

# Bid Ask Spreads and Market Microstructure: Are narrow spreads always feasible? **Very Preliminary Draft**

Michael Schwarz  
UC Berkeley and NBER

December 30, 2005

## **Abstract**

This paper describes a simple example of an institution that can maintain narrow bid ask spreads even in a market for a security where an uninformed specialist frequently trades with investors who have private information about the security's value.

Key words: bid ask spreads, market depth, information arrival, liquidity, market microstructure, price impact

## **1. Introduction**

Information asymmetries among investors influence bid ask spreads.<sup>1</sup> Glosten and Milgrom (1985) explicitly solves for the bid ask spread necessary to compensate the specialist for losses incurred in the trades with informed investors. However, "[t]he Glosten Milgrom model is relatively intractable if one wants to solve for a full equilibrium in which informed traders optimize their times of trade" (Back Baruch (2002) page 1).

The main insight of this paper is two fold, this paper considers a minimax strategy of the specialist who executes a sequence of trades, as opposed to standard

---

<sup>1</sup>For example Glosten (1987), Glosten Harris (1988), George, Kaul and Nimalendran (1991), Affleck-Graves, Hegde and Miller (1994) and Brockman and Chung (1999) consider decomposition of bid ask spread.

equilibrium analysis on the level of individual trades. Combining these two ideas I derive a robust upper bound on the contribution of adverse selection to spreads without having to solve an intractable problem of finding the impact of adverse selection on an individual trade.

**Example 1.** *Consider a market for contingent claims that pay one dollar if a particular event obtains and zero otherwise. Many identical claims are owned by investors at time zero. Investors can trade at any time until the state of the world is realized and claims are cashed at time  $T$ . Agents update their beliefs about the state of the world based on public and private signals that arrive over time. No assumptions are made about the process of information arrival and about what and when agents learn about each other's signals. Consider a specialist who is contractually obligated to keep the spread between the bid and the ask price no wider than  $s$  at all times. The prices quoted by the specialist are good for trades of standard size normalized to one share. After each trade a specialist can revise the price. Proposition 2 shows that the upper bound on the cost of hiring a specialist who agrees to guarantee the bid ask spread of size  $s$  is  $\frac{1}{8s}$ . The specialist is exposed to an arbitrarily large number of trades with agents who possess private information, yet the bound holds no matter how severe the information asymmetry is and no matter how risk averse the specialist is.*

This is not a probabilistic bound, the net trading loss of the specialist never exceeds the bound of Proposition 2. To understand the logic behind this result consider a strategy of the specialist that calls for revising the price by  $s$  after each trade. A pair of trades in opposite directions does not change the payoff of the specialist. As a result, the loss of the specialist only depends on the difference between the number of buy and sell orders that he executes. The loss is bounded because the price must remain in the interval from zero to one and the price quoted by the specialist increases by  $s$  for every buy order.

The viability of narrow bid ask spreads does not hinge on the assumption that price process is bounded. In Section 2.2 a contingent claim that can take any liquidation value at time  $T$  is considered. Let  $V$  denote the variance of price change between the beginning and the end of the interval for which the specialist is committed to maintain narrow spreads. Proposition 6 establishes that a payment of  $\frac{V}{2s}$  is sufficient for compensating a risk neutral specialist for providing bid ask spreads no wider than  $s$  during a time interval from zero to  $T$ .

Consequently, regardless of the amount of private information in the market it is feasible to hire a specialist who will provide narrow bid ask spreads for small

investors.

The specialists in NYSE have information privileges, in contrast this paper considers the role of an uninformed specialist who is hired to maintain narrow spreads. It is unusual but not unheard of for specialists without information privileges to have a contractual obligation to provide liquidity. The Paris Bourse is a limit order market that introduced specialists to improve liquidity in a subset of stocks. The specialists on the Bourse do not have privileged information about the order flow<sup>2</sup>. Consistent with the theory developed herein, Venkataraman, Waisburd and Mann (2004) document that introduction of non-monopolist specialist improved liquidity and increased the price levels at the Paris Bourse.

In Section 3 the Liquid Exchange Mechanism (LEM) is introduced and analyzed. LEM calls for auctioning off the privilege of being the monopolist specialist for a one year term. The monopolist specialist is obligated to quote bid and ask prices that are good for some standard trade size. Prospective specialists submit sealed bids indicating the bid ask spread that they are willing to commit to for a year. The lowest bidder becomes the specialist. Proposition 8 offers an estimate of the upper bound on the bid ask spreads under LEM. Example 9 suggests that LEM may reduce spreads relative to other trading institutions.

The viability of narrow bid ask spreads is important for a number of reasons. First, narrow spreads attract small uninformed investors and make it easier for arbitrageurs to provide liquidity, this in turn could reduce potential for excess volatility of the price process, see Schwarz (2005). Second, transaction costs and in particular the magnitude of the bid ask spreads may have a significant impact on long term price levels. The connection between asset prices and transaction costs is explored in Amihud and Mendelson (1986), Constantinides (1986) and more recently Luttmer (1996) and Vayanos and Vila (1999), Amihud (2002) and Brennan and Subrahmanyam (1996).

## 2. Cost of Maintaining Narrow Spreads

Consider a market for contingent claims during a time interval from zero to  $T$ . The uncertainty about the value of claims is fully resolved at time  $T$  when claims are cashed. Many identical contingent claim are held by investors at time zero.

---

<sup>2</sup>The specialists on the Paris Bourse are compensated by receiving investment banking business of the company that they make markets in. The fee for assuming the obligation to make markets is built into investment banking fees.

Investors update their beliefs about the value of claims based on public and private signals that arrive over time. Investors may choose to trade due to liquidity needs, private information and a variety of other factors. No assumptions are made about the process of information arrival and about what and when agents learn about each other's signals. No structure on the distribution of liquidity shocks is imposed, and rationality on the part of investors need not be assumed. Investors can trade with each other or with specialists. At any time during the interval from zero to  $T$  the specialist posts bid and ask prices. The prices posted by the specialist are good for transactions of up to some standard size. Without loss of generality the depth quoted by the specialist is normalized to unity. Trades smaller than the unit size are fractional trades. After each transaction the specialist can revise the price. What is the upper bound on the cost of hiring a specialist committed to the bid ask spread no wider than  $s$  at all times? Perhaps surprisingly, the upper bound does not depend on how many market participants are better informed than the specialist and how many trades the specialist will have to execute.

Before proceeding to results let us introduce some notations. An inventory  $h$  corresponding to a particular history of trades is the change in the specialist's holding during that history. (If the specialist executed ten sell orders and twelve buy orders of unit size the resulting inventory  $h = -2$  the same as for a specialist who executed two buys of unit size). Net (undiscounted) loss is the sum of net revenues received by the specialist during trading and the cash value of inventory of  $h$  contingent claims at time  $T$ .

A strategy that sets the price based solely on the (imbalance in) specialist's inventory is clearly feasible for an uninformed specialist. Such strategy can be represented by a function  $p(h)$  where  $p$  is the average of the bid and ask price, the bid and ask prices are  $p^{bid}(h) = p(h) - \frac{s}{2}$  and  $p^{ask}(h) = p(h) + \frac{s}{2}$  where  $s$  is the spread.

Consider a strategy  $B^*$  that maintains a bid ask spread of  $s$  at all times and calls for price  $p(h) = \pi_0 - hs$ , in the case of a contingent claim with value bounded on the interval from zero to one define  $\pi_0 = \frac{1}{2}$  (Section 2.1). In the case of a contingent claim with unbounded value  $\pi_0$  is defined as the expected value of the contingent claim computed based on the information set of the specialist at time time zero (Section 2.2). The following sections identify the upper bound on the net loss of a specialist who follows strategy  $B^*$ .

## 2.1. Spreads for Securities with Bounded Value

Consider a market for contingent claims that mature at time  $T$ . The value of claims at time  $T$  is bounded, the terminal value of the claim is normalized to be within the interval  $[0, 1]$ . A bet, or a security in an information market is an example of such a claim.

**Proposition 2.** *Consider an uninformed specialist following strategy  $B^*$  during a time interval  $[0, T]$ . The net loss of the specialist never exceed  $\frac{1-(2\alpha s-s)^2}{8s}$ , where  $\alpha = \frac{1-s}{2s} - INT(\frac{1-s}{2s})$  and ( $INT$  is a function that rounds a number down to the nearest integer).*

The bound established in this proposition holds regardless of the behavior of other traders and the number of trades that the specialist executes.

**Corollary 3.** *The net loss of a specialist who maintains bid ask spread of  $s$  at all times by following strategy  $B^*$  never exceeds  $\frac{1}{8s}$ .*

Let us summarize the intuition behind this result. An uninformed specialist can condition his price on the history of trades that he executed. The specialist does not have privileged access to information about the order flow. The specialist may not observe trades in which he does not participate. For the sake of the argument suppose all trades are of the same size, normalized to one. In this case a specialist following the strategy  $B^*$  would increase (decrease) his price by  $s$  each buy (sell) order of the unit size.<sup>3</sup> Round trip is defined as a pair of consecutive trades in opposite directions that leaves the specialist's inventory unchanged. A round trip does not change the payoff of the specialist. Using this observation one can show that the maximum losses of the specialist are realized in the state of the world where the security is worthless and the specialist receives a sequence of sell orders of unit size that continues until the sell price quoted by the specialist is smaller or equal to zero. Since trading stops when the price reaches zero and the specialist lowers the price by  $s$  after each trade the total (net) number of buy

---

<sup>3</sup>Strategy  $B^*$  may call for a specialist to quote a negative price even if it is know with certainty that the value of the security is not negative. The results does not depend on the ability of the specialist to quote prices outside of the interval of security values. For instance, consider a strategy  $B'$  according to which  $p(h) = \frac{1}{2} - hs$  if  $\frac{1}{2} - hs - \frac{s}{2} \geq 0$  and  $\frac{1}{2} - hs + \frac{s}{2} \leq 1$ . If  $\frac{1}{2} - hs - \frac{s}{2} < 0$  or  $\frac{1}{2} - hs + \frac{s}{2} > 1$  the specialist sets  $p^{bid} = 0$ ,  $p^{aks} = s$  or  $p^{bid} = 1 - s$ ,  $p^{aks} = 1$  respectively. With strategy  $B'$  the price always remains within the interval zero one. It is easy to see that the worst case payoff of strategy  $B^*$  is the same as that from strategy  $B'$ .

orders is no more than  $\frac{1}{2s}$ , the prices paid by the specialist form an arithmetic progression  $\frac{1-s}{2}, \frac{1-s}{2} - s, \frac{1-s}{2} - 2s, \frac{1-s}{2} - 3s \dots$  thus the average price paid for (worthless) shares converges to  $\frac{1}{4}$  for small  $s$ . For small  $s$  the inventory of the specialist at time  $T$  converges to  $\frac{1}{2s}$  with average loss per trade of  $\frac{1}{4}$ . This results in the net loss of  $\frac{1}{8s}$ . The above argument relies on the assumption that all trades are of the same size, relaxing this assumption considerably complicates the proof but the result continues to hold.

The proof of Proposition 2 is relegated to the appendix, let us sketch the main steps of the proof. We will say that a history is unfavorable (with respect to strategy  $B^*$ ) if any other history that results in the same inventory leads to higher or the same payoff for a specialist who follows strategy  $B^*$ . We will first show that there exists an unfavorable history containing only trades in one direction (Lemma 10), then we will show that there is at most one fractional trade in such a history (Lemma 11). It is easy to see that the true value of the security (zero) falls between the bid and ask prices quoted by the specialist after a history that maximizes the losses of the specialist. Combining this and the results of the lemmas implies that for a specialist who follows strategy  $B^*$  the losses are maximized in the state of the world where the value of the security is either zero or one. In the state of the world where the security is worthless the history consisting of  $INT(\frac{1-s}{2s}) + 1$  consecutive sell orders of unit size maximizes the losses of the specialist. This will allow us to compute the upper bound on the losses of the specialist.

Proposition 2 establishes the upper bound on the losses of the specialist who follows strategy  $B^*$ . Is it a tight upper bound? Is there another strategy that maintains bid ask spread no wider than  $s$  and guarantees that the worst case loss of the specialist is lower than that from strategy  $B^*$ ? If the answer to this question is no we could say that  $B^*$  is a minimax strategy and hence the bound of proposition Proposition 2 is tight.

**Proposition 4.** *Strategy  $B^*$  is a minimax strategy of the specialist.*

The proof is relegated to the Appendix.

**Remark 5.** *The upper bound on the specialist's loss obtained in Proposition 2 continues to hold in a world with discounting.*

The proof is relegated to the Appendix.

## 2.2. Contingent claims with unbounded values

Bounds derived in Section 2.1 rely on the assumption that the value of contingent claims is bounded. The same logic is used here to derive bounds on the expected loss of an uninformed specialist who trades a contingent claim with unbounded value. Let  $\pi$  denote the value of the contingent claim realized at time  $T$ , refer to  $\pi$  as the terminal value. At time zero a specialist views the terminal value as a random variable drawn from some distribution  $\rho(\cdot)$ , denote the expectation and variance of change in value of the claim computed at time zero by  $\pi_0 = E[\pi]$  and  $V = E[(\pi - \pi_0)^2]$ .

Consider a specialist who is committed to maintaining a bid ask spread of  $s$  by following strategy  $B^*$  where  $p(h) = \pi_0 - hs$  and  $p^{bid}(h) = p(h) - \frac{s}{2}$  and  $p^{ask}(h) = p(h) + \frac{s}{2}$ . As before information arrives over time and the specialist is exposed to arbitrarily large number of trades with traders who may have superior information. The following proposition provides an upper bound on the expected net loss of the specialist who maintains bid ask spreads of  $s$  by following strategy  $B^*$ .

**Proposition 6.** *The expected net loss of a specialist who maintains bid ask spreads of  $s$  at all times by following strategy  $B^*$  is bounded above by  $\frac{V}{2s}$ .*

*Proof.* First note that Lemma 11 continues to hold, consequently, it is sufficient to consider the histories consisting of unit size transactions in the same direction where the specialist continues to receive buy (sell) orders until his ask (bid) price exceeds (less than)  $\pi$ .

First consider the case of  $\pi > \pi_0$ , in this case the specialist continues to receive buy orders as long as  $p^{ask} = \pi_0 + \frac{s}{2} + hs < \pi$  thus  $h = INT(\frac{2(\pi - \pi_0) - s}{2s}) = \frac{2(\pi - \pi_0) - s - 2s\alpha}{2s}$  where  $\alpha = \frac{2(\pi - \pi_0) - s}{2s} - INT(\frac{2(\pi - \pi_0) - s}{2s}) \in [0, 1]$

The average price received by the specialist is  $\frac{1}{2}(\pi_0 + \frac{s}{2} + \pi_0 + \frac{s}{2} + \frac{2(\pi - \pi_0) - s - 2s\alpha}{2})$

The average loss per trade is  $\pi - \frac{1}{2}(2\pi_0 + s + \frac{2(\pi - \pi_0) - s - 2s\alpha}{2}) = \pi - \pi_0 - \frac{2s + 2(\pi - \pi_0) - s - 2s\alpha}{4} =$

$$\frac{1}{2}(\pi - \pi_0) - \frac{s + 2(\pi - \pi_0) - 2s\alpha}{4} = \frac{1}{2}(\pi - \pi_0) - \frac{s - 2s\alpha}{4}$$

Multiplying the average loss per trade by the number of trades we obtain the total loss

$$\begin{aligned} & (\frac{1}{2}(\pi - \pi_0) - \frac{s - 2s\alpha}{4})(\frac{2(\pi - \pi_0) - s - 2s\alpha + 2s}{2s}) = \frac{1}{8s}(2(\pi - \pi_0) - (s - 2s\alpha))(2(\pi - \pi_0) + \\ & (s - 2s\alpha)) = \\ & \frac{1}{8s}(4(\pi - \pi_0)^2 - (s - 2s\alpha)^2) \leq \frac{(\pi - \pi_0)^2}{2s} \end{aligned}$$

Since the strategy of the specialist is symmetric it is easy to see that for  $\pi < \pi_0$  the loss of the specialist does not exceed  $\frac{(\pi-\pi_0)^2}{2s}$ . Consequently the upper bound on the expected loss of a specialist following strategy  $B^*$  is  $\int_{-\infty}^{\infty} \frac{(\pi-\pi_0)^2}{2s} \rho(\pi) d\pi = \frac{V}{2s} \square$

**Example 7.** *Example.* Consider a stock valued at \$10 a share. The standard deviation is  $\sqrt{V} < \$10$ . The cost of hiring a specialist willing to offer a bid ask spread of \$0.05 for lots of up to 100 shares is less than  $\frac{100}{0.05} * 100 = \$200,000$ .

### 3. Main Result: Liquid Exchange Mechanism (LEM)

The previous sections demonstrated that it is feasible to hire an uninformed non-monopolist specialist who commits to maintaining narrow bid ask spreads. Now consider a monopolist specialist.<sup>4</sup> Let us describe a set of rules that in principle could be used by a stock exchange to guarantee narrow bid ask spreads.

Liquid Exchange Mechanism (LEM). Each stock is traded by a monopolist specialist. Once a year the privilege of being the specialist is auctioned off. Before the auction the exchange announces the smallest market depth  $K$  that the monopolist specialist is obligated to offer, the depth could be  $K = 100$  shares for small companies and larger for big corporations. Prospective specialists submit sealed bids indicating the bid ask spread that they are willing to commit to for a period of one year. The lowest bidder becomes the specialist.

Here we continue to use the model from Section 2.2. Now suppose that the specialist is a monopolist and all trades go via a specialist<sup>5</sup>. Suppose that the specialist is committed to a bid ask spread  $s$  for lots of size  $K$ , we will refer to a trade for  $K$  shares as a standard trade. For a given spread  $s$  the specialist anticipates trading volume of  $M$  shares over the course of a year. There is no discounting. All trades are of standard size so that the number of trades is  $N = \frac{M}{K}$  (the assumption that all trades are of standard size is without loss of generality according to the argument analogous to that in Lemma 11 and the assumption that there is not discounting could be easily relaxed in light of Remark 5). What

---

<sup>4</sup>Glosten (1989) shows that the monopolist specialist may be better in providing liquidity than competing specialist. The intuition is that when liquidity dries up due to large concentration of informed investors in the market a monopolist is willing to provide liquidity and sustained losses that could be recouped by future monopoly profit, in contrast in the competitive market specialists have a much weaker incentive to suffer losses in order to restore liquidity. In this paper the monopolist specialist does not receive monopoly profits because he is contractually committed to a particular value of the bid ask spread.

<sup>5</sup>A modifications of LEM where large trades bypass the specialist is probably more realistic.



is the size of bid ask spread necessary to assure positive expected profit for the specialist?

**Proposition 8.** *If  $s \geq 2\sqrt{\frac{V}{N}}$  there exists a strategy available to an uninformed specialist that yields a positive expected payoff for the specialist. In particular, a strategy that calls for increase (decrease) in price by  $\Delta = \sqrt{\frac{V}{N}} = \frac{s}{2}$  after each buy (sell) order yields a non-negative expected payoff.*

Technically speaking the above bounds are derived for a risk neutral specialist, in a world without discounting. However, the bid ask spread necessary to compensate a risk averse specialist is of the same order of magnitude.<sup>6</sup>

**Proof.** Consider a strategy of a specialist where he increases the price by the amount  $\Delta$  per share after each buy order of standard size and reduces the price by  $\Delta$  after each sell order of standard size. The bid ask spread is constant,  $s$  at all times. As before  $\pi$  is the price per share that the specialist will receive when he liquidates his position after a year and  $\pi_0$  is the specialist's expectation at time zero regarding the value of  $\pi$  that is  $\pi_0 = E[\pi]$ .

Using the intermediate result from the proof of Proposition 6 the expected loss of the specialist does not exceed  $E[K\frac{(\pi-\pi_0)^2}{2\Delta} - (s-\Delta)\frac{M-K\frac{\pi-\pi_0}{\Delta}}{2}]$ .

Note that  $E[(\pi - \pi_0)^2] = V$  and  $E[(\pi - \pi_0)] \leq \sqrt{V}$ , thus the upper bound on the specialist's loss is

$$\text{Specialist's loss} \leq \frac{K}{2} \left( \frac{V}{\Delta} - (s - \Delta) \left( N - \frac{\sqrt{V}}{\Delta} \right) \right) \quad (1)$$

Minimizing the above expression with respect to  $\Delta$  yields the F.O.C.

$$\begin{aligned} -\frac{V}{\Delta^2} + \left( N - \frac{\sqrt{V}}{\Delta} \right) - (s - \Delta) \left( \frac{\sqrt{V}}{\Delta^2} \right) &= 0 \\ -V + (N\Delta^2 - \Delta\sqrt{V}) - (s - \Delta)\sqrt{V} &= 0 \text{ thus } -V + N\Delta^2 - s\sqrt{V} = 0 \text{ thus} \\ \Delta = \sqrt{\frac{V+s\sqrt{V}}{N}} \text{ for } s \ll \sqrt{V} \text{ we have } \Delta \approx \sqrt{\frac{V}{N}}. \end{aligned}$$

Substituting  $\Delta = \sqrt{\frac{V}{N}}$  into

---

<sup>6</sup>The uninformed specialist loses money if there is a sufficiently large price change (in either direction). This risk can be partially hedged. Even if the specialist is unable to hedge the risk averse specialist may be willing to participate in LEM at relatively low spreads. Roughly speaking, the expected profit of the specialist who offers spreads of  $s = 2\sqrt{\frac{V}{N}}$  is positive if the annual price change does not exceed one standard deviation. Quadrupling the bid ask spread bound of Proposition 8 would result in positive profit as long as the price does not move by more than two standard deviations.

Equation 1 yields the lower bound on the specialist payoff, for  $s \geq 2\Delta = 2\sqrt{\frac{V}{N}}$  the lower bound on expected payoff of the specialist is positive.  $\square$

The obvious advantage of LEM is that it eliminates a possibility that liquidity dries up due to widening spreads.<sup>7</sup> What magnitude of spreads can one expect from LEM? Obviously that depends on many factors including the amount of information asymmetry. Proposition 8 implies that there is a mechanical strategy available to an uninformed specialist that yields positive expected payoff for a sufficiently large bid ask spread. If the only cost of the specialist is the cost due to losses from trades with informed investors the bid ask spread under LEM would be bounded above by  $2\sqrt{\frac{V}{N}}$ , where  $\sqrt{V}$  is a standard deviation of the annual change in the stock price and  $N$  is the annual volume of trade divided by the depth quoted by the specialist. Generally speaking the number of trades is a decreasing function of spreads, consequently, the volume of small trades at bid ask spread  $s_0$  can be interpreted as a lower bound on the volume of trade in a world with lower bid ask spread. The following example uses this principle to estimate spreads under LEM. The parameter values are typical for a NYSE stock in 1990 (fifth decile in terms of trading volume), the data is from Easley, Kiefer, O'Hara, and Paperman (1996).

**Example 9.** *Consider a stock with annual trading volume of 13.8 million shares that trades for \$24 a share. Assume that the standard deviation in the annual price change is less than the price (this assumption holds for almost any stock) that is  $\sqrt{V} < \$24$ . Suppose the specialist posts bid and ask for trades of up to 100 shares, this is a fairly standard trade size for an individual investor. Consequently, we assume that trading volume can be divided into at  $N = 138000$  trades of "standard" size. Applying Proposition 8 we obtain  $2\sqrt{\frac{V}{N}} \approx \frac{48}{\sqrt[2]{138000}} = \$0.12$ . The average spread observed in the data is more than two times greater.*

A few disclaimers are in order. First, plugging in actual variance and volume into Proposition 8 does not give an estimate of the spreads under LEM. It merely provides an upper bound on spreads in a frictionless world. If trading volume is sensitive to spreads the upper bound may be much higher than the spreads necessary for a specialist to make a positive profit (because if LEM were used in

---

<sup>7</sup>Of course, the specialist will have to have a sufficiently deep pocket in order to insure that he can fulfil his commitment. It is straightforward to estimate the upper bound on the wealth of the specialist so that he can survive a five standard deviation event.

practice, prospective specialists will bid lower than monopoly spreads in anticipation that low spreads will attract sufficient volume for them to make a profit). On the other hand, our example completely neglects the contribution of variable costs to bid ask spreads. In other words, Example 9 is not an empirical exercise, it is merely an illustration of the theorem. The bound of Proposition 8 is far from being a precise estimate of LEM's consequences. However, LEM is a theoretically interesting benchmark that could conceivably be useful in practice.

## 4. Concluding Remarks

Although contingent claims considered in this paper are reminiscent to securities traded in information markets such as the Iowa Electronic Market, the results are applicable to any security. The properties of the minimax strategy of an uninformed specialist might shed some light on behavior of specialists. In particular, specialists tend to aggressively adjust the price in order to balance their position. This is often explained by high inventory costs, see for instance seminal work of Stoll (1978). An uninformed specialist who follows a minimax strategy adjusts the price as if he has a high inventory cost. Consequently, an uninformed specialist with low inventory cost may be difficult to distinguish from an informed specialist with high inventory cost.

Proposition 8 and Example 9 offer a framework that allows to estimate when using Liquid Exchange Mechanism (LEM) can lead to lower spreads than other trading institutions. One can make an economic efficiency argument that bid ask spreads are the most relevant measure of liquidity because for a typical investor spreads are the relevant measure of transaction costs. Hence, narrow spreads are important for optimal risk sharing. Consequently, the magnitude of the bid ask spreads may have a significant impact on long term price levels as have been shown in Amihud and Mendelson (1986), Constantinides (1986) and more recently Luttmer (1996) and Vayanos and Vila (1999). For recent empirical evidence see Amihud (2002) and Brennan and Subrahmanyam (1996).

Other measures of liquidity are also important. In particular, the excess volatility of the stock price on the short run may hurt agents with a short investment horizon. Schwarz (2005) shows that narrow bid ask spreads promote price stability (by encouraging arbitrage). From the perspective of large institutional investors the price impact is probably the most relevant measure of liquidity. An upper bound on price impact of large trade under LEM follows as an immediate corollary from Proposition 8. However, this upper bound is of limited interest because

it might by far exceed the actual price impact of large trades. It is not obvious if using LEM would lead to lower or higher price impact of large trades relative to other trading institutions. This is an important question for future research.

## 5. References

Affleck-Graves, J., SP Hegde, RE Miller, 1994. Trading Mechanisms and the Components of the Bid-Ask Spread. *Journal of Finance*.

Amihud, Yakov, 2002, Illiquidity and stock returns: cross-section and time-series effects, *Journal of Financial Markets* 5, 31-56.

Amihud, Yakov and Haim Mendelson, Asset Pricing and the Bid-Ask Spread, *Journal of Financial Economics*, Vol. 17, (1986), pp. 223-249.

Brennan, M. J., and A. Subrahmanyam, 1996, Market microstructure and asset pricing: On the compensation for illiquidity in stock returns." *Journal of Financial Economics* 41 441-464.

Brockman, P; Chung, D.Y, 1999. Bid-ask spread components in an order-driven environment. *Journal of Financial Research*

Constantinides, George M., 1986, Capital market equilibrium with transaction costs, *Journal of Political Economy* 94, 842-862

Easley, D., N. Kiefer, M. O'Hara, and J. Paperman, 1996, Liquidity, Information, and Infrequently Traded Stocks, *Journal of Finance*, 51, 1405-1436.

Hagerty, Kathleen Equilibrium Bid-Ask Spreads in Markets with Multiple Assets *Review of Economic Studies*, 1991, vol. 58, issue 2, pages 237-57

George, TJ, G. Kaul, and M. Nimalendran 1991. Estimation of the bid-ask spread and its components: a new approach. *Review of Financial Studies*

Glosten, Lawrence R, 1989, Insider Trading, Liquidity, and the Role of the Monopolist Specialist, *Journal of Business*, University of Chicago Press, vol. 62(2), pages 211-35.

Glosten, Lawrence R, 1987. Components of the bid-ask spread and the statistical properties of transaction prices. *Journal of Finance*.

Glosten, L., and P. Milgrom, 1985, Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, *Journal of Financial Economics*, 13, 71-100.

Glosten, Lawrence R., and Lawrence E. Harris, 1988, Estimating the components of the bid/ask spread, *Journal of Financial Economics* 21, 123-142.

Back, Kerry and Shmeuel Baruch, 2004, Information in Securities Markets: Kyle Meets Glosten and Milgrom *Econometrica*, Vol. 72, No. 2 (March, 2004),

433–465.

Kyle, A. S. 1985, Continuous Auctions and Insider Trading, *Econometrica* 53, 1315–1335.

Luttmer, Erzo, 1996, Asset Pricing in Economies with Frictions, *Econometrica*, Vol. 64, No. 6, pp. 1439-1467.

Schwarz, Michael, 2005, Bid Ask Spreads Volatility and Arbitrage, Unpublished Manuscript

Stoll, Hans R. The Supply of Dealer Services in Securities Markets, *Journal of Finance*, Vol. 33, No. 4 (Sep., 1978) , pp. 1133-1151

Vayanos, Dimitri and Vila, Jean Luc, 1999, Equilibrium Interest Rate and Liquidity Premium With Transaction Costs, *Economic Theory*, 13, 509-539.

Venkataraman, Kumar, Andrew Waisburd and Steven Mann, 2004, The Value of the Non-Monopolist Specialist, NBER Market Microstructure Conference, Los Angeles 2004.

## 6. Appendix

Preliminaries for the proof of Proposition 2.

**Lemma 10.** *For any inventory level there exists an unfavorable history where all trades are in the same direction.*

Let us first show that there exists an unfavorable history where a sell order never immediately follows a buy order. Suppose the least favorable history contains a pair of trades where buy order immediately precedes a sell order. Suppose that at inventory  $h$  a buy order for  $\nu_B$  shares is placed followed by a sell order of size  $\nu_S$ . After these orders the seller is left with an inventory of  $h - \nu_B + \nu_S$ . Let us show that replacing this pair of trades with a single trade of size  $\nu_S - \nu_B$  will reduce the payoff for the specialist (and hence the least favorable history does not contain buy orders immediately followed by a sell order). Let us consider the case of  $\nu_B \geq \nu_S$ . We would like to show that replacing this pair of trades with a single buy order of  $\nu_B - \nu_S$  will reduce the payoff of a specialist. The payment that a specialist with inventory  $h$  receives for executing a buy order for  $(\nu_B - \nu_S)$  shares is

$$(\nu_B - \nu_S)\left(\frac{1}{2} - hs + \frac{s}{2}\right) \tag{2}$$

the payment for executing two consecutive trades is

$$\nu_B\left(\frac{1}{2} - hs + \frac{s}{2}\right) - \nu_S\left(\frac{1}{2} - hs + \nu_B s - \frac{s}{2}\right) \tag{3}$$

rearranging the terms taking into account that  $\nu_B \leq 1$  shows that the Expression 2 is smaller or equal to Expression 3. Consequently, replacing a buy order followed by a sell order with a single buy order reduces the payoff of the specialist in the case where  $\nu_B \geq \nu_S$ .

Now let us consider the case where  $\nu_B < \nu_S$ . The payment made by the specialist with inventory  $h$  for a sell order of size  $\nu_S - \nu_B$  is

$$(\nu_S - \nu_B)\left(\frac{1}{2} - hs - \frac{s}{2}\right) \quad (4)$$

the net payment made by the specialist after a buy order for  $\nu_B$  followed by a sell of  $\nu_S$  is

$$-\nu_B\left(\frac{1}{2} - hs + \frac{s}{2}\right) + \nu_S\left(\frac{1}{2} - hs + \nu_B s - \frac{s}{2}\right) \quad (5)$$

It is easy to see that Expression 4 is greater than Expression 5.

Due to the symmetry of the problem an identical argument procedure allows us to construct an unfavorable history where sell orders are never immediately followed by buy orders and buy orders are never immediately followed by sell orders. Consequently for any inventory there exists an unfavorable history where all trades are in the same direction.

**Lemma 11.** *For any inventory level there exists an unfavorable history consisting of trades in the same direction where all trades with a possible exception of the last trade are of unit size.*

From the previous lemma we know that for any inventory level there exists an unfavorable sequence of trades consisting only of trades in the same direction. Let us first show that such history contains at most one trade smaller than unit size. Indeed suppose at some inventory level  $h$  the specialist receives a sell order of size  $\nu_S$  followed by another sell order of size  $\nu'_S$  where both orders are of less than unit size. In this case the payment that the specialist makes for acquiring  $\nu_S + \nu'_S$  shares is  $\nu_S\left(\frac{1}{2} - hs - \frac{s}{2}\right) + \nu'_S\left(\frac{1}{2} - hs - \nu_S s - \frac{s}{2}\right)$ . We will show that at least one of these orders is of unit size.

It is obvious that in an unfavorable sequence  $(\nu_S + \nu'_S) > 1$  (otherwise the trades can be merged into a single trade of size  $\nu_S + \nu'_S$ ).

Consider the case of  $\nu_S + \nu'_S > 1$ . For an unfavorable history  $\nu$  that maximizes the following program must equal to  $\nu_S$

$\max_{\nu} \nu(\frac{1}{2} - hs - \frac{s}{2}) + (\nu_S + \nu'_S - \nu)(\frac{1}{2} - hs - \frac{s}{2} - \nu s)$  subject to  $\nu$  and  $(\nu_S + \nu'_S - \nu) \in [0, 1]$ . Differentiating this expression twice with respect to  $\nu$  we find that the second derivative is always positive hence there is no interior solution. Thus either  $\nu_S$  or  $\nu'_S$  are equal to one.

Now let us show that for two consecutive trades where one of the trades has unit size reversing the order of trades leaves the payoff of the specialist unchanged.

We want to show that a sell order of size 1 followed by a sell order of size  $\nu_S + \nu'_S - 1$  yields the same payoff for the specialist as in the case where the order of trades is reversed. It remains to show that

$$(\frac{1}{2} - hs - \frac{s}{2}) + (\nu_S + \nu'_S - 1)(\frac{1}{2} - hs - s - \frac{s}{2}) = (\nu_S + \nu'_S - 1)(\frac{1}{2} - hs - \frac{s}{2}) + (\frac{1}{2} - hs - (\nu_S + \nu'_S - 1)s - \frac{s}{2})$$

Simplifying the above expression shows that the equality holds.

Now let us consider an unfavorable history consisting of trades in the same direction and show that it contains at most one trade smaller than unity. Suppose there is more than one such trade. We already established that these trades could not immediately follow each other. Then there must exist some trades of size  $\nu$  and  $\nu'$  both smaller than unit size such that all trades between these trades are of unit size. Let  $n$  denote the number of trades between these trades and suppose trade of size  $\nu$  comes earlier in the trading sequence than the trade of size  $\nu'$ . Note that by reversing  $n$  times the order of trades between the trade of size  $\nu$  and a unit size trade that follows one creates a sequence of trades that leads to the same payoff for the specialist as the original unfavorable history, however, the resulting unfavorable history contains two subsequent trades of less than unit size. Hence a contradiction. Finally note that provided there is only one trade of less than unit size in the sequence of trades the payoff of the specialist is the same regardless of the position of that trade in the history of trades.  $\square$

**Proof** of Proposition 2.

Let us find the upper bound on the losses of a specialist following strategy  $B^*$ . Due to the symmetry of the problem we can limit consideration to the state of the world where the value of the security is less than 0.5. From Lemma 11 follows that the specialist incurs the maximum loss in the state of the world where the security is worthless and the inventory of the specialist at time  $T$  is  $INT(\frac{1-s}{2s}) + 1$ .

The loss of a specialist who follows strategy  $B^*$  after a sequence of  $INT(\frac{1-s}{2s}) + 1$  consecutive sell orders equals to  $\frac{1-(2\alpha s-s)^2}{8s}$ , where  $\alpha = \frac{1-s}{2s} - INT(\frac{1-s}{2s})$

The specialist following strategy  $B^*$  executes trades at prices  $\frac{1}{2} - \frac{s}{2}, \frac{1}{2} - \frac{s}{2} - s, \frac{1}{2} - \frac{s}{2} - 2 \times s \dots \frac{1}{2} - \frac{s}{2} - s \times INT(\frac{1-s}{2s})$

Thus the maximum loss of the specialist is given by the sum

$$\begin{aligned} & \frac{\frac{1}{2} - \frac{s}{2} + \frac{1}{2} - \frac{s}{2} - s \times INT(\frac{1-s}{2s})}{2} (INT(\frac{1-s}{2s}) + 1) \\ = & \frac{1-s-\frac{1-s-2s\alpha}{2}}{2} (\frac{1-s-2s\alpha}{2s} + 1) = \frac{(1-s+2s\alpha)(1+s-2s\alpha)}{8s} = \frac{1-(s-2\alpha s)^2}{8s} \leq \frac{1}{8s}. \square \end{aligned}$$

**Proof** of Proposition 4

Let us identify some properties of a minimax strategy. Since we are interested in the worst possible loss (not the expected loss) we can limit consideration to pure strategies. Note that there must exist a minimax strategy such that the price depends only on the inventory of the specialist<sup>8</sup>. Such minimax strategy must yield a non-negative payoff from a round trip trade. This means that as a result of a sell order a price can never decline buy more than  $s$  (otherwise a sequence containing an arbitrarily large number of round-trips trades would expose the specialist to an arbitrarily large loss, from Proposition 2 we know that the loss of the specialist following minimax strategy is finite). Due to the symmetry of the problem we can assume without loss of generality that minimax strategy calls for  $p(0) \geq \frac{1}{2}$ , this corresponds to bid price of at least  $\frac{1}{2} - \frac{s}{2}$  for a specialist with zero inventory. Combining the above observations implies that in the worst case scenario a specialist following the minimax strategy would loose at least  $(\frac{1}{2} - \frac{s}{2}) + (\frac{1}{2} - \frac{s}{2} - s) + (\frac{1}{2} - \frac{s}{2} - 2s) + \dots + (\frac{1}{2} - \frac{s}{2} - INT(\frac{1-s}{2s})s)$  computing this sum we obtain the same expression as in Proposition 2, thus we showed that there does not exist a strategy that yields a lower loss than  $\frac{1-(2\alpha s-s)^2}{8s}$  for all histories. Hence  $B^*$  is a minimax strategy.

**Proof** of Remark 5.

Suppose the discount rate is  $\beta \in [0, 1]$  let's denote the time of  $i$ -th trade by  $t_i$  and the payment received (or made) by the specialist at that time is denoted by  $m_i$  denote by  $h(T)$  the inventory of the specialist at a terminal history let  $\theta$  equal to zero in the state of the world where the value of the claim is zero and one otherwise. Then the present value of the specialist's payoff computed at time  $T$  is given by  $\sum_i m_i e^{\beta(T-t_i)} + \theta h(T)$

Consider a strategy  $B^\beta$  of the specialist where  $p^{bid}(h) = (\frac{1}{2} - \frac{s}{2} - hs)e^{-\beta(T-t)}$  and  $p^{bid}(h) = (\frac{1}{2} + \frac{s}{2} - hs)e^{-\beta(T-t)}$  if  $\frac{1}{2} - hs - \frac{s}{2} \geq 0$  and  $(\frac{1}{2} - hs + \frac{s}{2})e^{-\beta(T-t)} \leq 1$ . If  $\frac{1}{2} - hs - \frac{s}{2} < 0$  or  $(\frac{1}{2} - hs + \frac{s}{2})e^{-\beta(T-t)} > 1$  the specialist sets  $p^{bid} = 0$ ,  $p^{aks} = s$  or  $p^{bid} = (1-s)e^{-\beta(T-t)}$ ,  $p^{aks} = e^{-\beta(T-t)}$  respectively. Given this strategy for any history of trades the present value of the specialist's payoff computed at time  $T$

---

<sup>8</sup>Note that minimax strategy is not unique.



is exactly the same as the undiscounted payoff from strategy  $B^*$  consequently, the discounted payoff of the specialist who follows strategy  $B^\beta$  is bounded by the expression derived in Proposition 2.