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BIDDER COST REVELATION IN ELECTRIC POWER AUCTIONS

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Abstract

Competitive auctions for electric power sources whose operation will be based upon economic dispatch raise new challenges for auction designers. The efficient selection and operation of such generation sources requires revelation of bidder types over two-dimensions, fixed and variable costs. The way in which fixed and variable prices are combined into a net score, which determines the winning bids, plays a key role in influencing bidders behavior. This paper analyzes bidder strategies and develops necessary conditions of bid scoring systems for the existence of equilibrium strategies that will result in efficient operations. Existing and proposed bid scoring systems are examined using our results.

1. Introduction

Over the last decade, the electric power industry has evolved from a collection of natural monopolies into an environment that is increasingly competitive. Competition has expanded primarily in the generation segment of the industry. In the early eighties, this was mostly due to producers of power who, as "Qualifying Facilities"(QFs), began selling wholesale to utilities that were required by law to buy the power at "avoided cost." More recently, the contracts offered to independent producers have been rationed through different forms of competitive bidding.

There has been much speculation about the effects the newly introduced competitive forces will have on both the cost and reliability of electric power (Summerton and Bradshaw, 1991). At issue is whether the same level of operational efficiency achieved by a vertically integrated electric utility can be maintained in a system where some generation is acquired through competitive auctions. Competition may restrict the flow of information required for efficient system operations.

We examine auctions in which at least some operational control of the independent resource is granted to the purchasing utility. Such auctions are increasingly common, having been adopted in New Jersey and New York, as well as forming an important component of upcoming auctions in California. A utility that has full control over an independent resource will presumably operate it on the basis of the energy[1] price paid to that resource. It is therefore crucial for operational efficiency that the energy price paid reflect the true variable cost of the resource. In other words, it is desirable, for purposes of social efficiency, that bidders adopt a strategy of *truth-telling of variable costs.* This implies that any strategic behavior on the part of bidders be confined to the fixed cost component of the auction. We present conditions under which such strategies are feasible and discuss the implications for auction design that such conditions present.

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[1]The terms variable cost and energy cost are equivalent for the purposes of this paper.

The general concepts of multi-dimensional revelation and the reduction of strategic behavior to one of these dimensions seem to have many important applications in the regulatory arena. Logging rights (Wood, 1989), mineral leases (Rothkopf & Engelbrecht-Wiggans, 1989), and other natural resource auctions often share the characteristic of multi-dimensional or vector based bids. The goal of achieving efficient marginal cost payments by forcing agents to collect their information rents in the form of a non-distortionary fixed payment has appeal in the regulation of monopolies (Baron & Myerson, 1982) as with the practice of holding auctions for franchise rights (Williamson, 1976).

Section two presents some background on auctions for electric generation and discusses the value of operational control of a QF to a purchasing utility. In sections three and four we formulate two dimensional auction models in order to analyze strategies involving truth-telling of variable costs. In section five we discuss two existing or proposed scoring systems. Section six gives an example of a second price auction using the scoring systems from section five. In section 7 we discuss the potential impacts of uncertainty.

2. Background

Under the Public Utilities Regulatory Policies Act (PURPA) of 1978 , utilities have been legally obligated to purchase power from producers who met certain criteria, namely QFs. In states such as California, where regulators created a favorable environment for QFs, the response was much larger than anticipated (Hulett, 1989). A glut of QF capacity has led to a rationing of independent generation through competitive bidding processes.

Competitive bidding represents a middle ground between strictly utility owned generation and the unlimited private supply initially offered in PURPA. It is hoped that the innovation and efficiency benefits of competition can be realized while simultaneously the quantity of capacity for which rate payers are obligated to pay is constrained.

The basic concept employed in all auctions is simple. A planning process supervised by regulators determines a desired capacity addition and a Request For Proposals (RFP) is issued. If the capacity of offered bids exceeds the desired capacity, bids are accepted in order of increasing "cost" until the desired capacity is reached. Many difficult policy questions have arisen in the process of trying to implement this concept in practice. Factors such as the discrete amounts of capacity offered by bidders made reaching exact capacity targets difficult. The financial viability of a winning bid's project affects the reliability of a system that is counting on the added capacity. The most contentious and difficult issue is how to define the cost or benefit of a project.

Beyond the prices bid by potential suppliers, many factors such as site location and transmission access directly affect the value of a project. In addition, Public Utilities Commissions are attempting to include "social" benefits such as fuel diversity and environmental factors into the selection process. These elements, which are not directly related to price, are generally referred to as *non-price factors*.

This paper focuses on another key non-price factor, concerning the level of operational control the purchasing utility is allowed to exercise over the new resource. Operational control takes many forms and names, but for the remainder of this paper we refer to operational control issues as *curtailability*. In it's simplest interpretation, curtailability (also known as dispatchability) is the right of the purchasing utility to refuse to purchase power from the QF in any given hour.

In California, earliest concern over curtailability of QFs arose from an unexpectedly high response to supply programs. The combination of attractive long term guaranteed rates and no capacity limits had led to a "gold rush" of suppliers in the mid 1980's (Kahn, 1988, ch. 6). With California utilities experiencing a glut of capacity, the ability to refuse expensive QF purchases in off-peak hours became a valuable prerogative. We now discuss methods of quantifying the value of curtailability.

Economics of curtailment

Curtailability, as a bid characteristic, has the unique quality of not being entirely a non-price factor. In a way, curtailability is both a price and a non-price factor. Before we can describe how this is so, a brief overview of electric utility system economics is necessary.

Electricity is a non-storable product. As such, capacity must be enough to cover demand in those hours of highest need. An electric utility system therefore consists of a collection of generating resources whose usage over a year can vary greatly. Generally the system will consist of technologies ranging from *base-load* units, nuclear and hydro power, to *peaking* units, usually gas or oil fired combustion turbines.

With some notable exceptions, the optimal way for a utility to commit its units is simply in ascending order of energy costs. This ordering is often called the *merit order*. As demand levels from hour to hour change, so does the mix and output levels of the units committed. Thus, the least expensive (in energy costs) units will run the most. The *marginal unit* is the last one in the merit order to be called upon in that particular hour (e.g. for that level of demand). System marginal cost in that hour is thus the energy cost of the marginal unit, or of the next unit in the merit order if the marginal unit is dispatched at full output.

Electricity demand over a time period such as a week or year is often represented in a *load duration curve* (LDC). An LDC is essentially a right cumulative demand distribution that displays load level (kilo-watts, kw) on one axis and on the other axis the number (or %) of hours in that given time period for which demand is at or above that kw level (Figure 1). From the LDC one can roughly determine the number of hours a particular unit can be called upon to operate by "stacking" the merit order under the LDC.

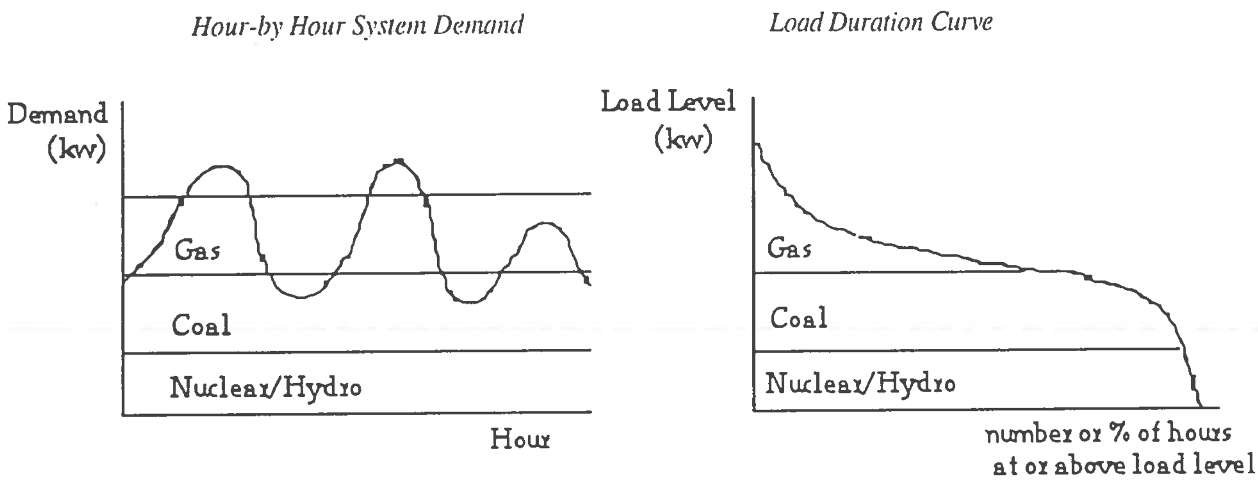

Figure 1: Load Duration Curve

This remarkably simple analysis is facilitated by the convenient property that optimal commitment in any hour is determined by moving up the merit order. The limitation of this analysis is that load duration curves treat all hours with the same load level as the same. Unfortunately, when inter-temporal constraints such as unit minimum down times and hydro depletion are considered, the ascending cost merit order may no longer be optimal. The degree of significance of these constraints is an active area of debate. It is generally considered - perhaps out of computational necessity - that for long range planning sophisticated versions of LDC analyses suffice.

Curtailability as a price factor

The obvious benefit to the utility of curtailability of a generating resource is the ability to not purchase power when it can generate it for less than the QF energy price. This is how curtailability can be viewed as a price factor. The cost in an hour during which the utility can't curtail is easily quantifiable - simply subtract system marginal cost from QF price. This concept can be extended to a full year using duration curves. Just as demand levels can be represented by a load duration curve, a cost duration curve displaying hours vs. level of cost can be constructed. From this curve, the per kw cost to the utility of a non-curtailable resource is the difference between the utility's system cost and the QF's energy price in those hours in which system cost is below that energy price (Figure 2). The benefit to the QF depends primarily upon the relation between payment price and the QF's true energy cost.

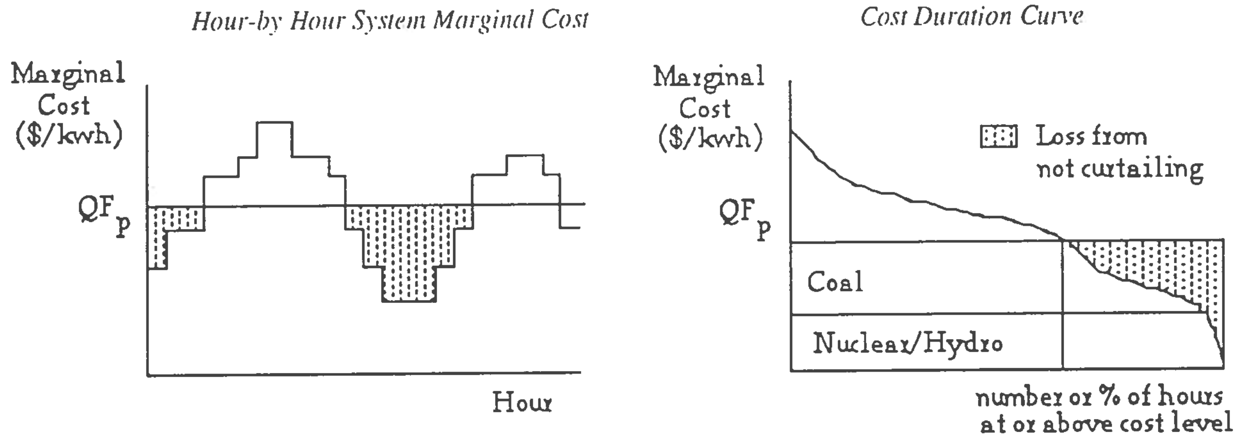

Figure 2: Electric System Marginal Cost

Curtailability as a non-price factor

The shortcomings of the LDC-based analysis which we briefly discussed earlier also cloud the issue of curtailability and produce effects that are truly non-price factors. Operational constraints such as minimum down times and ramp rates[2] make responding to rapid demand changes with base load units impractical. Such units are also much less efficient when operated at their minimum load levels than when at full output. Thus, it is sometimes the case that to optimize over the week's dispatch, a controller must shut down peaking units even when system marginal cost is above those units' operating cost in that hour. Full dispatchability gives central dispatchers the flexibility to optimize system operations and thus yields benefits that are difficult to quantify analytically. Kahn, Stoft, Berman, and

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[2]A ramping rate refers to the amount of time needed to bring a generating unit's output up to full capacity.

Grahame (1991) study the use of electric utility simulation models for estimating the non-price benefits of curtailability.

When the capacity contribution of the QFs represents a small portion of overall utility system capacity, the non-price effects become negligible. Under those conditions, LDC-type analysis of the benefits of potential capacity additions is considered sufficient. In this paper we will focus on such situations and concentrate on prices and their relationship to curtailability.

Electric Power Auctions

Previous literature on electric power auctions has often focused on the prudency of using first or second price auctions. The California Public Utilities Commission (CPUC, 1990) has embraced the second price auction due to its' belief, based on the analysis by Vickrey (1961), that a second price auction will preclude strategic behavior by bidders. The CPUC hopes that "truth-telling" will be a dominant strategy for bidders, therefore eliminating the danger that asymmetric beliefs amongst bidders might lead to an inefficient allocation of supply contracts.

Rothkopf, Tsieberg, and Kahn (1990A) show that in situations where bidders need to acquire inputs from third parties that possess some market power, those third parties may be able to extract any observed windfalls from the auction. The presence of such third parties provides strong incentive for *not* revealing true costs in the context of a second price auction. Simulation of an asymmetric bidding game has been used (Kahn, et. al. 1990) to study potential efficiency losses. It was shown that a bidder's choice of capacity size, and thus of production cost when economies of scale are present, mitigates the efficiency losses that might occur in a one dimensional, first price, asymmetric auction. Einhorn (1988) argues that both first and second price sealed auctions have severe shortcomings. He proposes a series of two-part tariffs, negotiated between suppliers and the purchasing utility with the knowledge that a "fair" contract is an available option to all suppliers.

In all of the above literature, the power purchased by the utility is "must take." Therefore, the debate over the efficiency of different auction forms in these models considers only the efficient *acquisition* of low-cost capacity; the operational efficiency of the resulting system is not considered. Bolle (1991) shows that QF auctions can theoretically be efficient in this generalized sense, but he does not specifically address the issues that curtailability brings to evaluating bids. In fact, none of this work addresses specific industry practices. A recent study by Stoft and Kahn (1991) investigated the existence of scoring "bias" in auctions that did require curtailability. They show that the practice of "ratio" scoring - using percentage of avoided production cost as a basis for ordering bids - favors peak-load (high energy cost) generation.

3. First Price Auctions with Fixed and Variable Price Bids

For an electric power auction requiring full curtailability of bidders to be both operationally and acquisitionally efficient, bidders must be ranked over two dimensions, fixed and energy costs. In this section we examine characteristics of bidding systems that incorporate both a fixed and a variable (energy) price into a net score. We specifically wish to examine what conditions are necessary for inducing bids in which energy prices are the same as costs. In other words, under which bid-scoring systems is it reasonable to expect bidders to state their actual operating costs as their energy price?

We have discussed some literature concerned with the efficient acquisition of generation capacity. Electric power auctions may also lead to operating inefficiencies. For example, a QF may have higher costs than the utility's own resources, but may bid an energy price lower than both its own costs and those of the utility. The bidder could subsidize this operating loss by padding its fixed cost bid. The utility would then operate the QF before its own since that resource is "cheaper" from the utility's perspective. From a social efficiency perspective, however, such a merit order would be sub-optimal. When the independent resources are not curtailable, the problem is exacerbated. In such instances the utility is no longer free to dispatch on the basis of energy payments, which themselves may not reflect costs.

The mixing of independent resources with the utility's own resources necessitates that the revelation of bidders' energy costs be cardinal, as opposed to ordinal. In the common one dimensional auction paradigm it does not matter from an efficiency standpoint if the bid-taker knows exactly what bidders' costs are as long as she knows the order in which those costs fall. Thus strategic behavior that preserves order is allowable. In the electric power context, however, the utility must know not only where a bidder's costs lay in relation to other bidders, but with the utility's own resources. To achieve this, revelation of the actual cost value is required.

Little work has been done on auctions with multiple dimensions of bid components. One context in which it has arisen is in the sale of logging rights to timber tracks (Wood, 1989). The main issue in the forestry application is uncertainty of the estimates of timber quantities in the tracts in question. Bidders bid a vector of unit prices, one for each type of timber. The suspicion that quantity estimates are inaccurate may lead bidders to skew their price bids in favor of timber types believed to be over represented in the estimates. Such a practice is known as "unbalanced bidding." It is a problem that also arises in the context of construction contracts (Stark, 1974). These examples seem to be a linear case of the general problem studied below, although the paradigm is one of *unknown* common values and the issue of concern is the maximization of the benefit of the bid taker. Our focus, by contrast is on the efficiency aspects of the revelation of *private* cost information.

Rothkopf and Engelbrecht-Wiggans (1989) examine auctions for mineral right leases with payment forms that may take on many components. To eliminate the potential for unbalanced bidding, they propose that the bid taker explicitly link these elements so that the bid of a single component uniquely determines the value of the other components. Crew and Harstad (1992) characterize a symmetric equilibria that could develop if such a procedure was adopted in the context of bidding for monopoly franchise rights. The Crew and Harstad model allows demand to be endogenous, as in our model. Auctions with endogenous quantities over one dimension, unit cost, were also studied by Hansen (1988).

<u>Bidder Characteristics</u>

Bidders in this model are characterized by their fixed and variable costs, (k,c) which at this point are assumed to be the private knowledge of each bidder.

$k_i \in \Gamma$	True fixed cost of bidder i (\$/kw)
$c_i \in X$	True variable cost of bidder i (\$/kwh)

Bidders are assumed to develop a two dimensional, vector valued strategy function *B* from a strategy space β. *B* maps from private cost information (*k,c*) into a two-dimensional bid,

(b_k,b_c), that reflects the unit fixed and variable price bids, respectively. The space of allowable bids is assumed bounded in the intervals [0,K] and [0,C] respectively.

We will be examining conditions necessary for the existence of desirable symmetric equilibria, where symmetry is defined by the property that any bidders with the same costs will submit the same bids.

$$B : \Gamma \times X \rightarrow [0,K] \times [0,C], \text{ assume } b \in C^2$$

and

$$(b_k,b_c) = B(k,c)$$

Auction Characteristics

The purchasing utility combines the two bid components into a scalar score in order to resolve the auction. We refer to this mapping from multiple dimensional bid components to a scalar as the auction's *Scoring System*. The scoring system is public information and is thus known to all bidders.

Let the function $S(b_k,b_c)$: $[0,K] \times [0,C] \rightarrow \Re_+$ represent the scoring system of a given auction. We assume that $S(b_k,b_c)$ is at least once differentiable. Without loss of generality, we assume that auction winners are the bidders with the lowest score. This keeps our framework compatible with the actual bid scoring systems analyzed below. Since both bid elements are prices paid by the utility to successful bidders, $S(b_k,b_c)$ will be increasing in both b_k and b_c.

The generation capacity (size) of bids is not explicitly treated in this model. The purchasing utility selects bidders in increasing order of score until that utility's capacity need is met. There is a serious problem with the "lumpiness" of capacity - i.e. the discrete capacities offered by bidders is likely to either over or under fill the utilities capacity need, depending on where the cutoff amongst bidders is made. This issue is a difficult one and is discussed in (Rothkopf, et. al., 1990B). For tractability of this problem and to focus on the aspect of bidder cost revelation, we suppress the capacity problem by assuming that the purchasing utility adopts the arbitrary rule of accepting bids in increasing order of score until the next accepted bid would exceed the capacity need.

Bidder Payoffs

Through some dispatch method, the utility estimates that a resource with energy price b_c will be dispatched (not curtailed) for $\rho(b_c)$ hours during a year. $\rho(b_c)$ can be considered a demand function that gives the quantity sold by a successful bid with variable price b_c. The function $\rho(b_c)$ is assumed known to all bidders as well as to the utility. We assume that $\rho(b_c)$ is monotonically decreasing in b_c and that $\rho(b_c)$ itself will not be substantially altered as a result of the auction. This will be the case when the amount of capacity added as a result of the auction is small in relation to the total system capacity.

We assume the bidders to be risk neutral, an assumption whose consequence we discuss in section 7. The payoff per kw from the auction to bidder i, if i has a winning bid is therefore $b_k - k_i + (b_c - c_i)\rho(b_c)$, where we have suppressed the arguments of b_k and b_c.

The expected payoff per kw from the auction to bidder i is the quantity above multiplied by bidder i's subjective probability of winning the auction. We now offer a general representation of bidder i's subjective probability of being successful. Namely that it is some general function, $P(S,k_i,c_i) \in [0,1]$, of the scalar score $S(b_k,b_c)$ of i's bid and of (k_i,c_i), the private cost information of bidder i. The specific form which $P(S,k_i,c_i)$ may take depends upon the assumptions on the distribution of k and c amongst the population of competing bidders and on the information available to bidder i. In general, we assume that $\partial P/\partial S < 0$. This general framework therefore encompasses both the independent, private values model and accommodates assumptions of "linkage" of cost parameters between bidders. For the independent values case, $\partial P/\partial k = \partial P/\partial c = 0$, while in the case of strictly affiliated costs P would be increasing in k and c.

Bidder i's expected payoff per kw is therefore

$$E(\pi(b_k,b_c,k_i,c_i)) = [b_k - k_i + (b_c-c_i)\rho(b_c)]P(S(b_k,b_c),k_i,c_i).$$

We now have the machinery to express our first result. As argued previously, one goal of a scoring system is to induce truthful energy cost bidding, thereby permitting efficient electric system dispatch. We therefore examine conditions for which symmetric strategies involving bidding true energy costs are feasible.

<u>Proposition 1</u>: *A necessary condition for a symmetric strategy involving truth-telling of energy costs to be a Bayesian-Nash equilibrium is*

$$(1) \qquad \frac{\partial S/\partial b_c}{\partial S/\partial b_k} = \rho(b_c) \text{ at } b_c = c \text{ for all possible values of } c.$$

A proof is provided in the appendix. Condition (1) states that the bidders' marginal rates of substitution between fixed and variable components in the scoring has to be the same as the rates of substitution reflected in the payments made to winners at the point of truth-telling of energy costs. This condition gives insight into price scoring methods that should be adopted by utilities and regulators. Auction designers should also be alert for non-price factors that distort the relative value of fixed and energy payments to the bidders. Such distortions can destroy the possibility of equilibria in which energy costs are truthfully bid.

4. Second Price Auctions with Fixed and Variable Price Bids

We now turn our attention to second price auctions. In one dimensional auctions it is well known that when the winning bidders are paid an amount equal to the highest loosing bid, it is a dominant strategy for all bidders to bid their true costs (Vickrey, 1961). In an auction with both fixed and variable price components, however, it is not immediately obvious what the second price is. What form of payment, if any, leads to an equilibrium where actual costs are bid for both fixed and variable components? At the same time, it is important that the auction mechanism that induces truth-telling bids of both parameters uses that information in an efficient manner. The payoff to a bidder must therefore reflect a level of curtailment that coincides with the efficient operation of a generation unit possessing that bid's variable cost.

These questions are in part motivated by bidding procedures under consideration by the California Public Utilities Commission (CPUC, 1990). The proposed scoring system will include (among other non-price factors) fixed and energy cost components. Curtailment based on energy price is expected from accepted suppliers. Successful bidders will be paid their bid energy price and a fixed price based on the total price of the lowest losing bid.

The CPUC hopes this system will lead to bids that equal costs for both fixed and variable cost components.

The analysis of this problem requires us to more explicitly model the probability of success of a specific bidder. In order to do this we must somewhat restrict the generality of the framework described in section 3. We assume that a bidder's cost parameters are drawn from a joint probability distribution $F(k,c)$ with support over the domain [0,K] x [0,C]. In the case of a symmetric, independent private values model, each bidder's costs would be drawn independently from identical distributions. This construction may also account for affiliation of cost parameters between bidders by making $F(k,c)$ conditional on private cost information. In general, a bidder who follows a truth telling strategy and assumes that his opponents are also following such a strategy would employ the following machinery to calculate his probability of success in the auction.

$$\Pr(\text{i's bid is a winner}) = \Pr(S(k_i,c_i) \text{ is lowest}) = \Pr(S(k_i,c_i) < S(k_j,c_j))$$

where bidder j is notation used for the lowest losing bidder. In the case of m winners, bidder j represents the bid with the $(m+1)$th lowest score. Once again the lumpiness of capacity of the bids may make m an unknown value, but we are forced to assume that m (as well as the number of bidders) is known to bidders *ex ante*. Given that all bidders know the scoring system $S(k,c)$, the probability of winning can be simplified to be expressed directly as a function of the score.

$$\text{Let } \Pr(\text{bidder j's score} \geq s) = \overline{G}(s,k_i,c_i) = \int_{\{S(k_j,c_j)\geq s\}} dF(k_j,c_j \mid k_i,c_i)$$

$$\text{Then } \Pr(\text{i's bid is a winner}) = \overline{G}(S(k_i,c_i),k_i,c_i)$$

In the expression for $\overline{G}(s,k_i,c_i)$ we have made the distribution $F(k_j,c_j)$ conditional upon the realizations of bidder i's costs. In this way affiliations between bidders costs may be accounted for. When there are more than two bidders and m winners, the $F(k_j,c_j)$ will be an order statistic reflecting the distribution of costs of the $(m+1)$th lowest score.

We now return to the goal of designing a second price auction that provides for full revelation of cost parameters as well as implementing efficient operations. As described in section 2, a purchasing utility with freedom to curtail will operate a QF with variable price b_c as if it were one of the utility's own units possessing variable cost b_c. Therefore if bidders truthfully bid their costs so that $b_c = c$, it is essential for socially efficient operation that the variable price paid by the utility also equals c. Any payments of a "second price" nature should therefore be confined to the fixed payment made to successful bidders.[3] This is in fact the policy that is being adopted for California PURPA auctions. The remaining design considerations are thus (1) what form the second price fixed payment should take, and, (2) what should the scoring system be.

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[3] A referee has pointed out to us that there are a large number of formats that a second price auction can take when multiple dimensions are considered. Indeed, the number of possible formats seems to grow geometrically with the dimensions of the auction. While it may be possible to prove that many of these formats have less desirable efficiency results than the policy we are analyzing, this is beyond the scope of this paper. We have restricted attention to the format above primarily because it is the format being adopted in California. We are grateful for this insight.

Define K_{sp} as the second price fixed payment made to successful bidders. In order to express the desired value of K_{sp}, we need to introduce the inverse function $S^{-1}(s; b_c)$.

$$\text{Define } S^{-1}(s; b_c) = \min\{b_k \mid S(b_k, b_c) = s\}.$$

$S^{-1}(s; b_c)$ can be interpreted as the lowest fixed price that, when combined with the variable price b_c, results in a net score of s. We are now able to present the following result.

Proposition 2: *Consider a second price auction where bid j is the best losing bid and a successful bid i is paid its bid variable price* b_{ci} *and a fixed price* $K_{sp} = S^{-1}(S(b_{kj}, b_{cj}); b_{ci})$, *then:*

1. *Condition (1) is necessary for truth telling to be a Bayes-Nash equilibrium.*
2. *If condition (1) is satisfied, then truth telling meets first and second order sufficiency conditions for local individual rationality under the assumption that all opponents tell the truth.*

Proof of this Proposition also is provided in the appendix. The value of K_{sp} given in Proposition 2 is the fixed payment that, when combined with a variable payment equal to that of a winning bidder, yields a score equal to that of the lowest losing bid. A payment of this type agrees with the analysis of Bolle (1991), who argues qualitatively for permitting the first best bidder to receive a fixed payment such that the net surplus of the utility is the same as if the second best bid had been selected.

Some interesting observations arise from Proposition 2. If there is more than one winner, the payments made to those bidders need not be identical. In fact, only those successful bidders with the same variable bid will receive the same fixed payments. Thus the terms *uniform price*, or *non-discriminatory*, that have been applied to one dimensional second price auctions do not apply here. A second interesting corollary in the independent values case is that bidders need not guess the costs (or bids) of their opponents to calculate their expected payoffs. They only need to estimate the distribution of their opponents *scores*. In other words bidders only need to estimate the level sets of their opponents with respect to the scoring system and not their exact location in the two dimensional cost space.

5. Industry Scoring Systems

We now examine two existing bid scoring systems. We must stress that the analysis below discusses only the scoring of fixed and variable prices. We present a simplified version of these systems to focus on how the two prices are combined into a net score and, in the case of California, on the "second price" payment that is paid to successful bidders. There are many additional non-price factors that affect the total net score of a bid. These other aspects of the scoring system, such as environmental benefits and availability-based capacity payments, make the actual scoring considerably more complicated than in the models presented here.

Consolidated Edison System

Consolidated Edison (CE 1988) used a scoring system that was additive in the two cost components. It estimated the total cost of providing for a kw of demand over the duration of the contract. Therefore scores were in dollars per unit of capacity ($/kw).

CE allowed bidders to state a "range" of allowable curtailment hours and adjusted price scoring accordingly. In this analysis, we assume that the number of curtailment hours preferred by the purchasing utility will fall within the range offered by the bidder. The CE system computed a projected cost for providing a kw-year of power from the combined sources of the bid project and the utility system. The cost of power from the bid project was simply bid fixed price plus bid variable price times the projected optimal number of dispatch hours. To this annual price was added the cost of providing power in "make-up hours" (i.e. those hours during which the bid project would be curtailed). The cost of make-up hours is represented by the shaded area on the right of Figure 3.

The total contribution to score due to the variable component is the sum of the two shaded areas below. To this amount the annual fixed cost is added to arrive at the total score. It should be pointed out again that the *low score* wins in this format.

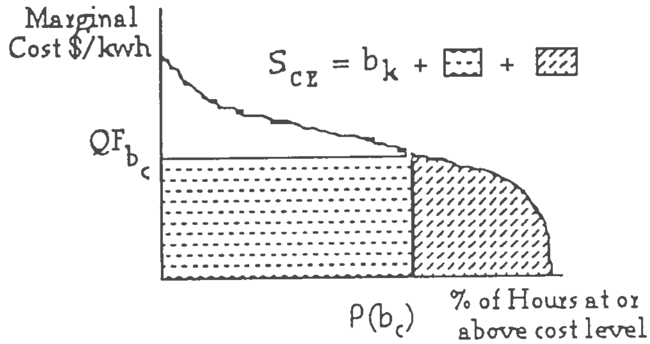

Figure 3. CE Scoring System

Using the notational conventions of section 3, the total score of a bid is thus

$$S_{CE}(b_k,b_c) = b_k + b_c\rho(b_c) + \int_{\rho(b_c)}^{\bar{\rho}} c(\rho)d\rho$$

where $c(\rho)$ is the inverse function and $\bar{\rho}$ is the maximum value of $\rho(b_c)$. We now verify that the CE system satisfies the condition of Proposition 1. For the CE scoring system given above, we have that:

$$\frac{\partial S_{CE}(b_k,b_c)}{\partial b_k} = 1$$

and

$$\frac{\partial S_{CE}(b_k,b_c)}{\partial b_c} = \rho(b_c) + b_c\rho'(b_c) - c(\rho(b_c))\rho'(b_c)$$

but $c(\rho(b_c))$ is simply b_c, leaving

$$\frac{\partial S_{CE}/\partial b_c}{\partial S_{CE}/\partial b_k} = \rho(b_c)$$

An interesting question at this point is whether the CE scoring system is unique or whether we can identify a more general class of scoring formulas that satisfy condition (1). In addressing this question we will restrict ourselves to scoring systems of the form $S(b_k,b_c)$

$= b_k + S_c(b_c)$ of which the CE system is a special case. We further require that condition (1) be satisfied over the entire interval of possible costs. This type of score can be interpreted as an adjusted capacity price whose units are ($/kw).

Proposition 3:

Scoring systems of the form $S(b_k,b_c) = b_k + S_c(b_c)$ that satisfy condition (1) for all values $c \in [0, C]$ *are unique up to an additive constant. Furthermore, these scores are equivalent with respect to the outcome of a first or second price auction of the types described in sections 3 and 4.*

proof:
For such a scoring system, $\partial S/\partial b_k = 1$. Then in order to meet condition (1) the scoring system must have $\partial S/\partial b_c = \rho(b_c)$. Solutions of this differential equation may vary only by a constant. Clearly an additive constant in the scoring formula will not change the subjective probabilities of winning. Furthermore, in first price auctions the payments to winners depend directly on b_k and b_c and not on the score and, therefore, an additive constant does not affect the outcomes of the auction. In the second price auction described in section 4, the invariance of the payoff to an additive constant in the score follows from the invariance of the inverse function $S^{-1}(s; b_c)$ to such a constant so again the outcome of the auction is unaffected. •

Keeping in mind the results of Proposition 3, we propose a scoring system that is equivalent to and perhaps more intuitive than the CE system. Our proposed system subtracts the surplus gained by the utility due to the energy purchase from the fixed price bid. The amount of surplus is equal to the white area above the line QF_{b_c} in figure 3 above. Total score using this system would be

$$S^*(b_k,b_c) = b_k - \int_0^{\rho(b_c)} (c(\rho) - b_c)d\rho$$

This differs from the CE score by an additive constant which equals the total area under the cost duration curve in figure 3.

California Utilities System

The proposed California system (PG&E 1992) scores bids in ($/kwh) units, rather than the ($/kw) units used by Consolidated Edison above.

$$S_{Cal}(b_k,b_c) = b_c + \frac{b_k}{\rho(b_c)}$$

where $\rho(b_c)$ represents the estimated number of hours in which the resource will *not* be curtailed.[4] Note that in this auction also the *low* bid wins. California plans to institute a second price auction in which the fixed payment will be $K_{sp} = [b_{kj}/\rho(b_{cj}) + b_{cj} - b_c]\rho(b_c)$,

4 In actuality, independent resources, upon being asked to curtail, are only required to curtail to 30% of their capacity. Energy above 30% of capacity during curtailment hours could be sold, but at the utility's avoided cost, rather than the resource's bid price. These options may cause further difficulties for energy cost revelation, but do not affect the incentive problem shown below, so for ease of exposition we have left them out of our representation of the scoring system.

where bidder j is the lowest losing bidder. Winning bidders will be paid their variable bid price.

The second price payment is set in agreement with the analysis of second price auctions given above. The California value for K_{sp} is the amount that equates the winning bids score's with the lowest losing score. However, the bidders' marginal rates of substitution between fixed and variable prices are not correctly reflected in the scoring. This causes the truth-telling properties of the second price auction to break down.

Proposition 4: *Using the proposed California auction format, there cannot exist a symmetric equilibrium in which both fixed and variable costs are truthfully bid.*

Proof:
From part 1 of Proposition 2, we have that if $K_{sp} = S^{-1}(S(b_{kj},b_{cj});\, b_{ci})$, it is necessary for a truth telling equilibrium that condition (1) be satisfied. Thus we need only to show that condition (1) is not met in the California system.

$$\frac{\partial S_{Cal}(b_k,b_c)}{\partial b_k} = \frac{1}{\rho(b_c)}$$

and

$$\frac{\partial S_{Cal}(b_k,b_c)}{\partial b_c} = 1 - \frac{b_k\rho'(b_c)}{\rho(b_c)^2}$$

Therefore

$$\frac{\partial S_{Cal}/\partial b_c}{\partial S_{Cal}/\partial b_k} = \rho(b_c)\left\{1 - \frac{b_k\rho'(b_c)}{\rho(b_c)^2}\right\}$$

$$\neq \rho(b_c) \text{ for all } b_k \text{ unless } \rho'(b_c) = 0. \bullet$$

Note that since condition (1) is not met by the California scoring system, a *first price* auction using this scoring formula will also not lead to an equilibrium in which variable costs are truthfully bid. Obviously there are many other factors that affect a bidder's decision. We readily admit that our representation of the California scoring system is a simplified one that ignores factors unrelated to curtailability. However, the decision to adopt a second price auction in California was based in large part upon models even simpler than the ones presented here. We therefore argue that these results call into doubt the expectation that the newly adopted scoring system will result in truthful cost bidding by participants.

6. Second Price Auction Example

We now present a simple example to illustrate bidder payoffs. We model second price auctions as in Section 4 . Successful bidders receive a variable payment equal to their variable bid price and a fixed payment that equalizes their score with that of the best losing bid i.e., $K_{sp} = S^{-1}(S(b_{kj},b_{cj}):\, b_c)$. The specific form of the function $S(\ )$ corresponds to either the Consolidated Edison or the California scoring systems which are examined below.

A two-bidder, symmetric independent values game is presented. Both cost parameters are assumed to be distributed with constant density with support over $c \in [0,.5]$, $.5 \leq k+c \leq 1$. In this way, higher fixed investment yields lower variable cost on average.[5] The shape

[5]We are grateful to an anonymous referee for suggesting this distribution.

of the cost-duration curve is assumed to be triangular with range and domain [0,1]. The dispatch function $\rho(b_c)$, interpreted here as the percentage of hours at or above c, is therefore $1 - b_c$.

Summary of 2nd price auction example

$k,\ c \in$	$[0,1] \times [0,.5]$
$f(k,c)$	1 $0 \le c \le .5$, $.5 \le k + c \le 1$ 0 otherwise
$\rho(b_c)$	$1 - b_c$
$S_{CE}(b_k,b_c)$	$b_k + b_c\rho(b_c) + \frac{1}{2}b_c^2$
$S_{CAL}(b_k,b_c)$	$b_c + \dfrac{b_k}{\rho(b_c)}$

Probability of Winning

For this example bidder i's subjective probability of winning the auction is

$$\int_{\underline{S}}^{S(b_k,b_c)} d\overline{G}(s) = \int_0^C \int_{S^{-1}(S(b_k,b_c):c)}^K f(k,c)\,dk\,dc$$

$$= \int_0^{.5} \int_{Max(.5-c,\,Min[S^{-1}(S(b_k,b_c):c),1-c])}^{1-c} dk\,dc$$

Consolidated Edison System

The variable price bid's contribution to total score is the area below b_c and below the cost-duration curve (figure 4). Total score is the fixed price plus the variable price contribution. Thus, we have $S(b_k,b_c) = b_k + b_c(1-b_c) + \frac{1}{2}b_c^2$ and it follows that $S^{-1}(s;b_c) = s - b_c(1-b_c) + \frac{1}{2}b_c^2$.

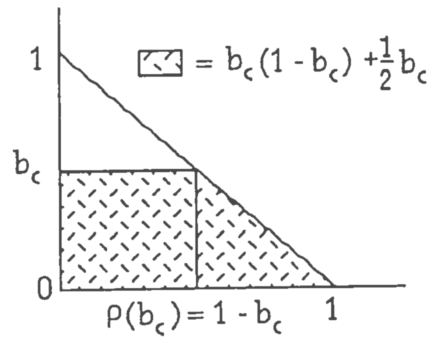

Figure 4: Consolidated Edison: Variable Price Contribution to Score

We now calculate what the expected payoff, as a function of b_k and b_c, would be for a bidder with costs $k_i = .5$, $c_i = .25$ when bidder i expects his opponents to tell the truth. Under a second price auction with payment set at K_{sp} described above, the expected payoff of bidder i, given his opponent with costs (k,c) follows the truth telling strategy, is:

$$E(\pi_i(b_k,b_c)) =$$

$$\int_0^{.5}\int_{Max(.5-c,Min[S^{-1}(S(b_k,b_c);c),1-c])}^{1-c} \{k-[b_c(1-b_c)+\tfrac{1}{2}b_c^2]+[c(1-c)+\tfrac{1}{2}c^2]-k_i+(b_c-c_i)(1-b_c)\}\,dk\,dc$$

This function is plotted in Figure 5 below for values of $k_i = .5$, $c_i = .25$. We know from the preceding analysis that first order conditions are met where $b_k = .5$ and $b_c = .25$. Figure 5 shows that this point (truth-telling) is indeed the optimum over the feasible range. Thus, while in general we have not shown that a truth telling equilibrium exists, for this example truth-telling is shown to be globally individually rational and therefore it is a Bayes-Nash equilibrium.

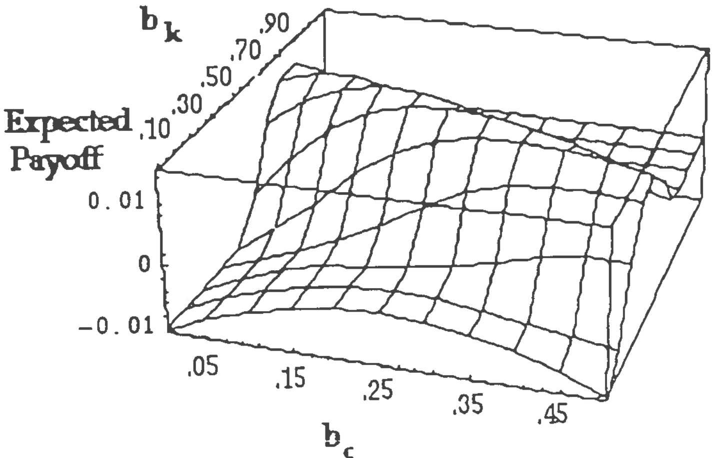

Figure 5: Expected Profit of Bidder i
$k_i = .50$, $c_i = .25$, opponent expected to truth tell

CPUC system

Once again, the California scoring procedure is as follows.

$$\text{Total score} = p_i + \frac{K_i}{\rho(p_i)} = p_i + \frac{K_i}{1-p_i} \quad \text{for this example.}$$

The fixed payment to winners is based upon the lowest losing score and equals $K_{sp} = S^{-1}(S(k_j,c_j);b_c) = [k_j/(1-c_j) + c_j - b_c](1-b_c)$. Once again, winners receive variable payment equal to the variable price that they bid.

For bidder i, expecting his opponent with costs (k,c) to bid truthfully, expected profits as a function of his own bid, (b_k,b_c), are:

$$E(\pi_i(b_k,b_c)) =$$

$$\int_0^{.5}\int_{Max(.5-c,Min[S^{-1}(S(b_k,b_c);c),1-c])}^{1-c} \{[k/(1-c) + c - b_c](1-b_c) - k_i + (b_c-c_i)(1-b_c)\}\,dk\,dc$$

The values of this function for $k_i = .5$, $c_i = .25$ are plotted below in Figure 6. We have indicated that truth-telling cannot be an equilibrium in this model. Figure 6 shows that it is indeed optimal for bidder i to deviate from truth telling if he expects his opponents to tell the truth.

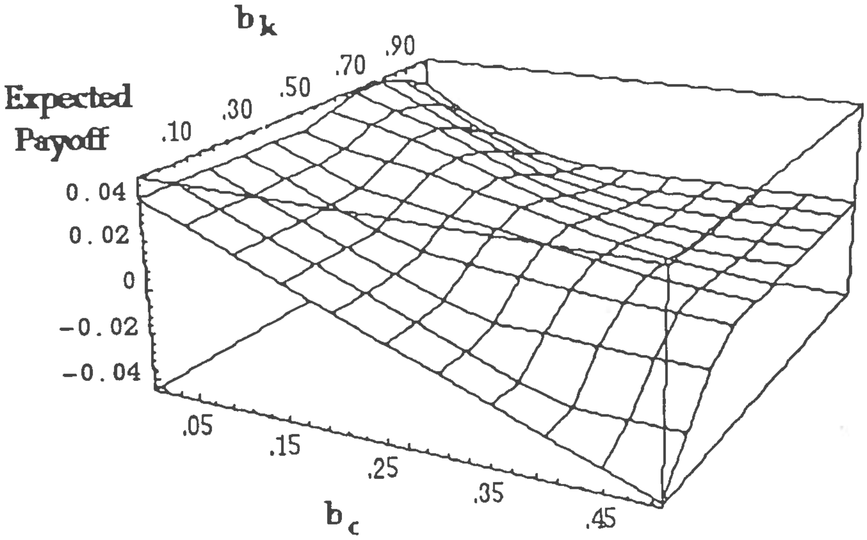

Figure 6: Expected Profit of Bidder i, California System
k_i = .5, c_i =.25, opponent expected to tell truth

7. The Role of Uncertainty and Risk Aversion

In our analysis we have assumed that the utility's dispatch function $\rho(b_c)$ as well as the individual costs of bidders were known with certainty. We now discuss the implications of these assumptions.

If all participants are, as we have assumed, risk neutral, the key complication that may arise from uncertainty in $\rho(b_c)$ is the danger of "skewed" or "unbalanced" bidding. This would imply however, that a bidder feels he has a more accurate estimation of $\rho(b_c)$ than the utility. This seems unlikely as long as the participants feel that the utility is not intentionally falsifying it's estimate. We can think of no reason why the utility would want to intentionally falsify it's estimate, except perhaps if the utility (or a subsidiary) were itself bidding in the auction.

When bidders are risk averse, the analysis grows more complicated. With risk aversion and uncertainty about $\rho(b_c)$, there is added incentive to tell the truth about variable costs since, for example, understating variable cost amounts to a gamble that true generation hours will be less that $\rho(b_c)$. Truth telling of variable cost is essentially the "no risk" position with respect to $\rho(b_c)$. In the presence of a scoring system such as the CPUC's that does not meet the conditions of Proposition 1, the end result will depend upon the *nature* of the risk aversion. The two effects, risk in $\rho(b_c)$ and a skewed scoring system, will work contrary to each other so the level of accuracy in the variable price bid will depend upon which effect dominates, and by how much.

With regards to uncertainty on the part of bidders about their own costs, it is less clear how the analysis is affected. The revelation problem we have outlined becomes one of inducing bidders to reveal their expected costs. In practice, bid prices are linked to fuel price or GNP inflators to reduce bidder uncertainty. Uncertainty and the practice of cost indexing itself add interesting complications to this problem by affecting the relative values of the two payment components. A risk averse bidder that has more uncertainty about one cost component than another may have incentive to collect information rents in the more certain component. In the case of multi-year contracts, which are in fact standard practice, the escalation of payments by different cost indices may once again change the rates of substitution between the two payment components. For all these reasons, a multi-period model allowing for some kind of risk aversion by bidders would be a very useful extension.

8. Conclusions

The restructuring of the electric power industry into one in which generation is competitively supplied will most likely lead to informational problems that did not exist when the industry was fully vertically integrated. As we have argued, some of these informational problems may arise from strategic behavior of bidders competing in auctions to sell power to utilities. The fact that informational constraints can produce negative reliability and efficiency effects should be recognized by policy makers.

At the same time, informational problems can be mitigated by carefully considered auction design. In the electric power context as well as others, such as natural resource auctions and franchise bidding, it may be advantageous to reduce the bidders' strategic behavior to one dimension or bid component. It is recommended that bid scoring systems meet the condition that the marginal rates of substitution between price components be the same in the payments made to winners as in the method of scoring bids if the reduction of bid shading to fixed prices (i.e. truthful bidding of energy costs) is a goal of auction designers.

Given our informational assumptions, the scoring system is the only direct tool with which a bid-taker may affect bidder strategies in first price auctions. Second price auctions, however, provide more tools for auction designers while at the same time presenting more ways of encouraging distortionary behavior on the part of bidders. Efficiency may be facilitated, as it is in the electric power context, by providing the "second price" payment in only one dimension while other components of payment remain as first price ones.

The condition for scoring systems provides useful insight into scoring provisions involving non-price factors as well. Any scoring provision that alters the relative values placed on fixed and variable payments to suppliers has the potential to destroy equilibria in which energy costs are accurately revealed. Future work analyzing industry scoring systems should examine the effects of such composite scoring on bidder strategies. The expansion of bidders strategy spaces to include a capacity decision would also be a valuable extension.

Appendix

Proof of Proposition 1:

If the strategy $\{b_k(k,c),\ b_c(k,c)\}$ is an equilibrium one, no bidder can find it optimal to unilaterally deviate from it. From first order conditions, $\partial E(\pi(b_k,b_c,k,c))/\partial b_k = \partial E(\pi(b_k,b_c,k,c))/\partial b_c = 0$ is therefore a necessary condition for an interior strategy $\{b_k,b_c\}$ to be a symmetric equilibrium.

We now show that the above first order condition can be satisfied at $b_c = c$ only if $\frac{\partial S/\partial b_c}{\partial S/\partial b_k} = \rho(b_c)$ at that value of b_c.

Once again, the expected payoff to a bidder following $\{b_k,b_c\}$ is

$$E(\pi(b_k,b_c,k_i,c_i)) = (b_k - k_i + (b_c-c_i)\rho(b_c))P$$

As argued above, for a strategy $\{b_k,b_c\}$ with $b_c = c$ to be an equilibrium one, we must have

$$\text{(2)}\qquad \frac{\partial E(\pi)}{\partial b_k} = P((S(b_k,b_c),k_i,c_i) + (b_k - k_i + (b_c-c_i)\rho(b_c))\frac{\partial P}{\partial S}\frac{\partial S}{\partial b_k} = 0$$

and

$$\text{(3)}\qquad \frac{\partial E(\pi)}{\partial b_c} = [(b_c-c)\rho'(b_c) + \rho(b_c)]P((S(b_k,b_c),k_i,c_i) + (b_k - k_i + (b_c-c_i)\rho(b_c))\frac{\partial P}{\partial S}\frac{\partial S}{\partial b_c} = 0$$

at the point $b_c = c_i$. Rearranging (2) and (3) and setting $b_c = c_i$ gives

$$\frac{\partial S}{\partial b_k} = -\frac{P(S(b_k,b_c),k_i,c_i)}{(b_k - k_i)\partial P/\partial S} = \frac{1}{\rho(b_c)}\frac{\partial S}{\partial b_c} \quad at\ b_c = c_i$$

which imply

$$\frac{\partial S/\partial b_c}{\partial S/\partial b_k} = \rho(b_c) \text{ at } b_c = c_i.$$

Since the same scoring rule applies to all bidders, this condition must hold for all possible values of c_i.

Proof of Proposition 2:

In order to prove Proposition 2, we need to establish an identity concerning the function $S^{-1}(s;\, b_c)$, defined in section 4.

Lemma 1:

$$\frac{\partial S^{-1}(s;\, b_c)}{\partial b_c} = -\frac{\partial S(k,b_c)/\partial b_c}{\partial S(k,b_c)/\partial k}\Big|_{k=S^{-1}(s;b_c)\forall s,b_c}$$

proof:

By definition, $S(S^{-1}(s;b_c),b_c) = s$. The result follows directly from differentiating both sides of this identity with respect to b_c.

Proposition 2:

The expected payoff to bidder i as a function of bid prices when he expects his opponents to adopt a strategy of bidding true cost is

$$E(\pi(b_k,b_c,k_i,c_i)) = \int_{\overline{S}}^{S(b_k,b_c)} \{ S^{-1}(s;\, b_c) - k_i + \rho(b_c)(b_c - c_i)\}\frac{\partial \overline{G}(s,k_i,c_i)}{\partial s}ds$$

where $\overline{S}$ is the upper bound on possible scores.[6] The truth-telling strategy can be an equilibrium only if first order conditions for a maximum of $E(\pi)$ with respect to b_k and b_c are met at $b_k = k_i$, $b_c = c_i$, i.e. if the following conditions are met.

(4) $$\frac{\partial E(\pi)}{\partial b_k} = \{ S^{-1}(S(b_k,b_c);b_c)-k_i+\rho(b_c)(b_c-c_i)\}\frac{\partial S}{\partial b_k}\frac{\partial \overline{G}(S(b_k,b_c),k_i,c_i)}{\partial s}$$
$$= 0 \,\Big|_{b_c=c_i,\,b_k=k_i}$$

(5) $$\frac{\partial E(\pi)}{\partial b_c} = \{ S^{-1}(S(b_k,b_c);b_c)-k_i +\rho(b_c)(b_c-c_i)\}\frac{\partial S}{\partial b_c}\frac{\partial \overline{G}(S(b_k,b_c),k_i,c_i)}{\partial s}$$
$$+\int_{\overline{S}}^{S(b_k,b_c)} \{\frac{\partial S^{-1}(s;\, b_c)}{\partial b_c} +\rho'(b_c)(b_c-c_i)+\rho(b_c)\}\frac{\partial \overline{G}(s,k_i,c_i)}{\partial s}ds$$
$$= 0\Big|_{b_c=c_i,\,b_k=k_i}$$

By definition $S^{-1}(S(b_k,\, b_c);\, b_c) = b_k$, therefore condition (4) is met when $b_k = k_i$, $b_c = c_i$. Similarly, the first term in (5) equals zero while the second term in (5) vanishes if and only if

$$\frac{\partial S^{-1}(s;\, b_c)}{\partial b_c} = -\,\rho(b_c)$$

The above reduces to condition (1) by applying Lemma 1 and the fact that in equilibrium the above condition must be satisfied for all bidders. This completes the proof of part 1.

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6 The order of the bounds of integration result from the fact that $\partial \overline{G}/\partial s$ is negative.

For part 2, we need to prove that truth telling by bidder i meets second order sufficiency conditions for a local maximum of $E(\pi)$ assuming that i's opponents tell the truth. We have already shown that the first order conditions, (4) and (5), hold under the hypothesis of the proposition. Now we verify the second order condition by using the determinant test to show that the Hessian of $E(\pi)$ with respect to b_k and b_c is negative definite at truth telling.

The second derivatives of the payoff function with respect to b_k, and b_c are

$$(6) \qquad \frac{\partial^2 E(\pi)}{\partial b_k^2}\Big|_{b_c=c_i,b_k=k_i} = \frac{\partial S}{\partial b_k}\frac{\partial \overline{G}(S(b_k,b_c),k_i,c_i)}{\partial s}$$

$$(7) \qquad \frac{\partial^2 E(\pi)}{\partial b_c \partial b_k}\Big|_{b_c=c_i,b_k=k_i} = \rho(b_c)\frac{\partial S}{\partial b_k}\frac{\partial \overline{G}(S(b_k,b_c),k_i,c_i)}{\partial s} = \frac{\partial S}{\partial b_c}\frac{\partial \overline{G}(S(b_k,b_c),k_i,c_i)}{\partial s}$$

$$(8) \qquad \frac{\partial^2 E(\pi)}{\partial b_c^2}\Big|_{b_c=c_i,b_k=k_i} = \rho(b_c)\frac{\partial S}{\partial b_c}\frac{\partial \overline{G}(S(b_k,b_c),k_i,c_i)}{\partial s} + \rho'(b_c)\overline{G}(S(b_k,b_c),k_i,c_i)$$
$$= \frac{\partial}{\partial b_c}(\overline{G}\rho)$$

The above terms can be condensed using the notation $\partial\overline{G}/\partial b_k = (\partial\overline{G}/\partial s)(\partial S/\partial b_k)$ and $\partial\overline{G}/\partial b_c = (\partial\overline{G}/\partial s)(\partial S/\partial b_c)$. We note that the first principal minor, $\partial\overline{G}/\partial b_k$, is negative as an increase in fixed bid will decrease the probability of winning. The determinant of the Hessian matrix is

$$\partial\overline{G}/\partial b_k\,[\,\rho(c)\,\partial\overline{G}/\partial b_c + \rho'(c)\overline{G}(S(b_k,\,b_c))] - [\rho(c)\,\partial\overline{G}/\partial b_k\,]^2$$

$$= \quad \partial\overline{G}/\partial b_k\,[\,\rho(c)\,\partial\overline{G}/\partial b_c + \rho'(c)\overline{G}(S(b_k,\,b_c))] - [\rho(c)\,\partial\overline{G}/\partial b_k\,\partial\overline{G}/\partial b_c]$$

$$= \quad \partial\overline{G}/\partial b_k\,\rho'(c)\overline{G}(S(b_k,\,b_c),k_i,c_i) > 0,\ \text{since } \rho \text{ is decreasing in } c.$$

The matrix of second partial derivatives is therefore negative definite at truth telling. It is therefore locally optimal for bidder i to truth tell if bidder i expects his opponents to truth tell. We should emphasize, however, that this does not show existence of a truth telling Bayes-Nash equilibrium as we have not excluded the possibility of a large deviation by bidder i from truth telling which may result in a globally optimal expected payoff. Proving such a result would require showing that $E(\pi)$ is globally concave in b_k and b_c.

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