The Length of Contracts and Collusion

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Many commodities (including energy, agricultural products and metals) are sold both on spot markets and through long-term contracts which commit the parties to exchange the commodity in each of a number of spot market trading periods. This paper shows how the length of forward contracts affects the possibility of collusion in a repeated price-setting game. We find that as the duration of contracts increases, collusion becomes harder to sustain. Nevertheless, firms with low discount factors that would not be able to sustain collusion without contracts, can always sustain some collusive prices above marginal cost, provided that they sell enough contracts. Hence long-term contracts have an ambiguous impact on collusion. Such ambiguity is due to the interaction of two effects, the gain-cutting effect, which reduces the immediate gain from defection, and the protection effect, which reduces the amount of punishment that deviators can receive.

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1 Introduction

Many commodities are sold both through long-term contracts and on spot markets. The seminal result on the interaction between spot and forward markets (Allaz and Vila, 1993) is that the presence of a forward market will make the spot market more competitive – firms are competing only over the unsold portion of the overall demand, and face a more elastic residual demand curve. More recently, however, some authors have suggested ways in which forward markets could be used to make the spot market less competitive. One possibility is that firms could buy in the forward market to increase their exposure to the spot market, and make their residual demand less elastic (Mahenc and Salanié, 2004). Another is that contracts could increase the likelihood and severity of collusion (Ferreira, 2003, Le Coq, 2004, Liski and Montero, 2004). The key intuition of these papers is that a firm which defects from a collusive agreement will not be able to capture the demand already covered by contract sales. Compared to the case with no contracts, this reduces the gains from defection without changing the punishment path, and therefore makes collusion easier to sustain. We show later that the profits on the punishment path will be increased by contracts that last for more than one spot period, and that these contracts can make collusion harder to sustain. Nevertheless, firms can sustain *some* collusive price above marginal cost, however long their contracts last, if they sell enough of them.

Empirical evidence suggests that most forward contracts last for many spot market trading periods. For example, two-thirds (by volume) of the gas trades reported in Britain on December 3, 2004 were for deliveries spread over a month or more, as were more than 90% of the electricity trades (Heren, 2004a,b). Fifty percent of California winegrape producers have contracts of more than one year with an average length of 3.5 years, the most frequent contract lengths being 3, 5 and 10 years (Goodhue, Dale Heien and Hyunok, 1999). In the U.S. agricultural industry, many contracts are signed for 3 months (20% for cattle and 35% for poultry) or for 3-12 months (80% and 49% respectively) (USDA, 1993). For non-ferrous metals traded on the London Metal Exchange similarly, contracts range from one day to many years (Slade and Tille, 2004 or www.lme.com).

The terminology used for short- and long-term markets can differ between industries. In some industries there is no obvious spot market, but short-term and long-term contracts coexist. Our model is applicable to the interaction between sales of short-term and long-term contracts in

these industries, as well as to those in which there is a (more or less) organised spot market. However, we will always use the term "spot" market to refer to the short-term market and forward contract to refer to the longer-term commitments.

While the papers we have cited focus on the way that some types of contracts can reduce the gains from defection and make collusion easier, we argue that when forward sales last for more than one spot market period, they have a second effect on collusion. If the contract has locked in the price received for part of the firm's output, it cannot be punished on these sales after a deviation. This will tend to make collusion harder to sustain. The aim of this paper is to investigate how the length of the forward contract affects the ability to collude on the spot market. We show that the longer the contract lasts, the more difficult it is to sustain collusion. Some firms, however, which would not be able to collude in the absence of contracts, are able to sustain some collusive prices, whatever the length of contracts.

The mechanism works as follows. Assume that firms compete in price repeatedly on the spot market. There is a contract round, which takes no time, before the first spot period. Firms can offer a contract that stipulates a quantity, a price, and a length. A contract is an agreement where a firm (buyer) commits to sell (to buy) a quantity at a price in each of a number of spot market periods. The length of the contract is defined as the number of spot periods for which the agreement stands, and is set exogenously. As soon as these contracts expire, another contract round is held. Assume also that firms collude on a price above marginal cost with a trigger strategy. Any contract reduces the demand to be met in the spot market, reduces the short-term gain from defection, and therefore makes collusion more attractive. The payoff associated with the punishment path however depends on the length of the contract. As soon as deviation occurs, the punishment is to offer the competitive price in the spot market in every period afterwards, and sell contracts at this price from the next contract round onwards. This punishment immediately affects profits in the spot market, but the deviating firm still receives profits from its contracts, until the next contract round. While the defecting firm cannot take away the sales its rival has covered by contracts in the period in which it defects, its rival cannot take away the sales the firm has covered by contracts made before the defection. This reduces the severity of the punishment that can be inflicted for defection. The longer the contracts last for, the greater the reduction of the punishment. The contracted quantities act as a "protection" during the length of the contract. The subject of the paper is how these two effects – the lower gain from deviation

and the reduced punishment afterwards – interact as we vary the length of the contracts. We call the first effect "gain-cutting" and the second the "protection effect".

We are not aware of any theoretical literature that has paid attention to the length of the contract.[1] Allaz and Vila allow the number of rounds of forward trading, N, to vary, but all the trades concern a single production period. We have a single round of trading for each contract, but allow it to hold for repeated production periods. Our model is thus an infinite repetition of theirs, in the special case of $N=1$, but with price instead of quantity competition in the spot market. Liski and Montero focus on repeated short-term contracting when contract rounds alternate with spot trading periods and the contracts are only valid for a single spot period, although they may be traded in many rounds before they take effect. We consider contracts of any length, with α spot periods between each contact round, making our paper a more general version of one of their models.[2]

With different contract lengths and an infinitely repeated game we offer a general setup described in the next section. In section 3, we give the general conditions for collusion to be sustainable. Section 4 shows how the discount factor needed to sustain the collusive price varies with the proportion of the collusive output covered by contracts, and the duration of those contracts. This allows us to demonstrate the two effects at work. Section 5 discusses how the collusive price varies with the discount factor, the proportion of output sold through contracts, and their duration, and shows that firms can maintain some collusive price above marginal cost for *any* positive discount factor, given appropriate contracts. Section 6 illustrates our results by using a linear demand example. Section 7 concludes.

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[1] One exception is Guriev and Kvasov (2006), who consider how the contract duration affects incomplete contract theory, which is a completely different setting to ours. Moreover, to the extent that contracting is a form of vertical integration, our work is related to Baldursson and von der Fehr (2005) who focus on risk and hedging motives to vertically integrate.

[2] We do not however consider forward buying as they do. If a firm is able to enhance its market power by buying its product in the forward market, we would expect the antitrust authorities to take action if they ever observed such behaviour.

2 The model

Consider two firms, *1* and *2*, who produce a homogeneous good with a constant marginal cost *c* and no capacity constraints. They can sell their good on either the contract market or the spot market. The firms have a common discount factor, δ.

Timing. Firms compete in price repeatedly on the spot market (taking place in all periods *t* =1,2,3,...). There is a contract round, which takes no time, before the first spot period. The contracts sold in this round will last for $\alpha > 0$ spot periods. As soon as they expire, there is another contract round, followed by α spot periods, and so on.

Demand. In each period, the demand is given by *D(p)*, which is a decreasing and continuous function of the price *p*. This demand can be met either by sales in the spot market or by commitments made under forward contracts – the forward sales are subtracted from the total demand when determining the demand remaining in the spot market.

Contract market. In each contract round, the two firms simultaneously choose the amount of forward contracts they want to sell in the forward market that call for delivery of an equal volume in each of the next α spot periods. Payment is made at the time(s) of delivery, avoiding the need to use different amounts of discounting for spot and contract sales. It is convenient to work in terms of the proportion of output sold in the contract market, rather than its volume. We define $x \in [0,1]$ as the proportion of contract sales, relative to the total output the firm would sell in the collusive equilibrium (spot and forward sales).[3] Note that α and *x* are fixed exogenously, but neither the contracted quantity nor the contracted price is.

No arbitrage. We assume that the contracts will only be accepted by purchasers if the contract price is equal to the expected price in the spot market. In other words, the firms will be able to sell contracts at a price of p^c, if and only if they will be able to sustain this price in the spot market, given their discount factors and contract sales. The firms' contract positions become common knowledge at the end of each contract round (i.e., before the first spot period for which those contracts apply).

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[3] We follow Liski and Montero's notation, but we do not allow for over-contracting or forward purchases, requiring that $x \in [0,1]$.

Spot market. In each spot period, the firms simultaneously bid prices, and the firm with the lower price serves the entire spot demand – the total demand at that price, less the forward commitments by each firm. If the firms submit the same price bid, they share the spot demand equally. We assume an efficient-rationing rule.

General profit functions. Aggregate profits are given by $\pi(p) = (p-c)D(p)$ and are assumed to be single peaked with a unique maximum at $\pi(p^m)$. Let $\pi^m (= \pi(p^m))$ denote the per-period profit earned by a monopoly firm. Let $\pi^c (= \pi(p^c))$ denote the per-period profit earned by each firm from collusion at the price p^c. And finally π^d denotes the firm's profit during the deviation period, taking its spot and contracted output together.

3. Sustaining Collusion

We assume that the firms will attempt to collude by using (symmetric) trigger strategies:

In the first contract round, each firm offers long-term contracts to sell $xD(p^c)/2$ at the collusive price p^c, where $D(p^c)/2$ is the output it would produce if it shared the (overall) market equally at this price. If both firms have followed the collusive equilibrium so far, then in each spot period, the firm will bid a spot price of p^c, sharing the spot market demand equally with the other firm. If at any point in time anyone is detected cheating in any previous contract round or spot period, then both firms will bid marginal cost, c, in each subsequent spot period. In every subsequent contract round after a deviation has occurred, the firms will sell an arbitrary volume of contracts, at the same price, c, as long as their combined sales do not exceed the market demand at marginal cost, $D(c)$.[4]

We now investigate whether these strategies can sustain collusion against an optimal deviation. In principle, a firm can deviate by either increasing its forward sales on the contract round or undercutting its spot price. Deviating during the contract round is never optimal, as shown by Liski and Montero (2004). Contracts are fully observable, and buyers know the firms' strategies. If one firm deviates in a contract round, the spot price in every succeeding spot period

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[4]Players revert to the static Nash equilibrium and remain there forever after any deviation. Exactly as in a repeated Bertrand competition, unrelenting trigger strategies are "optimal punishments" in our setting, since the players are at

will be equal to c. Knowing this, no buyer would pay more than c in the forward market to a seller that is attempting to deviate from its collusive strategy. The profit from deviation will thus be zero. Any collusive strategy that gives a profit of more than zero is thus proof against collusion occurring in a contract round. We can then state:

Lemma 1. *It is never optimal to deviate during a contract round.*

Thus, we need only to concentrate on deviations in the spot market. If there were no contracts, then the optimal deviation from any $p^c \in (c, p^m]$ would be to charge p^c-ε. Given that at the opening of the spot market in period t there is an already secured supply of $xD(p^c)$ units coming from firms' forward obligations signed in the most recent contract round, this may not be the optimal deviation in our model. If the residual demand in the spot market is too low, p^c may be above the monopoly price associated with that demand, and hence too high to be an optimal deviation.

Define p^{Rm} as the monopoly price associated with the residual demand, and given by

$$p^{Rm} = p^{Rm}(x, p^c) \equiv \arg\max (p - c)(D(p) - xD(p^c)) \tag{1}$$

The optimal deviation price is then defined as

$$p^d = p^d(x, p^c) \equiv \min\left[p^c - \varepsilon \,,\, p^{Rm} \right] \tag{2}$$

The firm's spot market profits from charging the optimal deviation price are given by π^{sd}, where

$$\pi^{sd} = \pi^{sd}(x, p^c, p^d) = (p^d - c)(D(p^d) - xD(p^c)) \tag{3}$$

The firm's total profits in the spot period during which it deviates are thus equal to

$$\pi^d = \pi^d(x, p^c, p^d) = \pi^{sd} + x\pi^c \tag{4}$$

In the punishment phase, starting just after the deviation period, the firms offer c on the spot market forever and offer forward contracts at that price in all following contract rounds. As a result, every subsequent spot profit is equal to zero. However, until the current contracts expire, firms still sell their contracted quantities at the collusive price, p^c. The punishment does not affect those quantities until the next contract round starts.

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their security levels. Expressed differently, no complex punishment mechanism can enlarge the set of supportable equilibria (Abreu, 1986).

This affects the optimal timing of a deviation. The present value of the profits from collusion continued forever equals $\pi^c/(1+\delta)$ in any spot market period. This includes profits of $x\pi^c$ from forward sales and $(1-x)\pi^c$ from sales in the spot market. The present value of the profits from a deviation depends on the number of spot market periods remaining before the next contract round. Consider a deviation with $\tau \in \{1,2,...\alpha\}$ periods remaining until the next contract round. We can write the profits during the punishment phase starting after the deviation period, discounted to the deviation period, as $x\pi^c \sum_{t=1}^{\tau-1} \delta^t$. The present value of the overall profits from deviation is thus equal to

$$PV^{deviation} = \pi^d + x\pi^c \frac{\delta - \delta^\tau}{1-\delta} \tag{5}$$

This allows us to state:

Lemma 2. *A deviation is the most profitable when it occurs in the spot period immediately after a contract round.*

Proof: Immediate, from inspection of equation (5), which is maximised when τ is at the highest possible value, implying $\tau = \alpha$. Since the profits from continued collusion do not depend on time, choosing the highest level of deviation profits is sufficient to choose the best time to deviate. ∎

The collusive agreement described above is sustainable as a subgame-perfect equilibrium as long as neither firm has an incentive at any period to defect unilaterally from the collusive agreement. The punishment phase (i.e., reversion to marginal cost forever) is subgame perfect, so we need to find the condition under which deviation from the collusive path is not profitable for either firm. Setting $\tau = \alpha$ in equation (5), this condition is equivalent to

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta - \delta^\alpha}{1-\delta} x\pi^c = \pi^{sd} + \frac{1-\delta^\alpha}{1-\delta} x\pi^c \tag{6}$$

The first part of the RHS of the inequality (6) gives the deviation profit that includes profit from the residual spot demand and the forward sales from the deviation period. The second part of the

RHS gives the profits that the firm will obtain from its forward sales after the deviation occurs until the period immediately before the next contract round.

Equation (6) can be rewritten as:

$$[1 - (1 - \delta^\alpha)x]\pi^c \geq (1 - \delta)\pi^{sd} \tag{7}$$

Let us define

$$h(x, \alpha, \delta, p^c) = [1 - (1 - \delta^\alpha)x]\pi^c - (1 - \delta)\pi^{sd} \tag{8}$$

We define the minimum discount factor as $\underline{\delta} = \underline{\delta}(x, \alpha, p^c)$ such that $h(x, \alpha, \underline{\delta}, p^c) = 0$. Hence we can state

Proposition 1. *The grim trigger strategies described above constitute a subgame perfect equilibrium if and only if $\delta \geq \underline{\delta}(x, \alpha, p^c)$, where $h(x, \alpha, \underline{\delta}, p^c) = 0$.*

Proof. The collusion is sustainable for any discount factor satisfying inequality (7). From the derivation of h, we know that this is equivalent to saying that $h(x, \alpha, \underline{\delta}, p^c) \geq 0$. We also have:

$$\frac{\partial h(x, \alpha, \delta, p^c)}{\partial \delta} = \alpha \delta^{\alpha - 1} x \pi^c + \pi^{sd} > 0 \quad \forall p^c > c \tag{9}$$

This means that any discount factor greater than $\underline{\delta}$ will give $h > 0$, and hence satisfy inequality (6), making collusion sustainable. ∎

Proposition 1 sets out the sufficient condition for a collusive equilibrium, but does not characterise that equilibrium. The following sections will do that, concentrating first on the interaction between the minimum discount factor and the length of contracts, and second on the price that can be sustained through collusion.

4.   The critical discount factor

Why does the level of forward contracting affect the critical discount factor at which collusion can just be sustained? There are two effects playing in opposite directions. First there is a pro-collusive effect (captured by the variable π^d in the inequality (6) that we call the *gain-cutting effect*. Second there is a pro-competitive effect (captured by the term $(1 - \delta^\alpha/1 - \delta)x\pi^c$ in the inequality (6)) which we call the *protection effect*.

Assume that firms have signed contracts ($x > 0$). A firm which defects from a collusive agreement will not be able to capture the demand already covered by contract sales. This reduces the gains from defection, for a given collusive price and discount factor, allowing the firms to sustain a higher collusive price, or making collusion possible with a lower discount factor. We say that the forward sales have a *gain-cutting effect* that increases with the amount of forward sales (because we have $\partial \pi^{sd}/\partial x < 0$). The more forward sales there are, the less the deviation profit is, which tends to make it easier to collude.

Forward sales can have a second impact on collusion, however, if they last for more than one period. While the defecting firm cannot take away the sales its rival has covered by contracts in the period in which it defects, its rival cannot take away the sales the firm has covered by contracts made before the defection. Hence if $\alpha > 1$, a firm that defects in the first spot period after a contract round will continue to receive the fixed forward price for part of its output, until the next contract round occurs. This reduces the severity of the punishment that can be inflicted for defection. The longer the length of the contracts, the greater this reduction, and the more protected are the firms. This *protection effect* increases with the length of the contract (since the coefficient $(1 - \delta^\alpha/1 - \delta)$ increases in α) as well as with the contracted quantity.

It is important to notice that the two effects do not play at the same time. The gain-cutting effect appears (during the deviation period) before the protection effect. Moreover the gain-cutting effect always exists as soon as $x > 0$, while the protection effect only appears when $\alpha > 1$. The overall impact on the sustainability of collusion depends upon the interaction of the two effects.

We now explore how the minimum discount factor $\underline{\delta}$ varies with the length of the contract (α) and with the contracted quantity (x). Before exploring the general results of Proposition 1, we discuss two simple examples that allow us to relate to previous literature.

No contracting case (Repeated Bertrand game). Assume that firms hold no contracts ($x = 0$) and agree to collude on the monopoly price; the game is identical to the traditional repeated price game. By sticking to the agreement, a firm receives $\pi^m / 2$ which corresponds to half the aggregate monopoly profit. When $x = 0$, the optimal deviation is to undercut the collusive price by a

fractional amount, capturing the entire market demand at this price, and obtaining twice the collusive profit in that period. In all future periods, the unilateral deviation triggers retaliation from the other firm. As a result, in all subsequent periods firm i earns the static Nash equilibrium profits, that is $\pi^N = \pi(c) = 0$. We can rewrite the inequality (7) as $\pi^m/(1-\delta) \geq 2\pi^m$. From Proposition 1, if there are no contracts, firms can sustain the monopoly price if their discount factor is ½ or more, and cannot sustain any price above marginal cost for any lower discount factor (Tirole, 1988). With no contracting, neither the gain-cutting effect nor the protection effect exists.

One period contracting case. Assume firms offer one-period contracts ($x > 0$ and $\alpha = 1$) and agree to collude on the monopoly price p^m, so that $\pi^c = \pi^m/2$. It is straightforward to show that whenever $x>0$ and $p^c = p^m$ the optimal deviation price is the residual monopoly price p^{Rm} defined by equation (1). Since contracts only last for one period, the punishment phase starts immediately after the deviation period. As a result, in all subsequent periods firm i earns the static Nash equilibrium profits, that is $\pi^N = \pi(c) = 0$, and there is no protection effect. Collusion is sustainable whenever $\pi^m/(1-\delta) \geq \pi^{sd} + x\pi^m$. Recall that from equation (3) we have $\pi^{sd} = 2(1-x)\pi^m$, and so the condition can be rewritten as $1/(1-\delta) \geq 2 - x$. Since the gain-cutting effect becomes stronger as the level of contracting rises, the critical discount factor is strictly decreasing in the level of one-period contracting. This explains the main result with one-period contracting (studied first by Liski and Montero, 2004): forward trading allows firms to sustain collusive profits that otherwise would not be possible. When we allow for longer contracting this result does not always hold.

Proposition 2. *The critical discount factor is strictly increasing in the length of the contract.*

Proof. We have $h_\delta = \alpha\delta^{\alpha-1}x\pi^c + \pi^{sd}$, which is clearly positive for prices above marginal cost. We also have $h_\alpha = x\pi^c\delta^\alpha \ln(\delta) < 0$, since $\delta < 1$. By the implicit function theorem, we have $d\underline{\delta}/d\alpha = -h_\alpha/h_\delta$. Using the above inequalities we can conclude that the critical discount factor $\underline{\delta}$ is strictly increasing in the length of the contract ($d\underline{\delta}/d\alpha > 0$).∎

With one period contracting, the firm knows that in every period after its defection its profits will be reduced to zero. If the contract lasts for more than one period ($\alpha > 1$), however, a firm that defects in the first spot period after a contract round will continue to receive the fixed forward price for part of its output, until the next contract round occurs. This reduces the severity of the punishment that the firm will receive – the more contracts, and the longer they last, the less the firm's profits will be reduced. Hence the greater the length of the contract, the longer the firm is protected against harsh punishment. This increases the temptation to deviate.

Proposition 3. *The critical discount factor may rise or fall as the proportion of contracted output increases.*

Proof. We have $h_x = -(1 - \delta^\alpha)\pi^c - (1 - \delta)\partial\pi^{sd}/\partial x$, and note that $\partial\pi^{sd}/\partial x < 0$, since increases in contract cover reduce the residual demand available in the spot market to a deviating firm. By the implicit function theorem, we have $d\underline{\delta}/dx = -h_x/h_\delta$. While $h_\delta > 0$, the sign of h_x is however not obvious, and the relation between the critical discount factor $\underline{\delta}$ and the proportion of contracted output x is ambiguous. ∎

The reason for an ambiguous effect of the contracted quantity on the minimum discount factor is that the amount of contracted quantity is present in the two effects. When the contracted quantity increases, the residual demand decreases and therefore the gains from deviation are lower (the gain-cutting effect becomes stronger). At the same time, when the contracted quantity increases, the forward profit increases, implying that the protection effect becomes stronger. This gives more incentive to deviate from the collusive agreement. Since the two effects are working in opposite directions, we get an ambiguous overall impact on the sustainability of collusion.

5. The level of the collusive price

To explore the impact of the level of the collusive price upon the sustainability of collusion, we start by asking whether firms would deviate with a small or a large reduction in the spot price. We already defined $p^d(x, p^c)$ as the optimal deviation price given p^c and x where

$$p^d(x, p^c) = \min\left\{p^c - \varepsilon, p^{Rm}(x, p^c)\right\}.$$

Note that the residual monopoly price falls with x and rises with p^c. By continuity, it follows that there exists a unique $x^*(p^c) \in [0,1]$ such that $p^{Rm}\left(x^*(p^c), p^c\right) = p^c$. Moreover $p^{Rm}(0, p^c) = p^m$, $p^{Rm}(1, p^c) < p^c$, and $p^{Rm}(1, c) = c$. Assume that $p^{Rm}(x, p^c)$ is continuous and differentiable with

$$\frac{\partial p^{Rm}(x, p^c)}{\partial x} < 0 \quad \forall x \in [0,1) \tag{10}$$

and

$$\frac{\partial p^{Rm}(x, p^c)}{\partial p^c} > 0 \quad \forall p^c \in [c, p^m) \tag{11}$$

Furthermore:

$$\begin{cases} p^c \le p^{Rm}(x, p^c) & \forall x \in [0, x^*] \\ p^c > p^{Rm}(x, p^c) & \forall x \in (x^*, 1] \end{cases} \tag{12}$$

If the collusive price is high and the level of contracting is low, the residual monopoly price will be greater than the collusive price. In that case, the optimal deviation will be to undercut the collusive price by ε, as when firms are trying to collude on the monopoly price with no contracts. We can rewrite the definition of the optimal deviation price as follows:

$$p^d = p^d(x, p^c) = \begin{cases} p^c & \text{if} \quad x \le x^*(p^c) \\ p^{Rm}(x, p^c) & \text{if} \quad x > x^*(p^c) \end{cases} \tag{13}$$

where $x^*(p^m) = 0$, $x^*(c) = 1$ and $\dfrac{\partial x^*(p^c)}{\partial p^c} < 0$.

Figure 1 shows the two regions, in which deviation by ε, and by more than ε, is optimal. We call these region 1 and region 2, respectively. We plot the frontier between these two kinds of

deviation, by equating p^c and p^{Rm}. It leaves the vertical axis at p^m, and meets the demand curve at a price of c. We had already seen that a firm would undercut the monopoly price by more than ε unless there were no forward contracts. The right-hand end of the line implies that when the collusive price is close to marginal cost, it will be optimal to undercut it by only ε, unless there is a very high degree of contracting. Covering more output with forward sales makes the residual demand in the spot market more elastic, favouring a lower deviation price. Setting a lower collusive price makes the residual demand at that price less elastic, favouring a higher deviation price.

Figure 1 about here.

The distinction between these two regions is important, because the firm's spot market profits after deviation are different. In both regions, the firm's forward profits are equal to $\pi^f = x\pi^c$, and its spot market profits from collusion are equal to $\pi^s = (1-x)\pi^c$. In region 2, there is no simple relationship between the spot market profits from deviation and from continued collusion. In region 1, however, the firm will optimally deviate by setting a price of p^c-ε and earn spot market profits of $\pi^{sd} = 2(1-x)\pi^c$, double the spot market profits it would gain from collusion. We use this straightforward relationship in the proof of:

Proposition 4: *For any positive discount factor and any length of contracts, a firm can sustain collusion at some price above marginal cost, given an appropriate level of forward contracting.*

Proof. The firm must choose a combination of p^c and x that places it in region 1. In this region, the optimal deviation price is to offer $p^c - \varepsilon$ and the firm's spot market profits from charging the optimal deviation price are given by $\pi^{sd} = 2(1-x)\pi^c$. From equation (7), collusion is sustainable if and only if $\dfrac{\pi^c}{1-\delta} \geq (2-x)\pi^c + \dfrac{\delta - \delta^\alpha}{1-\delta}x\pi^c$. We can rewrite this inequality as $1 - (1-\delta)(2-x) - (\delta - \delta^\alpha)x > 0$, which is equivalent to $(2\delta - 1)(1-x) + x\delta^\alpha > 0$. The last inequality is always true for $\delta > 1/2$ or if

$$x\delta^\alpha > (1-2\delta)(1-x) \tag{14}$$

Note that as *x* goes to 1, the above inequality is true for any $\delta > 0$, whatever the value of α. ∎

This proposition shows that forward contracts make it possible for firms to maintain some level of collusion at a price above marginal cost, however long those contracts last for, and however low their discount factor. Without contracts, collusion would only be possible for a discount factor of ½ or more, and so this result implies that contracts can have a pro-collusive effect, just as for Liski and Montero (2004). It is worth pointing out, however, that if the discount factor is low, or α is large, a high level of contract cover will be required. The proof, which is based on a sufficient rather than a necessary condition, requires that the firm is in region 1, implying that p^c must not be too high for the level of *x*, and that the highest p^c falls as *x* rises.

Within region 1, the level of the price does not affect the sustainability of collusion, implying that the highest sustainable price must be (weakly) within region 2. This in turn implies that the deviation price is set by maximising profits, given the residual demand in the spot market, rather than by simply offering $p^c - \varepsilon$. We will use this in the proof of:

Proposition 5: *Assume that $\partial^2 D(p)/\partial p^2 < 0$. The maximum sustainable collusive price is increasing in the discount factor.*

Proof. We have $h_{p^c} = \left(1 - (1 - \delta^\alpha)x\right)\dfrac{\partial \pi^c}{\partial p^c} - (1 - \delta)\dfrac{\partial \pi^{sd}}{\partial p^c}$. Since $h(x, \alpha, \underline{\delta}, p^c) = 0$, we have

$1 - (1 - \underline{\delta}^\alpha)x = (1 - \underline{\delta})\dfrac{\pi^{sd}}{\pi^c}$, and so $h_{p^c} = (1 - \underline{\delta})\left(\dfrac{\pi^{sd}}{\pi^c}\dfrac{\partial \pi^c}{\partial p^c} - \dfrac{\partial \pi^{sd}}{\partial p^c}\right)$.

With $\dfrac{\partial \pi^c}{\partial p^c} = \dfrac{(p^c - c)}{2}\dfrac{\partial D(p^c)}{\partial p^c} + \dfrac{D(p^c)}{2}$ and $\dfrac{\partial \pi^{sd}}{\partial p^c} = -\left(p^d - c\right)x\dfrac{\partial D(p^c)}{\partial p^c}$, we then have

$h_{p^c} = (1 - \underline{\delta})\left(p^d - c\right)\left(\dfrac{D\left(p^d\right)}{D(p^c)}\dfrac{\partial D(p^c)}{\partial p^c} + \dfrac{D\left(p^d\right) - xD(p^c)}{p^c - c}\right)$.

Since we are in region 2, and π^{sd} has been maximised with respect to p^d, we know that

$D\left(p^d\right) - xD(p^c) + (p^c - c)\dfrac{\partial D(p^c)}{\partial p^c} = 0$

This gives us:

$h_{p^c} = (1 - \underline{\delta})\left(p^d - c\right)\dfrac{\partial D(p^c)}{\partial p^c}\left[\dfrac{D\left(p^d\right)}{D(p^c)} - \dfrac{p^d - c}{p^c - c}\dfrac{\partial D\left(p^d\right)}{\partial p^d}\middle/\dfrac{\partial D(p^c)}{\partial p^c}\right]$

Since $p^d \leq p^c$, the first fraction in the square brackets is greater than one, and the second fraction is less than one. Since $\partial D(p)/\partial p < 0$ and $\partial^2 D(p)/\partial p^2 \geq 0$, the ratio of the two derivatives is also less than one. The term in square brackets is therefore positive, and $h_{p^c} < 0$. By the implicit function theorem, we have $\partial p^c / \partial \underline{\delta} = -h_\delta / h_{p^c}$. Recalling that $h_\delta > 0$, we can conclude that the maximum sustainable price is strictly increasing in the critical discount factor $\underline{\delta}$. ∎

We can also prove:

Proposition 6: *Assume that $\partial^2 D(p)/\partial p^2 \geq 0$. The maximum sustainable collusive price is decreasing in the length of contracts.*

Proof. By the implicit function theorem, we have $\partial p^c / \partial \alpha = -h_\alpha / h_{p^c}$. Recalling that $h_\alpha < 0$ and $h_{p^c} < 0$, we can conclude that the maximum sustainable price is strictly decreasing in the length of contracts α. ∎

Proposition 7: *The relationship between the maximum sustainable collusive price and the proportion of output covered by contracts is ambiguous.*

Proof. By the implicit function theorem, we have $\partial p^c / \partial x = -h_x / h_{p^c}$. Recalling that the sign of h_x might be either positive or negative, we are unable to draw conclusions on the relationship between the maximum sustainable price and the proportion of output covered by contracts. ∎

In the next section, we discuss this ambiguity and some other results, using a linear example

6. A linear demand example

We give an example of the interaction of the level of contract cover, the contract length, the discount factor, and the collusive price, by considering the same linear demand as Liski and Montero (2004). The total demand for the good is given by $a - p$, where p is the price in the spot market. In addition, we denote the price, quantity and profit associated with the one-period monopoly solution by $p^m = (a+c)/2$, $q^m = (a-c)/2$ and $\pi^m = (p^m - c)\, q^m = (a - c)^2/4$, respectively. Assume first that the firms collude on the monopoly price, so that $p^c = p^m$. It is easy to show that the monopoly price associated with the residual demand is given by $p^{Rm} = \dfrac{a + c - xq^m}{2}$.

We can insert these, and the other relevant formulae, into equation (7) and obtain:

Proposition 8: *Collusion at the monopoly price can be sustained through the use of grim trigger strategies, if and only if* $2x\delta^{\alpha} + \delta\,(2 - x)^2 \geq 1 + (1 - x)^2$.

It is interesting to note that with full contracting ($x = 1$), the condition becomes $2\delta^{\alpha} + \delta \geq 1$. For $\alpha = 1$, this gives us $\delta \geq 1/3$, as in Liski and Montero (2004). With $\alpha = 2$ and $x = 1$, the critical discount factor is again ½, just as if no contracts had been sold. The gain-cutting effect is exactly compensated by the protection effect. Figure 2 shows how the critical discount rate varies with x and the duration of the forward contracts. The two lowest lines, for contracts lasting one or two periods, are everywhere weakly below ½, confirming that these short-lived contracts make collusion easier to sustain. The lines for contracts lasting for three or more periods, however, rise above ½ at their right-hand ends. The top line shown, representing contracts lasting for 30 spot periods (which might imply a month-long contract overlying a spot market repeated each day), shows that the critical discount rate rises to more than 0.9 if the firms are fully contracted.

Assume now that the firms collude on $p^c \in (\,c, p^m\,]$. It is also possible, with some algebra, to calculate the maximum sustainable collusive price for values of x, δ and α. We insert expressions for the collusive profits and the deviation profits in the spot market into equation (8):

$$h\left(x, \alpha, \underline{\delta}, p^c\right) = \frac{a - p^c}{2}\left(p^c - c\right)\left(1 - x + x\underline{\delta}^\alpha\right) - \left(\frac{a - c - x(a - p^c)}{2}\right)^2 \left(1 - \underline{\delta}\right) = 0 \qquad (15)$$

As this is a quadratic equation in p^c, we can solve it to get the maximum sustainable collusive price, p^c*:

$$p^c* = c + (a - c)\left[\frac{1}{2} - \frac{x(2 - x)(1 - \underline{\delta}) - 2\sqrt{\left(1 - x + x\underline{\delta}^\alpha\right)\left(x\underline{\delta}^\alpha - (1 - x)(1 - 2\underline{\delta})\right)}}{4\left(1 - x + x\underline{\delta}^\alpha\right) + 2(1 - \underline{\delta})x^2}\right] \qquad (16)$$

We plot the results in Figures 3 to 5. These show how the maximum sustainable prices vary with the absolute volume of contracted sales, for different discount factors and three contract lengths – one period, two periods, and four periods. The figures are calibrated for $a = 8$ and $c = 0$, so that the monopoly price is equal to 4. The horizontal dashed line shows this price. Because we are plotting the absolute volume of sales, rather than x, on the horizontal axis, we can also show the firm's total output at a given collusive price, assuming that it is sharing the market equally. This is given by the dotted line, which can also be described as a half-demand curve (it shows half of the market demand). The line with dots and dashes is the border between region 1 (below the line - undercut the collusive price by ε) and region 2 (above the line – undercut by charging the residual monopoly price). With linear demand and constant costs, this border is also linear.

The solid lines give the frontier linking the highest sustainable collusive price and the level of contracted output, for a range of discount factors. In each case, prices below and to the right of the frontier (and to the left of the half-demand curve) are also sustainable.

Figure 3 shows that when contracts last for a single spot period, firms with a discount factor of ½ or more can sustain collusion at the monopoly price, whether or not they sell any contracts. Firms with a discount factor of between ½ and ⅓ can sustain the monopoly price if they have covered enough of their output with forward contracts. For example, a firm with a discount factor of 0.4 will be able to sustain the monopoly price if it sells one unit in the forward market, at point A, but it would not be able to sustain a price of 1, at point B. At this lower price, the single unit sold in advance would be too small a proportion of the firm's overall sales. From equation (14), with $\delta = 0.4$ and $\alpha = 1$, collusion is sustainable if $x \geq \frac{1}{3}$. Contract sales of 1 unit would give $x = \frac{1}{2}$ at a price of 4 (for the firm would have total sales of 2), but $x = 0.29$ at a price of 1, when the firm would be selling 3.5 units. Finally note that firms with a discount factor

of less than ⅓ cannot sustain the monopoly price, but will be able to sustain prices above marginal cost if they sell in the forward market.[5]

Figure 3 about here.

Figure 4 shows that firms with a discount factor of more than ½ can sustain collusion at the monopoly price for any level of contract cover, when the contracts last for two spot market periods. With a discount factor of ½ or less the frontiers are shifting downwards and to the right, showing that more cover is needed to make collusion sustainable, and that the maximum sustainable prices are falling. Firms with discount factors of close to ½ can still sustain the monopoly price for some levels of contract cover, however, and a frontier for a firm with $\delta = 0.47$ has been added to illustrate this.

Figure 4 about here.

Figure 5 illustrates the impact of contracts that last for four spot periods. With these contracts, the maximum sustainable prices for firms with discount factors of less than ½ are much lower, and only one case is shown on the figure. Without contracts, however, collusion would not be possible at all. A firm with a discount factor of ½ could sustain the monopoly price if it had no contracts, and could sustain a slightly higher price with a low level of contract cover – although it should not want to, as the higher price would reduce its profits! As its contract cover increases, however, the maximum sustainable price for this firm falls – if it covers all of its output in the forward market, the maximum price is less than half of the monopoly price. A firm with a discount factor of just over ½ will also have a non-monotonic frontier – it might maximise its sustainable price with a small amount of contract sales. Firms with higher discount factors, however, maximise their sustainable price if they do not sell any contracts, and the price that they can sustain falls as their contract sales increase. A firm with a discount factor of 0.645 is (just) able to sustain the monopoly price with full contracting.

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[5] Note that in region 1 (the area below the line of dots and dashes) the frontiers are bounded by straight segments – if extended, these would all converge on the vertical axis at a price of 8, the value taken by *a*

Figure 5 about here.

Figures 3-5 clearly show propositions 2 – 7 in action. The frontiers rise with higher discount factors, and fall as the length of the contracts increases. As contracts become longer, the protection effect becomes more important, relative to the gain-cutting effect, and collusion becomes harder to sustain for firms which would be able to sustain the monopoly price in the absence of contracts.

Firms with higher discount factors can sustain higher collusive prices. As the length of contracts increases, the highest sustainable price falls, or a higher discount factor is required to maintain a given collusive price. As the proportion of output covered by contracts increases, however, the highest sustainable price will sometimes rise, although it falls for high levels of contract cover when $\alpha > 1$.

7. Conclusion

We have shown that long-term contracts have an ambiguous impact on collusion. In some cases, they make collusion on a price above marginal cost possible when it would not be possible without them. In other cases, collusion would be possible without contracts, but becomes impossible (at a given price) as the level of contracting, or their length, becomes too great. We have shown that this ambiguity is due to the interaction of two effects, the gain-cutting effect, which reduces the immediate gain from defection, and the protection effect, which reduces the amount of punishment that deviators can receive.

We have taken the length of contracts, and the level of contracting, as exogenous. This was to allow us to explore the impact of changing these variables, but does beg the question of how they would be chosen. Contracts will often last for a "natural" period of calendar time, such as a week, a month or a year. In a competitive market, a particular contract design will only last if it meets a need, and for the length of a contract, this implies a trade-off between the

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in our example. This is also the case in figures 4 and 5, and would be for any other value of *T*.

convenience of not having to trade too frequently, and the ability to match a contractual position with a physical one. For example, month-long contracts will be well-placed in a market where demand does not change significantly from week to week, but does vary predictably over the course of a year.

The more interesting question is why firms would offer contracts if they make it harder for the firms to collude. For some firms, the premise of the question is incorrect, for collusion is only possible if they have sold contracts, and in these circumstances, they would wish to sell the amount of contracts that maximised the collusive price that they could achieve. Other firms, however, will potentially lose out by selling contracts, assuming that the possibility of collusion is in fact an attractive one for them. In a different model, with uncertain demand, covering some output with forward contracts would reduce the variability of the firms' profits. Hedging their profits in this way might be sufficient motivation for them to sell contracts. That remains an area for further research.

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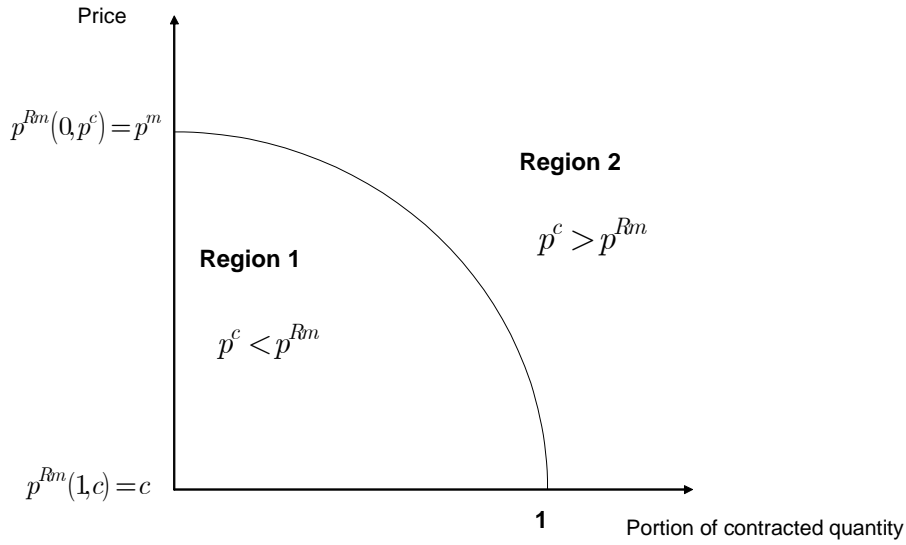USDA (1993) Farm Cost & Returns Survey.

ANNEX

Figure 1: Deviation types

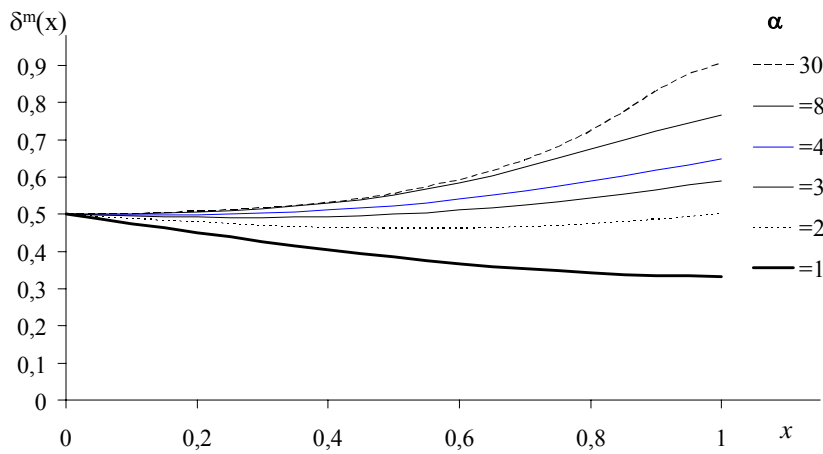**Figure 2: Critical discount factors to sustain collusion at the monopoly price**

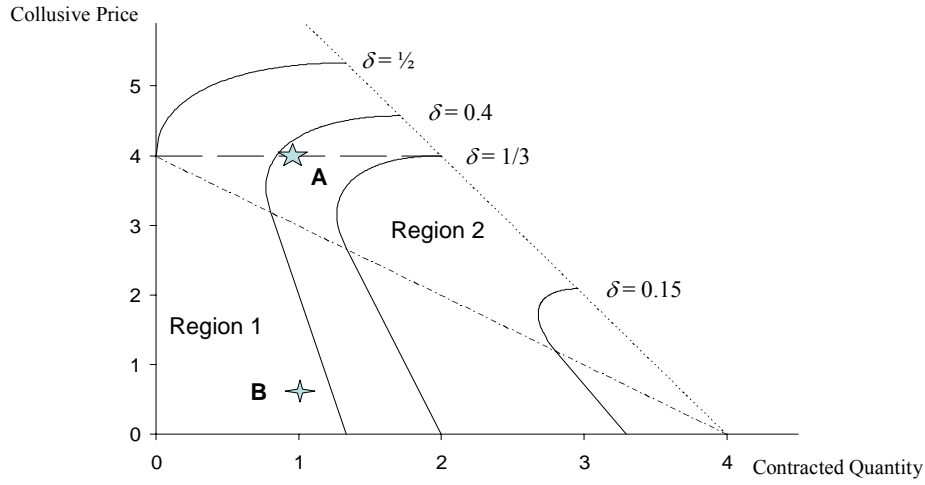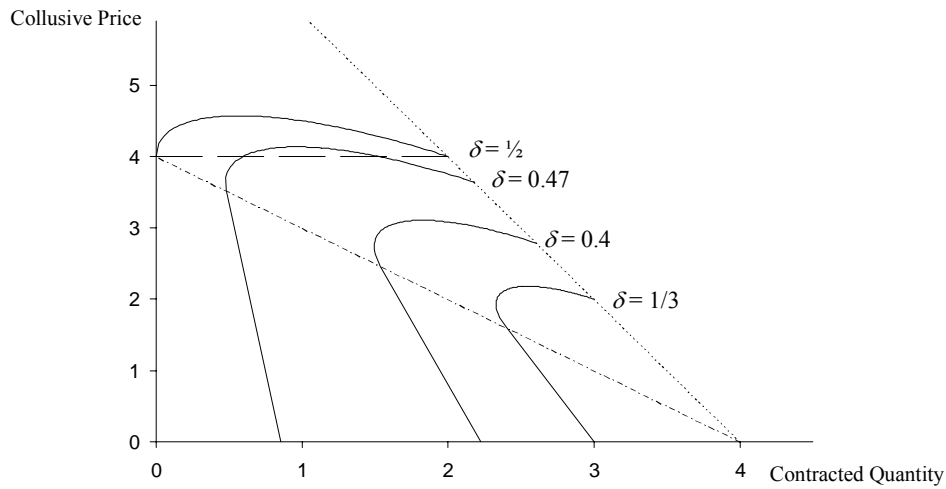**Figure 3: Sustainable collusive prices with α = 1**

Figure 4: Sustainable collusive prices with α = 2

Figure 5: Sustainable collusive prices with α = 4