

A dynamic theory of the corporate finance based upon repeated signaling*

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Abstract

We examine the effect of repeated hidden information regarding the marginal product of capital on the dynamics of financing and investment. The model features endogenous investment, debt, default, dividends, equity flotations and share repurchases. Since signaling costs are necessarily high if net worth is low, forward-looking shareholders have a precautionary motive to hoard cash and capital. In each period's least-cost separating equilibrium, firms signal positive information with high leverage and overinvest in order to reduce associated default costs. Firms with negative information save (rather than borrow) and raise external funds with equity. The need for costly signals is increasing in a firm's financing gap. If net worth is sufficiently low, extreme costs of signaling induce a switch to pooling equilibria where firms with positive (negative) information underinvest (overinvest). The model is rich in testable predictions and consistent with a broad set of stylized facts regarding leverage ratios and announcement effects.

1 Introduction

Three decades have elapsed since Ross (1977) and Leland and Pyle (1977) introduced the “incentive signaling” approach to corporate finance. However, we still lack a dynamic (infinite-horizon) quantitative theory of financing and investment in economies with hidden information.¹ This was recently noted by Ross (2005) himself, who stated, “The introduction of the issues raised by the presence of asymmetric information in the determination of the capital structure and the integration of these issues into the intertemporal neoclassical model are a major challenge.” Recently, a number of researchers have tried to analyze the

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¹In contrast, substantial progress has been made on dynamic models with taxes and hidden action. For recent examples, see Gamba and Triantis (2006) and DeMarzo and Fishman (2007), respectively.

effect of hidden information on investment by adopting the intuitively plausible assumption that it gives rise to linear-quadratic costs of external equity.² Recognizing the potential problems associated with this approach, Gomes (2001) states, “Ideally we would prefer to model financial intermediation endogenously.” In this paper, we start from primitives and develop a dynamic quantitative model of corporate financing and investment when insiders have private information about the marginal product of capital. We argue that the proposed theory can compete with structural trade-off theoretic models in terms of explaining a broad range of stylized facts.

We start with a neoclassical investment framework. The firm has a stochastic concave profit function $\theta_t \varepsilon_t k_t^\alpha$. The shock ε_t is public and observed simultaneously by all agents at the end of the period. The shock $\theta_t \in \{\theta_L, \theta_H\}$ is privately observed by an insider at the start of the period, with outsiders observing the private shock with a one-period lag. Using his private information, the insider maximizes the cum-dividend value of his equity stake. The insider is forward-looking and recognizes that his decisions affect the severity of future adverse selection problems.

Anticipation of signaling costs necessitated by repeated hidden information causes risk-neutral insiders to exhibit *pseudo-risk-averse* preferences with equity value concave in net worth.³ To see this, suppose the insider has positive information about the marginal product of capital, but net worth is low. Here the firm would need to raise a large amount of funds in order to implement first-best investment. However, the issuance of a large block of equity would create a strong temptation for firms with negative information to mimic, since they stand to gain a great deal from security mispricing. In order to separate, the insider with positive information must utilize costly signals such as defaultable debt. For the firm with low net worth, the marginal value of internal funds is high, since they reduce the need for external funds and concomitant signaling costs. At the opposite extreme, a firm with high net worth does not require external funding. Such a firm will use a marginal dollar to pay dividends, implying that internal funds have a shadow value of one.

The least-costly separating equilibrium (LCSE) minimizes the temptation of insiders with negative information to mimic. Since insiders are pseudo-risk-averse, firms reporting low expected profitability are financed with external equity, while still holding a cash buffer stock. In this way, outside investors provide the firm with insurance in the form of financial slack. In contrast, firms can credibly signal positive information by substituting debt for equity. Using the optimal mix of real and financial signals, investment of the high type is distorted above first-best, as the firm tries to lower expected default costs by increasing capital. As we show, similar signaling effects are also present in the static analog of our dynamic model, but precautionary motives are absent. Thus, in a setting with repeated hidden information, firms hoard more cash, issue less debt, and invest more in physical capital. The precautionary savings incentives arising from repeated hidden information offer a potential explanation for the debt conservatism puzzle posited by Graham (2000).

The possibility of pooling equilibria is addressed in an extension of the dynamic model. In order to deter

²For examples, see Gomes (2001), Cooley and Quadrini (2001), and Hennessy and Whited (2006).

³Biais, Mariotti, Plantin and Rochet (2007) and DeMarzo and Fishman (2007) show that repeated *hidden action* generates concave continuation values in infinite-horizon models of the firm.

imitation, high types must incur significant signaling costs in order separate when the financing gap is large. Absent such costly signals, the low type would gain from overvaluation of a large block of securities. As intuition would suggest, a pooling equilibrium Pareto dominates the LCSE if net worth is sufficiently low. In the pooling equilibrium, firms with positive (negative) private information underinvest (overinvest) relative to first-best. In the model, firms pool in low net worth states and separate via costly signals at intermediate levels of net worth. If net worth is sufficiently high, the external funding requirement is low, and firms with negative information have no motivation to mimic high types in order to gain from securities mispricing. In such states, policies resemble the full-information first-best, but saving and investment are somewhat higher due to endogenous precautionary motives.

A technical contribution of the paper is to develop a tractable algorithm to solve for equilibrium in a dynamic neoclassical setting with hidden information. This is important inasmuch as analogous symmetric information models are central in macroeconomics (e.g. Lucas and Prescott (1971)), production-based asset pricing (e.g. Cochrane (1991)), and corporate finance (e.g. Gomes (2001)). The solution method is logically the same as that used in static signaling models, e.g. Milgrom and Roberts (1986). The first additional source of complexity is that budget and incentive constraints involve endogenous value functions, while single-period models involve terminal cash flows. The second additional source of complexity is determining the endogenous default threshold accounting for the option value inherent in continuation. The two hurdles are readily cleared using recursive methods (e.g. Stokey and Lucas (1989)). Following Maskin and Tirole (1992) and Tirole (2006), a Pareto optimality criterion allows us to determine the net worth levels at which pooling is possible and when the LCSE is a unique equilibrium. Again, recursive methods must be exploited since changes in equilibria lead to changes in value functions, and vice versa.

The model can be mapped directly to real-world data since investment, debt, equity flotations, dividends, share repurchases, and default are all endogenous. The behavior of the low type in the model is consistent with empirical observation. In particular, Leary and Roberts (2006) document that “more often than not, when firms issue equity, they do so before turning to the debt market or in lieu of issuing debt.” Such behavior is inconsistent with the pecking-order proposed by Myers (1984). In our model, investors reward the firm for revealing negative information by providing it with ample equity financing and a cash buffer stock that can be used if investment becomes profitable in the future.

The model is also consistent with stylized facts regarding financial structure dynamics documented by Fama and French (2002). In simulated data, the leverage ratio is inversely related to lagged profitability and exhibits mean-reversion. Since the shock ε is public, it can be viewed as a proxy for macroeconomic shocks.⁴ Following low realizations of ε , firms have low net worth. In such states, simulated leverage ratios are particularly high for firms with positive private information regarding the marginal product of capital since they must send strong signals in conjunction with their major fund-raising efforts. Thus, the model predicts that firms will have higher average leverage ratios at the tail-end of recessions. This prediction is

⁴In contrast, there is no reason to assume insiders are privately informed about the macroeconomy. Hence, θ cannot proxy for the macroeconomy.

consistent with evidence presented by Korajczyk and Levy (2004).

Our theory of the cyclical behavior of leverage is complementary to that proposed by Choe, Masulis and Nanda (1993). They develop a static model in which firms finance a project of fixed size with debt or equity, but not both. In their model, equity issuance increases during expansions due to cyclicalities in the investment opportunity set. Our theory is distinct since we hold the investment opportunity set constant over the business cycle. In our model, cyclical effects are caused by endogenous fluctuations in firms' internal funds.

The baseline model also generates predictions consistent with observed patterns of abnormal equity returns surrounding corporate announcements. Such announcement effects do not emerge in trade-off theoretic models with common knowledge.⁵ Simulated share prices exhibit positive (negative) abnormal returns in response to increases (decreases) in capital expenditures. This is consistent with the findings of McConnell and Muscarella (1985), who document raw equity returns of 1.21% following announced increases in capital budgets and -1.52% for decreases. The model also predicts that a high debt percentage in total external finance leads to positive abnormal returns, even when the investment rate is included as a conditioning variable. Consistent with this prediction, Masulis (1983) documents a 13.97% primary announcement return in debt for equity exchanges and a -9.91% return in equity for debt exchanges.

In the most closely related paper, Constantinides and Grundy (1990) show that a firm with variable investment scale can implement first-best if bankruptcy is costless. First-best is achieved with the firm issuing debt in excess of the amount needed to fund investment. Excess funds are then used for share repurchases. Our static model, which is of independent interest, alters that of Constantinides and Grundy (1990) by introducing default costs. Here the firm chooses the least costly mix of real and financial policies to signal positive private information. This entails issuing risky debt and increasing capital in order to reduce default costs. In our dynamic model, the hidden information problem repeats and net worth evolves endogenously. The extension to a dynamic setting adds a number of insights. First, repetition of the hidden information problem encourages saving and capital accumulation relative to what occurs in the static model. Second, the continuation value inherent in the firm reduces default costs associated with debt, since the firm can issue securities against this continuation value in order to meet prior debt obligations. Third, allowing for endogenous fluctuations in net worth allows the model to explain mean-reverting and countercyclical leverage ratios, as well as the observed negative relationship between leverage and lagged profitability.⁶ Finally, by allowing for endogenous fluctuations in net worth, we can evaluate how the nature of equilibrium (pooling v. separating) varies endogenously.

Our static model is related to a number of papers that analyze static multi-dimensional signaling. Milgrom and Roberts (1986) consider multi-dimensional signaling in product markets. Ambarish, John

⁵In such environments, only changes in commonly observed state variables (e.g. earnings) induce changes in stock prices. Given these state variables, investors can predict firm actions, so policy announcements have no effect on prices.

⁶A single-period model also generates this prediction if one treats net worth as an exogenous parameter in the comparative statics. However, net worth is actually an endogenous variable, being determined by lagged policies, e.g. retention decisions. Endogeneity invalidates comparative statics treating net worth as a parameter.

and Williams (1987) and Williams (1988) consider settings in which the firm signals private information through investment and taxable dividends. There is no debt in these models. In terms of corporate finance models with multi-dimensional signaling, our static model is most similar to that of Viswanathan (1995), who also allows signaling through the issuance of debt and investment in a setting where default is costly. Viswanathan considers a setting where the asymmetric information does not concern the marginal product of new investment. For example, one may interpret his model as one in which asymmetric information concerns cash flows from assets in place. In contrast, we consider a setting in which asymmetric information concerns the marginal product of capital.

Our paper is also related to that of Myers and Majluf (1984), despite the fact that we predict departures from their asserted pecking-order. Myers and Majluf present a simple model illustrating that a firm constrained to finance with equity may rationally forego positive NPV projects if the lemons problem is sufficiently acute. They briefly (and less formally) consider the choice between debt and equity. They argue that firms should minimize the informational sensitivity of their securities, which generally implies a preference for debt over equity.

Nachman and Noe (1994) develop a theoretical foundation for the pecking-order hypothesis. They assume investment scale is fixed and attention is confined to securities with payoffs that increase monotonically with cash flow. Under these assumptions, there can be no separating equilibrium, as firms would always benefit from reporting the highest type, receiving the highest security value (due to monotonicity), and paying the additional funds as a dividend. Nachman and Noe derive sufficient conditions such that debt will be the unique source of financing in the pooling equilibrium. The essential idea is that debt minimizes cross-subsidies. The central difference between our setting and that considered by Nachman and Noe is that we allow the firm to signal using investment scale and share repurchases. The broader, and arguably more realistic, action space creates the possibility for separating equilibria.

Finally, there has been little work done on models of investment and financing under repeated hidden information. Lucas and McDonald (1990) develop a dynamic model of investment under hidden information. However, they constrain the firm to finance with equity. Sannikov (2006) presents a model of optimal security design when there is one-time hidden information *ex ante* and repeated moral hazard *ex post*. Biais, Mariotti, Plantin and Rochet (2007) and DeMarzo and Fishman (2007) analyze optimal security design when facing repeated moral hazard. Our focus here is on repeated hidden information.

The remainder of this paper is organized as follows. Section 2 presents our static signaling model. Section 3 characterizes the least-cost separating equilibrium in a dynamic setting. Section 4 uses simulated data to evaluate empirical implications of the model. Section 5 analyzes the possibility of pooling equilibria. Section 6 extends the model to incorporate a tax advantage to debt. Section 7 concludes.

2 Static Signaling Model

2.1 Technology and Timing

In order to convey some of the economic intuition of our dynamic model, we first present the single-period analog. Comparison of the static and dynamic models sheds light on economic effects and complexities arising from the problem of repeated hidden information. The static model is also of independent interest in that it analyzes the optimal mix of costly signals when insiders have private information regarding the marginal product of capital. This analysis complements the model of Viswanathan (1995) where inside information concerns cash flows that are additively separable from new investment, e.g. cash flows from assets in place.

Unless stated otherwise, technological assumptions are identical in the static and dynamic models. There is a risk-free asset paying interest rate $r > 0$. Agents are risk-neutral and share the discount factor $\beta \equiv (1 + r)^{-1}$. Capital (k) decays exponentially at rate $\delta \in [0, 1]$. The following two assumptions describe the production technology and timing of information revelation.

Assumption 1. Operating profits are $\theta_t \varepsilon_t k_t^\alpha$ where $\alpha \in (0, 1)$. The private shock θ takes values in $\Theta \equiv \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H$. The public shock ε has a density function $f : [\underline{\varepsilon}, \infty) \rightarrow [0, 1]$ that is continuously differentiable with $f(\varepsilon) > 0$ for all $\varepsilon \geq \underline{\varepsilon} > 0$. In the dynamic model, the private shock follows a first-order Markov process and the public shock is independently and identically distributed.

Assumption 2. The shock θ_t is privately observed by an insider-shareholder at the start of the period. Financing and investment are then determined. At the end of the period, the realized values of ε and net worth are simultaneously observed by all agents. Net worth is verifiable in a court.

In contrast to a moral hazard model, e.g. DeMarzo and Fishman (2006), the internal resources of the firm are verifiable by a court. However, we assume the court cannot determine whether a low realization of operating profits is due to low ε (bad luck) or low θ (negative private information). Thus, it is impossible to write a θ -contingent financial contract. Despite θ being nonverifiable, Assumption 2 ensures outside investors can infer the realized value of θ_t at the end of period t when they observe ε_t and net worth. In a dynamic setting, knowing the lagged value of θ is useful to outside investors since it determines transition probabilities. Effectively, Assumption 2 ensures insiders have a one-period informational advantage relative to outsiders each period.⁷

The firm can raise funds by borrowing or issuing new shares. The borrowing technology is a one-period debt contract with the lender senior in the event of default. The face value of debt for the type- i is denoted b_i .⁸ Although we do not analyze optimal security design, we here note that if the bankruptcy process is inherently costly, it would be more efficient to transfer ownership in low profit states using equity warrants to dilute current shareholders. However, security design should account for tax advantages of debt. If the

⁷We argue that permanent informational advantages are empirically implausible since numerous forces induce revelation of information about public firms.

⁸With some abuse of terminology, we call a firm that has realized the shock θ_i a type- i firm. However, it must be stressed that the type refers to a temporary shock.

debt tax shield is sufficiently large relative to bankruptcy costs, debt will dominate equity warrants as a device for eliciting truthful revelation. Consistent with this argument, McDonald (2004) shows that written put options, which have signal value, entail large tax costs relative to synthetic equivalents with explicit borrowing. Based on this discussion, we incorporate a tax advantage to debt in Section 6.

In the static model, the firm enters the model with an exogenous level of net worth w at the start of the period. The variable \tilde{w} denotes the end-of-period internal resources of the firm provisional on delivering the promised debt payment. Formally, *provisional net worth* (\tilde{w}) is the sum of capital net of depreciation plus operating profits less the promised debt payment

$$\tilde{w}(b, k, \varepsilon, \theta) \equiv (1 - \delta)k + \theta\varepsilon k^\alpha - b. \quad (1)$$

In a separating equilibrium, each period's true θ can be inferred by the investor. To establish separation it is necessary to consider the payoffs to an insider of type- i who receives the allocation of type- j where j does not necessarily equal i . Let ε_{ij}^d denote the critical value of ε such that type- i would default given that he has received the capital stock and taken on the debt obligation of type- j . In the static model, shareholders default when $\tilde{w} < 0$ and ε_{ij}^d is determined by

$$\tilde{w}(b, k, \varepsilon_{ij}^d, \theta) = 0 \Rightarrow \varepsilon_{ij}^d = \frac{b_j - (1 - \delta)k_j}{\theta_i k_j^\alpha}. \quad (2)$$

If $\varepsilon_{ij}^d \leq \underline{\varepsilon}$, there is zero probability of default for type- i that has received the allocation of type- j .

In the event of default, the lender seizes the firm's physical assets and operating profits. In the dynamic model, the lender also captures the going-concern value of the firm, which is determined endogenously. In the static model, the lender incurs a legal cost $\Phi > 0$ in the event of bankruptcy. In the dynamic model, bankruptcy costs are expressed as a percentage of going-concern value. The latter assumption captures the idea that distress costs are low for tangible assets and high for intangible assets.

As argued by Shyam-Sunder and Myers (1999) and Cooley and Quadrini (2001), there must be some cost to holding cash or long-lived firms facing financial market imperfections would never pay dividends. Two real-world costs of hoarding cash are the relatively low after-tax return to saving within the corporate shell (see Graham (2000)) or agency costs of giving managers access to discretionary cash (see Jensen (1986)). Presently, we model the latter friction since it is useful to distinguish the proposed theory from trade-off theory. Taxes are modeled in Section 6. Letting χ be an indicator for $b < 0$, agency costs of discretionary cash are equal to $\chi\gamma b^2/2$.

Assumption 3 summarizes the firm's borrowing technology.

Assumption 3. The debt contract is single-period. Default is endogenous and the lender is senior. In the static model, default costs are $\Phi > 0$. In the dynamic model, default costs are equal to ϕ times going-concern value. The yield on corporate saving is r . The firm incurs agency costs equal to $\chi\gamma b^2/2$ when it holds cash.

Let Ω^{ij} denote the expected discounted end-of-period value of shareholders' equity for a type- i that has taken the type- j allocation:

$$\Omega^{ij} = \beta \int_{\varepsilon_{ij}^d}^{\infty} [(1 - \delta)k_j + \theta_i \varepsilon k_j^\alpha - b_j] f(\varepsilon) d\varepsilon. \quad (3)$$

Let ρ^i denote the price of debt issued by type- i in equilibrium:

$$\rho^i \equiv \beta \left[b_i \int_{\varepsilon_{ii}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon_{ii}^d} [(1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha - \Phi] f(\varepsilon) d\varepsilon \right]. \quad (4)$$

Shares are issued and repurchased ex dividend. The insider holds $m > 0$ shares of stock. The number of shares outstanding at the start of the period, inclusive of insider shares, is $c > m$. Both c and m are exogenous. Insider objectives, stated as Assumption 4, are identical to those assumed by Constantinides and Grundy (1990).⁹

Assumption 4. The choice of signals is made by a risk-neutral insider-shareholder who maximizes the cum-dividend value of his equity stake given private information. The insider does not buy additional shares in the event of an equity flotation and does not tender shares in the event of a share repurchase.

The number of new shares issued is n , with $n < 0$ indicating a share repurchase. Let $s \equiv n/(c + n)$ represent the percentage equity stake sold (repurchased). Dividends are denoted d and are constrained to be nonnegative. Absent this constraint, the firm could avoid signaling costs by having shareholders inject their own funds directly into the firm. The insider receives a fraction m/c of total dividends and holds an equity stake of $m/(c + n)$ at the end of the period. Therefore, the cum-dividend value of the insider's equity stake is

$$\left(\frac{m}{c}\right)d + \left(\frac{m}{c+n}\right)\beta \int_{\varepsilon^d}^{\infty} [(1 - \delta)k + \theta_i \varepsilon k^\alpha - b] f(\varepsilon) d\varepsilon. \quad (5)$$

It is convenient to think of the insider as proposing an *allocation*, $a \equiv (b, d, k, s)$, to the investor. Suppose the insider observes θ_i at the start of the period. Using the definition of s , the objective function for the insider (5) simplifies to $(m/c)u(b, d, k, s)$, where

$$u(b, d, k, s) \equiv d + (1 - s)\beta \int_{\varepsilon^d}^{\infty} [(1 - \delta)k + \theta_i \varepsilon k^\alpha - b] f(\varepsilon) d\varepsilon. \quad (6)$$

Conveniently, the multiplicative term m/c has no effect on incentive constraints, since the vector a contains all relevant choice variables.

⁹Myers and Majluf (1984) also adopt the assumption of controlling shareholder passivity.

2.2 The Signaling Game

The insider moves first and offers a feasible allocation to the investor. The set of feasible allocations is \mathcal{A} ¹⁰

$$\mathcal{A} \equiv \{(b, d, k, s) : d \geq 0, k \geq 0, s \leq 1\}.$$

The investor then updates his beliefs and either accepts or rejects the allocation. If the investor accepts, the allocation is implemented. If the offer is rejected, the firm is constrained to finance with internal funds (i.e. $s = 0$ and $b \leq 0$). The equilibrium concept is perfect Bayesian equilibrium (PBE). A PBE imposes the following requirements: 1) For any feasible offer, the investor has some belief. 2) The investor's beliefs are derived from Bayes' rule for any offer made in equilibrium. 3) The investor accepts an offer only if it yields weakly positive profits given his updated beliefs. 4) Each type's offer is optimal given the lender's actions. This formulation of the signaling game assumes investment is contractible, as in the model of Constantinides and Grundy (1990). In reality, the terms of financing are routinely linked to firms' stated use of funds. For example, a secured debt contract links financing to a specific capital investment.

The LCSE depends upon net worth at the start of the period (w). Intuitively, the need for costly signaling increases in the amount of external funds raised and thus decreases in net worth at the start of the period. For example, if net worth is sufficiently high, the high type funds investment with internal funds. Therefore, the low type cannot benefit from mimicry and security mispricing. In such states, there is no need for the high type to burn money on costly signals.

To derive the LCSE we first solve Program L.

$$\text{PROGRAM L: } \max_{a \in \mathcal{A}} d_L + (1 - s_L)\Omega^{LL}$$

subject to the budget constraint

$$BC_L : d_L + k_L + \chi\gamma b_L^2/2 - w \leq \rho^L + s_L\Omega^{LL}.$$

The left side of BC_L measures the amount of funding provided by the investor and the right side represents the true value of the claims received by the investor. Thus, BC_L ensures the investor makes weakly positive profits in expectation. It should be noted that the budget constraint also accounts for share repurchases. To see this, assume the type- i insider reveals the firm's true type by choosing a separating allocation (b_i, d_i, k_i, s_i) . If the firm issues new shares ($n > 0$), the equity flotation raises $s_i\Omega^{ii}$. In the event of a share repurchase, each outsider-shareholder must be indifferent between tendering or not. In this case, $s_i\Omega^{ii}$ also represents the cash outflow from a share repurchase. To demonstrate this, consider a simple example. Suppose $\Omega^{ii} = 100$, $c = 50$ and $n = -10$. After the share repurchase, the ex dividend value of each remaining

¹⁰An additional constraint is that the firm cannot repurchase more than $c - m$ shares. That is, $s \geq -(c/m - 1)$. This constraint never binds for c/m sufficiently large.

share is $100/(50 - 10) = 2.5$. Therefore, in order to induce ten shareholders to sell, the firm must pay 25 ($= 10 \times 2.5$). Note $s_i \Omega^{ii} = -(10/40) \times 100 = -25$.

We define λ_i , η_i and ψ_i as the multipliers on the budget (BC_i), dividend non-negativity, and equity constraints ($s_i \leq 1$), respectively. The solution to Program L is denoted $a_L^* \equiv (b_L^*, d_L^*, k_L^*, s_L^*)$. We next solve a related Program H.

$$\text{PROGRAM H: } \max_{a \in \mathcal{A}} d_H + (1 - s_H) \Omega^{HH}$$

subject to the budget constraint

$$BC_H : d_H + k_H + \chi \gamma b_H^2 / 2 - w \leq \rho^H + s_H \Omega^{HH}$$

and a nonmimicry constraint

$$NM_{LH} : d_L^* + (1 - s_L^*) \Omega^{LL} \geq d_H + (1 - s_H) \Omega^{LH}.$$

The solution is denoted $a_H^* \equiv (b_H^*, d_H^*, k_H^*, s_H^*)$. The multiplier on the nonmimicry constraint is μ .

In solving Program L we do not impose a nonmimicry constraint. Lemma 1 shows this is without loss of generality.¹¹

Lemma 1. *Assume that a_H^* and a_L^* solve Program H and Program L, respectively, and that $s_L^* \geq 0$. Then the high type prefers a_H^* to a_L^* .*

Proof. See Appendix A.

Let $\hat{\theta}(a)$ denote the type inferred by the investor conditional upon receiving an arbitrary offer $a \in \mathcal{A}$. As shown in Appendix B, a fully-revealing PBE with allocations (a_L^*, a_H^*) can be supported by the following investor beliefs.

$$\begin{aligned} \hat{\theta}(a_H^*) &= \theta_H, \\ \hat{\theta}(a_L^*) &= \theta_L, \\ \hat{\theta}(a) &\in \arg \min_{\theta \in \Theta} s \int_{\varepsilon_{\theta\theta}^d}^{\infty} [(1 - \delta)k + \theta \varepsilon k^\alpha - b] f(\varepsilon) d\varepsilon + \\ &\quad b \int_{\varepsilon_{\theta\theta}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon_{\theta\theta}^d} [(1 - \delta)k + \theta \varepsilon k^\alpha - \Phi] f(\varepsilon) d\varepsilon, \\ \forall a &\notin \{a_L^*, a_H^*\}. \end{aligned} \tag{9}$$

On the equilibrium path, the beliefs in (9) are consistent with Bayes' rule. Off the equilibrium path, the investor imposes worst-case beliefs in the sense of Brennan and Kraus (1987). In particular, the investor

¹¹In solving Program L, we may confine attention to $s_L \geq 0$ without loss of generality. To see this, note that a share repurchase by the low type can be replaced with a dividend without affecting the value of the objective function.

attaches the lowest possible valuation to any package of securities not issued in equilibrium. For example, if the firm were to issue shares and/or debt, a worst-case belief imputes type θ_L . If the firm were to repurchase shares (and issue no debt), then a worst-case belief imputes type θ_H .

2.3 Single-Crossing Conditions in Static Model

The marginal effect of b on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned}\Omega_b^{HH} &= -\beta \int_{\varepsilon_{HH}^d}^{\infty} f(\varepsilon) d\varepsilon \\ \Omega_b^{LH} &= -\beta \int_{\varepsilon_{LH}^d}^{\infty} f(\varepsilon) d\varepsilon.\end{aligned}\tag{10}$$

The marginal effect of k on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned}\Omega_k^{HH} &= \beta \int_{\varepsilon_{HH}^d}^{\infty} [1 - \delta + \alpha\theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \\ \Omega_k^{LH} &= \beta \int_{\varepsilon_{LH}^d}^{\infty} [1 - \delta + \alpha\theta_L \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon.\end{aligned}\tag{11}$$

In order to obtain indifference curves over alternative allocations, we compute the total differential of the insider's objective function u as defined in equation (6). Evaluated at the high type allocation, indifference curves are defined by

$$du(a_H; \theta_i) = \Delta d + (1 - s_H) \Omega_k^{iH} \times \Delta k + (1 - s_H) \Omega_b^{iH} \times \Delta b - \Omega^{iH} \times \Delta s = 0.\tag{12}$$

From (12), the insider's willingness to exchange equity for reductions in the face value of debt is

$$\frac{ds}{db}(a_H; \theta_i) \equiv \frac{(1 - s_H) \Omega_b^{iH}}{\Omega^{iH}}.\tag{13}$$

In general, the relative slope of indifference curves in (s, b) space is ambiguous, reflecting competing forces. On one hand, the low type is more willing to give up equity, since he knows his equity is less valuable. On the other hand, the high type views servicing debt as more costly since he is less likely to default.

Next, define the left-truncated mean function for the public shock ε :

$$M(x) \equiv E[\varepsilon | \varepsilon \geq x].$$

The function M plays an important role in the signal content of alternative financial policies. Rearranging

equation (13) yields

$$\frac{ds}{db}(a_H; \theta_i) = \frac{-(1 - s_H)}{(1 - \delta)k_H - b_H + k_H^\alpha \theta_i M(\varepsilon_{iH}^d)}. \quad (14)$$

Equation (14) tells us that the signal content of a debt-for-equity substitution boils down to which firm has higher expected equity value, conditional upon survival (denominator). This is ambiguous. On one hand, the high type has higher equity value for any given draw of ε . On the other hand, the expected value of ε on the no-default region is higher for the low type. Based on this analysis, we can impose a technical assumption which is sufficient to ensure satisfaction of a single-crossing condition for financial policy in the static model.

Assumption 5 (Static Model). The left-truncated mean function for the public shock, $E[\varepsilon | \varepsilon \geq x]$, has elasticity less than one.

As shown below, Assumption 5 is a sufficient condition for indifference curves in (s, b) space to cross once, with the low type having steeper indifference curves. This is shown as Panel A in Figure 1. Returning to the discussion above, by limiting the growth rate of the left-truncated mean of ε , Assumption 5 ensures the high type has higher expected equity value conditional upon survival. This ensures the high type is less willing to exchange his equity for a debt reduction. Anticipating, other economic effects are present in the dynamic model, making it more difficult to determine the signal content of debt-for-equity substitutions.

From (12), the insider's willingness to exchange equity for capital is determined by

$$\frac{ds}{dk}(a_H; \theta_i) \equiv \frac{(1 - s_H)\Omega_k^{iH}}{\Omega^{iH}}. \quad (15)$$

Rearranging terms in equation (15) it is possible to show that in the static model

$$\frac{ds}{dk}(a_H; \theta_i) = \frac{(1 - s_H)\alpha}{k_H} + [(1 - \alpha)(1 - \delta) + \alpha b_H/k_H] \times \left| \frac{ds}{db}(a_H; \theta_i) \right|. \quad (16)$$

Under Assumption 5, the low type has steeper indifference curves in (s, b) space in the static model. It follows from equation (16) that the low type is more willing to exchange equity for a unit of capital. That is, equity-funded capital accumulation is a negative signal in the static model. This is shown as Panel B in Figure 1. In general, the relative slope of indifference curves in (s, k) space is ambiguous, reflecting competing factors. On one hand, the low type is more willing to give up equity, since he knows his equity is less valuable. On the other hand, the high type has a higher marginal product of capital. Under Assumption 5 the first effect dominates in the static model. Anticipating, other economic effects are present in the dynamic model, making it more difficult to determine the signal content of equity-funded investment.

We summarize the results of this subsection with the following lemma.

Lemma 2. *In the static model, Assumption 5 implies debt-for-equity substitutions are a positive signal and*

equity-financed investment is a negative signal with

$$\begin{aligned} \left| \frac{ds}{db}(a; \theta_L) \right| &> \left| \frac{ds}{db}(a; \theta_H) \right| \\ \frac{ds}{dk}(a; \theta_L) &> \frac{ds}{dk}(a; \theta_H) \text{ for all } a \in \mathcal{A} \text{ such that } b \geq 0. \end{aligned}$$

Proof. From equation (14) it is sufficient to show

$$\begin{aligned} \theta_H M(\varepsilon_{HH}^d) &> \theta_L M(\varepsilon_{LH}^d) \Leftrightarrow \theta_H M[\theta_H^{-1} k_H^{-\alpha} (b_H - (1 - \delta)k_H)] > \theta_L M[\theta_L^{-1} k_H^{-\alpha} (b_H - (1 - \delta)k_H)] \\ &\Leftrightarrow \theta_H M(\theta_H^{-1} \kappa) > \theta_L M(\theta_L^{-1} \kappa) \quad \forall \kappa > 0. \end{aligned} \quad (17)$$

Now define a family of functions $g(\theta) \equiv \theta M(\kappa/\theta)$ parameterized by $\kappa > 0$. We have

$$g'(\theta) = M(\kappa/\theta) - (\kappa/\theta)M'(\kappa/\theta).$$

Assumption 5 ensures $g' > 0$. This establishes the last $>$ sign in (17). ■

2.4 Equilibrium of Static Signaling Game

The full-information first-best investment policy is

$$k_i^{FB} \equiv \left[\frac{\alpha \theta_i E(\varepsilon)}{r + \delta} \right]^{1/(1-\alpha)}. \quad (18)$$

The following remark provides a description of the full-information economy.

Remark. If the profit shock θ_i is public information, the firm implements first-best investment (k_i^{FB}) using a combination of equity and default-free debt. The firm does not hold cash.

The above remark shows that bankruptcy costs are insufficient to induce distortions away from first-best. If there is symmetric information, the firm can still achieve first-best by financing entirely with equity. However, if there is hidden information, financing entirely with equity becomes problematic since low types have an incentive to mimic high types and capture gains from security mispricing.

It is easily verified that the solution to Program L entails $k_L^* = k_L^{FB}$ and $b_L^* = 0$. That is, in a static LCSE there is no need to distort policies of the low type away from the full-information first-best. The financing of the high type in the LCSE satisfies

$$\beta \frac{\partial \varepsilon_{HH}^d}{\partial b_H} f(\varepsilon_{HH}^d) \Phi = \frac{\mu \Omega^{LH}}{\lambda_H} \left[\left| \frac{ds}{db}(a_H; \theta_L) \right| - \left| \frac{ds}{db}(a_H; \theta_H) \right| \right]. \quad (19)$$

The left side of equation (19) measures marginal bankruptcy costs from debt. The right side measures the signal content of a debt-for-equity substitution, which was shown to be positive in Lemma 2.

The optimality condition for the investment of the high type is

$$1 = \int_{\underline{\varepsilon}}^{\infty} (1 - \delta + \alpha \theta_H \varepsilon k_H^\alpha) f(\varepsilon) d\varepsilon - \beta \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d) \Phi + \frac{\mu \Omega^{LH}}{\lambda_H} \left[\frac{ds}{dk}(a_H; \theta_H) - \frac{ds}{dk}(a_H; \theta_L) \right]. \quad (20)$$

The left side of equation (20) measures the marginal cost of investment. The first term on the right side measures the marginal revenue product of capital. The next term measures the benefit created by capital investment in terms of reducing bankruptcy costs. The final term on the right side captures the signal content of equity-funded investment, which was shown to be negative in Lemma 2. Given the competing effects, it is impossible to determine analytically whether the high type invests more or less than first-best in the static model.

The following proposition summarizes this discussion and follows directly from the first-order conditions in Programs L and H.

Proposition 1. *In the least-cost separating equilibrium of the static model, the low type chooses first-best policies, with $k_L^* = k_L^{FB}$ and $b_L^* = 0$. If net worth is sufficiently high, the nonmimicry constraint is slack and the high type also chooses first-best policies, with $k_H^* = k_H^{FB}$ and issues safe debt. For lower levels of net worth, the high type issues defaultable debt and distorts investment away from first-best.*

3 Dynamic Signaling Model

There are numerous reasons to consider a dynamic model. The foremost reason for doing so is that it gives rise to new economic effects. As shown below, repeated hidden information results in an endogenous precautionary motive for reducing debt, retaining cash and investing in more physical capital. This same precautionary motive also alters the signal content of alternative policies. A second motive is technical, since we here develop a tractable algorithm for incorporating hidden information into a dynamic neoclassical investment framework. A third motive for considering a dynamic setting is increased realism. For example, it is shown below that the static model overstates the probability of default by ignoring firms' ability to roll-over debt. Fourth, we can calibrate the dynamic model to real-world panel data in order to infer the magnitude of the costs arising from asymmetric information and to assess the model's ability to match moments. Fifth, the dynamic calibrated model represents a robust laboratory for generating null-hypotheses (see Hennessy and Whited (2005)). Finally, the dynamic model allows us to analyze the effect of shock persistence.¹²

¹²A two-period model would achieve some of these objectives. However, such a model is equally complex as an infinite-horizon model since both involve continuation value functions with no closed-form solutions.

3.1 Technology

For the dynamic model, we retain Assumptions 1-4, but drop Assumption 5. Anticipating, we impose a stronger assumption than Assumption 5 in the dynamic model in order to obtain a single-crossing condition. We let π_{ij} denote the probability of θ_i conditional on lagged type θ_j . The shock θ is persistent with $1 > \pi_{HH} \geq \pi_{HL} > 0$.

At the end of each period, after the public shock is observed, the firm either defaults or delivers the promised debt payment. Default occurs if and only if current shareholders will be unable to raise a sufficient amount of internal and external funds in the next round of financing in order to deliver the promised amount b_i . Consistent with the absolute priority rule, in the event of default the lender demands a cash payment that pins existing shareholders to their reservation value of zero. Anticipating, this is accomplished by forcing existing shareholders to sell off the firm and having them deliver to the lender the cash received in consideration for the equity. Cooley and Quadrini (2001) consider a similar debt contract, but they allow shareholders to inject cash directly into the firm. Anticipating, hidden information substantially alters the analysis of endogenous default.

The variable \tilde{w} again denotes provisional net worth, i.e. the net worth of the firm if shareholders deliver the promised debt payment. Following Cooley and Quadrini (2001), we assume capital investment is perfectly reversible. Thus, we need to keep track of total internal resources, but do not need to maintain separate state variables for cash and capital. There are two state variables in the model: the lagged value of θ and *net worth* (w). In general, there are two *equity value functions*, with $v_i(w)$ defined as the market value of shareholders' equity given that the firm began the period with type θ_i and ended the period with net worth w . The type at the beginning of the period is relevant to equity value since the lagged type determines transition probabilities. If the shock θ is i.i.d. there is no need to keep track of the lagged type as a state variable and $v_H = v_L \equiv v$. It is worth stressing that the equity value functions reflect the value of shareholders' equity after observing net worth but prior to observing the *next* realized value of θ .

There exists an endogenous cutoff $w^d < 0$ such that shareholders must default if $\tilde{w} < w^d$. If $\tilde{w} \geq 0$, default does not occur since the promised debt payment can be delivered using internal resources. If $\tilde{w} \in [w^d, 0)$, shareholders deliver the promised debt payment using a portion of the external funds raised in the next financing round. In fact, if $\tilde{w} = w^d$, shareholders can only deliver the promised debt payment by selling off all the equity in the firm ($s = 1$). As verified below, the model has an endogenous default condition of the form

$$v_L(w^d) = v_H(w^d) = 0. \tag{21}$$

If $\tilde{w} < w^d$, default occurs and the lender demands the maximum cash payment consistent with limited liability. From equation (21) we know the lender will demand a cash payment that leaves current shareholders

with net worth equal to w^d in the event of default. The resulting law of motion for net worth is

$$w(b, k, \varepsilon, \theta) = \max\{w^d, \tilde{w}(b, k, \varepsilon, \theta)\}. \quad (22)$$

The domain for the two equity value functions is $W \equiv [w^d, \infty)$. We heuristically speak of $|w^d|$ as representing the firm's *going-concern value*. More precisely, it is shown below that $|w^d|$ is equal the amount for which the firm can be sold to an uninformed outside investor.

In the dynamic model shareholders default when $\tilde{w} < w^d$, implying that the default-inducing shock ε_{ij}^d is determined by

$$\tilde{w}(b, k, \varepsilon_{ij}^d, \theta) = w^d \Rightarrow \varepsilon_{ij}^d \equiv \frac{b_j - (1 - \delta)k_j + w^d}{\theta_i k_j^\alpha}. \quad (23)$$

If $\varepsilon_{ij}^d \leq \underline{\varepsilon}$, there is zero probability of default for type- i that has received the allocation of type- j . As discussed above, an important difference between the static and dynamic models is that the option value inherent in the firm makes default less likely in the latter, *ceteris paribus*. This can be seen by comparing the default thresholds in equations (2) and (23).

In the event of default the lender demands a revised debt payment from shareholders, call it $b_i^r < b_i$, that leaves them with net worth equal to w^d . Therefore, we compute b_i^r using

$$(1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha - b_i^r = w^d \Rightarrow b_i^r = (1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha + |w^d|. \quad (24)$$

Equation (24) indicates that the lender seizes the firm's physical assets, operating profits, and going-concern value. Since a fraction ($\phi > 0$) of going-concern value is spent on legal fees, the lender's net default recovery is

$$b_i^r - \phi |w^d| = (1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha + (1 - \phi) |w^d|. \quad (25)$$

Let Ω^{ij} denote the expected discounted end-of-period value of shareholders' equity for a type- i that has taken the type- j allocation. In a dynamic setting

$$\Omega^{ij} \equiv \beta \int_{\varepsilon_{ij}^d}^{\infty} v_i [(1 - \delta)k_j + \theta_i \varepsilon k_j^\alpha - b_j] f(\varepsilon) d\varepsilon. \quad (26)$$

A comparison of equations (26) and (3) reveals the fundamental difference between the dynamic and static models. In the dynamic model, the value of equity is determined by some endogenous transformation (v_i) of internal resources. In a static model, equity value is linear in internal resources.

The market price of the debt issued by type- i in the dynamic equilibrium is

$$\rho^i \equiv \beta \left[b_i \int_{\varepsilon_{ii}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon_{ii}^d} [(1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha + (1 - \phi) |w^d|] f(\varepsilon) d\varepsilon \right]. \quad (27)$$

The informed insider maximizes the cum-dividend value of his equity stake. In the dynamic model, the

objective function of the informed insider of type- i is

$$\left(\frac{m}{c}\right)d + \left(\frac{m}{c+n}\right)\beta \int_{\varepsilon^d}^{\infty} v_i[(1-\delta)k + \theta_i \varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon. \quad (28)$$

Using the definition of s , the objective function for the insider (28) simplifies to $(m/c)u(b, d, k, s)$ where we have redefined u as:

$$u(b, d, k, s) \equiv d + (1-s)\beta \int_{\varepsilon^d}^{\infty} v_i[(1-\delta)k + \theta_i \varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon. \quad (29)$$

Comparison of equations (29) and (6) reveals the key difference between the static and dynamic models in terms of an insider's objective function. In the dynamic model, the equity value and objective function is determined by an endogenous transformation (v_i) of internal resources. The insider is forward-looking by construction. In particular, the equity value functions (v_L, v_H) entering the insider's objective function are constructed to correctly capitalize the outcome of all future signaling games.

3.2 Repeated Signaling Model

In the repeated signaling game, the insider again moves first and offers an allocation to the investor. The investor then updates his beliefs and either accepts or rejects the offer. If the investor accepts, the allocation is implemented. If the offer is rejected, the firm is then constrained to finance with internal funds (i.e. $s = 0$ and $b \leq 0$). To derive the LCSE we again solve Programs L and H as defined in Section 2. However, the dynamic model is more complex since the budget and nonmimicry constraints involve unknown equity value functions. Conveniently, the equity value functions can be defined recursively. In order to derive the end-of-period value of total shareholders' equity, we begin by noting that "next period's insider" holds a fraction equal to m/c of all shares. Therefore, the total value of all shares outstanding is simply c/m times the expectation (over types) of u given in equation (29). Recalling that the realized type is a Markov process, we have

$$\begin{aligned} v_j(w) \equiv & \pi_{Hj} \left[d_H^* + (1-s_H^*)\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H^* + \theta_H \varepsilon (k_H^*)^\alpha - b_H^*]f(\varepsilon)d\varepsilon \right] \\ & + (1-\pi_{Hj}) \left[d_L^* + (1-s_L^*)\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L^* + \theta_L \varepsilon (k_L^*)^\alpha - b_L^*]f(\varepsilon)d\varepsilon \right]. \end{aligned} \quad (30)$$

Although w is omitted as an argument in the allocation vectors, it is worth stressing that the equilibrium allocations $(a_L^*, a_H^*)(w)$ depend upon net worth at the start of the period.

The final step in the construction of the dynamic LCSE is the determination of the minimal level of net worth, denoted w^d . We first note that BC_L is binding in the LCSE. Substituting the budget constraint into

the maximand, we can rewrite Program L as

$$\text{Program L : } \max_{b_L, k_L} w + \rho^L(b_L, k_L) + \Omega^{LL}(b_L, k_L) - k_L - \chi\gamma b_L^2/2.$$

Since the objective function for Program L is linear in net worth, there exists a \hat{w} at which the maximized objective in Program L is just equal to zero. For any $w < \hat{w}$, the limited liability of shareholders would be violated. Next note that the maximized objective in Program H is necessarily equal to zero at this same level of net worth \hat{w} . To see this, note that any allocation with $d_H > 0$ or $s_H < 1$ would violate the nonmimicry constraint. It follows that regardless of the realized type, shareholders achieve a continuation value of zero if they start a financing round with net worth equal to \hat{w} . Therefore, in the baseline model, where attention is confined to LCSE, the default threshold is $w^d = \hat{w}$, where

$$\hat{w} \equiv - \max_{b_L, k_L} \rho^L(b_L, k_L) + \Omega^{LL}(b_L, k_L) - k_L - \chi\gamma b_L^2/2. \quad (31)$$

As discussed above, equation (31) shows that the default threshold for net worth is equal to the value for which ownership of the firm can be sold to an uninformed investor. Finally, we note that deadweight signaling costs shift down the equity value functions and limit the amount of external equity the firm can raise in order to meet prior debt obligations. Thus, our model predicts early defaults relative to structural models of endogenous default that adopt the assumption of symmetric information.

3.3 Dynamic Least-Cost Separating Equilibrium

In order to express the optimality conditions compactly, we redefine some variables. The marginal effect of b on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned} \Omega_b^{HH} &\equiv -\beta \int_{\varepsilon_{HH}^d}^{\infty} v'_H[(1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H]f(\varepsilon)d\varepsilon \\ \Omega_b^{LH} &\equiv -\beta \int_{\varepsilon_{LH}^d}^{\infty} v'_L[(1-\delta)k_H + \theta_L \varepsilon k_H^\alpha - b_H]f(\varepsilon)d\varepsilon. \end{aligned} \quad (32)$$

The marginal effect of k on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned} \Omega_k^{HH} &\equiv \beta \int_{\varepsilon_{HH}^d}^{\infty} v'_H[(1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H][1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon \\ \Omega_k^{LH} &\equiv \beta \int_{\varepsilon_{LH}^d}^{\infty} v'_L[(1-\delta)k_H + \theta_L \varepsilon k_H^\alpha - b_H][1 - \delta + \alpha \theta_L \varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon. \end{aligned} \quad (33)$$

Let $\mu(\cdot)$ denote the wealth-contingent multiplier on the NM_{LH} constraint in Program H.

In order to interpret the optimality conditions, it is useful to determine the shadow value of internal

resources. This is the subject of Lemma 3.

Lemma 3. In the dynamic model, there exists a level of net worth, w_{slack} , such that

$$\begin{aligned} v'_i(w) &= 1 + \pi_{Hi} \times \mu(w) \times [\Omega^{HH}(w) - \Omega^{LH}(w)] / \Omega^{HH}(w) > 1 \text{ for } w \in [w^d, w_{slack}) \\ &= 1 \text{ for } w \geq w_{slack}. \end{aligned} \quad (34)$$

Proof. See Appendix A.

The intuition for Lemma 3 is as follows. If the realized type in the subsequent period is θ_L , a dollar of internal funds is just worth a dollar, as the firm essentially obtains financing on fair terms. However, if the realized type in the subsequent period is θ_H , a dollar of internal funds can be worth more than a dollar if it allows the firm to avoid signaling costs. This explains why the shadow value of internal resources depends upon the probability (π_{Hi}) of transitioning to the high type. Note also that the adverse selection problem is manifest in equation (34) with the term $(\Omega^{HH} - \Omega^{LH}) / \Omega^{HH}$ measuring relative equity valuations. Lemma 3 indicates that repeated hidden information results in pseudo-risk-aversion, since the shadow value of internal funds is above one for low net worth levels and is equal to one if net worth is sufficiently high.¹³ Intuitively, if net worth is low and the probability of transitioning to a high type is high, then the marginal value of internal funds is very high since internal funds reduce expected signaling costs in the next financing round.

The next proposition follows from the first-order conditions for the debt and capital of the low type. The calculations are included in Appendix C.

Proposition 2. In the least-cost separating equilibrium of the dynamic model, the low type chooses second-best policies. For all $w \in [w^d, \infty)$, $k_L^*(w) = k_L^{SB} > k_L^{FB}$ and $b_L^*(w) = b_L^{SB} < 0$ where

$$b_L^{SB} = -\frac{\beta}{\gamma} \left[\int_{\underline{\varepsilon}}^{\infty} [v'_L((1-\delta)k_L^{SB} + \theta_L \varepsilon (k_L^{SB})^\alpha - b_L^{SB}) - 1] f(\varepsilon) d\varepsilon \right]. \quad (35)$$

Dividends and equity issuance for the low type are contingent upon net worth with

$$\begin{aligned} w < k_L^* - \beta b_L^* + \gamma (b_L^*)^2 / 2 &\Rightarrow d_L^* = 0 \text{ and } s_L^* > 0 \\ w \geq k_L^* - \beta b_L^* + \gamma (b_L^*)^2 / 2 &\Rightarrow d_L^* = w - (k_L^* - \beta b_L^* + \gamma (b_L^*)^2 / 2) \text{ and } s_L^* = 0. \end{aligned}$$

The intuition for the low type policies is as follows. In order to discourage imitation by the low type, the LCSE makes the low type as well off as possible. In the static signaling model, the optimal policy gives the low type the full-information first-best allocation. In the dynamic model, the low type is given a second-best allocation which accounts for the fact that the shadow value of internal resources exceeds one. Consequently, the low type overinvests relative to first-best and maintains costly financial slack. It is also

¹³We verify global concavity of the value functions using numerical simulations.

worth emphasizing that both b_L^* and k_L^* are invariant to net worth. Therefore, the low type satisfies BC_L by varying dividends and equity issuance only. Note, this is the exact opposite of the static pecking-order hypothesis that debt (and only debt) is used to achieve budget balance.

In Appendix C it is shown that the high type will not pay a dividend if the nonmimicry constraint (NM_{LH}) binds. Intuitively, when the nonmimicry constraint binds, the high type must engage in costly signaling in order to raise external funds. In order to minimize the external funding requirement, zero dividends are paid when the nonmimicry constraint binds. In the LCSE, b_H^* is determined by

$$\begin{aligned} & \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H) - 1] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial b_H} f(\varepsilon_{HH}^d) \phi |w^d| \right] + \chi \gamma b_H \quad (36) \\ & = \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}} \right]. \end{aligned}$$

The term $v'_H - 1$ on the left side of (36) captures a novel deadweight cost of debt service in a dynamic setting with repeated hidden information. The bondholder values a dollar of debt service at one dollar. However, anticipation of future signaling costs causes shareholders to view the cost of debt service as weakly greater than one. The next term on the left side of the equation measures marginal default costs. The third term measures the marginal cost of saving. The right side measures the signal content of debt-for-equity substitutions, discussed in greater detail below. From (36) and Lemma 3 it follows that $\mu = 0 \Rightarrow b_H < 0$. Condition (36) therefore leads to a key implication of the model, that signaling must be a concern if the firm issues debt.¹⁴

The optimality condition pinning down k_H^* is

$$\begin{aligned} 1 & = \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H)] [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] \quad (37) \\ & + \beta \left[\int_{-\infty}^{\varepsilon_{HH}^d} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d) \phi w^d \right] \\ & + \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) (1 - s_H) \left[\frac{\Omega_k^{HH}}{\Omega^{HH}} - \frac{\Omega_k^{LH}}{\Omega^{LH}} \right]. \end{aligned}$$

Equation (37) states that the optimal investment policy equates marginal benefits with the unit price of capital. The first term on the right side of the equation measures the benefit to shareholders from an additional unit of installed capital. Notice that installed capital has an added precautionary benefit when $v'_H > 1$. Intuitively, capital investment has added value in that it increases the pool of future internal resources. The second term measures the benefit of investment accruing to bondholders. The last term measures the signal content of equity-financed investment, which is discussed in greater detail below.

Proposition 3 spells out some implications of the first-order conditions when the nonmimicry constraint is slack.

¹⁴This result no longer holds once we introduce tax advantages to debt finance.

Proposition 3. *In the least-cost separating equilibrium of the dynamic model, for all $w \geq w_{slack}$, the high type implements second-best policies with $k_H^*(w) = k_H^{SB} > k_H^{FB}$ and $b_H^*(w) = b_H^{SB} < 0$ where*

$$b_H^{SB} = -\frac{\beta}{\gamma} \left[\int_{\underline{\varepsilon}}^{\infty} [v'_H((1-\delta)k_H^{SB} + \theta_H \varepsilon (k_H^{SB})^\alpha - b_H^{SB}) - 1] f(\varepsilon) d\varepsilon \right]. \quad (38)$$

For $w \geq w_{slack}$, dividends and equity issuance are contingent upon net worth with

$$\begin{aligned} w \in [w_{slack}, k_H^{SB} - \beta b_H^{SB} + \gamma (b_H^{SB})^2 / 2] &\Rightarrow d_H^* = 0 \text{ and } s_H^* > 0 \\ w \in [k_H^{SB} - \beta b_H^{SB} + \gamma (b_H^{SB})^2 / 2, \infty) &\Rightarrow d_H^* = w - [k_H^{SB} - \beta b_H^{SB} + \gamma (b_H^{SB})^2 / 2] \text{ and } s_H^* = 0. \end{aligned}$$

Consider next $w < w_{slack}$, where the nonmimicry constraint is binding. In terms of indifference curves, the debt optimality condition for the high type (36) can be rewritten as

$$\begin{aligned} &\beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H) - 1] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial b_H} f(\varepsilon_{HH}^d) \phi |w^d| \right] + \chi \gamma b_H \quad (39) \\ &= \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) \left[\left| \frac{ds}{db}(a_H; \theta_L) \right| - \left| \frac{ds}{db}(a_H; \theta_H) \right| \right]. \end{aligned}$$

Equation (39) tells us that the high-type's borrowing depends upon the signal content of debt-for-equity substitutions, as measured by the difference in the slope of the indifference curves in (s, b) space. If debt has positive signal content, the high type borrows more/saves less in order to separate from the low type.

Intuition suggests that four factors determine the relative slopes of the types' indifference curves in (s, b) space. First, the low type knows his equity is less valuable, which increases his willingness to exchange equity for debt reductions. Second, the low type may have a stronger precautionary motive for avoiding debt since he will have lower net worth for all possible realizations of the public shock. However, persistence of the private shocks may induce the higher type to have a stronger precautionary motive. Finally, the high type will service debt in more states of nature, increasing his marginal cost of debt service.

A bit of calculation allows us to write

$$\frac{ds}{db}(a_H; \theta_i) = \frac{-(1-s_H) \left[1 + \int_{\varepsilon_{iH}^d}^{\infty} (v'_i - 1) \frac{f(\varepsilon)}{1-F(\varepsilon_{iH}^d)} d\varepsilon \right]}{(1-\delta)k_H - b_H + k_H^\alpha \theta_i M(\varepsilon_{iH}^d) + \int_{\varepsilon_{iH}^d}^{\infty} (v_i - w) \frac{f(\varepsilon)}{1-F(\varepsilon_{iH}^d)} d\varepsilon}. \quad (40)$$

Comparison of equations (40) and (14) reveals that the signal content of debt-for-equity substitutions is more complicated in the dynamic model since the slope of the indifference curves depend upon the properties of endogenous value functions. We see that the numerator of (40) involves an added term $(v'_i - 1)$ capturing the marginal precautionary value of internal resources. The denominator of (40) also involves an added term $(v_i - w)$ capturing the *enterprise value* of the firm. The enterprise value of a firm is traditionally defined as

the difference between firm value and its cash balance. Since physical capital is perfectly reversible in the model, a dollar in cash and a dollar in capital have equivalent implications for equity's continuation value. Thus, enterprise value in our model is equal to $v_i - w$. The presence of additional terms in (40) is intuitive. Since the numerator of (40) measures the marginal cost of debt service, it must account for the precautionary value of internal funds. Since the denominator measures equity value, it must account for enterprise value.

For the remainder of the paper, we adopt Assumption 5'.

Assumption 5' (Dynamic Model). The public shock is exponentially distributed.

Under the exponential distribution $f(\varepsilon) \equiv \xi e^{-\varepsilon\xi}$. Assumption 5' is stronger than Assumption 5, since the elasticity of the left-truncated mean function at an arbitrary cutoff point x is equal to $x/(x+\xi^{-1}) < 1$. Under the exponential distribution, we obtain unambiguous results regarding the signal content of debt-for-equity substitutions allowing for arbitrary correlation in the private shocks.

We begin by rewriting the expression for the marginal cost of debt (32) as follows

$$\Omega_b^{ij} = \frac{-\beta}{\theta_i k_j^\alpha} \int_{\varepsilon_{ij}^d}^{\infty} f(\varepsilon) \left(\frac{\partial}{\partial \varepsilon} v_i((1-\delta)k_j + \theta_i \varepsilon k_j^\alpha - b_j) \right) d\varepsilon. \quad (41)$$

Using integration by parts it follows that

$$\Omega_b^{ij} = \frac{-\xi \Omega^{ij}}{\theta_i k_j^\alpha}. \quad (42)$$

It follows that a debt-for-equity substitution is a positive signal, with

$$\left| \frac{ds}{db}(a; \theta_L) \right| - \left| \frac{ds}{db}(a_H; \theta_H) \right| = (1 - s_H) \left[\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}} \right] = \frac{(1 - s_H)\xi(\theta_L^{-1} - \theta_H^{-1})}{k_H^\alpha}. \quad (43)$$

We have thus established Proposition 4. The Corollary follows directly from the optimality condition (39) and tells us that the high type will issue more debt when $w \in [w^d, w_{slack})$ than when $w \geq w_{slack}$.

Proposition 4. *If the public shocks have the exponential distribution, debt-for-equity substitutions are a positive signal in the dynamic model with*

$$\left| \frac{ds}{db}(a_H; \theta_L) \right| > \left| \frac{ds}{db}(a_H; \theta_H) \right|.$$

Corollary. *For $w \in [w^d, w_{slack})$, the high type chooses $b_H^*(w) > b_H^{SB}$.*

We can also rewrite the optimality condition for the high-type capital stock (37) using indifference curve notation as

$$1 = \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H)][1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] \quad (44)$$

$$+ \beta \left[\int_{-\infty}^{\varepsilon_{HH}^d} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d) \phi w^d \right] + \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) \left[\frac{ds}{dk}(a_H; \theta_H) - \frac{ds}{dk}(a_H; \theta_L) \right].$$

The optimality condition above tells us that the high-type's capital stock increases with the signal content of equity-financed investment, as measured by the difference in the slope of the two type's indifference curves in (s, k) space. Intuition suggests a number of factors determine the relative slopes of indifference curves in (s, k) space. First, the high type generates more future cash than a low type for a given level of capital, which increases his willingness to exchange equity for capital. Second, the low type has less valuable equity, increasing his willingness to trade equity for capital. In a static setting, the second effect dominates when ε has the exponential distribution. However, precautionary motives complicate the analysis. Persistence in θ implies that a high type has a high probability of being a high type in the subsequent period. For such a firm, the future internal cash generated by installed capital may be more valuable since internal funds reduce exposure to signaling costs. This effect serve to increase the slope of the high-type indifference curves. However, the low type may place a higher shadow value on a marginal dollar at the end of the period, since it necessarily realizes lower net worth.

An efficient mix of real and financial signals equates the ratio of distortion to signal content at the margin. In particular, equations (44) and (39) imply that in the LCSE

$$\begin{aligned}
& \frac{1 - \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} (v'_H) * [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(d\varepsilon) + \int_{-\infty}^{\varepsilon_{HH}^d} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(d\varepsilon) + \frac{\partial \varepsilon_{HH}^d}{\partial k} f(\varepsilon_{HH}^d) \phi w^d \right]}{\frac{ds}{dk}(a_H; \theta_H) - \frac{ds}{dk}(a_H; \theta_L)} \\
&= \frac{\beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [(v'_H) - 1] f(d\varepsilon) + \frac{\partial \varepsilon_{HH}^d}{\partial b_H} f(\varepsilon_{HH}^d) \phi |w^d| \right] + \chi \gamma b_H}{\left| \frac{ds}{db}(a_H; \theta_L) \right| - \left| \frac{ds}{db}(a_H; \theta_H) \right|}. \tag{45}
\end{aligned}$$

The numerator of the previous equation measures the deviation of capital from second-best (where second-best accounts for precautionary value and default costs). The numerator of the right side measures the deviation of debt from second-best. The denominators measure the marginal information content of real and financial signals, respectively.

4 Simulation of Baseline Model

The following parameter values are used in the simulations: $\beta = 1/1.065$; $\alpha = 0.60$; $\delta = 0.10$; $\gamma = 0.05$; $\phi = 0.20$; $\theta_H = 0.8$; $\theta_L = 0.6$; and $\pi_{HL} = \pi_{HH} = 1/2$. The public shocks are exponentially distributed with $f(\varepsilon) = 2 \exp(-2\varepsilon)$. The procedure used to solve the model numerically is described in Appendix D. Once the model is solved, the wealth-contingent policy functions $(a_L^*, a_H^*) : W \rightarrow \mathcal{A} \times \mathcal{A}$ are used to generate a simulated panel data set. We draw 3000 sample paths of public and private shocks consisting of 31 draws for each. All firms start with zero net worth and the initial period is dropped from the sample. We then use the policy functions generated by the model to determine shock-contingent policy and wealth paths. The simulated panel data set is similar in size to those commonly used in empirical testing.

To illustrate concavity of the equity value function, or pseudo-risk-aversion, we begin by plotting the firm's enterprise value. In the present setting, with independent draws of θ , the enterprise value (e) is computed

as $e(w) = v(w) - w$. The slope of the enterprise value function is a direct measure of the precautionary motive for saving, since $e' = v' - 1$. The slope of the enterprise value function measures the marginal net gain current shareholders would capture if they could inject cash directly into the firm. We can express the enterprise value as

$$e(\bar{w}) = |w^d| + \int_{w^d}^{\bar{w}} [v'(w) - 1]dw \quad \forall \bar{w} > w^d. \quad (46)$$

Thus, the enterprise value, at any point in the wealth space is equal to the going-concern value plus the cumulative precautionary value of internal resources. Figure 2 plots the enterprise value function. Consistent with concavity of the equity value function (v), the slope of the enterprise value function is positive but decreasing in net worth. The enterprise value has a slope of zero for high levels of net worth where the nonmimicry constraint is slack. Examining figure 2, we see that starting at the default threshold, the value gained from a one unit cash infusion is roughly equal to 1.05. This magnitude is sensitive to the parameterization of the model. For example, the gain from cash infusions increases in the probability of the high type.

Figure 3 plots the capital allocations of each type relative to first-best. In the LCSE, the low type invests slightly above first-best regardless of realized net worth. This overinvestment reflects the fact that informational asymmetries create a precautionary motive for capital accumulation. The high type also invests more than first-best, with the extent of the distortion decreasing in net worth. When net worth is low, the nonmimicry constraint binds and the high type overinvests by a larger amount in order to reduce default costs. If net worth is sufficiently high, the nonmimicry constraint is slack, but the high type still overinvests due to precautionary motives.

Figure 4 plots the wealth-contingent financing policies for each type. Consistent with Proposition 2, the low type uses dividends and equity issuance as the sole means of achieving budget-balance, while retaining a wealth-invariant level of savings. When net worth is low, the low type sets the dividend to zero and issues a large amount of equity. Equity issuance for the low type then declines monotonically in net worth.

The debt of the high type declines monotonically in net worth. Intuitively, increases in net worth reduce the need for external financing. This allows the high type to cut back on costly signals. Therefore, after a high realization of the public (ε) shock, the model predicts that firms will choose low leverage ratios. Thus, the model is consistent with the empirical observation that leverage ratios are countercyclical (e.g. Korajczyk and Levy (2004)). This is also consistent with the observation that leverage ratios are decreasing in lagged profitability (e.g. Fama and French (2002)).

It is worth noting that when the high type issues equity, it almost always conducts a joint offering which combines equity with debt. In contrast, the low type issues equity without any debt. This is consistent with existing studies. Asquith and Mullins (1986) document negative abnormal returns in a sample of pure common stock offerings. Masulis and Korwar (1986) find that seasoned equity offerings are associated with negative price changes on average. However, the announcement return is positively related to leverage changes.

Turning next to dividend policy, we see that both types pay dividends only if net worth is sufficiently high. This prediction is consistent with the empirically observed positive relation between dividends and internal funds. It is also apparent that the low type initiates dividends at a lower net worth threshold than the high type. This difference reflects the fact that the low type has inferior investment opportunities.

Table 1 reports reduced-form leverage regressions similar to those reported in the empirical literature. In the first three regressions, the dependent variable is the book leverage ratio. In the last regression, the dependent variable is the change in the book leverage ratio. The first row utilizes a specification from Shyam-Sunder and Myers (1999). Shyam-Sunder and Myers use this regression to test the pecking-order hypothesis that debt is used to fill all financing gaps. In our model, the financing gap is $k + d - w$. According to the pecking-order as traditionally specified, the predicted coefficient on the (capital-normalized) financing gap is one. Inspecting Table 1, we see that the simulated firms fill only 24% of the financing gap with debt. Leary and Roberts (2006) show that the test constructed by Shyam-Sunder and Myers is prone to Type II errors. Based on this simulated regression, we argue that the statistical test proposed by Shyam-Sunder and Myers is also prone to a form of Type I error. In our simulated data, the null hypothesis that “hidden information influences financing decisions” would be incorrectly rejected if the econometrician were to assume that the coefficient on the financing gap should be one.

The next two regressions are similar to those reported by Fama and French (2002). Fama and French report a negative relationship between leverage and lagged profits. In the simulated data, we also find that leverage is declining in lagged measures of profitability. In the model, high realizations of the public shock ε result in high lagged profitability and high net worth. Firms with positive information use the increase in net worth to reduce their reliance on debt. Also consistent with the estimates of Fama and French, the coefficient on the market-to-book ratio is positive in the simulated data.

Shyam-Sunder and Myers (1999) and Fama and French (2002) both estimate variants of the following equation to test for mean-reversion in leverage

$$LEV_t - LEV_{t-1} = B_0 + B_{TA} \times [LEV^* - LEV_{t-1}] + u_t. \quad (47)$$

We follow Shyam-Sunder and Myers in using the firm-specific sample average leverage ratio as an estimate of the “target” leverage ratio. Shyam-Sunder and Myers report a significant estimate of $B_{TA} = 0.41$. Fama and French (2002) estimate smaller, yet significant, coefficients with B_{TA} in the range of 0.07 to 0.10. These regressions are of interest since the empirical literature often treats mean-reverting leverage as prima facie evidence in favor of trade-off theory with transactions costs. For example, Fama and French (2002) state that, “the simple pecking order predicts that... the speed of adjustment is indistinguishable from zero, whereas the trade-off model says it is reliably positive.” In the fourth regression presented in Table 2, the estimated value of B_{TA} is 0.18. Thus, the simulated firm exhibits mean-reversion speeds consistent with those actually observed—absent taxes or direct transactions costs.

The model provides a laboratory for analyzing the cross-sectional determinants of abnormal returns associated with investment and financing announcements. We begin by noting that this analysis is not always directly comparable to existing empirical studies. In the simulated data, we can measure the stock price after the market has fully incorporated information about lagged earnings. This *ex ante stock price* is equal to $v(w)/c$. We can then measure the stock price just after the firm announces its financing plans. This *ex post stock price* is equal to $[d + (1 - s)\Omega]/c$. The pure abnormal return (AR) associated with the announced financing is the ex post price less the ex ante price normalized by the ex ante price

$$AR_t \equiv \frac{d_{it} + (1 - s_{it})\Omega_t^{ii} - v(w_t)}{v(w_t)}. \quad (48)$$

In contrast, empirically observed abnormal returns may reflect market inferences about lagged earnings in addition to inferences about future prospects.

Table 2 analyzes the cross-sectional determinants of abnormal returns by treating the simulated AR_t as a dependent variable. Such regressions are common to the event study literature. A common theme running through the regressions is that high investment rates are statistically and economically significant predictors of abnormal returns. This is consistent with the empirical evidence presented by McConnell and Muscarella (1985) that increases in capital expenditures are associated with positive abnormal returns. In the first reported regression, the abnormal return is positively related to the investment rate. In the second and third regressions we see a high percentage of debt in total external financing is also associated with positive abnormal returns. This is consistent with Masulis' (1983) finding that debt-for-equity exchanges are associated with positive abnormal returns. In the fourth regression we see that, unconditionally, the abnormal return is increasing in leverage. However, the fifth regression reveals that the significance of leverage is sensitive to the regression specification. In particular, the leverage ratio becomes insignificant once we include the investment rate as a regressor. This is because the high type always invests much more than the low type in our model. However, the high type does not necessarily issue much more debt. For example, when net worth is sufficiently high both firms actually save. Therefore, capital expenditures are the best predictor of abnormal returns.

5 Pooling Equilibrium

The objective of this section is to determine whether it is possible for the two firm types to pool at the same allocation, say $a^P(w)$, at some points in the state (w) space. To this end, we rely on results derived by Maskin and Tirole (1992) and Tirole (2006), who consider an option contract game.¹⁵ The utility of the option contract game is that it allows one to narrow the set of possible equilibria.

The timing and equilibrium concept in the option contract game are the same as that for the dynamic signaling game. However, the initial offer is a menu rather than an allocation. In particular, the game begins

¹⁵The exposition closely follows Tirole's Section 6.4.

with the insider proposing to the investor an *option contract* consisting of a pair $(a_1, a_2) \in \mathcal{A} \times \mathcal{A}$. Formally, the contract represents an agreement by the two parties to enter into a direct revelation mechanism. If he accepts, the investor has entered into a binding agreement to fund the firm regardless of which option on the menu the insider subsequently chooses. In reality, one can view a shelf-registration as resembling an option contract since a shelf-registration narrows the menu of securities from which a firm may (costlessly) select after it acquires new information.

We look for PBE at each point in the net worth space. Importantly, each point in the net worth space has measure zero. Therefore, at each point in the state space the agents can treat the value functions as *given*. Similarly, we the modelers can treat the value functions as fixed even as we vary the equilibrium at particular points. Having said this, all discussions regarding equilibrium at particular points in the net worth space should be understood as being conditioned on internally consistent value functions. Recursive methods are used to ensure internal consistency.

We begin by noting that for all $w \geq \hat{w}$, the LCSE entailed $s_L^*(w) \geq 0$. Therefore, for all $w \geq \hat{w}$, the investor would earn nonnegative profits if the high type took the low type's allocation. Thus, Tirole's weak monotonic profit condition is satisfied and the pair $(a_L^*, a_H^*)(w)$ represents the Rothschild-Stiglitz-Wilson allocation (RSW). For $w < \hat{w}$, the constraint BC_L cannot be satisfied by any $a \in \mathcal{A}$. Thus, for $w < \hat{w} < 0$, both types get RSW payoffs of zero. We refer to these RSW payoffs as the "separating payoffs."

The following is a restatement of Tirole's Proposition 6.1.

Lemma 4. *The separating payoffs are always in the equilibrium set. The payoffs from pooling at an option contract (a^P, a^P) are in the equilibrium set only if the investor earns a weakly positive profit in expectation (based upon his prior beliefs) and the payoffs Pareto dominate the separating payoffs from the perspective of both insider types.*

Recall that when attention was confined to separating equilibria, the default threshold was given by $w^d = \hat{w}$ as defined in equation (31). We conjecture, and verify, that the default threshold in the present setting, which allows for pooling, is $w^d = \hat{w}_P < \hat{w}$. That is, the possibility of pooling increases the continuation region relative to what is possible if only separating equilibria are considered. The intuition is simple. At a net worth level $\hat{w} - \epsilon$ both types receive zero in the LCSE because a low type, revealed as such, cannot satisfy his budget constraint. At this point, both types would be better off pooling and achieving budget balance in expectation (across types).

Proposition 5 states that it is possible to support equilibria in which pooling occurs for low net worth states while separation occurs for higher net worth states.

Proposition 5. *In the dynamic option contract game, it is possible to support a default threshold $w^d < \hat{w}$. There exists $\bar{w}^P > \hat{w}$ such that for $w \in [w^d, \bar{w}^P]$ the payoffs resulting from pooling at an allocation $a^P(w)$ are in the equilibrium set. For $w > \bar{w}^P$ there is no pooling equilibrium.*

Proof. We begin by establishing the existence of a pooling region. Choose ϵ arbitrarily small and let $w \equiv \hat{w} - \epsilon$. The RSW payoffs at this point are zero. However, if the types were to pool using $d = 0$, $b_L^*(\hat{w})$ and $k_L^*(\hat{w})$, they could achieve a strictly positive payoff at some $s < 1$ while the investor earns zero expected profit based upon priors. The claimed result then follows from Lemma 4. Next recall that there exists a $w \equiv w_{slack}$ at which the nonmimicry constraint in Program H is slack. For $w \geq w_{slack}$, the solution to Program H yields a greater payoff for the high type than any pooling allocation that gives the investor weakly positive expected profits based upon prior beliefs. From Lemma 4 it follows that no pooling equilibrium can exist on this interval. ■

Although pooling extends the continuation region, the firm must default if net worth is sufficiently low. In particular, the attempt to pool would be unsuccessful if the investor could not break even at $s = 1$. Therefore, we define the default threshold in the dynamic option contract game as $w^d = \hat{w}_P$, where

$$\hat{w}_P \equiv - \max_{b,k} \pi[\rho^H(b,k) + \Omega^H(b,k)] + (1 - \pi)[\rho^L(b,k) + \Omega^L(b,k)] - k - \chi\gamma b^2/2. \quad (49)$$

Based upon Lemma 4, we can use the following Program P to determine whether pooling is possible at a particular level of net worth. However, it is worth stressing that pooling equilibria need not be unique. For example, at some points on the net worth space there are pooling equilibria where only equity finance is employed. Program P makes the high type as well off as possible subject to: (1) The pooling allocation is improving for the low type relative to his separating allocation (LTI). (2) Provision of outside funding is profitable in expectation (PIE) based upon the investor's prior beliefs.

$$\text{PROGRAM P: } \max_{a \in \mathcal{A}} d + (1 - s)\Omega^H$$

subject to

$$LTI : d + (1 - s)\Omega^L \geq d_L^* + (1 - s_L^*)\Omega^{LL}$$

$$PIE : \pi[\rho^H(b,k) + \Omega^H(b,k)] + (1 - \pi)[\rho^L(b,k) + \Omega^L(b,k)] \geq k + \chi\gamma b^2/2 - w.$$

We denote the solution to Program P as $a^P \equiv (b^P, d^P, k^P, s^P)$. From Lemma 4 we know that pooling is only possible at point w if $d^P + (1 - s^P)\Omega^H \geq d_H^* + (1 - s_H^*)\Omega^{HH}$.

We define equilibrium allocations in the option contract game (a_L^{**}, a_H^{**}) point-wise on the net worth space with

$$d^P(w) + [1 - s^P(w)]\Omega^H \geq d_H^*(w) + [1 - s_H^*(w)]\Omega^{HH} \Rightarrow (a_L^{**}, a_H^{**})(w) \equiv (a^P(w), a^P(w)) \quad (51)$$

$$d^P(w) + [1 - s^P(w)]\Omega^H < d_H^*(w) + [1 - s_H^*(w)]\Omega^{HH} \Rightarrow (a_L^{**}, a_H^{**})(w) \equiv (a_L^*(w), a_H^*(w)).$$

The final step in the construction is to use a recursive equation to pin down the equity value functions

$$v_j(w) \equiv \pi_{Hj} \left[d_H^{**} + (1 - s_H^{**})\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1 - \delta)k_H^{**} + \theta_H \varepsilon (k_H^{**})^\alpha - b_H^{**}] f(\varepsilon) d\varepsilon \right] \\ + (1 - \pi_{Hj}) \left[d_L^{**} + (1 - s_L^{**})\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1 - \delta)k_L^{**} + \theta_L \varepsilon (k_L^{**})^\alpha - b_L^{**}] f(\varepsilon) d\varepsilon \right]. \quad (52)$$

Before turning to the results of the numerical simulation, we present the optimality conditions from Program P. Letting λ denote the multiplier on the PIE constraint, the optimal financing policy in the pooling equilibrium satisfies

$$\pi(\Omega_b^H + \rho_b^H) + (1 - \pi)(\Omega_b^L + \rho_b^L) + \chi\gamma b = \frac{(1 - \lambda\pi)\Omega^H(1 - s)}{\lambda} \left[\left| \frac{\Omega_b^H}{\Omega^H} \right| - \left| \frac{\Omega_b^L}{\Omega^L} \right| \right] \quad (53)$$

and the optimal investment rule satisfies

$$\pi(\Omega_K^H + \rho_K^H) + (1 - \pi)(\Omega_K^L + \rho_K^L) - 1 = \frac{(1 - \lambda\pi)\Omega^H(1 - s)}{\lambda} \left[\frac{\Omega_k^H}{\Omega_k^H} - \frac{\Omega_k^L}{\Omega_k^L} \right]. \quad (54)$$

From the optimality conditions on d and s it is also possible to show that $1 > \lambda\pi$. The left sides of the two optimality conditions represent the average deadweight loss associated with increases in debt and capital, respectively. Since the high type is pooling, he is concerned about the average level of efficiency. The right side of the optimality conditions capture the cross-subsidy provided by the high type to the low type in any pooling equilibrium.

Figure 5 presents the results from simulations under the same parameterization as in Section 4, but allows for the possibility of pooling. Consistent with Proposition 4, the firms pool when net worth is low and switch to the LCSE when net worth is sufficiently high. On the pooling interval, the high type underinvests and the low type overinvests relative to first-best. Once there is a switch to the LCSE, the high type sharply increases investment and the low type sharply cuts investment. Figure 6 depicts equilibrium financing policies. On the pooling region, investment is financed primarily with debt, since this reduces cross-subsidies from the high type to the low type. Once again, we see that the switch from pooling to separating equilibrium leads to non-monotonic policies. For example, the debt of the high type spikes upward once there is a switch from pooling to separating equilibrium. The debt of the high type then begins falling as increases in net worth reduce the need for costly signaling in the separating equilibrium.

A unique feature of our structural model is that it differentiates between dividends and share repurchases as modes of distributing cash to shareholders. From this perspective, it is interesting to note the share repurchases occur in equilibrium once we allow for pooling. This did not occur when we only considered LCSE. The intuition is simple. When we allow for pooling, the continuation region is larger and default costs are lower. In this case, the logic of the model more closely approximates that of Constantinides and Grundy (1990), where firms issue debt in excess of the amount needed for investment and use the residual

funds for share repurchases. In other words, when bankruptcy becomes less costly, the firm places greater reliance on financial signals (as distinct from real investment signals).

6 Integration with Trade-off Theory

The model can be extended to include a corporate income tax and the associated tax shield benefit of debt. There are a number of compelling rationales for extending the model to include taxes. First, there is no reason why theories of financing based upon hidden information and the trade-off theory should be mutually exclusive. It is noteworthy that this point was stressed by Myers (1984) himself. Second, including tax advantages of debt will help the model to explain an even broader set of stylized facts. In particular, Graham (1996) presents empirical evidence in support of the hypothesis that increases in the value of the debt tax shield result in marginal increases in the propensity of firms to use debt finance.

Following the convention in the literature, we assume the corporate income tax rate is a constant $\tau \in (0, 1)$. The base of the corporate income tax is operating income less economic depreciation less interest expense plus interest income. Let y denote the promised yield to maturity on debt. From the identity $\rho \equiv b/(1 + y)$, it follows that interest expense is $y\rho = b - \rho$. The same expression represents taxable interest income if $b < 0$. In the U.S. payments to lenders are treated as principal first. To capture this rule, we assume interest deductions are disallowed in the event of default. Finally, we note that the inclusion of a corporate income tax creates a tax penalty to financial slack since shareholders can earn the rate of return r by saving on their own account while the corporation earns $r(1 - \tau)$ after-tax. To isolate this tax effect, below we set agency costs of financial slack equal to zero ($\gamma = 0$).

To summarize, this subsection replaces Assumption 3 with Assumption 3'.

Assumption 3'. Corporate income is taxed at rate $\tau \in (0, 1)$. In the event of default, interest deductions are disallowed. Default is endogenous. In the event of default, the lender has strict seniority. The deadweight default cost is a fraction (ϕ) of going-concern value $|w^d|$. The pre-tax yield on corporate saving is r .

For simplicity, we return to the baseline model that considers only separating equilibria. Accounting for the corporate income tax, provisional net worth is

$$\tilde{w} \equiv (1 - \delta)k + \theta \varepsilon k^\alpha - b - \tau[\theta \varepsilon k^\alpha - \delta k - (b - \rho)] = (1 - \delta(1 - \tau))k + (1 - \tau)\theta \varepsilon k^\alpha - b + \tau(b - \rho). \quad (55)$$

From (55) it follows that the sum of net worth and the debt payment is just equal to the internal resources of an unlevered firm plus the value of the debt tax shield. The default-inducing shock must be modified to account for the effect of taxes, with

$$[1 - \delta(1 - \tau)]k_j + (1 - \tau)\theta_i \varepsilon_{ij}^d k_j^\alpha - b_j + \tau(b_j - \rho_j) \equiv w^d \Rightarrow \varepsilon_{ij}^d \equiv \frac{b_j - [1 - \delta(1 - \tau)]k_j + w^d - \tau(b_j - \rho_j)}{(1 - \tau)\theta_i k_j^\alpha}. \quad (56)$$

Accounting for the fact that interest deductions are disallowed in default, lender recoveries in the event of default are equal to

$$(1 - \delta(1 - \tau))k_i + (1 - \tau)\theta_i \varepsilon k_i^\alpha + (1 - \phi) |w^d|. \quad (57)$$

That is, in the event of a default, the lender receives the value of the physical capital, earnings, and going-concern value net of the corporate income tax bill.

The expected end-of-period equity value must also be adjusted to account for the new definition of net worth

$$\Omega^{ij} \equiv \beta \int_{\varepsilon_{ij}^d}^{\infty} v_i [(1 - \delta(1 - \tau))k_j + (1 - \tau)\theta_i \varepsilon k_j^\alpha - b_j + \tau(b_j - \rho_j)] f(\varepsilon) d\varepsilon, \quad (58)$$

and the bond value function must account for the new definition of lender recoveries in default

$$\rho^i \equiv \beta \left[b_i \int_{\varepsilon_{ii}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon_{ii}^d} [(1 - \delta(1 - \tau))k_i + (1 - \tau)\theta_i \varepsilon k_i^\alpha - (1 - \phi)w^d] f(\varepsilon) d\varepsilon \right]. \quad (59)$$

In the interest of brevity, we focus on financial policies. Once again, the low type allocation is derived by solving Program L. Program L is unaffected save the new definitions for the equity value and debt value which account for taxes. The optimality condition for b_L^* is

$$\frac{\partial}{\partial b} (y_L \rho_L) \times \tau \int_{\varepsilon_{LL}^d}^{\infty} f(\varepsilon) d\varepsilon = f(\varepsilon_{LL}^d) \frac{\partial \varepsilon_{LL}^d}{\partial b} \phi |w^d| + \left[1 - \tau \frac{\partial}{\partial b} (y_L \rho_L) \right] \left[\int_{\varepsilon_{LL}^d}^{\infty} [v'_L - 1] f(\varepsilon) d\varepsilon \right]. \quad (60)$$

The left side of equation (60) is the marginal tax shield benefit provided by debt. The first term on the right side of the equation is the marginal bankruptcy cost. The second term on the right side is a measure of the precautionary cost of debt service. Since asymmetric information induces $v'_L > 1$ for low net worth states, the low type is predicted to have low debt relative to traditional trade-off theory. Thus, the precautionary effect offers a potential explanation for what would appear to be conservative debt policies.

The high type allocation again solves Program H. In the interest of brevity, we redefine the marginal cost of debt service to account for taxes. The marginal effect of b on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned} \Omega_b^{HH} &\equiv - \left[1 - \tau \frac{\partial y \rho}{\partial b} \right] \beta \int_{\varepsilon_{HH}^d}^{\infty} v'_H [(1 - \delta(1 - \tau))k_H + (1 - \tau)\theta_H \varepsilon k_H^\alpha - b_H + \tau(b_H - \rho_H)] f(\varepsilon) d\varepsilon \quad (61) \\ \Omega_b^{LH} &\equiv - \left[1 - \tau \frac{\partial y \rho}{\partial b} \right] \beta \int_{\varepsilon_{LH}^d}^{\infty} v'_L [(1 - \delta(1 - \tau))k_H + (1 - \tau)\theta_L \varepsilon k_H^\alpha - b_H + \tau(b_H - \rho_H)] f(\varepsilon) d\varepsilon. \end{aligned}$$

The optimality condition for b_H^* is

$$\begin{aligned} &\frac{\partial}{\partial b} (y_H \rho_H) \times \tau \int_{\varepsilon_{HH}^d}^{\infty} f(\varepsilon) d\varepsilon + \left(\frac{\mu \Omega^{LH}}{\beta \lambda_H} \right) \left[\left| \frac{ds}{db} (a_H; \theta_L) \right| - \left| \frac{ds}{db} (a_H; \theta_H) \right| \right] \\ &= f(\varepsilon_{HH}^d) \frac{\partial \varepsilon_{HH}^d}{\partial b} \phi |w^d| + \left[1 - \tau \frac{\partial}{\partial b} (y_H \rho_H) \right] \left[\int_{\varepsilon_{HH}^d}^{\infty} [v'_H - 1] f(\varepsilon) d\varepsilon \right]. \quad (62) \end{aligned}$$

The financing of the high type equates marginal tax and signaling benefits of debt with bankruptcy and precautionary costs. If net worth is sufficiently high, the nonmimicry constraint is slack and the precautionary effect induces the high type to have low leverage relative to that predicted by traditional trade-off theory. This offers another explanation for Graham's (2000) finding that large-liquid firms have conservative debt policies.¹⁶ By way of contrast, if net worth is low, the signaling motive is operative. This induces high leverage relative to that predicted by trade-off theory.

7 Conclusions

The objective of this paper was to take a first step in constructing dynamic structural models of corporate financing when insiders have private information. Beginning with a static model, we showed that firms with positive information will signal through the issuance of defaultable debt and increase investment in order to reduce associated default costs. These two effects are also present in our dynamic model. The dynamic model delivers a number of added insights. First, anticipation of future signaling costs converts a risk-neutral insider into a pseudo-risk-averse insider. The implied precautionary value of internal funds discourages debt and encourages capital accumulation relative to what one obtains in a static setting. Second, default costs are lower in a dynamic setting, since continuation value allows the firm to roll-over prior debt obligations. Finally, we showed that the nature of equilibrium is contingent on the firm's net worth. In particular, if net worth is sufficiently low, the costs of separation are extremely high and firms should find their way to a Pareto dominating pooling equilibrium.

The most obvious direction to take this line of research is to analyze optimal security design. In this paper, we took the set of securities as given, and derived equilibria given this set. We anticipate that the securities emerging as optimal under repeated hidden information will be similar to those that are optimal under repeated moral hazard. Whether the problem is one of hidden information or hidden action, low reported values of operating profits must be punished. In our model, low reported profits are punished with transfers of ownership in default. A more general model, with long-term debt, would allow for intermediate punishments in the form of higher interest rates.

¹⁶This explanation differs fundamentally from that offered by Hennessy and Whited (2005), which is based upon taxes on distributions and direct flotation costs. In the present model, we have shut off those two channels.

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Appendix A: Proofs

Proof of Lemma 1.

The allocation a_L^* was in the feasible set for Program H. To verify, the NM_{LH} constraint would be trivial as the allocations would be type-independent. Since BC_L is satisfied at a_L^* and $s_L^* \geq 0$ we know BC_H would be slack. The fact that a_H^* solves Program H tells us the high type's maximand must be at least as high at a_H^* as at a_L^* .

Proof of Lemma 3.

First note that the nonmimicry constraint must be binding at w^d since the high type gets a zero payoff here but would achieve a strictly positive payoff if he needed to satisfy only BC_H but could ignore NM_{LH} . Thus, the nonmimicry constraint binds for sufficiently low net worth. Next consider high levels of net worth and solve Program H ignoring the nonmimicry constraint. For w sufficiently high the solution to that program entails $s_H = 0$ and $b_H < 0$. Thus, the high type is not floating any securities and the low type cannot gain from mimicry since there is no gain from securities mispricing.

Applying the Envelope Theorem, the value of internal funds if the low type is realized is $\lambda_L = 1$. The value of a dollar of internal funds to the high type is given by $\lambda_H + \mu$. To see this, it must be noted that the low type's equilibrium payoff (which enters the NM_{LH} constraint) can be expressed as $w + \kappa^*$. This explains why the shadow value of internal funds to the high type is not simply λ_H . Taking a probability weighted average of these values yields (34). ■

Appendix B: Verification of PBE

We now verify that the solutions to Programs L and H in conjunction with beliefs (9) constitute a PBE. First note that any offer $a_0 \notin \{a_L^*, a_H^*\}$ that would be acceptable to the investor must satisfy both BC_L and BC_H , because the beliefs minimize the value of the package of securities. We next verify that it is optimal for insiders observing θ_L and θ_H to offer to the investor a_L^* and a_H^* , respectively. Consider first the low type's incentive to offer some allocation $a_0 \notin \{a_L^*, a_H^*\}$ that would be acceptable to the investor. Since a_0 is acceptable, it must satisfy BC_L . Thus, a_0 was in the feasible set for Program L and optimality implies the low type prefers a_L^* to a_0 . This same argument shows the low type will not make an offer that is rejected, since such an offer induces zero outside funding, and zero outside funding is always acceptable to the investor. Therefore, the low type prefers a_L^* to all offers other than a_H^* . Finally, NM_{LH} ensures the low type prefers a_L^* to a_H^* .

Consider next the high type's incentive to offer some allocation $a_0 \neq a_H^*$ that would be accepted by the investor. Note that the allocation a_0 was in the feasible set for Program H. To see this, note that BC_H is necessarily satisfied, since the offer is acceptable even with worst-case beliefs. Since a_0 also satisfies BC_L , we know a_0 was in the feasible set for Program L. The optimality of a_L^* in Program L implies that NM_{LH} would be satisfied if the high type were to offer a_0 . Thus, a_0 was in the feasible set for Program H, and the

optimality of a_H^* for this program implies the high type prefers a_H^* to a_0 . This same argument shows that the high type will not make an offer that is rejected, since such an allocation is equivalent to zero outside funding, and zero outside funding is always acceptable to the investor.

Appendix C: Optimality Conditions in Dynamic Model

The Lagrangian for Program L is

$$L = d_L + (1 - s_L)\Omega^{LL} + \lambda_L[w - d_L - k_L - \chi\gamma b_L^2/2 + s_L\Omega^{LL} + \rho^L] + \eta_L d_L + \psi_L(1 - s_L).$$

The first-order conditions for d_L and s_L are

$$1 - \lambda_L + \eta_L = 0 \tag{63}$$

$$(\lambda_L - 1)\Omega^{LL} - \psi_L = 0. \tag{64}$$

From equations (63) and (64) it follows that $\eta_L\Omega^{LL} = \psi_L$. Since ε has unbounded support, $\psi_L(w) > 0$ cannot occur for $w > w^d$. To see this, note that the proposed equilibrium would entail the low type getting zero. But then the low type would always opt for the high type allocation unless $s_H = 1$ and $d_H = 0$. Of course, this implies both types get a payoff of zero which contradicts being on shareholders' continuation region.

The first-order condition pinning down b_L^* is

$$\beta \left[\int_{\varepsilon_{LL}^d}^{\infty} [v'_L((1 - \delta)k_L + \theta_L \varepsilon k_L^\alpha - b_L) - 1] f(\varepsilon) d\varepsilon \right] = -\chi\gamma b_L + \beta \frac{\partial \varepsilon_{LL}^d}{\partial b_L} f(\varepsilon_{LL}^d) \phi w_L^d. \tag{65}$$

From Lemma 2 it follows that the left side of (65) is positive. It follows that b_L must be negative. The optimality condition for k_L takes a similar form, with

$$\beta \left[\int_{\underline{\varepsilon}}^{\infty} [v'_L((1 - \delta)k_L + \theta_L \varepsilon k_L^\alpha - b_L)][1 - \delta + \alpha\theta_L \varepsilon k_L^{\alpha-1}] f(\varepsilon) d\varepsilon \right] = 1.$$

The Lagrangian for Program H is

$$\begin{aligned} L = & d_H + (1 - s_H)\Omega^{HH} + \lambda_H[w - d_H - k_H - \chi\gamma b_H^2/2 + s_H\Omega^{HH} + \rho^H] \\ & + \mu[d_L^* + (1 - s_L^*)\Omega^{LL} - d_H - (1 - s_H)\Omega^{LH}] + \eta_H d_H + \psi_H(1 - s_H). \end{aligned}$$

The first-order conditions for d_H and s_H are

$$1 - \lambda_H - \mu + \eta_H = 0 \tag{66}$$

$$(\lambda_H - 1)\Omega^{HH} + \mu\Omega^{LH} - \psi_H = 0 \quad (67)$$

Substituting (66) into (67), we obtain

$$\eta_H\Omega^{HH} = \mu(\Omega^{HH} - \Omega^{LH}) + \psi_H. \quad (68)$$

It is straightforward to establish $\psi_H(w) = 0$ on the continuation region ($w > w^d$). To see this, suppose to the contrary that $\psi_H > 0$. It follows from (68) that $\eta_H > 0$ and $d_H = 0$. Thus, the high type gets a payoff of zero. However, the low type would then also receive an allocation with a payoff of zero since the NM_{HL} constraint demands

$$d_H + (1 - s_H)\Omega^{HH} \geq d_L + (1 - s_L)\Omega^{HL} \geq d_L + (1 - s_L)\Omega^{LL}. \quad (69)$$

But this contradicts being on shareholders' continuation region. Without loss of generality we shall treat $\psi_H = 0$ as we solve for optimal policies on the continuation region.

Rearranging (68) we obtain

$$\eta_H = \mu[(\Omega^{HH} - \Omega^{LH})/\Omega^{HH}]. \quad (70)$$

Condition (70) tells us that whenever NM_{LH} binds, the dividend is zero.

Appendix D: Details of the Numerical Algorithm

The solution procedure is based on value function iteration. The individual steps are as follows. The idiosyncratic shock ε is implemented by discretizing its domain using N possible values. Each maximization is implemented by discretizing the domain of the decision variables.

1. Guess going-concern value w^d .
2. Guess the end-of-period equity value functions (v_j) which are vectors on the net worth space.
3. For each point in the net worth grid, find the allocation a_L that maximizes the objective function of the low type subject to its budget constraint. Since the dividend is not unique, pick the allocation in the optimal set that minimizes the dividend.
4. For each point in the net worth grid, find the allocation a_H that maximizes the high type's objective subject to the budget and nonmimicry constraints.
5. Using the solutions from steps 3 and 4, compute new value functions v'_j using the recursive equation

$$v'_j = \pi_{Hj} \left[d_H + \beta(1 - s_H) \sum_{n=1}^N f(\varepsilon_n) v_H [(1 - \delta)k_H + \theta_H \varepsilon_n k_H^\alpha - b_H] \right] \\ + \pi_{Lj} \left[d_L + \beta(1 - s_L) \sum_{n=1}^N f(\varepsilon_n) v_L [(1 - \delta)k_L + \theta_L \varepsilon_n k_L^\alpha - b_L] \right].$$

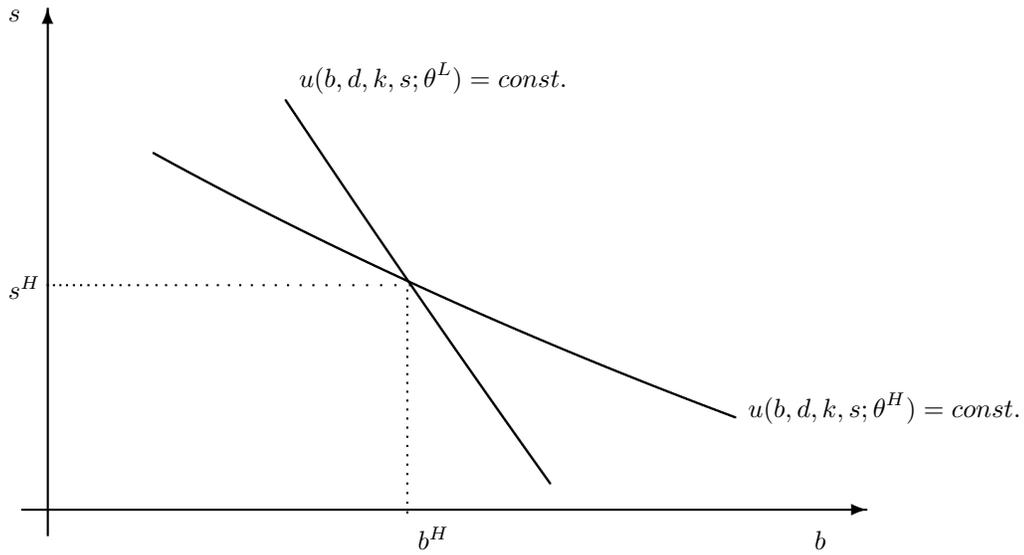
6. The functions v_j' from the previous step are the new guesses for v_j . The procedure is then restarted from step 2 until convergence.

7. Check that w^d satisfies the endogenous default condition $v_j(w^d) = 0$. If these conditions are not satisfied, update the initial guess w^d and restart the procedure from step 1 until convergence.

Figure 1: **Single-crossing conditions**

This figure presents single-crossing conditions. Panel A shows the indifference curves drawn under the assumption that the low type is more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type. Panel B shows the indifference curves drawn under the assumption that the low type has a greater willingness to exchange equity for capital. In this case, higher capital investment provides a negative signal which encourages underinvestment relative to first-best.

Panel A: Debt-for-Equity Substitution as Positive Signal



Panel B: Equity-Financed Investment as Negative Signal

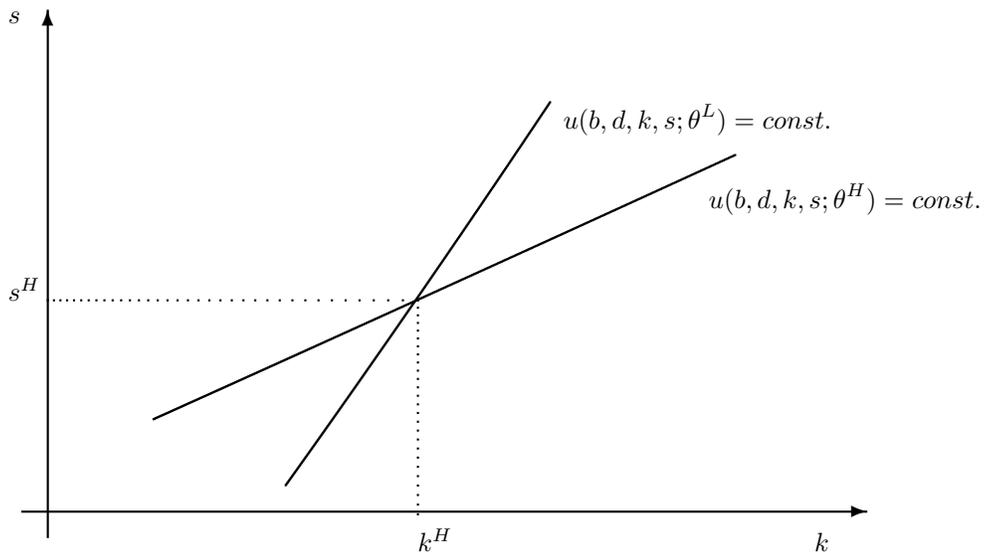


Figure 2: **Enterprise value of the firm**

The enterprise value of the firm $v - w$ is plotted as a function of the realized net worth, w . The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

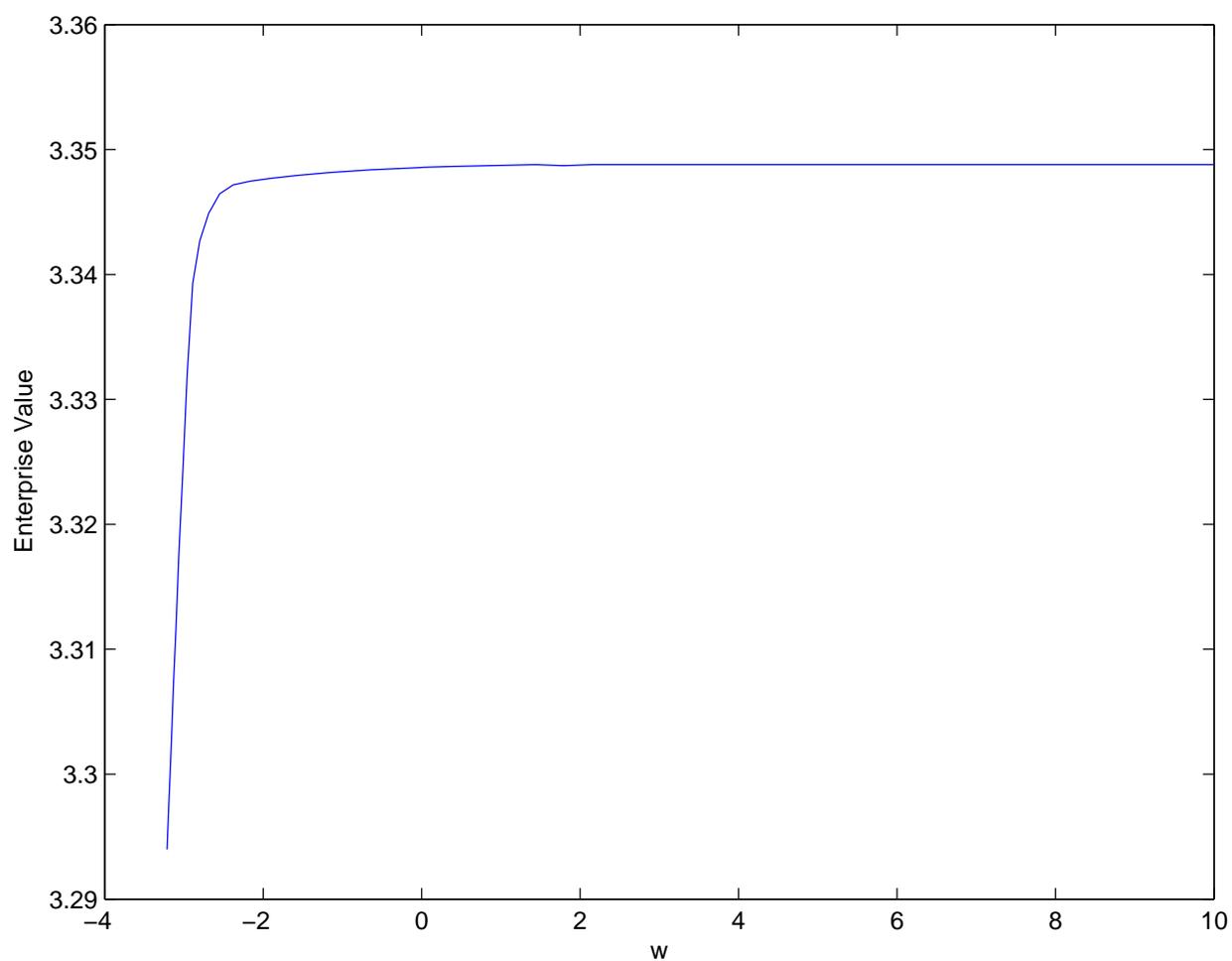


Figure 3: **Equilibrium capital allocations**

Equilibrium capital allocations, k_i^* , scaled by the first-best allocations, k_i^{FB} , are plotted as a function of the realized net worth, w , for both high and low values of θ . The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

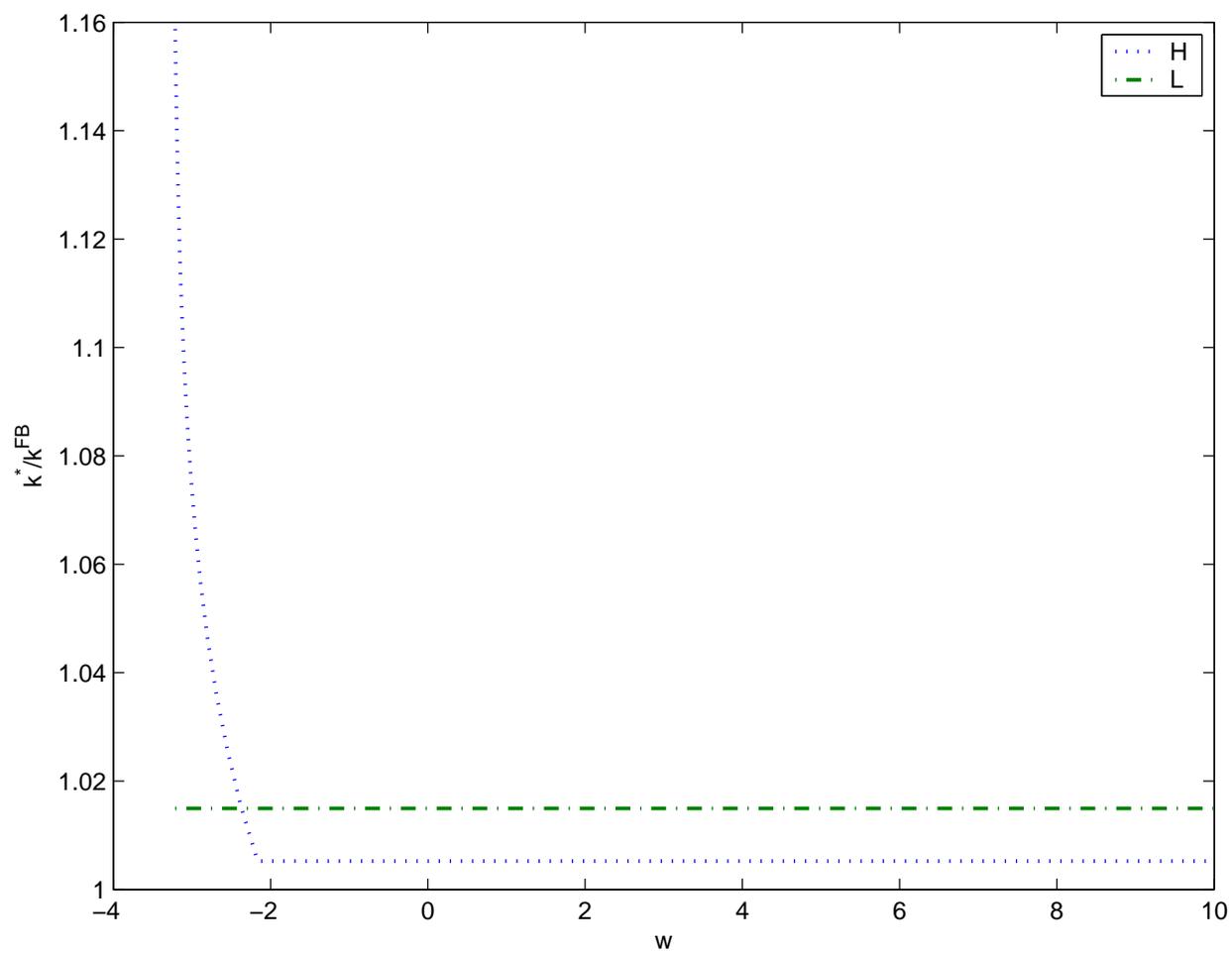


Figure 4: **Equilibrium financing policies**

Equilibrium financing policies: debt, ρ_i , and equity, $s_i^* \Omega^{ii}$, as well as equilibrium dividend policy, d_i^* , are plotted as functions of the realized net worth, w . Panel A presents the case of high value of θ , while Panel B presents the case of low value of θ . The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

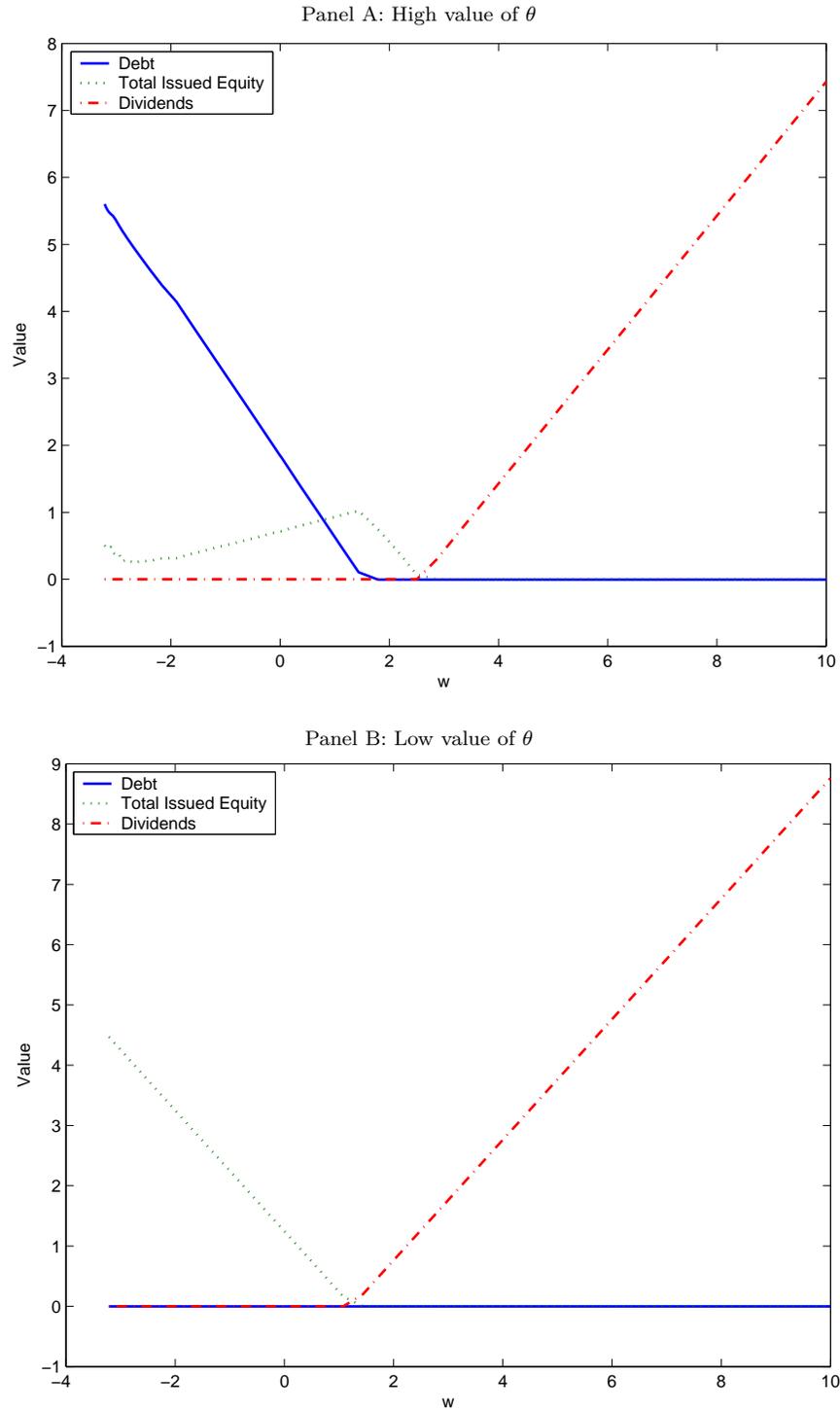


Figure 5: **Equilibrium capital allocations in option contract game**

Equilibrium capital allocations, k_i^* , scaled by the first-best allocations, k_i^{FB} , are plotted as a function of the realized net worth, w , for both high and low values of θ . The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

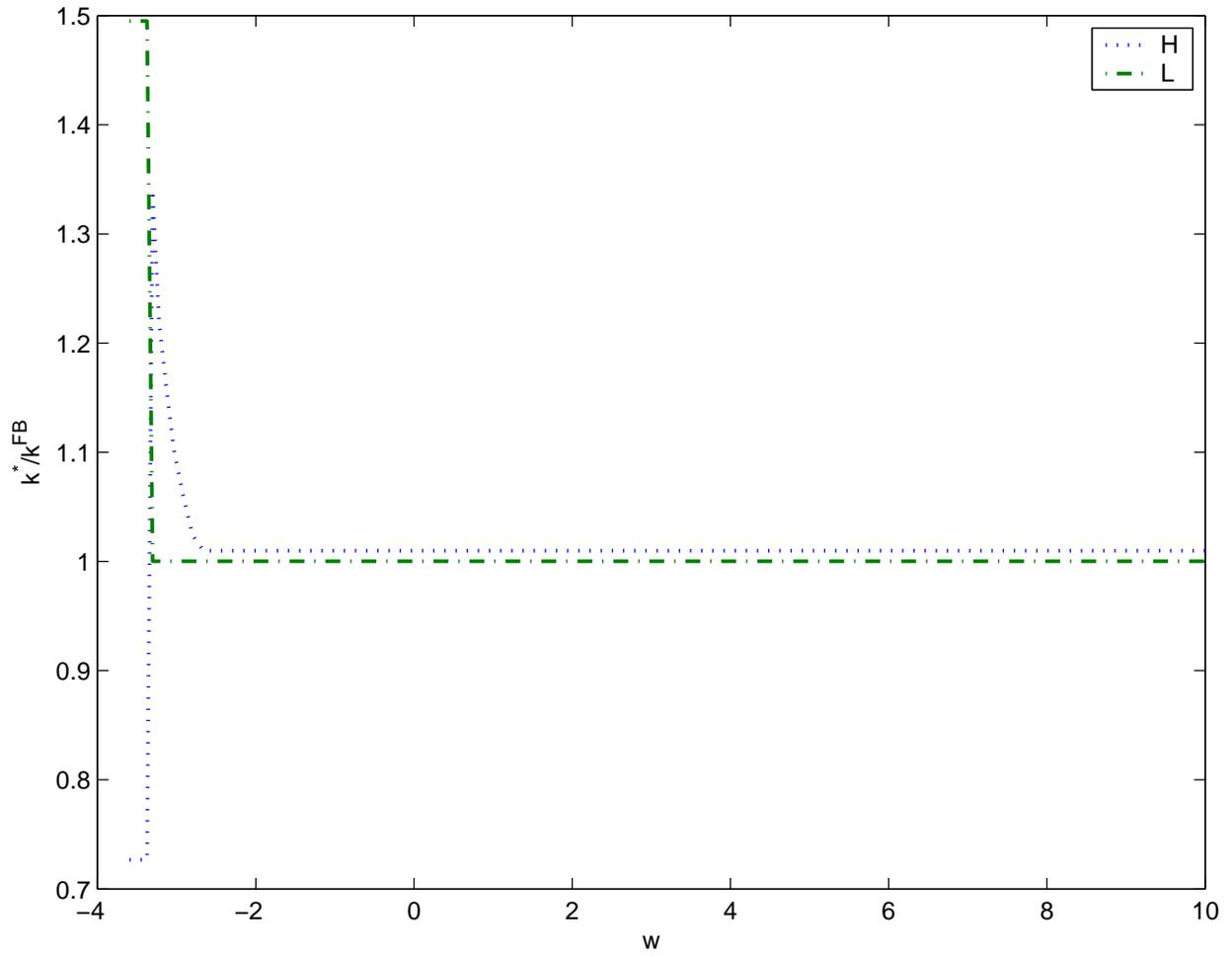


Figure 6: **Equilibrium financing policies in option contract game**

Equilibrium financing policies: debt, ρ_i , and equity, $s_i^* \Omega^{ii}$, as well as equilibrium dividend policy, d_i^* , are plotted as functions of the realized net worth, w . Panel A presents the case of high value of θ , while Panel B presents the case of low value of θ . The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

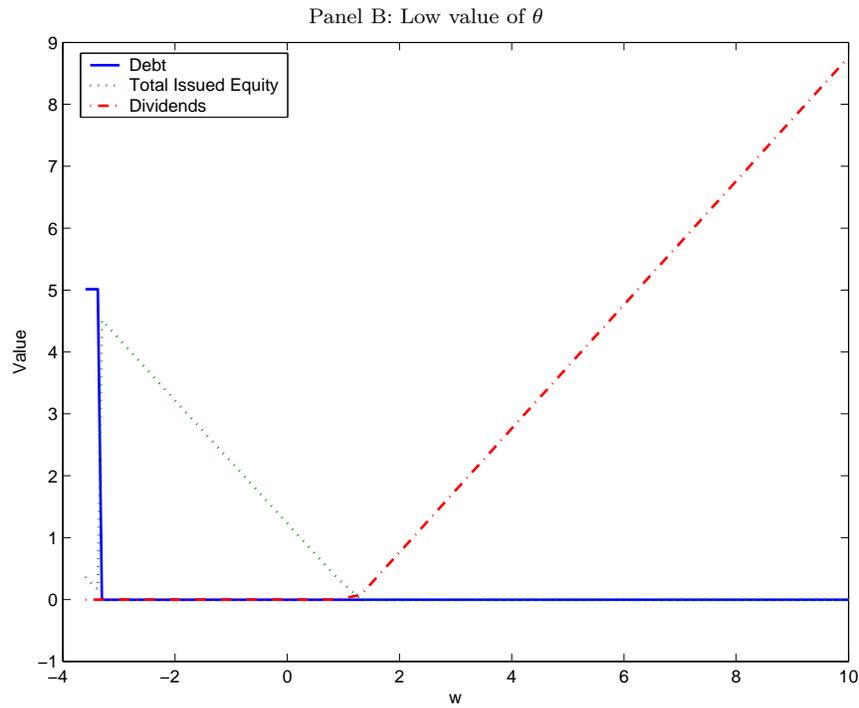
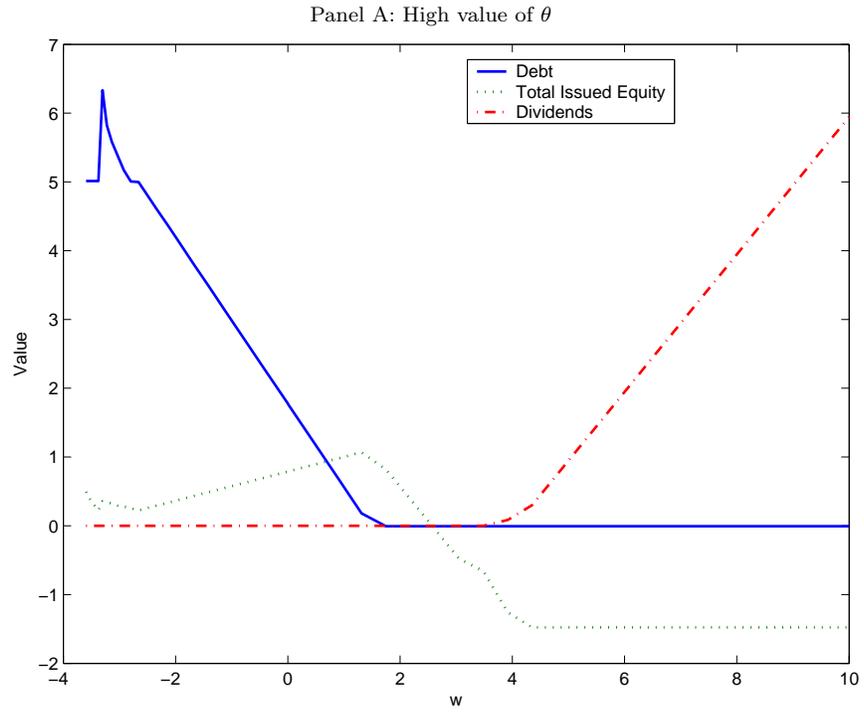


Table 1: **Leverage Regressions**

This table reports results of several regressions on the simulated data. The first three regressions have book leverage, $\frac{\rho_t}{k_t}$, as the dependent variable. The last regression has $\frac{\rho_t}{k_t} - \frac{\rho_{t-1}}{k_{t-1}}$ as the dependent variable. The financing gap is defined as $\frac{d_t + k_t - w_t}{k(t)}$ and operating profits are defined as $\theta_t \varepsilon_t k_t^{\alpha-1}$. The simulated panel of firms contains 3,000 firms over 31 time periods, where the initial period has been dropped for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

Financing Gap	Lagged Operating Profits	Book-to-Market	$E[\frac{\rho}{k}] - \frac{\rho_{t-1}}{k_{t-1}}$
0.2405 (0.0912)	-	-	-
-	-0.0114 (0.0044)	-	-
-	-0.0075 (0.0067)	0.0103 (0.0090)	-
-	-	-	0.1813
-	-	-	(0.0194)

Table 2: **Announcement Effect Regressions**

This table reports results of several regressions on the simulated data with the abnormal return on the announcement day, $AR_t = \frac{d_{it} + (1-s_{it})\Omega_t^{i,t} - v(w_t)}{v(w_t)}$, as the dependent variable. The investment rate is defined as $\frac{i_t}{k_t}$. Notation a^\pm means conditioning on the positive(negative) values of a . The simulated panel of firms contains 3,000 firms over 31 time periods, where the initial period has been dropped for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

Investment Rate	$\frac{\rho_t^+}{\rho_t^+ + s_t^+ \Omega_t}$	$\frac{\rho_t}{k_t}$	$\frac{s_t \Omega_t}{k_t}$
0.0163 (0.0012)	-	-	-
-	0.0752 (0.0251)	-	-
0.0155 (0.0017)	0.0103 (0.0094)	-	-
-	-	0.1307 (0.0595)	-
0.0162 (0.0052)	-	0.0222 (0.0512)	-0.0042 (0.0277)