

Individual Preferences, Monetary Gambles and the Equity Premium

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Abstract

We present evidence on typical attitudes to monetary gambles with both large and small stakes and ask what kinds of intertemporal preferences can capture these attitudes. We find that among a wide range of utility functions, including all expected utility and many non-expected utility specifications, the only ones that can easily capture these attitudes are those exhibiting both “first-order risk aversion” and “narrow framing.” We apply our result to understanding the kinds of equity premia generated by various preference specifications in a simple representative agent economy.

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1 Introduction

In the first part of this paper, we present some evidence on typical attitudes to simple monetary gambles and ask what kinds of intertemporal utility functions can best explain this evidence. In the second part, we argue that our results may shed light on an issue of particular interest to financial economists, namely the magnitude of the equity premia generated by various preference specifications in a simple representative agent economy.

We start by gathering evidence on how people react to the simplest imaginable form of risk – monetary gambles with just two outcomes, a gain and a loss, with stakes both large and small. We find that across a range of wealth levels, people typically reject gambles with small to moderate stakes in which the potential gain is less than twice the potential loss. For example, the majority of subjects we surveyed turned down a 50:50 bet offering a \$550 gain against a \$500 loss. Turning to gambles with large stakes, we find that the majority of subjects accept a 50:50 bet offering a \$20 million gain against a \$10,000 loss.

We then ask what kinds of intertemporal utility functions can best explain these attitudes to risk. While the observations we are trying to explain seem very reasonable, we find that a surprisingly wide range of utility functions, covering all expected utility and many non-expected utility specifications, have great difficulty capturing them. Among utility functions commonly used by financial economists, the only ones that *can* address these observations with any degree of satisfaction are those exhibiting both *first-order risk aversion* and *narrow framing*. First-order risk aversion means that a utility function is locally risk averse, unlike many standard preferences that are smooth and therefore locally risk-neutral; a utility function with a kink at the agent’s current wealth, for example, would exhibit this feature. Narrow framing means that to some extent, the agent evaluates new gambles she is offered in isolation from other risks that she faces. More formally, her utility function depends on the outcomes of specific gambles she faces over and above what those outcomes mean for her aggregate wealth or consumption risk.

To see why these two features are necessary, consider first a utility function without first-order risk aversion, in other words, one which is locally smooth. Since the agent is locally risk-neutral, she will normally be very happy to accept a small, actuarially attractive gamble like the coin flip bet to win \$550 or lose \$500. In order to explain the commonly observed rejection of such gambles, then, we need to push risk aversion up to very high levels. However, risk aversion will then be *so* high as to make the agent reject some apparently very favorable gambles with larger stakes, such as the 20 million/10,000 bet. To avoid such counterfactual predictions, we need utility functions that are locally risk averse, not locally risk-neutral; in other words, utility functions exhibiting first-order risk aversion.

This argument for first-order risk aversion has already appeared in various guises in the literature. Our more novel contribution, however, is to show that to explain attitudes to monetary gambles and in particular, aversion to small gambles, we need not only first-order

risk aversion but narrow framing as well. The intuition for why first-order risk aversion is not enough is straightforward. Suppose that an investor with first-order risk aversion, but who does *not* engage in narrow framing, is offered a small, independent gamble to be resolved at some point in the future, and that the gamble is actuarially attractive. Now also make the reasonable assumption that the investor faces some pre-existing risks: labor income risk perhaps, house price risk, or other financial market risk. The investor will decide whether to take on the new gamble by merging it with her pre-existing risk and checking to see if the combination is attractive. It turns out that the combination *is* almost always attractive: since the new gamble is independent of the agent’s other risks, it brings her useful diversification benefits. Even though she is first-order risk averse, she happily accepts it. The only way to get the agent to reject the gamble is, once again, to set risk aversion to extraordinarily high levels. However, this again implies, counterfactually, that the agent would reject some attractive gambles with larger stakes.

In order to explain the commonly observed aversion to a small gamble, then, it must be that the investor does *not* fully merge it with pre-existing risks, but that to some extent, she evaluates it in isolation. Put differently, her decision utility must depend on the outcome of the gamble over and above what that outcome means for the risk exposure of her aggregate wealth; more simply, her utility function must exhibit narrow framing.

While this result may be interesting in and of itself, we argue that it may also shed light on a topic of particular interest to financial economists, namely the magnitude of the equity premia generated by various kinds of intertemporal utility functions in simple representative agent economies.

To illustrate this, we consider an endowment economy with a representative agent in which consumption growth is i.i.d. and has low volatility. We then consider a range of preference specifications, drawn from a number of different classes of utility functions, and compute the equity premia they deliver. Following the standard practice of financial economists, we restrict ourselves to parameterizations that are “reasonable” in the sense that they make sensible predictions about attitudes to large gambles – for example, acceptance of the 20 million/10,000 gamble.

We find that when utility functions are restricted in this way, those exhibiting first-order risk aversion and narrow framing deliver by far the largest equity premium among all those considered. The preferences without narrow framing that we consider, whether expected utility or non-expected utility, all deliver equity premia of the same, smaller, order of magnitude.

We argue that our equity premium results can be understood in terms of our earlier findings. The idea is to think of “equity” playing the same role in the equity premium calculations as the 550/500 gamble did in the earlier analysis. At first sight, this may seem odd – the stock market is nowhere near as small a gamble as 550/500, nor is it independent of

investors' marginal utility, as the 550/500 gamble was. However, *in the simple representative agent economy we consider*, stocks are *small enough* a gamble and *uncorrelated enough* with marginal utility that our earlier results continue to hold, at least approximately, when "equity" is substituted for the 550/500 gamble.

In other words, just as utility functions with first-order risk aversion and narrow framing are best able to explain an aversion to 550/500 at the same time as a liking for 20 million/10,000, so such utility functions are also best able to explain aversion to the stock market – and hence a large equity premium – at the same time as a liking for 20 million/10,000.

Moreover, the intuition for why narrow framing is helpful for generating an aversion to stocks is similar to the intuition for why it is helpful for generating an aversion to the 550/500 gamble. In the absence of narrow framing, people with pre-existing risks are happy to accept a stock gamble because after merging it with those other risks, they find themselves to be better diversified. If they are to be averse to the stock market then, it must be that they do not fully merge the stock gamble with their pre-existing risks, but rather that they evaluate stocks in isolation, to some extent.

Our research builds on earlier work investigating how, in static settings, individuals with various preference specifications react to monetary gambles of different sizes. Kandel and Stambaugh (1990) point out that power utility functions have trouble simultaneously explaining attitudes to both large and small scale gambles, while Rabin (2000) shows that this problem extends to all expected utility functions. One contribution of our research is to check whether these static arguments apply to the intertemporal setting used by financial economists – we find that they do, albeit with some interesting caveats. Another is to show that even more general types of preferences, including those exhibiting first-order risk aversion, also have difficulty capturing attitudes to monetary gambles.

Barberis, Huang and Santos (2001), building on Benartzi and Thaler (1995), show that in a representative agent economy, preferences with first-order risk aversion and narrow framing can generate a substantial equity premium even when the volatility of consumption growth is very low. This result alone does not necessarily distinguish these preferences from others. In this paper, we make a more forceful case in favor of such preferences, by showing the advantages they have over other utility functions in explaining the equity premium and attitudes to monetary gambles at the same time.

Epstein and Zin (1990) and Epstein (1992) argue that in representative agent economies, stocks are effectively a small risk, making it important that the preferences we use to model attitudes to stocks do a good job capturing attitudes to small gambles. They use this reasoning as a way of motivating an investigation of first-order risk averse preferences. We agree with this line of thinking, but show that to capture attitudes to small gambles successfully, both first-order risk aversion *and* narrow framing are required, not first-order risk aversion

alone.

In Section 2, we discuss common attitudes to simple monetary gambles and introduce various classes of utility functions whose ability to match those attitudes we are interested in. In Section 3, we show that without first-order risk aversion, it is hard to match these attitudes. In Section 4, we show that even first-order risk aversion is not enough and that narrow framing is required as well. In Section 5, we apply our results to understanding what kinds of utility functions are best able to generate substantial equity premia in certain simple representative agent economies. Section 6 concludes.

2 Attitudes to Monetary Gambles

Consider the small-stakes gamble¹

$$G_S = (550, \frac{1}{2}; -500, \frac{1}{2}),$$

to be read as “gain \$550 with probability $\frac{1}{2}$ and lose \$500 with probability $\frac{1}{2}$.” It is the premise of this paper that people find this gamble unattractive. We base this premise on dozens of experimental studies, some with real money and some based on hypothetical questions, showing that people typically reject gambles with two equiprobable outcomes of small to moderate size when the potential gain is less than twice the potential loss.

To check that this premise holds even for very wealthy individuals, and not just for the college students often used in earlier studies, we conducted some additional experiments of our own. Our study is based on four different groups of subjects: (i) 68 part-time MBA students at the University of Chicago; (ii) 30 financial advisors at a mid-size U.S. brokerage firm; (iii) 19 Chief Investment Officers and Directors of quantitative equity research at large asset management firms; and (iv) 34 clients of the private wealth management division of a U.S. bank. The median wealth of the subjects in this last group exceeds \$10 million.

We asked each group for their reaction to G_S and other similar gambles. We did not play the gambles for real money but simply asked subjects to think hard about how they would choose. Table 1 presents the results. It confirms that for a majority of subjects in all four groups, G_S was indeed unattractive. While the wealthiest group, group (iv), did accept G_S more often than other groups, the majority of subjects in even that group continued to reject it.

Since economists are often skeptical of answers to hypothetical questions, we also conducted a real-money experiment with one last group of subjects, group (v), consisting of 41 part-time MBA students at the University of Chicago. They were asked whether they would play $(110, \frac{1}{2}; -100, \frac{1}{2})$ for real money. To be specific, they were told that if they wished to

¹The “S” subscript in G_S stands for Small stakes.

accept this gamble, they should indicate so on the experimental form, and then come to class the following week with the \$100 they would need in case they lost the gamble. They were informed that if they won, they would be paid immediately in cash. Of the 41 students that participated, only 4, or 10%, were willing to accept the gamble. Given that 24% of the MBA students in group (i) were willing to accept a hypothetical version of this gamble, it appears that playing for real money only makes gambles like G_S even more unattractive.²

In Sections 3 and 4, we investigate what kinds of utility functions can capture the commonly observed aversion to gambles like G_S . Of course, many utility functions can explain this evidence simply by assuming sufficiently high risk aversion. To provide a reasonable upper bound on individual risk aversion, we introduce a new gamble involving larger stakes,³

$$G_L = (20,000,000, \frac{1}{2}; -10,000, \frac{1}{2}).$$

It is the premise of this paper that this bet is typically *accepted*. To confirm this, we presented groups (i) to (iv) with this bet as well, and found that the vast majority of subjects in all four groups was indeed willing to accept.

In summary, then, we are interested in what kinds of preference specifications can explain the two observations, “ G_S is rejected” and “ G_L is accepted.” We do not insist that a utility function be able to explain these observations at *all* wealth levels. Rather, we make the weaker demand that they explain them over a reasonable range of wealth levels – neither too high nor too low. To be precise, we check whether:

- I. G_S is rejected for wealth levels $W \leq \$1,000,000$
- II. G_L is accepted for wealth levels $W \geq \$100,000$.

When we check utility functions’ ability to explain these observations, it can make a difference, for certain utility specifications, whether the gambles are “immediate” or “delayed.” A gamble is immediate if its uncertainty is resolved at once, before any further consumption decisions are made. A delayed gamble, on the other hand, might be played out as follows: in the case of G_S , the subject is told that at some point in the next few months, she will be

²Our results are not unique, and are in fact highly representative of previous results from the experimental literature. For example, a recent paper by Holt and Laury (2002) supports two of our findings. First, they find that subjects are even more risk averse for real choices than for hypothetical choices. Second, they find substantial risk aversion over gambles with stakes of similar magnitude to G_S : over sixty percent of their subjects preferred the safe bet (180, 0.6; 144, 0.4) to the risky bet (346, 0.6; 9, 0.4) even though the risky bet has an expected value that is \$45.60 higher (see also Binswanger, 1980). Similar behavior is observed in real world choices. For example, Grgeta and Thaler (2003) find that during the 1994-96 period, more than half of the purchasers of collision insurance in their sample elected a deductible of \$250 or less. The typical consumer could save about \$80 a year by increasing the deductible from \$250 to \$500. To justify the lower deductible people would have to file claims one year in three, but in fact the probability of a claim is less than one in ten.

³The L subscript on G_L stands for Large stakes.

contacted and informed either that she has just won \$550 or that she has lost \$500, the two outcomes being equally probable and independent of other risks.

Although certain utility functions can make a distinction between immediate and delayed gambles, we think that in reality, people do not treat the two kinds of bets very differently. To test this intuition, we asked the subjects in group (v) one additional hypothetical question: whether they would accept a 110/100 gamble if it were played out on a day picked at random during 2003. (The survey was conducted in October 2002.) The subjects largely shared our intuition: only 9 of the 41 subjects, or 22%, were willing to accept the delayed gamble, a fraction very similar to the fraction of MBA students in group (i) willing to accept the immediate version of the gamble, namely 24%.

In view of this evidence, we insist that the preference specifications we consider be able to capture observations I and II in both cases, immediate and delayed. For the first part of our analysis, we will only need to work with the computationally simpler immediate gambles: it turns out that many classes of utility functions have trouble explaining attitudes even to those gambles. In cases where utility functions *are* able to capture attitudes to immediate gambles, we challenge them with delayed gambles as well.

2.1 Utility Functions

We now introduce the different classes of utility functions whose ability to capture observations I and II we are interested in. We list them in increasing order of sophistication, along with the abbreviations we use to refer to them. Our list is in no way intended to be an exhaustive list of preference specifications. We focus only on the classes of utility functions that appear regularly in the asset pricing literature.

Expected utility preferences [EU]

Non-expected utility preferences:

Recursive utility with EU certainty equivalent [R-EU]

Recursive utility with non-EU, second-order risk averse certainty equivalent [R-SORA]

Recursive utility with non-EU, first-order risk averse certainty equivalent [R-FORA]

Expected utility preferences are familiar enough. What about non-expected utility specifications? In the intertemporal setting favored by researchers in the field of asset pricing, non-expected utility is typically implemented via a recursive structure in which time t utility, V_t , is defined through

$$V_t = W(C_t, \mu(\tilde{V}_{t+1}|I_t)). \quad (1)$$

Here $\mu(\tilde{V}_{t+1}|I_t)$ is the certainty equivalent of the distribution of future utility \tilde{V}_{t+1} conditional on time t information, and W is an aggregator function which aggregates current consumption C_t with the certainty equivalent of future utility to give current utility.

We consider three kinds of recursive utility. They differ in the properties they impose on μ . One property that plays an important role is the *order* of risk aversion built into μ , and in particular whether μ exhibits “second-order” risk aversion or “first-order” risk aversion, terms originally coined by Segal and Spivak (1990). An agent’s utility function exhibits second-order risk aversion if the premium the agent pays to avoid an actuarially fair gamble $k\tilde{\varepsilon}$ is, as $k \rightarrow 0$, proportional to k^2 . In simple terms, such utility functions are smooth and the investor is almost risk-neutral for small risks. First-order risk averse utility functions, on the other hand, are preferences where the premium paid to avoid an actuarially fair gamble $k\tilde{\varepsilon}$ is, as $k \rightarrow 0$, proportional to k . In this case, the investor is risk averse even over infinitesimal bets. A simple example of a utility function with this property is one exhibiting loss aversion, or a kink at the agent’s current wealth.

Utility functions in the expected utility class can generically only exhibit second-order risk aversion: an increasing, concave utility function can only have a kink at a countable number of points. Non-expected utility functions, on the other hand, can exhibit either second-order or first-order risk aversion, and it is important to consider the two cases separately.

We now describe the three kinds of recursive utility in more detail. First, we look at recursive preferences in which the certainty equivalent function μ has the expected utility form,

$$\mu(\tilde{X}) = h^{-1}Eh(\tilde{X}).$$

As noted above, we denote preferences in this class as R-EU.⁴ Most implementations of recursive utility that have appeared in the asset pricing literature, including those of Epstein and Zin (1991a), Campbell (1996) and Campbell and Viceira (1999), are of the R-EU form. Researchers have made use of such preferences primarily because they offer a simple way of separating risk aversion and intertemporal elasticity of substitution, something which cannot be done satisfactorily within the expected utility class (Epstein, 1992).

Next we consider recursive utility in which μ is in the non-expected utility class and exhibits second-order risk aversion (R-SORA). Such preferences appear more rarely in the asset pricing literature: since the usual objective when using recursive utility – the separation of risk aversion and intertemporal substitution – can be accomplished with the simpler R-EU preferences, fewer studies adopt R-SORA preferences. One exception is Epstein and Zin (1991b).

Finally, we consider recursive utility in which μ is again non-expected utility, but now exhibits first-order risk aversion (R-FORA). Such preferences have again only rarely appeared

⁴Note that even though μ is in the expected utility class, intertemporal utility V_t is still non-expected utility.

in the asset pricing literature, although they have been studied by Epstein and Zin (1990), Bekaert, Hodrick and Marshall (1997) and Ang, Bekaert and Liu (2002) among others.

In Section 3, we show that utility functions without first-order risk aversion – in other words, the EU, R-EU and R-SORA classes – have difficulty explaining the attitudes to monetary gambles listed in observations I and II. In Section 4, we show that even utility functions with first-order risk aversion, namely those in the R-FORA class, have a hard time explaining the observations, and that a second ingredient, narrow framing, is required.

3 The Importance of First-order Risk Aversion

3.1 Expected Utility

In the expected utility framework, preferences are generally defined over an intertemporal consumption stream,

$$E(U(\tilde{C}_0, \tilde{C}_1, \dots, \tilde{C}_T)), \quad (2)$$

where U is increasing and concave in each argument. Under mild conditions, one can show that optimizing expected utility over consumption leads to an indirect value function over wealth,

$$J(W_t; I_t, C_{-t}) = \max E_t(U(C_0, \dots, C_t, \tilde{C}_{t+1}, \dots, \tilde{C}_T)), \quad (3)$$

where $C_{-t} \equiv \{C_0, C_2, \dots, C_{t-1}\}$ denotes the individual's past consumption history and I_t denotes information available at time t about the state of the economy. We make the reasonable assumption that the outcomes of our monetary gambles do not affect I_t and are independent of all other economic uncertainty.

We now ask whether the expected utility preferences in (2) can explain the attitudes to large and small gambles listed in observations I and II. The following proposition establishes that *no* utility function in this class can do so.

Proposition 1.

(a) Consider an individual with an expected utility preference in which future utility does not depend on past consumption, so that her value function is $J(W_t; I_t)$. Suppose that for given I_t , she rejects G_S at wealth levels below \$1,000,000. Then she rejects G_L at all wealth levels below \$1,000,000.

(b) Consider an individual with an expected utility preference in which future utility depends on past consumption, so that her value function is $J(W_t; I_t, C_{-t})$. Suppose that for given I_t and C_{-t} , she rejects G_S at wealth levels below \$1,000,000. Then she rejects G_L at all wealth levels below \$1,000,000.

Proof: See Appendix.

In words, the proposition says that any utility function able to explain observation I – the rejection of G_S , the 550/500 bet – will inevitably fail to explain observation II, namely the acceptance of G_L , the 20,000,000/10,000 bet.

The proposition covers a wide range of utility specifications, including most of those used in asset pricing. Part (a) of the proposition includes time-separable and state-independent utility of the form

$$U(C_0, \dots, C_T) = \sum_{t=0}^T u_t(C_t),$$

as well as external habit dependence (Abel 1990, Campbell and Cochrane 1999). Part (b) covers cases with non-time-separable preferences, including internal habit dependence (Constantinides 1990, Sundaresan 1989).

Proposition 1 can be thought of as an intertemporal generalization of a recent result of Rabin (2000), who shows that in a static one-period setting, no EU specification with an increasing, concave utility function defined over wealth can explain both observations I and II. The intuition for Rabin’s finding, and hence also for Proposition 1, is straightforward. An individual with the preferences in Proposition 1 is locally risk-neutral; since gamble G_S involves small stakes, she would normally take it without hesitating. To get her to reject it, in accordance with observation I, we need to make her locally risk averse. Moreover, since she must reject G_S over a wide range of wealth levels, she must be locally risk averse over a wide range of wealths. Proposition 1 simply states that this immediately implies a level of global risk aversion so high that she even rejects the apparently favorable large gamble, G_L .

The proof of Proposition 1 depends crucially on a property of the expected utility preferences in (2) and (3), namely that for fixed I_t and C_{-t} , the utility difference between two wealth levels does not depend on current wealth: the increase in utility from having \$21,000 rather than \$20,000 is the same, whether current wealth is \$10,000 or \$20,000. Therefore, knowing that someone will turn down a small gamble like G_S at a wealth level of \$20,000 provides valuable information about how, at a wealth level of \$10,000, she would react to a large risk like G_L that might bring her into the neighbourhood of \$20,000.

At first sight, it might seem from Proposition 1 that Rabin’s (2000) argument transfers easily to the intertemporal setting. However, this is not completely true. The argument works much better for certain types of utility functions than for others. As is reasonable in a one-period context, Rabin (2000) considers utility functions that are defined over wealth alone. In an intertemporal setting, value functions often depend not only on wealth but, as shown in (3), on state variables I_t and past consumption C_{-t} as well. In order to apply Rabin’s argument, then, we need the assumption given in each part of the proposition, namely that *keeping these other variables fixed*, G_S is rejected at a range of wealth levels. The problem is that this assumption may sometimes be difficult to verify.

Consider an individual with internal habit preferences, covered in part (b) of the proposition. There, we assume that for fixed C_{-t} , the investor rejects G_S at a range of wealth levels. To provide evidence that this assumption actually holds, we would want to ask people with different wealth levels, but the *same* past consumption, how they feel about G_S . The problem is now clear: it is very hard to find a group of subjects to do this experiment on, because people with different wealth levels will tend to have different past consumption. Since it is difficult to show that the premise of the proposition holds for internal habit preferences, using the proposition to dismiss such preferences may be too harsh. This caveat does not let habit-based preferences off the hook though, because they are still subject to more general criticisms that we make later of all utility functions displaying second-order risk aversion, whether expected utility or non-expected utility.⁵

Initial indications of the problem with EU preferences appear in Kandel and Stambaugh (1990), who show that in a one-period setting, power utility preferences

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

have trouble simultaneously capturing attitudes to both small- and large-scale risks. Whatever value of γ is chosen, Kandel and Stambaugh (1990) show that the resulting preferences make counterintuitive predictions either about large-scale or about small-scale wealth gambles. Rabin (2000) and Proposition 1 above show that this problem arises not only for power utility functions but for all expected utility specifications. When they are calibrated to fit attitudes to small-scale gambles, they are unable to fit attitudes to large-scale gambles.

Example

We illustrate the proposition with a simple example. Consider an investor with power utility preferences

$$U(C_0, \dots, C_T) = \sum_{t=0}^T \rho^t \frac{C_t^{1-\gamma}}{1-\gamma} \quad (4)$$

and with i.i.d. investment opportunities. It is a standard result that the investor's value function is then given by

$$J(W_t) = \Gamma \frac{W_t^{1-\gamma}}{1-\gamma} \quad (5)$$

for some constant Γ .

⁵Rubinstein (2001) points out that Rabin's (2000) argument applies only when utility is defined over wealth, not when it is defined over wealth changes, say. In general, this critique is not relevant to our analysis. Financial economists define utility over consumption streams and as discussed in the main text, such utility functions lead quite generally to value functions defined over wealth, not changes in wealth. However, there is a sense in which the difficulty we raise in the case of internal habit preferences is very similar to the difficulty raised by Rubinstein (2001). In the case of internal habit, the value function $J(W_t; I_t, C_{-t})$ comes close to being a function of wealth changes, since past consumption C_{-t} is likely to be closely related to past wealth.

We now check that any γ able to explain observation I will be unable to explain observation II. To see this, note that the investor rejects an immediate gamble \tilde{v} iff

$$E(J(W_t + \tilde{v})) < J(W_t).$$

For $\gamma > 1$ and $\tilde{v} = (x, \frac{1}{2}; -y, \frac{1}{2})$, this reduces to

$$(W_t + x)^{1-\gamma} + (W_t - y)^{1-\gamma} > 2W_t^{1-\gamma}. \quad (6)$$

Suppose that the investor's preferences fit observation I, so that she rejects G_S , the 550/500 gamble, at any wealth level below \$1,000,000. Then, in particular, she rejects G_S at a wealth level *equal* to \$1,000,000. A simple computation shows that the lowest integer value of risk aversion γ that satisfies (6) for

$$W_t = 1,000,000, x = 550, y = 500$$

is $\gamma = 181$. But at this level of risk aversion, the investor rejects G_L , the 20,000,000/10,000 bet: for $\gamma \geq 181$, inequality (6) is violated when

$$x = 20,000,000, y = 10,000,$$

whatever the investor's initial wealth. In other words, if the investor's risk aversion γ is high enough to explain the rejection of G_S in observation I, it will be so high as to make it impossible to explain observation II.

For the sake of tractability, we have illustrated Proposition 1 for the simple case of power utility. It is important to note, however, that precisely the same problem applies for any expected utility specification, including the habit-based preference of Campbell and Cochrane (1999), say.

3.2 Non-expected Utility

Having shown in Proposition 1 that EU preferences are unable to explain observations I and II, we turn to non-expected utility specifications.

Recursive utility with expected utility certainty equivalent [R-EU]

We begin with the following proposition, which shows that the first type of non-EU utility preference – R-EU – *cannot* explain observations I and II.

Proposition 2. Suppose that an individual has the recursive utility preferences

$$V_t = W(C_t, \mu(\tilde{V}_{t+1}|I_t))$$

with $\mu(\tilde{V}_{t+1}|I_t)$ the certainty equivalent of the distribution of future utility \tilde{V}_{t+1} conditional on information at time t , and where μ has the expected utility form

$$\mu(\tilde{X}) = h^{-1} E h(\tilde{X})$$

for some increasing, concave h , so that the value function is

$$J(W_t; I_t) = \max W(C_t, \mu(\tilde{V}_{t+1}|I_t)).$$

Suppose that for given I_t , the individual rejects G_S at wealth levels below \$1,000,000. Then she also rejects G_L at wealth levels below \$1,000,000.

Proof: See Appendix.

In words, the proposition says that if an R-EU preference specification is calibrated to match observation I – the rejection of G_S , the 550/500 bet – it fails to match observation II, in that it predicts the rejection of G_L , the 20,000,000/10,000 bet.

In proving the proposition, we have to take a stand on how an investor with the recursive preferences in (1) evaluates immediate gambles. We simply adopt the method proposed by Epstein and Zin (1989), who lay out a careful exposition of recursive utility. They propose that to evaluate an immediate gamble \tilde{v} , the agent inserts an infinitesimal time step Δt at time t and checks whether the utility from taking the gamble,

$$W(0, \mu(V_{t+\Delta t})) = W(0, \mu(J(W_{t+\Delta t}))) = W(0, \mu(J(W_t + \tilde{v}))), \quad (7)$$

is greater than the utility from *not* taking the gamble

$$W(0, \mu(V_{t+\Delta t})) = W(0, \mu(J(W_{t+\Delta t}))) = W(0, \mu(J(W_t))). \quad (8)$$

The decision therefore comes down to comparing $\mu(J(W_t + \tilde{v}))$ and $\mu(J(W_t))$.

The idea behind the proof is now easy to see. Since

$$\mu(J(W_t + \tilde{v})) = h^{-1} E(h(J(W_t + \tilde{v}))),$$

attitudes to risk are determined by the expected utility function $E(h(J(\cdot)))$, even if the preferences in Proposition 2 are non-expected utility. Therefore just as expected utility functions cannot explain observations I and II – our result in Proposition 1 – so recursive utility with an expected utility functional μ cannot explain them either.

Example

To illustrate Proposition 2, consider an investor with the following preferences, which belong to the class studied in the proposition,

$$W(C, \mu) = ((1 - \beta)C^\rho + \beta\mu^\rho)^{\frac{1}{\rho}}, \quad \rho < 1, \quad \rho \neq 0 \quad (9)$$

$$\mu(\tilde{V}) = (E(\tilde{V}^{1-\gamma}))^{\frac{1}{1-\gamma}}, \quad (10)$$

and with i.i.d. investment opportunities. Epstein and Zin (1989) show that in this case, the investor's value function takes the form

$$J(W_t) = \Gamma W_t \tag{11}$$

for some constant Γ .

We now check that any calibration of (9)-(10) able to explain observation I is unable to explain observation II. Given the value function in (11), the utility from taking the gamble is

$$W(0, \mu(J(W_t + \tilde{v}))) = W(0, \mu(\Gamma(W_t + \tilde{v}))) = W(0, \Gamma\mu(W_t + \tilde{v})), \tag{12}$$

and the utility from *not* taking it is

$$W(0, \mu(J(W_t))) = W(0, \mu(\Gamma W_t)) = W(0, \Gamma\mu(W_t)). \tag{13}$$

The decision therefore comes down to comparing $\mu(W_t + \tilde{v})$ and $\mu(W_t)$. Given the form of μ in (10), an investor with $\gamma > 1$ therefore rejects an immediate gamble $(x, \frac{1}{2}; -y, \frac{1}{2})$ at wealth level W_t iff

$$(W_t + x)^{1-\gamma} + (W_t - y)^{1-\gamma} > 2W_t^{1-\gamma},$$

exactly the condition that emerged in the example following Proposition 1. Precisely the same reasoning shows that the preferences in (9)-(10) cannot simultaneously explain observations I and II.

Recursive Utility with second-order risk averse certainty equivalent [R-SORA]

We now turn to the second kind of non-EU preference, R-SORA. In this case, it is impossible to prove that such preferences can *never* explain observations I and II. In particular, the Rabin (2000) argument can no longer be applied to the same extent as before. As mentioned earlier, the expected utility preferences in (2) and (3) have a very useful property, namely that the utility difference between two wealth levels does not depend on current wealth. As a result, attitudes to small risks at one wealth level provide very valuable information about attitudes to larger risks at other wealth levels. Without the EU assumption, however, this logic fails: the knowledge that someone rejects G_S at a wealth of \$20,000 provides little information about their attitudes to G_L at \$10,000 wealth.

While R-SORA preferences can, in principle, explain observations I and II, we argue that they can only do so for extreme parameterizations. One reason for this is already well-known in the literature. R-SORA preferences are locally smooth, which means that the investor is locally risk-neutral: she will accept an infinitesimally small, actuarially fair gamble. While G_S is not literally infinitesimal, it is virtually so; R-SORA preferences therefore need to adopt extreme parameters to explain its rejection.

This argument is a standard one. But we can go further. While it is certainly a challenge for R-SORA preferences to explain observation I, it is a much greater challenge for them to

explain *both* observations I and II. The reasoning is essentially a “local” version of Rabin’s argument. Suppose that R-SORA preferences are calibrated to explain the rejection of G_S , the 550/500 bet. They must therefore exhibit a very high level of risk aversion, making it very likely that the individual will reject, at the *same* wealth level, a more attractive gamble with larger stakes, such as G_L .

Example

To see the difficulties faced by R-SORA preferences, consider a simple example of a utility function in this class,

$$W(C, \mu) = ((1 - \beta)C^\rho + \beta\mu^\rho)^{\frac{1}{\rho}}, \rho < 1, \rho \neq 0 \quad (14)$$

where μ takes a form suggested by Chew and MacCrimmon (1979) and Chew (1983), namely “weighted utility”. Given a gamble

$$\tilde{X} = (x_1, p_1; x_2, p_2; \dots; x_n, p_n),$$

μ is defined as

$$\mu(\tilde{X}) = \left(\frac{p_1 x_1^{1-\gamma+\delta} + \dots + p_n x_n^{1-\gamma+\delta}}{p_1 x_1^\delta + \dots + p_n x_n^\delta} \right)^{1/(1-\gamma)}, \gamma \neq 1. \quad (15)$$

Risk aversion increases as γ increases or as δ falls. When $\delta = 0$, these preferences reduce to the standard power utility specification.

When investment opportunities are i.i.d., Epstein and Zin (1989) show that the individual’s value function is given by

$$J(W_t) = \Gamma W_t.$$

An investor with such preferences therefore decides whether or not to take on an immediate gamble in the way laid out in equations (7) and (8). In particular, she accepts a gamble $\tilde{v} = (x, \frac{1}{2}; -y, \frac{1}{2})$ iff

$$\mu(W_t + \tilde{v}) > \mu(W_t),$$

or if

$$\left(\frac{(W_t + x)^{1-\gamma+\delta} + (W_t - y)^{1-\gamma+\delta}}{(W_t + x)^\delta + (W_t - y)^\delta} \right)^{1/(1-\gamma)} > W_t. \quad (16)$$

The area shaded with “+” signs in Figure 1 shows the range of values of γ and δ consistent with observation I, in other words, with the investor rejecting G_S , the 550/500 bet, at all wealth levels below \$1,000,000. Mathematically, these are the values of γ and δ for which inequality (16) fails for all $W_t < 1,000,000$ when

$$x = 550, y = 500.$$

We set $\beta = 0.9$ and $\rho = -1$, and the results depend little on these choices.

The diagram shows, as predicted, that extreme values of γ and δ are needed just to explain observation I. For example, when $\delta = 0$, so that the preferences in (14)-(15) collapse to power utility preferences, we see, consistent with our earlier evidence, that $\gamma \geq 181$ is needed to explain the rejection of G_S . The shaded area is concentrated in the bottom-right of the picture because risk aversion increases as we move towards the south-east, or as γ increases and δ falls.

The figure also shows, now with “x” signs, the values of γ and δ consistent with observation II, i.e., for which the investor would accept G_L , the 20,000,000/10,000 bet, at all wealth levels above \$100,000. Mathematically, these are the values of γ and δ for which inequality (16) holds when $W_t > 100,000$ and

$$x = 20,000,000, y = 10,000.$$

This region is located in the top-left corner of the picture: once risk aversion climbs too high, the investor is no longer willing to accept G_L .

The picture shows that while it is difficult to explain just observation I, explaining both observations I and II is even harder, with only a thin sliver of parameter values in the upper right-hand corner able to do the task. The intuition, once again, is that if the investor is so risk averse as to reject the 550/500 bet at some wealth level, she will probably reject more attractive gambles with larger stakes, such as G_L .

Recursive Utility with first-order risk averse certainty equivalent [RU-FORA]

Finally, we turn to the last class of recursive utility preferences, R-FORA, in which the certainty equivalent μ is non-expected utility and exhibits first-order risk aversion. Such preferences certainly do a better job addressing observations I and II than the other specifications we have seen so far. In particular, they have no trouble explaining the attitudes in observations I and II, *so long as the gambles are played out immediately*, a critical caveat we return to shortly.

The intuition for why R-FORA preferences can explain attitudes to immediate gambles is straightforward. The essence of the difficulty with EU, R-EU, and R-SORA preferences is that the investor is risk-neutral to small gambles, forcing us to push risk aversion up to dramatically high levels in order to explain the rejection of G_S , the 550/500 bet. An agent with R-FORA preferences, on the other hand, is by definition, locally risk averse. Risk aversion over large gambles does not, therefore, need to be increased very much to ensure that G_S is rejected.

Example

To illustrate, consider an investor with the following specific R-FORA preferences:

$$W(C, \mu) = ((1 - \beta)C^\rho + \beta\mu^\rho)^{1/\rho}, \quad (17)$$

where μ takes a form developed by Gul (1991),

$$\mu(\tilde{V}_{t+1})^{1-\gamma} = E(\tilde{V}_{t+1}^{1-\gamma}) + (\lambda - 1)E((\tilde{V}_{t+1}^{1-\gamma} - \mu(\tilde{V}_{t+1})^{1-\gamma})1(\tilde{V}_{t+1} < \mu(\tilde{V}_{t+1}))). \quad (18)$$

These preferences are often referred to as “disappointment aversion” preferences. The investor gets disutility if the outcome of the gamble \tilde{V} falls below its certainty equivalent μ , where the degree of disutility is governed by λ . This parameter effectively controls how sensitive the agent is to losses as opposed to gains. Any $\lambda > 1$ implies first-order risk aversion. For i.i.d. investment opportunities, Epstein and Zin (1990) show that the investor’s value function is given by

$$J(W_t) = \Gamma W_t. \quad (19)$$

We now check that the utility function in (17)-(18) can easily be parameterized to explain both observations I and II for the case of *immediate* gambles. As in our earlier examples of recursive utility, the investor evaluates an immediate gamble \tilde{v} by inserting an infinitesimal time step Δt at time t and checking whether the utility from taking the gamble, given in (7), is greater than the utility from not taking the gamble, given in (8), which again reduces to comparing $\mu(W_t + \tilde{v})$ and $\mu(W_t)$. Given the functional form of μ , a little algebra shows that observations I and II can be simultaneously explained if there exist γ and λ such that

$$((W_t + 550)^{1-\gamma} + \lambda(W_t - 500)^{1-\gamma})^{\frac{1}{1-\gamma}} < (1 + \lambda)^{\frac{1}{1-\gamma}} W_t \quad (20)$$

holds for all wealth levels below \$1,000,000, and

$$((W_t + 20,000,000)^{1-\gamma} + \lambda(W_t - 10,000)^{1-\gamma})^{\frac{1}{1-\gamma}} < (1 + \lambda)^{\frac{1}{1-\gamma}} W_t \quad (21)$$

holds for all wealth levels above \$100,000. Condition (20) ensures that G_S is rejected and condition (21) that G_L is accepted, all at the appropriate wealth levels. A quick computation confirms that both (20) and (21) can be satisfied with $\gamma = 2$ and $\lambda = 2$. The intuition is that since λ controls sensitivity to losses as opposed to gains, we need λ to exceed 1.1 so that the 550/500 bet is rejected.

4 The Importance of Narrow Framing

At first sight, then, it appears that preferences with first-order risk aversion can explain observations I and II. However, we have only shown that they can do so in a very special case, namely when the monetary gambles are *immediate*. We now show that in the more realistic and general setting where the gambles are played out with some delay, they have a much harder time explaining observations I and II. In other words, while they can easily explain aversion to small, immediate gambles, they have great difficulty – in a sense that we make precise below – capturing aversion to small, *delayed* gambles. This is a serious concern because as we saw in Section 2, people seem to be just as averse to the 550/500 bet when

it is played out quickly as when it is played out with delay. More generally, most real-world risks are delayed, making it important to get attitudes to such gambles right.

Before giving a precise statement of the difficulty with R-FORA preferences, we give a very informal example to illustrate the idea. Consider a simple one-period utility function exhibiting first-order risk aversion,

$$w(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2x & \text{for } x < 0 \end{cases} .$$

It is easy for such a utility function to explain why someone might reject the small, *immediate* gamble $(550, \frac{1}{2}; -500, \frac{1}{2})$: the individual would assign the gamble a value of $550(\frac{1}{2}) - 2(500)(\frac{1}{2}) = -225$, the negative outcome signalling that the gamble should be rejected. But how would this individual deal with the more realistic case of a small, *delayed* gamble?

In answering this, it is important to recall the essential difference between an immediate and a delayed gamble. The difference is that over the time interval that the uncertainty surrounding the delayed gamble is being played out, the individual is likely to be facing *other* sources of risk at the same time, such as labor income risk, house price risk, or risk from other financial investments. This is not true for the immediate gamble.

For the R-FORA preferences in (17)-(18), this distinction can have a big impact on whether a gamble is accepted. Suppose that the individual is facing the pre-existing risk $(30,000, \frac{1}{2}; -10,000, \frac{1}{2})$, to be resolved at the end of the period, and is wondering whether or not to take on an independent delayed gamble $(550, \frac{1}{2}; -500, \frac{1}{2})$, whose uncertainty is also to be resolved at the end of the period. The correct way for him to think about this is to merge the new gamble with the pre-existing gamble, and to check whether the combined gamble offers a high utility. Since the combined gamble is

$$(30, 550, \frac{1}{4}; 29, 500, \frac{1}{4}; -9, 450, \frac{1}{4}; -10, 500, \frac{1}{4}),$$

the comparison is between

$$30,000(\frac{1}{2}) - 2(10,000)(\frac{1}{2}) = 5000$$

and

$$30,550(\frac{1}{4}) + 29,500(\frac{1}{4}) - 2(9450)\frac{1}{4} - 2(10,500)\frac{1}{4} = 5037.5.$$

The important point here is that the combined gamble offers *higher* utility. In other words, the investor would want to *accept* the small delayed gamble, even if she would reject an immediate gamble with the same stakes. The intuition is that since the investor is already facing some pre-existing risks, adding a small, independent gamble represents a form of diversification, which she willingly takes on.

This simple example suggests that even if the certainty equivalent μ exhibits first-order risk aversion, it may be very difficult to explain the rejection of G_S , the 550/500 bet. In Proposition 3 below, we make the nature of this difficulty precise. In brief, while an individual with R-FORA utility acts in a first-order risk averse manner toward immediate gambles, she acts in a *second-order* risk averse manner towards independent, delayed gambles, so long as she is already facing other pre-existing risks.

This immediately reintroduces the same two difficulties that we saw in Section 3 when discussing preferences with second-order risk aversion (R-SORA). First and foremost, since the agent is second-order risk averse over delayed gambles, and since the delayed gamble G_S is virtually infinitesimal, she will be very keen to accept it. In order to explain why it is typically rejected, extreme parameterizations are required.

Second, if the agent rejects the delayed gamble $(550, \frac{1}{2}; -500, \frac{1}{2})$ at some wealth level, the local risk aversion will need to be so large that she will probably also reject, at the same wealth level, a more attractive gamble with larger stakes, such as G_L , the 20,000,000/10,000 bet. We illustrate both of these difficulties in an example following the proposition.

While Proposition 3 is proven for just one implementation of first-order risk aversion, namely the Gul (1991) implementation through the recursive utility, the argument used in the proof is very general and applies readily to other formalizations.

Proposition 3. Suppose an individual has first-order risk aversion preferences as implemented through the recursive utility framework of Gul (1991) laid out in (17)-(18) above, where the function W is strictly increasing and twice differentiable with respect to both arguments.

Suppose that the individual is offered an actuarially favorable gamble $k\tilde{\varepsilon}$ to pay off between time t and $t+1$, and that the payoffs do not affect, and are independent of, I_t and the future economic uncertainties. Finally, suppose that prior to taking the gamble, the distribution of the agent's $t+1$ utility value \tilde{V}_{t+1} does not have finite mass at μ .

Then, the individual will be second-order risk averse over the new gamble; in other words, for sufficiently small k , she will accept it.

Proof: See Appendix.

An important part of the proof is an assumption as to how an agent with pre-existing risk would evaluate a delayed monetary gamble. Epstein and Zin (1989), who lay out a careful exposition of recursive preferences, do not suggest a specific methodology. We therefore adopt the most natural one, which is that the agent merges the delayed gamble with other risks she is already taking and checks whether the combination offers higher utility. In other words, she compares the utility from not taking it,

$$W(C_t, \mu(V_{t+1})) = W(C_t, \mu(J(\tilde{W}_{t+1}))) = W(C_t, \mu(J((W_t - C_t)R_{t+1}))), \quad (22)$$

to the utility from taking it,

$$W(\hat{C}_t, \mu(V_{t+1})) = W(\hat{C}_t, \mu(J(\tilde{W}_{t+1}))) = W(\hat{C}_t, \mu(J((W_t - \hat{C}_t)R_{t+1} + \tilde{v}))), \quad (23)$$

where R_{t+1} is the return on invested wealth between time t and $t + 1$. The hat over \hat{C}_t is a reminder that if the investor takes on the gamble, her optimal consumption choice will be different from what it is when she does not take the gamble.

Example

We now illustrate the difficulties faced by R-FORA preferences with the help of a more formal example. The analysis closely mirrors the computations we did for the example of R-SORA preferences in Section 3. We first show that it is very difficult for the R-FORA preferences to explain the rejection of the delayed gamble $G_S = (550, \frac{1}{2}; -500, \frac{1}{2})$; and that it is even more difficult to explain both the rejection of G_S and the acceptance of the delayed gamble G_L .

To do the computations, we again consider an investor with the R-FORA preferences in (17)-(18). We assume, for simplicity, that the only investment opportunity available to the investor is a risky asset with gross return \tilde{R} , where \tilde{R} has a log-normal distribution

$$\log(\tilde{R}) \sim N(0.04, 0.03),$$

i.i.d. over time. As before, the investor's value function takes the form

$$J(W_t) = \Gamma W_t.$$

The utility from taking the gamble is therefore

$$W(\hat{C}_t, \mu(J((W_t - \hat{C}_t)\tilde{R} + \tilde{v}))) = W(\hat{C}_t, \Gamma\mu((W_t - \hat{C}_t)\tilde{R} + \tilde{v}))$$

and the utility from not taking it,

$$W(C_t, \mu(J((W_t - C_t)\tilde{R}))) = W(C_t, \Gamma\mu((W_t - C_t)\tilde{R} + \tilde{v})).$$

Figure 2, which has the same structure as Figure 1, presents the results. We again set $\beta = 0.9$ and $\rho = -1$. The area shaded with “+” signs shows the range of values of γ and λ for which the agent rejects the delayed gamble $G_S = (550, \frac{1}{2}; -500, \frac{1}{2})$. The figure confirms that extreme values are required to explain this rejection.

The figure also shows, this time marked with “x” signs, the range of values of γ and λ for which the agent accepts G_L . As the figure shows, across the wide range of parameter values checked in the figure, there is no overlap at all between the two shaded regions; in other words, there are no parameter values for which these R-FORA preferences can explain both observations I and II.

The intuition bears repeating: in the presence of pre-existing risk, the investor acts in a second-order risk averse manner towards small, delayed gambles. It therefore takes huge risk aversion to explain why a delayed gamble is rejected, so much, in fact, that she rejects more attractive gambles with larger stakes, such as G_L , at that wealth level.

4.1 Incorporating Narrow Framing

So far, we have shown that two simple observations – I and II – pose considerable difficulties for almost every utility specification that has appeared in the asset pricing literature. What, then, *can* satisfy these observations? Clearly, first-order risk aversion is a necessary ingredient: we need it to explain why small gambles like G_S , played out immediately, are rejected. However, the analysis earlier in this section shows that first-order risk aversion is not enough. Its weakness is that when an agent evaluates a small, delayed gamble, she merges it with pre-existing risks and since the resulting diversification is attractive, she is happy to accept it. To explain the *rejection* of such delayed gambles, then, it must be that the agent does *not* fully merge the gamble with pre-existing risks, but that to some extent, she evaluates it in isolation. More formally, her preferences must depend on the outcome of the gamble over and above what the outcome implies for aggregate wealth risk, a feature we call *narrow framing*.⁶

We now check that preferences incorporating both first-order risk aversion and narrow framing can easily explain observations I and II, whether the gambles are played out immediately or with delay. Preferences of this type were originally proposed by Kahneman and Tversky (1979) and have been used in the context of asset pricing by Benartzi and Thaler (1995) and Barberis, Huang and Santos (2001). Here, we adopt a more tractable specification proposed by Barberis and Huang (2002), in which time t utility is given by

$$V_t = W \left[C_t, \mu(V_{t+1}) + b_0 E_t \left(\sum_i \bar{v}(G_{i,t+1}) \right) \right] \quad (24)$$

where

$$W(c, y) = ((1 - \beta)C^{1-\gamma} + \beta y^{1-\gamma})^{1/(1-\gamma)} \quad (25)$$

⁶An equivalent definition of narrow framing is that an agent frames narrowly when she evaluates a gamble in isolation even when the uncertainty about the gamble is resolved over a finite period of time over which the agent is also facing other risks. Strictly speaking, the rejection of the delayed 550/500 gamble does not prove that that people are framing narrowly, because the uncertainty about the outcome is resolved over a single instant. The rejection of the gamble could then be due to an agent with first-order risk aversion inserting an infinitesimal time interval Δt around the moment that the delayed gamble's uncertainty is resolved: since the pre-existing risk is constant over the interval, she rejects the gamble even without narrow framing. If the uncertainty about the delayed gamble is resolved over some finite interval, however, narrow framing is immediately required to explain its rejection.

$$\begin{aligned}\bar{v}(x) &= \begin{cases} x & \text{for } x \geq 0 \\ \lambda x & \text{for } x < 0 \end{cases} \\ \mu(\tilde{V}) &= (E(\tilde{V}^{1-\gamma}))^{1/(1-\gamma)},\end{aligned}$$

and where $G_{i,t+1}$ are specific gambles faced by the investor whose uncertainty will be resolved between time t and $t + 1$.⁷

The term prefixed by b_0 in (24) shows that relative to the usual recursive specification in (1), we now allow utility to depend on outcomes of gambles $G_{i,t+1}$ over and above what those outcomes mean for aggregate wealth risk. In other words, we allow for narrow framing, with the parameter b_0 controlling the degree of narrow framing.

Barberis and Huang (2002) show that just like an investor with the R-FORA preferences in (17)-(18), an investor with the preferences in (24)-(25) is first-order risk averse over immediate gambles. This time, however, the first-order risk aversion does not come from μ itself: note that the μ in (25) is of the expected utility class, and therefore second-order risk averse. This time, first-order risk aversion comes from the kink in \bar{v} at the origin.

Barberis and Huang (2002) also show that an investor with the preferences in (24)-(25) is first-order risk averse not only to immediate gambles but to delayed gambles as well. The intuition is that when considering a delayed gamble, the investor does not fully merge it with pre-existing risks, but because of the \bar{v} term in (24), evaluates it in isolation to some extent. The piecewise linearity of \bar{v} then induces first-order risk aversion over the gamble.

The fact that the preferences in (24)-(25) predict first-order risk aversion over both immediate and delayed gambles means that they should have little trouble explaining the rejection of G_S , the 550/500 bet, whether in its immediate or delayed form. This stands in stark contrast to the preferences in (17)-(18): in that case, we saw that while the agent is first-order risk averse over an immediate small gamble, he is second-order risk averse over a delayed gamble, thereby predicting the rejection of the former but the *acceptance* of the latter.

We now check that the preferences in (24)-(25) can indeed explain the attitudes in observations I and II without difficulty, whether the gambles are immediate or delayed. We consider the same context as earlier in this section. The investor's only investment opportunity is a risky asset, with a gross return \tilde{R} , distributed log-normally as

$$\log(\tilde{R}) \sim N(0.04, 0.03),$$

i.i.d. over time. Barberis and Huang (2002) show that in this case, the investor's value function is given by

$$J(W_t) = \Gamma W_t.$$

⁷The specification in Barberis and Huang (2002) has numerous advantages over earlier formulations like that in Barberis, Huang and Santos (2001): it allows for a more tractable partial equilibrium analysis, offers a natural way of checking attitudes to monetary gambles and individual preferences no longer depend on an aggregate consumption scaling term.

The agent will therefore accept an immediate gamble \tilde{x} iff the utility from not taking it on,

$$W(0, \Gamma\mu(W_t))$$

is less than the utility from taking it,

$$W[0, \Gamma\mu(W_t + \tilde{x}) + b_0 E_t(\bar{v}(\tilde{x}))].$$

Analogously, she will take a delayed gamble \tilde{x} iff the utility from not taking it,

$$W(C_t, \Gamma\mu(W_{t+1}))$$

is less than the utility from taking it,

$$W[\hat{C}_t, \Gamma\mu(W_{t+1} + \tilde{x}) + b_0 E_t(\bar{v}(\tilde{x}))],$$

with the hat over \hat{C}_t again indicating that if the investor takes the gamble, her consumption choice will be different from what it is in the case where she doesn't take on the gamble.

We set β , which has little direct influence on attitudes to risk, to 0.9, and b_0 , which controls the degree of narrow framing, to 0.001. The top panel in Figure 3 shows the range of values of γ and λ consistent with observation I when the gamble, G_S , is delayed, while the bottom panel shows the range of values consistent with observation II when the gamble, G_L , is again delayed. The figure shows clearly that there is a wide range of parameter values for which the preferences in (24)-(25) can explain observations I and II for delayed gambles. Moreover, the figure is identical when the gambles are played out immediately. This contrasts with R-FORA preferences, which lack narrow framing, and which predict very different attitudes to immediate and delayed gambles.

5 Narrow Framing and the Equity Premium

So far, we have shown that a very simple observation about attitudes to risk, namely the widespread aversion to bets like G_S , can only satisfactorily be explained by narrow framing. While this result may be interesting in its own right, it may also shed light on an issue of particular interest to financial economists, namely the magnitude of the equity premia generated by various utility functions in simple representative agent economies.

To illustrate this, we take an endowment economy with an infinite number of identical investors, and two assets: a risk-free asset in zero net supply, with gross return $R_{f,t}$ between time t and $t + 1$, and a risky asset – the stock market – in fixed positive supply, with gross return R_{t+1} between time t and $t + 1$. The stock market is a claim to a perishable stream of dividends $\{D_t\}$, where

$$\frac{D_{t+1}}{D_t} = e^{g_D + \sigma_D \varepsilon_{t+1}}, \quad (26)$$

and where each period's dividend can be thought of as one component of a consumption endowment C_t , where

$$\frac{C_{t+1}}{C_t} = e^{g_C + \sigma_C \eta_{t+1}}, \quad (27)$$

and

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix} \right), \text{ i.i.d. over time.} \quad (28)$$

In our quantitative analysis, we use the endowment process parameters listed in Table 2. These parameters are estimated from U.S. data spanning the 20th century and are standard in the literature.

We now consider a number of different preference specifications and ask how large an equity premium they generate in this simple economy, when parameterized in a “reasonable” way. Following the standard practice of financial economists, we take “reasonable” to mean that the preferences make sensible predictions about attitudes to *large* monetary gambles – that an individual with those preferences would accept G_L , for example.

The four preference specifications we consider are: (a) the power utility form in (4); (b) recursive utility with second-order risk averse certainty equivalent, as in (14)-(15); (c) recursive utility with first-order risk averse certainty equivalent, as in (17)-(18); and finally, (d) the preferences in (24)-(25) exhibiting both first-order risk aversion and narrow framing.

We assume that in our economy, narrow framing means that investors frame their stock market investments narrowly: they get direct utility from fluctuations in the value of their stock market holdings even though these holdings are only one part of their total wealth. In mathematical terms, the variable $G_{1,t+1}$ in specification (24) equals $\tilde{R}_{t+1} - 1$. We discuss possible interpretations of this assumption more fully in Section 5.1. For now, note that given this assumption, utility is

$$V_t = W [C_t, \mu(V_{t+1}) + b_0 E_t(\bar{v}(R_{t+1} - 1))]. \quad (29)$$

In order to compute equity premia, we need the Euler equations of optimality for each preference specification. For power utility preferences, the Euler equation is well-known to be

$$1 = \rho E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right].$$

Epstein and Zin (1991) show that the Euler equations for the R-SORA and R-FORA preferences are given by

$$E_t \left[\phi \left(\left(\frac{\rho}{1-\alpha} \right)^{1/\theta} \frac{C_{t+1}}{C_t} \right) \right] = 0 \quad (30)$$

$$E_t \left[\phi' \left(\left(\frac{\rho}{1-\alpha} \right)^{1/\theta} \frac{C_{t+1}}{C_t} \right) (R_{t+1} - R_{f,t}) \right] = 0 \quad (31)$$

$$E_t \left[\phi' \left(\left(\frac{\rho}{1-\alpha} \right)^{1/\theta} \frac{C_{t+1}}{C_t} \right) \left(\frac{1}{1-\alpha} \frac{C_{t+1}}{C_t} - R_{f,t} \right) \right] = 0, \quad (32)$$

where α is the constant fraction of wealth consumed each period, and where

$$\phi(x) = x^\delta(x^{1-\gamma} - 1)$$

for R-SORA and

$$\phi(x) = \begin{cases} \frac{x^{1-\gamma}-1}{1-\gamma} & \text{for } x \geq 1 \\ \lambda \frac{x^{1-\gamma}-1}{1-\gamma} & \text{for } x < 1 \end{cases}$$

for R-FORA. In both cases, equation (30) determines α , equation (32) determines the risk-free rate R_f and (31) determines the expected stock return, thereby giving the equity premium.

Finally, Barberis and Huang (2002) show that the Euler equations for the narrow framing preferences in (24)-(25) are given by

$$\beta^{\frac{1}{1-\gamma}}(1-\alpha)^{-\frac{\gamma}{1-\gamma}}R_f E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right)\left(E\left(\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right)\right)^{\frac{\gamma}{1-\gamma}} = 1 \quad (33)$$

$$\frac{E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(R_{t+1} - R_f)\right)}{E\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right)} + b_0 R_f^{-1} \left(\frac{\beta}{1-\beta}\right)^{\frac{1}{1-\gamma}} \left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} E(\bar{v}(R_{t+1} - R_f)) = 0 \quad (34)$$

where α is the fraction of wealth consumed by the investor. Given a risk-free rate R_f , α is obtained from (33). With α in hand, (34) can then be used to compute the equity premium.

Given the assumptions about the endowment process in (26) and (27), the expectations in the Euler equations can often be computed explicitly. The reduced forms are given in the Appendix.

We find that for power utility preferences, the largest equity premium generated by any “reasonable” parameterization is 0.6%. It obtains for $\gamma = 7$, the largest integer value of γ for which the agent accepts the large gamble G_L at a wealth of \$100,000. Figures 4-6 present our results for the other three preference specifications. Throughout, we take $\beta = 0.9$, $\rho = -1$ and $b_0 = 0.0001$. In all these figures, the “+” signs indicate the parameter values for which the preferences deliver an equity premium greater than some fixed number, while the “x” signs show the parameters for which, given her equilibrium holdings of equity, the investor accepts the 20,000,000/10,000 gamble. This latter calculation is performed exactly as in Sections 3 and 4. For example, for the investor whose preferences exhibit narrow framing, utility from taking a gamble \tilde{x} is given by

$$V_t = W [C_t, \mu(V_{t+1}) + b_0 E_t \bar{v}(R_{i,t+1} - 1) + b_0 E(\bar{v}(\tilde{x}))]. \quad (35)$$

The figures show that when we constrain the four preference specifications to make sensible predictions about large gambles, the one exhibiting both first-order risk aversion and narrow framing generates by far the largest equity premium. For the other preference specifications, there is no intersection between the two shaded regions for the wide range of parameter values we search across.

We argue that our equity premium results can be understood in terms of our earlier findings. The idea is to think of “equity” playing the same role in the equity premium calculations as the 550/500 gamble did in the earlier analysis. At first sight, this may seem odd – the stock market is nowhere near as small a gamble as 550/500, nor is it independent of investors’ marginal utility, as the 550/500 gamble was. However, *in the simple representative agent economy we consider*, stocks are *small enough* a gamble and *uncorrelated enough* with marginal utility that our earlier results continue to hold, at least approximately, when “equity” is substituted for the 550/500 gamble.

In other words, just as utility functions with first-order risk aversion and narrow framing are best able to explain an aversion to 550/500 at the same time as a liking for 20 million/10,000, so such utility functions are also best able to explain aversion to the stock market – and hence a large equity premium – at the same time as a liking for 20 million/10,000.

Moreover, the intuition for why narrow framing is helpful for generating an aversion to stocks is similar to the intuition for why it is helpful for generating an aversion to the 550/500 gamble. In the absence of narrow framing, people with pre-existing risks are happy to accept a stock gamble because after merging it with those other risks, they find that it diversifies them. If they are to be averse to the stock market then, it must be that they do not fully merge the stock gamble with their pre-existing risks, but rather that they evaluate stocks in isolation, to some extent.

But why can we say that in our representative agent economy, stocks are “small enough” or “uncorrelated enough” a gamble? In any reasonable representative agent economy, stocks *must* be a relatively small gamble, because the model should match the fact that in aggregate, stock market wealth is a small fraction of total wealth, defined to include human capital, housing and bond holdings.⁸ Moreover, since stocks have a low correlation with consumption growth, they are also likely to have a low correlation with the marginal utility of consumption – and in our examples, they do.

Our results for the equity premium have a natural counterpart in partial equilibrium. Many researchers have noted that a large proportion of investors hold no equities at all, a puzzling fact given that any second-order risk averse utility function would tell investors to allocate some positive dollar amount to stocks, so long as the expected return on stocks exceeds the risk-free rate. Our results from Sections 3 and 4 suggest that when the four utility functions described earlier are parameterized to accept the 20,000,000/10,000 gamble, the one exhibiting both first-order risk aversion and narrow framing will have the easiest time explaining a zero allocation to stocks. In particular, first-order risk aversion on its own may not be enough: in the presence of background risk, and if the correlation between stocks and

⁸In our model, this is reflected in the low correlation of stock returns and consumption growth: if stocks were a large part of total wealth, a decline in stock values would lead to a decline in total wealth and consumption, and consumption growth and stock returns would be highly correlated.

other risks is sufficiently low, the investor will be happy to hold some position in stocks as a way of diversifying his other risks.

5.1 First Order Risk Aversion and Narrow Framing: Evidence and Explanations

In this paper, we have tried to argue that preferences with first-order risk aversion and narrow framing offer financial economists some attractive properties. Is there any evidence, though, that people actually display these features in their decision making? And, if so, why? We address the former issue first.

As we have argued in this paper, the commonly observed aversion to small gambles, whether in the form of laboratory gambles like G_S or real-life gambles involving automobile collision risks, is in itself evidence of first-order risk aversion and narrow framing. But there is much other evidence for these two phenomena, and in particular, for a kind of first-order risk aversion known as loss aversion, where people are more sensitive to losses than to gains.

For example, there is what Thaler (1980) calls the endowment effect, which is loss aversion in the absence of uncertainty. Kahneman, Knetsch and Thaler (1990) conducted a series of experiments in which subjects were either given some object such as a coffee mug and then asked if they would be willing to sell it, or not given the mug and offered a chance to buy one. They found that mug owners demanded more than twice as much to sell their mugs as non-owners were willing to pay to acquire one.

Other evidence of loss aversion in combination with narrow framing comes from Thaler, Kahneman, Tversky and Schwarz (1990) and Benartzi and Thaler (1999). Both these papers ask subjects how they would allocate between a risk-free asset and a risky asset over a long time horizon, thirty years, say. The key manipulation is that some subjects are given the distribution of asset returns over short horizons – monthly returns, say – while others are given long-term return distributions – the distribution of 30-year returns, say. In principle, the two groups of subjects should make similar allocation decisions, since they have the same decision problem: those subjects given shorter-term return distributions should simply use them to infer the more directly relevant longer-term distributions. In fact, these subjects allocate substantially less to the risky asset, suggesting that they persist in using the “narrow” frame of short-term returns to make a long-term allocation decision. Since losses occur more frequently over short horizons, their loss aversion leads them to allocate less to risky assets.⁹

Narrow framing is a very general phenomenon, and need not occur in combination with

⁹Gneezy and Potters (1997) obtain very similar results, while a follow-up study by Gneezy, Kapteyn and Potters (2002) finds that manipulating the type of information provided can affect prices in an experimental asset market.

loss aversion. Shortly after publishing their work on prospect theory, Tversky and Kahneman (1981) introduced the idea of decision framing and demonstrated narrow framing with examples such as the following:

Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer:

Choice (I) Choose between:

A. a sure gain of \$240

B. 25% chance to gain \$1000 and 75% chance to gain nothing

Choice (II) Choose between:

C. a sure loss of \$750

D. 75% chance to lose \$1000 and 25% chance to lose nothing

A large majority of subjects choose A and D; however, this choice is dominated by B and C. This suggests that subjects are framing their decision narrowly, treating each choice separately as if it had no connection to the other; indeed in situations where subjects are asked to choose *only* between A and B, or *only* between C and D, they typically choose A and D, respectively. Conversely, if they are asked to choose between options E and F, where $E = A+D$ and $F=B+C$, then every subject chooses F. Subjects do not knowingly choose a dominated alternative, but when the dominance is not transparent, they make choices one at a time.

Now that we have presented a few examples of narrow framing, we can ask why it is that people might have this feature embedded in their preferences, particularly when in some cases, as in the example above, it can lead to violations of invariance and dominance. Our best answer to this question is that narrow framing is a simplifying strategy for dealing with a very complex world. Suppose that someone is presented with clear information about a gamble whose outcome will be resolved at some point in the future and is wondering whether or not to accept it. Even if she knows that the right thing to do is to integrate the gamble with other gambles she is already facing, and then to check whether the resulting overall risk is preferable to the overall risk she previously faced, it may be difficult to do this from a computational standpoint. The individual may not be sure about the probability distribution of the outcomes for her other gambles, nor about the correlation between the gamble under consideration and previously accepted risks. As a result, she may adopt the general rule of deciding each gamble she faces in isolation, as if it is the only risk she faces in the world – in other words, she may use narrow framing. Computational difficulties would certainly seem to underlie the use of narrow framing in Tversky and Kahneman's (1981) example above.

In Section 5., we explained the equity premium by saying that agents get utility from

the outcome of their stock market investments even if those investments are only one part of their overall wealth; in other words, they frame the stock market narrowly. Does it seem plausible that narrow framing might indeed apply in the case of the stock market?

The bounded rationality view of narrow framing suggests that it might. When people think about how much to invest in the stock market, they typically spend substantial time thinking about the distribution of outcomes for the stock market gamble itself. However, integrating this risk with other risks they face, such as labor income risk, house price risk, and proprietary business risk, is much more difficult. Few people have a good quantitative sense of the size of the other risks, nor of the correlation between them and stock market risk. Given these complications and uncertainties, it is possible that people frame stock market risk narrowly to some extent, ignoring the other risks they face.

All this is not to say that people never take a broader view. If the stakes are very large, and the relation between various contingencies are obvious then a broader frame may be adopted. An example might be when an employee reaches retirement and is considering what to do with the proceeds of a defined contribution pension plan. In such circumstances, other sources of income and wealth are more likely to be considered in making a choice than, say, when the asset allocation decision was last revisited while the employee was still working.

6 Conclusion

We present evidence on typical attitudes to monetary gambles with both large and small stakes and ask what kinds of intertemporal preferences can capture these attitudes. We find that among a wide range of utility functions, including all expected utility and many non-expected utility specifications, the only ones that can easily capture these attitudes are those exhibiting both first-order risk aversion and narrow framing. We apply our result to understanding the kinds of equity premia generated by various preference specifications in a simple representative agent economy.

7 Appendix

Proof of Proposition 1. (We prove part (b) here. The argument for part (a) is very similar). With expected utility, and under the assumption that the outcome of a gamble does not affect I_t and is independent of all other economic uncertainty, the individual's preference to the gamble is evaluated through $E_t[J(W_t + \tilde{v}; I_t, C_{-t})]$, where not taking the gamble corresponds to $\tilde{v} = 0$. The argument in Rabin (2000), which applies to one-period utility functions defined over wealth, can therefore be applied to $J(W_t; I_t, C_{-t})$, giving the result.

Proof of Proposition 2. Epstein and Zin (1989) propose that an individual with recursive utility preferences evaluates an immediate gamble by inserting an infinitesimal time step Δt at time t and applying the recursive utility calculation over this time step. Under the assumption that the outcome of a gamble does not affect I_t and is independent of all other economic uncertainty, the individual's preference to the gamble is evaluated through

$$W(0, \mu(J(W_t + \tilde{v}; I_t))) = W(0, h^{-1}[E(h \cdot J(W_t + \tilde{v}; I_t))])$$

where not taking the gamble corresponds to $\tilde{v} = 0$. In this case, immediate gambles again are ranked by expected utility over wealth, with utility function given by $h \cdot J(\cdot)$. The argument in Rabin (2000), which applies to one-period utility functions defined over wealth, can therefore be applied to $h \cdot J(\cdot)$.

Proof of Proposition 3. We prove the proposition for the first-order risk averse preferences

$$\begin{aligned} W(C, \mu) &= ((1 - \beta)C^\rho + \beta\mu^\rho)^{1/\rho} \\ u(\mu(\tilde{V}_{t+1})) &= E(u(\tilde{V}_{t+1})) + (\lambda - 1)E((u(\tilde{V}_{t+1}) - u(\mu(\tilde{V}_{t+1})))1(\tilde{V}_{t+1} < \mu(\tilde{V}_{t+1}))) \end{aligned}$$

where u is strictly increasing with a positive first derivative and a negative second derivative. When

$$u(\tilde{x}) = E(\tilde{x}^{1-\gamma})^{\frac{1}{1-\gamma}},$$

this reduces to the first-order risk averse preferences in the text.

For a small change in the period $t + 1$ value function $\Delta\tilde{V}_{t+1} = \Delta\tilde{V}(\tilde{W}_{t+1}, I_{t+1})$, the certainty equivalent changes by

$$\Delta\mu = \frac{E(u'(\tilde{V}_{t+1})\Delta\tilde{V}_{t+1}) + (\lambda - 1)E(u'(\tilde{V}_{t+1})\Delta\tilde{V}_{t+1}1(\tilde{V}_{t+1} < \mu))}{u'(\mu)(1 + (\lambda - 1)\Pr(\tilde{V}_{t+1} < \mu))} + o(\|\Delta V_{t+1}\|)$$

where $\|x\| = E(|x|)$ and $\lim_{x \rightarrow 0}(o(x)/x) = 0$ by definition.

Assume for now that the agent does not optimally adjust his optimally chosen consumption and portfolio strategy if he decides to take the gamble. Then we have

$$\Delta\tilde{V}_{t+1} = V_W(\tilde{W}_{t+1}, I_{t+1})\tilde{v} + o(\|\tilde{v}\|).$$

which implies

$$\Delta\mu = \frac{E(u'(\tilde{V}_{t+1})V_W(\tilde{W}_{t+1}, I_{t+1})\tilde{v}) + (\lambda - 1)E(u'(\tilde{V}_{t+1})V_W(\tilde{W}_{t+1}, I_{t+1})\tilde{v}1(\tilde{V}_{t+1} < \mu))}{u'(\mu)(1 + (\lambda - 1)\Pr(V_{t+1} < \mu))} + o(\|\tilde{v}\|).$$

Given our assumption that v is independent of other economic uncertainties, we have

$$\Delta\mu = E(\tilde{v}) \frac{E(u'(\tilde{V}_{t+1})V_W(\tilde{W}_{t+1}, I_{t+1})) + (\lambda - 1)E(u'(\tilde{V}_{t+1})V_W(\tilde{W}_{t+1}, I_{t+1})1(\tilde{V}_{t+1} < \mu))}{u'(\mu)(1 + (\lambda - 1)\Pr(V_{t+1} < \mu))} + o(\|\tilde{v}\|).$$

So to first order, the certainty equivalence value of \tilde{V}_{t+1} depends only on $E(\tilde{v})$, not on its standard deviation.

Finally, the aggregator function $W(\cdot, \cdot)$ will not generate any first-order dependence on the standard deviation of the gamble \tilde{v} . In addition, assuming that the agent will adjust his consumption and portfolio choice optimally while taking into account the additional gamble, should he choose to take it on, will only introduce terms of second order of \tilde{v} .

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Table 1: Acceptance rates for monetary gambles across five groups of subjects. Groups (i) and (v) consist of part-time MBA students at the University of Chicago, group (ii) of financial advisors at a mid-size U.S. brokerage firm, group (iii) of Chief Investment Officers and Directors of quantitative equity research at large asset management firms and group (iv) of clients of the private wealth management division of a U.S. bank. The first four gambles are immediate – their uncertainty is resolved at the time of the experiment – while the last gamble is delayed – its uncertainty is resolved a few months later. The fourth gamble listed was played for real money, while the others are hypothetical. Gamble “X/Y” is a 50:50 bet to win \$X and lose \$Y.

	(i)	(ii)	(iii)	(iv)	(v)
110/100	24%	33%	31%	41%	-
550/500 (G_S)	10%	23%	16%	29%	-
1,100/1,000	4%	20%	5%	15%	-
110/100 real money	-	-	-	-	10%
110/100 delayed	-	-	-	-	22%
No. of subjects	68	30	19	34	41

Table 2: Parameter values for a simple consumption-based model. g_C and σ_C (g_D and σ_D) are the mean and standard deviation of log consumption (dividend) growth; ω is the correlation of log consumption growth and log dividend growth.

Parameter	
g_C	1.84%
σ_C	3.79%
g_D	1.5%
σ_D	12.0%
ω	0.15

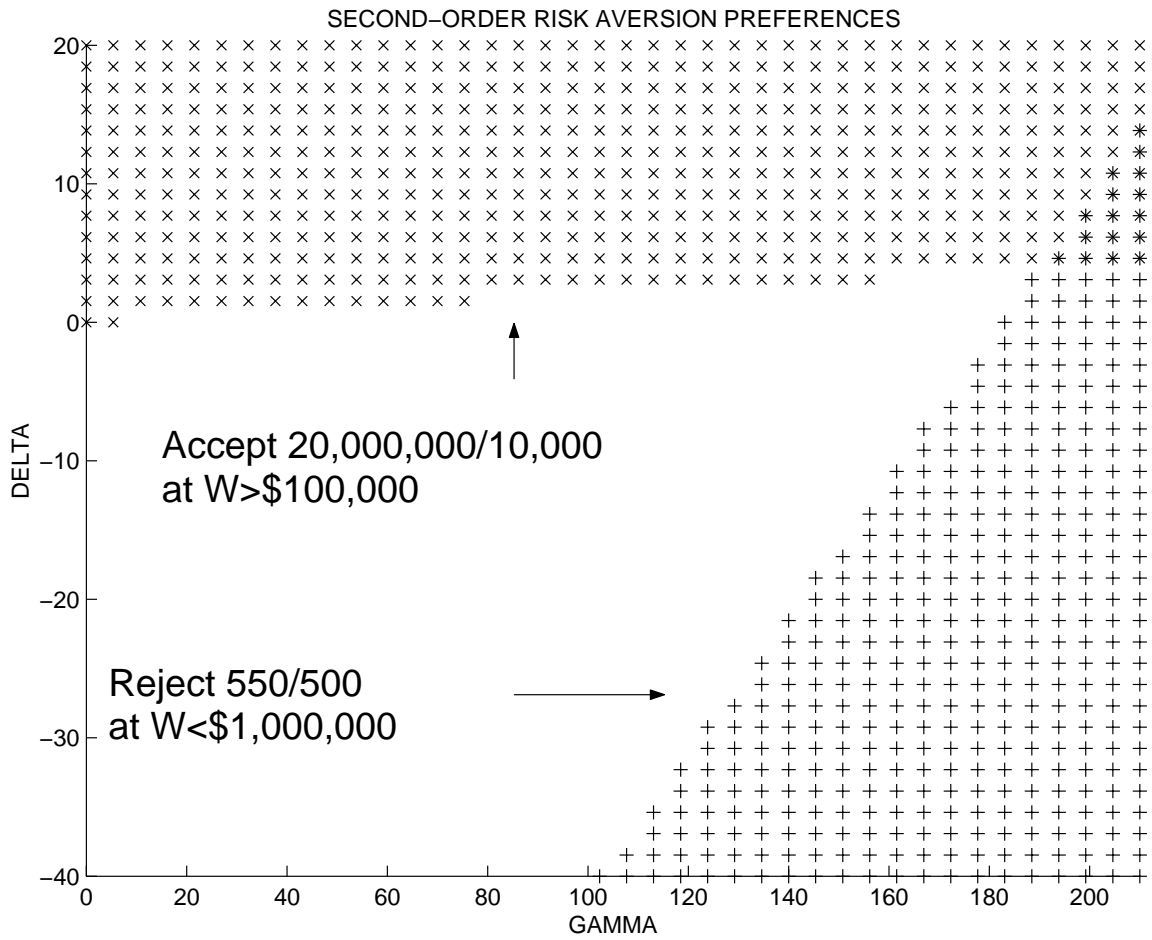


Figure 1. The “+” signs show the range of parameter values for which an agent with a recursive utility function with second-order risk averse certainty equivalent rejects a 50:50 bet to win \$550 or lose \$500. The “x” signs show the parameter values for which the agent accepts a 50:50 bet to win \$20,000,000 or lose \$10,000.

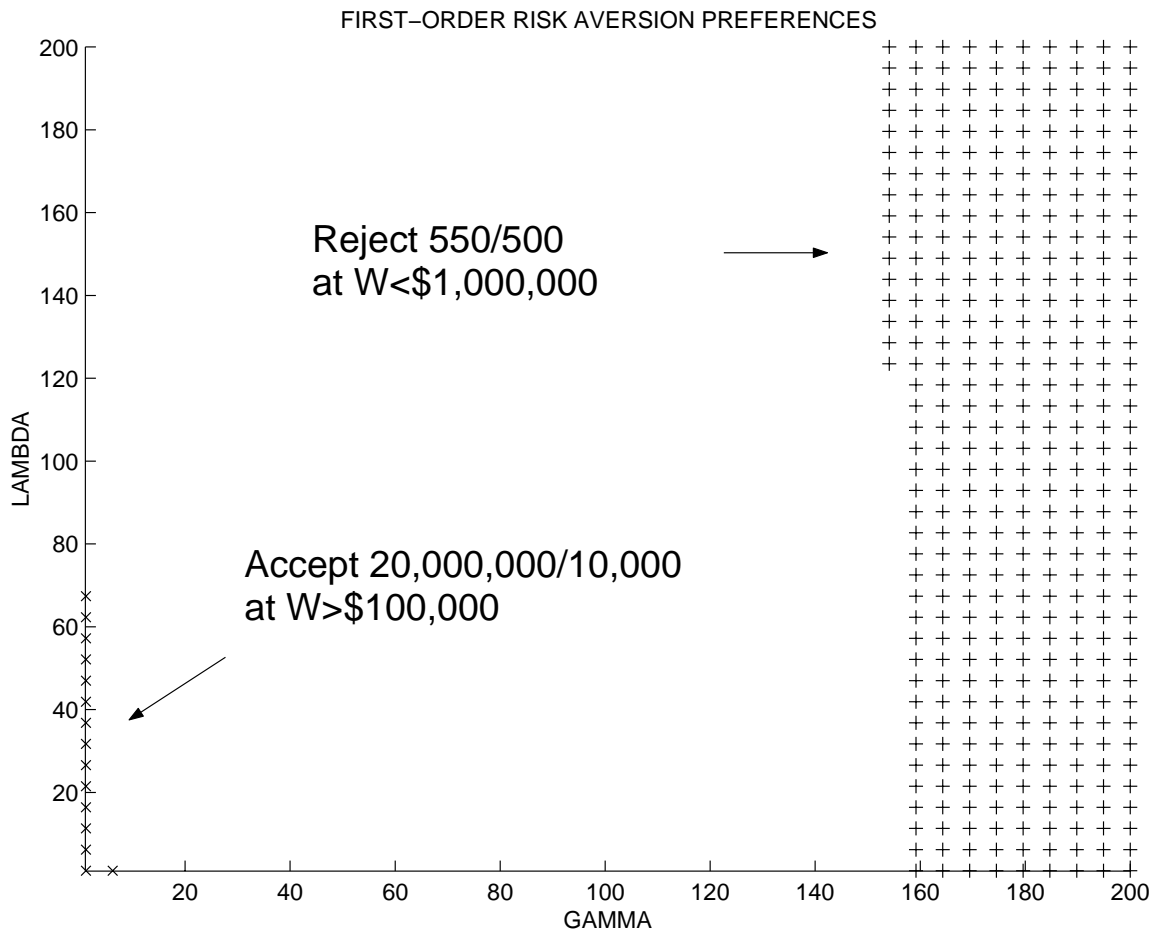


Figure 2. The “+” signs show the range of parameter values for which an agent with a recursive utility function with first-order risk averse certainty equivalent rejects a 50:50 bet to win \$550 or lose \$500. The “x” signs show the parameter values for which the agent accepts a 50:50 bet to win \$20,000,000 or lose \$10,000.

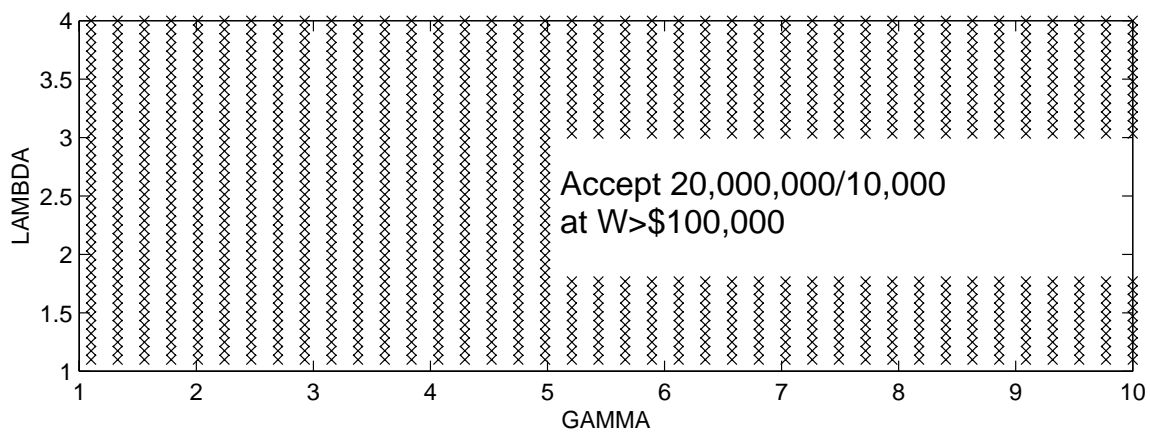
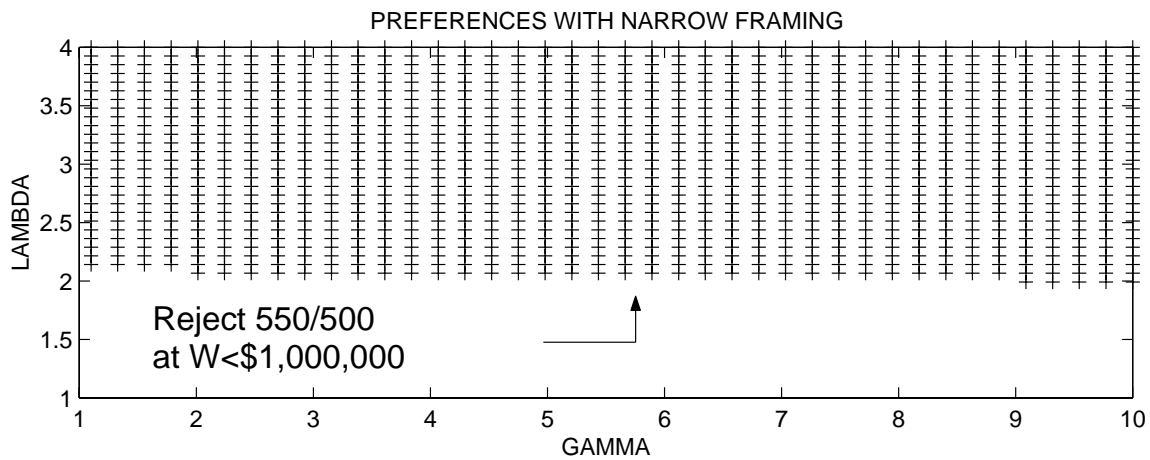


Figure 3. The “+” signs show the range of parameter values for which an agent with a recursive utility function with first-order risk averse certainty equivalent and narrow framing rejects a 50:50 bet to win \$550 or lose \$500. The “x” signs show the parameter values for which the agent accepts a 50:50 bet to win \$20,000,000 or lose \$10,000.

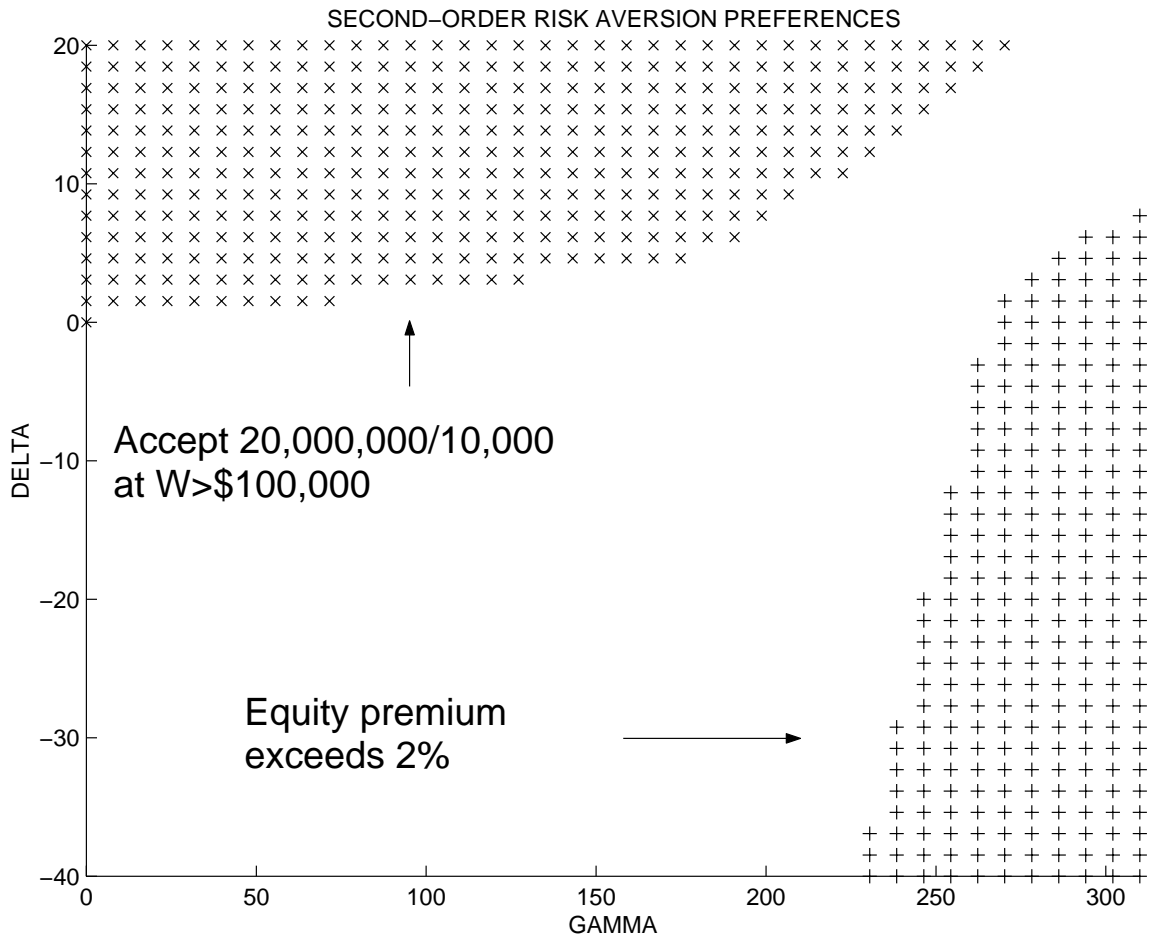


Figure 4. The “+” signs show the parameter values for which a representative agent with a recursive utility function with second-order risk averse certainty equivalent would require an equity premium greater than 2 percent. The “x” signs show where the agent would accept a 50:50 bet to win \$20,000,000 or lose \$10,000.

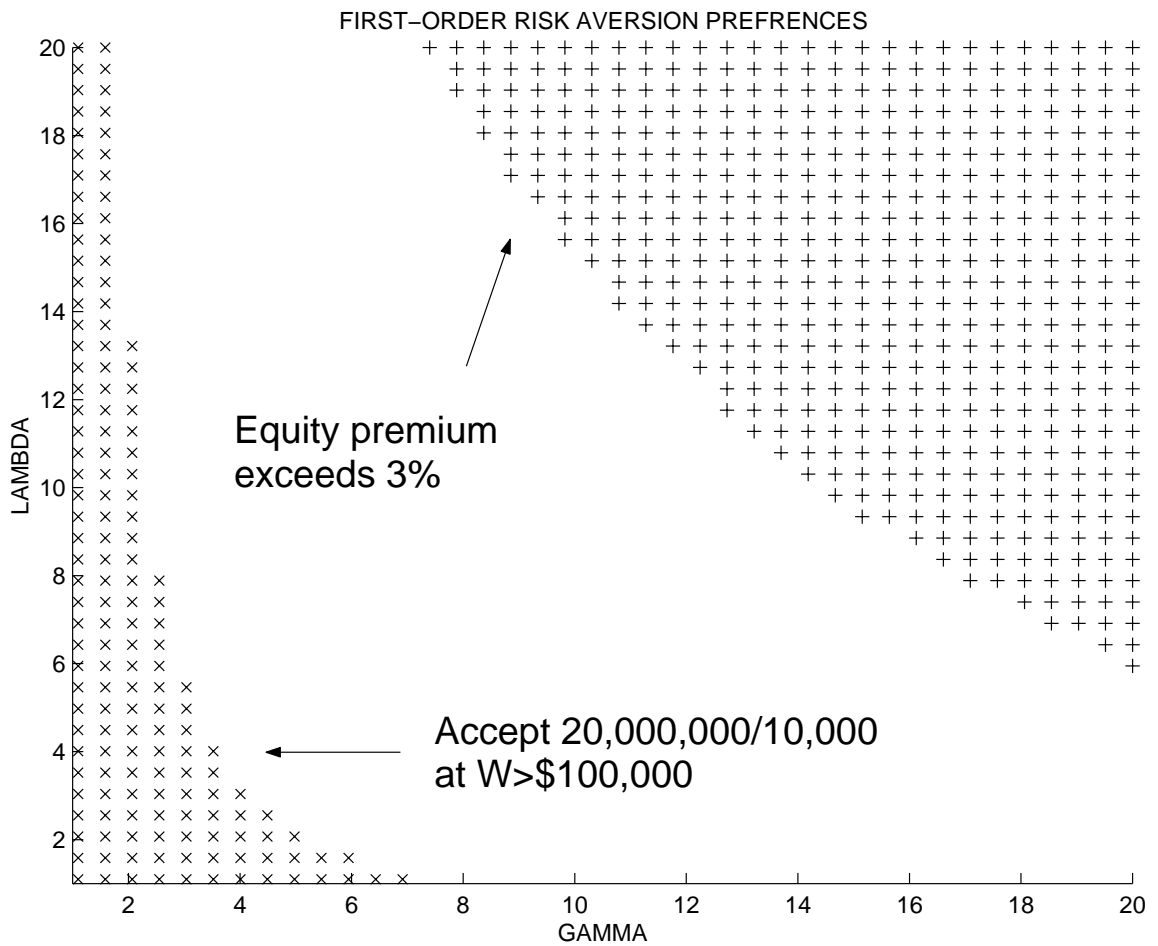


Figure 5. The “+” signs show the parameter values for which a representative agent with a recursive utility function with first-order risk averse certainty equivalent would require an equity premium greater than 3 percent. The “x” signs show where the agent would accept a 50:50 bet to win \$20,000,000 or lose \$10,000.

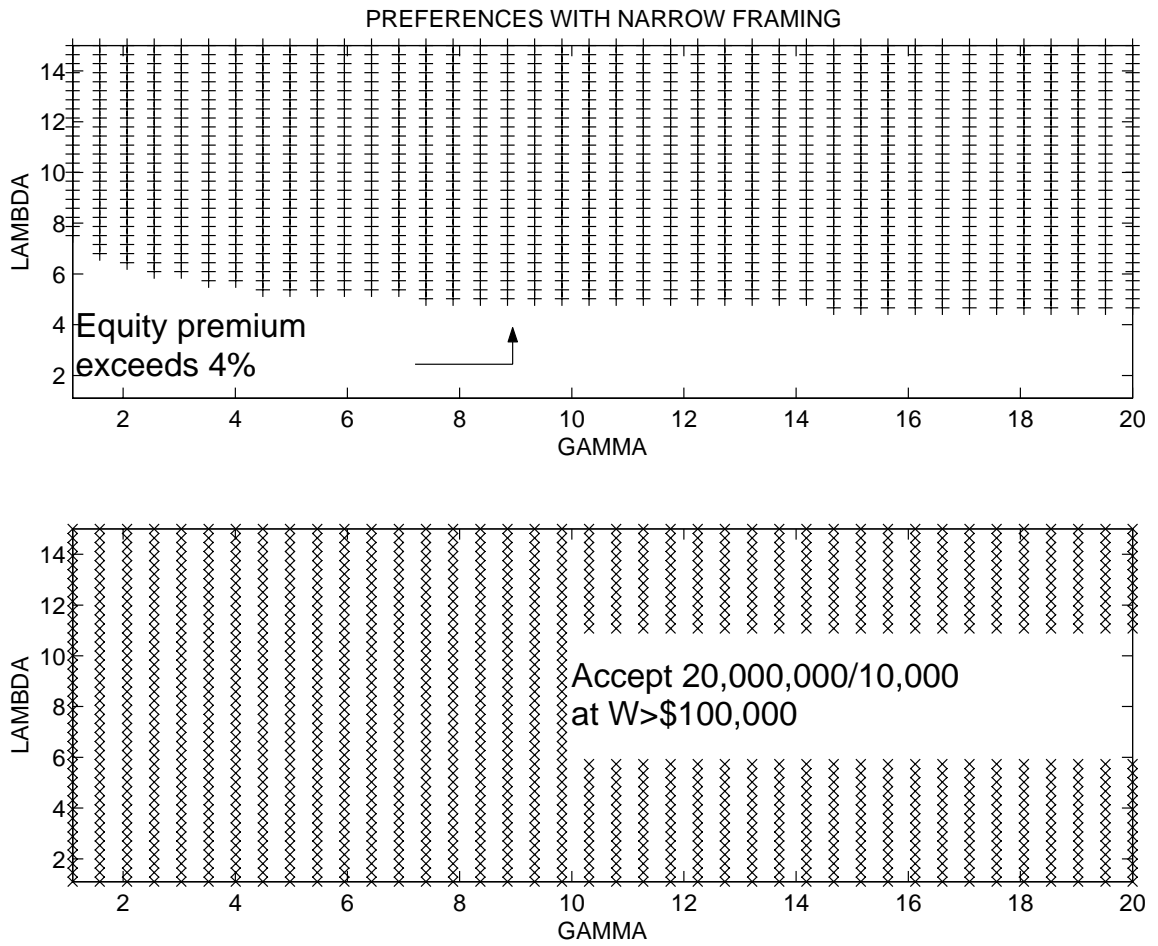


Figure 6. The “+” signs show the parameter values for which a representative agent with a recursive utility function with first-order risk averse certainty equivalent and narrow framing would require an equity premium greater than 4 percent. The “x” signs show where the agent would accept a 50:50 bet to win \$20,000,000 or lose \$10,000.