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Linear Exchangeable Permits for the Efficient Control of Multiple Pollutants

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Abstract

In this paper we describe a mechanism for the decentralized control of emissions and other problems with nonlinear externality effects. These are a generalization of tradable emission permits to the multivariate context. These *linear exchangeable permits* guarantee a bound on environmental damage, while allowing additional flexibility to the regulated industry when compared to standard tradable emissions rights. Thus, industry efficiency can be increased with no added risk of environmental damage.

1 Introduction

One of the fundamental policy decisions in environmental regulation concerns the choice between quantity mechanisms such as emission limits and price based mechanisms such as effluent taxes. As first analyzed in a general context by Weitzman (1974) , price mechanisms assured economic efficiency in the sense that emission levels would be set such that the marginal cost of control equals the effluent tax, which presumably is set equal to the marginal damage to the environment. In the presence of uncertainty (or private information), however, price mechanisms presented a risk as to the actual level of environmental damage that may result, while the quantity mechanism at least insured a certain level of emissions, even though that level may not turn out to be an economically efficient one. As argued by Weitzman, the quantity mechanism has clear advantages in the presence of uncertainty when potential environmental damage is a highly non-linear function of emissions. Such is the case when damage becomes severe once emissions cross some threshold level. Roberts and Spence (1976) later showed that a quantity mechanism combined with linear penalties can make a regulatory system more robust in the face of uncertainty. Outside of economic arguments, quantity mechanisms held an advantage in terms of their political acceptability (Cook, 1988).

There has been much recent interest in a specific type of quantity mechanism, the tradable emission permit. Tradable emissions permits represent an attempt to combine the efficiency benefits of price mechanisms with the certainty of output of the quantity mechanism (Hahn and Noll, 1982) . Producers would reduce (or increase) emissions to a level such that their marginal cost of abatement was equal to the price of the emission permit. A sort of hybrid emissions trading system has existed for some time in jurisdictions which have established ceilings on emissions of pollutants. Often a new polluter is required to purchase an "off-set" from an existing polluter prepared to reduce its emissions. The off-set essentially becomes a tradeable right to pollute. A more prototypical tradable emission permit scheme is the much heralded SO_2 emissions permit program that has been formed under the provisions of the 1990 Clean Air Act Amendments.

One issue that has not been widely addressed is the interaction of multiple pollutants in contributing to such environmental problems as ozone depletion and global warming. In 1982, Roberts commented that setting ratios allowing permits for different types of pollutants to offset one another, could facilitate trades. Hahn and Noll (1990) noted that rights to emit HCFCs and CFCs could be traded for one another at some pre-set ratio under the program to prevent ozone depletion.

Most recently, two electric utilities in the US have reached an agreement to exchange SO_2 emissions rights for the rights to Carbon Dioxide emissions that were created under a voluntary greenhouse gas reduction program (Passell, 1994). In this paper, we develop a theoretical foundation for cross-pollutant exchanges and present a regulatory scheme that takes these proposals one step further; a system of linear exchangeable (LEX) permits under which the rights to emit one of several different pollutants can be exchanged for one another at different exchange rates, depending on the permit held by the parties to the exchange. We demonstrate that in the presence of abatement cost uncertainties, this system will be superior to both standard tradable emissions rights and a system where pollutants are exchanged at a constant ratio. Section 2 of this paper provides an intuitive explanation of the LEX permit scheme and why it is superior to the aforementioned alternatives. In Section 3, we describe our model and define the LEX permit. In Section 4, we characterize the equilibrium that would result from efficient trading of LEX permits. Section 5 describes the set of LEX permits that are optimal from the regulator's standpoint and Section 6 discusses conclusions and extensions of this scheme.

2 Optimal Control of Two Interacting Pollutants

Consider a situation in which an instance of environmental degradation is influenced by the interaction of two types of pollutants, X_1 and X_2. Environmental damage is measured by the convex function $D(X_1, X_2)$, which is increasing in both parameters. The key informational assumption is that the individual producers' costs of controlling emissions is not well known by regulators. Regulators are, however, assumed able to derive a reasonable estimate of the damage done to the environment. We now qualitatively examine the effects of different regulatory mechanisms on both the environment and the relative costs of cleanup.

Figure 1 will help illustrate our arguments. The axes of figure 1 correspond to the levels of pollutants X_1 and X_2 which are emitted. The concave curve represents a level set of the environmental damage function $D(X_1, X_2)$. The convex curves represent one possible description of the level sets of the aggregate costs of controlling the pollution levels.

First, consider a price mechanism such as an effluent tax on both pollutants. As Weitzman first argued, there is a risk that the marginal costs of reducing these pollutants may be higher than estimated by regulators. Under such a scenario, when producers reduce pollution to the level where their marginal cost of reduction equals the tax, the resulting level of pollution will be much greater than anticipated. For example, the resulting emissions might be at point A of figure 1.

To insure against such a scenario, a quantity mechanism such as tradable permits may be employed. At one level, there could be predetermined quantities of both X_1 and X_2 with the corresponding number of permits for each pollutant. Producers would be allowed to freely exchange permits, but these could be applied only towards the emission of the permits' corresponding pollutants. Under this policy, the level of pollution is guaranteed to be at, for example, point B of figure 1. The amount of environmental damage is also guaranteed to be equal to that of the level set which point B falls on. A potential disadvantage to this approach when producer costs are not well estimated is that the marginal cost of reducing one pollutant could be much lower than the marginal cost of reducing the other. Thus the same degree of environmental damage can be obtained at lower cost by shifting the mix of pollutants X_1 and X_2 to point C of figure 1, which lies on the same level set of D as point B.

As a compromise solution intended to mitigate the difficulties of the previous two policies, it has been suggested that producers be allowed to exchange permits for different pollutants at some pre-specified exchange rate. This would allow more flexibility in exchanges and allow individual producers with varying rates of technical substitution for reducing pollutants to gain extra cost savings, but overall pollution levels would again remain at point B of figure 1. An alternative solution is to issue *LEX* permits that allow, for example, one unit of X_1 or as many units of X_2 as specified by the exchange rate. This solution would result in the less expensive, more efficient pollution mix represented by point D of figure 1, but at a level of higher environmental damage than allowed by the conventional tradable permits plan.

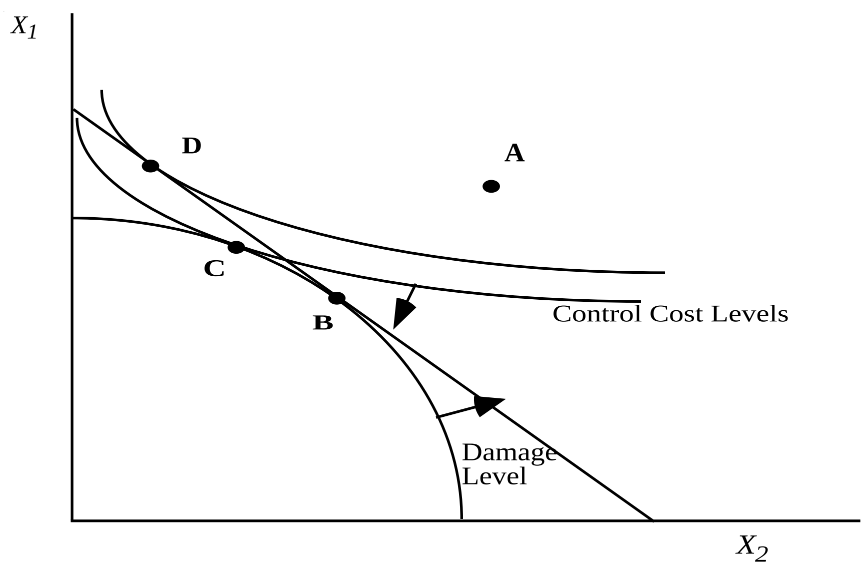


Figure 1: Effects of regulatory policies on environmental damage

In this paper, we propose a system of exchangeable permits that allow for trade between pollutant types at *multiple exchange rates*. This is accomplished by issuing some permits that allow either a unit of X_1 or an amount X_2 set according to an exchange rate recorded on the permit. Other permits would allow either the unit of X_1 or some other amount of X_2. We will show that each successive issuing of permits at different exchange rates will establish a linear bound over the possible resulting amount of pollution. These linear bounds will increasingly resemble the actual shape of the corresponding level set of D.

By using an infinite number of exchange rates the frontier of possible pollution levels will exactly mirror the level set of D. In this case the advantages of such a system becomes readily apparent. The mix of the aggregate levels of X_1 and X_2 may now be set at the point of maximum efficiency; where the aggregate rates of technical substitution of the costs of control are equal to the marginal rates of substitution between pollutants in the damage function. At the same time, the total damage incurred due to the pollution is fixed equal to the value of the respective level set of D. Thus the efficiency aspects of permits are enhanced, while strict limits on environmental damage are maintained.

3 Model

Let $X \in \Re^m$ be a vector of m pollutants. We define the regulator's objective as:

$$\min_X D(X) - E[V(X;\theta)] \tag{1}$$

where $D(X)$ is the environmental damage due to polluting X and $V(X;\theta)$ is the value to industry of emitting X under private information θ, which is not known to the regulator, and may not be known with certainty by the firm, ex-ante. We assume that $D(X)$ is convex and increasing, and that $V(X,\theta)$ is concave and increasing in X.

Note that we think of $V(X;\theta)$ as arising from the efficient operation of a diverse industry with n producers.

$$V(X;\theta) = \max \sum_{i=1}^{n} V_i(X^i;\theta^i) \;\text{ s.t.} \sum_i x_j^i = X_j \tag{2}$$

where X^i is the vector of m pollutants emitted by producer i and x_j^i is the amount of pollutant j emitted by producer i. $V_i(X^i;\theta^i)$ arises from a firm's individual maximization problem.

Since the objective function is convex, the first best (perfect information) optimum for this problem is characterized by the first order conditions

$$\frac{\partial D(X)}{\partial X_j} = \frac{\partial V(X;\theta)}{\partial X_j} \;\forall\; j \tag{3}$$

and

$$\frac{\partial V_k(X^k;\theta^k)}{\partial x_j^k} = \frac{\partial V_i(X^i;\theta^i)}{\partial x_j^i} \;\forall\; i,k,j. \tag{4}$$

The two traditional methods of regulating an optimum from equation 1 are by imposing prices or setting quantities. When regulating by prices, the regulator chooses a price vector $p \in \Re^m$ such that when the industry maximizes

$$\max_X \; V(X;\theta) - p \cdot X \tag{5}$$

the resulting emissions will be optimal. However, as θ is not known to the regulator, the resulting emissions could differ greatly from the desired level, as the price $p_j = \frac{\partial V(X;\theta)}{\partial X_j}$ probably will not equal $\frac{\partial D(X)}{\partial X_j}$ at the resulting X. Weitzman demonstrated that problems with the price approach will be most severe when the damage function is sharply curved or when producer value functions are relatively flat.

When regulators control by quantity they set the x_j^i explicitly. However, the distribution of emissions *within* industry (condition 4) is therefore not likely to be efficient in the absence of perfect information. A theoretically more efficient alternative is distributing permits s where each permit allows the holder to emit one unit of a specific pollutant, thereby fixing the aggregate quantities X. These permits may be traded (or sold) within the industry to allow the industry to achieve efficiency. Again, as θ is not known to the regulator, this X may not be optimal; however, permits have the desirable property that they bound the amount of environmental damage. Thus even if the regulator has very little knowledge of θ the amount of environmental damage is limited, which is not the case for price mechanisms. The presence of multiple interacting pollutants creates additional efficiency problems for the standard regulatory mechanisms. Although linear prices or tradable emissions permits both achieve the efficient allocation of a pollutant X_j between producers (condition 4), since condition (3) is unlikely to hold ex-post, it is unlikely that

$$\frac{\frac{\partial D(X)}{\partial X_j}}{\frac{\partial D(X)}{\partial X_k}} = \frac{\frac{\partial V(X;\theta)}{\partial X_j}}{\frac{\partial V(X;\theta)}{\partial X_k}} \; j,k \in \{1,\ldots,m\}. \tag{6}$$

Thus under any of these policies for regulating the level of environmental damage, it is likely that there will exist a more efficient mix of pollutants that will result in the same amount of net damage.

We now present a method that for this model is a strict improvement on conventional tradable permits in that it promotes the efficient distribution of pollutants for a given level of environmental damage. Define a LEX permit by $s = (s_1, s_2, \ldots, s_m)$ which allows, for example, a firm the option of emitting s_1 of pollutant 1, *or* s_2 of 2, *or* s_m of m, and so on. However, we assume that permits are infinitely divisible. Therefore the feasible emissions allowed by permit s are

$$\mathcal{F}(s) = \{X \in \Re^m_+ \mid \sum_{j=1}^{m} X_j/s_j \leq 1\}. \tag{7}$$

and given a set of permits S the feasible net emissions are

$$\mathcal{F}(S) = \sum_{s \in S} \mathcal{F}(s), \tag{8}$$

where the sum is interpreted as the Minkowsky sum.[1] Note that $\mathcal{F}(S)$ is convex as it is the sum of convex sets.

In the next section we will study the use of a set of tradable LEX permits within the industry. In the section after we will show how to construct a set of permits S so that

$$\mathcal{F}(S) = \{X \in \Re^m_+ \mid D(X) \leq d\} \tag{9}$$

where D is any convex function and d is the bound on environmental damage. Such a set S of LEX permits would be superior to direct control by conventional permits, since it increases the options of producers without increasing environmental damage.

4 Efficient Use of LEX Permits

Given a set of permits it is not obvious how they will be valued and used. For example it seems plausible that a $(10, 2)$ permit would be more valuable than a $(10, 0)$ permit. However, this will not be true if, at equilibrium, both permits are applied only to the emission of the first pollutant. In that case, the fact that the (10,2) permit could be put to a *less* efficient use does not increase

[1]The Minkowsky sum of two convex sets is defined as $A + B = \{x \mid \exists\ a \in A, b \in B, \quad x = a + b\}$. See Rockafellar, 1970.

it's value. [2]

In this section we show that at equilibrium there exist a set of prices p which determine the value of the LEX permits.

Theorem 1 *Let X^* be the optimal solution of*

$$\max_X V(X;\theta) \ \ s.t. \ X \in \mathcal{F}(S) \tag{10}$$

for a set of permits S and a specific realization of θ. Then there exists a price vector p such that each permit will be used to maximize its value. e.g. given $s \in S$ then only those pollutants i such that $i \in \arg\max_j p_j s_j$ will be emitted under this permit.

Proof: First note that this problem consists of maximizing a concave function over a convex set, therefore a solution is completely determined by Kuhn-Tucker conditions using equation (7) to define $\mathcal{F}(s)$. The lagrangian for this problem is

$$L = V(X;\theta) - \sum_{j=1}^{m} p_j (X_j - \sum_{l=1}^{r} x_{jl}) - \sum_{l=1}^{r} \lambda_l (\sum_{j=1}^{m} \frac{x_{jl}}{s_{jl}} - 1) + \sum_j \sum_l \mu_{jl} x_{jl} \tag{11}$$

where s_{jl} is the j'th component of permit s_l and x_{jl} is the amount of pollutant j emitted under permit l. The Kuhn-Tucker conditions for optimality are:

$$\frac{\partial V(X;\theta)}{\partial X_j} = p_j \tag{12}$$

$$p_j + \mu_{jl} = \lambda_l / s_{jl} \tag{13}$$

$$p_j, \lambda_l, \mu_{jl} \geq 0 \tag{14}$$

$$\mu_{jl} x_{jl} = 0 \tag{15}$$

[2] Before an equilibrium is reached and all information about θ is revealed, this will not be true. At intermediate stages a (10,2) permit would have more value than a (10,0) permit, even if both are used for the first pollutant. The difference in value at this stage could be viewed as an option value on the two units of the second pollutant.

The first equation defines the prices to be the marginal values of not decreasing pollutants.

Now given a permit s_l let $I = \{j \mid x_{jl} > 0\}$. Therefore by (15) $\mu_{jl} = 0$ for all $j \in I$. This implies that $p_j s_{jl} = \lambda_l$ for all $j \in I$ and $p_j s_{jl} \leq \lambda_l$ for all $j \notin I$. Thus $I = \arg\max_j p_j s_{jl}$. $\Box$

Theorem 1 describes how permits will be used by industry at equilibrium under this system. A single m-vector of prices will form based upon the marginal costs of reducing pollutants. These prices are not the value of individual permits, but rather the price of a unit of an individual pollutant under this system. Based upon these prices, individual permits will be used (barring ties) for the single pollutant that maximizes the value of the permit. For example, in the two pollutant case, if the aggregate marginal cost of reducing pollutant x_1 is twice that of reducing x_2 at the equilibrium point, all permits which allow trading a unit of x_1 for 2 units or less of x_2 will be applied toward the emission of x_1. λ_l represents the value of a permit of type l in equilibrium. It's value will be determined, from equation (13), by the price of the pollutant j for which the permit would be used, and the option allowance for j, s_{jl}.

Note that we have assumed that permits are infinitely divisible for the purpose of trading. In practice such a system could be implemented through a contractual mechanism. A simple method would be to issue permits that are small enough so that discreteness is not an issue. For example, instead of issuing one permit $(s_1, \ldots, s_m)$, L permits of type $(s_1/L, \ldots, s_2/L)$ could be issued, where L is some large integer.

5 Efficient Design of LEX Permits

In this section we describe the construction of a set of LEX permits that guarantee a bounded level of environmental damage which is superior to a set of conventional permits that allow the emission of L_i of pollutant i. In this section we index individual permit types with the superscript ω, so that $s^\omega \in S$ for all $\omega \in \Omega$, an index set.

Recall that $\mathcal{F}(S) = \sum_{\omega\in\Omega} \mathcal{F}(s^\omega)$. Now our goal is: given the convex set

$$G = \{X \in \Re^m_+ \mid D(X) \leq d\} \tag{16}$$

find a set of permits S such that $\mathcal{F}(S) \subseteq G$ and $\{L_i\} \in \mathcal{F}(S)$. Thus with this set of LEX permits the resulting environmental damage is guaranteed to be no greater than the damage resulting from the set of conventional permits, L_i. The construction of one such set can be accomplished by fixing $m-2$ pollutants at their corresponding L_i's and constructing two LEX permits for the remaining two pollutants (see figure 2).

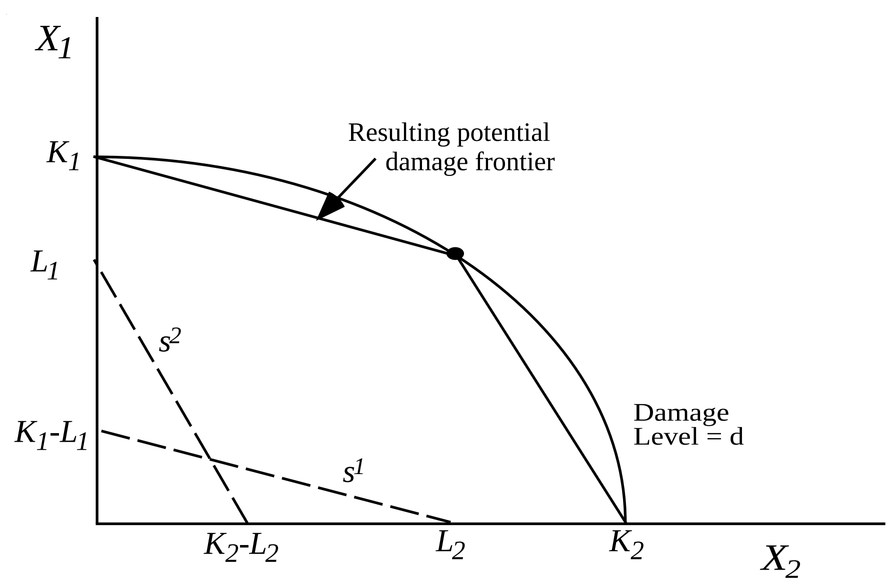


Figure 2: A simple two-pollutant LEX permit structure

Theorem 2 *There exists a finite set of permits S such that $\mathcal{F}(S) \subseteq G$ and $\{L_i\} \in \mathcal{F}(S)$.*

Proof: It is easy to construct a simple example using m permits [3] where $\omega = \{1, 2, \ldots, m\}$. Let $s^\omega = L_\omega e_\omega$ for $\omega \geq 3$ where e_ω is the ω'th unit vector. Now let

$$K_1 = \max \ X_1 \ \text{ s.t. } \ D(X_1, 0, L_3, \ldots, L_m) \leq D(L_1, \ldots, L_m) \tag{17}$$

and

$$K_2 = \max \ X_2 \ \text{ s.t. } \ D(0, X_2, L_3, \ldots, L_m) \leq D(L_1, \ldots, L_m). \tag{18}$$

Let $s^1 = (K_1 - L_1, L_2, 0, \ldots, 0)$ and $s^2 = (L_1, K_2 - L_2, 0, \ldots, 0)$. This set of permits satisfies the theorem as is easily verified. □

Note that there are many possible selections of permits that satisfy the above theorem. There is, however, a maximal set of permits. This occurs when $\mathcal{F}(S) = G$. If G is a polytope than this is possible with a finite set of permits. However; if G is not a finite collection of half spaces (or equivalently, the convex hull of a finite number of points) then it is necessary that S not be a finite set.

In this case we will assume that G is strictly convex and smooth. If $\mathcal{F}(S)$ is to be equivalent to G, this requires that S contain an infinite number of permits. Thus we describe the quantity of each type of permit by a density function $\rho(\omega)$, which for simplicity we assume to be smooth. Given some set $\{s^\omega\}$ we will construct the density $\rho(\omega)$ and define

$$\mathcal{F}(S, \rho) = \int_\Omega \mathcal{F}(s^\omega)\rho(\omega)d\omega. \tag{19}$$

[3] The set of permits constructed in the proof is not very efficient. It is straightforward to compute a 'good' set by solving a linear program.

where Ω is a measurable space and $d\omega$ a canonical measure. Thus the problem is reduced to finding a density function $\rho(\omega)$ so that $\mathcal{F}(S, \rho) = G$.

Consider a point X on the boundary of $\mathcal{F}(S, \rho)$. The supporting hyperplane at X is unique by our assumptions, and can be represented by the vector $c(X)$. This vector must have been generated by some permit s with $s_i \propto 1/c(X)_i$. We define $EB(G)$, the efficient boundary of G, as

$$EB(G) = \{X \in G \mid \nexists X' \in G \text{ s.t. } X' > X\} \tag{20}$$

In order to set $G = \mathcal{F}(S)$ we must have permits such that for all X on the efficient boundary of G, there exists an s^ω such that for all j, $s_j^\omega = 1/(\frac{\partial D(X)}{\partial X_j})$. Thus a natural method for choosing our set of permits is to set $\Omega = EB(G)$ and let $s_j^\omega = (\frac{\partial D(\omega)}{\partial X_j})^{-1}$, for all $\omega \in \Omega$.

We define $\Omega_j(p)$ as the subset of permits in Ω which will be used for emitting pollutant j when efficiently traded in a market that produces price vector p.

$$\Omega_j(p) = \{\omega \in \Omega \mid p_j s_j^\omega > p_k s_k^\omega \quad \forall k \neq j\} \tag{21}$$

Our next step is to reduce the problem to ordinary integrals.

Theorem 3 *Let $\{s^\omega\}$ be a set of permits and $\rho(\omega)$ a density such that*

$$\sum_j \frac{\partial D(X)}{\partial X_j} X_j = \sum_j \frac{\partial D(X)}{\partial X_j} \int_{\Omega_j(\nabla D(X))} s_j^\omega \rho(\omega) d\omega \tag{22}$$

for all $X \in EB(G)$, where all derivatives are evaluated at X. Then $\mathcal{F}(S, \rho) = G$.

Proof: For a convex set the support functional

$$\delta_G(p) = \max_{X \in G} \quad p \cdot X \tag{23}$$

uniquely defines the set. Also $\delta_{G+G'}(p) = \delta_G(p) + \delta_{G'}(p)$ (Rockafellar, 1970). Equation 22 simply shows the equivalence of the support functionals for G and $\mathcal{F}(S, \rho)$.

Consider the point $X \in EB(G)$. Given the price vector $p = \nabla D(X)$, the value of the support function will be $\nabla D(X) \cdot X$. Now we compute $\delta_{\mathcal{F}(S,\rho)}(\nabla D(X))$.

First note that

$$\delta_{\int_\Omega \mathcal{F}(s^\omega)\rho(\omega)d\omega}(p) = \int_\omega \delta_{\mathcal{F}(s^\omega)}(p)\rho(\omega)d\omega \tag{24}$$

by additivity of the support function and note that $\delta_{\mathcal{F}(s^\omega)}(p)$ is simply $\max_j p_j s_j^\omega$. From the definition of $\Omega_j(p)$ we see that

$$\int_\omega \delta_{\mathcal{F}(s^\omega)}(p)\rho(\omega)d\omega = \int_{\Omega_j(p)} s_j^\omega \rho(\omega)d\omega \tag{25}$$

proving the theorem. The remaining values of p are not necessary to check since both sets are guaranteed to lie in the positive quadrant. □

Note that the condition (22) for $\mathcal{F}(s, \rho) = G$ is in terms of a linear operator on ρ. If we take l points on $EB(G)$, X^i, and the permits with $s_j^i = (\frac{\partial D(X^i)}{\partial X_j})^{-1}$ then the problem reduces to a system of linear equalities over the positive orthant which could be solved using linear programming.

We also note that equation (22) shows us the correct equilibrium price, given an allocation $X \in EB(G)$.

Corollary 1 *Let $X \in EB(G)$ be the equilibrium allocation of permits. Then the equilibrium price vector for pollutants is $\nabla D(X)$.*

Note that this is consistent with the market clearing price in Theorem 1.

5.1 Construction of LEX Permits

In the case of two pollutants the solution of (22) is straightforward.

Theorem 4 *Let* $G = \{(X_1, X_2) \mid D(X_1, X_2) \leq d \;\; and \;\; X_1, X_2 \geq 0\}$ *with the boundary of* G *defined by* $(x, X_2(x))$ *for* $x \in [0, L_1]$*, where* $D(L_1, 0) = d$ *. If* $\Omega = [0, L_1]$*,* $s_x = (1/\frac{\partial D(x, X_2(x))}{\partial X_1}, 1/\frac{\partial D(x, X_2(x))}{\partial X_2})$ *and* $\rho(x) = \frac{\partial D(X_1, X_2)}{\partial X_1}$*, then* $\mathcal{F}(S, \rho) = G$*.*

Proof: Note that $\Omega_1(\nabla D(X)) = [0, X]$ and $\Omega_2(\nabla D(x)) = [X, L_1]$ by convexity. Therefore equation (22) reduces to

$$\frac{\partial D(X)}{\partial X_1} X_1 + \frac{\partial D(X)}{\partial X_2} X_2 = \frac{\partial D(X)}{\partial X_1} \int_0^X d\omega + \frac{\partial D(X)}{\partial X_2} \int_X^{L_1} \frac{\partial D(X)}{\partial X_1} / \frac{\partial D(X)}{\partial X_2} d\omega \tag{26}$$

Now the first integral is simply X_1 and noting that $\frac{\partial D(X)}{\partial X_1} / \frac{\partial D(X)}{\partial X_2} = dX_2/dX_1$ the second is X_2. $\square$

In the two pollutant case the amount of one pollutant uniquely determines the amount of the other for a point on a level set of the damage function $D(X_1, X_2)$. We can therefore index permits according to one pollutant. The 'number' of permits of a given type can thus be expressed as the partial derivative (according to the indicator pollutant) of the damage function at the point on the level curve determined by the permits' index.

EXAMPLE: Let the damage function $D(x, y) = x^2 + xy + y^2$ and let

$$G = \{x \in \Re_+, y \in \Re_+ \mid D(x, y) \leq 1\}. \tag{27}$$

Since $\arg\max_x D(x, 0)$ s.t. $D(x, 0) \leq 1$ is $x = 1$, we let $\Omega = [0, 1]$ and

$$s^\omega = (1/D_x(\omega), 1/D_y(\omega)) = (\frac{2}{3\omega + \sqrt{4 - 3\omega^2}}, \frac{1}{\sqrt{4 - 3\omega^2}}). \tag{28}$$

Then the density $\rho(\omega) = \frac{3\omega+\sqrt{4-3\omega^2}}{2}$ will produce a set of permits $\{S\}$ such that $\mathcal{F}(S) = G$.

When there are more than two pollutants, finding the density is significantly more complicated. We describe a methodology for computing the density in the appendix and here give an example.

EXAMPLE: Let the damage function $D(x, y, z) = x^2 + y^2 + z^2$ and let

$$G = \{x, y, z \in \Re_+ \mid D(x) \leq 1\}. \tag{29}$$

Let $\Omega = \{\omega \in \Re^2_+ | \omega_1^2 + \omega_2^2 \leq 1\}$ and $s^{(\omega_1,\omega_2)} = (1/\omega_1, 1/\omega_2, 1/\sqrt{1-\omega_1^2-\omega_2^2})$. Then the density $\rho(\omega) = 3\omega_1\omega_2$ will produce a set of permits $\{S\}$ such that $\mathcal{F}(S) = G$.

6 Conclusions

We have developed a system of tradable LEX permits that regulates the emission of a vector of pollutants. The focus of the permit mechanism has been shifted from controlling the quantity of pollutants to controlling the amount of resulting environmental damage. By manipulating the exchange rates of permits and the number of permits of various exchange rates in circulation, the possible outcomes can be constrained to be within a level set of environmental damage. Within the surface of that level set, however, the exact combination of pollutants can be determined from a market in which an implicit price of each pollutant is formed. We have shown that any outcome that arises from this system of permits can potentially reduce mitigation costs while at the same time guaranteeing that environmental damage does not exceed a given level.

Such a system of permits would be beneficial when the trade-offs between pollutants in terms of environmental damage are well understood, but the trade-offs in terms of mitigation costs, which are likely to be the private information of the affected producers, are not. In the case where the level set of an environmental damage function is estimated as the convex hull of a finite number of points, a finite number of permit types can produce a frontier of feasible outcomes that is identical to the damage level set. In the case of a smooth level surface of damage, an infinite number of permit types, issued according to a distribution, is necessary for exactly reproducing the damage level set. In this case, a finite number of permit types can still provide great flexibility with regards to the outcome while still insuring that outcome does not lie beyond the desired level of damage. Another advantage of LEX permits is their flexibility in the face of innovations in control cost technology. Such innovation could disproportionately reduce the costs of controlling certain pollutants. If such a change did occur, the permit system would, without regulatory intervention, allow for a shift in the mix of emissions that would be weighted towards the relatively expensive pollutants.

Finally, we note that LEX permits are quite general and can be applied in many other contexts. Of course many applications of the ideas in this article arise in areas for which the concepts of environmental and resource economics have frequently been applied. These include areas affected by the various common forms of market failure–the lack of defined property rights, common resources, externalities of production and consumption, as well as free ridership. Markets that are plagued by one or more of these troubles have long been thought of as the natural battleground in the prices vs. quantities debate. It is possible that some of these areas, such as fishing and logging, cost sharing, and water rights, for which the multi-dimensional production or damage function is nonlinear, could benefit from a LEX permit approach.

In addition to controlling multiple pollutants, LEX permits could be adapted to the problem of controlling single pollutants whose effects on the environment vary with geographic location. This has been identified as a key weakness of some applications of tradable emissions permits (Laffont & Tirole, 1994). Emissions at various locations can interact in a non-linear fashion and result in geographically divergent environmental damage. Using the LEX permit approach, a desired damage level for each population center could be established and a net damage level set could then be formed from the intersection of the locational damage limits. If the locational damage functions were convex, the resulting net damage level set would also be convex and thus adaptable to the techniques we have presented here.

Appendix: Explicit Construction of Densities

In general we can explicitly compute the density for LEX permits using

$$X_j(p) = \int_{\Omega_j(p)} s_j^{\omega} \rho(\omega) d\omega \tag{30}$$

which can be seen directly from the definition of $\Omega_j(p)$.

We now describe the procedure for computing the density when $m = 3$; the higher dimensional computations follow the same procedure with significantly more technical detail.

Let $D(X_1, X_2, X_3)$ be the damage function, $F = \{(X_1, X_2, X_3) \mid D(X_1, X_2, X_3) \leq 1\}$. Note that a point on the efficient boundary of F is uniquely defined by X_1 and X_2. Let $\Omega = \{\omega \in \Re_+^2 \mid D(\omega_1, \omega_2, 0) \leq 1\}$, the projection of $EB(F)$ onto the X_1, X_2 plane, and

$$s^{(\omega)} = (1/\frac{\partial D}{\partial X_1}, 1/\frac{\partial D}{\partial X_2}, 1/\frac{\partial D}{\partial X_3}) \tag{31}$$

where all derivatives are evaluated at $(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))$ and $\zeta(\omega_1, \omega_2)$ is defined by

$D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2)) = 1.$

Consider the point $\omega^0 \in \Omega$ and the corresponding price vector $p_0 = (\frac{\partial D}{\partial X_1}, \frac{\partial D}{\partial X_2}, \frac{\partial D}{\partial X_3})$ evaluated at $(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))$. We now compute $\rho(\omega^0)$ using Equation (30).

First note that

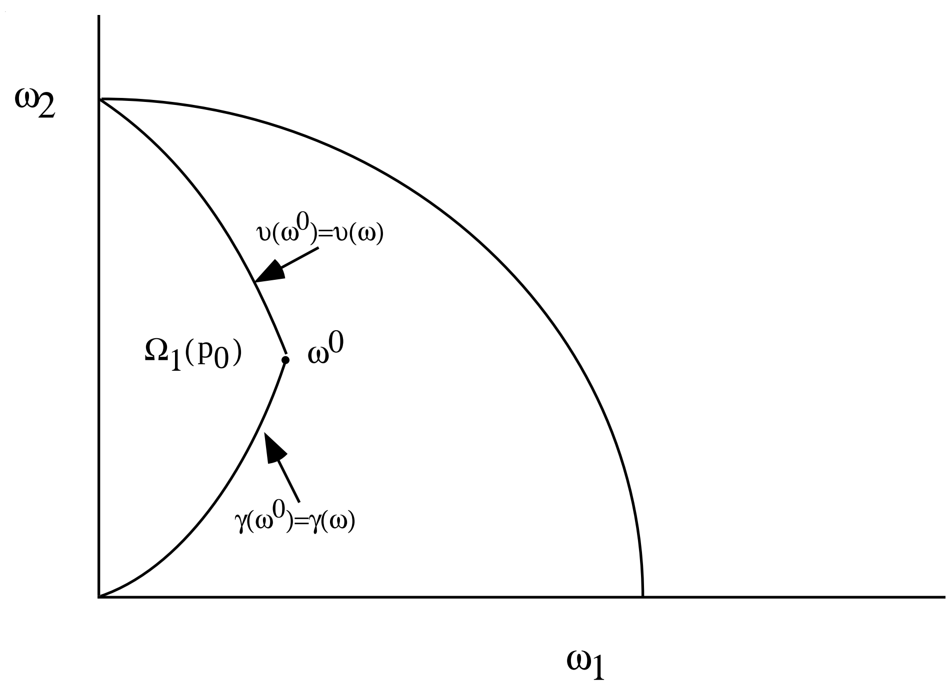
$$\Omega_1(p_0) = \{\omega \mid \gamma(\omega_1^0, \omega_2^0) \geq \gamma(\omega_1, \omega_2),\ \nu(\omega_1^0, \omega_2^0) \geq \nu(\omega_1, \omega_2)\} \tag{32}$$

where

$$\gamma(\omega_1, \omega_2) = \frac{\partial D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))/\partial X_1}{\partial D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))/\partial X_2} \tag{33}$$

and

$$\nu(\omega_1, \omega_2) = \frac{\partial D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))/\partial X_1}{\partial D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))/\partial X_3}. \tag{34}$$



Now for $\delta > 0$ consider the point $\omega^1 \in \Omega$ where $\omega_1^1 = \omega_1^0 - \delta$ and ω_2^1 is the (unique) solution of $\nu(\omega_1^0, \omega_2^0) = \nu(\omega_1^1, \omega_2^1)$. Let p_1 be the corresponding price vector for ω^1. Similarly define ω^2 where

$\omega_1^2 = \omega_1^0 - \delta$ and ω_2^2 solves $\gamma(\omega_1^0, \omega_2^0) = \gamma(\omega_1^2, \omega_2^2)$ and has corresponding price vector p_2. Thirdly define ω^3 to be the (unique) solution of $\gamma(\omega_1^0, \omega_2^0) = \gamma(\omega_1^3, \omega_2^3)$ and $\nu(\omega_1^2, \omega_2^2) = \nu(\omega_1^3, \omega_2^3)$ with p_3 as the corresponding price.

Define the region

$$A = \{\omega \in \Omega \mid \nu(\omega_1^0, \omega_2^0) \geq \nu(\omega_1, \omega_2) \geq \nu(\omega_1^2, \omega_2^2),\ \gamma(\omega_1^0, \omega_2^0) \geq \gamma(\omega_1, \omega_2) \geq \gamma(\omega_1^1, \omega_1^1)\}.$$

Using Equation (30), we have

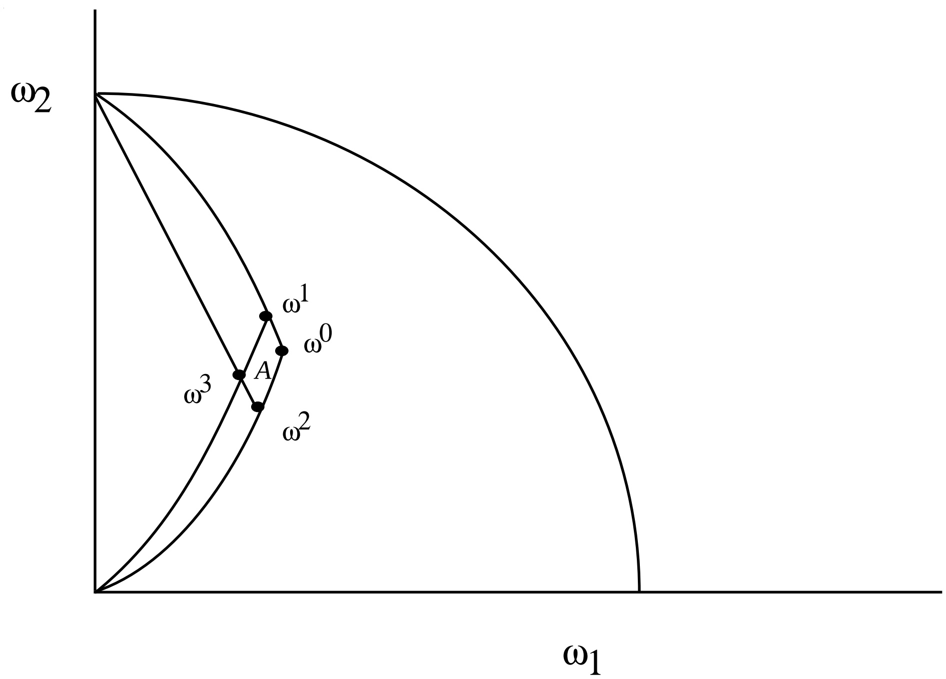
$$\begin{aligned} X_1(p_0) - X_1(p_1) - X_1(p_2) + X_1(p_3) &= \omega_1^0 - \omega_1^1 - \omega_1^2 + \omega_1^3 \\ &= \int_A \frac{\rho(\omega_1, \omega_2)}{\partial D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))/\partial X_1} d\omega_1 \omega_2 \end{aligned} \tag{35}$$

Since $\rho(\omega)$ is continuous (by our assumptions on $D(X)$) for small δ Equation (35) can be approximated by

$$\frac{\rho(\omega^0) Area(A)}{\partial D(\omega_1, \omega_2, \zeta(\omega_1, \omega_2))/\partial X_1}. \tag{36}$$

This can be solved for $\rho(\omega^0)$ after computing the X_1^i's and the area of A. [4] In the limit as $\delta \to 0$ this approximation is exact.

[4] Note that the area of A for small δ is $O(\delta^2)$ as is the left hand side of Equation (35).



However, the exact values for the needed quantities are not readily computable and must be approximated. The first step in this process is to consider the ω's as functions of δ. Then solve for this function to second order in δ.

For example consider the equation for ω^1

$$\nu(\omega^0) - \nu(\omega^1(\delta)) = 0 \tag{37}$$

One way to solve this is to write

$$\omega_2^1(\delta) = \omega_2^1(0) + \frac{d\omega_2^1}{d\delta}\delta + \frac{d^2\omega_2^1}{d\delta^2}\frac{\delta^2}{2} + O(\delta^3) \tag{38}$$

and recall that $\omega_1^1(\delta) = \omega_1^0 - \delta$. Now combing this with the above equation yields

$$\nu(\omega^0) - \nu(\omega_1^0 - \delta, \omega_2^1(0) + \frac{d\omega_2^1}{d\delta}\delta + \frac{d^2\omega_2^1}{d\delta^2}\frac{\delta^2}{2}) = 0. \tag{39}$$

The Taylor expansion of the left hand side of (39) to second order in δ is

$$-\frac{\partial \nu}{\partial \omega_1}\delta + \frac{\partial \nu}{\partial \omega_2}(\frac{d\omega_2^1}{d\delta}\delta + \frac{d^2\omega_2^1}{d\delta^2}\frac{\delta^2}{2}) + \frac{\partial^2 \nu}{\partial(\omega_1)^2}\delta^2 + \frac{\partial^2 \nu}{\partial(\omega_2)^2}(\frac{d\omega_2^1}{d\delta}\delta)^2 - \frac{\partial^2 \nu}{\partial\omega_1\partial\omega_2}(\frac{d\omega_2^1}{d\delta}\delta^2). \tag{40}$$

Note that this equation has six terms, of which two are first order in δ and four are second order. Thus, there are two equations to solve. The first order terms are independent of the second order and can be solved directly, giving

$$\frac{d\omega_2^1}{d\delta} = \frac{\partial \nu}{\partial \omega_1} / \frac{\partial \nu}{\partial \omega_2} \tag{41}$$

This result can be substituted into the second order equation, and solved for $\frac{d^2\omega_2^1}{d\delta^2}$. The remaining ω^i can be solved for similarly. It now remains to compute $Area(A)$. This can be approximated to second order as a parallelegram, and the area computed directly. [5]

This solution was seen to be impractical by hand, and thus we were forced to resort to using Mathematica©. The solution, as computed symbolically, was uniformative, and since it is approximately five pages long we do not include it here. [6] Nevertheless, it is straightforward to evaluate it for a specific damage function, and was used to generate the examples in this paper.

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[5] When $m > 3$, the analogue of A can be approximated as an $m-1$ dimensional parallelipiped which has volume of order δ^{m-1} and therefore all prior computations must be done to order δ^{m-1}. This significantly increases the algebraic difficulty of solving for the density.

[6] Readers interested in using this formula may obtain it directly from the authors.

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