Are Jumps in Corporate Bond Yields Priced? Modeling Contagion via the Updating of Beliefs[1]

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Abstract

If jumps (possibly to default) in yields of individual corporate bonds are priced, then there must be a market-wide response at the jump-event. Casual observation suggests that the jump in spreads of an individual firm can generate a change in the *perception* of risk inherent in the bonds of other firms, even in the absence of direct counterparty risk. Below, we propose a general reduced-form model which captures this notion of contagion. In contrast to existing counterparty risk models (e.g., Jarrow and Yu (2001)), our framework provides simple analytic solutions for the credit spreads on risky debt even when the cross-dependence involves an arbitrarily large number of firms. Furthermore, our framework can be used to capture contagion within a 'structural framework' by generalizing the model of Duffie and Lando (2001) so that the default of one firm generates revision in the beliefs about the 'quality of accounting information' of other firms. Empirically, we investigate how much of the credit spread can be attributable to the jump risk premium. While we find that credit spread jumps of large firms do have a market-wide impact, a calibration exercise suggests that the premium this risk commands does not explain a significant portion of observed credit spreads.

1 Introduction

If jumps in the yield spread of an individual corporate bond is priced risk, then there must be a market-wide response (i.e., a jump in the pricing kernel) at that jump-event. If not, then as emphasized by Jarrow, Lando and Yu (2000), such a risk is conditionally diversifiable, and hence does not command a risk premium.

One explanation for a market-wide response is that of counterparty-risk. This occurs when the default of one firm causes financial distress on other entities that had close business ties to the defaulting firm. The default of these firms then lead to the possible defaults of other firms via a snowball effect. As a recent example, the concern that the default of LTCM would lead to a defaults of other major investment banks provided an incentive for the Federal Reserve to intervene. Jarrow and Yu (2001) investigate the pricing of risky debt under such counterparty risk. Unfortunately, due to the recursive nature of the cross-dependence, their framework is not very tractable, even for their general two-firm model.

Another mechanism by which the default of one firm can trigger a market-wide response is through Bayesian updating of the 'perception' of risk. A recent example includes Enron, where inaccurate accounting data of one firm led to concern about the accounting numbers of other firms (e.g., GE, Tyco), even though there was no direct ties between these companies. Another example includes the leveraged buyout of RJR, where market participants assumed beforehand that such a firm was too large for an LBO to occur. The announcement of the proposed LBO sent shock waves throughout the corporate bond market, apparently due to the revised beliefs that all corporate debt, no matter how large the underlying firm, was subject to the risk of substantial losses due to a leveraged buyout.

Below, we propose a framework where a jump (possibly to default) in the yield spread of one firm can lead to the widening of yield spreads across an arbitrarily large number N of other firms. The number of firms N that share this cross-dependence can be chosen to be small enough as to be diversifiable, or can be chosen large enough so that this jump-risk cannot be diversified away. In this latter case, this jump-risk commands a risk-premium, as we demonstrate below within a simple general equilibrium framework.

While our proposed framework is consistent with a counterparty-risk interpretation, it is most naturally interpreted as an updating of beliefs due to an unexpected event. For example, using Enron as motivation, we generalize the framework of Duffie and Lando (2000) by assuming that the quality of accounting information throughout an economy is unknown, but shared across firms. Hence, a surprise default of one firm causes market participants to update their beliefs that the economy is in a poor accounting-quality state, forcing credit spreads to widen in other firms, even though there is no direct counter-party risk. Because what 'jumps' in our framework is the probability of being in the various risk states of nature, and because risky bond prices are linear in these probabilities, we obtain simple analytic solutions for risky bond prices, even when the possible contagion-event involves an arbitrarily large number of firms. This contrasts with Jarrow and Yu, who cannot obtain analytic solutions even for their two-firm model.

The existence of a jump-premium over and above the 'standard' premia (i.e., those associated with 'diffusion risk sources') offers a potential explanation for why structural models of default (whose dynamics are specified without jumps) generate credit spreads that are counterfactually low. Such a mechanism is potentially important in light of the findings of Huang and Huang (2002), who report that all structural models of default investigated generate counterfactually low yield spreads once they are calibrated to match historical default- and recovery-rates. As such, the existence of a sufficiently large jump-premium could possibly explain the 'credit spread puzzle'. Driessen (2002) uses this motivation to estimate the size of such a jump-premium. We note, however, that his approach effectively *defines* any risk-premium not captured by other factors of his model to be jump-risk premium. As such, his approach is essentially a joint hypothesis about i) the size of the jump risk premium, and ii) the specification of his proposed model. That is, if his model is misspecified, then this misspecification could lead to a bias in his estimates of the jump-risk premia. Below, we emphasize that while the existence of a jump-risk premium is only one of many potential explanations for the *cross-sectional* observation that yield spreads are 'too high',[1] the feature that uniquely identifies whether or not a jump-risk premium exists is the *time-series* implication that, at the jump-date, there is a market-wide response. Hence, while the empirical approach used by Driessen finds the jump-risk premium to be quite large, a back-of-the-envelope estimation suggests that it cannot explain a sizeable portion of observed yield spreads. Indeed, the additional excess return on bonds due to the existence of a jump-premium is the product of three variables: 1) the average size of the bond return loss $\left(\frac{\Delta P}{P}\right)$ at the jump, the probability (i.e., intensity) of such an event occurring (λ), and 3) the average jump in the pricing kernel $\left(\frac{\Delta \xi}{\xi}\right)$ at the time of the yield-spread jump event. Casual observation suggests that an upper bound to this additional excess return is on the order of one basis point per year, which cannot, it seems, explain the 'credit spread puzzle'.

[1] Other explanations for the 'credit spread puzzle' include liquidity, and the correct specification for the standard risk premia. Indeed, we are currently examining the size of a risk premium within a habit-formation model.

Using the returns of equity and corporate bond indices as proxies for the 'market portfolio', we estimate the size of the jump risk-premium. Consistent with intuition, (and with either a counter-party risk explanation or a 'change in perception or risk' explanation), we find that jumps in yield spreads of large firms have a greater effect on the market portfolio than do jumps in yield spreads of small firms. Furthermore, and consistent with our general equilibrium framework, such jumps lead to 'flights to quality' in that the risk-free rate jumps downward during these events. Consistent with our back-of-the envelope estimation, however, the size of the jump risk-premia are relatively small.

Many other papers examine correlated default risk. Duffie and Singleton (2000) present various approaches to estimate correlation risk within reduced form models where default correlation is the result of correlated intensities. Das, Freed, Geng and Kapadia (2002) provide an empirical investigation of this type of intensity-based default correlation. Davis and Lo (2000) study a static model of 'infectious' default, which shares some of the notion of contagion present in our framework. However, their model is a purely static one-period model in that the default of one firm may trigger the default of other firms, but there is no dynamic updating of default probabilities. Zhou (2001) and Cathcart and El-Jahel (2002) consider structural models that allow for jumps in firm value. Their models are not very tractable for more than two firms, however. Giesecke (2001) is closest to our paper in that investors learn over time about the probability of default of one firm from the defaults of other firms. A major difference is that, in his model, firms can only default if they are currently at an all-time low price. In other words, there does not exist an intensity for the default arrival time. This feature contrasts considerably with our framework, where defaults can occur at any time. Our paper is also very close in spirit to recent work by Schönbucher and Schubert (2001) who provide an approach to correlate default risks in a reduced-form model using a Copula. Since the Copula approach is inherently static, however, they rely on a clever use of the 'doubly stochastic' Cox-process construction. Our model does not require such assumption and the updating equation for intensities is more flexible and the resulting model more tractable.[2]

The paper is also closely related to the learning and contagion literature. In contrast to existing learning models such as Detemple (1986), Feldman (1989), Veronesi (2000) and David (1997) who use results on filtering theory for diffusion from Lipster and Shiryaev (1974), we derive results for continuous time updating based on observation of realization of point processes. In contrast to the pure diffusion setting, which seems to be tractable only for specific

[2] It would seem plausible, but we could not prove, that the framework of Schönbucher and Schubert (2001) is a special case of our general model.

(e.g., Gaussian) dynamics, our updating framework appears to be much less restrictive.

The mechanism for contagion modeled in this paper is closely linked to the information-based channel for linkages across international financial markets as introduced in King and Wadhwani (1990) and further discussed in Kodres and Pritsker (2002). There is also a large empirical literature that studies contagion in equity markets (lang and Stulz (1992)) and in international finance (see e.g., Bae, Karolyi and Stulz (2000)).

The rest of the paper is as follows. In Section 2, we specify a reduced-form model of credit spreads, and show that it is consistent with either an updating of beliefs or a counterparty-risk mechanism. We then generalize the framework of Duffie and Lando (2001) to show how the uncertainty of the quality of accounting earnings within a structural model generates predictions similar to the reduced form framework. In Section 3, we propose a general equilibrium framework which captures this same effect. In addition to predicting jumps in the yields of risky debt, this framework also predicts that there are also 'flights-to-quality' at these credit event dates in that the risk-free rates drop. The form of the jump-risk premia are identified. In Section 4, we provide empirical support for the existence of a jump-to-default premium. We conclude in Section 5. Many of the proofs are relegated to the appendices.

2 Models of Credit Risk with Contagion

We first present a general reduced-from framework of correlated default which is consistent with either a 'counterparty-risk' or an 'updating-of-beliefs' interpretation. In particular, the default of one firm will lead to a jump in the intensity of default of other firms. In contrast to the framework of Jarrow and Yu (2001), this framework is tractable even when the contagion involves an arbitrarily large number of firms. The tractability of this framework follows from the fact that the common factors driving individual firm's default intensities can be interpreted as probabilities of being in various default regimes. Then we demonstrate that the reduced-from framework can be generated from a structural framework by generalizing the incomplete-information model of Duffie and Lando (2001) to multiple firms that share 'quality of accounting information'.

2.1 A reduced-form model of contagion risk and changing risk-perception

Consider a reduced form model of default for n firms. We assume that the default time for each firm $i \in 1, \ldots, n$ is modeled by an unpredictable stopping time τ_i with risk-neutral intensity $\overline{\lambda}_i(t)$. It follows from the martingale characterization of intensity (Brémaud (1981) theorem 9) that the indicator of the default event satisfies $\mathrm{E}_t^Q\left[d\mathbf{1}_{\{\tau_i\le t\}}\right] \equiv \overline{\lambda}_i(t)\mathbf{1}_{\{\tau_i>t\}}dt$. In other words, with sufficient regularity on the intensity process the process $M_i(t)$ specified via its dynamics $dM_i(t) \equiv \left(d\mathbf{1}_{\{\tau_i\le t\}} - \overline{\lambda}_i(t)\mathbf{1}_{\{\tau_i>t\}}dt\right)$ is a Q-martingale.

We consider the functional form of the intensity $\overline{\lambda}_i(t)$ as

$$\overline{\lambda}_i(t) = \sum_{j=1}^{J} \lambda_i^j(t)p^j(t), \tag{1}$$

where, for each firm-i, the $\lambda_i^j(t)$ are positive integrable processes that can be ordered in the sense that:

$$0 \le \lambda_i^1(t) < \lambda_i^2(t) < \ldots < \lambda_i^J(t). \tag{2}$$

Further, we assume that the *initial* values of the "perception of risk" state variables p^j are positive $p^j(t_0) > 0$ and sum to unity: $\sum_{j=1}^J p^j(t_0) = 1$. The dynamics of the p^j are specified as:

$$\begin{aligned} dp^j(t) &= \sum_{i=1}^{N}\left[p^j(t^-)\left(\frac{\lambda_i^j(t^-)}{\overline{\lambda}_i(t^-)} - 1\right)\right]\left(d\mathbf{1}_{\{\tau_i\le t\}} - \overline{\lambda}_i(t)\mathbf{1}_{\{\tau_i>t\}}dt\right) \\ &\equiv \sum_{i=1}^{N} \alpha_i^j(t^-)\, dM_i(t). \end{aligned} \tag{3}$$

The p^j have a natural interpretation of being probabilities. In fact, the next two lemmas show that the $p^j(t)$ can be interpreted as the (risk-neutral) probabilities, conditional on the information $\mathcal{F}(t)$, that the economy is in state-j, and hence, that firm-i has a default intensity of λ_i^j.

Lemma 1 *The p^j satisfy $\left(1 > p^j(t) > 0\right)$ and $\sum_{j=1}^J p^j(t) = 1$ a.s. $\forall\,(t,\omega)$.*

<u>Proof:</u>

Since 0 is an absorbing barrier for $p^j(t)$, $p^j(t)$ cannot cross zero continuously. At a time of jump we have $p^j(\tau_i) = p^j(\tau_i^-)\frac{\lambda_i^j(\tau_i^-)}{\overline{\lambda}_i(\tau_i^-)}$ which is greater than zero as long as $\overline{\lambda}_i(\tau_i^-) > 0$. given the initial condition $p^j(t_0) > 0$,$\forall j$, we deduce that $p^j(t) > 0$ $a.s.\,\forall t \ge t_0$. Further, we have

$$d\left(\sum_{j=1}^{J} p^j(t)\right) = \sum_{i=1}^{N}\left(1 - \sum_{j=1}^{J} p^j(t)\right) dM_i^j(t). \tag{4}$$

Hence, 1 is an absorbing barrier for the process $\sum_{j=1}^J p^j(t)$. Given the initial condition $\sum_j p^j(t_0) = 1$ we conclude $\sum_j p^j(t) = 1$ $a.s.$ $\forall t \ge t_0$. Since $p^j(t) > 0$ $a.s.$ this implies $p^j(t) < 1$ $a.s.\forall t \ge t_0$. □

For what follows, it is convenient to define M_i^j by:

$$dM_i^j(t) = d(p^j(t)\mathbf{1}_{\{\tau_i \le t\}}) - \lambda_i^j(t)p^j(t)\mathbf{1}_{\{\tau_i > t\}}dt. \tag{5}$$

We claim:

Lemma 2 *$M_i^j(t)$ is a Q-martingale.*

Proof:

Using the generalized Ito formula for jump processes, we have:

$$\begin{aligned}
d(p^j(t)\mathbf{1}_{\{\tau_i \le t\}}) &= \mathbf{1}_{\{\tau_i \le t^-\}}dp^j(t) + p^j(t^-)d\mathbf{1}_{\{\tau_i \le t\}} + \Delta p^j(t)\Delta\mathbf{1}_{\{\tau_i \le t\}} \\
&= \mathbf{1}_{\{\tau_i \le t^-\}}dp^j(t) + p^j(t^-)\left(dM_i(t) + \overline{\lambda}_i(t)\mathbf{1}_{\{\tau_i > t\}}dt\right) + \sum_{k=1}^{N}\alpha_k^j(t^-)d\mathbf{1}_{\{\tau_k < t\}}d\mathbf{1}_{\{\tau_i < t\}} \\
&= \mathbf{1}_{\{\tau_i \le t^-\}}dp^j(t) + \left(p^j(t^-) + \alpha_i^j(t^-)\right)\left(dM_i(t) + \overline{\lambda}_i(t)\mathbf{1}_{\{\tau_i > t\}}dt\right) \\
&= \mathbf{1}_{\{\tau_i \le t^-\}}dp^j(t) + p^j(t^-)\left(\frac{\lambda_i^j(t^-)}{\overline{\lambda}_i(t^-)}\right)\left(dM_i(t) + \overline{\lambda}_i(t)\mathbf{1}_{\{\tau_i > t\}}dt\right).
\end{aligned} \tag{6}$$

Thus it follows that

$$\begin{aligned}
dM_i^j(t) &\equiv d(p^j(t)\mathbf{1}_{\{\tau_i \le t\}}) - \lambda_i^j(t)p^j(t)\mathbf{1}_{\{\tau_i > t\}}dt \\
&= \mathbf{1}_{\{\tau_i \le t^-\}}dp^j(t) + p^j(t^-)\left(\frac{\lambda_i^j(t^-)}{\overline{\lambda}_i(t^-)}\right)dM_i(t).
\end{aligned} \tag{7}$$

which is clearly a martingale. □

Note that if the $p^j(t)$ are interpreted as probabilities, then equation (3) shows that, after a default, the market updates their beliefs so that the probability of those states of nature associated with higher intensities of default, that is, the probability of those states-j with

$$\sum_{i=1}^{N}\left(\frac{\lambda_i^j(t^-)}{\overline{\lambda}_i(t^-)} - 1\right) > 0,$$

are revised upward after a default, whereas the probabilities of those states of nature associated with lower intensities of default are revised downward. In contrast, during times of no defaults, the drift component of equation (3) will generate a downward revision in the probabilities of those states associated with high default intensities. Note that this framework captures the idea of contagion in that, if firm-k defaults, then the intensity of default of firm $i \neq k$ jumps by $\Delta\overline{\lambda}_i(\tau_k) = \sum_{j=1}^{J}\lambda_i^j(\tau_k^-)\alpha_k^j(\tau_k^-)$.

Note that the dynamics of the 'conditional intensities' $\lambda_i^j(t)$ can be stochastic processes. For example, a very tractable framework obtains if the $\lambda_i^j(t)$ are specified as $\lambda_i^j(t) = a_i^j + b_i^j \cdot X(t)$, where $X(t)$ is an affine state vector (Duffie and Kan (1996)). Note that such a specification guarantees that the $\{\lambda_i^j\}$ maintain their ordering as in equation (2). Furthermore, it guarantees that our framework is econometrically identifiable in that, for each date-t and each risky bond, only one state variable needs to be identified. In fact, all our results hold if the $\lambda_i^j(t)$ processes themselves exhibit jumps, provided the following condition holds:

No-common jump (NCJ) condition :

Define $I_i^j(t) = \mathrm{E}_t^Q\left[e^{-\int_t^T \lambda_i^j(s)ds}\right]$. *We say* $\lambda_i^j(t)$ *satisfies the NCJ condition provided* I_i^j *does not jump at the time of default of a counterparty, i.e.* $\Delta I_i^j(\tau_k) = 0\ \forall k, i = 1, \dots n$ *and* $j = 1, \dots, J$. Indeed we show that:

Proposition 1 *If the dynamics of the 'conditional intensities'* $\lambda_i^j(t)$ *satisfy the NCJ condition, then the risk-neutral probability of default for counterparty* i *prior to maturity* T *is given by:*

$$Q_t(\tau_i \le T) \equiv 1 - \mathrm{E}_t^Q\left[\mathbf{1}_{\{\tau_i > T\}}\right], \tag{8}$$

where

$$\mathrm{E}_t^Q\left[\mathbf{1}_{\{\tau_i > T\}}\right] = \sum_{j=1}^{J} p^j(t)\, \mathrm{E}_t^Q\left[e^{-\int_t^T \lambda_i^j(s)ds}\right] \mathbf{1}_{\{\tau_i > t\}}. \tag{9}$$

In particular, if the $\lambda_i^j(t)$ *follow an affine process, then the risk-neutral probability of bankruptcy is a weighted average of exponential affine functions.*

<u>Proof</u>:

Note that since for all dates-t we have $\sum_{j=1}^J p^j(t) = 1$ a.s., it follows that:

$$\mathrm{E}_t^Q\left[\mathbf{1}_{\{\tau_i > T\}}\right] = \sum_{j=1}^{J} \mathrm{E}_t^Q\left[p^j(T)\, \mathbf{1}_{\{\tau_i > T\}}\right]. \tag{10}$$

It is convenient to define $q_i^j(t) \equiv p^j(t)\mathbf{1}_{\{\tau_i > t\}} = p^j(t)(1 - \mathbf{1}_{\{\tau_i \le t\}})$, and $I_i^j(t) = \mathrm{E}_t^Q[e^{-\int_t^T \lambda_i^j(s)ds}]$. It is sufficient to prove that $q_i^j(t) I_i^j(t) = \mathrm{E}_t^Q[q_i^j(T)]$. Note that $e^{-\int_0^t \lambda_i^j(s)ds} I_i^j(t)$ is a Q-martingale. It therefore follows that:

$$dI_i^j(t) - \lambda_i^j(t) I_i^j(t)dt \equiv dm_t$$

for some Q-martingale $m(t)$. Further, using the previous lemma we have

$$dq_i^j(t) = -\lambda_i^j(t) q_i^j(t)dt - dM_i^j(t) + dp^j(t). \tag{11}$$

Combining these two results we obtain

$$\begin{aligned} d\left(I_i^j(t)q_i^j(t)\right) &= q_i^j(t^-)dI_i^j(t) + I_i^j(t^-)dq_i^j(t) + \Delta I_i^j(t)\Delta q_i^j(t) \\ &= q_i^j(t^-)dm_t - I_i^j(t^-)dM_i^j(t) + I_i^j(t)\, dp^j(t), \end{aligned}$$

where we have used the fact that because of the NCJ condition $\Delta I_i^j(t)\Delta q_i^j(t) = 0$. We thus find that $q_i^j(t)I_i^j(t)$ is a Q-martingale. Therefore, it follows that:

$$q_i^j(t)\, I_i^j(t) = \mathrm{E}_t^Q\left[q_i^j(T)\, I_i^j(T)\right] = \mathrm{E}_t^Q\left[q_i^j(T)\right], \tag{12}$$

(since $I_i^j(T) = 1$), which is the desired result. □

Note that the probability of default $\pi^Q(\tau_i \leq T)$ experiences a jump at the default time τ_i in this framework. Because of this feature, the standard approach of discounting at the risk-adjusted rate does not directly apply in this framework, in contrast to standard reduced-from models. Indeed, this illustrates the importance of the 'no-jump' assumption discussed in, for example, Duffie and Singleton (1999). Indeed, in contrast to standard reduced-form models, proposition 1 implies that:

Corollary 1 *The probability of no default prior to T defined by $\mathrm{E}_t^Q[\mathbf{1}_{\{\tau_i > T\}}]$ is* **not** *equal to $\mathrm{E}_t^Q[e^{-\int_t^T \overline{\lambda}_i(s)ds}]\mathbf{1}_{\{\tau_i > t\}}$.*

For the case of constant interest rates, this proposition provides a solution to the price of a defaultable bond. For the more general case where interest rates are stochastic, we need to ammend slightly the no-common jump condition.

No-common jump condition (NCJ2):

Define $Y_i^j(t) = \mathrm{E}_t^Q\left[e^{-\int_t^T(r(s)+\lambda_i^j(s))ds}\right]$. We say $\lambda_i^j(t)$ satisfies the NCJ2 condition provided Y_i^j does not jump at the time of default of a counterparty, i.e. $\Delta Y_i^j(\tau_k) = 0\ \forall k, i = 1, \ldots n$ and $j = 1, \ldots, J$.

We show the following:

Proposition 2 *If the interest rate and intensity processes satisfy the NCJ2 condition, then the forward-neutral probability of default of counterparty i prior to maturity T can be computed from:*

$$\mathrm{E}_t^Q\left[e^{-\int_t^T r(s)ds}\,\mathbf{1}_{\{\tau_i > T\}}\right] = \sum_{j=1}^{J} p^j(t)\mathrm{E}_t^Q\left[e^{-\int_t^T(r(s)+\lambda_i^j(s))ds}\right]\mathbf{1}_{\{\tau_i > t\}}. \tag{13}$$

In particular, if both $r(t)$ and $\lambda_i^j(t)$ follow an affine process, then the forward-neutral probability of bankruptcy is a weighted average of exponential affine functions.

__Proof__:

Note that since $\sum_{j=1}^J p^j(t) = 1$ a.s., we have:

$$\mathrm{E}_t^Q\left[e^{-\int_t^T r(s)ds}\mathbf{1}_{\{\tau_i>T\}}\right] = \sum_{j=1}^J \mathrm{E}_t^Q\left[e^{-\int_t^T r(s)ds}p^j(T)\mathbf{1}_{\{\tau_i>T\}}\right]. \tag{14}$$

As before, we define $q_i^j(t) := p^j(t)\mathbf{1}_{\{\tau_i>t\}}$. In addition, it is convenient to introduce $Y_i^j(t) \equiv \mathrm{E}_t^Q\left[e^{-\int_t^T (r(s)+\lambda_i^j(s))ds}\right]$. Therefore, it suffices to prove that

$$\mathrm{E}_t^Q\left[e^{-\int_t^T r(s)ds}\, q_i^j(T)\right] = q_i^j(t)\, Y_i^j(t). \tag{15}$$

Clearly, $e^{-\int_0^t (r(s)+\lambda_i^j(s))ds}\, Y_i^j(t)$ is a Q-martingale. Thus

$$dY(t) = (r(t) + \lambda_i^j(t))\, Y_i^j(t)\, dt + dm^*(t) \tag{16}$$

for some Q-martingale $m^*(t)$. Further, recall that we have

$$dq_i^j(t) = \lambda_i^j(t)\, q_i^j(t)\, dt - dM_i^j(t) + dp^j(t).$$

Combining the two results we obtain

$$\begin{aligned} & d\left(e^{-\int_0^t r(s)ds}\, Y_i^j(t)\, q_i^j(t)\right) \\ & \quad = e^{-\int_0^t r(s)ds}\left(-r(t)\, Y_i^j(t)\, q_i^j(t)\, dt + q_i^j(t^-)\, dY(t) + Y_i^j(t^-)\, dq_i^j(t) + \Delta Y_i^j(t)\, \Delta q_i^j(t)\right) \\ & \quad = e^{-\int_0^t r(s)ds}\left(q_i^j(t^-)\, dm_t - Y_i^j(t^-)\, dM_i^j(t) + Y_i^j(t^-)\, dp^j(t)\right), \end{aligned} \tag{17}$$

where we have used the fact that because of NCJ2 $\Delta Y_i^j(t)\Delta q_i^j(t) = 0$. Hence, it follows that $e^{-\int_0^t r(s)ds}\, Y_i^j(t)\, q_i^j(t)$ is a Q-martingale, in turn implying that:

$$e^{-\int_0^t r(s)ds}\, Y_i^j(t)\, q_i^j(t) = \mathrm{E}_t^Q\left[e^{-\int_0^T r(s)ds}\, Y_i^j(T)\, q_i^j(T)\right].$$

Rearranging and using the fact that $Y_i^j(T) = 1$ we obtain the desired result:

$$Y_i^j(t)\, q_i^j(t) = \mathrm{E}_t^Q\left[e^{-\int_t^T r(s)ds}\, q_i^j(T)\right]. \tag{18}$$

□

2.2 An Application of the Framework

To illustrate the implications of the framework we consider the following example. Consider a portfolio of n=100 risky zero-coupon bonds. Suppose there are two possible states of nature (i.e., $J = 2$) corresponding to a high intensity of default (λ^H) and low intensity of default (λ^L). For simplicity, we make the following assumptions:

- Both ($\lambda^H = 0.01$) and ($\lambda^L = .002$) are constants.
- All N firms are in one of these two states of nature.
- The current estimate that the economy is in the high-intensity state is $p = .5$.
- The recovery rate is zero in the event of default.

Then, the price of the portfolio is

$$V(t) = 100 \left(p^H(t)e^{-\lambda^H(T-t)} + (1 - p^H(t))e^{-\lambda^L(T-t)} \right) \mathrm{E}_t^Q[e^{-\int_t^T r(s)ds}]. \tag{19}$$

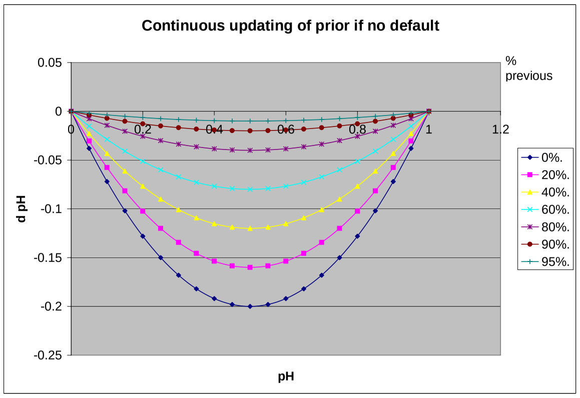
Further, since all firms are perfectly symmetric, the updating equation 3 can be written in terms of the counting process $N(t) = \sum_{i \geq 1} \mathbf{1}_{\{\tau_i \leq t\}}$ as follows:

$$dp^H(t) = p^H(t)(1 - p^H(t)) \frac{\lambda^H - \lambda^L}{\overline{\lambda}(t)} \left(\sum_{i=1}^{n} d\mathbf{1}_{\{\tau_i \geq t\}} - (n - N(t))\overline{\lambda}(t)dt \right), \tag{20}$$

where the intensity of a default in the pool at time t is given by $\overline{\lambda}(t) = p^H(t)\lambda^H + (1 - p^H(t))\lambda^L$.

Figure 1 below shows the drift- and jump-components of the updating equation for the probability that the pool be in the high risk state for various levels of default history ($N(t)$) and prior ($p(t)$). The pictures show that updating is more significant the larger the remaining pool size, which indicates that the larger the remaining data set, the more information is conveyed by either observing no default, or by observing an event. We also see that the continuous updating is convex with a minimum at $p(t) = 0.5$, which indicates that the informativeness of not observing any event is highest for the investor in states where he is most confused about which state is 'true.' As long as no default is observed investors revise their beliefs continuously downward. On the other hand, investors revise their prior that the intensity if high upward at the time of occurrence of a default. Note that the revision is more dramatic for priors close to, but below $p^H = 0.5$.

Figure 2 shows the risk-neutral probability density of observing $n = 1, \ldots 25$ defaults at a five year horizon within this portfolio. It also shows the probability density that would be



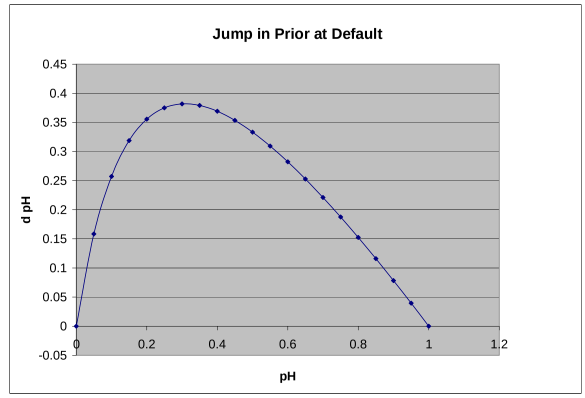


Figure 1: Updating of priors conditional on number of previously observed defaults and either no default (continuous part) or default (jump).

estimated if one were to assume firms had constant risk-neutral intensity $\widehat{\lambda}$ estimated to match the value of the bond portfolio, i.e., such that $100e^{-\widehat{\lambda}(T-t)}\mathrm{E}_t^Q[e^{-\int_t^T r(s)ds}] = V(t)$. For the example this yields $\widehat{\lambda} = 0.006$.

The figure shows that the contagion-model puts more weights into the tails of the distribution than does the contagion-free model. In general, the effect of the contagion-model is similar to a 'mean-preserving' spread in that it takes weight from the center of the distribution and redistributes it into the tails. In particular, as is apparent from the graph, the contagion-model puts a much higher weight on multiple borrowers defaulting (similar to the infectious default model of Davis and Lo (2000)). This indicates that the model would predict lower valuation for high-rated tranches in typical CBO or CDO deals (Duffie and Garleanu (2002)). Note further that, since the sum of the value of the tranches must equal the sum of the values of the underlying bonds, this model also suggests a higher valuation for the so-called 'toxic waste' tranches relative to a model based on iid defaults calibrated to prices of bonds in the initial pool.

We emphasize that, while our framework is most naturally interpreted as a Bayesian updating of beliefs, one need not interpret the state variables p^j in such a manner. As such, this framework is also consistent with a 'counterparty-risk' interpretation. In the next section, we show that an extension of the structural framework of Duffie and Lando (2000) to multiple firms gives rise to a special case of the reduced-from model presented above, effectively providing an economic 'justification' for this framework.

2.3 A structural model of contagion

Following the standard analysis of Merton (1974), Leland (1994), and Duffie and Lando (2001), we assume that each firm in the economy has total asset value given by $X_i(t)$ which follows a

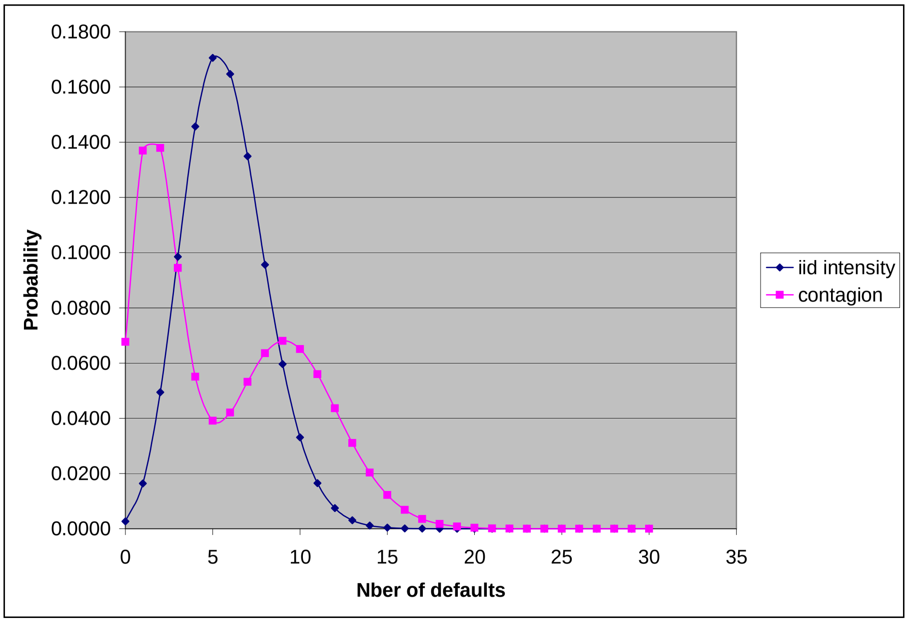


Figure 2: Ten year probability of bankruptcy assuming constant default intensity vs. model with contagion.

geometric Brownian motion:

$$\frac{dX_i(t)}{X_i(t)} = \mu_i \, dt + \sigma_i \, dz_i(t). \tag{21}$$

Here, the drift μ_i and volatility σ_i are constant, and $dz_i(t)$ is a standard Brownian motion. For simplicity, and to emphasize our point that firms do not need to have economic-ties for their spreads to have common jumps, we assume that Brownian motions are uncorrelated across firms: $dz_i(t)\ dz_j(t) = dt\, \mathbf{1}_{i=j}$.[3] For simplicity, we assume each firm has previously issued a perpetuity which pays a constant coupon rate. Further, each firm will default on its debt payments when the firm value reaches a known threshold level X_i^B.[4]

Following Duffie and Lando (2001), we assume that investors do not observe the actual firm value. DL have shown that if investors can only imperfectly observe the level of firm value, then default effectively becomes unpredictable, i.e. similar to a Poisson process. A similar result holds in our framework even though we assume a somewhat simpler information mechanism than DL. In particular, we assume that investors observe a signal which corresponds to some lagged firm value $x_i(t) = X_i(t - \ell_i)$ where the lag ℓ is not known perfectly. For simplicity, we

[3]This assumption implies that upon default of firm-i, there is no updating on the current values of other firms *conditional* upon whether the economy is in state-H or state-L. Rather, it only affects the belief whether the economy is in state-H or state-L.

[4]Following Leland (1994) and Duffie and Lando (2001), it is straightforward to determine the threshold boundary and optimal coupon endogenously. However, in order to focus on the relevant issues, after the issuance-date, we can think of the perpetuity payments $C\,dt$ and the default threshold level X_i^B as just some given exogenous constants.

assume that investors know that for each firm ℓ_i can take on only two values, $\ell_i^H > \ell_i^L$. Further we assume that the 'accounting quality' of all firms is perfectly correlated in that all firms are either in the high-delay state or the low-delay state. This captures the idea that all firms are using similar information technology (e.g., accounting 'techniques'). The longer the delay, the less is known about how close the current cash flows are to the default boundary. We note that within this framework, the most recently observed data point is a 'sufficient statistic' to price the risky debt. In particular, there is no need to Bayesian update on past values.

At date-t, the prior belief about this probability is defined as:

$$p^H(t) \equiv \pi\left(\ell_i = \ell_i^H \ \forall i = (1, N) \middle| \mathcal{F}_t\right). \tag{22}$$

Define $y_i(t) \equiv \log \frac{x_i(t)}{X_i^B}$ and $Y_i(t) \equiv \log \frac{X_i(t)}{X_i^B}$. Default occurs at the first time that $Y_i(t)$ reaches zero: $\tau_i = \inf\{t : Y_i(t) = 0\}$. Before default occurs, $y_i(t)$ (and $Y_i(t)$ for that matter) follows the process

$$\begin{aligned} dy_i(t) &= \left(\mu_i - \frac{\sigma_i^2}{2}\right) dt + \sigma_i \, dz_i(t) \\ &\equiv m_i \, dt + \sigma_i \, dz_i(t). \end{aligned} \tag{23}$$

Below, we first derive the default intensity for a single firm assuming that the delay is known and then solve the more complicated problem when delay is unknown.

2.3.1 Default intensity for a given known information lag.

Let us consider a single firm with a known delay of ℓ (we drop subscripts in this section for simplicity). Following, DL (2001) one can show that because firm value is imperfectly observed, its default time is unpredictable conditional on the information available to investors. Here we provide a proof of this proposition because it is much simpler to demonstrate in our framework than in DL (2001).

Proposition 3 *If the information lag is known to be ℓ, then each firm's default-date is an unpredictable stopping time with a default intensity defined on the set $\tau > t$ by:*

$$\lambda(t) = \frac{\frac{1}{\sqrt{2\pi\sigma^2\ell}}\left(\frac{y(t)}{\ell}\right) e^{-\frac{(m\ell + y(t))^2}{2\sigma^2\ell}}}{N\left(\frac{y(t)+m\ell}{\sigma\sqrt{\ell}}\right) - e^{\frac{-2y(t)\,m}{\sigma^2}} N\left(\frac{-y(t)+m\,\ell}{\sigma\sqrt{\ell}}\right)}. \tag{24}$$

<u>Proof</u>:

We can apply the general result of Duffie and Lando (2001) which shows that the intensity is given by:

$$\lambda(t) = \frac{1}{2}\sigma^2 F_Y(0\,|\,y(t),\ell), \tag{25}$$

where $F(Y_t\,|\,Y_0,t)$ is the density of a (m,σ) arithmetic Brownian motion, Y_t conditional on having not hit zero prior to t and being equal to Y_0 at time 0. It is given by:

$$F(Y_t|Y_0,t) \equiv \pi\left(Y_t\,|\,Y_0,\tau>t\right) = \frac{\pi\left(Y_t,\tau>t\,|\,Y_0\right)}{\pi\left(\tau>t\,|\,Y_0\right)}. \tag{26}$$

Both the numerator and denominator can be derived using the well-known results for given that Y is normally distributed $Y \sim N(m,\sigma^2)$ (See, for example, Borodin and Salminen (1996):)

$$\begin{aligned}\pi\left(Y_t,\tau>t\,|\,Y_0\right) &\equiv \pi\left(Y_t,\ \min_{s\in(0,t)} Y_s>0\,\middle|\,Y_0\right)\\ &= \frac{1}{\sqrt{2\,\pi\,\sigma^2 t}}\left[e^{-\frac{(Y_t-Y_0-mt)^2}{2\sigma^2 t}} - e^{\frac{-2Y_0\,m}{\sigma^2}}e^{-\frac{(Y_t+Y_0-mt)^2}{2\sigma^2 t}}\right].\end{aligned} \tag{27}$$

$$\begin{aligned}\pi(\tau>t\,|\,Y_0) &\equiv \pi(\min_{s\in(0,\ell)} Y_s>0\,|\,Y_0)\\ &= \mathrm{N}\left(\frac{Y_0+mt}{\sigma\sqrt{t}}\right) - e^{\frac{-2Y_0\,m}{\sigma^2}}\,\mathrm{N}\left(\frac{-Y_0+m\,t}{\sigma\sqrt{t}}\right).\end{aligned} \tag{28}$$

Using these two results we can explicitly compute the default intensity in (24) above.

Alternatively, this result can be demonstrated by noting that a default intensity defined on the set $t>\tau$ is given by:

$$\begin{aligned}\lambda(t) &\equiv \left(\frac{1}{dt}\right)\pi\left(\tau\in(t,t+dt)\,\middle|\,\tau>t,\,Y(t-\ell)\right)\\ &= \left(\frac{1}{dt}\right)\frac{\pi\left(\tau\in(t,t+dt)\,\middle|\,Y(t-\ell)\right)}{\pi\left(\tau>t\,\middle|\,Y(t-\ell)\right)},\end{aligned} \tag{29}$$

where the denominator is obtained from equation (28) and the numerator is simply:

$$\begin{aligned}\left(\frac{1}{dt}\right)\pi\left(\tau\in(t,t+dt)|\,Y(t-\ell)\right) &= -\frac{\partial}{\partial\ell}\pi\left(\min_{s\in(t,t+\ell)} y(s)>0\,\middle|\,y(t)\right)dt\\ &= \frac{1}{\sqrt{2\pi\sigma^2\ell}}\left(\frac{y(t)}{\ell}\right)e^{-\frac{(m\ell+y(t))^2}{2\sigma^2\ell}}.\end{aligned} \tag{30}$$

□

As in DL, even though the actual default is a predictable stopping time, conditional upon the information available to investors, default arrivals are unpredictable. Hence, bond prices will take on the characteristics of those in a reduced form model. For example, even short maturity bonds will command a yield spread. As in standard reduced-form models, the survival probability is given by the expectation $\mathrm{E}_t[e^{-\int_t^T \lambda(s)ds}]$. In this framework this is easily computed however:

Corollary 2 *For a given default lag ℓ, the survival probability $\pi_t(\tau > T)$ is given by:*

$$\pi_t\left(\tau > T \middle| \tau > t,\, Y(t-\ell)\right) = \mathrm{E}_t[e^{-\int_t^T \lambda(s)ds}] = \frac{\pi\left(\tau > T \,|\, Y(t-\ell)\right)}{\pi\left(\tau > t \,|\, Y(t-\ell)\right)}. \tag{31}$$

__Proof:__

Since for a given lag the intensity follows a diffusion process, the first equation follows directly from for example Lando (1998). The explicit solution can be derived either by solving the expectation or by noting that:

$$\pi\left(\tau > T \,|\, Y(t-\ell), \tau > t\right) = \frac{\pi\left(\tau > T \,|\, Y(t-\ell)\right)}{\pi\left(\tau > t \,|\, Y(t-\ell)\right)}$$

This result follows because, if $\tau_i > T$ then necessarily $\tau_i > t$. Note that both the numerator and denominator can be expressed in terms of equation (28). □

Given that the functional form of the intensity is highly non-linear, it may seem surprising that the expectation has a closed form solution. The latter is evident given the structure of the model. Thus, conditional on knowing the information lag, default intensity and survival probabilities are simply derived. Investors thus know that firms will default with probability $\lambda_i^H(t)$ or $\lambda_i^L(t)$ depending on whether the information delay is high or low. Note that these intensities are continuous, since they are functions of continuous state variables. Thus if the information lag was common-knowledge, then there would be no jump in credit spreads (outside of the jump to default itself). However, in our model there is additional uncertainty about the quality of information, which we discuss next.

2.3.2 Uncertainty about information quality and the updating of beliefs

We assume that investors do not know whether the information quality (i.e., the lag) is high or low. The filtration $\mathcal{F}_t$ representing the information available to agents is generated by the observation of the paths of signals $\{y_i(s), i \in (1,N)\}_{s\leq t}$ and the history of defaults $\{q_i(s), i \in (1,N)\}_{s\leq t}$. We assume agents start with an initial prior about the quality of the information at date 0. Since the information quality is shared by all firms, investors update their priors conditional on observed default. The more unexpected a default, the more likely that the priors were incorrect. More formally, we assume investors update $p^H(t) \equiv \pi\left(\ell = \ell^H \middle| \mathcal{F}_t\right)$ conditional on the observed history of defaults, where we define the event $\{\ell = \ell^H\} \equiv \{\ell_i = \ell_i^H \ \forall i\}$. Indeed as shown above, associated with each $i = 1, \ldots, N$ Poisson jump process $dq_i(t)$ are two intensity processes $\lambda_i^H(t)$, $\lambda_i^L(t)$ defined via

$$\lambda_i^H(t) \equiv \left(\frac{1}{dt}\right) \mathrm{E}_t \left[dq_i(t) | \mathcal{F}_t, \ell = \ell^H \right] \tag{32}$$

$$\lambda_i^L(t) \equiv \left(\frac{1}{dt}\right) \mathrm{E}_t \left[dq_i(t) | \mathcal{F}_t, \ell = \ell^L \right]. \tag{33}$$

While each firm may have different default intensities, i.e. $\lambda_i^H(t) \neq \lambda_j^H(t)$ for $i \neq j$, if $\ell_i = \ell_i^H$, then $\ell_j = \ell_j^H$, and similarly for the low intensity case. This assumption insures that the default of one firm will carry information about the quality of the accounting information for firms that share this risk.[5]

With these assumptions, investors will compute the intensity of firm i defaulting on the set $t < \tau_i$ as:

$$\overline{\lambda}_i(t) = p^H(t)\lambda_i^H(t) + \left(1 - p^H(t)\right)\lambda_i^L(t). \tag{34}$$

In appendix A we show the following:

Proposition 4 *Investors update the quality of beliefs conditional on the observed history of defaults according to:*

$$dp^H(t) = p^H(t)\left(1 - p^H(t)\right) \sum_{i=1}^{N} \frac{\lambda_i^H(t) - \lambda_i^L(t)}{\overline{\lambda}_i(t)} (1 - q_i(t)) \left(dq_i(t) - \overline{\lambda}_i(t)dt\right). \tag{35}$$

Clearly $p^H(t)$ is a $P, \mathcal{F}$- quadratic pure jump martingale which experiences only positive jumps. Further, both 0 and 1 are natural absorbing boundaries for the process. The interpretation is as follows. As long as no jump are observed, agents update their beliefs towards the more optimistic scenario of a low intensity (hence the negative drift in the updating process). When a jump occurs, agents will tilt their beliefs towards the high lag (i.e., poor accounting information quality) scenario.

An immediate consequence of this proposition is that the $\mathcal{F}_t$ conditional default intensities $\overline{\lambda}_i$ of all firms experience common jumps at times of defaults of individual firms. As a result, we expect to find common factors in credit spreads representing these probabilities of common jumps. We note that this equation is a special case of the general reduced form model presented in the first section. Thus all the results derived there apply to this framework. In particular, we derive the survival probabilities:

[5] This assumption could be weakened. It only needs to be the case that there is a market-wide correlation so that one can correctly update beliefs about the quality of accounting information by observing the time series of a finite number of defaults.

Corollary 3 *The survival probability with incomplete information is given by:*

$$
\begin{aligned}
P(\tau > T) &= p^H(t)\mathrm{E}_t[e^{-\int_t^T \lambda^H(s)ds}] + (1-p^H(t))\mathrm{E}[e^{-\int_t^T \lambda^L(s)ds}] \\
&= p^H(t)\frac{\pi\left(\tau > T \,|\, Y(t-\ell^H)\right)}{\pi\left(\tau > t \,|\, Y(t-\ell^H)\right)} + (1-p^H(t))\frac{\pi\left(\tau > T \,/\, Y(t-\ell^L)\right)}{\pi\left(\tau > t \,/\, Y(t-\ell^L)\right)}. \qquad (36)
\end{aligned}
$$

Proof: The result follows immediately from proposition 1 and corollary 2. □

We next show how this model allows to get simple tractable solutions for risky bonds in the presence of this interdependence of default risk.

2.3.3 Risky bonds and Credit Spreads

For simplicity, we consider the valuation of a risky zero-coupon bond which promises to pay \$1 at maturity T. In case of default we assume a recovery at maturity of a fraction $R < 1$ of the principal, or, equivalently recovery at the default time of a fraction of a risk-free zero-coupon bond with the same maturity. Further, we assume markets are frictionless and that it is possible to borrow or lend at a constant risk-free rate r. As usual, we assume the existence of an equivalent measure Q under which discounted self-financing portfolios are martingales (see, for example, Duffie (1996)). Then the date-t price of the risky debt is given by:

$$
\begin{aligned}
D(t) &= \mathrm{E}_t^Q\left[e^{-r(T-t)}\left(\mathbf{1}_{\{\tau_i > T\}} + R\mathbf{1}_{\{\tau_i \le T\}}\right) \,|\, \mathcal{F}_t\right] \\
&= e^{-r(T-t)}\left(R + (1-R)\mathrm{E}^Q\left[\mathbf{1}_{\{\tau_i > T\}} \,|\, \mathcal{F}_t\right]\right). \qquad (37)
\end{aligned}
$$

Under the risk-neutral measure the default time admits an $\mathcal{F}_t$-intensity $\overline{\lambda}_i^Q(t)$ (see Artzner and Delbaen (1995) and Kusuoka (1998)). The crucial issue for pricing is how to risk-adjust the intensity, i.e. identify $\overline{\lambda}_i^Q$. Following our derivation for the historical measure it may appear natural to write: $\overline{\lambda}_i^Q(t) = q^H(t)\lambda_{Q,i}^H(t) + (1-q^H(t))\lambda_{Q,i}^L(t)$ where q^H would be the risk-neutral measure probability of being in the high state (i.e., $q^H(t) = \pi^Q\left(\ell = \ell^H \,|\, \mathcal{F}_t\right)$) and,

$$
\lambda_{Q,i}^H(t) \equiv \left(\frac{1}{dt}\right)\mathrm{E}^Q\left[dq_i(t)|\,\mathcal{F}_t,\, \ell = \ell^H\right] \qquad (38)
$$

$$
\lambda_{Q,i}^L(t) \equiv \left(\frac{1}{dt}\right)\mathrm{E}^Q\left[dq_i(t)|\,\mathcal{F}_t,\, \ell = \ell^L\right]. \qquad (39)
$$

However, as we show in the next section, the typical form of the intensity is: $\overline{\lambda}_i^Q(t) = \zeta(t)\overline{\lambda}_i^P(t)$, where $\zeta(t)$ denotes the jump risk-premium which may be different from one if the state price density experiences a jump at τ_i. [6] In such a case, jump-risk is priced.

[6] Effectively $\zeta(\tau^-) = \mathrm{E}\left[\frac{\xi(\tau)}{\xi(\tau^-)}|\mathcal{F}(\tau^-)\right]$, where $\xi(t) = \mathrm{E}^Q[\frac{dQ}{dP}|\mathcal{F}(t)]$ is the conditional likelihood ratio.

The partial equilibrium model does not identify the nature of the adjustment for systematic jump risk. If the number of firms N that share the 'accounting quality' is a finite proportion of the economy, then we would expect such jump-risks to be priced. Here, however, we assume that such risks are conditionally diversifiable, and hence that $\zeta(t) = 1$. Hence, it follows that

$$\overline{\lambda}_i^Q(t) = p^H(t)\lambda_i^H(t) + (1 - p^H(t))\lambda_i^L(t). \tag{40}$$

In the next section, we investigate a general equilibrium framework which investigates this issue more carefully. Here we note however, that even with a choice of non-priced jump-risk, the default intensity process will still change under the Q-measure, since both λ^H and λ^L are functions of the firm value process $y(t)$, whose dynamics change under the risk-neutral measure (its drift is risk-adjusted if the Brownian motion risk is systematic).[7] Indeed suppose that under the risk-neutral measure the assets of firm i have the following dynamics:

$$\frac{dX_i(t)}{X_i(t)} = (r(t) - \delta_i)dt + \sigma_i dz_i^Q(t). \tag{41}$$

Then, it follows that

Proposition 5 *The risk-neutral measure survival probability is given by*

$$\mathrm{E}^Q\left[\mathbf{1}_{\{\tau_i > T\}} \mid \mathcal{F}_t\right] = p^H(t)\, Q_i^H(y(t), t) + (1 - p^H(t))\, Q_i^L(y(t), t), \tag{42}$$

where $Q_i^j(y(t), t) \equiv \mathrm{E}_t^Q\left[e^{-\int_t^T \lambda_i^j(s)ds}\right]$ $\forall j = H, L$, *each of which solve a partial differential equation of the form*

$$\frac{1}{2}\sigma_i^2 \frac{\partial^2}{\partial y^2} Q_i^j(y,t) + \frac{\partial}{\partial y} Q_i^j(y,t)(r - \delta_i - \frac{1}{2}\sigma^2) + \frac{\partial}{\partial t} Q_i^j(y,t) - \lambda_i^j(t) Q_i^j(y,t) = 0, \tag{43}$$

subject to the boundary condition $Q_i^j(y, T) = 1$ $\forall y$, *and where* $\lambda_i^{H,L}(s)$ *is given in proposition 3.*

Proof:

Applying Proposition 1 we have

$$\mathrm{E}^Q\left[\mathbf{1}_{\{\tau_i > T\}} \mid \mathcal{F}_t)\right] = p^H(t)\, E_t^Q[e^{-\int_t^T \lambda_i^H(s)ds}] + (1 - p^H(t))\, E_t^Q[e^{-\int_t^T \lambda_i^H(s)ds}]. \tag{44}$$

Using the fact that $y(t)$ is a Markov process and that $\{\lambda^H(s), \lambda^L(s)\}$ are deterministic functions of $y(s)$ as shown in Proposition 3, we can write $E_t^Q[e^{-\int_t^T \lambda_i^j(s)ds}] = Q_i^j(y(t), t)$ $j = L, H$ for some deterministic function $Q_i^j(\cdot, \cdot)$. An application of Feynman-Kac's theorem gives PDE satisfied by Q_i^j. $\square$

[7] See also the discussion in Jarrow, Lando and Yu (2000) and Collin-Dufresne and Hugonnier (2000).

3 Jump Risk Premia in a Simple Production Economy

Since the perception of information quality is a common factor driving all risky bonds, one would expect it should command a systematic risk-premium (assuming bonds are a substantial part of the market portfolio for example). In this section we propose a simple 'general equilibrium' production economy to investigate the risk-premium (if any) that jump risk should carry (e.g., $\zeta \gtreqless 1$). We use a simple Cox, Ingersoll and Ross (1985) production economy with many identical agents (i.e., there exists a representative agent) who maximize their expected utility of consumption

$$J(t) = \max_{C,\Pi} \mathrm{E}\left[\int_t^{\infty} ds\, e^{-\delta s}\, \frac{C_s^{1-\gamma}}{1-\gamma} \mid \mathcal{F}_t\right] \tag{45}$$

by investing in a single production technology which may experience bad shocks, modeled as rare events as in Ahn and Thomson (1988). The return on the risky technology is

$$\frac{d\eta}{\eta} = \mu\, dt + \sigma\, dz_t - \alpha\, dq_1(t), \tag{46}$$

where q_1 is a counting process associated with a poisson process which represents the rare 'bad' events. Similar to our partial equilibrium setup we assume the agents have imperfect information about the probability of such events. They know that the jump intensity is either λ_1^H or λ_1^L with $\lambda^H > \lambda^L$, and form priors represented by $p^H(t) = \mathrm{E}[\lambda = \lambda^H \mid \mathcal{F}_t]$ where $\mathcal{F}_t$ represents all the information available to agents a t. In addition to observing the return to the technology we assume investors observe an independent source of information modeled as a point process q_2 which has intensity λ_2^H or λ_2^L with $\lambda_2^H > \lambda_2^L$. While information arrival is independent of the technology, i.e., $dq_1 dq_2 = 0$, we assume, as in the partial equilibrium model, that investors know that the vector of point processes $q = \{q_1, q_2\}$ has intensity vector $\lambda^H = \{\lambda_1^H, \lambda_2^H\}$ or $\lambda^L = \{\lambda_1^L, \lambda_2^L\}$. Thus the arrival of information provides information about the actual intensity of the technological risk, similarly to our partial equilibrium model. This source of information could be interpreted as observed defaults on small firms in the economy which do not affect aggregate technology.[8] It allows the agents to update their beliefs about the riskiness of the technology, which in turn affects their consumption-investment decisions. Information arrival thus has real effects and the associated belief process commands a systematic risk-premium even though it does not directly affect the real return to the production technology. A direct implication is that the historical measure estimate of the default intensity should be

[8]Alternatively we can think of this as modeling investment in a new technology, such as Nuclear energy, whose riskiness is not known perfectly. Agents are performing experiments which allow them to learn about the riskiness of the technology, but they themselves are independent from the return on the technology.

modified to compute prices under the risk-neutral measure in order to account for systematic risk in credit spread jumps.

As before agents update their beliefs about the intensity of jumps by conditioning on information and actual observed jumps in the risky technology. This leads to the following updating of beliefs:

$$dp^H(t) = p^H(t)\left(1 - p^H(t)\right)\sum_{i=1}^{2}\frac{\lambda_i^H(t) - \lambda_i^L(t)}{\overline{\lambda}_i(t)}\left(dq_i(t) - \overline{\lambda}_i(t)dt\right). \quad (47)$$

where the $\mathcal{F}_t$-conditional intensity of an event $dq_i(t) = 1$ is defined as

$$\overline{\lambda}_i(t) = \lambda_i^H p^H(t) + \lambda_i^L(1 - p^H(t)). \quad (48)$$

The agents choose to optimally allocate a proportion θ_t of their wealth to production, to consume at rate c_t and save the remainder by investing at the risk-free rate r. As customary in these types of models, since agents are identical in equilibrium, net saving is zero, and r really represents the shadow borrowing and lending rate. Each agents dynamic wealth equation is:

$$dW_t = r_t W_t\, dt + \theta_t W_t\left(\frac{d\eta_t}{\eta_t} - r_t\, dt\right) - c_t\, dt \quad (49)$$

Using an extended Bellman equation to include jumps, and looking for a solution of the form $J(t) = e^{-\delta t} J(W_t, p^H(t))$, we obtain

$$\begin{aligned} 0 \quad = \quad & \max_{c,\theta}\Bigg\{\frac{c_t^{1-\gamma}}{1-\gamma} - \delta J + J_W\left(rW + \theta W(\mu - r) - c\right) + \frac{1}{2}J_{WW}W^2\theta^2\sigma^2 \\ & -J_{p^H}\left(p^H\left(1-p^H\right)\sum_{i=1}^{2}\left[\lambda_i^H - \lambda_i^L\right]\right) + \overline{\lambda}_1(p^H)\left[J\left(W(1-\alpha\theta), p^H\frac{\lambda_1^H}{\overline{\lambda}_1}\right) - J\left(W, p^H\right)\right] \\ & + \overline{\lambda}_2(p^H)\left[J\left(W, p^H\frac{\lambda_2^H}{\overline{\lambda}_2}\right) - J\left(W, p^H\right)\right]\Bigg\} \end{aligned} \quad (50)$$

We look for a solution of the form:

$$J(W, p^H) = A(p^H)^{-\gamma}\frac{W^{1-\gamma}}{1-\gamma} \quad (51)$$

Using the HJB equation we obtain the first order conditions:

$$c \quad = \quad (J_W)^{-1/\gamma} = A(p^H)W \quad (52)$$

$$\mu - r \quad = \quad -\frac{W J_{WW}}{J_W}\theta\sigma^2 + \overline{\lambda}_1\alpha\frac{J_W\left(W(1-\theta\alpha), p^H\frac{\lambda_1^H}{\overline{\lambda}_1}\right)}{J_W(W, p^H)} \quad (53)$$

Plugging these back into the HJB Equation and using the proposed value function confirms our guess.

Further the equilibrium condition in the bond market imposes $\theta = 1$. We thus find that the function $A(p^H)$ satisfies the following ordinary differential equation:

$$\begin{aligned} -\frac{A'}{A} p^H (1-p^H)\gamma \sum_i (\lambda_i^H - \lambda_i^L) &= \gamma A - \delta + (1-\gamma)(\mu - \gamma \frac{\sigma^2}{2}) \\ &+ \overline{\lambda}_1(p^H)\left((1-\alpha)^{1-\gamma} \frac{A(p^H \frac{\lambda_1^H}{\overline{\lambda}_1})^{-\gamma}}{A(p^H)^{-\gamma}} - 1 \right) + \overline{\lambda}_2(p^H)\left(\frac{A(p^H \frac{\lambda_2^H}{\overline{\lambda}_2})^{-\gamma}}{A(p^H)^{-\gamma}} - 1 \right) \quad (54) \end{aligned}$$

We note that by definition p^H has absorbing boundaries at 0 and 1. When $p^H = 0$, $\overline{\lambda}_i = \lambda_i^L$ and when $p^H = 1$ we have $\overline{\lambda}_i = \lambda_i^H$. We find:

$$A(0) = \frac{\delta - (1-\gamma)(\mu - \gamma\frac{\sigma^2}{2}) + \lambda_1^L \left(1 - (1-\alpha)^{1-\gamma}\right)}{\gamma} \quad (55)$$

$$A(1) = \frac{\delta - (1-\gamma)(\mu - \gamma\frac{\sigma^2}{2}) + \lambda_1^H \left(1 - (1-\alpha)^{1-\gamma}\right)}{\gamma} \quad (56)$$

Since the value function of agents is decreasing in the prior probability that shocks occur with high frequency we have the following:

Lemma 3 *The function $A(p)$ satisfies:*
$$\begin{cases} A'(p) \geq 0 & \text{if} \quad 0 < \gamma < 1 \\ A'(p) \leq 0 & \text{if} \quad \gamma > 1 \\ A(p) = \delta & \text{if} \quad \gamma = 1 \end{cases}$$

Proof: This lemma follows from $\frac{\partial J(p^H, W)}{\partial p^H} \leq 0$. We have no proof of this result yet. We have only verified it numerically. □

In equilibrium it is well-known (Cox, Ingersoll and Ross (1985)) that the state price density is

$$\Pi(t) = \frac{J_W(W_t, p^H(t))}{J_W(W_0, p^H(0))} \equiv e^{-\int_0^t r_s ds} \xi_t, \quad (57)$$

where we have defined the Radon-Nykodim density $\xi_t = \frac{dQ}{dP}\Big|_{0 \leq s \leq t}$. Applying Itô's lemma we find that:

$$\frac{d\xi_t}{\xi_t} = -\gamma\sigma dz_t + ((1-\alpha)^{-\gamma} \frac{A(p^H \frac{\lambda_1^H}{\overline{\lambda}_1})^{-\gamma}}{A(p^H)^{-\gamma}} - 1) dM_1(t) + (\frac{A(p^H \frac{\lambda_2^H}{\overline{\lambda}_2})^{-\gamma}}{A(p^H)^{-\gamma}} - 1) dM_2(t), \quad (58)$$

where we have defined the $(P, \mathcal{F}_t)$ martingales $M_i(t) = q_i(t) - \int_0^t \overline{\lambda}_i(s) ds \ \ i = 1, 2$. Thus we find the risk-adjustment for both q_1 and q_2 risks.

Proposition 6 *The risk-neutral measure intensity for information risk is given by:*

$$\overline{\lambda}_2^Q = \overline{\lambda}_2 \frac{A(p^H \frac{\lambda_2^H}{\overline{\lambda}_2})^{-\gamma}}{A(p^H)^{-\gamma}}. \tag{59}$$

The intensity for production jump risk is:

$$\overline{\lambda}_1^Q = \overline{\lambda}_1 (1-\alpha)^{-\gamma} \frac{A(p^H \frac{\lambda_1^H}{\overline{\lambda}_1})^{-\gamma}}{A(p^H)^{-\gamma}}. \tag{60}$$

Proof: We apply Itô's lemma to show that $\mathrm{E}_t[d(\xi_t M_i(t))] = \xi_t \left(\overline{\lambda}_i^Q(t) - \overline{\lambda}_i(t)\right) dt$. The result then follows from the definition of M_i, Girsanov's theorem for point processes and the martingale characterization of intensity (see Brémaud (1981)). □

We see that even if $\lambda_1^H = \lambda_2^H$ and $\lambda_1^L = \lambda_2^L$ the risk-adjustments for the two point processes q_1 and q_2 will be different reflecting the fact that only the first one affects equilibrium wealth. In fact, in the absence of uncertainty about the quality of information (i.e., when $p^H = 0$ or $p^H = 1$) we see that q_2 risk carries no risk-adjustment whereas q_1 still does. The proposition shows that for an economy populated by agents more risk-averse than the log-investors $\gamma > 1$ the risk-adjustment tends to increase the intensity under the risk-neutral measure. In fact, production jump risk may even face a positive jump risk-premium when investors are less risk-averse than log (due to the factor $(1-\alpha)^{-\gamma} > 1$).

Proposition 7 *The equilibrium risk-free interest rate is given by*

$$r(t) = \mathrm{E}_t^Q \left[\frac{d\eta(t)}{\eta(t)}\right] = \mu - \gamma\sigma^2 - \lambda_1^Q(t)\alpha = \mu - \gamma\sigma^2 - \overline{\lambda}_1(t)\alpha(1-\alpha)^{-\gamma} \frac{A(\frac{p^H \lambda_1^H}{\overline{\lambda}_1})^{-\gamma}}{A(p^H)^{-\gamma}} \tag{61}$$

We see that the risk-free rate experiences shocks in response to arrival of new information dq_2 through dp^H. We conjecture that: *For $\gamma > 1$ interest rates decrease when bad information arrives, i.e. $dr(t)dq_2(t) \leq 0$.*

4 Empirical Analysis

In the following sections, we investigate whether a jump in the yield spread of one firm is associated with a market-wide response. If such jumps are conditionally-diversifiable, then these jumps will have no effect on the returns of the 'market-portfolio'. However, if such jumps are priced, then they will be associated with poor returns in the corporate bond indices, and possibly also in the equity indices. Furthermore, our general equilibrium model predicts that if such jumps are priced then they will be associated with 'flights-to-quality', i.e., a downward movement in risk-free rates.[9]

In contrast to the implications of standard reduced-form models, where defaults and jumps are synonymous, here we do not equate the number of defaults to the number of yield-jumps for two reasons. First, many firms 'limp' to default, experiencing a number of smaller spread increases over several years before finally defaulting (e.g., Western Union). Second, many corporate bonds experience a large jump in their yield spreads without ever defaulting (e.g., RJR).

Regardless of whether the contagion is due to 'counterparty-risk' or to 'updating-of-beliefs', one would expect that jumps in the yields of larger, 'safer' firms would produce a greater impact on the 'market portfolio' than would 'riskier' firms.[10] As such, we limit our empirical investigation to investment-grade bonds.

4.1 Data

In order to gather a sufficiently large number of credit events in the investment-grade market, we use the Warga Fixed Income Database (FID), which contains trader quotes provided by Lehman Brothers. This database extends over nearly 25 years and contains reasonably good quality bond price data. The bond prices are month-end quotes and these data constitute the basis for the calculation of net asset values (NAVs) by mutual funds and other money managers.

We obtain spreads on bonds from the FID, which contains month-end bond price for the period January 1973-March 1998. The FID also contains the history of Lehman Brothers' corporate bond index and Treasury bond index over the January 1973-October 1997 period. In order to examine the effect on the bond market as a whole, we restrict our analysis to the months for which the corporate bond index is available. Corporate bond spreads are calculated as the difference between the bond's yield to maturity (YTM) and the interpolated YTM on a Treasury bond with a similar maturity. We obtain the interpolated YTM by using Nelson-Siegel (1987) estimates of the yield curve from the Federal Reserve's Constant Maturity Treasury (CMT) daily series. The CMT series is essentially a database of yield estimates for the on-the-run Treasuries, but occasionally the CMT will continue to estimate a yield when

[9]Note that we do not investigate incidences of major spread decreases, since the effect of these jumps on the 'market portfolio' is not expected to be symmetric.

[10]For example, when the LBO of the drugstore chain Revco led to a default and bankruptcy in August 1988, some of the firms' bonds lost considerable value, depending on such factors as where they stood in the priority in the bankruptcy case. Yet, the LBO was always understood to be risky and the default in August 1988 can be considered a random draw that was unlucky, yet well within the parameters of the deal.

the bonds are no longer auctioned. We only use yields from the CMT series in time periods when the bond is still being auctioned.

The corporate bond yield spread is the difference between the bond's yield and the estimated yield on the interpolated Treasury bond with the nearest maturity. Rather than estimates, we use actual CMT yields on corporate bonds with the same maturity as the on-the-run bonds.

Corporate bond prices are known to be inaccurate (see Warga (1991) and Warga and Welch (1993)). Few databases report transactions, and none of those contain a lengthy time-series of dealer market transactions. The Lehman Brothers database is not a transaction database, but rather a database of quotes on individual bonds supplied at month-end. Unlike many other quote sources, the FID distinguishes between matrix prices and trader quotes. The latter are prices quoted by the Lehman employees who trade the bond, whereas the matrix price is inserted when a trader has no quote. We only use yields that are based on trader quotes, deleting matrix prices from the analysis.

We consider spreads on all corporate bonds of investment-grade in the FID as long as they are not private placements, medium term notes, or Euro-bonds. By corporate bonds, we mean bonds issued by US firms that are not government-sponsored enterprises or supranational organizations, and which are not mortgage-backed or other asset-backed securities. Bonds convertible into preferred stock are also excluded.

We define a credit event as a major change in a bond's credit spread from one month to the next. Among the set of bonds that experience such shocks, we exclude bonds with less than two years until maturity.[11] We also exclude those bonds which have already fallen below a flat price of $80 (which would be the second piece of bad news about the firm). We also exclude bonds where the post-credit-shock price is above $95. The intention of this exclusion is to avoid identifying a coding-error as a credit-event.

The FID does not describe a bond as either floating or fixed in the same way that the Securities Data Corporation (SDC) does in its database. The only indication in the FID that a bond has a floating rate coupon is that the coupon variable changes from one month to the next. If a researcher uses the stated maturity on a floating rate bond to determine the appropriate yield on the riskfree rate, errors in calculated spreads are likely. As such, we eliminate floating rate bonds from the analysis. In the final sample, we cross-check our bonds with descriptions of the bonds in the SDC issuance database and Moody's bond record to make

[11] We do this for two reasons. First, the trading of short maturity bonds is rather illiquid. Hence, it is conceivable that a large drop in credit spreads could occur without a credit-event. Second, a large jump in the yield spread does not necessarily imply a large negative return when maturities are sufficiently low.

sure we have eliminated all floating rate bonds.

With these qualifications, we obtain 53,579 spread-widenings, 209,282 reductions, and 1,238 with no change in credit spread, for a total of 264,099 events. The fact that there are so many more reductions than widenings may at first seem surprising. First, the expected return on a corporate bond should be higher than the riskfree rate, which implies a relatively smaller fraction of positive spread widenings. Second, it is well documented that yield spreads are increasing functions of maturity on average. Hence, the average monthly yield spread change is negative.

Table 1 shows the distribution of reported spread increases on corporate bonds in the FID over the sample period. The vast majority of the increased spreads are quite small. Indeed, only $9,207$ out of 264,099 ($< 4\%$) of events involve credit spread jumps of more than 25 bp.

We are looking for events that are truly rare, yet we would like a database that does not cause small sample problems. Thus, we consider all spread widenings of 200 bp or more, which includes 164 bonds.

One concern is that out of 264,009 observations, the FID can include some data input errors that suggest the occurrence of a credit event where none exists. To ascertain that we are not analyzing a database of price-related typos, we investigate each of our sample's spread changes using Lexis-Nexus and Standard and Poor's Creditweek. If we see evidence that the bonds could have lost considerable value, we include it in our analysis. Evidence consistent with a major bond price movement includes news of a bond rating downgrade, dividend cut, major losses or other negative information in an earnings announcement, depressed stock prices, a major lawsuit or accident that could cause insolvency, a subsequent default or bankruptcy, or a leverage-increasing merger such as an LBO. If we could not find evidence of a credit shock to the bond, we also checked the bond price recorded in Moody's Bond Record to see if a sale price was recorded at a level close to our bond price. If the price was not similar and there was no news to indicate a problem with the firm, we assumed it was a typo in the FID. We have determined that 112 of the 164 bonds suffered wider spreads as a result of a credit event. These bonds belonged to 40 firms, two of which suffered two episodes of credit risk (Chrysler and RJR Nabisco). The large number of bonds relative to the number of firms reflects the fact that many of the firms had numerous bonds outstanding, and several of these bonds had spreads widen by over 200 bp.

The various credit events are listed in Table 2. The largest number of bonds affected by a credit shock can be classified as being the result of economic hardship. Many of these bonds belong to Chrysler, which suffered in the 1970s and again in 1990. Often the economic hardship events involve news indicating that a company is having difficulty producing operating profits, and some of these occur during recession years. Others problems, such as Kmart's, transcend the general economy. Another common event involves an industrial firm that is the target of a leveraged buyout or similar event (e.g., RJR Nabisco or Marriott). The third largest category is news that a bank has lent funds that it is unlikely to recover. Many of these banks bet badly on real estate loans in the 1990-1991 recession, leading to a number of separate companies having trouble in the same months in the Fall of 1990. A few banks appear in the early 1980s because of loans to Latin America. The accidents sometimes involve lawsuits, but the news we identified often said little about specific lawsuits.

Some of the corporate bonds in our sample lose substantial value in one month (i.e., they meet our definition of a credit shock), and then go on to lose more value in ensuing months. Because it is conceivable that such bonds were no longer considered 'investment-grade' in the minds of the marketplace after the first event, we choose to use only the first credit shock in a series of episodes in the analysis. By a series of episodes, we mean more than one credit shock for a bond over the course of a year. By using only the first episode of a firm, we identify 25 months over the sample period in which a credit event first occurs. That leaves 273 months in which no credit event occurred.

Consistent with either a contagion via counter-party risk hypothesis or a contagion via updated-expectations hypothesis, we investigate whether the size of the firm affects the impact that a credit event has on the stock and bond indices. We measure size by two methods. The first method aggregates the amount of debt outstanding in the FID for the issuer's six-digit cusip. Note that this measure of 'size' reflects the firm's presence in the corporate bond market. Such a proxy for size could be important if the supply of bonds affects prices. We note that this measure can be noisy because six-digit cusips do not always capture all of the bond issues of a firm, either because bonds are issued by subsidiaries, or because the parent issued bonds under old cusips (that differ as a result of mergers or reorganizations).

The second proxy we use for measuring issuer size is the total assets that the firm has in the year in which the credit event occurs. To identify asset size, we looked for the firm using the first six digits of the bond's cusip in Compustat. If that search failed, we searched for the firm's ticker using the look-up window in Compustat. If we could not find a ticker by that method, we then looked for its ticker in Standard & Poor's Stock Guide in the month in which the credit event occurred. Although this often required a crosscheck with the matched CRSP/Compustat file's company number, by this method we were able to obtain asset sizes on Compustat for all but one firm. For this remaining firm, which was private, we found assets reported in Moody's Transportation Manual.

4.2 Results

Once the true credit events were identified, we then examined how these events affected returns of other assets. We investigate the impact on i) the Lehman corporate bond index, ii) the NYSE index, and iii) the Lehman Treasury index. In particular, we identified the average returns for these three markets during months when a credit event occurred. These average returns were then compared to the average returns during those months when no event occurred. We then determined the t-statistics for the difference between the mean returns for months where a credit event did and did not occur. Since some of the changes in the corporate bond index reflect changes in Treasury rates (this is especially true of the late 1970s when Treasury market volatility was extremely high), we focus on returns to the corporate bond index in excess of Treasury returns. The Treasury returns are aggregate returns on the market captured by the Lehman Brothers aggregate Treasury market index.

Our events are quite rare and usually involve only a handful of bonds in any given month. Only three of the months include credit shocks to more than a dozen bonds. The month with the largest number of affected bonds is September 1990, during which 27 bonds experienced a spread widening of 200 bp or more. In estimating the effect of the credit shocks on the bond market as a whole, a major concern is whether these 27 bonds (fewer bonds in other months) constitute such a large fraction of the corporate bond index that there own price movements drive the returns on the bond index. This is not the case, as the Lehman corporate bond index is typically based on thousands of bonds. For example, in September 1990, 3811 corporate bonds were included in the index.

The results of the t-tests are reported in Tables 3 and 4. Table 3 reports the bond returns according to size, where size is measured as the amount of debt outstanding in the bond market. Table 4 reports bond returns according to total assets.

Table 3a shows that in the months in which a credit shock occurred the average excess return on the corporate bond index over Treasuries is negative and significantly less than the return in the other 273 months. In particular, the excess return on the corporate bond index was -33 bp for months with a credit event, and +6bp for months without a credit event. The t-stat for the difference in means is 1.74. The difference in excess returns are mainly driven by the largest bond issuers in the sample. Indeed, the mean excess return for the corporate bond index when the largest firms suffered a yield jump was -105 bp, versus +7bp during months with no credit event. It is worth noting that, according to the results in Table 3b, that about one-half of the -105 bp is due to a 50bp drop (122 - 73) in Treasuries, indicating a flight-to-quality during months where a large firm suffered a credit shock. Interestingly, unlike the corporate results shown in 3a, the Treasury returns seem to be affected just as strongly whether a small or a large firm suffers a credit shock. Finally, as shown in Table 3c, returns in the NYSE index appear to be mostly unaffected by these credit shocks.

As shown in Table 4, similar results are obtained when size is measured via assets rather than amount of debt outstanding. Again, the corporate bond index is mostly affected only when large firms suffer a credit shock. However, now the Treasury returns do appear to be more affected when a large firm rather than a small firm suffers a shock. Again, the NYSE index is mostly unaffected by such shocks.

In the t-tests conducted thus far, we are implicitly assuming that only credit shocks affect monthly asset returns, as no other factors are taken into account. In Table 5 we report the results of regression analysis of the three portfolios. In each set of regressions, we include factors that represent changes in the state of the economy. Except for the Treasury index regression, we include the slope of the term structure, as Estrella and Mishkin (1998) show it predicts recessions. To control for the effect of changes in real rates, we also include the current month's change in the actual Federal Funds rate. For the Treasury return regression, we prefer not to include the slope of the Treasury curve as an explanatory variable, as it may cause a spurious relationship in the estimation.

Table 5 shows similar results to the t-tests. The corporate returns are significantly lower in months in which a credit event occurs. The level of significance and the size of the coefficient are similar to those seen in the t-tests. Meanwhile, the signs of the control variables are as expected: a steep yield curve implies a strong economy and low chance of default, while Fed tightening and increases in industrial production are indicators that the peak in the current business cycle may be near, with defaults increasing soon.

The Treasury index regression shows some evidence of flight to quality from credit shocks, but the coefficient is smaller than that suggested by the t-tests and the p-value on the indicator for credit event months is not actually within the bounds of standard significance levels. We should keep in mind that other crises occurred in the months we describe as "normal," such as the October 1987 crash. Indeed, if we include an indicator for the stock market crash of 1987 in the Treasury regression, the coefficient on our credit event month indicator is virtually unchanged, but the p-value falls to just below the 10 percent level.

4.3 Calibration

Here we investigate the size of the risk-premia associated with jump-risk. Let us assume that there exist a pricing kernel $\Pi(t) = e^{-\int_0^t r_s ds}\xi(t)$, where ξ is a radon-Nikodym density which defines an equivalent martingale measure. Then for any risky asset we have $\Pi(t)P(t) = \mathrm{E}_t[\Pi(T)P(T)]$, or equivalently $\mathrm{E}[d(\xi(t)P(t))] = \xi(t)P(t)r(t)dt$. But using Itô's lemma (see Jacod and Shiryaev (1980) theorem 4.52) we have $d\xi(t)P(t) = \xi(t^-)dP(t) + P(t^-)d\xi(t) + d < P^c, \xi^c >_t + \Delta P(t)\Delta\xi(t)$, where $X^c(t)$ denotes the continuous martingale part of the semi-martingale X and $< X, Y >$ is the predictable quadratic co-variation process (see Jacod and Shiryaev (1980)). We thus find

$$\mathrm{E}_t[\frac{dP(t)}{P(t)}] - r(t) = d < P^c, \xi^c >_t + \mathrm{E}_t[\frac{\Delta P(t)}{P(t^-)}\frac{\Delta\xi(t)}{\xi(t^-)}]$$

Note that a jump in an asset price will command a systematic jump risk-premium only if the pricing kernel shares that common jump, i.e. if $\Delta\xi\Delta P \neq 0$. Below, we assume that asset prices and pricing kernel follow simple jump diffusion processes with constant coefficients:

$$\frac{d\Pi}{\Pi} = -r\,dt - \sigma_\xi\,dZ_\xi(t) - \Gamma_\xi\left(dq_i - \lambda^P\,dt\right) \tag{62}$$

$$\frac{dP_i(t)}{P_i(t)} = \mu_i\,dt + \sigma_i dZ_i(t) - \Gamma_i\left(dq_i - \lambda^P\,dt\right). \tag{63}$$

The $\{\sigma_\xi\}$ are the risk-premia associated with diffusion-risks dZ, and Γ_ξ is the risk premium on the jump-risk dq_i. Indeed, the risk-neutral default intensity λ^Q differs from the physical intensity λ^P only if $\Gamma_\xi \neq 0$:

$$\lambda^Q = (1 - \Gamma_\xi)\,\lambda^P. \tag{64}$$

In particular, the excess return that security-i commands can be written as

$$\mathrm{E}_t\left[\frac{dP_i + \delta_i\,dt}{P_i}\right] - r\,dt = -\sigma_\xi\,\sigma_i\,\rho_{\xi,i} - \lambda^P\Gamma_\xi\Gamma_i. \tag{65}$$

Clearly, it is the second term which captures the risk-premia associated with jump-risk. Note it is a product of three terms:

1) the intensity (or probability) of a jump
2) The jump in the pricing-kernel at the time of the jump
3) The return on security-i at the time of the jump.

Concentrating only on the jump-premium, and subtracting the returns on corporate bond-i in excess of Treasuries, we obtain

$$\left(\mu_i - \mu_{Treasury}\right)_{jump} = -\lambda^P \Gamma_\xi \left(\Gamma_i - \Gamma_{Treasury}\right) \quad (66)$$

Now, from Table 1 we see that the excess return of corporate bond-i over Treasuries conditional on a 200 bp change is on the order of -10%. Further, note that such events occurred 164 times out of 200,000 events (including the credit spread drops not reported in Table 1). Annualizing this number leads to an estimate of λ^P of about $\frac{164\times 12}{200{,}000} \approx .01$. Hence, only about 1 out of 100 corporate bonds face a credit-risk jump of this magnitude per year. Finally, note that the average annualized volatility of the pricing kernel needed to explain the historical equity premium is on the order of 0.5. Clearly, the great majority of the historical deviation is not due to credit-jumps. Hence, an upper bound for the jump in the pricing kernel due to these credit jumps would seem to be around 0.1. (Indeed, for small companies, our empirical results seem to indicate that the jump in the pricing kernel is approximately zero!!). Together, these estimates put the size of the jump-premium at

$$\left(\mu_i - \mu_{Treasury}\right)_{jump} = (.1)(.1)(.01), \quad (67)$$

or about one basis point per year. This calibration seems to suggest that the jump-premium does not have much explanatory power for the observed yield-spreads. Obviously this is a very rough calibration that does not account for any time variation in coefficients. In ongoing work we are planning to account explicitly for time variation in drift and diffusion of the pricing kernel to obtain a better measure of the component of credit spread due to systematic jump risk and credit risk premium.

5 Conclusion and Future Work

We have introduced a framework that provides a tractable solution to the contagion problem. Although consistent with the counterparty-risk hypothesis, our framework is most naturally interpreted as an updating of beliefs hypothesis, where the upward jump in the yield spread of one firm increases the perception of risk in the risky bonds of other firms. While the most tractable framework is in a reduced-form setting, we generalize the structural model of Duffie and Lando (2001) so that the default of one firm affects the market's perception of the quality of accounting numbers of other firms. Within a general equilibrium framework, we identify the structure of the jump-premia. Our empirical analysis suggests that, while such jumps appear to be priced for large firms, the size of the premia cannot explain a significant portion of the observed credit spreads.

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6 Appendix 1

Following an example used in Duffie and Lando (2001), we assume that the accounting numbers are not noisy, but rather, delayed. This simplifies the updating procedure and lets us focus on the relevant issues at hand. Clearly, the longer the lag, the more uncertainty there is about the present financial situation of the firms in the economy. We assume that the length of the delay can take on only two values: high and low. Further, we assume that all firms are either in the low-delay state or high-delay state. The justification for this assumption was discussed above. As shown in Duffie and Lando (2000), such a framework will generate what appears to be jumps to default to the outside market participants, even though the actual firm value dynamics follows a diffusion process.

Let us define $P_H(t) \equiv \pi\left(\text{Lag} = H | \mathcal{F}_t\right)$. Now, associated with each of $i = 1, \ldots, N$ Poisson jump processes dq_t are two intensity processes $\lambda_i^H(t)$, $\lambda_i^L(t)$ defined via

$$
\begin{aligned}
\lambda_i^H(t) &\equiv \mathrm{E}_t\left[dq_i(t) | \mathcal{F}_t, \text{Lag} = H\right] && (68) \\
\lambda_i^L(t) &\equiv \mathrm{E}_t\left[dq_i(t) | \mathcal{F}_t, \text{Lag} = L\right] && (69)
\end{aligned}
$$

Below, we will model the intensity processes explicitly. For now we only emphasize that in general for two different firms i and j, $\lambda_i^H(t) \neq \lambda_j^H(t)$. However, if $\text{Lag}_i = H$, then $\text{Lag}_j = H$, and similarly for $H \Rightarrow L$.[12]

We use the bold-face notation $\mathbf{dq}_t$ to represent the vector of jump processes. Using Bayes' rule, we find:

$$
\begin{aligned}
\pi\left(\text{Lag} = H, \mathbf{dq}(t) = 0 | \mathcal{F}_t\right) &= \pi\left(\mathbf{dq}(t) = 0 | \mathcal{F}_t, \text{Lag} = H\right) \pi\left(\text{Lag} = H, | \mathcal{F}_t\right) \\
&= P_H(t) \prod_{i=1}^{N} \left(1 - \lambda_i^H(t)\, dt\right) \\
&= P_H(t) \left[1 - \left(\sum_{i=1}^{N} \lambda_i^H(t)\right) dt\right] && (70)
\end{aligned}
$$

$$
\begin{aligned}
&\pi\left(\text{Lag} = H, dq_{i\neq j}(t) = 0, dq_j(t) = 1 | \mathcal{F}_t\right) \\
&\quad = \pi\left(dq_{i\neq j}(t) = 0, dq_j(t) = 1 \middle| \mathcal{F}_t, \text{Lag} = H\right) \pi\left(\text{Lag} = H, | \mathcal{F}_t\right) \\
&\quad = P_H(t)\, \lambda_j^H(t)\, dt \prod_{i\neq j} \left(1 - \lambda_i^H(t)\, dt\right) \\
&\quad = P_H(t)\, \lambda_j^H(t)\, dt && (71)
\end{aligned}
$$

[12] This assumption is stronger than necessary: it only needs to be the case that there is a market-wide correlation so that one can correctly update beliefs about the quality of accounting information by observing the time series of a finite number of defaults.

$$\pi\left(\text{Lag} = H,\, dq_{i\neq j,k}(t) = 0,\, dq_j(t) = 1,\, dq_k(t) = 1\,|\mathcal{F}_t\right)$$
$$= \mathcal{O}\left(dt^2\right). \tag{72}$$

Using similar arguments, we obtain:

$$\begin{aligned}
\pi\left(\text{Lag} = L,\, \mathbf{dq}(t) = 0|\,\mathcal{F}_t\right) &= (1 - P_H(t))\left[1 - \left(\textstyle\sum_{i=1}^N \lambda_i^L(t)\right) dt\right] \\
\pi\left(\text{Lag} = L,\, dq_{i\neq j}(t) = 0, dq_j(t) = 1\middle|\,\mathcal{F}_t\right) &= (1 - P_H(t))\ \lambda_j^L(t)\, dt \\
\pi\left(\text{Lag} = L,\, dq_{i\neq j,k}(t) = 0, dq_j(t) = 1,\, dq_k(t) = 1\middle|\,\mathcal{F}_t\right) &= \mathcal{O}\left(dt^2\right).
\end{aligned} \tag{73}$$

Summing over these terms, we find:

$$\begin{aligned}
\pi\left(\mathbf{dq}(t) = 0|\,\mathcal{F}_t\right) &= P_H(t)\left[1 - \left(\textstyle\sum_{i=1}^N \lambda_i^H(t)\right) dt\right] + (1 - P_H(t))\left[1 - \left(\textstyle\sum_{i=1}^N \lambda_i^L(t)\right) dt\right] \\
&= 1 - \left[P_H(t)\textstyle\sum_{i=1}^N \lambda_i^H(t) + (1 - P_H(t))\left(\textstyle\sum_{i=1}^N \lambda_i^L(t)\right)\right] dt \\
\pi\left(dq_{i\neq j}(t) = 0, dq_j(t) = 1\middle|\,\mathcal{F}_t\right) &= \left[P_H(t)\,\lambda_i^H(t) + (1 - P_H(t))\ \lambda_j^L(t)\right] dt.
\end{aligned} \tag{74}$$

Hence, using Bayes' rule again, we find

$$\begin{aligned}
\pi\left(\text{Lag} = H|\,\mathbf{dq}(t) = 0,\, \mathcal{F}_t\right) &= \frac{P_H(t)\left[1 - \left(\sum_{i=1}^N \lambda_i^H(t)\right) dt\right]}{1 - \left[P_H(t)\sum_{i=1}^N \lambda_i^H(t) + (1 - P_H(t))\left(\sum_{i=1}^N \lambda_i^L(t)\right)\right] dt} \\
&= P_H(t) - P_H(t)\,(1 - P_H(t))\sum_{i=1}^N \left[\lambda_i^H(t) - \lambda_i^L(t)\right] dt.
\end{aligned} \tag{75}$$

Equivalently, we can write

$$\begin{aligned}
dP_H(t)\big|_{\mathbf{dq}(t)=0} &= P_H(t + dt) - P_H(t) \\
&= -P_H(t)\,(1 - P_H(t))\sum_{i=1}^N \left[\lambda_i^H(t) - \lambda_i^L(t)\right] dt.
\end{aligned} \tag{76}$$

Intuitively, this states that the probability that the economy is in a high-lag state drifts downward over dates where no default has occurred. Similarly:

$$\begin{aligned}
\pi\left(\text{Lag} = H|\,dq_{i\neq j}(t) = 0,\, dq_j(t) = 1,\, \mathcal{F}_t\right) &= \frac{P_H(t)\,\lambda_j^H(t)\,dt}{\left[P_H(t)\,\lambda_i^H(t) + (1 - P_H(t))\ \lambda_j^L(t)\right] dt} \\
&= \frac{P_H(t)}{\left[P_H(t) + (1 - P_H(t))\ \frac{\lambda_j^L(t)}{\lambda_j^H(t)}\right]}
\end{aligned} \tag{77}$$

Note that if $\frac{\lambda_j^L(t)}{\lambda_j^H(t)} = 1$, then the denominator of equation (77) would be unity, and $P_H(t+dt) = P_H(t)$. However, since by definition $\frac{\lambda_j^L(t)}{\lambda_j^H(t)} < 1$, it follows that $P_H(t+dt) > P_H(t)$. That is, there is a jump in the belief that the economy is in a high-lag state. We can write this as (using $dq_j(t) = 1$)

$$\begin{aligned} dP_H(t)|_{dq_{i\neq j}(t)=0,\, dq_j(t)=1} &= P_H(t+dt) - P_H(t) \\ &= dq_j(t)\left[\frac{P_H(t)}{\left[P_H(t) + (1-P_H(t))\ \frac{\lambda_j^L(t)}{\lambda_j^H(t)}\right]} - P_H(t)\right]. \end{aligned} \tag{78}$$

Combining equations (76) and (78), we find

$$dP_H(t) = -P_H(t)\left(1-P_H(t)\right)\sum_{i=1}^{N}\left[\lambda_i^H(t) - \lambda_i^L(t)\right] dt + \sum_{i=1}^{N}\left[\frac{P_H(t)}{\left[P_H(t) + (1-P_H(t))\ \frac{\lambda_i^L(t)}{\lambda_i^H(t)}\right]} - P_H(t)\right] dq_i(t). \tag{79}$$

It is straightforward to demonstrate that $P_H = \mathrm{E}_t\left[\mathrm{Lag} = H\right]$ is a martingale in that

$$\mathrm{E}_t\left[dP_H(t)\right] = 0. \tag{80}$$

	Frequency	Cumulative Frequency	Percentage	Cumulative Percentage	Average Return	Excess Return
Over 10 percentage points	7	7	0.01%	0.01%	-31.33%	-31.37%
5 to 10 percentage points	7	14	0.01%	0.03%	-16.64%	-16.79%
3 to 5 percentage points	34	48	0.06%	0.09%	-12.42%	-12.91%
2 to 3 percentage points	116	164	0.22%	0.31%	-7.70%	-8.93%
1.5 to 2 percentage points	156	320	0.29%	0.60%	-5.13%	-6.69%
1.25 to 1.5 percentage points	143	463	0.27%	0.86%	-3.83%	-5.50%
1 to 1.25 percentage points	291	754	0.54%	1.41%	-2.66%	-4.08%
0.75 to 1 percentage points	597	1351	1.11%	2.52%	-1.88%	-3.63%
0.5 to 0.75 percentage points	1970	3321	3.68%	6.20%	-1.03%	-2.84%
0.25 to 0.5 percentage points	5886	9207	10.99%	17.18%	-0.54%	-1.18%
Less than 0.5 percentage points	44372	53579	82.82%	100.00%	-0.09%	-0.23%

Table 1: Distribution of Spread increases

	Frequency	Cumulative Frequency	Percentage	Cumulative Percentage
Accidents (e.g., toxic train crash, nuclear power plant accident)	5	5	4.46%	4.46%
Economic hardship of firm (e.g, declining revenue, higher costs)	36	41	32.14%	36.61%
Major lawsuit against firm (e.g., asbestos, tobacco)	5	46	4.46%	41.07%
LBO or other leverage increasing event	34	81	31.25%	72.32%
Liquidity and lack of access to new funds	1	82	0.89%	73.21%
Nonperforming bank loans or leases	30	112	26.79%	100.00%

Table 2: Description of Event Types

Table 3a	Corporate Bond Index Returns					
	Number of months in which an event occurs	Mean excess return in months when an event occurs	Mean excess return in other months	Difference in mean returns	t-test statistic	t-test p-value
All events	25	-0.33	0.06	0.39	1.74	0.084
Largest firms	11	-1.05	0.07	1.12	3.42	0.001
Smallest firms	14	0.24	0.02	-0.22	-1.50	0.149

Table 3b	Treasury Bond Index Returns					
	Number of months in which event occurs	Mean return in months when a event occurs	Mean return in other months	Difference in mean returns	t-test statistic	t-test p-value
All events	25	1.28	0.70	-0.59	-1.70	0.090
Largest firms	11	1.22	0.73	-0.49	-0.96	0.336
Smallest firms	14	1.33	0.72	-0.62	-1.37	0.173

Table 3c	Stock market (NYSE) Returns					
	Number of months in which event occurs	Mean return in months when a event occurs	Mean return in other months	Difference in mean returns	t-test statistic	t-test p-value
All events	25	0.01	0.01	0.00	0.36	0.721
Largest firms	11	0.00	0.01	0.01	0.57	0.578
Smallest firms	14	0.01	0.01	0.00	-0.25	0.807

Table 3: Effects of Credit Events on Corporate Bond, Treasury and Stock Indices Size of firm measured by bonds outstanding Monthly returns from January 1973 to October 1997 (298 months)

Table 4a	Corporate Bond Index Returns				
	Number of months in which event occurs	Mean excess return in months when an event occurs	Mean excess return in other months	Difference in mean returns	t-test statistic	t-test p-value
All events	25	-0.33	0.06	0.39	1.74	0.084
Largest firms	13	-0.53	0.06	0.58	1.91	0.057
Smallest firms	12	-0.11	0.04	0.15	0.47	0.641

Table 4b	Treasury Bond Index Returns					
	Number of months in which event occurs	Mean return in months when a event occurs	Mean return in other months	Difference in mean returns	t-test statistic	t-test p-value
All events	25	1.28	0.70	-0.59	-1.70	0.090
Largest firms	13	1.55	0.71	-0.84	-3.05	0.008
Smallest firms	12	1.00	0.74	-0.26	-0.54	0.593

Table 4c	Stock market (NYSE) Returns					
	Number of months in which event occurs	Mean return in months when a event occurs	Mean return in other months	Difference in mean returns	t-test statistic	t-test p-value
All events	25	0.01	0.01	0.00	0.36	0.721
Largest firms	13	0.00	0.01	0.01	0.72	0.474
Smallest firms	12	0.01	0.01	0.00	-0.24	0.811

Table 4: Effects of Credit Events on Corporate Bond, Treasury and Stock Indices Size of firm measured by total assets. Monthly returns from January 1973 to October 1997 (298 months)

	Corporate Bonds		Treasury Bonds			NYSE Stocks	
Constant	-0.13	-0.13	0.79	0.78	0.78	0.01	0.01
	(0.088)	(0.089)	(0.000)	(0.000)	(0.000)	(0.005)	(0.005)
Slope of the treasury curve	0.31	0.32	-	-	-	0.01	0.01
	(0.000)	(0.000)				(0.102)	(0.097)
Change in industrial production	-0.02	-0.03	-0.43	-0.47	-0.46	0.00	0.00
	(0.860)	(0.782)	(0.004)	(0.002)	(0.002)	(0.728)	(0.676)
Change in federal funds rate	-0.25	-0.24	-0.40	-0.39	-0.39	-0.01	-0.01
	(0.007)	(0.008)	(0.003)	(0.004)	(0.004)	(0.012)	(0.013)
Indicator for months with shocks	-0.36	-	0.54	0.55	-	0.00	-
	(0.098)		(0.105)	(0.100)		(0.732)	
Indicator for months with shocks		-0.59			0.71		-0.01
to large firms	-	(0.047)	-	-	(0.115)	-	(0.425)
Indicator for months with shocks		-0.12			0.37		0.00
to small firms	-	(0.701)	-	-	(0.425)	-	(0.751)
Indicator for 1987 stock market crash	-	-	-	3.84	3.83	-0.23	-0.23
				(0.016)	(0.016)	(0.000)	(0.000)
Adjusted R-square	0.11	0.11	0.08	0.10	0.10	0.12	0.12

Table 5: Regression of corporate bond and portoflio returns on Credit Event indicators. Size of firm measured by assets (p-values in parentheses)