

Optimal Exercise of Executive Stock Options and Implications for Firm Cost¹

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Abstract

The cost of executive stock options has become a focus of investor attention. The difficulty is that option cost depends on the exercise policies of executives. This paper analyzes the optimal policy for a general utility-maximizing executive holding a nontransferable option. We show analytically how the policy varies with risk aversion, wealth, and dividend rate, and when the policy is characterized by a single stock price boundary. We also provide an example with a split continuation region. In CRRA examples, option value decreases with risk aversion, increases with wealth, increases with outside hedging opportunities, but can actually decline with volatility.

With the explosive growth of executive stock options in corporate compensation, the cost of these options to shareholders has become a focus of attention in finance and accounting. Recent regulation requiring firms to recognize option expense after 2005 has intensified the demand for suitable valuation methods. The difficulty is that the value of these options depends crucially on the exercise policies of the option holders, but, because these options are nontransferable, the usual theory does not apply.

In the case of an ordinary call, the holder can sell the option at any time, so his goal is presumably to maximize the option's present value. The value-maximizing exercise policy in a Black-Scholes world has been researched extensively (see Merton (1973), Van Moerbeke (1976), Roll (1977), Geske (1979), Whaley (1981), Kim (1990)). It calls for exercising the option once the stock price rises above a critical level. This critical level is increasing in the riskless rate, the stock return volatility, and the time remaining to maturity, and it is decreasing in the dividend rate, with no early exercise if the dividend rate is zero.

By contrast, the holder of an executive stock option must bear the risk of the option payoff, so simply maximizing the option's present value is generally not optimal. Indeed, evidence indicates that executives systematically exercise options on non-dividend paying stocks well before expiration. The executive presumably chooses an option exercise policy as part of a greater utility maximization problem that includes other decisions, such as portfolio and consumption choice and managerial strategy.

This paper studies the optimal exercise policy for an executive stock option under simple but appealing assumptions about the executive's choice set. We analyze the optimal exercise policy for a general utility-maximizing executive and indicate when the policy is completely characterized by a single stock price boundary. We also provide an example with a split continuation region, in which the executive exercises at low and high stock prices but not in between. We show how the policy varies with risk aversion, wealth, and volatility, and explore implications for option value. For example, we provide conditions under which the continuation region is larger the greater executive risk aversion and outside wealth, and the lower the stock dividend rate. On the other hand, the size of the continuation region is not monotonic in stock volatility, and option cost to shareholders can actually decline as volatility rises.

When optimal trading of outside wealth in the market is possible, our examples suggest that in the absence of constraints on the outside portfolio weight in the market, the exercise boundary and option value increase with the magnitude of the correlation between the

stock return and the market return, independent of the sign of the correlation, converging to their levels under the value maximizing policy. Imposing a bound on the magnitude of the market weight in the outside portfolio reduces boundaries and option values, and when the market risk premium is nonzero, the magnitude of the effect of the portfolio constraint depends on the sign of the correlation. When the market risk premium and stock beta are both nonnegative, increasing the stock beta increases option value.

The intuition that the need for diversification can lead an executive to sacrifice some option value by exercising it early is well understood in the literature, but explicit theory of the optimal exercise of ESOs is still developing. Huddart (1994), Marcus and Kulatilaka (1994), and Carpenter (1998) build binomial models of the utility-maximizing exercise decision with exogenous assumptions about how non-option wealth is invested. Detemple and Sundaresan (1999) extend these to allow for simultaneous option exercise and portfolio choice decisions. These papers establish the economic approach to ESO valuation, focusing on the optimality of early exercise (and the fact that this makes ESOs worth less than their Black-Scholes value), rather than an in-depth analysis of the exercise policy itself. In a continuous-time framework, Ingersoll (2006) develops a subjective option valuation methodology, assuming the option is a marginal component of the executive's portfolio. More recently, several papers have solved versions of the problem we describe here for the case of constant absolute risk averse utility, where the optimal exercise policy is independent of the executive's wealth. Leung and Sircar (2006) solve the finite horizon problem, and include the risk of job termination and the possibility of partial option exercise. Kadam, Lakner, and Srinivasan (2003) model the optimal exercise policy for an infinite horizon option, but the model links the manager's consumption date to the option exercise date, which can distort the exercise decision, even in the absence of trading restrictions. Henderson (2004) also models the optimal exercise policy for an infinite horizon real option and links the manager's consumption date to the option exercise date, but uses a specialized utility function so that this link does not distort the exercise policy.

A number of papers model option value using exogenous specifications of the exercise policy. Jennergren and Näslund (1993), Carr and Linetsky (2000), and Cvitanić, Wiener, and Zapatero (2004) derive analytic formulas for option value assuming exogenously specified exercise boundaries and forfeiture rates. Hull and White (2004) propose a binomial model in which exercise occurs when the stock price reaches an exogenously specified multiple of the stock price and forfeiture occurs at an exogenous rate. Rubinstein (1995) and Cuny and Jorion (1995) also compute option value under exogenous assumptions about

the timing of exercise.

Other authors have focused on the executive's private valuation of the option using certainty equivalents rather than on the market value of the option from the viewpoint of shareholders. These include Lambert, Larcker, and Verrecchia (1991), Hall and Murphy (2002), Cai and Vihj (2005), and Miao and Wang (2005).

1 General framework

This section lays out the general model of the executive's optimal exercise problem and defines the resulting option cost to shareholders.

1.1 The executive's option exercise and portfolio choice problem

The executive has n finite-lived options with strike price K and expiration date T and additional wealth that can be invested subject to a prohibition on short sales of the stock.¹ The investment set includes riskless bonds with constant riskless rate r , the underlying stock with price S_t , and a market portfolio with price M_t . These prices satisfy

$$\frac{dS_t}{S_t} = (\lambda - \delta) dt + \sigma dB_t, \quad (1)$$

$$\frac{dM_t}{M_t} = \mu dt + \sigma_m dB_t, \quad (2)$$

where B_t is a standard two-dimensional Brownian motion on a probability space equipped with the natural filtration and σ and σ_m are two-dimensional row vectors. The stock return volatility, σ , the stock dividend rate δ , and the mean and volatility of the market return, μ and σ_m are constant, and the mean stock return λ is equal to the normal return for the stock given its correlation with the market,

$$\lambda = r + \frac{\sigma \sigma'_m}{\|\sigma_m\|^2} (\mu - r). \quad (3)$$

In particular, in the absence of the option, an optimal portfolio would contain no stock position beyond what is implicitly included in the market portfolio.

¹The use of zero-cost collars and equity swaps by corporate insiders documented by Bettis et al. (2001) suggests that insiders may have some scope for hedging their incentive compensation. However, evidence that the vast majority of options are exercised well before expiration, even when no dividend is present, suggests that option holders still face significant hedging constraints.

The executive simultaneously chooses an option exercise time τ , which is a stopping time of the filtration generated by the Brownian motion, and an investment strategy in the market and the stock, $\pi_t \equiv (\pi_t^m, \pi_t^s)$ satisfying $E \int_{t=0}^T \|\pi_t\|^2 dt < \infty$. His goal is to maximize the expected utility of time T wealth:

$$\max_{\{\tau \leq T, \pi^m, \pi^s \geq 0\}} E\{V(W_\tau + n(S_\tau - K)^+, \tau)\} \quad (4)$$

subject to

$$dW_t = rW_t dt + \pi_t^m((\mu - r) dt + \sigma_m dB_t) + \pi_t^s((\lambda - r) dt + \sigma dB_t), \quad (5)$$

where

$$V(W_t, t) \equiv \max_{\pi^m} E_t\{U(W_T)\} \text{ s.t. } dW_u = rW_u du + \pi_u^m((\mu - r) du + \sigma_m dB_u), \quad (6)$$

and the utility function U is strictly increasing, strictly concave, and twice continuously differentiable.

This formulation entails a number of simplifications. The executive's portfolio does not include a position in restricted shares of stock (see Kaul, Liu, and Longstaff (2003) for a model of portfolio choice with restricted stock). It allows only for a single block exercise of the option, although the executive would probably prefer to exercise the options at a stochastic rate over time. The model also considers only a single grant of options when in practice, executives are granted new ten-year options every year and typically build up large inventories of options with different strikes and expiration dates. It would be useful to understand which options are most attractive to exercise first and how the anticipation of future grants of options and other forms of compensation affects current exercise decisions. In addition, the model does not account for any control the executive has over the underlying stock price process through the exertion of effort and through project and leverage choices; these choices may interact with the exercise decision. Despite these simplifications, we believe this formulation captures the essence of the executive stock option problem.

Intuition suggests that the optimal outside position in the stock in problem (4) is $\pi^s \equiv 0$, however this remains to be proved. The example in Evans, Henderson, and Hobson (2005) shows that results from traditional portfolio theory may fail to hold in the presence of an optimal stopping problem.

If the optimal investment policy π_t and the indirect utility function V satisfy, respectively, linear and polynomial growth conditions in W and S , then Theorem 3.1.8 of Krylov (1980) implies that the value function for the executive's problem,

$$f(W_t, S_t, t) \equiv \max_{\{t \leq \tau \leq T, \pi^m, \pi^s \geq 0\}} \mathbf{E}_t \{V(W_\tau + n(S_\tau - K)^+, \tau)\} \quad (7)$$

subject to

$$dW_u = rW_u dt + \pi_u^m((\mu - r) du + \sigma_m dB_u) + \pi_u^s((\lambda - r) du + \sigma dB_u), \quad (8)$$

is continuous and satisfies $f(W_t, S_t, t) \geq V(W_t + n(S_t - K)^+, t)$ and $f(W_T, S_T, T) = U(W_T + n(S_T - K)^+)$.

1.2 Option cost to shareholders

The solution to the executive's optimal exercise problem, that is, the optimal exercise policy τ , defines the option payoff, $(S_\tau - K)^+$ that occurs at time τ . The cost of the option to shareholders who can trade freely is the present value, or replication cost, of that payoff, which can be represented as the risk-neutral expectation of its risklessly discounted value,

$$\mathbf{E}^* \{e^{-r\tau} (S_\tau - K)^+\}, \quad (9)$$

where \mathbf{E}^* means the expectation is taken with respect to the probability measure under which the expected returns on both the market and the stock are equal to the riskless rate.

Standard theory for tradeable options assumes the option holder chooses the exercise policy to maximize the option's present value, because when the option is tradeable, maximizing present value is consistent with maximizing expected utility. When the option is nontransferable these objectives are different, and the utility-maximizing payoff typically has a lower present value.

In addition, when the option is nontransferable, its value to the executive is different from its present value or cost to shareholders. The value to the executive may be defined as the amount of freely investable cash that would make the executive equally happy as having the nontransferable option. Having the option present value tied up in the option can be no better for the executive than having that value invested freely. Therefore, the amount of freely investable cash that would make the executive as happy as having the

option can be no more than the option's present value, and is typically less. That discount in the executive's private valuation of the option relative to its cost to shareholders is the price shareholders pay for any performance incentive benefits the option creates relative to cash compensation. While the executive's private valuation is an important consideration in the more general problem of optimal contracting, our focus in this paper is on the option cost to shareholders.

2 Special case with no portfolio choice

We start by analyzing the case in which the stock appreciates at the riskless rate,

$$\frac{dS_t}{S_t} = (r - \delta) dt + \sigma dB_t, \quad (10)$$

and there is no other risky asset available. In this case, it is clear that after the executive exercises the options, his optimal portfolio contains only riskless bonds, so

$$V(W_t, t) = U(W_t e^{r(T-t)}). \quad (11)$$

Intuition suggests that even before the option is exercised, the executive's optimal outside portfolio contains no stock, since he would choose to short stock in the absence of a short sale constraint. We proceed with the assumption that investing outside wealth in bonds is optimal here. The executive's problem at each time $t < T$ then becomes

$$f(S_t, t) \equiv \max_{\{t \leq \tau \leq T\}} E_t \{U(n(S_\tau - K)^+ e^{r(T-\tau)} + W)\}, \quad (12)$$

where the constant W is outside wealth at time T with $W > nK e^{rT}$ and $f : (0, \infty) \times [0, T] \rightarrow \mathcal{R}$ is a continuous function satisfying $f(S_t, t) \geq U(n(S_t - K)^+ + W)$ and $f(S_T, T) = U(n(S_T - K)^+ + W)$.

Note that

$$E[\sup_{0 \leq t \leq T} U(n(S_t - K)^+ e^{r(T-t)} + W)] = E[U(\max_{0 \leq t \leq T} (n(S_t - K)^+ e^{r(T-t)} + W))] \quad (13)$$

$$\leq U(E[\max_{0 \leq t \leq T} (n(S_t - K)^+ e^{r(T-t)} + W)]) \quad (14)$$

$$< \infty, \quad (15)$$

so Theorem D.12 of Karatzas and Shreve (1998) implies that an optimal exercise time is

$$\tau^* \equiv \inf\{t \in [0, T] : f(S_t, t) = U(n(S_t - K)^+ e^{r(T-t)} + W)\} \quad (16)$$

and the continuation region for the problem is

$$D \equiv \{(s, t) \in (0, \infty) \times [0, T] : f(s, t) > U(n(s - K)^+ e^{r(T-t)} + W)\}. \quad (17)$$

2.1 Existence of a critical stock price boundary

This section explores whether a single critical stock price boundary $\bar{s}(t)$ separates the continuation region below from the exercise region above, as is the case for ordinary American calls. This is often assumed to be true in executive stock option models with exogenously specified exercise policies, however, it remains to be proved that the utility-maximizing policy has this structure.

To formalize intuition about the various effects of waiting to exercise, let $g(s, t) \equiv U(n(s - K)^+ e^{r(T-t)} + W)$ denote the payoff function for the optimal stopping problem and note that on $(K, \infty) \times [0, T]$, g is $C^{2,1}$ and Itô's lemma implies that g has drift equal to $H(S_t, t)$ where

$$H(s, t) \equiv U'(h(s, t))(rK - \delta s)ne^{r(T-t)} + \frac{1}{2}U''(h(s, t))n^2e^{2r(T-t)}\sigma^2s^2 \quad (18)$$

and $h(s, t) \equiv n(s - K)e^{r(T-t)} + W$ is total time T wealth given exercise at time t and stock price s . This expression shows that when the option is in the money, the effects of waiting to exercise include the benefits of delaying payment of the strike price, the cost of losing dividends, and the cost of bearing stock price risk.

Proposition 2.1 *Suppose that H is nonincreasing in the stock price s . Then for each time $t \in [0, T]$, if there is any stock price at which exercise is optimal, then there exists a critical stock price $\bar{s}(t)$ such that it is optimal to exercise the option if and only if $S_t \geq \bar{s}(t)$.*

Proof Fix $t \in [0, T]$. Suppose (s_1, t) is a continuation point. We show that if $s_2 < s_1$ then (s_2, t) is also a continuation point. First note that it must be optimal to continue holding the option if $S_t \leq K$. Stopping then would guarantee a reward of $U(W)$, which is less than the expected utility of continuing, for example, until the first time the stock price rises to $K + c$, for some $c > 0$, or until expiration T .

So assume $s_1 > s_2 > K$. For $u \geq t$, let $S_u^{(i)}$ denote the stock price process starting from s_i at time t and note that $S_u^{(1)} > S_u^{(2)}$. Finally, let τ be the optimal stopping time given $S_t = s_1$. Since τ is a feasible strategy if $S_t = s_2$,

$$\begin{aligned}
f(s_2, t) - f(s_1, t) &\geq \mathbb{E}_t\{U(n(S_\tau^{(2)} - K)^+ e^{r(T-\tau)} + W) - U(n(S_\tau^{(1)} - K)^+ e^{r(T-\tau)} + W)\} \\
&\geq \mathbb{E}_t\{U(n(S_\tau^{(2)} - K) e^{r(T-\tau)} + W) - U(n(S_\tau^{(1)} - K) e^{r(T-\tau)} + W)\} \\
&= g(s_2, t) - g(s_1, t) + \mathbb{E}_t \int_t^\tau (H(S_u^{(2)}, u) - H(S_u^{(1)}, u)) du \\
&\geq g(s_2, t) - g(s_1, t) .
\end{aligned} \tag{19}$$

Therefore, $f(s_2, t) - g(s_2, t) \geq f(s_1, t) - g(s_1, t) > 0$.

Remark The hypothesis is satisfied for constant relative risk averse utility functions with relative risk aversion less than or equal to one. Similarly, in the value maximization problem for an ordinary option, the second order term in H does not appear, the drift is nonincreasing in the stock price, and it follows that it is optimal to exercise if and only if the stock price has risen above a critical level. For executive stock options however, the risk aversion of the option holder gives rise to the second order term, and the drift need no longer be monotonic in the stock price.

Example with a split continuation region Figure 1 shows the optimal exercise policy for utility function

$$U(W) = \frac{W^{1-A}}{1-A} + cW \tag{20}$$

with $A = 10, c = 0.0001, K = 1, T = 10, r = 0.05, \sigma = 30\%$, and $\delta = 0$. The utility function is strictly increasing and strictly concave. As the figure shows, the executive continues for low and high stock prices, but exercises the option for intermediate stock prices. It is not clear, however, how much valuation error would be created by erroneously assuming the existence of a single critical exercise boundary. That would depend on how that single boundary was determined. In this example, if we ignore the presence of the upper boundary, the option cost is 0.408 instead of the correct value of 0.432.

Ahn and Wilmott (2003) find an example with a disconnected continuation region using a specialized HARA utility function and no outside wealth, but it requires that the appreciation rate on the stock exceed the riskless rate. This can distort the exercise policy when the model does not permit the executive to buy stock, because he may choose to continue holding the option as a substitute, even when doing so is value-destroying.

2.2 Dependence of the continuation region on the parameters

Understanding how executive stock option value varies with stock return volatility, executive wealth, and other parameters requires an understanding of how these parameters affect the exercise policy. With an ordinary American call option, the exercise boundary, or in other words, the set of stock prices at which the option holder would continue at a given point in time, is increasing with the stock volatility and the time to expiration and decreasing with the dividend rate. With executive stock options, the dependence of the continuation region on the parameters is less clear cut. This section describes how the continuation region changes with executive risk aversion and wealth, the stock dividend rate, and the stock return volatility.

2.2.1 Monotonicity with respect to risk aversion and wealth

Intuition suggests that less risk averse managers are likely to continue longer. Similarly, one would expect that managers with decreasing absolute risk aversion will continue longer if they have more nonoption wealth. The following results verify this intuition and hold regardless of the actual shape of the continuation region.

Proposition 2.2 *An executive with less absolute risk aversion has a larger continuation region.*

Proof If U_1 and U_2 are utility functions and U_2 has everywhere less absolute risk aversion than U_1 , then by Theorem 5 on page 40 of Ingersoll (1987),

$$U_2(W) = G(U_1(W)) \tag{21}$$

where the function G satisfies $G' > 0$ and $G'' > 0$. Now suppose a given state (s, t) is in the continuation region with utility U_1 and let τ be the optimal stopping time for U_1 . Let $f_i(s, t)$ and $g_i(s, t)$ denote the value and payoff functions for the problem with utility U_i . Since τ is feasible for the problem with U_2 ,

$$\begin{aligned} f_2(s, t) - g_2(s, t) &\geq E_t\{U_2(n(S_\tau - K)^+ e^{r(T-\tau)} + W)\} - U_2(n(S_t - K)^+ e^{r(T-t)} + W) \\ &= E_t\{G(U_1(n(S_\tau - K)^+ e^{r(T-\tau)} + W))\} - G(U_1(n(S_t - K)^+ e^{r(T-t)} + W)) \\ &\geq G(E_t\{U_1(n(S_\tau - K)^+ e^{r(T-\tau)} + W)\}) - G(U_1(n(S_t - K)^+ e^{r(T-t)} + W)) \\ &= G(f_1(s, t)) - G(g_1(s, t)) \\ &> 0 \end{aligned} \tag{22}$$

Therefore, (s, t) is also in the continuation region for U_2 .

Corollary 2.1 *If the executive has decreasing absolute risk aversion, then the continuation region is larger with greater wealth.*

Proof Let $W_2 > W_1$ and note that $U(w + W_2 - W_1) = G(U(w))$ for some function G satisfying $G' > 0$ and $G'' > 0$.

Figures 2 and 3 illustrate these results and their implications for option value using examples with constant relative risk averse utility. All of the examples in this section are generated using an implicit finite difference method to solve the partial differential equations describing the executive value function and option cost. In all of the numerical examples, the dividend rate is set to zero so that the only motive for early exercise is the ability to transfer the option value to a more efficient portfolio, in this case, the riskless asset. In addition, the number of options and initial stock price are each normalized to one. Even in examples in which the coefficient of relative risk aversion, A , is greater than one, we find that the continuation region is characterized by a single critical stock price boundary.

Figure 2 plots the exercise boundaries and option values for various levels of risk aversion. The exercise boundary is a plot of the critical stock price $\bar{s}(t)$ vs. time t . The option value labeled “ESO” is the present value or cost of the option under the executive’s exercise policy, described by the boundary. Shown for comparison, the option value labeled “Max” is the value of the option under the present value-maximizing policy, which in this zero-dividend case is to hold the option to maturity. The figure shows that option cost to shareholders is greater the less risk averse the executive. Figure 3 plots exercise boundaries and option values for various levels of executive wealth. It shows that option cost to shareholders is greater the wealthier the executive.

2.2.2 Monotonicity with respect to the dividend rate

This section shows that the executive’s continuation region is larger the smaller the dividend rate on the stock, as is the case for an ordinary American option. Again, this result holds regardless of the shape of the continuation region.

Proposition 2.3 *The executive’s continuation region is larger the smaller the dividend rate on the stock.*

Proof Suppose a given state (s, t) is in the continuation region when the dividend rate is δ_1 and let $\delta_2 < \delta_1$. Let $f(s, t; \delta)$ denote the value function and $S_t^{(\delta)}$ denote the stock price process when the dividend rate is δ . Let τ be the optimal stopping time for the problem with δ_1 . Then, since τ is a feasible choice for the problem with δ_2 and $S_\tau^{(\delta_2)}/S_t^{(\delta_2)} > S_\tau^{(\delta_1)}/S_t^{(\delta_1)}$,

$$\begin{aligned} f(s, t; \delta_2) - f(s, t; \delta_1) &\geq \mathbb{E}\{U(n(S_\tau^{(\delta_2)} - K)^+ e^{r(T-\tau)} + W) - \\ &\quad U(n(S_\tau^{(\delta_1)} - K)^+ e^{r(T_1-\tau)} + W) | S_t^{(\delta_1)} = S_t^{(\delta_2)} = s\} \\ &\geq 0 \end{aligned} \tag{23}$$

Therefore, $f(s, t; \delta_2) \geq f(s, t; \delta_1) > g(s, t)$ so (s, t) is in the continuation region for δ_2 .

2.2.3 Non-monotonicity with respect to the stock return volatility

A basic result in standard option pricing theory is that option value is increasing in volatility. This is also typically the case in executive stock option models with an exogenously specified exercise boundary that does not change with volatility (see, for example, Cvitanic, Wiener, and Zapatero (2004)). However, the utility-maximizing continuation region can shrink considerably with volatility and this can lead to option value declining in volatility.

Figure 4 illustrates these effects using examples with constant relative risk averse utility and a zero dividend rate. Again, in all examples, even those in which the coefficient of relative risk aversion, A , is greater than one, the continuation region is characterized by a single critical stock price boundary. Figure 4 plots the exercise boundaries and option values for various levels of stock return volatility.

As volatility rises from 10% to 200%, the exercise boundary tends to fall first and then rise slightly. This is shown most clearly in Figure 4a, with risk aversion coefficient $A = 0.5$. The risk averse utility of the option payoff, as a function of the stock price, has both a convex region and a concave region, so in principle, an increase in volatility could either lead the executive to continue longer or exercise sooner. Apparently the concave portion dominates at low levels of volatility, making the executive exercise sooner as volatility rises. At higher levels of volatility, the convex portion seems to dominate and the boundary rises slightly. Empirically, Bettis, Bizjak, and Lemmon (2005) find that options are exercised earlier at higher volatility firms.

At the lower levels of risk aversion shown in Figures 4a and 4b, executive stock option

value is generally increasing in volatility. However, at the higher levels of risk aversion shown in Figures 4c and 4d, executive stock option value is decreasing in volatility at low levels of volatility. Here the negative effect on value of the drop in the boundary of offsets the positive effect of extreme stock prices becoming more likely.

3 General case with outside portfolio choice

This section examines the general problem with nontrivial outside portfolio optimization described in Section 1. Numerical examples characterize optimal exercise policies and option cost using an explicit finite difference method to solve the partial differential equations for the executive value and option cost functions.

Unreported results confirm that the wealth, risk aversion, and volatility effects from the last section still hold in the presence of optimal trading in a market portfolio with a nonzero risk premium and correlation with the stock return. In particular, option value is still increasing with executive wealth, decreasing with executive risk aversion, and non-monotonic with respect to stock return volatility. In addition, the optimal exercise policy appears to be characterized by a critical stock price for each possible date and wealth level, above which it is optimal to exercise and below which it is optimal to continue. We also note that when the market risk premium and the stock return correlation with the market are set to zero, then optimal market portfolio weight in the outside portfolio is zero, and the results are the same as those from the one-factor model of the last section.

The remainder of this section focuses on the dependence of the exercise policy and option value on the degree of correlation between the stock return and the return on the market. Our solution method requires that we place a bound on the magnitude of the weight on the market in the outside portfolio, in order to bound the portfolio volatility. When outside wealth is large relative to option wealth, that constraint is not binding, but the constraint impacts the results when outside wealth is small. The constraint is potentially relevant empirically because it corresponds to constraints on borrowing and short-selling. The results below disentangle the correlation effects from those of the portfolio constraint.

3.1 Correlation effects with zero market risk premium

Intuition suggests that when the market risk premium μ is zero, the only reason to hold a market position in the outside portfolio is to hedge the option position. Furthermore, all that should matter for the option exercise policy and value is the magnitude of the correlation, ρ , not the sign, since that is what determines how much stock risk can be hedged away.

Figures 5a and 5b confirm this intuition. Exercise boundaries and option values for a given value of ρ are the same as for $-\rho$. To ease comparison with the last section, the figures show exercise boundaries across time for wealth equal to its initial value. The examples fix the riskless rate at 5%, the market return volatility at 20%, the stock return volatility at 50%, and the stock dividend at zero.

Figure 5a illustrates exercise boundaries and option values in the case that outside wealth is large relative to the option position. Initial wealth is six times the value of shares under option, so that the portfolio constraint $|\pi^m| \leq 2$ is very loose. The level of executive risk aversion is also set high, $A = 10$, to make the amount of absolute risk aversion, A/W , comparable to that in some of the examples in the previous section. In this case, as the magnitude of correlation approaches one, the continuation region grows large and the option value approaches its maximized value, i.e., its Black-Scholes value. Indeed, if the executive can hedge the option position perfectly, there is no reason for a value-destroying early exercise.

Figure 5a also shows the effect of tightening the portfolio constraint to $|\pi^m| \leq 0.25$ or $|\pi^m| \leq 0.10$. Tightening the constraint uniformly reduces exercise boundaries and option values because it leaves the option holder exposed to more unwanted option risk. The impact of tightening the constraint is greater for larger correlation, because the larger the correlation, the larger market position the option holder would like to hold to hedge the option. Still, the effect of the portfolio constraint depends only on the magnitude of the correlation, not the sign. It has no impact when correlation is zero, because the optimal market position is zero in that case.

Figure 5b illustrates exercise boundaries and option values in the case that outside wealth is small relative to the option position. Here, initial wealth is 1.2 times the value of shares under option, which is the same as in the previous section in cases where terminal wealth is set equal to 2, and risk aversion $A = 2$. Because outside wealth is smaller relative to option wealth, relatively larger proportional market positions are necessary to hedge the option. Therefore, the portfolio constraint, $|\pi^m| \leq 2$, which was loose before, is

tight here, and the exercise boundaries and option values are similar to those in the case of larger wealth but tighter portfolio constraint.

3.2 Correlation effects with nonzero market risk premium

When the market risk premium is positive, the executive would optimally choose a long position in the market in the absence of the option position. The risk premium on the stock is equal to its normal level, i.e., the stock's market beta times the market risk premium. Thus the stock can be regarded as a portfolio containing the market and a risky idiosyncratic component that carries zero risk premium. Intuition suggests that in the absence of a constraint on the outside portfolio market weight, the executive will set the weight so as to hedge away the market component in the option position, and then incrementally increase the weight to the desired market exposure. In particular, at the optimum, the executive's net exposure to the market and the idiosyncratic risk of the option position should depend only on the magnitude of the correlation, not the sign. Therefore, in the absence of a portfolio constraint, correlation effects on exercise boundaries and option values should be the same as in the case of zero market risk premium. That is, boundaries and option values should rise with the magnitude of correlation, independent of the sign, as more and more of the option risk can be hedged away, and in the limit, option value should converge to its maximized value.

Consistent with this intuition, the left plot of Figure 5c, in which wealth is high, shows that when the portfolio constraint is loose, $|\pi^m| \leq 2$, the correlation effects on option value are virtually the same as in the case of zero market risk premium depicted in Figure 5a. Now, however, the impact of tightening the portfolio constraint is not symmetric in ρ . It is still the case that tightening the constraint uniformly reduces boundaries and option values. But the effect is greater when the correlation is negative than when it is positive. There are two reasons for this. First, the natural net market position is positive, so the constraint leaves less room for the incremental long position needed for hedging when the correlation is negative than it leaves for the incremental short position needed for hedging when the correlation is positive. Second, the unhedged market risk inherent in the option position carries a negative risk premium when the correlation, and thus beta, is negative, while it carries a positive risk premium when the correlation is positive. The effect of the negative risk premium associated with negative correlation and unhedgeable market risk can ultimately outweigh the benefits of partially hedging, so that when the portfolio

constraint is very tight, option value can even decrease as correlation falls from zero to minus one. This is illustrated in the left plot of Figure 5c for the case $|\pi^m| \leq 0.10$.

In the right plot of Figure 5c, outside wealth is lower relative to option wealth, which effectively makes the portfolio constraint $|\pi^m| \leq 2$ tighter. Thus, the correlation effect is comparable to that in the case of larger wealth but a tighter portfolio constraint. In particular, option value is now monotonically increasing in correlation, as in the case of initial wealth equal 6 and $|\pi^m| \leq 0.10$ shown in the left plot.

Figure 5d supports this reasoning by verifying that when the market risk premium is negative, the portfolio constraint is tighter for positive correlation. Indeed, the option values for a given ρ when the market risk premium is -8% are exactly the same as for $-\rho$ when the risk premium is 8% .

To summarize, the results of this section suggest that in the absence of constraint on the outside portfolio weight in the market, option values and exercise boundaries increase with the magnitude of the correlation between the stock return and the market return, independent of the sign of the correlation, and converge to their levels under a value-maximizing policy. Imposing a bound on the magnitude of the market weight reduces boundaries and option values because it leaves the executive exposed to more option risk and thus precipitates earlier exercise. When the market risk premium is positive, the impact of the bound is greater for negative correlation than positive correlation, so much so that option value can change from a U-shaped function of correlation to a monotonic increasing function of correlation. This is because when the natural market position is positive, the bound places a tighter constraint on the incremental long position needed to hedge in the case of negative correlation than the constraint on the incremental short position needed for hedging when correlation is positive. Moreover, the unhedged market position in the option carries a negative risk premium when the stock beta is negative, making the option position even less tolerable. In the empirically relevant range with nonnegative market risk premium and nonnegative correlation between the stock return and the market return, we find that option value is unambiguously increasing in correlation.

4 Summary and Conclusions

This paper seeks to advance the theory of executive stock option valuation with an in-depth study of the optimal exercise policy of a risk averse executive. Recent valuation

models for executive stock options set the exercise policy exogenously, assuming a single critical stock price boundary. This paper shows that the optimal exercise policy need not be in that form. However, when riskless bonds are the only investment available and the stock underlying the option appreciates at the riskless rate, we provide a sufficient condition for the existence of a single critical boundary. This condition is satisfied by constant relative risk averse utility functions with risk aversion coefficient less than or equal to one and we find no counterexamples among our numerical results for constant relative risk averse utility functions with risk aversion coefficient greater than one.

We also prove that the continuation region is larger for executives with less absolute risk aversion, larger for wealthier executives with decreasing absolute risk aversion, larger the lower the dividend rate on the stock, and these results hold regardless of the exact shape of the continuation region. Our numerical examples with constant relative risk aversion show that option cost to shareholders is increasing in executive nonoption wealth and decreasing in executive risk aversion.

The examples also show how the exercise boundary and option value vary with volatility. In contrast to results from standard option theory, or from executive stock option valuation models with a fixed exercise boundary, executive stock option value can decline in stock return volatility when increases in volatility cause the optimal exercise boundary to drop sufficiently.

Finally we show how exercise boundaries and option values vary with stock beta and portfolio constraints when trading outside wealth in the market is possible. When the market risk premium and stock beta are nonnegative, option value increases with the stock beta and decreases as portfolio constraints are tightened.

These results underscore the importance of accurately characterizing the exercise policy for option valuation and yield testable predictions about option exercise behavior.

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Figure 1: Exercise Policy with Split Continuation Region

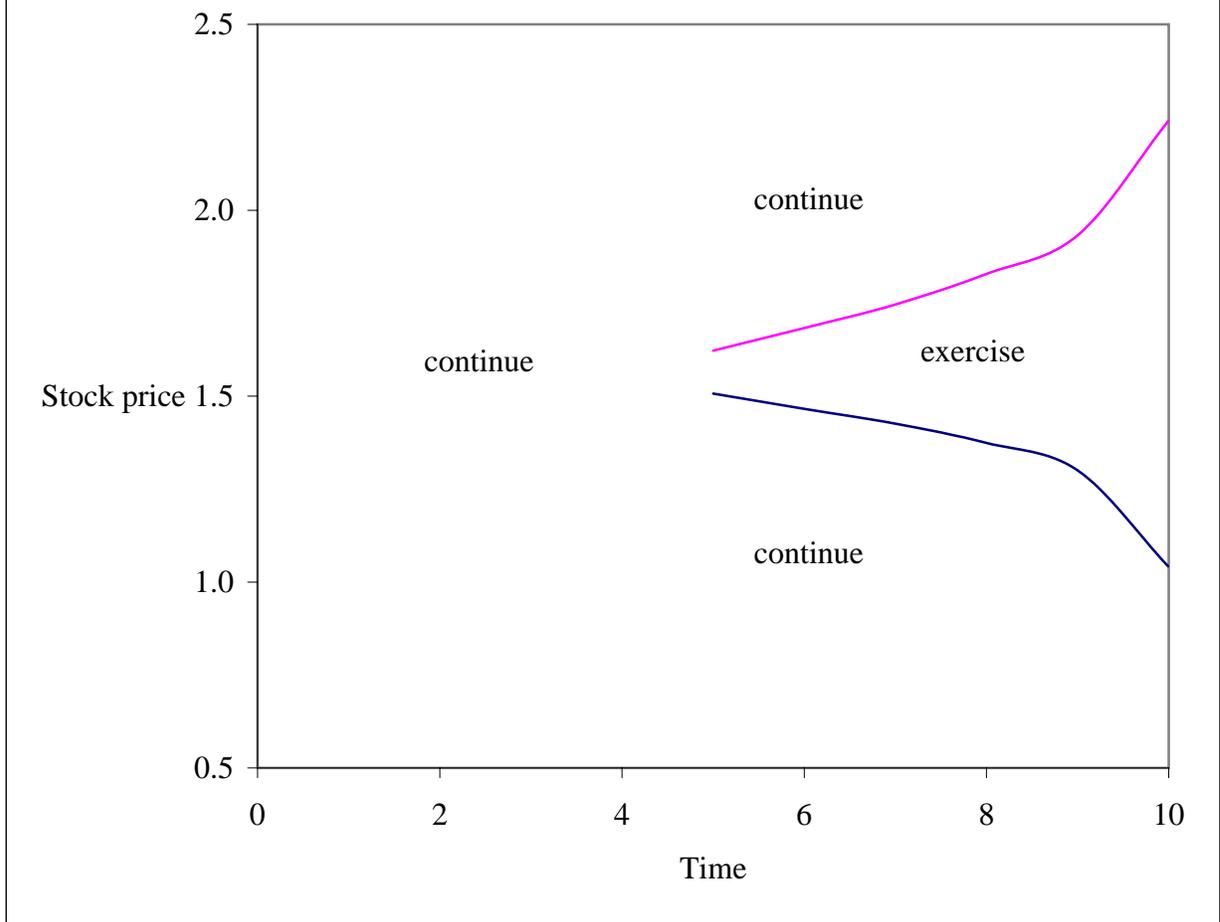
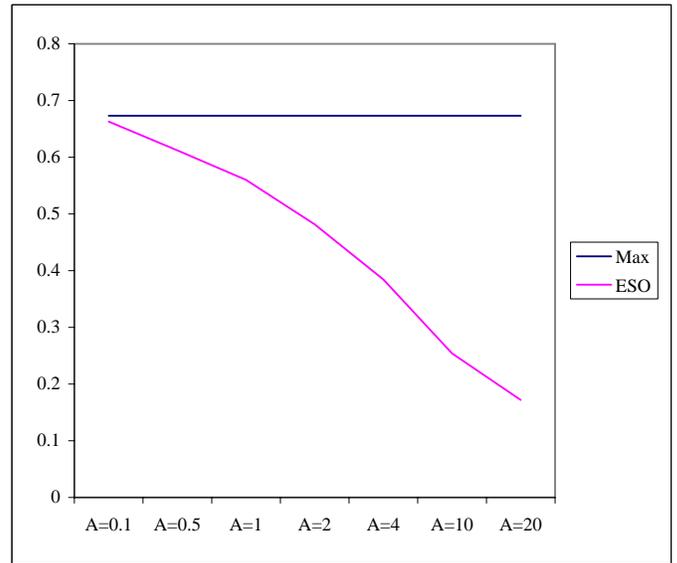
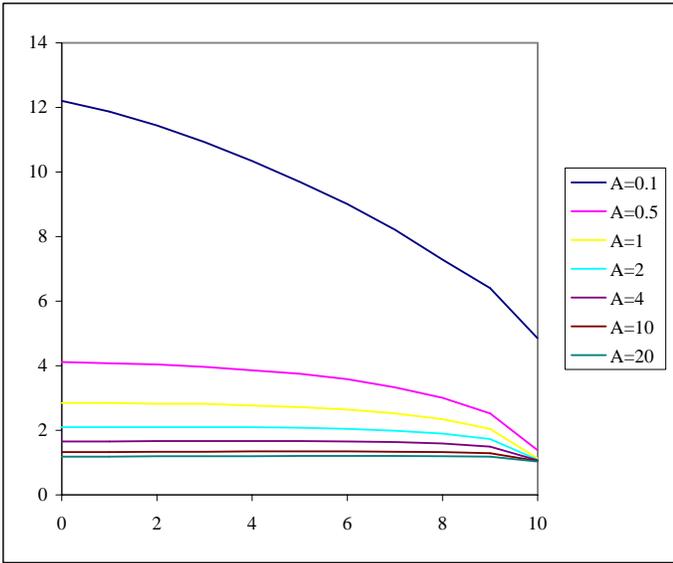


Figure 2: Exercise Boundaries and Option Values for Various Levels of Risk Aversion

a. Wealth = 2, Volatility = 50%



b. Wealth = 0.5, Volatility = 50%

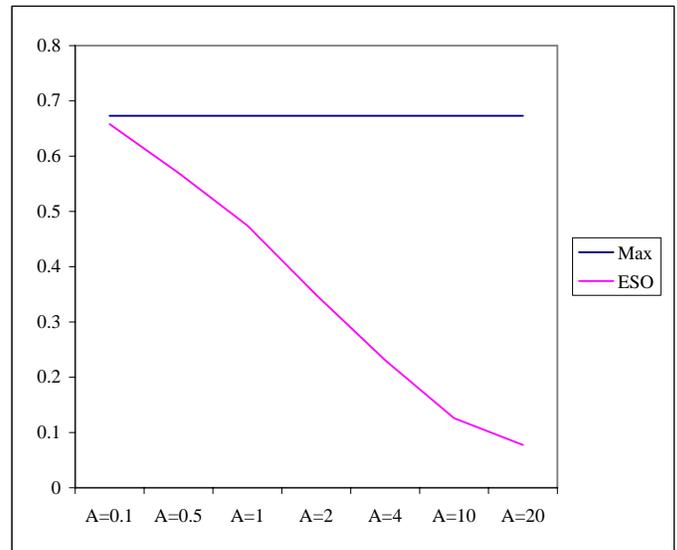
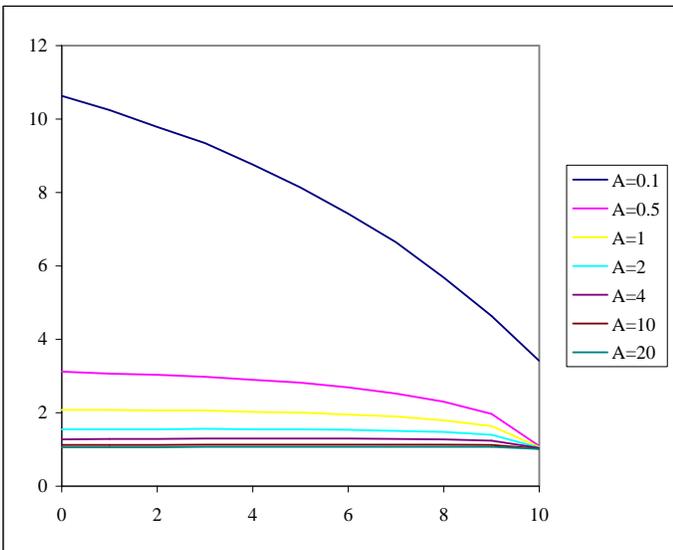
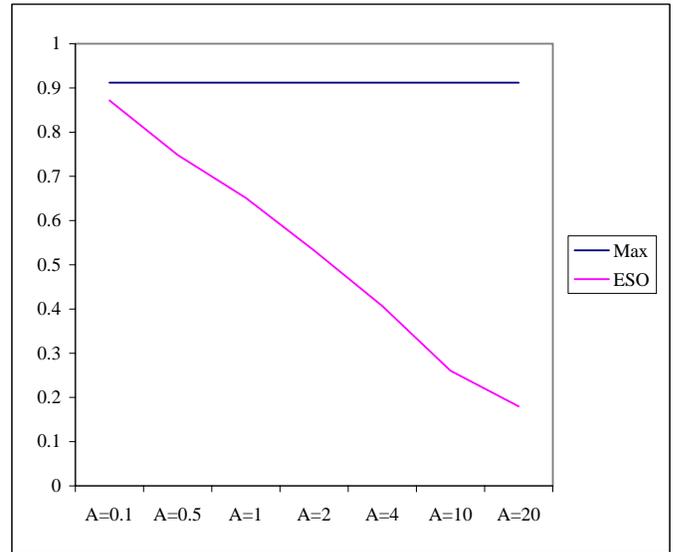
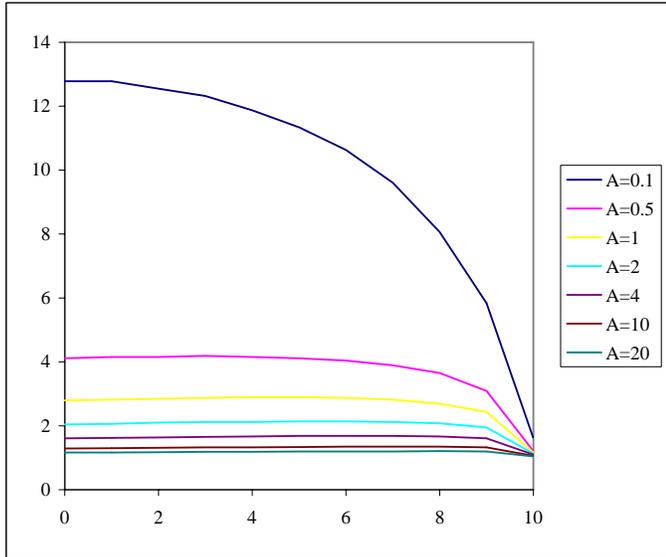


Figure 2 cont'd: Exercise Boundaries and Option Values for Various Levels of Risk Aversion

c. Wealth = 2, Volatility = 100%



d. Wealth = 0.5, Volatility = 100%

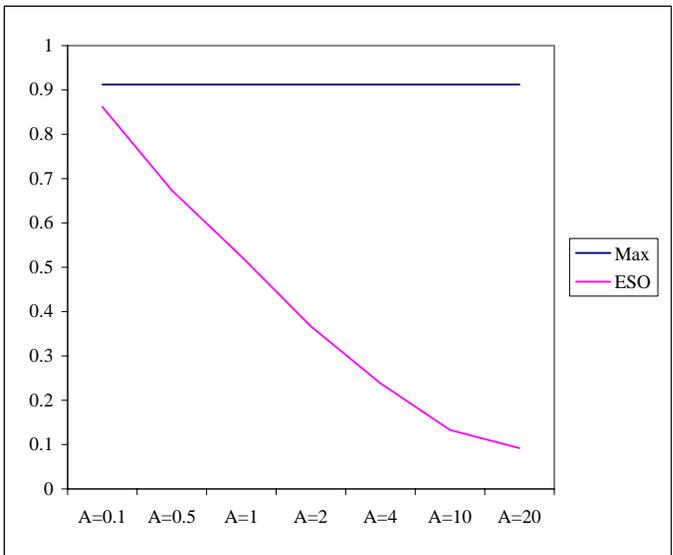
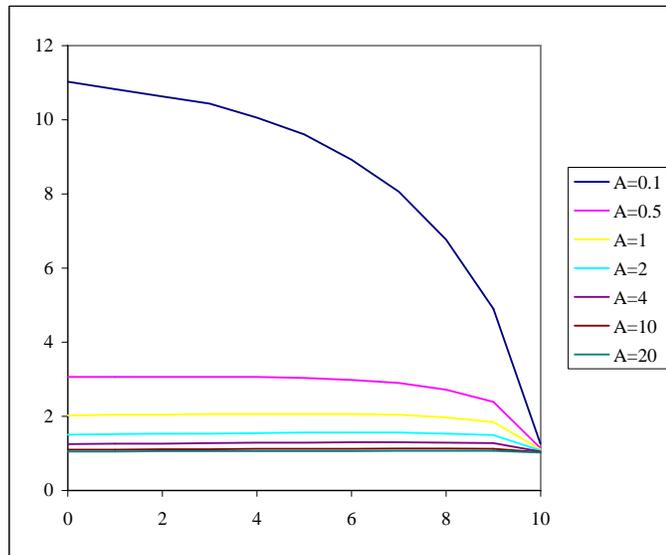
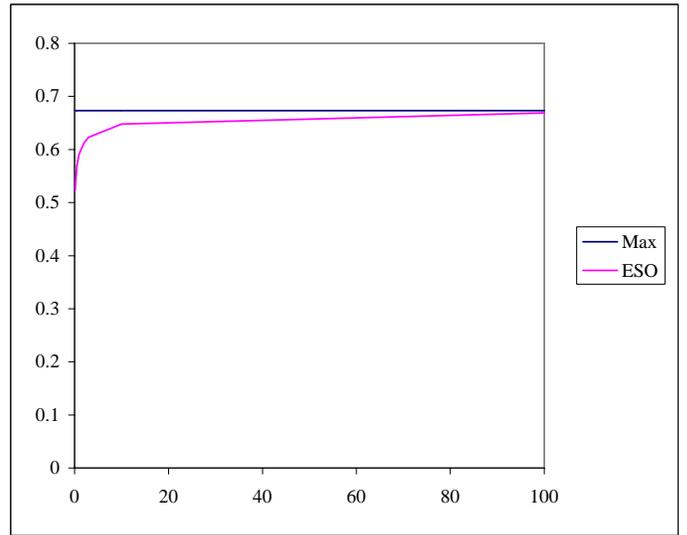
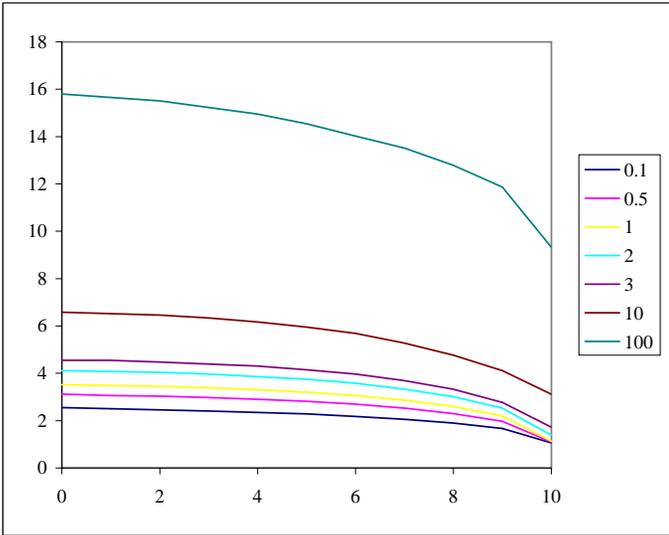


Figure 3: Exercise Boundaries and Option Values for Various Levels of Wealth

a. Risk aversion coefficient = 0.5, Volatility = 50%



b. Risk aversion coefficient = 2, Volatility = 50%

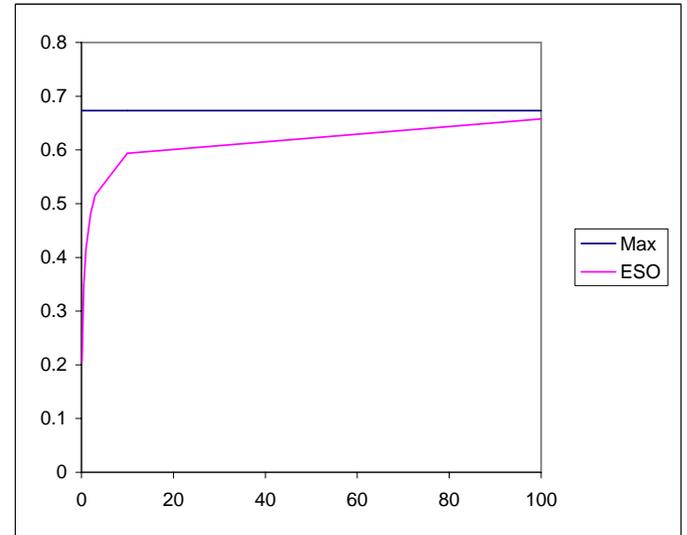
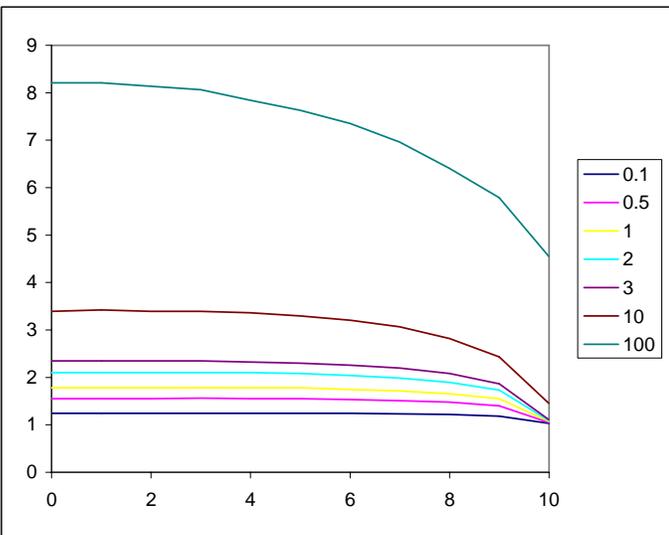
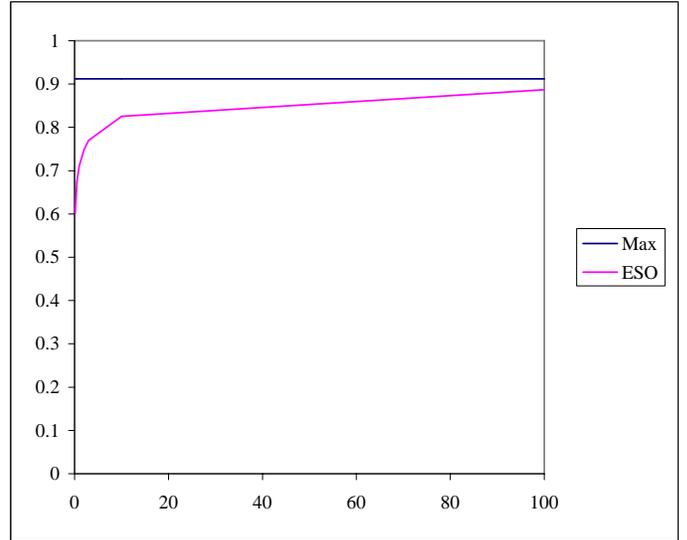
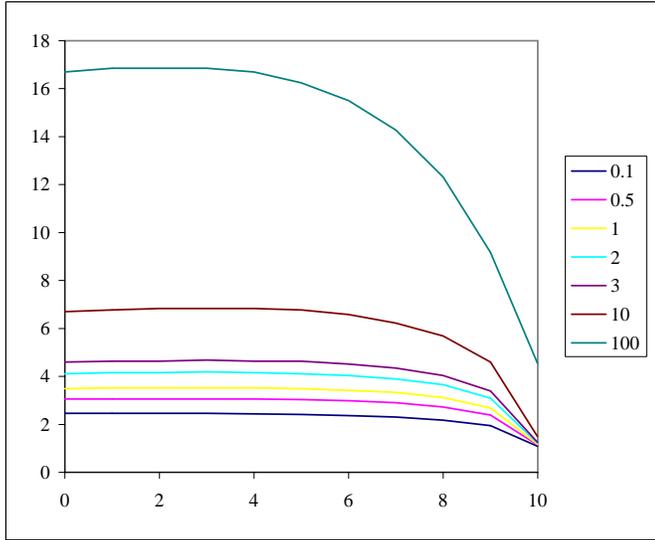


Figure 3 cont'd: Exercise Boundaries and Option Values for Various Levels of Wealth

c. Risk aversion coefficient = 0.5, Volatility = 100%



d. Risk aversion coefficient = 2, Volatility = 100%

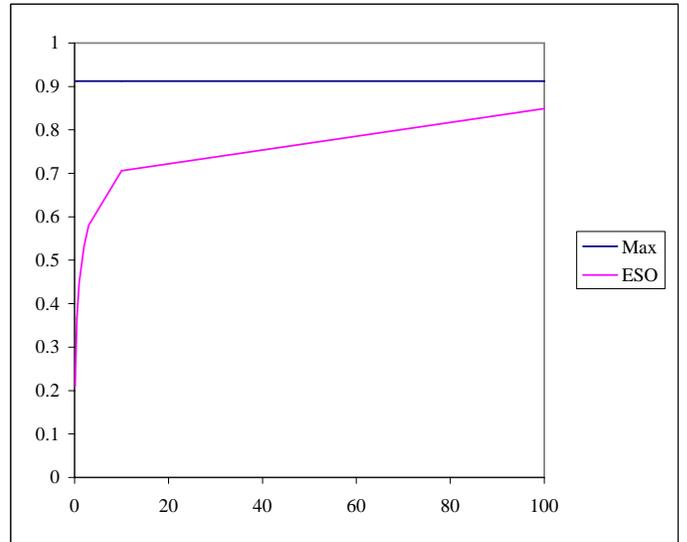
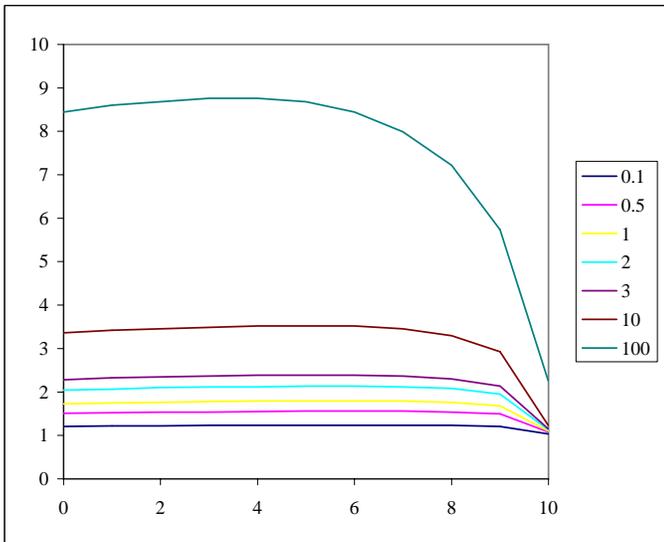
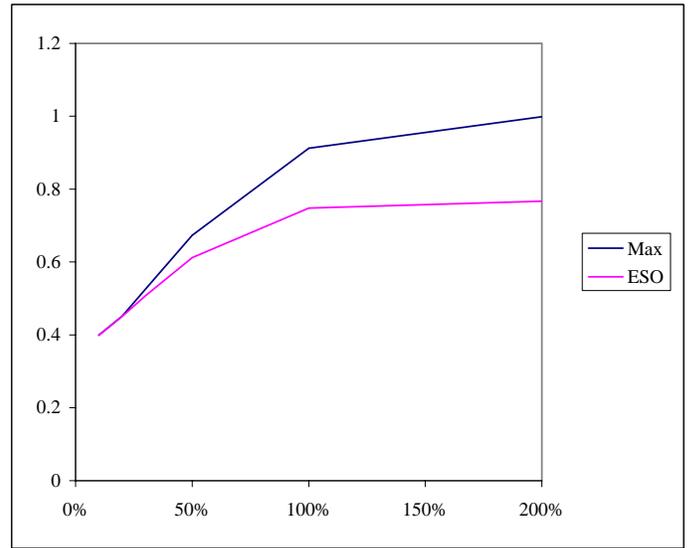
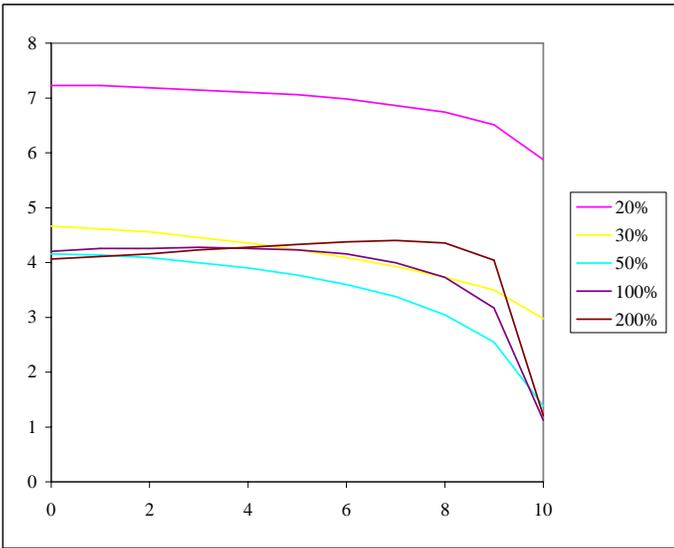


Figure 4: Exercise Boundaries and Option Values for Various Levels of Stock Volatility

a. Risk aversion coefficient = 0.5



b. Risk aversion coefficient = 2

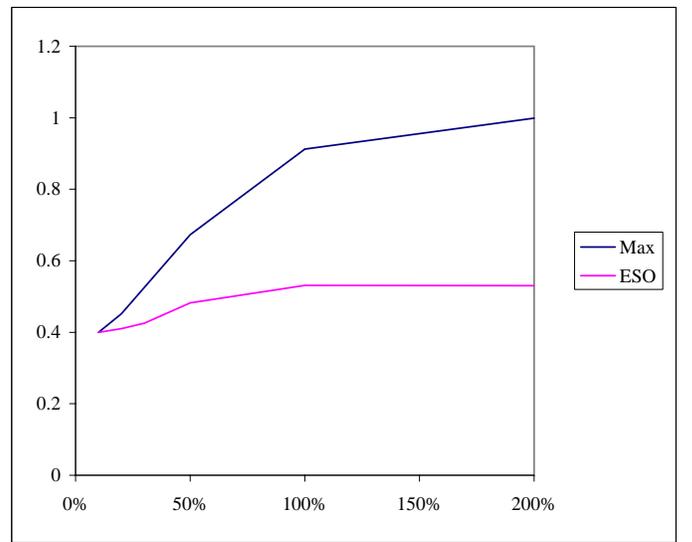
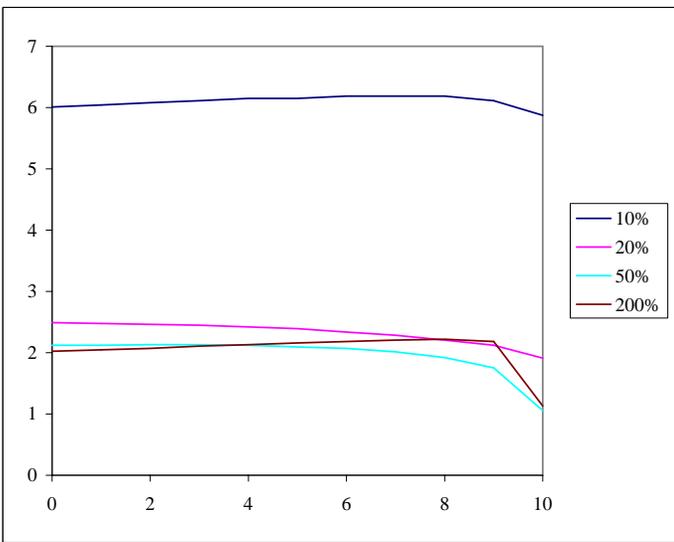
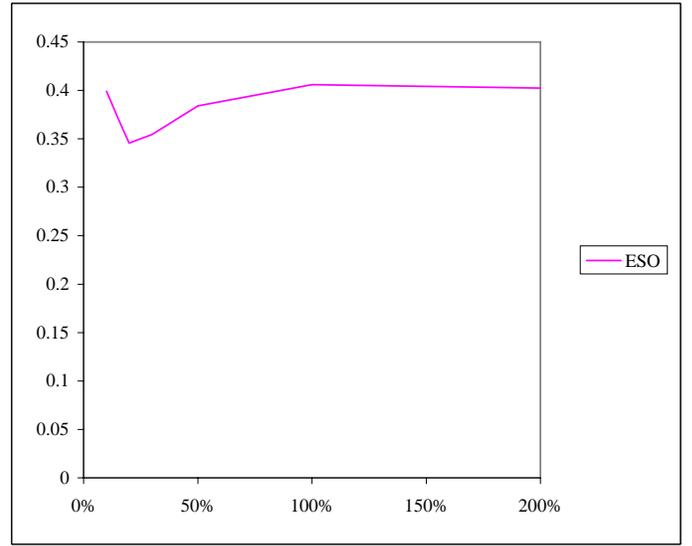
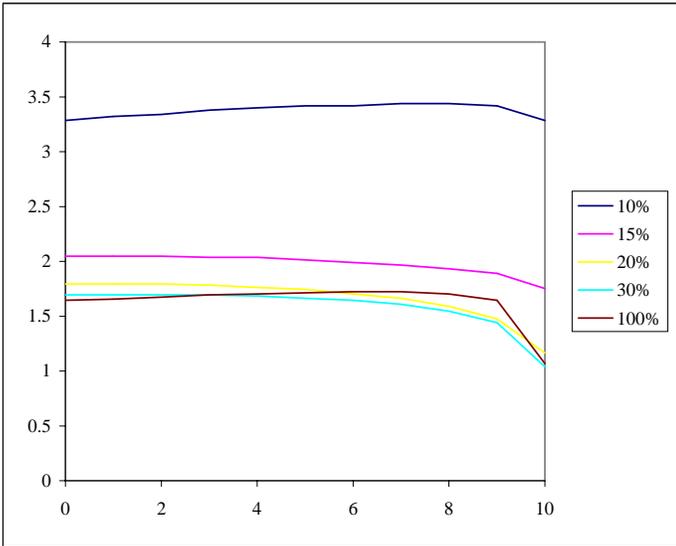


Figure 4 cont'd: Exercise Boundaries and Option Values for Various Levels of Stock Volatility

c. Risk aversion coefficient = 4



d. Risk aversion coefficient = 10

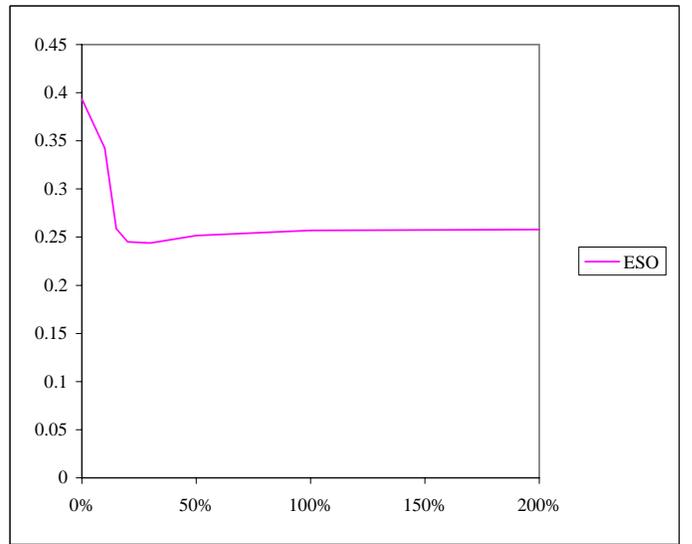
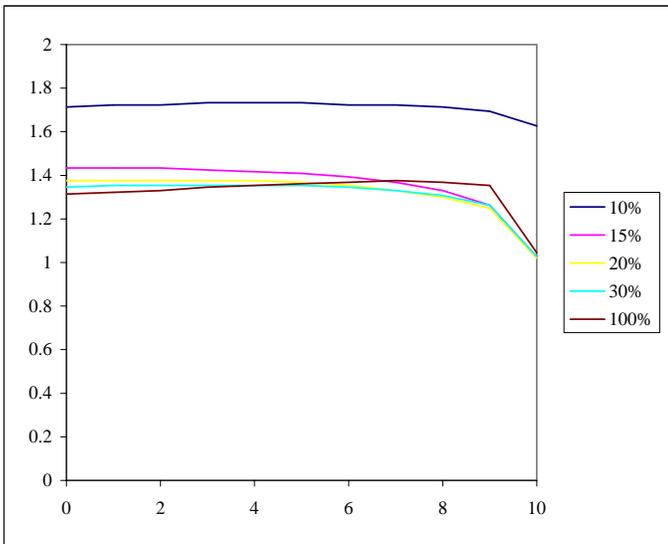


Figure 5: Exercise Boundaries and Option Values for Various Levels of Stock-Market Correlation

a. Market risk premium = 0, Initial wealth = 6, Risk aversion = 10

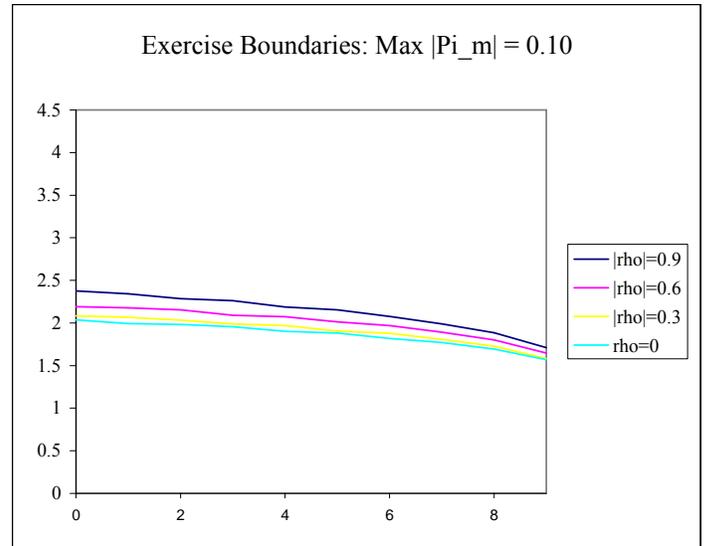
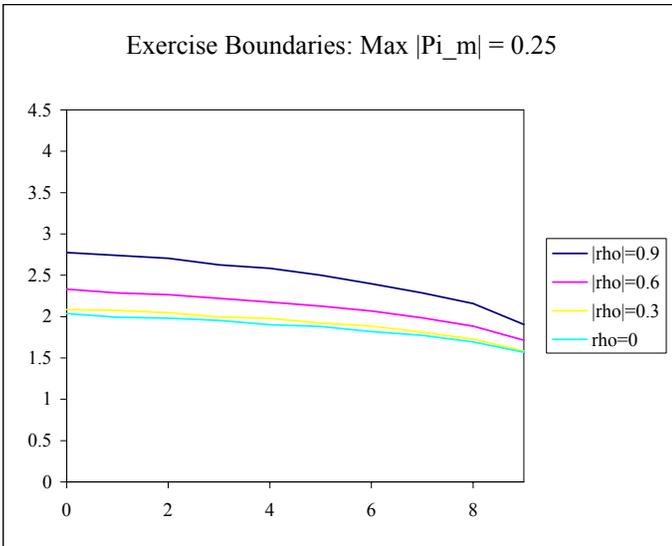
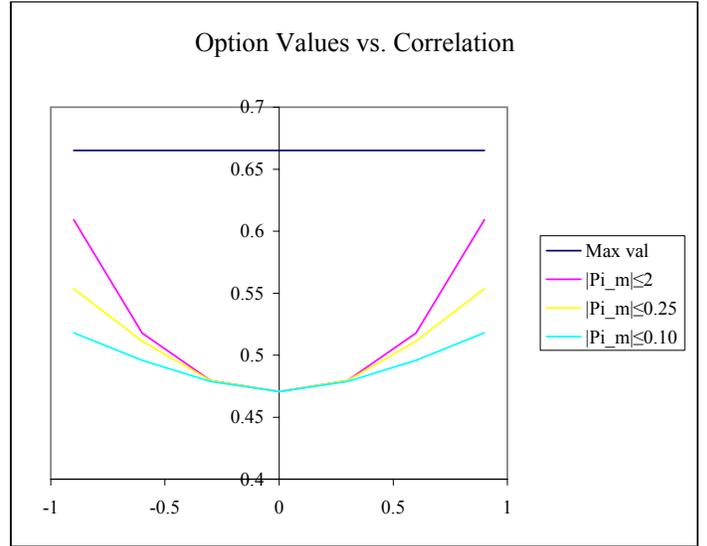
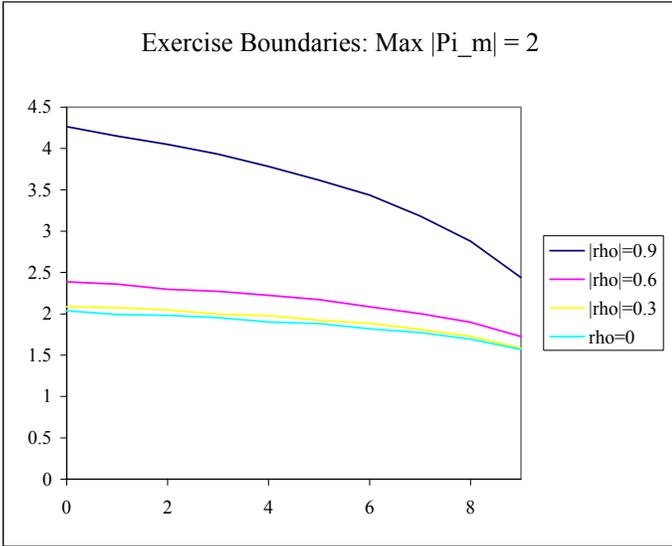
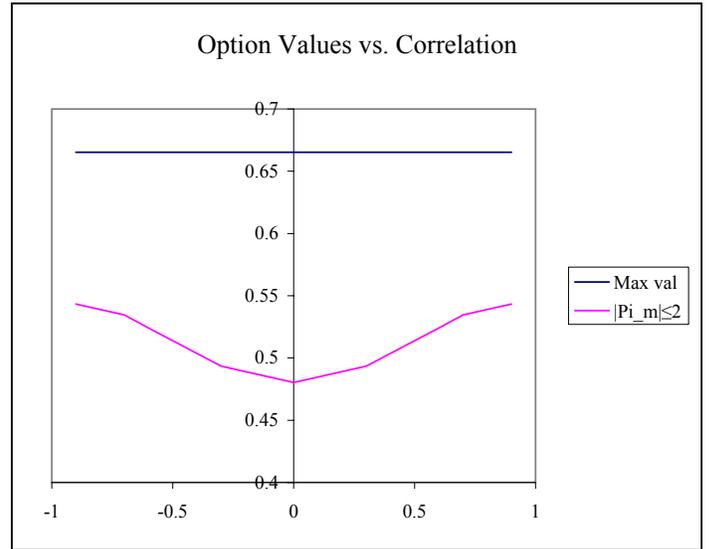
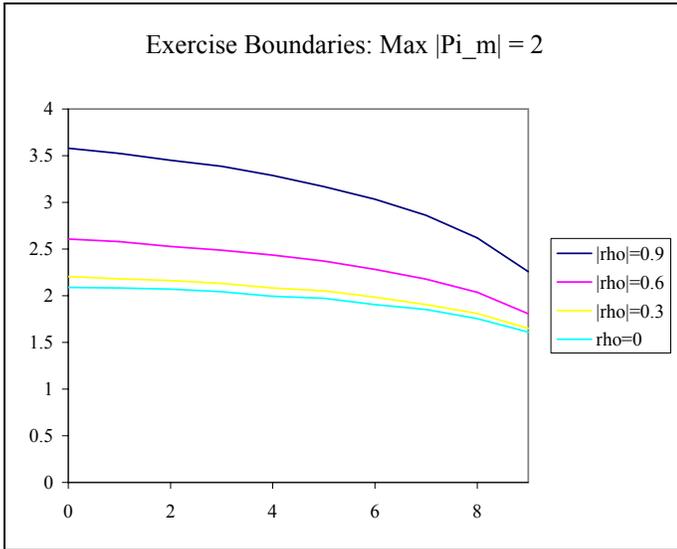


Figure 5 cont'd: Exercise Boundaries and Option Values for Various Levels of Correlation

b. Market risk premium = 0, Initial wealth = 1.2, Risk aversion = 2



c. Market risk premium = 8%

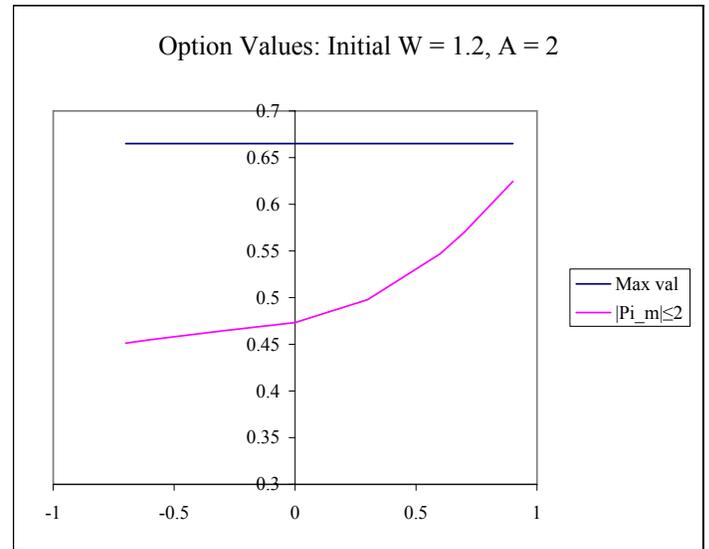
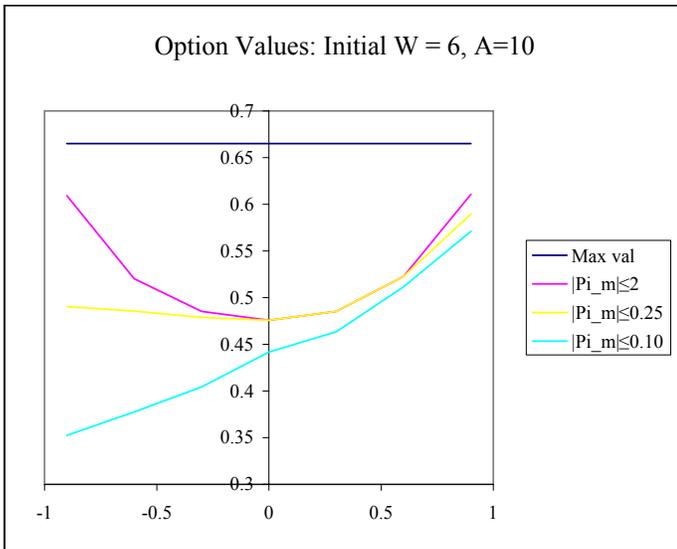


Figure 5 cont'd: Exercise Boundaries and Option Values for Various Levels of Correlation

d. Market risk premium = -8%

