

Government intervention and information aggregation by prices¹

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Abstract

Governments intervene in firms' lives in a variety of ways. However, efficient interventions depend on economic conditions, about which a government often has only limited information. Consequently, it is efficient for governments to at least partially "follow the market" and make interventions based on observed stock market prices. We analyze the implications of governments' reliance on market information for market prices and government decisions. A key point is that price informativeness is endogenous to government policy. In some cases, it is optimal for the government to commit to limited reliance on market prices in order to avoid harming traders' incentives to trade and the concomitant aggregation of information into market prices. For similar reasons, it is optimal for the government to limit transparency in some dimensions.

1 Introduction

Governments play a key role in the lives of firms and financial institutions. They provide numerous subsidies, set regulations, and then decide how to apply these regulations. These decisions have large effects on firm cash flows. An important motivation for such government intervention is the externalities that firms and financial institutions impose on the rest of the economy. Such externalities played a prominent role in the recent crisis and the wave of intervention and regulation that followed it. For example, government programs to stimulate bank lending (e.g., TARP and TALF) were motivated by concerns that a decrease in lending would hurt firms and deepen the recession. The bailouts of large financial institutions such as AIG and Bear Stearns were driven by fears that the failure of these institutions would bring down the financial system due to the connections across different institutions. The intervention in the auto industry was motivated by fears that bankruptcies of large automakers such as General Motors would have devastating implications for their employees, suppliers, and customers.

At the same time, a long-standing concern is that governments have only limited information about true economic conditions. This lack of information prevents governments from intervening in the most efficient way. In the context described above, lack of information stops governments from correctly assessing the externalities generated by a given firm or financial institution, and so leads to inefficient intervention decisions. Hence, many researchers and policy makers call for government reliance on market prices of financial securities when making intervention decisions. Indeed, a basic tenet in financial economics is that market prices contain a great deal of information. As such, market prices can provide valuable guidance for government decisions. Policy proposals calling for governments to make use of market prices are numerous, particularly in the realm of bank supervision.¹ Such policy proposals are increasingly prominent in the wake of the recent crisis and the

¹See, e.g., Evanoff and Wall (2004) and Herring (2004).

perceived failure of financial regulation prior to it.² These proposals come on top of existing research establishing that government actions are already affected by market prices to a significant extent.³

In this paper, we analyze government use of information in security prices in intervention decisions. We consider both positive and normative dimensions. What happens to prices and decisions when the government “follows the market” and makes decisions in response to price movements? What policies allow the government to most efficiently make use of the information in prices? (While we focus here on government intervention, many of our results apply more generally to other contexts in which individuals incorporate information from market prices into their decisions. We explicitly develop an application where the manager learns from the price when making an effort decision.)

Our analysis is centered around a point that is often overlooked: security prices are endogenous, and their information content is affected by government policy. Indeed, in the recent crisis, government actions were not only perceived to be reactions to market prices, but expectations about government actions were often a major driver of changes in asset prices. For example, market activity in the weeks leading up to the eventual announcement of government support for Fannie Mae and Freddie Mac, for Citigroup, and for General Motors was largely driven by speculation about the government’s behavior.⁴ Hence, government actions affect prices, and consequently also affect the ability of the government to learn from prices. This affects the desirability of market-based intervention. To study these effects, we consider the process by which information gets aggregated into the price. Our paper analyzes the effect of market-based government policy on the trading incentives of speculators and hence on the ability of the financial market to aggregate speculators’ dispersed information.

A particularly important application of our analysis relates to the effects of government

²For example, Hart and Zingales (2011).

³See Feldman and Schmidt (2003), Krainer and Lopez (2004), Piazzesi (2005), and Furlong and Williams (2006).

⁴Baker, Bloom, and Davis (2012) and Kelly, Lustig, and Van Nieuwerburgh (2012) provide evidence on the large effect of expectations about government policy on security prices.

transparency. For example, a current policy question is how much information governments should release about the outcomes of bank stress tests. Our analysis implies that governments should release only the dimensions of information in which market speculators have strictly worse information than the government.

The canonical model of information aggregation was developed by Grossman (1976), Hellwig (1980), and Admati (1985). Speculators possess heterogenous information about the payoffs of an asset and trade on it in a market that is subject to noise/liquidity shocks. The equilibrium price of the asset then reflects the aggregated information of speculators with noise. In the existing literature, the cash flows produced by the asset are exogenous. However, if the government (or some other decision maker) uses the information in prices when intervening in the firm's operations, the cash flows are instead endogenous. We extend the canonical model of information aggregation to account for this. The informativeness of the price in this model is determined by the trading incentives of speculators, i.e., the aggressiveness with which they trade on their information, and this is affected by the government's policy and the extent to which it relies on the market price.

We identify two opposing effects of the government's reliance on stock prices on price informativeness. We call the first effect the *Information Importance Effect*. When the government puts more weight on the price and less weight on its own information in the intervention decision, it makes speculators' information, conditional on the price, less important in predicting the government's action and hence the value of the security. This reduces speculators' incentives to trade on their information, and hence it reduces price informativeness. The second effect is the *Residual Risk Effect*. When the government puts more weight on the price and less weight on its own information in the intervention decision, it reduces the uncertainty that speculators are exposed to when they trade. Being risk averse, speculators then trade more aggressively on their information, and this leads to an increase in price informativeness.

Overall, which effect dominates depends on the parameters of the model. The residual

risk effect is weakened when the risk for speculators is driven mostly by exogenous risk (i.e., risk from an unforecastable and exogenous cash flow shock) rather than by endogenous risk (i.e., risk due to the unknown government action), and so in this case price informativeness is decreasing in the extent to which the government relies on market prices. We show that this effect is strong enough to make it optimal for the government to commit to underweight market prices in its intervention decision. Overall, our model implies a “Limit of Informativeness” result: when prices are informative, endogenous risk is smaller in comparison to exogenous risk (because the government’s action can be forecasted pretty well based on market prices), and this is exactly when the government is better off committing to reduce its learning from the price. Similarly, when the government’s own information is relatively precise, the government would gain from committing to underweight it.

Recall that such commitments are desirable due to the endogeneity of market prices. While papers by Rochet (2004), Hart and Zingales (2011) and others suggest that the government should commit to follow a pre-determined rule based on publicly observable prices, our paper highlights another consideration that needs to be taken into account when thinking about the costs and benefits of these proposals: the effect that such proposals may have on the informativeness of the price. In particular, in the circumstances mentioned above, commitment to a market-based rule reduces the informativeness of the price that the rule makes use of and hence also the value of the government’s objective function. In addition, in such circumstances, our model also implies that the government’s own information has more value than its direct effect on the efficiency of the government’s decision. When the government has more precise information, it relies less on the market price, and this makes the market price more informative. Hence there are complementarities between the government’s own information and the market’s information, and so it is not advisable for the government to rely completely on market information.

Another important aspect of government policy is transparency. Should the government reveal its own information publicly? This issue has been hotly debated recently in relation

to the stress tests that regulators are conducting for financial institutions. There are various views on whether the results of such stress tests should be publicly disclosed (see Goldstein and Sapra (2012)). Our model sheds light on this debate from a new angle: is disclosure of information to the market desirable when the government is trying to learn from the market? In our framework, the answer to this question depends on the type of information being disclosed. If the government discloses information about a variable about which speculators have at least some additional information, it reduces the value of its objective function because the disclosed information reduces the incentives of speculators to trade on their information (due to the information importance effect) and reduces the ability of the government to learn. If instead the government discloses information about a variable about which speculators know less than the government (i.e., their information is a coarsening of the government's), it increases the value of its objective function because the disclosed information reduces the risk that speculators face (due to the residual risk effect) incentivizing them to trade and increasing the ability of the government to learn about those variables on which it can benefit from market information. This distinction is new to the literature on transparency.⁵ In practice, it seems likely that individual bank conditions are an area in which speculators have substantial information not possessed by a government. At the opposite extreme, a government knows its own policy objectives, so there is no room for speculators to have useful information in this dimension. Consequently, transparency about policy objectives is useful for the government, but transparency about stress-test findings on individual bank conditions is harmful.

Our paper adds to a growing literature on the informational feedback from asset prices to real decisions; see, for example, Fishman and Hagerty (1992), Leland (1992), Khanna, Slezak, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997), Subrahmanyam and

⁵There are recent papers showing that transparency might be welfare reducing, e.g., Morris and Shin (2002) and Angeletos and Pavan (2007). In these papers, the source for the result is the existence of coordination motives across economic agents. In contrast, such coordination motives do not exist in our model, where, conditional on the price (which is observed to all), speculators do not care about what other speculators do. Importantly, the above-mentioned papers do not explore the implications of transparency about different types of information, as we do here.

Titman (1999), Fulghieri and Lukin (2001), Foucault and Gehrig (2008), and Bond and Eraslan (2010). In particular, it complements papers such as Bernanke and Woodford (1997), Goldstein and Guembel (2008), Bond, Goldstein and Prescott (2010), Dow, Goldstein and Guembel (2010), and Lehar, Seppi and Strobl (2010), which analyze distinct mechanisms via which the use of price information in real decisions might reduce the informational content of the price. For a recent review of this literature, see Bond, Edmans, and Goldstein (2012).

Relative to these papers, our focus is on the efficiency of aggregation of dispersed information by market prices. This topic, which has long been central in economics and finance (e.g., Hellwig (1980)), has not been analyzed in any of the related papers.

The remainder of the paper is organized as follows. Section 2 describes the basic model. The analysis and solution of the model are contained in Section 3. Section 4 analyzes how a government should optimally use market information. Section 5 looks at the importance of the government's own information. Section 6 analyzes the costs and benefits of transparency. Section 7 discusses and compares alternative notions of price informativeness. Section 8 analyzes an extension of the basic model and deals with some robustness issues. Section 9 concludes. Most proofs are relegated to an appendix.

2 The model

We focus on one firm (a financial institution, for example), whose stock is traded in the financial market. In $t = 0$, speculators obtain signals about a variable that affects the government's incentive to intervene, and trade on them. In $t = 1$, the government, which learns information from the price of the stock and from an additional private signal, makes a decision about its intervention. In $t = 2$, cash flows are realized and speculators get paid.

2.1 Cash flows and government intervention

The cash flow of the firm is given by:

$$X = \delta + T. \tag{1}$$

Here, δ is exogenous and is distributed normally with mean $\bar{\delta}$ and variance $var[\delta]$. For the purposes of our analysis below, the mean $\bar{\delta}$ can vary across firms, states of the world, and time; all that matters is that it is publicly known as of $t = 0$. We denote the precision of prior information about δ by $\tau_\delta \equiv var[\delta]^{-1}$. The component T of (1) denotes the cash flow effect of the government's intervention in the firm. The basic premise behind government intervention is that the firm's cash flows affect not only the shareholders (or other security holders) of the firm, but generate externalities to other agents in the economy. Hence, the government may want to intervene in the firm's operations and change its cash flows. The government can do this in a number of ways, including: direct transfers of cash to or from a firm, i.e., subsidies and taxes; liquidity support in the form of loans at below-market rates; changes in regulation and/or the application of existing regulation. Note that T can be positive or negative. In Section 8, we extend our analysis to consider cases in which the intervention T is correlated with other determinants of firm cash flow.

In Section 2.2 below, we elaborate on possible motives for government intervention with a few examples. For now, consider a general formulation that nests all the examples discussed below. A fully informed government would choose T to maximize the objective

$$v(T - \theta) - \mu T.$$

Here, v is a concave function. That is, the marginal social benefit from capital injected to the firm is decreasing in the amount of capital injected. Moreover, the social benefit differs across states of the world, as represented by the state variable θ . In particular, the marginal

benefit of intervention is higher when θ is higher. Hence, θ captures the extent to which the firm exerts positive externalities to the rest of the economy, and so when it is higher, the desirability of injecting capital to the firm increases. Finally, the term $-\mu T$ represents the cost of the intervention to the government, where the parameter μ captures the marginal cost of intervention. A leading special case is $\mu = 1$, corresponding to an opportunity cost of each dollar injected being one dollar. To guarantee a finite solution to the government's optimization problem, we assume $v'(-\infty) > \mu > v'(\infty)$.

The central problem faced by the government in our model is that it has incomplete information about the state variable θ . Consequently, the government chooses T to maximize the objective

$$E[v(T - \theta) | I_G] - \mu T, \quad (2)$$

where I_G denotes the information available to the government when making the decision. We assume that θ is distributed normally with mean $\bar{\theta}$ and variance $\text{var}[\theta]$. We denote the precision of prior information about θ by $\tau_\theta \equiv \text{var}[\theta]^{-1}$.

In more detail, the government's information I_G comes from two sources. First, the government observes the price P of firm equity, which in equilibrium is an informative signal of θ , as we will show. Second, the government privately observes a noisy signal s_G of θ , where

$$s_G = \theta + \varepsilon_G, \quad (3)$$

and the noise term ε_G is drawn from a normal distribution with mean 0 and variance $\text{var}[\varepsilon_G]$. We denote the precision of the government's signal by $\tau_G \equiv \text{var}[\varepsilon_G]^{-1}$.

The focus of our analysis is on how the government learns from the price of the firm's stock when it makes the decision about T . In our framework, market participants have noisy signals about the state variable θ that determines the externalities exerted by the firm on the rest of the economy. Speculators trade on this information, since the government itself also observes a noisy signal of θ , and so the value of θ is correlated with cash flows. But given

that speculators trade on their private information, the equilibrium price conveys some of the information possessed by speculators. The government can then use this information, which results in its intervention decisions being affected by the stock price. As a result, there is a feedback loop between the market price and the government's action. We analyze the equilibrium outcomes of this feedback loop. Before we describe the financial market and information environment in more detail in Section 2.3, we now present a few possible micro foundations for the government's objective function (2).

2.2 Micro foundations for the policy rule

The first two motivations presented in this subsection involve government intervention in financial institutions. The third one involves government intervention in general firms. Finally, in the fourth one, we shift attention from government intervention to managerial action to demonstrate that our analysis applies more generally than to government interventions. Still, we want to maintain the focus of the paper on government intervention, and so we do not mention managerial actions outside Section 2.2.4.

2.2.1 Intervention in bank lending

Banks are thought to perform an important role in the economy by providing funds to firms. The literature has established that relationships are important in banking, and so many firms rely on particular banks for funding their operations. Clearly, bank lending to firms generates externalities beyond the profit captured by the bank itself. When firms borrow, they can fund projects, employ workers, sell goods, etc. All these actions benefit other agents in the economy. Thus, to the extent that the government cares about these externalities, it may want to intervene in bank lending, e.g., by transferring resources into and out of banks. For example, in the recent credit freeze, it was argued that banks' lending is important to stimulate the economy and banks are not lending due to a bad coordination failure (see Bebchuk and Goldstein (2011)). Indeed, the recent intervention during the credit

freeze with programs like TARP, TALF, etc. was primarily motivated by the desire of the government to change the equilibrium in the credit market.

Formally, let $s(x - \theta)$ be the marginal social surplus created by the x th dollar lent by the bank. Here, s is a decreasing function, reflecting diminishing marginal social returns to lending. As discussed above, θ is a state variable that affects the social surplus from loans extended by the bank. Write L for the amount the bank would lend in the absence of intervention (i.e., $T = 0$), which we assume to be publicly known. Write v for the anti-derivative of s . Consequently, the social surplus associated with an intervention T is:⁶

$$\int_0^{L+T} s(x - \theta) dx = v(L + T - \theta) - v(-\theta).$$

Since L is known and $v(-\theta)$ is outside the government's control, this is consistent with the objective function (2).

We assume that each loan made with the funds T generates a gross return of r for the bank, so that the bank's cash flow is of the form $rT + \delta$, where the mean $\bar{\delta}$ of δ absorbs all publicly-known bank-specific factors. For expositional ease, we focus below on the case of $r = 1$.⁷

Note that the optimal value of T may be negative. This is the case when the positive externality generated by the cash injected is below the government's marginal cost of funds μ . Moreover, the optimal value of T is certainly negative if a bank is too big absent intervention: formally, if externalities are negative, so that $v'(-\theta) < 0$. For example, there is increasing concern about the negative consequences of credit booms (see Lorenzoni (2008) for a formal model).

⁶Note that negative values of $L + T$ correspond to the bank absorbing funds at the expense of projects. So negative values of $L + T$ generate social surplus $-\int_{L+T}^0 s(x - \theta) dx$, which equals $\int_0^{L+T} s(x - \theta) dx$.

⁷See also Section 8 below.

2.2.2 Actions to reduce systemic risk

Another important rationale for intervention by the government in the banking sector is that failing banks might hurt each other and the financial system as a whole, causing systemic risk. The government may want to reduce such systemic risk by transferring resources across banks. However, the extent to which the government wants to do so depends on the externalities exerted by the banks involved. Consider for example the behavior of the government around the failures of leading financial institutions such as Bear Stearns, Lehman Brothers, AIG, etc. The decision whether to bail them out or not depended to a large extent on assessing whether their failure is going to have a strong negative effect on the rest of the system.

The general formulation described above can be used to capture the motives for intervention to reduce systemic risk. Injecting capital to the bank reduces the negative externality imposed by the bank on the system as a whole. The marginal effect of capital in decreasing systemic risk is higher when the bank is more connected, which is formally captured in the general formulation by a larger θ .⁸

2.2.3 Intervention in firms with externalities

Intervention of the government in firms goes beyond the financial sector. In the recent crisis, the government provided help to the big three automakers. The government also provided liquidity in the market for commercial paper attempting to ease the financial constraints of firms in the real sector. In normal times, the government often provides subsidies to firms that improve infrastructure, increase employment, etc. The rationale for such intervention is again rooted in the externalities that firms exert on other agents in the economy. For example, in the case of automakers, there was major concern about the possibility that they go bankrupt, as this would imply loss of jobs by many employees and the collapse of many

⁸Moreover, and as in the previous subsection, it is possible that a bank has negative externalities absent intervention, and the government would like to make the bank smaller. This is one component of the much discussed Volcker rule. See Boyd and Heitz (2012) for one calculation of the costs of large banks.

firms connected to the big three automakers in the production and distribution chain. This scenario is again captured by our general formulation, where the variable θ summarizes the strength of positive externalities generated by a firm.

2.2.4 Managerial action

Finally, while our paper focuses on government intervention in the banking sector (or in the real sector), our model is more general and can fit other applications, such as a firm's manager choosing an action with regard to the firm he is running. We will elaborate on this possibility in this section, but then go back to the main application of government intervention which is compatible to what we discussed thus far.

Suppose that a firm manager has to choose the level of effort T he exerts in developing a new line of business. The value of the firm is then increasing in managerial effort and equal to $\delta + T$, where δ is a random variable as in the main model, capturing the cash flow the firm derives from activities unrelated to managerial effort (i.e., from existing lines of business). Suppose that the manager gets a fraction $\beta \in (0, 1]$ of the value of the firm. The manager incurs a private cost of effort which is given by the convex function $v^M(T - \theta)$. Hence, the manager chooses T to maximize

$$E [\beta (\delta + T) - v^M(T - \theta) | \text{manager information}],$$

which is consistent with the objective (2) above with $v(T - \theta) = -v^M(T - \theta)$ and $\mu = -\beta$.⁹ The variable θ here determines the marginal cost of effort the manager has to incur. As θ increases, the marginal cost decreases, and so the manager would like to increase his effort. We can think of θ as something that is not perfectly known to the manager, and so he can gain some information about it from the stock price. For example, suppose that a firm considers starting operations in a foreign country, and the manager has to decide on the

⁹Of course, here we need to assume that $v'(T - \theta)$ is always negative, which fits within the structure of the general model.

capacity of operations in this country. When making his decision, he may not know the exact amount of time he will need to dedicate to building these operations because he has no prior experience in this. Market traders who have been following other firms engaging in such operations may have some insights about this, and these insights may be reflected in their trading and the resulting stock price.

2.3 Trading in the financial market

We now return to the description of the main model. There is a continuum $[0, 1]$ of speculators in the financial market with constant absolute risk aversion (CARA) utility, $u(c) = -e^{-\alpha c}$, where c denotes consumption and α is the absolute risk aversion coefficient. Each speculator i receives a noisy signal about the state θ :

$$s_i = \theta + \varepsilon_i, \tag{4}$$

where the noise term ε_i is independently and identically distributed across speculators. It is drawn from a normal distribution with mean 0 and variance $\text{var}[\varepsilon_i]$. We use $\tau_\varepsilon \equiv \text{var}[\varepsilon_i]^{-1}$ to denote the precision of speculators' signals.

It is natural to assume that speculators have pieces of information about the externalities that the firm generates, and that the government learns from the aggregation of these pieces of information in the price. These externalities are not easily observed or measured. Different speculators have different assessments of the extent to which a firm affects the rest of the economy, and the government can benefit from their combined knowledge. For example, at the time when Bear Stearns and Lehman Brothers were close to failure, it was not clear to anyone—including the government—how much damage such a failure would inflict on the economy.

Each speculator chooses a quantity x_i to trade to maximize his expected utility given his

private signal s_i and the price P that is set in the market for the firm's stock:

$$x_i(s_i, P) = \arg \max_{\tilde{x}} E \left[-e^{-\alpha \tilde{x}(\delta + T - P)} | s_i, P \right]. \quad (5)$$

Here, trading a quantity x_i , the speculator will have an overall wealth of $x_i(\delta + T - P)$, where $\delta + T$ is the cash flow from the security after intervention, and P is the price paid for it. The speculator's information consists of his private signal s_i and the market price P .

In addition to the informed trading by speculators, there is a noisy supply shock, $-Z$, which is distributed normally with mean 0 and variance $\text{var}[Z]$. We again use the notation $\tau_z \equiv \text{var}[Z]^{-1}$. In equilibrium, the market clears and so:

$$\int x_i(s_i, P) di = -Z. \quad (6)$$

3 Analysis

In equilibrium, individual speculators' demands maximize their utilities given s_i and P (according to (5)), the market clearing condition (6) holds, and the government's choice of T maximizes its objective in (2) given its signal s_G and the price P . As is standard in almost all the literature, we focus on linear equilibria in which the price P is a linear function of the average signal realization—which equals the realization of the state θ —and the supply shock $-Z$.¹⁰ The complication in our model relative to the existing literature arises because of the endogenous government intervention T which affects the value of the firm. However, as we now show, there is an equilibrium, where not only the price function is linear in the primitive random variables, but the intervention of the government is linear in the primitive random variables as well.

Let us conjecture that in equilibrium T is indeed a linear function of the primitive random

¹⁰In a linear equilibrium, an individual speculator's demand is linear in his own signal, as we show below. Consequently, total speculator demand is a linear function of the average signal realization, which equals θ .

variables. In the proof of Proposition 1 below, we show that this leads to a linear price function. Then, given that the government learns based on the price and its own signal and given that all the primitive random variables in our model are normally distributed, the conditional distribution of θ given the government's information I_G is also normal.¹¹ Consequently, we can apply the following useful result, which confirms our conjecture that T is a linear function.

Lemma 1 *If the conditional distribution of the state variable θ given government information I_G is normal, then there exists a function g such that the intervention T that maximizes the government's objective (2) is of the linear form*

$$T = E[\theta|I_G] + g(\mu, \text{var}[\theta|I_G]). \quad (7)$$

The proof of Lemma 1 is short, and we give it here. The intervention T that maximizes the government's objective (2) satisfies the first-order condition:

$$E[v'(T - \theta)|I_G] - \mu = 0. \quad (8)$$

The government knows T , and so, by hypothesis, the conditional distribution of $T - \theta$ given I_G is normal; consequently, it is fully characterized by its first two moments. Hence, the expectation $E[v'(T - \theta)|I_G]$ can be written as a function of the first two moments of the conditional distribution of $T - \theta$, i.e., there exists some function $G : \Re^2 \rightarrow \Re$ such that

$$E[v'(T - \theta)|I_G] = G(E[T - \theta|I_G], \text{var}[T - \theta|I_G]).$$

¹¹A recent paper by Breon-Drish (2012) relaxes the normality assumptions in the canonical model. The key step in his generalization is to place enough structure on distributions so that the demand of an informed speculator is still linear in the informed speculator's signal. However, his model does not feature a feedback effect from the price of the security to the cash flows it generates.

Hence the solution T to (8) solves

$$G(E[T - \theta|I_G], \text{var}[\theta|I_G]) = \mu.$$

Given conditional normality, $\text{var}[\theta|I_G]$ is a constant. Define the inverse function $g(y, x)$ by $G(g(y, x), x) = y$.¹² Consequently,

$$T - E[\theta|I_G] = g(\mu, \text{var}[\theta|I_G]),$$

completing the proof of Lemma 1.

Proposition 1 below uses Lemma 1 to establish the existence of a linear equilibrium, and to characterize the associated level of price informativeness. Before stating the formal result, we give an informal derivation of the price informativeness characterization.

In a linear equilibrium, the price can be written as

$$P = p_0 + p_Z(\rho\theta + Z), \tag{9}$$

for some parameters p_0 , p_Z and ρ . In particular, $\rho^2\tau_Z$ measures the informativeness of the price, since the informational content of the price is the same as the linear transformation $\frac{1}{\rho p_Z}(P - p_0) = \theta + \rho^{-1}Z$, which is an unbiased estimate of the state θ with precision $\rho^2\tau_Z$. Intuitively, the price of the security is affected by both changes in the state θ and changes in the noise variable Z ; price informativeness is greater when ρ , the ratio of the effect of the state on the price and the effect of noise on the price, is greater. Because we typically take τ_Z as fixed in our comparative statics and policy analysis, we often refer to ρ itself as price informativeness.

It is worth highlighting that our measure of informativeness relates to the state θ , and not the cash flow $T + \delta$. This is because the government is attempting to learn the state θ

¹²The concavity of v implies that G is strictly decreasing in first argument. Hence $g(y, x)$ is well-defined.

from the price, and so the informativeness about θ is the relevant object for the government's ability to take an appropriate action attempting to maximize its objective. We discuss this distinction in more detail in Section 7 below.

Given normality of the state θ and the supply shock $-Z$, the price P is itself normal. Consequently, given normality of the error term ε_G , the government's posterior of the state θ is normal. Moreover, the government's estimate of the state is linear in its own signal, $s_G = \theta + \varepsilon_G$, and in the price P . The government's estimate of the state is consequently

$$E[\theta|s_G, P] = K_\theta(\rho)\bar{\theta} + K_P(\rho)\frac{1}{\rho p_Z}(P - p_0) + w(\rho)s_G, \quad (10)$$

where $K_\theta(\rho)$, $K_P(\rho)$ and $w(\rho)$ are weights that sum to one. In particular, $w(\rho)$ is the weight the government puts on its own signal in estimating the state, which depends on the information available in the price. By the standard application of Bayes' rule to normal distributions,

$$\begin{aligned} K_P(\rho) &\equiv \frac{\rho^2\tau_z}{\tau_\theta + \rho^2\tau_z + \tau_G} \\ w(\rho) &\equiv \frac{\tau_G}{\tau_\theta + \rho^2\tau_z + \tau_G}. \end{aligned} \quad (11)$$

The weight that the government puts on its own signal is the precision of this signal (τ_G) divided by the sum of precisions of the government's signal, the prior information (τ_θ) and the signal from the price ($\rho^2\tau_z$). As one would expect, the government puts more weight on its own signal when it is precise (τ_G is high) and less when the price is informative (ρ is high). Given the policy rule (7), the intervention is

$$T(s_G, P) = w(\rho)(\theta + \varepsilon_G) + K_P(\rho)\frac{1}{\rho p_Z}(P - p_0) + K_\theta(\rho)\bar{\theta} + g(\mu, \text{var}[\theta|I_G]). \quad (12)$$

Similar to the government, each speculator assigns a normal posterior (conditional on his own signal s_i and price P) to the state θ . Then, from (12), each speculator also assigns a

normal posterior to the size of the intervention T . Consequently, the well known expression for a CARA individual's demand for a normally distributed stock applies,

$$x_i(s_i, P) = \frac{1}{\alpha} \frac{E[T|s_i, P] + \bar{\delta} - P}{\text{var}[T|s_i, P] + \text{var}[\delta]}. \quad (13)$$

Thus, the amount traded is the difference between the expected value of the security and the price, divided by the variance of the expected value multiplied by the risk aversion coefficient. Intuitively, speculators trade more when there is a large gap between the expected security value and the security price, but, due to risk aversion, this tendency is reduced by the variance in expected security value.

To characterize the equilibrium informativeness of the stock price, consider simultaneous small shocks of φ to the state θ and $-\varphi\rho$ to Z . By construction (see (9)), this shock leaves the price P unchanged. Moreover, the market clearing condition (6) must hold for all realizations of θ and Z . Consequently,

$$\varphi \frac{\partial}{\partial \theta} \int x_i(s_i, P) di = \varphi \rho.$$

Substituting in (12) and (13) yields equilibrium price informativeness:

$$\rho = \frac{1}{\alpha} \frac{\frac{\partial}{\partial s_i} E[T|s_i, P]}{\text{var}[T|s_i, P] + \text{var}[\delta]} = \frac{1}{\alpha} \frac{w(\rho) \frac{\partial}{\partial s_i} E[\theta|s_i, P]}{w(\rho)^2 (\text{var}[\theta|s_i, P] + \text{var}[\varepsilon_G]) + \text{var}[\delta]}. \quad (14)$$

The informativeness of the price is determined by how much speculators trade on their information about θ . This is determined by two factors: the relation between the information and the value of the asset, which appears in the numerator, and the variance in the value of the asset, which appears in the denominator.

Proposition 1 *A linear equilibrium exists. Equilibrium price informativeness ρ satisfies (14). For $\text{var}[\varepsilon_G]$ sufficiently small, there is a unique linear equilibrium, which is continuous as a function of $\text{var}[\varepsilon_G]$.*

(All proofs are in the appendix.)

The case $\text{var}[\varepsilon_G] = 0$ corresponds to the standard case analyzed by the prior literature. Here, the government observes the state variable θ directly, and so puts full weight on its own signal, i.e., $w(\rho) \equiv 1$. Consequently, it ignores the price in choosing its intervention, and so speculators treat the firm's cash flow as exogenously given. In this case, (14) reduces to

$$\rho = \frac{1}{\alpha} \frac{\frac{\partial}{\partial s_i} E[\theta|s_i, P]}{\text{var}[\theta|s_i, P] + \text{var}[\delta]}. \quad (15)$$

Since $\text{var}[\theta|s_i, P] = \frac{1}{\tau_\theta + \rho^2 \tau_z + \tau_\varepsilon}$ and $\frac{\partial}{\partial s_i} E[\theta|s_i, P] = \tau_\varepsilon \text{var}[\theta|s_i, P]$, it is easy to see that the right-hand side of (15) is decreasing in ρ , and the equality has a unique solution. Consequently, there is a unique (linear) equilibrium.

However, due to the effect of the informativeness of the price on the weight that the government puts on its information in the intervention decision, our model sometimes exhibits multiple equilibria. This is because, as we see in (14), the informativeness ρ affects the weight w , causing the overall effect of ρ on the right-hand side of (14) to be unclear, and sometimes generating multiple solutions. Economically, as price informativeness increases, the government reduces the weight it puts on its own signal in the intervention decision, reducing the residual risk that speculators are exposed to, which may induce them to trade more aggressively resulting in a more informative price. Our paper is not the first to show that the uniqueness of equilibrium in Grossman and Stiglitz (1980) and Hellwig (1980) is not robust to extensions of the model. For example, Ganguli and Yang (2008) show that introducing private information about the aggregate liquidity shock may lead to multiplicity of equilibria.

3.1 Empirical implications

We conclude this section with a few comparative-statics results that provide empirical implications of the basic model:

Corollary 1 *The equilibrium weight $K_P(\rho) = \frac{\rho^2 \tau_Z}{\tau_\theta + \tau_G + \rho^2 \tau_Z}$ that the government attaches to the price in decisions is decreasing in risk-aversion, α ; the variance of the supply shock, $\text{var}[Z]$; the noise in speculator signals, $\text{var}[\varepsilon_i]$; and the unforecastable component of cash flows, $\text{var}[\delta]$. However, the effect of the noise in government signals, $\text{var}[\varepsilon_G]$, has an ambiguous effect on both price informativeness and the weight the government attaches to the price.¹³*

Testing these results empirically requires empirical proxies. The weight that the government attaches to the price in its decision can be assessed by measuring the sensitivity of government intervention to price changes; see, for example, Chen, Goldstein, and Jiang (2007) for an implementation of this in the context of corporate investment. The important but hard-to-observe terms relating to the volume of noise trading, $\text{var}[Z]$, and the information of speculators, $\text{var}[\varepsilon_i]$, can be proxied using microstructure measures such as the probability of informed trading (PIN) or price non-synchronicity, which are often deployed for this purpose. Alternatively, one could proxy $\text{var}[Z]$ and $\text{var}[\varepsilon_i]$, along with risk-aversion α , by using characteristics of the base of investors who trade in the stock: who they are, how informed they are, how much they trade due to hedging and liquidity needs (i.e., noise), etc.

The results in Corollary 1 are mostly straightforward and have the sign one would expect (and for the reasons one would expect). This includes the results for the parameters α , $\text{var}[Z]$, $\text{var}[\varepsilon_i]$, $\text{var}[\delta]$. When traders trade less aggressively due to risk aversion, when there is more noise trading, when traders have less precise information, and when there is more unforecastable uncertainty regarding the value of the firm, then the price ends up being less informative and the government relies less on it. The one result that is more surprising is the comparative static with respect to $\text{var}[\varepsilon_G]$. As the noise in the government signal increases, the government puts less weight on its own signal. Under many circumstances, this decreases price informativeness. (We discuss this point in much greater detail in the

¹³In cases of multiple equilibria, these statements should all be understood as applying to the equilibrium set.

next section, when we consider exogenous perturbations of w .) When this effect is large enough, the weight the government puts on the price drops.

4 Government policy and price informativeness

In the preceding section, we analyzed equilibrium outcomes when the government uses information in a way that is optimal ex post, i.e., applies Bayes' rule. As a benchmark throughout this section, let ρ^* denote such an equilibrium. However, given that the use of market prices may affect their informativeness, the government may benefit if it is able to commit ex ante to put a different weight on the price than is ex-post optimal. In this section, we analyze the best policy rule for a government with such commitment ability.

4.1 Preliminaries

Formally, when the government acts ex-post optimally it follows the linear intervention rule (12) with $\rho = \rho^*$. Accordingly, we consider the class of linear policy rules defined by weights \tilde{w} , \tilde{K}_P and the constant \bar{T} ,

$$\tilde{T}(s_G, P; \tilde{w}, \tilde{K}_P, \bar{T}) \equiv \tilde{w}s_G + \tilde{K}_P \frac{1}{\rho p_Z} (P - p_0) + \bar{T}. \quad (16)$$

The government's ex post optimal rule is nested in the class (16), and is given by $\tilde{w} = w(\rho^*)$, $\tilde{K}_P = K_P(\rho^*)$, and $\bar{T} = K_\theta(\rho^*)\bar{\theta} + g(\mu, \text{var}[\theta|I_G])$. We will study whether the government can benefit from deviating from the ex-post optimal rule to any other rule characterized by weights \tilde{w} , \tilde{K}_P and the constant \bar{T} .

As a first step, we need to determine price informativeness under rules that are different from the ex-post optimal rule. A straightforward adaptation of the proof of Proposition 1 implies that, given a policy rule of the form (16), equilibrium price informativeness is given

by the unique solution to¹⁴

$$\rho = \frac{1}{\alpha \tilde{w}^2} \frac{\tilde{w} \frac{\partial}{\partial s_i} E[\theta | s_i, P]}{(\text{var}[\theta | s_i, P] + \text{var}[\varepsilon_G]) + \text{var}[\delta]}. \quad (17)$$

Note that uniqueness here follows from the fact that the government uses fixed weights rather than adjusting the weights in an ex-post optimal way based on informativeness ρ .

We denote the expected value of the government's objective function when adopting a rule $(\tilde{w}, \tilde{K}_P, \bar{T})$ by $V(\tilde{w}, \tilde{K}_P, \bar{T})$; formally, this is equal to

$$E \left[v \left(\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \rho^{-1} Z) + \bar{T} - \theta \right) - \mu \left(\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \rho^{-1} Z) + \bar{T} \right) \right], \quad (18)$$

where the expectation depends on price informativeness ρ , which in turn depends on the rule $(\tilde{w}, \tilde{K}_P, \bar{T})$ by (17).

Below, we analyze how price informativeness ρ changes as the government departs from its ex post optimal rule ρ^* . From (17) it is clear that informativeness ρ depends on the government's policy rule only via the component \tilde{w} , i.e., the weight the government puts on its own information s_G . Our results below characterize how choices of \tilde{w} that over- or underweight the government's own information relative to the ex post optimum $w(\rho^*)$ affect price informativeness. Note, however, that perturbations of \tilde{w} away from $w(\rho^*)$ are accompanied by perturbations in the weight attached to prices, \tilde{K}_P , in the *opposite* direction. As such, if the government overweights (respectively, underweights) its own signal s_G , it also underweights (respectively, overweights) the price. The formal result is:

Lemma 2 *Let $\tilde{\rho}$ be price informativeness given rule $(\tilde{w}, \tilde{K}_P, \bar{T})$, and let \tilde{K}_P be optimal for the government given \tilde{w} . Then $\tilde{K}_P - K_P(\tilde{\rho})$ has the opposite sign to $\tilde{w} - w(\tilde{\rho})$ provided that $|\tilde{w} - w(\tilde{\rho})|$ is sufficiently small.*

While we analyze the effect of the government's rule on price informativeness, our main

¹⁴In the following expression, $\text{var}[\theta | s_i, P] = (\tau_\varepsilon + \tau_\theta + \rho^2 \tau_Z)^{-1}$ and $\frac{\partial}{\partial s_i} E[\theta | s_i, P] = \tau_\varepsilon \text{var}[\theta | s_i, P]$.

interest is in the effect that the rule has on the value of the government's objective function. Our next result shows that higher informativeness ρ is indeed related to higher value of the government's objective function. Specifically, for small departures from the ex post optimal rule, the value of the objective function moves in the same direction as price informativeness.

Proposition 2 *If a small exogenous increase in \tilde{w} away from the ex-post optimal weight $w(\rho^*)$ increases (respectively, decreases) price informativeness, it also increases (respectively, decreases) the value of the government's objective function.*

In general, changing the weight \tilde{w} away from the ex-post optimal weight $w(\rho^*)$ has two effects on the government's objective function. First, there is a direct reduction in the value of the government's objective function due to the deviation from the ex-post optimal rule. Second, there is an indirect effect on the government's objective function via the effect on price informativeness ρ . Proposition 2 says that the second indirect effect is the dominant one. This proposition is a straightforward application of the envelope theorem. By definition, a small perturbation of \tilde{w} away from the ex-post optimal weight $w(\rho^*)$ has only a second-order direct effect on the government's objective function; but the perturbation has a first-order impact on informativeness ρ .

Note that Proposition 2 is only a local result. In particular, even if we conclude that the government is better off committing to somewhat reduce the weight it puts on market prices in its intervention decision, this certainly does not imply that the government should reduce the weight on prices all the way to zero (i.e., set $\tilde{K}_P = 0$). While in many cases this policy rule would improve price informativeness relative to ρ^* , this increase in price informativeness is irrelevant since the government does not use the information in the price and so the value of the government's objective function is actually decreased. Hence, some reliance on market prices is always optimal.

4.2 The effect of the government's rule on price informativeness

With the above results in hand, we are now ready to characterize the effect of the government's rule on price informativeness. As a preliminary, observe that the effect of an exogenous change in \tilde{w} is determined by its effect on the right-hand side of (17):

Lemma 3 *An exogenous change in the weight the government puts on its own signal from $w(\rho^*)$ to $\tilde{w} \neq w(\rho^*)$ strictly increases (respectively, decreases) price informativeness relative to ρ^* if it strictly increases (respectively, decreases) the right-hand side of (17).*

Economically, Lemma 3 illustrates the following two offsetting effects of the government increasing the weight \tilde{w} it puts on its own information s_G .

Information importance effect: On the one hand, increasing \tilde{w} increases the importance of a speculator's signal s_i in forecasting the cash flow. This effect increases price informativeness, since, when their signals are more relevant, speculators trade more aggressively on their private signals. This is represented by the numerator in the right-hand side of (17). Note that the importance of the private signal s_i depends on the extent to which the government relies on its own signal s_G . To see this, consider the case where the government puts no weight on its own information ($\tilde{w} = 0$). Then, its intervention is a function of prices only, and each speculator's signal contains no information about cash flows beyond that contained in the price. As the government increases the weight on its own information to positive levels ($\tilde{w} > 0$), each speculator's signal contains information about cash flows because it contains information about the component θ of the government's signal $s_G = \theta + \varepsilon_G$.

Residual risk effect: On the other hand, the more weight the government puts on its own information s_G , the more residual risk speculators are exposed to. Because speculators are risk averse, this effect decreases the aggressiveness of speculative trading, and so decreases price informativeness. This is represented by the denominator in the right-hand side of (17). Note that this risk is composed of both the risk in the uncertainty about the state variable θ and the risk in the noisy component ε_G of the government's signal.

To identify the dominant effect out of the changes in information importance and residual risk, it is useful to decompose residual risk into its two sources: *exogenous* risk, namely $\text{var}[\delta]$, which is determined solely by the unforecastable and exogenous cash flow shock δ ; and *endogenous* risk,

$$N(w, \rho) \equiv w^2 (\text{var}[\theta|s_i, P] + \text{var}[\varepsilon_G]),$$

which depends both on the government's rule w and on equilibrium price informativeness. Using this decomposition, the information importance effect dominates if and only if the majority of residual risk is exogenous risk:

Proposition 3 *Underweighting the price—that is, a small exogenous increase in \tilde{w} away from the ex-post optimal weight $w(\rho^*)$ —increases price informativeness if and only if exogenous risk exceeds endogenous risk, i.e., $\text{var}[\delta] > N(w(\rho^*), \rho^*)$.*

To see the intuition, note that without any exogenous risk, the residual risk effect would always dominate the information importance effect, implying that it is always optimal for the government to reduce the weight it puts on its own signal below the ex-post optimal level. This is simply due to the fact that the weight \tilde{w} affects the endogenous risk in the denominator via \tilde{w}^2 while it has a linear effect on the importance of information in the numerator. However, as exogenous risk increases, the effect of \tilde{w} on the total residual risk that speculators are exposed to weakens, so that when exogenous risk is greater than endogenous risk, the information importance effect dominates the residual risk effect, implying that it is optimal for the government to increase the weight it puts on its own signal above the ex-post optimal level.

From Proposition 3, we can derive implications on the effect of various parameters of the model on the desirability of increasing or decreasing the weight the government puts on market prices (or on its own information). First, when the price is highly informative (i.e., ρ is high and/or $\text{var}[Z]$ is low), most of the residual risk is exogenous. This is strengthened by the fact that $w(\rho^*)$ is low when the price is informative, reducing the share of endogenous

risk even more. Hence, Proposition 3 implies that the government should underweight the market price (and overweight its own information) relative to the ex-post optimal weight precisely when the price is highly informative.

Second, a similar “limits of informativeness” result holds with respect to the government’s own information: the government should underweight its own signal s_G precisely when it is accurate ($\text{var}[\varepsilon_G]$ is low). To see this, note that here there are two effects. The first effect is that a low $\text{var}[\varepsilon_G]$ reduces endogenous risk directly, while the second effect is that it increases the weight w the government puts on its own information, which increases the share of endogenous risk. The second effect dominates (see proof of Corollary 2), and so overall the government should reduce the weight it puts on its own information, and increase the weight on the market price.

Both of these “limits of informativeness” results are somewhat surprising. Not only is the government better off deviating from the ex-post optimal weights on the different sources of information, but it should do it in a way that underweights sources of information that are highly informative, and hence especially valuable. These results are formalized in the following corollary:

Corollary 2 *Underweighting the price—that is, a small exogenous increase in \tilde{w} away from the ex-post optimal weight $w(\rho^*)$ —increases price informativeness if either:*

(I) *Risk-aversion α is low and/or the variance of supply $\text{var}[Z]$ is low, so that equilibrium price informativeness is high.*

(II) *The government’s own information is imprecise, i.e., $\text{var}[\varepsilon_G]$ is high.*

Given Proposition 2, these results translate automatically to statements about the desirable government’s rule. That is, due to the effect that the government’s reliance on the price has on the informativeness of the price, the government is better off committing to change its reliance on the price relative to what would be ex-post optimal.

A popular idea in some policy circles is that the government should commit to follow a pre-determined rule based on publicly observable prices: see, e.g., Rochet (2004) and Hart

and Zingales (2011). This suggestion is motivated by a number of concerns, some of which are outside our model—in particular, a concern that, absent clear rules, the government will act too softly *ex post*. However, our analysis highlights another consideration that needs to be taken into account when thinking about the costs and benefits of these proposals: the effect that such proposals may have on the informativeness of the price. In particular, our analysis shows that in some circumstances, as characterized in Corollary 2, commitment to a market-based rule reduces the informativeness of the price that the rule makes use of. In such a case, the government’s objective function is negatively impacted in two ways: the government inefficiently neglects its own information s_G , and moreover, this neglect makes public information—i.e., market prices—less informative.

While in Corollary 2 we studied the effect of parameters that change the endogenous risk $N(w, \rho)$, we now analyze the effect of exogenous risk $\text{var}[\delta]$:

Corollary 3 *Underweighting the price—that is, a small exogenous increase in \tilde{w} away from the ex-post optimal weight $w(\rho^*)$ —increases price informativeness if exogenous risk $\text{var}[\delta]$ is sufficiently high.*

Note that the result is not quite as straightforward as it may appear. When exogenous risk $\text{var}[\delta]$ is high, price informativeness ρ is low because speculators bear a lot of risk when trading. Consequently, when $\text{var}[\delta]$ is high, both exogenous and endogenous risk are high. However, the upper bound on endogenous risk is simply the unconditional variance of government’s signal, $\text{var}[s_G]$, and high levels of exogenous risk eventually dominate. Note that for firms with high exogenous risk, the government’s reliance on the price is low to begin with, since the price is a noisy indicator of what the government is trying to learn. However, Corollary 3 implies that the reliance on the price should decrease further, given the effect that reliance on the price has on price informativeness.

4.3 Empirical implications

Finally, while we emphasized the normative implications of the results in Corollaries 2 and 3, they also have positive implications that can be tested empirically. Governments may deviate from the ex-post optimal rule for various reasons. For example, an overconfident policymaker or a policymaker who is overly committed to his agenda may underweight the information from the market and put excessive weight on his own information. Corollaries 2 and 3 tell us that such behavior will increase price informativeness when risk-aversion α is low, the variance of supply $var[Z]$ is low, the noise in the government's own information $var[\varepsilon_G]$ is high, or exogenous risk $var[\delta]$ is large. These empirical predictions complement those in Corollary 1: While the predictions in Corollary 1 apply to cases where the government acts ex-post optimally, the predictions coming out of Corollaries 2 and 3 apply to the consequences of the government's deviation from the ex-post optimal rule.

5 The importance of the government's own information

It is tempting to interpret policy proposals to use market information as implying that governments do not need to engage in costly collection of information on their own. For example, in the context of banking supervision, one might imagine that the government could substantially reduce the number of bank regulators. Our framework enables analysis of this issue when the usefulness of market information is endogenous and affected by the government's use of this information. We find that whenever an exogenous increase in the weight the government puts on its own information, \tilde{w} , increases informativeness, then an increase in the quality of the government's information also increases price informativeness—even when the government acts ex post optimally. Conversely, a decrease in the quality of the government's information worsens price informativeness. Hence, the usual argument that market information can easily replace the government's own information is incorrect.

Formally, suppose that the precision of the government's information, τ_G , is a choice variable. What would be the benefits of increasing τ_G ? Given that the price aggregates speculators' information imperfectly, the government is using both the price and its private information s_G when making its intervention decision. Then, an increase in the precision of its private signal has a direct positive effect on the quality of the government's overall information about the fundamental θ . More interesting, however, is that an increase in τ_G may also have a positive indirect effect, in that more accurate government information leads to more informative prices. The logic follows the previous results on the effect of the government's use of market information on the quality of this information: An increase in τ_G increases the weight w that the government puts on its own information, which, in the cases characterized by Proposition 3, increases the equilibrium price informativeness. Hence, in these cases, the government should be willing to spend more on producing its own information than the direct contribution of this information to its decision making would imply.

The result is summarized in the following proposition.¹⁵

Proposition 4 *Suppose that a small exogenous increase in \tilde{w} away from $w(\rho^*)$ raises price informativeness. Then a small exogenous increase in the quality of the government's information, τ_G , results in an equilibrium with price informativeness strictly above ρ^* when the government acts ex post optimally.*

¹⁵Bond, Goldstein, and Prescott (2010) also note that the government's own information helps the government make use of market information. However, in that model, the market price perfectly reveals the expected value of the firm, and the problem is that the expected value does not provide clear guidance as to the optimal intervention decision. Hence, the government's information can complement the market information in enabling the government to figure out the optimal intervention decision. Here, on the other hand, the fact that the government is more informed encourages speculators to trade more aggressively, and thus leads the price to reflect the expected value more precisely.

6 Transparency

Governments are often criticized for not being transparent enough about their information and policy goals. But is government transparency actually desirable when the government itself is trying to elicit information from the price? Does the release of information by the government increase or decrease speculators' incentives to trade on their information? We analyze these questions using our model where the government is taking an action based on its own information and the information in the price. We find that the results are very different depending on the type of transparency in question. In particular, it matters whether the government is releasing information about a variable it is able to learn more about.

6.1 Transparency about a variable the government can learn more about

In our model, the government directly observes a signal s_G about θ , but also tries to extract further information about θ from the market price. To analyze the effect of transparency in this case, we consider what happens if the government is fully transparent about its own information, s_G . Specifically, suppose the government announces s_G before speculators trade. Transparency of this type has an extreme effect in our model, as we now show.

Recall that the price is determined by the market-clearing condition

$$\frac{1}{\alpha} \frac{E[T(s_G, P) | s_i, s_G, P] + \bar{\delta} - P}{\text{var}[T(s_G, P) | s_i, s_G, P] + \text{var}[\delta]} + Z = 0,$$

where note that, because of transparency, the government's signal s_G now enters speculators' information sets. Conditional on the price and s_G , there is now no uncertainty about the government's intervention T : hence $\text{var}[T(s_G, P) | s_i, s_G, P] = 0$, and moreover,

$E [T (s_G, P) | s_i, s_G, P] = T (s_G, P)$. Hence the market-clearing condition collapses to

$$\frac{1}{\alpha} \frac{T (s_G, P) + \bar{\delta} - P}{var [\delta]} + Z = 0. \quad (19)$$

From this identity, it is clear that the equilibrium price contains no information about θ beyond that available directly in s_G , and so transparency leads the government to ignore the price in making its intervention decision. Consequently:

Proposition 5 *If the government discloses its own information s_G , then the price ceases to be a useful source of information for the government, and the value of the government's objective function decreases.*

Note that by rearranging (19) we obtain an explicit expression for the price under transparency,

$$P (s_G, Z) = T (s_G) + \bar{\delta} + \alpha var [\delta] Z. \quad (20)$$

It is worth highlighting that the price mapping in (20) *does* provide an *uninformed* outsider¹⁶ with information about the fundamental θ , since it is still a noisy signal of s_G . But, as expressed in Proposition 5, the price contains no information that the government does not already have. In contrast, absent transparency the price represents a noisy signal of the fundamental θ that is conditionally independent of the government's own noisy signal $s_G = \theta + \varepsilon_G$. It follows directly that the government is worse off by revealing its signal s_G . When revealing its signal, the government makes the information in the price useless, and so has to make the intervention decision with less precise information overall.

¹⁶Specifically, an uninformed outsider who somehow missed the government's announcement of s_G .

6.2 Transparency about a variable the government cannot learn more about

We next consider the effects of transparency about a variable the government cannot learn more about. To do so, suppose now that the government's benefit from intervention is

$$v(T - \psi - \theta),$$

where ψ is a normally distributed variable that is uncorrelated with θ . As before, both the government and speculators observe independent noisy signals (s_G and s_i) of θ . In contrast, the government observes a signal of ψ , $\sigma_G = \psi + \zeta_G$, but speculators observe only noisy signals of σ_G , $\sigma_i = \psi + \zeta_G + \zeta_i$. Consequently, the government is unable to learn anything from market prices about the realization of ψ , for the simple reason that speculators do not have any information about ψ beyond that which the government already has. Parallel to before, we assume ζ_G and ζ_i are independently distributed random variables.

A leading interpretation of the state variable ψ is that it represents the government's policy objectives. In this case, it is natural to assume $\sigma_G \equiv \psi$, and so it is impossible for speculators to have information that the government does not already have. A second possible interpretation is that ψ is the aggregate state of the economy, while θ is a bank-specific state variable; and that a speculator's information about the aggregate state is weak coarsening of the government's own information.

Our main result is:

Proposition 6 *Disclosure of the government's information σ_G about ψ increases equilibrium price informativeness and hence the expected value of the government's objective function.*¹⁷

The economics behind Proposition 6 is easiest to give in the limiting case in which speculators know nothing at all about the government signal σ_G , i.e., $\text{var}[\zeta_i] = \infty$. In

¹⁷In the case in which there exist multiple linear equilibria, the result says that the least (respectively, most) informative equilibrium under disclosure is more informative than the least (respectively, most) informative equilibrium without disclosure.

this case, a linear equilibrium exists that takes the same form as before, and equilibrium price informativeness again satisfies the first equality in (14). Moreover, by Lemma 1, the government's intervention is

$$T = E[\psi|\sigma_G] + E[\theta|s_G, P] + g(\mu, \text{var}[\psi|\sigma_G] + \text{var}[\theta|s_G, P]).$$

In this case, transparency about σ_G has no effect on information importance term $\frac{\partial}{\partial s_i} E[T|s_i, P]$, while it unambiguously reduces the residual risk term $\text{var}[T|s_i, P]$. Consequently, price informativeness is increased.

The general case of $\text{var}[\zeta_i] < \infty$ is handled in full in the Appendix. The first complication is that the equilibrium price now depends on the government signal σ_G , which is known collectively by speculators. Consequently, the price is of the form $P = p_0 + \rho p_Z \theta + \xi \rho p_Z \sigma_G + p_Z Z$, for some parameters p_0 , p_Z , ρ and ξ . The existence of an equilibrium of this type is proved in the Appendix.

Because the government observes σ_G , the price conveys the same information to the government as does $\theta + \rho^{-1}Z$. Consequently, ρ remains the relevant measure of price informativeness. Moreover, the same argument as before implies that equilibrium price informativeness ρ still satisfies the first equality in (14).

The second complication is that speculator i 's signal s_i now has two distinct effects on speculator i 's expectation of the intervention T . One effect is as before: s_i allows speculator i to forecast s_G . The second effect is that s_i , combined with the price P and signal σ_i , allows the speculator to forecast σ_G , which also affects the government's intervention. The Appendix establishes that the effect of disclosing σ_G is to increase the information importance of s_i . Loosely speaking, disclosure of σ_G makes it easier for a speculator to forecast the intervention T , and hence $E[T|P, s_i, \sigma_i]$ becomes more sensitive to s_i . Moreover, and as in the limit case, disclosing σ_G decreases residual risk, so that price informativeness is again unambiguously increased.

Proposition 6 captures what is perhaps the usual intuition about transparency and the reason why it is strongly advocated. The idea is that when the government reveals its information (e.g., about its policy goal), it reduces uncertainty for speculators. This encourages them to trade more aggressively on their information, resulting in higher price informativeness. The government is then better off as it can make more informed decisions.

6.3 Discussion

In general, when the government reveals information to speculators, there are two effects on speculators' incentives to trade. On the one hand, making the government's information public might reduce speculators' incentive to trade because it reduces the informational advantage that brings them to the market in the first place. On the other hand, making the government's information public may increase speculators' incentive to trade because it reduces the overall uncertainty that speculators are exposed to and so allows them to trade more aggressively on the information they have. In our analysis in the previous subsections, if the government releases information about θ , about which speculators have information that the government does not, it eliminates its ability to learn from speculators about θ . This is the first effect above. However, if the government releases information about ψ , about which the speculators have strictly coarser information than the government, it enhances its ability to learn from the speculators about θ . This is the second effect above.

Hence, our model provides justification for disclosing the government's policy goal, as this is a variable about which the government is unlikely to have anything to learn about from the speculators. But, the government should be more cautious when disclosing information about the state of the bank, as this is a variable the government may want to learn more about and speculators may be informed about. A little more speculatively, disclosure of information about economic aggregates may be desirable, as the government is less likely to be able to learn something about them from the public.

7 The appropriate measure of informativeness

We now revisit our definition of price informativeness. So far, we have defined price informativeness as the information the price provides about the state variable θ . In our view, this is the natural definition of price informativeness in the context of our model. The reason is that the state variable θ is the variable that is relevant for the decision made by the government. Indeed, we showed that the informativeness of the price about θ is directly linked to the government's objective function. Hence, to the extent that we care about price informativeness because of its effect on the efficiency of the intervention decision made by the government, then the relevant notion of informativeness is one that captures the amount of information provided by the price for the intervention decision; this is the informativeness of the price about θ .

In contrast, the traditional definition of price informativeness in the literature looks at the information in the price about the cash flows generated by the firm. Specifically, this is captured by the inverse of the variance of cash flows conditional on observing the price, but no other information (for example, Brunnermeier (2005) and Peress (2010)). Going back to our main model, this measure is given by

$$\varsigma \equiv \left(\text{var} [\delta] + w^2 (\text{var} [\theta|P] + \text{var} [\varepsilon_G]) \right)^{-1}. \quad (21)$$

The idea behind this measure is to see how good of a job the market is doing in predicting future cash flows, i.e., how “efficient” the market is. In this section, we show that the measure ς is disconnected from real efficiency, which in our model is the efficiency of the intervention decision made by the government. The measure ς is built on the premise that the market is a side show that predicts future cash flows, rather than provide information that guides future cash flows. Hence, focusing on the measure ς might lead to very misleading answers if we care about the efficiency of the actions that are guided by market information.

To make this point, we provide two examples:

Example 1, Pure price-based intervention: If the government makes intervention decisions based purely on the price, the weight w on its own information is 0. On the one hand, pure price-based intervention maximizes the informativeness measure ς : by (21), ς reaches its upper bound of $\text{var}[\delta]^{-1}$ (which is determined by the exogenous component of cash flow that is impossible to forecast). But on the other hand, pure price-based intervention minimizes the relevant notion of price informativeness since from (17) (and provided $\text{var}[\delta] > 0$), $\rho = 0$. Hence, for the purpose of achieving greater efficiency in government intervention, it would be a mistake to focus on ς instead of on ρ when deciding on the weight w that the government puts on its own information

Example 2, Transparency about s_G : From the preceding section, if the government publicly announces s_G then the price ceases to be a useful source of information. However, at least when $\text{var}[\delta]$ is small, transparency increases the informativeness measure ς .¹⁸ This can be seen from (20): when $\text{var}[\delta]$ is small, the price forecasts intervention T very well, so that ς approaches its upper bound $\text{var}[\delta]^{-1}$.¹⁹ Again, if we care about the efficiency of the real action—the government intervention decision—then releasing the government signal s_G is a mistake even if it may increase the traditional measure of price informativeness.

As a specific application of Example 2, consider evaluating the success of government stress tests, which can be viewed as disclosures of government information. It may seem tempting to evaluate stress tests by asking whether they increase the traditional measure ς of market efficiency. But if the reason we care about price informativeness is that prices can guide decisions, this is the wrong metric to use.

Our overall point in this discussion is that it is hard, and probably impossible, to talk meaningfully about price informativeness in a completely theory-free way. Instead, one must specify why one cares about price informativeness in the first place, and allow this to

¹⁸Here, ς captures the information about cash flow conditional on the price, but not conditional on the government signal.

¹⁹In contrast, when the government does not announce s_G (no transparency), then ς does not approach $\text{var}[\delta]^{-1}$ even as $\text{var}[\delta] \rightarrow 0$.

inform the appropriate definition.²⁰ In our model, we care about the efficiency of the real decision—the government intervention decision—and so it is the informativeness of the price about the relevant state variable θ that matters. As we mentioned before, our efficiency criterion ignores the welfare of traders in the financial market; in that, we fall short of a full welfare analysis. Incorporating such considerations requires endogenization of the motives of noise traders, which is beyond the scope of our paper, and most likely beyond the scope of government policy. If these motives were part of the overall welfare function, then one would want to consider a weighted average of the government objective and trader welfare. Our analysis approximates the case where the weight on the government objective is large.

8 Extension

In this section, we consider an extension of our basic model to accommodate two features that are potentially relevant for some applications. First, while the basic model assumes that the value of equity of the firm increases by the *full* amount that the government injects into the firm, we allow here for the possibility that the change in equity value is different from the size of the injection. This enables us to discuss the extent to which the government should subsidize capital injections: this is an issue that came up often in policy debates surrounding government intervention in the recent crisis.

Second, while the basic model assumes that the state variable θ that affects the government's incentive to intervene has no direct effect on the value of the firm, we allow here for the possibility that θ enters the expression for firm value directly. Indeed, in the recent crisis, the government's incentive to inject capital to banks depended in many cases on the bank's balance sheet position. This allows us to address the differences between *corrective* government intervention and *amplifying* government intervention. In the case of corrective government interventions, the government helps weak firms at the expense of strong firms,

²⁰See, for example, Paul (1992) and Bresnahan, Milgrom and Paul (1992) for related discussions about price informativeness. We ourselves discuss a closely related point in Bond, Edmans and Goldstein (2012); see also other references cited therein.

e.g., because weak firms pose a risk for systemic stability. This is often the case with a bailout policy. In the case of amplifying government interventions, the government takes an action that hurts weak firms further, e.g., when liquidating a weak bank, the government might impose additional losses on its shareholders (in order to benefit depositors).²¹

Formally, in the extended version of our model, the firm's cash flow is

$$X = \delta + bT + c\theta. \quad (22)$$

Here, the parameter b captures the extent to which government interventions are not fully captured by equityholders. The parameter c captures the extent to which the state variable *directly* affects cash flow, even without any government intervention. We continue to assume the same objective function for the government as in (2).

It is straightforward to extend our analysis to cover the more general cash flow specification (22) and show that an equilibrium exists in which prices are a linear function of the state variable θ and the supply shock Z . As before, equilibrium price informativeness when the government acts optimally ex post is given by the solution to

$$\rho = \frac{1}{\alpha} \frac{\left| \frac{\partial}{\partial s_i} E[X|s_i, P] \right|}{\text{var}[X|s_i, P]},$$

which implies

$$\rho = \frac{1}{\alpha} \frac{|bw(\rho) + c| \frac{\partial}{\partial s_i} E[\theta|s_i, P]}{(bw(\rho) + c)^2 \text{var}[\theta|s_i, P] + (bw(\rho))^2 \text{var}[\varepsilon_G] + \text{var}[\delta]}. \quad (23)$$

The equilibrium condition (23) generalizes equation (14). Note that our extended model nests the basic model as a special case, where $b = 1$ and $c = 0$. We now consider the implications that arise from varying b and c from these benchmark levels.

²¹In Bond, Goldstein, and Prescott (2010), we conduct extensive analysis of the differences between corrective actions and amplifying actions in a different model.

8.1 Injection subsidies

A central policy question associated with government interventions is the extent to which equityholders should benefit from these interventions. In particular, there are often public calls for government cash injections to be accompanied by taxing equityholders, such that they do not fully benefit from the injection of capital. Clearly many different factors affect the appropriate degree of tax/subsidy. We can use the extended version of our model to consider the effects of tax/subsidy on information aggregation.

We set $b = 1 - R$, where $R \in [0, 1]$ is the gross interest rate that the government charges on its cash injection T . The basic version of our model above assumes $R = 0$, so that the cash injection is a pure subsidy. The case $R = 1$ corresponds to no subsidy. We can use the model to analyze the effect of the financing subsidy on the informativeness of the price. To focus attention on the effect of the financing subsidy, we set $c = 0$ as in the main model, so that the state variable θ has no direct effect on firm cash flows. The equilibrium condition (23) then simplifies to

$$\rho = \frac{1}{\alpha} \frac{(1 - R) w(\rho) \frac{\partial}{\partial s_i} E[\theta | s_i, P]}{(1 - R)^2 w(\rho)^2 (\text{var}[\theta | s_i, P] + \text{var}[\varepsilon_G]) + \text{var}[\delta]}. \quad (24)$$

We can see that the no-subsidy case $R = 1$ results in completely uninformative prices. Since the government completely neutralizes the effect of intervention such that equityholders are indifferent to it, equityholders have no incentive to trade on information they have about the state of the world θ that determines the incentives for government intervention. At the opposite extreme of $R = 0$, we can see that a small increase in the rate R affects price informativeness in the opposite direction to a small increase in the weight w that the government puts on its own information. This is because in (24) R and w enter only via the term $(1 - R) w$. Then, increasing the dilution factor R reduces the exposure of speculators to the state of the world θ when they trade the firm's share. On the one hand, this reduces the value of their information (i.e., information importance is reduced) reducing their incentive

to trade. On the other hand, it reduces the uncertainty they are exposed to when trading (i.e., residual risk is reduced), increasing their incentive to trade. From our earlier analysis, we know the net effect depends on particular parameter values. In particular, Corollaries 2 and 3 imply that increasing the interest rate R charged on government injections reduces price informativeness when risk-aversion α is low; or the variance of supply $\text{var}[Z]$ is low; or government information is imprecise (i.e., $\text{var}[\varepsilon_G]$ high); or exogenous risk $\text{var}[\delta]$ is high.

8.2 Direct cash flow effects

Next, consider the case in which the state variable θ directly affects firm cash flows, so that the firm's cash flow absent intervention is $\delta + c\theta$. Recall that, under full information, the government would choose $T = \theta + \text{constant}$. Hence, positive values of c correspond to *amplifying* government actions, in the sense that the intervention amplifies the direct effect of θ on cash flow. Likewise, negative values of c correspond to *corrective* government actions. As we mentioned above, in the context of banks, we often think of the bailout of a weak bank as a corrective action, and the liquidation of a weak bank as an amplifying action. To focus on the implications of direct cash flow effects, we set $b = 1$ as in our main model, so that when the government injects capital to the bank, it does so as a pure subsidy. Also, for conciseness, we focus on the case $\text{var}[\delta] = 0$, so that, absent intervention, there is only one source of uncertainty in the firm's cash flow. Price informativeness is then given by

$$\rho = \frac{1}{\alpha} \frac{|w(\rho) + c| \frac{\partial}{\partial s_i} E[\theta | s_i, P]}{(w(\rho) + c)^2 \text{var}[\theta | s_i, P] + (w(\rho))^2 \text{var}[\varepsilon_G]}. \quad (25)$$

We now revisit our central results on how a commitment by the government to overweight its own information—or equivalently, to underweight the price—affects price informativeness. The effect is determined by the same two forces as before: overweighting the government's own information affects information importance, as represented by the numerator of the RHS of (25), and also residual risk, as represented by the denominator of the RHS of (25). The

following result summarizes our findings. To understand the result, note that parameter values $c \in [-1, 0)$ correspond to strongly corrective interventions: the intervention that a fully informed government would implement more than outweighs the direct cash flow effect of θ .

Proposition 7 *Let $\text{var}[\delta] = 0$, and assume $c \neq -w(0)$.²² Let ρ^* be an equilibrium when the government acts ex post optimally. Then underweighting the price (i.e., a small exogenous increase in \tilde{w} away from the ex-post optimal weight $w(\rho^*)$):*

(I) *Decreases price informativeness if the intervention is either amplifying ($c > 0$) or strongly corrective ($0 > c \geq -1$).*

(II) *Increases price informativeness if the intervention is moderately corrective (formally, if $c \leq \bar{c}$, where $\bar{c} \leq -1$).*

From Proposition 3 in the basic version of our model, we know that underweighting the price decreases informativeness when $\text{var}[\delta] = 0$. From Proposition 7, this result continues to hold when θ directly affects the cash flow in the same direction as it affects the government's intervention, i.e., the intervention is amplifying ($c > 0$). Again, the reason is that increasing the weight on the government's signal increases both information importance and residual risk, and the residual risk effect is dominant.

When θ directly affects the cash flow in the opposite direction from the effect via the government's intervention, i.e., the intervention is corrective ($c < 0$), overweighting the government's information sometimes has the opposite effect: it reduces both information importance and residual risk, leading to an overall increase in informativeness. For this to be the case, c must be significantly below 0, i.e., the intervention is only moderately corrective. If instead the intervention is strongly corrective, then residual risk is increased (as in the amplifying case), and overweighting the government's intervention again reduces price informativeness.

²²The case of $c = -w(0)$ is non-generic of course.

9 Concluding remarks

Our paper analyzes how market-based government policy affects the trading incentives of risk-averse speculators in a rational-expectations model of financial markets. Increasing the reliance of government intervention on market information affects the risk-return tradeoff faced by speculators, and hence changes their incentive to trade. Our analysis shows that the use of market prices as an input for policy might not come for free and might damage the informational content of market prices themselves. We characterize cases in which the government would be better off limiting its reliance on market prices and increasing their informational content. However, the government always benefits from some reliance on market prices. Also, and counter to common belief, transparency by the government might be a bad idea in that it might reduce trading incentives and price informativeness, leading to a lower value for the government's objective function. While we focus in this paper on market-based government policy, our analysis and results apply more generally to other non-governmental actions based on the price. For example, our framework naturally covers the case of a manager or board of directors making decisions that affect firm cash flows. Hence, our paper contributes more generally to the understanding of the interaction between financial markets and corporate decisions, and in particular, the ways in which secondary financial market have real effects.

10 References

[TO BE COMPLETED]

Appendix

We start by defining some notation, which we use throughout the appendix:

$$v_\varepsilon(\rho) \equiv \tau_\theta + \rho^2 \tau_z + \tau_\varepsilon \quad (\text{A-1})$$

$$v_G(\rho) \equiv \tau_\theta + \rho^2 \tau_z + \tau_G \quad (\text{A-2})$$

$$F(w, \rho) = \frac{w \tau_\varepsilon v_\varepsilon(\rho)^{-1}}{w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1}}. \quad (\text{A-3})$$

Proof of results other than Proposition 6

Proof of Proposition 1 [existence]: To establish existence, we show that there exist p_0 , ρ and p_Z such that the price function (9), i.e., $P = p_0 + \rho p_Z \theta + p_Z Z$, is an equilibrium. Specifically, we show that there exist p_0 , ρ and p_Z such that market clearing (6) holds.

First, and as discussed in the main text, when the price function is of the form (9), observing the price is equivalent to observing $\tilde{P} \equiv \frac{1}{\rho p_Z} (P - p_0) = \theta + \rho^{-1} Z$, which is an unbiased estimate of θ , is normally distributed, and has precision $\rho^2 \tau_z$. Hence by a standard application of Bayes' rule to normal distributions, the conditional distributions of θ given the information of, respectively, the government and a speculator i are normal, with conditional variances $\text{var}[\theta|s_G, P] = v_G(\rho)^{-1}$ and $\text{var}[\theta|s_i, P] = v_\varepsilon(\rho)^{-1}$, where $v_G(\rho)$ and $v_\varepsilon(\rho)$ are as defined in (A-1) and (A-2). Likewise, the conditional expectations of θ are

$$E[\theta|s_G, P] = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_z \tilde{P} + \tau_G s_G}{v_G(\rho)}$$

$$E[\theta|s_i, P] = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_z \tilde{P} + \tau_\varepsilon s_i}{v_\varepsilon(\rho)}.$$

Substituting $E[\theta|s_G, P]$ and $\text{var}[\theta|s_G, P]$ into (7), and using the definition (11) of $w(\rho)$, the government's intervention is

$$T = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_z \tilde{P} + \tau_G s_G}{v_G(\rho)} + g(\mu, v_G(\rho)^{-1}) = w(\rho) s_G + \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_z \tilde{P}}{v_G(\rho)} + g(\mu, v_G(\rho)^{-1}).$$

So a speculator i 's conditional expectation and conditional variance of T are

$$\begin{aligned} E[T|s_i, P] &= w(\rho) E[\theta|s_i, P] + \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}}{v_G(\rho)} + g(\mu, v_G(\rho)^{-1}) \\ \text{var}[T|s_i, P] &= w(\rho)^2 (\text{var}[\theta|s_i, P] + \text{var}[\varepsilon_G]). \end{aligned}$$

Substituting in (13) and $\int s_i di = \theta$, the market-clearing condition (6) is

$$\frac{1}{\alpha} \frac{w(\rho) E[\theta|s_i = \theta, P] + \bar{\delta} + \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}}{v_G(\rho)} + g(\mu, v_G(\rho)^{-1}) - P}{w(\rho)^2 (\text{var}[\theta|s_i, P] + \text{var}[\varepsilon_G]) + \text{var}[\delta]} + Z = 0.$$

This is a linear expression in the random variables θ and Z . Consequently, market clearing (6) is satisfied for all θ and Z if and only if the intercept term and the coefficients on θ and Z all equal zero. Written explicitly, these three conditions are

$$w(\rho) \frac{\tau_\theta \bar{\theta}}{v_\varepsilon(\rho)} + \bar{\delta} + \frac{\tau_\theta \bar{\theta}}{v_G(\rho)} + g(\mu, v_G(\rho)^{-1}) - p_0 = 0 \quad (\text{A-4})$$

$$w(\rho) \frac{\rho^2 \tau_Z + \tau_\varepsilon}{v_\varepsilon(\rho)} + \frac{\rho^2 \tau_Z}{v_G(\rho)} - \rho p_Z = 0 \quad (\text{A-5})$$

$$w(\rho) \frac{\rho^2 \tau_Z}{v_\varepsilon(\rho)} \rho^{-1} + \frac{\rho^2 \tau_Z}{v_G(\rho)} \rho^{-1} - p_Z + \alpha (w(\rho)^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1}) = 0. \quad (\text{A-6})$$

To complete the proof of existence, we must show that this system of three equations in p_0 , ρ and p_Z has a solution. Observe that $\rho \times (\text{A-6}) - (\text{A-5})$ is

$$-w(\rho) \frac{\tau_\varepsilon}{v_\varepsilon(\rho)} + \alpha \rho (w(\rho)^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1}) = 0,$$

which using the definition (A-3) of F can be rewritten as

$$\rho = \frac{1}{\alpha} \frac{w(\rho) \tau_\varepsilon v_\varepsilon(\rho)^{-1}}{w(\rho)^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1}} = \frac{1}{\alpha} F(w(\rho), \rho).$$

(Note that this equation coincides with the equilibrium condition (14) given in the main text.) Hence to show existence, we show that there exist p_0 , ρ and p_Z with $\rho \neq 0$ that

satisfy (A-4), (A-5) and $\alpha\rho = F(w(\rho), \rho)$. Since p_0 appears only once, in (A-4), and p_Z appears only once, in (A-5), existence is established if $\alpha\rho = F(w(\rho), \rho)$ has a non-zero solution. This is indeed the case: $F(w(\rho), \rho)$ is continuous in ρ , and by Lemma A-1 below, $F(w(0), 0) > 0$ and $\lim_{\rho \rightarrow \infty} F(w(\rho), \rho) < \infty$. This completes the proof of equilibrium existence.

Lemma A-1 (I) $F(w(0), 0) > 0$ and $\lim_{\rho \rightarrow \infty} F(w(\rho), \rho) < \infty$. (II) For any $\tilde{w} > 0$, $F(\tilde{w}, 0) > 0$ and $\lim_{\rho \rightarrow \infty} F(\tilde{w}, \rho) < \infty$.

Proof of Lemma A-1: We prove part (I); part (II) is similar, but more straightforward.

The fact that $F(w(0), 0) > 0$ follows from $w(0)\tau_\varepsilon > 0$ and $w(0)^2(1 + v_\varepsilon(0)\tau_G^{-1}) + v_\varepsilon(0)\tau_\delta^{-1} < \infty$.

Both $v_\varepsilon(\rho)$ and $v_G(\rho)$ are strictly increasing in ρ . So certainly $\lim_{\rho \rightarrow \infty} w(\rho)\tau_\varepsilon v_\varepsilon(\rho)^{-1} < \infty$. If $\tau_\delta^{-1} > 0$, it is then immediate that $\lim_{\rho \rightarrow \infty} F(w(\rho), \rho) < \infty$. If instead $\tau_\delta^{-1} = 0$ and $\tau_G^{-1} > 0$, then note that

$$F(w(\rho), \rho) = \frac{\tau_\varepsilon v_\varepsilon(\rho)^{-1}}{\tau_G v_G(\rho)^{-1} (v_\varepsilon(\rho)^{-1} + \tau_G^{-1})},$$

and hence $\lim_{\rho \rightarrow \infty} F(w(\rho), \rho) < \infty$ because $\lim_{\rho \rightarrow \infty} v_G(\rho)/v_\varepsilon(\rho) = 1$. Finally, if $\tau_G^{-1} = \tau_\delta^{-1} = 0$, then $F(w(\rho), \rho) = \tau_\varepsilon$ for all ρ . **QED**

Proof of Proposition 1 [uniqueness]: The equilibrium is unique within the class of linear equilibria if $\alpha\rho = F(w(\rho), \rho)$ has a unique positive solution. Equilibrium uniqueness at $\tau_G^{-1} = 0$ follows from the fact that, in this case, $w(\rho) = 1$, and so $F(w(\rho), \rho)$ is decreasing in ρ , and so has a unique positive solution.

For $\tau_G^{-1} > 0$, we deal separately with the cases $\tau_\delta^{-1} > 0$ and $\tau_\delta^{-1} = 0$:

Case: $\tau_\delta^{-1} > 0$: Differentiation of $F(w(\rho), \rho)$ by τ_G^{-1} gives

$$\frac{\partial F(w(\rho), \rho)}{\partial (\tau_G^{-1})} = \frac{\partial w(\rho)}{\partial (\tau_G^{-1})} F_w(w, \rho) - \frac{w^3 \tau_\varepsilon v_\varepsilon(\rho)^{-1}}{(w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1})^2},$$

where

$$\begin{aligned}
F_w(w, \rho) &= \tau_\varepsilon v_\varepsilon(\rho)^{-1} \frac{w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1} - 2w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1})}{(w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1})^2} \\
&= \tau_\varepsilon v_\varepsilon(\rho)^{-1} \frac{\tau_\delta^{-1} - w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1})}{(w^2 (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1})^2} \tag{A-7}
\end{aligned}$$

and

$$\frac{\partial w(\rho)}{\partial (\tau_G^{-1})} = \frac{\partial w(\rho)}{\partial \tau_G} \frac{\partial \tau_G}{\partial (\tau_G^{-1})} = -\frac{v_G(\rho) - \tau_G}{(v_G(\rho))^2} \tau_G^2 = -\frac{\tau_\theta + \rho^2 \tau_Z}{(\tau_G + \tau_\theta + \rho^2 \tau_Z)^2} \tau_G^2 = -\frac{\tau_\theta + \rho^2 \tau_Z}{(1 + \tau_\theta \tau_G^{-1} + \rho^2 \tau_Z \tau_G^{-1})^2}.$$

For $\tau_G^{-1} > 0$, it follows straightforwardly that there exists some constant κ such that for all $\rho \in [0, \infty)$, $\left| \frac{\partial F(w(\rho), \rho)}{\partial (\tau_G^{-1})} \right| \leq \kappa$. At $\tau_G^{-1} = 0$, note that $w(\rho) \equiv 1$, and hence

$$\left. \frac{\partial F(w(\rho), \rho)}{\partial (\tau_G^{-1})} \right|_{\tau_G^{-1}=0} = -(\tau_\theta + \rho^2 \tau_Z) \frac{\tau_\varepsilon v_\varepsilon(\rho)^{-1} (\tau_\delta^{-1} - v_\varepsilon(\rho)^{-1})}{(v_\varepsilon(\rho)^{-1} + \tau_\delta^{-1})^2} - \frac{\tau_\varepsilon v_\varepsilon(\rho)^{-1}}{(v_\varepsilon(\rho)^{-1} + \tau_\delta^{-1})^2},$$

and so again there exists some constant κ such that for all $\rho \in [0, \infty)$, $\left| \frac{\partial F(w(\rho), \rho)}{\partial (\tau_G^{-1})} \right| \leq \kappa$. By standard arguments, the fact that $\alpha\rho = F(w(\rho), \rho)$ has a unique solution at $\tau_G^{-1} = 0$ then implies that it also has a unique solution for all τ_G^{-1} sufficiently small.

Case: $\tau_\delta^{-1} = 0$: In this case, $F(w(\rho), \rho)$ simplifies to

$$F(w(\rho), \rho) = \frac{\tau_\varepsilon v_\varepsilon(\rho)^{-1}}{w(\rho) (v_\varepsilon(\rho)^{-1} + \tau_G^{-1})} = \tau_\varepsilon \frac{1 + \tau_G^{-1} (\tau_\theta + \rho^2 \tau_Z)}{1 + \tau_G^{-1} (\tau_\varepsilon + \tau_\theta + \rho^2 \tau_Z)},$$

which is decreasing in τ_G^{-1} . Hence for all $\tau_G^{-1} \geq 0$, we know the equilibrium lies in the compact set $[0, \frac{\tau_\varepsilon}{\alpha}]$. Because $F(w(\rho), \rho)$ is well-behaved over this compact set, equilibrium uniqueness at $\tau_G^{-1} = 0$ implies equilibrium uniqueness for all τ_G^{-1} sufficiently small. **QED**

Proof of Corollary 1: Substituting into (14), the equilibrium value of ρ is determined by

the solution to

$$\rho = \frac{1}{\alpha} \frac{\tau_G v_G(\rho)^{-1} \tau_\varepsilon v_\varepsilon(\rho)^{-1}}{\tau_G^2 v_G(\rho)^{-2} (v_\varepsilon(\rho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1}}. \quad (\text{A-8})$$

The statement about τ_G is established by a numerical example: a particularly simple example is that if $\tau_Z = \tau_\delta = \tau_\theta = \tau_\varepsilon = \alpha = 1$, then $K_P(\rho)$ is first increasing then decreasing in τ_G .

To establish the other comparative statics, we show that $\rho^2 \tau_Z$ is increasing in $\tau_\delta, \tau_\varepsilon, \tau_Z$ and decreasing in α ; given the expression for $K_P(\rho)$, the result then follows.

From the existence proof (Proposition 1), we know that at the least- and most-informative equilibria, the RHS of (A-8) has a slope below 1. The implication that the equilibrium value of ρ is increasing in τ_δ and decreasing in α is then immediate from the fact that the RHS of (A-8) is increasing in τ_δ and decreasing in α . The implication that the equilibrium value of ρ is increasing in τ_ε also follows from the fact the RHS of (A-8) is increasing in τ_ε ; to see this, multiply both the numerator and denominator of the RHS by $v_\varepsilon(\rho)$. Finally, the comparative static with respect to τ_Z follows by a change of variables: defining $\varrho = \rho^2 \tau_Z$, the equilibrium equation is

$$\varrho^{1/2} = \frac{1}{\alpha} \tau_Z^{1/2} \frac{\tau_G^{-1} (\tau_\theta + \tau_G + \varrho)^{-1} \tau_\varepsilon (\tau_\theta + \tau_\varepsilon + \varrho)^{-1}}{\tau_G^2 (\tau_\theta + \tau_G + \varrho)^{-2} ((\tau_\theta + \tau_\varepsilon + \varrho)^{-1} + \tau_G^{-1}) + \tau_\delta^{-1}},$$

and the same argument as above implies that the equilibrium value of ϱ is increasing in τ_Z .

QED

Proof of Lemma 2: The proof exploits the fact that, by definition, $w(\tilde{\rho})$ and $K_P(\tilde{\rho})$ maximize $V(\tilde{w}, \tilde{K}_P, \bar{T})$ if price informativeness ρ is artificially held fixed at $\tilde{\rho}$. The derivative $V_{\tilde{K}_P}(\tilde{w}, \tilde{K}_P, \bar{T})$ with respect to \tilde{K}_P is

$$E \left[(\theta + \tilde{\rho}^{-1} Z) v' \left(\tilde{w}(\theta + \varepsilon_G) + \tilde{K}_P(\theta + \tilde{\rho}^{-1} Z) + \bar{T} - \theta \right) - \mu(\theta + \tilde{\rho}^{-1} Z) \right].$$

(Recall that \tilde{K}_P has no effect on equilibrium price informativeness.) A further round of differentiation implies $V_{\tilde{K}_P \tilde{K}_P} < 0$. By the law of iterated expectations the cross-derivative

$V_{\tilde{K}_P \tilde{w}}(\tilde{w}, \tilde{K}_P, \bar{T})$ with respect to \tilde{w} and \tilde{K}_P , holding $\tilde{\rho}$ fixed, can be written as

$$E \left[E \left[(\theta + \tilde{\rho}^{-1} Z) (\theta + \varepsilon_G) v'' \left(\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \tilde{\rho}^{-1} Z) + \bar{T} - \theta \right) \mid \theta + \tilde{\rho}^{-1} Z, \theta + \varepsilon_G \right] \right]. \quad (\text{A-9})$$

Because θ, ε_G and Z are distributed normally, the expectation

$$E \left[v'' \left(\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \tilde{\rho}^{-1} Z) + \bar{T} - \theta \right) \mid \theta + \tilde{\rho}^{-1} Z, \theta + \varepsilon_G \right] \quad (\text{A-10})$$

can be written as a function of $E \left[\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \tilde{\rho}^{-1} Z) + \bar{T} - \theta \mid \theta + \tilde{\rho}^{-1} Z, \theta + \varepsilon_G \right]$ and $\text{var} \left[\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \tilde{\rho}^{-1} Z) + \bar{T} - \theta \mid \theta + \tilde{\rho}^{-1} Z, \theta + \varepsilon_G \right]$. By normality, the conditional variance is independent of the realizations of θ, ε_G and Z ; while by the definition of $w(\tilde{\rho})$ and $K_P(\tilde{\rho})$, $E \left[w(\tilde{\rho}) (\theta + \varepsilon_G) + K_P(\tilde{\rho}) (\theta + \tilde{\rho}^{-1} Z) - \theta \mid \theta + \tilde{\rho}^{-1} Z, \theta + \varepsilon_G \right]$ is a constant. Consequently, when evaluated at $\tilde{w} = w(\tilde{\rho})$ and $\tilde{K}_P = K_P(\tilde{\rho})$, expression (A-10) is a constant, and is strictly negative since v is concave. This in turn implies that expression (A-9) is strictly negative when evaluated at $\tilde{w} = w(\tilde{\rho})$ and $\tilde{K}_P = K_P(\tilde{\rho})$. Consequently, $V_{\tilde{K}_P \tilde{w}}(\tilde{w}, \tilde{K}_P, \bar{T})$ is strictly negative for all (\tilde{w}, \tilde{K}_P) in the neighborhood of $(w(\tilde{\rho}), K_P(\tilde{\rho}))$. Hence if $\tilde{w} < w(\tilde{\rho})$ and $|\tilde{w} - w(\tilde{\rho})|$ is small then $V_{\tilde{K}_P}(\tilde{w}, K_P(\tilde{\rho}), \bar{T})$ is strictly positive, implying $\tilde{K}_P > K_P(\tilde{\rho})$. Likewise, $\tilde{w} > w(\tilde{\rho})$ and $|\tilde{w} - w(\tilde{\rho})|$ small imply $\tilde{K}_P < K_P(\tilde{\rho})$.

QED

Proof of Proposition 2: By definition, $\tilde{w} = w(\rho^*)$ maximizes expression (18) holding $\rho = \rho^*$ fixed. So by the envelope theorem, the derivative of this expression with respect to \tilde{w} is

$$\frac{\partial \rho}{\partial \tilde{w}} \frac{\partial}{\partial \rho} E_{\theta, \varepsilon_G, Z} \left[v \left(\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \rho^{-1} Z) + \bar{T} - \theta \right) - \mu \left(\tilde{w} (\theta + \varepsilon_G) + \tilde{K}_P (\theta + \rho^{-1} Z) + \bar{T} \right) \right].$$

Since v is concave, an increase ρ increases the expected value of the government's objective function by reducing the variance and so creating a second-order stochastically dominant gamble. Hence, this expression is strictly positive if $\frac{\partial \rho}{\partial \tilde{w}} > 0$ and strictly negative if $\frac{\partial \rho}{\partial \tilde{w}} < 0$.

QED

Proof of Lemma 3: Let F be as defined in (A-3) at the start of the appendix, which coincides with the RHS of (17). We know $\alpha\rho^* = F(w(\rho^*), \rho^*)$. Since $v_\varepsilon(\rho)$ is increasing in ρ , it follows that $F_\rho \leq 0$. The unique equilibrium for an exogenous \tilde{w} is given by the solution to $\alpha\rho = F(\tilde{w}, \rho)$. Hence (by Lemma A-1) if $F(\tilde{w}, \rho^*) > F(w(\rho^*), \rho^*)$ the equilibrium at \tilde{w} has $\rho > \rho^*$, while if $F(\tilde{w}, \rho^*) < F(w(\rho^*), \rho^*)$ the equilibrium at \tilde{w} has $\rho < \rho^*$. **QED**

Proof of Proposition 3: By Lemma 3, we must sign the derivative of the RHS of (17) with respect to \tilde{w} . Hence we must sign $\frac{\partial}{\partial \tilde{w}} \frac{\tilde{w}}{N(\tilde{w}, \rho) + \text{var}[\delta]}$, or equivalently, $N(\tilde{w}, \rho) + \text{var}[\delta] - \tilde{w} \frac{\partial N(\tilde{w}, \rho)}{\partial \tilde{w}}$. Using $\frac{\partial N(\tilde{w}, \rho)}{\partial \tilde{w}} = \frac{2N(\tilde{w}, \rho)}{\tilde{w}}$, this expression equals $\text{var}[\delta] - N(\tilde{w}, \rho)$, completing the proof. **QED**

Proof of Corollary 2: For both statements we show that endogenous risk $N(w(\rho^*), \rho^*)$ approaches 0 under the conditions stated, and then apply Proposition 3.

Part (I): We claim, and show below, that $\rho^{*2}\tau_Z \rightarrow \infty$ as either $\alpha \rightarrow 0$ or $\tau_Z \rightarrow \infty$. The result follows easily from the claim, since $\rho^{*2}\tau_Z \rightarrow \infty$ implies that both $v_\varepsilon(\rho^*) \rightarrow \infty$ and $v_G(\rho^*) \rightarrow \infty$, so that $w(\rho^*) \rightarrow 0$ and endogenous risk $N(w(\rho^*), \rho^*) \rightarrow 0$.

To prove the claim for $\alpha \rightarrow 0$, note first that $F(w(\rho), \rho) > 0$ for all ρ , and that $F(w(\rho), \rho)$ is independent of the risk-aversion parameter α . Hence as $\alpha \rightarrow 0$, the minimum solution to $\alpha\rho = F(w(\rho), \rho)$ grows unboundedly large.

For $\tau_Z \rightarrow \infty$, write $F(w(\cdot), \cdot; \tau_z)$ to emphasize the dependence on τ_Z , and observe that the equality $\alpha\rho = F(w(\rho), \rho; \tau_Z)$ is equivalent to the equality $\alpha\tau_Z^{-1/2} \left(\rho\tau_Z^{1/2}\right) = F\left(w\left(\rho\tau_Z^{1/2}\right), \rho\tau_Z^{1/2}; \tau_Z = 1\right)$. So by exactly the same argument as for $\alpha \rightarrow 0$, it follows that as $\tau_z \rightarrow \infty$, $\rho^2\tau_Z$ grows unboundedly large.

Part (II): Rewriting, endogenous risk $N(w(\rho^*), \rho^*)$ equals $w(\rho^*) (w(\rho^*) \text{var}[\theta|s_i, P] + w(\rho^*) \text{var}[\varepsilon_G])$. Observe that $w(\rho^*) \rightarrow 0$ as $\text{var}[\varepsilon_G] \rightarrow \infty$, regardless of how equilibrium informativeness ρ^* changes. Moreover, $w(\rho^*) \text{var}[\varepsilon_G] = 1/v_G(\rho^*)$, which is bounded above by $1/\tau_\theta$; likewise, $\text{var}[\theta|s_i, P] = 1/v_\varepsilon(\rho^*)$ is bounded above $1/\tau_\theta$. Hence $N(w(\rho^*), \rho^*) \rightarrow 0$ as $\text{var}[\varepsilon_G] \rightarrow \infty$.

QED

Proof of Corollary 3: Endogenous risk is bounded above by $\text{var}[\theta + \varepsilon_G]$. Consequently,

for sufficiently large $var[\delta]$ exogenous risk exceeds endogenous risk, and the result follows from Proposition 3. **QED**

Proof of Proposition 4: Let F be defined as in (A-3) at the start of the appendix. A small exogenous change in τ_G affects $F(w(\rho^*), \rho^*)$ according to

$$\frac{\partial F(w(\rho^*), \rho^*)}{\partial \tau_G} + \frac{\partial w(\rho)}{\partial \tau_G} F_w(w(\rho^*), \rho^*).$$

By hypothesis, and from Lemma 3, we know $F_w(w(\rho^*), \rho^*) > 0$. Moreover, $\frac{\partial w(\rho)}{\partial \tau_G} > 0$ and $\frac{\partial F(w(\rho^*), \rho^*)}{\partial \tau_G} > 0$. Hence if τ_G is slightly increased, $F(w(\rho^*), \rho^*) > \rho^*$. By Lemma A-1, $\lim_{\rho \rightarrow \infty} F(w(\rho), \rho)$ is finite, so it follows that at the new τ_G , there exists an equilibrium strictly above ρ^* . **QED**

Proof of Proposition 7: It is straightforward to verify that Lemma 3 continues to hold, so that the effect of a small deviation away from $w(\rho^*)$ is determined by the sign of $F_w(w(\rho^*), \rho^*)$, where $F(w, \rho)$ is defined analogously to (A-3) at the start of the appendix: specifically,

$$F(w, \rho) = \frac{|w(\rho) + c| \tau_\varepsilon v_\varepsilon(\rho)^{-1}}{(w(\rho) + c)^2 v_\varepsilon(\rho)^{-1} + (w(\rho))^2 \tau_G^{-1} + var[\delta]}.$$

As a preliminary, note that $w(\rho^*) + c \neq 0$: if instead $w(\rho^*) + c = 0$, then the $\rho^* = 0$, but by assumption $w(0) + c \neq 0$.

Next, note that if $w(\rho^*) + c > 0$, then $F_w < 0$ since $\frac{w^2}{w+c}$ is increasing in w , and by assumption $var[\delta] = 0$.

Most of the proof deals with the case $w(\rho^*) + c < 0$. In this case, an increase in w away from $w(\rho^*)$ decreases the numerator of $F(w, \rho^*)$. Hence F_w is certainly negative if increasing w also increases the denominator (i.e., residual risk): formally, if $2(w+c)v_\varepsilon^{-1} + 2w\tau_G^{-1} > 0$. Substituting in $w(\rho) = \tau_G v_G(\rho)^{-1}$, this inequality is equivalent to $1 > v_\varepsilon^{-1}(-c v_G - \tau_G)$. Substituting in for v_ε and v_G , the inequality is in turn equivalent to $1 > \frac{(-c)(\tau_\theta + \rho^2 \tau_Z) - (1+c)\tau_G}{\tau_\theta + \rho^2 \tau_Z + \tau_\varepsilon}$.

This inequality is certainly satisfied for $c \geq -1$, completing the proof of part (I).

Finally, for part (II), observe that when $w(\rho^*) + c < 0$,

$$\begin{aligned} \text{sign}(F_w) &= -((w+c)^2 v_\varepsilon^{-1} + w^2 \tau_G^{-1} + \text{var}[\delta]) + (w+c)(2(w+c)v_\varepsilon^{-1} + 2w\tau_G^{-1}) \\ &= (w+c)^2 v_\varepsilon^{-1} - (w^2 - 2w(w+c))\tau_G^{-1} - \text{var}[\delta]. \end{aligned}$$

Since $w^2 - 2w(w+c) = c^2 - (w+c)^2$,

$$\text{sign}(F_w) = (w+c)^2 (v_\varepsilon^{-1} + \tau_G^{-1}) - c^2 \tau_G^{-1} - \text{var}[\delta].$$

We consider the case $c \ll -1$. As a preliminary, observe that equilibrium price informativeness is bounded above by

$$\frac{1}{\alpha} \frac{|w(\rho) + c| \frac{\partial}{\partial s_i} E[\theta | s_i, P]}{(w(\rho) + c)^2 \text{var}[\theta | s_i, P]} = \frac{\tau_\varepsilon}{\alpha} \frac{1}{|w(\rho) + c|} \leq \frac{\tau_\varepsilon}{\alpha} \frac{1}{-1 - c}.$$

Consequently, as $c \rightarrow -\infty$, ρ and hence v_ε remain bounded above. Since

$$\text{sign}(F_w) = \left(\frac{w+c}{c}\right)^2 \left(\frac{\tau_G}{v_\varepsilon} + 1\right) - 1 - \frac{\text{var}[\delta] \tau_G}{c^2},$$

it follows that $F_w > 0$ for all c sufficiently small. **QED**

Proof of Proposition 6

Equilibrium existence

To establish existence, we show that there exist p_0, ρ, ξ, p_Z such that the price function $P = p_0 + \rho p_Z \theta + \xi \rho p_Z \sigma_G + p_Z Z$ is an equilibrium. Specifically, we show that there exist p_0, ρ, ξ, p_Z such that market clearing (6) holds.

First, when the price function is of this form, for the government observing the price is equivalent to observing $\tilde{P}_G \equiv \frac{1}{\rho p_Z} (P - p_0) - \xi \sigma_G = \theta + \rho^{-1} Z$, which is an unbiased estimate of θ , is normally distributed, and has precision $\rho^2 \tau_Z$. Hence by a standard application of Bayes'

rule to normal distributions, the conditional distribution of θ given the information of the government is normal, with conditional variance $\text{var} [\theta|s_G, \tilde{P}_G] = v_G(\rho)^{-1}$ and conditional expectation

$$E [\theta|s_G, \tilde{P}_G] = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}_G + \tau_G s_G}{v_G(\rho)}.$$

Substituting $E [\theta|s_G, \tilde{P}_G]$ and $\text{var} [\theta|s_G, \tilde{P}_G]$ into (7), the government's intervention is

$$T(s_G, \sigma_G, \tilde{P}_G) = \frac{\tau_\theta \bar{\theta} + \rho^2 \tau_Z \tilde{P}_G + \tau_G s_G}{v_G(\rho)} + E[\psi|\sigma_G] + g(\mu, v_G(\rho)^{-1} + \text{var}[\psi|\sigma_G]).$$

For speculator i , when σ_G remains hidden, observing the price P has the same information content as observing $\tilde{P}_H \equiv \frac{1}{p_Z}(P - p_0) = \rho\theta + \xi\rho\sigma_G + Z$. Observe that $\tilde{P}_G = \frac{1}{\rho}\tilde{P}_H - \xi\sigma_G$. So a speculator i 's conditional expectation and conditional variance of T are

$$\begin{aligned} E [T|s_i, \sigma_i, \tilde{P}_H] &= \frac{1}{v_G(\rho)} \left(\tau_\theta \bar{\theta} + \rho^2 \tau_Z E [\tilde{P}_G|s_i, \sigma_i, \tilde{P}_H] + \tau_G E [\theta|s_i, \sigma_i, \tilde{P}_H] \right) \\ &\quad + E \left[E[\psi|\sigma_G] | s_i, \sigma_i, \tilde{P}_H \right] + g(\mu, v_G(\rho)^{-1} + \text{var}[\psi|\sigma_G]) \\ \text{var} [T|s_i, \sigma_i, \tilde{P}_H] &= \text{var} \left[\frac{\rho^2 \tau_Z (-\xi\sigma_G) + \tau_G s_G}{v_G(\rho)} + E[\psi|\sigma_G] | s_i, \sigma_i, \tilde{P}_H \right]. \end{aligned}$$

Speculator i 's demand is

$$x_i(s_i, \sigma_i, \tilde{P}_H) = \frac{1}{\alpha} \frac{E [T|s_i, \sigma_i, \tilde{P}_H] - (p_Z \tilde{P}_H + p_0)}{\text{var} [T|s_i, \sigma_i, \tilde{P}_H] + \text{var} [\delta]}.$$

Since all variables are normally distributed, $x_i(s_i, \sigma_i, \tilde{P}_H)$ is linear in $s_i, \sigma_i, \tilde{P}_H$. Hence $\int x_i(s_i, \sigma_i, \tilde{P}_H) di = x_i(s_i = \theta, \sigma_i = \sigma_G, \tilde{P}_H)$, and the market-clearing condition is simply $x_i(s_i = \theta, \sigma_i = \sigma_G, \tilde{P}_H(\theta, \sigma_G, Z)) + Z = 0$. This is a linear function of θ, σ_G, Z . Hence we must show that there exist p_0, ρ, ξ, p_Z such the intercept and the coefficients on θ, σ_G, Z are all zero, i.e., show that the system of four equations in p_0, ρ, ξ, p_Z has a solution. Note that p_0 only enters the intercept term, so can be freely chosen to set the intercept to 0. The

conditions that the coefficients on θ , σ_G , Z are zero can be written, respectively, as

$$\frac{\partial x_i}{\partial s_i} + \rho \frac{\partial x_i}{\partial \tilde{P}_H} = 0 \quad (\text{A-11})$$

$$\frac{\partial x_i}{\partial \sigma_i} + \xi \rho \frac{\partial x_i}{\partial \tilde{P}_H} = 0 \quad (\text{A-12})$$

$$1 + \frac{\partial x_i}{\partial \tilde{P}_H} = 0. \quad (\text{A-13})$$

To show that this system of equations has a solution, we show that (A-12) together with

$$\xi = \frac{\partial x_i}{\partial \sigma_i} / \frac{\partial x_i}{\partial s_i} \quad (\text{A-14})$$

$$\rho = \frac{\partial x_i}{\partial s_i} \quad (\text{A-15})$$

has a solution. (Note that (A-14) and (A-12) imply (A-11), while (A-14), (A-12) and (A-15) together imply (A-13).)

Note that $\frac{\partial x_i}{\partial \sigma_i}$ and $\frac{\partial x_i}{\partial s_i}$ are independent of p_Z , while $\frac{\partial x_i}{\partial \tilde{P}_H}$ takes the form

$$\frac{\partial x_i}{\partial \tilde{P}_H} = \frac{\frac{\partial}{\partial \tilde{P}_H} E [T | s_i, \sigma_i, \tilde{P}_H] - p_Z}{\alpha \text{var} [T + \delta | s_i, \sigma_i, \tilde{P}_H]}.$$

Hence, given ξ and ρ satisfying (A-14) and (A-15), p_Z can be freely chosen to satisfy (A-12).

Hence it remains to show that there exist ξ and ρ satisfying (A-14) and (A-15).

There exist functions $w(\rho)$ and $q(\xi, \rho)$ such that the derivatives $\frac{\partial x_i}{\partial s_i}$ and $\frac{\partial x_i}{\partial \sigma_i}$ have the form

$$\begin{aligned} \frac{\partial x_i}{\partial s_i} &= \frac{\frac{\partial}{\partial s_i} E [w(\rho) \theta + q(\xi, \rho) \sigma_G | s_i, \sigma_i, \tilde{P}_H]}{\alpha \text{var} [T + \delta | s_i, \sigma_i, \tilde{P}_H]} \\ \frac{\partial x_i}{\partial \sigma_i} &= \frac{\frac{\partial}{\partial \sigma_i} E [w(\rho) \theta + q(\xi, \rho) \sigma_G | s_i, \sigma_i, \tilde{P}_H]}{\alpha \text{var} [T + \delta | s_i, \sigma_i, \tilde{P}_H]}. \end{aligned}$$

In particular, $w(\rho) = \frac{\tau_G}{v_G(\rho)}$ as before, while there exist $\kappa \in [0, 1]$ such that $q(\xi, \rho) = \kappa - \frac{\rho^2 \xi \tau_Z}{v_G(\rho)}$. A standard (though tedious) application of the properties of multivariate normal distributions yields the following result; the proof is at the end of the paper:

Lemma A-2

$$\begin{aligned}
E \left[w\theta + q\sigma_G | \tilde{P}_H, s_i, \sigma_i \right] &= w(\rho^2 \xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i]) \frac{\text{var}[\theta]}{|\Sigma_{22}|} s_i \\
&+ q(-\rho \rho \xi \text{var}[\zeta_i] \text{var}[\sigma_G]) \frac{\text{var}[\theta]}{|\Sigma_{22}|} s_i \\
&+ w(-\rho \rho \xi \text{var}[\theta] \text{var}[\varepsilon_i]) \frac{\text{var}[\sigma_G]}{|\Sigma_{22}|} \sigma_i \\
&+ q(\rho^2 \text{var}[\theta] \text{var}[\varepsilon_i] + \text{var}[Z] \text{var}[s_i]) \frac{\text{var}[\sigma_G]}{|\Sigma_{22}|} \sigma_i \\
&+ \text{terms independent of } s_i, \sigma_i
\end{aligned}$$

where

$$|\Sigma_{22}| = \text{var}[Z] \text{var}[s_i] \text{var}[\sigma_i] + \rho^2 \text{var}[\theta] \text{var}[\varepsilon_i] \text{var}[\sigma_i] + \rho^2 \xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] \text{var}[s_i].$$

Applying Lemma A-2 yields

$$\frac{\frac{\partial x_i}{\partial \sigma_i}}{\frac{\partial x_i}{\partial s_i}} = \frac{w(\rho)(-\rho^2 \xi \text{var}[\theta] \text{var}[\varepsilon_i]) + q(\xi, \rho)(\rho^2 \text{var}[\theta] \text{var}[\varepsilon_i] + \text{var}[Z] \text{var}[s_i])}{w(\rho)(\rho^2 \xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i]) + q(\xi, \rho)(-\rho^2 \xi \text{var}[\zeta_i] \text{var}[\sigma_G])} \frac{\text{var}[\sigma_G]}{\text{var}[\theta]}.$$

We first show that, for any given ρ , there exists ξ satisfying

$$\xi = \frac{w(\rho)(-\rho^2 \xi \text{var}[\theta] \text{var}[\varepsilon_i]) + q(\xi, \rho)(\rho^2 \text{var}[\theta] \text{var}[\varepsilon_i] + \text{var}[Z] \text{var}[s_i])}{w(\rho)(\rho^2 \xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i]) + q(\xi, \rho)(-\rho^2 \xi \text{var}[\zeta_i] \text{var}[\sigma_G])} \frac{\text{var}[\sigma_G]}{\text{var}[\theta]}.$$

To see this, note that rearrangement of this equality yields

$$\begin{aligned}
&w(\rho) \xi \text{var}[\theta] (\rho^2 \xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i]) + w(\rho) \text{var}[\sigma_G] \rho^2 \xi \text{var}[\theta] \text{var}[\varepsilon_i] \\
&= q(\xi, \rho) \text{var}[\sigma_G] (\rho^2 \text{var}[\theta] \text{var}[\varepsilon_i] + \text{var}[Z] \text{var}[s_i]) + q(\xi, \rho) \text{var}[\theta] \rho^2 \xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G],
\end{aligned}$$

so that

$$\frac{q(\xi, \rho)}{\xi w(\rho)} = \frac{\rho^2 \xi^2 \text{var}[\theta] \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[\theta] \text{var}[Z] \text{var}[\sigma_i] + \rho^2 \text{var}[\sigma_G] \text{var}[\theta] \text{var}[\varepsilon_i]}{\rho^2 \text{var}[\sigma_G] \text{var}[\theta] \text{var}[\varepsilon_i] + \text{var}[\sigma_G] \text{var}[Z] \text{var}[s_i] + \rho^2 \xi^2 \text{var}[\theta] \text{var}[\zeta_i] \text{var}[\sigma_G]}.$$

(A-16)

The RHS is continuous as a function of ξ , is finite at $\xi = 0$, and converges to 1 as $\xi \rightarrow \infty$. Since $q(\xi, \rho) / \xi = \kappa / \xi - \frac{\rho^2 \tau_Z}{v_G(\rho)}$, continuity implies that for any ρ there exists $\xi > 0$ such that this equality is satisfied. Denote the smallest such solution by $\xi(\rho)$. Observe that $\xi(\rho)$ is continuous in ρ , and remains bounded away from both 0 and ∞ both as $\rho \rightarrow 0$ and as $\rho \rightarrow \infty$.

To complete the proof of existence, we must show there exists ρ satisfying $\rho = \frac{\partial x_i}{\partial s_i}$, where $\frac{\partial x_i}{\partial s_i}$ is evaluated using ρ and $\xi(\rho)$. Again, this follows from continuity, as follows. As $\rho \rightarrow 0$, the government ignores the price, and $\frac{\partial x_i}{\partial s_i}$ remains bounded away from 0 since the signal s_i gives speculator i information about the government's signal s_G . As $\rho \rightarrow \infty$, the government ignores its own signal s_G , and the price conveys the same information to speculator i as does $\theta + \xi \sigma_G$; consequently, a change in s_i changes the speculator's expectation about σ_G , but the effect is finite (and moreover, $\text{var}[T|s_i, \sigma_i, \tilde{P}_H]$ remains bounded away from 0).

Increased informativeness under transparency

Writing $\rho = \frac{\partial x_i}{\partial s_i}$ explicitly, price informativeness when the government does not disclose σ_G is given by the solution to

$$\alpha \rho = \frac{(w(\rho) (\rho^2 \xi(\rho))^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i]) + q(\xi(\rho), \rho) (-\rho^2 \xi(\rho) \text{var}[\zeta_i] \text{var}[\sigma_G])) \frac{\text{var}[\theta]}{|\Sigma_{22}|}}{\text{var}[T|s_i, \sigma_i, \tilde{P}_H] + \text{var}[\delta]}.$$

(A-17)

If instead the government makes its signal σ_G public, i.e., transparency, price informativeness is determined by the solution to

$$\alpha \rho = \frac{w(\rho) \frac{\partial}{\partial s_i} E[\theta|s_i, P]}{\text{var}[T|s_i, P] + \text{var}[\delta]}.$$

(A-18)

To establish the result, we show that, for any ρ , the RHS of (A-17) is strictly less than the RHS of (A-18). It is immediate that, for any ρ , the denominator in (A-18) is less than the denominator in (A-17). The main step in the proof is to show that the numerator in (A-17) is less than the numerator in (A-18).

Expanding, the numerator in (A-18) is

$$\frac{w(\rho)\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \rho^2\tau_Z} = \frac{w(\rho)\text{var}[\theta]\text{var}[Z]}{\text{var}[\varepsilon_i]\text{var}[Z] + \text{var}[\theta]\text{var}[Z] + \rho^2\text{var}[\theta]\text{var}[\varepsilon_i]}.$$

Define

$$\begin{aligned} D(\rho) &= \text{var}[\varepsilon_i]\text{var}[Z] + \text{var}[\theta]\text{var}[Z] + \rho^2\text{var}[\theta]\text{var}[\varepsilon_i] \\ &= \text{var}[s_i]\text{var}[Z] + \rho^2\text{var}[\theta]\text{var}[\varepsilon_i]. \end{aligned}$$

Using this definition,

$$|\Sigma_{22}| = D(\rho)\text{var}[\sigma_i] + \rho^2\xi^2\text{var}[\zeta_i]\text{var}[\sigma_G]\text{var}[s_i].$$

Hence we must show

$$\begin{aligned} & \frac{w(\rho)\text{var}[\theta]\text{var}[Z]}{D(\rho)} \\ & > \frac{w(\rho)(\rho^2\xi(\rho)^2\text{var}[\zeta_i]\text{var}[\sigma_G] + \text{var}[Z]\text{var}[\sigma_i]) + q(\xi(\rho), \rho)(-\rho^2\xi(\rho)\text{var}[\zeta_i]\text{var}[\sigma_G])}{D(\rho)\text{var}[\sigma_i] + \rho^2\xi(\rho)^2\text{var}[\zeta_i]\text{var}[\sigma_G]\text{var}[s_i]}\text{var}[\theta], \end{aligned}$$

or equivalently,

$$\begin{aligned} & w(\rho)\text{var}[Z](D(\rho)\text{var}[\sigma_i] + \rho^2\xi(\rho)^2\text{var}[\zeta_i]\text{var}[\sigma_G]\text{var}[s_i]) \\ & > D(\rho)(w(\rho)(\rho^2\xi(\rho)^2\text{var}[\zeta_i]\text{var}[\sigma_G] + \text{var}[Z]\text{var}[\sigma_i]) - q(\xi, \rho)\rho^2\xi(\rho)\text{var}[\zeta_i]\text{var}[\sigma_G]) \end{aligned}$$

or equivalently,

$$\begin{aligned} & w(\rho) \operatorname{var}[Z] \rho^2 \xi(\rho)^2 \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G] \operatorname{var}[s_i] \\ & > D(\rho) \left(w(\rho) \rho^2 \xi(\rho)^2 \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G] - q(\xi(\rho), \rho) \rho^2 \xi(\rho) \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G] \right), \end{aligned}$$

or equivalently,

$$w(\rho) \rho^2 \xi(\rho)^2 \operatorname{var}[Z] \operatorname{var}[s_i] > D(\rho) \left(w(\rho) \rho^2 \xi(\rho)^2 - q(\xi(\rho), \rho) \rho^2 \xi(\rho) \right),$$

or equivalently,

$$w(\rho) \rho^2 \xi(\rho)^2 \left(\operatorname{var}[Z] \operatorname{var}[s_i] - \left(1 - \frac{q(\xi(\rho), \rho)}{w(\rho) \xi(\rho)} \right) D(\rho) \right) > 0,$$

or equivalently (using $w(\rho) > 0$),

$$\frac{q(\xi(\rho), \rho)}{w(\rho) \xi(\rho)} > 1 - \frac{\operatorname{var}[Z] \operatorname{var}[s_i]}{D(\rho)} = \frac{\rho^2 \operatorname{var}[\theta] \operatorname{var}[\varepsilon_i]}{D(\rho)}.$$

Substituting in (A-16), along with the definition of $D(\rho)$, this inequality is in turn equivalent to

$$\begin{aligned} & \frac{\rho^2 \xi(\rho)^2 \operatorname{var}[\theta] \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G] + \operatorname{var}[\theta] \operatorname{var}[Z] \operatorname{var}[\sigma_i] + \rho^2 \operatorname{var}[\sigma_G] \operatorname{var}[\theta] \operatorname{var}[\varepsilon_i]}{\operatorname{var}[\sigma_G] D(\rho) + \rho^2 \xi(\rho)^2 \operatorname{var}[\theta] \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G]} \\ & > \frac{\rho^2 \operatorname{var}[\theta] \operatorname{var}[\varepsilon_i]}{D(\rho)}, \end{aligned}$$

or equivalently,

$$\frac{\rho^2 \xi(\rho)^2 \operatorname{var}[\theta] \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G] + \operatorname{var}[\theta] \operatorname{var}[Z] \operatorname{var}[\sigma_i]}{\rho^2 \xi(\rho)^2 \operatorname{var}[\theta] \operatorname{var}[\zeta_i] \operatorname{var}[\sigma_G]} > \frac{\rho^2 \operatorname{var}[\theta] \operatorname{var}[\varepsilon_i]}{D(\rho)}.$$

This is indeed the case since the LHS is above 1 and the RHS is below 1, completing the proof.

Standard analysis of multivariate normal

Proof of Lemma A-2: Define

$$\Sigma_{22} = \begin{pmatrix} \text{var} [\tilde{P}_H] & \text{cov} [\tilde{P}_H, s_i] & \text{cov} [\tilde{P}_H, \sigma_i] \\ \text{cov} [s_i, \tilde{P}_H] & \text{var} [s_i] & \text{cov} [s_i, \sigma_i] \\ \text{cov} [\sigma_i, \tilde{P}_H] & \text{cov} [\sigma_i, s_i] & \text{var} [\sigma_i] \end{pmatrix} = \begin{pmatrix} \text{var} [\tilde{P}_H] & \rho \text{var} [\theta] & \rho \xi \text{var} [\sigma_G] \\ \rho \text{var} [\theta] & \text{var} [s_i] & 0 \\ \rho \xi \text{var} [\sigma_G] & 0 & \text{var} [\sigma_i] \end{pmatrix}.$$

and

$$\Sigma_{12} = \begin{pmatrix} \text{cov} [\theta, \tilde{P}_H] & \text{cov} [\theta, s_i] & \text{cov} [\theta, \sigma_i] \\ \text{cov} [\sigma_G, \tilde{P}_H] & \text{cov} [\sigma_G, s_i] & \text{var} [\sigma_G] \end{pmatrix} = \begin{pmatrix} \rho \text{var} [\theta] & \text{var} [\theta] & 0 \\ \rho \xi \text{var} [\sigma_G] & 0 & \text{var} [\sigma_G] \end{pmatrix}.$$

Then

$$E [w\theta + q\sigma_G | \tilde{P}_H, s_i, \sigma_i] = \begin{pmatrix} w & q \end{pmatrix} \Sigma_{12} \Sigma_{22}^{-1} \begin{pmatrix} \tilde{P}_H \\ s_i \\ \sigma_i \end{pmatrix}.$$

Evaluating

$$\Sigma_{22}^{-1} = \frac{1}{|\Sigma_{22}|} \begin{pmatrix} \text{var} [s_i] \text{var} [\sigma_i] & -\rho \text{var} [\theta] \text{var} [\sigma_i] & -\rho \xi \text{var} [\sigma_G] \text{var} [s_i] \\ -\rho \text{var} [\theta] \text{var} [\sigma_i] & \text{var} [\tilde{P}_H] \text{var} [\sigma_i] - (\rho \xi \text{var} [\sigma_G])^2 & \rho \text{var} [\theta] \rho \xi \text{var} [\sigma_G] \\ -\rho \xi \text{var} [\sigma_G] \text{var} [s_i] & \rho \text{var} [\theta] \rho \xi \text{var} [\sigma_G] & \text{var} [\tilde{P}_H] \text{var} [s_i] - (\rho \text{var} [\theta])^2 \end{pmatrix},$$

where

$$\begin{aligned} |\Sigma_{22}| &= \text{var} [\tilde{P}_H] \text{var} [s_i] \text{var} [\sigma_i] - (\rho \text{var} [\theta])^2 \text{var} [\sigma_i] - (\rho \xi \text{var} [\sigma_G])^2 \text{var} [s_i] \\ &= \text{var} [Z] \text{var} [s_i] \text{var} [\sigma_i] + \rho^2 \text{var} [\theta] \text{var} [\varepsilon_i] \text{var} [\sigma_i] + \rho^2 \xi^2 \text{var} [\zeta_i] \text{var} [\sigma_G] \text{var} [s_i]. \end{aligned}$$

So

$$\begin{aligned}
\Sigma_{12}\Sigma_{22}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} &= \frac{1}{|\Sigma_{22}|} \Sigma_{12} \begin{pmatrix} -\rho \text{var}[\theta] \text{var}[\sigma_i] \\ \text{var}[\tilde{P}_H] \text{var}[\sigma_i] - (\rho\xi \text{var}[\sigma_G])^2 \\ \rho \text{var}[\theta] \rho\xi \text{var}[\sigma_G] \end{pmatrix} \\
&= \frac{\text{var}[\theta]}{|\Sigma_{22}|} \begin{pmatrix} -\rho^2 \text{var}[\theta] \text{var}[\sigma_i] + \text{var}[\tilde{P}_H] \text{var}[\sigma_i] - (\rho\xi \text{var}[\sigma_G])^2 \\ -\rho\rho\xi \text{var}[\sigma_i] \text{var}[\sigma_G] + \rho\rho\xi \text{var}[\sigma_G]^2 \end{pmatrix} \\
&= \frac{\text{var}[\theta]}{|\Sigma_{22}|} \begin{pmatrix} \rho^2\xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i] \\ -\rho\rho\xi \text{var}[\zeta_i] \text{var}[\sigma_G] \end{pmatrix}
\end{aligned}$$

where the final inequality follows from

$$\begin{aligned}
&-\rho^2 \text{var}[\theta] \text{var}[\sigma_i] + \text{var}[\tilde{P}_H] \text{var}[\sigma_i] - (\rho\xi \text{var}[\sigma_G])^2 \\
&= -\rho^2 \text{var}[\theta] \text{var}[\sigma_i] + (\rho^2 \text{var}[\theta] + \rho^2\xi^2 \text{var}[\sigma_G] + \text{var}[Z]) \text{var}[\sigma_i] - (\rho\xi \text{var}[\sigma_G])^2 \\
&= (\rho^2\xi^2 \text{var}[\sigma_G] + \text{var}[Z]) \text{var}[\sigma_i] - (\rho\xi \text{var}[\sigma_G])^2 \\
&= \rho^2\xi^2 \text{var}[\zeta_i] \text{var}[\sigma_G] + \text{var}[Z] \text{var}[\sigma_i].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Sigma_{12}\Sigma_{22}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} &= \frac{1}{|\Sigma_{22}|} \Sigma_{12} \begin{pmatrix} -\rho\xi \text{var}[\sigma_G] \text{var}[s_i] \\ \rho \text{var}[\theta] \rho\xi \text{var}[\sigma_G] \\ \text{var}[\tilde{P}_H] \text{var}[s_i] - (\rho \text{var}[\theta])^2 \end{pmatrix} \\
&= \frac{\text{var}[\sigma_G]}{|\Sigma_{22}|} \begin{pmatrix} -\rho \text{var}[\theta] \rho\xi \text{var}[s_i] + \text{var}[\theta] \rho \text{var}[\theta] \rho\xi \\ -\rho\xi \text{var}[\sigma_G] \rho\xi \text{var}[s_i] + \text{var}[\tilde{P}_H] \text{var}[s_i] - (\rho \text{var}[\theta])^2 \end{pmatrix} \\
&= \frac{\text{var}[\sigma_G]}{|\Sigma_{22}|} \begin{pmatrix} -\rho\rho\xi \text{var}[\theta] \text{var}[\varepsilon_i] \\ \rho^2 \text{var}[\theta] \text{var}[\varepsilon_i] + \text{var}[Z] \text{var}[s_i] \end{pmatrix}
\end{aligned}$$

where the final inequality follows from

$$\begin{aligned}
& -\rho\xi\text{var}[\sigma_G]\xi\rho\text{var}[s_i] + \text{var}[\tilde{P}_H]\text{var}[s_i] - (\rho\text{var}[\theta])^2 \\
= & -\rho^2\xi^2\text{var}[\sigma_G]\text{var}[s_i] + (\rho^2\xi^2\text{var}[\sigma_G] + \rho^2\text{var}[\theta] + \text{var}[Z])\text{var}[s_i] - \rho^2\text{var}[\theta]^2 \\
= & (\rho^2\text{var}[\theta] + \text{var}[Z])\text{var}[s_i] - \rho^2\text{var}[\theta]^2 \\
= & \rho^2\text{var}[\theta]\text{var}[\varepsilon_i] + \text{var}[Z]\text{var}[s_i].
\end{aligned}$$

The result follows. **QED**