Political Selection and Persistence of Bad Governments*

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Abstract

We study the dynamic selection of governments. A government consists of a subset of the individuals in the society. The competence level of the government in office determines collective utilities (e.g., by determining the amount and quality of public goods), and each individual derives additional utility from being part of the government (e.g., corruption or rents from holding office). We characterize the dynamic evolution of governments and determine structure of stable governments, which arise and persist in equilibrium. Our main focus is on the impact of different political institutions on the selection of governments. Perfect democracy, where current members of the government do not have an incumbency advantage or special powers, always leads to the emergence of the most competent government. However, any deviation from perfect democracy destroys this result. There is always at least one other, less competent government that is also stable and can persist forever. In addition, even the least competent government can persist forever in office. When there are stochastic shocks to the competence levels of different governments or to the rules determining the election of new governments, political institutions with a greater degree of democracy (less power for incumbents) are shown to perform better, because they can adapt to changes more successfully. This suggests that a particular advantage of democratic regimes is their relative flexibility. We also show that, in the presence of stochastic shocks, "royalty-like" dictatorships may be more successful than "junta-like" dictatorships, because they might also be more adaptable to change.

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1 Introduction

There is now a relatively broad consensus that government policies have important implications for economic outcomes, development, and the welfare of citizens. This has motivated a large literature focusing on how a society can provide the right incentives to its politicians (see, among others, Barro, 1973, Ferejohn, 1986, Besley and Case, 1995, Persson, Roland and Tabellini, 1997) and on how undesirable economic outcomes follow when the society is unable to do so (e.g., Niskanen, 1971, Shleifer and Vishny, 1993, Acemoglu, Robinson and Verdier, 2004, Padro-i-Miquel, 2007). In practice, the selection of the right (honest, competent, motivated) politicians and the right types of governments may be as important as the provision of incentives. Besley (2005, p. 43), for example, quotes James Madison, to emphasize the importance of the selection of politicians for the success of a society:

> "The aim of every political Constitution, is or ought to be, first to obtain for rulers men who possess most wisdom to discern, and most virtue to pursue, the common good of society; and in the next place, to take the most effectual precautions for keeping them virtuous whilst they continue to hold their public trust."

Motivated by these concerns, a recent and small literature investigates how a society can ensure the selection of the right politicians, as well as incentivizing them (e.g., Banks and Sundaram, 1998, Diermeier, Keane, and Merlo, 2005, Besley, 2005). The main challenge facing the society and the design of political institutions in these papers is that the ability and motivations of politicians are not observed by voters or outside parties. While such information-related selection issues are undoubtedly important, in this paper we argue that there is no guarantee that the society will select the right government even when information is perfect and common.

We construct a dynamic model of government formation. Each government has a different level of competence, affecting the collective utility it provides to citizens (e.g., the level of public goods). Each individual also receives rents from being part of the government (additional income, utility of office, or rents from corruption). New governments are put in place by a combination of "votes" from the citizens and "consent" from current government members. The importance of the consent of current government members is a measure of the degree of democracy. For example, a perfect democracy can be thought of as a situation in which current incumbents have no special power. Many political institutions provide additional decision-making or blocking power to current government members, however. For example, in many democracies, there is incumbency advantage, so that a government in power is harder to oust than it would have been to institute had it been out of power (e.g., Cox and Katz, 1996, for a discussion of such incumbency advantage in mature democracies). In nondemocratic societies, this advantage of current government members is more pronounced, even palpable. For instance, only new governments that include some members of previous governments might be feasible or much more than a simple majority of voters may be necessary to oust the government (as witnessed, for example, by the recent elections in Zimbabwe).

The first contribution of our paper is to provide a general framework for the study of such dynamic selection issues and a characterization of equilibria. Our second contribution is to investigate the structure (and efficiency) of the dynamic selection of politicians under different political institutions. Perfect democracies always ensure the emergence of the best (most competent) government. In contrast, under any other arrangement, incompetent and bad governments can emerge and persist despite the absence of information-related challenges to selecting good politicians. For example, even a small departure from perfect democracy, whereby only one member of the current government needs to consent to a new government, may make the *worst* possible government persist forever. The intuitive explanation for why even a small degree of incumbency advantage might lead to such outcomes is as follows: improvements away from a bad (or even the worst) government might lead to another potential government that is itself unstable and will open the way for a further round of changes. If this process ultimately leads to a government that does not have any common members with the initial government, then it may fail to get the support of any of the initial government members. In this case, the initial government survives even though it has low, or even possibly the lowest, level of competence.

This discussion highlights the important dynamic interactions in the process of selecting politicians and governments. Another important implication of this analysis is that there is no obvious ranking among different shades of imperfect democracy and dictatorships. Either of these different regimes may lead to the emergence of better governments in the long run. This result is consistent with the empirical findings in the literature that show no clear-cut relationship between democracy and economic performance (e.g., Przeworski and Limongi, 1997, Barro, 1997, Minier, 1999). In fact, both under imperfect democracies and extreme dictatorships, the competence of the equilibrium government and the success of the society depend on the identity of the initial members of the government. This is consistent with the emphasis in the recent political science and economics literatures on the role that leaders may play under weak institutions (see, for example, Brooker, 2000, or Jones and Olken, 2004, who show that the death of an autocrat leads to a significant change in growth, and this does not happen with democratic leaders).

Our third contribution is to show how stochastic shocks affect the selection of politicians and efficiency of government formation. This is of methodological as well as substantive interest, since the analysis of this class of dynamic models under uncertainty is a challenging problem. We characterize the structure of equilibria when stochastic shocks are sufficiently infrequent. Using this characterization, we show how the quality (competence level) of governments evolves in the presence of stochastic shocks. It is a potential paradoxical feature of our nonstochastic analysis that, beyond perfect democracy, a greater degree of democracy does not necessarily guarantee a better government. In contrast, in the presence of stochastic shocks, we show that a greater degree of democracy typically leads to better outcomes in the long run. This is because a greater degree of democracy enables greater adaptability to changes in conditions (either in the form of changes in competence as or changes in rules). Consequently, our analysis suggests that a particular long-run advantage of more democratic regimes is their greater *flexibility*.[1]

Our final sets of results show that under certain conditions, "royalty-like" nondemocratic regimes, where some individuals must always be in the government, may lead to better long-run outcomes than "junta-like" regimes, where a subset of the current members of the junta can block change even though no member is essential. The royalty-like regimes might sometimes allow greater adaptation to change because one of the members of the initial government is secure in her position. In contrast, as discussed above, without such security the fear of further changes might block all competence-increasing reforms in government.

We now illustrate some of the basic ideas with a simple example.

Example 1 Suppose that the society consists of n individuals. Assume that any $k = 3$ individuals could form a government. A change in government requires the support of the majority of the citizens and in addition, the consent of $l = 1$ member of the government, so that there is an imperfect democracy, with some degree of incumbency advantage. Suppose that each individual has a level of competence, denoted by γ_j for individual j, and order them, without loss of any generality, in descending order according to their competence, so $\gamma_1 > \gamma_2 > ... > \gamma_n$. The competence of a government is the sum of the competences of its three members. Each individual obtains instantaneous utility from the competence level of the government and also a large rent from being in office, so that each prefers to be in office regardless of the competence level of the government. Suppose also that individuals have a sufficiently high discount factor, so that the

[1] The stochastic analysis also shows that random shocks to the identity of the members of the government may also lead to better governments in the long run because they destroy incompetent governments. Besley (2005) writes: "History suggests that four main methods of selection to political office are available: drawing lots, heredity, the use of force and voting." Our model suggests why, somewhat paradoxically, drawing lots, which was used in Ancient Greece, might sometimes lead to better long-run outcomes than the alternatives.

future matters a lot relative to the present.

It is straightforward to determine the stable governments that will persist and remain in power once formed. Evidently, $\{1,2,3\}$ is a stable government, since it has the highest level of competence, so neither outsiders nor members of the government would like to initiate a change (some outsiders may want to initiate a change: for example, 4, 5, and 6 would prefer government $\{4,5,6\}$, but they do not have the power to enforce such a change). In contrast, governments of the form $\{1,i,j\}$, $\{i,2,j\}$, and $\{i,j,3\}$ are unstable (for $i,j > 3$), meaning that starting with these governments, there will necessarily be a change. In particular, in each of these cases, $\{1,2,3\}$ will receive support from both one current member of government and from the rest of the population, who would be willing to see a more competent government.

Consider next the case where $n = 6$ and suppose that society starts with the government $\{4,5,6\}$. This is also a stable government, even though it is the worst possible option for the society as a whole. This is because any change in government will take the individuals to a new government of the form of either $\{1,i,j\}$, or $\{i,2,j\}$, or $\{i,j,3\}$, but we know that all of these are unstable. Therefore, any of the more competent governments will ultimately take us to $\{1,2,3\}$, which does not include any of the members of the initial government. Since individuals are assumed to be relatively patient, none of the initial members of the government would support (consent to) a change that will ultimately exclude them. As a consequence, the initial worst government persists forever. This example illustrates how the worst possible government can be stable.

Our paper is related to a number of different literatures. We have already mentioned work on models of political selection as well as the literature on providing incentives to politicians and how such incentives may fail in societies with weak institutions. In addition, our results are also related to recent work on the persistence of bad governments and inefficient institutions, including Acemoglu and Robinson (2008), Acemoglu, Ticchi, and Vindigni (2006), and Egorov and Sonin (2004).

Besley and Coate (1997, 1998), Caselli and Morelli (2004), Messner and Polborn (2004), and Mattozzi and Merlo (2006) provide alternative and complementary "theories of bad governments/politicians". For example, Caselli and Morelli (2004) suggest that voters might be unwilling to replace the corrupt incumbent by a challenger whom they expect to be equally corrupt. Mattozzi and Merlo (2006) argue that more competent politicians have higher opportunity costs of entering politics. None of these papers develop the potential persistence in bad governments resulting from dynamics of government formation. We are also not aware of other papers providing a comparison of different political regimes in terms of the selection of politicians under nonstochastic and stochastic conditions. McKelvey and Reizman (1992) suggest that seniority rules in the Senate and the House may be playing the role of creating an endogenous incumbency advantage, and when this is the case, current members of these bodies will indeed have an incentive to introduce such seniority rules.

More closely related to our work are prior analyses of dynamic political equilibria in the context of club formation as in Roberts (1997) and Barbera, Maschler, and Shalev (2001), as well as dynamic analyses of choice of constitutions and equilibrium political institutions as in Acemoglu and Robinson (2006), Barbera and Jackson (2004), Matthias and Polborn (2004), and Lagunoff (2006). Our recent work, Acemoglu, Egorov, and Sonin (2008), provided a general framework for the analysis of the dynamics of constitutions, coalitions and clubs. The current paper is a continuation of this line of research. It differs from our previous work in a number of important dimensions. First, it allows for richer dynamics in the formation of the governments (in particular, the structure of preferences here implies that our previous results cannot be applied). Second, it enables an analysis of the impact of different political institutions on the selection of politicians and governments. Third, it extends our previous work by allowing for stochastic shocks both to the competence levels of individuals who might make up governments and also to the rules determining the formation of new governments.

Finally, our paper is in the tradition of citizen-candidate models as in Besley and Coate (1997, 1998) and Osborne and Slivinski (1996), since individuals run for office and act both as potential government officials and citizens voting in favor of or against other governments.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 provides the general characterization result. Section 4 contains a brief discussion of cycles, whereby the same government comes to power at different points in time, and how these could be ruled out, so that starting from any initial conditions, a stable government emerges and persists forever. Section 5 provides our main results on the comparison of different regimes in terms of selection of governments and politicians. Section 6 extends the analysis to allow for stochastic changes in the competences of the members of the society and in the rules governing elections. It also contains the comparison of different regimes in the presence of stochastic shocks. Section 7 concludes, while the Appendix contains all of the proofs of the results stated in the text and some examples.

2 Model

2.1 Agents and Preferences

We consider a dynamic game in discrete time indexed by $t = 0, 1, 2, \ldots$. The population is represented by the set $\mathcal{I}$ and consists of $n < \infty$ individuals. We refer to non-empty subsets of $\mathcal{I}$ as *coalitions* and denote the set of coalitions by $\mathcal{C}$. Most importantly, we also designate a subset coalitions $\mathcal{G} \subset \mathcal{C}$ as the set of *feasible governments*. For example, the set of feasible governments could consist of all groups of individuals of size k_0 (for some integer k_0) or all groups of individuals of size greater than k_0 and less than some other integer k_1. To simplify the discussion, we define $\bar{k} = \max_{G \in \mathcal{G}} |G|$, so $\bar{k}$ is the upper bound for the size of any feasible government: for any $G \in \mathcal{G}$, $|G| \leq \bar{k}$.

In each period, the society is ruled by one of the feasible governments $G^t \in \mathcal{G}$. The initial government G^0 is given as part of the description of the game and G^t for $t > 0$ is determined in equilibrium as a result of the political process described below. The government in power at any date affects three aspects of the society:

1. It influences collective utilities (for example, by providing public goods or influencing how competently the government functions).

2. It determines individual utilities (members of the government may receive additional utility because of rents of being in office or corruption).

3. It indirectly influences the future evolution of governments by shaping the distribution of political power in the society (for example, by creating incumbency advantage in democracies or providing greater decision-making power or veto rights to members of the government under other political institutions).

We now describe each of these in turn. The influence of the government on collective utilities is modeled via its *competence*. In particular, at each date t, there exists a function

$$\Gamma^t : \mathcal{G} \to \mathbb{R}$$

designating the competence of each feasible government $G \in \mathcal{G}$ (at that date). We refer to $\Gamma^t_G \in \mathbb{R}$ as government G's competence, with the convention that higher values correspond to greater competence. In Section 5, we will assume that each individual has a level of competence and the competence of a government is a function of the competences of its members. For now, this additional assumption is not necessary. Note also that the function Γ^t depends on time.

This generality is introduced to allow for stochastic shocks, changing the relative competences of different individuals and governments over time.

Individual utilities are determined by the competence of the government that is in power at that date and by whether the individual in question is herself in the government. More specifically, each individual $i \in \mathcal{I}$ at time t has discounted (expected) utility given by

$$U_i^{\tau} = \mathbb{E} \sum\nolimits_{t=\tau}^{\infty} \beta^{(t-\tau)} u_i^t,$$

where $\beta \in (0,1)$ is the discount factor and u_i^t is individual's instantaneous utility, given by

$$u_i^t = w_i \left(G^t, \Gamma_{G^t}^t \right) = w_i \left(G^t \right),$$

where in the second equality we suppress dependence on $\Gamma_{G^t}^t$ to simplify notation. We will therefore drop the second argument to economize on notation unless special emphasis is necessary. We assume that w_i satisfies the following properties.

Assumption 1 *The function w_i satisfies the following properties for each $i \in \mathcal{I}$ and any $G, H \in \mathcal{G}$ such that $\Gamma_G^t > \Gamma_H^t$: if $i \in G$ or $i \notin H$, then $w_i(G) > w_i(H)$.*

This assumption is a relatively mild restriction on payoffs. It implies that all else equal, more competent governments give higher instantaneous utility. In particular, an individual belongs to both governments G and H and G is more competent than H, then the individual prefers G. The same conclusion also holds when the individual is not a member of either of these two government or when she is only a member of G (and not of H). Therefore, Assumption 1 implies that the only situation in which an individual may prefer a less competent government to a more competent one is when she is a member of the former, but not of the latter. This simply captures the presence of rents from holding office or additional incomes from being in government due to higher salaries or corruption. This assumption is important for our analysis, since it will be the main source of conflict of interest in this society; individuals typically prefer to be in government even when this does not benefit the rest of the society.

We next provide an example that makes some of these notions slightly more concrete.

Example 2 Suppose that the competence of government G, Γ_G, is the amount of public good produced in the economy under feasible government G. Then, we can write $w_i(G)$ as

$$w_i(G) = v_i(\Gamma_G) + b_i \mathbf{I}_{\{i \in G\}}, \tag{1}$$

where $v_i : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function (for each $i \in \mathcal{I}$) corresponding to the utility from public good for individual i, b_i is a measure of the rents that individual i obtains from being in office, and $\mathbf{I}_X$ is the indicator of event X. If $b_i \geq 0$ for each $i \in \mathcal{I}$, then (1) satisfies Assumption 1. In addition, if b_i is sufficiently large for each i, then each individual prefers to be a member of the government, even if this government has a very low level of competence.

Finally, the government in power also influences the distribution of political power and the potential evolution of governments in the future. We represent the distribution of political power at time t in the society by the set of *winning coalitions* denoted by $\mathcal{W}_G^t$. In particular, consider the mapping

$$\mathcal{W}^t : \mathcal{G} \to \mathcal{P}(\mathcal{C})$$

from the set of feasible government to the power set of the set of coalitions of individuals. $\mathcal{W}_G^t$ is the set of winning coalitions in period t when the current government is G. The set of winning coalitions determines which subsets of the society are able to force (or to block) a change in government. We only impose a minimal amount of structure on the set of winning coalitions.

Assumption 2 *The mapping $\mathcal{W}^t$ is such that $\mathcal{W}_G^t$ at any $t \geq 0$ and for any feasible government $G \in \mathcal{G}$ is given by*

$$\mathcal{W}_G^t = \left\{ X \in \mathcal{C} : |X| \geq m_G^t \text{ and } |X \cap G| \geq l_G^t \right\},$$

where l_G^t and m_G^t are integers satisfying $0 \leq l_G^t \leq |G| \leq \bar{k} < m_G^t \leq n - \bar{k}$ (where recall that $\bar{k}$ is the maximal size of the government and n is the size of the society).

The minimal restrictions imposed in Assumption 2 are intuitive. In particular, they amount to requiring that a new government can be instituted if it receives a sufficient number of votes from the entire society (m_G^t total votes) *and* if it receives support from some subset of the members of the current government (l_G^t of the current government members need to support such a change). This definition allows l_G^t to be any number between 0 and $|G|$. We can then think of the case where $l_G^t = 0$ as *perfect democracy*, where current members of the government have no special power, and the case where $l_G^t = |G|$ as *extreme dictatorship*, where unanimity among government members is necessary for any change. In between these extremes are imperfect democracies (or less strict forms of dictatorships), which may arise either because there is some form of (strong or weak) incumbency advantage in democracy or because current government (junta) members are able to block the introduction of a new government. Note also that we have imposed some mild assumptions on m_G^t. In particular, less than $\bar{k}$ individuals is insufficient for a change to take place. This ensures that that a rival government cannot take the power without any support from other individuals (recall that $\bar{k}$ denotes the maximum size of the government, so the rival government must have less than $\bar{k}$ members), and $n - \bar{k}$ individuals are sufficient to implement the change provided that l_G^t members of the current government are among them (though less may be sufficient as well). These assumptions (on $\bar{k}$) are quite mild, particularly since we will often consider the case where n is large.

In addition to Assumptions 1 and 2, we also impose the following *genericity* assumption, which ensures that different governments have different competences. This assumption simplifies the notation and is without much loss of generality, since if it were not satisfied for a society, any small perturbation of competence levels would restore it.

Assumption 3 *For any $t \geq 0$ and any $G, H \in \mathcal{G}$ such that $G \neq H$, $\Gamma_G^t \neq \Gamma_H^t$.*

2.2 Dynamic Game

We have so far specified the set of winning coalitions and preferences over feasible governments and from being in office. To complete the description of the environment, we need to specify the order in which moves are made and how a government can be replaced by the alternative.

We first introduce an additional state variable, denoted by v^t, which determines whether the current government can be changed. In particular, v^t takes two values: $v^t = s$ corresponds to a "sheltered" political situation (or "stable" political situation, though we will use the term stable for another purpose below) and $v^t = u$ designates an unstable situation. The government can only be changed during unstable times. A sheltered political situation destabilizes (becomes unstable) with probability r in each period, that is, $\mathbf{P}\left(v^t = u \mid v^{t-1} = s\right) = r$. These events are independent across periods and we also assume that $v^0 = u$. An unstable situation becomes sheltered when an incumbent government survives a challenge or is not challenged (as explained below).

We next described the procedure for challenging an incumbent government. We start with some government G^t at time t. If at time t the situation is unstable, then all individuals $i \in \mathcal{I}$ are ordered according to some sequence η_{G^t}. Then each individual, in this order, nominates a subset of alternative governments $A_i^t \subset \mathcal{G} \backslash \left\{G^t\right\}$ that will be part of the primaries. An individual may choose not to nominate any alternative government, in which case he may choose $A_i^t = \varnothing$. All nominated governments (except the incumbent) make up the set $\mathcal{A}^t$, so

$$\mathcal{A}^t = \left\{G \in \mathcal{G} \setminus \left\{G^t\right\} : G \in A_i \text{ for some } i \in \mathcal{I}\right\}. \tag{2}$$

If $\mathcal{A}^t \neq \varnothing$, then all alternatives in $\mathcal{A}^t$ take part in the *primaries* at time t. The primaries take place as follows. All of the alternatives in $\mathcal{A}^t$ are ordered $\pi_{G^t}^{\mathcal{A}^t}(1), \pi_{G^t}^{\mathcal{A}^t}(2), \ldots, \pi_{G^t}^{\mathcal{A}^t}\left(\left|\mathcal{A}^t\right|\right)$ according to some pre-specified order (depending on $\mathcal{A}^t$ and the current government G^t). We refer to this order as the *protocol*, $\pi_{G^t}^{\mathcal{A}^t}$. The primaries are then used to determine the challenging government $G' \in \mathcal{A}^t$. In particular, we start with G'_1 given by the first element of the protocol $\pi^{\mathcal{A}^t}_{G^t}$, $\pi^{\mathcal{A}^t}_{G^t}(1)$. At the second step, G'_1 is voted against the second element, $\pi^{\mathcal{A}^t}_{G^t}(2)$. We assume that all votings are sequential (and show in the Appendix that the sequence in which votes take place does not have any affect on the outcome). If more than $n/2$ of individuals support the latter, then $G'_2 = \pi^{\mathcal{A}^t}_{G^t}(2)$; otherwise $G'_2 = G'_1$. Proceeding in order, G'_3, G'_4, ..., and $G'_{|\mathcal{A}^t|}$ are determined, and G' is equal to the last element of the sequence, $G'_{|\mathcal{A}^t|}$. This ends the primary.

After the primary, the challenger G' is voted against incumbent government G^t. G' wins if and only if a winning coalition of individuals (i.e., a coalition that belongs to $\mathcal{W}^t_{G^t}$) supports G'. Otherwise, we say that the incumbent government G^t wins. If $\mathcal{A}^t = \varnothing$ to start with, then there is no challenger and the incumbent government is again the winner.

If the incumbent government wins, it stays in power, and moreover the political situation becomes sheltered, that is, $G^{t+1} = G^t$ and $v^{t+1} = s$. Otherwise, the challenger becomes the new government, but the situation remains unstable, that is, $G^{t+1} = G'$ and $v^{t+1} = v^t = u$. All individuals receive instantaneous payoff $w_i\left(G^t\right)$ (we assume that the new government starts acting from the next period on).

More formally, the exact procedure is as follows.

- Period $t = 0, 1, 2, ...$ begins with government G^t in power. If the political situation is sheltered, $v^t = s$, then each individual $i \in \mathcal{I}$ receives instantaneous utility $u^t_i\left(G^t\right)$; in the next period, $G^{t+1} = G^t$, $v^{t+1} = v^t = s$ with probability $1-r$ and $v^{t+1} = u$ with probability r.

- If the political situation is unstable, $v_t = u$, then the following events take place:

  1. Individuals are ordered according to η_{G^t}, and in this sequence, each individual i nominates a subset of feasible governments $A^t_i \subset \mathcal{G} \setminus \left\{G^t\right\}$ for the primaries. These determine the set of alternatives $\mathcal{A}^t$ as in (2).

  2. If $\mathcal{A}^t = \varnothing$, then we say that the incumbent government wins, $G^{t+1} = G^t$, $v^{t+1} = s$, and each individual receives instantaneous utility $u^t_i\left(G^t\right)$. If $\mathcal{A}^t \neq \varnothing$, then the alternatives in $\mathcal{A}^t$ are ordered according to protocol $\pi^{\mathcal{A}^t}_{G^t}$.

  3. If $\mathcal{A}^t \neq \varnothing$, then the alternatives in $\mathcal{A}^t$ are voted against each other. In particular, at the first step, $G'_1 = \pi^{\mathcal{A}^t}_{G^t}(1)$. If $\left|\mathcal{A}^t\right| > 1$, then for $2 \leq j \leq \left|\mathcal{A}^t\right|$, at step j, alternative G'_{j-1} is voted against $\pi^{\mathcal{A}^t}_{G^t}(j)$. Voting in the primary takes place as follows: all individuals vote *yes* or *no* sequentially according to some pre-specified order, and $G'_j = \pi^{\mathcal{A}^t}_{G^t}(j)$ if and only if the set of the individuals who voted *yes*, $\mathcal{Y}^t_j$, is a simple

majority (i.e., if $\left|\mathcal{Y}_j^t\right| > n/2$); otherwise, $G'_j = G'_{j-1}$. The challenger is determined as $G' = G'_{|\mathcal{A}^t|}$.

4. Government G' challenges the incumbent government G^t, and voting in the election takes place. In particular, all individuals vote *yes* or *no* sequentially according to some pre-specified order, and G' wins if and only if the set of the individuals who voted *yes*, $\mathcal{Y}^t$, is a winning coalition in G^t (i.e., if $\mathcal{Y}^t \in \mathcal{W}_{G^t}^t$); otherwise, G^t wins.

5. If G^t wins, then $G^{t+1} = G^t$, $v^{t+1} = s$; if G' wins, then $G^{t+1} = G'$, $v^{t+1} = u$. In either case, and each individual gets instantaneous utility $u_i^t\left(G^t\right)$.

There are several important features about this dynamic game that are worth emphasizing. First, the set the winning coalitions, $\mathcal{W}_{G^t}^t$ when the government is G^t, determines which proposals for governmental change are accepted. Second, to specify a well-defined game we had to introduce the pre-specified order η_G in which individuals nominate alternatives for the primaries, the protocol $\pi_G^{\mathcal{A}^t}$ for the order in which alternatives are considered, and also the order in which votes are cast. Ideally we would like these orders not to have a major influence on the structure of equilibria, since they are not an essential part of the economic environment and we do not have a good way of mapping the specific orders to reality. We will discuss this issue further below. Finally, the rate at which political situations become unstable, r, has an important influence on payoffs by determining the rate at which opportunities to change the government arise. In what follows, we will typically suppose that r is relatively small, so that political situations are not unstable most of the time. Here, it is also important that political instability ceases after the incumbent government withstands a challenge (or if there is no challenge). This can be interpreted as the government having survived a "no-confidence" motion. We will also focus on situations in which the discount factor β is large.

2.3 Strategies and Definition of Equilibrium

We define strategies and equilibria in the usual fashion. In particular, let h^{t,Q^t} denote the history of the game up to period t and stage Q^t in period t (there are many stages in period t if $v^t = u$). This history includes all governments, all proposals, votes and stochastic events up to this time. The set of histories is denoted by $\mathcal{H}^{t,Q^t}$. A history h^{t,Q^t} can also be decomposed into two parts. We can write $h^{t,Q^t} = \left(h^t, Q^t\right)$ and correspondingly, $\mathcal{H}^{t,Q^t} = \mathcal{H}^t \times \mathcal{Q}^t$, where h^t summarizes all events that have taken place up to period $t-1$ and Q^t is the list of events that have taken place within the time instant t when there is an opportunity to change the government.

A strategy for individual $i \in \mathcal{I}$, denoted by σ_i, maps $\mathcal{H}^{t,Q^t}$ (for all t and Q^t) into a proposal when i nominates an alternative government (i.e., at the first stage of the period where $v^t = u$) and a vote for each possible proposal at each possible decision node (recall that the ordering of alternatives is automatic and is done according to a protocol). A Subgame Perfect Equilibrium (SPE) is a strategy profile $\{\sigma_i\}_{i\in\mathcal{I}}$ such that the strategy of each i is the best response to the strategies of all other individuals for all histories.

Throughout the rest of the paper, we focus on the Markovian subset of SPEs, and *equilibrium* refers to Markov Perfect Equilibrium (MPE) in pure strategies. More formally:

Definition 1 *A* Markov Perfect Equilibrium *is a SPE profile of strategies* $\{\sigma_i^*\}_{i\in\mathcal{I}}$ *such that* σ_i^* *for each* i *in each period* t *depends only on* G^t, Γ^t, $\mathcal{W}^t$, *and* Q^t *(previous actions taken in period* t*).*

MPEs are natural in such dynamic games, since they enable individuals to condition on all of the payoff-relevant information, but rule out complicated trigger-like strategies, which are not our focus in this paper. It turns out that even MPEs potentially lead to a very rich set of behavior. For this reason, it is also useful to consider subsets of MPEs, in particular, *acyclic MPEs* and *order-independent MPEs*. Loosely speaking, an equilibrium is acyclic if cycles (changing the initial government but then reinstalling it at some future date) do not take place along the equilibrium path. Cyclical MPEs are both less realistic and also more difficult to characterize, motivating our main focus on acyclic MPEs. Formally, we have:

Definition 2 *An MPE* σ^* *is* cyclic *if the probability that there exist* $t_1 < t_2 < t_3$ *such that* $G^{t_3} = G^{t_1} \neq G^{t_2}$ *along the equilibrium path is positive. An MPE* σ^* *is* acyclic *if it is not cyclic.*

We also refer to acyclic MPEs as *acyclic equilibria*. Another relevant subset of MPEs, order-independent MPEs or simply order-independent equilibria, is introduced by Moldovanu and Winter (1995). These equilibria impose that strategies should not depend on the order in which certain events, in particular here the order of proposal-making, unfold. Here we generalize (and slightly modify) their definition for our present context. For this purpose, let us denote the above-described game when the set of protocols is given by $\pi = \left\{\pi_G^{\mathcal{A}^t}\right\}_{G\in\mathcal{G},\mathcal{A}^t\in\mathcal{P}(\mathcal{G}),G\notin\mathcal{A}^t}$ as $GAME[\pi]$ and denote the set of feasible protocols by Π.

Definition 3 *Consider* $GAME[\pi]$. *Then* σ^* *is an* order-independent equilibrium *for* $GAME[\pi]$ *if for any* $\pi' \in \Pi$, *there exists an equilibrium* σ'^* *of* $GAME[\pi']$ *such that* σ^* *and* σ'^* *lead to the same distributions of equilibrium governments* $G^\tau \mid G^t$ *for* $\tau > t$.

We will establish the relationship between acyclic and order-independent equilibria in Theorem 2.

3 The Structure of Equilibria: The Nonstochastic Case

In this section, we provide general existence and characterization results for the environment described above when there are no stochastic changes in the competences or in the set of winning coalitions. In particular, in this section, and in fact until Section 6, we assume that Γ_G^t and $\mathcal{W}_G^t$ do not depend on time, and thus

$$\Gamma_G^t = \Gamma_G \text{ and } \mathcal{W}_G^t = \mathcal{W}_G \text{ for all } t.$$

In describing the structure of equilibrium, we will make use of a mapping

$$\phi : \mathcal{G} \to \mathcal{G}.$$

This mapping will designate a government $\phi\left(G^0\right)$ that will eventually emerge and persist when the initial government is G^0. The most intuitive way of developing our analysis is to first construct the mapping ϕ, which will play this role, and then show that, first of all, there exist equilibria corresponding to this mapping and characterize the equilibrium strategies leading to this type of behavior, and second, that equilibria leading to this behavior are more likely, in a sense that we precisely define below. We will also discuss what other types of equilibria can emerge and how these other equilibria can be ruled out.

To construct the mapping ϕ, let us first enumerating the (finite) set of feasible governments $\mathcal{G}$ as: $G_1, G_2, \ldots, G_{|\mathcal{G}|}$ so that $\Gamma_{G_x} > \Gamma_{G_y}$ whenever $x < y$. With this enumeration, G_1 is the most competent ("best") government. The mapping ϕ will now be constructed inductively. Take government G_1 and consider, which government can successfully challenge G_1 (we are leaving the question of whether such government will make its way through the primaries aside). Naturally, there is no other government G' which is preferred to G_1 by a winning coalition in G_1. Indeed, Assumption 1 implies that all all members of G_1 prefer G_1 to G' and so do individuals who are not members of G_1 or G'. Assumption 2 then ensures that no proposal will generate a set of votes that is sufficient to change this government. This reasoning holds even if individuals anticipate that alternative G' will eventually lead to government G'' to persist: indeed, we can apply this reasoning to G'' instead of G'. Therefore, G_1 will remain as the equilibrium government forever and thus $\phi\left(G_1\right) = G_1$ according to our desired mapping ϕ.

Next suppose the game starts with government G_2 in power. By the same argument as above, governments $G_3, G_4, \ldots, G_{|\mathcal{G}|}$ will not generate sufficient votes to come to power starting with G_2. So, the only choice for the society is between G_1 and G_2. In this comparison, G_1 will be preferred if it has sufficiently many supporters, that is, if the set of individuals preferring G_1

to G_2 forms a winning coalition within G_2, or more formally if

$$\{i \in \mathcal{I} : w_i(G_1) > w_i(G_2)\} \in \mathcal{W}_{G_2}.$$

If this is the case, we write $\phi(G_2) = G_1$; otherwise, $\phi(G_2) = G_2$.

Next suppose that we start with government G_3. Consider the choice between G_1, G_2, and G_3. To move to G_1, it suffices that a winning coalition within G_3 prefers G_1 to G_3. (If some winning coalition also prefers G_2 to G_3, then G_1 should still be chosen over G_2, because only members of G_2 who do not belong to G_1 prefer G_2 to G_1, but again according to Assumption 2, they are not sufficient to block the choice of G_1 over G_2 within G_3, and so G_1 will be chosen during the primaries). However, even if G_1 is not preferred to G_3 by a winning coalition in G_3 but G_2 is, this does not guarantee that the transition to G_2 will take place. In particular suppose that $\phi(G_2) = G_1$. Then a transition to G_2 will lead to a transition to G_1 in the next period, since the newly-formed government G_2 is in a politically unstable situation. But this may not be desirable. In particular, we may have that

$$\{i \in \mathcal{I} : w_i(G_2) > w_i(G_3)\} \in \mathcal{W}_{G_3},$$

but

$$\{i \in \mathcal{I} : w_i(G_1) > w_i(G_3)\} \notin \mathcal{W}_{G_3}.$$

If so, the transition from G_3 to G_2 may be blocked with the anticipation that it will rapidly lead to the government G_1 which does not receive the support of a winning coalition within G_3. This reasoning illustrates that for a transition to take place, not only the target government should be preferred to the current one by a winning coalition (within the current government), but also that the target government should be "stable".

The discussion above provides the intuitive justification for our definition of the mapping ϕ provided next. Theorem 1 then provides a proof that this mapping will indeed represent the structure of (acyclic) equilibria. Suppose we have defined ϕ for G_j with $j < q$ where $q > 1$. Let

$$\mathcal{M}_q = \left\{j : 1 \leq j < q,\ \{i \in \mathcal{I} : w_i(G_j) > w_i(G_q)\} \in \mathcal{W}_{G_q}, \text{ and } \phi(G_j) = G_j\right\}. \tag{3}$$

Then we define

$$\phi(G_q) = \begin{cases} G_q & \text{if } \mathcal{M}_q = \varnothing; \\ G_{\min\{j \in \mathcal{M}_q\}} & \text{if } \mathcal{M}_q \neq \varnothing. \end{cases} \tag{4}$$

We have now constructed mapping ϕ. It can be verified easily that this mapping is well-defined and unique. We are now in a position to establish the following key theorem. This theorem will then enable us to obtain various characterization results on the structure of governments and selection of politicians, including the implications of perfect democracies, imperfect democracies and dictatorships for the selection of politicians that we discussed in the Introduction.

Theorem 1 *Consider the game described above. Suppose that Assumptions 1-3 hold and let* $\phi : \mathcal{G} \to \mathcal{G}$ *be the mapping defined by* (4)*. Then there exists* $\beta_0 < 1$ *such that for any discount factor* $\beta > \beta_0$*, for any protocol* $\pi \in \Pi$ *and for* r *sufficiently small:*

1. *There exists an acyclic MPE in pure strategies* σ^**.*
2. *Take an acyclic MPE in pure or mixed strategies* σ^**. Then under* σ^**, we have that:*
   - *if* $\phi\left(G^0\right) = G^0$*, then there are no transitions; and*
   - *if* $\phi\left(G^0\right) \neq G^0$*, then with probability* 1 *there exists a period* t *where the government* $\phi\left(G^0\right)$ *is proposed, wins the primaries, and wins the power struggle against* G^t*. After that, there are no transitions, so* $G^\tau = \phi\left(G^0\right)$ *for all* $\tau \geq t$*.*

Proof. See the Appendix. ■

The intuition for this theorem is provided by the heuristic construction of the mapping ϕ preceding the theorem. The hypothesis that r is sufficiently small ensures that stable political situations are sufficiently stable, so that if the government passes a "no-confidence" voting, it stays for some time. As it turns out, this is crucial to ensure that an equilibrium in pure strategies exists (which in turn allows us to obtain a characterization of equilibria). Example 8 in Appendix B illustrates the potential problem. We also assumed that the discount factor β is sufficiently large; the reason for this assumption is that we want the individuals to be forward-looking and take into account the transitions that will happen further along the equilibrium path.

We should note at this point that the acyclicity requirement is not redundant. Example 3 in the next section shows that there may be cyclic equilibria. for a certain set of protocols $\pi \in \Pi$. Cyclic equilibria are less natural than the equilibria we focus on and this motivates our focus on acyclic equilibria, while the discussion in the previous paragraph motivates us to consider the case with high β and small r.

It is also noteworthy that part 2 of Theorem 1 ensures that in any acyclic equilibrium there will eventually be a transition to government $\phi\left(G^0\right)$, but such a transition can be slow and in multiple steps. In the next section, we establish that order-independent equilibria not only ensure acyclicity, but also that all transitions take place "rapidly," i.e., at the first unstable period (which is period $t = 0$ by assumption).

4 Cycles, Acyclicity and Order-Independent Equilibria

We first show that cyclic MPEs are possible. The next example illustrates this.

Example 3 Consider a society consisting of five individuals ($n = 5$). The only feasible governments are $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}$. Suppose that there is "full democracy," so that in terms of Assumption 2 $l_G = l = 0$ for $G \in \mathcal{G}^k$, and that voting takes the form of a simple majority rule, so that again with the same notation $m_G = m = 3$ for all G. Suppose also that the competences of different feasible governments are given by

$$\Gamma_{\{i\}} = 5 - i,$$

so $\{1\}$ is the best government.

Assume also that instantaneous utilities are given as in Example 2. In particular,

$$w_i(G) = \Gamma_G + 100\mathbf{I}_{\{i \in G\}}.$$

These utilities imply that each individual receives a high value from being part of the government relative to the utility she receives from government competence.

Finally, we define the protocols $\pi_G^{\mathcal{A}}$ as follows. If $G = \{1\}$, then $\pi_G^{\mathcal{G}\setminus\{G\}} = \pi_{\{1\}}^{\{\{2\},\{3\},\{4\},\{5\}\}} = (\{3\}, \{4\}, \{5\}, \{2\})$ and $\pi_{\{1\}}^{\mathcal{A}}$ for $A \neq (\{3\}, \{4\}, \{5\}, \{2\})$ is obtained from $\pi_{\{1\}}^{\{2,3,4,5\}}$ by dropping governments which are not in A: for example, $\pi_{\{1\}}^{\{\{2\},\{3\},\{5\}\}} = (\{3\}, \{5\}, \{2\})$. For other governments, we define $\pi_{\{2\}}^{\{\{1\},\{3\},\{4\},\{5\}\}} = (\{4\}, \{5\}, \{1\}, \{3\})$, $\pi_{\{3\}}^{\{\{1\},\{2\},\{4\},\{5\}\}} = (\{5\}, \{1\}, \{2\}, \{4\})$, $\pi_{\{4\}}^{\{\{1\},\{2\},\{3\},\{5\}\}} = (\{1\}, \{2\}, \{3\}, \{5\})$ and $\pi_{\{5\}}^{\{\{1\},\{2\},\{3\},\{4\}\}} = (\{2\}, \{3\}, \{4\}, \{1\})$, and for other A again define $\pi_G^{\mathcal{A}}$ by dropping the governments absent in A. Then there exists an equilibrium where the governments follow a cycle of the form $\{5\} \to \{4\} \to \{3\} \to \{2\} \to \{1\} \to \{5\} \to \cdots$.

To verify this claim, consider the following nomination strategies by the individuals. If the government is $\{1\}$, two individuals nominate $\{2\}$ and other three nominate $\{5\}$; if it is $\{2\}$, two individuals nominate $\{3\}$ and three nominate $\{1\}$; if it is $\{3\}$, two nominate $\{4\}$ and three nominate $\{2\}$; if it is $\{4\}$, two nominate $\{5\}$ and three nominate $\{3\}$; if it is $\{5\}$, two nominate $\{1\}$ and three nominate $\{4\}$.

Let us next turn to voting strategies. Here we appeal to Lemma 1 from Acemoglu, Egorov and Sonin (2008), which shows that in this class of games, it is sufficient to focus on strategies in which individuals always vote for the alternative yielding the highest payoff for them at each stage. Then, note that in equilibrium, any alternative government which wins the primaries, on or off equilibrium path, subsequently wins against the incumbent government. In particular, in such an equilibrium supporting the incumbent government will break a cycle, but only one person (the member of the incumbent government) is in favor of it. We next show if only one individual deviates at the nomination stage, then next government in the cycle still wins in the primaries. Suppose that the current government is $\{3\}$ (other cases are treated similarly). Then by construction, governments $\{2\}$ and $\{4\}$ are necessarily nominated, and perhaps $\{1\}$ or $\{5\}$ also are. Notice that if the last voting in the primaries is between $\{2\}$ and $\{4\}$, then $\{2\}$ wins: indeed, all individuals know that both alternatives can take over the incumbent government, but $\{2\}$ is preferred by individuals 1, 2, and 5 (because they would want to be government members earlier rather than later). If, however, the last stage involves voting between $\{4\}$ on the one hand and either $\{1\}$ or $\{5\}$ on the other, then $\{4\}$ wins for similar reason. Now, if either $\{1\}$ or $\{5\}$ is nominated, then in the first voting it is voted against $\{2\}$. All individuals know that accepting $\{2\}$ will ultimately lead to a transition to $\{2\}$, whereas supporting $\{1\}$ or $\{5\}$ will lead to $\{4\}$. Because of that, at least three individuals (1, 2, 5) will support $\{2\}$. This proves that $\{2\}$ will win against the incumbent government $\{3\}$, provided that $\{2\}$ and $\{4\}$ participate in the primaries, which is necessarily the case if no more than one individual deviates. This, in turn, implies that nomination strategies are also optimal in the sense that there is no profitable one-shot deviation for any individual. We can easily verify that this holds for other incumbent governments as well.

We have thus proved that the strategies we constructed form a SPE; since they are also Markovian, it is a MPE as well. Along the equilibrium path, the governments follow a cycle $\{5\} \to \{4\} \to \{3\} \to \{2\} \to \{1\} \to \{5\} \to \cdots$. We can similarly construct a cycle that moves in the other direction: $\{1\} \to \{2\} \to \{3\} \to \{4\} \to \{5\} \to \{1\} \to \cdots$ (though this would require different protocols). Hence, for some protocols, cyclic equilibria are possible.

Intuitively, a cycle enables different individuals that will not be part of the limiting (stable) government to enjoy the benefits of being in power. This example, and the intuition we suggest, also highlight that even when there is a cyclic equilibrium, an acyclic equilibrium still exists (see, in particular, Theorem 2). Moreover, the acyclic and order-independent equilibria have additional desirable properties.

Example 3 shows that the order in which proposals are made may be crucial for supporting the cyclic equilibrium. In particular, it is straightforward to verify that the equilibrium does not survive if we take $\pi_G^{\mathcal{A}^t}$ to be the same for all G's. We can also construct examples in which there are no cycles, but the emergence of a stable government takes several transitions. This is illustrated in the next example. Theorem 2 below will show that when we focus on order-independent MPE, both cyclic equilibria and equilibria with multi-step transitions will be ruled out.

Example 4 Take the setup of Example 3, with the exception that $l_{\{1\}} = 1$ (so that consent of individual 1 is needed to change the government when the government is $\{1\}$). It is then easy to check that the strategy profile constructed in Example 3 is a MPE in this case as well. However, since individual 1 will vote against any alternative which wins the primaries, the difference is that alternative $\{5\}$ will not be accepted in equilibrium and government $\{1\}$ will persist. Hence, in equilibrium, the transitions are as follows: $\{5\} \to \{4\} \to \{3\} \to \{2\} \to \{1\}$.

We now establish that order-independent equilibria always exist, are always acyclic, and lead to rapid (one-step) equilibrium transitions. As such, this theorem will be the starting point of our more detailed characterization of the influence of political institutions on the selection of politicians and governments in the next section. The proof of this theorem also requires a slightly stronger version of Assumption 3, which we now introduce.

Assumption $3'$ *For any $i \in \mathcal{I}$ and any sequence of feasible governments, $H_1, H_2, \ldots, H_q \in \mathcal{G}$ (for $q \geq 2$), we have*

$$w_i(H_1) \neq \frac{\sum_{j=2}^{q} w_i(H_j)}{q-1}.$$

Recall that Assumption 3 imposed that no two feasible governments have exactly the same competence. Assumption $3'$ strengthens this and requires that the competence of any government should not be the average of the competences of other feasible governments. Like Assumption 3, Assumption $3'$ is satisfied "generically," in the sense that if it were not satisfied for a society, any small perturbation of competence levels would restore it.

Theorem 2 *Consider the game described above. Suppose that Assumptions 1, 2 and $3'$ hold and let $\phi : \mathcal{G} \to \mathcal{G}$ be the mapping defined by (4). Then there exists $\beta_0 < 1$ such that for any discount factor $\beta > \beta_0$, for any protocol $\pi \in \Pi$ and for r sufficiently small:*

1. *There exists an order-independent MPE in pure strategies σ^*.*

2. *Any order-independent MPE in pure strategies σ^* is acyclic.*

3. *In any order-independent MPE σ^*, we have that:*

   - *if $\phi(G^0) = G^0$, then there are no transitions and government $G^t = G^0$ for each t;*
   - *if $\phi(G^0) \neq G^0$, then there is a transition from G^0 to $\phi(G^0)$ in period $t = 0$, and there are no more transitions: $G^t = \phi(G^0)$ for all $t \geq 1$.*

4. In any order-independent MPE σ^*, the payoff of each individual $i \in \mathcal{I}$ is given by

$$u_i^0 = w_i\left(G^0\right) + \frac{\beta}{1-\beta} w_i\left(\phi\left(G^0\right)\right).$$

Proof. See the Appendix. ■

5 Characterization of Non-Stochastic Transitions

In this section, we provide the comparison of different political regimes in terms of their ability to select governments with high levels of competence. To simplify the exposition and focus on the more important interactions, we make a number of additional assumptions. First, we strengthen Assumption 1, so that individuals always prefer governments that they are part of to those that exclude them. In particular:

Assumption 1′ *Assumption 1 holds and also*

$$\textit{for any } G, H \in \mathcal{G} \textit{ and any } i \in G \setminus H,\ w_i(G) > w_i(H).$$

We also assume that all feasible governments have the same size, $k \in \mathbb{N}$, where $k < n/2$. More formally, let us define

$$\mathcal{C}^k = \{Y \in \mathcal{C} : |Y| = k\}.$$

Then, $\mathcal{G} = \mathcal{C}^k$. In addition, we assume that for any $G \in \mathcal{G}$, $l_G^t = l \in \mathbb{N}$ and $m_G^t = m \in \mathbb{N}$, so that the set of winning coalitions can be simply expressed as

$$\mathcal{W}_G = \{X \in \mathcal{C} : |X| \geq m \text{ and } |X \cap G| \geq l\}, \tag{5}$$

where $0 \leq l \leq k < m \leq n-k$. This specification implies that given n, k, and m, the number l corresponds to an inverse measure of democracy. If $l = 0$, then all individuals have equal weight and there is no incumbency advantage, thus we have a *perfect democracy.* In contrast, if $l > 0$, the consent of some of the members of the government is necessary for a change, thus we have an *imperfect democracy.* We thus have strengthened Assumption 2 to the following.

Assumption 2′ *We have that $\mathcal{G} = \mathcal{C}^k$, and that there exist integers l and m such that the set a winning coalitions is given by (5).*

Given this additional structure, equations (3) and (4) that determine the mapping ϕ can be written in a simpler form. Recall that governments are still ranked according to their level of competence, so that G_1 denotes the most competent government. Then we have:

$$\mathcal{M}_q = \{j : 1 \leq j < q,\ |G_k \cap G_q| \geq l, \text{ and } \phi(G_j) = G_j\}, \tag{6}$$

and, as before,

$$\phi(G_q) = \begin{cases} G_q & \text{if } \mathcal{M}_q = \varnothing; \\ G_{\min\{j \in \mathcal{M}_q\}} & \text{if } \mathcal{M}_q \neq \varnothing. \end{cases} \tag{7}$$

Naturally, the mapping ϕ is again well-defined and unique. Finally, let us also define

$$\mathcal{D} = \{G \in \mathcal{G} : \phi(G) = G\}$$

as the set of stable governments (the fixed points of mapping ϕ). If $G \in \mathcal{D}$, then $\phi(G) = G$, and this government will persist forever if it is the initial government of the society.

Given this more specific environment, we now investigate the structure of stable governments and how it changes as a function of the underlying political institutions, in particular, the degree of democracy. Throughout this section, we assume that Assumptions $1'$, $2'$ and $3'$ hold and we focus on order-independent MPE as characterized in Theorem 2. We do not add these qualifiers to any of the propositions to economize on space.

Our first result establishes some useful technical results. More importantly, it shows that perfect democracy ensures the emergence of the best (most competent) government, but any departure from perfect democracy destroys this result and enables the emergence of highly incompetent/inefficient governments.

Proposition 1 *The set of stable feasible governments $\mathcal{D}$ satisfies the following properties.*

1. *If $G, H \in \mathcal{D}$ and $|G \cap H| \geq l$, then $G = H$. In other words, any two distinct stable governments may have at most $l - 1$ common members.*

2. *Suppose that $l = 0$, so that the society is a* perfect democracy*. Then $\mathcal{D} = \{G_1\}$. In other words, starting from any initial government, the society will transition to the most competent government.*

3. *Suppose $l \geq 1$, so that the society is an* imperfect democracy *or a* dictatorship*. Then there are at least two stable governments, i.e., $|\mathcal{D}| \geq 2$. Moreover, the least competent governments may be stable.*

4. *Suppose $l = k$, so that the society is an* extreme dictatorship*. Then $\mathcal{D} = \mathcal{G}$, so any feasible government is stable.*

Proof. See the Appendix. ■

Proposition 1 shows the fundamental contrast between perfect democracy, where there is no incumbency advantage, and other political institutions, which provide some additional power to "insiders" (current members of the government). With perfect democracy, the best government will necessarily emerge. With any deviation from perfect democracy, there will necessarily exist at least one other stable government (by definition less competent than the best) and even the worst government might be stable. The next example supplements Example 1 from the Introduction by showing a richer environment in which the least competent government is stable.

Example 5 Suppose $n = 9$, $k = 3$, $l = 1$, and $m = 5$, so that a change in government requires support from a simple majority of the society, including at least one member of the current government. Suppose $I = \{1, 2, \ldots, 9\}$, and that instantaneous utilities are given by (1) in Example 2. Assume also that $\Gamma_{\{i_1,i_2,i_3\}} = 1000 - 100i_1 - 10i_2 - i_3$, provided that $i_1 < i_2 < i_3$.

Then $\{1, 2, 3\}$ is the most competent government, and is therefore stable. Any other government that includes 1 or 2 or 3 is unstable. For example, the government $\{2, 5, 9\}$ will transit to $\{1, 2, 3\}$, as all individuals except 5 and 9 prefer the latter. However, government $\{4, 5, 6\}$ is stable: any more competent government must include 1 or 2 or 3, and therefore is either $\{1, 2, 3\}$ or will immediately transit to $\{1, 2, 3\}$, which means that any such transition will not get support by any of the members of $\{4, 5, 6\}$. Now, proceeding inductively, we find that any government other than $\{1, 2, 3\}$ and $\{4, 5, 6\}$ that contains at least one individual $1, 2, \ldots, 6$ is unstable. Consequently, government $\{7, 8, 9\}$, which is the least competent government, is stable.

The next example shows that, starting with the same government, the long-run equilibrium government may be worse when political institutions are more democratic (as long as we are not in a perfect democracy).

Example 6 Take the setup from Example 5 ($n = 9$, $k = 3$, $l = 1$, and $m = 5$), and suppose that the initial government is $\{4, 5, 6\}$. As we showed there, government $\{4, 5, 6\}$ is stable, and will therefore will persist. Suppose, however, that $l = 2$ instead. In that case, $\{4, 5, 6\}$ is unstable, and $\phi(\{4, 5, 6\}) = \{1, 4, 5\}$, therefore, there will be a transition to $\{1, 4, 5\}$. Since $\{1, 4, 5\}$ is more competent than $\{4, 5, 6\}$, this is an example where the long-run equilibrium government is worse under $l = 1$, when the institutions are more democratic, than under $l = 2$, when they are less democratic. Note that if $l = 3$, $\{4, 5, 6\}$ would be stable again.

When the number of individuals, n, is sufficiently large, we can provide a tighter characterization of the structure of stable governments. This is done in the next proposition.

Proposition 2 *Suppose $l \geq 1$.*

1. *If*

$$n \geq 2k + k\,(k-l)\,\frac{(k-1)!}{(l-1)!\,(k-l)!}, \tag{8}$$

then there exists a stable government $G \in \mathcal{D}$ that contains no members of the ideal government μ_1.

2. *Take any $x \in \mathbb{N}$. If*

$$n \geq k + x + x\,(k-l)\,\frac{(k-1)!}{(l-1)!\,(k-l)!}, \tag{9}$$

then for any set of individuals X with $|X| \leq x$ there exists a stable government $G \in \mathcal{D}$ such that $X \cap G = \varnothing$ (so no member of set X belongs to G).

Proof. See the Appendix. ■

Let us provide the intuition for Proposition 2 when $l = 1$. Recall that G_1 is the most competent government. Let G be the most competent government among those that do not include members of G_1 (such G exists, since $n > 2k$ by assumption). In this case, Proposition 2 implies that G is stable, that is, $G \in \mathcal{D}$. The reason is that if $\phi(G) = H \neq G$, then $\Gamma_H > \Gamma_G$, and therefore $H \cap G_1$ contains at least one element by construction of G. But then $\phi(H) = G_1$, as implied by (7). Intuitively, if $l = 1$, then once the current government contains a member of the most competent government G_1, this member will consent to (support) a transition to G_1, which will also receive the support of the population at large. She can do so, because G_1 is stable, thus there is no threats that a further round of transitions will harm her. But then, as in Example 1 in the Introduction, G itself becomes stable, because any reform away from G will take us to an unstable government.[2]

We will next strengthen our characterization results by putting some more structure on the competences of governments. For this reason, suppose that each individual $i \in \mathcal{I}$ has a level of ability (or competence) given by $\gamma_i \in \mathbb{R}_+$ and suppose that the competence of the government is a strictly increasing function of the abilities of its members. More formally, we impose the following assumption throughout the rest of the analysis (again without explicitly stating it in each proposition).

Assumption 4 *Suppose $G \in \mathcal{G}$, and individuals $i, j \in \mathcal{I}$ are such that $i \in G$, $j \notin G$, and $\gamma_i \geq \gamma_j$. Then $\Gamma_G \geq \Gamma_{(G \setminus \{i\}) \cup \{j\}}$.*

[2] It is also interesting to note that the upper bound on X in Part 2 of Proposition 2 is $O(x)$, meaning that, increasing x does not require an exponential increase in the size of population n for Proposition 2 to hold.

Assumption 3 now implies that $\gamma_i \neq \gamma_j$ whenever $i \neq j$. The canonical form of the competence function consistent with Assumption 4 is

$$\Gamma_G = \sum\nolimits_{i \in G} \gamma_i, \tag{10}$$

though for most of our analysis, we do not need to impose the specific functional form.

Given Assumption 4, it is also useful to enumerate individuals according to their abilities, so that $\gamma_i > \gamma_j$ whenever $i < j$. Recall also that $\lfloor x \rfloor$ denotes the integer part of a real number x.

Proposition 3 *The set of stable governments $\mathcal{D}$ takes the following forms.*

1. *If $l = 0$, then $\mathcal{D}$ is a singleton and the unique stable government is $\{1, 2, ..., k\}$.*
2. *If $l = 1$, then $\mathcal{D}$ consists of $\lfloor n/k \rfloor$ elements of the form $\{qk + 1, qk + 2, \ldots, qk + k\}$, where q is an integer such that $0 \leq q \leq \lfloor n/k \rfloor - 1$.*

Proof. See the Appendix. ■

Naturally, the results that when $l = k$ all governments are stable still applies from Proposition 1. In addition, when either $k = 1$ or $k = 2$, we have:

Proposition 4 1. *If $k = 1$, then either $l = 0$ (perfect democracy), in which case $\phi(G) = \{G_1\} = \{1\}$ for any $G \in \mathcal{G}$, or $l = 1 = k$ (extreme dictatorship), in which case $\phi(G) = G$ for any $G \in \mathcal{G}$.*

2. *If $k = 2$, and $l = 0$ (perfect democracy), then $\phi(G) = G_1 = \{1, 2\}$ for any $G \in \mathcal{G}$. If $k = 2$ and $l = 1$ (imperfect democracy), then if $G = \{p, q\}$ with $p < q$, then $\phi(G) = \{p - 1, p\}$ if p is even and $\phi(G) = \{p, p + 1\}$ if p is odd; in particular, $\phi(G) = G$ if and only if p is odd and $q = p + 1$. If $k = 2$ and $l = 2$ (perfect dictatorship), then $\phi(G) = G$ for any $G \in \mathcal{G}$.*

Proof. See the Appendix. ■

Proposition 4, though simple, provides an important insight about the structure of stable governments that will be further exploited in the next section. In the case of an extreme dictatorship, all governments are stable, so the initial government persists forever. In contrast, a perfect ensures the emergence of the competent feasible government. The case of imperfect democracy, highlighted in the case $k = 2$, lies somewhere in between. In that case, the competence of the limiting government is determined by the more able of the two members of the initial government. This means that, with rare exceptions, the initial quality of government will be improved to some degree, but large improvements are not possible, because one of the two members must still be part of the ultimate government. Therefore, summarizing these three cases, we can say that with a perfect democracy, the best government will arise; with an extreme dictatorship, there will be no improvement in the initial government; and with an imperfect democracy, there will be some limited improvements in the quality of the government.

When $k \geq 3$ and $l \geq 2$, the structure of stable governments is more complex, and this is illustrated in the next example.

Example 7 Suppose that $k = 3$ and $l = 2$. Then the following are stable governments that include the most able individual, 1: $\{1, 2, 3\}$, $\{1, 4, 5\}$, $\{1, 6, 7\}$, and so on. Similarly, the following are stable governments that include individual 2 but not 1: $\{2, 4, 6\}$, $\{2, 5, 7\}$, and so on. The most competent government that does not include 1 and 2 might then be either $\{3, 4, 7\}$ or $\{3, 5, 6\}$, depending on which one is more competent.

6 The Structure of Equilibria: The Stochastic Case

In this section, we generalize the environment studied so far by allowing for stochastic shocks. Our objective is to compare the implications of different political institutions for the selection of governments in (stochastically) changing environments. In particular, we would like to understand whether certain institutional features ensure better long-run outcomes by providing greater "flexibility". The main source of stochasticity we have in mind is changes in the structure of appropriate governments. For example, changes in economic, political, or social environment may necessitate a different type of government to deal with the newly-emerging problems. In the context of our framework, this can be modeled as changes in the function $\Gamma_G^t : \mathcal{G} \to \mathbb{R}$, which determines the competence associated with each feasible government. We also allow the mapping $\mathcal{W}_G^t$, which determines the set on winning coalitions as a function of the incumbent government, to change stochastically, though this is less important for our substantive results.

Formally, we assume that at each t, with probability $1 - \delta$, there is no change in Γ_G^t and $\mathcal{W}_G^t$ from Γ_G^{t-1} and $\mathcal{W}_G^{t-1}$, and with probability δ, there is a shock and Γ_G^t and $\mathcal{W}_G^t$ will change. In particular, following such a shock we assume that there exist two (sets of) distribution functions $F_\Gamma\left(\Gamma_G^t \mid \Gamma_G^{t-1}\right)$ and $F_\mathcal{W}\left(\mathcal{W}_G^t \mid \mathcal{W}_G^{t-1}\right)$ that give the conditional distribution of Γ_G^t and $\mathcal{W}_G^t$ at time t as functions of Γ_G^{t-1} and $\mathcal{W}_G^{t-1}$. We will focus on the case where δ is small. In addition, throughout this section, we simplify the discussion by assuming that Assumption 4 holds and by focusing on order-independent MPEs. The next theorem generalizes Theorem 2 to this stochastic environment.

Theorem 3 *Consider the above-described stochastic environment. Suppose that Assumptions 1, 2,* $3'$*, and 4 hold. Let* $\phi^t : \mathcal{G} \to \mathcal{G}$ *be the mapping defined by* (4) *for* Γ_G^t *and* $\mathcal{W}_G^t$*. Then there exists* $\beta_0 < 1$ *such that for any discount factor* $\beta > \beta_0$*, for any protocol* $\pi \in \Pi$*, and for* r *and* δ *sufficiently small (meaning that shocks are sufficiently infrequent), we have the following results.*

1. *There exists an order-independent MPE in pure strategies.*

2. *Suppose that between periods* t_1 *and* t_2 *there are no shocks. Then in any order-independent MPE in pure strategies, the following results hold:*

    - *if* $\phi\left(G^{t_1}\right) = G^{t_1}$*, then there are no transitions between* t_1 *and* t_2*;*
    - *if* $\phi\left(G^{t_1}\right) \neq G^{t_1}$*, then alternative* $\phi\left(G^{t_1}\right)$ *is accepted during the first period of instability (after* t_1*).*

Proof. See the Appendix. ■

In the rest of this section, we provide further characterization results and the comparisons of this and regimes in this environment. In the results that follow, we always assume that Assumptions $1'$, $2'$, $3'$, and 4 hold, and we focus on order-independent MPEs. We do not add these qualifiers to economize on notation in the statement of the propositions.

First consider the case where the set of winning coalitions, $\mathcal{W}_G^t$, does not change, so that $\mathcal{W}_G^t = \mathcal{W}_G^0$ for all t. In addition, let us also put some additional structure on the distribution $F_\Gamma\left(\Gamma_G^t \mid \Gamma_G^{t-1}\right)$. In particular, let us assume that any shock corresponds to a rearrangement of the abilities of different individuals (and thus $F_\Gamma\left(\Gamma_G^t \mid \Gamma_G^{t-1}\right)$ gives the induced distribution of government competences according to Assumption 4). Put differently, there is a fixed vector of abilities, say $a = \{a_1, ..., a_n\}$, and the actual distribution of abilities across individuals at time t, $\left\{\gamma_j^t\right\}_{j=1}^n$ is given by some permutation of this vector a. We adopt the convention that $a_1 > a_2 > ... > a_n$. Intuitively, this captures the notion that a shock will change which individual is the best placed to solve certain tasks and thus most effective in government functions.

The next proposition shows the difference in flexibility of different regimes.

Proposition 5 *Suppose that* $\mathcal{W}_G^t = \mathcal{W}_G^0$ *for all* t*. Suppose also that there is a fixed vector of abilities* $a = \{a_1, ..., a_n\}$ *and the distribution of abilities at time* t*,* $\left\{\gamma_j^t\right\}_{j=1}^n$*, is given by a permutation* φ^t *of* a*. Then, the following are true.*

1. *If* $l = 0$ *(i.e., perfect democracy), then a shock immediately leads to the replacement of the current government by the new most competent government.*

2. *If $l = 1$ (i.e., imperfect democracy), the competence of the government following a shock never decreases further and will increase with probability no less than*

$$1 - \frac{(k-1)!\,(n-k)!}{(n-1)!} = 1 - \frac{1}{\binom{n-1}{k-1}}.$$

The probability that the most competent government will ultimately come to power as a result of a shock is

$$1 - \frac{\binom{n-k}{k}}{\binom{n}{k}} < 1.$$

This probability tends to 0 for fixed k as $n \to \infty$.

3. *If $l = k$ (i.e., extreme dictatorship), then a shock never leads to a change in government. The probability that the most competent government is in power at any given period after the shock is*

$$\frac{1}{\binom{n}{k}}.$$

This probability is strictly less than the corresponding probabilities when $l = 0$ and $l = 1$.

Proof. See the Appendix. ■

Proposition 5 contains a number of important results. A perfect democracy does not create any barriers against the installation of the best government at any point in time, thus every shock is, flexibly, met by a change in government according to the wishes of the population at large (which here means that the most competent government will come to power). As we know from the analysis of Section 5, this is no longer true as soon as members of the governments have incumbency advantage. In particular, we know that without stochastic shocks, arbitrarily incompetent governments may come to power and remain in power. However, in the presence of shocks there are new forces affecting the structure of equilibrium governments.

Even though the immediate effect of a shock may be a deterioration in government competence, there are forces that increase government competence in the long run. This is most clearly illustrated in the case where $l = 1$. With this set of political institutions, there is zero probability that there will be a further decrease in government competence following a shock. Moreover, there is a positive probability that competence will improve and in fact a positive probability that, following a shock, the most competent government will be instituted. This is intuitive: a shock may make the current government unstable, and in this case, there will be a transition to a new stable government. A transition to a less competent government would never receive support from the population. The change in abilities may be such that the only stable government after the shock, starting with the current government, may be the best government.

Nevertheless, the probability of the most competent government coming to power, though positive, may be arbitrarily low. Proposition 5 also shows that when political institutions take the form of an extreme dictatorship, there will never be any transition, thus the current government can deteriorate following shocks and can do so significantly.

Perhaps most importantly, Proposition 5 also shows that imperfect democracy has a higher degree of *flexibility* than dictatorship, ensuring better long-run outcomes (and naturally perfect democracy has the highest degree of flexibility). This unambiguous ranking between imperfect democracy and dictatorship in the presence of stochastic shocks contrasts with the results in Section 5, which showed that general comparisons between imperfect democracy and dictatorship are not possible in the nonstochastic case. This highlights that a distinct advantage of more democratic regimes might be their flexibility in the face of changing environments.

The next proposition strengthens the conclusions of Proposition 5. In particular, it establishes that the probability of having the most competent government in power is increasing in the degree of democracy (i.e., it is decreasing in l).[3]

Proposition 6 *Suppose that $\mathcal{W}_G^t = \mathcal{W}_G^0$ for all t. Suppose also that there is a fixed vector of abilities $a = \{a_1, ..., a_n\}$ and the distribution of abilities at time t, $\left\{\gamma_j^t\right\}_{j=1}^n$, is given by a permutation φ^t of a. Then, probability of having the most competent government is decreasing in l; i.e., more democratic regimes are more likely to produce the most competent government.*

Proof. See the Appendix. ■

The results of Propositions 5 and 6 can be strengthened further, when shocks are limited so that only the abilities of two (and in the second part of the proposition x) individuals and society are swapped.

Proposition 7 *Suppose that $\mathcal{W}_G^t = \mathcal{W}_G^0$ for all t. Suppose also that there is a fixed vector of abilities $a = \{a_1, ..., a_n\}$ and any shock swaps the abilities of x individuals in the society.*

1. *If $x = 2$ (so that the abilities of two individuals are swapped at a time), then the competence of the government in poweris nondecreasing in time. Moreover, if the probability of swapping of abilities between any two individuals is positive, then the most competent government will be in power as $t \to \infty$ with probability 1.*

[3]This conclusion is not true for "expected competence" of the government, since we have not made "cardinal" assumptions on abilities. In particular, it is possible that some player is not a member of any stable government for some l and becomes part of a stable government for some $l' < l$. If this player has very low ability, then expected competence under l' may be lower. In the Appendix, we illustrate this with an example, and we also show that expected competence of government is monotone in l when l is close to 0 or to k.

2. *If $x > 2$, then the results in part 1 hold provided that $l \leq k - \lfloor x/2 \rfloor$.*

Proof. See the Appendix. ■

An interesting application of Proposition 7 is that when shocks are (relatively) rare and limited in their scope, relatively democratic regimes will gradually improve over time and institute the most competent government in the long run. This is not true for the most autocratic governments, however. This proposition, therefore, strengthens the conclusions of Propositions 5 and 6 in highlighting the flexibility benefits of more democratic regimes.

We next briefly discuss the results when shocks only affect the set of winning coalitions. In particular, we look at shocks that affect the "degree of democracy". Although such shocks may be of limited practical importance, the following proposition is useful in highlighting a number of forces present in our model and will be useful for what follows.

Proposition 8 *Suppose that $\Gamma_G^t = \Gamma_G^0$ for all t. Then:*

1. *Any shock to democracy parameter (i.e., any shocks such that $l^t \neq l^{t-1}$) does not decrease the competence level of the government in power and may increase it.*

2. *Suppose that conditional on a shock in period t, $\Pr\left(l^t = 0 \mid l^{t-1}\right) > 0$ (i.e., there is positive probability of moving to perfect democracy). Then, the most competent government will be in power as $t \to \infty$ with probability 1.*

Proof. See the Appendix. ■

This result is intuitive. When a shock occurs, either the current government continues to be stable or it is destabilized and replaced by a better government. When a change in government happens, the new government is better in absolute terms, since the competence of the current government is unchanged as a result of the shock (by the assumption that $\Gamma_G^t = \Gamma_G^0$ for all t). The somewhat surprising result is that this is true even when the shock reduces the degree of democracy. Intuitively, a decrease in the degree of democracy may destabilize a stable government, and since the next government cannot have lower competence, the shock will have led to an increase in the equilibrium competence level of the government. In addition, Proposition 8 shows that if there is a positive probability of perfect democracy emerging, the best government will be instituted at some point and this will never be reversed (even when the degree of democracy falls subsequently). Nevertheless, we should also note that the possibility of decreases in the degree of democracy leading to potentially increases in the competence of the government, highlighted in Proposition 8, is a consequence of the assumption that $\Gamma_G^t = \Gamma_G^0$

for all t. As Propositions 5-7 show, when shocks also affect the competence levels of alternative governments, a greater degree of democracy leads to better long-run quality of government, because it facilitates adaptation to changes. It is precisely because there are no such changes in the rankings of governments requiring adaptation that, in Proposition 8, a decrease in the degree of democracy can improve the quality of government by destabilizing an incompetent, but previously stable government.

The political institutions considered so far are "junta-like" in the sense that no single member is essential. Incumbency advantage takes the form of the requirement that some members of the current government must consent to change. The alternative is a "royalty-like" environment where one or several members of the government are irreplaceable. All else equal, this can be conjecture to be a negative force, since it would mean that a potentially low ability person must always be part of the government. However, the situation is more complex, because such an irreplaceable member (the member of the "royalty") is also unafraid of changes, whereas, as we have seen, junta members would resist certain changes because of the further transitions that these will unleash.

More formally, we change Assumption 2 and the structure of the set of winning coalitions $\mathcal{W}_G$ to accommodate "royalty-like" situations. We assume that there are l royalty individuals whose votes are *always* necessary for a transition to be implemented (regardless of whether they are current government members). We denote the set of these individuals by Y. So, the new set of winning coalitions becomes

$$\mathcal{W}_G = \{X \in \mathcal{C} : |X| \geq m \text{ and } Y \subset X\}.$$

We also assume that all royal individuals are members of the initial government, that is,$Y \subset G^0$. The next proposition characterizes the structure of equilibrium in this case.

Proposition 9 *Suppose that $\mathcal{W}_G^t = \mathcal{W}_G^0$ for all t. Suppose also that there is a fixed vector of abilities $a = \{a_1, ..., a_n\}$ and the distribution of abilities at time t, $\left\{\gamma_j^t\right\}_{j=1}^n$, is given by a permutation φ^t of a. In addition, suppose that we have a royalty-system with $1 \leq l < k$ and competences of governments are given by (10). Then the l royalty individuals are never removed from the government. Moreover, if $\{a_1, ..., a_n\}$ is sufficiently "convex" meaning that $\frac{a_1 - a_2}{a_2 - a_n}$ is sufficiently large, then the expected competence of the government under the royalty-system is greater than under the original, junta-like system. The opposite conclusion holds if $\frac{a_1 - a_{n-1}}{a_{n-1} - a_n}$ is sufficiently low and $l = 1$.*

Proof. See the Appendix. ■

Proposition 9 shows that royalty-like systems perform better in the long run than junta-like systems when $\{a_1, ..., a_n\}$ is highly "convex," which increases the premium of having the highest ability individual in government. Juntas are unlikely to lead to such an outcome because of the fear of a change leading to a further round of changes, excluding all initial members of the junta. Royalty-like systems avoid this and may therefore perform better even though they may keep a low ability royalty as part of the government. This reasoning also clarifies why when $\{a_1, ..., a_n\}$ is sufficiently "concave," the opposite conclusion holds. This result is interesting because it suggests that different types of dictatorships may have quite distinct implications for long-run quality of government and performance, and regimes that provide security to certain members of the incumbent government may be better at dealing with changes and in ensuring relatively high-quality governments in the long run.

7 Conclusion

In this paper, we provided a tractable dynamic model of political selection. The main barrier to the selection of good politicians and to the formation of good governments in our model is not the difficulty of identifying competent or honest politicians, but the incumbency advantage of current governments. Our framework shows how a small degree of incumbency advantage can lead to the persistence of highly inefficient and incompetent governments. The intuition for this pattern is that incumbency advantage implies that one of (potentially many) members of the government needs to consent to a change in the composition of government. However, all current members of the government may recognize that any change may unleash a further round of changes, ultimately unseating themselves. In this case, they will all oppose any change in government, even if such changes can improve welfare significantly for the rest of the society, and highly incompetent governments can remain in power.

Using this framework, we study the implications of different political institutions for the selection of governments both in nonstochastic and stochastic environments. A perfect democracy corresponds to a situation in which there is no incumbency advantage, thus citizens can nominate alternative governments and vote them to power without the need for the consent of any member of the incumbent government. In this case, we show that the most competent government will always come to power. However, interestingly, any deviation from perfect democracy breaks this result and the long-run equilibrium government can be arbitrarily incompetent (relative to the best possible government). In extreme dictatorship, where any single member of the current government has a veto power on any change, the initial government always remains in power and this can be arbitrarily costly for the society. Perhaps more surprisingly, the same is true for any political institution other than perfect democracy. Moreover, there is no obvious ranking between different sets of political institutions (other than perfect democracy and extreme dictatorship) in terms of what they will imply for the quality of long run government. Even though no such ranking across political institutions is possible, we provide a fairly tight characterization of the structure of stable governments in our benchmark nonstochastic society.

When we turn to stochastic environments, however, a distinct advantage for more democratic institutions emerges. In stochastic environment, either the abilities and competences of individuals or the needs of government functions change, shuffling the ranking of different possible governments in terms of their competences. A greater degree of democracy then ensures greater "adaptability" or flexibility. Perfect democracy is most flexible and immediately adjusts to any shock by instituting the new government that has the greatest competence after the shock. Extreme dictatorship is at the other extreme and again leads to no change in the initial government. Therefore, shocks that reduce the competence of the individuals currently in power can lead to significant deterioration in the quality of government. Perhaps most interestingly, more democratic political institutions allow some degree of flexibility in response to shocks. In particular, shocks cannot lead to the emergence of a worse government (relative to the competence of the government the power following the shock). They may, however, destabilize the current government and induce the emergence of a more competent government. We show that political institutions with a greater degree of democracy have higher probability of improving the competence of the government following a shock and ultimately instituting the most competent government.

Finally, we also compare "junta-like" and "royalty-like" regimes. The former is our benchmark society, where change in government requires the consent or support of one or multiple members of the current government. The second corresponds to situations in which one or multiple individuals are special and must always be part of the government (hence the title "royalty"). If royal individuals have low ability, royalty-like regimes can lead to the persistence of highly incompetent governments. However, we also show that in stochastic environments royalty-like regimes may lead to the emergence of higher quality governments in the long run than junta-like regimes. This is because royal individuals are not afraid of changes in governments, because their powers are absolute. In contrast, members of the junta may resist changes in government even if this increases its quality because such changes may lead to another round of changes, ultimately excluding all members of the initial government.

There are various interesting research areas highlighted by our analysis. Most important is an extension of this framework that combines the dynamic considerations emphasized here with the asymmetric information issues emphasize in the previous literature. For example, we can generalize the environment in this paper such that the ability of an individual is not observed until he becomes part of the government. In this case, to institute high quality governments, it is necessary to first "experiment" with different types of governments. The dynamic interactions highlighted by our analysis will then become a barrier to such experimentation. In this case, the set of political institutions that will ensure high quality governments must exhibit a different type of flexibility, whereby some degree of "churning" of governments can be guaranteed even without shocks. Another interesting area is to introduce additional instruments, so that some political regimes can provide incentives to politicians to take actions in line with the interests of the society at large. In that case, successful political institutions must ensure both the selection of high ability individuals and the provision of incentives to to these individuals once they are in government. We view these directions as interesting and important challenges for future research.

Appendix A

In the appendix, we use the following notation. First, we introduce the following binary relation on the set of feasible governments $\mathcal{G}$. For any $G, H \in \mathcal{G}$ we write

$$H \succ G \text{ if and only if } \{i \in \mathcal{I} : w_i(H) > w_i(G)\} \in \mathcal{W}_G, \tag{A1}$$

in other words, $H \succ G$ if and only if a winning coalition in G prefers H to G. Also, define set $\mathcal{D}$ as

$$\mathcal{D} = \{G \in \mathcal{G} : \phi(G) = G\}.$$

The next two lemmas summarize the properties of payoff functions and mapping ϕ.

Lemma 1 *Suppose that $G, H \in \mathcal{G}$ and $\Gamma_G > \Gamma_H$. Then:*

1. *If for $i \in \mathcal{I}$, $w_i(G) < w_i(H)$, then $i \in H \setminus G$.*
2. *$H \not\succ G$.*
3. *$|\{i \in \mathcal{I} : w_i(G) > w_i(H)\}| > n/2 \geq \bar{k}$.*

Proof of Lemma 1. Part 1. If $\Gamma_G > \Gamma_H$ then, by Assumption 1, $w_i(G) > w_i(H)$ whenever $i \in G$ or $i \notin H$. Hence, $w_i(G) < w_i(H)$ is possible only if $i \in H \setminus G$ (note that $w_i(G) = w_i(H)$ is ruled out by Assumption 3). At the same time, $i \in H \setminus G$ implies $w_i(G) < w_i(H)$ by Assumption 1, hence the equivalence.

Part 2. We have $|H \setminus G| \leq |H| \leq \bar{k} \leq m_G$; since by Assumption 2 $\bar{k} \leq m_G$, then $H \setminus G \notin \mathcal{W}_G$, and $H \not\succ G$ by definition (A1).

Part 3. We have $\{i \in \mathcal{I} : w_i(G) > w_i(H)\} = \mathcal{I} \setminus \{i \in \mathcal{I} : w_i(G) < w_i(H)\} \supset \mathcal{I} \setminus (H \setminus G)$, hence, $|\{i \in \mathcal{I} : w_i(G) > w_i(H)\}| \geq n - \bar{k} \geq n - n/2 = n/2 \geq \bar{k}$. ■

Lemma 2 *Consider the mapping ϕ constructed in Section 3 and let $G, H \in \mathcal{G}$. Then:*

1. *Either $\phi(G) = G$ (and then $G \in \mathcal{D}$) or $\phi(G) \succ G$.*
2. *$\Gamma_{\phi(G)} \geq \Gamma_G$.*
3. *If $\phi(G) \succ G$ and $H \succ G$, then $\Gamma_{\phi(G)} \geq \Gamma_H$.*
4. *$\phi(\phi(G)) = \phi(G)$.*

Proof of Lemma 1. Straightforward. ■

Proof of Theorem 1. Part 1. In this proof, we use of Lemma 1. We take β_0 such that for any $\beta > \beta_0$ the following inequalities are satisfied:

$$\begin{aligned} \text{for any } G, G', H, H' &\in \mathcal{G} \text{ and } i \in \mathcal{I} : w_i(G) < w_i(H) \text{ implies} \\ \left(1-\beta^{|\mathcal{G}|}\right) w_i\left(G'\right) + \beta^{|\mathcal{G}|} w_i(G) &< \left(1-\beta^{|\mathcal{G}|}\right) w_i\left(H'\right) + \beta^{|\mathcal{G}|} w_i(H). \end{aligned} \tag{A2}$$

For each $G \in \mathcal{G}$, define the following mapping $\chi_G : \mathcal{G} \to \mathcal{G}$:

$$\chi_G(H) = \begin{cases} \phi(H) & \text{if } H \neq G \\ G & \text{if } H = G \end{cases}.$$

Take any protocol $\pi \in \Pi$. Now take some node of the game in the beginning of some period t when $\nu^t = u$. Consider the stages of the dynamic game that take place in this period as a finite game by assigning the following payoffs to the terminal nodes:$\frac{1+r\beta}{1-\beta(1-r)}$

$$v_i(G, H) = \begin{cases} w_i(H) + \frac{\beta}{1-\beta} w_i(\phi(H)) & \text{if } H \neq G \\ \frac{1+r\beta}{1-\beta(1-r)} w_i(G) + \frac{r\beta^2}{(1-\beta)(1-\beta(1-r))} w_i(\phi(G)) & \text{if } H = G \end{cases}, \tag{A3}$$

where $H = G^{t+1}$ is the government that is scheduled to be in power in period $t+1$, i.e., the government that defeated the incumbent G^t if it was defeated and G^t itself if it was not. For any such period t, take a SPE in pure strategies $\sigma_G^* = \sigma_{G^t}^*$ of the truncated game, such that this SPE is the same for any two nodes with the same incumbent government; the latter requirement ensures that once we map these SPEs to a strategy profile σ^* of the entire game $GAME[\pi]$, this profile will be Markovian. In what follows, we prove that for any $G \in \mathcal{G}$, (a) if σ_G^* is played, then there is no transition if $\phi(G) = G$ and there is a transition to $\phi(G)$ otherwise and (b) actions in profile σ^* are best responses if continuation payoffs are taken from profile σ^* rather than assumed to be given by (A3). These two results will complete the proof of part 1.

We start with part (a); take any government G and consider the SPE of the truncated game σ_G^*. First, consider the subgame where some alternative H has won the primaries and challenges the incumbent government G. Clearly, proposal H will be accepted if and only if $\phi(H) \succ G$. This implies, in particular, from the construction of mapping ϕ, that if $\phi(G) = G$, then no alternative H may be accepted. Second, consider the subgame where nominations have been made and the players are voting according to protocol $\pi_G^{\mathcal{A}}$. We prove that if $\phi(G) \in \mathcal{A}$, then $\phi(G)$ wins the primaries regardless of π (and subsequently wins against G, as $\phi(\phi(G)) = \phi(G) \succ G$. This is proved by backward induction: suppose that $\phi(G)$ has number q in the protocol, let us show that if it makes its way to jth round, where $q \le j \le |\mathcal{A}|$, and then it will win this round. The base is evident: if $\phi(G)$ wins in the last round, players will get $v(G,\phi(G)) = \chi_G(\phi(G)) = \frac{1}{1-\beta} w(\phi(G))$ (we drop the subscript for player to refer to w and v as vectors of payoffs), while if it loses, they either get $v(G,H)$ for $H \neq \phi(G)$. Clearly, voting for $\phi(G)$ is better for a majority of population, and thus $\phi(G)$ wins the primaries and defeats G. The step is proven similarly, hence, in the subgame which starts from qth round, $\phi(G)$ will defeat the incumbent government. Since this holds irrespective of what happens in previous rounds, this concludes the second step. Third, consider the stage where nominations are made, and suppose, to obtain a contradiction, that $\phi(G)$ is not proposed. Then, in the equilibrium, players get a payoff vector $v(G,H)$, where $H \neq \phi(G)$. But then, clearly, any member of $\phi(G)$ has a profitable deviation, which is to nominate $\phi(G)$ instead of or in addition to what he is nominating in profile σ_G^*. Since in a SPE there should be no profitable deviations, this competes the proof of part (a).

Part (b) is fairly obvious. Suppose that the incumbent government is G. If some alternative H defeats government G, then from part (a), the payoffs that players get starting from next period are given by $\frac{1}{1-\beta} w_i(H)$ if $\phi(H) = H$ and $w_i(H) + \frac{\beta}{1-\beta} w_i(\phi(H))$ otherwise; in either case, the payoff is exactly equal to $v_i(G,H)$. If no alternative defeats government G, then the $\nu^{t+1} = s$ (the situation becomes stable), and after that, government G stays until the situation becomes unstable, and government $\phi(G)$ is in power in all periods ever since; this again gives the payoff $\frac{1+r\beta}{1-\beta(1-r)} w_i(G) + \frac{r\beta^2}{(1-\beta)(1-\beta(1-r))} w_i(\phi(G))$. This implies that the continuation payoffs are indeed given by $v_i(G,H)$, which means that if in the entire game profile σ^* is played, no player has a profitable deviation. This proves part 1.

Part 2. Suppose σ^* is an acyclic MPE. Take any governement $G = G^t$ at some period t in some node on or off the equilibrium path. Define binary relation $\rightarrow$ on set $\mathcal{G}$ as follows: $G \rightarrow H$ if and only if either $G = H$ and G has a positive probability of staying in power when $G^t = G$ and $\nu^t = u$, or $G \neq H$ and $G^{t+1} = H$ with positive probability if $G^t = G$ and $\nu^t = u$. Define another binary relation $\mapsto$ on $\mathcal{G}$ as follows: $G \mapsto H$ if any only if there exists a sequence (perhaps empty) of different governments $H_1, \ldots, H_q$ such that $G \rightarrow H_1 \rightarrow H_2 \rightarrow \cdots \rightarrow H_q = H$ and $H \rightarrow H$. In other words, $G \mapsto H$ if there is an on-equilibrium path that involves a sequence of transitions from G to H and stabilization of political situation at H. Now, since σ^* is an acyclic equilibrium, there is no sequence that contains at least two different governments $H_1, \ldots, H_q$ such that $H_1 \rightarrow H_2 \rightarrow \cdots \rightarrow H_q \rightarrow H_1$. Suppose that for at least one $G \in \mathcal{G}$, the set $\{H \in \mathcal{G} : G \mapsto H\}$ contains at least two elements. From acyclicity it is easy to derive the existence of government G with the following properties: $\{H \in \mathcal{G} : G \mapsto H\}$ contains at least two elements, but for any element H of this set, $\{H' \in \mathcal{G} : H \mapsto H'\}$ is a singleton.

Consider the restriction of profile σ^* on the part of the game where government G is in power, and call it σ^*_G. The way we picked G implies that some government may defeat G with a positive probability, and for any such government H the subsequent evolution prescribed by profile σ^* does not exhibit any uncertainty, and the political situation will stabilize at the unique government $H' \neq G$ (but perhaps $H' = H$) such that $H \mapsto H'$ in no more than $|\mathcal{G}| - 2$ steps. Given our assumption (A2) and the assumption that r is small, this implies that no player is indifferent between two terminal nodes of this period which ultimately lead to two different governments H'_1 and H'_2, or between one where G stays and one where it is overthrown. But players act sequentially, one at a time, which means that the last player to act on the equilibrium path when it is still possible to get different outcomes must mix, and therefore be indifferent. This contradiction proves that for any G, government H such that $G \mapsto H$ is well-defined. Denote this government by $\psi(G)$.

To finish the proof, we must show that $\psi(G) = \phi(G)$ for all G. Suppose not; then, since $\psi(G) \succ G$ (otherwise G would not be defeated as players would prefer to stay in G), we must have that $\Gamma_{\phi(G)} > \Gamma_{\psi(G)}$. This implies that if some alternative H such that $H \mapsto \phi(G)$ is nominated, it must win the primaries; this is easily shown by backward induction. If no such alternative is nominated, then, since there is a player who prefers $\phi(G)$ to $\psi(G)$ (any member of $\phi(G)$ does), such player would be better off deviating and nominating $\psi(G)$. A deviation is not possible in equilibrium, so $\psi(G) = \phi(G)$ for all G. By construction of mapping ψ, whis implies that there are no transitions if $G = \phi(G)$ and one or more transitions ultimately leading to government $\phi(G)$ otherwise. This completes the proof. ■

Proof of Theorem 2. Part 1. In part 1 of Theorem 1, we proved that for any $\pi \in \Pi$ there exists a MPE in pure strategies, and from part 2 of Theorem 1 it follows that these MPE constructed for different $\pi \in \Pi$ have the same equilibrium path of governments. The existence of an order-independent equilibrium follows.

Part 2. Suppose, to obtain a contradiction, that order-independent MPE in pure strategies σ^* is cyclic. Define mapping $\chi : \mathcal{G} \to \mathcal{G}$ as follows: $\chi(G) = H$ if for any node on equilibrium path which starts with government $G^t = G$ and $\nu^t = u$, the next government $G^{t+1} = H$. Since the equilibrium is in pure strategies, this mapping is well-defined and unique. The assumption that equilibrium σ^* is acyclic implies that there is a sequence of pairwise different governments $H_1, H_2, \ldots, H_q$ (where $q \geq 2$) such that $\chi(H_j) = H_{j+1}$ for $1 \leq j < q$ and $\chi(H_q) = H_1$. Without loss of generality, assume that H_2 has the least competence of all governments $H_1, H_2, \ldots, H_q$. If $q = 2$, then the cycle has two elements, of which H_2 is the worse government. However, this implies that H_2 cannot defeat H_1 even if it wins the primaries, since all players, except, perhaps, those in $H_2 \setminus H_1$, prefer H_1 to an eternal cycle of H_1 and H_2. This immediate contradiction implies that we only need to consider the case $q \geq 3$.

If $q \geq 3$, then, by the choice of H_2, $\Gamma_{H_1} > \Gamma_{H_2}$ and $\Gamma_{H_3} > \Gamma_{H_2}$. Without loss of generality, we may assume that the protocol is such that if the incumbent government is H_1, H_3 is put at the end (if H_3 is nominated); this is possible since σ^* is an order-independent equilibrium. By definition, we must have that proposal H_2 is nominated and accepted in this equilibrium along the equilibrium path.

Let us first prove that alternative H_3 will defeat the incumbent government H_1 if it wins the primaries. Consider a player i who would have weakly preferred H_2, the next equilibrium government, to win over H_1 if H_2 won the primaries; since H_2 defeats H_1 on the equilibrium path, such players must form a winning coalition in H_1. If $i \notin H_2$, then H_2 brings i the lowest utility of all governments in the cycle; hence, i would be willing to skip H_2; hence, such i would be strictly better off if H_3 defeated H_1. Now suppose $i \in H_2$. If, in addition, $i \in H_1$, then he prefers H_1 to H_2. Assume, to obtain a contradiction, that i weakly prefers that H_3 does not defeat H_1; it is then easy to see that since he prefers H_1 to H_2, he would strictly prefer H_2 not to defeat H_1 if H_2 won the primaries. The last case to consider is $i \in H_2$ and $i \notin H_1$. If β is sufficiently close to 1, then, as implied by Assumption $3'$, player i will either prefer that both H_2 and H_3 defeat H_1 or that none of them does. Consequently, all players who would support H_2 also support H_3, which proves that H_3 would be accepted if nominated.

Let us prove that in equilibrium H_3 is not nominated. Suppose the opposite, i.e., that H_3 is nominated. Then H_2 cannot win the primaries: indeed, in the last voting, H_2 must face H_3, and since, as we showed, only members of H_2 may prefer that H_2 rather than H_3 is the next government, H_3 must defeat H_2 in this voting. This means that in equilibrium H_3 is not nominated.

Consider, however, what would happen if all alternatives were nominated. Suppose that some government G then wins the primaries. It must necessarily be the case that G defeats H_1: indeed, if instead H_1 would stay in power, then $G \neq H_3$ (we know that H_3 would defeat H_1), and this implies that in the last voting of the primaries, H_3 would defeat G. Let us denote the continuation utility that player i gets if some government H comes to power as $v_i(H)$. If there is at least one player with $v_i(G) > v_i(H_2)$, then this player has a profitable deviation during nominations: he can nominate all alternatives and ensure that G wins the primaries and defeats H_1. Otherwise, if $v_i(G) \leq v_i(H_2)$ for all players, we must have that $v_i(G) < v_i(H_3)$ for a winning coalition of players, which again means that G cannot win the primaries. This contradiction proves that for the protocol we chose, H_2 cannot be the next government, and this implies that there are no cyclic order-independent equilibria in pure strategies.

Part 3. The proof is similar to the proof of part 2. We define mapping χ in the same way and choose government H such that $\chi(\chi(H)) \neq \chi(H)$, but $\chi(\chi(\chi(H))) = \chi(\chi(H))$. We then take a protocol which puts government $\chi(\chi(H))$ at the end whenever it is nominated and come to a similar contradiction.

Part 4. This follows straightforwardly from part 3, since the only transition may happen at period $t = 0$. ■

Proof of Proposition 1. Part 1. Suppose, to obtain a contradiction, that $|G \cap H| \geq l$, but $G \neq H$; by Assumption 3 we need to have $\gamma_G < \gamma_H$ or $\gamma_G > \gamma_H$; without loss of generality assume the former. Then $H \succ G$ by Lemma 1, since $|G \cap H| \geq l$. Note that $G = G_q$ for some q and $H = G_j$ for some j such that $j < q$. Since H is stable, $\phi(G_j) = G_j$, but then $\mathcal{M}_q \neq \varnothing$ by (3), and so $\phi(G_q) \neq G_q$, as follows from (4). However, this contradicts the hypothesis that $G_q = G \in \mathcal{D}$, and thus completes the proof.

Part 2. By definition of mapping ϕ, $\phi(G_1) = G_1$, so $G_1 \in \mathcal{D}$. Take any government $G \in \mathcal{D}$; since $|G \cap G_1| \geq 0 = L$, we have $G = G_1$ by part 1. Consequently, $\mathcal{D} = \{G_1\}$, so $\mathcal{D}$ is a singleton.

Part 3. As before, the most competent government, G_1, is stable, i.e. $G_1 \in \mathcal{D}$. Now consider the set of governments which intersect with G_1 by fewer than l members:

$$\mathcal{B} = \{G \in \mathcal{G} : |G \cap G_1| < l\}.$$

This set is non-empty, because $n > 2k$ implies that there exist a government which does not intersect with G_1; obviously, it is in $\mathcal{B}$. Now take the most competent government from $\mathcal{B}$, μ_j where

$$j = \min\{q : 1 \leq q \leq |G| \text{ and } G_q \in \mathcal{B}\}.$$

Obviously, $G_j \neq G_1$, because $G_1 \notin \mathcal{B}$. Let us show that G_j is stable. Note that any government G_q such that $\gamma_{G_q} > \gamma_{G_j}$ does not belong to $\mathcal{B}$ and therefore has at least l common members with stable government G_1. Hence, $\phi(G_q) = G_1$ (see (4)), and therefore G_q is unstable, except for the case $q = 1$. Now we observe that set $\mathcal{M}_j$ is empty: for each government μ_q with $1 < q \leq j$ either the first condition in (6) is violated (if $q = 1$) or the second one (otherwise). But this implies that $\phi(\mu_j) = \mu_j$, so μ_j is stable. This proves that if $l \geq 1$, $\mathcal{D}$ contains at least two elements. Finally, note that this boundary is achieved: for example, if $l = 1$ and $n < 3k$.

Part 4. Take $l = 1$. If $n = ak$ where a is an integer, then every individual is part of a stable government. More precisely, there are exactly a stable governments in this case. Indeed, part 1 implies that no two stable governments can intersect, so the number of individuals who belong to some stable government is a multiple of k. If it is less than n, then there are at least k individuals who are not part of any stable government, so we can form at least one government consisting of such individuals. Pick among such governments the most competent one; it is straightforward to show (like we did in part 3) that this government is stable, which is a contradiction to the assumption that these individuals belong to no stable government. Now assume that n is not a multiple of k. Then again, any two stable governments must intersect, and therefore there must be a individual who is not part of a stable government. This completes the proof of Proposition 1 ■

Proof of Proposition 2. Part 1. We prove the more general part 2, then the statement of part 1 will be a corollary: to obtain (8), one only needs to substitute $b = k$ into (9).

Part 2. Let us prove the existence of such stable government. Define a set-valued function $\chi : \mathcal{C}^L \to \mathcal{C}^{K-L} \cup \{\varnothing\}$ by

$$\chi(S) = \begin{cases} G \setminus S & \text{if } G \in \mathcal{D} \text{ and } S \subset G; \\ \varnothing & \text{if there exists no } G \in \mathcal{D} \text{ such that } S \subset G. \end{cases} \tag{A4}$$

In words, for any coalition of l individuals, function χ assigns a coalition of $k - l$ individuals such that their union is a stable government whenever such other coalition exists or an empty set when it does not exist. Note that $\chi(S)$ is a well-defined single-valued function: indeed, there cannot be two different stable governments G and H which contain S, for this would violate Proposition 1 (part 1).

Let Y_{l-1} be the coalition of $l-1$ individuals such that $X \cap Y_{l-1} = \varnothing$; denote these individuals by $i_1, \ldots, i_{l-1}$. We now add $k - l + 1$ individuals to this coalition one by one. Let $X_{l-1} = X$, and let

$$X_l = X \cup Y_{l-1} \cup \left(\bigcup_{i \in X} \chi(Y_{l-1} \cup \{i\}) \right). \tag{A5}$$

Intuitively, we take the set of individuals which are either forbidden to join the government under construction (X) or are already there (Y_{l-1}), and add all individuals which can be in the same government with all individuals from Y_{l-1} and at least one individual from $X_{l-1} = X$. Now take some individual $i_l \in \mathcal{I} \setminus X_l$ (below we show that such individual exists) and let $Y_l = Y_{l-1} \cup \{i_l\}$. At each subsequent step z, $l + 1 \leq z \leq k$, we choose zth individual for the government under construction as follows. We first define

$$X_z = X \cup Y_{z-1} \cup \left(\bigcup_{S \subset Y_{z-1} : |S| = l-1; i \in X} \chi(S \cup \{i\}) \right) \tag{A6}$$

and then take

$$i_z \in \mathcal{I} \setminus X_z \tag{A7}$$

(we prove that we can do that later) and denote $Y_z = Y_{z-1} \cup \{i_z\}$. Let the last government obtained in this way be denoted by $Y = Y_k$.

We now show that $\phi(Y) \cap X = \varnothing$. Suppose not, then there is individual $i \in \phi(Y) \cap X$. By (4) we must have that $|\phi(Y) \cap Y| \geq l$; take the individual i_j with the highest j of such individuals. Clearly, $j \geq l$, so individual i_j could not be a member of the initial Y_{l-1} and was added at a later stage. Now let S be a subset of $(\phi(Y) \cap Y) \setminus \{i_j\}$ such that $|S| = l - 1$. Since government $\phi(Y)$ is stable and contains the entire S as well as $i \in X$ (and $i \notin S$ because $S \subset Y$ and $X \cap Y = \varnothing$), we must have $\chi(S \cup \{i\}) = \phi(Y)$. Consequently, if we consider the right-hand side of (A6) for $z = j$, we will immediately get that $\phi(Y) \subset X_j$, and therefore $i_j \in X_j$. But we picked i_j such that $i_j \in \mathcal{I} \setminus X_j$, according to (A7). We get to a contradiction, which proves that $\phi(Y) \cap X = \varnothing$, so $\phi(Y)$ is a stable government which contains no member of X.

It remains to show that we can always pick such individual, we need to show that the number of individuals in X_z is less than n for any $z : l \leq z \leq k$. Note that the union in the inner parentheses of (A6) consists of at most

$$(k-l)\binom{z-1}{l-1}b \leq (k-l)\binom{k-1}{l-1}b$$

individuals, while $z - 1 \leq k - 1$. Therefore, it is sufficient to require that

$$\begin{aligned} n &> b + k - 1 + (k-l)\binom{k-1}{l-1}b \\ &= b + k - 1 + b(k-l)\frac{(k-1)!}{(l-1)!(k-l)!}. \end{aligned}$$

Because we are dealing with integers, this implies (9), which completes the proof. ■

Proof of Proposition 3. Part 1. Note that the most competent government is the one containing individuals $i_1, i_2, \ldots, i_k$, so $G_1 = \{i_1, i_2, \ldots, i_k\}$. The result now follows from 1, part 2.

Part 2. If suffices to prove that governments of the form $H_q = \{i_{qk+1}, i_{qk+2}, \ldots, i_{qk+K}\}$ for $q \in \mathbb{Z} : 0 \leq q \leq \lfloor n/k \rfloor - 1$ are stable; then the fact that no other stable government exists will follow from Proposition 1, part 1, since $l = 1$ and any other government G (consisting of k members) has a non-empty intersection with at least one of these governments. The most competent government $G_1 = \{i_1, i_2, \ldots, i_k\}$, is stable by construction of mapping ϕ. Suppose we have proved that governments $H_1, \ldots, H_{q-1}$ are stable. Consider the set of governments that do not intersect with any of $H_1, \ldots, H_{q-1}$; obviously, H_q is the most competent among them. Let us follow the construction of mapping ϕ. If H_q is the jth most competent government, i.e., $H_q = G_j$, then $\mathcal{M}_j = \varnothing$, because the only governments which are more competent than H_q and stable are $H_1, \ldots, H_{q-1}$, and these do not intersect with H_q. Hence, $\phi(G_j) = G_j$, which implies that $G_j = H_q$ is a stable government. This completes the induction step and thus the proof.

Part 3. The most competent government G_1 is stable. If $l = k$, then for any $q > 1$, $\mathcal{M}_q = \varnothing$, because $|G_q \cap G_j| < k = l$ for $j < q$. By construction of mapping ϕ, government G_q is stable. ■

Proof of Proposition 4. Part 1. By Assumption 2, $0 \leq l \leq k$, so either $l = 0$ or $l = 1$. The result follows from 3 1 and 3: if $l = 0$, $\mathcal{D} = \{\{i_1\}\}$, so since $\phi(G) \in \mathcal{D}$ for any $G \in \mathcal{G}$, we must have $\phi(G) = \{i_1\}$; if $l = 1$, any G is stable.

Part 2. In this case, either $l = 0$, $l = 1$, or $l = 2$. If $l = 0$ or $l = 2$, we again can apply 3 (parts 1 and 3) to obtain the result. If $l = 1$, then 3 (part 2) fully characterizes the set of stable states. By the construction of mapping ϕ, either $\phi(G) = G$ or $|\phi(G) \cap G| = 1$. If $G = \{i_p, i_q\}$ with $p < q$, then $\phi(G)$ will include either i_p or i_q. Now it is evident that $\phi(G)$ will be the stable state which includes i_p, because it is more competent than the one which includes i_q if the latter exists and is different. Full characterization follows. ■

Proof of Theorem 3. Part 1. If δ is sufficiently small, then the possibility of shocks does not change the ordering of continuation utilities at the end of any period any for any player. Hence, the equilibrium constructed in the proof of part 1 of Theorem 1 proves this statement as well.

Part 2. If δ is sufficiently small, the proof of Theorem 2 (parts 2 and 3) may be applied here with minimal changes, which are omitted. ■

Proof of Proposition 5. Part 1. If $l = 0$, then by Proposition 1 for any G, $\phi^t(G) = G_1^t$, where G_1^t is the most competent government $\{i_1^t, \ldots, i_k^t\}$.

Part 2. Suppose $l = 1$, then Proposition 1 provides a full characterization. There are $\lfloor n/k \rfloor \leq n/k$ stable governments. Each consists of k individuals, so the probability that a random new government coincides with any given stable government is $1/\binom{n}{k} = \frac{k!(n-k)!}{n!}$. The probability that it coincides with any stable government is $\lfloor n/k \rfloor / \binom{n}{k} \leq \frac{n}{k}\frac{k!(n-k)!}{n!} = \frac{(k-1)!(n-k)!}{(n-1)!} = 1/\binom{n-1}{k-1}$. The government will change to a more competent one if and only if it is unstable, which happens with probability greater than or equal to $1 - 1/\binom{n-1}{k-1}$.

The most competent government will be installed if and only if after the shock, the government contains at least 1 of the k most competent members. The probability that it does not contain any of these equals $\binom{n-k}{k}/\binom{n}{k}$ (this is the number of combinations that do not include

k most competent members divided by the total number of combinations. We have

$$\begin{aligned}\binom{n-k}{k}\Big/\binom{n}{k} &= \frac{(n-k)!k!\,(n-k!)}{k!\,(n-2k)!n!} \\ &= \frac{(n-k)!}{(n-2k)!}\frac{(n-k!)}{n!} \\ &= \prod_{j=0}^{k}\frac{n-k-j}{n-j}.\end{aligned}$$

Since each of the k factors tends to 1 as $n \to \infty$, so does the product. Hence, the probability that the most competent government will arise, $1-\binom{n-k}{k}/\binom{n}{k}$, tends to 0 as $n \to \infty$.

Part 3. If $l = k$, then $\phi^t(G) = G$ for any t and G. Hence, the government will not change. It will be the most competent if it contains k most competent individuals, which happens with probability $1/\binom{n}{k}$. This is less than 1, which is the corresponding probability for $l = 0$. If $k > 2$, it is also less than the corresponding probability for $l = 1$: in the latter case, there are at least two governments which will lead to the most competent one: $\{i_1^t, \ldots, i_k^t\}$ and $\{i_1^t, \ldots, i_{k-1}^t, i_{k+1}^t\}$, which completes the proof. ■

Proof of Proposition 6. The probability of having the most competent government G_1^t is the probability that at least l members of G^t are members of G_1^t. This probability equals (from hypergeometric distribution)

$$\frac{\sum_{q=0}^{l}\binom{k}{q}\binom{n-k}{k-q}}{\binom{n}{k}},$$

and therefore is strictly increasing in l. ■

Proof of Proposition 7. Part 1. Any such swapping (or, more generally, any transposition σ, where $\sigma(i)$ is the individual whose former competence individual i now has) induces a one-to-one mapping that maps government G to government $\rho(G)$: $i \in \rho(G)$ if and only if $\sigma(i) \in G$. By construction, $\Gamma_G^{t-1} = \Gamma_{\rho(G)}^t$, and, by construction of mapping ϕ, $\phi^{t-1}(G) = \phi^t(\rho(G))$ for all G. If all transitions occur in one stage, and a shock triggers a period of instability, then with probability 1 all shocks arrive at times t where government G^{t-1} is ϕ^{t-1}-stable.

If abilities of only two individuals are swapped, then $|G \cap \rho(G)| \geq k-1 \geq l$. But G is ϕ^{t-1}-stable with probability 1, hence, $\rho(G)$ is ϕ^t-stable. Consider two cases. If $\Gamma_G^t \geq \Gamma_{\rho(G)}^t$, then $\Gamma_{\phi^t(G)}^t \geq \Gamma_G^t \geq \Gamma_{\rho(G)}^t$. If $\Gamma_G^t < \Gamma_{\rho(G)}^t$, then again $\Gamma_{\phi^t(G)}^t \geq \Gamma_{\rho(G)}^t$, since there is a ϕ^t-stable government $\rho(G)$ which has with G at least l common members and the competence of which is $\Gamma_{\rho(G)}^t$. Hence, $\phi^t(G)$ is either $\rho(G)$ or a more competent government. Hence, the competence of government cannot decrease. However, it may increase, unless G contains k most competent members. Indeed, in that case there exist $i, j \in \mathcal{I}$ with $i < j$ such that $i \notin G$ and

$j \in G$. Obviously, swapping the abilities of these individuals increases the competence of G: $\Gamma_G^t > \Gamma_G^{t-1}$, and thus the stable government that will evolve will satisfy $\Gamma_{\phi(G)}^t \geq \Gamma_G^t > \Gamma_G^{t-1}$. Since the probability of this swapping is non-zero, eventually the competence of government will improve. Since there is a finite number of possible values of current government's competence, then with probability 1 the most competent government will emerge.

Part 2. This follows from an argument of part 1, taking into account that if abilities of x individuals changed, then $|G \cap \rho(G)| \geq k - \lfloor x/2 \rfloor \geq l$. Indeed, if $|G \cap \rho(G)| < k - \lfloor x/2 \rfloor$, we would have $|G \cap \rho(G)| \leq k - \lfloor (x+1)/2 \rfloor$ since the numbers of both sides are integers, and thus $|(G \setminus \rho(G)) \cup (\rho(G) \setminus G)| > 2\lfloor (x+1)/2 \rfloor \geq x$. However, all individuals in $(G \setminus \rho(G)) \cup (\rho(G) \setminus G)$ changed their abilities, so the last inequality contradicts the assumption that no more than x individuals did. This contradiction completes the proof. ■

Proof of Proposition 8. Part 1. Suppose that when the shock happens, government $G^t = G$. If G^t is ϕ^t-stable, then it will not change until a new shock arrives. If G^t is not ϕ^t-stable, it will change to $\phi^t(G^t)$ at the first unstable period. Since $\Gamma_{\phi^t(G^t)} > \Gamma_{G^t}$, the result follows.

Part 2. When $l^t = 0$, $\phi^t(G^t) = G_1$ for any G^t. When the government becomes G_1, it stays thereafter, as follows from part 1. The conclusion follows. ■

Proof of Proposition 9. The probability of having the most able player in the government under the royalty system is 1. Indeed, government $\phi^t(G^t)$ will for any t and any G^t consist of l irreplaceable members and $k - l$ most competent members. Since $l < k$, this always includes the most competent player. In the case of junta-like system, there is a positive probability that a government that does not include player i_1^t is stable. If $\frac{a_1 - a_2}{a_2 - a_n}$ is sufficiently large, any government that includes player i_1^t is more competent that a government that does not. The first part follows. Now consider the probability that the least competent player, i_n^t, is a part of the government. In a royalty system, this can happen if and only if the irreplaceable member is the least competent, i.e., with probability $1/n$. In a junta-like system, the probability that the most competent government is installed is the probability that one of the players $i_1^t, \ldots, i_k^t$ is in the government immediately after the shock. This probability is higher than the probability that any given player is among $i_1^t, \ldots, i_k^t$, which is $k/n \geq 2/n$. This completes the proof. ■

Appendix B

Example 8 Suppose that the society consists of five individuals $1, 2, 3, 4, 5$ ($n = 5$). Suppose each government consists of two members, so $k = 2$. There is "almost perfect" democracy ($l = 1$), and suppose $m = 3$. Assume

$$\Gamma_{\{i,j\}} = 30 - \min\{i, j\} - 5\max\{i, j\}.$$

Moreover, assume that all individuals care a lot about being in the government, and about the competence if they are not in the government; however, if a individual is a member of two different governments, he is almost indifferent. In this environment, there are two fixed points of mapping ϕ: $\{1, 2\}$ and $\{3, 4\}$.

Let us show that there is no MPE in pure strategies if $v^t = u$ for all t (so that the incumbent government is contested in each period). Suppose that there is such equilibrium for some protocol π. One can easily see that no alternative may win if the incumbent government is $\{1, 2\}$: indeed,if in equilibrium there is a transition to some $G \neq \{1, 2\}$, then in the last voting, when $\{1, 2\}$ is challenged by G, both 1 and 2 would be better off rejecting the alternative and postpone transition to the government (or a chain of governments) that they like less. It is also not hard to show that any of the governments that include 1 or 2 (i.e., $\{1, 3\}$, $\{1, 4\}$, $\{1, 5\}$, $\{2, 3\}$, $\{2, 4\}$, and $\{2, 5\}$) lose the contest for power to $\{1, 2\}$ in equilibrium. Indeed, if $\{1, 2\}$ is included in the primaries, it must be the winner (intuitively, this happens because $\{1, 2\}$ is the Condorcet winner in simple majority votings). Given that, it must always be included in the primaries, for otherwise individual 1 would have a profitable deviation and nominate $\{1, 2\}$. We can now conclude that government $\{3, 4\}$ is stable: any government which includes 1 or 2 will immediately lead to $\{1, 2\}$ which is undesirable for both 3 and 4, while $\{3, 5\}$ and $\{4, 5\}$ are worse than $\{3, 4\}$ for 3 and 4 as well; therefore, if there is some transition in equilibrium, then 3 and 4 are better off staying at $\{3, 4\}$ for an extra period, which makes a profitable deviation.

We now consider the governments $\{3, 5\}$ and $\{4, 5\}$. First, we rule out the possibility that from $\{3, 5\}$ the individuals move to $\{4, 5\}$ and vice versa. Indeed, if this was the case, then in the last voting when the government is $\{3, 5\}$ and the alternative is $\{4, 5\}$, individuals $1, 2, 3, 5$ would be better off blocking this transition (i.e., postponing it for one period). Hence, either one of governments $\{3, 5\}$ and $\{4, 5\}$ is stable or one of them leads to $\{3, 4\}$ in one step or $\{1, 2\}$ in two steps. We consider these three possibilities for the government $\{3, 5\}$ and come to a contradiction; the case of $\{4, 5\}$ may be considered similarly and also leads to a contradiction.

It is trivial to see that a transition to $\{1, 2\}$ (in one or two steps) cannot be in an equilibrium. If this was the case, then in the last voting, individuals 3 and 5 would block this transition, since they are better off staying in $\{3,5\}$ for one more period (even if the intermediate step to $\{1,2\}$ is a government that includes either 3 or 5). This is a profitable deviation which cannot happen in an equilibrium. It is also trivial to check that $\{3,5\}$ cannot be stable. Indeed, if this was the case, then if alternative $\{3,4\}$ won the primaries, it would be accepted, as individuals $1,2,3,4$ would support it. At the same time, any alternative that would lead to $\{1,2\}$ would not be accepted, and neither will be alternative $\{4,5\}$, unless it leads to $\{3,4\}$. Because of that, alternative $\{3,4\}$ would make its way through the primaries if nominated, for it is better than $\{3,5\}$ for a simple majority of individuals. But then $\{3,4\}$ must be nominated, for, say, individual 4 is better off if it were, since he prefers $\{3,4\}$ to $\{3,5\}$. Consequently, if $\{3,5\}$ were stable, we would get a contradiction, since we proved that in this case, $\{3,4\}$ must be nominated, win the primaries and take over the incumbent government $\{3,5\}$.

The remaining case to consider is where from $\{3,5\}$ the individuals transit to $\{3,4\}$. Note that in this case, alternative $\{1,2\}$ would be accepted if it won the primaries: indeed, individuals 1 and 2 prefer $\{1,2\}$ over $\{3,4\}$ for obvious reasons, but individual 5 is also better off if $\{1,2\}$ is accepted, for he is worse off under $\{3,4\}$ than under $\{1,2\}$, even if the former grants him an extra period of staying in power (recall that the discount factor β is close to 1). Similarly, any alternative which would lead to $\{1,2\}$ in the next period must also be accepted in the last voting. This implies, however, that such alternative ($\{1,2\}$ or some other one which leads to $\{1,2\}$) must necessarily win the primaries if nominated (by the previous discussion, $\{4,5\}$ may not be a stable government, and hence the only choice the individuals make is whether to move ultimately to $\{3,4\}$ or to $\{1,2\}$, of which they prefer the latter). This, in turn, means that $\{1,2\}$ must be nominated, for otherwise, say, individual 1 would be better off doing that. Hence, we have come to a contradiction in all possible cases, which proves that for no protocol π there exists a MPE in pure strategies. Note that we have proven that both cyclic and acyclic equilibria do not exist.

Example 9 We prove this by constructing an example. Suppose $n = 9$, $k = 4$, $l_1 = 3$, $m = 5$. Let the individuals be denoted $1,2,3,4,5,6,7,8,9$, with decreasing ability. Namely, suppose that abilities of individuals $1,\ldots,8$ are given by $\gamma_i = 2^{8-i}$, and $\gamma_9 = -10^6$. Then the 14 stable governments, in the order of decreasing competence, are given in the table.

1234	2358
1256	2367
1278	2457
1357	2468
1368	3456
1458	3478
1467	5678

(Note that this would be the list of stable governments for any decreasing sequence $\{\gamma_i\}_{i=1}^{9}$, except for that, say, $\Gamma_{\{1368\}}$ may become than $\Gamma_{\{1458\}}$.) Now consider the same parameters, but take $l_2 = 2$. Then there are three stable governments 1234, 1567, and 2589. For a random initial government, the probability that individual 9 will be a part of the stable government that evolves is $\frac{9}{126} = \frac{1}{16}$: of $\binom{9}{4} = 126$ feasible governments there are 9 governments that lead to 2589, which are 2589, 2689, 2789, 3589, 3689, 3789, 4589, 4689, and 4789. Clearly, the expected competence of government for $l_2 = 2$ is negative, whereas for $l_1 = 3$ it is positive, as no stable government includes the least competent individual 9.

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