Excess Volatility of Corporate Bonds

Jack Bao and Jun Pan*

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Abstract

This paper examines the connection among corporate bonds, stocks, and Treasury bonds under the Merton model with stochastic interest rate, focusing in particular on the volatility of corporate bonds and its connection to the equity volatility of the same firm and the Treasury bond volatility. For a broad cross-section of corporate bonds from 2002 through 2006, empirical measures of bond volatility are constructed using bond returns over daily, weekly, and monthly horizons. Comparing the empirical volatility with its model-implied counterpart, we find an overwhelming degree of excess volatility that is difficult to be explained by a default-based model. This excess volatility is found to be the strongest at the daily and weekly horizons, indicating a more pronounced liquidity component in corporate bonds at short horizons. At the monthly horizon, the excess volatility tapers off but remains significant. Moreover, we find that variables known to be linked to bond liquidity are important in explaining the cross-sectional variations in excess volatility, providing further evidence of a liquidity problem in corporate bonds. Finally, subtracting the equity and Treasury exposures from corporate bond returns, we find a non-trivial systematic component in the bond residuals that give rise to the excess volatility.

*Bao is at the MIT Sloan School of Management, jackbao@mit.edu. Pan is at the MIT Sloan School of Management and NBER, junpan@mit.edu. We benefit from discussions with Jiang Wang; seminar participants at the MIT Finance Lunch and Boston University (Economics). We thank Duncan Ma for assistance in gathering Bloomberg data, and financial support from the outreach program of J.P. Morgan.

1 Introduction

We examine the connection between corporate bonds and stocks under the structural model of Merton with stochastic interest rate. In particular, we focus on the volatility of corporate bonds and its connection to the equity volatility of the same firm, as well as the Treasury volatility. Using daily returns on bonds and stocks, we find an overwhelming amount of excess volatility in corporate bonds. In annualized terms, the difference between the empirical volatility $\hat{\sigma}_D$ and its model-implied counterpart σ_D^{Merton} is on average 12.58% with a robust t-stat of 34. Moving from daily returns to weekly and monthly horizons, this excess volatility tapers off quite dramatically, indicating a liquidity component in corporate bonds that is more pronounced at the short horizon.

The motivation for this empirical study is two-folded. First, while the structural models pioneered by Merton (1974) have a direct impact on our conceptual understanding of the connection between bonds and stocks of the same firm, their empirical reach remains somewhat limited. And the limited access to quality bond data in the past certainly is an important factor. The recently available TRACE data, however, greatly improves the situation by offering transaction-level data with both price and volume information. The second, and perhaps more important motivation for this study is the severe liquidity issue in the corporate bond market. While the corporate bond market is as large as the Treasury bond market, the difference in liquidity between these two markets can offer a stark contrast. By the first quarter 2007, the total amount outstanding is \$4.45 trillions in Treasury and \$5.45 trillions in corporate debt. By comparison, in January 2007, the average daily trading volume is around \$492 billions in Treasury and \$16.7 billions in corporate debt, In other words, one can have active trading in Treasury and hardly any trading in corporates. Liquidity issue of this magnitude is bound to find its way to corporate bond pricing.

In this paper, we tackle these two issues by introducing and comparing two measures of corporate bond volatility: the empirically estimated versus the model implied. We use the Merton model to take into account of the two main drivers of corporate bond volatility: the firm's asset volatility and the Treasury volatility. Measuring Treasury volatility using Treasury bond returns and inferring asset volatility from the firm's equity volatility through the Merton model, we feed these these estimates collected from equity and Treasury markets back to the Merton model with stochastic interest rate to obtain model-implied return volatilities for the respective corporate bonds. We then compare this model-implied bond volatility σ_D^{Merton} with the empirically observed bond return volatility $\hat{\sigma}_D$. The discrepancy between the two volatility measures sets the stage for an empirical evaluation of the Merton model, and more interestingly, an empirical investigation on the nature of illiquidity in the corporate bond market.

For a broad cross-section of corporate bonds from July 2002 through December 2006, we find that the annualized bond volatility $\hat{\sigma}_D$ is on average 18.03% using daily bond returns, 9.61% using weekly returns, and 7.17% using monthly returns. This pattern of decreasing annualized volatility with increasing measurement horizon is found to be unique only in corporate bonds. In particular, when daily, weekly, and monthly equity returns are used for the same pool of bond issuers in our sample, the estimated annualized equity volatilities are similar in magnitudes across measurement horizons. The same is true when daily, weekly, and monthly Treasury bond returns are used to estimate the Treasury bond volatility. Absent of a structural model, this result along the measurement horizon by itself has an immediate implication on the liquidity of corporate bonds. It quantifies and contrasts the illiquidity of corporate bonds in relation to that in the equity and Treasury markets.

To connect the diverging information in the three markets – corporate bond, equity and Treasury bond — into one unified framework, we adopt the Merton model with stochastic interest rate. In the model, the corporate bond volatility has two contributions: random fluctuations in firm value and riskfree interest rate. While the Treasury bond volatility can be directly estimated using Treasury bond returns, we can only infer, via the Merton model, the firm's asset volatility from estimates of the equity volatility of the firm. Consequently, the two important inputs of the model comes from the volatility estimates of equity and Treasury bond returns. In addition, we also rely on the firm's balance sheet information to estimate the other firm-level characteristics that are important in the model. We find that, for the same pool of bonds and for the same time periods, the annualized bond volatility is on average 5.45% using daily equity and Treasury returns, 5.18% using weekly returns, and 5.38% using monthly returns. Comparing this set of model-implied volatilities against the ones estimated empirically, we see a clear pattern of excess volatility in corporate bonds that is most severe at the short horizon, but remains significant, both statistically and economically, even at the longer measurement horizon.

To further shed light on the economic origin of the excess bond volatility, we examine its cross-sectional determinants. Our result paints a general picture that links the degree of excess bond volatility to the illiquidity of a bond. For example, our result shows that excess volatility is higher for smaller bonds, which are typically less liquid. More interestingly, our result also shows that, after controlling for bond characteristics including maturity, rating and size, older bonds have higher excess volatility. As newly issued bonds are typically more liquid, while the old bonds are more likely to be locked away in some safety boxes, our result is consistent with a liquidity explanation. Finally, we also find interesting connections to the trading related variables. For example, we find more excess volatility in corporate bonds whose average trade size is small. This is consistent with the possibility that, after controlling for bond size, the bonds with smaller average trade size are more likely to be traded in less liquid bond trading platforms and therefore has a larger liquidity component. In addition to this cross-sectional explanation, the monthly time-series variation in excess volatility and the bond trading variable could also contribute to the result. In particular, it is consistent with the assumption that the month during which the average trade size of a particular bond is small is a less liquid month, resulting in higher excess volatility for that bond during that month.

Compared with the existing literature that examines the empirical performance of the Merton model from the angle of the first moment, our particular focus on the second moment of corporate bond returns sets us apart and provides us with a unique and fresh angle. Our motivation, however, is very much aligned with this literature. In particular, we would like to be able to add to the debate that is central to this literature and has been raised, among others, by Huang and Huang (2003): how much of the market-observed corporate bond yield spreads is due to the firm's credit risk? And our empirical results point in the direction of an illiquidity component in corporate bond returns. One might argue that with the reliance of a model, the conclusion is always a joint hypothesis of the empirical performance, or, in this case, the lack of empirical performance, of the Merton model. Nevertheless, it would be highly implausible for any default-based model to generate the observed pattern of increasing bond volatility with decreasing measuring horizon. A pattern like this is more reminiscent of a microstructure model with bid-ask bounce playing a more important role at the short horizon. The fact that we find this pattern only in corporate bonds but not in equity and Treasury bonds is indicative of a liquidity problem beyond simple bid-ask bounce. Indeed, using the quoted bid-ask spreads of corporate bonds, we find the effect of bid-ask bounce to be rather minor in addressing the excess volatility puzzle. In summary, the empirical pattern of bond volatility documented in this paper stands on its own to provide a clear and unambiguous support of the importance of liquidity in corporate bond prices.

To the extent that the structural model of Merton is important in our analysis, it provides a set of benchmark numbers of corporate bond volatility, incorporating the firm's balance sheet information as well as information from the equity and Treasury bond markets. A formal and extensive empirical evaluation of the Merton model constitutes as another important motivation of our paper. In relation to the important work of Eom, Helwege, and Huang (2004), who examine the empirical performance of structural models of default including the Merton model using only 182 data points, our contribution is to performance an empirical analysis of the Merton model on a much larger scale of corporate bonds. And more importantly, by contrasting the model-implied bond volatility against the empirical volatility measures, we are able to shed light on the empirical performance of the Merton model from a perspective that have not been looked at before.

Our result indicates that while at the daily and weekly measurement horizons, the Merton model cannot even begin to generate the kind of bond volatility observed in the data due to the liquidity problems in corporate bonds, at the monthly return horizon, the model is able to generate an average volatility of 5.38% that is relatively close to the empirically observed bond volatility of 7.17%. This excess volatility of 1.79%, however, is still statistically significant with a robust t-stat of 2.15, and, perhaps more importantly, still accounts for a quarter of the observed empirical bond volatility. Whether or not this is due to liquidity or model mis-specification remains an interesting question. We find that even at the monthly measurement horizon, the cross-sectional determinants of the excess volatility are still closely related to liquidity variables such as the age of a bond and its average trade size.

On the other hand, we can look for potential model mis-specifications by examining the model-implied bond volatility more closely. In constructing the model-implied volatility, the corporate bond's sensitivities to its firm's asset and the Treasury bond are the two basic building blocks. As such, whether or not the model-implied sensitivities match their empirical counterparts provides a more detailed test of the model. In fact, Schaefer and Strebulaev (2004) show the Merton model provides quite accurate predictions of the sensitivity of corporate bond returns to changes in the value of equity. Checking the model-implied numbers against their empirical counterparts for our sample, we find that indeed the Merton model does a reasonable job in capturing the equity sensitivity, but the empirical sensitivity to Treasury bond is found to be significantly lower than those prescribed by the model. This result indicates that the excess volatility puzzle documented in this paper is somewhat understated. Given the importance of Treasury bond volatility in generating the model-implied corporate bond volatility, the magnitude of the excess volatility puzzle would have been more severe had we not used the model-implied sensitivity measures.

Indeed, this is confirmed in our finding of an even more exacerbated volatility puzzle when excess bond returns are used to avoid explicitly modeling the riskfree interest rate. Given the importance of interest rate risk in corporate bonds, the stochastic interest rate component of the model should perhaps be subject to the most severe scrutiny. In particular, the bonds in our sample have a median maturity close to 7 years. To properly account for the riskfree volatility, we employ the Vasicek (1997) model for its simplicity, but calibrate the volatility coefficient of the model so that the model generates the empirically observed level of volatility for a 7-year Treasury coupon bond. Missing in this simple one-factor term structure model is the potentially rich term-structure of interest rate volatility. To account for this, we work with excess bond returns to avoid relying on a term structure model. Specifically, we calculate excess bond returns by subtracting, from the corporate bond returns, the contemporaneous Treasury bond returns of a similar maturity. Comparing the volatility measured by bond excess returns to the model-implied excess bond volatility, we find that, in annualized terms, the excess volatility is 17.29% using daily returns, 8.10% using weekly returns, and 4.83% using monthly returns. In other words, the excess volatility puzzle is more severe in this treatment.

Our paper is related to Collin-Dufresne and Goldstein (2001), who regress monthly changes in corporate bond yields to variables that should in theory determine credit spread changes, and find very low R^2's in their regressions. We are able to put a horizon dimension to this illuminating and intuitive result. Specifically, regressing daily corporate bond returns on daily equity returns on its firm equity and on a Treasury bond of a similar maturity, we find a cross-sectional average R^2 of 18.28%, which increases to 46.38% when monthly returns are employed. More importantly, by working in a structural setting, we are able to contrast the data to the model more closely. Instead of simply comparing the magnitudes of R^2, we are able to construct formal empirical tests linking the empirical volatility to the model-implied volatility. In the liquidity dimension, our paper is related to the papers by Houweling, Mentink, and Vorst (2003), Downing, Underwood, and Xing (2005), Chacko (2006), de Jong and Driessen (2005), and Chen, Lesmond, and Wei (2007), which examine the liquidity impact in corporate through a liquidity premium component in bond yields. Our empirical evaluation of the Merton model is closely related to the papers by Crosbie and Bohn (2003), Leland (2004) and Bharath and Shumway (2004), which use the structural models of default to forest default probability. Finally, also related are the papers by Vassalou and Xing (2004) and Campbell, Hilscher, and Szilagyi (2007), which use default probability to examine the expected equity returns of the same firm.

The rest of the paper is organized as follows. Section 2 outlines the empirical specification.

Section 3 summarizes the data and the empirical volatility estimates. Section 4 details the model implied volatility. Section 5 summarizes the main empirical results of our paper. Section 6 reports the cross-sectional determinants of excess volatility. Section 7 supplements with a time-series analysis of corporate bond returns. Section 8 concludes.

2 Empirical Specification

2.1 The Merton Model

We use the Merton (1974) model to connect the equity and corporate bonds of the same firm. Let V be the total firm value, whose risk-neutral dynamics is assumed to be

$$\frac{dV_t}{V_t} = (r_t - \delta)\, dt + \sigma_v\, dW_t^Q\,, \tag{1}$$

where W is a standard Brownian motion, and where the payout rate δ and the asset volatility σ_v are assumed to be constant.

We adopt a simple extension of the Merton model to allow for stochastic interest rate. This is important for our purpose because a large component of the corporate bond volatility comes from the Treasury market. Specifically, we model the riskfree rate using the Vasicek (1997) model:

$$dr_t = \kappa\,(\theta - r_t)\ dt + \sigma_r\, dZ_t^Q\,, \tag{2}$$

where Z is a standard Brownian motion independent of W, and where the mean-reversion rate κ, long-run mean θ and the diffusion coefficient σ_r are assumed to be constant.

Following Merton (1974), let's assume for the moment that the firm has, in addition to its equity, a single homogeneous class of debt, and promises to pay a total of K dollars to the bondholders on the pre-specified date T. The equity then becomes a call option on V:

$$S_t = V_t\, e^{-\delta T}\, N(d_1) - K\, e^{a(T)+b(T)\, r_t}\, N(d_2), \tag{3}$$

where $N(\cdot)$ is the cumulative distribution function for a standard normal, $d_1 = d_2 + \sigma_v\sqrt{T}$, and

$$d_2 = \frac{\ln(V/K) - a(T) - b(T)\, r_t - \left(\delta + \frac{\sigma_v^2}{2}\right) T}{\sigma_v \sqrt{T}}\,, \tag{4}$$

and where $a(T)$ and $b(T)$ are the exponents of the discount function of the Vasicek model:

$$b(T) = \frac{e^{-\kappa T} - 1}{\kappa}\,; \quad a(T) = \theta\left(\frac{1 - e^{-\kappa T}}{\kappa} - T\right) + \frac{\sigma^2}{2\kappa^2}\left(\frac{1 - e^{-2\kappa T}}{2\kappa} - 2\frac{1 - e^{-\kappa T}}{\kappa} + T\right)\,. \tag{5}$$

2.2 From Equity Volatility to Asset Volatility

We first use the Merton model to link the firm's asset volatility to its equity volatility. Let σ_E be the volatility of instantaneous equity returns. In our model, the equity volatility is affected by two sources of random fluctuations:

$$\sigma_E^2 = \left(\frac{\partial \ln S_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln S_t}{\partial r_t}\right)^2 \sigma_r^2 \,. \tag{6}$$

Using equation (3), we can calculate the sensitivities of equity returns to the random shocks in asset returns and riskfree rates:

$$\frac{\partial \ln S_t}{\partial \ln V_t} = \frac{1}{1-\mathcal{L}} \quad \text{and} \quad \frac{\partial \ln S_t}{\partial r_t} = \frac{b(T)\,\mathcal{L}}{1-\mathcal{L}}\,,$$

where

$$\mathcal{L} = \frac{K}{V}\,\frac{N(d_2)}{N(d_1)}\,\exp\left(\delta\, T + a(T) + b(T)\, r_t\right)\,.$$

Combining the above equations, we have

$$\sigma_E^2 = \left(\frac{1}{1-\mathcal{L}}\right)^2 \sigma_v^2 + \left(\frac{\mathcal{L}}{1-\mathcal{L}}\right)^2 b(T)^2\, \sigma_r^2 \,. \tag{7}$$

As expected, the firm's equity volatility σ_E is closely related to its asset volatility σ_v. In addition, it is also affected by the Treasury volatility σ_r through the firm's borrowing activity in the bond market. And this is reflected in the second term of equation (7), with $b(T)\,\sigma_r$ being the volatility of instantaneous returns on a zero-coupon riskfree bond of the same maturity T. The actual impact of these two random shocks is further amplified through $\mathcal{L}$, which, for lack of a better expression, we refer to as the "modified leverage." Specifically, for a firm with a higher $\mathcal{L}$, one unit shock to its asset return is translated to a larger shock to its equity return. Of course, this is the standard leverage effect. Moreover, as shown in the second term of equation(7), for such a highly "levered" firm, its equity return also bears more interest rate risk. Conversely, for an all-equity firm, $\mathcal{L} = 0$, and the interest-rate component diminishes to zero.

The first step of our empirical implementation is to estimate asset volatility σ_v using equation (7). In adopting the Merton model, however, we inherit a simple capital and debt maturity structure: the firm issues a single zero-coupon bond with face value K and maturity T. In the actual empirical implementation, we need, at a minimum, adapt to a more complex maturity structure. To the extent the Merton model is important in our empirical implementation, it is in deriving the analytical expressions that enter equation (7). In other words, we rely on the Merton model to tell us how the sensitivities or elasticities vary as functions of the key parameters of the model including leverage K/V, asset volatility σ_v, payout rate δ, and debt maturity T. When it comes to the actual calculations of these key parameters, we deviate from the Merton model as follows.

In calculating the leverage ratio K/V, we let S be the market value of equity, K be the book value of its debt, and let the firm value be $V = S + K$. Here, we deviate from the zero-coupon structure of the Merton model in order to take into account of the fact that firms typically issue bonds at par. Effectively, we approximate the market value of its debt by its book value. And to further improve on this approximation, we collect, for each firm, all of its bonds in our sample and calculate an issuance weighted market-to-book ratio and multiply K by this ratio.[1] For the purpose of backing out asset return volatility σ_v from σ_E, we need to pick a firm-level debt maturity T. Taking into account the actual maturity structure of the firm, we collect, for each firm, all of its bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. Finally, in calculating the payout ratio δ, we aggregate the firm's equity dividends, repurchase, and issuance and the debt coupon payments and scale the total payout by $V = S + K$, with the details of calculating S and K summarized above.

Like it is in many empirical studies before us, a structure model such as the Merton model plays a crucial role in connecting the asset value of a firm to its equity value. And ours is not the first empirical exercise to back out asset volatility using observations from the equity market.[2] While conceptually similar, our empirical implementation differs from the existing approaches in one important way. Specifically, the existing approach follows the one pioneered and popularized by Moody's KMV and uses the Merton model to calculate $\partial S/\partial V$ as well as to infer the firm value V through equation (3). By contrast, we use the Merton model to derive the entire piece of the sensitivity or elasticity function $\partial \ln S/\partial \ln V$, as opposed to using only $\partial S/\partial V$ from the model and then plugging in the market observed equity value S for the scaling component. At a conceptual level, we believe that taking the entire piece of the sensitivity function from the Merton model is a more consistent approach. At a practical level, while the Merton model might have its limitations in the exact valuation of bond and equity, it is still valuable in providing insights on how a percent change in asset value propagates to percent changes in equity value for a levered firm. Our reliance on the Merton model centers on this sensitivity measure.

2.3 Model-Implied Bond Volatility

The second step of our empirical implementation is to calculate, bond by bond, the volatility of its instantaneous returns, taking the inferred asset volatility $\hat{\sigma}_v$ from the first step as

[1] By adopting this empirical implementation, we have to live with one internal inconsistency with respect to the interpretation of of K. Central to this inconsistency is the problem of applying a model designed for zero-coupon bonds to coupon bonds. For our empirical results, the sole implication is on the firm's actual leverage. Our choice is to adopt the conventional calculation of a firm's leverage: $K/(S + K)$, and interpret K as the market value of its debt.

[2] See, for example, Crosbie and Bohn (2003), Eom, Helwege, and Huang (2004), Bharath and Shumway (2004), and Vassalou and Xing (2004).

a key input. Again, we have to take a compromising approach to the Merton model to accommodate the bonds of varying maturities issued by the same firm. Specifically, we rely on the Merton model to tell us, for any given time τ, the risk-neutral survival probability up to time τ: $P^\tau = N(d_2)$, where d_2 is as defined in equation (4) with T replaced by τ. Instead of taking the Merton model literally, which would imply no default between time 0 and the maturity date T, We find this to be a more realistic adoption of the model.[3]

Equipped with the term structure of default probabilities implied by the Merton model, we can now price defaultable bonds issued by each firm. Consider a τ-year bond paying semi-annual coupons with an annual rate of c. Assuming a face value of \$1, the time-t price of the bond is

$$B_t = \sum_{i=1}^{2\tau} \frac{c}{2}\, e^{a(i/2)+b(i/2)\, r_t} P^{i/2} + e^{a(\tau)+b(\tau)\, r_t} P^\tau + \sum_{i=1}^{2\tau} \mathcal{R}\, e^{a(i/2)+b(i/2)\, r_t} \left(P^{(i-1)/2} - P^{i/2}\right) , \quad (8)$$

where $\mathcal{R}$ is the risk-neutral expected recovery rate of the bond upon default. The first two terms in equation (8) collects the coupon and the principal payments taking into account the probabilities of survival up to each payment. The third term collects the recovery of the bond taking into account the probability of default happening exactly within each six-month period, which is $P^{(i-1)/2} - P^{i/2}$ for the i-th six-month period. The terms involving a and b are the respective riskfree discount functions implied by the Vasicek model as defined in equation (5).

Let σ_D^{Merton} be the volatility of the instantaneous returns of the defaultable bond. The model-implied bond volatility can be calculated as

$$\left(\sigma_D^{\text{Merton}}\right)^2 = \left(\frac{\partial \ln B_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln B_t}{\partial r_t}\right)^2 \sigma_r^2 \, . \quad (9)$$

Using the bond priced in equation (8) as an example, we can see that its asset-sensitivity, $\partial \ln B_t/\partial \ln V_t$, arises from the sequence of risk-neutral default probabilities, $P^{i/2}$ for $i = 1, \ldots, 2\tau$, while the Treasury-sensitivity, $\partial \ln B_t/\partial r_t$ arises both explicitly from the sequence of Vasicek discount functions and implicitly from the sequence of risk-neutral default probabilities.

It might be instructive to consider a τ-year zero-coupon bond, since its calculation can be further simplified to

$$\frac{\partial \ln B_t}{\partial \ln V_t} = \frac{n(d_2)\,(1-\mathcal{R})}{N(d_2) + (1-N(d_2))\,\mathcal{R}} \frac{1}{\sigma_v \sqrt{\tau}} \quad \text{and} \quad \frac{\partial \ln B_t}{\partial r_t} = b(\tau) \left(1 - \frac{\partial \ln B_t}{\partial \ln V_t}\right) ,$$

where $n(\cdot)$ is the probability distribution function of a standard normal. As expected, with full recovery upon default, $\mathcal{R} = 1$, the bond is equivalent to a treasury bond and its asset-sensitivity is zero and its Treasury-sensitivity becomes $b(\tau)$. The asset-sensitivity becomes more important with increasing loss given default, $1 - \mathcal{R}$, as well as with increasing firm

[3] A more self-consistent approach is to use the Black and Cox (1976) model, which generates a term structure of default probability that is the complementary first passage time distribution.

leverage K/V. From this example, we can also see the importance of allowing for a stochastic riskfree rate, as the Treasury volatility is an important component in the defaultable bond volatility.

In calculating the model-implied bond volatility, we take advantage of the model-implied term structure of survival probabilities but avoid treating the defaultable bond as a one large piece of zero-coupon bond with face value of K and maturity of T. This calculation is similar to the reduced-form approach of Duffie and Singleton (1999), except for the fact that our term structure of survival probabilities comes from a structural model while theirs derives from a stochastic default intensity.

3 Data and Construction of Volatility Estimates

3.1 The TRACE Data Set

The bond pricing data for this paper were obtained from FINRA's TRACE (Transaction Reporting and Compliance Engine). This data set is a result of recent regulatory initiatives to increase the price transparency in the secondary corporate bond markets. FINRA, formerly NASD,[4] is responsible for operating the reporting and dissemination facility for over-the-counter corporate trades. Trade reports are time-stamped and include information on the clean price and par value traded, although the par value traded is truncated at \$1 million for speculative grade bonds and at \$5 millions for investment grade bonds.

The cross-sections of bonds in our sample vary with the expansion of coverage by TRACE. On July 1, 2002, the NASD began Phase I of bond transaction reporting, requiring that transaction information be disseminated for investment grade securities with an initial issue of \$1 billion or greater. At the end of 2002, the NASD was disseminating information on approximately 520 bonds. Phase II, implemented on April 14, 2003, expanded reporting requirements, bringing the number of bonds to approximately 4,650. Phase III, implemented on February 7, 2005, required reporting on approximately 99% of all public transactions.

3.2 The Bond Sample

We use the transaction-level data from TRACE to construct bond return volatility for non-financial firms. First, we construct daily bond returns as follows. For any day t, we keep the last observation of the day for the bond and calculate the log return on day t as:

$$R_t = \ln\left(\frac{P_t + AI_t + C_t}{P_{t-1} + AI_{t-1}}\right),$$

where P_t is the clean price as reported in TRACE, AI_t is the accrued interest, and C_t is the coupon paid at t if day t is a scheduled coupon payment day. We use FISD to get bond-level information on coupon rates and payment dates. Accrued interest is calculated using the

[4] In July 2007, the NASD merged with the regulation, enforcement, and arbitration branches of the New York Stock Exchange to form the Financial Industry Regulatory Authority (FINRA).

Table 1: **Bond Sample Summary Statistics**

	2002			2003			2004			2005			2006		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std
Our Sample															
#Bonds	184			341			467			773			784		
Maturity	8.51	7.05	7.97	8.34	6.49	7.67	8.26	6.19	7.51	8.67	6.48	7.61	8.55	6.12	7.64
Amt	1,686	1,368	976	1,233	1,082	944	1,033	895	903	782	521	775	799	579	746
Rating	6.58	7.00	2.88	6.31	6.92	2.82	6.89	6.96	3.14	8.79	8.75	4.13	9.27	9.00	4.31
Age	2.02	1.58	1.69	2.74	2.07	2.31	3.28	2.59	2.53	3.82	3.13	2.92	4.04	3.56	3.03
#Trades	534	241	941	326	171	582	213	131	285	208	116	345	150	101	151
Volume	245	169	255	172	101	257	113	57	227	75	36	145	58	33	82
Turnover	13.69	11.12	9.50	11.79	9.02	9.77	9.14	6.68	8.34	8.44	5.73	8.10	7.31	4.81	7.42
%Traded	96.12	98.44	6.17	95.18	98.82	7.02	93.68	98.43	8.00	92.93	96.48	8.34	91.79	95.31	8.84
Trd Size	781	610	606	651	484	571	541	392	519	418	279	462	450	269	521
#Firms	60			92			129			227			236		
Mkt Cap	35.90	17.61	52.78	36.56	19.05	50.76	34.67	20.33	51.65	25.23	10.46	46.65	26.96	11.50	48.22
US Corporates in FISD															
#Bonds	21,465			22,305			24,203			26,590			28,710		
Maturity	7.11	4.33	8.97	7.24	4.43	8.88	7.43	4.55	8.74	7.45	4.65	8.61	7.23	4.39	8.57
Amt	172	68	328	174	57	336	171	48	337	164	31	335	164	25	339
Rating	7.49	6.00	4.26	7.41	6.58	4.25	7.02	6.17	4.10	6.77	6.00	4.03	6.51	5.58	4.11
Age	4.47	3.84	3.82	4.15	3.11	3.86	3.76	2.18	3.89	3.56	2.04	3.85	3.55	2.29	3.79

#Bonds and *#Firm* are the average numbers of bonds and firms per month. *Maturity* is the bond's time to maturity in years. *Amt* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Age* is the time since issuance in years. *#Trades* is the bond's total number of trades in a month. *Volume* is the bond's total trading volume in a month in millions of dollars of face value. *Turnover* is the bond's monthly trading volume as a percentage of the amount oustanding. *%Traded* is the percentage of business days in a month when the bond is traded. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *Mkt Cap* is the equity market capitalization in billions of dollars. The reported std and median are the time-series averages of cross-sectional values.

standard 30/360 convention. Returns are only calculated for day t if there is a price available for both t and $t-1$. Given the daily return data, we next construct time-series of monthly bond volatilities by taking the standard deviation of the daily bond returns (if there are at least 10 bond returns in a month) and annualizing. The sample of bonds that survive this calculation form the basis of our bond sample. In addition, to exclude highly infrequently traded bonds, we include bonds for which we can construct monthly volatilities for at least 75% of its presence in TRACE.

Table 1 summarizes our bond sample. The number of bonds increases throughout our sample period largely due to the coverage expansion of the TRACE. Compared with the universe of U.S. corporate bonds documented in FISD, our sample contains only a small number of bonds. In terms of size, however, these bonds are orders of magnitude larger than the median size bond in FISD. For example, at the beginning of our sample in 2002, the median bond size is $1,368 millions in our sample, compared with $68 millions in FISD. In the early sample, this is largely due to the limited coverage of TRACE , but overall, our sample construction biases toward picking more frequently traded bonds, which are typically larger.

The average maturity of the bonds in our sample is about 8.5 years, similar in magnitude but slightly higher than the average maturity of 7.3 years for the bond universe in FISD. While the cross-sectional median maturity is close to 7 years in our sample, it is only around 4.5 years in the FISD sample. These observations are consistent with a relatively higher degree of cross-sectional dispersion of bond maturity in FISD. In the early sample period, the bonds in our sample are noticeably younger than those in the FISD sample, although this difference diminishes toward the later sample period. The representative bonds in our sample are investment grade, with a median rating of roughly 7 (Moody's A3) during the early sample and 9 (Moody's Baa2) during the later sample. By contrast, the median rating in the FISD sample remains stable.

Given that TRACE is a transaction-level data, we can further collect trading information for the bonds in our sample. For example, in 2002, an average bond is traded on average 534 times a month with $245 millions of average trading volume and 13.69% turnover. Over time, this set of numbers decrease quite significantly, reflecting the coverage expansion of TRACE to include smaller and less frequently traded bonds. By 2006, an average bond was traded on average 150 times a month with $58 millions of average trading volume and 7.31% turnover. Also, the average trade size is $781 thousands in 2002 and $450 thousands in 2006, reflecting the inclusion of smaller trades. Overall, compared with the entire TRACE sample, our sample is biased toward bonds that are more frequently traded. For example, on over 95% of the business days, a median bond in our sample is traded at least once on that day.

Merging our bond sample with CRSP and COMPUSTAT by bond issuer, the firm-level summary statistics are reported in Table 1. The average number of firms in our sample is 60 in 2002 and grows to 236 in 2006. By equity market capitalization, the firms whose bonds are in our sample are typically large, with an average market capitalization of $35.90 billions in 2002 and $26.96 billions in 2006.

3.3 Bond Return Volatility $\hat{\sigma}_D$

The direct outcome of our sample construction is a monthly time-series of bond return volatility, $\hat{\sigma}_D$, for cross-sections of bonds. Building on the same bond sample, we also use weekly bond returns to construct a quarterly time-series of bond volatility, and monthly bond returns to construct a yearly time-series.[5]

The first panel of Table 2 summarizes the empirically estimated bond volatility, $\hat{\sigma}_D$, using daily, weekly, and monthly returns. Moving across the three return horizons, the magnitude of $\hat{\sigma}_D$'s, all annualized, decreases markedly. Specifically, the sample mean of $\hat{\sigma}_D$ is 18.03% when estimated using daily returns, contrasted with 9.61% using weekly returns, and 7.17% using monthly returns. The time-series averages of the cross-sectional median of $\hat{\sigma}_D$ exhibit a similar pattern: 15.74% at daily, 8.47% at weekly, and 6.36% at monthly frequency.

Implicitly in this pattern are strong negative auto-covariances of daily and weekly bond returns. Given that we are using transaction prices to construct bond returns, bid-ask bounce could be a natural candidate for such negative autocorrelations.[6] One could use the volatility estimate proposed by French, Schwert, and Stambaugh (1987) to take out this autocorrelation and therefore construct a volatility estimate that is more closely linked to the fundamental movements. For our purpose, however, it is more appropriate to use the simple measure of volatility that include the fundamental component as well as the potential liquidity component. As we move on next to construct equity return volatility from daily, weekly and monthly stock returns, we will adopt the same treatment.

To emphasize the cross-sectional variation of the empirical bond volatility, we sort $\hat{\sigma}_D$, at the appropriate frequencies, by a set of bond- and firm-level variables into quartiles, and report the means for each quartile. As reported in Table 3, bonds with smaller issuance, longer maturity, and lower rating are more volatile. Bonds issued by firms with higher equity volatility and higher leverage are also more volatile. The relation of $\hat{\sigma}_D$ to the bond trading variables such as turnover and the frequency of its trading is not as clear, nor is there a clear pattern linking $\hat{\sigma}_D$ to the firm payout ratio. Finally, it is interesting to notice that moving across measurement horizons from daily to monthly returns, the cross-sectional patterns hold quite well, while the overall magnitude decreases in a dramatic fashion.

3.4 Equity Return Volatility $\hat{\sigma}_E$

The equity return volatility is one key input to our structural model, from which the asset volatility for the firm can be backed out. The equity sample used to construct the equity volatility mirrors the bond sample summarized in Table 1. For each firm whose bonds enter our bond sample, we use CRSP daily, weekly, and monthly returns to form monthly, quarterly, and yearly estimates of equity volatility.

The second panel of Table 2 summarizes the empirical equity return volatility $\hat{\sigma}_E$. When average across firm and time, the annualized equity volatility of our sample is 26.78% when

[5]Like the case for daily returns, we require at least 10 weekly bond returns in a quarter to form a quarterly estimate of bond volatility, and at least 10 monthly bond returns in a year to form a yearly estimate.

[6]See, for example, Niederhoffer and Osborne (1966) and Roll (1984).

Table 2: **Volatility Estimates**

	Daily Returns			Weekly Returns			Monthly Returns		
	mean	med	std	mean	med	std	mean	med	std
Empirical Bond Volatility $\hat{\sigma}_D$									
2003	21.13	18.29	13.09	11.43	10.01	7.09	8.79	8.42	4.50
2004	17.34	14.05	12.30	8.82	7.50	6.26	6.21	6.07	3.14
2005	17.48	13.87	13.55	9.30	7.40	8.10	6.44	5.28	4.66
2006	16.06	13.30	10.99	8.63	6.99	8.99	7.50	5.67	5.85
Full	18.03	15.74	13.52	9.61	8.47	8.43	7.17	6.36	4.54
Empirical Equity Volatility $\hat{\sigma}_E$									
2003	27.07	25.29	11.47	25.88	25.09	11.13	25.46	23.27	10.73
2004	21.99	18.61	12.36	23.18	19.57	13.27	18.73	16.43	8.20
2005	26.01	21.04	17.29	26.71	21.88	17.73	26.16	21.89	19.88
2006	26.17	21.47	17.69	27.06	21.58	20.01	24.87	19.61	18.50
Full	26.78	24.46	16.04	27.39	25.51	16.70	24.27	20.30	14.33
Empirical 7-Year Treasury Bond Volatility									
2003	6.83	7.11	1.15	6.98	7.06	1.20	7.96	NA	NA
2004	5.77	5.61	1.16	5.36	5.33	1.73	6.15	NA	NA
2005	4.65	4.81	0.62	4.18	4.01	0.72	5.00	NA	NA
2006	3.93	4.12	0.54	3.43	3.41	0.50	3.35	NA	NA
Full	5.51	5.20	1.55	5.30	5.19	1.93	5.62	5.58	1.94
Model-Implied Asset Volatility σ_v^{Merton}									
2003	18.14	17.50	10.02	17.22	16.41	10.19	17.11	16.27	8.34
2004	15.13	13.36	10.57	15.87	13.32	11.16	13.27	12.37	6.29
2005	16.74	13.45	13.15	17.17	13.86	13.15	17.11	14.08	16.71
2006	16.97	13.94	13.51	17.39	14.21	14.66	16.39	13.34	13.00
Full	17.78	16.54	13.55	18.08	16.88	13.89	16.17	14.02	11.08
Model-Implied Bond Volatility σ_D^{Merton}									
2003	7.56	8.16	2.55	7.47	8.23	2.52	8.28	9.13	2.62
2004	5.86	6.49	2.17	5.47	5.96	2.18	6.02	6.77	2.17
2005	4.88	5.04	2.84	4.43	4.52	2.59	4.99	5.29	2.62
2006	4.05	4.12	2.36	3.76	3.65	2.68	4.03	3.69	2.80
Full	5.45	6.37	2.72	5.18	6.24	2.84	5.38	6.22	2.55

All volatility estimates are annualized and expressed in percentages. The empirical bond and equity return volatilities are constructed using daily, weely, and monthly bond and equity returns, respectively. The model-implied asset and bond return volatilities are backed out from equations (7) and (9), respectively, using the equity return volatility $\hat{\sigma}_E$ as inputs. The reported med and std are the time-series averages of cross-sectional medians and standard deviations. For empirical Treasury bond volatility, the reported numbers are time-series medians and standard deviations. The full sample includes data from July 2002 through 2006.

Table 3: **Volatility Estimates by Firm or Bond Characteristics**

	Daily Returns				Weekly Returns				Monthly Returns			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Empirical Bond Volatility $\hat{\sigma}_D$												
Bond Amt	36.58	21.60	18.10	17.79	20.70	11.90	9.80	9.88	12.77	7.99	7.34	7.30
Bond Maturity	13.83	19.04	24.96	36.15	7.09	10.32	14.33	20.50	4.82	7.12	9.78	14.27
Rating	15.75	17.02	24.67	33.85	7.82	8.82	13.65	19.76	5.32	6.26	7.79	14.63
Equity Volatility	16.85	17.43	23.81	35.93	8.73	9.07	16.57	18.66	5.78	6.32	8.61	15.29
Firm Leverage	17.47	20.16	19.76	37.03	9.52	10.40	11.45	21.87	6.61	7.14	7.46	15.05
Firm Payout	20.05	17.84	29.53	25.59	10.64	9.76	17.66	13.56	7.40	7.22	11.83	8.91
Bond Turnover	25.84	21.45	20.40	25.83	14.16	11.53	10.95	15.31	9.76	7.17	7.53	11.43
%Day Traded	23.61	24.92	23.81	20.71	13.24	13.68	13.16	11.44	10.93	8.23	8.36	8.24
Empirical Equity Volatility $\hat{\sigma}_E$												
Equity Mkt Cap	36.64	24.28	23.32	20.91	39.99	25.82	23.19	21.52	38.59	24.37	20.68	18.81
Firm Leverage	23.81	24.25	24.13	39.77	23.60	24.56	25.39	41.03	20.19	23.86	21.06	37.81
Model-Implied Asset Volatility σ_v^{Merton}												
Equity Mkt Cap	21.12	17.98	17.48	18.44	22.52	18.84	17.11	18.10	19.65	17.27	15.85	15.56
Firm Leverage	21.59	19.33	15.67	17.48	21.89	19.63	15.73	18.15	19.02	17.90	15.39	16.04
Equity Vol	10.98	15.90	19.40	28.66	10.69	15.35	20.53	31.42	8.81	13.50	17.78	28.54
Rating	15.65	17.19	17.57	21.64	15.65	16.85	17.85	22.33	14.13	14.86	14.37	22.62
Model-Implied Bond Volatility σ_D^{Merton}												
Bond Amt	5.96	5.22	5.41	5.60	5.76	4.83	5.08	5.36	6.05	5.38	4.86	5.45
Firm Leverage	5.01	5.59	5.30	6.81	4.60	5.07	5.12	6.48	4.85	5.19	5.38	6.35
Bond Maturity	3.16	5.50	6.63	7.00	2.99	5.15	6.34	6.61	2.75	5.34	6.56	7.13
Equity Volatility	4.94	5.22	5.57	6.63	4.65	4.75	5.07	6.98	4.95	4.96	5.13	6.72
Firm Payout	5.62	5.34	5.61	5.60	5.19	5.13	5.31	5.32	5.53	5.05	6.13	5.02
Bond Volatlity $\hat{\sigma}_D$	3.68	5.35	6.19	7.29	3.39	4.90	5.86	7.11	2.95	5.07	6.12	7.62
Rating	5.11	5.31	5.82	6.05	4.76	5.02	5.47	5.79	5.02	5.33	5.40	6.00

All volatility estimates are annualized and expressed in percentages. The empirical bond and equity return volatilities are constructed using daily, weely, and monthly bond and equity returns, respectively. The model-implied asset and bond return volatilities are backed out from equations (7) and (9), respectively, using equity return volatility $\hat{\sigma}_E$ as inputs. The sample is sorted by the respective variables from low to high into quartiles: Q1 (low), Q2, Q3, and Q4 (high). For bond ratings: Q1=Aaa&Aa, Q2=A, Q3=Baa and Q4=Junk.

estimated using daily equity returns, 27.39% using weekly returns, and 24.27% using monthly returns. Compared with the dramatic pattern of decreasing empirical bond volatility with increasing measurement horizon, it is a strong indication of the relative importance of liquidity in the bond and equity markets. The cross-sectional variation of the empirical equity volatility is reported in the second panel of Table 3. As expected, smaller firms are more volatile, so are more leverage firms.

Overall, our volatility measures are lower than those reported for U.S. equities, in part due to the fact that firms in our sample are typically larger firms. Another important driver is that our sample period, from July 2002 through 2006, is a relatively low volatility period. We maintain a contemporaneous sample of empirical bond and equity volatilities so as to capture the time-variation in asset volatility and its impact on both the bond and equity volatilities. Nevertheless, our model is set in a constant volatility setting. So a lower than average equity volatility would have a more permanent impact on our estimate of the firm asset volatility than it otherwise would in a stochastic volatility setting. We will consider the robustness of our result with respect to this limitation of our model.

4 Model-Implied Volatility

4.1 Parameter Calibration of the Merton Model

The parameters that govern the dynamics of the riskfree rate are calibrated as follows. First, we use the daily time-series of three-month Tbill rates from 1982 through 2006 to calibrate the long-run mean parameter θ and the rate of mean reversion parameter κ. Specifically, we set $\theta = 5.46\%$, so that the long-run mean equals its time-series average; $\kappa = 0.2443$, so that the daily autocorrelation of the model matches the sample autocorrelation.[7] Second, we set the volatility parameter σ_r so that, for a 7-year Treasury coupon bond, the model-implied volatility of its instantaneous returns matches the sample volatility. More specifically, to parallel our treatment of the empirical bond and equity volatilities, we use daily 7-year Treasury coupon-bond returns to form monthly estimates of Treasury bond volatilities, and weekly returns to form quarterly estimates and monthly returns to form yearly estimates. The Treasury volatility estimates are reported in the third panel of Table 2. When averaged across time, the annualized volatility estimates remain stable regardless of the measurement horizons, again, contrasting with the pattern in corporate bonds.

The random fluctuation of Treasury rate is an important component in corporate bonds. Our choice of the riskfree parameters, particularly σ_r, has a big impact on the model-implied bond volatility. The median maturity of the bonds in our sample is closer 7 years. We choose σ_r to match the volatility of a 7-year Treasury coupon bond so that on average, the Treasury component of the bond volatility is matched to its sample counterpart. While σ_r varies over time in our calibrations, its time-series average is around 2.3%. By contrast, the volatility coefficient σ_r estimated from the time-series of three-month Tbill rates is only at 1.3%, which

[7] To be more precise, we should match the risk-neutral value of κ since we are using the model for pricing purpose.

would severely under-estimate the Treasury component of corporate bond volatility. Implicit in this difference in σ_r is the fact that the simple one-factor model of Vasicek cannot match well the term structure of Treasury bond volatility. Our approach is to force the model to match well near the 7-year maturity, which is close to the median maturity of our bond sample.

Apart from the asset volatility σ_v, which is to be inferred from the model, the firm-level parameters to be calibrated are the payout ratio δ, leverage K/V, and maturity T. For each firm, its leverage K/V is calculated as follows. First, we calculate the book value of the firm's debt to be the sum of long-term debt and debt in current liability using COMPUSTAT data. Given that firms typically issue bonds at par, the market value of the debt should be close to the book value. To improve on this approximation, however, we collect, for each firm, all of its bonds covered by TRACE and calculate an issuance weighted market-to-book ratio. We set the parameter K in the Merton model to be the book value of the firm's debt multiplied by market-to-book ratio. We set $V = S + K$ to be the firm value with S equaling the market value of the equity.

In calibrating the firm-level debt maturity T, we take into account the actual maturity structure of the firm and collect all of the firm's bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. Finally, we calculate the firm's payout ratio δ by adding its annual dividends plus repurchases minus issuances and plus annual coupon payments and scaled the total dollar payout by the firm value V.

Table 4: **Firm-Level Parameters of the Merton Model**

	Firm T			Leverage K/V			Modified Leverage $\mathcal{L}$			Payout Ratio δ		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std
2003	6.32	6.25	2.70	28.45	27.13	18.82	27.55	24.34	18.95	3.68	3.18	2.57
2004	6.22	6.00	2.41	25.86	22.35	16.36	24.51	19.65	16.91	3.74	3.37	2.62
2005	6.25	5.77	3.06	30.63	27.01	19.73	28.34	23.68	19.69	3.87	4.25	3.05
2006	5.97	5.64	2.62	31.56	28.01	20.04	28.49	25.66	18.70	4.46	4.47	3.27
Full	6.15	5.79	2.75	29.88	26.43	19.27	27.66	23.49	18.80	4.03	4.03	3.02

With the exception of leverage K/V, the other firm-level parameters are constant in the model. We do, however, take its time-variation into account and update the firm level parameters at the appropriate frequencies. Table 4 summarizes the firm-level parameters. The average leverage K/V of the firms in our sample is about 29.88%, the average firm T is 6.15 years, and the average average payout ratio 4.03%. Overall, these firm-level variables are stable over time, and are only mildly skewed.

4.2 Model-Implied Asset Return Volatility $\sigma_v^{\textbf{Merton}}$

For each firm in our sample, we back out its asset volatility σ_v^{Merton} using the Merton model via equation (7). The riskfree parameters as well as the firm-level model parameters including leverage K/V, payout ratio δ and firm T are calibrated as described in Section 4.1. The fourth panel of Table 2 summarizes the model-implied asset volatility using the firm-level parameters, the estimated equity volatility $\hat{\sigma}_E$, and the bond volatility σ_r as inputs. Among the inputs, however, the key variable is equity volatility, which remains quite stable across measurement horizon. The model-implied asset volatility inherits this pattern. In addition, it also inherits a relatively low asset volatility from the relatively low equity volatility.

The cross-sectional variation of the model-implied asset volatility is reported in the third panel of Table 3. It shows that smaller firms have higher asset volatility, It is interesting that firms with lower leverage have higher asset volatility, consistent with the possibility of leverage being an endogenous variable. On the other hand, firms whose bonds are speculative grades have markedly higher volatility that the investment grades. Within investment grade, there is some evidence of increasing asset volatility with decreasing credit rating, although this pattern is not robust. Finally, the relation of asset volatility to equity volatility is monotonically increasing, indicating that the cross-sectional variation in leverage, or more precisely the modified leverage $\mathcal{L}$, does not break the link between the two.

Finally, we should mention one bias in our sample regarding the calculation of model-implied asset volatility. Although it is clear from equation (7) that, for each firm with a fixed set of parameters, the equity volatility will be the deciding factor in backing out the asset volatility, the interest rate volatility component does plays a role. For firms with high leverage, its equity volatility should have a component tied to the volatility of the riskfree rate. But for some highly levered firms in our sample, their empirical equity volatility is to low to account for the riskfree interest-rate volatility component. In such cases, an asset volatility cannot be backed out from equation (7),and we exclude the firm and their bonds from our sample. The frequency of such incidents is not rare, and happens for about 10% of the firms and 20% of the bonds in our sample. Had we used a zero asset volatility in the model, the bond volatility would be identical to the Treasury bond volatility of the same maturity. In practice, however, these bonds' volatilities are higher than their Treasury counterparts because of the default risk. By excluding these bonds from our sample, we effective create an downward bias in the difference between empirical and model-implied bond volatility.

4.3 Model-Implied Bond Return Volatility $\sigma_D^{\textbf{Merton}}$

For each bond in our sample, we calculate its model-implied volatility σ_D^{Merton} using the Merton model via equation (9). The riskfree parameters as well as the firm-level model parameters are the same as before, except that we are taking the model-implied asset volatility σ_v^{Merton} as a key put. Moreover, we no longer need the firm maturity T. Instead, the respective bond maturity is used in calculating the return volatility for coupon bonds and the loss given default is set at 50%.

The last panel of Table 2 summarizes the model-implied bond volatility, σ_D^{Merton}. It is interesting to note that while the sample mean of $\hat{\sigma}_D$ is 18.03%, 9.61%, and 7.17% when estimated using daily, weekly, and month bond returns, the sample mean of σ_D^{Merton} is 5.45%, 5.18%, and 5.38%, respectively. Of course, this lack of variation across horizon is not surprising given that a key input in estimating σ_D^{Merton} is the equity return volatility $\hat{\sigma}_E$, which is relatively stable when various horizon returns are used. It does, however, reflect an interesting disconnect between the bond and equity market. We will compare $\hat{\sigma}_D$ and σ_D^{Merton} more closely in the next section.

The cross-sectional variation of σ_D^{Merton} is summarized in the last panel of Table 3. Quite intuitively, bonds with higher firm leverage, longer maturity, and lower rating are more volatile. In fact, among all variables used in the sorting, bond maturity is among the most effective variables in generating a spread in σ_D^{Merton}. In other words, the duration risk is an important component in the cross-sectional determinants of σ_D^{Merton}. The second most effective variable is the bond volatility $\hat{\sigma}_D$ estimated directly from the data. The fact that $\hat{\sigma}_D$ and σ_D^{Merton} line up in the expected direction is encouraging for our model. In addition, the fact that the spread is even wider when sorted by the empirical bond volatility σ_D^{Merton} estimated using monthly bond returns is even more telling. It indicates that the model-implied bond volatility, which includes no information about the potential liquidity problems in corporate bonds, line up better cross-sectionally with the empirical bond volatilities that are estimated using longer horizon returns and are less subject to liquidity contaminations. Sorting by equity volatility $\hat{\sigma}_E$ generates a cross-sectional variation in σ_D^{Merton}, although the effect is somewhat muted.[8] Finally, the relation of σ_D^{Merton} to the size of the bond, however, is not as clear as it is the case for $\hat{\sigma}_D$.

5 Bond Volatility: Empirical vs. Model

5.1 The Main Result

Table 5 summarizes the main result of our paper. Specifically, there is a strong discrepancy between the bond volatility $\hat{\sigma}_D$ estimated directly using bond return data and the bond volatility σ_D^{Merton} implied by the Merton model. Using daily returns to construct the volatility estimates, the sample means of $\hat{\sigma}_D$ and σ_D^{Merton} are 18.03% and 5.45%, respectively. The full sample mean of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ is 12.58% with a robust t-stat of 34.47 (cluster by month and by bond). The economic magnitude of such a discrepancy is quite large, and it indicates a volatility component in corporate bonds that is disconnected from the equity volatility of the same issuer or the interest rate volatility in Treasury bonds. Figure 1 paints a very similar picture by reporting the cross-sectional distribution the time-series averages of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$, bond by bond. In fact, 691 out of the 695 bonds in our sample have $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ that are

[8] It should be noted that σ_D^{Merton} is not necessarily monotonic in σ_E. For example, while the first term equation (9), which measures the defaultable bond's exposure to firm risk, is clearly increasing in σ_v, the second term, however, is decreasing in σ_v. For long duration bonds, the second term, which captures the riskfree interest rate exposure, could outweight the first.

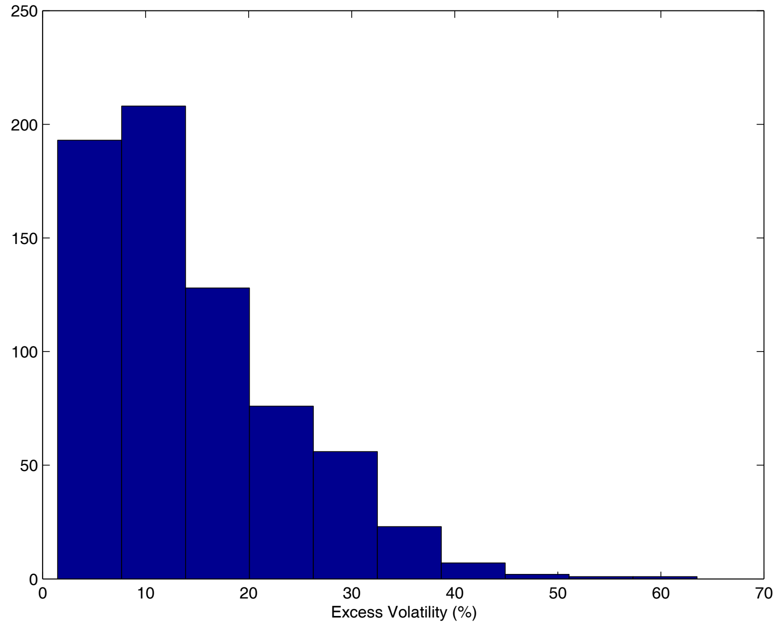

Figure 1: The cross-sectional distribution of the time-series means (bond by bond) of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$

positive with t-stat's greater than 1.96.

To better understand this large excess volatility component, we examine $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ across various measurement horizons.[9] When the volatility estimates are constructed using weekly returns, the sample mean of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ shrinks to 4.44% with a robust t-stat of 15.01. Moving to monthly horizon, the difference is further reduced to 1.79% with a robust t-stat of 2.15. Putting aside the fact that even at the monthly level the discrepancy is still significant statistically and large economically, the dramatic reduction in $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ across measurement horizons indicates that the disconnect is most severe at shorter horizons. Indeed, this horizon result is driven almost entirely by the short-term behavior of corporate bond returns. Specifically, the sample means of the empirical bond volatility $\hat{\sigma}_D$ are 18.03%, 9.61%, 7.17%, respectively, when measured using daily, weekly, and monthly bond returns. By contrast, the empirical equity volatility $\hat{\sigma}_E$ remains stable across the different measurement horizons. So does the model-implied bond volatility. Implicit in this unique horizon result is a high degree of negative auto-covariances in short-horizon bond returns, accentuating a severe liquidity component in the corporate bond market.

To exclude the possibility that our results are driven by a few bonds with extreme values of $\hat{\sigma}_D$, we examine the time-series average of the cross-sectional medians of the volatility estimates. Specifically, the medians of $\hat{\sigma}_D$ are 15.74%, 8.47%, and 6.36% for daily, weekly, and monthly measurement horizons respectively; the medians of σ_D^{Merton} are 6.37%, 6.24%,

[9] It should be mentioned that the model-implied volatility σ_D^{Merton} is derived for instantaneous returns. As such, when we move on to calculate volatility of bond returns over monthly horizons, the approximation error would increase.

Table 5: **Data Estimated vs. Model Implied Bond Volatility**

	$\hat{\sigma}_D - \hat{\sigma}_D^{\mathbf{Merton}}$								
	Daily Returns			Weekly Returns			Monthly Returns		
	#obs	mean	t-stat	#obs	mean	t-stat	#obs	mean	t-stat
Full Sample*	20,416	12.58	34.47	7,053	4.44	15.01	1,691	1.79	2.15
Straight	7,031	11.56	21.47	2,427	3.74	11.66	577	1.22	1.96
Callable Only	13,385	13.11	29.41	4,626	4.80	12.85	1,114	2.09	2.30
Convertible	1,848	27.32	26.40	624	13.81	13.23	133	8.55	7.72
Putable Not Conv.	117	23.75	6.03	41	12.75	3.64	10	4.59	3.49
By Year									
2003	3,185	13.58	23.41	1,032	3.96	7.36	277	0.51	*2.59*
2004	4,129	11.48	23.44	1,441	3.34	6.18	319	0.18	*1.49*
2005	5,948	12.61	26.06	2,021	4.88	9.15	481	1.45	*9.83*
2006	6,335	12.01	32.19	2,274	4.87	11.83	614	3.47	*19.07*
By Rating									
Aaa	2,087	10.67	12.12	684	3.20	7.33	154	0.40	0.91
Aa	2,258	10.22	12.79	751	2.88	7.85	166	0.19	0.42
A	7,049	10.36	19.86	2,387	3.07	10.23	571	0.51	1.25
Baa	5,266	13.58	19.82	1,827	4.93	10.44	419	1.81	3.41
Ba	1,660	17.25	15.39	648	7.80	8.25	202	6.14	5.19
B	1,220	17.05	17.22	444	6.83	8.78	96	3.18	6.06
By Duration									
≤ 2	3,389	7.45	19.54	1,208	2.48	9.69	317	0.92	2.54
$2-4$	4,430	9.00	22.72	1,553	2.42	8.16	366	0.34	0.62
$4-6$	5,108	11.37	28.10	1,752	3.90	8.49	441	1.15	1.41
$6-8$	3,898	14.36	27.28	1,311	5.16	12.26	267	2.89	1.83
> 8	3,142	22.69	21.52	1,089	9.47	18.53	270	4.87	4.24
Using Excess (Bond - Treasury) Returns									
	19,014	17.29	42.67	6,661	8.10	31.01	1,667	4.83	19.46
Factor in Bid/Ask Spreads									
	17,671	11.57	30.73	6,118	3.93	14.25	1,487	1.70	2.00

The full sample includes straigt and callable bonds, excluding convertibles and putables. Convertibles and putables are excluded from all tests except those reported under Covertible, and Putable Not Covertible. $\hat{\sigma}_D$ is estimated using daily, weekly, and monthly bond returns, respectively. $\sigma_D^{\mathrm{Merton}}$ is the model-implied bond volatility. #obs is bond month for daily, bond quarter for weekly, and bond year for monthly. The t-stat's are calculated using robust standard errors, clustered by time and by bond, with the by-year results at the monthly horizon being the only exception.

and 6.22% respectively; and the medians of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ are 9.79%, 2.54%, and 0.43%, respectively. The patterns are similar to those reported for the sample mean results, with the exception that the median discrepancy in volatility estimates is only 43 basis points when measured with monthly returns. In other words, the excess volatility in corporate bond is most severe when measured with short-horizon returns, and then tapers off near monthly returns.

As shown in Table 5, at the daily and weekly measurement horizons, the pattern of excess volatility is quite robust, whether the sample is split by bond type, by year, by credit rating, or by bond duration. While our full sample includes only straight bonds and callable bonds, we also report the results for convertible and putable bonds separately. It is expected that, except for straight bonds, the model-implied bond volatility will be off in capturing the real bond volatility. In particular, the model would under-estimate the volatility for a convertible bond. Indeed, we find a much higher level of excess volatility for convertible bonds. The fact that the putable bonds also have higher excess volatility, however, is puzzling, although the sample is quite small. Given that the callability feature is more tied to the random fluctuation of interest rate and effectively shortens the duration of a callable bond, one would expect the model to over-estimate the volatility in a callable bond, and therefore generates a lower degree of excess volatility. Table 5, however, shows that the excess volatility is slightly higher for callable bonds than for straight bonds. This comparison, however, fails to factor in bond-level characteristics such as maturity and rating, which could be important in driving the excess volatility results. Indeed, the average maturity of the callable bonds in our sample is 8.82 years compared with 5.31 years for straight bonds. Their credit ratings are also on average one or two notches below the straight bonds. We will revisit this issue in Section 6, when we examine the difference of these two samples after controlling for other bond level characteristics including maturity and rating.

It is not surprising that the model performs the best at the monthly return horizon. Specifically, the model is able to generate an average volatility of 5.38% that is relatively close to the empirically observed bond volatility of 7.17%. This excess volatility of 1.79%, however, is still statistically significant with a robust t-stat of 2.15, and, perhaps more importantly, still accounts for a quarter of the observed empirical bond volatility. The subsample results at the monthly measurement horizon, however, are not as robust as those at the shorter horizons. For example, at the monthly measurement horizon, excess volatility is not significant for A and above rated bonds, but remains to be important for bonds rated Baa and below. The U-shaped pattern of excess volatility by bond duration is also interesting. At the monthly measurement horizon, the average empirical volatility is 3.14% for bonds with duration less than 2 years, and the corresponding excess volatility is 0.92%, which is close to a third of the observed empirical volatility and is statistically significant. For median duration bonds, however, excess volatility becomes less significant. But as we move to the category of bonds with the longest duration, excess volatility becomes significant again. The average empirical volatility for bonds with duration longer than 8 years is 11.78%, and the corresponding excess volatility is 4.87%, which is about 40% of the observed empirical volatility.

Overall, our results indicate that while at the daily and weekly measurement horizons, the Merton model cannot even begin to generate the kind of bond volatility observed in the data due to the liquidity problems in corporate bonds, the model is doing a much better job at the monthly horizon. Whether or not the remaining amount of excess volatility is due to liquidity or model mis-specification remains an interesting question. We will pay close attention to this aspect of our result in the next few Sections.

5.2 Further Considerations

5.2.1 Stochastic Interest Rate

The simple term-structure model employed in this paper is one issue of concern. A proper account of the riskfree volatility is important because small fluctuations in the riskfree interest rate will be magnified by the duration of the bond to a sizeable volatility. Because of this, we calibrate the volatility coefficient σ_r in the riskfree rate model so that the model generates the empirically observed level of volatility for a 7-year Treasury coupon bond. Effectively, we force the model to match well near the 7-year maturity, which is close to the median maturity of our bond sample. This, however, still does not full capture the entire term-structure of interest rate volatility.

To account for this, we work with excess bond returns to avoid relying on a term structure model. We calculate excess bond returns by subtracting contemporaneous Treasury bond returns of a similar maturity from the corporate bond returns.[10] Comparing the volatility measured by bond excess returns to the model-implied excess bond volatility, we find that the results are similar to our main result. For the daily, weekly, and monthly measurement horizons, the sample means of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ are respectively 17.29%, 8.10%, and 4.83% with robust t-stat of 42.67, 31.01, and 19.46; the time-series averages of the cross-sectional medians are 14.62%, 6.50%, and 3.89%.

Overall, the excess bond volatility puzzle is somewhat deepened here with the adoption of the model-free approach. The main reason is that in working with the Merton model with stochastic interest rate, we inherit the model-implied correlation between the corporate bond and Treasury bond. In the model-free approach, the empirical correlation is used. As it is implicit in the current result, the empirical link is weaker than that prescribed by the model, consequently making the excess volatility puzzle even larger in magnitude. In Section 7.1, we will explicitly examine the empirical sensitivity of the corporate bond to Treasury bond and compare it with the model implied sensitivity.

5.2.2 Bid-Ask Bounce

Given the importance of liquidity as a potential explanation of our results, we further try to correct the bond volatility measure by factoring in the effect of bid-ask bounce. Following Roll (1984) and assuming that transactions at bid and ask are equally likely, we can map an observed bid-ask spread to its impact on return volatility. For all the bonds in our sample,

[10] The return horizons are matched at daily, weekly, and monthly, respectively. We use 1-, 2-, 5-, 7-, 10-, 20-, and 30-year Treasury returns as the basis of our extrapolation to get the target maturity.

we collect monthly bid-ask spreads from Bloomberg Terminals, and calculate the associated "bid-ask bounce" contribution to the bond return volatility.[11]

The last panel in Table 5 reports $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$, where $\hat{\sigma}_D$ is the square-root of the difference between the empirical bond variance minus the variance generated by the bid-ask bounce. While decreasing somewhat from the previous results, the magnitude of the discrepancy remains similar to our main result. In other words, the excess bond volatility documented here cannot be explained by bid-ask bounces alone.

5.2.3 Firms with Missing Asset Volatility

We use the Merton model to back out asset volatility from equity and Treasury volatilities via equation (7). For most firms, the key input of this calculation is equity volatility, with interest rate volatility relegated to playing only a minor role. It is for this reason, many of the existing studies do not include Treasury volatility in the calculation. For firms with higher than usual leverages, however, this Treasury component becomes too large to be ignored. If the high leverage is further coupled with a higher than usual payout ratio δ, it would result in a high level of modified leverage $\mathcal{L}$. And from equation (7), we see that for such a firm, its equity volatility would collect a component amplified by $\mathcal{L}/(1-\mathcal{L})$ from the Treasury volatility. If in practice, the equity volatility for such a firm is not large enough to even account for this component, then we run into the problem of not being able to back out asset volatility from equation (7).

Indeed, for 10% of the firms summarized in Table 1, we run into this problem of missing asset volatility. This pool of firms have an average leverage of 82%, more than two standard deviations from the sample average of 30%. They have an average payout ratio of 12.85%, also more than two standard deviations from the sample average of 4%. On the other hand, their firm-T is on average 5 years, not that different from and slightly lower than the sample average. Their equity volatility is on average 34.77%, which is indeed higher than the sample average of 24.27%, but not high enough to account for their interest rate exposure.

This problem of missing asset volatility affects 20% of the bonds in our sample that is summarized in Table 1. Using monthly bond returns, we construct empirical bond volatility for this sample of bonds and find an average bond volatility of 14.57% and a median volatility of 10.86%, which are markedly higher than the average of 7.17% and median of 6.36% for the sample of bonds without the missing asset volatility problem. The exclusion of such firms and their bonds effectively creates a downward bias in our main result. In other words, the magnitude of excess volatility reported in our main result would have been a lot bigger had we included such bonds in our analysis.

[11]Bloomberg typically provides bid-ask quotes from various dealers and we use the Bloomberg Generic (BGN) Quote, which reflects consensus market quotes. BGN quotes are available for a larger number of bonds in our subsample and typically have a longer time series than quotes by other dealers. It should also be noted that our adjustment is one-sided, since the bid-ask spread in the equity market might also have an impact as it finds its way to σ_D^{Merton}.

5.2.4 Conditional vs. Unconditional Asset Volatility

One limitation of our modeling approach is the tension between conditional versus unconditional volatility. Using conditional volatility sharpens the contemporaneous connection among the equity, Treasury, and corporate bond volatilities, capturing the link both across firms and across time. Indeed, our approach leans heavily toward this conditional approach by constructing monthly estimates of volatility using daily returns, and yearly estimates using monthly returns. The limitation, however, is that when the Merton model is used to calculate the sensitivity coefficients in equations (6) and (9), the conditional rather than unconditional volatility is plugged into the model. This has the wrong implication that if the current volatility is low, it will stay low for the entire life of the firm or the entire maturity of the bond. Given that our sample of July 2002 through December 2006 falls under a low equity volatility period, this tension could be important. The best resolution of this tension is to have a stochastic volatility model. Given that it will dramatically increase the complexity of the problem without the benefit of additional insights, we examine the robustness of our main result by using the following "hybrid" approach.

We first obtain unconditional estimates of equity and Treasury bond volatiles using monthly equity and Treasury bond returns going as far back into the history as possible. We then plug in the unconditional volatilities to equation (6) to obtain an unconditional version of the asset volatility. Averaged across all firms, this unconditional asset volatility is 23.69% with a cross-sectional median of 21.65%, which are indeed higher than the the model-implied asset volatility reported for our sample period. Armed with the unconditional asset volatility, we can now fix the problem with respect to the model-implied sensitivity coefficients in equations (6) and (9). Given that the horizon of the firm is typically long (firm-T is on average 6 years), and the rate of mean reversion of equity volatility is relatively fast, calculating the sensitivity coefficients using an unconditional approach seems reasonable. Applying this hybrid approach, we find only minor affect on our main result. For the monthly measurement horizon, the excess volatility measure $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ is on average 1.35% with a t-stat of 3.49. In other words, excess volatility remains significant both statistically and economically even at the monthly horizon.

6 Cross-Sectional Determinants of Excess Volatility

To shed light on the discrepancy measure $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$, we examine its cross-sectional determinants. Using the daily measurement horizon, we have monthly time-series of $\hat{\sigma}_D$ and σ_D^{Merton} for cross-sections of bonds. Table 6 reports the Fama and MacBeth (1973) cross-sectional regressions at monthly frequency with $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ as the dependent variable.

We find that, after controlling for other bond characteristics such as maturity and rating, excess volatility is more severe in smaller bonds. Given that smaller bonds are typically less liquid, this could potentially be a liquidity explanation. Similarly, the result on age is also quite interesting. In terms of the level of bond volatility, there is no theoretical link between the age of a bond and its volatility, keeping bond maturity fixed. By contrast, we find that

Table 6: **Cross-sectional Determinants of $\hat{\sigma}_D - \hat{\sigma}_D^{\textbf{Merton}}$**

Constant	17.34	14.88	12.14	3.20	23.86	23.73	19.97	8.77
	[6.98]	[6.99]	[5.38]	[1.74]	[6.81]	[10.10]	[8.31]	[3.06]
ln(Amt)	-2.53	-1.51	-3.40	-1.81	-3.53	-1.11	-1.81	-2.86
	[-7.30]	[-5.08]	[-8.49]	[-5.43]	[-7.56]	[-3.67]	[-5.43]	[-7.76]
Matutiry	0.61	0.63	0.60	0.64	0.63	0.65	0.64	0.61
	[17.15]	[20.06]	[16.93]	[20.30]	[19.94]	[20.40]	[20.30]	[17.91]
Age	0.60	0.45	0.64	0.34	0.57	0.27	0.34	0.60
	[4.67]	[3.42]	[4.69]	[2.86]	[4.79]	[2.47]	[2.86]	[4.66]
Rating	0.33	0.35	0.42	0.54	0.26	0.51	0.54	0.38
	[4.07]	[5.03]	[4.87]	[7.38]	[4.33]	[7.36]	[7.38]	[4.36]
Leverage	3.81	3.98	3.25	2.96	4.73	3.22	2.96	3.68
	[6.04]	[6.61]	[5.53]	[6.08]	[7.27]	[6.24]	[6.08]	[6.20]
Equity Vol	0.053	0.072	0.025	0.041	0.063	0.059	0.041	0.048
	[1.66]	[2.46]	[0.77]	[1.37]	[1.97]	[1.95]	[1.37]	[1.49]
ln(Volume)		-1.21		-2.43				
		[-5.61]		[-11.55]				
ln(#Trd)			2.26	3.77			1.34	
			[10.28]	[13.28]			[6.72]	
Turnover					-0.014			
					[-1.54]			
ln(Trd Sz)						-2.76	-2.43	
						[-14.68]	[-11.55]	
%Day Trd								0.11
								[6.33]
Callable	-0.46	-0.46	-0.37	-0.38	-0.25	-0.43	-0.38	-0.48
	[-1.12]	[-1.20]	[-0.93]	[-1.08]	[-0.71]	[-1.20]	[-1.08]	[-1.18]
R-sqd	29.87	31.18	31.45	34.52	31.39	33.90	34.52	30.63

Reported are monthly Fama-MacBeth cross-sectional regressions with $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ as the dependent variable, where σ_D and σ_D^{Merton}, both in %, are estimated using daily returns. The Fama-MacBeth t-stats are corrected for autocorrelation using Newey-West. The reported R-sqd's are the time-series averages of cross-sectional R^2's. Convertible and putable bonds are excluded from the regression, and Callable is one for a callable bond and zero otherwise. Age is year since issuance, Amt is in \$m, Ratings are coded as 1 for Aaa and 21 for C, Leverage is in decimals, Volume is monthly bond trading volume in \$m, Trd Sz is average trade size in \$thousand, and Turnover, %Day Trd and Equity vol are all in %. See Table 1 for the summary statistics of the independent variables.

$\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ is higher for those bonds that have aged over time. Specifically, controlling for other bond characteristics including maturity, rating and size, a bond that is one-year older has an additional excess volatility of 60 basis points, and the t-stat is 4.67. As newly issued bonds are typically more liquid, while the old bonds are more likely to be locked away in some safety boxes, our result is consistent with a liquidity explanation.

Table 6 shows that both the bond maturity and rating play an important role in explaining $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$. More specifically, longer maturity bonds and lower rated bonds exhibit more excess volatility. This effect, however, could be confounded with the fact that longer maturity bonds and lower rated bonds are more volatile in general. The liquidity connections in terms of rating and maturity are not apparent, although it is probably true that investment grades are more liquid than speculative grades, and certain maturity cohorts are more liquid. Given the importance of these two variables in bond volatility, we add them to serve more as controls.

The results using the firm-level variables are mixed. After controlling for rating, bonds that are issued by firms with higher leverage have higher excess volatility. Specifically, a 10% increase in a firm's leverage increases the excess volatility by 38 basis points. The firm's equity volatility is found to be positively related to the cross-sectional variation of $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$, but the result is not statistically significant. Given the inherent connection between the corporate bonds and equity of the same firm, one would expect a positive link between the empirical bond volatility $\hat{\sigma}_D$ and the equity volatility $\hat{\sigma}_E$. Indeed, Table 3 shows that the cross-sectional variation in empirical bond volatility is closely connected to the cross-sectional variation in equity volatility. The fact that the equity volatility can no longer explain, with statistical significance, the cross-sectional variation in $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ indicates that the model-implied bond volatility σ_D^{Merton} is doing a good job in capturing the cross-sectional variation in $\hat{\sigma}_D$ that was previously proxied by $\hat{\sigma}_E$.

The trading related variables also show some interesting results. We find more excess volatility in corporate bonds whose average trade size is small. Controlling for other bond characteristics including bond size, maturity, rating, and age, a bond with an average trade size of \$100,000 face value would have an additional excess volatility of $2.76\ln(10)$=6.36% than a bond with an average trade size of \$1,000,000 face value.[12] One could argue that, after controlling for bond size, the bonds with smaller average trade size are more likely to be traded in less liquid bond trading platforms and therefore have a larger liquidity component. It should be mentioned that the monthly time-series variation of the variables could also contribute to the result. That is, for each bond, the month during which the average trade size is small is a less liquid month, therefore resulting in a higher $\hat{\sigma}_D - \sigma_D^{\text{Merton}}$ during that month. The results from the other trading variables are somewhat mixed. For example, bonds with higher trading volume yield lower excess volatility. The turnover result points in the same direction, but is not statistically significant. On the other hand, bonds with more number of trades yield higher excess volatility and is statistically significant. While trading volume and number of trades are positively related, their implications for liquidity might

[12]It should be mentioned that TRACE truncates the trade size at \$1 million for speculative grade bonds and at \$5 millions for investment grade bonds.

be different, and this observation is consistent with our result from average trade size.[13] Similarly, we find that bonds with higher percentage days of trading exhibit more excess bond volatility. [14]

Finally, using the monthly measurement horizon, we have yearly time-series of the empirical and model-implied bond volatilities, and the Fama-MacBeth cross-sectional regression could be done at yearly frequency. It is plausible that the liquidity problem is less prominent at this measurement horizon. Indeed, we find that the size of the bond is no longer important in explaining the excess volatility. The age of a bond remains important: older bonds have more excess volatility, but the importance diminishes when the average trade size of a bond is used. Overall, the average trade size of a bond remains to be important: bonds trading in smaller average sizes have higher excess volatility. The leverage of the firm, however, is no longer important in explaining the cross-sectional variation in excess volatility.

7 Time-Series Determinants of Bond Returns

7.1 Exposure to Equity and Treasury: Empirical vs. Model

Let $\mathcal{R}^D$, $\mathcal{R}^E$, and $\mathcal{R}^T$ denote the instantaneous returns of a corporate bond, its equity, and a treasury bond (of a similar maturity as the corporate bond). According to our model,

$$\mathcal{R}_t^D = a + h^E \mathcal{R}_t^E + h^T \mathcal{R}_t^T \,,$$

where $h^E = (\partial \ln B / \partial \ln V)(\partial \ln S / \partial \ln V)^{-1}$ is the sensitivity of bond to equity. Because equity returns are correlated with Treasury bond returns, h^T has two components. The first term is the sensitivity of bond to Treasury: $(\partial \ln B / \partial r)(\partial \ln G / \partial r)^{-1}$ with G being the price of a Treasury coupon bond. The second term is $-h^E (\partial \ln S / \partial r)(\partial \ln G / \partial r)^{-1}$, which corrects for the fact that, in our model, equity is a function of riskfree interest rate.

Following Schaefer and Strebulaev (2004), we refer to h^E and h^T as hedge ratios, which effectively measure the instantaneous exposure of corporate bond returns to equity and Treasury shocks. Using our earlier derivations, the model-implied hedge ratios can be calculated as follows. The equity hedge ratio h^E can be derived by taking advantage of our earlier calculations of $\partial \ln B / \partial \ln V$ and $\partial \ln S / \partial \ln V$. Likewise, the Treasury hedge ratio can be calculated by using $\partial \ln B / \partial r$ and $\partial \ln G / \partial r$, where r is the instantaneous interest rate.

The empirical counterparts of the model-implied hedge ratios can be estimated as follows. For each bond in our sample, we regress the daily bond returns on the daily equity returns of the same firm and the daily Treasury bond returns of a similar maturity:

$$R_t^D = \alpha + \beta^E R_t^E + \beta^T R_t^T + \epsilon_t \,. \tag{10}$$

[13] A bond with high number of trades from investors trading with small average trade size is different from a bond with low number of trades from investors trading large average trade size.

[14] Overall, it should be mention that our bond sample selection biases toward more liquid bonds. So to the extend that we would like to interpret some of the trading variables as proxying for bond liquidity, we are working within the domain of relatively liquid bonds.

For each bond in our sample, this regression is run every month (at least 10 daily observations are required). This exercise results in monthly estimates of β^E and β^T, which can be thought of as empirical hedge ratios. We also repeat the exercise using monthly returns, by running the regression using the entire time-series data for each bond. And this exercise results in one cross-sectional of empirical hedge ratios.

An alternative specification, used in Schaefer and Strebulaev (2004), is to run the regression in equation (10) with the equity and bond returns interacted with model-implied hedge ratios:

$$R_t^D = \alpha + \gamma^E h_{t-1}^E R_t^E + \gamma^T \, h_{t-1}^T \, R_t^T + \epsilon_t \,,$$

where h^E and h^T are the model-implied hedge ratios and where the regression coefficients γ^E and γ^T should be close to one if the model works according to the data. This specification has the advantage of capturing the time-variation in hedge ratios directly in the regression. Our analyses, however, yield some infrequent but extremely large values of γ^E. This is largely caused by bonds with very low model-implied hedge ratios but non-trivial empirical exposure to equity shocks. For this reason, we adopt the approach of comparing the model-implied and empirical hedge ratios.

The model-implied and empirical hedge ratios are reported in Table 7. For this analysis, we exclude all bonds with convertible and putable features, but keep the callable bonds. We focus first on the equity hedge ratio. When daily returns are used to estimate the hedge ratios, the full-sample mean of the empirical ratios is 1.09%, while that for the model implied is 2.67%. The difference between the two is -1.58% with a t-stat of -4.30 (clustered by bond and by month). Splitting the sample into straight bonds and callable bonds, we find that this discrepancy is mainly driven by callable bonds.[15] Moving on to the results at the monthly return level, we find that the empirical hedge ratio is on average 3.69% while the model-implied hedge ratio is on average 2.79%. The difference of the two is on average 0.91% with a t-stat of 2.72. These results point to the possibility that the model under-estimate the link between corporate bond and equity.[16] Focusing only on straight bonds, however, the difference is not statistically significant. Overall, our results on the equity hedge ratio are somewhat mixed, and point to the possibility that the model does a reasonable job in capturing the empirical hedge ratios for equity risk. This conclusion is similar to that in Schaefer and Strebulaev (2004).

For Treasury exposures, the result consistently shows that the link between the corporate bond and Treasury bond is weaker than that prescribed by the model. At daily return horizon, the full-sample mean of the Treasury hedge ratio is 60.71% empirically and 95.72% implied by the model. The average difference between the two is -35.01% with a t-stat of -13.6 (clustered by bond and month). Moving from daily to monthly horizons, this disconnect is less severe but remains important. The average hedge ratio is 76.78% empirically and 95.67% implied by the model. The average difference between the two is -18.89% with a t-stat of

[15] For most bonds, it is safe to assume that the callable option is tied more closely to interest rate. It is therefore puzzling as to why the model would over-estimate the equity hedge ratio for the callable bonds.

[16] It should be mentioned, however, our model-implied hedge ratios are calculated for instantaneous returns. While this approximation works at the daily return horizon, it might be off for the monthly horizon results.

Table 7: **Bond Exposure to Equity and Treasury, Empirical vs. Model Implied**

		Equity Exposure				Treasury Exposure				R-sqd
		Emp	Model	Difference		Emp	Model	Difference		Emp
	#obs	mean	mean	mean	t-stat	mean	mean	mean	t-stat	mean
(A) Using Daily Returns										
Both	17,663	1.09	2.67	-1.58	-4.30	60.71	95.72	-35.01	-13.6	17.84
Straight	5,779	1.21	1.66	-0.44	-0.85	71.54	97.24	-25.70	-5.88	16.33
Callable	11,884	1.03	3.16	-2.13	-5.18	55.44	94.98	-39.54	-13.2	18.58
By Rating										
Aaa,Aa	3,724	0.15	0.93	-0.77	-1.42	75.57	98.59	-23.02	-7.40	16.16
A	6,196	-0.01	1.50	-1.51	-3.44	73.23	97.68	-24.44	-8.94	18.43
Baa	4,586	1.88	2.87	-0.99	-1.42	60.63	95.61	-34.98	-10.09	19.84
Junk	2,771	3.52	7.37	-3.84	-4.10	17.56	87.48	-69.93	-7.86	15.42
By Maturity										
≤ 3	3,478	0.87	0.75	0.11	0.46	67.00	99.31	-32.31	-4.35	15.43
3-6	4,926	0.88	1.71	-0.83	-2.30	68.13	97.60	-29.47	-8.30	17.52
6-9	5,000	1.24	3.35	-2.11	-3.97	58.53	94.32	-35.79	-14.36	18.22
> 9	4,259	1.33	4.54	-3.21	-3.44	49.54	92.26	-42.72	-15.41	19.74
(B) Using Monthly Returns										
Both	582	3.69	2.79	0.91	2.72	76.78	95.67	-18.89	-9.38	44.49
Straight	189	2.20	1.51	0.69	1.26	87.30	97.71	-10.41	-3.19	47.04
Callable	393	4.41	3.40	1.01	2.42	71.72	94.69	-22.97	-9.15	43.26
By Rating										
Aaa,Aa	102	1.12	0.48	0.63	1.28	98.98	99.21	-0.22	-0.07	54.48
A	191	2.10	1.43	0.67	1.87	89.78	97.87	-8.09	-3.46	54.13
Baa	144	4.80	2.41	2.39	2.83	79.59	96.32	-16.73	-5.68	43.73
Junk	136	6.84	6.84	-0.01	-0.01	40.75	89.19	-48.43	-7.98	25.59
By Maturity										
≤ 3	174	2.01	1.09	0.93	2.88	97.91	98.86	-0.95	-0.21	44.24
3-6	143	2.45	2.58	-0.14	-0.24	75.86	96.19	-20.33	-6.22	45.37
6-9	161	5.61	3.80	1.80	3.13	68.71	93.75	-25.04	-8.65	45.98
> 9	103	5.33	4.38	0.95	0.70	56.09	92.50	-36.41	-8.36	41.70

Corporate bond returns are regressed on returns on equity of the same firm and Treasury bond of similar maturity to obtain the empirical hedge ratios and R-squared. Covertible and putable bonds are excluded. In Panel A, daily returns are used to obtain monthly estimates, #obs are bond month, and the t-stats are clustered by month and bond. In Panel B, monthly returns are used to obtain one estimate per bond for the entire time period, and #obs is the number of bond. All mean and R-sqd are reported in percentage.

-9.38. Given that the callability in callable bonds is closely related to the fluctuations in interest rate, it is instructive to split the sample into straight and callable bonds. Empirically, we find that callable bonds have lower sensitivity to the Treasury bonds of similar maturities. For example, at the daily frequency, the callable bonds have an average hedge ratio of 55.44% compared with 71.54% for straight bonds. This is expected since the callability feature effectively shortens the duration of a bond. Given that our model does not take into account of callability, however, the model-implied Treasury hedge ratios should not vary much across the two samples.[17] We find that the model-implied Treasury hedge ratio is 97.24% for straight bonds and 94.98% for callable bonds. Overall, even for straight bonds alone, the model-implied Treasury hedge ratio is statistically higher than the empirical hedge ratio.

Finally, we also report in Table 7 the hedge ratios by rating and maturity. In general, the empirical variations in hedge ratios by rating and maturity are consistent with those implied by the model. For example, the empirical link between equity and corporate bond increases moving from Aaa to Junk bonds, and the model can produce such a pattern. In general, our results are consistent with the conclusion of Schaefer and Strebulaev (2004). For a model as simple as the Merton model, it seems to be doing a quite reasonable job in producing the empirically observed hedge ratios for the equity component. Judging from the magnitudes of the hedge ratios, however, it is clear that Treasury is the more important component. And the model is weak in that dimension. A better riskfree term-structure model alone will not be the solution. At the long horizon, there needs to be some non-trivial interaction between the Treasury interest rates and the capital structure of the firm. At the short horizon, the liquidity of the corporate bond market and its interaction with the Treasury market are important in explaining our results.

7.2 A Decomposition of Systematic and Idiosyncratic Volatility

In this section, we focus on the random shocks that give rise to the excess volatility puzzle documented in this paper. In particular, we are interested in knowing if these random shocks exist only at the individual bond level, and whether or not they aggregate to a systematic component.

For this, we apply the method of Campbell, Lettau, Malkiel, and Xu (2001) to decompose the volatility associated with the random shocks into systematic and idiosyncratic components. The results are summarized in Figure 2. Given that uneven panels might introduce time variations in the relative magnitude of idiosyncratic and systematic volatility, we perform our analysis for the sample period after April 14, 2003, when Phase II of TRACE was introduced and the coverage was broader. Moreover, we exclude bonds that entered only after Phase III. The stock sample mirrors this bond sample. The bond residuals are the regression residuals from equation (10). Intuitively, it captures the variation of bond returns after taking out the exposure to the equity of the same firm and the exposure to Treasury bond of a similar maturity.

[17] Unless, of course, the sample of callable bonds are different in characteristics from the sample of straight bonds. The difference in model-implied equity hedge ratio between these two samples seems to indicate these

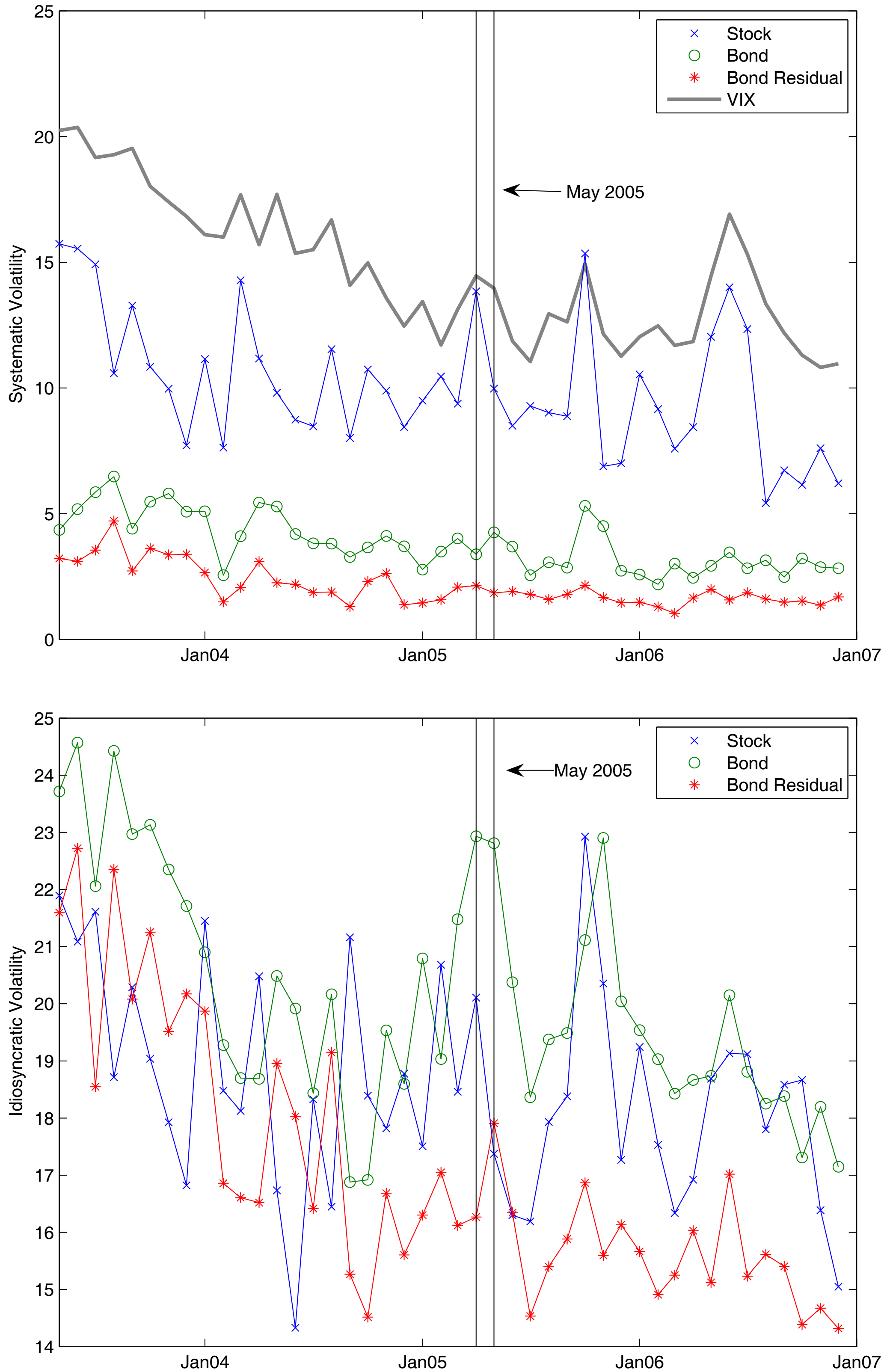

Figure 2: Systematic (top panel) and idiosyncratic (bottom panel) volatility of daily returns on stock, bond, and bond residuals.

As shown in the top panel of Figure 2, the systematic volatility of the corporate bond is on average 3.82%, which is low compared with the equity market. The bond residual has an even lower systematic volatility at 2.11%. According to the model, however, this systematic volatility should be zero. In practice, however, over 50% of the systematic volatility in the corporate bond market actually arises from this systematic component of the residuals that should have been zero according to the model. Moreover, as shown in Figure 2, the systematic volatility of bond moves quite closely with the systematic volatility of bond residual, with a correlation of 85%. Interestingly, they also co-move with the CBOE VIX index: the correlation is 71.62% with bond and 76.21% with bond residual.

The bottom panel of Figure 2 plots the idiosyncratic volatility of stock, bond, and bond residual. Not surprising, the idiosyncratic volatility of the residuals is disproportionally large compared with that in the equity market. The average idiosyncratic volatility is 18.52% for equity, 20.11% for bond, and 17.02% for bond residual. Given that the equity market in aggregate is several times more volatile than the aggregate bond market, the similar magnitudes of their idiosyncratic volatility indicate a disproportionally large idiosyncratic component in corporate bond returns. Moreover, the fact that the idiosyncratic volatility of the bond residual remains high, at a level of 17.02%, indicates that this large idiosyncratic component cannot be explained by our model. Interestingly, this idiosyncratic volatility has a correlation of 80% with the systematic volatility of the residuals and a correlation of 83.56% with the CBOE VIX index.

8 Conclusions

In recent years, we've seen increasing research activities on the empirical performance of structural models of default. Much attention has been focused on the model's ability or inability to match credit spreads. And our knowledge of the empirical performance of structural models, while intriguing and informative, is formed in large part by calibrations at the level of credit ratings or by applying the model to only a handful of bond observations. With the availability of the high frequency data from TRACE, which offers better data quality for a broad cross-section of corporate bonds, it is perhaps an opportune time to examine the structural models of default more closely.

Our paper's contribution is to provide an alternative angle from which the empirical performance of the structural models can be evaluated. In addition, by taking advantage of the high frequency data from TRACE, we can evaluate the empirical performance of the Merton model from varying measurement horizons. Applying the model at the firm level for a relatively broad cross-section of bonds, we are also able to get a better sense of how the Merton model actually performs at the individual firm and bond level and examine the cross-sectional and time-series determinants of the discrepancies between model and data.

For a broad cross-section of corporate bonds that extend from July 2002 through December 2006, we find an overwhelming amount of excess volatility in corporate bonds that

two samples are not identical.

cannot be explained by the Merton model of default. In fact, perhaps no structural model of default can explain our results at the daily and weekly measurement horizon: the magnitudes of the discrepancy too large and the patterns too unique to be contributed by default risk. At these horizons, the issue of liquidity is unambiguously important. Moreover, we find that variables known to be linked to bond liquidity are important in explaining the cross-sectional variations in excess volatility, providing further evidence of a liquidity problem in corporate bonds.

At the monthly measurement horizon, excess volatility becomes less severe, although on average it still accounts for a quarter of the observed empirical bond volatility. At this horizon, it becomes interesting to question whether or not the documented excess volatility is due to liquidity or model mis-specification. Our additional analyses show that it cannot be attributed to the lack of a sophisticated term-structure model or to the bid-ask bounce in the quoted bid-ask spread for corporate bonds. Moreover, comparing the model-implied equity and Treasury hedge ratios against their empirical counterparts, we find that while the model does a reasonable job in capturing the equity exposure, it consistently over-estimates the empirical exposures to Treasury bond. This indicates that the excess volatility puzzle documented in this paper would have been more severe, had we used the empirical instead of the model-implied Treasury exposures in calculating the model-implied bond volatility. Finally, even at the monthly measure horizon, we still find interesting connections between variables known to be linked to bond volatility and the cross-sectional variations in excess volatility,

Overall, the main result of this paper is the pattern excess volatility in corporate bonds and its connection to the liquidity of the corporate bond market. In future research, we plan to link this excess volatility more closely to the micro-structure of corporate bonds so as to understand the economic driver of the illiquidity in corporate bonds.

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