POWER

PWP-045

Price Driven Coordination in a Lossy Power Grid

Bart McGuire

February 1997

This paper is part of the working papers series of the Program on Workable Energy Regulation (POWER). POWER is a program of the University of California Energy Institute, a multicampus research unit of the University of California, located on the Berkeley campus.

University of California Energy Institute
2539 Channing Way
Berkeley, California 94720-5180
www.ucei.berkeley.edu/ucei

PRICE-DRIVEN COORDINATION IN A LOSSY POWER GRID

Bart McGuire
University of California Energy Institute
and
Graduate School of Public Policy UCB

Abstract

In a lossy electric power grid complex interactions exist among the decisions of power producers, power consumers, and the operator of the transmission grid. Must coordination be carried out centrally, or can some form of decentralization achieve an equally good result? In particular, can price-quantity dialogues between producers and consumers on the one hand and power grid managers on the other provide sufficient dissemination of information to achieve an economically efficient outcome? We examine this question in the context of a simple DC lossy system with rather simply characterized producers and consumers. An effective iterative price-quantity adjustment process is proposed and demonstrated in a simulation model. Decision interactions (i.e., externalities) are carefully specified and internalized in the economist's traditional way by imposition of transmission tolls. Emphasis is placed on the use of prices to drive— not merely to reflect— socially good decisions.

PRICE-DRIVEN COORDINATION IN A LOSSY POWER GRID[1]

Bart McGuire

University of California Energy Institute

> *Analysis... can make gestures which carry some promise of system. [Here] I shall make such a gesture and hope that the intimation of a distant theory will compensate for the sketchiness of my account.*
>
> — Roger Scruton

1. INTRODUCTION

A reading of the current discussion on changes in the organization of the electricity industry suggests that most of the participants view restructuring as inseparable from increased decentralization of decision making. Indeed, if by restructuring one means breaking up the industry into coordinated smaller functioning parts, are not restructuring and decentralization the same thing? Not quite, I want to argue. Even given the extent of break-up, decentralization is a matter of degree. The locus of decision-making can remain quite central— if well informed— and the widely dispersed players merely order-followers (albeit perhaps for their own good). Conversely, the individual players— less widely informed— can retain decision power and coordinate their dispersed actions by means of iterated communications. In this paper I examine the workability of this last option in the context of a proposed decentralized coordination mechanism for a *lossy* power grid responding to requests for transmission of bilaterally contracted amounts of power.

Much argument centers on the degree of decentralization that remains compatible with the myriad complexities of electric generation systems and transmission networks. Much argument also relates to the relative market power afforded to players by one or another scheme of decentralization. But such questions cannot be addressed until alternative mechanisms

[1] An earlier version of this paper was presented at the University of California Energy Institute's First Annual Research Conference on Electricity Industry Restructuring in March 1996.

of coordination are specified in closer detail than is common in current discussion. In this paper we shall study coordinating mechanisms for electrical power systems in terms of what have come to be called adjustment processes. To avoid misunderstanding, a prefatory word on the style of analysis to be employed may be helpful. The behavior of theoretical concepts is investigated in the context of numerical examples complicated only to the extent required to catch some of those features thought to be most important in real-life systems. The resulting simple examples are, indeed, simple: examples, not portraits. Behavior of proposed adjustment processes is investigated by means of computer simulation both of (rational) player behavior and of system computation. The question— Does a proposed process "work" or not?— is addressed by a MATLAB program for the simulation. The program substitutes, imperfectly, for mathematical proof. But just as the examples are not to be reqarded as system portraits, neither should the underlying design of the MATLAB routine— useful as it is for small examples— be seen as the basis for computing in a real system. Effective implementation would involve big problems of numerical analysis and system programming.

Section 2 describes the notion of an adjustment process. Section 3 sets the stage for the analyses of *lossy* power grids. Section 4 outlines the formal optimum against which any decentralized system is measured. Section 5 defines the link-impact matrix which plays a central role (if that makes sense) in our decentralized system. The adjustment process of that system is described in Section 6, and its behavior in several examples in Section 7.

2. MODELING COORDINATION AS AN ADJUSTMENT PROCESS

Very generally, an adjustment process specifies the pattern of communications among the players of a system whose actions are to be coordinated. In a discrete process we imagine the environment to which the system must respond changes periodically— for concreteness (not to be taken seriously!), say, once an hour on the hour. After each such change, the players begin a sequence of message exchanges in accord with the pattern specified by the process— one set of exchanges per minute, say. The content of a player's message depends upon his own special knowledge about his part of the new environment, and upon what he has learned from the messages he has received from other players. The exchanges continue until some termination criterion (specified by the adjustment process) is satisfied. Actions that are determined by the terminal messages are finally implemented. To be even minimally useful these final actions must be appropriate for the changed environment and, of course, the process must come to an end rather early in the 60-minute duration of this environment.

This general adjustment-process model is a wild idealization of what happens in real organizations, but it does provide a framework for discussing a number of important aspects of organizational coordination. The process described is a "lose-while-you-choose" process: last hour's actions stay in effect while the organization "computes" the newly appropriate actions. In the economic literature such a process has been called *tâtonnement*: the blind man tapping his cane before taking a step. A different style of process— an "earn-while-you-learn" process— would follow each message exchange with new actions— imperfect, but better than nothing.

A process may be judged by a number of criteria in addition to the aforementioned qualities of speed and near optimality of the final actions. How much computational capacity and learning is required for a player to construct his next message? What demands are made by the process on communication channels? To what extent is the privacy of locally held knowledge compromised? Can the system be exploited by players for private gain? How difficult is it to audit player behavior?

One very special adjustment process is worth noting at the outset. In the first "minute" every player but Number One submits the totality of his relevant information to Number One who now computes the final array of actions. Done. This one-step process is the extreme of centralization. But notice that unless the steps are made longer, the demands on computation at the center are magnified greatly; no advantage is taken of the possible economies of parallel computation. Communication requirements are similarly changed: big messages instead of repeated small ones. Privacy is completely gone, but auditing is probably easier.

Another feature of this centralized process relates to its ease of design. We know more about one-processor algorithms than we do about parallel-processor algorithms. For many problems we simply do not yet know how to reach a near-optimal solution in a significantly decentralized fashion. It can be argued that some writers in the electricity area have tried to have it both ways. Many have proposed that properly constructed nodal prices will enable coordinated decentralized power-dispatch decisions (a view we endorse). But where do these prices come from? In examples intended to demonstrate the proposition, the desired nodal prices are commonly shown to be derived from the shadow prices arising in a completely centralized optimization. If the responses of players to these prices are identical to the actions centrally computed, this only confirms that the information on which the prices were based is correct. To call such a process decentralized is a charade. [2]

Perhaps the simplest example of a truly decentralized process is the familiar cobweb mechanism in which a demander and a supplier of a commodity exchange price and quantity messages in an effort to find that price-quantity pair where demand equals supply. Numerous variations suggest themselves. In one, the supplier communicates to the demander a price at which he will sell. The demander responds with a quantity he will buy at that price. The supplier comes back with the price he would ask for that quantity. *Etc., etc.,* until the responses are nearly stable *or* until patience is exhausted. Sometimes this process works (i.e., converges), sometimes it doesn't work (i.e., explodes). For situations where it does not work satisfactorily, it can sometimes be altered to work by "tuning": the demander may respond with a quantity that is only, say, half the distance from the old demand to the true demand at the new price. Or the process could be altered to a reversal of roles: the demander announces a price and the supplier responds with a quantity; where the one

[2] In a marvelous short paragraph Schweppe *et al* (1982) recognize this quandary, attempt to disarm the critic a bit, and drop the subject like a hot potato: "In the deregulated world...the generators are not centrally dispatched. Instead the Market Coordinator ... sends each generator a spot price and each generator self-dispatches itself by generating if the spot prices paid for electric energy exceeds the plant's marginal operating costs. A perceptive reader might say, 'Such generator self-dispatch and central utility dispatch are theoretically equivalent.' Such a reader would be right." The spot prices they refer to arise in a centrally computed example based on the totality of relevant information.

converges the other explodes. All of this is only to suggest that the variations in design and consequent performance of an adjustment process for a specific context are limited only by imagination.[3]

Granularity. "Dispatch of the many generators in a complicated power grid is much too delicate and little-understood an operation to be handled in a routinized decentralized fashion." "Leave it to the experienced dispatchers who know how to deal with minute-by-minute changes without the very costly results that inappropriate actions can bring about." Such frequently voiced opinions— more often than not those of experts— demand respect, but perhaps not capitulation.

With some little difficulty, a talented analyst could formulate demand and supply functions and a fast decentralized adjustment process that would produce a decision about whether, on a certain Tuesday after a rainstorm, northbound freight train A carrying cattle should enter a siding to allow passenger train B to pass unimpeded or vice-versa. If that is carrying theoretical decentralization to a ridiculous extreme, how far should one go?

The question can equally well be asked from the other side. Is complete centralization the only feasible organization? Of course not: an equally ridiculous extreme. The appropriate question is whether the concepts and the variables in a proposed decentralization scheme and its adjustment process fit into a plausible "coarsened" or aggregated model of the organization. To be specific: node voltages (or voltage phase angles) play a role in the adjustment process described in Section 6. Such messages sound perfectly appropriate for a literal (but practically ridiculous?) minute-by-minute adjustment process. Are they still appropriate for an hour-by-hour process? Day-by-day? Marginal generation costs and consumption values also play a role. Here, one guesses, the appropriateness changes in the opposite direction with aggregation.

---

[3] The cobweb example given is a price-quantity adjustment process, but except to economists there is nothing sacred about these particular forms of message. Wu and Varaiya (1995), for example, have proposed an adjustment process for a power grid which does not rely on price-quantity exchanges.

The model presented below merely accepts without question the version of appropriate "granularity" proposed by a number of other writers— Schweppe,*et al* (1988), Hogan (1992), and Wu and Varaiya (1995). It would be most interesting to see some specific adjustment-process proposals at both finer and coarser levels.

3. LOSSLESS NETWORKS, LOSSY NETWORKS, AC AND DC

A number of instructive papers initiating the study of coordination in decentralized power grids have for simplicity restricted their studies to lossless networks, using the lossless pseudo-DC model popular in many engineering papers. In such models, the only short-run costs of transmission are opportunity costs: the cost of transmitting one watt on a line with limited capacity is the value lost by the watt displaced. This cost is has come to be called, quite properly, congestion cost and its source is *thermal* capacity, which for a given line is taken to be a specific numerical ceiling on power transmitted. In a lossy network there is another form of congestion— my transmission of power on a line increases the power loss incurred by your transmission on that line. This is a *real* cost not an opportunity cost.[4]

The present analysis encompasses both of these types of congestion costs; they are different and must be dealt with differently. The generalization to this broader notion of congestion is, in our opinion, significant and new but nonetheless somewhat of a side issue. It must not obscure the paper's main concern with specifying and studying adjustment processes in detail— an investigation no less relevant in a lossless model.

In one way or another, discussions of the *capacity* and *economic employment* of an electric power grid always involve questions of power lost in transmission. Real power losses on a tranmission line manifest themselves in heat which at high levels may impair the line.

And, like the uncovered Central Valley tomato truck on a winding road, the shipment

[4] Comparison to vehicular traffic is instructive. Thermal congestion is analogous to competition for the limited seats on a scheduled airline flight. Power loss congestion is analogous to speed degradation on a crowded road.

dispatched at A headed to B doesn't all get there. The role in capacity and employment of the first feature of losses— heat— has has been studied in numerous papers.[5] The role of the second feature of losses— the "tomato feature"— has received much less attention than I think it merits, especially in the context of designing decentralized coordination mechanisms for the grid.

In the present paper I outline a decentralized coordination mechanism for a *lossy* power grid responding to requests for transmission of bilaterally contracted amounts of power. In the interest of conveying the central ideas most simply, I have sacrificed radically in terms of representing existing systems: *The grid is assumed to be a strictly-DC system.* Some apology is perhaps required. This assumption is not as restrictive as it might appear. Much of the arithmetic below carries through unchanged with complex variables, and for the most part the important economic problems remain essentially the same. The challenge remains to work out this extension with suitable reinterpretations of variables in terms of real and reactive power, voltages, currents, admittances, *etc.*

The system described envisages bilateral contracts between producers and consumers. A POOLCO or unilateral system in which producers sell to the grid and consumers buy from it at their respective node prices would be slightly different in detail.

Important matters not addressed here include contract enforcement, non-performance, contingency power generation, and all long-run matters such as investment in generation and grid capacity. But a better understanding of decentralization — the focus of this paper— will surely provide a better basis for study of many of these questions.

---

[5] Oddly— and ingeniously— most of these papers study the effects of losses in a *lossless(!)* system, making use of a pseudo-DC model. Are lines so fragile that losses imperceptible to buyers and sellers are large enough to cause damage?

4. OPTIMUM POWER DISPATCH

Oh, you press the first valve down
And the music goes down and around,
Oh-ho,ho-ho,ho-ho,
And it comes out here.
—Saxie Dowell, 1935

We first consider an n-node strictly-DC lossy network with admittance (i.e., conductance) matrix A and thermal capacity matrix C. Some of the nodes are producer-nodes and some are consumer-nodes, others are passive; their roles as such are given. Cost and value functions are assumed to be quadratic with given parameters. Write injections as positive numbers and ejections as negative numbers. Then a producer, say i, who injects total power x_i at node i incurs a total cost $r_i x_i + q_i x_i^2$. A consumer, say j, who "injects" total power x_j at node j enjoys a total value $-(r_j x_j + q_j x_j^2)$. The total social surplus S can be written as a function of a (possibly infeasible) vector of node power injections x:

$$S = -(r'x + x'Qx) \tag{1}$$

where $r = (r_1, .., r_n)', x = (x_1, .., x_n)'$ and Q is a diagonal matrix with $Q_{ii} \equiv q_i$.

A node injection vector x is said to be *feasible* if voltage constraints are satisfied and thermal capacities are not exceeded.

The voltage constraint demands that: There exists a vector $v = (v_1, ..v_n)'$ of non-negative-yet-not-too-large node voltages which give rise to x, that is that

$$0 \leq v_i \leq vtop \tag{2}$$

for $i = 1, .., n$ and for some fixed number $vtop$; and[6]

$$x = v \times Av. \tag{3}$$

[6] Here and elsewhere the "times" and "divides" signs will be used to indicate element-by-element operations on same-shaped arrays.

The thermal capacity constraint is satisfied if power losses on network links do not exceed thermal capacities defined by C:

$$(v_i - v_j)a_{ij}(v_i - v_j) \leq c_{ij} \tag{4}$$

on link ij for $i = 1, .., n$ and $j = 1, .., n$.

The Optimum Power Dispatch (OPD) problem can now be stated: Choose x and non-negative v to maximize (1) subject to constraints (2),(3), and (4) and $x_i \geq (\leq) 0$ if i is a producer (consumer) node.

In the next sections we reformulate this OPD problem to make it more numerically tractable and economically interesting.

5. THE LINK-IMPACT MATRIX

Let an origin-destination (O-D) current flow from node i to node j be denoted y_{ij}. Current flow on link ij will be denoted z_{ij}. The superposition theorem of circuit theory[7] enables us to calculate precisely the unit impact $m_{(ij),(kl)}$ of O-D current flow y_{kl} on the link current flow z_{ij}. The $n^2 \times n^2$ matrix M of these impact coefficients we shall call the *link-impact matrix*. M is a function solely of the network admittance matrix A and is constructed in the following fashion. Define I^h to be column h of the $n \times n$ identity matrix. Let matrix A^h be A with I^h replacing row h and column h. It then can be shown that

$$m_{(ij),(kl)} = g_k - g_l \tag{5}$$

where

$$g \equiv A_h^{-1} I^i \times a_{hi}.$$

Successive solutions of (5) finally yield the link-impact matrix M. Since M is highly redundant, it will suffice for our purposes to deal only with a submatrix (for short, also

[7] Desoer & Kuh, p. 658*ff*

called M) formed by eliminating all but those columns $m_{(..),(kl)}$ for which k is a producer and l is a consumer. (Passive nodes can play both roles.)

The link-impact matrix can now be employed to write link current flows as a function of O-D current flows:

$$My = z, \tag{6}$$

where y, z and a (below) are "ravelled" from matrices in the fashion

$$y = (y_{11}, .., y_{1n}, .., y_{n1}, .., y_{nn}).$$

Voltage constraint (3) can be rephrased in terms of link currents as follows: voltage drop on link ij must equal link current divided by link admittance: $v_j - v_i = z_{ij}/a_{ij}$. Using (6) we then get

$$My = a \times D'v \equiv Ev \tag{7}$$

where D is the $n \times n^2$ incidence matrix of the form (shown for the case $n = 3$):[8]

$$D = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix}.$$

The relation between node power injections and O–D currents can be written

$$x = v \times Dy \equiv By. \tag{8}$$

Finally, the thermal constraint (4) becomes

$$My \times D'v \leq c$$

or, more conveniently for our purposes,

$$My \leq c/D'v \equiv c^*. \tag{9}$$

[8] D is the transpose of what Schweppe *et al*, p. 323, call the "network incidence matrix before reduction".

We can now write the rephrased OPD problem: Choose x, y, and v to maximize (1) subject to the constraints

$$\begin{bmatrix} I & B & 0 \\ 0 & M & E \\ 0 & M & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ v \end{bmatrix} \begin{matrix} = \\ = \\ \leq \end{matrix} \begin{bmatrix} 0 \\ 0 \\ c^* \end{bmatrix} \tag{10}$$

with $0 \leq v \leq vtop$ and $y_{ij} \geq (=) 0$ if ij is (is not) a producer-consumer pair.

But for one serious complication (10) is a standard linearly constrained quadratic programming problem for which efficient solution routines exist. The difficulty is that the voltages v occur as constraint coefficients *and* as variables in the objective function. In the next sections we describe a procedure which attempts (successfully in most of the cases we have explored) to circumvent this trouble.

6. A PRICE/QUANTITY ADJUSTMENT PROCESS

The main objective of this paper is concerned with organization design. We wish to discover a decentralized adjustment process (a *tâtonnement*) which leads step-by-step to implementation of the solution of the OPD problem by means of "minimal" communications among the participants— producers, consumers, and the grid manager. Producers know costs, consumers know values, and the grid manager knows the network, the voltage constraints, and the link admittances and thermal limits. Ideally, we want this local information to stay local— to the extent consistent with a successful termination of the process.

Price/quantity exchanges have not been shown generally to be these desired "minimal" effective communications, but they are among the leading contenders for this title. More important for our purposes, they are such a familiar coordinating device that little rationale or indoctrination need be offered to the players in the game. Where the organizational context is sufficiently simple (i.e., decomposable), we (in western economies, at least) customarily employ price/quantity communications. Where it is too complex, we resort to more ad hoc, bureaucratic, and centrally controlled procedures. The question addressed in this paper is whether the efficient dispatching of a power grid is too complex for coordination by means of price/quantity exchanges. We argue that it is not. The analysis that follows suggests that a market organization, employing bi-lateral price/quantity communications and relying where possible on competition and private profit incentives can indeed bring about effective coordination in a power grid. The model examined, that of Section 1, is of course extremely simple and perhaps elements of complexity it ignores could affect the validity of our thesis. We would point out, however, that most of the studies that call for more centralized direction, less minimal communication, or more "multilateral" action, base their conclusions on a model no more comprehensive than the one employed here.

The context of the proposed adjustment process is a discretely changing *state of the world* (e.g., cost and value functions, network admittances, etc.). For each context some set of socially appropriate actions (e.g., producer–consumer contracts, prices, etc.) is to be computed and implemented. As in most *tâtonnement* models we will assume here that the context changes slowly enough that proper implementation can be achieved before it is obsolete. The computation is a recursive process which consists of a sequence of communications representing tentative actions. The sequence continues until the communications stop changing (approximately). Upon termination, actions are implemented.

The recursion is conveniently regarded as a series of *innings*, each with a first-half and a second-half, as in baseball. A first-half begins with a broadcast announcement from the grid manager to all the participants of tentative node voltages and O–D transmission *tolls*. Since a unit of power directed from node i to node j results in delivery of v_j/v_i units at node j, announcement of tentative node voltages informs producers and consumers about apparent real power losses in proposed contracts. The O–D tolls represent other less explicitly visible opportunity costs imposed on the system by the relevant O–D flows. The "outside players"— the producers and consumers— now form tentative bilateral contracts for point-to-point purchase and sale of power taking into account apparent real losses (as specified by the node voltages) and transmission tolls. We make the strong assumption that these first-half negotiations among buyers and sellers are carried to efficient completion— no apparent Pareto improvements remain.

The second-half begins with communications from the sellers to the grid manager of the

O–D power flows implicit in the proposed contracts. Using the old node voltages and the link-impact matrix, the grid manager translates the proposed O–D power flows into link O–D current flows and then into link current flows. These in turn, with the admittance matrix, yield new link-voltage drops, and hence new node voltages. Current flows also identify for the manager those links where thermal limits are violated, and hence give him a basis for construction of one— but only one!— component of the new vector of O–D tolls. The second component of tolls captures the sum of the invisible spillover costs imposed by each O–D flow on the power losses suffered by all O–D flows. Let $t_{ij} = t^c_{ij} + t^s_{ij}$ denote the unit toll— along with its two components— on O–D current flow from i to j.[9]

The manager computes the first component, approximating congestion shadow costs, as follows. He first finds the link current flows y implicit in the tentative O–D power flow proposals. He next determines the extent of thermal capacity violations. Using the appropriate column of the link-impact matrix, he then attributes shares to the responsible O–D current flows. Finally, he adjusts t^c from its old value:

$$t^c_{ij} \leftarrow t^c_{ij} + \alpha m'_{(..),(ij)}(My - c*). \tag{12}$$

The positive scalar adjustment multiplier $\alpha \leq 1$ is a parameter intended to be tuned for appropriate convergence of the process.

The manager computes the second toll component, representing spillover power losses, by the formula

$$t^s_{ij} = \sum_{kl} (m_{(kl),(ij)} y_{kl}/a_{kl}) MC_k \tag{13}$$

where MC_k denotes marginal cost at producer node k. The motivation for (12) is as follows. The power loss associated with power injected at node k and directed toward node l is $L_{kl} \equiv (v_k - v_l)y_{kl}$. The derivative of link voltage drop with respect to link current is $1/a_{kl}$ so we have $dL_{kl}/dz_{kl} = y_{kl}/a_{kl}$. The portion of this loss attributable to

[9] For purposes of analysis, if not practice, it is slightly more convenient to attach tolls to current rather than power because we don't have to specify which party in a lossy bilateral transaction is to pay the bill. The conversion to power tolls is of course straightforward.

O–D current flow y_{ij} is given by the link-impact coefficient $m_{(kl),(ij)}$. Multiplication by MC_k gives the dollar value. Finally, summing over producer-consumer links gives (13): the unit spillover toll to be assessed on O–D current flow from i to j.[10]

7. COMPUTATION, CONVERGENCE, AND OPTIMALITY

The description of the proposed adjustment process is now complete. How does it behave? Since we cannot prove convergence we resort to computation and examination of simple numerical examples.

Except in one respect, the earlier discussion specifically defines the steps to be followed in computing the path of the adjustment process. That exception involves the Pareto-optimal contracting activities of buyers and sellers in the first half of an inning. The end result of these activities in a given inning is the solution to the simple quadratic programming problem (Compare to the much more complicated Problem (10)): Choose x and y to maximize $-(r'x + x'Qx) - t'y$ subject to $Ix + By = 0$, where the matrix B, it will be remembered, is a function of the announced tentative voltages from the grid manager. The operations involved in computing the solution to this quadratic program are not intended to be interpreted as a simulation of the detailed contracting operations of buyers and sellers; only the end result is meant to represent the outcome of an efficient market.[11]

For any given example, the MATLAB program STEP1 runs the adjustment process through a single inning, resetting voltages and tolls for the next inning, and outputting tentative O-D flows, node power injections, and the social surplus S. Repeated applications of STEP1 allow us to investigate convergence. If convergence obtains, we can next inspect

---

[10] As it stands, (13) does not quite meet the limitations we have imposed on communication: according to the rules of the process nobody is supposed to inform the grid manager specifically about marginal costs. It should be possible, however, to infer relative marginal costs from other information in possession of the manager. This remains to be explored.

[11] How markets compute is well beyond the scope of this paper!

for optimality. The set of feasible node injections is probably convex,[12] so a local optimum is probably a global optimum.

In the simple examples presented next we find rapid convergence to optimal solutions for starting points not wildly distant from optimum. These interim results encourage us to believe that the proposed adjustment process may work generally in the desired fashion. Of course, without additional numerical analysis in the context of more realistic systems, not much can be said about speed of convergence. We would emphasize that the main purpose of these simple examples— of little intrinsic interest in themselves— is to confirm that the proposed adjustment process is behaving as it is intended to do. Such experimentation can give some guidance while we await a deeper mathematical understanding.

The following pages display the data and results for three very simple examples: EX34, EX34a, and EX34b. For each example, the first paragraph, "PARAMETERS" defines the example, and the second paragraph sets startup and tuner parameters. Then the outcomes of six or seven STEP1 iterations are displayed. The reader can trace through the entire sequence of message exchanges. In each case the final iteration is a near equilibrium as well as an optimum.

EX34 and EX34a are both triangular grids, identical in all respects except thermal capacity on link (1,3). None of the capacity constraints in EX34 are binding, and only (1,3) in EX34a. In EX34 we see (for the first time in the literature?) an example of substantial transmission tolls in the *absence* of any binding thermal constraint— in the absence, that is to say, of what is usually called *congestion*. Comparison of *surplus* in the two examples shows that the tighter constraint hurts, of course. Congestion tolls increase as expected with the new constraint, but spillover tolls diminish— perhaps less obviously. EX34b adds a more severe thermal capacity constraint to EX34a. The spillover tolls are reduced; the congestion tolls change significantly causing a shift of production from node 2 toward node 1. The (1,3) constraint is no longer binding.

---

[12] A property widely believed to hold but, so far as we know, not yet proved.

The adjustment process proposed and demonstrated here is only one of many that would be instructive to explore and compare. Readers of earlier drafts of this paper have already come up with a number of interesting variants, and new readers are invited to do the same. The design of such a process forces attention to matters that we have argued are important— locally held knowledge, privacy, communication and computation, speed of adjustment to new environments, and approximation to optimal actions. Further such design activity is crucial to the understanding of effective coordination in genuinely decentralized power systems.

8. CONCLUSIONS

Nodal spot prices derived from optimal power flow models do not a power market make— to borrow a phrase and sentiment from Joskow. Unless these prices are derived from and in turn influence local decisions they are merely financial cosmetics.

Price-quantity adjustment processes that honor the privacy of local information *can* serve to coordinate decisions in a power grid— but much remains to be learned about their behavior in various contexts. Other well-specified adjustment processes should be designed and studied.

Loss (or "spillover") congestion may be *the* most important contributor to short-run transmission cost— dominating the effect of the more widely recognized phenomenon of thermal congestion. And loss congestion, unlike thermal congestion, is *always* present. Properly constructed transmission tolls force each decision maker to take account of the impact of his actions on the losses of others.

REFERENCES

Desoer,C.A., and E.S.Kuh, *Basic Circuit Theory*, McGraw-Hill (1969)

Hogan, W. W., "Contract Networks for Electric Power Transmission", *Journal of Regulatory Economics* 4:211-242 (1992)

Schweppe,F.C., M.C.Caramanis, R.D.Tabors, and R.E.Bohn, *Spot Pricing of Electricity*, Kluwer (1988)

Wu, F., and P. Varaiya, “Coordinated Multitlateral Trades for Electric Power Networks: Theory and Implementation”, University of California Energy Institute Working Paper, PWP-031 (June 1995)

APPENDIX

Iterations of EX34, EX34a, and EX34b

Partial simulation results are presented here for the three examples described in Section 7. In each case the parameters for the example are given first, and then the results for the second, fourth, and sixth iterations. The last iteration is very close to equilibrium (and social optimum) in each case.

```
ex34
y =
   -27    15    12
    15   -23     8
    12     8   -20
c =
           0        1000        1000
        1000           0        1000
        1000        1000           0
v =
     0     0   100
H =
    0.0500   0.1500   0.1000
id =
    -1    -1     1
***************Iteration 2***************
SURPLUS =
  1.7246e+004
VOLTAGES =
    9.9929   10.0000    7.0921
NODEINJECTIONS =
  215.7065
  115.1085
 -172.2556
MARGINALCOSTS =
   10.7853   17.2663   82.7744
POWERFLOWS =
         0   13.8741  201.8325
  -14.0979         0  129.2064
 -105.6780  -66.5777         0
SPILLOVERTOLLS =
   14.1568   13.3688
CONGESTIONTOLLS =
     0     0
BOTHTOLLS =
   14.1568   13.3688
***************Iteration 4***************
SURPLUS =
  2.6030e+004
VOLTAGES =
   10.0000    9.6741    6.8198
NODEINJECTIONS =
  431.7124
  165.5722
 -408.1251
MARGINALCOSTS =
   21.5856   24.8358   59.1875
POWERFLOWS =
         0   53.3862  378.3262
  -51.8862         0  217.4585
```

```
 -256.4555 -151.6696         0
SPILLOVERTOLLS =
   18.6602   17.1114
CONGESTIONTOLLS =
     0     0
BOTHTOLLS =
   18.6602   17.1114
***************Iteration 6***************
SURPLUS =
  2.6048e+004
VOLTAGES =
   10.0000    9.6460    6.8420
NODEINJECTIONS =
  432.5295
  164.0719
 -412.0488
MARGINALCOSTS =
   21.6265   24.6108   58.7951
POWERFLOWS =
         0   53.8112  378.7184
  -51.9443         0  216.0162
 -259.0058 -153.0431         0
SPILLOVERTOLLS =
   18.5557   17.0501
CONGESTIONTOLLS =
     0     0
BOTHTOLLS =
   18.5557   17.0501
*******************************************
ex34a
y =
   -27    15    12
    15   -23     8
    12     8   -20
c =
           0        1000         100
        1000           0        1000
         100        1000           0
v =
     0     0   100
H =
    0.0500    0.1500    0.1000
id =
    -1    -1     1
***************Iteration 2***************
SURPLUS =
  2.0014e+004
VOLTAGES =
    9.8620   10.0000    7.3186
NODEINJECTIONS =
  292.2649
  137.1244
 -291.8220
MARGINALCOSTS =
   14.6132   20.5687   70.8178
POWERFLOWS =
         0   30.4179  261.8470
  -32.6057         0  169.7301
 -181.8533 -109.9687         0
SPILLOVERTOLLS =
   16.7960   15.0249
CONGESTIONTOLLS =
    5.9808    3.8467
BOTHTOLLS =
   22.7768   18.8716
```

```
***************Iteration 4***************
SURPLUS =
  2.5870e+004
VOLTAGES =
   10.0000    9.7488    7.0984
NODEINJECTIONS =
  401.8231
  162.0665
 -406.8870
MARGINALCOSTS =
   20.0912   24.3100   59.3113
POWERFLOWS =
         0   46.9094  354.9137
  -46.1646         0  208.2311
 -254.9138 -151.9733         0
SPILLOVERTOLLS =
   17.5420   16.0913
CONGESTIONTOLLS =
    2.5613    1.7134
BOTHTOLLS =
   20.1033   17.8047
***************Iteration 6***************
SURPLUS =
  2.5948e+004
VOLTAGES =
   10.0000    9.7176    7.1127
NODEINJECTIONS =
  390.2110
  160.0844
 -394.7787
MARGINALCOSTS =
   19.5105   24.0127   60.5221
POWERFLOWS =
         0   43.4496  346.7614
  -42.2700         0  202.3544
 -246.7615 -148.0173         0
SPILLOVERTOLLS =
   17.4890   16.1932
CONGESTIONTOLLS =
    2.9989    2.0126
BOTHTOLLS =
   20.4879   18.2058
*****************************************
ex34b
y =
   -27    15    12
    15   -23     8
    12     8   -20
c =
           0        1000         100
        1000           0          45
         100          45           0
v =
     0     0   100
H =
    0.0500    0.1500    0.1000
id =
    -1    -1     1
***************Iteration 2***************
SURPLUS =
  2.2055e+004
VOLTAGES =
   10.0000    9.8259    7.5017
NODEINJECTIONS =
  376.3742
   58.3791
```

```
 -319.8983
MARGINALCOSTS =
   18.8187    8.7569   68.0102
POWERFLOWS =
         0   88.8436  287.5305
  -93.5196         0  151.8987
 -212.9996 -106.8987         0
SPILLOVERTOLLS =
   17.1535   13.1421
CONGESTIONTOLLS =
    6.8451   12.5394
BOTHTOLLS =
   23.9986   25.6815
***************Iteration 4***************
SURPLUS =
  2.5624e+004
VOLTAGES =
   10.0000    9.5949    7.2252
NODEINJECTIONS =
  406.6206
  116.5247
 -384.2567
MARGINALCOSTS =
   20.3310   17.4787   61.5743
POWERFLOWS =
         0   68.4553  338.1653
  -66.2087         0  182.7334
 -246.5234 -137.7334         0
SPILLOVERTOLLS =
   17.3519   15.1558
CONGESTIONTOLLS =
    3.8913    7.3000
BOTHTOLLS =
   21.2432   22.4559
***************Iteration 6***************
SURPLUS =
  2.5604e+004
VOLTAGES =
   10.0000    9.5683    7.1966
NODEINJECTIONS =
  402.4508
  118.9373
 -379.3809
MARGINALCOSTS =
   20.1225   17.8406   62.0619
POWERFLOWS =
         0   65.4965  336.9543
  -62.7172         0  181.6545
 -242.7263 -136.6545         0
SPILLOVERTOLLS =
   17.4267   15.3809
CONGESTIONTOLLS =
    3.5968    6.7664
BOTHTOLLS =
   21.0235   22.1473

```