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Ramsey Pricing in a Congested Network with Market Power in Generation: A Numerical Illustration for Belgium

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Abstract: This paper derives the socially optimal transmission prices in a congested electricity network when there is imperfect competition in generation, and when the budget constraint of the network operator is binding. The results which we derive are a generalization of the standard Ramsey prices and also of the locational marginal prices (LMP). The model is illustrated with a numerical model based on the Belgian electricity data.

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1. INTRODUCTION

This paper introduces a model that calculates socially optimal transmission prices in situations where the transmission capacity on the network is limited, the network operator faces a budget constraint and competition in generation is imperfect. Its focus is on the transmission prices. These prices can be regarded as a generalization of three cases.

First, they are a generalization of the well known Ramsey prices, which are the optimal prices when the network operator faces a budget constraint, when competition in generation is perfect and when there is no congestion on the network. Second, the prices are a generalization of the marginal nodal spot prices (Schweppe, Caramanis *et al.* (1988)). These latter prices result in an efficient use of a congested network, when generators are perfectly competitive and when the network operator does not face a budget constraint. Finally, even in the absence of network congestion and a budget constraint, transmission prices can become non-zero. With imperfect competition, a welfare maximizing network operator will use transmission prices to move the market outcome towards the first best outcome. Some of the prices will become negative, reflecting subsidies to generation plants, in order to stimulate them to increase production. This result can be found in most undergraduate textbooks, see also Tirole (1988) pg 68.

The model is illustrated with numerical simulations that capture the major features of the Belgian electricity system, both in terms of its technical characteristics of generation and transmission, and in terms of the demand for electricity. These characteristics, such as generation plants, grid layout, and $n-1$ security constraints will be described in more detail in section 4.

Four scenarios are considered in the numerical simulations. In the first scenario (“First Best”), we assume that the network operator does not face a budget constraint, and that generation is perfectly competitive. Given the presence of congestion in the network, the network operator will set transmission prices equal to network congestion charges.

In the second scenario (“Second Best”), we assume that the network operator faces a budget constraint and has to cover his fixed costs by increasing the transmission prices. Network congestion is still assumed to be present.

The third and the fourth scenarios still have a budget constraint and limited transmission capacity, but now imperfect competition in generation is added. The third scenario considers a monopoly in the generation market. As the Belgian electricity market is currently highly concentrated (the largest generator owns 83 % of the production capacity), this scenario could be considered as the current situation. The fourth scenario considers three Cournot players in generation. This can be interpreted as a market where some of the monopolist's production capacity is virtually auctioned, a mechanism that currently is implemented in Belgium, or as a market that could emerge when some new generators would enter the market.

The structure of the paper is as follows: the next section starts with a general discussion of socially optimal transmission prices under a number of assumptions and continues with a review of relevant literature on strategic behavior in the electricity market. It starts with a description of models of imperfect competition in electricity generation without transmission constraints, and continues with a discussion of some Cournot models in which transmission constraints are present.

Sections 3 and 4 describe the structure of the model and the data, respectively. Section 5 discusses the simulation results and, finally, section 6 presents conclusions and some extensions for future research.

2. MODELING THE ELECTRICITY MARKET

2.1. Transmission pricing in the network

As stated in the introduction, this paper develops a model for calculating the social optimal transmission prices when there is congestion on the network, when the network operator is faced with a budget constraint and when competition in generation is imperfect. This subsection starts with a very simple reference case where (1) there are no binding transmission constraints, (2) the network operator has no budget constraint, and (3) generators are perfectly competitive. Then we will look at how *each* of these three assumptions affects socially optimal transmission prices.

1 Reference case

It is assumed that electricity transport is costless[1]. In that case, optimal transmission prices are zero, as any non-zero transmission price would create distortions in the market and would decrease welfare. As competition in generation is perfect, there will be perfect arbitrage between the different nodes in the network, and the price for electricity will be the same for all generators and for all consumers, independent of their location in the network.

2 Network congestion[2]

If transmission capacity is scarce, then the optimal price for transmission is no longer zero. Transmission prices should become positive and will be set according to the peak load pricing rule, as discussed by Schweppe, Caramanis *et al.* (1988)[3]. The network operator sets the price of transmission equal to the opportunity cost of using the transmission line, i.e., congestion charges or marginal nodal congestion charges.

Another way of looking at network capacity is to consider it as a public good which is used by the different users of the network. As long as the network is not congested, the use of the network does

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1 We make abstraction of the thermal losses on the network in this paper.
2 We will call this scenario the first best later on in the text.
3 In the US, this system is also part of the standard market design of the FERC and is called Locational Marginal Prices.

not create an externality and it is not priced. When the network becomes congested, the optimal tariff for using the network is equal to the external cost that one unit of transportation produces. This is the well known Pigouvian taxation rule (Pigou (1932)).

3 Network operator with a budget constraint

The operation of the transmission network is a natural monopoly, featured by decreasing average costs as transmission output increases[4]. As a simplification we assume that the network operator has a fixed cost, independent of the network use, and zero marginal costs.

Given zero marginal transmission costs, the optimal use of the network requires the transmission price to be zero. But a zero price will not generate sufficient revenue to cover the costs of the network operator, and the network operator will lose money.

In order to cover costs, the network operator will set positive transmission prices. A welfare maximizing network operator will do this such that total deadweight loss is minimized, i.e. using Ramsey prices (Ramsey (1927)). Transmission tariffs are set inversely proportional to the demand and supply elasticities at the different nodes.

In this paper, we implicitly assume that the network operator cannot use a two-part tariff. This assumption is not in line with common practice, as in most countries the transmission fee has a fixed and a variable component. However, in most cases the 'fixed' part is not completely fixed, and depends on the actual use of the network, so there is still creation of deadweight loss[5]. As long as there are no distortion-free instruments, it remains optimal to use a form of Ramsey prices.

4 Imperfect competition in generation

Suppose that there is a monopoly in generation. The monopolist will generate less electricity than the social optimal quantity. In such a case, the optimal transmission tariff is not equal to zero. Instead, the welfare maximizing network operator will subsidize the monopolist for using the transmission line[6]. This transmission tariff will change the monopolist's incentives to generate electricity in such a way that we obtain the first best outcome.

Overview

The results of the previous discussion are summarized in Table 1. Analytic expressions for the social optimal transmission prices for the cases 2 to 4 are derived in Appendix B :.

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[4] This *output* includes the transport of electricity, but also other functions, such as the provision of reliable electricity supply (operation of reserve power and balancing markets) and of qualitative power (*e.g.* voltage level, frequency stability).

[5] The fixed part often depends on the maximal usage of the network over a certain time period, or the average use of the network over a time period. As the fixed part is a function of the actual use of the network, users will take it into account when they make their decision about how much transmission they want to use. They will transport less electricity than the welfare optimum, and deadweight loss is created.

[6] Note that we assume that the Network Operator does not have a budget constraint.

Case	Budget constraint	Binding network constraints	Imperfect competition	Welfare maximizing pricing rule
1	-	-	-	Zero price for transmission, uniform price for electricity
2	-	√	-	Optimal Nodal pricing
3	√	-	-	Standard Ramsey Pricing
4	-	-	√	Subsidize Monopolist until marginal willingness to pay is equal to marginal costs

Table 1: Welfare maximizing prices in the reference case and when one of the imperfections is added.

Several market imperfections

In the numerical simulation in section 5, we will gradually add extra assumptions to the model.

In the first scenario we look at the *first best*. (Model 2 in Table 1) In the second scenario we add the budget constraint and study the *second best* (combination of the models 2 and 3) and in the last two scenarios we also add imperfect competition (combination of models 2, 3 and 4).[7]

Before we describe the model itself, we review some relevant literature of imperfect competition in generation. We start with a description of models of imperfect competition in electricity generation without transmission constraints, and continue with a discussion of some Cournot models in which transmission constraints are present.

2.2. Imperfect competition in generation

Due to the non-storability of electricity and its highly variable demand, electricity systems tend to feature a mix of base load plants and peak load plants. Peak load plants are typically characterized by high marginal production costs and low investment costs, while base load plants typically have low marginal costs and high investment costs. Peak load plants are only used in periods of high demand, base load plants are used at all times. Peak load power and base load power could therefore be considered as two different goods. Two types of equilibrium have been developed in the specialized literature to model imperfect competition in such a multi-good market: the multi-unit auction and the supply function equilibrium.

Multi-unit auction

In the multi-unit auction, generators bid a price for each plant at which they are willing to supply given capacities[8]. The equilibrium price, determined as the price that clears the market, is applied to all infra-marginal units[9]. In this setting, bidders, offering more than one unit of capacity, have an incentive to increase their bids for those plants that are likely to be marginal. For example Wolfram (1998) uses the multi-unit auction approach to find empirical evidence that, in the England & Wales market, large

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7 Buchanan (1969) and Barnett (1980) study the combination of model 2 and 4, and discuss the optimal taxation of externalities when there is market power in the output market.

8 See for instance the models of von der Fehr and Harbord (1993).

9 Other types of auctions can also be considered.

players effectively try to use their market power in this way. A drawback of multi-unit auctions is that they are particularly hard to model, and do not always have a Nash equilibrium.

Supply Function Equilibrium

The supply function equilibrium concept is based on Klemperer and Meyer (1989). Generators choose a continuous and differentiable supply function, which, for each price, specifies the quantity they are willing to generate. Again, the electricity price is established as the market clearing price. Klemperer and Meyer show that an infinite number of Nash equilibriums exist when electricity demand is known with certainty. The reason is that only one point on the supply function is required to determine the market clearing price, the remainder of the supply function can be chosen more or less free.

However, if electricity demand is uncertain when generators decide about their supply function, then the latter function has to be appropriate for several situations, and the number of equilibriums is reduced[10]. Klemperer and Meyer even show that, under certain conditions, the differentiable supply function equilibrium becomes unique.

An example of the supply function equilibrium approach is Green and Newbery (1992). These authors apply the Klemperer and Meyer model to the two largest generators in the English market[11]. By adding an output constraint for each generator, they can further reduce the set of equilibriums. Furthermore, Green and Newbery assume that generators will coordinate on the equilibrium that maximizes total profit. Their model predicts that, in the absence of a threat of entry, the two generators are able to sustain a non-collusive equilibrium in which prices are well above operating costs.

One of the major drawbacks of the two types of models discussed above is that the spatial structure of the electricity market, and therefore the impact of transmission constraints, is often omitted. Applying these two approaches in a market with transmission constraints is quite difficult[12].

Most researchers therefore opt for some kind of Cournot market, while dropping some of the multi-good aspects of the actual market. This option is supported by an empirical study of Wolak and Patrick (2001) who suggest that Cournot competition is an appropriate representation of the electricity generation market[13]. This paper will also follow the Cournot approach for the electricity generation market.

2.3. Cournot in generation - Price taking in transmission

Even Cournot models become quite cumbersome when simulations are made for larger networks with transmission constraints. This is the case because generators realize that, with scarce transmission capacity, transmission prices can be influenced, and congestion can be created. Cournot-Nash equilibriums are then no longer guaranteed to exist, and rationing rules or arbitrageurs need to be

[10] Klemperer and Meyer consider horizontal shifts in demand.

[11] Other studies using this model are Bolle (1992), Newbery (1998), Green (1996) and Rudkevich, Duckworth *et al.* (1998).

[12] One notable exception is the work of Hobbs, Metzler *et al.* (2000) who restrict themselves to linear supply functions.

[13] Other studies using Cournot competition are Oren (1997), Stoft (1997), Borenstein, Bushnell *et al.* (2000), Borenstein, Bushnell *et al.* (1999), Borenstein and Bushnell (1999), Hogan (1997), Cardell, Hitt *et al.* (1997).

added to the model. Papers looking at this issue are Hogan (1997), Borenstein, Bushnell *et al.* (2000), Willems (2002b), Willems (2002a).

Therefore, this paper assumes that generators behave *à la* Cournot in the energy market (buying and selling of electricity), but are price takers in the transmission market. This approach is inspired by the model of Smeers and Wei (1997) and Wei and Smeers (1999), two models to be discussed in the next section.

2.4. Network operation

In addition to the question of how generators perceive transmission prices, there is also the question of how the transmission firms set prices. With respect to the latter question, different assumptions can be made. Examples are *congestion pricing* (Smeers and Wei (1997)), *regulated pricing* (Wei and Smeers (1999)), and *strategic price setting* (this paper). A short discussion follows.

Congestion pricing

Smeers and Wei (1997) assume that the network operator sets prices for using the network on the basis of congestion charges (CC)[14]. As long as a line is not used at full capacity, the transmission tariff equals zero. If the line becomes congested, the transmission tariff is increased until demand for transmission equals supply. This can be illustrated for a network with one line of capacity $\bar{Q}$. With Q the demand on the line, and T the transmission tariff, we have:

$$\begin{aligned} &\text{if} \quad \mathrm{T} > 0 \quad \text{then} \quad Q = \bar{Q} \\ &\text{if} \quad \mathrm{T} = 0 \quad \text{then} \quad Q \leq \bar{Q} \end{aligned} \tag{1}$$

Congestion pricing can be interpreted as the result of assuming that the network operator behaves perfectly competitive. Thus, the network operator acts as a price taker in the transmission market and appears to be unaware of his market power in that market. Congestion charges can be implemented when the network operator is forced to sell *all* transmission capacity in an auction, is not allowed to withhold capacity from the market, and is not allowed to set a minimal reservation price for the transmission rights. Hobbs (2001) extended the framework of Smeers and Wei, by adding a DC flow model and allowing for arbitrage between the nodes. (See Appendix B : for a discussion of arbitrage).

Regulated prices

Wei and Smeers (1999) study *regulated transmission prices*. Here, the transmission charge is the sum of two parts: a congestion charge T^{CC}, and a regulated charge T^{R} [15]:

$$\mathrm{T}(Q) = \mathrm{T}^{CC}(Q) + \mathrm{T}^{R}(Q) \tag{2}$$

The term $\mathrm{T}^{R}(Q)$ is set according to a regulatory rule which depends on the use of the line. They study two types of regulatory rules: *marginal cost pricing* and *average cost pricing*. The average cost

[14] This type of pricing is also called Location Marginal Pricing (LMP) or nodal spot pricing. (Schweppe, Caramanis *et al.* (1988))

[15] Wei and Smeers (1999) give a different interpretation to the congestion charges than we do here. They look for a Generalized Nash Equilibrium where transmission constraints are internalized. In that case, congestion charges are internal multipliers.

pricing rule sets the regulated charge for a transmission line equal to the average cost of building a new transmission line. With the marginal cost pricing rule, the regulated charges are set according to the marginal cost of building transmission lines.

The congestion charge $\mathrm{T}^{CC}(Q)$ is required to clear the market when the demand for transmission is in excess of available capacity at a transmission price equal to the regulated charge $\mathrm{T}^{R}(Q)$. Wei and Smeers assume that congestion charges are *not* used to refund the network operator for building new transmission capacity[16].

Both models of Smeers and Wei assume no strategic behavior from the side of the transmission firm. This paper drops this assumption and assumes the transmission firm to behave strategically.

Welfare maximizing network operator

Smeers and Wei (1997) assume that transmission is priced according to marginal cost. Underlying this pricing rule are the assumptions that (A) marginal cost pricing is optimal, and (B) the network operator can be perfectly regulated.

In this paper we drop the first assumption and look at the two cases where marginal cost pricing is not optimal[17].

First, with increasing returns to scale in network operation, marginal cost pricing does not guarantee sufficient revenue for cost recovery. Network operation is featured by decreasing average costs, implying marginal costs are below average costs at relevant transmission output levels. Competition in network services is not sustainable, and, for that reason, most regions opted for a single regulated transmission firm operating the whole regional network[18]. If the losses of this transmission firm cannot be subsidized via transfers (i.e. the transmission firm has a binding budget constraint), then other pricing rules, generating prices above marginal cost, will have to be applied.

Second, it is optimal for the transmission firm to deviate from marginal cost pricing if competition in generation is imperfect. Imperfect competition would result in output levels below the welfare optimum and a welfare maximizing network operator will subsidize the generator(s) in order to increase output and, hence, to decrease dead-weight losses.

This paper models the transmission firm's decision process as a two stage Stackelberg game. In the first stage, the network operator sets the transmission charges at each consumption and generation node. The network operator maximizes welfare, taking into account the effect of its pricing decision on the strategic behavior of the players in the second stage.

In the second stage, generators behave à la Cournot, taking the nodal transmission surcharges and the congestion charges as given. The current model differs from the *regulated prices* model of Wei and Smeers (1999) in that the transmission charges are set by the network operator, rather than by a regulatory rule.

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[16] It has often been argued that congestion payments should not go to the network operator as this could give the wrong incentives.

[17] In a companion paper we will also drop the second assumption that the network operator can be regulated perfectly.

[18] See for example Train (1991) for a discussion of the conditions for a natural monopoly.

3. THE MODEL

Define the sets F and G as the sets of generation firms and generation plants. Let G_f be the set of generation plants owned by generation firm $f \in F$. With I being the set of network nodes, G_i denotes the generation plants at node $i \in I$, and G_{fi} the generation plants at node i owned by firm f. Furthermore, let A be the set of transmission lines in the network.

For notational simplicity, the model will be further described as if it concerned a one period model, *i.e.* a model that does not distinguish between peak and off-peak periods. However, the numerical simulations discussed in section 5 differentiate between peak and off-peak demand in a 4-period model.

The model distinguishes three types of players: consumers, generation firms and the network operator.

Consumers are price takers. At node i, they consume s_i units of electricity. Their inverse demand for electricity, denoted as $p_i\left(s_i\right)$, is downward sloping and concave. Consumer prices include compensation for both the generation and the transmission of electricity.

Generation firm $f \in F$ maximizes profits, while acting as a price taker in transmission. At node i, it owns the generation plants $g \in G_{fi}$.

Electricity generation in *plant* g is q_g and the generation cost is $C_g\left(q_g\right)$. Total generation costs are convex, with fixed generation costs normalized to zero. The generation capacity of plant g is labeled $\overline{q}_g$. Output should be nonnegative, and cannot exceed available generation capacity. Therefore, we have

$$0 \leq q_g \leq \overline{q}_g$$

The *network operator* or *transmission company* maximizes social welfare and sets a nodal transmission charge τ_i^c for consumers and τ_i^p for generators. This is the per unit payment generators have to make for injecting power, and that consumers have to pay for taking power from the grid. These charges can be different. For instance, a generator who generates electricity in node i and sells electricity in node j will pay $\tau_i^p + \tau_j^c$. Only the sum of the consumer and generation transmission charge is important, and therefore one of the charges can be set equal to zero without loss of generality.

As explained before, the model has two stages. In the first stage, the transmission operator sets transmission prices. In the second stage, generation firms play a Cournot game in which transmission prices and their competitor's quantities are assumed as given. The next subsection describes the second stage of the game.

3.1. The second stage

Each firm f observes the transmission charges τ_i^p and τ_i^c as set by the network operator and plays a Cournot game. A firm f collects revenue by selling s_{fi} units of electricity at node i at the per unit price p_i. Firms also set the production level q_g $\left(g \in G_f\right)$ at each of their plants. Their competitor's sales in node i, denoted by $\tilde{s}_{-fi}$, are taken as given. Apart from generation costs, firms also pay a transmission cost τ_i^p for injecting electricity to the network at node i, and τ_i^c for the delivery of electricity to node i. This results in the following profit function for generation firm f,

$$\Pi_f^{Gen} = \sum_{i \in I} (p_i - \tau_i^c) \cdot s_{fi} - \sum_{i \in I} \sum_{g \in G_{fi}} \left[C_g(q_g) + \tau_i^p q_g \right] \tag{3}$$

The nodal price p_i that is received by generator f depends on the total sales in that node, *i.e.*

$$\begin{aligned} p_i &= p_i(s_i) \\ s_i &= s_{fi} + \tilde{s}_{-fi} \end{aligned}$$

where a tilde indicates that the variable is considered as given. In equation (3), the first term reflects revenues from electricity sales net of transmission charges paid at the consumption nodes. The second term reflects generation costs and transmission charges to put the electricity on the network. Summarizing, we have the following maximization problem for a generator[19]:

$$\begin{aligned} \underset{s_{fi}, q_g(g \in G_f)}{\text{Max}} \quad & \Pi_f^{Gen} = \sum_{i \in I} (p_i - \tau_i^c) \cdot s_{fi} - \sum_{i \in I} \sum_{g \in G_{fi}} \left[C_g(q_g) + \tau_i^p q_g \right] \\ s.t. \quad & 0 \le q_g \le \overline{q}_g \qquad \left(\underline{\mu}_g, \overline{\mu}_g \right) \quad \forall g \in G_f \\ & \sum_{i \in I} s_{fi} = \sum_{g \in G_f} q_g \quad \left(\lambda_f^p \right) \\ & s_i = s_{fi} + \tilde{s}_{-fi} \qquad\qquad \forall i \in I \end{aligned} \tag{4}$$

As noted before, the first constraints reflect generation capacity constraints. The second constraint represents the energy balance at the firm level, *i.e.* total output should equal total sales. The last constraint represents demand. This constraint has no multiplier as it is substituted into the objective function and the other constraints before derivatives are taken.

The following first order conditions are then derived:

$$\frac{\partial C_g(q_g)}{\partial q_g} + \tau_i^p + \underline{\mu}_g - \overline{\mu}_g = \lambda_f^p \quad \forall g \in G_{fi}, \forall i \in I \tag{5}$$

$$p_i + \frac{\partial p_i(s_i)}{\partial s_i} s_{fi} - \tau_i^c = \lambda_f^p \quad \forall i \in I \tag{6}$$

These are the standard first-order conditions for profit maximization, *i.e.* as long as generation constraints are not binding, marginal revenue equals marginal cost in all market segments. The Lagrange multiplier of the energy balance constraint λ_f^p, is the value of energy *in the network* for generation firm f. This value is different for every firm.

[19] Note that in this formulation nodal sales of the generators can become negative. Generators can act like arbitrageurs that buy electricity in one region and sell it in another. They will, however, still take into account the effect on the marginal revenue in both regions. Therefore, with a limited number of firms, not all price differences will be arbitraged away. As the number of generation firms increases, arbitrage will improve.

Cost minimization requires that each firm equalizes the sum of the marginal cost and the generation charge at all generation plants. Profit maximization requires that marginal revenues net of consumption charges are equalized.

Note that each firm's reaction function with respect to the sales s_{-fi} and the transmission charges, τ_i^c and τ_i^p can be derived from the equations (4), (5) and (6).

The multipliers $\underline{\mu}_g$ and $\overline{\mu}_g$ are positive, and satisfy the complementarity conditions:

$$\begin{aligned} &\underline{\mu}_g \geq 0 \qquad \underline{\mu}_g \cdot q_g = 0 \\ &\overline{\mu}_g \geq 0 \qquad \overline{\mu}_g \cdot \left(\overline{q}_g - q_g\right) = 0 \end{aligned} \tag{7}$$

Electricity transmission

The model captures the technical features of the electricity system, especially at the level of electricity transport. Electricity transport is subject to physical constraints. These constraints have an impact on the power flow through the network and therefore potentially also on the pricing of transmission services. In this paper we concentrate on active power and we adopt a simplified DC flow model without losses.[20]

Each line $a \in A$ in the network is characterized by a transmission capacity Q_a. Denoting the flow over the line a as Q_a, we must have

$$Q_a \leq \overline{Q}_a \quad \forall a \in A \tag{8}$$

Transmission must not be larger than the available transmission capacity. This is also called the thermal constraint of line a, because the line's temperature increases too much if the line carries larger flows.

The flow over a line a depends on the injection and the extraction of electrical energy in *all* the nodes of the network. The flow on line a is equal to:

$$\sum_{i \in I / \{\text{swing node}\}} \theta_{a,i}(q_i - s_i) = Q_a \tag{9}$$

with s_i, q_i the total consumption and generation in node i, respectively:

$$s_i = \sum_{f \in F} s_{fi} \quad \forall i \in I \tag{10}$$

$$q_i = \sum_{g \in G_i} q_g \quad \forall i \in I \tag{11}$$

The flow over a transmission line is a linear function of the net injections at all nodes. The variables $\theta_{a,i}$ are the power transmission distribution factors (PTDFs). They describe how much a change of net

[20] Such a model assumes that line resistance is small relative to reactance, that voltage magnitudes are the same at all nodes, and that voltage angles between nodes at opposite ends of a transmission line are small. Engineers often use the linearised model of the network for long term planning. See Schweppe, Caramanis *et al.* (1988).

The alternative, AC-power flow, was used in a previous version of the program, but did not give fundamentally different results.

injection in node i will change the flow on line a. The PTDFs are determined by the physical properties of all the lines and the layout of the network.

Equations (8)-(11) describe the transmission possibilities of the network, *i.e.* they define the production feasibility set of the network operator.

Security of supply

The network operator also needs to secure the supply of electricity. A minimal requirement for this is that, if unexpectedly a line goes out of service the remaining lines should still be able to transport all supplied electricity. This is the "$n-1$" rule. The network operator will check that for all contingencies $k \in K$ the network is still capable of accommodating all flows.

For instance, if during contingency k the line $\alpha \in A$ breaks down, then the set of the remaining lines $A \setminus \{a\}$ should be able to transport the power over the network. After a contingency, the flows redistribute themselves over the network, and these new flows should still be feasible given the thermal constraints of the remaining lines.

Taking into account the security of supply for all contingencies *K*, the following equation needs to be added to the network Equations (8)-(11):

$$\sum_{i \in I / \{\text{swing node}\}} \theta^k_{a,i}(q_i - s_i) \leq \bar{Q}_a \qquad \forall a \in A, \forall k \in K \tag{12}$$

For notational simplicity, we will assume that the set of contingencies *K* also includes the case where all lines are available, such that we can drop equations (9) and (8). Equations (10)-(12) describe the feasibility set of transmissions on the security constrained network.

3.2. *The first stage*

The network operator maximizes welfare subject to a budget constraint. He sets the consumption and generation transmission charges (τ_i^c and τ_i^p), which can be differentiated over the nodes. It is assumed that the cost of providing transmission services is separable into operating costs and capacity costs. In the present model, operating costs and network losses are neglected. Therefore, only the capacity costs B remain.

The profit of the network operator is then equal to:

$$\Pi^{tr} = \sum_{i \in I} (\tau_i^c s_i + \tau_i^p q_i) - B \tag{13}$$

The first term between brackets is the revenue of selling transmission services to consumers at node i. The second term is the revenue of selling transmission to the generators at node i. The last term represents capacity costs. By assumption, capacity costs are fixed.

The network operator maximizes

$$W = \sum_{i \in I} \int_0^{s_i} p_i(t)\,dt - \sum_{g \in G} C_g(q_g) \tag{14}$$

subject to the *energy balance* at the firm level,

$$\sum_{i \in I} s_{fi} = \sum_{g \in G_f} q_g \quad \forall f \in F \tag{15}$$

the *Cournot behavior (Sales - Production),*

$$p_i(s_i) + \frac{\partial p_i(s_i)}{\partial s_i} s_{fi} - \tau_i^c = \lambda_f^p \quad \forall i \in I, \forall f \in F \tag{16}$$

$$\frac{\partial C_g(q_g)}{\partial q_g} + \tau_i^p + \underline{\mu}_g - \overline{\mu}_g = \lambda_f^p \quad \forall i \in I, \forall f \in F, \forall g \in G_{fi} \tag{17}$$

$$\begin{array}{lll} \underline{\mu}_g \geq 0 & \underline{\mu}_g \cdot q_g = 0 & q_g \geq 0 \\ \overline{\mu}_g \geq 0 & \overline{\mu}_g \cdot \left(\overline{q}_g - q_g\right) = 0 & q_g \leq \overline{q}_g \end{array} \tag{18}$$

the *network equations*

$$\sum_{i \in I / \{\text{swing node}\}} \theta_{ai}^k (q_i - s_i) \leq Q_a^k \quad \forall a \in A, \forall k \in K \tag{19}$$

$$s_i = \sum_{f \in F} s_{fi} \quad \forall i \in I \tag{20}$$

$$q_i = \sum_{g \in G_i} q_g \quad \forall i \in I \tag{21}$$

and the budget constraint:

$$\sum_{i \in I} (\tau_i^c s_i + \tau_i^p q_i) - B = \Pi^{tr} \geq 0 \tag{22}$$

This latter constraint is added in order to avoid that the network operator goes bankrupt.

4. DATA AND CALIBRATION

Before continuing with the simulations, we discuss the data that have been used as an input for the model. Also, the calibration procedure will be described. The choice of the technical features of the transmission grid and of the available generation plants is inspired by the Belgian electricity system.

4.1. The Network

Figure 1 shows the network that has been modeled. It consists of 55 nodes and 92 lines and includes all the Belgian 380 kV and 220 kV transmission lines, but also some 380 kV lines in The Netherlands and France because they are important for the flows inside the Belgian network. The full lines on the graph are 380 kV lines, the dotted lines are 220 kV lines. The line between Gouy and Avelgem represents several lines of the 110 kV network that connect both nodes. [21]

[21] Network data was kindly provided by Peter Van Roy and Konrad Purchala of the K.U.Leuven Electrical Engineering Department. More detailed information on origin and destination, voltage level, admittance, thermal capacity… is available upon request.

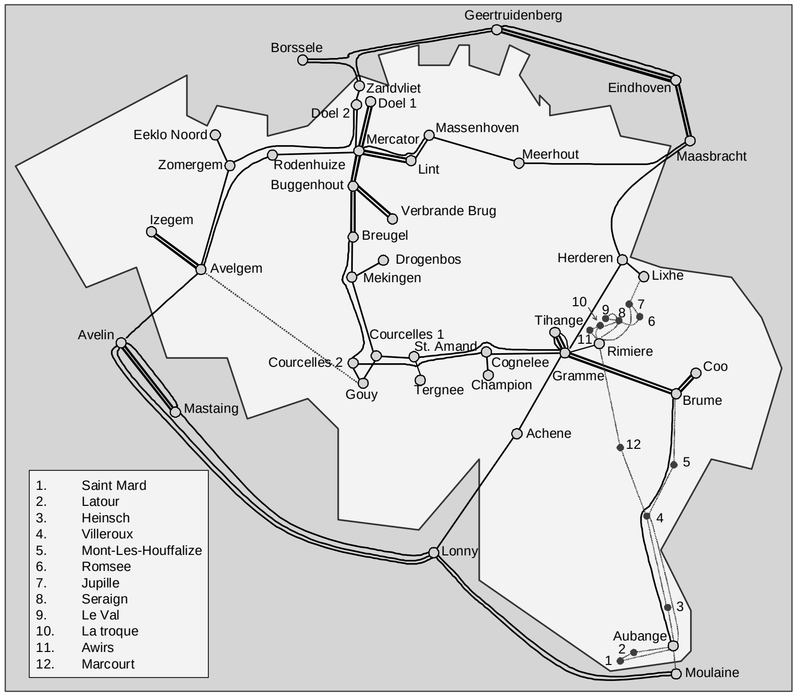

Figure 1: The Belgian high voltage network, situation in 2002.

The $n-1$ rule holds for all Belgian 380 kV lines, except for some loose ends. Using the $n-1$ rule for these lines makes no sense when we do not include lower voltage levels. Also, the $n-1$ rule is not imposed for the interconnections with France and The Netherlands and for the lines within these countries, because sufficient or adequate information is lacking.

4.2. Electricity generation

The total generation capacity connected to the grid is 14 475 MW. Of this capacity, approximately 1070 MW consists of smaller generation plants which are not included in the model. These are mainly combined heat and power generation units (970 MW), and some small hydro units (90 MW). We assume that in any time period, 50 % of these plants produce electricity. The remaining production capacity (13405 MW), is spread over 51 generation units, which are modeled in the paper based upon data of the year 2002[22]. Each generator is assumed to have constant marginal production costs C_g.

In the simulations, two alternative scenarios are considered with respect to market power of the generators. First, we assume a generation monopoly, *i.e.* all generation units are owned by one profit maximizing generator. The second scenario considers three Cournot players, having a market share in generation capacity of 43%, 34% and 23%, respectively.

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[22] Some of the data was kindly provided by Leonardo Meeus and Kris Voorspools of the Departments of Electrical and Mechanical Engineering, respectively. Data was also taken from several editions of the BFE statistical yearbook, the annual report of Electabel. Data are available from the authors upon request.

Each player maximizes profit, taking into account plant characteristics. Generation decisions are described by the first order conditions (5) and the complementarity conditions (7). The player's complementarity conditions are highly non-linear at the zero production level and at the maximal capacity of each plant. This makes the model in this paper a Mathematical Program with Equilibrium Constraints (MPEC). MPECs are a class of problems which are known to be difficult to solve (Luo, Pang *et al.* (1996)). This paper uses a pragmatic approach[23] to solve them and relaxes the complementarity conditions (18) to

$$\begin{aligned} \underline{\mu}_g \cdot \left(q_g + \beta_g\right)^{\phi_g} &= \alpha_g \\ \overline{\mu}_g \cdot \left(\overline{q}_g - q_g + \beta_g\right)^{\phi_g} &= \alpha_g \end{aligned} \tag{23}$$

The parameters α_g, β_g and ϕ_g are the relaxation parameters of the complementarity conditions. When α_g and β_g become equal to zero, we obtain again the exact complementarity conditions.

After solving for the Lagrange multipliers $\underline{\mu}_g$ and $\overline{\mu_g}$ in equation (23), the optimal sales decision (5) can be rewritten as

$$\frac{\partial C_g^{sm}\left(q_g\right)}{\partial q_g} + \tau_i^p = \lambda_f^p \quad \forall i \in I, \forall f \in F, \forall g \in G_{fi} \tag{24}$$

with $\frac{\partial C_g^{sm}\left(q_g\right)}{\partial q_g}$ the smoothed marginal cost function:

$$\frac{\partial C_g^{sm}\left(q_g\right)}{\partial q_g} \equiv C_g + \frac{\alpha_g}{\left(q_g + \beta_g\right)^{\phi_g}} - \frac{\alpha_g}{\left(\overline{q}_g - q_g + \beta_g\right)^{\phi_g}} \quad \forall g \in G \tag{25}$$

This is the approach followed in the numerical simulations. By smoothing the marginal cost functions of the generators, we make sure that the generators will choose an internal solution, and not one of the boundary generation levels 0 or $\overline{q}_g$.

Three plants are *pumped storage plants*, i.e. they can store energy in the form of a water reservoir. When generation costs are low, these plants consume electricity and pump water to a higher level. When generation costs are high, the reservoir is emptied and electricity is produced. The underlying decision process is not modeled in this paper. We assume that these plants generate electricity during peak periods at a marginal cost of €13 per MWh, and we count them as part of the consumption side during the off-peak periods.[24]

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[23] The paper does not try to develop new ways of solving MPECs. Instead it focuses on the economic intuition behind the model. We do not think that the use of more complex solvers would increase the economic insight in the model. For a comparison of different methods to solve MPEC problems see Fletcher and Leyffer (2002).

[24] A better modelling of the pumped storage plants would require to take into account the capacity constraint of the water reservoir, and to make the generation and consumption decisions endogenous.

4.3. *Electricity demand*

The model has been calibrated on the basis of Belgian data for electricity demand in 2002[25]. In that year the average demand was 9.52 GW. Total yearly demand in Belgium is 83.4 TWh per Year. Figure 2 presents a histogram of demand in Belgium. The histogram is based on periodical observations with a length of 15 minutes. The highest and lowest observed demand levels were 13.7 GW and 5.8 GW, respectively.

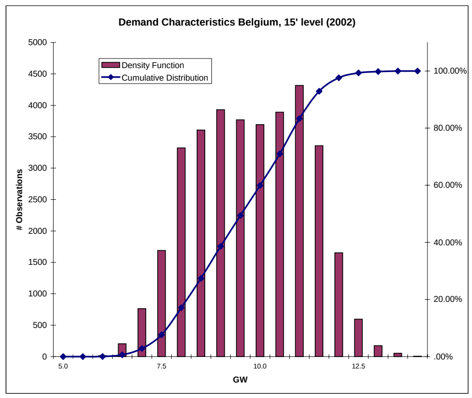

Figure 2: Histogram of electricity demand in Belgium in 2002 (Source Elia).

Extending the model from one period to multi periods

Obviously, the demand for electricity is not constant over time and in order to take this into account, the numerical one-period model has been extended to a 4-period model. For this, the model needs to be slightly extended, as one time period might have an effect on other time periods. In the present model, we distinguish four potential links.

First, cross-substitution can take place between time periods. For example, demand for electricity during the night will not only depend on the price in the night, but also on the price that is charged during the day. In this model it is assumed that these cross-substitution effects are zero. There is thus no intertemporal substitution.

Second, as was mentioned before, the consumption and generation decision of the pumped storage plants can be endogenized.

Third, there are intertemporal production constraints for generation. Generators can increase or decrease output only at a certain speed (ramping constraints). Starting up and shutting down generators is costly and requires time. These production constraints are not included in the model.

Finally, when the budget constraint of the network operator is binding, then this budget constraint creates a link between the different time periods. The marginal welfare cost of obtaining revenue for

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[25] The network of one part of Luxembourg forms an integral part of the Belgian network. Demand levels for that part are included in the model here.

the network operator should now be equal over each time period. In this paper, only the last link is taken into account.

4.4. Network operator

The network operator has total costs of $B = €649M$ per year (Source: Annual report ELIA, 2002). Capital costs are about 50% of the total costs, the other 50% being operating costs, such as wages and network maintenance costs. Wages and network maintenance costs are not directly related to the amount of MW transported over a line, they are inherent to the existence of that line. Therefore, as we could not find a more detailed description of the cost function of the network operator, we assume all costs to be fixed. Network losses are neglected in the model. Clearly, these would depend on the actual use of the network. With a total electricity demand of 83.4 TWh in 2002, the average cost of the network operator is €7.78 per MWh.

4.5. Calibration

The calibration of the model involves three steps. Each of these three steps is described below.

Fixing periodic aggregate demand and the length of each period

The first step is to decide about the level of electricity demand in each of the four periods, and about the length of each period in a standard year. This has been done on the basis of the data presented in Figure 2. This figure shows how often a certain demand level occurs in the Belgian market. We will consider 4 periods with average demand levels fixed at 8, 10, 11.5 and 12.5 GW. The length of each time period is then set such that the cumulative distribution function of the 4 periods approximates the observed cumulative distribution function (Table 2). As 500 MW of this demand is provided by small generators, the demand level as seen by the generators in our model is fixed 500 MW lower. Thus, the demand levels used to calibrate the demand functions are 7.5, 9.5, 11 and 12 GW.

Period	Observed Demand (GW)	Period Length (hr)	Period Length (%)	Model Demand (GW)	Reference price (€ per MWh)
1	12.5	208	2.4%	12.0	45.2
2	11.5	1759	20.1%	11.0	37.9
3	10.0	3410	38.9%	9.5	35.6
4	8.0	3383	38.6%	7.5	27.0
Average	***9.6***	***8760***	***100.0%***	***9.1***	***33.0***

Table 2: Calibration of the 4 time periods

Fixing a reference price for each period

Given the periodic electricity demand derived in the first step, we minimize the production costs to supply this demand. Here, it is assumed that pumped water storage can only be used in periods one and two. In periods three and four, pumped storage plants pump water into a reservoir.

Via this procedure, we obtain the marginal production cost for each period. The obtained values are increased with the average costs of the network operator (€7.78 per MWh) to obtain a reference price for each period (Table 2).

Fixing periodic electricity demand in the consumption nodes

In the third step, we derive for each node a linear demand function. The price elasticity of demand is assumed to be –0.2 in all nodes and all periods. Total demand is distributed proportionally over the different periods on the basis of the demand data in Van Roy (2001) and the reference prices are calculated in step 2. This information is sufficient to derive for each consumption node the parameters of the linear demand function.

4.6. Transit

The model also takes into account that the Belgian grid is used for relatively large transit flows. These flows are generally directed from France to The Netherlands and, as a first approximation, we impose an exogenous transit flow of 1000 MW from the south to the north. This transit is assumed to occur in all periods. The foreign generation and load nodes are summarized in Table 3.

Imports from France to Belgium are neglected. Without modeling the French generator(s), imports cannot be included in our model in a sound way. In practice, imports are about 400 MW.

Clearly, this is only an approximation. A more detailed and better modeling procedure should be the subject of further research.

	The Netherlands	
Node		
Maasbracht		-731 MW
Geertruidenberg		-368 MW
Borssele		99 MW
	TOTAL	***-1000 MW***
	France	
Node		
Avelin		543 MW
Lonny		34 MW
Moulaine		423 MW
	TOTAL	***1000 MW***

Table 3: Exogenous generation levels at the foreign nodes (Negative numbers are loads).

5. SIMULATION RESULTS

This section discusses some simulation results obtained from the model. Before starting the discussion, we first try to grasp some intuition on setting the transmission charges for consumers and generators. Subsection 5.2 discusses the simulation results in the first best case, with perfect competition in generation, and no budget constraint for the network operator.

The following subsections then will subsequently drop each of these assumptions. Subsection 5.3 adds the budget constraint to the network operator, and subsection 5.4 adds imperfect competition in generation. It considers two alternative market structures: monopoly competition and Cournot competition.

5.1. Interpreting transmission prices

It is assumed that the network operator is able to set a transmission price for generation and consumption in each node: τ_i^p and τ_i^c[26]. However, the optimal transmission charges are not uniquely defined. First, take a node i at which no consumers are connected. For that node, the consumer transmission price τ_i^c does not play a role and it can safely be set equal to zero. The same is true for nodes without generation. Here, τ_i^p is not uniquely defined and the charge is set equal to zero.

Second, note that a firm generating electricity in node i that is sold in node j, has to pay a per unit transmission charge equal to

$$\tau_{ij} = \tau_i^p + \tau_i^c$$

For the generation firm, only the total transmission charge is important, not its exact composition. The network operator has therefore one degree of freedom in setting the transmission charge components. This can easily be checked from the equations (16) and (17), and by noting that one can uniformly increase all generation tariffs with t and decrease all consumer tariffs with t without changing the sum of the charges. Indeed, the new tariffs $\tau_i^{c^*}$ and $\tau_i^{p^*}$ will then equal

$$\tau_i^{c^*} = \tau_i^c - t$$
$$\tau_i^{p^*} = \tau_i^p + t$$

but the total charge for transmission between any two nodes i, and j remains the same, *i.e.* $\tau_{ij} = \overset{*}{\tau_{ij}}$. We can therefore arbitrarily fix the consumers' transmission price in one node equal to zero. This is done for the consumption charge in the swing node:

$$\tau_i^c = 0 \quad i = \text{swing node}$$

Finally, note that the model implicitly assumes that the charges need to be paid for all consumption and generation, even if generation and consumption are located at the same node. A generator in node i who sells electricity locally does not use the transmission network, but will have to pay a transmission payment $\tau_{ii} = \tau_i^c + \tau_i^p$. We will call this charge the price wedge, because this charge creates a wedge between the consumer price and the generator price in node i[27].

The next subsection continues with a discussion of the simulation results for the first best case.

5.2. First Best

The first best case considers the situation where the network operator does not face a budget constraint and generators are competing competitively, but where the transmission capacity is limited.

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[26] Within the set of linear price structures, this is the most general assumption. It encompasses a number of 'price structure' options as special cases. For example, only charging consumption, only charging generation, a separate but uniform tariff for generation and consumption and, one uniform tariff for both generation and consumption as the most extreme case.

[27] With imperfect competition, the generator price is not defined.

Table 4 shows the simulation results in terms of welfare, the surpluses for the economic agents, the network operator costs, the generation level and the multiplier of the budget constraint (which in this case is zero by definition). The table looks at a representative hour in each period.

Peak demand is situated in period 1. The periods 2 and 3, have intermediate demand, and period 4 has off-peak demand. The first column in Table 4 gives the values for an average hour over the course of the year, taking into account the length of each period.

Table 5 shows the aggregate values for all time periods and for a whole year.

First Best		**Average**	**Period 1**	**Period 2**	**Period 3**	**Period 4**
Length of the period	**%**		**2,4%**	**20.1%**	**38.9**	**38.6%**
Welfare	(k€ per hr)	908.6	1,647.5	1,256.3	1,003.4	586.9
Consumer surplus	(k€ per hr)	772.4	1,301.8	1,031.7	837.1	539.8
Producer surplus	(k€ per hr)	170.8	305.2	230.6	194.1	107.9
Profit Network operator	(k€ per hr)	-34.5	40.5	-5.9	-27.8	-60.8
Revenue Network operator	(k€ per hr)	39.5	114.6	68.2	46.3	13.3
Fixed cost network operator	(k€ per hr)	74.1	74.1	74.1	74.1	74.1
Multiplier budget constraint	(€ per €)	-	-	-	-	-
Total generation	(MWh per hr)	9,108	11,657	10,880	9,403	7,734
Average price	(€ per MWh)	32.2	48.0	37.9	35.5	22.4
Minimum price	(€ per MWh)	18.5	26.1	23.2	17.6	15.4
Maximum price	(€ per MWh)	76.4	121.7	91.7	92.6	46.8

Table 4: First Best, results for a representative hour.

Consumption in the low and the high demand period are 7734 MW and 11664 MW, respectively. Total annual production is 79791 GWh.[28]

First Best		**Average**	**Period 1**	**Period 2**	**Period 3**	**Period 4**
Length of the period	**hrs**		**208**	**1,759**	**3,410**	**3,383**
Welfare	(M€)	7,959	342	2,210	3,422	1,986
Consumer surplus	(M€)	6,766	270	1,815	2,855	1,826
Producer surplus	(M€)	1,496	63	406	662	365
Profit Network operator	(M€)	-303	8	-10	-95	-206
Revenue Network operator	(M€)	346	24	120	158	45
Fixed cost network operator	(M€)	649	15	130	253	251
Multiplier budget constraint	(€ per k€)	-	-	-	-	-
Total generation	(GWh)	79,790	2,420	19,138	32,067	26,165

Table 5 First Best, aggregate results

In Table 4, the indicated prices are wholesale prices, covering generation as well as transmission. If all transmission charges would be set equal to zero, then in all periods the network capacity would be insufficient to satisfy the demand for transmission. Thus, congestion is an issue in the four periods, and network use must be charged in order to solve capacity problems. A welfare maximizing network operator will set charges such that distortions are minimized. The best way to do this is to tax the effective use of the network, but not the 'intra-nodal' trade, i.e. the network operator will set the price wedge equal to zero, *i.e.* $\tau_{ii} = 0 \left(\tau_i^c = -\tau_i^p\right)$. The reason for this is simple: setting a positive price wedge $\tau_{ii} = \tau_i^c + \tau_i^p > 0$, increases the distortion in the local market at node i, but only has an indirect effect on the network flows that cause the congestion. Therefore, it is best to set the price

[28] This is lower than the 83.4 TWh that was used to calibrate the demand functions. The reason for this is that we neglected the network constraints when we were calibrating. The network constraints reduce demand for electricity.

wedge equal to zero. Note that this only makes sense for nodes at which both generators and consumers are connected. If not, the price wedge does not play a role.

These congestion charges allow the network operator to collect a revenue equal to €13 300 per hour in the low demand period and €114 600 per hour in the peak demand period. Aggregated over the four periods, the network operator would cover 53% of its fixed costs by charging these congestion charges. Note that hourly congestion revenue is mainly collected in the periods 1, 2 and 3. (See Table 4). Congestion is low in period 4, so congestion revenue is rather low.

The congestion charges are node specific. They depend on how much consumption in a node affects the flows on the congested lines. In fact, the network operator uses the standard nodal pricing model[29]. As a consequence, consumers at different nodes will pay different prices for electricity. In the low demand period, prices range between €15 and €47 per MWh. In the peak period, the price range is €26 to €122 per MWh.

Table 6 shows which transmission constraints are binding in the optimum. If the line in the first column would break down, then the line in the second column would be loaded up to its thermal capacity. The last column shows the shadow price of the thermal constraint of the lines that become constrained. The shadow price expresses how much welfare would be increased if the transmission constraint would be increased with one MW for a period of one hour (€ per MWh).

The number of congested lines is highest during peak demand, and lowest during off-peak demand. This must not necessarily be the case, as will be discussed below. Finally, note that the results largely depend on the assumed distribution of demand. Better information is needed in order to get a more realistic prediction about congestion in practice.

First Best - Contingencies				
IF this line breaks		**THEN this line is at limit**		**Shadow price**
From	**To**	**From**	**To**	**(€ per MWh)**
		Period 1		
Aubange	Moulin (FR)	Aubange	Brume	155.7
Doel 2	Mercator	Doel 2	Zandvliet	11.8
Jupille	Lixhe	Gramme	Rimiere	24.0
Le Val	Seraing	Herderen	Lixhe	102.8
		Period 2		
Aubange	Moulin (FR)	Aubange	Brume	113.1
Doel 2	Mercator	Doel 2	Zandvliet	2.2
Jupille	Lixhe	Gramme	Rimiere	16.7
Le Val	Seraing	Herderen	Lixhe	66.0
		Period 3		
Aubange	Moulin (FR)	Aubange	Brume	38.6
Aubange	Moulin (FR)	Doel 2	Zandvliet	118.0
Le Val	Seraing	Herderen	Lixhe	37.2
		Period 4		
Aubange	Moulin (FR)	Aubange	Brume	21.5
Aubange	Moulin (FR)	Doel 2	Zandvliet	42.2

Table 6: Congested lines in the first best model

29 The optimal transmission prices are derived analytically in Appendix B :

Two remarks should be made before moving on to the second best case. The first remark has to do with the result that congestion is present in all periods. The second one discusses the role of the elasticity of demand.

Congestion in peak as well as in off peak periods

On first sight, one would expect that at times of low demand, the network is more likely to be uncongested. This is however not necessarily the case, as in periods of low demand only base-load plants are running and, therefore, the 'average distance' between consumption and generation nodes can increase compared to periods of peak demand. As a result the network is used more[30]. In the case of a contingency, flows are rerouted to a greater extent, as there is often no generation to provide the electricity locally.

On a larger scale this is exactly what happens on the French - Belgian interconnector during summer periods[31]. As demand is low in France, cheap French electricity, produced with nuclear power plants, is exported to Belgium and to the Netherlands. As a result, we observe congestion on the Belgian network during this period of low demand.

Elasticity of demand

The simulations in this paper assume a demand elasticity of -0.2. In general, changing the elasticity has a relatively large impact on electricity prices, but only a small impact on quantities generated and consumed. In this first best case, with congestion in all periods, the network operator will reschedule generation and consumption in order to relieve network congestion. With inelastic demand, generation will be rescheduled to a larger extent than consumption. With elastic demand, the network operator will mainly reschedule consumption.

5.3. Second Best

In the *second best* case, the network operator is faced with a budget constraint. Congestion charges now need to be increased above their first best level in order to obtain sufficient revenue to cover the remaining 47% (= 100%–53.%) of the network operator’s cost.

These increased transmission prices will result in increased wholesale prices and thus reduced demand in all periods. Aggregate demand (and generation) decreases by 1.5% to 78,592 GWh. Prices are now in the range of €18 and €47 per MWh in off-peak hours and of €30 and €123 in peak hours.

[30] Of course, this depends on the location of the base-load plants in the network. If base-load plants are not distributed homogenously in the network congestion is larger.

[31] In Northern Europe, peak demand is occurring in the cold winter months.

Second Best		Average	Period 1	Period 2	Period 3	Period 4
Length of the period	**%**		**2,4%**	**20.1%**	**38.9**	**38.6%**
Welfare	(k€ per hr)	908.3	1,647.1	1,256.0	1,003.0	586.7
Consumer surplus	(k€ per hr)	749.0	1,270.5	1,001.4	811.6	522.7
Producer surplus	(k€ per hr)	159.3	282.6	214.1	179.1	103.3
Profit Network operator	(k€ per hr)	0.0	94.0	40.5	12.4	-39.3
Revenue Network operator	(k€ per hr)	74.1	168.1	114.6	86.5	34.8
Fixed cost network operator	(k€ per hr)	74.1	74.1	74.1	74.1	74.1
Multiplier budget constraint	(€ per k€)	17.3	17.3	17.3	17.3	17.3
Total generation	(MWh per hr)	8,971	11,521	10,722	9,262	7,612
Average price	(€ per MWh)	34.8	50.8	40.7	38.3	24.7
Minimum price	(€ per MWh)	21.7	29.7	27.0	21.0	17.9
Maximum price	(€ per MWh)	76.8	122.8	93.0	92.6	46.8

Table 7 Second Best, results for a representative hour

Second Best		Average	Period 1	Period 2	Period 3	Period 4
Length of the period	**hrs**		**208**	**1,759**	**3,410**	**3,383**
Welfare	(M€)	7,957	342	2,209	3,421	1,985
Consumer surplus	(M€)	6,561	264	1,761	2,768	1,768
Producer surplus	(M€)	1,395	59	377	611	349
Profit Network operator	(M€)	0	20	71	42	-133
Revenue Network operator	(M€)	649	35	202	295	118
Fixed cost network operator	(M€)	649	15	130	253	251
Multiplier budget constraint	(€ per k€)	17.3	17.3	17.3	17.3	17.3
Total generation	(GWh)	78,590	2,392	18,861	31,585	25,753

Table 8 Second Best, aggregate results.

On average, the network operator needs to collect €74,100 per hour. Table 7 shows that the network operator collects €168,100 per hour during the peak period, and €34,800 per hour during base load periods. He makes sure that marginal deadweight loss of collecting revenue is equal in all the four time periods. Collecting €1,000 of extra revenue creates a deadweight loss of €17.3.

The network operator increases transmission tariffs, in order to cover his costs. The solution to this problem is known as Ramsey pricing. The basic idea is that prices should be increased in a way that minimizes distortions, which amounts to applying price increases that are inversely proportional to the demand elasticities.

The use of Ramsey prices has two effects. On the one hand, the higher transmission prices will decrease the total demand for transmission. On the other hand, the pattern of the flows over the network will change. In the first best, transmission quantities depend on the price levels and the marginal production costs. In the second best, transmission quantities will also depend on the demand and supply elasticities.

Table 9 shows the congested lines in the second best case. The results are very similar to the first best case.

Second best - Contingencies				
IF this line breaks		THEN this line is at limit		Shadow price
From	To	From	To	(€ per MWh)
		Period 1		
Aubange	Moulin (FR)	Aubange	Brume	156.0
Doel 2	Mercator	Doel 2	Zandvliet	10.4
Jupille	Lixhe	Gramme	Rimiere	23.2
Le Val	Seraing	Herderen	Lixhe	100.8
		Period 2		
Aubange	Moulin (FR)	Aubange	Brume	113.4
Doel 2	Mercator	Doel 2	Zandvliet	0.5
Jupille	Lixhe	Gramme	Rimiere	15.9
Le Val	Seraing	Herderen	Lixhe	63.9
		Period 3		
Aubange	Moulin (FR)	Aubange	Brume	34.1
Aubange	Moulin (FR)	Doel 2	Zandvliet	122.0
Le Val	Seraing	Herderen	Lixhe	35.2
		Period 4		
Aubange	Moulin (FR)	Aubange	Brume	15.7
Aubange	Moulin (FR)	Doel 2	Zandvliet	46.7

Table 9 Congested lines in the second best model

The role of the demand elasticities

In the case of a low elasticity of demand, demand functions will be steep and a price increase will result in a small deadweight loss because change in demand is relatively small. In the extreme case where demand is perfectly inelastic, there will be no deadweight loss because demand is insensitive to a price change. From a social point of view, covering the fixed cost of the network operator is less costly when demand functions are steeper, i.e. when demand elasticities are low.

Having congestion on top of the budget constraint does not fundamentally change the intuition. An elastic demand makes it easier to solve network congestion, but makes revenue collection more costly.

5.4. Strategic behavior of generators

The following two simulations look at how imperfect competition influences the second best model. Now, all three imperfections are present: transmission constraints, market power in generation and the budget constraint. We evaluate two scenarios for the market power in the generation market: Cournot competition and monopoly. Table 10 presents the results for both scenarios, but only presents values aggregated over the four periods. The results do not come as a surprise. Ceteris paribus, increasing competition in generation increases welfare.

The multiplier of the budget constraint of the network operator measures the net cost of giving one Euro to the network operator. The effect is about thirty times as large with monopoly as with perfect competition. There are two reasons for this. As total demand drops by almost 50% from perfect competition (= the second best) to the monopoly situation, the network operator needs to double the transmission tariffs in order to obtain the same amount of revenue. At the same time, transmission prices create a larger deadweight loss, as there is already a deadweight loss due to the strategic behavior of the monopolist.

The average price for electricity increases from €34.8 per MWh under perfect competition to €114 per MWh in a monopoly setting.

SCENARIO		First best	Second Best	Cournot	Monopoly
Welfare	(M€ per yr)	7959	7957	7310	5889
Consumer surplus	(M€ per yr)	6766	6561	3778	1827
Producer surplus	(M€ per yr)	1496	1395	3532	4061
Profit Network operator	(M€ per yr)	-303	0	0	0
Revenue Network operator	(M€ per yr)	346	649	649	649
Fixed cost network operator	(M€ per yr)	649	649	649	649
Multiplier budget constraint	(€ per k€)	-	17.3	30.6	59.8
Total generation	(GWh)	79,790	78,590	59,691	41,531
Average price	(€ per MWh)	32.2	34.8	75.7	114.4
Minimum price	(€ per MWh)	18.5	21.7	68.7	114.0
Maximum price	(€ per MWh)	76.4	76.8	92.1	114.6

Table 10: Aggregate results of the 4 scenarios

Table 11 shows the same results as Table 10, but now expressed relative to the second best scenario. In an oligopoly with 3 players, welfare is about 8% lower than with perfect competition. In a monopoly welfare is 26% lower. Consumer surplus drops by 42 % and 72 % in the oligopoly and the monopoly case, respectively.

In the Cournot model, the location of the firms in the grid might be important in determining the market power of the generators. If firms have geographically dispersed production capacities, the effect of congestion might be much smaller than when firms are geographically concentrated.

Also the ownership structure might affect the market outcome. If all firms own a diverse portfolio with base load and peak load plants, then each firm will have an incentive to withhold some production capacity at the margin, as it will increase the price it receives for the infra-marginal plants. In the opposite case where one firm only owns base load plants and another firm only peak load plants, there are fewer incentives to reduce production. These effects of location and ownership remain a topic for further research.

SCENARIO		First best	Second Best	Cournot	Monopoly
Welfare	(%)	100.03	**100.00**	91.87	74.01
Consumer surplus	(%)	103.12	**100.00**	57.58	27.85
Producer surplus	(%)	107.20	**100.00**	253.12	291.04
Profit Network operator	(%)	-	**-**	-	-
Revenue Network operator	(%)	53.37	**100.00**	100.00	100.00
Fixed cost network operator	(%)	100.00	**100.00**	100.00	100.00
Multiplier budget constraint	(%)	-	**-**	-	-
Total generation	(%)	101.53	**100.00**	75.95	52.84
Average price	(%)	92.42	**100.00**	217.38	328.57
Minimum price	(%)	85.27	**100.00**	316.25	524.63
Maximum price	(%)	99.55	**100.00**	119.97	149.36

Table 11: Relative performance of the 4 scenarios (second best = 100%).

6. CONCLUSIONS

This paper looks at the socially optimal transmission prices in a congested network when there is imperfect competition in electricity generation and when the network operator has a binding budget constraint. It shows that generators and consumers have to pay different transmission prices in the social optimum. These differences reflect the fact that the network operator needs to collect revenues and that the generation sector is not competitive.

The model in this paper has imperfect competition in the generation market, but assumes that generators are price takers in the transmission market. The network operator is a Stackelberg leader and sets the transmission price before generators decide about generation and sales. The model is illustrated with some simulation runs. It studies 4 scenarios: a first best scenario, a second best scenario, a second best scenario with Cournot competition and second best scenario with a monopoly in generation. The parameterization of the model is inspired by the technical characteristics of the Belgian electricity system. It includes the Belgian high voltage transmission grid and the lines in France and the Netherlands which are important for the Belgian network. The network is presented as a linearized DC-load flow model. Transmission is limited by the thermal constraints of the lines and $n-1$ security constraints are imposed.

The model provides an analytical framework that can help policy makers to think about transmission tariffs. It shows how prices should be adapted in response to changes in market power, and in which way revenue should be collected. The model links some of the standard regulation literature on Ramsey pricing with the electricity literature on optimal pricing of transmission networks. The qualitative results that come out of the model can be very informative, but some reservation is at place if one would consider implementing such a pricing scheme. There are a number of reasons for this.

We assume that generators are price takers in the transmission market, but, in practice, generators might abuse their locational market power. Other types of models are needed if there is such market behavior. (See for example Borenstein, Bushnell *et al.* (2000))

The model neglects entry in generation, and only derives short run optimal prices. These prices might not give the right long run incentives for investing in new generation capacity.

The network operator is assumed to have perfect information about generation costs and demand. In practice, this information is not readily available. Any mechanism to allocate transmission capacity will have to take into account this information asymmetry.

It can also be the case that the network operator does not maximize welfare, but rather profits or the interests of some political pressure groups. This is the reason why the network operator is regulated and is limited in setting transmission prices. A companion paper studies the strategic behavior of the network operator.

The previous list already provides a first set of possible extensions for the model, but with some other extensions, the model can be used to study many relevant policy issues concerning electricity markets. A short, but non-exhaustive survey of possible extensions and applications follows.

Many countries (such as Belgium) are located between low price countries (France) and high price countries (the Netherlands) and serve as a transit country. The model in the paper could be used to calculate the welfare impact of different levels of transit. For example, as most of the congestion on the Belgian network involves international transactions, it would be interesting to also include the Dutch, the German, and the French generation markets and networks, as in Day, Hobbs *et al.* (2002). The model could calculate how costs can be allocated between the different countries.

Consumers do not resell electricity and there is no arbitrage in the model. This could be included and its impact could be studied. Appendix A shows how arbitrage can be introduced.

The current paper assumes that generators are competing à la Cournot. One could consider generators to compete with conjectured supply functions, as shown in Day, Hobbs *et al.* (2002).

The model could be used to study the impact of vertical partial ownership structures in the electricity sector. This is done in a companion paper which studies a profit maximizing network operator when there are cross-ownership relations between the network operator and one of the generators.

Appendix A : Arbitrage

The model of Smeers and Wei (1997) has been extended by Metzler, Hobbs *et al.* (2003) and Hobbs (2001) to include arbitrage. This appendix shows how arbitrage can be introduced in the model presented in this paper.

An arbitrageur can be modeled as an extra generator with index $a \in F$ who has no generation capacity $(G_a = \varnothing)$ and who is price taker in both the energy and the transmission market.

The arbitrageur maximizes

$$\max_{s_{ai}} \quad \sum_{i \in I} (\, p_i - \tau_i^c \,) \cdot s_{ai}$$

Subject to

$$\sum_{i \in I} s_{ai} = 0 \quad (\lambda_a^p) \tag{26}$$

The arbitrageur's first order conditions are

$$(\, p_i - \tau_i^c \,) = \lambda_a^p \quad \forall i \in I \tag{27}$$

With arbitrage, the price difference between two nodes need to be equal to the differences in congestion charges for consumers.

$$(\, p_i - p_j \,) - (\, \tau_i^c - \tau_j^c \,) = 0 \tag{28}$$

Intuition: If the price p_i is too high, arbitrageurs will buy electricity from consumers in region j and sell in region i. The value of electricity for consumers in node j is p_j. Consumers reducing consumption with one unit in node j will save τ_j^c on their transmission bill. They will therefore sell their electricity to arbitrageur for a price $p_j - \tau_j^c$. The arbitrageur will resell the electricity to consumers in region i at a price $p_i - \tau_i^c$ as consumers in region i have to pay the congestion charge τ_i^c before consuming.

Appendix B : Optimal transmission prices

This appendix derives the socially optimal transmission tariffs under the scenarios discussed in 2.1. The same notation as in the body of the paper is used, but some simplifying assumptions are made in order not to mess up the results.

We assume that at each node there is exactly one generation plant, which is owned by one firm. Using the new assumptions we can drop the indices for generators g and for plants f. The plant at node i produces q_i units of electricity at a cost $C_i(q_i)$. In order to keep the network operator's problem tractable, we assume that the generation constraints are not binding. $\overline{\mu_i} = \underline{\mu_i} = 0$. Thus, we assume that firms will not choose corner solutions. This is the case when the marginal costs are zero for zero output and sufficiently large at full output. In each node i there are consumers with a inverse demand function $p_i(s_i)$.

Model 1: Reference model

We start with the reference case where there is ample transmission capacity, no generation market power and no budget constraint for the network operator.

The network operator maximizes welfare

$$W = \sum_{i \in I} \left(\int_0^{s_i} p_i(t)\,dt - C_i(q_i) \right) \tag{29}$$

subject to the energy balance

$$\sum_{i \in I} (s_i - q_i) = 0 \tag{30}$$

Socially optimal allocation

Using the first order conditions, one can easily derive the following condition for the social optimum

$$p_i(s_i) = p_j(s_j) = C'_i(q_i) = C'_j(q_j) \tag{31}$$

Consumers pay and generators receive the same price in all nodes. Equation (31) defines the socially optimal allocation of production and consumption.

Transmission prices

As there is perfect competition, arbitrage is perfect, meaning that the price difference between node i and the swing node equals τ_i^c (Remember that we normalized the consumers' transmission price to zero at the swing node, i.e. $\tau_{swing}^i = 0$).

$$\tau_i^c = p_i - p_{swing} \tag{32}$$

Perfect competition also implies that the difference of marginal cost and the price at the swing bus is equal to τ_i^p

$$\tau_i^p = p_{swing} - C_i'(q_i) \tag{33}$$

Given the socially optimal allocation (31), it is clear that all transmission tariffs are equal to zero:

$$\tau_i^c = \tau_j^p = 0 \tag{34}$$

The intuition is straight forward: any non-zero transmission price would create distortions in the market and would decrease welfare.

Model 2: Congestion (Optimal nodal spot price – First Best)[32]

The second model assumes scarce transmission capacity.. Setting transmission prices equal to zero would create a demand for transmission that outweighs available transmission capacity. In order to eliminate congestion, the network operator will price transmission at its opportunity cost.

The network operator now maximizes welfare subject to the energy balance and the network constraints

[32] This model is the first best described in section 5.2.

$$\sum_{n \in I/\{\text{swing node}\}} \theta_{a,n}(q_n - s_n) \leq \bar{Q}_a \qquad \forall a \in A \qquad (\mathrm{T}_a) \tag{35}$$

with $\theta_{a,i}$ the Power Transmission Distribution Factors, and T_a the Lagrange multiplier of the transmission constraint, i.e. the opportunity price of using line a.

Socially optimal allocation

The first order conditions impose that price equals marginal cost at each node i, i.e.

$$C_i\,'(q_i) = p_i(s_i) \tag{36}$$

and that the transmission price from the swing bus to node i reflects the opportunity costs of the transmission lines, i.e.

$$p_i = p_{swing} - \sum_a \theta_{a,i}\mathrm{T}_a \tag{37}$$

The opportunity price for transmission is equal to zero $\mathrm{T}_a = 0$, as long as the line is not congested, i.e. $Q_a \leq \overline{Q_a}$. With a congested line, the transmission price becomes positive:

$$\begin{aligned} &\text{if} \quad \mathrm{T}_a > 0 \quad \text{then} \quad Q_a = \bar{Q}_a \\ &\text{if} \quad \mathrm{T}_a = 0 \quad \text{then} \quad Q_a \leq \bar{Q}_a \end{aligned} \tag{38}$$

The equations (36) and (37) describe the socially optimal allocation and are the well known nodal spot prices (Schweppe, Caramanis *et al.* (1988)). Prices will be different in each node in the network, reflecting the regional differences in generation costs and demand functions and the scarcity of transmission capacity.

The first equation implies that there is no intra-nodal wedge between the generator's price and the consumer's price. The intuition behind the second equation is the following: If a consumer in node i buys one unit of electricity from the swing bus, then the flow on line a will increase with $\theta_{a,i}$. With T_a being the opportunity cost for using transmission line a, the total payment for a consumer at node i is equal to $\tau_i^c = -\sum_a \theta_{a,i}\mathrm{T}_a$.

Transmission prices

As there is perfect competition, equations (32) and (33) are still satisfied. Using the socially optimal allocation one can derive that the transmission prices satisfy the following equations:

$$\begin{aligned} \tau_i^c &= \tau_i^p \\ \tau_i^c &= -\sum_a \theta_{a,i}T_a \end{aligned} \tag{39}$$

The intuition for this was already mentioned in section 5.2. Setting a positive intra-nodal price wedge $(\tau_{ii} = \tau_i^c + \tau_i^p > 0)$, increases the distortion in the local market at node i, but only has an indirect effect on the network flows that cause the congestion.

Model 3: Budget constraint (Ramsey prices)[33]

The third model has ample transmission capacity, perfect competition in generation, but a binding budget constraint for the network operator. The network operator maximizes welfare, subject to the energy balance and the budget constraint

$$\sum_{i \in I} \left(\tau_i^c s_i + \tau_i^p q_i \right) \geq B \tag{40}$$

Using the energy balance and perfect competition allows to rewrite the budget constraint as

$$R \equiv \sum_{i \in I} \left(p_i\left(s_i\right) \cdot s_i - C_j^{'}(q_j) \cdot q_i \right) \geq B \qquad (\lambda) \tag{41}$$

with λ the multiplier of the budget constraint.

Socially optimal allocation

The first order condition of the network operator imposes for all nodes i and j that

$$-\frac{p_i(s_i) - C_j^{'}\left(q_j\right)}{\frac{\partial R}{\partial s_i} - \frac{\partial R}{\partial q_j}} = \lambda \tag{42}$$

with $\frac{\partial R}{\partial s_i}$ and $\frac{\partial R}{\partial q_i}$ the marginal revenue functions of the network operator. These can be expressed as a function of demand and supply elasticities $\varepsilon_i = \left|\frac{\partial p_i}{\partial s_i}\right| \frac{s_i}{p_i}$ and $\omega_i = \frac{C_i^{''}(q_i)}{C_i^{'}(q_i)} q_i$ in node i, i.e.

$$\begin{aligned} \frac{\partial R}{\partial s_i} &= p_i \frac{\varepsilon_i - 1}{\varepsilon_i} \\ \frac{\partial R}{\partial q_i} &= C_i^{'} \frac{\omega_i + 1}{\omega_1} \end{aligned} \tag{43}$$

Equation (42) reflects that for any trade between node *i* and node *j* the network operator needs to trade off his revenue with the dead weight loss that he creates by setting positive transmission prices. The social value of producing one unit of electricity in node j, transporting it, and consuming it in node i is $p_i - C_j^{'}$. But in order to transport one extra unit, the network operator has to lower the total transmission tariffs between the two nodes, which results in a revenue loss equal to $\frac{\partial R}{\partial q_j} - \frac{\partial R}{\partial s_i}$.

With a nonbinding budget constraint $(\lambda = 0)$, equation (42) simplifies to (34). The condition then imposes that price is equal to marginal cost in each node.

With a binding budget constraint $(\lambda > 0)$, there is a wedge between the consumer's price and the generator's price at a node i.

Transmission prices

[33] Note that the network is not congested, therefore it is not equal to the second best scenario in section 5.3.

As there is perfect competition in generation,, the arbitrage condition (32) is satisfied. Using the socially optimal allocation (42), this give the following transmission price for consumers:

$$\tau_i^c = \lambda\left(p_{swing}\frac{\varepsilon_{swing} - 1}{\varepsilon_{swing}} - p_i\frac{\varepsilon_i - 1}{\varepsilon_i}\right) \tag{44}$$

In the same way, we find the transmission charges for generators by using equation (33)

$$\tau_i^p = \lambda\left(C_j^{'}\frac{\omega_j + 1}{\omega_j} - p_{swing}\frac{\varepsilon_{swing} - 1}{\varepsilon_{swing}}\right) \tag{45}$$

With linear inverse demand functions given by $p_i = \alpha_i - \beta_i s_i$*, the consumer prices are the following* $\tau_i^c = \lambda\left(\alpha_{swing} - \alpha_i\right)$.

Model 4: Imperfect competition

In the fourth model, we assume that there is ample transmission capacity, no budget constraint, but imperfect competition. The network operator maximizes welfare, subject to the energy balance and taking into account the behavior of the monopolist. The latter is described by his first order conditions, i.e.

$$p_i\left(s_i\right) + \frac{\partial p_i\left(s_i\right)}{\partial s_i}s_i - \tau_i^c = \lambda^p \tag{46}$$

$$C_i^{'}\left(q_i\right) + \tau_i^p = \lambda^p \tag{47}$$

Socially optimal allocation

In the optimum the network operator subsidizes the monopolist such that the prices and marginal costs are equalized over all nodes:

$$p_i(s_i) = p_{swing} \tag{48}$$

$$C_i^{'}(q_i) = p_{swing} \tag{49}$$

Equations (48) and (49) describe the socially optimal allocation.

Transmission prices

Because of imperfect competition in generation, the equations (32) and (33) are no longer valid. In order to find the optimal transmission prices, we will need to take into account the behavior of the monopolist as described by the equations (46) and (47).

First, the monopolist will shift sales between the two regions until the difference in transportation costs is equal to the difference in marginal revenues in the two regions, i.e.

$$\tau_i^c = p_i\frac{\varepsilon_i - 1}{\varepsilon_i} - p_{swing}\frac{\varepsilon_{swing} - 1}{\varepsilon_{swing}} \tag{50}$$

Second, the monopolist will choose his production level by setting his marginal revenue at the swing bus equal to the marginal production costs augmented with the transmission component.

$$\tau_i^p = p_{swing} \frac{\varepsilon_{swing} - 1}{\varepsilon_{swing}} - C'_i \tag{51}$$

The equations (48) - (51) define the transmission tariffs.

For linear inverse demand, the imperfect arbitrage condition (50) *becomes*

$$\tau_i^c = (\alpha_i - 2\beta_i s_i) - (\alpha_{swing} - 2\beta_{swing} s_{swing}) \tag{52}$$

and the social optimality implies

$$\alpha_i - \beta_i s_i = \alpha_{swing} - \beta_{swing} s_{swing} \tag{53}$$

Solving equations (52) and (53) for the transmission price gives:

$$\tau_i^c = \alpha_{swing} - \alpha_i \tag{54}$$

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