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## Designing Electricity Auctions

Natalia Fabra, Nils-Henrik von der Fehr, and  
David Harbord

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2547 Channing Way  
Berkeley, California 94720-5180  
[www.ucei.org](http://www.ucei.org)

# Designing Electricity Auctions\*

Natalia Fabra  
Universidad Carlos III

Nils-Henrik von der Fehr  
University of Oslo

David Harbord  
Market Analysis Ltd

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## Abstract

Motivated by the new auction format introduced in the England and Wales electricity market, as well as the recent debate in California, we characterize bidding behavior and market outcomes in uniform and discriminatory electricity auctions. We find that uniform auctions result in higher average prices than discriminatory auctions, but the ranking in terms of productive efficiency is ambiguous. The comparative effects of other market design features, such as the number of steps in suppliers' bid functions, the duration of bids and the elasticity of demand are also analysed. We also consider the relationship between market structure and market performance in the two auction formats. Finally, we clarify some methodological issues in the analysis of electricity auctions. In particular, we show that analogies with continuous share auctions are misplaced so long as firms are restricted to a finite number of bids.

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\*Natalia Fabra: Department of Economics, Calle Madrid 126, 28903 Getafe (Madrid), Spain. E-mail: nfabra@eco.uc3m.es. Nils-Henrik von der Fehr: Department of Economics, PO Box 1095 Blindern, N-0317 Oslo, Norway. E-mail: nhfehr@econ.uio.no. David Harbord: Market Analysis Ltd, 4 Mill Street, Eynsham, Oxford OX29 4JS, UK. E-mail: manalysis@aol.com.

## 1 Introduction

Electricity wholesale markets differ in numerous dimensions, but until recently all have been organized as uniform, first-price auctions. Recent experience - and the perceived poor performance - of some decentralized electricity markets however, has led certain regulatory authorities to consider adopting new auction designs. In England and Wales a major overhaul of the electricity trading arrangements introduced in 1990 has recently taken place, and among the reforms implemented in March 2001, a discriminatory or ‘pay-your-bid’ auction format was adopted. The British regulatory authority (Ofgem) believed that uniform auctions are more subject to strategic manipulation by large traders than are discriminatory auctions, and expected the new market design to yield substantial reductions in wholesale electricity prices. Similarly, before its collapse, the California Power Exchange commissioned a report by leading auction theorists on the advisability of a switch to a discriminatory auction format for the Exchange’s day ahead market, due to the increasing incidence of price spikes in both on- and off-peak periods (see Kahn et al., 2001).

It is well-known that discriminatory auctions are not generally superior to uniform auctions. Both types of auction are commonly used in financial and other markets, and there is now a voluminous economic literature devoted to their study.<sup>1</sup> In multi-unit settings the comparison between these two auction forms is particularly complex. Neither theory nor empirical evidence tell us that discriminatory auctions perform better than uniform auctions in markets such as those for electricity, although this has become controversial.

Wolfram (1999), for instance, argues in favor of uniform auctions for electricity, and Rassenti, Smith and Wilson (2003) cite experimental evidence which suggests that discriminatory auctions may reduce volatility (i.e. price spikes), but at the expense of higher average prices. Other authors have come to opposite conclusions. Federico and Rahman (2003) find theoretical evidence in favor of discriminatory auctions, at least for the polar cases of perfect competition and monopoly, while Klemperer (2001, 2002) suggests that discriminatory auctions might be less subject to ‘implicit collusion’.<sup>2</sup> Kahn et al. (2001), on the other hand, reject outright the idea that switching to a discriminatory auction will result in greater competition or lower prices.

In Britain, Ofgem has credited the recent fall in electricity prices in England and Wales to the new market design, however this too is controversial.<sup>3</sup> Evans and Green

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<sup>1</sup>See Ausubel and Cramton (2002) and Binmore and Swierzbinski (2000) for the theory and empirical evidence. Archibald and Malvey (1998) and Belzer and Reinhart (1996) discuss the US Treasury’s experiments with these auction formats in more detail. See also Kremer and Nyborg (2003).

<sup>2</sup>In a theoretical model similar to that used in this paper, Fabra (2003) shows that tacit collusion is easier to sustain in uniform auctions than in discriminatory auctions.

<sup>3</sup>Ofgem reports a 19% fall in wholesale baseload prices from the implementation of the reforms in March 2001 to February 2002, and a 40% reduction since 1998 when the reform process began. Wholesale prices have since risen again so that they are now near their pre-reform levels.

(2002) present some supporting evidence,<sup>4</sup> but Bower (2002) and Newbery (2003) argue that the decline in prices is fully explained by the reduction in market concentration brought about by asset divestitures, an increase in imports and market excess capacity. Fabra and Toro (2003) suggest that all of these factors, including the change in market design, are significant in explaining the reduction in wholesale electricity prices.<sup>5</sup>

The purpose of this paper is to address this electricity market design issue in a tractable model designed to capture some of the key features of decentralized electricity markets.<sup>6</sup> We characterize equilibrium market outcomes in a discrete, multi-unit auction model for uniform and discriminatory electricity auctions under a variety of assumptions concerning costs and capacity configurations, bid formats, demand elasticities and the number of suppliers in the market. Our purpose is to gain an improved understanding of how these different auction formats affect suppliers' bidding incentives, the degree of competition and overall welfare in decentralized electricity markets.

Our analysis proceeds by first considering a 'basic duopoly model', similar to the discrete, multi-unit auction described in von der Fehr and Harbord (1993), which is then varied in several directions. In the basic duopoly model, two 'single-unit' suppliers with asymmetric capacities and (marginal) costs face a market demand curve which is assumed to be both perfectly inelastic and known with certainty when suppliers submit their offer prices. By 'single-unit' we mean that each supplier must submit a single price offer for its entire capacity (i.e. its bid function is horizontal). This assumption simplifies the analysis considerably, but in Section 4.1 we show that it is largely inessential. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs which are independent of the prices negotiated in the wholesale market, at least in the short run.<sup>7</sup> However, in order to evaluate some of the possible effects of real-time pricing or demand-side bidding, we consider downward-sloping demand functions in Section 4.2. In Section 4.3, we consider the oligopoly case in order to shed some light on the relationship between market concentration and market performance.

Finally, the assumption that suppliers have perfect information concerning market demand is descriptively reasonable when applied to markets in which offers are 'short-

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<sup>4</sup>Evans and Green argue that the new trading arrangements may have undermined opportunities for tacit collusion. Sweeting (2001) claims to have found evidence of collusion in the England and Wales market during the late 1990s, although this finding has been challenged by Newbery (2003).

<sup>5</sup>Another contributing explanation for the fall in prices may be that Ofgem staked its reputation on the market reforms delivering lower-cost electricity, and for more than a year after their introduction sought to expand its regulatory powers to police 'market abuses' by smaller generators. See Bishop and McSorely (2001) for a discussion.

<sup>6</sup>For a discussion of some methodological issues in modelling electricity markets, which has informed our choice of models, see von der Fehr and Harbord (1998) and Fabra, von der Fehr and Harbord (2002).

<sup>7</sup>See Wolak and Patrick (1997) and Wilson (2002) on this. In most electricity markets large industrial consumers can purchase electricity directly from suppliers or the wholesale market, but their demand comprises only a small fraction of the total volume traded.

lived', such as in Spain where there are 24 hourly markets each day (see García-Díaz and Marín, 2003). In such markets suppliers can be assumed to know the demand they face in any period with a high degree of certainty. In markets in which offer prices remain fixed for longer periods, e.g. a whole day, such as in Australia and in the original market design in England and Wales, on the other hand, it is more accurate to assume that suppliers face some degree of demand uncertainty or volatility at the time they submit their offers. Hence we allow for this type of uncertainty in Section 4.4.

Under each set of assumptions we characterize suppliers' equilibrium bidding behavior in uniform and discriminatory auctions, and compare the equilibrium outcomes in terms of prices and productive efficiency. Our main insights may be summarized as follows. Equilibrium outcomes in either auction format fall essentially into one of two categories, depending upon the level of demand. In low-demand realizations prices are competitive, in the sense that they cannot exceed the cost of the most efficient non-despatched supplier: in high-demand realizations, on the other hand, prices exceed the cost of even the most inefficient supplier. In high-demand states<sup>8</sup> there are multiple, price-equivalent pure strategy equilibria in the uniform auction, while in the discriminatory auction the equilibrium is in mixed strategies. With certain demand (i.e. short-lived bids), payments to suppliers (or average prices) are lower in the discriminatory auction and numerical examples suggest that the difference can be substantial.<sup>9</sup> The comparison in terms of productive efficiency, however, is ambiguous and depends on parameter values as well as which pure-strategy equilibrium is played in the uniform auction. The relative incidence of low-demand and high-demand states depends upon structural features of the market, such as the degree of market concentration, as well as on the market design, in particular the market reserve price and opportunities for demand-side bidding. Structural factors that reduce the incidence of high-demand states affect bidding strategies in the discriminatory, but not in the uniform, auction. Market design changes, on the other hand, affect bidding strategies in both types of auction.

## 2 The Model

In the basic duopoly model two independent suppliers compete to supply the market with productive capacities given by  $k_i > 0$ ,  $i = 1, 2$ . Capacity is assumed to be perfectly divisible. Supplier  $i$ 's marginal cost of production is  $c_i \geq 0$  for production levels less than capacity, while production above capacity is impossible (i.e. infinitely costly). The suppliers are indexed such that  $c_1 \leq c_2$ . Without further loss of generality we may normalize suppliers' marginal costs so that  $0 = c_1 \leq c_2 = c$ . The level of demand in any period,  $\theta$ , is a random variable which is independent of the market price, i.e. perfectly price inelastic. In particular,  $\theta \in [\underline{\theta}, \bar{\theta}] \subseteq (0, k_1 + k_2)$  is distributed according to some

<sup>8</sup>The terms 'state' and 'realization' are used interchangeably throughout this paper.

<sup>9</sup>With uncertain demand, at least in the symmetric case, payments to suppliers are equal in both auction formats.

known distribution function  $G(\theta)$ .

The two suppliers compete on the basis of bids, or offer prices, submitted to the auctioneer. The timing of the game is as follows. Having observed the realization of demand, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply the whole of its capacity,  $b_i \in [0, P]$ ,  $i = 1, 2$ , where  $P$  denotes the ‘market reserve price,’ possibly determined by regulation.<sup>10</sup> We let  $\mathbf{b} \equiv (b_1, b_2)$  denote a bid profile. On the basis of this profile the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier’s capacity is despatched first. If this capacity is not sufficient to satisfy the total demand  $\theta$ , the higher-bidding supplier’s capacity is then despatched to serve the residual demand, i.e. total demand minus the capacity of the lower-bidding supplier. If the two suppliers submit equal bids, then supplier  $i$  is ranked first with probability  $\rho_i$ , where  $\rho_1 + \rho_2 = 1$ ,  $\rho_i = 1$  if  $c_i < c_j$  and  $\rho_i = \frac{1}{2}$  if  $c_i = c_j$ ,  $i = 1, 2, i \neq j$ .

For a given bid profile  $\mathbf{b}$ , the quantities allocated to each supplier are thus independent of the auction format. The output allocated to supplier  $i$ ,  $i = 1, 2$ , denoted by  $q_i(\theta; \mathbf{b})$ , is given by

$$q_i(\theta; \mathbf{b}) = \begin{cases} \min\{\theta, k_i\} & \text{if } b_i < b_j \\ \rho_i \min\{\theta, k_i\} + [1 - \rho_i] \max\{0, \theta - k_j\} & \text{if } b_i = b_j \\ \max\{0, \theta - k_j\} & \text{if } b_i > b_j, \end{cases} \quad (1)$$

and is solely a function of demand and the bid profile (and costs when equal price bids are submitted).

The payments made by the auctioneer to the suppliers do depend upon the auction format, however. In the *uniform auction*, the price received by a supplier for any positive quantity despatched by the auctioneer is equal to the highest accepted bid in the auction. Hence, for a given value of  $\theta$  and a bid profile  $\mathbf{b} = (b_i, b_j)$ , supplier  $i$ ’s profits,  $i = 1, 2$ ,  $i \neq j$ , can be expressed as

$$\pi_i^u(\theta; \mathbf{b}) = \begin{cases} [b_j - c_i] q_i(\theta; \mathbf{b}) & \text{if } b_i \leq b_j \text{ and } \theta > k_i \\ [b_i - c_i] q_i(\theta; \mathbf{b}) & \text{otherwise,} \end{cases} \quad (2)$$

where  $q_i(\theta; \mathbf{b})$  is determined by (1).

In the *discriminatory auction*, the price received by supplier  $i$  for its output is equal to its own offer price whenever a bid is wholly or partly accepted. Hence for a given value of  $\theta$ , and a bid profile  $\mathbf{b}$ , supplier  $i$ ’s,  $i = 1, 2$ , profits can be expressed as

$$\pi_i^d(\theta; \mathbf{b}) = [b_i - c_i] q_i(\theta; \mathbf{b}), \quad (3)$$

where again  $q_i(\theta; \mathbf{b})$  is determined by (1).<sup>11</sup>

<sup>10</sup> $P$  can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

<sup>11</sup>Note that the discriminatory auction is essentially a Bertrand-Edgeworth game. See Deneckere and Kovenock (1996).

Both suppliers are assumed to be risk neutral and to maximize their expected profits in the auction.

### 3 Equilibrium Analysis: A Tale of Two States

We first characterize the Nash equilibria in weakly undominated strategies of the model described in the previous section and then compare equilibrium outcomes.<sup>12</sup>

**Lemma 1** *In any pure-strategy equilibrium, the highest accepted price offer is in the set  $\{c, P\}$ . Moreover, in the discriminatory auction, in a pure-strategy equilibrium all accepted units are offered at the same price.*

Based on this ancillary result, we can prove the main result of this section, namely that equilibrium outcomes essentially fall into one of two categories, depending upon the level of demand:

**Proposition 1** *There exists  $\hat{\theta} = \hat{\theta}(c, k_1, k_2, P)$  such that:*

(i) *(low demand) if  $\theta \leq \hat{\theta}$ , in the unique pure-strategy equilibrium the highest accepted price offer is  $c$ .*

(ii) *(high demand) if  $\theta > \hat{\theta}$ , all suppliers are paid prices that exceed  $c$ . A pure-strategy equilibrium exists in the uniform auction, with the highest accepted offer price equal to  $P$ , but not in the discriminatory auction.*

As is easily seen, in low-demand realizations the equilibrium is both unique and identical across the two auction formats. In equilibrium, both suppliers submit offer prices equal to  $c$  (i.e. the cost of the inefficient supplier) but only the most efficient supplier is despatched. Hence the equilibrium outcomes in both auctions are competitive in the sense that prices are constrained by the cost of the least efficient supplier. They are also cost efficient, i.e. overall generation costs are minimized.

In high-demand realizations the equilibrium outcomes are very different. In the uniform auction, any pure-strategy equilibrium involves one supplier bidding at the market reserve price  $P$ , while the other supplier submits an offer price sufficiently low so as to make undercutting unprofitable (c.f. von der Fehr and Harbord, 1993). The precise nature of the equilibrium depends upon parameter values. There are three possible cases: (a) if  $\hat{\theta}_2 \leq \theta \leq \hat{\theta}_1$ , or  $k_1 \leq \theta \leq k_2 + \frac{c}{P}k_1$ , only equilibria in which  $b_1 < b_2 = P$  exist; (b) if  $\hat{\theta}_1 < \theta \leq \hat{\theta}_2$ , or  $\frac{P}{P-c}k_2 < \theta \leq k_1$  only equilibria in which  $b_2 < b_1 = P$  exist; and (c) if  $\theta > \max\{\hat{\theta}_1, \hat{\theta}_2\}$ , or  $\theta > \max\{k_1, k_2 + \frac{c}{P}k_1\}$  both types of pure-strategy equilibria exist. Note that in Case (a) the equilibrium outcome is always cost efficient, while in Case (b) it is always inefficient. In Case (c) cost efficiency depends on which equilibrium is played.<sup>13</sup>

<sup>12</sup>All derivations of results are relegated to the Appendix.

<sup>13</sup>There are also mixed-strategy equilibria in the uniform auction. However since these are pay-off dominated by the pure strategy equilibria we do not consider them here. See the Appendix.

In the discriminatory auction only mixed-strategy equilibria exist in high-demand states. In particular, there exists a unique equilibrium in which the two suppliers mix over a common support which lies above the cost of the inefficient supplier and includes the market reserve price, i.e.  $b_i \in (c, P]$ ,  $i = 1, 2$ . This mixed strategy equilibrium is not efficient in general, as there is a positive probability that the inefficient supplier will submit the lowest offer price.

The relative likelihood of low-demand versus high-demand states depends upon structural characteristics of the industry and on the strictness of the regulatory regime. Straight-forward calculations show that

$$\hat{\theta} = \begin{cases} k_1 & \text{if } k_1 \leq \frac{P}{P-c}k_2 \\ \frac{P}{P-c}k_2 & \text{if } k_1 > \frac{P}{P-c}k_2 \end{cases} \quad (4)$$

From this expression it follows that, for a given ratio of supplier capacities, the incidence of low-demand states is increasing in aggregate capacity. The incidence of low-demand states is also greater when suppliers are more symmetrically sized; more precisely, given  $c, P$  and  $K$ , with  $k_1 + k_2 = K$ ,  $\hat{\theta}$  is maximized at  $k_1 = \frac{P}{P-c}k_2$ , which involves perfect symmetry if  $c = 0$ . Further, cost asymmetry tends to make low-demand states more likely, since the loss in profit from undercutting the inefficient rival relative to serving residual demand is smaller the higher is his cost. Finally, since pricing monopolistically and serving residual demand is more profitable the higher is the market reserve price, the incidence of high-demand states is greater the higher is  $P$ . If we think of the market reserve price as a regulatory price cap, it follows that stricter regulation can improve market performance, not only because market power is reduced in high-demand states, but also because the likelihood of high-demand states occurring is lowered.

In comparing market performance across the two auction formats we consider both total generation costs and the average price paid to suppliers. For auction format  $f = d, u$ , let  $C^f$  and  $R^f$  denote equilibrium levels of total generation costs and payments to suppliers, respectively, and let  $b_i^f$  and  $q_i^f$  denote supplier  $i$ 's equilibrium offer price and output, respectively. We have  $C^f = \sum_i c_i q_i^f$ ,  $f = u, d$ ,  $R^d = \sum_i b_i^d q_i^d$  in the discriminatory auction, and  $R^u = p^u \sum_i q_i^u = p^u \theta$ , where  $p^u = \max_i \{b_i^u \mid q_i^u > 0\}$  is the market price, in the uniform auction. From Proposition 1 the following result is immediate:

**Proposition 2** *Market performance:*

- (i)  $R^d = R^u$  if  $\theta \leq \hat{\theta}$  and  $R^d < R^u$  if  $\theta > \hat{\theta}$ .
- (ii)  $C^d = C^u$  if  $\theta \leq \hat{\theta}$ ,  $C^d > C^u$  if  $\hat{\theta}_2 < \theta \leq \hat{\theta}_1$ ,  $C^d < C^u$  if  $\hat{\theta}_1 < \theta \leq \hat{\theta}_2$ , and  $C^d \geq C^u$  otherwise, depending upon whether, in the uniform auction, an equilibrium is played in which Supplier 1 or Supplier 2 submits the higher offer price.

In other words, the discriminatory auction weakly outperforms the uniform auction in terms of payments (or the average price paid) to suppliers. In low-demand realizations the equilibrium outcomes are identical in both auctions. In high-demand realizations, the

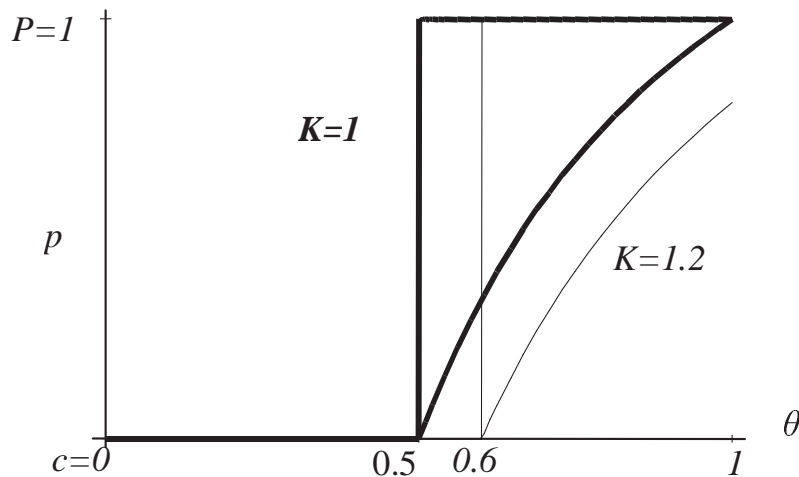


market price is at its maximum ( $P$ ) in the uniform auction, while prices in the discriminatory auction are below  $P$  with positive probability. Comparison of the auctions in terms of productive efficiency is more complex, however. In low-demand realizations costs are minimized in both auction formats. In high-demand realizations, the comparison is unambiguous in Cases (a) and (b) only. In the uniform auction production costs are minimized in Case (a) and maximized in Case (b), while in the mixed-strategy equilibrium of the discriminatory auction the more efficient supplier is undercut by the inefficient supplier with positive probability. Hence the cost performance in the uniform auction is superior to that of the discriminatory auction in Case (a), but worse in Case (b). In Case (c) the comparison depends upon which pure-strategy equilibrium is played in the uniform auction.

We conclude this section by considering how the performance of the two auction formats depends upon the parameters of the model. A change in parameter values affects outcomes in two distinct ways: first, by altering the relative incidence of high- versus low-demand states, and secondly by affecting the intensity of price competition in high-demand states. The importance of these two effects differ between the two auction formats. In the uniform auction, in high-demand realizations, price always equals the market reserve price, whereas in the discriminatory auction bidding strategies depend on the cost and capacity configuration, as well as on the level of demand and the market reserve price. An increase in the threshold  $\hat{\theta}$  has a profound effect on prices in the uniform auction, as prices jump down from the market reserve price to marginal cost over the relevant range of demand realizations. In the discriminatory auction, however, the effect of an increase in  $\hat{\theta}$  is much less pronounced. Since the equilibrium outcomes in high-demand realizations approach those of low-demand realizations as  $\theta \downarrow \hat{\theta}$ , a marginal increase in  $\hat{\theta}$  has no effect on the outcome *per se*.

The different ways in which outcomes are affected by changes in parameter values is illustrated in Figure 1 below. The figure is based on an example in which  $[\underline{\theta}, \bar{\theta}] = [0, 1]$ ,  $c = 0$ ,  $P = 1$  and  $k_1 = k_2 = \frac{K}{2}$ . The two solid lines show (expected) equilibrium prices for different realizations of demand for the two auction formats when  $K = 1$ . In both formats, price equals  $c = 0$  when  $\theta \leq \hat{\theta} = 0.5$ . When  $\theta > \hat{\theta}$ , price equals  $P = 1$  in the uniform auction, whereas it increases gradually with demand in the discriminatory format. The thin lines show the corresponding prices for the case  $K = 1.2$ , in which the critical threshold is now  $\hat{\theta} = 0.6$ . Whereas the increase in the relative incidence of low-demand realizations is the same in both auction formats, the effects on prices differ: in the uniform auction, prices jump from  $P = 1$  to  $c = 0$  for some demand realizations; in the discriminatory auction, the effect on prices is smoother but applies to a wider range of demand realizations.

Because of this fundamental differences in the way in which the equilibrium outcomes are affected, it is not possible in general to specify how a change in a particular parameter affects the relative performance of the two auction formats. In particular, changes in relative performance depend critically upon the distribution of demand  $G$ . In order to

Figure 1: Expected Equilibrium Price for Different Demand Realizations,  $\theta$ 

illustrate the possible effects, as well as the potential order of magnitudes involved, we proceed by considering a series of numerical examples. We maintain the parametrization introduced above, with the added assumption that  $G(\theta) = \theta$ , and define  $k_1 + k_2 = K \geq 1$ , with  $k_1 \geq k_2$ . Then expected payments to suppliers taken over all possible demand realizations (which are equal to expected profits in this case), become  $ER^d = \frac{K}{2} \frac{[1-k_2]^2}{k_1}$  and  $ER^u = \frac{1}{2} [1 - k_2] [1 + k_2]$ , respectively. Table 1 presents numerical results for different values of total installed capacity  $K$  for the case in which individual capacities are symmetric, i.e.  $k_1 = k_2 = \frac{K}{2}$ .

$K$	1	1.2	1.4	1.6	1.8	2
$ER^d$	0.250	0.160	0.090	0.040	0.010	0
$ER^u$	0.375	0.320	0.255	0.180	0.095	0
$\frac{ER^d}{ER^u}$	0.667	0.500	0.353	0.222	0.105	na

Table 1: Increasing Installed Capacity

At  $K = 1$ , total expected payments are 33% lower in the discriminatory auction. In the uniform auction, a similar reduction in average prices would require an excess capacity of 40% (i.e.  $K = 1.4$ ).<sup>14</sup> In both auctions, increasing the size of the players reduces both average prices and revenues. The pro-competitive effect on bidding strategies in the discriminatory auction is strong enough in this example so that its relative performance improves the higher is the capacity margin.

In Table 2 we present results for different distributions of a given total capacity  $K = 1$ :

<sup>14</sup>Since in both auctions the level of demand served in equilibrium is fixed at  $\theta$ , expected revenues can be taken as a proxy for the expected (average) price paid by consumers.

$k_1$	0.5	0.6	0.7	0.8	0.9	1
$k_2$	0.5	0.4	0.3	0.2	0.1	0
$ER^d$	0.250	0.300	0.350	0.400	0.450	0.5
$ER^u$	0.375	0.420	0.455	0.480	0.495	0.5
$\frac{ER^d}{ER^u}$	0.667	0.714	0.769	0.833	0.909	1

Table 2: Increasing Capacity Asymmetry

A more asymmetric distribution of capacities implies poorer performance in both types of auction, although the effect is stronger in the discriminatory auction. Reducing the size of the smaller supplier increases the incidence of high-demand states. In the discriminatory auction, the larger supplier faces a larger residual demand and hence has more to gain from submitting higher offer prices. Given this, the smaller supplier responds by increasing its offer prices also. Overall the result is that reallocating capacity from the larger to the smaller supplier (e.g. via capacity divestitures) improves the relative performance of the discriminatory auction over the uniform auction.

Finally, we consider how changes to the market reserve price  $P$  affect performance in the two auctions. Using the same example, we fix total capacity so  $K = 1$  and consider symmetric firms, i.e.  $k_1 = k_2 = 0.5$ .<sup>15</sup> Table 3 below presents the numerical results.

$P$	1	0.90	0.75	0.50	0.25	0
$ER^d$	0.250	0.225	0.188	0.125	0.063	0
$ER^u$	0.375	0.334	0.281	0.188	0.094	0
$\frac{ER^d}{ER^u}$	0.667	0.667	0.667	0.667	0.667	na

Table 3: Reducing the Market Reserve Price

Reducing the market reserve price reduces equilibrium price (and hence revenues) in both types of auction without affecting the comparison of their relative performance. This is because equilibrium revenues are proportional to the reserve price  $P$  in both auctions when  $c = 0$ .

## 4 Extensions and Variations

In the preceding sections we have analyzed electricity auctions for an asymmetric duopoly assuming that each supplier could submit only a single offer price for its entire capacity, and that demand was both known with certainty at the time offer prices were submitted

<sup>15</sup>This implies that the incidence of high versus low demand states is unaffected by changes in the market reserve price  $P$  in this example.

and perfectly inelastic. In the following subsections we relax each of these assumptions in turn.

#### 4.1 Multiple bids

We first extend the analysis by allowing suppliers to submit upward-sloping step offer-price functions instead of constraining them to submit a single bid for their entire capacity. An offer-price function for supplier  $i$ ,  $i = 1, 2$ , is then a set of price-quantity pairs  $(b_{in}, k_{in})$ ,  $n = 1, \dots, N_i$ ,  $N_i < \infty$ . For each pair, the offer price  $b_{in}$  specifies the minimum price for the corresponding capacity increment  $k_{in}$ , where  $b_{in} \in [0, P]$  and  $\sum_{n=1}^{N_i} k_{in} = k_i$ ,  $i = 1, 2$ . The following lemma states that the *equilibrium outcomes* - but not the *equilibrium pricing strategies* - are essentially independent of the number of admissible steps in each supplier's bid function (and whether the 'step sizes' are choice variables for suppliers). This implies that our comparisons between auction types remain valid in this setting.

**Lemma 2** (*Multiple-unit suppliers*) (i) *Uniform auction: the set of (pure-strategy) equilibrium outcomes is independent of the number of units per supplier (in particular, whether  $N_i = 1$  or  $N_i > 1$ ).* (ii) *Discriminatory auction: for low-demand realizations, there is a unique equilibrium outcome independent of the number of units per supplier. For high-demand realizations, there exists a set of mixed strategies that constitute an equilibrium independently of the number of units per supplier; when  $N_1 = N_2 = 1$ , these strategies constitute the unique equilibrium.*<sup>16</sup>

The existence of a unique, competitive equilibrium outcome in the uniform auction is in stark contrast to analyses which assume continuously differentiable supply functions, i.e.  $N_i = \infty$  (see e.g. Green and Newbery, 1993). As first shown by Wilson (1979), and further developed by Back and Zender (1993) and Wang and Zender (2002), in the uniform auction with continuous supply functions there exists a continuum of pure-strategy equilibria, some of which result in very low revenues for the auctioneer (or high payments to suppliers in procurement auctions). The latter are characterized by participants offering very steep supply functions which inhibit competition at the margin: faced with a rival's steep supply function, a supplier's incentive to price more aggressively is offset by the large decrease in price (the 'price effect') that is required to capture an increment in output (the 'quantity effect'). Since the 'price effect' always outweighs the 'quantity effect' for units of *infinitesimal size*, extremely collusive-like equilibria can be supported in the continuous uniform auction, even in a one-shot game.<sup>17</sup>

<sup>16</sup>The equilibrium offer price functions, however, do depend upon the number of units or admissible bids; for instance, there can be payoff-irrelevant units which are offered at higher prices as long as there are sufficiently many units priced at marginal cost.

<sup>17</sup>This type of equilibrium cannot be supported in a discriminatory auction. Klemperer (2002) provides a particularly clear discussion.

Discreteness of the bid functions rules out such equilibria however. When suppliers are limited to a finite number of price-quantity bids, a positive increment in output can always be obtained by just slightly undercutting the price of a rival's unit. Since the 'price effect' no longer outweighs the 'quantity effect', the collusive-like equilibria found in the continuous auction cannot be implemented. This observation casts some doubt on the relevance of applying the continuous share auction model to electricity markets in which participants are limited to a small number of offer prices per generating unit. The collusive-like equilibria obtained under the assumption that bid functions are continuous do not generalize to models in which offer increments are of positive size, no matter how small these are (see also Kremer and Nyborg, 2003). We conclude that the equilibrium outcomes for the two types of auction are independent of the number of admissible steps in the offer-price functions, so as long as this number is finite. Hence the characterization of the equilibrium outcomes provided in Section 3 would remain unchanged if we had instead assumed that suppliers submit offer-price functions rather than a single offer price for their whole capacity.

It is tempting to draw the conclusion that limiting the number of allowable bids in a uniform-price electricity auction would therefore improve market performance. Strictly speaking, our analysis does not support such a conclusion. What we have shown is that (i) moving from a continuous to a discrete-bid auction potentially improves market performance by eliminating the 'collusive-like' equilibria in the uniform auction, but (ii) market performance in a discrete-bid auction is independent of the number of allowable bids, so long as this number is finite. It could be argued, however, that since limiting the number of bids does not effectively restrict agents' opportunities, it might be desirable in the interests of market simplicity and transparency. Indeed, in equilibrium players may optimally choose not to differentiate their bids even when they are able to do so.

## 4.2 Price-elastic demand

Our next variation on the basic duopoly model considers the case of price-elastic demand. For this purpose we let the market demand function be represented by  $D(p, \theta)$ , which is assumed to satisfy the following standard assumptions: as a function of  $p$ ,  $D$  is continuous and bounded; there exists a price  $\bar{p}(\theta) > 0$  such that  $D(p, \theta) = 0$  if and only if  $p \geq \bar{p}(\theta)$ ;  $D$  is decreasing in  $p$ ,  $\forall p \in [0, \bar{p}(\theta)]$ ; and  $pD$  is strictly quasi-concave in  $p$ ,  $\forall p \in [0, \bar{p}(\theta)]$ .

Given a downward-sloping demand function, in either auction format the output allocated to supplier  $i$ ,  $q_i(\mathbf{b}, \theta)$ , as a function of the offer price profile  $\mathbf{b} = (b_i, b_j)$ , becomes:

$$q_i(\mathbf{b}, \theta) = \begin{cases} \min \{D(b_i, \theta), k_i\} & \text{if } b_i < b_j \\ \rho_i \min \{D(b_i, \theta), k_i\} \\ \quad + \rho_j \min \{\max \{0, D(b_i, \theta) - k_j\}, k_i\} & \text{if } b_i = b_j \\ \min \{\max \{0, D(b_i, \theta) - k_j\}, k_i\} & \text{if } b_i > b_j, \end{cases}$$

for  $i = 1, 2$ . Note that independently of the payments made to suppliers in either auction

format, it is implicitly assumed that consumers are charged the market-clearing price, i.e. the highest accepted offer price. Obviously, this leads to the market (auctioneer) running surpluses in the discriminatory auction. Assuming that such surpluses are dealt with via lump-sum transfers, total surplus (i.e. the sum of supplier profits and consumer surplus) will be determined solely by the market-clearing price and the allocation of output between suppliers.

From the above assumptions it follows that market demand is a continuous and decreasing function of price and that, whenever  $D(c_i) > k_j$ ,  $j \neq i$ , there exists a unique price  $p_i^r$  that maximizes a supplier's profits from serving the residual demand, i.e.  $p_i^r(\theta) = \arg \max_p \{p \min [D(p, \theta) - k_j, k_i]\}$ . The price  $p_i^r$  will be referred to as the 'residual monopoly price' of supplier  $i$ .

We further assume that the parameter  $\theta$  defines a family of demand functions such that if  $\theta_1 < \theta_2$ ,  $D(p, \theta_1) < D(p, \theta_2)$ . Intuitively,  $\theta$  is a shift parameter that affects the position, but not the slope, of the demand function (at least not to the extent that demand functions corresponding to different  $\theta$ 's cross). It follows that  $p_i^r(\theta)$  is increasing in  $\theta$ .

Let  $P_i^r = \min \{p_i^r, P\}$  be the effective residual monopoly price of supplier  $i$ . Then it should be clear that the argument of Lemma 1 goes through as before, with  $P_1^r$  and  $P_2^r$  substituted for  $P$ . Furthermore, we can extend the result of Proposition 1 that there exists a unique threshold  $\hat{\theta}$  such that equilibrium outcomes are of the low-demand and high-demand type, respectively, depending upon whether the shift parameter  $\theta$  is below or above the threshold. The performance comparison across auction formats is also essentially the same, with the following caveat: since the consumer price is generally lower in the discriminatory auction there is an allocative efficiency gain due to the corresponding increase in consumption.

Our main purpose of this section, however, is to relate the critical threshold  $\hat{\theta}$  to the price elasticity of demand. To this end we use the following definition: for two demand functions  $D^1$  and  $D^2$  with  $D^1(p, \theta) = D^2(p, \theta)$  at  $p = c$ , the demand function  $D^1$  is said to be more elastic than the demand function  $D^2$  if  $D^1(p, \theta) < D^2(p, \theta)$  for all  $p \geq c$ . If we let  $p_i^{r^t}$  denote the residual monopoly price of supplier  $i$  corresponding to the demand function  $D^t$ , it follows that  $p_i^{r^1} < p_i^{r^2}$  if  $D^1$  is more elastic than  $D^2$ . The following result is then immediate:

**Proposition 3** *The critical threshold  $\hat{\theta}$  is non-decreasing in the elasticity of the demand function  $D$ .*

In other words, the price elasticity of demand affects market performance in two distinct ways. First, given a high-demand realization, the distortion due to the exercise of market power is smaller when demand is more price-elastic (i.e. the residual monopoly price is lower). Second, the incidence of high-demand realizations is reduced the more elastic is the demand curve. With a downward-sloping demand function, the gain from exercising market power relative to residual demand is less and hence there is more incen-

tive to compete for market share by undercutting the rival, leading to a higher incidence of competitive outcomes.

We conclude this section by considering a numerical example. We maintain the assumptions introduced in the example considered in Section 3 above - with  $k_1 = k_2 = k$  - and in addition assume that  $D(p, \theta) = \theta - \beta p$ . It follows that  $\hat{\theta} = k$  and that (for  $\beta$  sufficiently small)  $P_1^r = P_2^r = \frac{\theta - k}{2\beta}$  for  $\theta < k + 2\beta$  and  $P_1^r = P_2^r = P = 1$  otherwise. Expected payments to suppliers become  $ER^d = \int_k^{k+2\beta} \frac{1}{2\beta} [\theta - k]^2 d\theta + 2 \int_{k+2\beta}^1 [\theta - \beta - k] d\theta$  and  $ER^u = \int_k^{k+2\beta} \frac{1}{4\beta} [\theta - k] [\theta + k] d\theta + \int_{k+2\beta}^1 [\theta - b] d\theta$ , respectively. In Table 4 we present results for different values of the slope of the demand function:<sup>18</sup>

$\beta$	0	0.025	0.050	0.075	0.100	0.125	0.150
$ER^d$	0.250	0.226	0.203	0.183	0.163	0.146	0.130
$ER^u$	0.375	0.350	0.327	0.304	0.282	0.260	0.240
$\frac{ER^d}{ER^u}$	0.667	0.646	0.621	0.602	0.578	0.562	0.542

Table 4: Increasing the Elasticity of Demand

As expected, a more elastic demand reduces payments to suppliers. In this example, the relative incidence of low-demand and high-demand states ( $\hat{\theta}$ ) is not affected, although more elastic demand does reduce the effective residual monopoly price. In the discriminatory auction we have the additional effect that bidding becomes more aggressive in high-demand states. Consequently, the relative performance of the discriminatory auction increases with the elasticity of demand here.<sup>19,20</sup>

### 4.3 Oligopoly

Our next variation on the basic duopoly model considers the case of oligopoly. This allows us to generalize some of the insights from the duopoly model as well as analyze the impact of changes in the number of suppliers on profits and pricing behavior.

Accordingly we now consider  $S$  suppliers, where  $k_s$  is the capacity and  $c_s$  the marginal cost of supplier  $s$ ,  $s = 1, 2, \dots, S$ . Suppliers are ordered by efficiency, so that  $0 = c_1 \leq c_2 \leq \dots \leq c_S = c$ . As before, the types of equilibria which arise in the different auction formats depend upon the value of the market demand  $\theta$  relative to suppliers' individual and aggregate capacities. In particular, we have the following result:

<sup>18</sup>Note that, for  $\beta$  sufficiently small,  $\beta$  approximates the price elasticity of demand at the peak (i.e.  $\theta = 1$ ) evaluated at the maximum admissible price  $P = 1$ .

<sup>19</sup>As pointed out above, the revenue comparison tends to understate the performance of the discriminatory auction relative to that of the uniform auction as far as consumer prices (and, indeed, consumer surplus) is concerned.

<sup>20</sup>The difference in total payments between the two auction formats in the case of perfectly inelastic demand ( $\beta = 0$ ) corresponds to the difference between the cases  $\beta = 0$  and  $\beta = 0.15$  in the uniform auction.



**Proposition 4** *There exists  $\hat{\theta}_s^-$  and  $\hat{\theta}_s^+$ ,  $\hat{\theta}_s^- \leq \hat{\theta}_s^+$ , such that, for  $s = 1, 2, \dots, S$ ,*

(i) *if  $\theta \leq \hat{\theta}_s^-$ , in any equilibrium the highest accepted price offer is at or below  $c_s$ ;*

(ii) *if  $\theta > \hat{\theta}_s^+$ , in any equilibrium suppliers are paid prices that are at least equal to  $c_s$*

*and strictly above  $c_s$  if  $s = S$  or  $c_s < c_{s+1}$ ,  $s = 1, 2, \dots, S - 1$ ;*

(iii)  *$\theta_s^- = \hat{\theta}_s^+ = \hat{\theta}_s$  if  $k_s \geq \max_{i < S} k_i$ .*

In other words, we have a series of demand threshold pairs, each pair corresponding to the cost of a particular supplier. When demand is below the lower of these two thresholds, equilibrium prices are limited by the cost of the corresponding supplier; when demand is above the upper threshold, equilibrium prices always exceed the cost of that same supplier. A sufficient condition for the two thresholds to be equal is that the capacity of the corresponding supplier is at least as large as that of any more efficient supplier.

To demonstrate that the two thresholds may in fact differ, and hence that there may be a range of demand outcomes for which competitive and non-competitive equilibria coexist, consider the following example. Let  $S = 3$ ,  $c_1 = 0$ ,  $c_2 = 0.5$ ,  $c_3 = 1$ ,  $k_1 = 1$ ,  $k_2 = 1$ , and  $k_3 = 0.25$ . Furthermore, let  $P = 1.75$  and  $\theta = 1.5$ . Then it is easily verified that the following equilibria exist in the uniform auction:  $\{b_1 = 1, b_2 = 0.5, b_3 = 1\}$  and  $\{b_1 = 0, b_2 = 1.75, b_3 = 1\}$ . Note that the first of these equilibria is competitive in the sense that price is limited by the cost of the inefficient supplier, whereas the second equilibrium is not. Note further that the both equilibria are inefficient in the sense that overall generation costs are not minimized: in particular, when the market outcome is competitive, inefficient dispatch nevertheless results.

In the discriminatory auction, no pure-strategy equilibria exists so long as  $\theta > \hat{\theta}_1^-$ . To see this, note that in any equilibrium in which more than one supplier is dispatched, profits of lower-pricing suppliers are strictly increasing in their offer prices below the offer price of the marginal supplier. Furthermore, for the marginal supplier, undercutting is always profitable so long as competing offer prices are sufficiently close. These opposing forces destroy any candidate pure-strategy equilibrium. We consequently have a similar dichotomy to that observed in the duopoly case, in which the comparison of outcomes between the two auction formats generally depends on which equilibrium is played in the uniform auction.

We end this section by considering the relationship between market structure and market performance. We take as our starting point a generalization of the ‘two-state’ result of the duopoly section, which follows as a corollary of the above equilibrium characterization:

**Corollary 1** *There exists  $\hat{\theta}^-$  and  $\hat{\theta}^+$ ,  $\hat{\theta}^- \leq \hat{\theta}^+$ , such that*

(i) *(low demand) if  $\theta \leq \hat{\theta}^-$ , in any equilibrium the highest accepted price offer is at or below  $c$ ;*

(ii) *(high demand) if  $\theta > \hat{\theta}^+$ , in any equilibrium suppliers are paid prices that exceed  $c$ ;*

(iii)  *$\hat{\theta}^- = \hat{\theta}^+ = \hat{\theta}$  if  $k_S \geq \max_{j < S} k_j$ .*



In low-demand realizations prices are limited by costs, whereas in high-demand realizations they are not. Low-demand equilibria are competitive in the sense that prices are limited by the cost of less efficient, non-despatched suppliers. However, unlike in the duopoly case, low-demand equilibria are not necessarily cost efficient. In the uniform auction there may exist pure-strategy equilibria in which less efficient suppliers are ranked before more efficient suppliers, while in the mixed-strategy equilibria of the discriminatory auction such outcomes occur with positive probability.

To highlight the relationship between market concentration and performance, we focus on the symmetric case, in which we readily obtain the following result that corresponds directly with the results obtained in the duopoly case:

**Proposition 5** *In the oligopoly model with symmetric suppliers, in particular,  $k_s = \frac{K}{S}$ ,  $s = 1, 2, \dots, S$ :*

- (i) *(low demand) if  $\theta \leq \hat{\theta} = \frac{S-1}{S}K$ ,  $R^d = R^u = 0$ .*
- (ii) *(high demand) if  $\theta > \hat{\theta} = \frac{S-1}{S}K$ ,  $R^d = PS[\theta - \frac{S-1}{S}K] < P\theta = R^u$ .*

Market structure affects equilibrium outcomes differently in the two auction formats. In both formats, the threshold that determines whether demand is ‘low’ or ‘high’ is increasing in the number of suppliers. In other words, pricing at marginal cost is more likely in a more fragmented industry. However, in the discriminatory auction (as opposed to the uniform auction), market structure also affects bidding strategies in high-demand realizations. In the discriminatory auction suppliers play symmetric mixed strategies, and in equilibrium these strategies strike a balance between a ‘price’ and a ‘quantity’ effect: lowering the price offer reduces the price received, but increases the likelihood of undercutting rivals and hence gaining a larger market share. For a given level of demand, the ‘quantity effect’ is more important the larger is the number of competitors. Hence in the discriminatory auction price competition will be more intense the less concentrated is the market structure.

To illustrate the above points, we again consider the numerical example introduced above, with the specification that  $k_s = \frac{K}{S}$  with  $K = 1$  and  $c_s = 0$ ,  $s = 1, 2, \dots, S$ . Expected payments to suppliers become  $ER^d = \frac{1}{2S}$  and  $ER^u = \frac{2S-1}{2S^2}$ , respectively. Numerical values for different numbers of suppliers are given in the following table:

Table 5: Increasing the Number of Suppliers

$S$	2	3	4	5	10	100	$\infty$
$ER^d$	0.250	0.167	0.125	0.100	0.050	0.005	0
$ER^u$	0.375	0.278	0.219	0.180	0.095	0.010	0
$\frac{ER^d}{ER^u}$	0.667	0.600	0.571	0.556	0.526	0.503	0.5

A more fragmented industry structure improves the performance of both auctions, as well as the relative performance of the discriminatory auction. For a given number of suppliers, the difference in payments between the two auctions roughly corresponds to the effect of doubling the number of suppliers in the uniform auction.

#### 4.4 Long-lived bids

Our final variation on the basic duopoly model considers the case in which suppliers face time-varying, or stochastic, demand. This is of particular relevance to electricity markets in which suppliers submit offer-prices that remain fixed for twenty-four or forty-eight market periods, such as in Australia and the original market in England and Wales. We therefore assume here that price offers must be made before the realization of demand (i.e.  $\theta$ ) is known. It is easy to verify that our previous analysis is robust to this change in the timing of decisions so long as the largest possible demand realization is low enough, or the lowest possible demand realization is large enough. For instance, when demand never exceeds the critical threshold  $\hat{\theta}$  defined in Proposition 1 equilibria correspond to those analyzed for low-demand realizations. The introduction of demand variability adds a new dimension to the problem only when *both* low and high demand realizations occur with positive probability. We therefore assume that demand  $\theta$  takes values in the support  $[\underline{\theta}, \bar{\theta}] \subseteq (0, k_1 + k_2)$ , with  $\underline{\theta} < \hat{\theta} < \bar{\theta}$ , according to some (commonly known) distribution function  $G(\theta)$ .

The equilibria of both the uniform and discriminatory auctions now differ significantly from the case in which demand is known with certainty before bids are submitted. Demand uncertainty, or variability, upsets all candidate pure-strategy equilibria in both types of auction (see von der Fehr and Harbord, 1993 and García-Díaz, 2000). We therefore consider equilibria in mixed strategies. For both the uniform and discriminatory auctions there exist unique mixed-strategy equilibria, and it is possible to derive explicit formulae for the suppliers' strategies:

**Lemma 3** *Assume  $[\underline{\theta}, \bar{\theta}] \subseteq (0, k_1 + k_2)$ , with  $\underline{\theta} < \hat{\theta} < \bar{\theta}$ . Then there does not exist an equilibrium in pure strategies in either auction. In the unique mixed-strategy equilibrium suppliers submit bids that strictly exceed  $c$ .*

In a mixed-strategy equilibrium in either type of auction, suppliers must strike a balance between two opposing effects: on the one hand, a higher offer price tends to result in higher equilibrium prices; on the other hand, pricing high reduces each suppliers' expected output, *ceteris paribus*. The first effect is less pronounced in the uniform auction than in the discriminatory auction. In the uniform auction, a higher offer price translates into a higher market price only in the event that the offer price is marginal, while in the discriminatory auction pricing higher always results in the supplier increasing the expected price it receives, conditional on being despatched. Consequently, there is a tendency for suppliers to price less aggressively in the discriminatory auction compared to a

uniform auction. This intuition is confirmed in the symmetric case (i.e. when  $k_1 = k_2 = k$  and  $c_1 = c_2 = 0$ ), in which the equilibrium mixed-strategy distribution function in the discriminatory auction first-order stochastically dominates the corresponding distribution function in the uniform auction, i.e.  $F_i^u(b) \geq F_i^d(b)$ .<sup>21</sup>

We have not been able to characterize in detail the relationship between the model parameters and suppliers' equilibrium strategies in the general case. In the case of symmetric capacities, however, we can show that in the limit, as  $\bar{\theta} \rightarrow k$  (or  $k \rightarrow \bar{\theta}$ ), so that demand is always less than the capacity of a single supplier, the mixed-strategy equilibrium outcome in either auction approaches the equilibrium outcome for a low-demand realization, with price equal to the marginal cost of the higher-cost supplier. Similarly, as  $\underline{\theta} \rightarrow k$  (or  $k \rightarrow \underline{\theta}$ ), so that demand always exceeds the capacity of a single supplier, the equilibrium outcomes approach those for a high-demand realization. Further, in the uniform auction the limiting equilibrium outcome is efficient, i.e. the more efficient supplier produces at capacity and the less efficient supplier supplies the residual demand. This is in contrast to the model with non-stochastic demand, in which there exist both efficient and inefficient pure-strategy equilibria in high-demand realizations in the uniform auction.<sup>22</sup> This suggests that the uniform auction performs better in efficiency terms than the discriminatory auction, although we have not been able to demonstrate that this result holds generally. Revenue comparisons also prove difficult, except in the symmetric case, where it is easily demonstrated that (in expected terms) total payments to suppliers are the same in both auction formats.

We end this section by comparing market performance under short-lived and long-lived bids, respectively. This comparison is difficult in the general case and hence we limit our attention to the symmetric case. Let  $ER_s^f$  and  $ER_l^f$  denote expected total supplier payments in auction format  $f = d, u$  in the case of short-lived and long-lived bids, respectively. We obtain the following result:

**Proposition 6** *In the symmetric duopoly model,  $ER_l^u < ER_s^u$  and  $ER_l^d = ER_s^d$ .*

In other words, while there is no difference in the discriminatory auction, in the uniform auction long-lived bids outperform short-lived bids. With short-lived bids, the poor performance of the uniform auction is caused by the extreme equilibrium outcome for high-demand realizations, in which suppliers are paid the market reserve price. This equilibrium is supported by the inframarginal supplier bidding sufficiently low so as to discourage undercutting by the high-bidding, price-setting supplier. With long-lived bids, however, the low-bidding supplier determines the market price in low-demand realizations, and hence has an incentive to increase its offer price. As a result, incentives for undercutting and

<sup>21</sup>The result follows from the observation that  $F_i^u(b) < F_i^d(b) \implies \pi_i^u > \pi_i^d$ , whereas in the symmetric case  $\pi_i^u = \pi_i^d$ .

<sup>22</sup>The fact that with uncertain demand the efficient outcome is unique might be viewed as a justification for treating this as a natural 'focal point' in the certain-demand case also.

competing for market share are increased, leading to more aggressive bidding and lower prices overall in the uniform auction.

## 5 Conclusions

In this paper we have characterized equilibrium pricing behavior in uniform and discriminatory auctions in a multi-unit auction model reflecting some key features of decentralized electricity markets. Equilibria in the two auction formats have been compared in terms of both average prices paid to suppliers and productive efficiency. In the case of certain demand (i.e. short-lived bids), we found that uniform auctions yield higher average prices than discriminatory auctions. Comparison of the auctions in terms of productive efficiency is more complex, however, as it depends on which equilibrium is played in the uniform price auction as well as on parameter values. When demand is uncertain (or bids are long-lived), at least in the perfectly symmetric case, expected payments to suppliers are the same in both auction formats.

Our theoretical model is obviously highly stylized, and while it does lead to a number of qualitative results, it does not allow us to draw conclusions about their quantitative importance. Nevertheless, numerical examples suggest that some of the effects identified may be significant. For example, moving from a uniform to a discriminatory auction format in the certain demand case may have a similar effect on average prices to either doubling the number of suppliers or increasing the capacity of two symmetric duopolists by almost 40%. Without overstating the importance of these findings, they suggest that the new market rules may have been responsible for at least part of the initial reduction in England and Wales wholesale electricity prices in 2001/2. The effects of these changes on productive efficiency remains a matter for speculation, however.<sup>23</sup>

A key determinant of market performance in our analysis is the relative incidence of low-demand and high-demand states, and this does not depend upon the auction format. Rather, it depends on other market design issues and on structural features of the market. In particular, the incidence of high-demand states is lower when there is more excess capacity in the industry, the market structure is more fragmented, suppliers have symmetric capacities, demand is price elastic and the market reserve price is low. These factors affect not only the relative incidence of low and high-demand states, but may also influence bidding strategies. Changes in total capacity, the capacity distribution and market structure (i.e. ‘structural factors’) have no effect on prices in the uniform auction *in high-demand states*, but can lead to more vigorous price competition in the discriminatory auction. Regulatory interventions to change the market rules, on the other hand, affect bidding strategies in both types of auction. A reduction in the market reserve price reduces average market prices in both auctions. Measures that increase the elasticity of demand (e.g.

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<sup>23</sup>See Kahn et al (2001), for instance, who argue against adopting discriminatory auctions on the grounds that they are likely to result in increased inefficiency.

the introduction of demand-side bidding) have similar effects. A change from short-lived to long-lived bids, however, which makes the demand state uncertain when suppliers' submit their bids, may have a greater effect on prices in the uniform auction.

From a methodological point of view, the paper has also contributed to the analysis of multi-unit electricity auctions in a number of ways.<sup>24</sup> First, we have shown that the set of equilibrium outcomes in uniform and discriminatory auctions is essentially independent of the number of admissible steps in suppliers' offer-price functions, so as long as this number is finite. This reduces the complexity involved in the analysis of multi-unit auctions as it allows us to focus on the single-unit case with no significant loss in generality. Secondly, we have demonstrated that the 'implicitly collusive' equilibria found in the uniform auction when offer prices are infinitely divisible are unique to this formulation of the auction (i.e. to share auctions), and do not arise when offer-price functions are discrete. Hence the concerns expressed in the literature that uniform auctions may lead to 'collusive-like' outcomes even in potentially competitive periods when there is considerable excess capacity, are likely misplaced.<sup>25</sup> Though we cannot conclude that simplifying the bidding format will typically improve market performance in electricity auctions, it appears that there may be little to lose from adopting such a measure.

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<sup>24</sup>See Fabra, von der Fehr and Harbord (2002) for a nontechnical discussion.

<sup>25</sup>This point has recently been made independently by Kremer and Nyborg (2003).

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## Appendix

### A Derivations of Results

#### Proof of Lemma 1

Let  $p$  denote the highest accepted price offer and let  $b_i = p$ . Clearly, we must have  $p \geq c_i$ . Let  $c_p = \max_{c_j \leq p} c_j$  and  $c^p = \{\min_{c_j > p} c_j \text{ if } p < c; \text{ and } P \text{ otherwise}\}$ . Suppose  $p > c_p$ . Then, for  $j \neq i$  with  $c_j < p$ , we must have  $b_j \leq p$  (with strict inequality if  $c_j = c_i$ ) since otherwise supplier  $j$  could gain by matching (undercutting)  $b_i$ . But then  $i$ 's profit is strictly increasing in  $b_i$  on  $[p, c^p]$ , proving the first part of the result. Lastly, in the discriminatory auction, in a pure-strategy equilibrium we cannot have  $b_j < p$ , given that supplier  $j$ 's profit is strictly increasing in  $b_j$  up to  $p$ .



### Proof of Proposition 1

Consider first the possibility of a pure-strategy equilibrium in which the highest accepted offer price equals  $c$ . Profits to Supplier  $i$  are given by  $[c - c_i] \min \{\theta - K_{i-1}, k_i\}$ , where  $K_i = \sum_{j=1}^i k_j$ ,  $i = 1, 2$  and  $K_0 = 0$ , while the profits from deviating to a higher price is at most  $[P - c_i] \max \{\theta - K_{-i}, 0\}$ , where  $K_{-i} = \sum_{j \neq i} k_j$ . A necessary (and, indeed, sufficient) condition for such an equilibrium to exist consequently is  $[c - c_i] \min \{\theta - K_{i-1}, k_i\} - [P - c_i] \max \{\theta - K_{-i}, 0\} \geq 0$ . Given that, for  $\theta \geq K_{-i}$ , the left-hand side of this expression is non-increasing in  $\theta$ , there exists a unique  $\hat{\theta}_i$  such that the condition is satisfied iff  $\theta \leq \hat{\theta}_i$ . Existence of the equilibrium then requires  $\theta \leq \min \hat{\theta}_i \equiv \hat{\theta}$ .

Consider next the possibility of an equilibrium in which supplier  $i$  submits the highest accepted price offer  $b_i = P$ . Clearly, for such an equilibrium to exist we must have  $\theta - K_{-i} > 0$ . By the argument in the proof of Lemma 1, it follows that  $i$ 's equilibrium profits are  $[P - c_i] [\theta - K_{-i}]$ . Obviously, any profitable deviation by  $i$  would involve undercutting the competitor so as to increase output (with a consequent fall in price). If the competitor prices at cost, the maximum gain from undercutting is given by  $[c_j - c_i] \min \{\theta - K_{i-1}, k_i\}$  when  $\theta \in (K_{j-1}, K_j]$ . Consequently, a necessary condition for such an equilibrium to exist is that  $[P - c_i] [\theta - K_{-i}] - [c_j - c_i] \min \{\theta - K_{i-1}, k_i\} \geq 0$ . By the monotonicity of the left-hand side of the condition, it follows that the condition is satisfied iff  $\theta \geq \hat{\theta}_i$ , implying that a monopolistic pure-strategy equilibrium can exist only if  $\theta \geq \hat{\theta}$ .

The existence of a monopolistic pure-strategy equilibrium in the uniform auction when  $\theta \geq \hat{\theta}_i$  for some  $i$  is straightforward and involves Supplier  $i$  pricing at  $P$  while the competitor prices sufficiently low so as to make undercutting by  $i$  unprofitable. In the discriminatory auction, by the result in Lemma 1 that in a pure-strategy equilibrium all accepted units are offered at the same price, it follows that there cannot exist an equilibrium in which accepted price offers exceed  $c$ , since then at least one supplier could increase output by (marginally) undercutting its competitor. When  $\theta \geq \hat{\theta}_i$ , Supplier  $i$ 's rival knows that a price offer of  $c$  being undercut is a probability-zero event, and hence will surely price above  $c$  also.

For further reference, we register the following results. Noting that we must have  $\hat{\theta}_1 \geq k_2$ ,  $\hat{\theta}_1$  is implicitly defined by the equation  $c \min \{\hat{\theta}_1, k_1\} = P [\hat{\theta}_1 - k_2]$ . It follows that  $\hat{\theta}_1 = \frac{P}{P-c} k_2$  if  $\hat{\theta}_1 \leq k_1$  and  $\hat{\theta}_1 = k_2 + \frac{c}{P} k_1$  if  $\hat{\theta}_1 > k_1$ . This may alternatively be stated as  $\hat{\theta}_1 = \frac{P}{P-c} k_2$  if  $\frac{P}{P-c} k_2 \leq k_1$  and  $\hat{\theta}_1 = k_2 + \frac{c}{P} k_1$  otherwise. Similar reasoning leads to the result that  $\hat{\theta}_2 = k_1$ . Consequently,  $\hat{\theta} = \frac{P}{P-c} k_2$  if  $\frac{P}{P-c} k_2 \leq k_1$  and  $\hat{\theta} = k_1$  otherwise.

### Mixed-strategy equilibria in the basic model

In this section we characterize the mixed-strategy equilibria of the uniform auction for the case in which there are multiple pure-strategy equilibria (i.e., for demand realizations  $\theta \geq \max \{\hat{\theta}_1, \hat{\theta}_2\}$ ), as well as for the corresponding discriminatory auction for high-demand realizations (i.e.,  $\theta > \hat{\theta} = \min \{\hat{\theta}_1, \hat{\theta}_2\}$ ).



### Uniform auction

Assume  $\theta \geq \max\{\widehat{\theta}_1, \widehat{\theta}_2\} = \max\{k_1, k_2 + \frac{c}{P}k_1\}$ . Let  $F_i^u(b) = \Pr\{b_i \leq b\}$  denote the equilibrium mixed-strategy of supplier  $i$ ,  $i = 1, 2$ , with density  $f_i^u(b) = F_i^u(b)$ , and let  $S_i^u$  be the support of  $F_i^u$ . Furthermore, let  $S^u = (\max\{\inf S_1^u, \inf S_2^u\}, \min\{\sup S_1^u, \sup S_2^u\})$ . Note first that  $F_i^u$  cannot have a mass point on  $S^u$ . To see this, suppose, for contradiction, that  $F_i^u$  has a mass point at some  $b' \in S^u$ . Then, for some interval  $[b', b' + \epsilon)$ ,  $\epsilon > 0$ ,  $i$ 's competitor would be better off by offering to supply at a price just below  $b'$  than to offer prices in this interval. But then  $i$ 's profit would be strictly increasing on  $[b', b' + \epsilon)$ , contradicting the assumption that  $b'$  is in the support of  $i$ 's strategy. A similar argument establishes that  $S_i^u$  is an interval (i.e. without 'holes'). Furthermore, since  $P$  must be in the support of at least one supplier's strategy, we have  $S^u = S_1^u \cap S_2^u = (\underline{b}, P)$ . We want to demonstrate that any mixed-strategy equilibrium has the form

$$\begin{aligned} F_1^u(b) &= \begin{cases} A_1 \left[ \frac{b-c}{P-c} \right]^{\frac{\theta-k_2}{k_1+k_2-\theta}} & \text{for } \underline{b} < b < P \\ 1 & \text{for } b = P \end{cases} \\ F_2^u(b) &= \begin{cases} A_2 \left[ \frac{b}{P} \right]^{\frac{\theta-k_1}{k_1+k_2-\theta}} & \text{for } \underline{b} < b < P \\ 1 & \text{for } b = P \end{cases} \\ \underline{b} &= c \end{aligned}$$

where either (i)  $A_1 = 1$  and  $0 < A_2 \leq 1$  or (ii)  $0 < A_1 \leq 1$  and  $A_2 = 1$ .

On  $(\underline{b}, P)$ , strategies must satisfy the following differential equations:

$$\begin{aligned} F_2^u(b) [\theta - k_2] - f_2^u(b) b [k_1 + k_2 - \theta] &= 0, \\ F_1^u(b) [\theta - k_1] - f_1^u(b) [b - c] [k_1 + k_2 - \theta] &= 0. \end{aligned}$$

On the interior of the support of the mixed strategies the net gain from raising the bid marginally must be zero. The first elements on the left-hand side of the above expressions represents the gain to a supplier from the resulting increase in the price received in the event that the rival bids below. The second element represents the loss from reducing the chance of being despatched at full capacity instead of serving the residual demand only (the difference being, for supplier  $i$ ,  $k_i - [\theta - k_j] = k_1 + k_2 - \theta$ ). The above expressions may alternatively be written:

$$\begin{aligned} f_2^u(b) - \frac{1}{b} \frac{\theta - k_2}{k_1 + k_2 - \theta} F_2^u(b) &= 0, \\ f_1^u(b) - \frac{1}{b - c} \frac{\theta - k_1}{k_1 + k_2 - \theta} F_1^u(b) &= 0, \end{aligned}$$

and have solutions

$$\begin{aligned} F_1^u(b) &= \widehat{A}_1 [b - c]^{\frac{\theta-k_2}{k_1+k_2-\theta}} \\ F_2^u(b) &= \widehat{A}_2 b^{\frac{\theta-k_1}{k_1+k_2-\theta}}, \end{aligned}$$

with  $\widehat{A}_i > 0$ ,  $i = 1, 2$ .

Since at most one supplier can play  $P$  with positive probability (i.e., either  $\Pr(b_1 = P) = 0$  or  $\Pr(b_2 = P) = 0$ ), we have either (i)  $\lim_{b \rightarrow P} F_2^u(b) \leq \lim_{b \rightarrow P} F_1^u(b) = 1$ , implying  $\widehat{A}_1 = \left[ \frac{1}{P-c} \right]^{\frac{\theta-k_2}{k_1+k_2-\theta}}$  and  $\widehat{A}_2 \leq \left[ \frac{1}{P} \right]^{\frac{\theta-k_1}{k_1+k_2-\theta}}$  or (ii)  $\lim_{b \rightarrow P} F_1^u(b) \leq \lim_{b \rightarrow P} F_2^u(b) = 1$ , implying  $\widehat{A}_1 \leq \left[ \frac{1}{P-c} \right]^{\frac{\theta-k_2}{k_1+k_2-\theta}}$  and  $\widehat{A}_2 = \left[ \frac{1}{P} \right]^{\frac{\theta-k_1}{k_1+k_2-\theta}}$ .

Note that, because there are no mass points on  $(\underline{b}, P)$  and  $\lim_{b \rightarrow c} F_1^u(b) = 0$ , we must have  $\underline{b} = c$ . Since  $\lim_{b \rightarrow c} F_2^u(b) = \widehat{A}_2 c^{\frac{\theta-k_1}{k_1+k_2-\theta}} > 0$ , while  $F_2^u$  cannot have a mass point at  $c$ , it follows that for a mixed-strategy equilibrium to exist it must involve, with positive probability, Supplier 2 offering to supply at prices below his own cost (note that this implies that there does not exist a mixed-strategy equilibrium in weakly undominated strategies). The only constraint that  $F_2(b)$  must satisfy for  $b \leq c$  follows from the condition that undercutting by Supplier 1 must be unprofitable; one solution satisfying this constraint is given by the above first-order condition, but a continuum of other solutions exist as well.

In a mixed-strategy equilibrium profits become:

$$\begin{aligned}\pi_1^u &= P \{ \Pr(b_2 = P) k_1 + [1 - \Pr(b_2 = P)] [\theta - k_2] \}, \\ \pi_2^u &= [P - c] \{ \Pr(b_1 = P) k_2 + [1 - \Pr(b_1 = P)] [\theta - k_1] \}.\end{aligned}$$

Note that, for the class of equilibria in which  $\lim_{b \rightarrow P} F_1^u(b) = 1$ , total industry profits are maximized in the limiting case  $\Pr(b_2 = P) = 1$  (which corresponds to  $A_1 = 1$  and  $A_2 = 0$ ), in which case we have  $\pi_1^u = P k_1$  and  $\pi_2^u = [P - c] [\theta - k_1]$ . Note that this is the same as in the corresponding pure-strategy equilibrium in which Supplier 2 is bidding high, implying that profits in this pure-strategy equilibrium dominate those in any mixed-strategy equilibrium. A corresponding result holds for the other class of mixed-strategy equilibria.

### Discriminatory auction

Assume  $\theta > \widehat{\theta}$ . From the proof of Proposition 1, there are two cases to consider;  $\frac{P}{P-c} k_2 \leq k_1$ , in which case  $\widehat{\theta} = \frac{P}{P-c} k_2$ , and  $\frac{P}{P-c} k_2 > k_1$ , in which case,  $\widehat{\theta} = k_1$ .

Let  $F_i^d(b) = \Pr\{b_i \leq b\}$  denote the equilibrium mixed strategy of supplier  $i$  and let  $S_i^d$  be the support of  $F_i^d$ . Standard arguments (see above) imply that  $S = (\underline{b}, P) \subseteq S_1^d$ ,  $S_2^d \subseteq [\underline{b}, P]$  and that  $F_i^d$  and  $F_j^d$  do not have mass points on  $[\underline{b}, P)$ . We want to show that there exists a unique equilibrium with,

$$\begin{aligned}F_1^d(b) &= \begin{cases} \frac{\min\{\theta, k_2\}}{\min\{\theta, k_1\} + \min\{\theta, k_2\} - \theta} \frac{b-b}{b-c} & \text{for } b < P \\ 1 & \text{for } b = P \end{cases}, \\ F_2^d(b) &= \begin{cases} \frac{\min\{\theta, k_1\}}{\min\{\theta, k_1\} + \min\{\theta, k_2\} - \theta} \frac{b-b}{b} & \text{for } b < P \\ 1 & \text{for } b = P \end{cases},\end{aligned}$$

where  $\underline{b} = c + [P - c] \frac{\theta-k_1}{\min\{\theta, k_2\}}$  if  $P k_2 > [P - c] k_1$  and  $\underline{b} = P \frac{\theta-k_2}{\min\{\theta, k_1\}}$  if  $P k_2 \leq [P - c] k_1$  (note that, in both cases,  $\underline{b} \geq c$ ).

Suppliers' profits may be written

$$\begin{aligned}\pi_1^d(b) &= b \left\{ F_2^d(b) \max \{ \theta - k_2, 0 \} + [1 - F_2^d(b)] \min \{ \theta, k_1 \} \right\}, \\ \pi_2^d(b) &= [b - c] \left\{ F_1^d(b) \max \{ \theta - k_1, 0 \} + [1 - F_1^d(b)] \min \{ \theta, k_2 \} \right\}.\end{aligned}$$

A necessary condition for supplier  $i$  to be indifferent between any price in  $S_i^d$  is that, for all  $b \in S_i^d$ ,  $\pi_i^d(b) = \bar{\pi}_i^d$ , implying

$$\begin{aligned}F_1^d(b) &= \frac{[b - c] \min \{ \theta, k_2 \} - \bar{\pi}_2^d}{[b - c] [\min \{ \theta, k_1 \} + \min \{ \theta, k_2 \} - \theta]}, \\ F_2^d(b) &= \frac{b \min \{ \theta, k_1 \} - \bar{\pi}_1^d}{b [\min \{ \theta, k_1 \} + \min \{ \theta, k_2 \} - \theta]},\end{aligned}$$

where we have used the fact that  $\max \{ \theta - k_i, 0 \} = \theta - \min \{ \theta, k_i \}$ .

Observe that the boundary condition  $F_1^d(\underline{b}) = F_2^d(\underline{b}) = 0$  implies

$$\begin{aligned}\bar{\pi}_1^d &= \underline{b} \min \{ \theta, k_1 \}, \\ \bar{\pi}_2^d &= [\underline{b} - c] \min \{ \theta, k_2 \}.\end{aligned}$$

Furthermore, we have

$$\lim_{b \rightarrow P} [F_1^d(b) - F_2^d(b)] = \frac{P - \underline{b}}{\min \{ \theta, k_1 \} + \min \{ \theta, k_2 \} - \theta} \left[ \frac{\min \{ \theta, k_2 \}}{P - c} - \frac{\min \{ \theta, k_1 \}}{P} \right].$$

If  $k_1 < \frac{P}{P-c}k_2$ , in which case  $\theta > k_1$ , we cannot have  $\lim_{b \rightarrow P} F_2^d(b) = 1$  since this would imply  $\lim_{b \rightarrow P} F_1^d(b) > 1$ . Consequently, we have the boundary condition  $\lim_{b \rightarrow P} F_1^d(P) = 1$ , which implies

$$\bar{\pi}_2^d = [P - c] [\theta - k_1],$$

and, together with the condition  $F_1^d(\underline{b}) = 0$ ,

$$\underline{b} = c + [P - c] \frac{\theta - k_1}{\min \{ \theta, k_2 \}} \geq c.$$

If, on the other hand,  $k_1 > \frac{P}{P-c}k_2$ , in which case  $\theta > \frac{P}{P-c}k_2$ , we have the boundary condition  $\lim_{b \rightarrow P} F_2^d(P) = 1$ , which implies

$$\bar{\pi}_1^d = P [\theta - k_2],$$

and, together with the condition  $F_2^d(\underline{b}) = 0$ ,

$$\underline{b} = P \frac{\theta - k_2}{\min \{ \theta, k_1 \}} \geq c.$$

Note that, in both cases,  $\underline{b} \rightarrow c$  as  $\theta \rightarrow \hat{\theta}$ , and so, in the limit,  $\pi_1 = c [\hat{\theta} - k_2]$  and  $\pi_2 = 0$ .

In the case  $k_1 < \frac{P}{P-c}k_2$  (similar results are obtained in the alternative case), equilibrium profits, expected costs and expected revenues may be written:

$$\pi_1^d = ck_1 + [P - c][\theta - k_1] \frac{k_1}{\min\{\theta, k_2\}} \text{ and } \pi_2^d = [P - c][\theta - k_1]$$

$$\begin{aligned} EC^d &= \Pr\{b_1 \leq b_2\} c[\theta - k_1] + \Pr\{b_1 > b_2\} c \min\{\theta, k_2\} \\ ER^d &= \pi_1^d + \pi_2^d + EC^d \end{aligned}$$

where

$$\Pr\{b_1 \leq b_2\} = \int_{\underline{b}}^P F_1^d(b) dF_2^d(b) + 1 - \frac{k_1}{k_1 + \min\{\theta, k_2\} - \theta} \frac{P - \underline{b}}{P}$$

With some algebra,

$$\int_{\underline{b}}^P F_1^d(b) dF_2^d(b) = \frac{k_1 \min\{\theta, k_2\}}{[k_1 + \min\{\theta, k_2\} - \theta]^2} \frac{\underline{b}}{c} \left[ \frac{P - \underline{b}}{P} - \frac{\underline{b} - c}{c} \ln \left( \frac{P - c \underline{b}}{\underline{b} - c P} \right) \right]$$

In the limit,

$$\begin{aligned} \lim_{c \rightarrow 0} \Pr\{b_1 \leq b_2\} &= 1 - \frac{1}{2} \frac{k_1}{\min\{\theta, k_2\}} \geq \frac{1}{2}, \\ \lim_{c \rightarrow P} \Pr\{b_1 \leq b_2\} &= 1, \end{aligned}$$

and hence

$$\begin{aligned} \frac{1}{2} &\leq \Pr\{b_1 \leq b_2\} \leq 1, \\ c[\theta - k_1] &\leq EC^d \leq \frac{c \min\{\theta, k_2\} + c[\theta - k_1]}{2}, \\ \pi_1^d + \pi_2^d + c[\theta - k_1] &\leq ER^d \leq \pi_1^d + \pi_2^d + \frac{c \min\{\theta, k_2\} + c[\theta - k_1]}{2}. \end{aligned}$$

Furthermore, we know that we cannot have  $ER^d = P\theta$ , since this would require both suppliers playing  $P$  with positive probability. Thus,  $ER^d < P\theta$ .

## Proof of Lemma 2

Verifying that the arguments of Lemma 1 and Proposition 2 go through with multiple bids is straightforward. Below we want to demonstrate that, in the discriminatory auction, the best response to a rival offering all of his capacity at the same price according to an equilibrium distribution function is to bid a flat bid function also. Under the assumption that  $b_{jn} = b_j$ ,  $n = 1, \dots, N_j$ , with  $b_j$  chosen according to the distribution function  $F_j$ , supplier  $i$ 's expected profits may be written

$$\begin{aligned} \pi_i(\mathbf{b}_i) &= \sum_{n=1}^{N_{i1}} [b_{in} - c_i] \left\{ F_j(b_{in}) \min \left\{ k_{in}, \max \left\{ \theta - k_j - \sum_{m=1}^{n-1} k_{im}, 0 \right\} \right\} \right\} \\ &\quad + [1 - F_j(b_{in})] k_{in}, \end{aligned}$$

where we have defined  $\sum_{m=1}^0 k_{im} \equiv 0$ . Suppose  $\mathbf{b}_i$  is set optimally, that  $N_i > 1$  and that  $b_{in} < b_{in+1}$  for some  $n = 1, 2, \dots, N_i - 1$  (i.e., there is at least two steps in  $i$ 's bid function). We want to show that this leads to a contradiction. Consider first the case that  $\theta > k_j$  and let  $\hat{n}$  be chosen such that  $0 < \theta - k_j - \sum_{m=1}^{\hat{n}-1} k_{im} < k_{i\hat{n}}$ . Clearly such an  $\hat{n}$  exists and is unique. Note that we have  $\theta - k_j - \sum_{m=1}^{n-1} k_{im} > k_{in}$  for  $n < \hat{n}$  and  $\theta - k_j - \sum_{m=1}^{n-1} k_{im} < 0$  for  $n > \hat{n}$ . Supplier  $i$ 's profit can then be rewritten as,

$$\begin{aligned} \pi_i(\mathbf{b}_i) &= \sum_{n=1}^{\hat{n}-1} [b_{in} - c_i] k_{in} \\ &\quad + [b_{i\hat{n}} - c_i] \left\{ F_j(b_{i\hat{n}}) \left[ \theta - k_j - \sum_{n=1}^{\hat{n}-1} k_{in} \right] + [1 - F_j(b_{i\hat{n}})] k_{i\hat{n}} \right\} \\ &\quad + \sum_{n=\hat{n}+1}^{N_i} [b_{in} - c_i] [1 - F_j(b_{in})] k_{in} \\ &= [b_{i\hat{n}} - c_i] \{ F_j(b_{i\hat{n}}) [\theta - k_j] + [1 - F_j(b_{i\hat{n}})] k_i \} \\ &\quad + \sum_{n=1}^{\hat{n}-1} [b_{in} - b_{i\hat{n}}] k_{in} \\ &\quad + \sum_{n=\hat{n}+1}^{N_i} \{ [b_{in} - c_i] [1 - F_j(b_{in})] - [b_{i\hat{n}} - c_i] [1 - F_j(b_{i\hat{n}})] \} k_{in}. \end{aligned}$$

The first term in the last expression equals the profit Supplier  $i$  would obtain if all of his units were bid in at the same price  $b_{i\hat{n}}$ . The second term is clearly negative: it is always profitable to increase offer prices on units that will be despatched with probability 1. The last term is negative also. To see this, note that if  $F_j$  is the mixed-strategy corresponding to an equilibrium in which supplier  $i$  offer all units at the same price, it must satisfy

$$\begin{aligned} \pi_i(b_i) &= [b_i - c_i] \{ F_j(b_i) \min \{ k_i, \max \{ \theta - k_j, 0 \} \} \\ &\quad + [1 - F_j(b_i)] \min \{ k_i, \theta \} \} = \bar{\pi}_i, \end{aligned}$$

where  $\bar{\pi}_i$  is some constant. Consider two offer prices  $\hat{b} > \tilde{b}$  on the support of  $F_j$ . Then

$$\begin{aligned} 0 &= [\hat{b} - c_i] \{ F_j(\hat{b}) \min \{ k_i, \max \{ \theta - k_j, 0 \} \} + [1 - F_j(\hat{b})] \min \{ k_i, \theta \} \} \\ &\quad - [\tilde{b} - c_i] \{ F_j(\tilde{b}) \min \{ k_i, \max \{ \theta - k_j, 0 \} \} + [1 - F_j(\tilde{b})] \min \{ k_i, \theta \} \} \\ &= \{ [\hat{b} - c_i] F_j(\hat{b}) - [\tilde{b} - c_i] F_j(\tilde{b}) \} \min \{ k_i, \max \{ \theta - k_j, 0 \} \} \\ &\quad + \{ [\hat{b} - c_i] [1 - F_j(\hat{b})] - [\tilde{b} - c_i] [1 - F_j(\tilde{b})] \} \min \{ k_i, \theta \} \\ &\geq \{ [\hat{b} - c_i] [1 - F_j(\hat{b})] - [\tilde{b} - c_i] [1 - F_j(\tilde{b})] \} \min \{ k_i, \theta \}, \end{aligned}$$

where the inequality follows from the observation that  $[b - c_i] F_j(b)$  is increasing in  $b$  (the inequality is strict if  $\theta > k_j$ ). In the case that  $\theta \leq k_j$ , supplier  $i$ 's profits simplify to

$$\pi_i(\mathbf{b}_i) = \sum_{n=1}^{N_i} [b_{in} - c_i] [1 - F_j(b_{in})] k_{in},$$

and so we can apply a similar argument to the one immediately above to demonstrate that profits are maximized for  $b_{i1} = b_{i2} = \dots = b_{iN_i} = b_i$ . We conclude that for supplier  $i$  to offer all capacity at a single price is a best response to  $F_j$ .

### Proof of Proposition 4

Let  $K_s = \sum_{i=1}^s k_i$  be the accumulated capacity of the  $s$  most efficient suppliers and  $K_s^{-i} = K_s - k_i$ ,  $i \leq s$ , the accumulated capacity of the  $s$  most efficient suppliers not including supplier  $i$ . Note first that accepted price offers cannot exceed  $c_s$  if  $\theta \leq \min_{i \leq s} \{K_s^{-i}\}$ . To see this, suppose that the highest accepted price offer were indeed  $b > c_s$ . Since at most one supplier will offer  $b$  with positive probability, all other suppliers  $i \neq s$ ,  $c_i < b$ , will price below  $b$ . But then, since  $\theta < \min_{i \leq s} \{K_s^{-i}\}$  a price offer of  $b$  will never be accepted. It follows that  $\min_{i \leq s} \{K_s^{-i}\}$  is a lower bound for  $\hat{\theta}_s^-$ .

Consider next events in which  $\theta \geq K_{s-1}$ . Then, since supplier  $s$  never price below  $c_s$ , any supplier  $i < s$  who offers  $b_i < c_s$  will be accepted with probability 1 and despatched at full capacity. It follows that there cannot exist an equilibrium in which some supplier accepts to be paid a price below  $c_s$ . Furthermore, if  $c_s < c_{s+1}$ , or  $s = S$  (so  $\theta \geq K_{S-1}$ ), supplier  $s$  will price above  $c_s$  with probability 1 and hence suppliers  $i < s$  will not accept to be paid prices equal to  $c_s$  either. Consequently,  $K_{s-1}$  is an upper bound for  $\hat{\theta}_s^+$ .

Lastly, we observe that  $\min_{i \leq s} \{K_s^{-i}\} = K_{s-1}$  if  $k_s = \max_{i \leq s} k_i$  (or  $k_s \geq \max_{i < s} k_i$ ), in which case we must have  $\hat{\theta}_s^- = \hat{\theta}_s^+$ .

### Proof of Lemma 3

We start by showing that a pure-strategy equilibrium does not exist in either auction format. To see this, note first that in a pure-strategy equilibrium all effective offer prices (i.e., offers that with positive probability affect the prices suppliers are paid) must be equal; if not, some supplier could profitably increase his offer price towards the next higher bid, thereby increasing profits in the event that this offer is effective without reducing output in any event. Next, observe that this common price cannot exceed  $c$ ; if it did, some supplier could profitably deviate to a slightly lower price, thereby increasing the expected quantity despatched with only a negligible effect on the expected price. Lastly, bidding at  $c$  cannot constitute an equilibrium either, since the supplier with costs equal to  $c$  could obtain positive profits in the event that demand exceeds the capacity of his rival by raising his offer price.

We next characterize the unique equilibrium for each auction format.

### Uniform auction

Let  $F_i^u(b) = \Pr \{b_i \leq b\}$  denote the equilibrium mixed-strategy of supplier  $i$ ,  $i = 1, 2$ , in the uniform auction, with  $f_i^u(b) = F_i^{u'}(b)$ , and let  $S_i^u$  be the support of  $F_i^u$ . Standard

arguments imply that  $S_1^u \cap S_2^u = [\underline{b}^u, P)$ ,  $\underline{b}^u \geq c$ , and that  $F_1^u$  and  $F_2^u$  do not have mass points on  $(\underline{b}^u, P)$ .

We focus on the case in which  $\underline{\theta} < \min\{k_1, k_2\} \leq \max\{k_1, k_2\} < \bar{\theta}$ . Supplier  $i$ 's profit, when bidding  $b$ , may then be written

$$\begin{aligned} \pi_i^u(b) = & F_j^u(b) \int_{k_j}^{\bar{\theta}} [b - c_i] [\theta - k_j] dG(\theta) \\ & + \int_b^P \left[ \int_{\underline{\theta}}^{k_i} [b - c_i] \theta dG(\theta) + \int_{k_i}^{\bar{\theta}} [v - c_i] k_i dG(\theta) \right] dF_j^u(v). \end{aligned}$$

The first term on the right-hand side represents supplier  $i$ 's profits in the event that the rival bids below  $b$ , in which case supplier  $i$  produces a positive quantity only when demand is above the capacity of the rival. The second term represents supplier  $i$ 's profits in the event that the rival bids above  $b$ . As given by the first element of this term, supplier  $i$  will then serve all demand at his own price when his capacity is sufficient to satisfy all of demand. On the other hand, and as given by the second element, supplier  $i$  will produce at full capacity and receive a price determined by the rival's bid in the event that demand exceeds his capacity.

On  $(\underline{b}^u, P)$ , strategies must satisfy the following differential equations:

$$\begin{aligned} & F_j^u(b) \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta) + [1 - F_j^u(b)] \int_{\underline{\theta}}^{k_i} \theta dG(\theta) \\ & - [b - c_i] f_j^u(b) \left\{ \int_{\underline{\theta}}^{k_i} \theta dG(\theta) + \int_{k_i}^{\bar{\theta}} k_i dG(\theta) - \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta) \right\} = 0 \end{aligned}$$

On the interior of the support of the mixed strategies the net gain from raising the bid marginally must be zero. The first element on the left-hand side represents the gain to a supplier from the resulting increase in the price received in the event that demand exceeds the capacity of the rival and the rival bids below. The second element represents the gain to a supplier from the resulting increase in the price in the event that demand is lower than his capacity and the rival bids above. Lastly, the third term represents the loss from being dispatched with a smaller output: in case demand falls below the supplier's capacity the loss of output equals total demand; in case demand exceeds the supplier's capacity the loss equals the difference between being dispatched at full capacity and serving residual demand only (i.e.,  $k_i - [\theta - k_j]$ ). The above expressions may alternatively be written

$$f_j^u(b) - \frac{\lambda_j}{b - c_i} F_j^u(b) = \frac{\beta_j}{b - c_i},$$

where

$$\begin{aligned} \lambda_j &= \frac{\int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta) - \int_{\underline{\theta}}^{k_i} \theta dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta) - \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta)} \\ \beta_j &= \frac{\int_{\underline{\theta}}^{k_i} \theta dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta) - \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta)} \end{aligned}$$

which have solutions

$$F_j^u(b) = \begin{cases} \beta_j \ln(b - c_i) + \Omega_1^j & \text{for } \lambda_j = 0 \\ \Omega_2^j [b - c_i]^{\lambda_j} - \frac{\beta_j}{\lambda_j} & \text{for } \lambda_j \neq 0 \end{cases}$$

where  $\Omega_1^j, \Omega_2^j$ ,  $j = 1, 2$ , are constants of integration. Note that, if  $k_i \leq k_j$ ,  $\beta_i \geq \beta_j$ . Furthermore,  $\beta_1 = \beta_2$  and  $\lambda_1 = \lambda_2$  when  $k_1 = k_2$ . Also, if  $k_i \leq k_j$ ,  $\beta_j \rightarrow 0$  as  $\underline{\theta} \uparrow k_i$  while  $\beta_j + \lambda_j \rightarrow 0$  as  $\bar{\theta} \downarrow k_j$ .

Given the boundary condition  $F_j^u(\underline{b}^u) = 0$ , these equations yield the mixed-strategy distribution functions for  $b \in [\underline{b}^u, P)$ :

$$F_j^u(b) = \begin{cases} \beta_j \ln\left(\frac{b - c_i}{\underline{b}^u - c_i}\right) & \text{for } \lambda_j = 0, \\ \frac{\beta_j}{\lambda_j} \left\{ \left[ \frac{b - c_i}{\underline{b}^u - c_i} \right]^{\lambda_j} - 1 \right\} & \text{for } \lambda_j \neq 0. \end{cases}$$

Suppose  $\lim_{b \uparrow P} F_2^u(b) \leq \lim_{b \uparrow P} F_1^u(b) = 1$  (in the opposite case, i.e. when  $\lim_{b \uparrow P} F_1^u(b) \leq \lim_{b \uparrow P} F_2^u(b) = 1$ , a corresponding argument can be applied). Then it is straightforward to verify that  $\underline{b}^u$  is given uniquely as

$$\underline{b}^u = \begin{cases} c_2 + [P - c_2] e^{-\frac{1}{\beta_1}} & \text{for } \lambda_1 = 0, \\ c_2 + [P - c_2] \left[ \frac{\beta_1}{\lambda_1 + \beta_1} \right]^{\frac{1}{\lambda_1}} & \text{for } \lambda_1 \neq 0. \end{cases}$$

Substituting for  $\underline{b}^u$ , we find

$$F_1^u(b) = \begin{cases} 1 + \beta_1 \ln\left(\frac{b - c_2}{P - c_2}\right) & \text{for } \lambda_1 = 0, \\ \frac{\beta_1}{\lambda_1} \left\{ \frac{\lambda_1 + \beta_1}{\beta_1} \left[ \frac{b - c_2}{P - c_2} \right]^{\lambda_1} - 1 \right\} & \text{for } \lambda_1 \neq 0, \end{cases}$$

while  $F_2^u(P) = 1$  and, for  $b \in [\underline{b}^u, P)$ ,

$$F_2^u(b) = \begin{cases} \beta_2 \ln\left(\frac{b - c_1}{[P - c_2] e^{-\frac{1}{\beta_1}} + c_2 - c_1}\right) & \text{for } \lambda_1 = \lambda_2 = 0, \\ \frac{\beta_2}{\lambda_2} \left\{ \left[ \frac{b - c_1}{[P - c_2] \left[ \frac{\beta_1}{\lambda_1 + \beta_1} \right]^{\frac{1}{\lambda_1}} + c_2 - c_1} \right]^{\lambda_2} - 1 \right\} & \text{for } \lambda_1, \lambda_2 \neq 0. \end{cases}$$

Equilibrium profits become

$$\pi_1^u = [P - c_1] \left\{ \Pr(b_2 < P) \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta) + \Pr(b_2 = P) \int_{\underline{\theta}}^{\bar{\theta}} \min(\theta, k_1) dG(\theta) \right\},$$

$$\pi_2^u = [P - c_2] \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta),$$

where

$$\Pr(b_2 < P) = \lim_{b \uparrow P} F_2^d(b).$$



*Symmetric Capacities:* When  $k_1 = k_2 = k$  and  $0 = c_1 \leq c_2 = c$ , one can show that we must have  $\lim_{b \uparrow P} F_2^u(b) \leq \lim_{b \uparrow P} F_1^u(b) = 1$  and so we find

$$\begin{aligned} \underline{b}^u &= \begin{cases} c + [P - c] e^{-\frac{1}{\beta}} & \text{for } \lambda = 0 \\ c + [P - c] \left[ \frac{\beta}{\lambda + \beta} \right]^{\frac{1}{\lambda}} & \text{for } \lambda \neq 0 \end{cases} \\ F_1^u(b) &= \begin{cases} 1 + \beta \ln \left( \frac{b-c}{P-c} \right) & \text{for } \lambda = 0 \\ \frac{\beta}{\lambda} \left\{ \frac{\lambda + \beta}{\beta} \left[ \frac{b-c}{P-c} \right]^\lambda - 1 \right\} & \text{for } \lambda \neq 0 \end{cases} \\ F_2^u(b) &= \begin{cases} \begin{cases} \beta \ln \left( \frac{b}{[P-c]e^{-\frac{1}{\beta}} + c} \right) & \text{for } \lambda = 0 \\ \frac{\beta}{\lambda} \left\{ \left[ \frac{b}{[P-c] \left[ \frac{\beta}{\lambda + \beta} \right]^{\frac{1}{\lambda}} + c} \right]^\lambda - 1 \right\} & \text{for } \lambda \neq 0 \end{cases} & , b \in [\underline{b}^u, P), \\ 1, b = P \end{cases} \end{aligned}$$

where

$$\begin{aligned} \lambda &= \frac{\int_k^{\bar{\theta}} [\theta - k] dG(\theta) - \int_{\underline{\theta}}^k \theta dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - 2 \int_k^{\bar{\theta}} [\theta - k] dG(\theta)} \\ \beta &= \frac{\int_{\underline{\theta}}^k \theta dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - 2 \int_k^{\bar{\theta}} [\theta - k] dG(\theta)} \end{aligned}$$

Furthermore,

$$\begin{aligned} \pi_1^u &= P \left\{ \Pr(b_2 < P) \int_k^{\bar{\theta}} [\theta - k] dG(\theta) + \Pr(b_2 = P) \int_{\underline{\theta}}^{\bar{\theta}} \min(\theta, k) dG(\theta) \right\}, \\ \pi_2^u &= [P - c] \int_k^{\bar{\theta}} [\theta - k] dG(\theta). \end{aligned}$$

Consequently, at equilibrium the low-cost supplier bids more aggressively than the high-cost supplier; in particular, the strategy of the low-cost supplier stochastically first-order dominates the strategy of the high-cost supplier.

Again,  $\beta \rightarrow 0$  (while  $\lambda \neq 0$ ) as  $\underline{\theta} \uparrow k$ . In particular,

$$\begin{aligned} \lim_{\underline{\theta} \uparrow k} \underline{b}^u &= c, \\ \lim_{\underline{\theta} \downarrow k} F_1^u(b) &= \left[ \frac{b-c}{P-c} \right]^\lambda \\ \lim_{\underline{\theta} \uparrow k} F_2^u(b) &= \begin{cases} 0, b < P \\ 1, b = P \end{cases} \\ \lim_{\underline{\theta} \uparrow k} \pi_1^u &= Pk, \\ \lim_{\underline{\theta} \uparrow k} \pi_2^u &= [P - c] [E\theta - k], \end{aligned}$$

where we have used the fact that  $\lim_{\bar{\theta} \uparrow k} \int_k^{\bar{\theta}} \theta dG(\theta) = E\theta$ . Consequently, as the probability that demand falls below the capacity of an individual supplier goes to zero, equilibrium approaches something with the flavour of the equilibrium found for high-demand realizations, with the high-cost supplier bidding at  $P$  and the low-cost supplier mixing over a range between  $c$  and  $P$  so as to make undercutting by the high-cost supplier unprofitable.

Also,  $\beta \rightarrow 1$  and  $\lambda \rightarrow -1$  as  $\bar{\theta} \downarrow k$ . In particular,

$$\begin{aligned} \lim_{\bar{\theta} \downarrow k} \underline{b}^u &= c, \\ \lim_{\bar{\theta} \downarrow k} F_1^u(b) &= 1 \\ \lim_{\bar{\theta} \downarrow k} F_2^u(b) &= 1 - \frac{c}{b}, \quad b < P \\ \lim_{\bar{\theta} \downarrow k} \pi_1^u &= cE\theta \\ \lim_{\bar{\theta} \downarrow k} \pi_2^u &= 0, \end{aligned}$$

where we have used the fact that  $\lim_{\bar{\theta} \downarrow k} \int_{\bar{\theta}}^k \theta dG(\theta) = E\theta$ . Consequently, as the probability that demand exceeds the capacity of an individual supplier goes to zero, equilibrium approaches something with the flavour of the Bertrand-like equilibrium found for low-demand realizations, with the low-cost supplier bidding at the cost of the high-cost supplier and the high-cost supplier mixing between  $c$  and  $P$  (with a mass point at  $P$ ).

*Symmetric costs:* When  $c_1 = c_2 = 0$  and  $k_1 \leq k_2$ , we again must have  $\lim_{b \uparrow P} F_2^u(b) \leq \lim_{b \uparrow P} F_1^u(b) = 1$  and so

$$\begin{aligned} \underline{b}^u &= \begin{cases} Pe^{-\frac{1}{\beta_1}} & \text{for } \lambda_1 = 0 \\ P \left[ \frac{\beta_1}{\lambda_1 + \beta_1} \right]^{\frac{1}{\lambda_1}} & \text{for } \lambda_1 \neq 0 \end{cases} \\ F_1^u(b) &= \begin{cases} 1 + \beta_1 \ln\left(\frac{b}{P}\right) & \text{for } \lambda_1 = 0 \\ \frac{\beta_1}{\lambda_1} \left\{ \left[ \frac{\lambda_1 + \beta_1}{\beta_1} \left[ \frac{b}{P} \right]^{\lambda_1} - 1 \right] \right\} & \text{for } \lambda_1 \neq 0 \end{cases} \\ F_2^u(b) &= \begin{cases} \begin{cases} \beta_2 \ln\left(\frac{b}{Pe^{\frac{-1}{\beta_1}}}\right) & \text{for } \lambda_1 = \lambda_2 = 0 \\ \frac{\beta_2}{\lambda_2} \left\{ \left[ \frac{\lambda_1 + \beta_1}{\beta_1} \right]^{\frac{\lambda_2}{\lambda_1}} \left[ \frac{b}{P} \right]^{\lambda_2} - 1 \right\} & \text{for } \lambda_1, \lambda_2 \neq 0 \end{cases} & , \quad b \in [\underline{b}^u, P) \\ 1, \quad b = P \end{cases} \\ \pi_1^u &= P \left\{ \Pr(b_2 < P) \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta) + \Pr(b_2 = P) \int_{\underline{\theta}}^{\bar{\theta}} \min(\theta, k_1) dG(\theta) \right\}, \\ \pi_2^u &= P \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta). \end{aligned}$$

Consequently, at equilibrium the smaller supplier bids more aggressively than the larger supplier; in particular, the strategy of the smaller supplier stochastically first-order dominates the strategy of the larger supplier.

In the limit,

$$\begin{aligned}\lim_{\theta \uparrow k_1} F_2^u(b) &= 0, \quad b < P \\ \lim_{\theta \uparrow k_1} \pi_1^u &= Pk_1 \\ \lim_{\theta \uparrow k_1} \pi_2^u &= P[E\theta - k_1],\end{aligned}$$

where we have used the fact that  $\lim_{\theta \uparrow k_1} \int_{k_1}^{\theta} \theta dG(\theta) = E\theta$ . Consequently, as the probability that demand falls below the capacity of the smaller supplier goes to zero, equilibrium approaches something with the flavour of the high-low bidding equilibrium found for high-demand realizations, with the larger supplier bidding at  $P$  and the smaller supplier mixing over a range below  $P$  so as to make undercutting by the larger supplier unprofitable.

*Symmetric costs and capacities:* When  $k_1 = k_2 = k$  and  $c_1 = c_2 = 0$ , we have  $F_1^u(b) = F_2^u(b)$  and so we find

$$\begin{aligned}\underline{b}^u &= \begin{cases} Pe^{-\frac{1}{\beta}} & \text{for } \lambda = 0 \\ P \left[ \frac{\beta}{\lambda + \beta} \right]^{\frac{1}{\lambda}} & \text{for } \lambda \neq 0 \end{cases} \\ F_1^u(b) = F_2^u(b) &= \begin{cases} 1 + \beta \ln \left( \frac{b}{P} \right) & \text{for } \lambda = 0 \\ \frac{\beta}{\lambda} \left\{ \frac{\lambda + \beta}{\beta} \left[ \frac{b}{P} \right]^{\lambda} - 1 \right\} & \text{for } \lambda \neq 0 \end{cases} \\ \pi_1^u = \pi_2^u &= P \int_k^{\bar{\theta}} [\theta - k] dG(\theta).\end{aligned}$$

### Discriminatory auction

Let  $F_i^d(b) = \Pr \{b_i \leq b\}$  denote the equilibrium mixed-strategy of supplier  $i$ ,  $i = 1, 2$ , in the discriminatory auction, and let  $S_i^d$  be the support of  $F_i^d$  and  $f_i^d(b)$  its density function. Standard arguments imply that  $S_1^d \cap S_2^d = [\underline{b}^d, P)$ ,  $\underline{b}^d \geq c$ , and that  $F_1^d$  and  $F_2^d$  do not have mass points on  $[\underline{b}^d, P)$ .

Again we focus on the case in which  $\underline{\theta} < \min \{k_1, k_2\} \leq \max \{k_1, k_2\} < \bar{\theta}$ . Supplier  $i$ 's profit, when bidding  $b$ , may then be written

$$\begin{aligned}\pi_i^d(b) &= [b - c_i] \left\{ F_j^d(b) \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta) \right. \\ &\quad \left. + [1 - F_j^d(b)] \left[ \int_{\underline{\theta}}^{k_i} \theta dG(\theta) + \int_{k_i}^{\bar{\theta}} k_i dG(\theta) \right] \right\}.\end{aligned}$$

A necessary condition for supplier  $i$  to be indifferent between any price in  $S^d$  is that, for all  $b \in S^d$ ,  $\pi_i^d(b) = \bar{\pi}_i^d$ , implying

$$F_j^d(b) = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta) - \frac{\bar{\pi}_i^d}{b - c_i}}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta) - \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta)}.$$

Observe that the boundary condition  $F_j^d(\underline{b}^d) = 0$  implies

$$\bar{\pi}_i^d = [\underline{b}^d - c_i] \left[ \int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta) \right],$$

and so

$$F_j^d(b) = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_i}^{\bar{\theta}} [\theta - k_i] dG(\theta) - \int_{k_j}^{\bar{\theta}} [\theta - k_j] dG(\theta)} \frac{b - \underline{b}^d}{b - c_i}.$$

We have

$$F_1^d(b) \geq F_2^d(b) \iff \frac{b - c_1}{b - c_2} \geq \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)}.$$

Suppose  $F_1^d(b) > F_2^d(b)$  (in the opposite case a corresponding argument to the following may be applied). Then we cannot have  $\lim_{b \uparrow P} F_2^d(b) = 1$  since this would imply  $\lim_{b \uparrow P} F_1^d(b) > 1$ . Consequently, we have the boundary condition  $\lim_{b \uparrow P} F_1^d(P) = 1$ , which implies

$$\bar{\pi}_2^d = [P - c_2] \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta),$$

and, together with the condition  $F_1^d(\underline{b}^d) = 0$ ,

$$\underline{b}^d = c_2 + [P - c_2] \frac{\int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)}.$$

Equilibrium profits become

$$\pi_1^d = [P - c_1] \left\{ \Pr(b_2 < P) \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta) + \Pr(b_2 = P) \int_{\underline{\theta}}^{\bar{\theta}} \min(\theta, k_1) dG(\theta) \right\},$$

$$\pi_2^d = [P - c_2] \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta),$$

where

$$\Pr(b_2 < P) = \lim_{b \uparrow P} F_2^d(b) = \frac{P - c_2 \int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta)}{P - c_1 \int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)}.$$

*Symmetric capacities:* When  $k_1 = k_2 = k$  and  $0 = c_1 < c_2 = c$ ,  $F_1^d(b) > F_2^d(b)$  and so we find

$$\underline{b}^d = c + [P - c] \frac{\int_k^{\bar{\theta}} [\theta - k] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_k^{\bar{\theta}} [\theta - k] dG(\theta)}$$

$$\pi_1^d = [P - c] \int_k^{\bar{\theta}} [\theta - k] dG(\theta) + c \left[ \int_{\underline{\theta}}^k \theta dG(\theta) + \int_k^{\bar{\theta}} k dG(\theta) \right],$$

$$\pi_2^d = [P - c] \int_k^{\bar{\theta}} [\theta - k] dG(\theta).$$

Consequently, at equilibrium the low-cost supplier bids more aggressively than the high-cost supplier; in particular, the strategy of the low-cost supplier first-order stochastically dominates that of the high-cost supplier.

In the limit, we find

$$\lim_{\underline{\theta} \uparrow k} \underline{b}^d = c + [P - c] \frac{E\theta - k}{k},$$

$$\lim_{\underline{\theta} \uparrow k} F_1^d(b) = \frac{k}{2k - E\theta} \frac{b - \underline{b}^d}{b - c}$$

$$\lim_{\underline{\theta} \uparrow k} F_2^d(b) = \begin{cases} \frac{k}{2k - E\theta} \frac{b - \underline{b}^d}{b} & , \quad b < P \\ 1 & , \quad b = P \end{cases}$$

$$\lim_{\underline{\theta} \uparrow k} \pi_1^d = [P - c] [E\theta - k] + ck,$$

$$\lim_{\underline{\theta} \uparrow k} \pi_2^d = [P - c] [E\theta - k].$$

Consequently, when the probability that demand falls below the capacity of any individual supplier goes to zero, equilibrium approaches the mixed-strategy equilibrium for high-demand realizations.

Furthermore,

$$\lim_{\bar{\theta} \downarrow k} \underline{b}^d = c,$$

$$\lim_{\bar{\theta} \downarrow k} F_1^d(b) = 1,$$

$$\lim_{\bar{\theta} \downarrow k} F_2^d(b) = \begin{cases} 1 - \frac{c}{b}, & b < P \\ 1, & b = P \end{cases}$$

$$\lim_{\bar{\theta} \downarrow k} \pi_1^d = cE\theta,$$

$$\lim_{\bar{\theta} \downarrow k} \pi_2^d = 0.$$

Consequently, as the probability that demand exceeds the capacity of an individual supplier goes to zero, equilibrium approaches the Bertrand-like equilibrium for low-demand realizations, with the low-cost supplier bidding at the cost of the high-cost supplier and the high-cost supplier mixing between  $c$  and  $P$  (with a mass point at  $P$ ).

*Symmetric costs:* When  $c_1 = c_2 = 0$  and  $k_1 < k_2$ ,  $F_1^d(b) > F_2^d(b)$  and so we find

$$\begin{aligned} \underline{b}^d &= P \frac{\int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)} \\ \pi_1^d &= P \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta) \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)} \\ \pi_2^d &= P \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta) \end{aligned}$$

In the limit,

$$\begin{aligned} \lim_{\underline{\theta} \uparrow k_1} \underline{b}^d &= P \frac{E\theta - k_1}{E\theta - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)} \\ \lim_{\underline{\theta} \uparrow k_1} F_1^d(b) &= \frac{E\theta - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)}{k_1 - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)} \frac{b - \underline{b}^d}{b} \\ \lim_{\underline{\theta} \uparrow k} F_2^d(b) &= \begin{cases} \frac{k_1}{k_1 - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)} \frac{b - \underline{b}^d}{b} & , \quad b < P \\ 1 & , \quad b = P \end{cases} \\ \lim_{\underline{\theta} \uparrow k_1} \pi_1^d &= P [E\theta - k_1] \frac{k_1}{E\theta - \int_{k_2}^{\bar{\theta}} [\theta - k_2] dG(\theta)} \\ \lim_{\underline{\theta} \uparrow k_1} \pi_2^d &= P [E\theta - k_1] \end{aligned}$$

Again, when the probability that demand falls below the capacity of any individual supplier goes to zero, equilibrium approaches the mixed-strategy equilibrium for high-demand realizations.

Furthermore,

$$\begin{aligned} \lim_{\bar{\theta} \downarrow k_2} \underline{b}^d &= P \frac{\int_{k_1}^{k_2} [\theta - k_1] dG(\theta)}{E\theta} \\ \lim_{\bar{\theta} \downarrow k_1} F_1^d(b) &= \frac{E\theta}{E\theta - \int_{k_1}^{k_2} [\theta - k_1] dG(\theta)} \frac{b - \underline{b}^d}{b} \\ \lim_{\bar{\theta} \downarrow k} F_2^d(b) &= \begin{cases} \frac{b - \underline{b}^d}{b} & , \quad b < P \\ 1 & , \quad b = P \end{cases} \\ \lim_{\bar{\theta} \downarrow k_2} \pi_1^d &= P \int_{k_1}^{k_2} [\theta - k_1] dG(\theta) \frac{E\theta - \int_{k_1}^{\bar{\theta}} [\theta - k_1] dG(\theta)}{E\theta} \\ \lim_{\bar{\theta} \downarrow k_2} \pi_2^d &= P \int_{k_1}^{k_2} [\theta - k_1] dG(\theta) \end{aligned}$$

Consequently, as the probability that demand exceeds the capacity of the larger supplier goes to zero, equilibrium approaches the mixed-strategy equilibrium for high-demand realizations, with the smaller supplier bidding more aggressively.

*Symmetric capacities and costs:* When  $k_1 = k_2 = k$  and  $c_1 = c_2 = 0$ ,  $F_1^d(b) = F_2^d(b)$  and so we find

$$\underline{b}^d = P \frac{\int_k^{\bar{\theta}} [\theta - k] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_k^{\bar{\theta}} [\theta - k] dG(\theta)},$$

$$F_1^d(b) = F_2^d(b) = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_k^{\bar{\theta}} [\theta - k] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - 2 \int_k^{\bar{\theta}} [\theta - k] dG(\theta)} \left[ 1 - \frac{P}{b} \frac{\int_k^{\bar{\theta}} [\theta - k] dG(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dG(\theta) - \int_k^{\bar{\theta}} [\theta - k] dG(\theta)} \right],$$

$$\pi_1^d = \pi_2^d = P \int_k^{\bar{\theta}} [\theta - k] dG(\theta).$$

### Proof of Proposition 6

*Uniform auction format:* With short-lived bids total payments to suppliers equal zero for low-demand realizations and  $P\theta$  for high-demand realizations, and so overall expected payments equal  $ER_s^u = PE\{\theta \mid \theta \geq k\}G(k)$ . With long-lived bids, for given demand realization  $\theta$ , total payments equal  $2P \max\{\theta - k, 0\}$ , and so in expected terms we have  $ER_l^u = 2P[E\{\theta \mid \theta \geq k\} - k]G(k)$ . From these expressions we find

$$ER_l^u - ER_s^u = P[E\{\theta \mid \theta \geq k\} - 2k]G(k) < 0.$$

*Discriminatory auction format:* With short-lived bids total payments to suppliers equal zero for low-demand realizations and  $2P[\theta - k]$  for high-demand realizations, and so overall expected payments equal  $ER_s^d = 2P[E\{\theta \mid \theta \geq k\} - k]G(k)$ . With long-lived bids, for given demand realization  $\theta$ , total payments equal  $2P \max\{\theta - k, 0\}$ , and so in expected terms we have  $ER_l^d = 2P[E\{\theta \mid \theta \geq k\} - k]G(k) = ER_s^d$ .