Can Idiosyncratic Cash Flow Shocks Explain Asset Pricing Anomalies?*

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ABSTRACT

Asset pricing anomalies appear in a dynamic framework if unpriced idiosyncratic cash flow shocks contain information about future priced risk. A positive idiosyncratic shock decreases the sensitivity of firm value to priced risk factors and simultaneously increases firm size. Therefore, a model with idiosyncratic shocks can explain value and size premia, as well as the negative relation between idiosyncratic volatility and stock returns. More generally, our results imply that any economic variable correlated with the history of idiosyncratic cash flow shocks can be successful in explaining expected stock returns.

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It is well established that, under standard asset pricing assumptions, only systematic cash flow risk is priced. In this paper, we argue that unpriced idiosyncratic cash flow shocks can also be essential for asset prices since they contain valuable conditioning information in a dynamic asset pricing framework. In particular, we show that the conditional beta with respect to any priced source of risk directly depends on the history of firm-specific shocks. We use this insight to provide risk-based explanations for several anomalies in the cross-section of equity returns, including the widely documented value and size effects, as well as the negative relation between idiosyncratic volatility and stock returns, and the post-investment (post-SEO) performance.[1]

To understand why firm-specific shocks are useful as conditioning information, consider a firm with two divisions. Suppose the profit of the first division depends exclusively on idiosyncratic profitability shocks and the profit of the second division is driven only by systematic shocks. The firm can thus be seen as a portfolio of a zero beta and a risky asset. When a positive idiosyncratic shock arrives, the size of the zero beta asset increases, making it a larger fraction of the total portfolio value. As a result, overall firm beta decreases, as do expected stock returns. Therefore, any firm characteristic correlated with the history of idiosyncratic cash flow shocks can be successful in explaining expected stock returns. In a more general setting, we show that beta is invariant with respect to idiosyncratic shocks only in the special case where profits are the product of idiosyncratic and systematic profitability shocks. Multiplicative value functions of this type are used extensively in the literature because of their tractability properties (see, e.g., Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Cooper (2006)). Therefore, without additional features such as operating leverage or time varying price of risk, market betas derived in these papers are independent of firm-specific shocks.[2]

Using this insight, we build a simple model of firms' investment decisions in which firm value is additive in two types of shocks and only systematic risk is priced. We first consider a

[1] In the cross-section, firms with small market capitalizations and high ratios of fundamentals to price tend to have high stock returns (Banz (1981), Graham and Dodd (1934)). Fama and French (1992) provide a detailed analysis of both value and size premium.

[2] Two earlier papers that obtain dependence of beta on idiosyncratic shocks are Brennan (1973) and Bossaerts and Green (1989), who model cash flows as conditionally linear in a systematic factor and allow the firm-specific intercept to vary over time.

firm comprised entirely of assets in place, and later add growth options and product market competition. We show that in the cross-section of returns generated by this benchmark model, firm characteristics are related to expected returns. This is because firms with a larger idiosyncratic share of value have larger market capitalizations and lower book-to-market ratios, but at the same time lower equity betas since the systematic component contributes a smaller fraction to firm value. As a result, firms with high market capitalizations and low book-to-market ratios have low expected returns.

Our model with systematic and idiosyncratic cash flow shocks is able to explain some of the regularities related to growth options. First, we contribute to the debate on whether more investment options, all else equal, imply higher or lower firm risk.[3] We show that the relation between growth options and risk crucially depends on the type of investment options. In particular, while growth options linked to the systematic profit component increase firm's risk, growth options linked to the idiosyncratic profitability shocks have the opposite effect. Somewhat surprisingly, exercises of both systematic and idiosyncratic growth options always lead to a decline in firm's systematic risk if the firm finances investment with new equity. Thus, our model accounts for the observed poor stock return performance following seasoned equity offerings (Loughran and Ritter (1995)).

Further, value and size premia are magnified in the model with growth options for two reasons. First, options make firm values and conditional betas more sensitive to profitability shocks, as irreversible investment options grow in value exponentially (Dixit and Pindyck (1994)). Second, firms optimally exercise investment options, and as a result lower risk, only when their market capitalizations are high. We also show that the nonlinearity of growth options in the underlying profitability shock results in a negative relation between price-to-earnings ratios and expected returns, although this relation is somewhat weaker than the one between market-to-book ratios and expected returns.

Our generalized model with a distinct role for idiosyncratic shocks also provides new insights about the asset pricing implications of product market competition. As economic

[3]Among others, Gomes, Kogan, and Zhang (2003), Petkova and Zhang (2005), Zhang (2005), Carlson, Fisher, and Giammarino (2006), and Novy-Marx (2011) show that the relation between growth options and firm risk can depend on operating leverage and investment irreversibility.

conditions improve, increased industry competition leads to an endogenous limit to growth in the systematic component of the profit. When systematic profitability is relatively high, any further increase in profitability is offset by entry of new competing firms. As a result, conditional betas are low when systematic profitability is high. Competition thus adds a dynamic component to our model, resulting in a low risk premium in "good times". This is consistent with empirical evidence in Bustamante and Donangelo (2012) that firms in competitive industries have low returns. Since all betas decline when competition is more intense, the value and size effects become less pronounced. This last result is consistent with empirical evidence provided in Hou and Robinson (2006), who find that the value effect is stronger in highly concentrated industries.

The intuition developed in this paper applies to any general setting with a single or multiple sources of priced risk. As in previous papers, size and value effects are not anomalous relative to the correctly specified asset pricing model and appear only when not all sources of priced risk are correctly accounted for, as in Berk (1995). Reconciling the predictions of our model with the empirical evidence on value and size premium relative to the CAPM thus relies on imperfect measurement of risk, and in particular differences between conditional and unconditional betas, as in Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Cooper (2006).[4] Betas are likely to be mismeasured either because asset pricing tests fail to use all conditioning information (Hansen and Richard (1987)) or because the proxy for the market portfolio is imperfect (Roll (1977)).

We use the analytical solutions from the model to generate firms' stock returns and examine the fit between the model-generated and empirically observed data. Our analysis of the simulated data indicates that the model can produce reasonable value and size effects in the cross-sectional Fama and MacBeth (1973) regressions even when we explicitly control for empirically estimated betas. For example, we find a value premium of 56 basis points per month for the decile of largest book-to-market ratios relative to the smallest decile. Similarly, stocks in the smallest size decile outperform the largest decile by approximately 54 basis points

[4]Lewellen and Nagel (2006) argue that the conditional CAPM is unable to match the magnitude of observed anomalies.

per month in the benchmark model. The decile of high price-earnings ratios underperforms the lowest decile by 17 basis points. Using the simulated data, we show that value and size anomalies are more pronounced in firms with highly valuable growth options and are abated by product market competition. These results are consistent with empirical evidence in Da, Guo, and Jagannathan (2012) and Grullon, Lyandres, and Zhdanov (2012), who argue that the poor empirical performance of the unconditional CAPM is mainly attributable to real options.

Finally, our model produces a negative relation between idiosyncratic volatility and expected stock returns (e.g., Ang, Hodrick, Xing, and Zhang (2006)). In our model, a history of favorable idiosyncratic shocks decreases the relative magnitude of the systematic profit component. This leads not only to a drop in firm equity beta, but also to an increase in the idiosyncratic stock return volatility. As a result, estimated idiosyncratic volatility is negatively correlated with firms' conditional betas and expected returns. For example, we find a monthly premium of approximately 56 basis points for the low idiosyncratic volatility decile over the top decile.

The paper is organized as follows. In the next section, we provide a concise summary of the related literature. Section II builds a simple example to develop the intuition. Section III presents the continuous-time model with investment and competition. Section IV discusses the model predictions, and Section V presents simulation results and compares them to observed empirical regularities. The last section concludes.

I. Literature

Early work by Brennan (1973) and Bossaerts and Green (1989) models dividends as sum of persistent idiosyncratic shocks and a single systematic shock. In particular, Bossaerts and Green (1989) derive two factor arbitrage pricing theory restrictions on dynamic equilibrium asset returns. They show that conditional expected returns depend inversely on the current stock price and apply this intuition to explain, in particular, the abnormally high January returns of small stocks. In our paper, we focus on the role of idiosyncratic shocks as valuable conditioning information and aim to explain a broad range of asset pricing anomalies. More importantly, we show that the asset pricing implications of Bossaerts and Green (1989) are general and not restricted to their additive dividend process. In other words, the effects of idiosyncratic profit components are relevant for *almost every* firm in the economy.

Berk (1995) shows that anomalies related to market capitalization arise due to differences in firms' unobservable discount rates. Instead, our results are driven by variation in firms' cash flows and are thus complementary to those in Berk (1995). In particular, in our model, firms with low dividends have a small idiosyncratic profit component and thus high systematic risk. Anomalies therefore show up in fundamentals, such as dividends or earnings, and not only in market values. Adding the feedback of discount rates to market valuation would strengthen our results.

Our paper contributes to the rapidly growing literature that links the theory of investment under uncertainty to determinants of the cross-section of stock returns. Berk, Green, and Naik (1999) were among the first to link firm investment options to risk. Gomes, Kogan, and Zhang (2003) build a general equilibrium model with perfect competition that generates value and size effects in cross-section. In their model, the cross-sectional differences in firms' risk are driven by the importance of growth options relative to assets in place. In contrast, our analysis focuses on the differences in dynamics of idiosyncratic shocks across stocks.

Carlson, Fisher, and Giammarino (2004) model a firm that can expand its business by investing in new projects. In their model, operating leverage makes assets in place more risky than growth options, giving rise to book-to-market and size anomalies. Zhang (2005) models costly investment reversibility and a countercyclical price of risk. Specifically, he shows that in bad times assets in place are riskier than growth options because they are difficult to reduce. This leads to an unconditional value premium since the price of risk is high in bad times. Cooper (2006) develops similar intuition in a model with lumpy investment and constant price of risk. Kogan and Papanikolaou (2012) develop an equilibrium model with two aggregate sources of risk that have different implications for growth options and assets in place. In our paper, value and size effects appear even when firms have no operating leverage, no growth options, and the price of the single source of risk is constant.

To provide a risk-based explanation for momentum, Sagi and Seasholes (2007) model a firm as a portfolio of risk-free and risky assets. They show that a long position in the risk-free asset results in a positive return autocorrelation, as high returns on the risky asset increase the weight of the risky asset in a portfolio and therefore increase portfolio risk. In contrast, we model a risky zero-beta asset instead of a risk-free asset. The volatility embedded in the zero-beta asset changes the intuition of Sagi and Seasholes since returns can now originate from either systematic or idiosyncratic sources and can thus either increase or decrease future risk. Therefore, our paper takes no stance regarding the time series properties of individual asset returns and focuses instead on the cross-sectional implications.

Our paper is also related to the literature examining the effect of product market competition on the value of real options and the optimal exercise timing. Our primary interest lies in identifying how competition affects the dynamics of conditional betas, whereas the majority of previous papers focused on the optimal timing of investment options exercises.[5] Aguerrevere (2009) and Bena and Garlappi (2012) also examine competition and its impact on systematic risk. The main difference is that in their papers, competition mainly works through its effect on growth options.

II. Idiosyncratic Shocks and Firm Risk

We now develop a simple example to highlight the main economic mechanism in the paper. Consider a firm with value $V(x_i, y)$ that depends both on idiosyncratic shock, x_i, and systematic shock, y. Without loss of generality, we assume that higher values of idiosyncratic shocks indicate better states of the world, i.e., $V_x > 0$, where the subscript denotes the first order derivative. We define beta as the sensitivity of relative changes in value to relative changes

[5] For example, Grenadier (2002) argues that competition erodes the value of real options, thereby reducing the advantage of waiting to invest. In contrast, Leahy (1993) and Caballero and Pindyck (1996) argue that, despite the fact that the option to wait is less valuable in a competitive environment, irreversible investment is still delayed because the upside profits are lowered by new firm entry. By considering a nonlinear production technology, Novy-Marx (2007) shows that firms in a competitive industry may delay irreversible investment even longer than suggested by a neoclassical framework.

in the systematic shock

$$\beta_i = \frac{V_y(x_i, y)\, y}{V(x_i, y)}. \tag{1}$$

By differentiating this expression with respect to x_i and setting the result to zero, we show that beta is independent of idiosyncratic shock if and only if the following partial differential equation for the value function is satisfied:

$$V(x_i, y) = \frac{V_x(x_i, y)\, V_y(x_i, y)}{V_{xy}(x_i, y)}. \tag{2}$$

One class of value functions that satisfies this equation is multiplicatively separable functions of the form

$$V(x_i, y) = f(x_i)\, g(y), \tag{3}$$

which have been used extensively in the previous literature. It is worth examining criterion (2) with care. Clearly, for the majority of value functions this condition will not be satisfied and betas will depend on the history of idiosyncratic shock realizations. This result implies that for the majority of firms, characteristics such as size, book-to-market, or volatility, must be correlated with expected returns.

For example, it can be instructive to consider a firm with the following additive value function

$$V(x_i, y) = f(x_i) + g(y) - c, \tag{4}$$

where the first term captures the value derived from idiosyncratic shocks, the second term value from systematic shocks, and c is the firm's long or short position in the risk-free asset.

From (1), the firm market beta is given by

$$\beta_i = \frac{g_y(y)\, y}{V(x_i, y)}. \tag{5}$$

It is easy to see that β_i in this case is decreasing in the idiosyncratic shock since the numerator in expression (5) is independent of x_i and the denominator is increasing in x_i. This result implies that a positive idiosyncratic shock simultaneously increases firm market value and decreases beta, giving rise to value and size effects in the cross-section of stock returns.

Finally, to link our results to prior literature, it is also convenient to rewrite (5) as follows

$$\beta_i = 1 - \frac{f(x_i)}{V(x_i, y)} + \frac{g_y(y)\,y - g(y)}{V(x_i, y)} + \frac{c}{V(x_i, y)}. \tag{6}$$

The first term is normalized to one. The second term directly generates the size effect since a higher value of shock x_i will simultaneously lead to higher firm value and lower beta. The third term in (6) appears only if function $g(y)$ is nonlinear in the systematic shock y. Whether this term increases or decreases the overall firm beta depends on the concavity/convexity of function $g(y)$. For example, growth options linked to systematic profitability shocks induce convexity in the value function, most prominently near the exercise threshold, and therefore tend to increase systematic risk. In contrast, product market competition which limits firm's profits induces concavity in the value function and decreases firm beta. The last term in (6) is proportional to c, and can represent operating or financial leverage ($c > 0$), or cash savings ($c < 0$). This term has been studied in the previous literature (see, e.g., Carlson, Fisher, and Giammarino (2004) and Sagi and Seasholes (2007)).

III. The Model

This section lays out a model that extends the simple example to a dynamic setting and allows us to incorporate effects of growth options, investment, and product market competition. The model facilitates comparison of our results to those in previous studies and enables us to evaluate the economic importance of asset pricing anomalies in simulated data. We deliberately do not model operating leverage or time varying price of risk since previous work has already shown that these features can help generating size and value premia.

Each firm in the economy generates profit P_i

$$P_i = x_i + \rho_i Y Q^{-\varepsilon}, \tag{7}$$

where x_i is the idiosyncratic demand shock (e.g., tastes for the differentiated firm's product), ρ_i is the firm's sensitivity to the systematic demand shock Y, Q is the mass of firms, and $1/\varepsilon$ is the positive price elasticity of demand. Time subscripts throughout are omitted. The profitability shocks follow geometric Brownian motions in the risk-neutral measure

$$dx_i/x_i = \mu_x dt + \sigma_x dz_i, \tag{8}$$

$$dY/Y = \mu_y dt + \sigma_y dz_y, \tag{9}$$

where dz_i and dz_y are increments of uncorrelated standard Wiener processes, $E[dz_i dz_y] = 0$ for all i. The idiosyncratic shocks have identical drifts and volatilities and are uncorrelated across firms, $E[dz_i dz_j] = 0$ for $i \neq j$.

We model product market competition through changes in the equilibrium number of firms Q. New firms can enter the market by paying a fixed cost R. The value of shock Y is common knowledge prior to entry, and we assume that all prospective entrants receive identical initial draws of idiosyncratic shocks x_i. All firms are risk-neutral and infinitesimally small.[6] Since for tractability purposes we do not model optimal firm exit, we assume that the number of firms decays over time with intensity λ,

$$dQ_t = -\lambda Q_t dt. \tag{10}$$

In a unit of time when there is no entry, the number of firms deterministically decreases as in (10). Hence, by denoting $y = YQ^{-\varepsilon}$ and using Ito's lemma, we can write the dynamics of the process y as

$$dy = \left(\mu_y + \varepsilon\lambda\right) y dt + \sigma_y y dz_y, \tag{11}$$

where the additional term in the drift, $\varepsilon\lambda$, appears because the expected decline in the mass of firms, Q, leads to a higher growth of y.

Since all prospective entrants are identical, they enter at the same threshold, $\overline{y}$. Thus, new entry endogenously limits growth of the product price associated with systematic innovations in firm profitability, similar to the exogenous limits to growth in, for example, Carlson, Fisher, and Giammarino (2004). As a result, the process (11) has a reflecting barrier at $\overline{y}$, formally

[6]The last assumption allows us to treat firms as price-takers and to ignore the effect of firm's own output on the equilibrium price. Aguerrevere (2009) presents a more general model of competition, in which firms also take into account the effect of their own output on product price.

defined as

$$dy = \begin{cases} 0, & \text{for } y = \overline{y} \text{ and } dz_y > 0 \\ \widehat{\mu}_y y dt + \sigma_y y dz_y, & \text{otherwise} \end{cases} \tag{12}$$

where $\widehat{\mu}_y = \mu_y + \varepsilon\lambda$.[7]

We model investment options in reduced form. In addition to receiving continuous cash flows (7), each firm has an opportunity to irreversibly expand production or improve technology by paying a fixed cost. For tractability purposes, we assume that a firm can separately exercise growth options linked to idiosyncratic shocks (x-options) and to systematic shocks (y-options).[8] Specifically, by paying an investment cost of I_x a firm can increase the idiosyncratic component of its cash flows x_i by a factor $1+\gamma_x$, and by spending $\rho_i I_y$ it can increase the systematic component of cash flows $\rho_i y$ by a factor $1+\gamma_y$. Making the exercise cost of the $y-$option proportional to ρ_i ensures that the cost of exercising options scales up appropriately with the size of the firms' assets. The exercise cost of the systematic option is assumed to be sufficiently small relative to the cost of entry to ensure that option are optimally exercised prior to reaching the competition boundary. Investment is irreversible and indivisible, and, unlike Ai and Kiku (2012), we do not allow the investment cost to change with the state of economy.

The value of the firm with cash flows CF_t is given by

$$V(x_i, y) = E \int CF_t e^{-\widehat{r}} dt, \tag{13}$$

where $\widehat{r} = r + \lambda$. The discount rate is adjusted to reflect the risks of exogenous exit of each firm. Firm value is obtained from the following partial differential equation

$$\widehat{r}V = CF_t + V_x \mu_x x_i + V_y \widehat{\mu}_y y + \frac{1}{2} V_{xx} \sigma_x^2 x_i^2 + \frac{1}{2} V_{yy} \sigma_y^2 y^2, \tag{14}$$

with appropriate boundary conditions. To prevent arbitrage in the model, we require that as the value of the systematic shock y approaches the reflecting barrier, the firm value becomes

[7] As in Caballero and Pindyck (1996), the reflecting barrier $\overline{y}$ is time-invariant in our setting. A sufficient condition for this is a stationary distribution of the number of entering and exiting firms.

[8] Alternatively, growth options could be modeled to depend on both idiosyncratic and systematic profits, and increase *total* firm cash flows. Such approach does not change the intuition developed in this paper, but creates significant complications due to the two-dimensional option exercise policy.

insensitive to further changes in the shock

$$V_y(x_i, \overline{y}) = 0. \tag{15}$$

In addition, optimal option exercise requires the smooth-pasting conditions on the first derivatives to be satisfied at the point of exercise (Dumas (1991) and Dixit (1993)).

The solution for firm value is summarized by the following proposition.

Proposition 1. *Denote by ι_x and ι_y the indicator functions, equal to one if the respective growth option has been exercised. Then the market value of the firm is given by*

$$V(x_i, y) = V^{AX}(x_i) + V^{GX}(x_i) + \rho_i \left(V^{AY}(y) + V^{GY}(y) - V^C(y)\right), \tag{16}$$

where the value components $V^{AX}(x_i), V^{GX}(x_i), V^{AY}(y), V^{GY}(y)$, and $V^C(y)$ are given by

$$\begin{aligned}
V^{AX}(x_i) &= (1 + \iota_x \gamma_x) \frac{x_i}{\widehat{r} - \mu_x}, & (17)\\
V^{GX}(x_i) &= (1 - \iota_x) \frac{\gamma_x x^*}{(\widehat{r} - \mu_x) d_2} \left(\frac{x_i}{x^*}\right)^{d_2}, & (18)\\
V^{AY}(y) &= \left(1 + \iota_y \gamma_y\right) \frac{y}{\widehat{r} - \widehat{\mu}_y}, & (19)\\
V^{GY}(y) &= (1 - \iota_y) \frac{\gamma_y y^*}{\left(\widehat{r} - \widehat{\mu}_y\right) b_2} \left(\frac{y}{y^*}\right)^{b_2}, & (20)\\
V^C(y) &= \frac{\left(1 + \gamma_y\right) \overline{y}}{\left(\widehat{r} - \widehat{\mu}_y\right) b_2} \left(\frac{y}{\overline{y}}\right)^{b_2}, & (21)
\end{aligned}$$

and the constants $b_2 > 1$ and $d_2 > 1$, the option exercise thresholds x^* and y^*, and the entry threshold $\overline{y} > y^*$ are given in the Appendix.

Proof of Proposition 1. See Appendix B.

Having derived firm market values and optimal investment and entry strategies, we now turn attention to the analysis of systematic risk.

IV. Equity Betas

Since the systematic shock y represents aggregate uncertainty in the model, the firm's equity beta is the elasticity of the firm market value with respect to y.[9] In the following proposition, we derive factor betas.

[9] Appendix C discusses the relation between betas with respect to y derived here and betas with respect to the aggregate market.

Proposition 2. *Adopting the notation of Proposition 1, the market beta of the firm is given by*

$$\beta_i = 1 - \frac{V^{AX}(x_i)}{V(x_i, y)} - \frac{V^{GX}(x_i)}{V(x_i, y)} + \rho_i (b_2 - 1) \left(\frac{V^{GY}(y)}{V(x_i, y)} - \frac{V^{C}(y)}{V(x_i, y)} \right). \tag{22}$$

Proof of Proposition 2. See Appendix B.

The first term in (22) is normalized to one. The second term appears because part of firm value is derived from profits uncorrelated with aggregate demand uncertainty and thus reduces the overall firm's exposure to the systematic risk. The third term captures a negative impact of growth options linked to idiosyncratic profitability shocks ($x-$options). The fourth term shows that firm's risk increases with growth options linked to systematic shocks ($y-$options) since these options are more sensitive to aggregate uncertainty than assets in place. Finally, the last term appears because of the limiting effect of competition on firm's cash flows.

Next, we use Propositions 1 and 2 to discuss the ability of the model to explain asset-pricing anomalies. To facilitate discussion, we first present comparative statics at a point away from the option exercise threshold and then consider the implications of the option exercise for firms' betas.

A. Asset Pricing Anomalies in the Benchmark Model

Consider first the benchmark model with no real options and competition ($\gamma_x = \gamma_y = 0$, $\overline{y} = \infty$). From (16) and (22) we can derive the sensitivities of market values and firm betas to the idiosyncratic profitability shock x_i

$$\frac{\partial V}{\partial x_i} = \frac{1}{\widehat{r} - \mu_x} > 0, \tag{23}$$

$$\frac{\partial \beta_i}{\partial x_i} = -\frac{1}{V^2(x_i, y)} \frac{\rho_i y}{(\widehat{r} - \mu_x)(\widehat{r} - \widehat{\mu}_y)} < 0. \tag{24}$$

It follows that a positive shock simultaneously increases firm market value and decreases beta, leading to the size effect.

To relate the model to the value anomaly, we also need to specify the evolution of book values. We calculate book values based on the cost incurred per unit of installed capital. Since at option exercise, γ_x units of $x-$assets are added at cost I_x, and $\rho_i \gamma_y$ units of $y-$assets at cost $\rho_i I_y$, the initial book value is

$$B_i = \frac{I_x}{\gamma_x} + \rho_i \frac{I_y}{\gamma_y}. \tag{25}$$

Book values increase by I_x and $\rho_i I_y$ for corresponding idiosyncratic and systematic growth option exercises. Firms in our model have no leverage, and therefore the book-to-market ratio is $\frac{B_i}{V_i}$. Note that in the benchmark model, there is no time-series variation in book values, and therefore value and size anomalies are indistinguishable. This result changes, however, with the introduction of growth options, since book values are affected by option exercises.

Other anomalies also appear in our framework and are driven by shocks to the idiosyncratic profit component. In particular, a negative relation between stock returns and idiosyncratic volatility, documented in Ang, Hodrick, Xing, and Zhang (2006), follows directly from the benchmark model. A positive idiosyncratic shock implies a larger idiosyncratic share of profits and hence higher idiosyncratic volatility of stock returns, while at the same time it also lowers systematic firm risk.

Price-earnings ratios in the benchmark model can be shown to be constant if the effective risk-neutral drifts in x and y are identical. Price-earnings ratios are therefore unrelated to expected stock returns in this case. However, this result changes once we incorporate either growth options or product market competition into the model.

Finally, the predictive ability of past returns for future returns (reversals or momentum) depends on the parameterization of the model. On the one hand, return reversal arises naturally in the model and is caused by the evolution of idiosyncratic profitability shocks. Momentum, on the other hand, can potentially appear in a cross-section due to differential exposure of profits to the systematic shock (ρ_i). In particular, a positive systematic shock has a larger effect on stock returns of high sensitivity firms (high ρ_i). At the same time, this shock tends to increase the conditional betas of the high sensitivity firms, resulting in a positive relation between past and future stock returns. This effect is similar to the one described by Sagi and Seasholes (2007), except that the idiosyncratic profit component takes the role of firm's long position in the risk-free asset.

B. Asset Pricing Implications of Growth Options

The general form of our profit function naturally suggests a role for growth options that derive their value from the idiosyncratic profit component. We analyze the impact of growth options on firm risk and their importance for asset pricing anomalies.

Since growth options are levered claims on assets in place, the relation between the value of options and overall firm risk is typically positive.[10] We show that this intuition does not extend to general profit functions. Specifically, according to Proposition 1, both $x-$ and $y-$options increase firm market value, while Proposition 2 shows that the effects of these options on firm's beta go in opposite directions. In line with previous literature, a higher value of the systematic option increases the firm's exposure to systematic risk. However, larger idiosyncratic options imply a smaller overall beta. Therefore, depending on their nature, growth options can lead to either lower or higher firm risk.

In the context of prior empirical literature (see, e.g., Da, Guo, and Jagannathan (2012) and Grullon, Lyandres, and Zhdanov (2012)), it is also useful to discuss how growth options affect value and size anomalies. Again, we shut down the competition channel, and compute the effects of idiosyncratic shocks on betas and market values

$$\frac{\partial V}{\partial x_i} = \frac{1}{\widehat{r}-\mu_x}\left(1+\gamma_x\left(\frac{x_i}{x^*}\right)^{d_2-1}\right) > 0, \tag{26}$$

$$\frac{\partial \beta_i}{\partial x_i} = -\frac{1}{V^2\left(x_i, y\right)}\frac{\rho_i y}{\left(\widehat{r}-\mu_x\right)\left(\widehat{r}-\widehat{\mu}_y\right)}\left(1+\gamma_x\left(\frac{x_i}{x^*}\right)^{d_2-1}\right)\left(1+\gamma_y\left(\frac{y}{y^*}\right)^{b_2-1}\right) < 0. \tag{27}$$

By comparing expressions (23) and (26), it is easy to see that with $x-$options firm market values and hence the book-to-market ratios become more sensitive to the idiosyncratic shocks. The systematic options have no effect on this sensitivity. At the same time, the sensitivity of betas to shocks x_i increases with both systematic and idiosyncratic options. In particular, it follows from (27) that, conditional on the same total firm value, a firm that derives more value from the growth options will have a higher sensitivity to the idiosyncratic profitability shocks. Overall, these results imply that growth options, particularly those linked to idiosyncratic

[10]Note that operating or financial leverage can change this relation (see the discussions in Gomes, Kogan, and Zhang (2003), Petkova and Zhang (2005), Zhang (2005), Carlson, Fisher, and Giammarino (2006), and Novy-Marx (2011)).

shocks, magnify value and size effects in the model.

Finally, we show that growth options add to our understanding of the predictive ability of price-earnings ratios. While a constant in the benchmark model, the price-earnings ratio fluctuates with idiosyncratic and systematic shocks, and in particular changes discontinuously at option exercise. The price-earnings ratios are given in our model by the ratio of market value $V(x_i, y)$ to the current earnings of the firm $E(x_i, y)$

$$E(x_i, y) = (1 + \iota_x \gamma_x) x_i + \left(1 + \iota_y \gamma_y\right) \rho_i y. \tag{28}$$

To study the evolution of price-earnings ratios, we write the sensitivity of P/E ratios to shock x_i prior to exercise of growth options as

$$\frac{\partial}{\partial x_i} \frac{V(x_i, y)}{E(x_i, y)} = \frac{1}{E^2(x_i, y)} \left(\frac{\mu_x - \widehat{\mu}_y}{(\widehat{r} - \mu_x)\left(\widehat{r} - \widehat{\mu}_y\right)} \rho_i y + V^{GX}(x_i) \left(d_2 - 1 + d_2 \frac{\rho_i y}{x_i} \right) - \rho_i V^{GY}(y) \right). \tag{29}$$

The relation between price-earnings ratios and firm risk is ambiguous. While betas always decrease with idiosyncratic shocks, price-earnings ratios can either increase or decrease. If the effective discount rates are equal ($\mu_x = \widehat{\mu}_y$), the negative relation observed empirically obtains when idiosyncratic options outweigh the systematic ones. Also note that price-earnings ratios drop at option exercise, since the relative increase in earnings is larger than the growth in firm value.

C. Asset Pricing Implications of Competition

In this section, we exploit the idea that firms compete in the common systematic component of profits. We now analyze the effect of competition on cross-sectional anomalies by looking at the derivatives of value function and beta with respect to idiosyncratic profits. The sensitivity of firm market values to idiosyncratic shocks is independent of competition and is given by (26). However, the sensitivity of betas to shocks is attenuated by competition

$$\begin{aligned} \frac{\partial \beta_i}{\partial x_i} \quad = \quad & -\frac{1}{V^2(x_i, y)} \frac{\rho_i y}{(\widehat{r} - \mu_x)\left(\widehat{r} - \widehat{\mu}_y\right)} \left(1 + \gamma_x \left(\frac{x_i}{x^*}\right)^{d_2 - 1}\right) \cdot \\ & \left(1 + \gamma_y \left(\frac{y}{y^*}\right)^{b_2 - 1} - (1 + \gamma_y) \left(\frac{y}{\overline{y}}\right)^{b_2 - 1}\right) < 0. \end{aligned} \tag{30}$$

This implies that the book-to-market and size effects weaken as a result of higher competition (lower $\overline{y}$). This result is consistent with Hou and Robinson (2006), who document larger book-to-market premium in highly concentrated industries.

Finally, competition further strengthens the negative relation between price-earning ratios and firm risk. It can easily be verified that competition increases the sensitivity of price-earnings ratios to idiosyncratic shocks by adding a positive term to Equation (29).

D. Option Exercise and Firm Risk

In this section, we argue that value and size anomalies generated by gradual changes in firm systematic risk are magnified by exercises of firm's real options. The following proposition shows that any option exercise (either $x-$ or $y-$type) leads to a decline in equity beta, provided that the new investment is financed by equity issuance.

Proposition 3. *The market beta of the firm declines at the exercise of the idiosyncratic or systematic growth options if investment is financed by new equity issuance.*

Proof of Proposition 3. See Appendix.

As in Carlson, Fisher, and Giammarino (2006), systematic risk decreases after the exercise of options linked to systematic shocks because such options are more sensitive to the priced factor than are assets in place. Although this effect gives a clear prediction for $y-$options, it does not by itself result in a lower beta for the $x-$option exercise because, as we have shown in the simple example, only the total value of the idiosyncratic component matters for beta. The proposition shows, however, that the firm beta is still lower after the $x-$option exercise because infusion of external financing increases the value of the idiosyncratic component. Finally, we find that competition tends to attenuate the effect of option exercises on betas but does not reverse it.[11]

To link the change in risk at option exercise to asset pricing anomalies, consider a cross-section of firms with identical systematic profit components. Firms with higher idiosyncratic

[11] If investment is financed by new debt issues or firm's cash reserves, then firm betas do not change at the time of growth option exercises.

shocks, which already have lower betas due to larger idiosyncratic component of value, are also more likely to exercise their options and experience a further decline in their betas.

V. Empirical Implications

In this section, we use simulations to evaluate the ability of our framework to reproduce the key features of stock return data and illustrate additional intuition from the model. Since closed form solutions to the model are available, we use them directly to generate a panel of firms over time. We first discuss calibrations of the parameters of the model and then turn to examining the properties of the generated data.

A. Calibration

Table I summarizes the parameters used in the calibration. We simulate monthly data for $N = 100$ economies and $n = 2,000$ firms over 45 years. The first five years are discarded to ensure sufficient variation in firm characteristics in the cross-section, resulting in time-series of $T = 40$ years, roughly in line with data used in recent empirical studies (e.g., Fama and French (1992)). Both profitability shocks have mean growth rates of $\mu_y = \mu_x = 0.03$, corresponding to annual earnings growth. Volatilities are $\sigma_y = 0.15$ and $\sigma_x = 0.25$. The risk-free rate is $r_f = 0.04$, and the price of risk associated with the $y-$factor is $\bar{r}_y = 0.15$. Under our parametrization, this price of risk implies an average equity risk premium of approximately 4%. The price elasticity of demand is set to $1/\varepsilon = 1/0.5 = 2$, similar to an elasticity of 1.6 used in Aguerrevere (2009).

The remaining parameters are chosen as follows: the number of firms decays at an annual rate of $\lambda = 0.02$. Firms' exposure to the systematic shock, ρ_i, is uniformly distributed on the interval $[0.5, 1.5]$. Exercises of systematic and idiosyncratic growth options double the corresponding cash flows, $\gamma_x = \gamma_y = 1$. The initial values as well as exercise and entry costs are available in Table I. They are selected such that firms exercise their options before competition suppresses further growth, and about 70% of all idiosyncratic and systematic options are exercised over the sample period. Across economies, entry cost ensures that the competition boundary is reached in approximately half of simulated economies.

We construct realized returns as follows. First, we simulate the model forward to obtain the full history of firm dividends and values given the initial conditions and realizations of the shocks $x_{i,t}$ and y_t. Second, using these values we compute the realized holding period returns in the risk-neutral measure. Finally, to these returns we add the risk premium estimated as individual firm's beta multiplied by the per-period risk premium. The following formula summarizes

$$r_{i,t} = \frac{V(x_{i,t}, y_t) + D(x_{i,t}, y_t) - V(x_{i,t-1}, y_{t-1}) - I_x \iota_x - \rho_i I_y \iota_y}{V(x_{i,t-1}, y_{t-1})} + \bar{r}_y \beta_{i,t}, \tag{31}$$

where $D(x_{i,t}, y_t)$ denotes the dividend process, and the returns are adjusted for external financing of investments.

B. Results

Figures 1 and 2 help to understand the dynamics of the main variables in the model. Specifically, in Panel A of Figure 1 we display a sample path of the systematic component of cash flows, y. Note that y bounces back whenever it reaches the upper reflective barrier, $\overline{y}$, where more firms enter the market.

In Panel B of Figure 1, we plot the evolution of the mass of firms, Q, corresponding to the path of y. The smooth downward adjustment in the mass of firms is due to the gradual decay in the number of firms, while the upward jumps are caused by entry of new firms. Note that firms tend to enter the market following favorable systematic shocks, and more firms tend to enter when there are fewer competitors in the market.

Panel A of Figure 2 displays three sample paths of x-shocks from the simulated economy in Figure 1. The vertical lines indicate the times of exercise of the idiosyncratic investment options. One of the three firms does not exercise it's idiosyncratic option over the observation period.

We next compute firm values at each point in time and plot them in Panel B. Observe that firm values jump at the point of option exercises. This is caused by inflow of external funds to finance firm expansion. Observe that firms with higher idiosyncratic shocks exercise their options sooner.

The book-to-market ratios in Panel C fluctuate as $x-$ and $y-$shocks evolve. Additionally, book-to-market ratios change discontinuously at exercise of investment options. At exercise, both the market value and the book value of firms increase by the same amount, the cost of investment. Since the book-to-market ratios in our model are typically below one, exercise generally increases the book-to-market ratio.

The price-earnings ratios in Panel D fluctuate with profitability shocks because the option-to-asset ratio changes over time. P/E ratios drop sharply at option exercises because options are replaced with newly installed assets in place. Further, observe that the limit to growth in the $y-$component depresses price-earnings ratios for all firms in the economy. This is particularly evident when y approaches the competition threshold.

Panel E illustrates that factor betas can change over time because of several effects. First, given a systematic shock, firms' betas decrease with idiosyncratic shocks. This effect drives most of the gradual changes in betas in the graph. Second, when systematic investment options (y-options) are exercised and converted into assets in place, there is an instant drop in beta because assets in place have a lower sensitivity to the value of the shock. As we have argued theoretically, betas also decline at the exercise of idiosyncratic options (x-options) because the part of firm value which is unrelated to market risk increases by the amount of new equity investment. Third, firm betas decline to zero in proximity of the competition threshold.

In Panel F, we display the corresponding market betas. Construction of market betas is described in Appendix C, and relies on the assumption that the market is the sum of all firms in the economy. By construction, average market betas are one, and in particular do not decline to zero in times of strong competition as factor betas do. Market betas are more dispersed when options are not exercised and when the systematic shock is small.

We now analyze the asset pricing implications of the simulated data. Every year, using the simulated panel of data, we form 20 portfolios based on ranked book-to-market ratios, market capitalizations, price-earnings ratios, and idiosyncratic volatilities at the beginning of the year. We weight stocks equally within each portfolio, and hold the portfolios for twelve months. Time-series average returns are calculated for each portfolio.

Panel A of Figure 3 provides a scatter plot of the relation between average returns of the 20 book-to-market portfolios and average log book-to-market ratios. The relation between book-to-market ratio and returns is positive and nearly linear. Similarly, Panel B documents that the average realized returns monotonically decrease in firm size.

Returns of price-earnings portfolios shown in Panel C exhibit a decreasing, but non-monotonic pattern. In our calibration, the exogenous decay in the number of firms implies a larger effective growth rate for systematic shocks, and therefore higher valuation ratios associated with $y-$earnings. As a result, in absence of growth options and competition, price-earnings ratios are positively related to returns. Growth options and competition reverse this relation, which is clearly visible in the tails of the graph.

Returns of idiosyncratic volatility sorted portfolios are shown in Panel D. Idiosyncratic volatility is estimated as residual standard deviation from time series regressions of stock returns onto changes in the systematic profitability shock over the 24 months prior to portfolio formation. Consistent with empirical evidence, high idiosyncratic risk is associated with low returns.

Figure 3 suggests that the model has a potential to explain four common asset pricing anomalies in a univariate setting. We now turn to cross-sectional Fama-MacBeth regressions to evaluate our model's multivariate performance.

Table II reports the average Fama-MacBeth coefficients across 100 simulated economies and the corresponding average $t-$statistics for each coefficient. In each month t, realized stock returns are regressed on theoretical betas (β_t), estimated betas ($\widehat{\beta}_t$), the log book-to-market ratio (B/M), log firm value ($Size$), the prior 12-month returns (MOM), log price-earnings ratio (P/E), and the log of idiosyncratic volatility ($IVol$). Betas and idiosyncratic volatility are estimated, respectively, as slope coefficient and residual standard deviation from time series regressions of stock returns onto changes in the systematic profitability shock from month $t-24$ to $t-1$.

As a reference, the first row in the table shows that theoretical conditional betas are, not surprisingly, highly significant and the factor risk premium is 1.16% per month. Specification II replaces true betas by their estimated counterparts. While empirical betas are also strongly related to returns, the estimated risk premium drops due to the measurement error in the explanatory variable. Specifications III and IV demonstrate that there is a significant positive relation between the realized returns and book-to-market ratios in the simulated data, and a negative relation between stock returns and firm size. Regression V shows that our choice of parameters does not result in stock return momentum. Price-earnings ratios and idiosyncratic volatility in specifications VI and VII are negatively related to returns. Specifications VIII shows that, when beta is estimated, both size and book-to-market ratio are significant return predictors. Since our model is a conditional one-factor model, true theoretical betas in regression IX drive out all other variables.

In Table III, we document the magnitudes of the cross-sectional effects generated by the model across 10 decile portfolios. For example, the difference in average stock returns between the top and bottom decile portfolios by book-to-market ratio amounts to 47 basis points per month. This premium is fully explained by the difference in betas of 0.37. Similarly, returns of size sorted portfolios differ by 45, price-earnings portfolios by 23, and idiosyncratic volatility portfolios by 46 basis points.

We next analyze how various features of the model contribute to the magnitude of return differences across portfolios. Specifically, we sort the model-generated data into deciles for the following model specifications: the full model (Panel A), the model without growth options ($\gamma_x = \gamma_y = 0$, Panel B), and the model without competition ($R \to \infty$, Panel C). Table IV shows the results. As we have argued, removing growth options tends to decrease the magnitude of the book-to-market, size, and idiosyncratic volatility premium. In contrast, shutting down competition increases the premia. While in the full model, stocks with low price-earnings ratios outperform high price-earnings stocks by approximately 17 basis points, this relationship reverses without competition. The limit to growth in the $y-$component depresses price-earnings ratios at times when the systematic shock is high. This coincides with times of high systematic risk and a high risk premium, as the $y-$shock constitutes a large component of firm value. Competition therefore induces a negative relation between price-earnings ratios and returns.

Table V shows that options are crucial to separate the predictive ability of book-to-market and size in multivariate regression. In particular, in the model without options, the book-to-market ratio significantly predicts returns, but size does not. Both effects are reduced by competition.

In Figure 4, we plot key firm characteristics around the time of growth option exercises, averaged across firms and economies. Panels A and B show the dynamics of firm characteristics during the 48-month period centered around the exercise of $y-$ and $x-$options, respectively. The two plots for market values show that options are typically exercised following high stock returns. Values jump at exercise due of the injection of additional cash in the firm. Importantly, the market capitalization increases with the issuance of new shares. There are no arbitrage opportunities as the price per share is unchanged. In contrast, book-to-market ratios tend to decrease prior to option exercise. At the time of investment, both the book and market values increase by the same amounts. Since the average book-to-market ratio, under the chosen parameters, is below one, the ratio typically increases at exercise.

Finally, we plot conditional factor betas around the exercise of systematic and idiosyncratic options. As we have argued theoretically, betas decrease at the time of $x-$ and $y-$option exercises under external equity financing. Exercising $y-$option lowers the sensitivity to the systematic profitability shock component since in our model assets in place are less risky than growth options. In contrast, financing exercise $x-$ options increases the value of assets derived from the idiosyncratic component, thereby lowering firm risk. The distinctive pattern in the dynamics of pre-exercise betas is informative. Consistent with prior literature, the average beta tends to increase prior to the $y-$option exercise since the latter is caused by increasing value of the systematic component. Conversely, the average beta decreases right before the $x-$option is exercised since this option depends on the idiosyncratic profit component.

To better understand the conditional nature of betas and the value premium, we plot in Figure 5 average firm betas, the market risk premium, a measure of the cross-sectional variation in factor and market betas, and the realized value premium against the level of the systematic shock y. The plots on the left (Panel A) show results in an economy without competition, while Panel B is the full model with competition. The first row plots the relation between the weighted average factor beta, β^y_M, and the factor y. As expected, β^y_M is monotonically increasing in y in absence of competition. A higher value of y implies that a larger part of total market value is systematic, leading to higher systematic risk. Introducing competition breaks this link. While the average firm beta initially increases in y, it is non-monotonic. As predicted by Proposition 2, firm betas decrease as y approaches the competition boundary. The second row shows the corresponding market risk premium, in percent annually. With constant factor risk premium, the market risk premium is just a rescaled version of average firm betas.

The asset pricing anomalies that arise in our model are a direct result of cross-sectional variation in firm betas. This variation is measured by the difference between the average betas of the first and tenth decile of beta-sorted portfolios. Variation in factor and market betas is shown in row three and four, respectively. For the factor betas, the spread $\beta^y_{10} - \beta^y_1$ is highest for intermediate values of y. If the systematic shock is very small relative to typical idiosyncratic shocks, most betas are close to zero, which in turn leads to a low cross-sectional variation. As y gets very large, in the case without competition, betas tend to one, again reducing variation. The results for the case with competition are similar, yet more pronounced and the economic reasoning is different. As y approaches the competition boundary, new entry makes firms less sensitive to systematic shocks, thereby decreasing all factor betas to zero. The effect of option exercise is visible in the slopes in both plots. In general, the effect of option exercise on risk is larger for high beta than low beta firm, leading to a decrease in the cross-sectional dispersion of risk. The slope in the plots is steeper in the regions of y where option exercise is common, and it flattens once all firms expanded.

The spreads in market betas in the fourth row show a different pattern. Since for a very small systematic shock all individual factor betas are close to zero, rescaling brings up a large cross-sectional dispersion in market betas. The spread then decreases as y increases.

Finally, the last two plots show that the asset pricing anomalies in our model are directly related to the cross-sectional dispersion in firm betas. The realized value premium is initially increasing in y and later decreasing, closely mimicking the behavior of the beta spread $\beta^y_{10} - \beta^y_1$.

VI. Conclusion

Cross-sectional anomalies in stock returns have long presented a challenge to standard asset pricing models. In this paper, we show that, under very general assumptions, firms' conditional market betas directly depend on the history of idiosyncratic shocks and vary over time. We show that firm value, even using accounting measures, negatively relates to risk. Our insight is that non-systematic shocks to cash flows simultaneously lead to an increase in firm value and a decrease in firms' systematic risk. Our results imply that any economic variable correlated with the history of idiosyncratic cash flow shocks can be successful in explaining expected stock returns.

We show that investment magnifies the anomalies in the model, allows us to separate value and size effects, and generates predictability by price-earnings ratios. The presence of growth options can increase or decrease firms' factor risk. Product market competition affects the magnitude of cross-sectional anomalies, and with the exception of predictability from price-earnings ratios, tends to attenuate them. Our analysis of the data generated by the model confirms that it can produce reasonable magnitudes of value, size, price-earnings, and idiosyncratic volatility anomalies both in portfolio sorts and Fama-MacBeth cross-sectional regressions.

References

Aguerrevere, Felipe L., 2009, Real options, product market competition, and asset returns, *Journal of Finance* 64, 957–983.

Ai, Hengjie, and Dana Kiku, 2012, Growth to value: Option exercise and the cross section of equity returns, Working Paper, University of Pennsylvania, Philadelphia, PA.

Ang, Andrew, Robert J. Hodrick, Yuhang H. Xing, and Xiaoyan Y. Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Banz, Rolf W., 1981, The relationship between return and market value of common-stocks, *Journal of Financial Economics* 9, 3–18.

Bena, Jan, and Lorenzo Garlappi, 2012, Corporate innovation and returns, Working Paper, University of British Columbia, Vancouver, Canada.

Berk, Jonathan B., 1995, A critique of size-related anomalies, *Review of Financial Studies* 8, 275–286.

——— , Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Bossaerts, Peter, and Richard C. Green, 1989, A general equilibrium model of changing risk premia: theory and tests, *The Review of Financial Studies* 2, 467 – 493.

Brennan, Michael J., 1973, An approach to valuation of uncertain income streams, *Journal of Finance* 28, 661–674.

Bustamante, M. Cecilia, and Andrés Donangelo, 2012, Product market competition and industry returns, Working Paper, University of Texas, Austin, Texas.

Caballero, Ricardo J., and Robert S. Pindyck, 1996, Uncertainty, investment, and industry evolution, *International Economic Review* 37, 641–662.

Carlson, Murray, Adlai J. Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.

——— , 2006, Corporate investment and asset price dynamics: implications for SEO event studies and long-run performance, *Journal of Finance* 61, 1009–1034.

Cooper, Ilan, 2006, Asset pricing implications of non-convex adjustment costs of investment, *Journal of Finance* 61, 139–170.

Da, Zhi, Re J. Guo, and Ravi Jagannathan, 2012, CAPM for estimating the cost of equity capital: Interpreting the empirical evidence, *Journal of Financial Economics* 103, 204–220.

Dixit, Avinash K., 1993, The art of smooth pasting, in Jacques Lesourne, and Hugo Sonnenschein, ed.: *Fundamentals of Pure and Applied Economics* (Harwood Academic Publishers: Reading, UK).

——— , and Robert S. Pindyck, 1994, *Investment under Uncertainty* (Princeton University Press: Princeton, New Jersey).

Dumas, Bernard, 1991, Super contact and related optimality conditions, *Journal of Economic Dynamics & Control* 15, 675–685.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium - empirical tests, *Journal of Political Economy* 81, 607–636.

Gomes, Joao F., Leonid Kogan, and Lu Zhang, 2003, Equilibrium cross section of returns, *Journal of Political Economy* 111, 693–732.

Graham, Benjamin, and David L. Dodd, 1934, *Security Analysis* (McGraw-Hill: New York).

Grenadier, Steven R., 2002, Option exercise games: An application to the equilibrium investment strategies of firms, *Review of Financial Studies* 15, 691–721.

Grullon, Gustavo, Evgeny Lyandres, and Alexei Zhdanov, 2012, Real options, volatility, and stock returns, *Journal of Finance*, Forthcoming.

Hansen, Lars Peter, and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587–613.

Hou, Kewei, and David T. Robinson, 2006, Industry concentration and average stock returns, *Journal of Finance* 61, 1927–1956.

Kogan, Leonid, and Dimitris Papanikolaou, 2012, A theory of firm characteristics and stock returns: The role of investment-specific shocks, Working Paper, Massachusetts Institute of Technology, Cambridge, MA.

Leahy, John V., 1993, Investment in competitive-equilibrium - the optimality of myopic behavior, *Quarterly Journal of Economics* 108, 1105–1133.

Lewellen, Jonathan, and Stefan Nagel, 2006, The conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics* 82, 289–314.

Loughran, Tim, and Jay R. Ritter, 1995, The new issue puzzle, *Journal of Finance* 50, 23–51.

Novy-Marx, Robert, 2007, An equilibrium model of investment under uncertainty, *Review of Financial Studies* 20, 1461–1502.

——— , 2011, Operating leverage, *Review of Finance* 15, 103–134.

Petkova, Ralitsa, and Lu Zhang, 2005, Is value riskier than growth?, *Journal of Financial Economics* 78, 187–202.

Roll, Richard, 1977, A critique of the asset pricing theory's tests part i: On past and potential testability of the theory, *Journal of Financial Economics* 4, 129–176.

Sagi, Jacob S., and Mark S. Seasholes, 2007, Firm-specific attributes and the cross-section of momentum, *Journal of Financial Economics* 84, 389–434.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.

Appendix

A. Notational Key

Q	Equilibrium mass of firms in the economy
λ	Decay intensity in mass of firms Q
$1/\varepsilon$	Price elasticity of demand
X_i, Y	Idiosyncratic and systematic profitability shock
ρ_x, ρ_y	Profit function constants
x_i, y	Idiosyncratic and systematic cash flow components
x^*, y^*	Exercise thresholds for x-option and y-option
$I_x, \rho_i I_y$	Cost of exercising the respective options
ι_x, ι_y	Indicators for exercising the options
γ_x, γ_y	Investment scale
R	Cost of entry
$\overline{y}$	Limit to growth in y from firm competition
$V(x_i, y)$	Value of firm i
$V^{AX}(x_i)$	Idiosyncratic component of firm value V due to assets in place
$V^{GX}(x_i), V^{GY}(y)$	Growth option components of firm value V
$V^C(y)$	Adjustment to firm value V due to competition

B. Proofs and Derivations

Proof of Proposition 1. Consider a trial solution

$$V(x_i, y) = v_1(x_i) + v_2(y). \tag{32}$$

Since equation (14) is additively separable in variables x_i and y, the general solution to (14) is equal to the sum of the ODE solution for $v_1(x_i)$ and the ODE solution for $v_2(y)$. Consider the "continuation" problem of the firm that exercised all its options. Its value is given by

$$\widehat{V}(x_i, y) = \frac{(1+\gamma_x)\, x_i}{\widehat{r} - \mu_x} + \frac{\left(1+\gamma_y\right) \rho_i y}{\widehat{r} - \widehat{\mu}_y} + A y^{b_2}, \tag{33}$$

where the last term is negative and appears because of the limiting effect of competition, and b_2 is the positive root of the quadratic equation

$$b^2 \sigma_y^2 + b\left(2\widehat{\mu}_y - \sigma_y^2\right) - 2\widehat{r} = 0. \tag{34}$$

Prior to the exercise of the option, the general solution for the firm value is given by

$$V(x_i, y) = \frac{x_i}{\widehat{r} - \mu_x} + \frac{\rho_i y}{\widehat{r} - \widehat{\mu}_y} + B y^{b_2} + C x_i^{d_2}, \tag{35}$$

where B and C are constants, b_2 is the positive root of (34), and d_2 is the positive root of a similar equation for x

$$d^2 \sigma_x^2 + d\left(2\mu_x - \sigma_x^2\right) - 2\widehat{r} = 0. \tag{36}$$

At the time of the exercise, the value of the firm is equal to the value after the exercise minus the investment cost (the value-matching conditions)

$$V(x^*, y) = \widehat{V}(x^*, y) - I_x, \tag{37}$$

$$V(x_i, y^*) = \widehat{V}(x_i, y^*) - \rho_i I_y, \tag{38}$$

where $\widehat{V}$ is given by (33). Note that since firm value separates in x and y component, the exercise of one option does not affect the exercise policy for another. For the exercise to be optimal, an additional condition known as smooth-pasting or high-contact condition (Dumas (1991), Dixit (1993)) has to be satisfied,

$$V_x(x^*, y) = \widehat{V}_x(x^*, y), \tag{39}$$

$$V_y(x_i, y^*) = \widehat{V}_y(x_i, y^*). \tag{40}$$

Using (15) and (37)-(40), we find constants A, B, and C and well as pre and post exercise firm values,

$$\widehat{V}(x_i, y) = \frac{(1+\gamma_x) x_i}{\widehat{r} - \mu_x} + \frac{(1+\gamma_y) \rho_i y}{\widehat{r} - \widehat{\mu}_y} - \frac{(1+\gamma_y) \rho_i \overline{y}}{(\widehat{r} - \widehat{\mu}_y) b_2} \left(\frac{y}{\overline{y}}\right)^{b_2}, \tag{41}$$

and

$$\begin{aligned} V(x_i, y) &= \frac{(1+\iota_x \gamma_x) x_i}{\widehat{r} - \mu_x} + \frac{(1-\iota_x)\gamma_x x^*}{(\widehat{r} - \mu_x) d_2} \left(\frac{x_i}{x^*}\right)^{d_2} \\ &\quad + \frac{(1+\gamma_y \iota_y)\rho_i y}{\widehat{r} - \widehat{\mu}_y} + \frac{(1-\iota_y)\gamma_y \rho_i y^*}{(\widehat{r} - \widehat{\mu}_y) b_2} \left(\frac{y}{y^*}\right)^{b_2} - \frac{(1+\gamma_y) \rho_i \overline{y}}{(\widehat{r} - \widehat{\mu}_y) b_2} \left(\frac{y}{\overline{y}}\right)^{b_2}. \end{aligned} \tag{42}$$

The thresholds for exercise x^* and y^* are then defined as

$$x^* = \frac{d_2}{d_2 - 1} \frac{(\widehat{r} - \mu_x) I_x}{\gamma_x}, \tag{43}$$

$$y^* = \frac{b_2}{b_2 - 1} \frac{(\widehat{r} - \widehat{\mu}_y) I_y}{\gamma_y}. \tag{44}$$

Note that y^* is identical for all firms since both investment benefits and costs are proportional to ρ_i. Since entry into the market is competitive, we follow Leahy (1993) and Caballero and Pindyck (1996) to require that expected profit at entry be zero,

$$V(x_0, \overline{y}) = R, \tag{45}$$

where x_0 is the expected idiosyncratic shock x_i and R is the cost of entry. We assume that the $y-$options are exercised prior to reaching the reflecting barrier. Since new firms enter industry at $\overline{y} > y^*$, they have no $y-$options but have investment options linked to the idiosyncratic profit component. Thus we can solve for the limit to growth parameter $\overline{y}$ from (45), which yields

$$\overline{y} = \left(R - \frac{x_0}{\widehat{r} - \mu_x} - \frac{\gamma_x x^*}{(\widehat{r} - \mu_x) d_2} \left(\frac{x_0}{x^*}\right)^{d_2}\right) \frac{b_2}{b_2 - 1} \frac{\widehat{r} - \widehat{\mu}_y}{(1+\gamma_y) \rho_0}, \tag{46}$$

where ρ_0 is the expected sensitivity to the systematic shock. It is clear from equation above that the entry threshold $\overline{y}$ is time-independent and increases in the cost of entry R. □

Proof of Proposition 2. The proof is by direct differentiation of (16). For easy exposition, we give a separate derivation for market beta before and after exercise. The pre-exercise firm i's beta is then given by differentiating Equation (42),

$$\beta_i \equiv \frac{dV(x_i,y)}{dy}\frac{y}{V(x_i,y)} = 1 - \frac{1}{V(x_i,y)}\cdot \tag{47}$$

$$\left[\frac{x_i}{\widehat{r}-\mu_x} + \frac{\gamma_x x^*}{(\widehat{r}-\mu_x)\,d_2}\left(\frac{x_i}{x^*}\right)^{d_2} - \frac{(b_2-1)\,\rho_i}{(\widehat{r}-\widehat{\mu}_y)\,b_2}\left(\gamma_y y^*\left(\frac{y}{y^*}\right)^{b_2} + (1+\gamma_y)\,\overline{y}\left(\frac{y}{\overline{y}}\right)^{b_2}\right)\right].$$

The firm's beta after it has invested, can be obtained in a similar manner from (41)

$$\widehat{\beta}_i = 1 - \frac{1}{V(x_i,y)}\left(\frac{(1+\gamma_x)\,x_i}{\widehat{r}-\mu_x} + (b_2-1)\,\rho_i\frac{(1+\gamma_y)\,\overline{y}}{(\widehat{r}-\widehat{\mu}_y)\,b_2}\left(\frac{y}{\overline{y}}\right)^{b_2}\right). \tag{48}$$

The formula in the proposition summarizes expressions (47) and (48) using indicator functions.

□

Proof of Proposition 3. We start by considering the effect of exercise of the option linked to idiosyncratic shocks. From (22), beta just *prior* to the exercise of x-option, $\beta_i(x^*-)$, is

$$\beta_i(x^*_-) = 1 - \frac{\frac{x^*}{\widehat{r}-\mu_x}\left(1+\frac{\gamma_x}{d_2}\right)}{V(x^*,y)} + (1-\iota_y)(b_2-1)\frac{\rho_i V^{GY}(y)}{V(x^*,y)} - (b_2-1)\frac{\rho_i V^C(y)}{V(x^*,y)}. \tag{49}$$

Similarly, beta just *after* the exercise of x-option, $\beta_i(x^*+)$, can be written as

$$\beta_i(x^*_+) = 1 - \frac{\frac{x^*}{\widehat{r}-\mu_x}(1+\gamma_x)}{V(x^*,y)+I_x} + (1-\iota_y)(b_2-1)\frac{\rho_i V^{GY}(y)}{V(x^*,y)+I_x} - (b_2-1)\frac{\rho_i V^C(y)}{V(x^*,y)+I_x}. \tag{50}$$

Taking the difference between post-exercise and pre-exercise betas, and substituting the investment threshold, x^*, from (43) and value function $V(x^*,y)$ from (16) we have

$$\beta_i(x^*_+) - \beta_i(x^*_-) = \frac{-I_x\rho_i\left(\frac{(1+\gamma_y\iota_y)y}{\widehat{r}-\widehat{\mu}_y} + \frac{(1-\iota_y)\gamma_y y^*}{\widehat{r}-\widehat{\mu}_y}\left(\frac{y}{y^*}\right)^{b_2} - \frac{(1+\gamma_y)\overline{y}}{\widehat{r}-\widehat{\mu}_y}\left(\frac{y}{\overline{y}}\right)^{b_2}\right)}{V(x^*,y)\left(V(x^*,y)+I_x\right)} < 0 \tag{51}$$

Since the expression in parentheses in the numerator is always positive when $y^* < \overline{y}$, we have the result that beta always decreases after exercise of x-option.

Similarly, we can write beta just *prior* to the exercise of y-option as

$$\beta_i(y^*_-) = 1 - \frac{V^{AX}(x_i)}{V(x_i,y^*)} - (1-\iota_x)\frac{V^{GX}(x_i)}{V(x_i,y^*)} + (b_2-1)\frac{\rho_i V^{GY}(y^*)}{V(x_i,y^*)} - (b_2-1)\frac{\rho_i V^C(y^*)}{V(x_i,y^*)}, \tag{52}$$

and beta just *after* the exercise of y-option

$$\beta_i(y^*_+) = 1 - \frac{V^{AX}(x_i)}{V(x_i,y^*)+\rho_i I_y} - (1-\iota_x)\frac{V^{GX}(x_i)}{V(x_i,y^*)+\rho_i I_y} - (b_2-1)\frac{\rho_i V^C(y^*)}{V(x_i,y^*)+\rho_i I_y}. \tag{53}$$

The difference in betas is given by

$$\beta_i(y^*_+) - \beta_i(y^*_-) = \frac{-\rho_i I_y \left(1+\gamma_y\right)}{V\left(x_i, y^*\right)\left(V\left(x_i, y^*\right) + \rho_i I_y\right)} \frac{\rho_i y^*}{\widehat{r} - \widehat{\mu}_y} \left(1 - \left(\frac{y^*}{\overline{y}}\right)^{b_2 - 1}\right) < 0 \qquad (54)$$

Again since $b_2 > 1$ and $y^* < \overline{y}$, we see that beta declines at the exercise of y-option. □

C. Additional Results: Aggregation

In the main text, we based our analysis on betas with respect to the common risk factor Y,

$$\beta_i^Y = \frac{dV_i}{dY}\frac{Y}{V_i}. \tag{55}$$

We now connect our analysis to the commonly used "market betas" in the context of the CAPM.

C.1. Relation between Factor and Market Betas

Assume a stochastic discount factor (SDF) m that prices all assets, i.e. $E(R_i m) = 1$. Alternatively, this can be written as

$$E(R_i) - R_f = -\frac{Cov(R_i, m)}{E(m)}. \tag{56}$$

If Y is the only priced source of risk, any stochastic discount factor can be decomposed into its projection onto Y and an orthogonal part, ε,

$$m = a + bY + \varepsilon. \tag{57}$$

We can then rewrite (56) as

$$E(R_i) - R_f = -\beta_i^Y \frac{Cov\,(Y, m)}{E(m)}, \tag{58}$$

where β_i^Y is defined by (55). The above expression suggests that the systematic shock is measured by β_i^Y and carries a positive premium as long as Y is negatively correlated with m.

In the context of CAPM, the SDF is

$$m = \frac{1}{R_f} - \frac{E(R_m) - R_f}{R_f \cdot Var(R_m)} R_m. \tag{59}$$

Combining (58) and (59), we obtain

$$E(R_i) - R_f = \beta_i^Y \frac{Cov(R_m, Y)}{Var(R_m)} \left(E(R_m) - R_f\right). \tag{60}$$

Comparing this relation to the well known CAPM pricing formula, we obtain a simple relation between the factor and market betas

$$\beta_i^M = \beta_i^Y \beta_Y^M, \tag{61}$$

where

$$\beta_Y^M = \frac{Cov(Y, R_m)}{Var(R_m)}. \tag{62}$$

C.2. Market Betas

We now aggregate individual firms to obtain the value of the stock market and compute market betas. Using Proposition 1, we write the value of the firm as

$$V_i = A(x_i) + \rho_i B(y), \tag{63}$$

where for simplicity all x-terms are summarized by $A(x_i)$ and

$$B(y) = \frac{(1+\iota_y\gamma_y)y}{\widehat{r}-\widehat{\mu}_y} + \frac{(1-\iota_y)\gamma_y y^*}{b_2\left(\widehat{r}-\widehat{\mu}_y\right)}\left(\frac{y}{y^*}\right)^{b_2} - \frac{\left(1+\gamma_y\right)\overline{y}}{b_2\left(\widehat{r}-\widehat{\mu}_y\right)}\left(\frac{y}{\overline{y}}\right)^{b_2}. \tag{64}$$

Since the market is the sum of values of N stocks,

$$M = \Sigma_i V_i = \Sigma_i A(x_i) + \Sigma_i \rho_i B(y), \tag{65}$$

the beta of y with respect to the market is $\beta_y^M = \frac{M}{y}\frac{dy}{dM}$, and can be found using implicit differentiation for $\frac{dy}{dM}$:

$$\beta_y^M = \frac{M}{y}\left[\left((1+\iota_y\gamma_y) + (1-\iota_y)\gamma_y\left(\frac{y}{y^*}\right)^{b_2-1} - \left(1+\gamma_y\right)\left(\frac{y}{\overline{y}}\right)^{b_2-1}\right)\frac{\Sigma_i\rho_i}{\left(\widehat{r}-\widehat{\mu}_y\right)}\right]^{-1}. \tag{66}$$

From Proposition 2, beta simplifies to

$$\beta_i^y = \frac{1}{V_i}\left(\frac{(1+\iota_y\gamma_y)\rho_i y}{\widehat{r}-\widehat{\mu}_y} - \frac{\left(1+\gamma_y\right)\rho_i\overline{y}}{\widehat{r}-\widehat{\mu}_y}\left(\frac{y}{\overline{y}}\right)^{b_2} + \frac{(1-\iota_y)\gamma_y\rho_i y^*}{\widehat{r}-\widehat{\mu}_y}\left(\frac{y}{y^*}\right)^{b_2}\right), \tag{67}$$

therefore, the equilibrium relation to the CAPM beta can be written as

$$\beta_i^M = \beta_i^y\beta_y^M = \frac{\Sigma_i V_i}{V_i}\frac{\rho_i}{\Sigma_i\rho_i} = \frac{\frac{\Sigma_i A(x_i)}{\Sigma_i\rho_i} + B(y)}{\frac{A(x_i)}{\rho_i} + B(y)}, \tag{68}$$

In particular, when all firms have the same sensitivity to the systematic shock, $\rho_i = \rho$, we have

$$\beta_i^M = \frac{\overline{V}}{V_i}, \tag{69}$$

where $\overline{V}$ is the average value of firms in the economy. Then, firms smaller than average will have betas above one, and firms larger than average will have betas below one. By construction, the weighted sum of market betas is equal to one, $\Sigma_i V_i \beta_i^M / \Sigma_i V_i = 1$.

C.3. Equilibrium Market Risk Premium and Value Premium

Here we show how the ideas developed in this paper can contribute to describing the time-series performance of value strategies. Time-series alphas of any asset relative to the CAPM can arise unconditionally even if the CAPM prices the asset correctly conditionally. For the value-minus-growth portfolio, we identify a positive covariance between portfolio risk, $\beta_V^M - \beta_G^M$, and market risk premium as one channel to generate unconditional alphas. We first show that $\beta_V^M - \beta_G^M$ is decreasing in the systematic shock. Second, the exposure of the market to the systematic profit shock y goes to zero as the systematic profit shock approaches the competition threshold. The positive covariance between market risk premium and value beta follows.

How is value different from growth in our model? Value stocks have low idiosyncratic cash flows x_i and hence low $A(x_i)$. We denote A_V for value and A_G for growth.

$$\begin{aligned} \beta_V^M - \beta_G^M &= \frac{\overline{A} + f(y)}{\frac{A_V}{\rho_V} + B(y)} - \frac{\overline{A} + f(y)}{\frac{A_G}{\rho_G} + B(y)} = \\ &= \frac{\overline{A} + B(y)}{\left(\frac{A_V}{\rho_V} + B(y)\right)\left(\frac{A_G}{\rho_G} + B(y)\right)} S, \end{aligned} \tag{70}$$

where constant $S = \frac{A_G}{\rho_G} - \frac{A_V}{\rho_V} > 0$ is independent of the aggregate dividend y. Since $B(y)$ is increasing in y, it is clear that as the systematic profitability increases the spread in betas decreases. Thus, the premium of value stocks relative to growth stocks is smaller in good times.

We now turn to the market risk premium. The exposure of the market to the systematic profit is given by

$$\beta_M^y = \frac{\frac{(1+\iota_y\gamma_y)y}{\widehat{r}-\widehat{\mu}_y} - \frac{(1+\gamma_y)\overline{y}}{\widehat{r}-\widehat{\mu}_y}\left(\frac{y}{\overline{y}}\right)^{b_2} + \frac{(1-\iota_y)\gamma_y y^*}{\widehat{r}-\widehat{\mu}_y}\left(\frac{y}{y^*}\right)^{b_2}}{\frac{\Sigma_i A(x_i)}{\Sigma_i \rho_i} + B(y)}, \tag{71}$$

In absence of competition, the market's exposure to $y-$risk, β_M^y, is increasing in y as the systematic shock contributes a greater part to market value. Competition, however, reverses this intuition, since systematic risk is close to zero near the competition threshold:

$$\beta_M^y = \frac{\frac{1+\gamma_y}{r-\mu_y}\left(y - \overline{y}\left(\frac{y}{\overline{y}}\right)^{b_2}\right)}{\frac{\Sigma_i A(x_i)}{\Sigma_i \rho_i} + \frac{1+\gamma_y}{r-\mu_y}\left(y - \frac{\overline{y}}{b_2}\left(\frac{y}{\overline{y}}\right)^{b_2}\right)} \tag{72}$$

It can easily be verified that the risk of the market goes to zero as the systematic profit shock approaches the competition boundary. Assuming a constant price of Y risk, competition thus generates endogenous time variation in the market risk premium. Crucially, near the competition threshold, both the market risk premium and the value beta decrease in profitability, yielding

$$\text{Cov}\left(R_M, \beta_V^M - \beta_G^M\right) > 0. \tag{73}$$

In the context of CAPM testing, this becomes important because covariances between conditional risk and the equity risk premium appear as pricing errors in unconditional tests.

Table I
Parameter Values Used in Simulations

This table lists the parameters used for the simulations.

Parameter	*Notation*	*Determination*	*Value*
Initial mass of firms	Q_0	normalized	1.5
Decay intensity for Q	λ	normalized	0.02
Price elasticity of demand	$1/\varepsilon$	adopted	1/0.5
Distribution of sensitivity	ρ_i		$U[0.5, 1.5]$
Initial profitability shocks	(X_0, Y_0)	normalized	$(1, 1)$
Cost of entry	R	normalized	200
Cost of exercising options	(I_x, I_y)	normalized	$(20, U[10, 30])$
Volatility of profitability shocks	(σ_x, σ_y)	empirical	$(0.25, 0.15)$
Drift of profitability shocks	(μ_x, μ_y)	empirical	$(0.03, 0.03)$
Investment scale	(γ_x, γ_y)	normalized	$(1, 1)$
Simulation horizon (in years)	T	adopted	40
Number of simulated firms	n	normalized	$2,000$
Number of economies simulated	N	normalized	100
Price of y risk	r_y	empirical	0.15
Market risk premium	$\beta_M^y \times \bar{r}_y$	calculated	4%
Risk-free rate	r_f	empirical	0.04

Table II
Fama-MacBeth Regressions on the Simulated Data

This table reports average Fama−MacBeth coefficients and average $t-$statistics from 100 simulations of a cross-section of 2,000 stocks over 45 years, the first 5 of which are discarded. In each month t, the realized stock returns are regressed on theoretical betas (β_t), estimated betas ($\widehat{\beta}_t$), the log book-to-market ratio (B/M), log firm value ($Size$), the prior 12-month returns (MOM), the log price-earnings ratio (P/E), and the log of idiosyncratic volatility ($IVol$). Betas and idiosyncratic volatility are estimated, respectively, as slope coefficient and residual standard deviation from time series regressions of stock returns onto changes in the systematic profitability shock from month $t-24$ to $t-1$.

	β_t	$\widehat{\beta}_t$	B/M	$Size$	MOM	P/E	$IVol$
I	1.16						
	(4.96)						
II		0.35					
		(4.60)					
III			0.23				
			(5.76)				
IV				-0.18			
				(-5.91)			
V					-0.04		
					(-0.52)		
VI						-0.36	
						(-3.23)	
VII							-0.24
							(-5.79)
VIII		0.13	0.11	-0.08			
		(3.81)	(3.71)	(-4.07)			
IX	1.16		-0.00	-0.00			
	(4.50)		(-0.03)	(-0.07)			

Table III
Fama-French Portfolio Sorts on Simulated Data

This table reports average returns, estimated unconditional portfolio betas, and characteristics of 10 portfolios formed by book-to-market ratio (B/M, Panel A), size ($Size$, B), price-earnings ratio (P/E, C), and idiosyncratic volatility ($IVol$, D). IVol is estimated as residual standard deviation from time series regressions of stock returns onto changes in the systematic profitability shock from month $t-24$ to $t-1$. The data consist of 100 simulations of a cross-section of 2,000 stocks over 45 years, the first 5 of which are discarded.

	Low	2	3	4	5	6	7	8	9	High	H−L
Panel A: Book-to-Market Portfolios											
Ret	0.61	0.70	0.78	0.84	0.90	0.95	1.01	1.06	1.11	1.17	0.56
$\widehat{\beta}$	0.10	0.17	0.23	0.29	0.33	0.38	0.42	0.45	0.49	0.54	0.45
B/M	-2.16	-1.47	-1.20	-1.02	-0.88	-0.75	-0.63	-0.51	-0.38	-0.17	1.99
Panel B: Size Portfolios											
Ret	1.16	1.09	1.06	1.02	0.97	0.91	0.84	0.77	0.70	0.62	-0.54
$\widehat{\beta}$	0.53	0.48	0.45	0.42	0.39	0.34	0.29	0.23	0.17	0.10	-0.43
$Size$	3.95	4.27	4.47	4.64	4.80	4.97	5.17	5.41	5.74	6.46	2.51
Panel C: Price-Earnings Portfolios											
Ret	1.00	0.92	0.88	0.88	0.89	0.92	0.94	0.94	0.92	0.83	-0.17
$\widehat{\beta}$	0.43	0.35	0.32	0.31	0.32	0.33	0.35	0.37	0.35	0.27	-0.16
P/E	3.25	3.28	3.31	3.34	3.37	3.42	3.46	3.51	3.56	3.63	0.38
Panel D: Idiosyncratic Volatility Portfolios											
Ret	1.21	1.13	1.06	0.99	0.93	0.87	0.81	0.76	0.71	0.65	-0.56
$\widehat{\beta}$	0.57	0.51	0.46	0.41	0.36	0.31	0.26	0.22	0.17	0.12	-0.45
$IVol$	-5.02	-4.36	-4.00	-3.73	-3.52	-3.34	-3.19	-3.06	-2.93	-2.76	2.26

Table IV

Returns of Fama-French Portfolio Sorts on Simulated Data

This table reports average monthly returns, betas, and characteristics of 10 portfolios formed by book-to-market (B/M), size ($Size$), price-earnings ratio (P/E), and idiosyncratic volatility ($IVol$) from our simulated data. IVol is estimated as residual standard deviation from time series regressions of stock returns onto changes in the systematic profitability shock from month $t-24$ to $t-1$. The data consists of 100 simulations of a cross-section of 2,000 stocks over 45 years, the first 5 of which are discarded. Panel A contains results for our benchmark model. In Panels B and C, we present results for the model without growth options and competition, respectively.

	Low	2	3	4	5	6	7	8	9	High	H–L
Panel A: Full model											
B/M	0.61	0.70	0.78	0.84	0.90	0.95	1.01	1.06	1.11	1.17	0.56
$Size$	1.16	1.09	1.06	1.02	0.97	0.91	0.84	0.77	0.70	0.62	-0.54
P/E	1.00	0.92	0.88	0.88	0.89	0.92	0.94	0.94	0.92	0.83	-0.17
$IVol$	1.21	1.13	1.06	0.99	0.93	0.87	0.81	0.76	0.71	0.65	-0.56
Panel B: Model without options											
B/M	0.61	0.70	0.76	0.82	0.87	0.92	0.96	1.00	1.05	1.10	0.48
$Size$	1.07	1.02	0.99	0.97	0.93	0.89	0.83	0.77	0.71	0.62	-0.45
P/E	1.11	1.05	1.01	0.96	0.91	0.86	0.81	0.76	0.70	0.62	-0.49
$IVol$	1.14	1.06	1.00	0.95	0.90	0.85	0.80	0.75	0.71	0.64	-0.50
Panel C: Model without competition											
B/M	0.90	1.14	1.25	1.33	1.39	1.44	1.49	1.53	1.57	1.63	0.73
$Size$	1.63	1.56	1.52	1.50	1.46	1.42	1.34	1.24	1.10	0.88	-0.75
P/E	0.85	1.05	1.21	1.35	1.47	1.52	1.51	1.52	1.56	1.61	0.76
$IVol$	1.71	1.64	1.58	1.51	1.44	1.36	1.27	1.18	1.06	0.89	-0.82

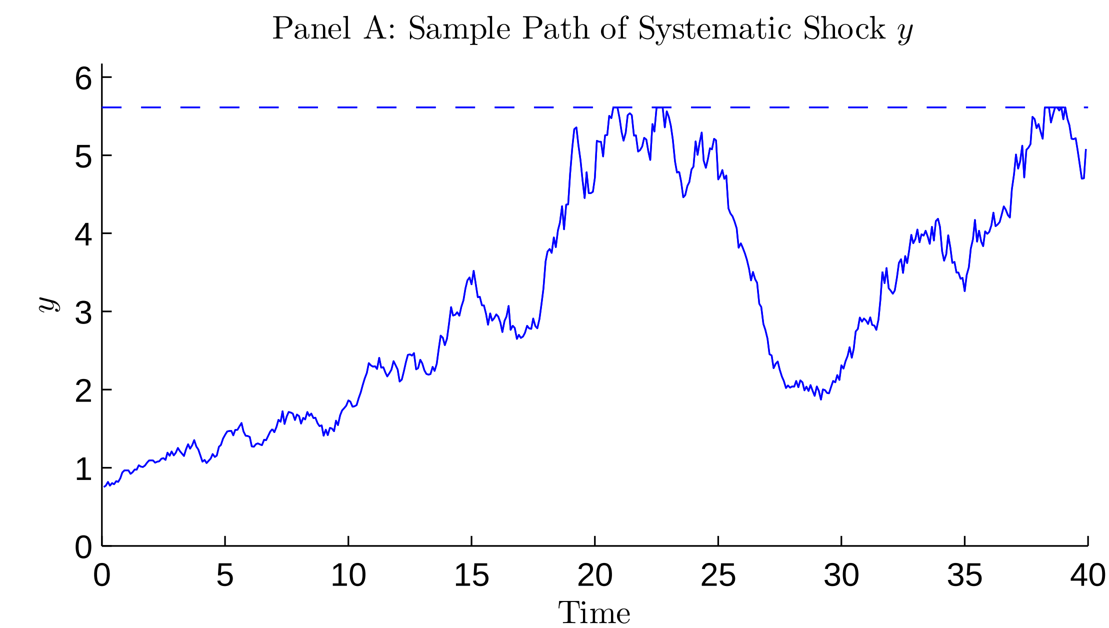
Table V
Separating Value and Size

This table reports average Fama−MacBeth coefficients and average t−statistics from 100 simulations of a cross-section of 2,000 stocks over 45 years, the first 5 of which are discarded. In each month t, the realized stock returns are regressed on estimated betas, log book-to-market ratios, and log firm values. Betas are estimated from time series regressions of stock returns onto changes in the systematic profitability shock from month $t-24$ to $t-1$.

	$\widehat{\beta_t}$	B/M	$Size$
Full model	0.13	0.11	-0.08
	(3.81)	(3.71)	(-4.07)
Model w/o options	0.12	0.20	0.00
	(3.34)	(4.77)	(0.03)
Model w/o competition	0.48	0.12	-0.12
	(5.20)	(3.76)	(-4.97)

Figure 1. Simulated Sample Economy

This figure shows one sample path for the systematic profitability shock y (Panel A) and the respective dynamics of the mass of firms in the competitive economy (Panel B). The horizontal dashed line indicates the reflection barrier due to competition. The parameters are described in the text and summarized in Table I.



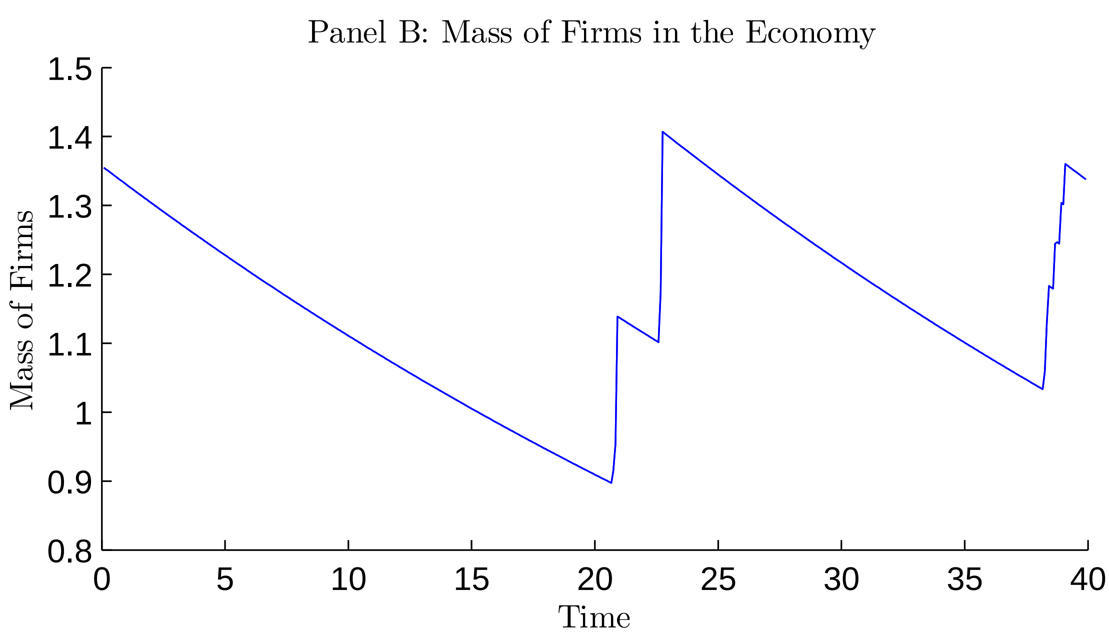
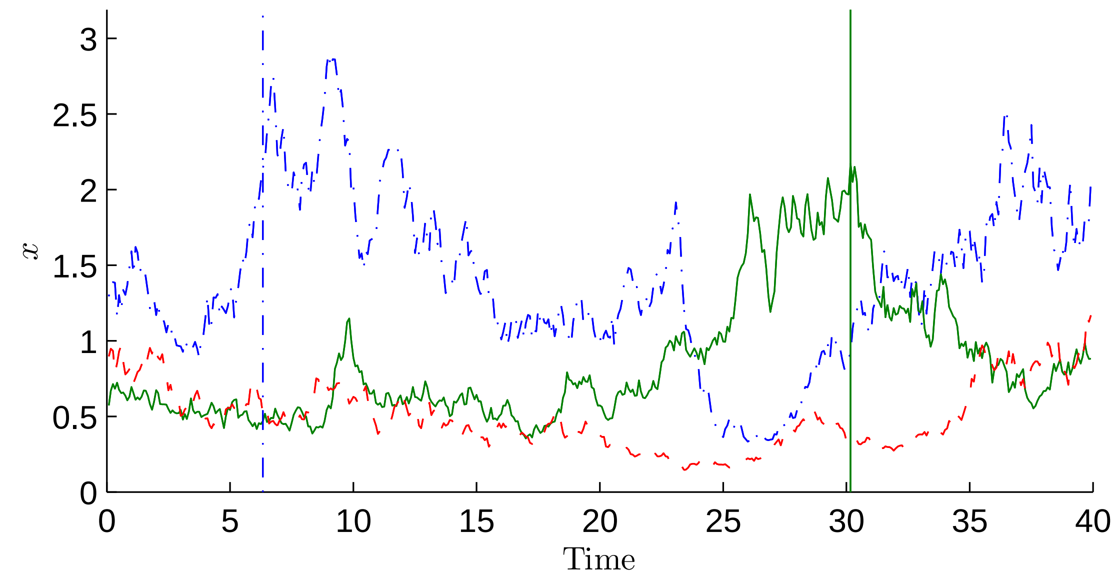


Figure 2. Sample Firms Dynamics

This figure shows, for three randomly selected firms from the simulation in Figure 1, sample paths of the $x-$shocks (Panel A), firm values (B), book-to-market ratios (C), price-earnings ratios (D), as well as betas with respect to y (E) and with respect to the aggregate market (F). The vertical lines indicate the times of exercise of the idiosyncratic options.

Panel A: Three Sample Paths of Idiosyncratic Shocks x_i



Panel B: Corresponding Firm Values

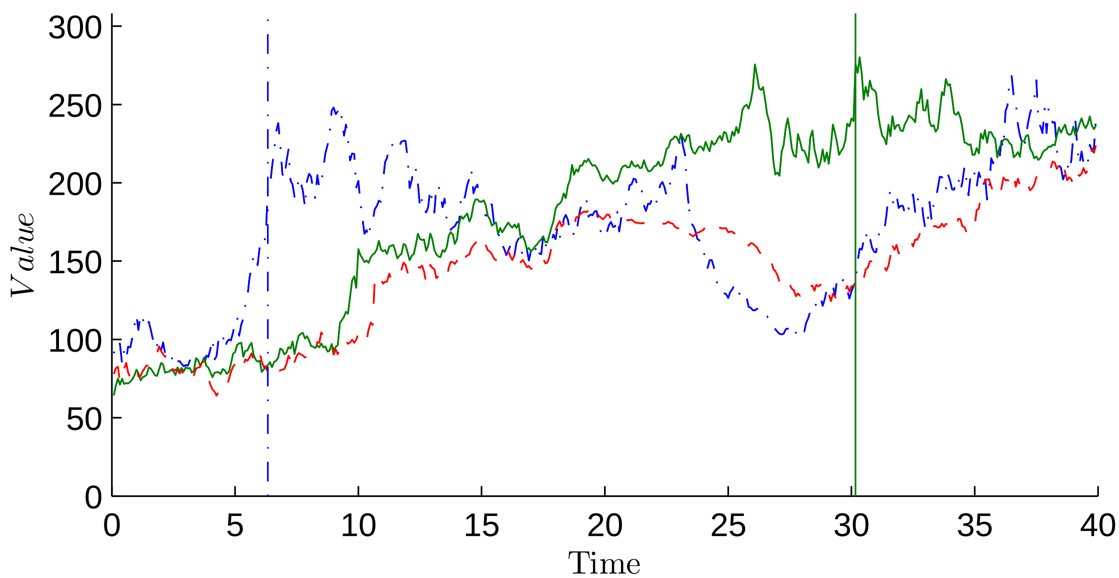
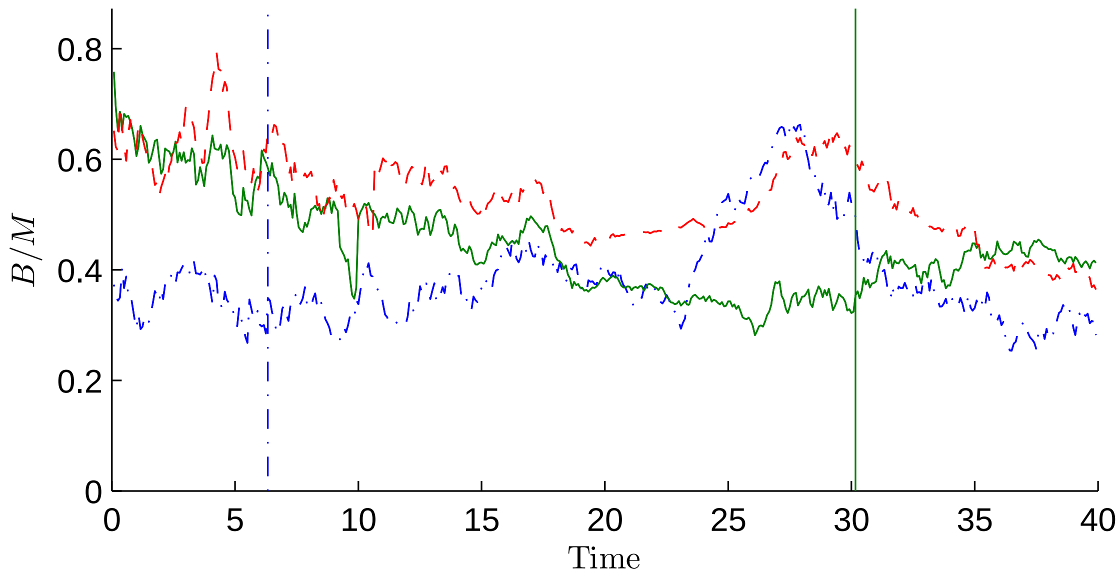


Figure 2. continued

Panel C: Corresponding Book-to-Market Ratios



Panel D: Corresponding Price-Earnings Ratios

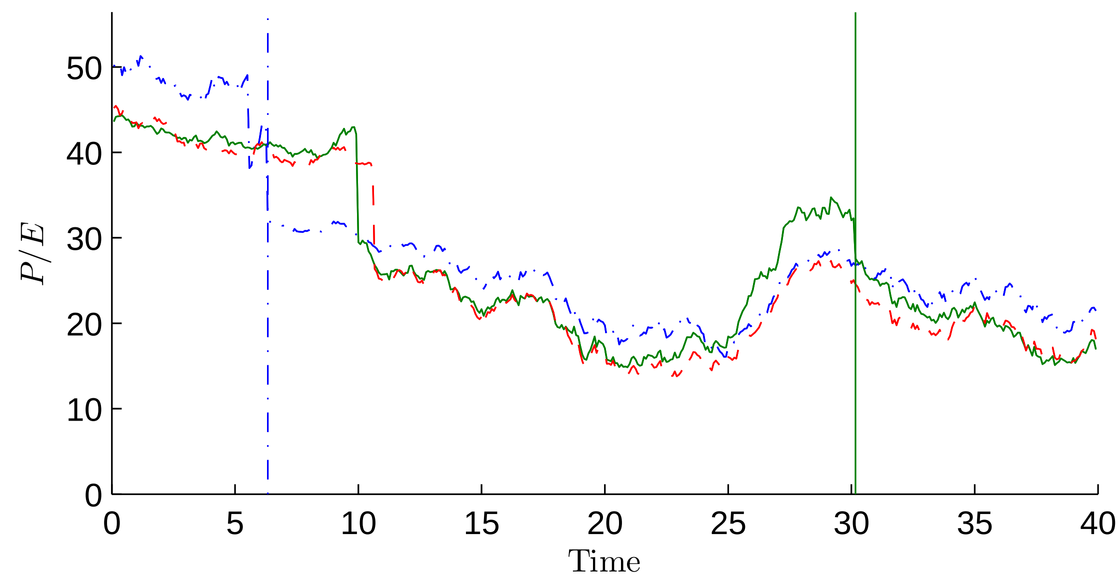
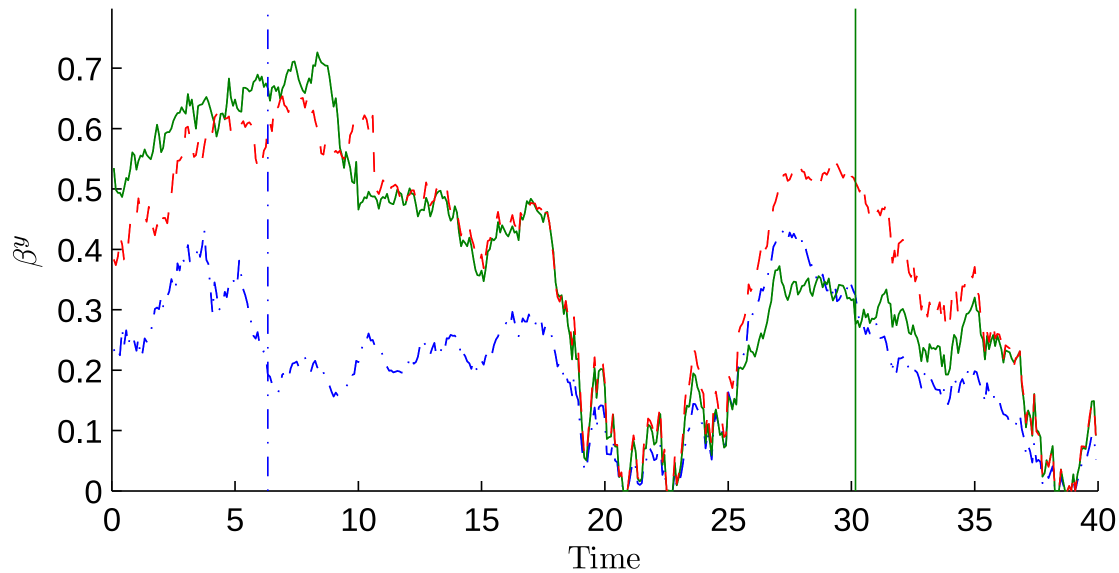


Figure 2. continued

Panel E: Corresponding Firm Factor Betas



Panel F: Corresponding Firm Market Betas

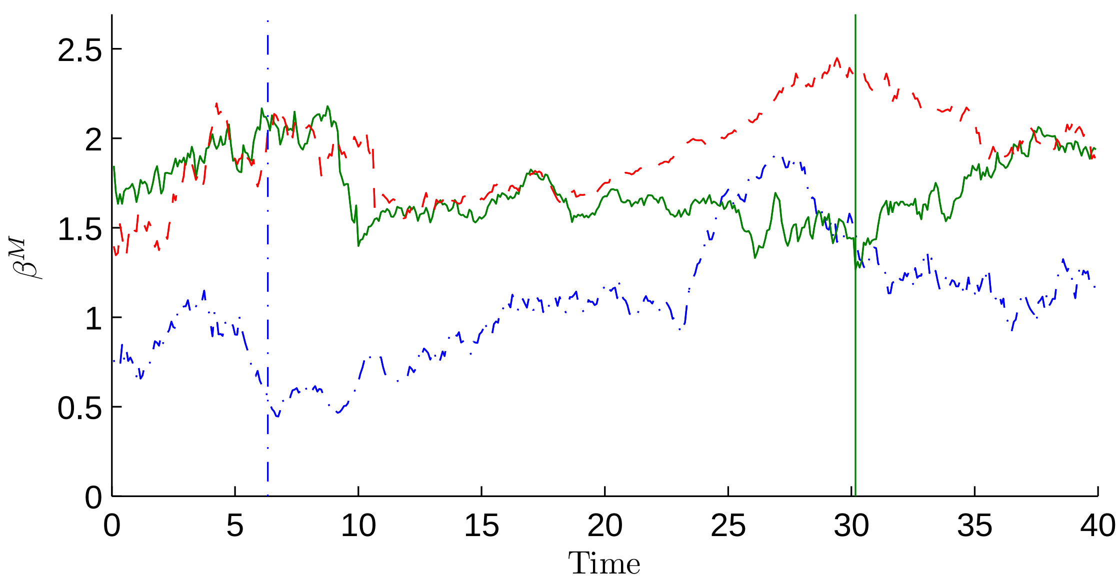
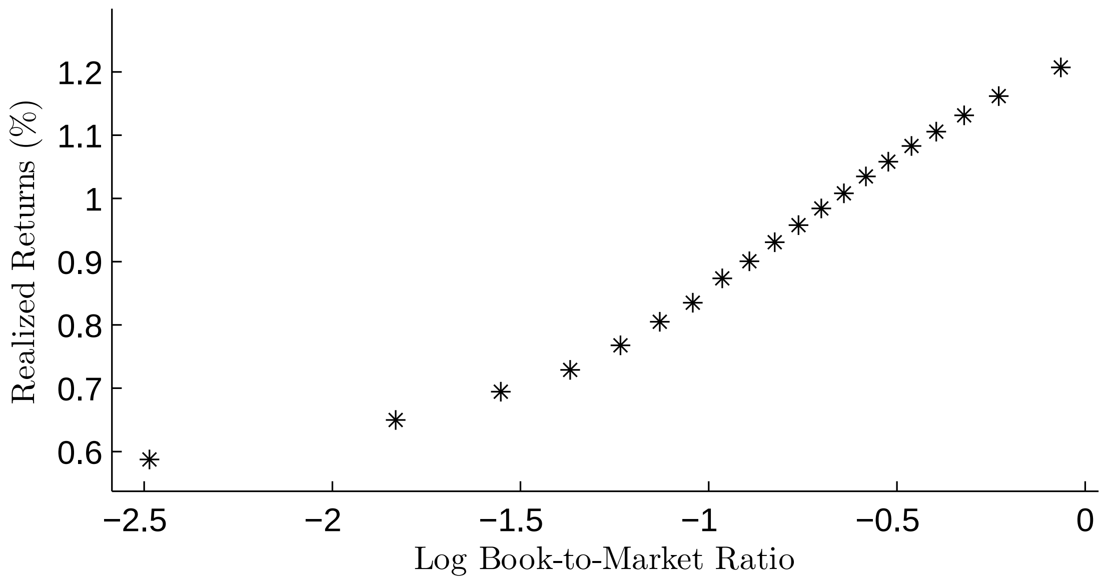


Figure 3. Book-to-market and Size Sorted Portfolio Returns

This figure plots average returns of characteristic sorted portfolios. In Panel A, 20 portfolios are formed based on book-to-market ratio, and returns are plotted against average log book-to-market characteristics. Panels B – D show the results for market capitalization, price-earnings ratio, and idiosyncratic volatility.

Panel A: Returns of Book-to-Market Portfolios



Panel B: Returns of Size Portfolios

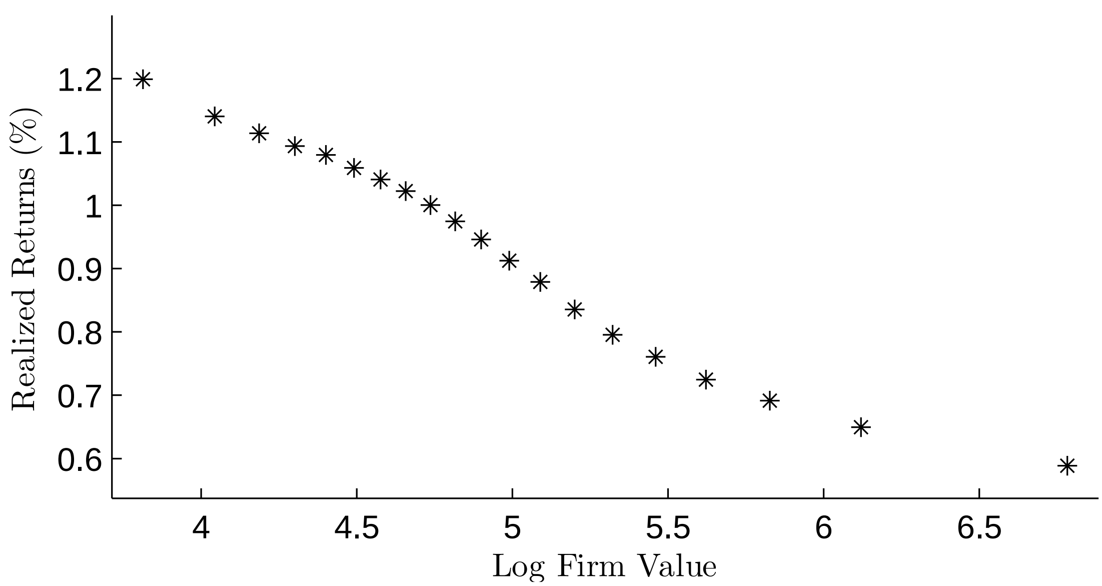
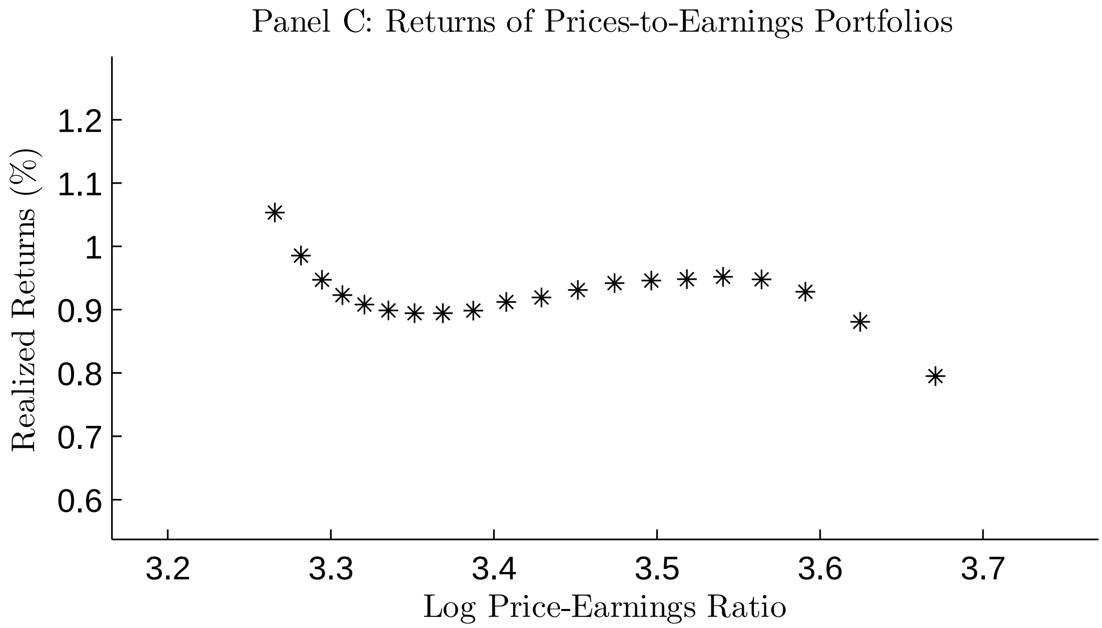


Figure 3. continued



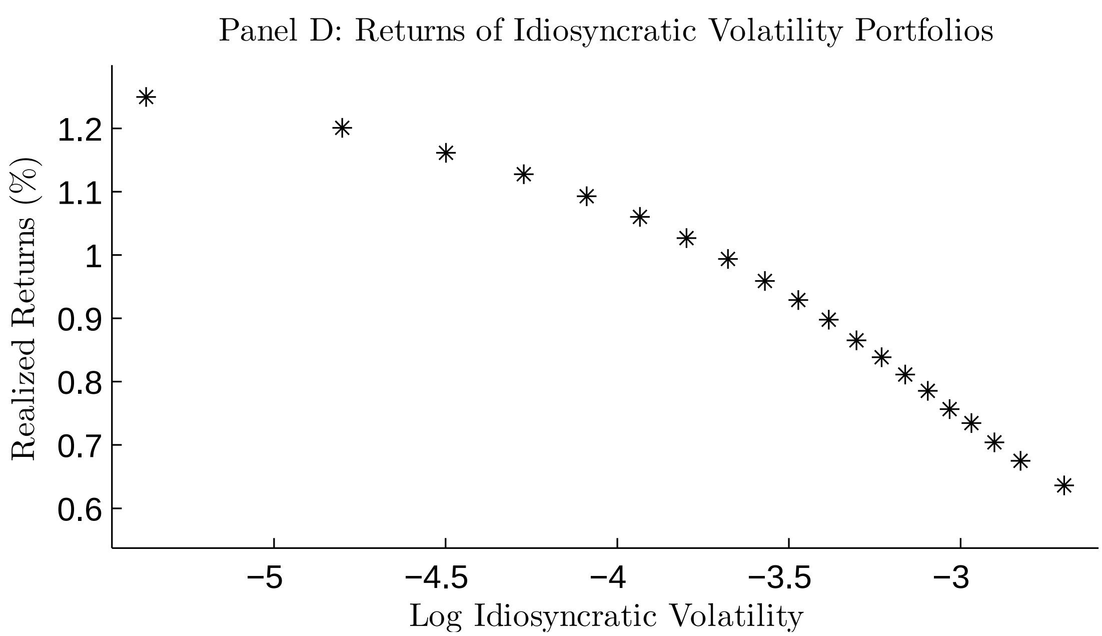


Figure 4. Firm Characteristics around Option Exercise

This figure plots average firm value (top row), book-to-market ratio (middle row), and betas (bottom row) in a 48 month window around exercise of the $y-$option (Panel A) and $x-$option (B).

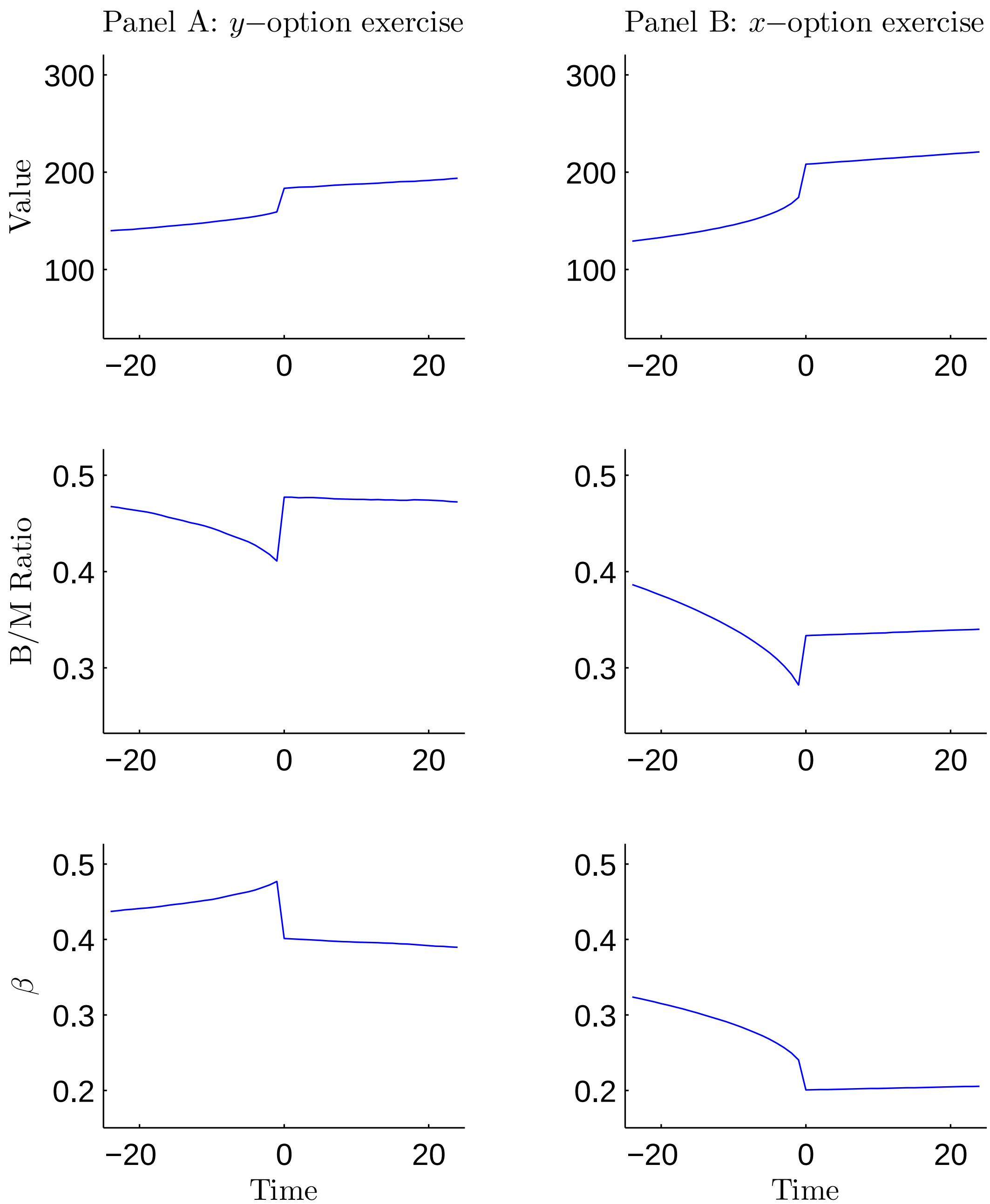


Figure 5. Time Variation in Risk and Value Premium

This figure plots average factor betas (β_M^y), the market risk premium (MRP) in percent annually, a measure of the cross-sectional variation in factor betas ($\beta_{10}^y - \beta_1^y$) and market betas ($\beta_{10}^M - \beta_1^M$), and the realized value premium (VP) against the level of the systematic shock y. The variation in betas is measured as the difference between the average betas of the first and tenth decile of beta-sorted portfolios. The realized value premium is computed as the difference in returns of the top and bottom decile of book-to-market sorted portfolios. Panel A shows the results for the case without competition and entry of new firms, Panel B is the fully specified case.