}_A^g)(1-\tilde{\sigma}_B^g) + (1-\mu)(1-\tilde{\sigma}_A^b)(1-\tilde{\sigma}_B^b).
\end{aligned} \tag{4}$$

[12] For simplicity, we sometimes omit the quality q, and denote this notion as $\sigma_j = (\sigma_j^g, \sigma_j^b)$ subsequently.

Time-line

The game proceeds in three stages. Stage 1 is the *advertising stage*. First, each firm is endowed with its own quality type q_j, which is drawn independently from a distribution $F(\cdot)$ on $[0, 1]$. Given their quality q_j, firms choose their levels of advertising $\tilde{\sigma}_j^s$ for *perceived* s-type segment ($s \in \{g, b\}$) with the information accuracy $\alpha \in (0, 1)$ based on the customer data. Each consumer receives advertising from either, both or none of firms.

Stage 2 is the *inference stage* in which each consumer makes inferences based on her advertising state θ_i and decides whether and which firm to visit first. If a consumer receives an advertising from firm j, the consumer can visit the firm by simply clicking on an interactive link or banner. Here, we assume that her first visit to a firm incurs zero cost to capture the effect of being a prominent firm. Also, when consumers receive ads from both firms, consumers randomly choose one of them, and visit it with a zero cost (if she decides to search another firm, she can do so in stage 3 which we describe below). If consumers receive no ads, they remain unaware of the new product category and thus, do not participate in the market. We will discuss consumers' decisions and their search behaviors more carefully in the next section.

Once a consumer visits firm j, the consumer learns her utility $u_{ij} \in \{0, 1\}$. However, she does not separately observe her exact match for the category (m_i), or the product value v_j, which is a function of the firm's unobserved quality q_j. If $u_{ij} = 1$, the consumer buys a product and leaves the market. On the other hand, if $u_{ij} = 0$, consumers makes inferences about two dimensions: their match for the category, m_i, and the product value of the other firm, v_k for $k \neq j$. Depending on these inferences, consumers decide whether or not to search further for another firm. Consumers' two-dimensional belief updating is influenced by firms' advertising strategy, consumer's advertising state, and targeting accuracy. The fact that consumers get targeted by advertising has a significant influence on their inferences and search behaviors.

Stage 3 is the *search stage* in which consumers decide whether to continue to search the other firm at a search cost t_i, which is drawn from a uniform distribution with support $[0, T]$. Here, a consumer compares her search costs t_i with her expected utility from search based on her updated beliefs from Stage 2. When she decides to search another firm, she incurs a search cost irrespective of whether a consumer has or has not received an ad in Stage 1. Even for the firm whose ad was delivered to a

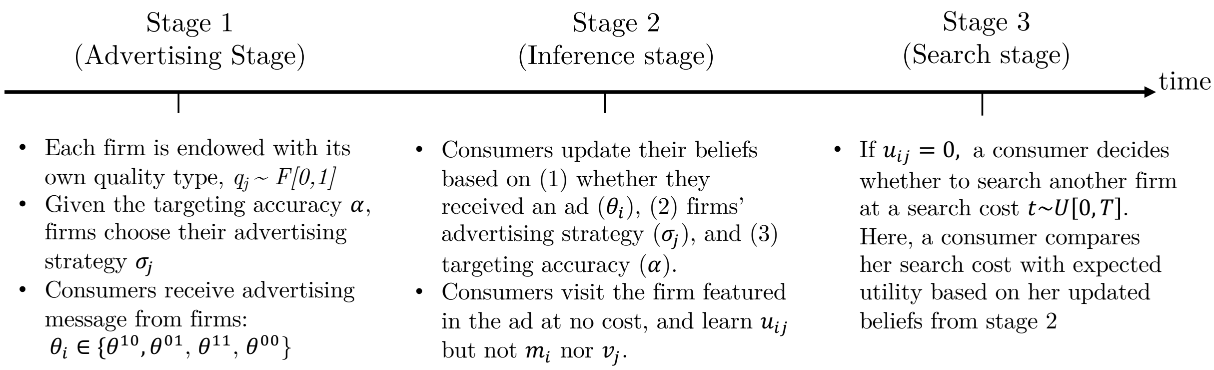

Figure 1: Timing of the Game

consumer in the past, she has to still incur extra time and effort to remember and find out the old ad that she once had overlooked.[13]

The entire sequence of the game is summarized in Figure 1. In the next section, we turn to consumer inference processes and their search behaviors.

Equilibrium

We use Bayesian Nash Equilibrium as our solution concept, which is defined as follows: (1) each firm's advertising strategy $\sigma_j(q)$ that maximizes its expected profit for a given quality $q_j \in [0,1]$, provided the other firm's advertising strategy and consumers' search and purchase decisions; (2) each consumer makes a search and purchase decision optimally, given firms' advertising strategies.

We consider a symmetric equilibrium in which both firms choose $\sigma^*(q) = \sigma_A^*(q) = \sigma_B^*(q)$ for any $q \in [0,1]$. We derive the symmetric equilibrium strategy in the next section. Before doing that, we establish a useful equilibrium property in the following lemma.

Lemma 1 *In any symmetric equilibrium, for any given q, if $\sigma^{g*}(q) < 1$, then $\sigma^{b*}(q) = 0$. In other words, firms do not advertise to perceived bad-type before they exhaust all the perceived good-type.*

Proof. See the Appendix. ■

[13] Cost from searching for a totally new firm and searching for a firm from which a consumer had received an ad in the past would be different. Probably, the latter will be lower than the former. This is a simplifying assumption that is still without loss of generality. What is crucial is the cost difference only between the first visit and subsequent visit. As long as the there is a small additional cost associated with any subsequent visit (it can be time or effort associated with finding out the other ad or using the search engine like Google), which makes this subsequent visit more costly than the very first visit that can be done rather effortlessly by clicking on the link, our results hold.

The lemma suggests that each firm would never send costly advertising to the perceived bad-types until they cover the entire perceived good-type consumers. Hence, when the advertising cost is sufficiently large, both firms choose $\sigma^*(q) = \sigma_A^*(q) = \sigma_B^*(q)$ where $\sigma^{b*}(q) = 0$. That is, firms concentrate their advertising efforts on the perceived good-types only. In Proposition 5, we characterize this equilibrium and identify conditions under which it uniquely exists. It is important to note that, in this equilibrium, observing a targeted advertising leads to the consumer's more optimistic inferences about her own true match-type with the category, because firms target only perceived good-types with advertising.

4 Analysis: Consumer inference and search behavior

We start with the consumer's problem where we examine the rational inference process when consumers observe targeted advertising. With the understanding of this micro-process of consumers' inference, we analyze their search and purchase decisions. After that, we examine the firms' advertising strategy which, in turn, alters consumer inference. Finally, we derive the equilibrium outcomes taking into account both consumers' inference and firms' optimal advertising strategy.

4.1 Consumer Inferences

Consumer i has prior beliefs about their own match value for the category and each firm's quality type. After a consumer i realizes advertising state $\theta_i^{a_A,a_B} \in \{\theta_i^{1,1}, \theta_i^{1,0}, \theta_i^{0,1}, \theta_i^{0,0}\}$, she updates her belief about her own match type m_i, as well as the firm's quality q_j, based on her prior beliefs ($\mu = \Pr(m_i = 1)$ and $q_0 = E[q]$), each firm's advertising strategy σ_j, and targeting accuracy α.

Belief updating about own category-match type

Given each firm j's advertising strategies $\sigma_j(q) = (\sigma_j^g(q), \sigma_j^b(q))$ for any given $q \in [0,1]$, the consumer's posterior belief about her own type after realizing advertising state $\theta_i^{a_A,a_B}$, where $\theta_i^{a_A,a_B} \in \{\theta^{1,1}, \theta^{1,0}, \theta^{0,1}, \theta^{0,0}\}$ is as the following:

$$\Pr(m_i = 1|\theta_i^{a_A,a_B}) = \frac{\Pr(m_i = 1) \cdot \big(\sum_{s\in\{g,b\}} \Pr(\theta_i^{a_A,a_B}|s) \cdot \Pr(s|m_i = 1)\big)}{\sum_{m_i\in\{0,1\}} \Pr(m_i)\big(\sum_{s\in\{g,b\}} \Pr(\theta_i^{a_A,a_B}|s) \cdot \Pr(s|m_i)\big)} \tag{5}$$

Here, $\Pr(\theta_i^{a_A,a_B}|s)$ is the distribution of realized advertising state conditional on the noisy signal attached to the consumer, $s \in \{g, b\}$. This depends on each firm's advertising strategy, which is a function of its private quality type. However, since consumers do not observe the firm's quality type q_j in updating beliefs, they account for the expected advertising. Let us define

$$E[\sigma_j^s(q)] := \int_0^1 \sigma_j^s(q)\, f(q)\, dq. \tag{6}$$

Then, the expected probability distribution over the advertising states, given the signal, s, generated for a consumer is: $\Pr(\theta_i^{1,1}|s) = E[\sigma_A^s(q)] \cdot E[\sigma_B^s(q)]$, $\Pr(\theta_i^{1,0}|s) = E[\sigma_A^s(q)] \cdot (1 - E[\sigma_B^s(q)])$, $\Pr(\theta_i^{0,1}|s) = (1 - E[\sigma_A^s(q)]) \cdot E[\sigma_B^s(q)]$, and $\Pr(\theta_i^{0,0}|s) = (1 - E[\sigma_A^s(q)])(1 - E[\sigma_B^s(q)])$.

As we can see from Equation (5), the consumer's posterior belief about the match type depends on the prior ($\mu = \Pr(m_i = 1)$), firms' equilibrium advertising strategy through $\Pr(\theta_i^{a_A,a_B}|s)$, and targeting accuracy (α).[14]

The next proposition characterizes the consumer's belief updating process about her match with the product category after receiving an targeted advertising.

Proposition 1 (Posterior Beliefs about Consumer's Match with Category) *When a consumer receives an ad, the consumer's posterior belief about her match with the product category improves:* $\Pr(m_i = 1|a_j = 1) - \Pr(m_i = 1|a_j = 0) > 0$. *Moreover, the marginal improvement in the posterior beliefs is increasing in targeting accuracy* α*:* $\frac{\partial[\Pr(m_i=1|a_j=1)-\Pr(m_i=1|a_j=0)]}{\partial\alpha} > 0$.

Proof. See the Appendix. ■

The result is intuitive. From Lemma 1, on average a firm covers a greater fraction of perceived good-type consumers than bad-type ones. Thus, consumers upon receiving an ad will make more optimistic inferences about their match with the product category. This marginal effect of advertising is greater if targeting is more accurate. On the flip side, if a consumer receives no advertising, then her posterior beliefs become more pessimistic under more accurate targeting.

[14] The targeting accuracy influences the belief updating in equation (5) through $\Pr(s|m_i)$ where $\Pr(s_i = g|m_i = 1) = \alpha + (1 - \alpha) \cdot \mu$, and $\Pr(s_i = b|m_i = 0) = \alpha + (1 - \alpha) \cdot (1 - \mu)$ as defined in equation (2).

Belief updating about firm's quality

Whether a consumer receives firm j's targeted advertising or not also influences her beliefs about the firm's quality level and its product value. The posterior distributions of firm j's unobserved quality depending on whether the consumer observed an advertising from the firm, $a_j \in \{0, 1\}$, is defined as follows:

$$\begin{aligned} h_j(q|a_j = 1) &= \frac{\Big(\mu \cdot \sigma_j^g(q) + (1-\mu) \cdot \sigma_j^b(q)\Big) f(q)}{\int_0^1 \Big(\mu \cdot \sigma_j^g(y) + (1-\mu) \cdot \sigma_j^b(y)\Big) f(y)\, dy}, \\ h_j(q|a_j = 0) &= \frac{\Big(\mu \cdot (1-\sigma_j^g(q)) + (1-\mu) \cdot (1-\sigma_j^b(q))\Big) f(q)}{\int_0^1 \Big(\mu \cdot (1-\sigma_j^g(y)) + (1-\mu) \cdot (1-\sigma_j^b(y))\Big) f(y)\, dy}. \end{aligned} \tag{7}$$

where $\sigma_j^g(q)$ and $\sigma_j^b(q)$ are firm j's advertising strategy to segment of consumers whose signal is $s \in \{g, b\}$ for a given quality level $q \in \{0, 1\}$.

In particular, consumers' inferences about firm j does not depend on whether they received an ad from the other firm. This is because each firm's quality type is independent, and therefore consumers have no additional information about firm j's type from the other firm's advertising strategy. The next proposition formally characterizes the consumer's belief updating about the firm's quality type upon receiving an ad.

Proposition 2 (Posterior Beliefs about Firm's Quality Type) *The posterior belief about firm's quality $h_j(q|a_j)$ satisfies the monotone-likelihood ratio property (MLRP): $\frac{h_j(q|a_j=1)}{h_j(q|a_j=0)}$ is increasing in q if and only if the total amount of advertising is increasing in q: $\mu \cdot \frac{d}{dq}(\sigma_j^g(q)) + (1-\mu) \cdot \frac{d}{dq}(\sigma_j^b(q)) > 0$.*

Proof. See the Appendix. ■

The monotone-likelihood ratio property implies that upon observing a targeted advertising ($a_j = 1$), the consumer's posterior distribution over the firm's unobserved quality type becomes more optimistic.[15] Intuitively, this should be true if indeed a higher quality firm advertises more than a lower quality one.

[15] The monotone-likelihood ratio property implies that $h_j(q|a_j = 1)$ has first-order stochastic dominance over $h_j(q|a_j = 0)$.

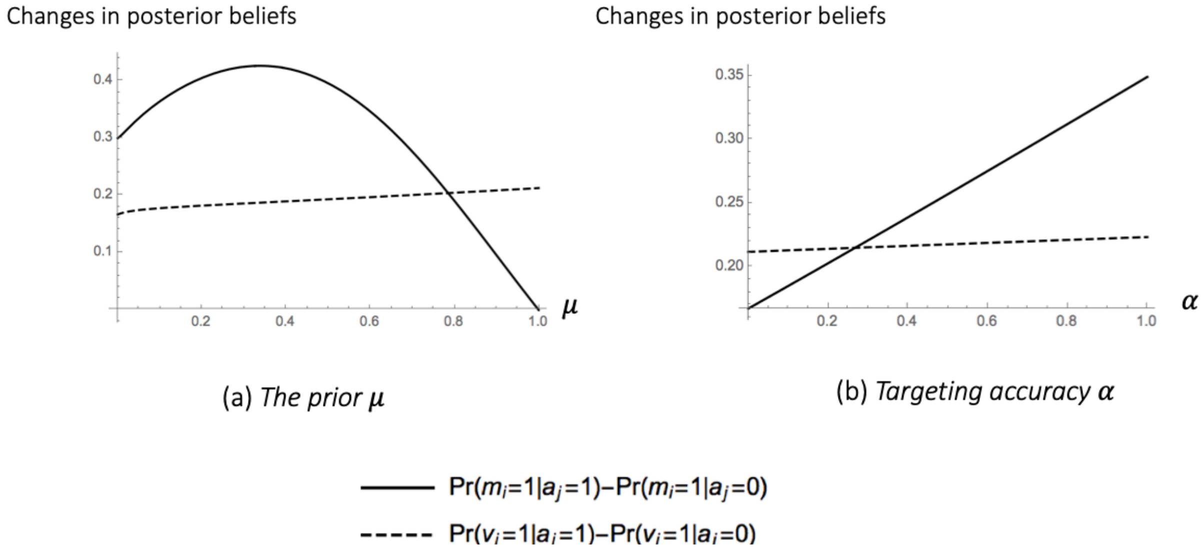

Figure 2: Marginal effect of targeting on the posterior beliefs as (a)μ and (b)α; $T = 0.4,\ k = 2$

Two-dimensional belief updating

In summary, consumers react to an targeted advertising based on firms' advertising strategy, prior beliefs, and information accuracy. After observing advertising, they update their beliefs about their own match type for the product category and about each firm's quality type. In particular, under quite general conditions a greater targeting accuracy can lead to more optimistic posterior beliefs about own matching type and firm's quality type.

Figure 2 plots two dimensional belief updating, more specifically $\Pr(m_i|a_j = 1) - \Pr(m_i|a_j = 0)$ and $\Pr(v_j = 1|a_j = 1) - \Pr(v_j = 1|a_j = 0)$. The former is the marginal effect of targeted advertising on consumer's posterior beliefs about the category type, and the latter the same for the firm's quality type. First, Figure 2-(a) demonstrates that μ (the prior belief about the consumer's category match-type), which can be considered as the measure of the potential size of the market, has always positive effects on the beliefs about quality type, and its effect increases monotonically. On the other hand, the effect on the beliefs about the consumer's own category match-type is non-monotonic because there is a ceiling for its effect as μ approaches one (then, there is little room for the belief to change). Note that in equilibrium, firms send advertising only to perceived good-type consumers of mass μ. Therefore, if μ is small, targeted advertising affects consumers' beliefs about their own category match-type more than the beliefs about the firm's unobserved quality type. However, this result can be the opposite if μ

is large. This implies that, for new or innovative product categories with a low μ, targeted advertising can stimulate their interests in the product category mainly because they come to believe that they may benefit from the product category in general. Nevertheless, for a product category which appeals to a large segment of customers, i.e., μ is large, targeted advertising can engage consumers in the product category primarily because they make inferences about product quality.

Second, Figure 2-(b) shows the nature of belief updating on these two dimensions along the targeting accuracy. Targeting accuracy has positive effects on consumer's beliefs about both the consumer's category type and the firm's quality. However, this positive effect is greater for the former. The targeting is based on the consumer's perceived category type, so the targeting accuracy has direct effects on the beliefs about the category type. On the other hand, its effect on the beliefs about the firm's quality type is indirect through the firm's advertising strategy. That is, given a higher targeting accuracy, a firm of higher quality may invest more, and therefore whether a consumer is targeted or not provides information about the firm's unobserved quality indirectly. This indirect effect is smaller than the direct effect on the beliefs about the category type.

4.2 Consumer Search and Demand

Once consumers observe advertising, they can visit a firm $j \in \{A, B\}$ featured in advertising at no cost, for example, by effortlessly clicking on the banner or link. Some of these consumers are satisfied with the firm (i.e., $u_{ij} = 1$) and make a purchase. We call this group of consumers firm j's *direct demand*, and denote it by D_j^{Dir}. On the other hand, there are also consumers who visit the other firm $k(\neq j)$ first, and then search firm j because they are not satisfied with firm k, i.e., $u_{ik} = 0$. If those consumers enjoy firm j's product, i.e., $u_{ij} = 1$, they purchase a product from firm j. This group of consumers is called *indirect demand*, denoted by D_j^{Ind}.

Direct Demand: Costless Consumer Visit to the Prominent Firm

If consumer i receives an advertising, she can visit the firm featured in the advertising first at no cost. Had she received an ad from only one firm $j \in \{A, B\}$ (i.e., $\theta^{1,0}$ or $\theta^{0,1}$), it is optimal for the consumer to visit firm j first. If a consumer is reached by both firms (i.e., $\theta^{1,1}$), she is indifferent between two firms. So, she visits one firm randomly with probability 1/2. This particular order of consumer visit provides the firm with an opportunity to preempt demand, which is the *prominence effect*. If she enjoys the product $u_{ij} = 1$, then she pays her willingness to pay.[16] On the other hand, if a consumer does not receive any advertising ($\theta^{0,0}$), then she is not even aware of the existence of the product category, such as scanning mobile app, and therefore does not participate in this market.

To identify a symmetric equilibrium advertising strategy, without loss of generality, we solve firm A's problem, given firm B's knwon advertising strategy. We denote firm A's choice of advertising levels by $\tilde{\sigma}_A = (\tilde{\sigma}_A^g, \tilde{\sigma}_A^b)$, given its private quality type q_A. Each firm's advertising strategy is denoted by $\sigma_A(q)$ and $\sigma_B(q)$. In equilibrium, both firm A and B choose their advertising levels based on their private quality types. But, from firm A's perspective, neither firm B's quality type, nor its choice of advertising, is observable. Instead, firm A forms expectation over firm B's advertising level by averaging firm B's advertising strategy over the distribution of quality types: $E[\sigma_B^{g*}(q)]$ and $E[\sigma_B^{b*}(q)]$. So, the expected direct demand is

$$\begin{aligned} D^{Dir}(\tilde{\sigma}_A; q_A) &= \Pr(m=1) \cdot \Pr(v_A = 1|q_A) \sum_{s \in \{g,b\}} \Pr(s|m=1)\big(\Pr(\theta^{1,0}|s) + \frac{1}{2} \cdot \Pr(\theta^{1,1}|s)\big) \\ &= \mu \cdot q_A \cdot \Bigg(\big(\alpha + (1-\alpha)\mu\big) \left(\tilde{\sigma}_A^g (1 - E[\sigma_B^{g*}(q)]) + \frac{\tilde{\sigma}_A^g \cdot E[\sigma_B^{g*}(q)]}{2} \right) \\ &\quad + (1-\alpha)(1-\mu) \left(\tilde{\sigma}_A^b (1 - E[\sigma_B^{b*}(q)]) + \frac{\tilde{\sigma}_A^b \cdot E[\sigma_B^{b*}(q)]}{2} \right) \Bigg) \end{aligned} \tag{8}$$

So, firm A's expected direct demand increases in its own advertising while decreasing in the competitor's advertising amount. This is the prominence effect (Armstrong et al., 2009) of advertising. When a firm advertises more, a consumer is more likely to see its advertising and visit the firm first instead of its competitor, which helps the advertising firm to preempt demand. This prominence effect provides incentives for firms to invest in costly advertising.

Lemma 2 (Prominence) *The direct demand $D^{Dir}(\tilde{\sigma}_A; q_A)$ increases in the firm's advertising amount, $\tilde{\sigma}_A$: $\frac{\partial D^{Dir}(\tilde{\sigma}_A; q_A)}{\partial \tilde{\sigma}_A} > 0$.*

Proof. See the Appendix. ■

[16] Here, we assume away the issue of pricing to focus on the consumer inference triggered by targeted advertising. In our model, we are considering the market where the supply side is short such that consumers need to bid for each firms product at their willingness to pay (Cabral 2000). This assumption simplifies the analysis significantly and allows us to set aside the issue of price signaling.

Indirect demand: costly consumer search beyond the prominent firm

Alternatively, some consumers may first visit firm B, and subsequently search for firm A after begin dissatisfied with firm B, i.e., $u_{iB} = 0$.[17] The decision depends on her updated beliefs about her match type with the product category ($\Pr(m = 1|\theta^{a_A,a_B}, u_{iB} = 0)$) and firm's quality type ($\Pr(v_A = 1|\theta^{a_A,a_B}, u_{iB} = 0)$). Among these consumers, some are now satisfied with firm A's product, which becomes the firm A *indirect demand.* There can be two distinct initial advertising states that can lead to firm A's indirect demand: (1) $\theta^{0,1}$ where a consumer received an ad only from firm B, or (2) $\theta^{1,1}$ where she received an ad from both firms.

We analyze the case of $\theta^{0,1}$ first. Upon realizing a low utility from firm B ($u_{iB} = 0$), the consumer i decides whether to continue searching for another firm. The consumer undertakes a costly search if the expected utility from visiting another firm exceeds her cost t_i:

$$E[u_{iA}|\theta^{0,1}, u_{iB} = 0] = \Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0) \cdot \underbrace{\Pr(v_A = 1|\theta^{0,1}, u_{iB} = 0)}_{=\int_0^1 q\, h(q|a_A=0)\, dq} > t_i. \tag{9}$$

Here, her expected utility is a product of the probability of having a good match with the product category, $\Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0)$, and the probability of finding firm A satisfactory , $\Pr(v_A = 1|\theta^{0,1}, u_{iB} = 0)$. For the probability of having a good match with the product category, the consumer updates her beliefs $\Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0)$ using Bayes rule:

$$\Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0) = \frac{\Pr(m_i = 1) \cdot \sum_{s\in\{g,b\}} \Pr(s|m_i = 1) \cdot \Pr(\theta^{0,1}|s) \cdot \Pr(u_{iB} = 0|s, \theta^{0,1})}{\sum_{m_i\in\{0,1\}} \Pr(m_i) \sum_{s\in\{g,b\}} \Pr(s|m_i) \cdot \Pr(\theta^{0,1}|s) \cdot \Pr(u_{iB} = 0|s, \theta^{0,1})}. \tag{10}$$

The denominator computes the total probability of a consumer realizing an advertising state $\theta^{0,1}$ and $u_{iB} = 0$. This is the sum of probabilities for two distinct cases, depending on the consumer's unknown category match, $m_i = 1$ or $m_i = 0$.[18] And, the numerator is the probability that the consumer has a good category match, $m_i = 1$, and realize advertising state $\theta^{0,1}$ and $u_{iB} = 0$.

[17] Here, we assume that a consumer finds out firm A as long as she engages in search. For example, Google search will show up the competitor's identity. However, if a consumer is unaware of the product category, she cannot engage in any product search.

[18] The denominator can be expressed in terms of model primitives as following: $\mu\left((\alpha + (1-\alpha)\mu)E[\sigma_B^{g*}](1 - E[\sigma_A^{g*}]) + (1-\alpha)(1-\mu)E[\sigma_B^{b*}](1 - E[\sigma_A^{b*}])\right)\left(1 - E[q_B|\theta^{0,1}]\right) + (1 - \mu) \cdot \left((\alpha + (1-\alpha)(1-\mu))E[\sigma_B^{b*}](1 - E[\sigma_A^{b*}]) + (1-\alpha)\mu E[\sigma_B^{g*}](1 - E[\sigma_A^{g*}])\right)$.

The fact that the consumer is not satisfied with firm B's product can lead to more pessimistic beliefs. However, if the targeting is accurate and firms send sufficient amount of advertising to the perceived good-types, then the consumer's beliefs are still optimistic enough that she may want to engage in a further search for firm A. This implies that a more accurate targeting, captured by a higher α, can increase amount of consumer search in the product category.

Given that each consumer's search cost is drawn from a uniform distribution on $[0, T]$, the fraction of consumers whose advertising state is $\theta^{0,1}$ and who subsequently find firm B dis-satisfactory is $E[u_{iA}|\theta^{0,1}, u_{iB} = 0]/T$. This amount of consumer search, conditional on being unhappy with the prominent firm, is decreasing in T.

The decision whether to further search for firm A also depends on the consumer's belief about whether firm A will deliver high value ($v_A = 1$) to the consumer. Firm A's private quality level affects its own strategy, but not firm B's strategy. Therefore, the posterior belief about firm A's unobserved quality level is determined by whether a consumer received an advertising from the firm, which are characterized in Equation (7). In particular, for $\theta^{0,1}$, the posterior belief corresponds to the second line of Equation (7) because the consumer did not receive an ad from firm A. Provided with these posterior beliefs about her own product match and about firm A's unobserved quality, the consumer makes the search decision following the rule specified in equation (9).

Similarly, we now turn to the second case in which the consumer receives an ad from both firms ($\theta^{1,1}$). Compared to the first case, $\theta^{0,1}$, in which the consumer only receives an ad from firm B, the consumer may have more optimistic beliefs about the category match, as well as about firm A's unobserved quality level. This would be true if firms' advertising strategy is to send more advertising to the perceived good-type consumers, and more advertising is sent by a higher quality firm. The consumer searches for firm A if and only if the expected utility is greater than the search cost, i.e., $E[u_{iA}|\theta^{1,1}, u_{iB} = 0] > t_i$.

$$E[u_{iA}|\theta^{1,1}, u_{iB} = 0] = \Pr(m_i = 1|\theta^{1,1}, u_{iB} = 0) \cdot \Pr(v_A = 1|\theta^{1,1}, u_{iB} = 0) > t_i. \tag{11}$$

Then, considering the distribution of consumer search cost, the fraction of consumers in advertising state $\theta^{1,1}$, who randomly visit firm B first and subsequently find it dis-satisfactory, is $E[u_{iA}|\theta^{1,1}, u_{iB} = 0]/T$, which again decreases in T.

Combining indirect demand generated through two advertising states $\theta^{0,1}$ and $\theta^{1,1}$, the total expected indirect demand is

$$
\begin{aligned}
D^{Ind}(\tilde{\sigma}_A; q_A) :=& \mu \cdot q_A \cdot \sum_{s \in \{g,b\}} \Pr(s|m=1) \cdot \Bigg(\Pr(\theta^{0,1}|s) \cdot \Pr(v_B = 0|\theta^{0,1}) \cdot \frac{E[u_{iA}|\theta^{0,1}, u_{iB} = 0]}{T} \\
&+ \frac{\Pr(\theta^{1,1}|s)}{2} \cdot \Pr(v_B = 0|\theta^{1,1}) \cdot \frac{E[u_{iA}|\theta^{1,1}, u_{iB} = 0]}{T} \Bigg)
\end{aligned} \tag{12}
$$

Lemma 3 (Advertising Spillover) *The indirect demand, $D^{Ind}(\tilde{\sigma}_A; q_A)$, decreases in the firm's advertising amount, $\tilde{\sigma}_A$, if and only if $\frac{\Pr(v_j=1|a_j=1)}{\Pr(v_j=1|a_j=0)} < 2$.*

Proof. See the Appendix. ■

Firm A's indirect demand can be decreasing in the firm's advertising amount, which indicates the *advertising spillover effect.* This is induced by consumer search beyond the prominent firm. Some consumers would eventually visit firm A after first visiting firm B, even if they did not see firm A's advertising. However, this result holds if the marginal effect of advertising on consumers' inferences about the firm's private quality type is not too large, i.e., $\frac{\Pr(v_j=1|a_j=1)}{\Pr(v_j=1|a_j=0)} < 2$. Otherwise, the indirect demand would increase in the firm's advertising amount. This is because the firm wants to show its advertising to consumers and improve their beliefs about its quality type significantly, even if they will visit the firm as the second stop. Later in our analysis, we verify that this condition ($\frac{\Pr(v_j=1|a_j=1)}{\Pr(v_j=1|a_j=0)} < 2$) is satisfied, and therefore indeed advertising spillover effects exist in this framework.

4.3 Firm's profit

Based on the above consumer search behaviors, a firm face a total demand, which consists of direct demand and indirect demand.

The exact configuration of the total demand depends on the prior beliefs (μ and q_0), firms' advertising strategies (σ_j^s), and the targeting accuracy ($\alpha \in (0,1)$).

For every sale, each firm makes revenue of 1. Therefore, the total expected revenue is equal to the total expected demand, which is the sum of direct and indirect demand, characterized in Equations (8) and (12), respectively. Therefore, firm A's expected profit, given the firm's choice of advertising level $\tilde{\sigma}_A = (\tilde{\sigma}^g_A, \tilde{\sigma}^b_A)$, firm B's equilibrium strategy $\sigma^* = (\sigma^{g*}, \sigma^{b*})$, is

$$\Pi_A(\tilde{\sigma}_A; \sigma^*, q_A) = D^{Dir}_A(\tilde{\sigma}_A; \sigma^*, q_A) + D^{Ind}_A(\tilde{\sigma}_A; \sigma^*, q_A) - c(\tilde{\sigma}_A) \tag{13}$$

5 Optimal advertising strategies

5.1 Untargeted advertising without using customer information

We start our analysis with a benchmark case of untargeted advertising. This benchmark helps to isolate the effect of *informative targeting* beyond the simple awareness effect of advertising by highlighting the role of consumer inferences based on the mere fact that they were targeted.

Under untargeted advertising where firms commit to relinquish customer data, each firm j's advertising strategy $\sigma^{un}_j(q)$ is a simple mapping from its own quality to the fraction of entire consumers who will receive advertising. Therefore, advertising is non-targeted and firms send an ad for the same fraction of consumers in both types, $\sigma^g = \sigma^b$. We can consider a special case where the targeting accuracy is zero or $\alpha = 0$ as one of such untargeted advertising cases.[19]

Since these are non-targeted ads, consumers do not update their beliefs about their match with the product category. Instead, they only update their beliefs about the firm's quality type following Equation (7):

$$h^{un}_j(q|a_j = 1) = \frac{\sigma^{un}(q)f(q)}{\int \sigma^{un}(y)f(y)\,dy}, \qquad h^{un}_j(q|a_j = 0) = \frac{(1-\sigma^{un}(q))f(q)}{\int (1-\sigma^{un}(y))f(y)\,dy}. \tag{14}$$

Let $\tilde{\sigma}_A$ be the actual advertising level chosen by the firm A while σ^{un*}_A is the equilibrium advertising strategy of firm A. Then, the direct demand from consumers who visit firm A first is

$$D^{Dir}_o(\tilde{\sigma}_A; \sigma^{un*}, q_A) = \mu \cdot \tilde{\sigma}_A \cdot (1 - \frac{E[\sigma^{un*}]}{2}) \cdot q_A \tag{15}$$

If consumer i visits the other firm B first, then the consumer searches firm A if the expected benefit from the search, $E[u_{iA}|\theta^{a_A,a_B}, u_{iB} = 0]$ is greater than her search cost t_i. Also, their realized

[19] Clearly, the case for $\alpha = 0$ is one of untargeted advertising cases because it is effectively the same whether the firm use the data or not. Firms cannot condition advertising on their noisy signals of each consumer because of the lack of customer data or precision of the information. However, untargeted advertising is possible even if $\alpha > 0$ as long as the firm can commit to ignore the customer information.

advertising state must be either $\theta^{0,1}$, or $\theta^{1,1}$. So,

$$E[u_A|\theta^{1,1}, u_B = 0] = \mu \cdot \Pr(v_A = 1|\theta^{1,1}) = \mu \cdot \int q\, h_A(q|a_A = 1)\, dq,$$
$$E[u_A|\theta^{0,1}, u_B = 0] = \mu \cdot \Pr(v_A = 1|\theta^{0,1}) = \mu \cdot \int q\, h_A(q|a_A = 0)\, dq, \tag{16}$$

where $h_A(q|a_A = 1)$ and $h_A(q|a_A = 0)$ are from (14).

Therefore, firm A's indirect demand is equal to

$$D_o^{Ind}(\tilde{\sigma}_A; \sigma^{un*}, q_A) = \mu \cdot q_A \cdot \Bigg\{ \frac{\tilde{\sigma}_A \cdot E[\sigma^{un*}]}{2} \cdot (1 - \Pr(v_B = 1|\theta^{1,1})) \cdot \frac{E[u_A|\theta^{1,1}, u_B = 0]}{T} \tag{17}$$
$$+ (1 - \tilde{\sigma}_A) \cdot E[\sigma^{un*}] \cdot (1 - \Pr(v_B = 1|\theta^{0,1})) \cdot \frac{E[u_A|\theta^{0,1}, u_B = 0]}{T} \Bigg\}.$$

Then, firm A's expected profit is

$$\Pi_A(\tilde{\sigma}_A; \sigma^{un*}, q_A) = D_o^{Dir}(\tilde{\sigma}_A; \sigma^{un*}, q_A) + D_o^{Ind}(\tilde{\sigma}_A; \sigma^{un*}, q_A) - c(\tilde{\sigma}_A), \tag{18}$$

where $c(\tilde{\sigma}_A) = \frac{k}{2} \cdot (\tilde{\sigma}_A)^2$.

Given this profit function, $\sigma^{un*}(\cdot)$ is an equilibrium strategy if the first order condition holds for $\tilde{\sigma}_A = \sigma^{un*}(q_A)$. Therefore,

$$\frac{\partial\, \Pi_A(\tilde{\sigma}_A; \sigma^{un*}, q_A)}{\partial\, \tilde{\sigma}_A}|_{\tilde{\sigma}_A = \sigma^{un*}(q_A)} = 0. \tag{19}$$

Differentiating the profit function with respect to the chosen advertising level, $\tilde{\sigma}_A$, gives:

$$\frac{\partial\, \Pi_A(\tilde{\sigma}_A; \sigma^{un*}, q_A)}{\partial\, \tilde{\sigma}_A} = \mu \cdot q_A \cdot \Bigg(\underbrace{1 - \frac{E[\sigma^{un*}]}{2} + \frac{E[\sigma^{un*}]}{2} \cdot (1 - \Pr(v_B = 1|\theta^{1,1})) \cdot \frac{E[u_A|\theta^{1,1}, u_B = 0]}{T}}_{\text{Net Prominence Effect}}$$
$$- \underbrace{E[\sigma^*] \cdot (1 - \Pr(v_B = 1|\theta^{0,1})) \cdot \frac{E[u_A|\theta^{0,1}, u_B = 0]}{T}}_{\text{Advertising Spill-over Effect}} \Bigg) - k \cdot \tilde{\sigma}_A \tag{20}$$

The first order condition is satisfied ($\frac{\partial \Pi_A}{\partial \tilde{\sigma}_A} = 0$) if $\tilde{\sigma}_A = \sigma^{un*}(q_A)$. Here, the firm balances the benefit of advertising considering both advantage of being the first (*the prominence effect*) and potential advantage of not being the first (*free-riding effect*) against the cost of advertising. After using the fact that $\Pr(v_B = 1|\theta^{1,1}) = \Pr(v_B = 1|\theta^{0,1})$, which is equal to $\Pr(v_B = 1|a_B = 1)$, we can rearrange the first order condition as following:

$$
\begin{aligned}
k \cdot \sigma^{un*}(q_A) = &\mu \cdot q_A \cdot \Bigg\{ 1 - \frac{E[\sigma^{un*}]}{2} + E[\sigma^{un*}] \times (1 - \Pr(v_B = 1 | a_B = 1)) \\
& \times \left(\frac{E[u_A | \theta^{1,1}, u_B = 0]}{2T} - \frac{E[u_A | \theta^{0,1}, u_B = 0]}{T} \right) \Bigg\}
\end{aligned}
\tag{21}
$$

This condition must hold for all values of $q_A \in [0, 1]$. It is important to note that, for any given strategy σ^{un*}, the right-hand side is equal to some constant times q_A. Therefore, this implies that the left-hand side must also be of the same form, and in particular,

$$
\sigma^{un*}(q_A) \equiv \lambda^{un} \cdot q_A \tag{22}
$$

for some constant λ^{un}. This linearity is obtained from Equation (21), which uses an assumption that the total cost of advertising is quadratic in the amount of advertising.[20]

To pin down the constant λ^{un}, we plug in $\sigma^{un*}(q) = \lambda^{un} \cdot q$ into Equation (20), and we impose additional simplification assumption that the quality types are drawn from a standard uniform distribution, i.e., $F(q) = q$ for $q \in [0, 1]$.

Proposition 3 (Equilibrium Strategy: Untargeting) *Under untargeted advertising, the symmetric equilibrium advertising is characterized by $\sigma^{un*}(q) = \lambda^{un}(\mu, T, k) \cdot q$. This equilibrium exists and is unique if advertising is sufficiently costly ($k \geq \overline{k} = \frac{3\mu}{4}$) and the average consumer search cost is not too small ($\frac{T}{2} > \frac{1}{36} \approx 0.028$).*

Proof. See the Appendix. ■

The proposition states an important point that in equilibrium, the amount of advertising is linearly increasing in firm's quality type ($\sigma^{un*}(q) = \lambda^{un} \cdot q$). This implies that a firm of a higher quality type advertises more aggressively than lower quality firms. Therefore, upon receiving an untargeted advertising, a consumer rationally infers that the advertising firm is more likely to be higher quality. In particular, it satisfies the condition in Proposition 2. On the other hand, because this advertising

[20] The linearity of equilibrium advertising strategy does not hinge on the assumptions about distributions from which each firm's quality type and consumer's search costs are drawn. For example, we assume that the search cost is uniformly distributed on $[0, T]$. For any distribution $G(\cdot)$, the second line of the equation would still be a constant of the form: $\frac{1}{2} \cdot G(E[u_A|\theta^{1,1}, u_B = 0]) - G(E[u_A|\theta^{0,1}, u_B = 0])$. Therefore, the linearity of the equilibrium strategy does not depend on this assumption. It depends on the assumption that the cost function is quadratic.

is not based on customer information, the consumer does not make inferences about her own match type with the product category.

The constant $\lambda^{un}(\mu, T, k)$ determines the equilibrium amount of advertising for each quality type, and therefore can be interpreted as the equilibrium *intensity* of advertising. The following proposition summarizes how $\lambda^{un}(\mu, T, k)$ depends the model primitives: μ, T, and k.

Proposition 4 (Comparative Statics: Untargeting) *If firms engages in non-targeted advertising where they do not condition their advertising strategy on customer data:*

1. *The equilibrium intensity of advertising, captured by λ^{un}, increases in the average consumers search cost ($\frac{T}{2}$), but it decreases in the cost for advertising (k): $\frac{\partial \lambda^{un}}{\partial T} > 0, \frac{\partial \lambda^{un}}{\partial k} < 0$.*

2. *Moreover, the equilibrium intensity of advertising increases in the proportion of good-type consumers in the product category (μ): $\frac{\partial \lambda^{un}}{\partial \mu} > 0$.*

Proof. See the Appendix. ■

It is intuitive that the equilibrium advertising intensity decreases in k, the cost of each unit of advertising. Firms reduce their investments in advertising if it is costly: $\frac{\partial \lambda^{un}}{\partial k} < 0$. If consumers' average search cost is high (a large T), then consumers are less likely to search beyond the prominent firm. Therefore, the free-riding effects reduce, whereas the prominence becomes more valuable. So, firms respond by competing more fiercely through advertising, i.e., $\frac{\partial \lambda^{un}}{\partial T} > 0$.

Furthermore, if μ is large, each consumer is more likely to have a good match with the product category. Therefore, firms invest more in advertising so that consumers visit them first as their prominent firm. Conditional on being dissatisfied with the prominent firm, consumers are more likely to search for another firm, i.e., the free-riding effects increase. However, an increase in prominence effects outweighs an increase in free-riding effects, thus leading to a net effect: $\frac{\partial \lambda^{un}}{\partial \mu} > 0$. So, for a product category characterized by a large μ such as a mass product category, firms compete aggressively over customers by increasing non-targeted advertising.

5.2 Targeted advertising using customer information

Now, we analyze our main model in which the firm can send targeted advertising. Based on the customer data, each consumer is perceived to be a good-type or bad-type in terms of her match with the product category. And, each firm of quality type q decides the advertising intensity, or the advertising coverage in terms of fraction for the perceived good-type and bad-type consumers, denoted by $\sigma^{g*}(q)$ and $\sigma^{b*}(q)$, respectively. Due to Lemma 1, we focus on a symmetric equilibrium in which the firm only targets perceived-good types with advertising, and none of the perceived-bad consumers, i.e., $\sigma^{g*}(q) \leq 1$ and $\sigma^{b*}(q) = 0$ for all $q \in [0, 1]$.

Without loss of generality, we take firm A's perspective. A symmetric strategy $\sigma^*(q) = (\sigma^{g*}(q), 0)$ is an equilibrium if it is indeed optimal for firm A to choose the advertising coverage that coincides with the strategy, i.e., $\tilde{\sigma}_A^g = \sigma^{g*}(q_A)$ and $\tilde{\sigma}_A^b = \sigma^{b*}(q_A) = 0$. To identify the conditions for a symmetric equilibrium, we differentiate the profit function in Equation (13) with respect to $\tilde{\sigma}_A^g$ and $\tilde{\sigma}_A^b$, and plugging in the symmetric equilibrium strategies:

$$\begin{aligned} \frac{\partial \Pi_A(\tilde{\sigma}_A; \sigma^*, q_A)}{\partial \tilde{\sigma}_A^g} &= 0 \\ \frac{\partial \Pi_A(\tilde{\sigma}_A; \sigma^*, q_A)}{\partial \tilde{\sigma}_A^b} &\leq 0 \end{aligned} \tag{23}$$

The first line of Equation (23) corresponds to the condition that it is optimal to choose a positive advertising level, $\tilde{\sigma}_A^g$, for the perceived-good consumers according to the equilibrium strategy. Also, the firm sends no advertising to the perceived-bad consumers, which is indeed optimal if the second line of Equation (23) holds.

Targeted advertising is based on each consumer's perceived types, $s_i \in \{g, b\}$, which provides noisy information about her true match type for the product category, $m_i \in \{g, b\}$. As noted in Proposition 1, consumers make inferences about their unknown match type based on targeted advertising. This is the effect of *informative targeting* based on the mere fact that consumers are targeted. The following proposition characterizes the equilibrium targeting strategy.

Proposition 5 (Equilibrium Strategy: Targeting) *Under targeted advertising with accuracy α, a symmetric equilibrium advertising is characterized by $\sigma^*(q) = \left(\sigma^{*g}(q), \sigma^{*b}(q)\right) = \left(\lambda^{tar} \cdot q, 0\right)$, for some constant $\lambda^{tar} \in (0, 1)$. This equilibrium exists and is unique if the cost of advertising (k) is sufficiently large and targeting accuracy (α) is not too small.*

Proof. See the Appendix. ■

This equilibrium exists and is unique if the cost of advertising (k) is sufficiently high so that the firm finds it optimal to cover perceived bad-types with advertising. Also, the targeting accuracy α should not be too small because otherwise perceived good-type and bad-type are not differentiated enough, and therefore firms would want to send advertising to both types.

Similar to the benchmark case for untargeted advertising, an equilibrium advertising strategy is characterized by an increasing linear function of the firm's private quality type. Firm of a higher quality invests more aggressively in targeted advertising, thus satisfying the condition in Proposition 2. Consequently, consumers make more optimistic inferences about the firm's quality type upon receiving the firm's ad.

However, in contrast to the case of untargeted advertising, firms concentrate their advertising efforts on the perceived good-types. Therefore, upon being targeted, a consumer makes more optimistic inferences about her own match with the product category. If the advertising cost or the targeting accuracy is very small, there is little reason for firms to restrict their adverting efforts to the subset of entire customers, i.e., perceived good-type consumers. Therefore, this equilibrium uniquely exists if the advertising cost and the targeting accuracy are sufficiently large.

The greater the targeting accuracy α, the more optimistic the inferences are. With the more optimistic updated beliefs, consumers may engage in costly search beyond their prominent firm if they are dissatisfied with it. Therefore, the amount of consumer search may increase in the targeting accuracy. On the other hand, given an accurate targeting, consumers are more likely to find their prominent firm satisfactory, in which case they make a purchase without further search. We investigate these opposing effects of targeting accuracy on consumer search in the following proposition:

Proposition 6 (Amount of Search) *The number of consumers who engage in costly search after first visiting firm B increases in the targeting accuracy, α, if α is sufficiently large.*

Proof. See the Appendix. ■

This result shows that the amount of consumer search can be non-monotonic in targeting accuracy. But, if α is large enough, then it always increases in α. This implies that a highly accurate targeting would induce more advertising spillover, and thus reduce firms' advertising amount.

Next, we look at the equilibrium amount of advertising given consumers' search behaviors. The following proposition states that the equilibrium amount of advertising, λ^{tar}, is non-monotonic in targeting accuracy. It also describes how λ^{tar} depends on other model parameters such as the average consumer search cost ($\frac{T}{2}$) and the size of the good-type consumers (μ).

Proposition 7 (Amount of Advertising) *Suppose the advertising cost is sufficiently large.*[21] *Under the targeted advertising,*

1. *The equilibrium amount of advertising, captured by the constant λ^{tar}, increases in the average search cost ($\frac{T}{2}$): $\frac{\partial \lambda^{tar}}{\partial T} > 0$.*
2. *λ^{tar} decreases in the piror belief (μ): $\frac{\partial \lambda^{tar}}{\partial \mu} < 0$.*
3. *Lastly, if T is sufficiently large so that consumer search is costly, then λ^{tar} monotonically increases in the targeting accuracy (α): $\frac{\partial \lambda^{tar}}{\partial \alpha} > 0$. However, if T is not sufficiently large, then λ^{tar} is non-monotonic in α. It first increases ($\frac{\partial \lambda^{tar}}{\partial \alpha} > 0$) and then decreases ($\frac{\partial \lambda^{tar}}{\partial \alpha} < 0$) in targeting accuracy.*

Proof. See the Appendix. ■

First two points are similar to the case of non-targeting, the amount of advertising increases in consumer search cost, T, because with fewer consumers searching between firms, free-riding effects for advertising are mitigated. This implies that the prominence is more valuable, which leads firms to invest more in targeted advertising, i.e., $\frac{\partial \lambda^{tar}}{\partial T} > 0$. Also, in this equilibrium, firms choose the advertising coverage only among the perceived good-type consumers of mass μ. Therefore, as μ increases, the firm spends more advertising expenditure. Therefore, the advertising level in equilibrium decreases in μ, i.e., $\frac{\partial \lambda^{tar}}{\partial \mu} < 0$.

What is unique about *targeted advertising* is the effect of accuracy. As explained above, a greater targeting accuracy brings about two opposing forces in terms of advertising incentives. Firms are able to reach the right consumers for the product category with a greater probability, and therefore, each advertising is more efficient. So, firms compete more fiercely to become prominent. On the other hand, consumers who are dissatisfied with the prominent firm are more willing to search for the second firm, because the greater targeting accuracy generates more positive inferences about their own match type with the product category. So, a more precise targeting induces more consumer search, which in turn increases free-riding effects in advertising and thus, reduces firms' incentives to advertise. An interplay between these two effects– *prominence* and *free-riding* –can result in a non-monotonic effect

[21]It is to ensure the existence of the advertising equilibrium, identified in Proposition 5, where $\sigma^*(q) = \left(\lambda^{tar} \cdot q, 0\right)$.

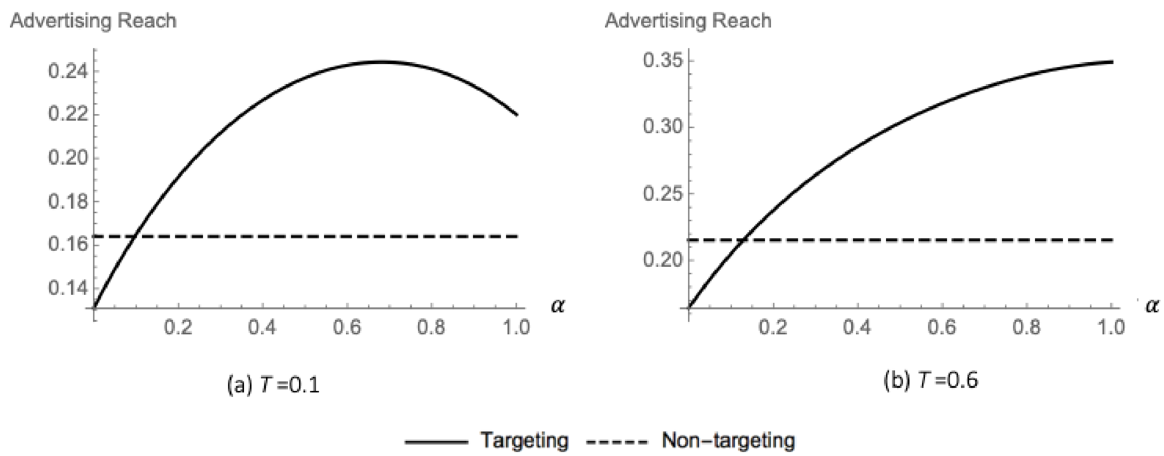

Figure 3: Advertising Reach for (a) $T = 0.1$ and (b) $T = 0.6$; $\mu = 0.2,\ k = 0.8$

of targeting accuracy on the equilibrium advertising amount. This is the case, as demonstrated in Figure 3-(a), if T is not too large so that there is enough consumer search beyond the prominent firm.

However, if T is sufficiently large where the average consumer search is costly, the prominence effect dominates the free-riding effect because fewer consumers will search for the second firm. Thus, attracting consumers to visit the firm first and preempting more demand becomes more important. Under such situations, the amount of advertising monotonically increases in targeting accuracy due to this prominence effect (see Figure 3-(b)).

6 Reach vs. Accuracy in Advertising: a Profit Analysis

Given the equilibrium advertising strategy, the total amount of advertising under targeting is $\mu \cdot \lambda^{tar} \cdot q$. So, $\mu \cdot \lambda^{tar}$ corresponds to the equilibrium extent of advertising reach under targeting. Comparing this with the reach under untargeting gives the following result:

Proposition 8 (Comparison: Advertising Reach) *The reach of targeted advertising, $\mu \cdot \lambda^{tar}$, is greater than that under un-targeting, λ^{un}, if and only if the targeting accuracy is sufficiently high.*

Proof. See the Appendix. ■

This proposition highlight the role of targeting accuracy. The total amount of advertising is greater under targeting than no targeting if and only if targeting is accurate enough. Targeting, by definition, is based on a smaller market of consumers who are likely to have a good match with the category. However, when targeting is very accurate, firms invest in advertising so aggressively that they may eventually spend more on advertising than the untargeting case.

This result implies that an highly accurate targeting may result in less profits because competition in targeted advertising amplifies. Therefore, firms may be better-off by relinquishing customer data altogether. Rather, it may be optimal for firms to execute non-targeted advertising, which allows for a greater reach in advertising. We analyze this tradeoff between *reach* and *accuracy* by comparing the equilibrium profits.

Proposition 9 (Comparison: Profits) *Suppose that T is sufficiently large. If k is sufficiently small,* $\Pi^{un*}(q) < \Pi^{tar*}(q)$ *when the targeting accuracy is very high. However, if k is sufficiently high,* $\Pi^{un*}(q) > \Pi^{tar*}(q)$ *for all* $\alpha \in [0, 1]$.

Proof. See the Appendix. ■

If T is sufficiently large, very few consumers search beyond the prominent firm. Therefore, there is little advertising spillover, and prominence becomes more valuable. Therefore, firms invest more aggressively in advertising. So, if k is sufficiently small, then the equilibrium profit can be greater under targeting than under untargeting. However, if k is large, then the opposite result holds, even when targeting accuracy is very high. This additional cost outweighs the benefits of an accurate targeting if the unit cost of advertising (k) is sufficiently large. In this case, firms are better-off forgoing any customer data to which they have access. Instead, they should engage in untargeted advertising, which would allow a greater reach and mitigate competition in advertising.

7 Conclusion

In this paper, we analyzed a model of competitive targeted advertising combined with a consumer model which captures micro-process of consumers' inferences and search behaviors. Firms have access to customer data, which allows them to imperfectly identify whether each consumer will benefit from a product category under consideration. Uncertain about their own benefit from the product category, as well as each firm's unobserved quality type, consumers make inferences about both unobservables based on the mere fact that they are targeted with advertising.

We identified a symmetric equilibrium in which firms focus their advertising efforts only on consumers who are, according to the customer data, likely to have a good match with the product category. We also identified the conditions under which this is the unique symmetric equilibrium. In particular, the advertising amount is increasing in the firm's quality type. Therefore, upon being targeted, consumers rationally make inferences that they are more likely to benefit from the product category, as well as the firm is more likely to be of higher quality.

We also provided an answer to whether an improved targeting accuracy will increase or decrease consumer search. As targeting technology improves, consumers are more likely to be satisfied with the first firm they visit, and therefore eliminate the need for further search. And yet, conditional on being dissatisfied, consumers are more likely to search because, from being targeted, they make more optimistic inferences about the category. Based on these trade-offs, we showed that targeting accuracy can sometimes increase the total amount of consumer search.

Lastly, we also show that, even though by definition targeted advertising is subject to a smaller market of consumers who are likely to make a purchase, if targeting is accurate enough firms invest in advertising so aggressively that they may end up spending more resources on advertising than under non-targeting case.

Appendix

Proof of Lemma 1

We adopt "proof by contradiction." Suppose there was an equilibrium in which $\sigma^{g*}(q) < 1$ and $\sigma^{g*}(b) > 0$ for some q. Then, it is profitable for a firm of this quality type to deviate from this strategy by shifting a very small amount of advertising from the perceived bad types to perceived good types, i.e., $\tilde{\sigma}^g = \sigma^{g*}(q) + \epsilon_g$ and $\tilde{\sigma}^b = \sigma^{b*}(q) - \epsilon_b$. This deviation does not affect consumers' inferences but improves the probability of the consumers newly targeted as a result of this deviation making a purchase, because they are more likely to be of good type. Moreover, we can find some $\epsilon_g > 0$ and $\epsilon_b > 0$ such that $\mu \cdot \epsilon_g - (1-\mu) \cdot \epsilon_b < 0$, which ensures that the firm does not incur extra costs of advertising. ■

Proof of Proposition 1

The proposition states the conditions for the marginal effect of advertising on consumers' posterior beliefs to be positive. This means that we only need to identify conditions for $\Pr(m_i = 1|\theta^{1,a_B}) - \Pr(m_i = 1|\theta^{0,a_B}) > 0$ for $a_B \in \{0,1\}$. First, for $a_B = 1$ $\Pr(m_i = 1|\theta^{1,1}) - \Pr(m_i = 1|\theta^{0,1})$ is

$$\frac{\alpha\,\mu\,(1-\mu)(E[\sigma_A^g] - E[\sigma_A^b])E[\sigma_B^g]E[\sigma_B^b]}{(\mu\,E[\sigma_A^g]E[\sigma_B^g] + (1-\mu)E[\sigma_A^b]E[\sigma_B^b]) \cdot (\mu\,(1-E[\sigma_A^g])E[\sigma_B^g] + (1-\mu)(1-E[\sigma_A^b])E[\sigma_B^b])}, \tag{24}$$

which is greater than zero if and only if $E[\sigma_A^g] - E[\sigma_A^b] > 0$, which is true from Lemma 1.

Similarly, for $a_B = 0$, $\Pr(m_i = 1|\theta^{1,0}) - \Pr(m_i = 1|\theta^{0,0})$ is

$$\frac{\alpha\,\mu\,(1-\mu)(E[\sigma_A^g] - E[\sigma_A^b])(1-E[\sigma_B^g])(1-E[\sigma_B^b])}{(\mu \cdot (1-E[\sigma_A^g])(1-E[\sigma_B^g]) + (1-\mu)\cdot(1-\sigma_A^b)(1-\sigma_B^b)) \cdot (\mu\,(1-E[\sigma_B^g])E[\sigma_A^g] + (1-\mu)(1-E[\sigma_B^b])E[\sigma_A^b])}, \tag{25}$$

which is also greater than zero if and only if $E[\sigma_A^g] - E[\sigma_A^b] > 0$. By symmetry, this proves the first part of the proposition. Also, from both equations above, it is straightforward that the marginal improvement in advertising is increasing in the targeting accuracy, α. This completes the proof. ■

Proof of Proposition 2

We need to show that $\frac{h_A(q|a_j=1)}{h_A(q|a_j=0)}$ is increasing in q. We note that $\frac{h_A(q|a_j=1)}{h_A(q|a_j=0)}$ is

$$\frac{\mu \cdot \sigma^g(q) + (1-\mu) \cdot \sigma^b(q)}{\mu \cdot (1-\sigma^g(q)) + (1-\mu) \cdot (1-\sigma^b(q))} \times \frac{\int_0^1 \mu \cdot (1-\sigma^g(y)) + (1-\mu) \cdot (1-\sigma^b(y))\, dy}{\int_0^1 \mu \cdot \sigma^g(y) + (1-\mu) \cdot \sigma^b(y)\, dy}. \tag{26}$$

Here, only the first fraction depends on q, and therefore, the ratio between two posterior beliefs is increasing in q if and only if $\frac{\mu \cdot \sigma^g(q)+(1-\mu)\cdot\sigma^b(q)}{\mu\cdot(1-\sigma^g(q))+(1-\mu)\cdot(1-\sigma^b(q))}$ is increasing in q. Moreover, it is easy to see that $\frac{d}{dq}(\frac{h_A(q|a_j=1)}{h_A(q|a_j=0)}) \geq 0 \Leftrightarrow \mu \cdot \frac{d\sigma^g(q)}{dq} + (1-\mu)\frac{d\sigma^b(q)}{dq} \geq 0$. This proves the proposition. ■

Proof of Lemma 2

This result directly follows from differentiating Equation (8) with respect to $\tilde{\sigma}^g_A$ and $\tilde{\sigma}^b_A$:

$$\frac{\partial\, D^{Dir}(\tilde{\sigma}_A; q_A)}{\partial\, \tilde{\sigma}^g_A} = \mu \cdot q_A \cdot \big(\alpha + (1-\alpha)\mu\big) \cdot \left(1 - \frac{E[\sigma^{g*}_B(q)]}{2}\right) > 0,$$
$$\frac{\partial\, D^{Dir}(\tilde{\sigma}_A; q_A)}{\partial\, \tilde{\sigma}^b_A} = \mu \cdot q_A \cdot (1-\alpha)(1-\mu) \cdot \left(1 - \frac{E[\sigma^{b*}_B(q)]}{2}\right) > 0.$$

■

Proof of Lemma 3

Differentiating Equation (12) with respect to $\tilde{\sigma}^g$ yields:

$$\begin{aligned}\frac{\partial\, D^{Ind}(\tilde{\sigma}_A; q_A)}{\tilde{\sigma}^g_A} :=& \mu \cdot q_A \cdot \Pr(s=g|m=1) \cdot \Bigg(- E[\sigma^{g*}] \cdot \Pr(v_B = 0|\theta^{0,1}) \cdot \frac{E[u_{iA}|\theta^{0,1}, u_{iB}=0]}{T} \\ &+ \frac{E[\sigma^{g*}]}{2} \cdot \Pr(v_B=0|\theta^{1,1}) \cdot \frac{E[u_{iA}|\theta^{1,1}, u_{iB}=0]}{T}\Bigg)\end{aligned} \tag{27}$$

Also, as we can see from Equation (7), the posterior distribution about the firm's type depends only on whether the consumer received the firm's advertising. So, $\Pr(v_B = 0|\theta^{0,1}) = \Pr(v_B = 0|\theta^{1,1})$ because in both cases firm B's advertising was received, i.e. $a_B = 1$. Therefore, the equation above is negative if and only if

$$-\frac{E[u_{iA}|\theta^{0,1}, u_{iB}=0]}{T} + \frac{E[u_{iA}|\theta^{1,1}, u_{iB}=0]}{2T} < 0.$$

$E[u_{iA}|\theta^{0,1}, u_{iB} = 0] = \Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0) \cdot \Pr(v_A = 1|\theta^{0,1}, u_{iB} = 0)$, and $E[u_{iA}|\theta^{1,1}, u_{iB} = 0] = \Pr(m_i = 1|\theta^{1,1}, u_{iB} = 0) \cdot \Pr(v_A = 1|\theta^{1,1}, u_{iB} = 0)$. Given that $\sigma^b = 0$, upon realizing any advertising, whether from an either firm or both firms, it reveals that the consumer is a perceived good-type. In other words, $\Pr(m_i = 1|\theta^{0,1}, u_{iB} = 0) = \Pr(m_i = 1|\theta^{1,1}, u_{iB} = 0) = \alpha + (1-\alpha)\mu$, as in Equation (**??**).

The condition above is equivalent to $\frac{1}{2}\cdot\Pr(v_A = 1|a_A = 1) < \Pr(v_A = 1|a_A = 0)\, l \Leftrightarrow \frac{\Pr(v_A=1|a_A=1)}{\Pr(v_A=1|a_A=0)} < 2$. ■

Proof of Proposition 3

Now, the quality types are drawn from a uniform distribution: $U(q) = q$ for $q \in [0, 1]$. Then,

$$
\begin{aligned}
&E[\sigma^{un*}] = \lambda \int x f(x)\, dx = \frac{\lambda}{2},\\
&E[q_A|a_A = 1] = \int x h_A^{un}(x|a_A = 1)\, dx = \frac{\int x^2 f(x)\, dx}{\int y f(y)\, dy} = \frac{2}{3}\\
&E[q_A|a_A = 0] = \int x h_A^{un}(x|a_A = 0)\, dx = \frac{\int x(1-\lambda x) f(x)\, dx}{\int (1-\lambda y) f(y)\, dy} = \frac{3-2\lambda}{3(2-\lambda)}\\
&\Pr(m_i = 1|\theta^{1,1}, u_B = 0) = \Pr(m_i = 1|\theta^{0,1}, u_B = 0) = \frac{\mu(1 - E[q|a_B = 1])}{\mu(1 - E[q|a_B = 1]) + 1 - \mu} = \frac{\mu}{3 - 2\mu}
\end{aligned}
$$

Also, the first order condition simplifies to

$$
q_A \cdot \mu \cdot \left(1 - \frac{E[\sigma^{un*}]}{2} + \frac{E[\sigma^{un*}]}{3} \cdot \frac{E[u_A|\theta^{1,1}, u_B = 0] - 2E[u_A|\theta^{0,1}, u_B = 0]}{2T}\right) - q_A \cdot k \cdot \lambda = q_A \times \Gamma^{un}(\lambda)
$$

where

$$
\Gamma^{un}(\lambda) := \mu \cdot \left(1 - \frac{\lambda}{4} - \frac{\mu \cdot \lambda}{18(3-2\mu)T} \cdot \frac{1-\lambda}{2-\lambda}\right) - k \cdot \lambda. \tag{28}
$$

Here, $\lambda = \lambda^{un}$ solves the equation $\Gamma^{un}(\lambda) = 0$.

At $\lambda = 0$, $\Gamma^{un}(\lambda = 0) = \mu > 0$. And at $\lambda = 1$, $\Gamma^{un}(1) = \frac{3\mu}{4} - k$. Then, if $k > \frac{3\mu}{4}$, then by continuity, there exists $\lambda^{un} \in (0, 1)$ that solves the equation.

For uniqueness, we need $\Gamma^{un}(\lambda)$ to be monotonically decreasing in λ.

$$\frac{\partial\,\Gamma^{un}(\lambda)}{\partial\,\lambda} = \mu \cdot \left(-\frac{1}{4} - \frac{\mu}{18 \cdot T \cdot (3-2\mu)} \cdot \frac{\lambda^2 - 4\lambda + 2}{(2-\lambda)^2} \right) - k < 0$$

$$\iff \quad k > -\frac{\mu}{4} - \frac{\mu}{18T} \cdot \frac{\mu}{3-2\mu} \cdot \frac{\lambda^2 - 4\lambda + 2}{(2-\lambda)^2},$$

where $-1 < \frac{\lambda^2-4\lambda+2}{(2-\lambda)^2} < \frac{1}{2}$. Therefore, the right-hand side is less than $-\frac{\mu}{4} + \frac{\mu}{36T} \cdot \frac{\mu}{3-2\mu}$. Therefore, for any $k > \frac{3\mu}{4}$, the condition holds if $\frac{T}{2} > \frac{\mu}{36(3-2\mu)}$. So, if $k > \frac{3\mu}{4}$ and $\frac{T}{2} > \frac{\mu}{36(3-2\mu)} \geq \frac{1}{36} \approx 0.028$, the equilibrium $\sigma^{g*}(q) = \lambda^{un} \cdot q$ and $\sigma^{b*}(q) = 0$ uniquely exsits. ■

Proof of Proposition 4

For comparative statics, we turn back to $\Gamma^{un}(\lambda)$ in the first-order condition (equation (28)), where $\lambda^{un}(\mu, k, T)$ depends on parameters, such as μ, k, and T. First, differentiating the $\Gamma^{un}(\lambda)$ with respect to T, the following must hold:

$$\frac{d\,\Gamma^{un}(\lambda^{un})}{d\,T} = \frac{\partial \Gamma^{un}(\lambda^{un})}{\partial\,T} + \frac{\partial \Gamma^{un}(\lambda^{un})}{\partial\,\lambda^{un}} \cdot \frac{\partial \lambda^{un}}{\partial\,T} = 0.$$

Moreover, $\frac{\partial}{\partial T}\left(\Gamma^{un}(\lambda^{un})\right) = \frac{\mu^2 \lambda^{un}(1-\lambda^{un})}{18(3-2\mu)(2-\lambda^{un})(T)^2}$, which is always positive, and $\frac{\partial \Gamma^{un}(\lambda^{un})}{\partial\,\lambda^{un}} < 0$ from the proof of Proposition 3 above. Therefore, $\frac{\partial\,\lambda^{un}}{\partial\,T} > 0$.

Second, differentiating both sides of equation (28) with respect to k, the following must hold;

$$\frac{d\,\Gamma^{un}(\lambda^{un})}{d\,k} = \frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,k} + \frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,\lambda^{un}} \cdot \frac{\partial\,\lambda^{un}}{\partial\,k} = 0.$$

Moreover, $\frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,k} = -\lambda^{un} < 0$, and $\frac{\partial \Gamma^{un}(\lambda^{un})}{\partial\,\lambda^{un}} < 0$ from the proof of Proposition 3 above. Therefore, it must be the case that $\frac{\partial\,\lambda^{un}}{\partial\,k} < 0$.

Finally, differentiating both sides of equation (28) with respect to μ, the following must hold.

$$\frac{d\,\Gamma^{un}(\lambda^{un})}{d\,\mu} = \frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,\mu} + \frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,\lambda^{un}} \cdot \frac{\partial\,\lambda^{un}}{\partial\,\mu} = 0.$$

Here,

$$\frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,\mu} = \left(1 - \frac{\lambda^{un}}{4} - \frac{\mu\cdot\lambda^{un}}{18(3-2\mu)T}\cdot\frac{1-\lambda^{un}}{2-\lambda^{un}}\right) - \mu\cdot\frac{\lambda^{un}(1-\lambda^{un})}{18T(2-\lambda^{un})}\cdot\frac{\mu^2-2\mu+3}{(3-2\mu)^2}$$

If T is sufficiently large, $\frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,\mu} > 0$, and therefore $\frac{\partial\,\lambda^{un}}{\partial\,\mu} > 0$ (again, because of $\frac{\partial\Gamma^{un}(\lambda^{un})}{\partial\,\lambda^{un}} < 0$). $\frac{\partial\,\Gamma^{un}(\lambda^{un})}{\partial\,\mu} > 0$ holds if and only if

$$\frac{T}{2} > \underbrace{\frac{\lambda^{un}(1-\lambda^{un})}{(2-\lambda^{un})(4-\lambda^{un})}}_{\leq\frac{5}{2}-\sqrt{6}}\cdot\underbrace{\frac{\mu}{9(3-2\mu)}\cdot\left(1+\frac{\mu^2-2\mu+3}{3-2\mu}\right)}_{\leq\frac{1}{3}}, \tag{29}$$

and therefore a sufficient condition is $\frac{T}{2} > \frac{1}{3}\left(\frac{5}{2}-\sqrt{6}\right) \approx 0.017$. But, for the existence and uniqueness of the equilibrium, we already assumed that $\frac{T}{2} > 1/36 \approx 0.028$. So, the sufficient condition is already met, and therefore $\frac{\partial\,\lambda^{un}}{\partial\,\mu} > 0$. ■

Proof of Proposition 5

The profit consists of three parts: direct demand, indirect demand, and the cost of advertising. We differentiate each part with respect to the firm's choice of advertising level for each perceived type of customers: $\tilde{\sigma}_A^g$ and $\tilde{\sigma}_A^b$. For direct demand,

$$\begin{aligned}\frac{\partial D_A^{Dir}(\tilde{\sigma}_A; q_A, \sigma^*(q))}{\partial\tilde{\sigma}_A^g} &= \mu\cdot q_A\cdot(\alpha+(1-\alpha)\mu)\cdot\left(1-\frac{E[\sigma^{g*}(q)]}{2}\right),\\ \frac{\partial D_A^{Dir}(\tilde{\sigma}_A; q_A, \sigma^*(q))}{\partial\tilde{\sigma}_A^b} &= \mu\cdot q_A\cdot(1-\alpha)(1-\mu)\cdot\left(1-\frac{E[\sigma^{b*}(q)]}{2}\right)\end{aligned} \tag{30}$$

For indirect demand,

$$\begin{aligned}\frac{\partial D_A^{Ind}(\tilde{\sigma}_A; q_A, \sigma^*(q))}{\partial\tilde{\sigma}_A^g} =& \mu\cdot q_A\cdot(\alpha+(1-\alpha)\mu)\cdot E[\sigma^{g*}(q)]\cdot(1-E[q_B|a_B=1])\\ &\times\frac{E[u_{iA}|\theta^{1,1}, u_{iB}=0]-2\cdot E[u_{iA}|\theta^{0,1}, u_{iB}=0]}{2T},\\ \frac{\partial D_A^{Ind}(\tilde{\sigma}_A; q_A, \sigma^*(q))}{\partial\tilde{\sigma}_A^b} =& \mu\cdot q_A\cdot(1-\alpha)\cdot(1-\mu)\cdot E[\sigma^{b*}(q)]\cdot(1-E[q_B|a_B=1])\\ &\times\frac{E[u_{iA}|\theta^{1,1}, u_B=0]-2\cdot E[u_{iA}|\theta^{0,1}, u_B=0]}{2T}\end{aligned} \tag{31}$$

In order for $\sigma^{g*}(q_A) > 0$ and $\sigma^{b*}(q_A) = 0$ to be an equilibrium, the first order conditions in Equation (23) must hold for $\tilde{\sigma}_A^g = \sigma^{g*}(q_A)$ and for any $\tilde{\sigma}_A^b \geq 0$

$$\begin{aligned} & -k \cdot \mu^2 \cdot \sigma^{g*}(q_A) + \mu \cdot (\alpha + (1-\alpha)\mu) \cdot \Bigg(1 - \frac{E[\sigma^{g*}(q)]}{2} \\ & + E[\sigma^{g*}(q)] \cdot (1 - E[q_B|a_B = 1]) \cdot \frac{E[u_A|\theta^{1,1}, u_B = 0] - 2 \cdot E[u_A|\theta^{0,1}, u_B = 0]}{2T} \Bigg) \cdot q_A \quad = 0, \end{aligned} \tag{32}$$

and

$$-k \cdot \mu(1-\mu) \cdot \sigma^{g*}(q_A) + q_A \cdot \mu(1-\mu) \cdot (1-\alpha) \quad \leq 0. \tag{33}$$

For any $\sigma^{g*}(q)$, $E[\sigma^{g*}(q)]$ is a constant. So, Equation (32) holds for all q_A if and only if $\sigma^{g*}(q_A) = \lambda \cdot q_A$ for some constant λ. The equilibrium strategy is pinned down by identifying a constant $\lambda = \lambda^{tar}$ which satisfies Equation (32).

By plugging in $\sigma^{g*}(q_A) = \lambda \cdot q_A$ into Equation (33), $\sigma^{b*}(q) = 0$ is part of this equilibrium if

$$k \geq \frac{q_A}{\sigma^{g*}(q_A)}(1-\alpha) = \frac{1-\alpha}{\lambda^{tar}}. \tag{34}$$

Intuitively, the firm sets $\sigma^{b*} = 0$, i.e., sends no advertising to the perceived bad types, if advertising is costly enough (a large k), or if targeting is highly accurate so that the perceived bad types are in fact bad types. It also depends on the equilibrium strategy through the constant λ, which can be interpreted as the advertising intensity for the perceived good-type consumers.

Now, we plug in $\sigma^{g*}(q) = \lambda \cdot q$ to re-write the first-order condition in terms of model parameters.

$$E[\sigma^{g*}(q)] = \int_0^1 \lambda q dq = \frac{\lambda}{2}$$

$$E[q_j|a_j = 1] = \frac{\int_0^1 \mu \cdot \lambda x^2 dx}{\int_0^1 \mu \cdot \lambda y dy} = \frac{2}{3}$$

$$E[q_j|a_j = 0] = \frac{\int_0^1 x \cdot \big(\mu(1-\lambda x) + 1 - \mu\big)dx}{\int_0^1 \big(\mu(1-\lambda y) + 1 - \mu\big)dy} = \frac{\mu(\frac{1}{2} - \frac{\lambda}{3}) + \frac{1-\mu}{2}}{\mu(1-\frac{\lambda}{2}) + 1 - \mu} = \frac{3 - 2\lambda\mu}{3(2-\lambda\mu)}$$

$$\begin{aligned}\Pr(m_i = 1|\theta^{0,1}, u_B = 0) &= \frac{\mu\big((\alpha + (1-\alpha)\mu)\frac{\lambda}{2}\frac{2-\lambda}{2}\big) \cdot \frac{1}{3}}{\mu\big((\alpha + (1-\alpha)\mu)\frac{\lambda}{2}\frac{2-\lambda}{2}\big) \cdot \frac{1}{3} + (1-\mu)(1-\alpha)\mu\frac{\lambda}{2}\frac{2-\lambda}{2}} \\ &= \frac{\alpha + (1-\alpha)\mu}{\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)} = \Pr(m_i = 1|\theta^{1,1}, u_B = 0)\end{aligned}$$

By plugging these expressions into Equation (32), the first order condition for the targeting case simplifies to $\Gamma(\lambda) = 0$, where

$$\begin{aligned}\Gamma(\lambda) :=& - k \cdot \mu^2 \cdot \lambda + \mu \cdot (\alpha + (1-\alpha)\mu) \cdot \Bigg(1 - \frac{\lambda}{4} \\ & - \frac{\lambda}{18T} \cdot \frac{\alpha + (1-\alpha)\mu}{\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)} \cdot \frac{1-\lambda\mu}{2-\lambda\mu}\Bigg).\end{aligned} \tag{35}$$

Differentiating with respect to λ,

$$\begin{aligned}\frac{\partial}{\partial \lambda}(\Gamma(\lambda)) =& - k \cdot \mu^2 + \mu \cdot (\alpha + (1-\alpha)\mu) \cdot \Bigg(- \frac{1}{4} \\ & - \frac{\alpha + (1-\alpha)\mu}{18T \cdot \big(\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)\big)} \cdot \underbrace{\big(1 - \frac{2}{(2-\lambda\mu)^2}\big)}_{\geq 0}\Bigg),\end{aligned} \tag{36}$$

which is non-positive for all parameters. So, $\Gamma(\lambda)$ decreases monotonically.

If $\lambda = 0$, then $\Gamma(0) = \alpha + (1-\alpha)\mu > 0$ for all q, and therefore cannot solve $\Gamma(\lambda) = 0$. If $\lambda = 1$, then

$$\Gamma(1) = -k \cdot \mu^2 + \mu \cdot \big(\alpha + (1-\alpha)\mu\big) \cdot \left(\frac{3}{4} - \frac{1}{18T} \cdot \frac{\alpha + (1-\alpha)\mu}{\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)} \cdot \frac{1-\mu}{2-\mu}\right),$$

which is negative if

$$
\begin{aligned}
k >& \frac{\alpha+(1-\alpha)\mu}{\mu}\left(\frac{3}{4}-\frac{1}{18T}\cdot\underbrace{\frac{\alpha+(1-\alpha)\mu}{\alpha+(1-\alpha)\mu+3(1-\mu)(1-\alpha)}}_{\leq 1}\cdot\underbrace{\frac{1-\mu}{2-\mu}}_{\leq 1/2}\right) \\
\geq& \frac{\big(\alpha+(1-\alpha)\mu\big)}{\mu}\left(\frac{3}{4}-\frac{1}{36T}\right) \qquad .
\end{aligned}
\tag{37}
$$

By the intermediate value theorem, this condition ensures the existence of this equilibrium. Together with the monotonicity of $\Gamma(\lambda)$, the uniqueness is guaranteed.

Therefore, if $k > \frac{\big(\alpha+(1-\alpha)\mu\big)}{\mu}\cdot\big(\frac{3}{4}-\frac{1}{36T}\big) > \frac{3\big(\alpha+(1-\alpha)\mu\big)}{4\mu}$ and Equation (34) holds, then there exists a unique $\lambda^{tar}\in(0,1)$ such that $\Gamma(\lambda^{tar})=0$. This condition is more likely to hold if targeting is accurate, and more crucially μ is not too small. In other words, the product category must appeal to enough consumers in the market in order for firms to focus only on the perceived good-type consumers. ■

Proof of Proposition 6

The mass of consumers who first visit firm B, and subsequently search for firm A, denoted by $\Sigma(q_B)$ is as follows:

$$
\begin{aligned}
\Sigma :=& \underbrace{\mu\cdot\frac{\lambda^{tar}}{2}}_{\Pr(a_B=1)}\cdot\left(\underbrace{1-(\alpha+(1-\alpha)\mu)\cdot\Pr(v_B=1|a_B=1)}_{=Pr(u_{iB}=0|a_B=1)}\right)\cdot\underbrace{\Pr(m_i=1|a_B=1)}_{=\frac{\alpha+(1-\alpha)\mu}{\alpha+(1-\alpha)\mu+3(1-\alpha)(1-\mu)}} \\
&\cdot\left((1-\lambda^{tar}q_A)\cdot\underbrace{\Pr(v_A=1|a_A=0)}_{=\frac{3-2\lambda^{tar}\mu}{6-3\lambda^{tar}\mu}}+\frac{\lambda^{tar}q_A}{2}\cdot\underbrace{\Pr(v_A=1|a_A=1)}_{=\frac{2}{3}}\right)\cdot\frac{1}{T} \qquad ,
\end{aligned}
\tag{38}
$$

which simplifies to:

$$
\mu\cdot\frac{\lambda^{tar}}{2}\cdot\frac{\alpha+(1-\alpha)\mu}{3}\cdot\left(\frac{3-2\lambda^{tar}\mu}{3(2-\lambda^{tar}\mu)}-\lambda^{tar}q_A\cdot\frac{1-\lambda^{tar}\mu}{3(2-\lambda^{tar}\mu)}\right)
\tag{39}
$$

The terms that directly depends on α is $\Big(1-(\alpha+(1-\alpha)\mu)\cdot\Pr(v_B=1|a_B=1)\Big)\cdot\Pr(m_i=1|a_B=1)$.

Differentiating this expression with respect to α yields:

$$\frac{(1-\mu)\big(1-2\mu(1-\alpha)(5-3\alpha-2\mu(1-\alpha))\big)}{3\big(4-3\alpha-2\mu(1-\alpha)\big)^2}, \tag{40}$$

which is non-negative if and only if $\mu \leq \frac{5-\sqrt{21}}{4}$, or if $\mu > \frac{5-\sqrt{21}}{4}$ and $\alpha \geq \frac{2(2-\mu)}{3-2\mu} - \sqrt{\frac{3}{2\mu}} \cdot \frac{1}{3-2\mu}$. So, in particular, the amount of search is increasing in α for α large enough.

Proof of Proposition 7

To understand how the equilibrium amount of advertising λ^{tar} depends on model parameters, we differentiate the first order condition with each respective parameter: k (cost of advertising), T (the average consumer search cost), α (amount of data on consumers), and μ (fraction of customers who will benefit from the product category).

First, with respect to k, $\frac{d(\Gamma(\lambda^{tar}))}{d\,k} = \frac{\partial(\Gamma(\lambda^{tar}))}{\partial k} + \frac{\partial(\Gamma(\lambda^{tar}))}{\partial\,\lambda^{tar}} \cdot \frac{\partial \lambda^{tar}}{\partial\,k} = 0$ must satisfy, where $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial k} = -\mu \cdot \lambda^{tar} < 0$, and $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\,\lambda^{tar}} < 0$ for a sufficiently large k. Therefore, it must be the case that $\frac{\partial \lambda^{tar}}{\partial\,k} < 0$.

We repeat similar exercise for other parameters. $\frac{d(\Gamma(\lambda^{tar}))}{d\,T} = \frac{\partial(\Gamma(\lambda^{tar}))}{\partial T} + \frac{\partial(\Gamma(\lambda^{tar}))}{\partial\,\lambda^{tar}} \cdot \frac{\partial \lambda^{tar}}{\partial\,T} = 0$

$$\frac{\partial(\Gamma(\lambda^{tar}))}{\partial T} = \mu \cdot \big(\alpha + (1-\alpha)\mu\big) \cdot \frac{\lambda}{18T^2} \frac{\alpha + (1-\alpha)\mu}{\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)} \cdot \frac{1-\lambda\mu}{2-\lambda\mu} > 0 \tag{41}$$

Also, from the previous proposition, $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\,\lambda^{tar}} < 0$. Therefore, in order for the equation above to hold, $\frac{\partial \lambda^{tar}}{\partial T} > 0$ must be true.

With respect to the targeting accuracy, $\frac{d(\Gamma(\lambda^{tar}))}{d\,\alpha} = \frac{\partial(\Gamma(\lambda^{tar}))}{\partial \alpha} + \frac{\partial(\Gamma(\lambda^{tar}))}{\partial\,\lambda^{tar}} \cdot \frac{\partial \lambda^{tar}}{\partial\,\alpha} = 0.$

$$\begin{aligned}
\frac{\partial(\Gamma(\lambda^{tar}))}{\partial \alpha} \quad &= \quad (1-\mu)\left(1 - \frac{\lambda}{4} - \frac{\lambda}{18T} \cdot \frac{\alpha + (1-\alpha)\mu}{\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)} \frac{1-\lambda\mu}{2-\lambda\mu}\right) \\
&\quad -\big(\alpha + (1-\alpha)\mu\big) \cdot \frac{\lambda}{18T} \cdot \frac{1-\lambda\mu}{2-\lambda\mu} \cdot \frac{3(1-\mu)}{\big(\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)\big)^2} \\
&= \quad (1-\mu)\left(1 - \frac{\lambda}{4} - \frac{\lambda}{18T} \frac{\big(\alpha + (1-\alpha)\mu\big) \cdot \frac{1-\lambda\mu}{2-\lambda\mu}}{\big(\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha)\big)^2} \big(\alpha + (1-\alpha)\mu + 3(1-\mu)(1-\alpha) + 3\big)\right)
\end{aligned}$$

If T is very large, then this partial derivative is approximately $(1-\mu)(1-\lambda/4)$, which is positive. The term in the last line of the equations above after $1-\lambda/4$, starting with $\lambda/(18T)$, is positive. Therefore,

$\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\alpha} > 0$ if and only if T is sufficiently large;

$$T > \underbrace{\frac{2\lambda}{9(4-\lambda)}}_{\leq 2/27} \cdot \underbrace{\frac{\big(\alpha+(1-\alpha)\mu\big)\cdot\big(\alpha+(1-\alpha)\mu+3(1-\mu)(1-\alpha)+3\big)}{\big(\alpha+(1-\alpha)\mu+3(1-\mu)(1-\alpha)\big)^2}}_{\leq 4} \cdot \underbrace{\frac{1-\lambda\mu}{2-\lambda\mu}}_{\leq 1/2}.$$

In particular, the right-hand side is less than $\frac{4}{27}$, so if $\frac{T}{2} > \frac{2}{27}$, then $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\alpha} > 0$. In this case, $\frac{\partial\lambda^{tar}}{\partial\alpha} > 0$.

But, if T is not too large, whether the Euqation above holds depends on α, because the right-hand side increases in α. Therefore, $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\alpha} > 0$ if and only if α is less than some threshold level. So, $\frac{\partial\lambda^{tar}}{\partial\alpha} > 0$ if and only if α is less than the same threshold level, and otherwise, $\frac{\partial\lambda^{tar}}{\partial\alpha} < 0$. That is, λ^{tar} is first increasing, and then decreasing in α.

Lastly, with respect to μ, $\frac{d(\Gamma(\lambda^{tar}))}{d\mu} = \frac{\partial(\Gamma(\lambda^{tar}))}{\partial\mu} + \frac{\partial(\Gamma(\lambda^{tar}))}{\partial\lambda^{tar}} \cdot \frac{\partial\lambda^{tar}}{\partial\mu} = 0$.

$$\begin{aligned}\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\mu} =& -2k\cdot\mu\cdot\lambda^{tar} + \big(\alpha+2(1-\alpha)\mu\big)\cdot\left(1-\frac{\lambda^{tar}}{4}-\frac{\lambda^{tar}}{18T}\cdot\frac{\alpha+(1-\alpha)\mu}{\alpha+(1-\alpha)\mu+3(1-\mu)(1-\alpha)}\cdot\frac{1-\lambda^{tar}\mu}{2-\lambda^{tar}\mu}\right)\\ &-\big(\alpha+(1-\alpha)\mu\big)\cdot\frac{\lambda^{tar}(1-\lambda^{tar}\mu)}{18T(2-\lambda^{tar}\mu)}\cdot\frac{3(1-\mu)}{\big(\alpha+(1-\alpha)\mu+3(1-\mu)(1-\alpha)\big)^2}\end{aligned} \tag{42}$$

So, if T is sufficiently large and k not too large, $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\mu} > 0$, so that $\frac{\partial\lambda^{tar}}{\partial\mu} > 0$. On the other hand, if T is small, or k is sufficiently large, $\frac{\partial(\Gamma(\lambda^{tar}))}{\partial\mu} < 0$, and therefore therefore $\frac{\partial\lambda^{tar}}{\partial\mu} < 0$. ■

Proof of Proposition 8

Recall that the equilibrium amount of untargeted advertising solves $\Gamma^{un}(\lambda) = 0$ where $\Gamma^{un}(\lambda)$ is defined in equation (28). For targeted advertising, the equilibrium advertising λ^{tar} solves $\Gamma(\lambda) = 0$, which is defined in equation (36). Both $\Gamma^{un}(\lambda)$ and $\Gamma(\lambda)$ decrease in λ. The total advertising coverage in two cases are $\mu\lambda^{tar}$ and λ^{un}. To compare the advertising coverage between the two cases, we plug $\lambda = \mu\lambda^{tar}$ into equation (28). $\mu\lambda^{tar} > \lambda^{un}$ if and only if $\Gamma^{un}(\mu\lambda^{tar}) < 0$.

$$\Gamma^{un}(\mu\lambda^{tar}) = \mu\cdot\left(1-\frac{\mu\lambda^{tar}}{4}-\frac{\mu\lambda^{tar}}{18(3-2\mu)T}\cdot\frac{\mu(1-\mu\lambda^{tar})}{2-\mu\lambda^{tar}}\right)-k\cdot\mu\cdot\lambda^{tar}, \tag{43}$$

We know λ^{tar} solves the first-order condition $\Gamma(\lambda^{tar}) = 0$ defined in Equation (36).

$$\mu(\alpha+(1-\alpha)\mu)\cdot\left(1-\frac{\lambda^{tar}}{4}-\frac{\lambda^{tar}}{18T}\cdot\frac{\alpha+(1-\alpha)\mu}{\alpha+(1-\alpha)\mu+3(1-\mu)(1-\alpha)}\cdot\frac{4-(3+\lambda^{tar})\mu}{2-\mu\lambda^{tar}}\right)-k\cdot\mu^2\cdot\lambda^{tar} = 0$$

If $\alpha = 0$, the equation above is equal to

$$\Gamma^{\lambda^{tar}}|_{\alpha=0} = \mu^2 \cdot \left(1 - \frac{\lambda^{tar}}{4} - \frac{\mu\lambda^{tar}}{18(3-2\mu)T} \cdot \frac{4-(3+\lambda^{tar})\mu}{2-\mu\lambda^{tar}}\right) - k \cdot \mu^2 \cdot \lambda^{tar} = 0,$$

where λ^{tar} here is evaluated at $\alpha = 0$.

Comparing $\frac{\Gamma(\lambda^{tar})|_{\alpha=0}}{\mu^2}$ and $\frac{\Gamma^{un}(\mu\lambda^{tar}|_{\alpha=0})}{\mu}$ in Equation (43), $\frac{\Gamma^{un}(\mu\lambda^{tar}|_{\alpha=0})}{\mu} - \frac{\Gamma(\lambda^{tar})|_{\alpha=0}}{\mu^2} =$

$$\frac{(1-\mu)\lambda^{tar}}{4} + \frac{\mu\lambda^{tar}}{18(3-2\mu)T} \cdot \frac{(1-\mu)(4-\mu\lambda^{tar})}{2-\mu\lambda^{tar}} > 0. \tag{44}$$

Therefore, for $\alpha = 0$, $\Gamma^{un}(\mu\lambda^{tar}) > 0$. Since $\Gamma^{un}(\cdot)$ is a decreasing function, this shows that $\mu\lambda^{tar} < \lambda^{un}$. In other words, for $\alpha = 0$, the equilibrium untargeted advertising amount exceeds that for targeted advertising.

If $\alpha = 1$, $\Gamma(\lambda^{tar})|_{\alpha=1} =$

$$\mu\left(1 - \frac{\lambda^{tar}}{4} - \frac{\lambda^{tar}}{18T} \cdot \frac{4-(3+\lambda^{tar})\mu}{2-\mu\lambda^{tar}}\right) - k \cdot \mu^2 \cdot \lambda^{tar} = 0,$$

where λ^{tar} here is evaluated at $\alpha = 1$. Then, directly subtracting equation (43) from the equation above, $\Gamma^{un}(\mu\lambda^{tar})|_{\alpha=1} - \frac{\Gamma(\lambda^{tar})|_{\alpha=1}}{\mu} =$

$$(1-\mu)\Big(\underbrace{\frac{\lambda^{tar}(1+\mu)}{4} - 1}_{<0}\Big) - \frac{\lambda^{tar}}{18(2-\mu\lambda^{tar})T}\Big(\underbrace{4-(3+\lambda^{tar})\mu - \frac{\mu^3(1-\mu\lambda^{tar})}{3-2\mu}}_{>0}\Big) < 0. \tag{45}$$

Therefore, $\Gamma^{un}(\mu\lambda^{tar})|_{\alpha=1} < 0$, which means $\mu\lambda^{tar} > \lambda^{un}$ because $\Gamma^{un}(\cdot)$ is a decreasing function. Therefore, if $\alpha = 1$, firms advertise more under targeting than under no targeting.

Proposition 7 shows that under targeting the equilibrium amount of advertising monotonically increases in α for a sufficiently high T, i.e., $\frac{\partial\lambda^{tar}}{\alpha} > 0$. Otherwise, if T is not sufficiently high, λ^{tar} first increases and then decreases in α. Also, $\mu\lambda^{tar} < \lambda^{un}$ at $\alpha = 0$, but $\mu\lambda^{tar} > \lambda^{un}$ at $\alpha = 1$. So, by the intermediate value theorem, there exists an intermediate $\overline{\alpha} \in (0,1)$ such that $\mu\lambda^{tar} > \lambda^{un}$ if and only if $\alpha > \overline{\alpha}$. That is, the equilibrium amount of targeted advertising is greater than that under un-targeting if and only if the targeting accuracy is sufficiently high.

■

Proof of Proposition 9

The equilibrium profits for a firm of quality q is obtained by choosing advertising level λ^{un} and λ^{tar}. From equation (18)

$$\Pi^{un*}(q) = \mu \cdot q\Bigg(\lambda^{un}\, q\big(1-\frac{\lambda^{un}}{4}+\frac{\mu\lambda^{un}}{18(3-2\mu)T}\big)+(1-\lambda^{un}\, q)\frac{\mu\,\lambda^{un}(3-2\lambda^{un})}{18(3-2\mu)(2-\lambda^{un})T}\Bigg)-\frac{k(\lambda^{un}\, q)^2}{2} \quad (46)$$

For the case of targeted advertising, the profit function from (13),

$$\begin{aligned}\Pi^{tar*}(q) = \mu \cdot q\big(\alpha+(1-\alpha)\mu\big)\cdot\Bigg(&\lambda^{tar} q\big(1-\frac{\lambda^{tar}}{4}+\frac{\lambda^{tar}}{18T}\cdot\frac{\alpha+(1-\alpha)\mu}{\alpha+(1-\alpha)\mu+3(1-\alpha)(1-\mu)}\big)\\ &+(1-\lambda^{tar} q)\cdot\frac{\lambda^{tar}}{18T}\cdot\frac{3-2\lambda^{tar}\mu}{2-\lambda^{tar}\mu}\cdot\frac{\alpha+(1-\alpha)\mu}{\alpha+(1-\alpha)\mu+3(1-\alpha)(1-\mu)}\Bigg)-k\cdot\frac{(\mu\,\lambda^{tar}\, q)^2}{2}\quad.\end{aligned} \quad (47)$$

For a sufficiently large T, in which case there is no search beyond the prominent firm, the profit function can be approximated by $\Pi^{tar*}(q) \to \mu\cdot q^2\big(\alpha+(1-\alpha)\mu\big)\cdot\lambda^{tar}\big(1-\frac{\lambda^{tar}}{4}\big)-\frac{k(\mu\,\lambda^{tar}\, q)^2}{2}$. Differentiating this profit function with respect to α results in

$$\begin{aligned}\lim_{T\to\infty}\frac{\partial\Pi^{tar*}(q)}{\partial\alpha} =& \mu\cdot q^2\cdot\Bigg((1-\mu)\cdot\lambda^{tar}\cdot(1-\frac{\lambda^{tar}}{4})+(\alpha+(1-\alpha)\mu)\cdot(1-\frac{\lambda^{tar}}{2})\cdot\frac{\partial\,\lambda^{tar}}{\partial\,\alpha}\\ &-k\cdot\mu\cdot\lambda^{tar}\cdot\frac{\partial\,\lambda^{tar}}{\partial\,\alpha}\Bigg)\end{aligned} \quad (48)$$

From Proposition 7, for T sufficiently large, $\frac{\partial\,\lambda^{tar}}{\partial\,\alpha} > 0$. Therefore, the equation above, $\frac{\partial\Pi^{tar*}(q)}{\partial\alpha} < 0$ if T is large and k is sufficiently large. So, if advertising is costly (a large k) and consumers do not search beyond the prominent firm (a large T, and no free-riding effects), then firms engage in a fierce competition through advertising. This competition hurts the firms' profits if the cost of advertising is high. Therefore, the firm's profit can decrease in the targeting accuracy.

For $\alpha = 0$, the profit function is equal to

$$\Pi^{tar*}(q)|_{\alpha=0} = \mu^2\cdot q\Bigg(\lambda^{tar} q\big(1-\frac{\lambda^{tar}}{4}+\frac{\mu\lambda^{tar}}{18(3-2\mu)T}\big)+(1-\lambda^{tar} q)\cdot\frac{\mu\lambda^{tar}\big(6-(3+2\lambda^{tar})\mu\big)}{18(2-\mu\lambda^{tar})(3-2\mu)T}\Bigg)-\frac{k(\mu\,\lambda^{tar}\, q)^2}{2}, \quad (49)$$

and for $\alpha = 1$,

$$\Pi^{tar*}(q)|_{\alpha=1} = \mu\cdot q\Bigg(\lambda^{tar} q\big(1-\frac{\lambda^{tar}}{4}+\frac{\lambda^{tar}}{18T}\big)+(1-\lambda^{tar} q)\cdot\frac{\lambda^{tar}\big(6-(3+2\lambda^{tar})\mu\big)}{18(2-\mu\lambda^{tar})T}\Bigg)-\frac{k(\mu\,\lambda^{tar}\, q)^2}{2}, \quad (50)$$

Then, we compare the firm's profits under untargeted case and perfect targeting case in the limit as $T \to \infty$ by computing $\lim_{T\to\infty} \Pi^{tar*}(q)|_{\alpha=1} - \Pi^{un*}(q)$:

$$\mu \cdot q^2 \cdot (\underbrace{\lambda^{tar} - \lambda^{un}}_{\geq 0})\left(1 - \frac{\lambda^{tar} + \lambda^{un}}{4}\right) - \underbrace{\frac{k \cdot q^2 \cdot (\mu\lambda^{tar} - \lambda^{un})(\mu\lambda^{tar} + \lambda^{un})}{2}}_{\geq 0} \tag{51}$$

At $\alpha = 1$, $\lambda^{tar} > \mu\lambda^{tar} > \lambda^{un}$. Therefore, $\lim_{T\to\infty} \Pi^{tar*}(q)|_{\alpha=1} - \Pi^{un*}(q) > 0$ if and only if k is sufficiently small.

If $\alpha = 0$, $\lim_{T\to\infty} \Pi^{tar*}(q)|_{\alpha=0} - \Pi^{un*}(q)$:

$$\mu \cdot q^2 \cdot \left(\underbrace{\mu\lambda^{tar}(1 - \frac{\lambda^{tar}}{4}) - \lambda^{un}(1 - \frac{\lambda^{un}}{4})}_{\leq 0}\right) - \underbrace{\frac{k \cdot q^2 \cdot (\mu\lambda^{tar} - \lambda^{un})(\mu\lambda^{tar} + \lambda^{un})}{2}}_{\geq 0} < 0 \tag{52}$$

At $\alpha = 0$, $\mu\lambda^{tar} < \lambda^{un}$. Therefore, $\mu\lambda^{tar} - \lambda^{un} - \frac{1}{4}\big(\mu(\lambda^{tar})^2 - (\lambda^{un})^2\big) < (\mu\lambda^{tar} - \lambda^{un}) - \frac{1}{4}(\mu\lambda^{tar} - \lambda^{un})(\mu\lambda^{tar} + \lambda^{un}) = (\mu\lambda^{tar} - \lambda^{un})\big(1 - \frac{\mu\lambda^{tar} + \lambda^{un}}{4}\big) < 0$. Therefore, $\lim_{T\to\infty} \Pi^{tar*}(q)|_{\alpha=0} - \Pi^{un*}(q) < 0$.

So, if T approaches infinity and k is sufficiently large, $\Pi^{un*}(q) > \Pi^{tar*}(q)$ for all q. But, if k is not sufficiently high, $\Pi^{un*}(q) < \Pi^{tar*}(q)$ for α close enough to 1.

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