PWP-053r

Multi-Unit Auctions with Complementarities: Issues of Efficiency in Electricity Auctions

Wedad J. Elmaghraby

April 1998

This paper is part of the working papers series of the Program on Workable Energy Regulation (POWER). POWER is a program of the University of California Energy Institute, a multicampus research unit of the University of California, located on the Berkeley campus.

University of California Energy Institute
2539 Channing Way
Berkeley, California 94720-5180
www.ucei.berkeley.edu/ucei

Multi-Unit Auctions with Complementarities: Issues of Efficiency in Electricity Auctions

Wedad J. Elmaghraby*

4/2/98

Abstract

As auction based mechanisms for electricity dispatch are emerging in previously regulated electricity supply industries, it is imperative to understand the effect of auction rules and structure on efficiency. In this paper, I analyze the ability of different auction structures to induce the efficient dispatch in a framework where generators know their own and opponents' costs with certainty. In particular, I am interested in identifying which, if any, rules in an auction structure are necessary and sufficient to guarantee the efficient dispatch in equilibrium. I find that the auction mechanism chosen for California cannot guarantee productive efficiency in equilibrium. The main failing of the California auction design is the way in which demand is bundled and hence the way bids are defined. I show that, while an auction mechanism which allows for more than one winner in an auction cannot guarantee efficiency, an auction where there is exactly one winner can guarantee that all pure-strategy equilibria are efficient.

1 Introduction

Many governments are beginning to believe that it is in their and their constituents' best interest to deregulate their electricity supply industries. The goal of the regulators is to introduce competition

*I would like to thank Severin Borenstein, Jim Bushnell, Micheal Katz and my advisor Shmuel Oren for helpful suggestions and discussions. Financial support from the University of California Energy Institute is gratefully acknowledged. U.C. Berkeley, University of California Energy Institute, 2539 Channing Way, Berkeley, CA 94720-5180, email: wood@cimsim.ieor.berkeley.edu

into their electricity supply industries and create the appropriate medium through which electricity buyers and sellers can actively trade electricity, in the hope that such a competitive market will promote efficiency.

As of January 1, 1998, California will join the ranks of deregulated electricity supply industries. At the center of California's deregulation effort is an electricity auction mechanism, the Power Exchange (PX), whose role is to facilitate the matching of supply with all or part of their electricity demand. Auction-based mechanisms for electricity dispatch have already been implemented in the United Kingdom and Australia. Each of these governments has chosen an auction mechanism which in this paper is identified as a uniform price, vertical, simultaneous auction. The design of an electricity auction which induces an efficient use of generation resources is complicated by the fact that electricity demand, which fluctuates from hour to hour, must be satisfied by many suppliers with different costs, and that the generators lack the ability to store electricity in inventory. Determination of the optimal dispatch is a computationally difficult problem even in a centralized model with known generator costs. It is an even greater challenge to design an auction where generators voluntarily reveal cost information so as to be efficiently dispatched.

There are two forms of inefficiency which may arise from an ill-designed market; productive inefficiency and allocative inefficiency. Productive inefficiency implies an inefficient use of resources in the production of goods, while allocative inefficiency occurs when the goods are priced above marginal cost, leading to an inefficient consumption of the goods. The purpose of this paper is to analyze the performance of different auction structures in guaranteeing productive efficiency, i.e., minimizing generation costs. In particular, I am interested in identifying which, if any, rules in an auction structure are necessary and sufficient to guarantee productive efficiency in equilibrium. I focus only on the generation(supply) side of the market and assume that the demand for electricity is both deterministic and inelastic. This is done in order to gain a better understanding of the incentives provided by different auction structures and the ability of each auction structure to induce the efficient[1] allocation, or *efficient dispatch*. Under these assumptions, the final allocation is efficient if the generators chosen to supply electricity (i.e., win dispatch) minimize *total* generation costs.

[1]I will assume that there exists a unique efficient dispatch. This assumption does not in any way alter the results of the paper.

I find that the auction mechanism chosen for California cannot *guarantee* productive efficiency in equilibrium. The main failing of the California auction design is the way in which demand is bundled and hence the way bids are defined. I show that, while an auction mechanism which allows for more than one winner in an auction cannot guarantee efficiency, an auction where there is exactly one winner can guarantee that all pure-strategy equilibria are efficient.

In section 2, I provide the reader with some background literature on multi-unit auctions. I then go on to characterize an electricity auction as a multi-unit auction with private valuations which are possibly dependent over several units, and outline the different auction mechanisms considered and the cost characteristics of generators. In section 3, I argue that bundling demand into lots which allow for more than one winner precludes guaranteeing the efficient dispatch in equilibrium. In section 4, I present an auction mechanism that does guarantee efficiency in equilibrium.

2 Electricity Auctions

2.1 Auction Literature

In order to appreciate the new and interesting questions posed by an electricity auction, it is important to examine its place in the existing auction literature. The largest portion of auction literature looks at models where bidders desire at most one object (McAfee and McMillan (1987) provide an excellent survey of the auction literature). As I shall explain in more detail in the next section, an electricity auction is a multi-unit demand, private valuations auction where there may exist complementarites across units[2]. In this paper, the objects for auction are 1 MWhs of energy. As many researchers are pointing out, it is not possible to carry over the results from single unit auctions and apply them to a multi-unit auction. Hence, studying auctions for electricity forces us to leave the well-studied and understood world of single-unit auctions and explore the performance of auctions in a more diverse setting.

There are a few papers that study multi-unit auctions with complementarites which were in-

[2]Two objects are said to be complements (have superadditive value, or exhibit synergies) when their valuation independently is less then when combined, (i.e., if $v[\omega]$ is a bidder's value for object ω, and $v[\tau]$ is its value for object τ, then $v[\omega] + v[\tau] < v[\omega + \tau]$).

spired by the recent FCC spectrum auction. In the FCC auctions, bidders, comprised of US telecommunication companies, cellular telephone companies, and cable-television companies, competed to win various spectrum licenses for different geographical area. The synergies arising from owning licenses in adjoining geographical area create dependencies in (some) bidders' valuations for individual licenses (see McMillan (1994), Cramton (1995) and McMillan and McAfee (1996) for further discussion of the FCC spectrum auctions). Using the FCC spectrum auctions as their motivation, Krishna and Rosenthal (1996) study auctions where there are two types of bidders, global and local. Global bidders desire more than one object and their valuation for multiple objects is greater than the sum of each individual object's valuation, while local bidders desire at most one object. They are able to identify equilibrium bidding strategies when individual licenses are auctioned individually and simultaneously. They remark, however, that the equilibrium bidding strategies need not necessarily result in allocative efficiency. Ausubel and Cramton (1996) question the superior allocative efficiency properties of uniform pricing rules using Wilson's (1979) "share" auction framework with private valuations. They find that the efficiency of 2^{nd} price (uniform) auctions in a single-unit auction do not carry over to a multi-unit framework. They conclude that when bidders desire more than one object, or a large share of the total objects being auctioned, they have an incentive to underbid or "shade" their bids, resulting in an inefficient allocation. Levin (1997) searches for optimal auction design, i.e., what is the optimal way to auction goods when there exist complementarities between the goods. He finds, in the case of two goods, that bundling the goods and auctioning them together increases the revenue of the seller, but does not necessarily lead to the efficient allocation.

Several other papers have addressed the issue of multi-unit auctions. Colwell and Yavas (1994) examine the relative revenues raised in the simultaneous and sequential auctions of adjacent land tracts. Hausch (1986) studies a two object auction, where there are two bidders with common valuations who desire both objects. Hausch finds that the seller's revenue is greater when both objects are sold simultaneously versus sequentially. Gandal (1997), in an empirical paper, looks at the sequential auctioning of cable television licenses in Israel, and finds that there may have existed some interdependencies among licenses' valuation. Rothkopf et al. (1995) identify several different structures of combinatorial bids, in particular a nested tree structure, for which finding the set of bids that maximizes the seller's revenue and do not sell any object twice, can be solved in polynomial time. Krishna(1993) examines the efficiency properties of a sequential auction of capacity to an incumbent and several potential new entrants. She finds that the sequential timing of the auctions leads to the benefits of aggregation not being realized. Bikhchandani (1996) establishes the allocative efficiency properties of first[3] and second[4] price auctions when several heterogeneous objects are sold simultaneously (one auction for each type of object) and bidders may desire more than one object. The bidders' are assumed to have no budget constraints, private valuations for the objects that are common knowledge. Bikhchandani finds that, for a first price auction, an efficient pure-strategy Nash Equilibrium exists if and only if a Walrasian equilibrium exists.

Von der Fehr and Harbord's (1993) analysis of the United Kingdom's Electricity Industry is the only other study I know of that identifies an electricity auction as a multi-unit auction with private valuations and attempts to study the strategic bidding behavior of generators. Von der Fehr and Harbord assume a framework with two generators who have (different) constant marginal costs of generation and whose costs are common knowledge. Demand for electricity is uncertain but its distribution is known. They show that the less efficient (higher marginal cost) generator may submit lower bids than the more efficient generator, and hence generation costs may not be minimized in equilibrium. Building on their analysis, I incorporate the presence of fixed "start-up" costs into generation costs and extend their study of bidding behavior to alternative auction mechanisms, relaxing their restrictive assumption of two generators.

2.1.1 Multi-Unit Dependent Valuations Auction

What separates designing an auction for electricity from the vast body of auction literature is the structure of generation costs. Generators have different types of costs (e.g., ramp-up costs, no-load costs, etc.) which must be recovered through their sales revenues. Generation costs can be broadly classified into two groups: fixed "start-up" costs are incurred when a generator is turned on to generate, and variable costs are incurred with each additional MWh generated.[5] Due to this cost

[3] A first price auction represents a first-price sealed bid auction.

[4] A second price auction represents a second-price sealed bid auction.

[5] Through out this paper, start-up costs refer to the operating cost that must be incurred each time a generation plant is turned-on. Construction costs are ignored since I address the issue of relative efficeincy given a certain mix of generation plants is in place.

structure, there exist cost *dependencies* in both time and quantity dimensions, i.e., the (average) cost to generate 1 MW during hour t depends upon the number of additional MW generated during hour t and other hours.

If the object being auctioned is defined as 1 MWh of demand for energy, then demand can be interpreted as a collection of identical objects, distinguished only by the time at which they occur. Generators are sellers of electricity who may wish to win many objects (by winning an object they win the right to supply that 1 MWh of demand at a price determined through the auction process) and hence have *multi-unit demand*.

A generator's profit from "winning" a MWh is the difference between the auction price it is paid and its own *private* cost for supplying the MWh. A generator is constrained from winning all objects (and hence supplying the entire demand the following day) by the presence of capacity constraints on its generation level at any point in time (i.e., if a generator has a capacity of K, the maximum level of MW at which it can generate at any point in time is K MW). For most hours in the day, demand is greater than the capacity of any one generator; hence several generators must be chosen to supply demand.

Hence, an electricity auction is a multi-unit auction where there exist dependent private valuations for the units and "purchasing" capacity constraints.

2.2 Auction Structures

In designing an electricity auction, the auctioneer must decide on such auction dimensions as: how to bundle demand into lots for auction, what the pricing rule will be and what the sequencing of the auction will be. The United Kingdom, Australia and California have decided to bundle demand into vertical lots (explained below), to pay the same uniform price to every winning generator in a lot, and to have generators submit their bids for all lots simultaneously. As will be shown in section 3.1, this auction structure does not guarantee efficiency in equilibrium. In order to understand why this is true and how the auction might be modified to remedy this problem, I identify the different auction dimensions and the possible alternatives within each dimension.

Bundling of Demand into Lots In the case of electricity, the basic object to be auctioned is 1 MWh of the forecasted daily demand. When there are several objects to be auctioned, the auctioneer must decide how to bundle the objects into lots for auction. Should the bidders submit one bid that applies equally to all objects, or should the object be divided into distinct lots for which bidders submit a separate bid for each lot?

In the context of an electricity auction, there exist two "natural" bundling forms: horizontal and vertical.

Definition 1 *In a horizontal auction (see Figure 1), demand lots are formed by partitioning daily demand according to its duration, i.e., a distinct lot for each duration* t*. Hence, generators submit a supply curve for each lot, indicating the price at which they are willing to generate* k *megawatts for a* **duration** *of* t *hours, where* $k, t > 0$.[6]

Definition 2 *In a vertical auction (see Figure 2), daily demand is divided into* T *hourly demand lots, where each demand lot contains all the demand in hour* t, $t = 1...T$*. For each hour* t*, generators bid the prices at which they are willing to generate* k *megawatts* **during** *hour* t*, where* $k, t > 0$.

While there exist countless ways to bundle demand, the two bundling forms identified here are the most practical and logical to examine in an electricity auction setting. As stated earlier, the electricity auctions in operation in the United Kingdom, Australia and California are vertical auctions, where generators must bid their generation into hourly markets. However, the decision to operate a plant is not made on an hourly basis. The physical characteristics of most generation plants require planning its scheduling for a duration of time. Therefore, allowing generators to bid for durations of operation is, I will argue, a more natural way to design an electricity auction.

Sequencing of Auctions When there is more than one demand lot to be auctioned, the auctioneer must decide how to sequence their sale. In a simultaneous auction, the bids are submitted, and allocation decisions for all demand lots are made simultaneously. Alternatively, in a sequential auction, demand lots are auctioned sequentially; before each new auction, the results of any previous auctions are made known.

[6] An alternative way to define a horizontal auction is in terms of start-up and stopping times. I am grateful to an anonymous referee for pointing this out.

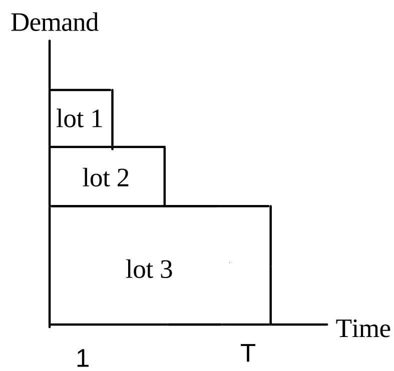

Figure 1: Horizontal auction

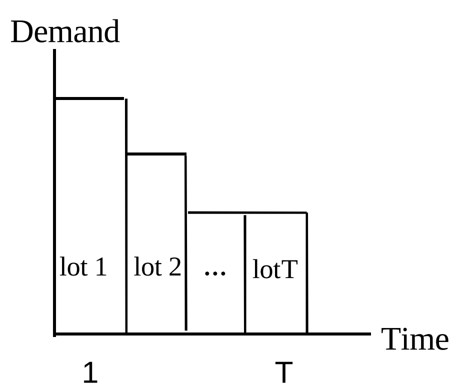

Figure 2: Vertical auction

Pricing Rule Before the bidders submit their bids, they must know how the prices at which the transactions take place are to be determined. If there is more than one winner per lot, each with different bids, in a lot, do the winners pay the same price, or different prices? The former pricing rule is called a *uniform pricing rule* and the latter a *discriminatory pricing rule*. Under a uniform pricing rule, all winners in a lot are paid the highest accepted bid price. [7] Under a discriminatory pricing rule, each winner is paid its own bid price. Both auction pricing rules implicitly determine the transaction price(s) using a 1^{st} price rule.

Figure 3 lists all the possible auction mechanisms given the dimensions and choices identified above.

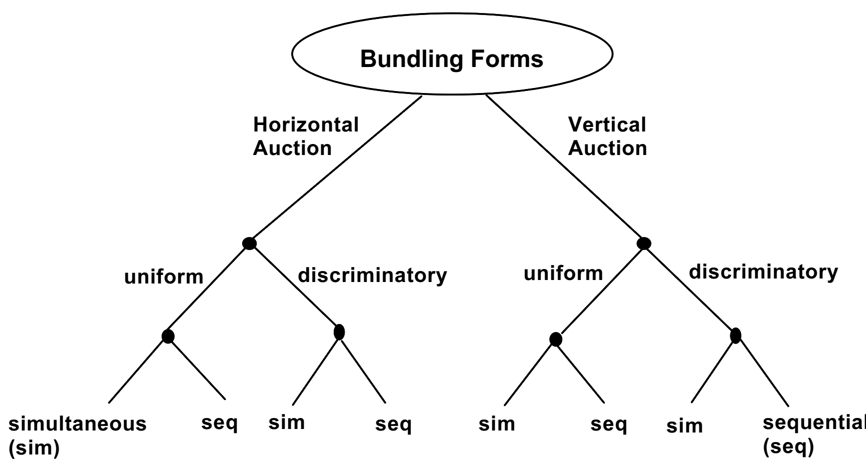

Figure 3: Possible auction mechanisms.

2.3 Model

In order to gain a better understanding of the incentives provided by different auction structures and the ability of each auction structure to induce the efficient dispatch, I assume that electricity demand is both deterministic and inelastic and that the auction is conducted only once.[8] For

[7]This is because an electricity auction is a procurement auction. In a procurement auction, the goal is solicit bids from suppliers for a service.

[8]While, in reality, the auction will be repeated daily, we can gain insight into some of the equilibrium strategies for the repeated game by examining its one-shot version.

simplicity, demand is always assumed to be a step function which is constant during each hour,[9] and is public information and hence known to all generators.

A generation company (referred to as a generator) typically owns several generating plants. Throughout this section I will assume that a generating plant can be one of n technology types. Unless otherwise stated, I will assume that each generator owns one generating plant. This assumption is without loss of generality for all of chapter 2, with the exception of section 2.3.1, where each generator must own at least two plants.[10] Assume that there exist M generators who each own one type i plant, denoted by G_{ij}, $i = 1...n$, $j = 1...M$.. Each plant has two costs associated with generation: a fixed "start-up" cost, f_i, and a variable cost per MWh, v_i, once the plant is up and running. The cost of generating a total of Q MWh in T hours from a type i plant is $C_i(Q) = f_i + Qv_i$,[11] where $Q = \sum_{t=1}^{T} q_t$ where q_t is the number of MW generated during hour t and $q_t \leq K$. The upper limit on the value of q_t, K, is a plant's capacity. The capacity constraint implies that a plant cannot supply more than K MW at any point in time, but places no restrictions on the duration for which it can generate. Assume that all generation plants have the same capacity constraint of K MW.

Figure 4 plots the stylized total costs of generation associated with different types of plant technology, assuming a generating plant is "switched on" only once (The horizontal axis measures the total number of MWh generated over time. Numerical examples of fixed and variable costs have been provided for convenience of the reader). While illustrative, this plot accurately captures the salient features of generation costs: Costs are such that each type i is the least-cost technology for some output level, in this case, i MWh. This is done so as to correctly reflect reality: most thermal generation plants are either nuclear, coal or gas-fired, and each fuel type is the most efficient source over some output range.

Assume that all cost and capacity information is publicly known.[12] The assumption of complete information may at first seem to be quite restrictive and unrealistic. However, in an industry such as

[9]Any daily demand can be approximated, to a 1^{st} order, by a step-function.

[10]The assumption that each generator owns one plant is without loss of generality since the identified inefficient equilibrium bidding strategies are supportable in a framework where generators own several, not necessarily identical, generation plants.

[11]A generator incurs no cost if it is not turned on to generate, i.e., $C_i(0) = 0$.

[12]Investor-Owned Utilities, which account for 77% of the total power generated in California (California Energy

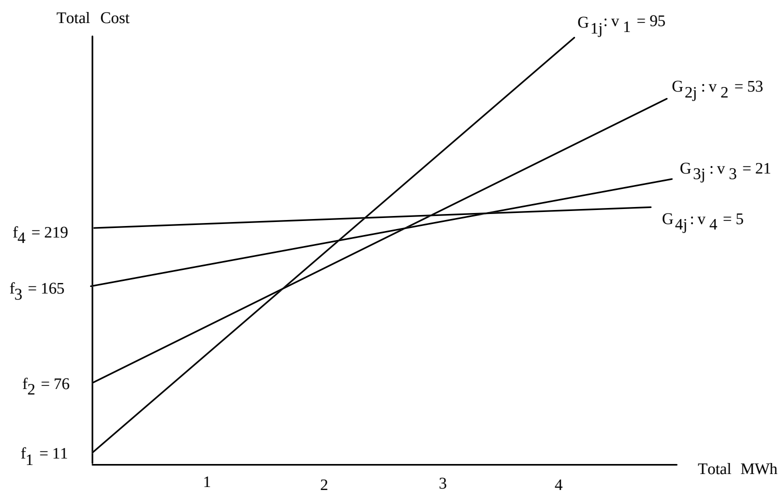

Figure 4: Total cost for plant technology types $n = 1, ..., 4$

the electricity supply industry, which has a long history of regulation (often with all the generation sources under government management as was the case in the United Kingdom before 1990), it is reasonable to assume that generators are aware of what types of plants their competitors own and what the costs associated with generation are. As a first attempt to understand the relative strengths and weaknesses of different auctions mechanisms, it is quite informative to examine generators' behavior in an environment with complete information.

When submitting bids, generators are restricted to submitting a single, uni-dimensional bid per demand lot per generating plant.[13] A generator's bid is binding once it has been submitted. A generator that is chosen for dispatch is committed to generate the specified load it won or incur a large penalty.[14] For a vertical auction, the bid indicates the minimum price, per MWh, to generate

Commission Report, November 1995, Table 6-2), have to annually submit a public report declaring their different generation plants, its associated costs, generation capacity, etc. This reporting requirement may change under the new deregulated regime.

[13] In reality, generators may own several units (gensets) and may submit a separate bid per unit. I shall to refer to a unit as a plant.

[14] The penalty is set high enough such that it is never in the interest of a generator to renege on its generation commitment.

1 MW during hour t; for a horizontal auction it indicates the minimum price to generate 1 MW for t hours. Generators are dispatched in increasing order of bids for a lot. A "winning" generator in a lot is dispatched at its full capacity or until demand in the lot is exhausted, i.e., a generator is not able to restrict its capacity availability.

3 Demand Lots with Many Winners

The question of interest to us is: In a complete information setting, does the proposed auction induce non-cooperative profit-maximizing generators to bid in a way that always results in an efficient dispatch in Nash equilibrium,[15] i.e., are all equilibria efficient for all demand scenarios? I find that the answer is negative. The demand bundling forms under consideration, horizontal and vertical, are such that there can be more than one winner in a lot. This, in turn, creates incentives for generators to not bid their true costs and can preclude attaining the efficient dispatch.

Theorem 1 *In a complete information setting with strictly concave generation costs, none of the auction mechanisms in figure 3 can guarantee the efficient dispatch in equilibrium.*

The proof to Theorem 1 is presented in this section via counterexamples. I address the failings of all the auction structures presented in figure 3 in sections 3.1 and 3.2. For each of the auction mechanisms in figure 3, I am able to find an instance of demand for which either 1) there exists an inefficient equilibrium or 2) the efficient dispatch is not supported in equilibrium. Hence, none of the auction mechanisms can guarantee efficiency in equilibrium. It is important to reiterate that the existence of an inefficient dispatch in equilibrium does not imply the absence of the efficient dispatch in equilibrium. Given the presence of both efficient and inefficient equilibrium dispatches, game theory does not allow us to conclude which one will occur. It is, therefore, the objective of this paper to establish the existence of inefficient equilibria, i.e., the potential for inefficiency in the proposed California auction design. While the examples in this paper will assume a cost function of the form "start-up" plus constant variable cost and this section assumes a setting with three

[15] Or a subgame perfect Nash Equilibrium in the case of a sequential auction.

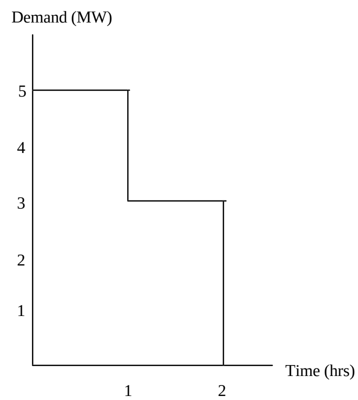

Figure 5: Daily demand

generators, the results hold for any strictly concave cost function and any number of generators (greater than 3) owning any number of plants.

Assume throughout section 3 that there are four technology types, i.e., $n = 4$, M generators of each type, and that each generator owns one plant with a capacity of $K = 2$ MW. In addition, suppose that forecasted demand is as in figure 5. This demand model, albeit a simple one, is rich enough to illustrate the failings of all the auctions in figure 3.

Given the assumed demand, cost, and capacity functions, the unique efficient dispatch is given in figure 6.[16] The efficient dispatch consists of the same type 4 generator (G_{4j}) supplying 2 MWh in both time periods, the same type 3 generator (G_{3j}) supplying 2 MWh in the 1^{st} period and 1 MWh in the second, and a type 1 generator (G_{1j}) supplying the top 1 MW. For expositional ease and without loss of generality, I will assume that the winning generators in the efficient dispatch are G_{11}, G_{31}, and G_{41}.

3.1 Vertical Auction

A vertical auction of the demand in figure 5 consists of the auction of two lots, as in figure 7. Each generator G_{ij} submits two bids, one for each lot k, defined to be b_{ij}^k (recall that each generator owns

[16]The efficient dispatch is always the same, regardless of the auction mechanism.

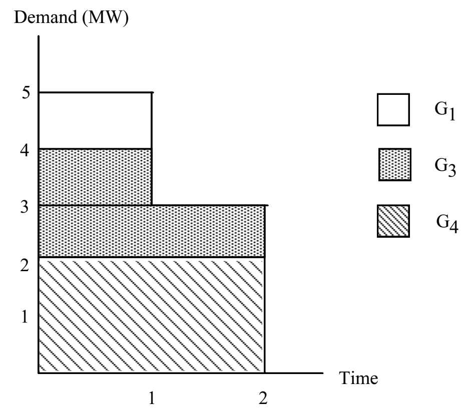

Figure 6: Efficient dispatch

only one plant and therefore only submits one bid per lot). A bid of b_{ij}^k is the minimum amount generator G_{ij} must be paid to generate 1 MWh in lot k. In order for the efficient dispatch to result from the submitted bids, G_{11}, G_{31}, and G_{41}[17] must submit the lowest bids in *lot 1* and G_{31} and G_{41} must submit the lowest bids in *lot 2*, regardless of the pricing rule or auction sequencing. In lots 1 and 2, the winning bids must be ordered as follows: $b_{41}^1 < b_{11}^1$, $b_{31}^1 < b_{11}^1$, and $b_{41}^2 < b_{31}^2$.

3.1.1 Vertical Uniform Auction

In a uniform price auction, all generators who "win" and are dispatched in a given demand lot are paid a uniform price equal to the highest accepted bid. I show in this section that when there is more than one winner per demand lot, a uniform pricing is unable to guarantee an efficient dispatch in equilibrium because: (1) The bid price is separated from received price for all except the marginally dispatched generators, (2) The same \$/MWh price is paid for each MWh generated in a demand lot, and (3) The total cost curve is strictly concave.

Given the demand in figure 5 and capacity assumptions, table 1 defines a set of equilibrium bids for generators G_{ij}, $i = 1...4$, $j = 1...M$, which constitute an *inefficient* dispatch for a vertical, uniform, simultaneous and a vertical, uniform, sequential auction (see figure 8 for the inefficient dispatch). The winning bids are followed by an asterisk (Note: Neither these bids nor dispatch

[17] The choice of generators G_{i1}, $i = 1, 3, 4$, as the lowest bidders is arbitrary and purely for expositional ease.

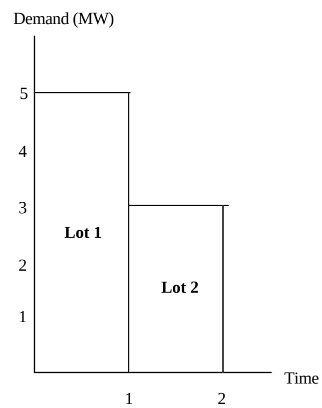

Figure 7: Vertical auction of demand

constitute the unique equilibrium for either auction. In particular, the efficient dispatch can also be supported in equilibrium). Define δ to be the smallest bid increment.

Given their opponents' strategies in table 1, no generator has an incentive to deviate from its bids. This profile of bids results in G_{21} being dispatched for 2 MWh in *lots 1* and *2*, G_{31} being dispatched for 2 MWh in *lot 1* and 1 MWh in *lot 2*, and G_{11} being dispatched for 1 MWh in *lot 1*. The clearing price received by all winning generators in *lot 1* is $f_1 + v_1$ per MWh and in *lot 2* is $v_1 - \delta$ per MWh. Since G_{21}'s payoff is not determined by her bid, she has every incentive to bid as low as possible to ensure dispatch. By submitting a bid of zero in lots 1 and 2, G_{21} is able to win dispatch at a positive profit. Although G_{41} is (one of) the least-cost producers of 4 MWh, he is unable to profitably undercut G_{21}'s bids of $b_{21}^1 = b_{21}^2 = 0$.

This simple example clearly illustrates why a vertical, uniform auction cannot guarantee efficiency in a multi-unit environment with strictly concave costs. If the demand in any hour t is not an integer multiple of K, then not all the generators will be dispatched at the same output level within that hour. In this scenario, there exists an opportunity for a relatively inefficient generator to accrue a positive profit by bidding zero and ensuring dispatch without fear of receiving its below-cost bid price. With the knowledge that in equilibrium the clearing price is guaranteed to be at least the cost of the marginal bidder in hour t, a relatively inefficient generator can "sneak-in" to the dispatch schedule by submitting a zero bid, get dispatched at a higher level in hour t than

Generator $(s = 2...M, y = 1...M)$	**Bid for Lot 1** $\left(\mathbf{b}_{ij}^{1}\right)$	**Bid for Lot 2** $\left(\mathbf{b}_{ij}^{2}\right)$
G_{11}	$f_1 + v_1^*$	v_1
G_{1s}	$f_1 + v_1 + \delta$	$f_1 + v_1$
G_{21}	0^*	0^*
G_{2s}	$f_2 + v_2$	$f_2 + v_2$
G_{31}	0^*	$v_1 - \delta^*$
G_{3s}	$f_3 + v_3$	$f_3 + v_3$
G_{4y}	$f_4 + v_4$	$f_4 + v_4$

Table 1: Inefficient dispatch in Vertical Uniform Auction

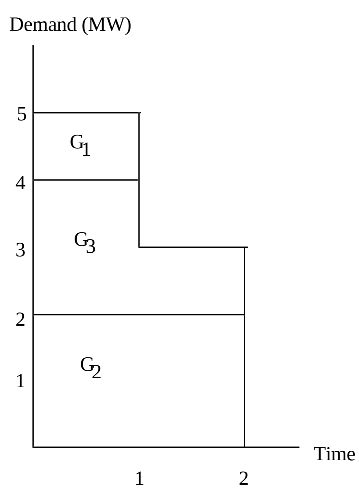

Figure 8: Inefficient equilibrium dispatch

the marginal price-setting bidder and accrue a positive profit due to the concavity of its cost curve. It is important to point out that this result also holds when (a) generators are allowed to submit a (finite) step bid function for a generating plant's entire capacity (see Appendix A for further discussion) and (b) when each generator owns several (not necessarily identical) generation plants.

This "zero" bid strategy creates a very similar effect to one identified by Back and Zender (1993) for the uniform auction of Treasury bills. Bidders are able to costlessly deter competitors from bidding more aggressively by submitting extremely steep demand curves. The low bids on inframarginal quantities have no chance of determining the clearing price, but act as a deterrent to competitors from bidding more aggressively.

This result bears directly on the auction mechanisms chosen in the UK, Australia and California. All three auctions are designed so as to have generators submit hourly (or half-hourly) bids: The winners in each time period are paid the highest accepted price. This is the structure of a vertical, uniform auction and hence we should not expect the auctions to be providing generators with the correct incentives so as to result in the efficient dispatch. It is important to emphasize that this result does not say that the auctions currently operating in the UK, Australia, and California will yield inefficient dispatches; only that there exists the potential for inefficiency to be supported in an equilibrium.[18]

3.1.2 Vertical Discriminatory Auctions

While a uniform-price vertical auction of the demand in figure 5 fails to guarantee efficiency in equilibrium because of the existence of inefficient dispatches in equilibrium, a discriminatory-price vertical auction fails because it cannot support the efficient dispatch in equilibrium. The following exposition is true for both a vertical, discriminatory, sequential and a vertical, discriminatory, simultaneous auction.

In a discriminatory-price auction, each generator chosen for dispatch receives its own bid. For the efficient dispatch to occur in equilibrium in a vertical auction, (as stated earlier) the three lowest

[18]In actuality, the auctions are conducted on a daily basis. The repeated nature of the auctions implies that the set of supportable equilibria is larger than that for the static auction (known from the Folk Theorem). However, the inefficient equilibria identified here will also be supportable in a repeated setting.

bids in *lot 1* must be from a type 4,3, and 1 generator (with the type 1 generator having the third lowest bid) and the two lowest bids in *lot 2* must be from a type 4 and 3 generator (with the type 3 generator having the second lowest bid). As there are many identical generators of type 1, in an efficient equilibrium, G_{11} must be earning zero profits. If G_{11} were earning a positive profit, then any generator G_{1j}, $j \neq 1$, would have the incentive to undercut b_{11}^1 and replace G_{11} as the third lowest bidder in *lot 1*. Therefore we know that, in an efficient equilibrium, G_{11} will bid $b_{11}^1 = f_1 + v_1$. Since each generator is paid what it bids, and each generator wishes to maximize its profits, in an efficient equilibrium, G_{31} and G_{41} will bid $b_{31}^1 = b_{41}^1 = f_1 + v_1 - \delta$, and all other generators must submit bids for *lot 1* greater than $f_1 + v_1$. At a bid of $b_{31}^1 = f_1 + v_1 - \delta$, G_{31} is dispatched at 2 MWh in *lot 1* and earns $2b_{31}^1 > f_3 + 2v_3$. There are, however, many other type 3 generators who are not being dispatched and are earning zero profits. Therefore, any G_{3j} , $j = 2...M$, has the incentive to undercut b_{31}^1 and replace G_{31}'s position in the bid ordering. But then we have just shown that the bids (and bid ordering) necessary to support an efficient dispatch are not equilibrium bidding strategies. Therefore, the efficient dispatch cannot be supported in a discriminatory-price, vertical, simultaneous or sequential auction.

The structure of vertical demand lots creates the need for more than one winner per lot, which in turn creates a barrier to guaranteeing efficiency. A generator's dispatch depends upon its bids' placement within in a lot. In this example, the efficient dispatch requires that different types of generators, with different cost structures, win within a lot. This fact, coupled with the winning generators' desire to maximize their profits, creates a situation where G_{31} and G_{41} bid above their costs. But in the presence of other identical generators, it cannot be an equilibrium for G_{31} and G_{41} to bid above their cost without other generators undercutting their bids. [Note: The inability to support the efficient dispatch in equilibrium exists even when generators own several generation plants.]

Given the concavity of total costs, it is not possible to design a vertical auction with unidimensional bids[19] which is able to provide generators with the proper incentives so as to guarantee the

[19] I have found that, unlike with a uniform pricing rule, an auction that has a discriminatory pricing rule in combination with a multi-part energy bid that fully captures the generators' cost structure is able to support the efficient dispatch in equilibrium. However, allowing for multi-part energy bids poses the combinatorial optimization problem for the auctioneer of identifying the least-cost manner of satisfying demand in each hour. In these simple

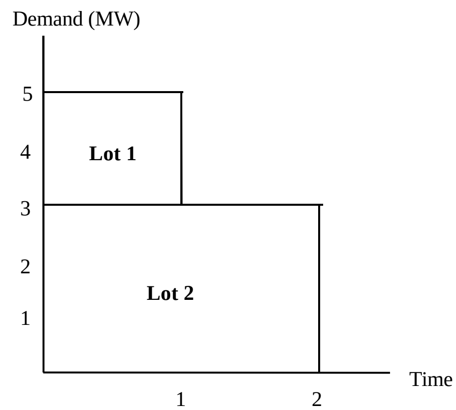

Figure 9: Horizontal auction of demand

efficient dispatch in equilibrium.

3.2 Horizontal Auction

A horizontal auction of the demand in figure 5 consists of auctioning two lots as in figure 9. Each generator G_{ij} submits two bids, one for each lot k, defined to be b_{ij}^{k} (recall that each generator owns only one plant and therefore only submits one bid per lot). The lot number corresponds, in this case, to the width of the slice. A bid of b_{ij}^{k} is the minimum amount generator G_{ij} must be paid to generator 1 MW for k time periods. In order for the efficient dispatch to result from the submitted bids, G_{31} and G_{41} must submit the lowest bids in *lot 2* and G_{31} and G_{41} must submit the lowest bids in *lot 1*, regardless of the pricing rule or auction sequencing. In lots 1 and 2, the winning bids must be ordered as follows: $b_{41}^{2} < b_{31}^{2}$ and $b_{31}^{1} < b_{11}^{1}$.

3.2.1 Horizontal Uniform Auction

The inefficiencies associated with a uniform price are not limited to vertical auctions. In a horizontal auction, demand is divided into lots by duration. There is no reason to believe that the number

examples, the generator could only generate at two possible output levels, while in reality the possible output levels are much larger.

Generators	**Bid for Lot 2**	**Bid for Lot 1**
$(\mathbf{s = 2...M, y = 1...M})$	$\left(\mathbf{b}_{ij}^{2}\right)$	$\left(\mathbf{b}_{ij}^{1}\right)$
G_{1y}	$f_1 + 2v_1$	$f_1 + v_1^*$
G_{21}	0^*	$f_2 + v_2$
G_{2s}	$f_2 + 2v_2$	$f_2 + v_2$
G_{31}	$f_3 + 3v_3 - (f_1 + v_1)^*$	0^*
G_{3s}	$f_3 + 3v_3 - (f_1 + v_1) + \delta$	$f_3 + v_3$
G_{4y}	$f_4 + 2v_4$	$f_4 + v_4$

Table 2: Inefficient dispatch in a Horizontal Uniform Auction

of MW of actual demand with duration t will be a multiple of the generators' capacity, i.e., it is possible to have more than one generator win dispatch in a given demand lot at different output levels. In such a scenario, a strategy similar to the one used in a vertical uniform auction can lead to an inefficient dispatch being supported in equilibrium, i.e., a relatively inefficient generator can ensure its dispatch by submitting a low-bid, while reaping a positive profit from the marginal price.[20] This section presents an example of an inefficient equilibrium in a uniform, horizontal auction. The strategies presented constitute an inefficient equilibrium for both a sequential and a simultaneous uniform, horizontal auction.

Given the demand in figure 5 and capacity assumptions, table 2 defines a set of equilibrium bids for generators G_{ij}, $i = 1...4$, $j = 1...M$, which constitute an *inefficient* dispatch in a horizontal, uniform, simultaneous and a horizontal, uniform, sequential auction (see figure 8 for the inefficient dispatch). The winning bids are followed by an asterisk. Note: the type 1 winner, G_{1y}, will be chosen randomly.

Given her opponents' bids for *lot 2*, G_{21} has the incentive to bid low so as to ensure dispatch. By submitting a bid of $b_{21}^2 = 0$, G_{21} wins a dispatch of 2 MW for 2 hours and wards off her opponents' from undercutting her bid. If any other generator, in particular the efficient generators G_{4j}, were to match G_2's bid and submit a zero bid for *lot 2*, the clearing price would be zero and the dispatched generators would generate at prices below their costs. Since the demand in *lot 2* is greater than the

[20] This strategy would not be profitable in equilibrium if all winning generators were dispatched the same amount in each demand lot.

capacity of G_{21}, G_{31} also wins 1 MW dispatch and sets the clearing price at $f_3 + 3v_3 - (f_1 + v_1)$. Due to the height of *lot 2* of 3 MW, G_{21} is dispatched at twice G_{31}'s level and receives twice the payment. Hence, the uniformity of bids and unequal dispatch of generators in a demand lot combined with the concavity of its cost function allow G_{21} to recoup its generation costs with a bid of zero and inefficient dispatch to be supported in equilibrium. As with a vertical uniform auction, this result also hold when generators are allowed to submit a (finite) step bid function for a generating plant's entire capacity and when generators own several generation plants.

3.2.2 Horizontal Discriminatory Auction

Given the demand in figure 5, in order for the efficient dispatch to occur from the submitted bids in a horizontal discriminatory auction, the lowest and second lowest bid in *lot 2* must be from G_{41} and G_{31}, respectively, and G_{31} and G_{11} must bid lower than all other generators in the auction for *lot 1*. As was shown in the case of a vertical, discriminatory auction, the need for more than one type of generator to win within a lot combined with generators' profit maximizing behavior bars the efficient dispatch from being attainable in equilibrium. The following exposition is true for both a horizontal, discriminatory, sequential and a horizontal, discriminatory, simultaneous auction.

The presence of several identical type 1 generators implies that in equilibrium the winning generator G_{11} must accrue zero profit. If he were to bid above his cost of $f_1 + v_1$ in *lot 1,* one of his opponents of the same technology would have an incentive to undercut his bid. Similar reasoning can be used to argue that a profit maximizing G_{31} would bid $f_1 + v_1 - \delta$ for *lot* 1. In an efficient equilibrium dispatch, G_{31}'s total profit must equal zero and hence he must bid $b_{31}^2 = f_3 + 3v_3 - (f_1 + v_1 - \delta)$ for *lot* 2. For the efficient dispatch, the bids in *lot 2* must be such that b_{41}^2 is below b_{31}^2 and all other generators (including all other type 4 generators) bid above b_{31}^2. Profit maximization will imply that $b_{41}^2 = f_3 + 3v_3 - (f_1 + v_1 - \delta) - \delta$: At such a bid, G_{41} is dispatched for 4 MWh and earns a positive profit. But then it is not an optimal response for all other type 4 generators to bid above b_{31}^2, not be dispatched and earn zero profit. Instead, they have the incentive to undercut b_{41}^2 and replace G_{41} as the lowest bidder. Hence, as was the case with the vertical, discriminatory auction, the efficient dispatch cannot be supported in equilibrium.[21],[22]

Thus it has been shown that none of the auctions in figure 3 can *guarantee* efficiency in equilibrium. While this section illustrated the failings of each auction design using examples that assumed three generators with identical capacities and one plant each, the results extend to a more general setting with $n \geq 3$ generators with non-identical capacities and diversified plant portfolios.

4 Demand Lots with One Winner

In section 3, I established the inability of auction mechanisms that bundle demand into horizontal and vertical lots and require unidimensional bids to guarantee efficiency in equilibrium in the presence of strictly concave generation costs. The main flaw with horizontal and vertical lots is the possibility of there being more than one winner per lot. Therefore, it is appropriate to search for an alternative bundling form which assures only one winner per lot. In an electricity auction, where demand is larger than the capacity of any one generator, it is not possible to achieve only one winner per lot when demand is partitioned vertically. However, it is possible with horizontal partitioning.

Definition 3 *In a* α**- horizontal auction***, demand lots are formed by partitioning daily demand into horizontal strips of height* α *MWs. Generators submit a bid for each lot, indicating the price at which they are willing to generate* α *megawatts for a* **duration** *of* t *hours, where* t *is the length of the strip.*

By partitioning demand into thin horizontal strips, whose height of α MW, is less than or equal to the capacity of generators, it is possible to limit the number of winners per lot to one. Figure 10 illustrates an $\alpha - horizontal$ auction with $\alpha = 1$. (For the remainder of this section, I will focus my discussion on a $1 - horizontal$ auction.) A $1 - horizontal$ auction is by definition, a discriminatory auction since there is only one winner per lot. There is still the possibility of conducting the auction

[21] If, however, the generators were allowed to submit a two-part that reflected the structure of their generation cost, the efficient dispatch can be supported in equilibrium.

[22] This result generalizes to the scenario when generators own several plants.

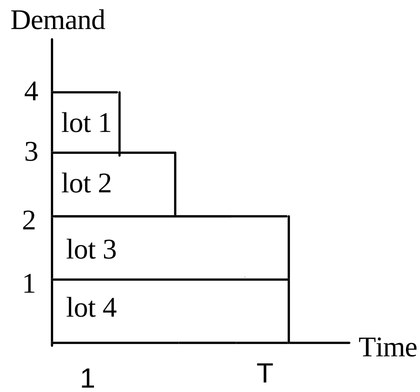

Figure 10: $1 - horizontal$ auction of demand

of the lots simultaneously or sequentially. I find that while the existence of only one winner per lot is necessary to guarantee efficiency, it is not sufficient. The final characteristic upon which efficiency depends is the sequencing of an auction. When bids are made sequentially and all plants of a particular technology type are owned by a few generators, a $1 - horizontal$ auction cannot guarantee efficiency. However, when all the bids are submitted simultaneously, the unique Nash Equilibrium dispatch is the efficient dispatch.

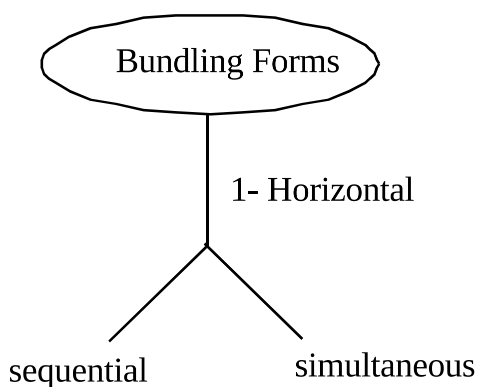

Figure 11: Alternative bundling forms and auction mechanisms

Section 4.1 contains an example of a sequential auction of demand in which the set of equilibrium dispatches contains an inefficient dispatch. Section 4.2 concludes with a proof stating that the unique equilibrium dispatch in a simultaneous *1 − horizontal* auction is the efficient dispatch.

4.1 1-Horizontal Sequential Auction

A *1 − horizontal* sequential auction cannot guarantee efficiency in equilibrium if the ownership of a particular technology type is concentrated in the hands of a few generators. In particular, if each of a small number of technology types has a single owner, a relatively inefficient generator is able to strategically bid so as to squeeze out an efficient competitor. The ability to support inefficient dispatches in equilibrium, however, critically rests on the distribution of technology types amongst generators. If plants of the same technology types are owned by many[23] generators, then all equilibria are efficient.

Theorem 2 *In a complete information framework with a concentrated distribution of generation resources, a 1-horizontal sequential auction cannot guarantee efficiency in equilibrium.*

Proof. Theorem 2 is proven by counterexample. Suppose that there are three generators, G_1, G_2, and G_3 who each own two identical generating plants, denoted by g_i, with $K = 1$ each (for convenience, we have dropped the second subscript since $M = 1$). The generation costs associated with plant g_i is given in figure 12, $i = 1, 2, 3$. Suppose the generation costs are such that $f_2 + 3v_2 - (f_3 + 3v_3) < f_3 + 2v_3 - (f_1 + 2v_1)$.

Assume that there exists a daily demand given by figure 13, which is to be auctioned via a *1 − horizontal*, sequential auction. The longest duration lot, *lot* 3, is auctioned first, followed by *lot* 2 and then *lot* 1. Figure 13 also depicts the unique efficient dispatch. Despite the simple structure of demand, it is possible to support an inefficient dispatch in equilibrium, in particular the dispatch given in figure 14.

The equilibrium strategies supporting this inefficient dispatch are given in Appendix B. I briefly summarize here the strategies used and incentives behind them. Recall that each generator owns

[23] Where "many" implies that there is at least one more generator of each type than demand for which it is the most efficient technology. That is, if in the efficient dispatch, x plants of technology type i are dispatched, "many" implies that there are at least $x + 1$ plants of type i, owned by more than one generator, participating in the auction .

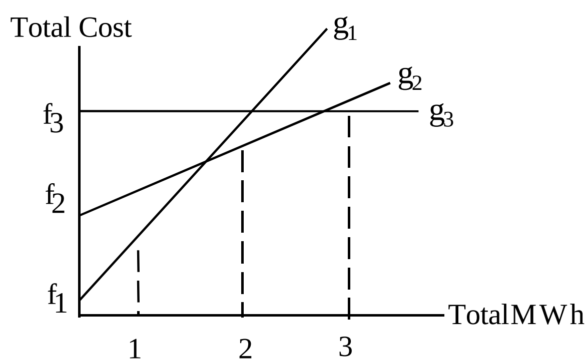

Figure 12: Generation costs for generation plant g_i, owned by generator G_i, for $i = 1, 2, 3$

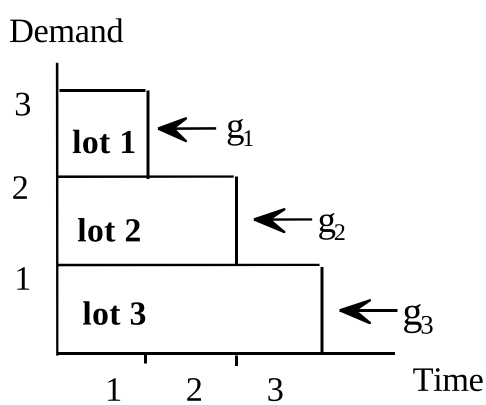

Figure 13: The unique efficient dispatch for a sequential $1 - horizontal$ auction

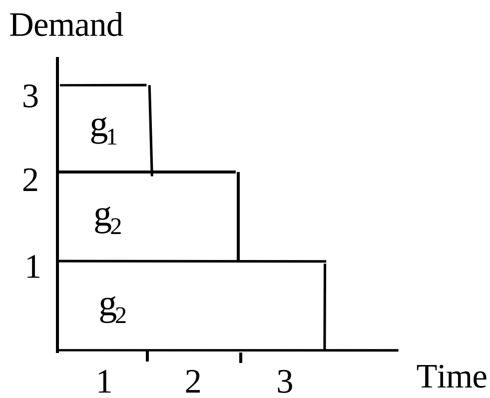

Figure 14: Inefficient dispatch for a sequential $1 - horizontal$ auction

two identical plants with $K = 1$. The dispatch in figure 14 is a result of equilibrium bids where G_2 submits a bid of $f_2 + 3v_2 - \delta$ for *lot* 3 for one of its g_2 plants. Such a bid is below both her and G_3's cost to supply 3 MWh and allows her to win *lot* 3. G_2 knows that by winning *lot 3*, she is committing herself to participate in only one of the two remaining auctions (due to capacity constraints). If all three generators were to participate in every auction, the upper bound on G_2's winning bid for *lot 2* is $f_1 + 2v_1 - \delta$, and the upper bound on G_1's winning bid for *lot 1* is $f_2 + v_2 - \delta$. Given G_2 has won *lot* 3, it is to G_1's advantage that G_2 win *lot* 2 and hence be removed from participating in the auction for *lot* 1. With G_2 no longer participating in the auction, G_1 is able to win *lot* 1 at a bid of $f_3 + v_3 - \delta$. Therefore, G_1's optimal response is to not undercut G_2's bid for *lot 2* and allow G_2 to win *lot 2* with a bid of $f_3 + 2v_3 - \delta$. Hence, G_2 is able to undercut G_3's lower costs for *lot 3* with the knowledge that it is in G_1's best interest to allow her to win *lot 2* with a large profit margin. ■

Bundling demand so that there is one winner per lot did not remove the incentives for relatively inefficient generators to bid below cost nor prevent the resulting inefficient dispatch. It is the ability to change the upper bounds on winning bids via strategic interactions that allows an inefficient dispatch to be supported in equilibrium. As a consequence of the sequential nature of the auction and the lack of competition from identical generators, G_1 knows that, given G_2 wins both *lot* 3 and *lot* 2, the upper bound on a winning bid for *lot* 1 is raised from $f_2 + v_2$ to $f_3 + v_3$. Similarly, the sole reason winning *lot 3* is part of an overall profitable strategy for G_2 is that G_2 is able to change the upper bound on her winning bid for *lot* 2 from $f_1 + v_1 - \delta$ to $f_3 + v_3 - \delta$. If, instead, there were to exist several generators identical to G_1 or G_2, then this strategy is no longer optimal in equilibrium.

For example, if instead the market structure satisfied the following assumptions:

Assumption 1 *No one generator owns all the plants of a specific technology type, and*

Assumption 2 *There exists at least one extra plant of each technology type participating in the auction, than is used in the efficient dispatch,*

then G_2 would be unable to win *lot 2* at any price higher than δ above its cost, $f_2 + 2v_2$. For if she were to bid above her cost for *lot 2*, the identical generator would have the incentive to undercut its bid, and G_2 would win only *lot 3* at a loss. A similar argument can be made to show that, if there were to exist another generator identical to G_1, the upper bound on G_1's winning bid for *lot 1* remains $f_1 + v_1$, regardless of the outcomes of the two previous auctions. G_1 will undercut any bid for *lot 2* that is above its own cost of $f_1 + 2v_1$. Therefore, when her competitors own plants similar to her own, it does not behoove G_2 (or any other generator) to bid below her cost for any plant, and the ability to support inefficient dispatches in equilibrium disappears.

Theorem 3 *In a complete information framework where Assumptions 1 and 2 are satisfied, all pure-strategy Nash Equilibria in a sequential 1-horizontal auction are efficient.*

Proof. The proof to Theorem 3 is identical to that of Theorem 4 and will be presented in the next section.

4.2 1-Horizontal Simultaneous Auctions

This section concludes our search for an auction which guarantees efficiency in equilibrium.

Theorem 4 *In a complete information framework, all pure-strategy Nash Equilibria in a simultaneous* $1-$ *horizontal auction are efficient.*

Proof. In order to prove Theorem 4 it must be shown that (Step 1) there do not exist any inefficient equilibrium dispatches and (Step 2) the efficient dispatch can be supported in equilibrium. To accomplish Step 1, no assumptions need to be made regarding market structure (i.e., the number of generators,[24] their plant portfolio mix and plant capacity). However, in order to identify equilibrium bidding strategies, it is necessary to make some assumptions with regards to market structure; that is, the equilibrium bidding strategies will change with variations in market structure.

(Step 1) Suppose that there are Ω lots for auction[25] in a 1-horizontal auction and P plants are participating in the auction. Let $D^* = (D_1, ..., D_\Omega)$ denote the efficient dispatch,[26] the components of which indicate which plant wins lot ω in the efficient dispatch. The total cost associated with the efficient dispatch is denoted by $C(D^*)$. Suppose that there *does* exist an inefficient dispatch, $\hat{D}$, in equilibrium.[27] Let $\hat{B} = \left(\hat{B}_1, ..., \hat{B}_\Omega\right)$ denote the winning bids and $C(\hat{D})$ to be the total generation costs associated with the inefficient dispatch $\hat{D}$. In equilibrium, each generator must be making a non-negative profit. Given that each generator is paid its bid and that all bids are submitted simultaneously, no generator has an incentive to bid a plant below its cost.[28] Summing over these constraints for the generators in dispatch $\hat{D}$ gives us,

$$C(\hat{D}) \leq \sum_{i=1}^{\Omega} \hat{B}_i \tag{1}$$

To constitute an equilibrium, for dispatch $\hat{D}$ and its associated bids , no generator must have an incentive to change its bid given its opponents' bids. An inefficient dispatch $\hat{D}$ can be one of two types; either (a) at least one of the plants in D^* is not dispatched in $\hat{D}$ or (b) all the plants in D^* are in $\hat{D}$, but their dispatch is inefficient.

If $\hat{D}$'s inefficiency is of type (a), then we know that for it to be an equilibrium, it must be unprofitable for the plants in D^* to undercut the bids in dispatch $\hat{D}$. Summing these constraints

[24] I do assume, as throughout this thesis, that attaining the efficient dispatch is feasible with the present generation mix.

[25] I do not assume that Ω is a multiple of plant capacity size.

[26] The efficient dispatch is not unique, since any two identical plants can switch schedules and the dispatch remain optimal. For expositional ease, I will refer to *a* particular efficient dispatch D^*.

[27] That is, the types of plants dispatched and their schedules in $\hat{D}$ are not the same as in D^*.

[28] See discussion in Appendix C.

for the plants in D^* yields,

$$\sum_{i=1}^{\Omega} \hat{B}_i < C(D^*) \qquad (2)$$

Combining equations (1) and (2) yields,

$$C(\hat{D}) < C(D^*)$$

which contradicts the assumption that D^* is the efficient (least-cost) dispatch.

If $\hat{D}$'s inefficiency if of type (b), a similar argument can be constructed to show that $\hat{D}$ cannot be an equilibrium. To illustrate this using a simple example, suppose that two plants, v and w win lots x and y in D^*, respectively. $\hat{D}$ is identical to D^* with the exception that plant x wins lot w and plant y wins lot v. For $\hat{D}$ to be an inefficient dispatch, lots v and w must not be of the same duration. Without loss of generality, suppose that lot v is of a shorter duration than lot w. Define $C_i(z)$ to be plant i's cost of supplying lot z. $\hat{D}$ is an equilibrium if each plant's profit is greater in dispatch $\hat{D}$ than D^*. That is,

$$\left[\hat{B}_v - C_y(v)\right] - \left[\hat{B}_w - C_y(w)\right] \geq 0$$

and

$$\left[\hat{B}_w - C_x(w)\right] - \left[\hat{B}_v - C_x(v)\right] \geq 0$$

which implies[29]

$$\underbrace{C_y(w) - C_y(v)}_{+} \leq \hat{B}_v - \hat{B}_w \leq \underbrace{C_x(v) - C_x(w)}_{-}$$

yielding a contradiction.

(Step 2) In proving Step 1, no assumptions were made regarding the number of generators, their generation plants' portfolios, or plant capacities. However, identifying the equilibrium bids that support an efficient dispatch critically depends upon these market structure dimensions. However, all efficient equilibrium bids have the following underlying structure: The bids of plants in D^* reflect the opportunity cost of their most efficient competitors. Outlined below are the bidding strategies that yield an efficient equilibrium.

[29] Since we know that $C_x(v) < C_y(v) < C_y(w) < C_x(w)$.

For plants $p = 1, ..., P$ and lots $l = 1, ..., \Omega$

if $D_l^* = p$ then plant p bids $\frac{\min_{i=1,\ldots,P;\, i\neq p}\left[C_i\left(MW(D_l^*)\right)+OC(i)\right]}{MW(D_l^*)} \times (T(l))$ for lot l,

else plant p bids $\frac{C_p(MW(D_l^*))+OC(p)}{MW(D_l^*)} \times (T(l)) + \delta$ for lot l,

where D_l^* is plant dispatched for lot l in the efficient dispatch, the $C_p(q)$ is plant p's total cost to generate q MWhs, $MW(p)$ is the total number of MWhs generated by plant p in D^*, $OC(p)$ is plant p's opportunity cost in the efficient dispatch D^*, and $T(l)$ is the total number of MWhs in lot l. Given the bidding strategies outlined above, no plant has the incentive to change its bid and hence they constitute an equilibrium. ■

For example, if we consider a market structure characterized by Assumptions (1) and (2), the following bidding strategies constitute an efficient equilibrium.[30]

For plants $p = 1, ..., P$ and lots $l = 1, ..., \Omega$

if $D_l^* = p$ then plant p bids $\frac{C_p(MW(D_l^*))}{MW(D_l^*)} \times (T(l))$ for lot l,

else plant p bids $\frac{C_p(MW(D_l^*))}{MW(D_l^*)} \times (T(l)) + \delta$ for lot l.

Note that the presence of "extra" generation resources (from Assumption (2)) limits the winning plants' bids to covering operating costs only. The numerator in the bidding equation is the cost of the plant. The opportunity cost of the "extra" plant is zero in the efficient dispatch (since it is not dispatched in D^* and therefore foregoes nothing by undercutting any bid above its costs) and hence the plant's earn zero profit.

The existence of many plants that are never dispatched is not a sustainable market structure. More than likely, these "extra" plants will be used to either meet unexpected changes in demand[31] or will be deemed unprofitable and be retired from operation. In either scenario, the equilibrium

[30] As a result of various regulatory programs, many states in the United States have entered into the deregulation process with an overcapitalized ESI. Except for in isolated geographical areas where, at certain times in the day due to transmission constraints there are capacity shortages, most states in the United States currently have ample generation resources to meet their local demand.

[31] Recall that the electricity auction is a day-ahead auction and hence there still exists some uncertainty with regards to actual demand levels.

bidding strategies of plants in D^* will reflect the positive opportunity cost of it competitors[32], and (some) plants will no longer be bid into the auction at cost.

The intuition behind the success of a sequential *1-horizontal* auction with diversified ownership of plant types or simultaneous *1-horizontal* auction is the following: Under discriminatory pricing, having a simultaneous auction or many generators that share the same plant types creates a strict upper bound on winning bids. When a generator is paid its bid and there is no possibility of changing the upper bound on a winning bid through strategic interaction, it will never operate one of its plants at a negative profit: A generator whose total bids for a plant are less than the plant's generation costs will always be better off by withdrawing its bid(s) and earning zero profit.

There exist alternative bundling forms which limit the number of winner per lot to one. For example, the auctioneer can auction each individual 1 MWh energy block separately, rather than an entire strip of energy demand, i.e., 1 MW over t hours. In this case, the unique equilibrium dispatch is efficient (and the proof to Theorem 4 remains the same). However, the auctioneer will be increasing the complexity of the auction (by increasing the number of auctions held) without gaining any additional efficiency benefits.

5 Conclusion

As auction based mechanisms for electricity dispatch are emerging in previously-regulated electricity supply industries, it is imperative to understand the effect of auction rules and structure on efficiency. This paper addresses exactly this relationship by asking which auction structures are sufficient to guarantee that demand is satisfied in a least-cost manner. What makes this an interesting and challenging question is the existence of electricity industry-specific characteristics such as the existence of start-up costs and the inability to store electricity, with its implications for the existence of cost dependencies in supplying MWh over both time and quantity dimensions and the desire of generators to supply several MWh of demand.

[32] See Appendix D for an example of equilibrium bidding strategies in a market structure where all but one plant are dispatched in D^*. Under the assumption of completely inelastic demand, it is necessary to have at least one unutilized plant; otherwise the resulting bids would be ∞.

Using a complete information framework, I found that the current California auction design has the potential to support inefficient dispatching in equilibrium. The structuring of the auction is such that there are ways for relatively inefficient generators to game the auction. Any auction design that allows for more than one winner per demand lot, as California's does, in an environment with strictly concave costs will not be able to guarantee efficiency in equilibrium. In addition to the short-run inefficiencies that arise from inefficient dispatching, there is the additional concern that the wrong investment signals will come out of the auction. This result is pertinent not only to California, but also to regulators in several other states who are currently contemplating deregulating their respective electricity supply industries. Alternative auction designs that better lend themselves to the particular cost structure of electricity generation should be explored. I have identified such an auction structure, a *1-horizontal* auction. By restricting there to be only one winner per demand lot, a *1-horizontal* auction guarantees efficiency in equilibrium.

Appendix A

It might be thought that the restricted bid structure, i.e., one in which generators are restricted to submitting one bid per unit per demand lot, is an additional culprit to inefficiency in uniform-price auctions. Since the average cost of generating 1 GW depends upon the total number of GW generated, restricting generators to submit one bid (per generating plant) regardless of quantity should open the door for inefficiencies to arise in equilibrium.

Alternatively, allowing generators to submit a bid which is contingent on quantity should help reduce the existence of inefficient equilibria. To test this hypothesis, I analyze the same uniform, vertical, simultaneous auction presented in sections 3.1, but change the bid structure such that generators are allowed to submit step supply functions. Similar examples for the other auction mechanisms in figure 3 can easily be constructed.

Two-part bid in a Uniform, Vertical, Simultaneous Auction

Assume the same framework of generators and demand as in section 3.1, and that the generators costs are given by figure 4. The bid structure is changed such that a generator is allowed to submit a two-part bid in each hour. Since each generator can only generate in increments of 1 MW and has a capacity of $K = 2$, a 2-part bid can sufficiently capture and reflect its cost structure. Generators submit a two-part bid which is of the form

$$\begin{Bmatrix} \text{minimum price to generate 1 MW during hour t} \\ \text{minimum price to generate 2 MW during hour t} \end{Bmatrix}$$

for each hour $t = 1, 2$. I found that the additional flexibility of a quantity-specific bid did not eliminate inefficient dispatches from the set of Nash Equilibria. For example, the following bid strategies support the same inefficient dispatch found in section 3.1 and constitute a Nash Equilibrium bidding strategy (asterisks follow winning bids):

Given the bids in table 3, the least-cost dispatch is to accept G_{21}'s bids for 2 MW during both hours, G_{31}'s bid for 2 MWh in hour 1 and 1 MWh in hour 2, and G_{1y}'s bid for 1 MW during hour 1, for some $y \in [1, M]$. This sets the clearing prices paid per MW in hours 1 and 2 to be $f_1 + v_1$ and $v_1 - \delta$, respectively.[33] At these clearing prices, G_{21} and G_{31} earns a positive profit and G_1 earn a zero profit. G_{21} and G_{31} have successfully submitted sufficiently low bids so as to make it impossible for any other generator to profitably undercut them in either time hour for either quantity level.

Not only does a richer bid structure not preempt inefficient dispatching in equilibrium, in addition it poses the combinatorial optimization problem for the auctioneer of identifying the least-cost manner of satisfying demand in each hour. In these simple examples, the generator could only generate at two possible output levels, while in reality the possible output levels are much larger.

Appendix B

The following subgame perfect Nash equilibrium result in an inefficient dispatch in equilibrium for a discriminatory, horizontal, sequential auction. Assume that all generators submit the same bids for both of their plants unless otherwise specified. Once a particular plant has won a lot, it can no longer participate in later auctions.

G_1's strategy:

[33] Note that G_{1y}'s bid for 1 MW is the highest accepted bid in hour 1 and G_{31}'s bid for 1 MW is the highest accepted bid for hour 2.

Generator ($s = 2...M$, $y = 1...M$)	**Bids for hour1**	**Bids for hour 2**
G_{1y}	$\binom{f_1+v_1*}{f_1+2v_1}$	$\binom{v_1}{2v_1}$
G_{21}	$\binom{0}{0*}$	$\binom{0}{0*}$
G_{2s}	$\binom{f_2+v_2}{f_2+2v_2}$	$\binom{f_2+v_2}{f_2+2v_2}$
G_{31}	$\binom{0}{0*}$	$\binom{v_1-\delta*}{2v_1-\delta}$
G_{3s}	$\binom{f_3+v_3}{f_3+2v_3}$	$\binom{f_3+v_3}{f_3+2v_3}$
G_{4y}	$\binom{f_4+v_4}{f_4+2v_4}$	$\binom{f_4+v_4}{f_4+2v_4}$

Table 3: Inefficient equilibrium 2-part bidding strategies

Bid for $f_3 + 3v_3 + \delta$ for *lot 3*

- If I win *lot 3*, bid $f_3 + 2v_3 - \delta$ for *lot 2*
  - If I win *lot 2*, bid ∞ for *lot 1*
  - If 2 wins *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*
  - If 3 wins *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*
- If 2 wins *lot 3*, bid $f_3 + 2v_3 + \delta$ for *lot 2*
  - If I win *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*
  - If 2 wins *lot 2*, bid $f_3 + v_3 - \delta$ for *lot 1*
  - If 3 wins *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*
- If 3 wins *lot 3*, bid $f_1 + 2v_1$ for *lot 2*
  - If I win *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*
  - If 2 wins *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*
  - If 3 wins *lot 2*, bid $f_2 + v_2 - \delta$ for *lot 1*

G_2's strategy:

Bid for $f_3 + 3v_3 - \delta$ for *lot 3*

If 1 wins *lot 3*, bid $f_3 + 2v_3 + \delta$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3 - \delta$ for *lot 1*

If I win *lot 2*, bid $f_2 + v_2$ for *lot 1*

If 3 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If I win *lot 3*, bid $f_3 + 2v_3 - \delta$ for *lot 2*

If 1 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If I win *lot 2*, bid ∞ for *lot 1*

If 3 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If 3 wins *lot 3*, bid $f_3 + 2v_3 - \delta$ for *lot 2*

If 1 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

If I win *lot 2*, bid $f_2 + v_2$ for *lot 1*

If 3 wins *lot 2*, bid $f_2 + v_2$ for *lot 1*

G_3's strategy:

Bid for $f_3 + 3v_3$ for *lot 3*

If 1 wins *lot 3*, bid $f_3 + 2v_3$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 3*, bid $f_3 + 2v_3$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 3*, bid $f_3 + 2v_3$ for *lot 2*

If 1 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If 2 wins *lot 2*, bid $f_3 + v_3$ for *lot 1*

If I win *lot 2*, bid $f_3 + v_3$ for *lot 1*

The result of these strategies is that G_2 wins *lot 3* and *lot 2*, and G_1 wins *lot 1*.

Appendix C

Under discriminatory pricing, having a simultaneous auction or a surplus of identical generators creates a strict upper bound on winning bids in a 1-*horizontal* auction. This in turn implies that, in equilibrium, generators will never dispatch a plant at a negative profit. It is only beneficial for a generator to bid one of its plants into the dispatch at a negative profit, if it believes that it can recover those losses by increasing its winning bid(s) on another plant. I argue that having a simultaneous auction or a surplus of identical generators implies that there is no credible way to change the upper bound on a winning bid.

The intuition behind this can easily be understood in the case where there are a surplus of identical generators. The presence of "too many" generators of the same type naturally creates an upper bound on the bids a generator may successfully place. In particular, the generator must bid its true cost to generate; otherwise, an identical generator that is not currently being dispatched has the incentive to undercut its opponent's bid and replace it in the dispatch schedule.

With a simultaneous auction, even in the extreme case where each generator is the sole owner of a particular technology type,[34] the simultaneity of the bid submission implies that there is no credible way to change the upper bounds on a winning bid, and therefore a generator will never find it advantageous to bid a plant below its total cost. The intuition behind this statement can easily be understood using a three *lot* auction.

Suppose that there are three *lots*, A, B, and C that are auctioned simultaneously and that there are $n > 2$ generators, G_i for $i = 1...n$, participating in the auctions. Suppose that in the efficient

[34] Recall, this was the exact market structure used when identifying an inefficient dispatch for a sequential, discriminatory, 1-horizontal auction.

dispatch G_1 wins *lots* A and B and G_2 wins *lot* C. Suppose that G_1 must turn on a new plant for each *lot*, i.e., there are no cost dependencies across *lots*. We know (from Step (2) of Theorem 4) that there exists an efficient equilibrium where all generators are earning a non-negative profit from each plant. Define this equilibrium to be E^*. I argue that there does not exist an alternative efficient equilibrium where at least one of the generators is operating a plant at a negative profit.

A generator would only bid one of its plants into dispatch below cost if a) it would not be dispatched otherwise *and* b) doing so results in increasing the upper bound on the winning bids for one of its other plants. Suppose that G_1 is considering bidding below her plant's generation costs for *lot* A. The only reason G_1 would find such a bidding strategy desirable is if doing so allowed her to submit a higher winning bid (due to a change in the upper bound) for *lot* B, and that this action would result in a higher profit overall than in dispatch E^*.

In order for the upper bound on G_1's winning bid for *lot* B to change, it must be that G_2's bid is defining that upper bound (that is, the upper bound must be defined by a generator that is currently dispatched). In addition, G_2 must experience an increase in the upper bound on its winning bid for *lot* C as a direct result of the change in G_1's change in bid for *lot* A. The only way the upper bound on the winning bid for *lot* C would change is if it is G_1's bid that is defining the upper bound.[35]

Suppose we are in the state where G_1 is winning *lot* A with a bid below plant cost and *lot* B at a bid price higher than in equilibrium E^*, and that G_2 is winning *lot* C at a bid price higher than in equilibrium E^*. Given the simultaneous nature of bid submission, it is not an equilibrium for G_1 to bid a plant below cost for *lot* A. Instead, G_1 has the incentive to undercut G_2's bid for *lot* C and withdraw its below-cost bid for *lot* A. Therefore, it is never profitable (and not sustainable in equilibrium) to bid below a plant's cost in a simultaneous *1-horizontal* (discriminatory) auction.

Appendix D

In this section, I illustrate an efficient equilibrium in the extreme market structure where all plants, save one, are dispatched. Suppose that there are four generators participating in the market, each

[35] Again, the upper bound for *lot* C must be defined by a generator that is currently dispatched.

of which owns one plant with a capacity of $K = 1$. Figure 15 plots $C_i(q)$, that is, generator i's cost to generate a total of q MWhs.

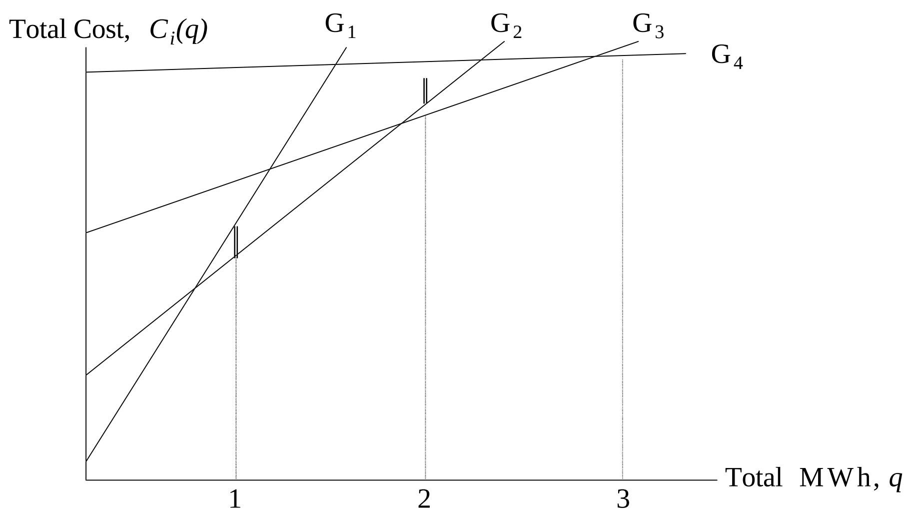

Figure 15: Generation costs for generators G_i, $i = 1, .., 4$.

Suppose that daily demand is as given in figure 16. The efficient dispatch, D^*, is also provided in figure 16.

Generator	**Bid for Lot 1**	**Bid for Lot 2**
G_1	$C_1(1) + \delta$	$C_1(2)$
G_2	$C_1(1)$	$C_2(2) + C_1(1) - C_2(1) + \delta$
G_3	$C_3(1)$	$C_2(2) + C_1(1) - C_2(1)$
G_4	$C_4(1)$	$C_4(2)$

Table 4: Generator bids for lots 1 and 2 in a 1-horizontal auction

Since G_1 is not scheduled to generate in D^*, he has an opportunity cost of zero associated with winning lots $1, 2$, or 3; that is, G_1 will have the incentive to undercut any bid that is above its operating cost for supplying the demand in the lot. G_1 is also the next most efficient generator for lot 1, and therefore defines the upper bound on G_2's winning bid for lot 1.

$$\text{Upper bound on } G_2\text{'s winning bid} = C_1(1)$$

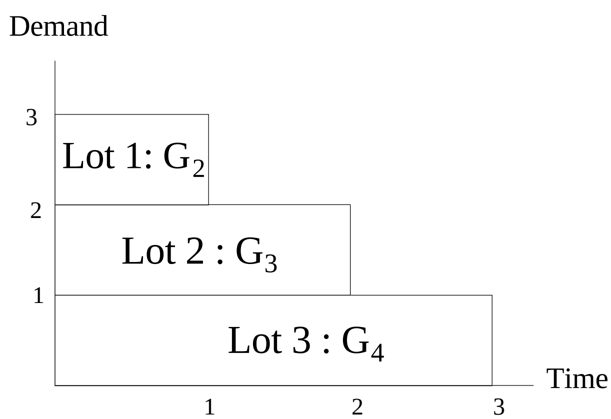

Figure 16: Daily demand, partitioned into three lots for a $1 - horizontal$ auction.

Generator	**Bid for Lot 3**
G_1	$C_1(3)$
G_2	$C_2(3)$
G_3	$C_3(3) + C_2(2) + C_1(1) - C_2(1) - C_3(2) + \delta$
G_4	$C_3(3) + C_2(2) + C_1(1) - C_2(1) - C_3(2)$

Table 5: Generator bids for lot 3 in a 1-horizontal auction

G_2, in turn, is the next most efficient generator for lot 2. However, G_2 does have a positive opportunity cost associated with winning lot 2; equal to $C_1(1) - C_2(1)$ (which is equal to her profit from winning lot 1 at a bid equal to G_1's cost minus her own operating costs) . Therefore, G_3's winning bid for lot 2 is constrained by G_2's operating cost *plus* her opportunity cost (in figure 15, the parallel lines denote G_2's positive opportunity cost).

$$\text{Upper bound on } G_3\text{'s winning bid} = C_2(2) + \underbrace{C_1(1) - C_2(1)}_{G_2\text{'s opportunity cost}}$$

Finally, G_3 is the next most efficient generator for lot 3 and also has a positive opportunity cost associated with winning lot 3. As a result, G_4's winning bid is constrained by G_3's cost plus his opportunity cost.

$$\text{Upper bound on } G_4\text{'s winning bid} = C_3(3) + \underbrace{C_2(2) + C_1(1) - C_2(1) - C_3(2)}_{G_3\text{'s opportunity cost}}$$

Note that, unlike the case where Assumptions (1) and (2) are satisfied, the generators submit winning bids above their operating costs.

References

Ausubel, L., and P. Cramton, 1996, "Demand Reduction and Inefficiency in Multi-unit Auctions," Working Paper No. 96-07, University of Maryland

Back, K., and J. Zender, 1993, "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment," *The Review of Financial Studies,* No. 4, pp. 733-764

Bikhchandani, S., 1996, "Auctions of Heterogeneous Objects," mimeo, The Anderson School, University of California at Los Angeles

Colwell, P. and A. Yavas, 1994, "The Demand for Agricultural Land and Strategic Bidding in Auctions." *Journal of Real Estate Finance and Economics*, Vol. 8, pp. 137-149

Cramton, P., 1995, "Money Out of Thin Air: The Nationwide Narrowband PCS Auction," *Journal of Economic& Management Strategy,* Vol. 4, No. 2, pp. 267-343

Gandal, N., 1997, "Sequential Auctions of Interdependent Objects: Israeli Cable Television Licenses," *The Journal of Industrial Economics*, Vol. 45, No. 3, pp. 227-243

Hausch, D., 1986, "Multi-Object Auctions: Sequential vs. Simultaneous Sales," *Management Science*, Vol.32, No.12, pp.1599-1612

Krishna, K., 1993, "Auctions with Endogenous Valuations: The Persistence of Monopoly Revisited," *The American Economic Review*, Vol. 83, No. 1,pp. 147-160

Krishna, V., and R. Rosenthal, 1996, "Simultaneous Auctions with Synergies," *Games and Economic Behavior*, 17, pp. 1-31

Levin, J., 1997, "An Optimal Auction for Complements," *Games and Economic Behavior*, Vol. 18, pp. 176-192

Maskin, E., and J. Riley, 1989, "Optimal Multi-unit Auctions," in Frank Hahn (ed) The Economics of Missing Markets, Information, and Games, Oxford: Oxford University Press, pp. 312-335

McAfee, P., and J. McMillan, 1987, "Auctions and Bidding," *Journal of Economic Literature*, Vol. 15, pp. 699-738

McAfee, P., and J. McMillan, 1996, "Analyzing the Airwaves Auction," *Journal of Economic Perspectives*, Vol. 10, No. 1, pp. 159-175

McMillan, J., 1994, "Selling Spectrum Rights," *Journal of Economic Perspectives,* Vol. 8, No. 3, pp. 145-162

Rothkopf, M., A. Pekec, and R. Harstad, 1995, "Computationally Manageable Combinatorial Auctions," Working Paper, Rutgers University

Von der Fehr, N. and D. Harbord, 1993, "Spot Market Competition in the UK Electricity Industry," *The Economic Journal,* 103, pp. 531-546

Wilson, R., 1979, "Auctions of Shares," *The Quarterly Journal of Economics,* Nov., pp. 675-689