

Selloffs, Bailouts, and Feedback: Can Asset Markets Inform Policy?*

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Abstract

We present a model in which a policymaker observes trade in a financial asset before deciding whether to intervene in the economy, for example by offering a bailout or monetary stimulus. Because an intervention erodes the value of private information, informed investors are reluctant to take short positions and selloffs are, therefore, less likely and less informative. The policymaker faces a tradeoff between eliciting information from the asset market and using the information so obtained. In general she can elicit more information if she commits to intervene only infrequently. She thus may benefit from imperfections in the intervention process or from being non-transparent about the costs or benefits of intervention.

JEL: C72, D82, E61, G01, G14.

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Central bankers naturally pay close attention to interest rates and asset prices, in large part because these variables are the principal conduits through which monetary policy affects real activity and inflation. But policymakers watch financial markets carefully for another reason, which is that asset prices and yields are potentially valuable sources of timely information about economic and financial conditions. Because the future returns on most financial assets depend sensitively on economic conditions, asset prices—if determined in sufficiently liquid markets—should embody a great deal of investors’ collective information and beliefs about the future course of the economy.

—Ben Bernanke¹

¹FRB: Speech, “What Policymakers Can Learn from Asset Prices,” April 15, 2004.

1 INTRODUCTION

Ever since the pioneering work of Mitchell and Burns (1938), economists have been aware of the power of asset markets to forecast business cycle fluctuations. In their influential article, Stock and Watson (2003) also found evidence that asset prices predicted inflation and output growth in some historical periods. Indeed the idea that securities prices convey information is a central tenet of the efficient markets hypothesis (Malkiel and Fama 1970), and it is taken as common wisdom that asset prices augur the health of individual firms, industries, and even the entire economy.

Given the prevalence of this idea, it is not surprising that policymakers would endeavor to use changes in securities prices to help inform the implementation of their policies. For instance, since the major financial market selloff of October 19, 1987 (known as Black Monday), it has become common practice for the Federal Reserve to respond to large drops in the stock market by injecting liquidity in to the economy either by reducing the federal funds rate (the so-called Greenspan Put) or by quantitative easing (the so-called Bernanke Put) (Brough 2013).²

There is, however, a problem with using declines in asset prices to inform policy. If privately-informed investors are aware that a selloff will trigger a corrective intervention, then they will have substantially less incentive to take short positions *in the first place*, and asset prices consequently will be less informative. In other words, the use of asset prices to actuate policy can undermine the informational content of the very prices in question.

We study this dilemma in a market micro-structure model wherein privately informed investors anticipate that their trades in an Arrow security may trigger an intervention by a policy maker, who can change the underlying state from *bad* to *good* at some cost. If trade in the asset market is sufficiently noisy, then the model admits a unique Perfect Bayesian equilibrium with some intriguing features. The policymaker never intervenes for small selloffs and intervenes randomly and with increasing probability for large ones. This generates a non-monotonicity in the equilibrium asset price: initially it falls with the magnitude of a selloff as the market becomes more convinced that the bad state will obtain and then it rises as the market anticipates the increasing probability that a corrective intervention will be triggered. Compared with a benchmark setting in which interventions are infeasible, the expected value of the asset price is higher in equilibrium and order flow is less (Blackwell) informative about the state precisely because informed investors are discouraged from taking short positions.

Moreover, in this type of equilibrium, the policymaker never benefits from the ability to intervene. It is sequentially rational for the policymaker to intervene whenever she believes that the state is sufficiently likely to be low. However, by doing so, she eliminates uncertainty about the state, depriving the informed trader of rent. Therefore, informed investors adopt less aggressive trading strategies that truncate beliefs at her point of indifference. The policymaker can stimulate trade and improve her payoff by committing to intervene less often than is sequentially rational or

²The regulatory case for using stock prices to assess the health of financial firms is documented by Curry, Elmer and Fissel (2003). Bernanke and Woodford (1997) suggest targeting long-run inflation to the level implicit in asset prices.

by maintaining private information about the cost or benefit of intervention.

If, for example, the probability of intervention is capped at some level less than one because of imperfections in the political process or the intervention technology, then traders are more willing to sell aggressively. Thus, large selloffs will be more informative and can trigger strictly beneficial interventions with positive probability. A limited ability to intervene in situations in which it is beneficial *ex post* allows the policymaker to acquire information that is both beneficial and actionable, benefitting her *ex ante*. Likewise, if the policymaker adopts a regime of secrecy rather than transparency regarding her expected benefit or cost of intervening, then large selloffs again will yield valuable information. These findings may seem provocative given the prevailing wisdom that policy should be conducted in an environment of minimal uncertainty (Eusepi 2005, Dincer and Eichengreen 2007). In the prevailing literature, however, no informational feedback exists between trade and policymaker intervention. In the presence of this feedback, the more certain the policymaker is to act on information, the more difficult it is to glean. A lack of transparency increases trader uncertainty about policymaker actions, strengthening the flow of information from the market. In these ways, institutions such as an unpredictable political process or opaque implementation protocol can benefit a policymaker by blurring investors' beliefs about the likelihood that any given selloff will trigger an intervention.

This paper touches on several related literatures. Our analysis is embedded in a novel market micro-structure model most closely related to Glosten and Milgrom (1985). The main difference in the settings is that the noise trades in our model are continuously distributed, which induces informed traders to adopt mixed strategies (supported on an interval) in equilibrium. Hence, order flow in our model is continuously distributed with more extreme orders conveying more information, as in Kyle (1985).

Our paper also contributes to the emerging theoretical literature on government bailouts of financial entities. Farhi and Tirole (2012) study a moral hazard setting in which borrowers engage in excessive leverage and banks choose to correlate their risk exposures so as to benefit from a bailout by the monetary authority. They characterize the optimal regulation of banks and the structure of optimal bailouts. Philippon and Skreta (2010) and Tirole (2012) investigate adverse selection in the financial sector, arriving at somewhat different conclusions. Philippon and Skreta (2010) find that simple programs of debt guarantee are optimal in their model and that there is no scope for equity stakes or asset purchases, while in Tirole (2012) the government clears the market of its weakest assets through a mixture of buybacks and equity stakes.

Relative to these papers, we take a more agnostic approach both on the need for and implementation of a bailout and focus on a somewhat different set of questions. Rather than moral hazard or adverse selection we suppose (for simplicity) that the need for a bailout arises randomly and exogenously but is unobserved by the policymaker. We also do not model the set of policy instruments available to the authority, assuming only that it has at its disposal an effective but costly intervention technology. Rather than the design of an optimal bailout mechanism, we are concerned with how a policymaker might extract information from asset markets to decide whether

a bailout is needed.

In this regard, our paper also relates to the literature on information aggregation in prediction markets (Cowgill, Wolfers and Zitzewitz 2009, Wolfers and Zitzewitz 2004, Snowberg, Wolfers and Zitzewitz 2012, Arrow et al. 2008), which are commonly viewed as efficient means of eliciting information. Our results suggest a caveat: prediction markets aggregate information effectively provided that policymakers (who watch the market in order to learn about the state) cannot take actions that affect the state; otherwise, market informativeness may be compromised. Ottaviani and Sorenson (2007) present a complementary analysis, which shows that the information aggregation properties of prediction markets may be undermined when *traders* can directly affect the state. These authors consider an internal corporate prediction market in which traders—who are also employees of the corporation—act as price takers in the prediction market but can manipulate the state through unobserved actions, characterizing the extent of outcome manipulation that arises in equilibrium and its impact on asset prices.³ Also related is Hanson and Oprea (2009) who consider information aggregation in a prediction market with a manipulative trader. Although he cannot affect the state directly, the manipulator can affect the market with his orders, seeking to match the market price to a specific target (unknown to other traders) for exogenous reasons. Paradoxically, these authors find that the presence of the manipulator may increase market informativeness. Unlike Hanson and Oprea (2009), in which the trader’s incentive for manipulation is exogenous, in our analysis, this incentive is endogenous. The trader internalizes the impact of his trade on the policymaker’s intervention decision, adjusting his trading behavior to avoid intervention; simultaneously, the strategic policymaker anticipates endogenous trader behavior when drawing inferences from trade. This *interaction* between trader behavior, market informativeness, and policymaker intervention is central to our analysis.

Finally, as the title of our paper suggests, we contribute to a burgeoning corporate finance literature on feedback between policy and asset prices. The works closest to ours in this vein are Bond, Goldstein and Prescott (2010), Edmans, Goldstein and Jiang (2011), and Bond and Goldstein (2012). Like ours, each of these papers features a decision maker who learns from asset prices in a setting where investors anticipate the impact of the decision maker’s ultimate choice of action on asset value. In all such environments there is an asymmetry between long positions (which confirm the decision maker’s prior) and short positions (which refute it). Several features distinguish our paper from the others, though – as we discuss below – we view them as largely complementary. Most importantly, we propose a novel market micro-structure model, which highlights different economic forces than those identified in the previous literature and underscores the important role of commitment.

In their insightful paper, Bond, Goldstein and Prescott (2010) consider intervention in a rational

³According to Cowgill, Wolfers and Zitzewitz (2009) many companies (e.g., Abbott Labs, Arcelor Mittal, Best Buy, Chrysler, Corning, Electronic Arts, Eli Lilly, Frito Lay, General Electric, Google, Hewlett Packard, Intel, InterContinental Hotels, Masterfoods, Microsoft, Motorola, Nokia, Pfizer, Qualcomm, Siemens, and TNT) incentivize their employees to trade assets in internal prediction markets designed to elicit information on a variety of concerns from forecasting demand to meeting cost and quality targets.

expectations framework. These authors find non-monotonic prices in the presence of intervention (as do we), though their model does not feature strategic trade (and sometimes can fail to possess an equilibrium). Bond and Goldstein (2012) consider government interventions in a Grossman and Stiglitz (1980) market framework; interventions change the risk profile of assets and can either generate or dampen trade. Edmans, Goldstein and Jiang (2011) focus on a firm’s decision to abandon an investment in response to trade in a binary version of the Kyle (1985) model. These authors find that in the presence of sufficient transaction costs, trade is dampened on the sell side. The central conflict of interest in our analysis is different from the ones previously identified. In our analysis the sell side trader wishes to avoid an intervention because it undermines the value of his private information. It is driven by strategic trade (unlike Bond, Goldstein and Prescott (2010)), and unlike Bond and Goldstein (2012), is not driven by traders’ risk preferences. Furthermore, our mechanism applies in the absence of transaction or opportunity costs of trade, required by Edmans, Goldstein and Jiang (2011).

We present our model and some preliminary observations in the next section. In Section 3 we analyze a benchmark setting in which bailouts are infeasible or are too expensive ever to implement. In Section 4 we fully characterize all Perfect Bayesian equilibria of the game. As mentioned above, a unique equilibrium exists when interventions are expensive or order flow is noisy. In the remainder of the parameter space an infinite number of equilibria exist, all of which are payoff equivalent for given parameter values. In these equilibria the policymaker intervenes with certainty unless she observes positive order flow. In fact, she may intervene after observing order flow of zero; lack of liquidity may signal that a bailout is needed. In Section 5 we show that the expected asset price is higher and that order flow is less informative when interventions are possible. In Section 6 we characterize the Pareto optimal intervention mechanism for the policymaker assuming that she has full power of commitment. Although this analysis is illuminating, commitment to the finely tuned optimal intervention mechanism seems implausible. Thus, in Section 7 we analyze two cruder forms of commitment: imperfections in the bailout process and a regime of secrecy, showing that the policy maker generally benefits from either of these. We recap our findings and provide brief concluding remarks in Section 8. Proofs and technical lemmas appear in Appendix A.

2 THE MODEL

We study a game with two active risk-neutral players, an investor and a policymaker, and a passive market maker who takes the other side of any trade with the investor. Two states of nature are possible: $\omega = 0$, which occurs with prior probability q , and $\omega = 1$, which occurs with $1 - q$. The investor may trade shares of an Arrow security that commits the seller to pay the buyer one if $\omega = 1$ and zero if $\omega = 0$.⁴ The asset price is equal to its expected payoff given all publicly available information, and it adjusts instantly to arrival of new public information.⁵ Investors can take either positive or negative positions in the asset: a negative position represents a sale and a

⁴It is formally equivalent to suppose that the asset has “fundamental” value ω in state ω and allow short sales.

⁵The standard justification for this argument is Bertrand competition among market makers.

positive position represents a purchase.

The investor is an *informed trader* with probability a and a *noise trader* with probability $1 - a$. An informed trader privately observes a signal realization (his type) $i \in \{0, 1\}$ which is perfectly correlated with ω , and he invests in an effort to maximize his expected return. A noise trader invests for exogenous reasons (e.g., a liquidity shock) and generates a random order flow, uniformly distributed on $[-1, 1]$.⁶

The policymaker cares intrinsically about the state. In particular, she receives payoffs normalized to one if $\omega = 1$ and zero if $\omega = 0$.⁷ The policymaker has a costly technology that allows her to “intervene” in the process that generates the state. If she intervenes, then she bears cost c but guarantees that the state is $\omega = 1$ with probability one.⁸ We focus on the case in which the intervention is sufficiently costly that the policymaker would not want to intervene under the prior, $c > q$. She may, however, find it optimal to intervene if the investor’s trade (e.g. a large sell order) reveals that the state is likely to be $\omega = 0$.⁹

The game proceeds in four stages. In the first stage the state and the trader’s type (zero, one or noise) are realized. In the second stage the trader submits an order $t \in \mathfrak{R}$ to the market, observed publicly. The market maker updates his beliefs based on the order and adjusts the price to equal the asset’s expected value and then fills the order. In the third stage, the policymaker observes the trade and decides whether to intervene. In the last stage the state is revealed and all payoffs are realized.

In this game, it is (weakly) dominated for a type-0 trader to buy the asset and a type-1 trader to sell it. There is, therefore, no loss of generality in treating all trades as non-negative numbers with the understanding that a sale of $t > 0$ units results in negative order flow. A mixed strategy by type $i \in \{0, 1\}$ is represented by the probability mass function $\phi_i(\cdot)$, defined over support $S_i \subset \mathfrak{R}_+$, with smallest element m_i .¹⁰ We often refer to m_i as trader i ’s minimum trade size. The random variable generated by the mixed strategy is τ_i and its realization is t . Denote the belief of the policymaker (and the market maker) that the state is zero conditional on observing order t by $\chi(t) \equiv \Pr\{\omega = 0 | \tau_i = t\}$, and denote the probability that the policymaker intervenes after observing order flow t by $\alpha(t)$. The solution concept is perfect Bayesian equilibrium, which consists of a trading strategy for each type of informed investor, $\phi_i(t)$, an intervention strategy for the policymaker, $\alpha(t)$ and a belief function $\chi(t)$. Each player’s strategy must be sequentially rational given the strategy of the other player, and for each order size on the equilibrium path the belief function

⁶The interval $[-1, 1]$ is a normalization. Investor payoffs are homogenous in the interval length. Also, the uniform distribution is assumed merely for analytic convenience. The model can accommodate any distribution with no qualitative changes in the results.

⁷This normalization is obviously linearly isomorphic to supposing that the policymaker suffers a loss if $\omega = 0$ and receives zero if $\omega = 1$.

⁸In section 7 we consider a case in which the intervention does not succeed with certainty.

⁹As presented, the policymaker cannot learn the state before deploying the intervention. If we allow the policymaker to perform a costly audit (based on the order flow) which allows her to learn the state before intervening, the results are formally equivalent. See the analysis in Appendix C.

¹⁰With the inclusion of the Dirac $\delta(\cdot)$ function, this definition also allows for pure strategies. This issue is irrelevant, because in equilibrium traders always play mixed strategies with no mass points.

must be consistent with Bayes' rule applied to equilibrium strategies.¹¹ Next, we perform some preliminary analysis, deriving expressions for prices, payoff functions, the incentive constraints, and beliefs. We then characterize some simple but significant properties of any equilibrium of the game.

Market Price. The asset pays one if $\omega = 1$ and zero if $\omega = 0$; its price must therefore equal the probability that $\omega = 1$ at the end of the game:

$$(1) \quad p(t) = (1 - \chi(t))(1 - \alpha(t)) + \alpha(t) = 1 - \chi(t) + \alpha(t)\chi(t)$$

If an intervention takes place, the asset is worth one for certain, but if the intervention does not take place, then the expected payoff of the asset is equal to $1 - \chi(t)$, the probability that the state is one, given the observed order. The price therefore incorporates information about both the “fundamental” (it is decreasing in $\chi(t)$) and about the anticipated intervention policy (it is increasing in $\alpha(t)$).

Trader Payoffs. Whether or not an intervention takes place, the type-1 trader expects the asset to be worth one. Thus, the type one trader's expected profit on each share purchased is just one minus the price $p(t)$. A type-1 trader's expected payoff from purchasing t shares of the asset is thus

$$(2) \quad \begin{aligned} u_1(t) &= t(1 - p(t)) \\ &= t\chi(t)(1 - \alpha(t)) \end{aligned}$$

Meanwhile, a type-0 trader collects the sale price, $p(t)$ on each share that he sells. In the absence of intervention he knows that the asset will be worth zero, allowing him to cover his short position at zero cost. If intervention occurs, however, he will owe one per share sold. Therefore, on each share sold the type-0 trader expects to collect a payoff equal to $p(t) - \alpha(t)$. A type-0 trader's expected payoff from selling t shares of the asset is thus

$$(3) \quad \begin{aligned} u_0(t) &= t(p(t) - \alpha(t)) \\ &= t(1 - \chi(t))(1 - \alpha(t)) \end{aligned}$$

In the absence of interventions ($\alpha(t) = 0$), either type of informed trader expects a positive rent, unless the order fully reveals his private information. Whenever the market maker is uncertain about the true state, the market maker mis-prices the asset, selling it too cheaply to a type-1 trader ($p(t) < 1$), and buying it too expensively from a type-0 trader ($p(t) > 0$). The possibility of intervention does not change the mis-pricing that arises from asymmetric information. However, if an intervention takes place, the state is known to be $\omega = 1$; asymmetric information vanishes, and with it, the trader's rent. Thus the trader's expected payoff when interventions occur with positive probability is simply his expected payoff in the absence of interventions, multiplied by the

¹¹Because all trades inside $[-1, 1]$ may be submitted by the noise trader, all possible orders in this interval are on the equilibrium path. No order outside this interval is submitted by a trader (noise or informed) in any equilibrium.

probability that no intervention takes place.

Policymaker Payoff. The policymaker's expected payoff from intervening with probability α is

$$(1 - \alpha)(1 - \chi(t)) + \alpha(1 - c) = 1 - \chi(t) + \alpha(\chi(t) - c).$$

If the policymaker intervenes, she ensures $\omega = 1$, but loses c ; if she does not intervene she receives payoff one whenever $\omega = 1$. From this, it follows that the policymaker's equilibrium intervention strategy must satisfy the following sequential rationality condition:

$$(4) \quad \alpha(t) = \begin{cases} 0, & \text{if } \chi(t) < c \\ [0, 1] & \text{if } \chi(t) = c \\ 1, & \text{if } \chi(t) > c \end{cases}$$

Thus, the policymaker intervenes whenever the probability that the state is low exceeds the intervention cost. This condition highlights the dilemma facing an informed trader. If he executes a trade t that reveals too much information, so that $\chi(t) > c$, then the market maker will anticipate an intervention and will set $p(t) = 1$, resulting in a payoff of zero for the trader. The investor must therefore be cognizant of precisely how his trades impact beliefs.

Equilibrium Beliefs. In equilibrium, beliefs are determined by Bayes' Rule applied to strategies. Because trading on the opposite side of the market is weakly dominated, a buy order either comes from a type-1 informed trader or a noise trader, so

$$(5) \quad \chi(t) = \frac{q(1-a)\frac{1}{2}}{(1-q)a\phi_1(t) + (1-a)\frac{1}{2}} \text{ if buy order } t \text{ is observed.}$$

Likewise, a sell order either comes from a type-0 informed trader or a noise trader, so

$$(6) \quad \chi(t) = \frac{q(a\phi_0(t) + (1-a)\frac{1}{2})}{qa\phi_0(t) + (1-a)\frac{1}{2}} \text{ if sell order of } t \text{ is observed}$$

Additional Notation. For ease of exposition, we define some additional notation. The following multivariate function turns out to be instrumental in the analysis:

$$(7) \quad Q(m, x|j, k) \equiv \left(\frac{xj^2}{1-x} + 1 \right) m^2 - 2(k+1)m + 1 - x.$$

It is also helpful to define the following transformations of the parameters of the model:

$$J(c, q) \equiv \frac{1-q}{1-c} \quad (c, q) \in [0, 1]^2$$

$$K_i(a) \equiv \frac{2a}{1-a}((1-i)q + i(1-q)) \quad (a, q) \in [0, 1]^2 \text{ and } i \in \{0, 1\}$$

We suppress arguments (c, q) and a , writing J, K_i whenever doing so does not create confusion. If the policymaker does not intervene under the prior $(c > q)$, parameter $J \in (1, \infty)$; it is a measure of how much information can be revealed without triggering an intervention, shrinking with q and growing with c . K_i is a measure of market informativeness that grows with a .

Simple Observations. A number of observations follow from the simple analysis presented so far, described in Lemma A.1 in the appendix. We highlight the most significant of these here. In any equilibrium in which a type- i informed trader expects a positive payoff, he must employ a mixed strategy representable by a probability density function $\phi_i(\cdot)$ with no mass points or gaps, supported on interval $[m_i, 1]$. In order to avoid revealing his private information to the market maker (or policymaker) the informed trader must “hide in the noise” generated by the uninformed trader’s order flow. Furthermore, any order that triggers an intervention for certain results in a zero payoff for the trader, and it therefore cannot be in the support of an equilibrium strategy with positive trader payoff. This observation underlies the central tension of our setting: if the trader expects to make money by participating in the market, he will never submit an order that is expected to trigger an intervention for certain. At the same time if the expected benefit of the intervention is positive, the policymaker would like to undertake it for certain; thus no beneficial intervention could be triggered by a trader’s order.

3 NO INTERVENTIONS

As a benchmark, we first present the case in which interventions are either infeasible or prohibitively expensive (an exact bound on the intervention cost will be derived). The following proposition describes the equilibrium and its comparative statics. In equilibrium, both types of trader must be indifferent over all trades inside the support of their mixed strategies. Using this observation together with the expressions for trader payoffs (see equations (2) and (3)) allows us to determine the mixing densities, parametrized by the minimum trade size m_i . We then invoke Lemma A.1 to determine the connection between the trader’s equilibrium payoff and his minimum trade size. Finally, we solve for the equilibrium minimum trade size by ensuring that the trader’s mixing density integrates to one (see appendix A for details).

Proposition 3.1 *(No interventions). The unique equilibrium when interventions are not possible is characterized as follows.*

- **Strategies.** A type- i trader submits an order of random size t , described by probability density function $\phi_i(t)$ over support $[m_i^*, 1]$, where

$$\phi_1(t) = \frac{t - m_1^*}{K_1 m_1^*} \text{ and } \phi_0(t) = \frac{t - m_0^*}{K_0 m_0^*},$$

and m_i^* is the smaller value satisfying $Q(m_i^*, 0|J, K_i) = 0$ (recall equation (7)).

- **Payoffs.** Informed trader payoffs are $u_1 = m_1^*q$ and $u_0 = m_0^*(1 - q)$
- **Comparative Statics.**
 - An increase in a causes 1) m_i^* to decrease, 2) both types to trade less aggressively in the sense of first order stochastic dominance, 3) a fall in type i 's equilibrium expected payoff u_i .
 - An increase in q causes 1) an increase in m_1^* and decrease in m_0^* , 2) type 1 to trade more aggressively and type 0 to trade less aggressively in the sense of first order stochastic dominance, 3) a rise in type 1's equilibrium expected payoff u_1 , and a fall in type 0's equilibrium expected payoff u_0 .

The informed investors' equilibrium mixing densities are increasing linear functions supported in an interval from a strictly positive trade m_i^* to one. Other things equal, an informed investor stands to gain more from extreme trades. Other things are, of course, not equal. Because they are tempting for the informed trader, large orders cause large movements in the beliefs of the market maker and hence, in the transaction price. Because the investor must be indifferent over all trades $t \in [m_i^*, 1]$, in equilibrium the effect on the price must exactly offset the informed trader's benefit of trading larger volume. The trader's payoff functions (equations (2) and (3)) reveal the exact connection between equilibrium beliefs and order flow.

$$\text{A sell order of size } t \geq m_0^* \Rightarrow \chi(t) = 1 - \frac{m_0^*(1 - q)}{t}$$

$$\text{A buy order of size } t \geq m_1^* \Rightarrow \chi(t) = \frac{m_1^*q}{t}$$

$$\text{Any other order } t \leq 1 \Rightarrow \chi(t) = q$$

In other words, large sell (buy) orders are associated with a relatively high (low) belief that $\omega = 0$ and correspondingly low (high) asset price. The most extreme beliefs derive from trades of size $t = 1$:

$$\bar{\chi} = 1 - m_0^*(1 - q) \text{ and } \underline{\chi} = m_1^*q.$$

Figure 1 illustrates the equilibrium beliefs as a function of order flow.

Values m_1^* and m_0^* represent respectively the largest buy and sell orders that a trader can make “for free”; that is, without revealing information that causes beliefs to change from the prior. Since an informed trader must be indifferent between all trades over which he mixes, it follows from (2) and (3) that equilibrium payoffs are $u_1 = m_1^*q$ and $u_0 = m_0^*(1 - q)$. The comparative statics in Proposition 3.1 are intuitive. When the probability that the trader is informed, a , is high, the market maker's beliefs (and hence prices) are very sensitive to order flow. Both types of informed trader mix over a wide range of orders using a relatively flat density approximating the noise trader's uniform one. Because the noise trader is unlikely to be present, this possibility provides

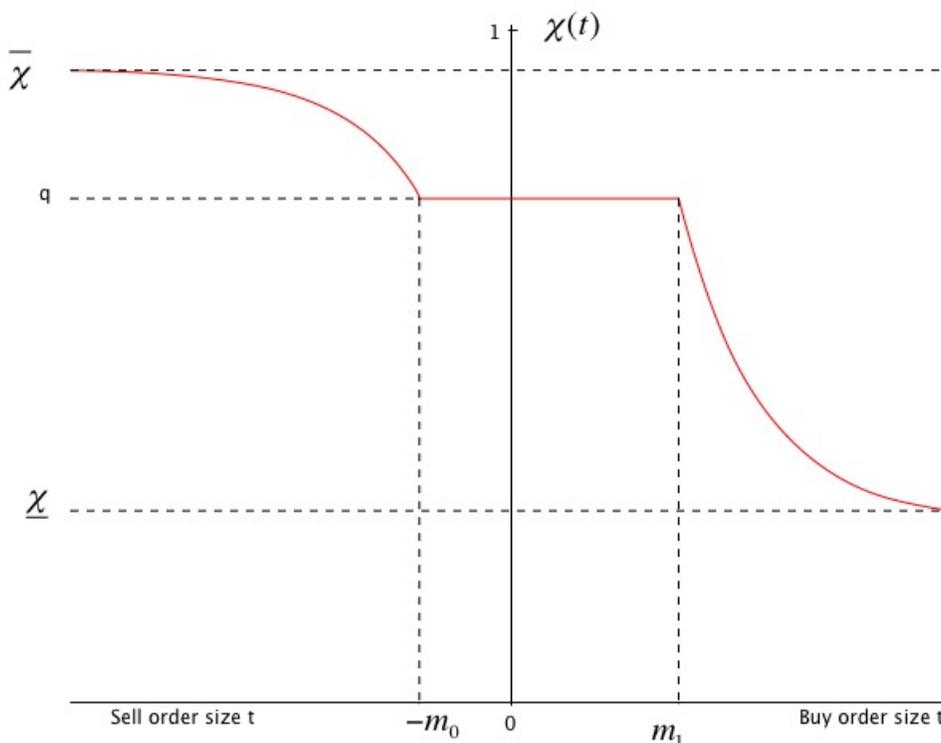


Figure 1: THE POSTERIOR BELIEF WITH NO POSSIBILITY OF INTERVENTION

weak “cover” for the informed trader, yielding him meager information rents. By contrast, when a is low, order flow is most likely generated by a noise trader and prices are, therefore, relatively insensitive. Hence, an informed investor can make large trades without causing large movements in the price, thereby securing substantial information rents.

Similarly, when the prior is more biased toward state zero, i.e. q is high, a sell order is relatively more likely to have come from an informed trader and a buy order is relatively more likely to have come from a noise trader. Thus, sell orders – which confirm the prior – generate large downward movements in the price and buy orders – which contradict the prior – generate small upward movements. Hence, an informed buyer can trade more aggressively without revealing his information and secure correspondingly higher rents than an informed seller. Of course, the reverse comparative statics (and intuition) hold when $1 - q$ is high.

4 INTERVENTIONS

With the benchmark of the preceding section in hand, we now turn to a setting in which interventions are possible. First, observe that the no intervention equilibrium of Proposition 3.1 remains the unique equilibrium when $c \geq \bar{\chi}$ (review (4)). This can happen for one of three reasons: either

c is too high, a is too low, or q is too low. In the first case interventions are simply too costly and in the second and third cases even a maximal selloff is not sufficiently informative to justify intervening. Hence, a necessary condition for interventions to occur in equilibrium is $c < \bar{\chi}$. Next, note that the possibility of intervention does not affect the type-1 trader's equilibrium behavior. Part 3 of Lemma A.1 shows that beliefs following a buy order must be (weakly) more optimistic, $\chi(t) \leq q$. Hence, a buy order never triggers an intervention and a type-1 trader's equilibrium behavior and payoff are as described in Proposition 3.1. For this reason, we focus in what follows on the equilibrium strategies of the type-0 trader and the policymaker.

As we show below, one of two classes of equilibria obtain depending on parameter values. To facilitate analysis, define the following notation:

$$\hat{\chi} \equiv \frac{K_0 + q}{K_0 + 1} \text{ and } f \equiv \frac{J - 1}{K_0}$$

The significance of $\hat{\chi}$ is discussed following the characterization of the second class of equilibria below. For now we note only that part 1 of Lemma A.2 establishes $q < \hat{\chi} < \bar{\chi}$.

Parameter f represents the value of the trader's mixing density $\phi_0(t)$ for which the belief function following a sell order (given in equation (6)) is equal to the intervention cost. Thus, sequential rationality for the policymaker (4) dictates that any trade for which $\phi_0(t) > f$ will certainly trigger an intervention, while any trade for which $\phi_0(t) < f$ will certainly not (see part 2 of Lemma A.2). Any trade that triggers an intervention with certainty (i.e., for which $\alpha(t) = 1$) yields the trader a payoff of zero by (3), and hence any equilibrium in which the trader expects a positive payoff requires $\phi_0(t) \leq f$.

The following proposition characterizes the unique equilibrium when interventions are feasible and $c \in (\hat{\chi}, \bar{\chi})$. It is derived in an analogous way to Proposition 3.1, with the additional condition that $\phi_0(t) \leq f$.

Proposition 4.1 (*Stochastic Interventions*). *If $c \in (\hat{\chi}, \bar{\chi})$ then the game has a unique equilibrium, characterized as follows.*

- **Strategies.**

- The type-0 trader places a sell order distributed according to continuous probability density function $\phi_0(t)$ over support $[m_0^\dagger, 1]$ defined piecewise:

$$\phi_0(t) = \begin{cases} \frac{t - m_0^\dagger}{K_0 m_0^\dagger} & \text{if } t \in [m_0^\dagger, \theta^\dagger] \\ f & \text{if } t \in [\theta^\dagger, 1], \end{cases}$$

where m_0^\dagger is the non-zero value satisfying $Q(m^\dagger, 1 - Jm^\dagger | J, K_0) = 0$, and $\theta^\dagger = Jm_0^\dagger$.

- The policymaker intervenes with probability

$$\alpha(t) = \begin{cases} 0 & \text{if } t \in [0, \theta^\dagger] \\ 1 - \frac{\theta^\dagger}{t} & \text{if } t \in [\theta^\dagger, 1]. \end{cases}$$

- **Payoffs.** The type-0 trader's expected payoff is $u_0 = m_0^\dagger(1 - q)$. Policymaker's expected payoff is the same as if interventions were not possible, $1 - q$.
- **Comparative Statics.** m_0^\dagger , θ^\dagger , and u_0 are increasing in c and decreasing in a and q . In addition, $c = \hat{\chi}$ implies $m_0^\dagger = \theta^\dagger = 0$, and $c = \bar{\chi}$ implies $m_0^\dagger = m_0^*$ and $\theta^\dagger = 1$.

When $c \in (\hat{\chi}, \bar{\chi})$, sell orders are partitioned into two intervals, a *safe* zone of modest trades ($t \leq \theta^\dagger$) that never trigger an intervention and a *risky* zone of larger trades ($t > \theta^\dagger$) that trigger an intervention with positive probability. Over the safe zone the type-0 trader mixes with an increasing linear density similar to Proposition 3.1, and larger trades reveal more information, increasing the posterior belief $\chi(t)$. At the critical trade $t = \theta^\dagger$, the policymaker is just indifferent about intervening, $\chi(\theta^\dagger) = c$ (see Figure 2). At this point beliefs must stop increasing with order flow because higher beliefs would induce the policymaker to intervene with certainty. In order to truncate beliefs at this level, the informed trader pools with the noise trader, mixing uniformly over the risky zone of trades. Although her beliefs are constant over the risky zone, the probability that the policymaker intervenes increases with order flow to offset the temptation of the investor to make larger trades.¹²

Unfortunately for the policymaker, when $c \in (\hat{\chi}, \bar{\chi})$, her expected equilibrium payoff is the same as if interventions were not possible. In the unique equilibrium, the policymaker either does not intervene, or mixes and is therefore indifferent between intervening and not. This is a direct consequence of the fundamental conflict of interest between the trader and the policymaker: the trader makes positive profit in equilibrium only if the policymaker does not benefit from the ability to intervene (we show that the converse is also true in Proposition 4.2 below). In Section 6, we investigate a version of the game in which the policymaker can commit to an intervention policy that violates her sequential rationality condition (4). This commitment mitigates the conflict of interest, allowing the policymaker to benefit from intervening while preserving positive profit for the investor.

The type-0 trader is worse off in the equilibrium of Proposition 4.1 than in the benchmark setting of Proposition 3.1: $m_0^\dagger(1 - q) < m_0^*(1 - q)$. Interventions, which occur with positive probability over the risky zone, harm the type-0 trader because they require him to pay the market maker one on each share sold, resulting in an *ex post* payoff of $t(p(t) - 1) < 0$ to the trader. We show in the next section that this effect induces him to trade less aggressively (in a formal sense) than he would if interventions were not possible.

As $c \rightarrow \bar{\chi}$, the equilibrium approaches the no-intervention benchmark presented in Proposition 3.1: the risky zone shrinks; the probability of an intervention goes to zero, and the investor's payoff approaches $m_0^*(1 - q)$. By contrast, as $c \rightarrow \hat{\chi}$: the risky zone expands; the probability of intervention goes to one, and the investor's payoff tends to zero. Indeed, at $c = \hat{\chi}$, the type-0 trader mixes uniformly on the entire interval $[0, 1]$, and the policymaker intervenes with probability

¹²A degree of unpredictability is often inherent in government intervention policies; examples include the differential treatments of Bear Stearns and Lehman Brothers, and the government's repeated refusal to delineate an explicit bailout policy for Fannie Mae and Freddie Mac, (Frame and White 2005).

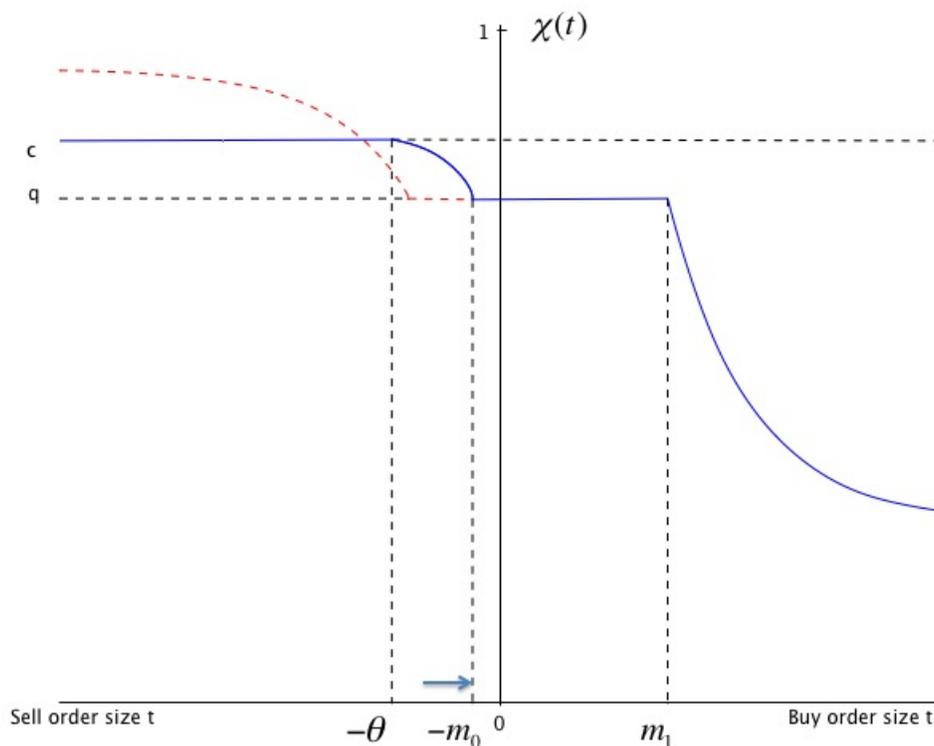


Figure 2: THE POSTERIOR BELIEF FOR THE EQUILIBRIUM WITH STOCHASTIC INTERVENTIONS

one after every sell order. This foreshadows equilibrium behavior over the remaining region of the parameter space.

Proposition 4.2 (*Certain Interventions*). *For any parameter values satisfying $c \in (q, \hat{\chi})$ there exists an infinite set of equilibria all of which are payoff equivalent.*

- **Strategies.**

- The type-0 trader places density of at least f on every trade $t \in (0, 1]$. A mass point may exist on any trade volume, including $t = 0$.
- Policymaker intervenes with probability one after any sell order $t > 0$ and intervenes after observing order flow $t = 0$ if a mass point exists on $t = 0$ or if $\phi_0(0) > f$.

- **Payoffs.** The type-0 trader's expected payoff is zero. The policymaker's expected payoff is

$$(1 - q) + \left(qa + (1 - a)\frac{1}{2} \right) (\hat{\chi} - c).$$

- **Comparative Statics.** The policymaker's expected payoff is decreasing in c and q and increasing in a .

When the cost of an intervention is low, or the prior is high, or order flow is very informative, then the policymaker strictly benefits from the ability to intervene. To understand this result, observe that

$$\hat{\chi} = \frac{K_0 + q}{K_0 + 1} = \frac{q(a + (1 - a)^{\frac{1}{2}})}{qa + (1 - a)^{\frac{1}{2}}}$$

This expression is the posterior belief associated with knowing that a buy order was *not* placed, revealing the investor to be either a type-0 trader or a noise trader. If this information alone is sufficient to induce an intervention, then in equilibrium the policymaker intervenes after observing *any* sell order. The type-0 trader cannot profit from his private information because any sell order will instantaneously drive the price to one. Thus, as mentioned earlier, the type-0 trader makes positive expected profit in equilibrium if and only if the policymaker does not benefit from the ability to intervene. Observe that there is a stark asymmetry between sell and buy orders in the equilibria characterized in proposition 4.2. A small buy order $t \leq m_1$ has no impact on the equilibrium price (i.e., $p(t) = 1 - q$), while even an infinitesimal sell order $t > 0$ causes the price to jump to one.

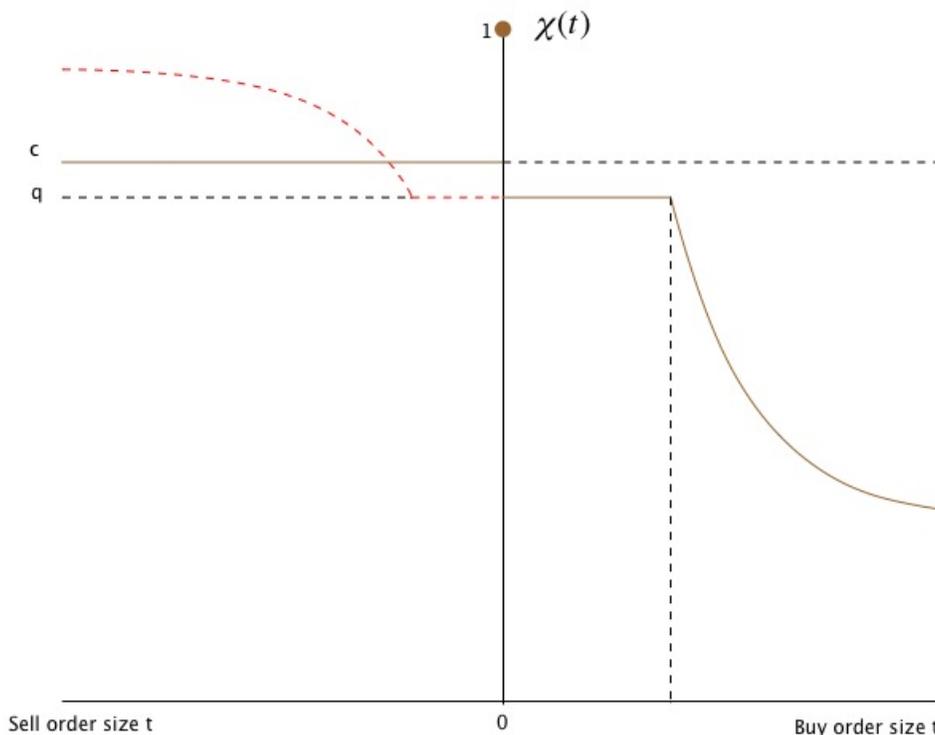


Figure 3: POSTERIOR BELIEFS IN A CERTAIN INTERVENTIONS EQUILIBRIUM.

An especially intriguing equilibrium is one in which the type-0 trader places the minimum

density $\phi_0(t) = f$ for all trades $t \in (0, 1]$ and accumulates the residual probability in a mass point on $t = 0$ (see Figure 3).¹³ In this equilibrium, the type-0 trader stays “out of the market” with positive probability $1 - f$, and perfectly mimics the noise trader (mixing uniformly on $[0, 1]$) if he enters.¹⁴ If the policymaker observes a sell order (with non-zero size), then she intervenes for certain, although she is indifferent about doing so. If however, she observes an order of zero, then she knows for certain that an intervention is necessary. In other words, no trade may convey worse news about the state than a large sell order. Although the policymaker always responds to a sell order with corrective action, in this equilibrium she benefits from doing so only in response to zero volume.

Propositions 3.1 (no interventions), 4.1 (stochastic interventions), and 4.2 (certain interventions) provide a full characterization of the equilibria of the game. In the remainder of the paper we suppose $c \in (\hat{\chi}, \bar{\chi})$, focussing on the case of stochastic interventions. We do so for two reasons. First, this seems the more interesting and plausible situation to study, and second, the fact that the policymaker does not benefit from the ability to intervene raises questions of how this might be remedied. We continue our analysis in the next section by investigating the consequences of interventions for the asset price, order flow, and the signal of the state generated by the market.

5 EQUILIBRIUM ASSET PRICE, ORDER FLOW, AND MARKET SIGNAL

In this section we describe the way that interventions affect the market price and order flow. We show that the possibility of an intervention increases the expected equilibrium price of the asset; at the same time, it softens incentives to trade on the sell side, inducing the type-0 trader to trade less aggressively. Because the equilibrium order flow acts as a public signal of the (pre-intervention) fundamental, less aggressive trade by the type-0 investor reduces the Blackwell-informativeness of this signal.

In the case of stochastic interventions, $c \in (\hat{\chi}, \bar{\chi})$, the equilibrium asset price is non-monotonic in order flow (review (1)). Over the safe zone, the price falls with larger sell orders as the market maker becomes more convinced that $\omega = 0$. Over the risky zone, however, the price rises as the market maker becomes ever more convinced that an intervention is forthcoming. In the first case $\chi(t)$ rises while $\alpha(t)$ remains constant at zero, and in the second case $\alpha(t)$ rises while $\chi(t)$ remains constant at c . In this sense, a large selloff can be regarded as “good news” for share holders because it may trigger a policy intervention. A natural question is whether the equilibrium price is consequently higher on average when interventions are possible. The following proposition answers this in the affirmative.

Proposition 5.1 (*Higher Mean Price*). *The expected asset price is higher in the equilibrium with stochastic interventions than in the no-intervention benchmark.*

¹³This is the equilibrium that is selected by considering a vanishing transaction or opportunity cost of trade.

¹⁴When $c < \hat{\chi}$, $f < 1$ by part 3 of Lemma A.2

The equilibrium price (see equation (1)) is composed of two distinct terms: the first term $1 - \chi(t)$ is the posterior belief about the state in the absence of intervention, while the second term $\alpha(t)\chi(t)$ reflects the impact of intervention, which is relevant when it transitions the state from zero to one. Because $\chi(t)$ is the posterior belief conditional on order flow t which is itself random (as it comes from a mixed strategy), Bayesian rationality requires that the expected value of the posterior $\chi(t)$ is equal to the prior. Intuitively, without the possibility of intervention, (so that $\alpha(t) = 0$) trade in the financial market *reveals information* about the state, but it does not *affect the state* directly. Hence, the ex ante expected value of the state is equal to its expected value under the prior belief, regardless of the (anticipated) trading strategy of the investor. When interventions are possible, this intuition no longer holds, because trade in the financial market can trigger an intervention and therefore can directly affect the state. Indeed, in the risky zone of trades, interventions are triggered with positive probability, increasing the probability of state one and with it, the asset price.

A number of empirical papers document a connection between the possibility of corrective intervention and high asset prices. Frame and White (2005) survey several papers which estimate that debt issued by Fannie Mae and Freddie Mac trades at interest rates 0.35-0.40% below its risk rating, resulting in a higher asset price. According to these authors “financial markets treat [Fannie and Freddie’s] obligations as if those obligations are backed by the federal government” despite the fact that the government is under no legal obligation to intervene in the event of trouble. In fact, financial markets correctly forecasted government policy: in September 2008 the federal government intervened to stabilize Fannie Mae and Freddie Mac (see Frame (2009) for more information). In a similar vein, O’Hara and Shaw (1990) show that congressional testimony by the Comptroller of the Currency that some banks are “too big to fail” caused equity prices to increase at several large banks. Interestingly, the increases were most significant for eleven banks named in media coverage of the story, which was not identical to the set of banks covered by the policy. Indeed, certain banks that the Comptroller intended to cover experienced price drops because they were excluded from the list reported in the media. Gandhi and Lustig (2010) also show that announcements in support of bailouts increase bank equity prices and present a broad range of evidence supporting the idea that equity prices of large banks and government sponsored enterprises benefit from implicit government guarantees.

In the equilibrium of the model with stochastic interventions, the type-0 trader must sometimes pay out on the shares of the security he sold. Therefore, his incentive to trade is softened relative to the no-intervention benchmark, and he trades less aggressively (in the sense of first order dominance) in order to mitigate the probability of an intervention.

Proposition 5.2 (*Less Aggressive Trade*). *The type-0 investor trades less aggressively in the equilibrium with stochastic interventions than in the no-intervention benchmark: his equilibrium mixed strategy in the absence of interventions first order stochastically dominates his equilibrium mixed strategy in the stochastic intervention equilibrium.*

Because the type-0 trader has strong incentives to avoid triggering an intervention, he stochastically

reduces his trading volume relative to the benchmark case. Our model, therefore, has a testable prediction that entities such as large banks and government sponsored enterprises should see less short selling than other similar institutions (for example, foreign banks) for which bailouts or other corrective interventions are perceived to be less likely. The standard event study methodology values the implicit government guarantee by comparing equity returns before and after a bailout announcement (O’Hara and Shaw 1990, Gandhi and Lustig 2010). Our model suggest that an alternative empirical approach is to contrast sell volume at institutions with otherwise similar characteristics.

Because the informativeness of the market signal is directly tied to trader aggressiveness, dampened selloffs limit the amount of information about the state (pre-intervention) that is released to the market.

Proposition 5.3 (*Less Information*). *Order flow is less Blackwell informative about the underlying state in the equilibrium with stochastic interventions than in the no-intervention benchmark.*

In both the equilibrium with stochastic interventions and the benchmark setting, order flow t generates a posterior belief $\chi(t)$. Because order flow is random in both cases, from an *ex ante* perspective, $\chi(t)$ is a random variable. In the proof of Proposition 5.3, we construct this random variable for each environment explicitly. We then appeal to results that show that if the posterior belief random variable generated by signal A is a mean preserving spread of the posterior belief random variable generated by signal B, then A is more Blackwell informative. Intuitively, if posterior beliefs under signal A are a mean preserving spread of posterior beliefs under signal B, then under signal A, the realization of the posterior belief is more likely to be “extreme,” which means that it is more likely to be close to zero or one; hence, signal A is more informative. The law of iterated expectations implies that the expected value of $\chi(t)$ is q in either case. Hence, the proof boils down to showing the distribution of beliefs under the equilibrium with stochastic interventions is more concentrated around the prior than the one generated in the absence of interventions. Although this is somewhat technical to establish, it is quite intuitive. The type-0 investor’s trading strategy truncates equilibrium beliefs at c , while beliefs under the no-intervention benchmark range up to $\bar{\chi}$. Moreover $m_0^\dagger < m_0^*$ implies that the type-0 investor puts more weight on small – less informative – trades when faced with the possibility of an intervention.

In the current model, social welfare corresponds to the policymaker’s expected payoff. To see this, note that the market maker earns expected profit of zero on every trade, and therefore the positive rents earned by informed traders (as usual) come out of the pockets of noise traders. From an *ex ante* perspective, the policymaker does not benefit from the ability to intervene, but because of its effect on informativeness, the possibility of interventions could have adverse consequences for resource allocation outside the scope of the current model.

6 PARETO IMPROVING INTERVENTIONS

As noted in Proposition 4.1, when $c \in (\hat{\chi}, \bar{\chi})$, the policymaker intervenes stochastically, but does not benefit relative to the no-intervention benchmark. The reason for this is her lack of commitment power. Specifically, it is sequentially rational for the policymaker to never intervene if $\chi(t) < c$ and to always intervene if $\chi(t) > c$. The type-0 trader therefore chokes off the information content of order flow at $t > \theta^\dagger$ to avoid triggering a certain intervention.

In this section we consider a hypothetical setting in which *ex ante* commitment to an intervention policy $\alpha(t)$ is possible. In particular, we derive the *ex ante* optimal intervention plan for the policymaker holding the type-0 investor's expected payoff at its equilibrium level, $u_0 = m_0^\dagger(1 - q)$. This restriction ensures that the expected payoffs of all market participants are unchanged from their equilibrium values so that the policy we derive represents a Pareto improvement over the equilibrium.¹⁵ Although we do not regard full commitment to an optimal random intervention policy (particularly with a continuum of possible trade sizes) as especially realistic, it is helpful to understand how the policy maker's optimal plan differs from her equilibrium strategy. Among other things, this comparison paves the way for exploring more plausible remedies in the next section.

To formulate the policymaker's optimal policy with commitment as a constrained programming problem, we adopt the standard approach of the principal-agent literature, allowing the policymaker to select both her own strategy and the type-0 trader's, imposing the equilibrium conditions for the trader's strategy as incentive compatibility constraints. We therefore imagine that the policymaker chooses $\alpha(t)$ and $\phi_0(t)$ in order to maximize her *ex ante* expected payoff

$$(8) \quad v = (1 - q) \left(a + (1 - a) \frac{1}{2} \right) + \int_0^1 \left(aq\phi_0(t) + (1 - a) \frac{1}{2} \right) (\alpha(t)(1 - c) + (1 - \alpha(t))(1 - \chi(t))) dt$$

The first term in this expression is the contribution to the policymaker's expected payoff from a buy order (after which it is never optimal to intervene). The first term of the integrand is the density of sell orders and the second term is the policymaker's expected payoff given the chosen intervention probability at order flow t . This maximization is also subject to feasibility conditions that ensure that the intervention probability and mixing density are valid, and incentive constraints, that ensure that the trader is willing to comply with the policymaker's recommended mixing density (see the proof of Proposition 6.1 for details).

Proposition 6.1 (*Pareto Optimal Interventions and Trade*). *If $\max\{\frac{1+q}{2}, \hat{\chi}\} < c < \bar{\chi}$, then there exists a constant $\lambda \in (0, 1)$ such that the policymaker's optimal choices of intervention policy and investor mixing density are characterized as follows.*

- *The type-0 trader places a sell order distributed according to continuous probability density*

¹⁵Recall that the market maker's payoff is zero, and informed trader rents come at the expense of noise traders. Other Pareto optimal policies exist (indexed by the informed trader's payoff) but these cannot be Pareto ranked.

function $\phi_0(t)$ over support $[m_0^\dagger, 1]$ defined piecewise:

$$\phi_0(t) = \begin{cases} \frac{t-m_0^\dagger}{K_0 m_0^\dagger} & \text{if } t \in [m_0^\dagger, \frac{\theta^\dagger}{1+\lambda}] \\ f + \frac{(1-\lambda)t-\theta^\dagger}{2K_0 m_0^\dagger} & \text{if } t \in [\frac{\theta^\dagger}{1+\lambda}, 1] \end{cases}$$

- *Policymaker intervenes with probability*

$$\alpha(t) = \begin{cases} 0 & \text{if } t \in [0, \frac{\theta^\dagger}{1+\lambda}] \\ \frac{1}{2} \left(1 + \lambda - \frac{\theta^\dagger}{t}\right), & \text{if } t \in [\frac{\theta^\dagger}{1+\lambda}, 1]. \end{cases}$$

When designing the *ex ante* optimal policy, the policymaker faces a delicate tradeoff. In order to induce the type-0 trader to reveal information, she must commit not to use the information too aggressively. On the other hand, acquiring information is pointless if she cannot act on it by making a beneficial intervention. To highlight this tension, note that (8) can be rewritten in the following form:¹⁶

$$v = (1 - q) + aq(1 - c) \int_0^1 \alpha(t)(\phi_0(t) - f) dt$$

Two things are evident from this formulation of the policymaker's objective. First, it is clear why she does not benefit in an equilibrium with stochastic interventions – namely, $\alpha(t) = 0$ for $t < \theta^\dagger$ and $\phi_0(t) = f$ for $t \geq \theta^\dagger$: so $v = 1 - q$ in this case. Second, an intervention at t is beneficial if and only if $\phi_0(t) > f$. The incentive constraints of the trader, however, imply that a high value of $\phi_0(t)$ necessitates a low value of $\alpha(t)$ and *vice versa*: acquiring valuable information requires a commitment to intervene with relatively low probability.

As is illustrated in Figure 4, compared with the policymaker's strategy in the stochastic interventions equilibrium, her *ex ante* optimal intervention policy (viewed as a function of the order flow) has a smaller intercept and is less steep.¹⁷ The policymaker therefore optimally begins intervening sooner (i.e. for smaller sell orders), but uses an intervention rule that is flatter and therefore less sensitive to order flow. The two intervention policies intersect at order flow $t = \frac{\theta^\dagger}{1-\lambda}$; for larger order flows the optimal policy lies below the equilibrium one (and is weakly above for smaller orders).

Facing the policymaker's *ex ante* optimal intervention policy, the type-0 trader mixes according to a piecewise linear density. In fact, this density is the same as his equilibrium strategy over the commitment safe zone, $[m_0^\dagger, \theta/(1 + \lambda)]$. However, the commitment safe zone is smaller than the equilibrium safe zone, ending at order flow $t = \theta^\dagger/(1 + \lambda)$ instead of $t = \theta^\dagger$. At $t = \theta^\dagger/(1 + \lambda)$ the type-0 trader's optimal density becomes flatter so that the policymaker's belief increases less rapidly as the order flow increases. The optimal density does not become completely flat (as in equilibrium), but continues increasing until it crosses the flat equilibrium density f at $\frac{\theta^\dagger}{1-\lambda}$. Thus,

¹⁶See the proof of proposition 6.1 for the derivation.

¹⁷Figure 4 corresponds to the following parameter values: $f = 2$, $k = 1$, $q = 1/10$. From these the following values are implied: $a = 1/12$, $\hat{\chi} = 11/20$, $c = 7/10$, $m_0^\dagger = 1/4$, $\theta^\dagger = 3/4$, $u_0 = 9/40$, $m_0^* = 2 - \sqrt{3} \approx 0.27$, $\bar{\chi} \approx 0.76$. This figure was generated from a numerical computation, not from an analytical solution.

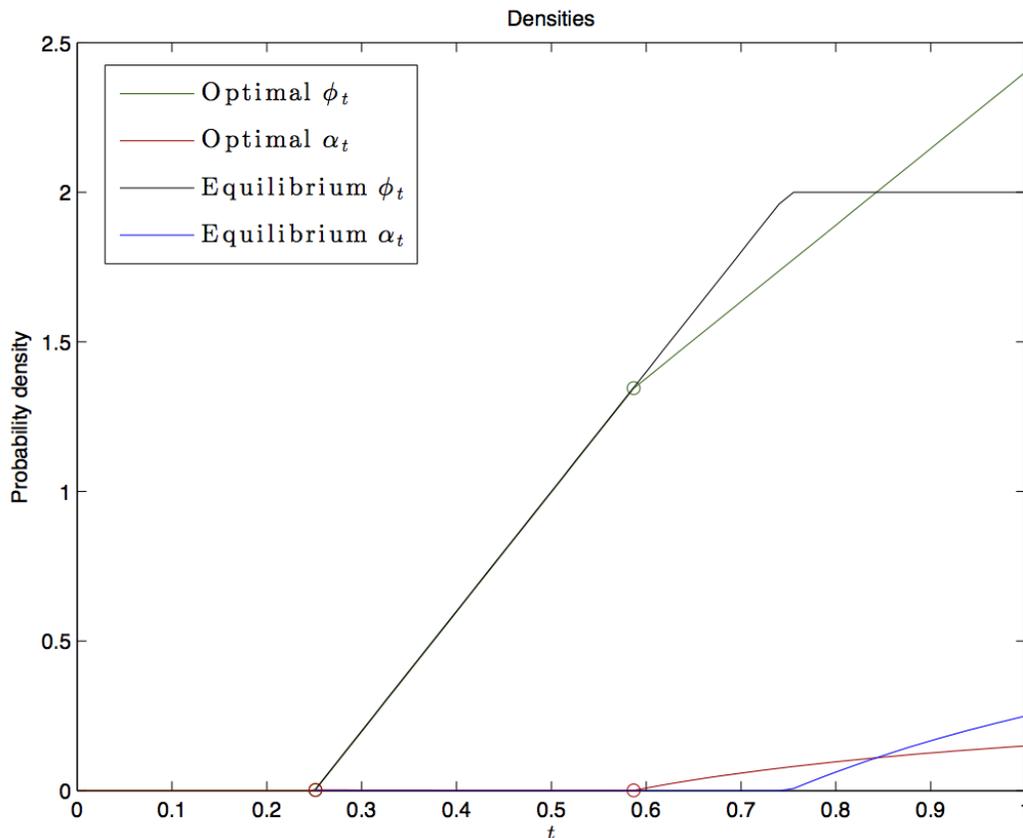


Figure 4: THE PARETO OPTIMAL INTERVENTION POLICY

for order flow $t \in \left(\frac{\theta^\dagger}{1+\lambda}, \frac{\theta^\dagger}{1-\lambda}\right)$ beliefs satisfy $\chi(t) < c$ and interventions – which occur with positive probability under the optimal policy – actually harm the policymaker in expectation. On the other hand, for order flow $t \in \left(\frac{\theta^\dagger}{1-\lambda}, 1\right]$, beliefs satisfy $\chi(t) > c$ and interventions strictly benefit the policymaker in expectation. The *ex ante* Pareto optimal policy therefore requires *both* kinds of commitment from the policymaker: for an intermediate range of order flow she must intervene with positive probability when she would prefer not to intervene at all, and for high order flow she must refrain from intervening with certainty, although she would benefit from doing so.

Of course, committing to an intervention policy that randomizes with precisely the correct probabilities at each order flow is implausible because it requires verification by the type-0 trader or some impartial third party. In the absence of such verification, the policymaker could simply implement her *ex post* preferred policy and claim that she randomized according to the *ex ante* optimal one. Even if the policymaker cannot commit to the precise *ex ante* optimal intervention policy, she may be able to verifiably commit to a suboptimal policy that raises her expected payoff. We investigate this possibility in the next section.

7 REMEDIES

As described in Proposition 4.1, when $c \in (\hat{\chi}, \bar{\chi})$, the policymaker does not benefit from the ability to intervene in equilibrium: she either does not intervene, or randomizes (and is therefore indifferent). In the previous section we derived the Pareto Optimal intervention policy supposing that the policymaker could verifiably commit not to abide by her sequential rationality condition (4). In fact, verification that the policymaker indeed abides by the finely-tuned optimal intervention policy presented in Proposition 6.1 seems quite implausible. Nevertheless, she may have access to other less precise, but more easily verified forms of commitment that allow her to relax sequential rationality at least to some extent. We explore two possibilities below.

IMPERFECT INTERVENTIONS

We consider an alternative environment in which the institution or technology that executes interventions is imperfect. The institution caps the intervention probability from above at some commonly known level $\bar{\alpha} < 1$. In a political context it could be that any attempted bailout is blocked with probability $1 - \bar{\alpha}$, so that the probability of actually intervening given that a bailout is attempted with probability $\beta(t)$ is $\alpha(t) = \bar{\alpha}\beta(t)$. Alternatively, the policymaker could employ a policy instrument that does not guarantee that the state is one for certain, but effects a transition to state one with some probability $\bar{\alpha} < 1$. If the cap is a feature of the institution or technology, it seems reasonable that its existence would be common knowledge among all parties.

As noted in Proposition 6.1, the Pareto optimal policy is flatter than the equilibrium policy. a cap on the intervention policy approximates this by forcing the policymaker to use a *completely* flat intervention policy for large orders (where the cap is binding). With the cap imposed, sequential rationality for the policymaker requires:

$$\alpha(t) = \begin{cases} 0 & \text{if } \chi(t) < c \\ [0, \bar{\alpha}] & \text{if } \chi(t) = c \\ \bar{\alpha} & \text{if } t > c \end{cases}$$

This condition suggests why the cap might be desirable for the policymaker. Without the cap, whenever the policymaker believes that intervention is strictly beneficial, she intervenes with probability one, depriving the trader of all rent (see equations (2, 3)). The trader therefore mixes in a way that chokes off information and avoids a certain intervention. With the cap, the policymaker cannot intervene with probability one and leave the trader with zero payoff. Therefore, the trader may be willing to place more weight on large orders, even if this leads the policymaker to believe that an intervention is strictly beneficial. This intuition is formalized in the following proposition.

Proposition 7.1 (*Imperfect Interventions*). *If $c \in (\hat{\chi}, \bar{\chi})$ and $\bar{\alpha} < 1 - \theta^\dagger$, the game has a unique equilibrium that is characterized as follows.*

- *Strategies.*

- The type-0 trader places a sell order distributed according to continuous probability density function $\phi_0(t)$ over support $[\bar{m}_0, 1]$ defined piecewise:

$$\phi_0(t) = \begin{cases} \frac{t-\bar{m}_0}{K_0\bar{m}_0} & \text{if } t \in [m_0(\bar{\alpha}), \theta_1] \\ f & \text{if } t \in [\theta_1, \theta_2] \\ \frac{(1-\bar{\alpha})t-\bar{m}_0}{K_0\bar{m}_0} & \text{if } t \in [\theta_2, 1] \end{cases}$$

where \bar{m}_0 is the smaller value that solves $Q(\bar{m}_0, \bar{\alpha}|J, K_0) = 0$, and $\theta_1 = J\bar{m}_0$, and $\theta_2 = \frac{\theta_1}{1-\bar{\alpha}}$.

- The policymaker intervenes with probability

$$\alpha(t) = \begin{cases} 0 & \text{if } t \in [0, \theta_1] \\ 1 - \frac{\theta_1}{t} & \text{if } t \in [\theta_1, \theta_2] \\ \bar{\alpha} & \text{if } t \in [\theta_2, 1] \end{cases}$$

- **Payoffs.** The type-0 trader's expected payoff is $u_0 = \bar{m}_0(1 - q)$. The policymaker's expected payoff is

$$(1 - q) + aq(1 - c)\bar{\alpha} \left(\frac{1 - \theta_1 - \bar{\alpha}}{2K_0m_0(\bar{\alpha})} \right).$$

- **Comparative statics.** \bar{m}_0 is decreasing in $\bar{\alpha}$. Also, $\bar{\alpha} = 0$ implies $\bar{m}_0 = m_0^*$; in addition $\bar{\alpha} = 1 - \theta^\dagger$ implies $\bar{m}_0 = m_0^\dagger$, $\theta_1 = \theta^\dagger$ and $\theta_2 = 1$. If $\bar{\alpha} > 1 - \theta^\dagger$, the intervention cap is non-binding, and the equilibrium is identical to the one in Proposition 4.1.

When interventions are imperfect, the risky zone of trades is split into two segments. For $t \in [\theta_1, \theta_2)$, the cap on the intervention probability does not bind, and equilibrium behavior for both players is similar to that given in Proposition 4.1. The investor chokes off information by mixing uniformly, and the policymaker intervenes with increasing probability. For the larger range of orders, $t \in [\theta_2, 1]$, the cap on intervention probability binds. Over this range, the type-0 investor mixes using an increasing density, thereby releasing more information than in the case with no cap; indeed over this range $\chi(t) > c$, and because interventions take place with positive probability over this range, the policymaker expects to benefit (see Figure 5).

In essence, the imperfect technology commits the policymaker not to intervene with certainty, and this induces the investor to reveal more information over the interval of extreme trades where the cap binds. Because large orders simultaneously convey more information and trigger interventions in expectation, the policymaker benefits relative to the case of stochastic interventions with no cap. In fact, both the policymaker and the type-0 trader prefer that the policymaker uses *any* (sufficiently) imperfect intervention technology

Corollary 7.2 (*Gains From Imperfect Interventions*). *If $c \in (\hat{\chi}, \bar{\chi})$, then the equilibrium payoffs of both the type-0 trader and the policymaker are strictly higher with a binding cap, $\bar{\alpha} \in (0, 1 - \theta^\dagger)$ than without a binding cap, $\bar{\alpha} \in [1 - \theta^\dagger, 1]$. Moreover, the trader's payoff is decreasing in $\bar{\alpha}$, while the policymaker's payoff is single peaked.*

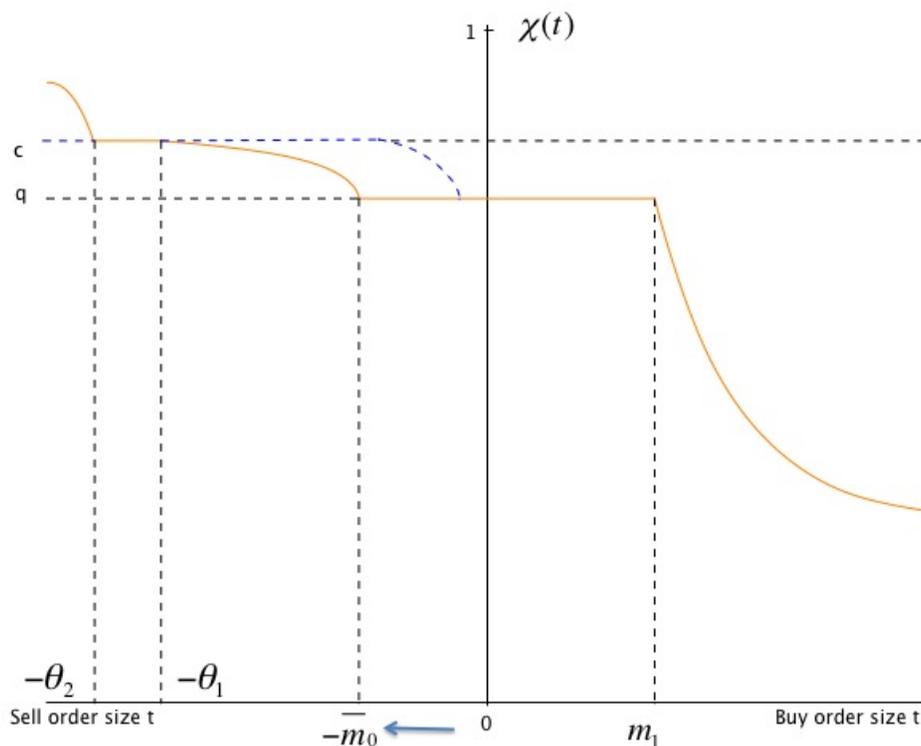


Figure 5: THE POSTERIOR BELIEF IN THE EQUILIBRIUM WITH A BINDING INTERVENTION CAP.

As $\bar{\alpha}$ ranges from 0 to $1 - \theta^\dagger$, the equilibrium moves continuously from the no-intervention benchmark of Proposition 3.1 to the stochastic intervention setting of Proposition 4.1. Since the type-0 trader prefers the former environment to the latter one, it is not surprising that his welfare increases as the cap decreases. Recall, however, that the policymaker's expected payoff is $1 - q$ in both the no-intervention and stochastic-intervention settings. Nevertheless, her expected equilibrium payoff is strictly higher for any intermediate case. The reason is that the cap $\bar{\alpha}$ influences her payoff in two ways. With a tighter cap, the investor reveals more information through his trades, but the policymaker is less able to use this information to execute a beneficial intervention. At one extreme $\bar{\alpha} = 0$, the investor reveals the most information, but the policymaker's hands are tied. At the other extreme $\bar{\alpha} = 1 - \theta^\dagger$, the policymaker is unconstrained, but the investor reveals no valuable information. For all intermediate values of the cap, the investor reveals *some* valuable information which the policymaker can *sometimes* exploit.

Unlike the Pareto optimal policy of Proposition 6.1 which improves the policymaker payoff while keeping the market participants' payoffs at their equilibrium levels, imperfect interventions benefit the informed trader as well as the policymaker. This benefit to the informed trader comes at the expense of the noise trader, however, so that the capped equilibrium does not Pareto dominate the stochastic intervention equilibrium. Imperfections in the political process or intervention technology

benefit the policymaker and increase utilitarian social welfare but also effect a “transfer” of utility from noise traders to informed ones.

LACK OF TRANSPARENCY

Finally, we consider a setting in which the policymaker possesses private information, so that some aspect of her decision environment is not transparent to the market. Formally, we model this as private information about her cost of intervening. Observe, however, that because the policymaker’s benefit is normalized to equal one, c is actually the cost-to-benefit ratio, and hence, modeling private information about the cost of intervening is equivalent to modeling private information about the benefit.¹⁸

Because of the lack of transparency, both the trader and market maker are uncertain about the intervention cost. Both believe that it is c_L with probability γ and c_H with probability $1 - \gamma$. We focus on the case in which $\hat{\chi} < c_L < c_H < \bar{\chi}$, so that an equilibrium with stochastic interventions (in which the ability to intervene is worthless) would obtain under either cost realization; any benefits for the policymaker in the resulting equilibrium can therefore be attributed to the lack of transparency. For $i \in \{L, H\}$ define

$$J_i \equiv \frac{1 - q}{1 - c_i}, f_i \equiv \frac{J_i - 1}{K_0}, \text{ and } R \equiv \frac{J_H^2 - 1}{J_L^2 - 1}$$

The first two parameters are analogous to the case of transparency allowing for different intervention costs, while the third parameter is a measure of the relative difference between the possible intervention costs (note that $R > 1$). Let m_i^\dagger and θ_i^\dagger be type-0 minimum trade size and intervention threshold that would obtain in an equilibrium with stochastic interventions if it were common knowledge that the policymaker’s intervention cost was c_i (described in Proposition 4.1); let $\bar{m}_i(\bar{\alpha})$ be the value of type-0’s minimum trade size that would obtain in an equilibrium with intervention cap $\bar{\alpha}$ and cost c_i (as described in Proposition 7.1). Finally, define

$$(9) \quad \bar{\gamma} = (1 - \theta_H^\dagger)R \text{ and } \hat{m}_0 \equiv \frac{R(1 - \gamma)}{R - \gamma} m_H^\dagger \quad \text{for } \gamma \in [0, \bar{\gamma}].$$

The equilibrium – while always unique – can be one of three types, depending on the prior belief about the policymaker’s cost. We characterize the three possible equilibrium configurations in the following propositions.

Proposition 7.3 (*Privately Informed Policymaker-I*). *Suppose $\hat{\chi} < c_L < c_H < \bar{\chi}$ and the policymaker is likely to have a high cost, $\gamma \in [0, \bar{\gamma}]$, then the unique equilibrium is characterized as follows.*

¹⁸In Appendix B we show that private information about cost is analogous to the policymaker receiving a private (imperfect) signal about the underlying state ω . Thus, learning that she has low intervention cost is the same as learning that she has high intervention benefit or learning that state zero is more likely to occur than under the prior.

• **Strategies.**

- The type-0 trader places a sell order distributed according to probability density function $\phi_0(t)$ over support $[\widehat{m}_0, 1]$ defined piecewise:

$$\phi_0(t) = \begin{cases} \frac{t - \widehat{m}_0}{K_0 \widehat{m}_0} & \text{if } t \in [\widehat{m}_0, \theta_1] \\ f_L & \text{if } t \in [\theta_1, \theta_2] \\ \frac{(1-\gamma)t - \widehat{m}_0}{K_0 \widehat{m}_0} & \text{if } t \in [\theta_2, \theta_3] \\ f_H & \text{if } t \in [\theta_3, 1] \end{cases}$$

where $\theta_1 = J_L \widehat{m}_0$, $\theta_2 = \frac{J_L \widehat{m}_0}{1-\gamma}$, $\theta_3 = \frac{J_H \widehat{m}_0}{1-\gamma}$.

- Each type of policymaker intervenes with probability

$$\alpha_L(t) = \begin{cases} 0 & \text{if } t \in [0, \theta_1] \\ \frac{1}{\gamma} \left(1 - \frac{\theta_1}{t}\right) & \text{if } t \in [\theta_1, \theta_2] \\ 1 & \text{if } t \in (\theta_2, 1], \end{cases}$$

$$\alpha_H(t) = \begin{cases} 0 & \text{if } t \in [0, \theta_3] \\ 1 - \frac{\theta_3}{t} & \text{if } t \in [\theta_3, 1]. \end{cases}$$

• **Payoffs.**

- The type-0 trader's expected payoff is $u_0 = \widehat{m}_0(1 - q)$.
- The type-H policymaker's expected payoff is $v_H = 1 - q$, and the type-L policymaker's expected payoff is

$$v_L = (1 - q) + aq(1 - c)(1 - \gamma) \left(\frac{(1 - \theta_2)^2 - (1 - \theta_3)^2}{2K_0 \widehat{m}_0} \right).$$

• **Comparative Statics.**

\widehat{m}_0 and θ_1 are decreasing in γ , while θ_2 and θ_3 are increasing in γ . Also, $\gamma = 0$ implies $\widehat{m}_0 = m_H^\dagger$, $\theta_1 = \theta_2$, and $\theta_3 = \theta_H^\dagger$, so the equilibrium with $\gamma = 0$ reduces to the equilibrium with cost commonly known to be c_H . In addition, $\gamma = \bar{\gamma}$ implies $\widehat{m}_0 = \bar{m}_0(\gamma)$ and $\theta_3 = 1$.

If the policymaker is likely to have the high intervention cost, then sell orders are broken into four regions. The first region ($t < \theta_1$) is completely safe: neither type of policymaker intervenes. Over this region increasing trade size reveals information, but not enough for even the low cost policymaker to intervene. In the second region, ($t \in (\theta_1, \theta_2)$) the low cost policymaker intervenes with increasing probability, but the high cost policymaker does not intervene. Information is choked off to keep the low cost policymaker from intervening for certain. At the right endpoint of the second region, the low cost policymaker intervenes for certain, while the high cost policymaker does not intervene. In the third region, (θ_2, θ_3) , the low cost policymaker intervenes for certain ($\chi(t) > c_L$), but the high cost policymaker does not intervene ($\chi(t) < c_H$). In this region an increase

in the trade size cannot increase the intervention probability because the low cost policymaker is already intervening with certainty and it is too expensive for the high cost policymaker to intervene. Therefore to offset the benefit of larger volume the trader's order must reveal more information to the market. Over this region both the trader density and the belief function grow. Because γ is relatively low, the posterior belief function reaches c_H at $t = \theta_3$. In the fourth region ($t \in (\theta_3, 1)$), the low cost policymaker intervenes for certain, and the high cost policymaker intervenes with increasing probability. Information flow is choked off to prevent the high cost policymaker from intervening for certain. Figure 6 depicts posterior beliefs in this equilibrium

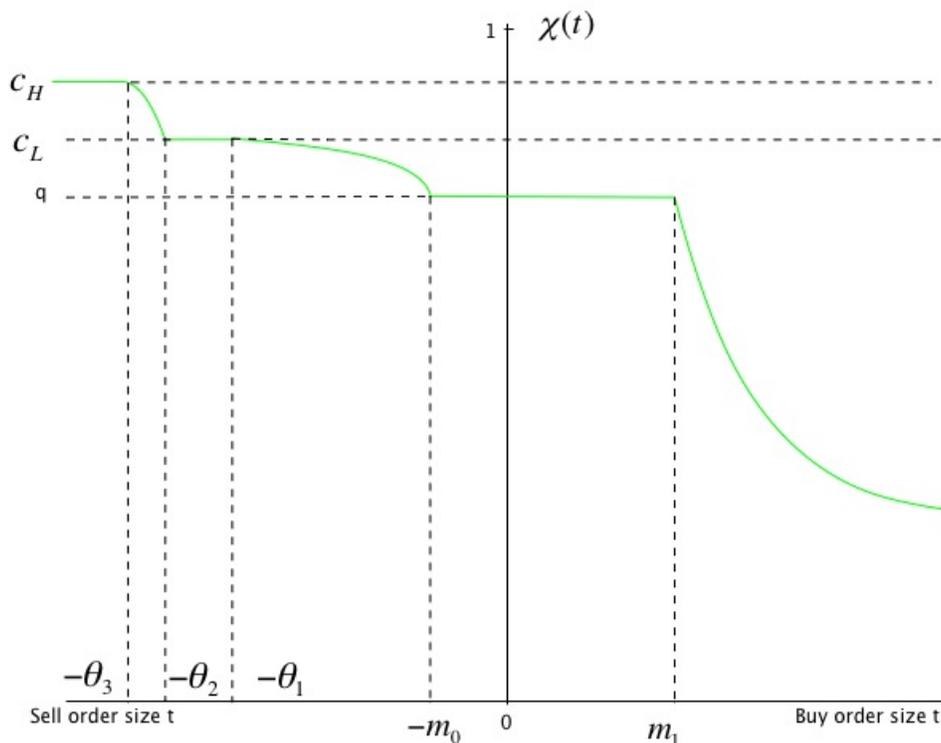


Figure 6: POSTERIOR BELIEFS IN THE PRIVATELY INFORMED POLICYMAKER-I EQUILIBRIUM.

When $\gamma = 0$ – so that the policymaker's high cost is common knowledge – region two is empty ($\theta_1 = \theta_2$), the posterior belief increases smoothly from q to c_H , and the equilibrium is identical to the case of stochastic interventions with $c = c_H$. As γ increases, region two ($t \in (\theta_1, \theta_2)$) expands and region four ($t \in (\theta_3, 1)$) shrinks until $\gamma = \bar{\gamma}$. At this point, region four disappears entirely, so that the high cost policymaker never intervenes. Indeed, for $\gamma \in [\bar{\gamma}, 1 - \theta_L^\dagger]$, the equilibrium closely resembles the case of a cap on the probability of intervention with $\bar{\alpha} = \gamma$.

Proposition 7.4 (*Privately Informed Policymaker-II*). For $\hat{\chi} < c_L < c_H < \bar{\chi}$, if the probability of low cost is intermediate, $\gamma \in [\bar{\gamma}, 1 - \theta_L^\dagger]$, the unique equilibrium is characterized as follows. The

high cost policymaker never intervenes. The type-0 trader plays the same strategy as in Proposition 7.1 (the equilibrium with an intervention cap) with $c = c_L$ and $\bar{\alpha} = \gamma$. The low cost policymaker's intervention policy is equal to the intervention policy of Proposition 7.1, multiplied by $1/\gamma$. The trader's expected payoff is $u_0 = \bar{m}_L(\gamma)(1 - q)$. The high cost policymaker's expected payoff is $v_H = 1 - q$, and the low cost policymaker's expected payoff is

$$v_L = aq(1 - c) \left(\frac{1 - \theta_1 - \gamma}{2K_0 m_0(\gamma)} \right).$$

For intermediate values of γ , the high cost policymaker never intervenes. Therefore, the probability of having low cost γ acts as a cap on the intervention probability. Up to a transformation (or relabeling) of the low cost policymaker intervention probability, the second equilibrium with private cost is identical to the capped equilibrium, in which the intervention cost is c_L and the intervention cap is γ . As γ continues to increase, it becomes more likely that the policymaker's cost is low, and the region in which the “cap” binds shrinks, disappearing at $\gamma = 1 - \theta_L^\dagger$. For larger values of γ , the “cap” never binds and the equilibrium parallels the case of stochastic interventions with $c = c_L$.

Proposition 7.5 (*Privately Informed Policymaker–III*). For $\hat{\chi} < c_L < c_H < \bar{\chi}$, if the probability of low cost is high, $\gamma \in (1 - \theta_L^\dagger, 1]$, the unique equilibrium is characterized as follows. The high cost policymaker never intervenes. The type-0 trader plays the same strategy as in Proposition 4.1 (the equilibrium with stochastic intervention) with $c = c_L$. The low cost policymaker's intervention strategy is the intervention strategy of Proposition 4.1, multiplied by $1/\gamma$, and is always strictly less than one. The trader's expected payoff is $u_0 = m_L^\dagger(1 - q)$. Both types of policymaker expect payoff $1 - q$ (and do not benefit from interventions).

Whenever costs are likely to be low, $\gamma > 1 - \theta_L^\dagger$, secrecy does not benefit the policymaker. In contrast, whenever costs are sufficiently likely to be high, (when $\gamma < 1 - \theta_L^\dagger$), there is a positive probability that $\chi(t) > c_L$ in equilibrium. In this case, the low cost policymaker may receive strictly beneficial information from a large selloff, something that never occurs in the absence of private information.

Corollary 7.6 (*Benefit of Private Information*). If $\tilde{\chi} < c_L < c_H < \bar{\chi}$ and $\gamma \in (0, 1 - \theta_L^\dagger)$, then the low cost policymaker strictly benefits in expectation from her private information, $v_L > 1 - q$.

Suppose that – before learning her type – the policymaker could choose a disclosure regime. That is, she could commit either to reveal her private information (transparency) or not to reveal it (secrecy). An implication of Corollary 7.6 is that she would strictly prefer secrecy if $\gamma < 1 - \theta_L^\dagger$ and would never strictly prefer transparency. Under transparency the resulting equilibrium would involve stochastic interventions and an expected payoff of $1 - q$ for either cost realization. Under secrecy, a positive probability exists that a low cost policymaker would observe a strong selloff,

allowing her to make a strictly beneficial (in expectation) intervention. Hence, the expected payoff to secrecy satisfies $\gamma v_L + (1 - \gamma)v_H > 1 - q$.¹⁹

It is worth considering a related variant of this environment. Suppose, the policymaker's cost is initially known to be c . She can undertake one of two measures that has the potential to alter the intervention cost. A conciliatory approach generates political support for interventions, reducing her cost to $c_L < c$ with probability ρ_S , otherwise leaving it unchanged. An adversarial approach generates political opposition to interventions, increasing her cost to $c_H > c$ with probability ρ_O , otherwise leaving it unchanged.²⁰ The market can observe the policymaker's adopted approach, but not the realization of her cost. Thus parameter ρ_i is the probability that the policy actually changes the policymaker's cost. Three observations follow from Corollary 7.6. First, both generating support for interventions (stochastically reducing cost), and generating opposition to interventions (stochastically increasing cost) can benefit the policymaker. Second, generating support for interventions raises the policymaker's expected payoff only when it is relatively *unlikely* to succeed (ρ_S small). If it is too likely to generate a low cost, no benefit of cost reduction is achieved. Third, generating opposition to interventions, on the other hand, benefits the policymaker in expectation when it is relatively *likely* to succeed. Thus whether drumming up support or opposition is beneficial depends on the extent to which the measure is likely to alter attitudes toward intervention.

Finally, briefly consider a setting in which n cost realizations are possible, satisfying $\hat{\chi} < c_1 < \dots < c_n < \bar{\chi}$. Arguments analogous to those used to prove Propositions 7.3, 7.4, 7.5 show that a regime of secrecy can strictly benefit all policymaker types except type- n . Much like an intervention cap, private information acts as a commitment on the probability of intervening and induces the type-0 trader to reveal more information than he would if costs were publicly disclosed.

8 CONCLUSION

In this paper we explore a setting in which privately informed investors trade an asset in an effort to profit from their knowledge of the underlying state and in which a policymaker who cares intrinsically about the state observes trading activity and decides whether to take a preemptive costly intervention; e.g., a bailout. We completely characterize the set of perfect Bayesian equilibria of the game and derive a number of results.

We show that there exists a region of the parameter space in which sales orders are partitioned into two sets: a safe zone of small trades that never trigger an intervention and a risky zone of large trades that induce random interventions by the policy maker in equilibrium. Although the probability of an intervention increases with sell volume over the risky zone, the policy maker does not expect to benefit from intervening. This occurs because investors employ equilibrium trading strategies that choke off information at the point where the policymaker is *just* indifferent about

¹⁹Note also that a regime of secrecy would be time-consistent because neither type of policymaker would have a strict incentive to reveal her cost after learning it.

²⁰ $c, c_L, c_H \in (\hat{\chi}, \bar{\chi})$

acting. The asset price is non-monotonic in sell volume, falling over the safe zone as the market becomes more convinced that the ‘bad’ state will obtain and rising over the risky zone as the market becomes more convinced that a bailout will be triggered resulting in the ‘good’ state. Indeed, the expected price of the asset is higher in equilibrium than if interventions were not possible. Moreover, to mitigate the probability of an intervention, informed sellers trade less aggressively and trades are, therefore, less Blackwell informative about the underlying state.

The primary tension facing the policymaker is that in order to induce investors to reveal information through their trades, she must commit not to intervene with high probability. Absent such commitment, traders employ strategies that reveal no useful information to her at all. While committing to a finely tuned intervention plan is generally not plausible for the policymaker, she may have access to institutions that provide some degree of commitment. For instance, we show that if the political process or intervention technology places a potentially binding cap on the probability of a successful bailout, then extreme selloffs do reveal valuable information. Similarly, the policymaker may induce investors to reveal useful information if she adopts a regime of secrecy regarding the actual cost or benefit of a particular bailout or her own imperfect signal of the underlying state. That is, the policy maker should *not* be transparent about revealing her own private information to investors.

While the model we analyze is admittedly stylized, it identifies some key tradeoffs and delivers novel insights regarding the use of financial markets to inform policy. A number of avenues remain open for future research. For instance, it would be instructive (albeit technically challenging) to extend the model presented here to a setting with many traders each possessing conditionally independent private information. Likewise, studying incentives for information acquisition by investors could also prove fruitful. Finally, the methods employed in this paper could be used to investigate a variety of similar settings such as a seller who learns the value of her object by observing bids in an auction and who may decide not to sell if the object is revealed to be highly valuable. Indeed, situations in which strategies of agents both inform and anticipate policy interventions are quite ubiquitous, and the question of how policymakers should act in such settings has never been more relevant.

REFERENCES

- Arrow, Kenneth J., Robert Forsythe, Michael Gorham, Robert Hahn, Robin Hanson, John O. Ledyard, Saul Levmore, Robert Litan, Paul Milgrom, Forrest D. Nelson, George R. Neumann, Marco Ottaviani, Thomas C. Shelling, Robert J. Shiller, Vernon L. Smith, Erik Snowberg, Cass R. Sunstein, Paul C. Tetlock, Philip E. Tetlock, Hal R. Varian, Justin Wolfers, and Eric Zitzewitz.** 2008. “The Promise of Prediction Markets.” *Science*, 320: 877–878.
- Bernanke, Ben, and Michael Woodford.** 1997. “Inflation Forecasts and Monetary Policy.” *Journal of Money, Credit, and Banking*, 29(4): 635–686.
- Bond, Philip, and Itay Goldstein.** 2012. “Government Intervention and Information Aggregation by Prices.” University of Minnesota Working Paper.
- Bond, Philip, Itay Goldstein, and Edward Simpson Prescott.** 2010. “Market Based Corrective Actions.” *Review of Financial Studies*, 23(2): 781–820.
- Brough, Wayne T.** 2013. “As Bernanke Doubles Down On ‘Greenspan Put,’ Larry Summers Exhales.” *Forbes*.
- Cowgill, Bo, Justin Wolfers, and Eric Zitzewitz.** 2009. “Using Prediction Markets to Track Information Flows: Evidence from Google.” Working Paper.
- Curry, Timothy J., Peter J. Elmer, and Gary S. Fissel.** 2003. “Using Market Information to Help Identify Distressed Institutions: A Regulatory Perspective.” *FDIC Banking Review*, 15(10).
- Dincer, N. Nergiz, and Barry Eichengreen.** 2007. “Central bank transparency: where, why, and with what effects?” No. w13003. National Bureau of Economic Research.
- Edmans, Alex, Itay Goldstein, and Wei Jiang.** 2011. “Feedback Effects and the Limits to Arbitrage.” NBER Working Paper 17582.
- Eusepi, Stefano.** 2005. “Central Bank Transparency Under Model Uncertainty.” Staff Report 199, Federal Reserve Bank of New York.
- Farhi, Emmanuel, and Jean Tirole.** 2012. “Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts.” *American Economic Review*, 102(1): 60–93.
- Frame, W. Scott.** 2009. “The 2008 Federal Intervention to Stabilize Fannie Mae and Freddie Mac.” Working Paper, Federal Reserve Bank of Atlanta.
- Frame, W. Scott, and Lawrence J. White.** 2005. “Fussing and Fuming over Fannie and Freddie: How Much Smoke, How Much Fire?” *Journal of Economic Perspectives*, 19(2): 159–184.

- Gandhi, Priyank, and Hanno Lustig.** 2010. "Size Anomalies in U.S. Bank Stock Returns: a Fiscal Explanation." NBER Working Paper No. 16553.
- Glosten, Lawrence R., and Paul R. Milgrom.** 1985. "Bid, ask and transaction prices in a specialist market with heterogeneously informed traders." *Journal of Financial Economics*, 14(1): 71–100.
- Grossman, Sanford J., and Joseph E. Stiglitz.** 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review*, 70(3).
- Hanson, Robin, and Ryan Oprea.** 2009. "A Manipulator Can Aid Prediction Market Accuracy." *Economica*, 76(302): 304–314.
- Kyle, Albert S.** 1985. "Continuous Auctions and Insider Trading." *Econometrica*, 53(6): 1315–1335.
- Malkiel, Burton G., and Eugene F. Fama.** 1970. "Efficient Capital Markets: A Review Of Theory And Empirical Work." *The Journal of Finance*, 25(2): 383–417.
- Mitchell, Wesley, and Arthur Burns.** 1938. "Statistical Indicators of Cyclical Revivals." In *Business Cycle Indicators.*, ed. Geoffrey Moore. Princeton University Press. NBER Bulletin 69, New York.
- O'Hara, Maureen, and Wayne Shaw.** 1990. "Deposit Insurance and Wealth Effects: The Value of Being "Too Big to Fail". " *The Journal of Finance*, 45(5): 1587–1600.
- Ottaviani, Marco, and Peter Norman Sorenson.** 2007. "Outcome manipulation in corporate prediction markets." *Journal of the European Economic Association*, 5.2(3): 554–563.
- Philippon, Thomas, and Vasiliki Skreta.** 2010. "Optimal Interventions in Markets with Adverse Selection." NBER Working Paper No. 15785.
- Snowberg, Erik, Justin Wolfers, and Eric Zitzewitz.** 2012. "Prediction Markets for Economic Forecasting." NBER Working Paper No. 18222.
- Stock, James H., and Mark W. Watson.** 2003. "Forecasting Output and Inflation: The Role of Asset Prices." *The Journal of Economic Literature*, 41(3): 788–829.
- Tirole, Jean.** 2012. "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning." *American Economic Review*, 102(1): 29–59.
- Wolfers, Justin, and Eric Zitzewitz.** 2004. "Prediction Markets." *Journal of Economic Perspectives*, 18(2): 107–126.

A APPENDIX, FOR ONLINE PUBLICATION

This appendix contains the proofs of all the propositions presented in the text as well as several technical lemmas and their proofs.

Lemma A.1 (*Equilibrium Properties*). *Any equilibrium in which a type- i trader receives a positive expected payoff must satisfy the following properties.*

1. Trader i plays a mixed strategy with no mass point.

Proof. If type- i 's equilibrium strategy contains a mass point on order size \hat{t} (or consists entirely of a mass point, i.e. a pure strategy), then the posterior belief $\chi(\hat{t})$ is equal to $1 - i$ (because the distribution of noise trades has no mass points). This implies that the payoff associated with pure strategy \hat{t} is zero for type i . Because \hat{t} is inside the support, type i 's equilibrium payoff must therefore be zero, which violates the hypothesis that the trader's expected payoff is strictly positive. ■

2. If an order triggers an intervention for certain, then it must be outside the support of the trader's mixed strategy.

Proof. If order \hat{t} triggers an intervention for certain then type i 's expected payoff of submitting order \hat{t} is zero (see (2) and (3)). If this order were inside the support of i 's mixed strategy, then i 's equilibrium payoff would also be zero, violating the maintained hypothesis. ■

3. The posterior belief associated with any sell (buy) order is weakly above (below) q .

Proof. Consider the posterior belief following a buy order of size t , $\chi(t) = \frac{(1-a)q/2}{a(1-q)\phi_1(t)+(1-a)/2}$. Observe that this posterior belief is decreasing in $\phi_1(t)$, and because $\phi_1(t) \geq 0$, the posterior belief is bounded from above by q , i.e. $\chi(t) \leq q$. The proof of the reverse case is analogous. ■

4. Equilibrium payoffs for the traders are $u_1 = m_1q$ and $u_0 = m_0(1 - q)$.

Proof. Because the equilibrium payoff of a type-1 trader is the same as the payoff from playing m_1 for certain, we find (from (2)) that $u_1 = m_1\chi(m_1)(1 - \alpha(m_1))$. Applying point 2. gives, $u_1 \leq m_1q(1 - \alpha(m_1))$. Next, consider an order just below m_1 . Because m_1 is the smallest element of the support of type 1's mixed strategy, an order just below m_1 could only be submitted by a noise trader and is therefore associated with posterior belief q . Hence, by submitting an order less than m_1 (and outside of the support), the type one trader can guarantee a payoff arbitrarily close to m_1q . In order for such a deviation to be unprofitable, $u_1 \geq m_1q$. Combining these inequalities gives $m_1q \leq u_1 \leq m_1q(1 - \alpha(m_1))$. These inequalities imply that $\alpha(m_1) = 0$ and $u_1 = m_1q$. The second part, $u_0 = m_0(1 - q)$ is established in the same way. ■

5. The probability of intervention at m_0 and m_1 is zero.

Proof. See proof of part 2. ■

6. The maximum element of S_i is 1.

Proof. Suppose that $t = 1$ is outside the support of type- i 's mixed strategy. Because $t = 1$ is only submitted by a noise trader $\chi(1) = q$. Hence, by deviating outside the support to $t = 1$ a type 1 trader can guarantee payoff q and a type 0 trader can guarantee payoff $1 - q$. Because the equilibrium payoff for the type 1 trader must be no less than the payoff of this deviation, $m_0(1 - q) \geq 1 - q \Rightarrow m_0 \geq 1$ and $m_1q \geq q \Rightarrow m_1 \geq 1$. Because any order above 1 reveals the trader's type, the maximum element of the support must be less than or equal to one, hence $m_0 = m_1 = 1$. If, however, the minimum and maximum of the support are both one, then the trader is employing a pure strategy on 1, ruled out in part 1. ■

7. The support of each type of trader's mixed strategy contains no gaps.

Proof. The argument is very similar to the one in part 4. Suppose a gap exists inside of the support. This means that intervals $[m_i, x]$ and $[y, 1]$ are inside the support of type i mixed strategy (where $m_i < x < y < 1$) but (x, y) is not inside the support. Because it is the center of $[x, y]$, $\hat{t} = \frac{x+y}{2}$ is outside the support. Hence by deviating to \hat{t} , a type 1 trader can ensure a payoff of $\hat{t}q$ and a type 0 trader can ensure a payoff of $\hat{t}(1 - q)$. Because $\hat{t} > x > m_i$, this is a profitable deviation $\hat{t}(1 - q) > m_0(1 - q) = u_0$ and $\hat{t}q > m_1q = u_1$. ■

8. If the probability of intervention is constant for all sell (buy) orders in an interval inside S_i , then $\chi(t)$ is increasing (decreasing) on this interval.

Proof. Suppose $[t_L, t_H] \subset S_i$, and for $t \in [t_L, t_H]$ the probability of intervention $\alpha(t) = \alpha \in [0, 1)$. By point 1. of this Lemma, $\alpha < 1$. Type 1's indifference condition requires that $u_1 = t\chi(t)(1 - \alpha)$, which immediately implies that $\chi(t)$ is decreasing. Similarly, type 0 indifference condition requires that $u_0 = t(1 - \chi(t))(1 - \alpha)$, which immediately implies that $\chi(t)$ is increasing. ■

9. If the posterior belief is constant for all sell (buy) orders in an interval inside S_i then $\alpha(t)$ is increasing on this interval.

Proof. Analogous to 7. ■

Proof. Proposition 3.1. If the trader uses a mixed strategy in equilibrium, he must be indifferent between all orders inside the support of his mixed strategy. In the absence of interventions, this condition requires (see equations (2) and (3))

$$u_1 = t\chi(t), \quad \forall t \in [m_1, 1] \text{ and } u_0 = t(1 - \chi(t)) \quad \forall t \in [m_0, 1].$$

Substituting the equilibrium payoffs derived in part 4 of Lemma A.1, $u_1 = m_1(1 - q)$ and $u_0 = m_0(1 - q)$ and the posterior beliefs given in (5) and (6) and then solving these equations gives expressions for the equilibrium mixing densities:

$$\phi_1(t) = \frac{t - m_1}{K_1 m_1} \text{ and } \phi_0(t) = \frac{t - m_0}{K_0 m_0}$$

The equilibrium value of m_i is computed by ensuring that the densities integrate to one. For the type-1 trader

$$\int_{m_1^*}^1 \frac{t - m_1^*}{K_1 m_1^*} dt = \frac{(1 - m_1^*)^2}{2K_1 m_1^*}$$

Setting this equal to 1 yields equation $Q(m_1^*, 0|K_1, J) = 0$ (see equation (7)). Hence,

$$m_1^* = K_1 + 1 - \sqrt{(K_1 + 1)^2 - 1}$$

which is evidently always inside $(0, 1)$.²¹ Moreover, replacing $(1 - q)$ with q in the above calculation yields

$$(A1) \quad m_0^* = K_0 + 1 - \sqrt{(K_0 + 1)^2 - 1}$$

We prove the comparative static claims for q ; those for a are proven analogously. Define

$$F(k) = k + 1 - \sqrt{(k + 1)^2 - 1}.$$

Observe that

$$F'(k) = -\frac{F(k)}{\sqrt{(k + 1)^2 - 1}} < 0$$

$$(A2) \quad \frac{\partial K_1}{\partial q} = -\frac{2a}{1 - a} < 0$$

1. Note that $\frac{\partial m_1^*}{\partial q} = F'(K_1) \frac{\partial K_1}{\partial q} > 0$.
2. We just established that the left end of the support of type-1's equilibrium mixed strategy increases with q . The right endpoint of the support is 1 for all q . First order stochastic dominance will follow if a rise in q causes $\phi_1(t)$ to become steeper at every t for which it is non-zero. Because $\phi_1(t)$ is linear in t , this amounts to demonstrating that its slope increases with q . Hence, showing that

$$\frac{d}{dq} \left(\frac{1}{K_1 m_1^*} \right) > 0$$

will prove the claim.

$$\frac{d}{dq} \left(\frac{1}{K_1 m_1^*} \right) = -\frac{\frac{\partial K_1}{\partial q} m_1^* - \frac{\partial m_1^*}{\partial q} K_1}{(K_1 m_1^*)^2}$$

This expression is positive if the numerator of the fraction is negative

$$\frac{\partial K_1}{\partial q} m_1^* - \frac{\partial m_1^*}{\partial q} K_1 < 0$$

²¹It is straightforward to verify that the larger root is greater than 1.

Because $\frac{\partial K_1}{\partial q} < 0$ and $\frac{\partial m_1^*}{\partial q} > 0$, as well as $m_1^* > 0$ and $K_1 > 0$, the result is evident.

3. Note that $u_1 = m_1^* q$. Because m_1^* and q both increase with q , the result is evident.

This completes the proof of Proposition 3.1 ■

Lemma A.2 (*Useful Relationships*) *The following relationships turn out to be analytically useful.*

1. $a > 0$ and $q > 0$ imply $\tilde{\chi} < \bar{\chi}$.

Proof. Consider the following string of equivalent expressions:

$$\begin{aligned} \hat{\chi} &< \bar{\chi} \\ \Leftrightarrow \frac{K+q}{K+1} &< 1 - (1-q)m_0^* \\ \Leftrightarrow m_0^* &< \frac{1}{K+1} \\ \Leftrightarrow K+1 - \sqrt{(K+1)^2 - 1} &< \frac{1}{K+1} \\ \Leftrightarrow (K+1)^2 - \sqrt{(K+1)^4 - (K+1)^2} &< 1 \end{aligned}$$

Let $z = (K+1)^2$. Then, the last line above holds iff

$$\begin{aligned} z-1 &< \sqrt{z^2 - z} \\ \Leftrightarrow 1 &< z \\ \Leftrightarrow K &> 0 \\ \Leftrightarrow a > 0 \text{ and } q > 0. \end{aligned}$$

■

2. $\phi_0(t) < f \Leftrightarrow \chi(t) < c$

Proof. Consider the following string of equivalent expressions:

$$\begin{aligned} \chi(t) &< c \\ \Leftrightarrow \frac{q(a\phi_0(t) + (1-a)\frac{1}{2})}{qa\phi_0(t) + (1-a)\frac{1}{2}} &< c \\ \Leftrightarrow \frac{K_0\phi_0(t) + q}{K_0\phi_0(t) + 1} &< c \\ \Leftrightarrow K_0\phi_0(t)(1-c) &< c - q \\ \Leftrightarrow \phi_0(t) &< f \end{aligned}$$

■

3. $\hat{\chi} < c \Leftrightarrow f > 1$ and $J - 1 > K$.

Proof. Consider the following string of equivalent expressions:

$$\begin{aligned}
 \hat{\chi} &< c \\
 &\Leftrightarrow \frac{K_0 + q}{K_0 + 1} < c \\
 &\Leftrightarrow 1 - \frac{1 - q}{K_0 + 1} < c \\
 &\Leftrightarrow 1 - c < \frac{1 - q}{K_0 + 1} \\
 &\Leftrightarrow K + 1 < J \\
 &\Leftrightarrow 1 < f.
 \end{aligned}$$

■

4. $c < \bar{\chi} \Leftrightarrow J < \frac{1}{m_0^*}$

Proof. Consider the following string of equivalent expressions:

$$\begin{aligned}
 c &< \bar{\chi} \\
 &\Leftrightarrow c < 1 - m_0^*(1 - q) \\
 &\Leftrightarrow J < \frac{1}{m_0^*}
 \end{aligned}$$

■

Lemma A.3 (Equilibrium structure) *In any equilibrium with non-zero expected payoff for the type-0 trader and with a positive probability of intervention on the equilibrium path, there exist values $0 < m_0 \leq \theta < 1$ such that the structure of player strategies is the following:*

- Policymaker intervenes with probability $\alpha(t) \in (0, 1)$ if she observes a sell order $t > \theta$, and does not intervene for smaller sell orders, $t \leq \theta$.
- Type-0 trader places a sell order distributed according to probability density function $\phi_0(t)$ over support $[m_0, 1]$ defined piecewise:

$$\phi_0(t) = \begin{cases} \phi_L(t) & \text{if } t \in [m_0, \theta) \\ \phi_H(t) & \text{if } t \in [\theta, 1] \end{cases}$$

where $\phi_L(t)$ is increasing and $\phi_H(t)$ is constant. In addition $\phi_L(\theta^-) \leq \phi_H(\theta)$.

Proof. We derive this result by combining several points.

Combining parts 1 and 6 of Lemma A.1 gives that all order flows between m_0 and 1 are inside the

support of the type 0 trader's mixed strategy. Because $u_0 = m_0(1 - q)$, and $u_0 > 0$, it must be that $m_0 > 0$. Thus the density can be represented as a piecewise function on $[m_0, 1]$.

Note that $t, t' \in S_0$, $\alpha(t) > 0$ and $t' > t$ imply that $\alpha(t') > 0$. This holds because any order flow that triggers intervention for certain cannot be part of the support of the trader's strategy, so $\alpha(t) \in (0, 1)$. Therefore $\chi(t) = c$ by (4). Substituting this into (3) and noting that the trader must be indifferent over all trades in S_0 yields $u_0 = t(1 - c)(1 - \alpha(t))$. Because $t' \in S_0$, we also have $u_0 = t'(1 - \chi(t'))(1 - \alpha(t'))$. Suppose $\alpha(t') = 0$. Then, $\chi(t') \leq c$ and $t' > t$ imply that $u_0 = t'(1 - \chi(t')) > t(1 - c) > t(1 - c)(1 - \alpha(t)) = u_0$, a contradiction. Thus, if $\alpha(t) > 0$, for some t , it is also strictly positive for all $t' > t$. The set of order flows that trigger stochastic intervention is thus an interval. If θ denotes the left endpoint of this interval, then the argument above establishes that the policymaker stochastically intervenes for all order flows above θ . This implies that for $t \in (\theta, 1]$, the posterior belief $\chi(t) = c$, and thus $\phi_0(t)$ is constant on this interval.

If $m_0 = \theta$, then $\chi(t) = c$ for all $t \in (m_0, 1]$. However, if $m_0 < \theta$, then on interval $[m_0, \theta)$ the probability of intervention is zero by definition of θ . Then, by part 8 of Lemma A.1, $\chi(t)$ is increasing on $[m_0, \theta)$, which implies that $\phi_0(t)$ is increasing on this interval. Furthermore, because for all order flows inside the support of the trader's mixed strategy the posterior belief is less than c , is equal to c for $t \in [\theta, 1]$ and $\chi(t)$ is an increasing function inside $[m_0, \theta)$, it follows that $\phi_L(\theta^-) \leq \phi_H$ ■

Proof. Proposition 4.1. The structure of the equilibrium strategies is derived in Lemma A.3. We prove the rest of the claims by first computing the density for the trader's mixed strategy.

Sequential rationality for the policymaker (i.e., (4) requires

$$\chi(t) = c \text{ if } t \in [\theta^\dagger, 1]$$

Substituting for $\chi(t)$ from (6) and solving yields

$$\phi_H(t) = f, \text{ if } t \in [\theta^\dagger, 1]$$

Moreover, the same argument as in the proof of Proposition 3.1 yields

$$\phi_L(t) = \frac{t - m_0}{K_0 m_0}$$

Next we establish the proposed relationship between the intervention threshold θ^\dagger and the minimum order size m_0 . Because $\chi(t) \leq c$ for all order flows inside of the support of the investor's mixed strategy, and $\chi(t)$ increases with $\phi_0(t)$, it must be that for all t

$$\frac{t - m_0}{K_0 m_0} \leq f$$

and thus for $t = \theta^\dagger$ we have $\theta^\dagger \leq Jm_0$. Next, consider the trader indifference condition at $t = \theta^\dagger$.

$$m_0(1 - q) = \theta^\dagger(1 - c)(1 - \alpha(\theta^\dagger))$$

Then $\alpha(\theta^\dagger) \geq 0$ implies $\theta^\dagger \geq Jm_0$. Hence, $\theta^\dagger = Jm_0$.

Now, because $\phi_0(t)$ is a density, it must integrate to 1

$$\int_{m_0^\dagger}^{Jm_0^\dagger} \frac{t - m_0^\dagger}{K_0 m_0^\dagger} dt + (1 - Jm_0^\dagger) \frac{J - 1}{K_0} = 1.$$

Integrating yields equation $Q(m_0^\dagger, 1 - Jm_0^\dagger | K_0, J) = 0$. Two solutions exist. One is zero and the other is

$$(A3) \quad m_0^\dagger \equiv \frac{2(J - K_0 - 1)}{J^2 - 1}.$$

This is a valid equilibrium only if $m_0^\dagger \geq 0$ and $\theta^\dagger \leq 1$. Part 3 of Lemma A.2 shows $m^\dagger > 0$ when $c > \hat{\chi}$. To verify $\theta^\dagger \leq 1$, we show that $\theta^\dagger = 1$ for $c = \bar{\chi}$ and that $\frac{\partial m_0^\dagger}{\partial c} > 0$. Note that by part 4 of Lemma A.2, $c < \bar{\chi} \Rightarrow J = 1/m_0^*$. In this instance

$$\theta^\dagger = Jm_0^\dagger = \frac{1}{m_0^*} \frac{2(\frac{1}{m_0^*} - K_0 - 1)}{\frac{1}{(m_0^*)^2} - 1}$$

Subtracting one from this expression and simplifying yields, $\theta^\dagger - 1 = Q(m_0^*, 0 | K_0, J) / (1 - m_0^2) = 0$ by the definition of m_0^* . Hence $c = \bar{\chi} \Rightarrow \theta^\dagger = 1$. Note that an immediate consequence is $c = \bar{\chi} \Rightarrow m_0^\dagger = m_0^*$. Next, we show $\frac{\partial m_0^\dagger}{\partial c} > 0$. Differentiation yields

$$\frac{\partial m_0^\dagger}{\partial c} = \left(\frac{2(1 - q)}{((1 - c)(J^2 - 1))^2} \right) (-J^2 + 2J(K + 1) - 1)$$

The first term in parentheses is evidently positive, so we must show

$$(A4) \quad -J^2 + 2J(K + 1) - 1 > 0$$

Observe that

$$\frac{\partial}{\partial J} (-J^2 + 2J(K + 1) - 1) = -2(J - K - 1)$$

This is negative by part 3 of Lemma A.2. Hence, the left side of (A4) is smallest when J is as large as possible. By part 4 of Lemma A.2 that $J < \frac{1}{m_0^*}$. Thus, the result will follow if

$$-(m_0^*)^{-2} + 2(m_0^*)^{-1}(K + 1) - 1 \geq 0,$$

or equivalently if

$$(m_0^*)^2 - 2(K + 1)m_0^* + 1 \leq 0.$$

But, the left side of this expression is zero by definition of m_0^* (i.e., it is $Q(m_0^*, 0|K_0, J)$ (see equation (7)). Thus, m_0^\dagger , and θ^\dagger are increasing in c . Thus, $c \in (\hat{\chi}, \bar{\chi})$ implies $m_0^\dagger \in (0, m_0^*)$ and $\theta^\dagger \in (m_0^\dagger, 1)$. (Showing $\frac{\partial m_0^\dagger}{\partial q} < 0$ follows the same lines as showing $\frac{\partial m_0^\dagger}{\partial c} > 0$. Also, $\frac{\partial m_0^\dagger}{\partial a} < 0$ follows directly from $\frac{\partial K}{\partial a} > 0$.)

Next, we derive policymaker's equilibrium strategy. The trader indifference conditions and policymaker's sequential rationality conditions (3) and (4) on $[\theta^\dagger, 1]$ require:

$$u_0 = t(1 - \chi(t))(1 - \alpha(t)) \text{ and } \chi(t) = c$$

Solving these gives

$$\alpha(t) = 1 - \frac{u_0}{t(1 - c)}$$

Because $u_0 = m_0^\dagger(1 - q)$ (by part 4 of Lemma A.1), we find that $u_0 = \theta^\dagger(1 - c)$.

$$\alpha(t) = 1 - \frac{\theta^\dagger}{t}$$

This is obviously a valid probability for $t \in [\theta^\dagger, 1]$. For $t \in [0, \theta^\dagger)$ $\chi(t) < c$ and so $\alpha(t) = 0$ (see equation (4)).

Finally, note that because policymaker does not intervene for $t \in [0, \theta^\dagger)$ and is indifferent about intervening for $t \in [\theta^\dagger, 1]$, her expected payoff must be the same as if she never intervened, namely $1 - q$. ■

Lemma A.4 (*Zero Trader Payoff*) *If $c \in (q, \hat{\chi})$, then the type-0 trader receives expected payoff of zero in any equilibrium.*

Proof. By way of contradiction, suppose $c < \tilde{\chi}$ and an equilibrium exists in which the type-0 trader makes positive expected payoff. By parts 1 and 6 of Lemma A.1 the investor mixes according to a density $\phi_0(t)$ (with no mass points) over some range $[m_0, 1]$. Part 2 of Lemma A.1 states that for any trade $t \in [m_0, 1]$ we have $\chi(t) \leq c$. Hence, by part 2 of Lemma A.2 $\phi_0(t) \leq f$. Therefore

$$\int_{m_0}^1 \phi_0(t) dt \leq \int_0^1 f dt = f.$$

But $f < 1$ by part 3 of Lemma A.2. So $\phi_0(t)$ is not a valid mixed strategy. ■

Lemma A.5 (*PM Always Intervenes*) *If $c \in (q, \hat{\chi})$, then in every equilibrium policymaker intervenes with certainty following all sell orders, $t > 0$.*

Proof. By way of contradiction, suppose $c \in (q, \hat{\chi})$ and consider an equilibrium in which there exists $\hat{t} > 0$ such that $\alpha(\hat{t}) < 1$. The sequential rationality condition for policymaker (4) implies $\chi(\hat{t}) \leq c$.

But, then placing a sell order of \hat{t} would earn the type-0 trader expected payoff $\hat{t}(1-\chi(\hat{t}))(1-\alpha(\hat{t})) > 0$ contradicting Lemma A.4. ■

Proof. Proposition 4.2. By Lemma A.5, $\alpha(t) = 1$ for all $t > 0$. For this to hold (4) requires $\chi(t) \geq c$. By part 2 of Lemma A.2 this is equivalent to $\phi_0(t) \geq f$. Note also by part 3 of Lemma A.2 that $c < \hat{\chi}$ is equivalent to $f < 1$. Hence, if (for instance) $\phi_0(t) = f$ for almost every $t \in (0, 1]$, then residual probability mass of $1 - f > 0$ exists. Since $\alpha(t) = 1$ for all $t > 0$ this probability may be distributed as mass points over any set of trades (including $t = 0$) without impacting the investor's payoff.

The claim regarding PM's strategy is established in Lemma A.5 except for $\alpha(0)$. As just noted, the investor may place density of at least f or positive probability mass on $t = 0$ leading to $\chi(0) > c$ and $\alpha(0) = 1$.

We now compute PM's equilibrium expected payoff. First, consider the portion of her expected payoff that corresponds to buy orders. Here $\chi(t)$ represents the posterior belief following a buy order of size t :

$$\int_0^1 (a(1-q)\phi_1(t) + (1-a)/2)(1-\chi(t)) dt = \int_0^1 (1-a+2a\phi_1(t))(1-q)/2 dt = \left(a + (1-a)\frac{1}{2}\right)(1-q)$$

Next, consider the portion of her expected payoff that corresponds to sell orders. Here $\chi(t)$ represents the posterior belief following a sell order of size t .

$$\int_0^1 (aq\phi_0(t) + (1-a)/2)(1-c) dt = \left(aq + (1-a)\frac{1}{2}\right)(1-c)$$

Adding these and performing straightforward algebra gives

$$\left(a + (1-a)\frac{1}{2}\right)(1-q) + \left(aq + (1-a)\frac{1}{2}\right)(1-c) = (1-q) + \left(aq + (1-a)\frac{1}{2}\right)(\hat{\chi} - c)$$

This is evidently decreasing in c . To see that it is decreasing in q , differentiate *wrt* q to get

$$-\frac{1+a}{2} + a(1-c).$$

This is negative for $a = 0$ and $a = 1$ and hence for all $a \in [0, 1]$. To see that the payoff is increasing in a , differentiate *wrt* a to get

$$\frac{1-q}{2} + \left(q - \frac{1}{2}\right)(1-c).$$

This is positive for $q = 0$ and $q = 1$ and hence for all $q \in [0, 1]$. ■

Proof. Proposition 5.1. Taking expectations in (1) gives

$$E[p(t)] = 1 - E[\chi(t)] + E[\chi(t)\alpha(t)].$$

Because $\chi(t)$ is the posterior belief that the state is zero *conditional on observed order flow t* , the law of iterated expectations guarantees that the expected value of $\chi(t)$ with respect to the order flow t is equal to the prior. Thus,

$$E[p(t)] = 1 - q + E[\chi(t)\alpha(t)].$$

In the no-intervention benchmark, $\alpha(t) = 0$ for all t , while in the equilibrium with stochastic interventions, $\alpha(t) > 0$ for sell orders $t \in (\theta^\dagger, 1]$. Since these orders occur with positive probability, the result follows. ■

Proof. Proposition 5.2. Let $\phi_0^*(\cdot)$ and $\phi_0^\dagger(\cdot)$ be the mixing densities of the type-0 trader given respectively in Propositions 3.1 and 4.1. Recall that $c \in (\tilde{\chi}, \bar{\chi})$ implies $m_0^\dagger < m_0^*$. Thus, $\phi_0^\dagger(t)$ starts sooner and is steeper than $\phi_0^*(t)$ for $t < \theta^\dagger$. Because both densities end at $t = 1$ and both must integrate to 1, $\phi_0^*(t)$ must cross the flat portion of $\phi_0^\dagger(t)$ at some $\hat{t} \in (\theta^\dagger, 1)$. That is

$$t \in [m_0^\dagger, \hat{t}) \Rightarrow \phi_0^\dagger(t) - \phi_0^*(t) > 0$$

$$t \in (\hat{t}, 1] \Rightarrow \phi_0^\dagger(t) - \phi_0^*(t) < 0$$

The first inequality immediately implies:

$$t \in [m_0^\dagger, \hat{t}] \Rightarrow \int_{m_0^\dagger}^t \phi_0^\dagger(t) - \phi_0^*(t) dt > 0$$

Next observe that because the top of the support of either mixed strategy is one,

$$\int_{m_0^\dagger}^1 \phi_0^\dagger(t) - \phi_0^*(t) dt = 0$$

and thus

$$t \in [\hat{t}, 1] \Rightarrow \int_{m_0^\dagger}^t \phi_0^\dagger(t) - \phi_0^*(t) dt + \int_t^1 \phi_0^\dagger(t) - \phi_0^*(t) dt = 0$$

and thus

$$t \in [\hat{t}, 1] \Rightarrow \int_{m_0^\dagger}^t \phi_0^\dagger(t) - \phi_0^*(t) dt = \int_t^1 \phi_0^*(t) - \phi_0^\dagger(t) dt$$

For $t > \hat{t}$ the right side is positive. Hence

$$t \in [m_0^\dagger, 1] \Rightarrow \int_{m_0^\dagger}^t \phi_0^\dagger(t) - \phi_0^*(t) dt \geq 0$$

■

Proof. Proposition 5.3. We begin by constructing the posterior belief random variables for the equilibria with and without interventions. We adopt a convention of labeling sell orders as negative and buy orders as positive.

No-intervention Construction. In the benchmark case, each trader type mixes according to densities:

$$\phi_1^*(t) = \frac{t - m_1^*}{K_1 m_1^*} \text{ and } \phi_0^*(t) = \frac{t - m_0^*}{K_0 m_0^*}$$

over supports $S_0 = [m_0^*, 1]$ and $S_1 = [m_1^*, 1]$. Viewed *ex ante*, the equilibrium order flow is a random variable t^* , with support on $[-1, 1]$, and the following density:

$$f^*(t) = \begin{cases} aq\phi_0^*(-t) + (1-a)/2 & \text{if } t \in [-1, -m_0^*] \\ (1-a)/2 & \text{if } t \in [-m_0^*, m_1^*] \\ a(1-q)\phi_1^*(t) + (1-a)/2 & \text{if } t \in [m_1^*, 1] \end{cases}$$

which simplifies in the following way:

$$f^*(t) = \begin{cases} \frac{(1-a)|t|}{2m_0^*} & \text{if } t \in [-1, -m_0^*] \\ (1-a)/2 & \text{if } t \in [-m_0^*, m_1^*] \\ \frac{(1-a)t}{2m_1^*} & \text{if } t \in [m_1^*, 1] \end{cases}$$

In the interval $[-1, -m_0^*]$ the order flow could be generated either by a noise trader or by a negatively informed trader. Thus the density is a weighted average of the densities of the negatively informed trader and the noise trader. In the interval $[-m_0^*, m_1^*]$ order flow is generated only by the noise trader, and therefore follows his density, and in $[m_1^*, 1]$ the order flow could again be generated by either the positively informed trader or the noise trader. The posterior belief as a function of the observed order flow is as follows:

$$\chi(t) = \begin{cases} 1 + \frac{m_0^*(1-q)}{t} & \text{if } t \in [-1, -m_0^*] \\ q & \text{if } t \in [-m_0^*, m_1^*] \\ \frac{m_1^*q}{t} & \text{if } t \in [m_1^*, 1] \end{cases}$$

Let x^* represent the posterior belief random variable for the no intervention equilibrium, i.e. $x^* = \chi(t^*)$. The support of x^* is clearly $[\underline{\chi}, \bar{\chi}] \equiv [m_1^*q, 1 - (1-q)m_0^*]$. Furthermore x^* clearly has a mass point on q , taking on this realization with probability $(m_0^* + m_1^*)(1-a)/2$. To calculate the density of x^* on intervals $[\underline{\chi}, q)$ and $(q, \bar{\chi}]$ apply the standard formula for transformation of density

to obtain $g^*(x)$, the density of x^* .

$$x \in [\underline{\chi}, q) \Rightarrow g^*(x) = f^*\left(\frac{m_1^*q}{x}\right) \left\| \frac{d}{dx} \left(\frac{m_1^*q}{x}\right) \right\| = \frac{m_1^*q^2(1-a)}{2x^3}$$

$$x \in (q, \bar{\chi}] \Rightarrow g^*(x) = f^*\left(\frac{m_0^*q}{1-x}\right) \left\| \frac{d}{dx} \left(\frac{m_0^*q}{1-x}\right) \right\| = \frac{m_0^*(1-q)^2(1-a)}{2(1-x)^3}$$

Integrating this density function (and remembering the mass point on q) gives the distribution function of x^* , $G^*(x)$

$$G^*(x) = \begin{cases} \frac{(1-a)(x^2 - q^2 m_1^{*2})}{4m_1^* x^2} & \text{if } x \in [\underline{\chi}, q) \\ \frac{(1-a)(m_1^{*2} + 2m_0^* m_1^* + 1)}{4m_1^*} + \frac{m_0^*(1-a)(x-q)(2-x-q)}{4(1-x)^2} & \text{if } x \in (q, \bar{\chi}] \\ 1 & \text{if } x \in [\bar{\chi}, \infty) \end{cases}$$

Stochastic Intervention Construction. Denote the posterior belief random variable for the stochastic intervention equilibrium as x^\dagger . In this equilibrium the type-1 trader places an order distributed in an identical fashion to the no-intervention equilibrium. Thus the distribution of x^\dagger over interval $[\underline{\chi}, q)$ is unaffected. The type-0 trader places a sell order distributed according to probability density function $\phi_0^\dagger(t)$ over support $[m_0^\dagger, 1]$ defined piecewise:

$$\phi_0^\dagger(t) = \begin{cases} \frac{t - m_0^\dagger}{m_0^\dagger K_0} & \text{if } t \in [m_0^\dagger, \theta^\dagger] \\ \frac{(c-q)}{(1-c)K_0} & \text{if } t \in [\theta^\dagger, 1] \end{cases}$$

Thus the order flow in an equilibrium with stochastic interventions has the following density:

$$f^\dagger(t) = \begin{cases} aq \frac{(c-q)}{(1-c)K_0} + (1-a)/2 & \text{if } t \in [-1, -\theta^\dagger] \\ aq \frac{(-t - m_0^\dagger)}{m_0^\dagger K_0} + (1-a)/2 & \text{if } t \in [-\theta^\dagger, -m_0^\dagger] \\ (1-a)/2 & \text{if } t \in [-m_0^\dagger, m_1^*] \\ a(1-q)\phi_1^*(t) + (1-a)/2 & \text{if } t \in [m_1^*, 1] \end{cases}$$

which simplifies to

$$f^\dagger(t) = \begin{cases} \frac{(1-a)(1-q)}{2(1-c)} & \text{if } t \in [-1, -\theta^\dagger] \\ \frac{(1-a)|t|}{2m_0^\dagger} & \text{if } t \in [-\theta^\dagger, -m_0^\dagger] \\ (1-a)/2 & \text{if } t \in [-m_0^\dagger, m_1^*] \\ \frac{(1-a)t}{2m_1^*} & \text{if } t \in [m_1^*, 1] \end{cases}$$

The posterior belief as a function of order t is given by

$$\chi(t) = \begin{cases} c & \text{if } t \in [-1, -\theta^\dagger] \\ 1 + \frac{m_0^\dagger(1-q)}{t} & \text{if } t \in [-\theta^\dagger, -m_0^\dagger] \\ q & \text{if } t \in [-m_0^\dagger, m_1^*] \\ \frac{m_1^*q}{t} & \text{if } t \in [m_1^*, 1] \end{cases}$$

From here, it is clear that the distribution of posterior beliefs greater than q is affected in a number of ways. First, a mass point on c exists.

$$\Pr(x^\dagger = c) = (1 - \theta^\dagger) \left(\frac{c - q}{(1 - c)K(a, q)} aq + (1 - a) \frac{1}{2} \right) = (1 - \theta^\dagger) \frac{(1 - a)(1 - q)}{2(1 - c)}$$

The mass point on q is smaller with intervention, because the bottom of the support with intervention $m_0^\dagger < m_0^*$, as the following calculation illustrates:

$$\Pr(x^\dagger = q) = (m_0^\dagger + m_1^*) \frac{1 - a}{2}$$

Inside interval (q, c) the density is defined by the same expression as with no intervention, however, the minimum trade size is different: instead of m_0^* substitute $m_0^\dagger = \theta^\dagger \frac{1-c}{1-q}$. For $x \in (q, c)$ the density of x^\dagger , denoted $g^\dagger(x)$ is as follows:

$$x \in (q, c) \Rightarrow g^\dagger(x) = \theta^\dagger(1 - c) \frac{(1 - q)(1 - a)}{2(1 - x)^3}$$

Hence, for $x \in (q, c)$ the distribution of the posterior belief is

$$x \in (q, c) \Rightarrow G^\dagger(x) = \frac{(1 - a)(1 - m_1^{*2})}{4m_1^*} + \frac{1 - a}{2}(m_0^\dagger + m_1^*) + \int_q^x \theta^\dagger(1 - c) \frac{(1 - q)(1 - a)}{2(1 - s)^3} ds$$

$$G^\dagger(x) = \frac{(1 - a)(1 - m_1^{*2})}{4m_1^*} + \frac{1 - a}{2}(m_0^\dagger + m_1^*) + \frac{m_0^\dagger(1 - a)(x - q)(2 - x - q)}{4(1 - x)^2}$$

Thus the distribution of the posterior belief random variable for the equilibrium with stochastic interventions is as follows:

$$G^\dagger(x) = \begin{cases} \frac{(1-a)(x^2 - q^2 m_1^{*2})}{4m_1^* x^2} & \text{if } t \in [\underline{\chi}, q) \\ \frac{(1-a)(1-m_1^{*2})}{4m_1^*} + \frac{1-a}{2}(m_0^\dagger + m_1^*) + \frac{m_0^\dagger(1-a)(x-q)(2-x-q)}{4(1-x)^2} & \text{if } x \in (q, c) \\ 1 & \text{if } x \in [c, \infty) \end{cases}$$

Comparison. Our goal is to show that the signal with interventions is less Blackwell informative than the signal without interventions. To do this it is sufficient to establish that the posterior random variable with no intervention x^* is a mean preserving spread of the posterior belief random variable with intervention, x^\dagger . See Theorem 2 of Ganuza and Penalva (2010) for a proof that this property is equivalent to Blackwell informativeness for binary states. Both posterior belief random variables must have the same mean (namely q) by the law of iterated expectations. Therefore, to show that x^* is a mean preserving spread we must establish that for all x

$$\int_{-\infty}^x G^*(s) - G^\dagger(s) ds \geq 0$$

Because $s < q \Rightarrow G^*(s) = G^\dagger(s)$, this property is trivially satisfied for $s < q$. Next, consider $s \in [q, c]$.

$$s \in (q, c) \Rightarrow G^*(s) - G^\dagger(s) = (m_0^* - m_0^\dagger) \left(\frac{1-a}{2} + \frac{(1-a)(s-q)(2-s-q)}{4(1-s)^2} \right)$$

This is positive because $m_0^* > m_0^\dagger$. Hence $G^*(s) - G^\dagger(s) \geq 0$ is positive for all $s \in (q, c)$, and therefore for any $x \in (q, c)$

$$\int_{-\infty}^x G^*(s) - G^\dagger(s) ds > 0$$

Next, consider $x \in (c, \bar{\chi})$. At $s = c$ distribution $G^\dagger(s)$ jumps to 1. Hence, for $x \in (c, \bar{\chi}]$ the difference in the integral of the CDFs is the following:

$$\int_q^c (m_0^* - m_0^\dagger) \left(\frac{1-a}{2} + \frac{(1-a)(s-q)(2-s-q)}{4(1-s)^2} \right) ds - \int_c^x 1 - G^*(s) ds$$

The second term becomes more negative as x increases. Therefore if the expression is positive for $x = \bar{\chi}$ then it is positive for all $x \in (c, \bar{\chi}]$. Thus, if the following condition holds, then the market is more Blackwell informative without interventions.

$$\begin{aligned} & \int_q^c (m_0^* - m_0^\dagger) \left(\frac{1-a}{2} + \frac{(1-a)(s-q)(2-s-q)}{4(1-s)^2} \right) ds \geq \int_c^{\bar{\chi}} 1 - G^*(s) ds \\ & \int_q^c (m_0^* - m_0^\dagger) \left(\frac{1-a}{2} + \frac{(1-a)(s-q)(2-s-q)}{4(1-s)^2} \right) ds = (m_0^* - m_0^\dagger) \frac{(1-a)(c-q)(2-c-q)}{4(1-c)} \\ & \int_c^{\bar{\chi}} 1 - G^*(s) ds = \left(1 - \frac{1-a}{4m_1^*} (m_1^{*2} + 2m_0^*m_1^* + 1)\right) (1 - m_0^*(1-q) - c) - \frac{1-a}{4(1-c)} (1-c - m_0^*(1-q))(1-q - m_0^*(1-c)) \end{aligned}$$

Substituting the equilibrium values of m_0^* , m_1^* , m_0^\dagger gives that the two terms are exactly equal. Thus the integral under the CDF of the posterior random variable without interventions is greater than the integral of the CDF of the posterior random variable with interventions. ■

Proof. Proposition 6.1. PM chooses $\alpha(t)$ and $\phi_0(t)$ to maximize her *ex ante* expected payoff:

(A5)

$$v = (1 - q) \left(a + (1 - a) \frac{1}{2} \right) + \int_0^1 \left(aq\phi_0(t) + (1 - a) \frac{1}{2} \right) (\alpha(t)(1 - c) + (1 - \alpha(t))(1 - \chi(t))) dt$$

Associated with these choices are feasibility constraints:

$$0 \leq \alpha(t) \leq 1 \quad \text{and} \quad \phi_0(t) \geq 0$$

The trader's equilibrium indifference condition requires that he is indifferent between all order flows inside the support of his mixed strategy. Therefore, if $\phi_0(t) > 0$ then $t(1 - \chi(t))(1 - \alpha(t)) = u_0$. We write this constraint in the following way:

$$\phi_0(t)[t(1 - \chi(t))(1 - \alpha(t)) - u_0] = 0.$$

A second equilibrium condition requires that no pure strategy outside of the support of the trader's mixed strategy would give the trader an expected payoff greater than u_0 . We write this condition in the following way:

$$t(1 - \chi(t))(1 - \alpha(t)) \leq u_0$$

The last equilibrium condition requires that the trader's density integrates to one over the unit interval.

$$\int_0^1 \phi_0(t) dt = 1$$

When formulating the policymaker's problem, we will include Lagrange multipliers for the following constraints:

$$-\alpha(t) \leq 0 \quad \text{multiplier: } \mu_{1t}$$

$$-\phi_0(t) \leq 0 \quad \text{multiplier: } \mu_{2t}$$

$$\phi_0(t)[t(1 - \chi(t))(1 - \alpha(t)) - u_0] = 0 \quad \text{multiplier: } \mu_{3t}$$

$$\int_0^1 \phi_0(t) dt - 1 = 0 \quad \text{multiplier: } \lambda$$

To proceed, we now simplify the policymaker's objective function:

$$v = (1 - q) \left(a + (1 - a) \frac{1}{2} \right) \int_0^1 \left(aq\phi_0(t) + (1 - a) \frac{1}{2} \right) ((1 - \chi(t)) + \alpha(t)(\chi(t) - c)) dt$$

Using the definition of $\chi(t)$, the first term of the product in the integrand can be reduced:

$$\left(aq\phi_0(t) + (1-a)\frac{1}{2}\right)(1-\chi(t)) = (1-a)(1-q)\frac{1}{2}$$

Hence,

$$v = (1-q) + \int_0^1 \alpha(t) \left(aq\phi_0(t) + (1-a)\frac{1}{2}\right) (\chi(t) - c) dt$$

Next, simplify by substituting the definition of $\chi(t)$.

$$v = (1-q) + \int_0^1 aq(1-c)\alpha(t)\phi_0(t) - \frac{1}{2}(c-q)(1-a)\alpha(t) dt$$

$$(A6) \quad v = (1-q) + aq(1-c) \int_0^1 \alpha(t)(\phi_0(t) - f) dt$$

Because the constant terms in front of the integral do not affect the choice of $(\alpha(t), \phi_0(t))$ we omit these from the Lagrangian:

$$\mathcal{L} = \int_0^1 \alpha(t)(\phi_0(t) - f) + \mu_{1t}\alpha(t) + \mu_{2t}\phi_0(t) + \mu_{3t}\phi_0(t) \left[t \frac{(1-q)}{1+K\phi_0(t)} (1-\alpha(t)) - u_0 \right] - \lambda(\phi_0(t) - 1) dt$$

The stationarity condition for $\alpha(t)$ requires:

$$(A7) \quad \frac{\partial \mathcal{L}}{\partial \alpha(t)} = 0 \Rightarrow \phi_0(t) - f + \mu_{1t} - \mu_{3t}(1-q) \frac{t\phi_0(t)}{1+K\phi_0(t)} = 0$$

The stationarity condition for $\phi_0(t)$ requires

$$\frac{\partial \mathcal{L}}{\partial \phi_0(t)} = 0 \Rightarrow \alpha(t) + \mu_{2t} + \mu_{3t} \left[t \frac{(1-q)}{1+K\phi_0(t)} (1-\alpha(t)) - u_0 - \phi_0(t)t(1-\alpha(t)) \frac{K(1-q)}{(1+K\phi_0(t))^2} \right] - \lambda = 0$$

Observe that

$$t \frac{(1-q)}{1+K\phi_0(t)} (1-\alpha(t)) - u_0 - \phi_0(t)t(1-\alpha(t)) \frac{K(1-q)}{(1+K\phi_0(t))^2} = \frac{t(1-\alpha(t))(1-q)}{(1+K\phi_0(t))^2} - u_0$$

Simplifying this stationarity condition gives:

$$(A8) \quad \alpha(t) + \mu_{2t} + \mu_{3t} \left[\frac{t(1-\alpha(t))(1-q)}{(1+K\phi_0(t))^2} - u_0 \right] - \lambda = 0$$

In addition to the primal feasibility conditions and the two stationarity conditions, dual feasibility requires that

$$\mu_{1t} \geq 0 \quad \mu_{2t} \geq 0$$

and complementary slackness requires that

$$\mu_{1t}\alpha(t) = 0 \quad \mu_{2t}\phi_0(t) = 0.$$

In an equilibrium with $u_0 > 0$, the trader cannot mix over the entire interval $[0, 1]$. A set of order flows must exist for which $\phi_0(t) = 0$. Consider some such order flow. The stationarity condition for $\alpha(t)$ implies that

$$\mu_{1t} = f > 0$$

Because $\mu_{1t} > 0$, complementary slackness implies that $\alpha(t) = 0$. Therefore $\phi_0(t) = 0 \Rightarrow \alpha(t) = 0$, that is, no intervention takes place outside of the support of the trader's mixed strategy. Therefore from a primal constraint, for any t outside of the support of the trader's mixed strategy it must be that

$$t(1 - q) \leq u_0$$

Recalling that $u_0 = m_0^\dagger(1 - q)$, this becomes $t \leq m_0^\dagger$. Hence if some t is outside of the support then all smaller t are also outside of the support. Thus, the set of order flows outside the support forms an interval. The supremum of points outside of the support is m_0^\dagger . Part 5 of Lemma A.1 implies that no intervention takes place at m_0^\dagger . This also implies that an interval exists in which $\phi_0(t) > 0$ and $\alpha(t) = 0$.

Consider an order flow for which $\phi_0(t) > 0$ and $\alpha(t) = 0$. Here the stationarity conditions require

$$\phi_0(t) - f + \mu_{1t} - \mu_{3t}(1 - q) \frac{t\phi_0(t)}{1 + K\phi_0(t)} = 0$$

$$\mu_{3t} \left[\frac{t(1 - q)}{(1 + K\phi_0(t))^2} - m_0^\dagger(1 - q) \right] - \lambda = 0$$

Primal feasibility then implies that

$$\phi_0(t) = \frac{t - m_0^\dagger}{Km_0^\dagger}$$

Solving these equations gives that

$$\mu_{3t} = - \frac{\lambda t}{m_0^\dagger(t - m_0^\dagger)(1 - q)}$$

$$\mu_{1t} = f + \frac{1}{K} - \frac{1 + \lambda}{Km_0^\dagger} t$$

Thus (recalling $J = 1 + Kf$ and $\theta^\dagger = Jm_0^\dagger$)

$$\mu_{1t} \geq 0 \Leftrightarrow t \leq \frac{\theta^\dagger}{1 + \lambda}$$

$$\phi_t \geq 0 \Leftrightarrow t \geq m_0$$

If no region in which $\alpha(t) > 0$ exists, then the policymaker never benefits from intervention. Hence, under the optimal policy, a region must exist with $\alpha(t) > 0$. In this region the stationarity conditions are

$$\begin{aligned} \phi_0(t) - f - \mu_{3t}(1-q)\frac{t\phi_0(t)}{1+K\phi_0(t)} &= 0 \\ \alpha(t) + \mu_{3t}\left[\frac{t(1-\alpha(t))(1-q)}{(1+K\phi_0(t))^2} - m_0^\dagger(1-q)\right] - \lambda &= 0 \end{aligned}$$

and primal feasibility implies that

$$\phi_0(t) = \frac{t(1-\alpha(t)) - m_0^\dagger}{Km_0^\dagger}$$

Solving these conditions gives that

$$\alpha(t) = \frac{1}{2} \left(1 + \lambda - \frac{\theta^\dagger}{t} \right)$$

$$\alpha(t) > 0 \Rightarrow t > \frac{\theta^\dagger}{1+\lambda}$$

$$\alpha(1) < 1 \Rightarrow \lambda < 1 + \theta^\dagger$$

Observe that when $\lambda = 0$ the commitment policy is exactly half of the equilibrium intervention policy (and starts from the same intervention threshold θ^\dagger). Therefore, if λ were zero, the density would integrate to more than one: hence, $\lambda > 0$ (this can also be established directly by differentiating the constraint and signing its derivative). When PM intervenes,

$$\phi_0(t) = \frac{t(1 - \frac{1}{2}(1 + \lambda - \frac{\theta^\dagger}{t})) - m_0^\dagger}{Km_0^\dagger} = f + \frac{(1-\lambda)t - (1+Kf)m_0}{2Km_0}$$

Observe first that at the intervention threshold, $t = \frac{\theta^\dagger}{1+\lambda}$ the value of $\phi_0(t)$ is

$$\phi_0(t) = f - \frac{\lambda J}{K(1+\lambda)}$$

which (because $\lambda > 0$) implies that in the constrained optimal program the policymaker begins to intervene at a *smaller order* than is profitable (i.e. when intervention negatively impacts her payoff). This result also implies that $\lambda < 1$, otherwise $\phi_0(t)$ is decreasing in the intervention region, which implies that $\alpha(t) > 0 \Rightarrow \phi_0(t) < f$, but this is dominated by the equilibrium intervention policy. An immediate consequence of $\lambda < 1$ is $\alpha(1) < 1$, so the upper feasibility constraint for $\alpha(t)$ never binds. In fact $\alpha(1)$ must be less than the corresponding equilibrium value. Because the intervention policies are functions of the form $k_1 - \frac{k_2}{t}$ they cross at most once (as a function of

t). For low t the optimal policy lies above the equilibrium policy; If they did not cross at all, then the optimal policy would always be above, generating a (weakly) smaller value of $\phi_0(t)$ for all t , certainly worse than the equilibrium policy. The value of λ can be found by solving the constraint:

$$\int_{m_0^\dagger}^{\frac{\theta^\dagger}{1+\lambda}} \frac{t - m_0^\dagger}{K m_0^\dagger} dt + \int_{\frac{\theta^\dagger}{1+\lambda}}^1 f + \frac{(1-\lambda)t - \theta^\dagger}{2K m_0} dt = 1$$

We know that the left side exceeds one when $\lambda = 0$ and that optimality requires $\lambda < 1$. In order for this solution to also be consistent with inequality constraints for order flows outside the support, it is necessary and sufficient that the intervention threshold exceeds m_0 , or that $\lambda < J - 1$. This condition is implied by $\lambda < 1$ whenever $J > 2$ or $c > \frac{1+q}{2}$. Otherwise, it is possible that interventions take place at m_0^\dagger , violating the inequality conditions for order flows below m_0^\dagger . These inequalities therefore bind in this case. As this possibility is quite technical and not especially illuminating, we omit it here, and assume $c > \frac{1+q}{2}$. ■

Proof. Proposition 7.1. Arguments analogous to those used in the proof of Lemma A.3 pin down the structure of the equilibrium strategies. We omit the specific arguments here for the sake of brevity (and tedium).

In the equilibrium with stochastic interventions the maximum probability of intervention is $\alpha(1) = 1 - \theta^\dagger$. Hence, a necessary condition for the cap to bind is $\bar{\alpha} < 1 - \theta^\dagger$.

By part 4 of Lemma A.1 the trader's expected payoff is $u_0 = m_0(1 - q)$. Hence, his indifference requires the following:

$$\begin{aligned} t \in [m_0, \theta_1] &\Rightarrow m_0(1 - q) = t(1 - \chi(t)) \\ t \in [\theta_1, \theta_2] &\Rightarrow m_0(1 - q) = t(1 - \chi(t))(1 - \alpha(t)) \\ t \in [\theta_2, 1] &\Rightarrow m_0(1 - q) = t(1 - \chi(t))(1 - \bar{\alpha}). \end{aligned}$$

Sequential rationality by PM requires the following:

$$\begin{aligned} t \in [m_0, \theta_1] &\Rightarrow \chi(t) < c \\ t \in [\theta_1, \theta_2] &\Rightarrow \chi(t) = c \text{ for } t \in [\theta_1, \theta_2] \\ t \in [\theta_2, 1] &\Rightarrow \chi(t) \geq c. \end{aligned}$$

Using (6) and solving these equations gives the following expressions:

$$\begin{aligned} \phi_0(t) &= \frac{t - m_0}{K_0 m_0} \text{ if } t \in [m_0, \theta_1] \\ \phi_0(t) &= f \text{ if } t \in [\theta_1, \theta_2] \\ \phi_0(t) &= \frac{(1 - \bar{\alpha})t - m_0}{K_0 m_0} \text{ if } t \in [\theta_2, 1] \end{aligned}$$

As in the case without the intervention cap, (and for the same reason) $\theta_1 = Jm_0$. Next, $t \in \{\theta_1, \theta_2\}$ implies $\chi(t) = c$. Trader indifference gives

$$\theta_1(1 - c) = \theta_2(1 - c)(1 - \bar{\alpha}) \Rightarrow \theta_2 = \frac{\theta_1}{1 - \bar{\alpha}}$$

Finally, for any $t \in (\theta_1, \theta_2)$ trader indifference and part 4 of Lemma A.1 yields

$$m_0(1 - q) = t(1 - c)(1 - \alpha(t))$$

or

$$\alpha(t) = 1 - \frac{\theta_1}{t}$$

Thus we have specified equilibrium strategies up to an arbitrary constant m_0 . To determine this constant, integrate $\phi_0(t)$ from m_0 to 1 and set the result equal to 1. This yields an equation $Q(\bar{m}, \bar{\alpha} | K_0, J) = 0$ (see eq. 7). The larger root of this equation is greater than 1 (proof available on request). Consider the smaller root

$$(A9) \quad \bar{m}_0 = \frac{1 - \bar{\alpha}}{\bar{\alpha}J^2 + 1 - \bar{\alpha}} \left(K_0 + 1 - \sqrt{(K_0 + 1)^2 - (\bar{\alpha}J^2 + 1 - \bar{\alpha})} \right)$$

Clearly if \bar{m}_0 is real, then it is positive. Hence to verify existence of the equilibrium we must show \bar{m}_0 is real and that $\theta_2 < 1$.

Now, \bar{m}_0 is real if

$$(K_0 + 1)^2 \geq \bar{\alpha}J^2 + 1 - \bar{\alpha}.$$

Because $J > 1$ the right side is largest when $\bar{\alpha}$ is as big as possible, namely when $\bar{\alpha} = 1 - \theta^\dagger$. Hence $m_0(\bar{\alpha})$ is real iff

$$(K + 1)^2 \geq (1 - \theta^\dagger)J^2 + \theta^\dagger = J^2 - Jm_0^\dagger(J^2 - 1).$$

Substituting the definition of m_0^\dagger from (A3) gives

$$(K + 1)^2 \geq J^2 - 2J(J - K - 1)$$

or

$$(J - (K + 1))^2 \geq 0.$$

By point 3 of Lemma A.2, $J - 1 > K$ when $c > \hat{\chi}$. Hence, \bar{m}_0 is real and positive.

Now, $\theta_2 \leq 1$ iff $\frac{J\bar{m}_0}{1-\bar{\alpha}} \leq 1$. Letting $Z \equiv \bar{\alpha}J^2 + 1 - \bar{\alpha}$ and substituting from (A9) we have

$$\begin{aligned} \frac{J}{Z} \left(K + 1 - \sqrt{(K + 1)^2 - Z} \right) &\leq 1 \\ \Leftrightarrow J(K + 1) - Z &\leq J\sqrt{(K + 1)^2 - Z} \\ \Leftrightarrow J^2(K + 1)^2 - 2J(K + 1)Z + Z^2 &\leq J^2(K + 1)^2 - J^2Z \\ \Leftrightarrow Z &\leq 2J(K + 1) - J^2 \\ \Leftrightarrow \bar{\alpha}J^2 + 1 - \bar{\alpha} &\leq 2J(K + 1) - J^2. \end{aligned}$$

Again, the left side of the last line is largest when $\bar{\alpha} = 1 - \theta^\dagger$ or

$$\begin{aligned} J^2 - Jm_0^\dagger(J^2 - 1) &\leq 2J(K + 1) - J^2 \\ \Leftrightarrow m_0^\dagger &\geq \frac{2(J - K - 1)}{J^2 - 1}. \end{aligned}$$

The last line holds with equality by definition of m_0^\dagger (A3). Hence, the specified strategies and beliefs constitute a valid equilibrium.

The type-0 trader's expected equilibrium payoff is $\bar{m}_0(1 - q)$ by part 4 of Lemma A.1. PM's expected payoff is found by using (A5)

$$v = 1 - q + aq(1 - c) \int_0^1 \alpha(t)(\phi_0(t) - f) dt$$

For $t \in [m_0(\bar{\alpha}), \theta_1]$, $\alpha(t) = 0$ and for $t \in [\theta_1, \theta_2]$, $\phi_0(t) = f$. Thus

$$v = 1 - q + aq(1 - c) \int_{\theta_2}^1 \bar{\alpha} \left(\frac{(1 - \bar{\alpha})t - m_0(\bar{\alpha})}{Km_0(\bar{\alpha})} - f \right) dt.$$

Performing the integration and collecting terms yields the claim.

To prove the comparative static claims, implicitly differentiate (7) to get

$$\frac{d\bar{m}_0}{d\bar{\alpha}} = \frac{1 - \theta_2^2}{2 \left(\left(\frac{\bar{\alpha}J^2}{1-\bar{\alpha}} + 1 \right) m_0(\bar{\alpha}) - K - 1 \right)}$$

Note that the numerator on the right is positive because $\theta_2 < 1$ for $\bar{\alpha} < 1 - \theta^\dagger$ was established above. Hence,

$$\frac{d\bar{m}_0}{d\bar{\alpha}} < 0 \Leftrightarrow \left(\frac{\bar{\alpha}J^2}{1-\bar{\alpha}} + 1 \right) \bar{m}_0 < K + 1.$$

Substituting from (A9), this is true iff

$$K_0 + 1 - \sqrt{(K_0 + 1)^2 - (\bar{\alpha}J^2 + 1 - \bar{\alpha})} < K_0 + 1$$

which clearly holds. To prove the last part of the claim, observe that the smaller root of $Q(m, 0|K_0, J)$ (in m) is m_0^* . Whereas the non-zero root of $Q(m, 1 - \theta^\dagger|K_0, J) = 0$ is m_0^\dagger . ■

Proof. Corollary 7.2. Applying the results of Proposition 7.1 we have the following.

Trader. $m_0^\dagger(1 - q) < \bar{m}_0(1 - q) < m_0^*(1 - q)$. Moreover \bar{m}_0 is decreasing in $\bar{\alpha}$.

Policymaker. Recall that

$$aq(1 - c)\bar{\alpha} \left(\frac{1 - \theta_1 - \bar{\alpha}}{2Km_0(\bar{\alpha})} \right) = aq(1 - c)\bar{\alpha} \int_{\theta_2}^1 \phi_0(t) - f dt.$$

Because $\phi_0(\theta_2) = f$ the integrand is strictly positive for $t > \theta_2$. Moreover, $\bar{\alpha} = 1 - \theta^\dagger$ implies $\theta_2 = 1$. ■

Lemma A.6 (Equilibrium Structure with Private Intervention Costs) *Suppose $\gamma \in [0, \bar{\gamma}]$. Then, there exist $m_0 < \theta_1 < \theta_2 < \theta_3$ such that the structure of player strategies is the following:*

- *The low cost PM intervenes with probability zero if she observes a sell order in (m_0, θ_1) , with probability $\alpha_L(t) \in (0, 1)$ if she observes a sell order $t \in (\theta_1, \theta_2)$, with probability 1 if she observes sell order $t \in (\theta_2, 1]$.*
- *The high cost PM intervenes with probability zero if she observes a sell order in (m_0, θ_3) , with probability $\alpha_H(t) \in (0, 1)$ if she observes a sell order $t \in (\theta_3, 1]$.*
- *The type-0 trader places a sell order distributed according to probability density function $\phi_0(t)$ over support $[m_0, 1]$ defined piecewise:*

$$\phi_0(t) = \begin{cases} \phi_L(t) & \text{if } t \in [m_0, \theta_1) \\ \phi_{M1}(t) & \text{if } t \in (\theta_1, \theta_2) \\ \phi_{M2}(t) & \text{if } t \in (\theta_2, \theta_3) \\ \phi_H(t) & \text{if } t \in (\theta_3, 1] \end{cases}$$

where $\phi_L(t)$ and $\phi_{M2}(t)$ are increasing and $\phi_{M1}(t)$ and $\phi_H(t)$ are constant.

Proof. First, note that all parts of Lemma A.1 apply. Now, any order flow for which $\alpha_H(t) > 0 \Rightarrow \chi(t) = c_H \Rightarrow \alpha_L(t) = 1$. If the high cost PM intervenes for certain for any order flow, then the low cost PM also intervenes for certain. In this case the probability of intervention is one, and therefore the order generates a zero payoff for the trader, and is therefore outside of the support. Therefore, if the high cost type intervenes with positive probability, in equilibrium, this probability must be less than 1. Suppose that for some order t the high cost PM intervenes $\alpha_H(t) \in (0, 1)$. For

this order $u_0 = t(1 - c_H)(1 - \gamma - (1 - \gamma)\alpha_H(t)) = t(1 - c_H)(1 - \gamma)(1 - \alpha_H(t))$. Consider $t' > t$. The expected payoff at t' is $u'_0 = t'(1 - \chi(t'))(1 - \gamma\alpha_L(t) - (1 - \gamma)\alpha_H(t))$. Because $\chi(t') \leq c_H$ for all order flows inside the support $u'_0 \geq t'(1 - c_H)(1 - \gamma\alpha_L(t') - (1 - \gamma)\alpha_H(t'))$. Therefore, if $\alpha_H(t') = 0$, then $u'_0 > u_0$. Hence, if $\alpha_H(t) \in (0, 1)$ for some t , then $\alpha_H(t') \in (0, 1)$ for $t' > t$. Therefore, if θ_3 represents the intervention threshold for the high-cost PM, then for all $t \in (\theta_3, 1]$ the high cost PM intervenes with non-zero probability, $\alpha_H(t') > 0$. In this interval, $\chi(t) = c_H$, and therefore $\phi_H(t)$ is constant.

Next we argue that $\alpha_L(t) > 0$ and $t' > t \Rightarrow \alpha_L(t') > 0$. As discussed above if $t' > \theta_3$ then $\alpha_L(t) = 1$. Therefore consider $t' < \theta_3$. Because $\alpha_L(t) \in (0, 1)$, the expected payoff of playing t is given by $u_0 = t(1 - c_L)(1 - \gamma\alpha_L(t))$. Because $t' < \theta_3$, the expected payoff of submitting order flow t' is equal to $u'_0 = t'(1 - \chi(t'))(1 - \gamma\alpha_L(t'))$. Suppose $\chi(t') < c_L$. In this case $u'_0 = t'(1 - \chi(t')) > u_0$, which contradicts trader indifference. Suppose $\chi(t') = c_L$. In this case $u'_0 = t'(1 - c_L)(1 - \gamma\alpha_L(t'))$. Because $t' > t$ and $u_0 = u'_0$, it must be that $\alpha_L(t') > \alpha_L(t) > 0$. Suppose $\chi(t') > c_L$. In this case, PM's sequential rationality condition requires $\alpha_L(t') = 1$. Hence, $\alpha_L(t) > 0$ and $t' > t \Rightarrow \alpha_L(t') > 0$. Therefore, if θ_1 denotes the minimum threshold at which the low cost PM begins to intervene, then for all $t > \theta_1$, $\alpha_L(t) > 0$.

We argue that there exists $\theta_2 < \theta_3$, such that $t > \theta_2 \Rightarrow \alpha_L(t) = 1$. Over interval $(\theta_3, 1]$ the high type intervenes with non-zero probability and therefore the low cost PM intervenes for certain. Hence, $t \geq \theta_3 \Rightarrow \alpha_L(t) = 1$, therefore $\theta_2 \leq \theta_3$. Next, we argue that $\theta_2 < \theta_3$. Suppose $\theta_2 = \theta_3 = \theta$. In this case, the expected payoff for $\theta^- = \theta - \epsilon$ is $u_0^- \equiv \theta^-(1 - c_L)(1 - \gamma\alpha_L(\theta^-))$, meanwhile the expected payoff for $\theta^+ \equiv \theta + \epsilon$ is $u_0^+ = \theta^+(1 - c_H)(1 - \gamma - (1 - \gamma)\alpha_H(\theta^+))$. Because $1 - c_L > 1 - c_H$ and $1 - \gamma\alpha_L(\theta^-) > 1 - \gamma - (1 - \gamma)\alpha_L(\theta^+)$, for sufficiently small ϵ , $u_0^- > u_0^+$, violating trader indifference. If $m_0 = \theta_1$, then by deviating just below m_0 , the trader obtains payoff $m_0(1 - q) > (m_0 + \epsilon)(1 - c_L)$ for small ϵ . Hence, if $\theta_1 = m_0$, then a deviation outside of the support increases his payoff.

Next, we characterize the structure of the trader strategy given the structure of PM's strategy described above. In interval $[m_0, \theta_1)$, intervention probability is zero for both types. Here $\chi(t) < c$ and is increasing in t by part 8 of Lemma A.1. Hence, $\phi_L(t)$ is increasing in t . In interval (θ_1, θ_2) the low cost PM mixes but the high cost does not intervene, hence $\chi(t) = c_L$. Because the posterior is constant in this interval $\phi_{M1}(t)$ is constant on this interval. In interval (θ_2, θ_3) the low cost PM intervenes for certain and the high cost PM does not intervene, hence the intervention probability is constant on this interval (and equal to γ). The posterior belief is therefore increasing on this interval by part 8 of Lemma A.1, which implies that $\phi_{M2}(t)$ is increasing. Finally, on interval $(\theta_3, 1]$ the high-cost PM mixes, which implies that the posterior belief is constant on this interval. Therefore $\phi_H(t)$ is constant. ■

Proof. Propositions 7.3, 7.4, 7.5. First we establish $0 < \bar{\gamma} < 1 - \theta_L^\dagger$. That $\bar{\gamma} = R(1 - \theta_H^\dagger) > 0$

is obvious. To show $\bar{\gamma} < 1$, consider the following string of equivalent expressions.

$$\begin{aligned}
 \bar{\gamma} &< 1 - \theta_L^\dagger \\
 &\Leftrightarrow R(1 - \theta_H^\dagger) < 1 - \theta_L^\dagger \\
 &\Leftrightarrow \frac{J_H^2 - 1}{J_L^2 - 1} \left(1 - J_H \frac{2(J_H - K_0 - 1)}{J_H^2 - 1}\right) < 1 - J_L \frac{2(J_L - K_0 - 1)}{J_L^2 - 1} \\
 &\Leftrightarrow \frac{J_H - J_L}{J_L^2 - 1} (2(K_0 + 1) - J_H - J_L) < 0
 \end{aligned}$$

where the last inequality holds by point 3 of Lemma A.2.

Case I. Suppose $\gamma \in [0, \bar{\gamma}]$. Then the structure of the equilibrium is derived in Lemma A.6. By part 4 of Lemma A.1 the trader's equilibrium payoff is $u_0 = m_0(1 - q)$. Hence, his indifference requires the following:

$$t \in [m_0, \theta_1] \Rightarrow m_0(1 - q) = t(1 - \chi(t))$$

$$t \in [\theta_1, \theta_2] \Rightarrow m_0(1 - q) = t(1 - c_L)(1 - \gamma\alpha_L(t))$$

$$t \in [\theta_2, \theta_3] \Rightarrow m_0(1 - q) = t(1 - \chi(t))(1 - \gamma)$$

$$t \in [\theta_3, 1] \Rightarrow m_0(1 - q) = t(1 - c_H)(1 - \gamma - (1 - \gamma)\alpha_H(t)) = t(1 - c_H)(1 - \gamma)(1 - \alpha_H(t))$$

Sequential rationality by each type of PM requires the following:

$$t \in [m_0, \theta_1] \Rightarrow \chi(t) < c_L$$

$$t \in [\theta_1, \theta_2] \Rightarrow \chi(t) = c_L$$

$$t \in [\theta_2, \theta_3] \Rightarrow c_L < \chi(t) < c_H$$

$$t \in [\theta_3, 1] \Rightarrow \chi(t) = c_H$$

We now establish the relationship between the intervention thresholds θ_i and the minimum informed sell size m_0 proposed in the proposition.

1: Consider $\theta_1^- = \theta_1 - \epsilon$ and $\theta_1^+ = \theta_1 + \epsilon$. The expected payoff of submitting order θ_1^- , is

$$u_0^- = \theta_1^-(1 - \chi(\theta_1^-))$$

and the expected payoff of submitting order θ_1^+ , is

$$u_0^+ = \theta_1^+(1 - c_L)(1 - \gamma\alpha_L(\theta_1^+)).$$

If either $\lim_{\epsilon \rightarrow 0} \chi(\theta_1^-) < c_L$ or $\lim_{\epsilon \rightarrow 0} \alpha_L(\theta_1^-) > 0$ then $u_0^- \neq u_0^+$ for a set of sufficiently small ϵ . Therefore, if the indifference condition is satisfied at θ_1 it must be that $m_0(1 - q) = \theta_1(1 - c_L)$, and therefore $\theta_1 = J_L m_0$.

2: Consider $\theta_2^- = \theta_2 - \epsilon$ and $\theta_2^+ = \theta_2 + \epsilon$. The expected payoff of submitting order θ_2^- , is

$$u_0^- = \theta_2^- (1 - c_L)(1 - \gamma \alpha_L(\theta_2^-))$$

and the expected payoff of submitting order θ_2^+ , is

$$u_0^+ = \theta_2^+ (1 - \chi(\theta_2^+))(1 - \gamma).$$

If either $\lim_{\epsilon \rightarrow 0} \chi(\theta_2^-) < c_L$ or $\lim_{\epsilon \rightarrow 0} \alpha_L(\theta_2^-) < 1$ then $u_0^- \neq u_0^+$ for a set of sufficiently small ϵ . Therefore, for the indifference condition to be satisfied at θ_2 it must be that $m_0(1 - q) = \theta_2(1 - c_L)(1 - \gamma)$, and therefore $\theta_2 = \frac{J_L m_0}{1 - \gamma}$.

3: Consider $\theta_3^- = \theta_3 - \epsilon$ and $\theta_3^+ = \theta_3 + \epsilon$. The expected payoff of submitting order θ_3^- , is

$$u_0^- = \theta_3^- (1 - \chi(\theta_3^-))(1 - \gamma)$$

and the expected payoff of submitting order θ_3^+ , is

$$u_0^+ = \theta_3^+ (1 - c_H)(1 - \gamma)(1 - \alpha_H(\theta_3^+)).$$

If either $\lim_{\epsilon \rightarrow 0} \chi(\theta_3^-) < c_H$ or $\lim_{\epsilon \rightarrow 0} \alpha_H(\theta_3^-) > 0$ then $u_0^- \neq u_0^+$ for a set of sufficiently small ϵ . Therefore, for the indifference condition to be satisfied at θ_3 it must be that $m_0(1 - q) = \theta_3(1 - c_H)(1 - \gamma)$, and therefore $\theta_3 = \frac{J_H m_0}{1 - \gamma}$.

Next, We derive expressions for the trader's mixing density and each type of PM's intervention probability. Using (6) to solve the trader indifference and PM's sequential rationality conditions gives the following:

$$t \in [m_0, \theta_1] \Rightarrow \phi_L(t) = \frac{t - m_0}{K_0 m_0}$$

$$t \in [\theta_1, \theta_2] \Rightarrow \phi_{M1}(t) = f_L \text{ and } \alpha_L(t) = \frac{1}{\gamma} \left(1 - \frac{\theta_1}{t} \right)$$

$$t \in [\theta_2, \theta_3] \Rightarrow \phi_{M2}(t) = \frac{(1 - \gamma)t - m_0}{K_0 m_0}$$

$$t \in [\theta_3, 1] \Rightarrow \phi_H(t) = f_H \text{ and } \alpha_H(t) = 1 - \frac{\theta_3}{t}$$

Notice that for the intervals in which the expressions given above hold, $\alpha_i(t) \in [0, 1]$.

Thus we have specified equilibrium strategies up to an arbitrary constant m_0 . To determine this constant, integrate $\phi_0(t)$ from m_0 to 1 and set the result equal to 1. Collecting terms then yields the quadratic equation

$$\left(\frac{J_H - \gamma J_L}{1 - \gamma} - 1 \right) m_0^2 - 2(J_H - K_0 - 1)m_0 = 0$$

The non-zero root of this is

$$\begin{aligned}\widehat{m}_0 &= \frac{2(J_H - K_0 - 1)}{\left(\frac{J_H - \gamma J_L}{1 - \gamma} - 1\right)} \\ &= \left(\frac{(1 - \gamma)(J_H^2 - 1)}{J_H^2 - 1 - \gamma(J_L^2 - 1)}\right) \frac{2(J_H - K_0 - 1)}{J_H^2 - 1} \\ &= \frac{(1 - \gamma)R}{R - \gamma} m_H^\dagger\end{aligned}$$

Because $c_H > \widehat{\chi}$, m_H^\dagger is positive; the denominator of the fraction positive because $R > 1 > \gamma$. Thus $\widehat{m}_0 > 0$.

Because $0 < \widehat{m}_0 < \theta_1 < \theta_2 < \theta_3$, to verify validity of the strategies we need only to show $\theta_3 < 1$. Consider the following string of equivalent expressions

$$\begin{aligned}\theta_3 &< 1 \\ \Leftrightarrow \frac{J_H}{1 - \gamma} \widehat{m}_0 &< 1 \\ \Leftrightarrow \frac{J_H}{1 - \gamma} \frac{R(1 - \gamma)}{R - \gamma} m_H^\dagger &< 1 \\ \Leftrightarrow \frac{J_H(J_H^2 - 1)}{J_H^2 - 1 - \gamma(J_L^2 - 1)} m_H^\dagger &< 1 \\ \Leftrightarrow \gamma < (1 - J_H m_H^\dagger) \frac{J_H^2 - 1}{J_L^2 - 1} \\ \Leftrightarrow \gamma < (1 - \theta_H^\dagger) R = \bar{\gamma}\end{aligned}$$

Note also that $\gamma = \bar{\gamma}$ implies $\theta_3 = 1$.

The type-0 trader's expected equilibrium payoff is $\widehat{m}_0(1 - q)$ by part 4 of Lemma A.1. The type- H PM does not intervene for $t \leq \theta_3$ and mixes for $t > \theta_3$ and hence has expected payoff $1 - q$. The type- L PM's expected payoff is found by using (A5)

$$v_L = 1 - q + aq(1 - c) \int_0^1 \alpha_L(t)(\phi_0(t) - f_L) dt$$

For $t \in [0, \theta_1]$, $\alpha_L(t) = 0$; for $t \in [\theta_1, \theta_2]$, $\phi_0(t) = f_L$; and for $t > \theta_2$, $\alpha_L(t) = 1$. Thus

$$(A10) \quad v_L = 1 - q + aq(1 - c) \int_{\theta_2}^{\theta_3} \left(\frac{(1 - \gamma)t - \widehat{m}_0}{K_0 \widehat{m}_0} - f_L \right) dt + aq(1 - c) \int_{\theta_3}^1 (f_H - f_L) dt.$$

Performing the integration and collecting terms yields the claim.

To prove the comparative static claims, first note that

$$\frac{d}{d\gamma} \left(\frac{J_H - \gamma J_L}{1 - \gamma} \right) = \frac{J_H - J_L}{(1 - \gamma)^2} > 0.$$

Hence, \hat{m}_0 and θ_1 are decreasing in γ . Next, for $i \in \{2, 3\}$

$$\theta_i = \frac{2J_i(J_H - K - 1)}{J_H^2 - J_L^2 + (1 - \gamma)(J_L^2 - 1)}.$$

Because $J_L^2 - 1 > 0$, θ_i is evidently increasing in γ . Next, observe that setting $\gamma = 0$ in (9) yields (A3) with $c = c_H$, and hence for $\gamma = 0$, $\hat{m} = m_H^\dagger$ and $\theta_3 = \theta_H^\dagger$.

Case II. Suppose $\gamma \in [\bar{\gamma}, 1 - \theta_L^\dagger]$. Then it was shown above that $\theta_3 = 1$, so that the type- H PM never intervenes. Integrating $\phi_0(t)$ from m_0 to 1, setting the result equal to 1, and collecting terms, shows that m_0 is the root in m of $Q(m, \gamma | K_0, J_L)$. Hence, the trader plays exactly as if it were common knowledge that $c = c_L$ and there was an intervention cap of γ . Letting $\bar{m}_0(\gamma)$ represent the minimum trade size in this equilibrium, trader's expected payoff is thus $\bar{m}_0(\gamma)(1 - q)$. The type- H PM's expected payoff is obviously $1 - q$. The type- L PM's expected payoff is found by using (A5)

$$v_L = 1 - q + aq(1 - c) \int_0^1 \alpha_L(t)(\phi_0(t) - f_L) dt$$

For $t \in [0, \theta_1]$, $\alpha_L(t) = 0$; for $t \in [\theta_1, \theta_2]$, $\phi_0(t) = f_L$; and for $t > \theta_2$, $\alpha_L(t) = 1$. Thus

$$v_L = 1 - q + aq(1 - c) \int_{\theta_2}^1 \left(\frac{(1 - \gamma)t - \bar{m}_0(\gamma)}{K_0 m_0(\gamma)} - f_L \right) dt$$

Performing the integration and collecting terms yields the claim.

Case III. Suppose $\gamma \in [1 - \theta_L^\dagger, 1]$. Then it was shown in the proof of Proposition 7.1 that $\theta_2 = 1$, so that the ‘‘cap’’ never binds. Integrating $\phi_0(t)$ from m_0 to 1, setting the result equal to 1, and collecting terms yields $Q(m_0, 0 | K - 0, J_L) = 0$, hence the solution is m_L^\dagger . The trader plays exactly as if it were common knowledge that $c = c_L$. His expected payoff is thus $(m_L^\dagger(1 - q))$ and the expected payoff of both types of PM is $1 - q$. ■

Proof. Corollary 7.6. Observe that $\phi_0(\theta_2) = f_L$. Hence the integrand in (A10) is positive at every point $t \in (\theta_2, 1]$. Moreover, $\gamma < 1 - \theta_L^\dagger$ implies $\theta_2 < 1$. ■

APPENDIX B

In this appendix we consider a case in which PM observes a private signal in addition to the order flow before making an intervention decision and show that this is formally equivalent to privately known intervention cost.

Although it does not impact the results, we focus on the following timing: PM observes the order flow, and then observes the realization $\sigma \in \{L, H\}$ of the following signal

$$\Pr(\Sigma = L|\omega = 0) = \Pr(\Sigma = H|\omega = 1) = b$$

$$\Pr(\Sigma = H|\omega = 0) = \Pr(\Sigma = L|\omega = 1) = 1 - b$$

This signal structure is a straightforward garbling of the true state. Parameter $b \geq 1/2$ represents the probability that the signal realization is a true indicator of the state: when b is high the signal is more-likely to realize H in state one and L in state 0. We refer to the privately observed signal realization as PM's type.

Given an interim belief for PM that the state is 0, $\chi(t)$, the Bayesian update given realization σ is

$$\chi_L(t) = \frac{\chi(t)b}{\chi(t)b + (1 - \chi(t))(1 - b)}$$

$$\chi_H(t) = \frac{\chi(t)(1 - b)}{\chi(t)(1 - b) + (1 - \chi(t))b}$$

Each type of PM intervenes when her private belief exceeds the intervention cost, $\chi_i(t) \geq c$, therefore

$$\chi(t) \geq c_L \equiv \frac{c(1 - b)}{c(1 - b) + b(1 - c)} \Leftrightarrow \chi_L(t) \geq c$$

$$\chi(t) \geq c_H \equiv \frac{cb}{cb + (1 - b)(1 - c)} \Leftrightarrow \chi_H(t) \geq c$$

Sequential rationality for each type of PM, $i \in \{L, H\}$ requires

$$\alpha_i(t) = \begin{cases} 0 & \text{if } \chi(t) < c_i \\ [0, 1] & \text{if } \chi(t) = c_i \\ 1 & \text{if } \chi(t) > c_i \end{cases}$$

Given prior $\chi(t)$ that the state is zero, the probability of observing either signal realization is

$$\Pr(\Sigma = L|\chi(t)) \equiv \gamma(\chi(t)) = \chi(t)b + (1 - \chi(t))(1 - b)$$

$$\Pr(\Sigma = H|\chi(t)) = 1 - \gamma(\chi(t)) = \chi(t)(1 - b) + (1 - \chi(t))b$$

The market maker sets the price of the asset equal to its expected payoff conditional on the

observed order:

$$\begin{aligned}
 p(t) &= \gamma(\chi(t))((1 - \alpha_L(t))(1 - \chi_L(t)) + \alpha_L(t)) + (1 - \gamma(\chi(t)))(1 - \alpha_H(t))(1 - \chi_H(t)) + \alpha_H(t) \\
 &= \gamma(\chi(t))(1 - \chi_L(t) + \alpha_L(t)\chi_L(t)) + (1 - \gamma(\chi(t)))(1 - \chi_H(t) + \alpha_H(t)\chi_H(t)) \\
 &= 1 - \chi(t)(1 - b\alpha_L(t) + (1 - b)\alpha_H(t))
 \end{aligned}$$

The model therefore reduces to the model with private costs, in which the intervention “costs” are given by the expressions above, and the probability of having a low cost is b instead of γ . In order for the analysis to apply we must have $\hat{\chi} < c_L < c_H < \bar{\chi}$. If $c \in (\hat{\chi}, \bar{\chi})$ and b is not too large, then both “costs” satisfy these requirements.

APPENDIX C

In the main text we consider interventions that must be deployed at an interim stage, without PM knowledge of the state. In a variety of instances, it is conceivable that through some type of auditing mechanism, the PM could determine the state before deploying the intervention. We focus on the case in which the principal can choose to either audit or intervene at the interim stage. We show that through a simple transformation of parameters, the equilibrium of the model with auditing reduces to the equilibrium of the model without auditing.

Imagine that after observing orderflow, the policymaker can intervene to guarantee state one at cost c , or can conduct an audit to determine the true state at cost k ; following the audit, it is sequentially rational for the policymaker to intervene in state zero. Let $\beta(t)$ represent the probability of auditing following orderflow t . The policymaker's interim payoff of intervening with probability $\alpha(t)$ and auditing with probability $\beta(t)$ is

$$(1 - \alpha(t) - \beta(t))(1 - \chi(t)) + \alpha(t)(1 - c) + \beta(t)(1 - \chi(t) + \chi(t)(1 - c) - k) = \\ 1 - \chi(t) + \alpha(t)(\chi(t) - c) + \beta(t)(\chi(t)(1 - c) - k)$$

Comparing these alternatives gives the policymaker incentive constraint. Observe that auditing gives higher expected payoff than intervening if and only if

$$\chi(t) < 1 - \frac{k}{c}.$$

Meanwhile the expected payoff of intervening is positive

$$\chi(t) > c$$

and expected payoff of auditing is positive whenever

$$\chi(t) > \frac{k}{1 - c}$$

Note that when $k \geq c(1 - c)$, the expected payoff of auditing is strictly less than the expected payoff of intervening for any order flow.

$$\chi(t)(1 - c) - k \leq \chi(t)(1 - c) - c(1 - c) \leq (1 - c)(\chi(t) - c) < \chi(t) - c$$

Hence, when $k \geq c(1 - c)$, auditing is never preferred to intervening, and is therefore irrelevant. Next, suppose that $k < c(1 - c)$. In this case the above thresholds are ordered as follows:

$$\frac{k}{1 - c} < c < 1 - \frac{k}{c} < 1.$$

Hence sequentially rational behavior for the policymaker is

$$\begin{aligned} \alpha(t) = \beta(t) = 0 & \quad \text{if } \chi(t) < \frac{k}{1-c} \\ \alpha(t) = 0, \beta(t) = 1 & \quad \text{if } \frac{k}{1-c} < \chi(t) < 1 - \frac{k}{c} \\ \alpha(t) = 1, \beta(t) = 0 & \quad \text{if } \chi(t) > 1 - \frac{k}{c} \end{aligned}$$

Next, consider the market price as a function of the order flow:

$$p(t) = (1 - \alpha(t) - \beta(t))(1 - \chi(t)) + (\alpha(t) + \beta(t))$$

From the perspective of the market, following either an intervention or an audit, the state will be one. Indeed, an intervention (without audit) guarantees state one, while an audit ensures that an intervention will take place whenever the state would be zero otherwise. Thus, following either audit or intervention, the state is guaranteed to be one. This immediately implies the following payoff functions for the trader:

$$u_0(t) = t(1 - \chi(t))(1 - \alpha(t) - \beta(t)) \text{ and } u_1(t) = t\chi(t)(1 - \alpha(t) - \beta(t))$$

Therefore, consider a “generalized intervention” to be either a direct intervention or an audit. The probability of a generalized intervention, $\alpha'(t) \equiv \alpha(t) + \beta(t)$. The incentive constraint for the policymaker for generalized intervention requires

$$\alpha'(t) = \begin{cases} 0 & \text{if } \chi(t) < \frac{k}{1-c} \\ [0, 1] & \text{if } \chi(t) = \frac{k}{1-c} \\ 1 & \text{if } \chi(t) > \frac{k}{1-c} \end{cases}$$

which is identical to the PM incentive constraint in the basic model with a cost of generalized intervention of $k/(1 - c)$. The trader incentive constraints become

$$u_0(t) = t(1 - \chi(t))(1 - \alpha'(t)) \text{ and } u_1(t) = t\chi(t)(1 - \alpha'(t))$$

also identical to their counterparts in the model without auditing. Hence, for equilibrium analysis, the model with auditing is outcome equivalent to the model without auditing, with a transformed cost.