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Power Auctions and Intertemporal Production Costs: Dealing with Unit Commitment

Bart McGuire

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University of California Energy Institute
2539 Channing Way
Berkeley, California 94720-5180
www.ucei.berkeley.edu/ucei

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Bart McGuire
University of California Energy Institute
and
Graduate School of Public Policy UCB

Abstract

How should generators ("units") with different energy costs, different minimum and maximum levels of operation, and different start-up costs be assigned to satisfy a time profile of electricity demand? Two very simple models of this "unit commitment problem" are studied— both in the context of uncertainty about future demand. The preliminary objective of the analysis is to find optimal commitment strategies for the manager of a system of generators. The main objective is to suggest models of power-generation technology sufficiently simple to make possible the exploration of efficient power-auction design. Small steps toward both goals are described.

POWER AUCTIONS AND INTERTEMPORAL PRODUCTION COSTS:
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Bart McGuire
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How should generators with different energy costs and start-up costs be assigned to satisfy a time profile of electricity demand? Problems of this sort have come to be known in the power engineering literature as "unit commitment problems", and many variations have been studied. This large literature is characterized by progressively more detailed and realistic assumptions and correspondingly more complex solution algorithms.[1] The scheduling of units (*i.e.,* generators) may be constrained by minimum up- and down-times, start-up costs may be time-dependent, ramp rates may be constrained, demand may be treated as given or uncertain and it may include reactive power and spinning reserve requirements, *etc.* Algorithms typically call for recursions of procedures involving *both* non-linear programming and dynamic programming.[2]

However useful these advances may be for day–to–day scheduling, they do not yield a portrait of technology simple enough to contribute to the current policy discussions concerning power auctions. Costs of running power systems are clearly *intertemporal*— that's what unit commitment is all about. Yet for lack of any simple model of intertemporal costs, the main power auction proposals are conflicted: they first pretend each hour is independent of any other, they hold simultaneous independent auctions for each hour, and then allow revision of bids upon revelation of the outcomes of these parallel auctions.[3] I am not aware

[1] See Baldick(1995) and references in Lee(1988) and Tseng, Li, and Oren(1997)

[2] Another literature— even larger and older— deals with unit commitment in a different fashion and under another name: "peak-load pricing." Crew, Fernando, and Kleindorfer(1995) is a useful survey of this economic literature, and Crew and Kleindorfer(1976) and McGuire and McGuire(1993) are pieces that tie in closely with the present discussion.

[3] The Victoria auction is such an example.

of any analysis of the performance of this patchwork procedure.

The unit commitment model studied here is a simple one. Forget reactive power and spinning reserves[4], and forget ramping constraints and minimum up- and down-times. In most of the literature the time profile is assumed to be known with certainty— an assumption that, in my view, makes for immense analytical difficulty without contributing much if any realism. Here we assume uncertainty. Demand is a Markov process: demand today (or this hour) is a random variable drawn from a distribution conditioned only by yesterday's (or last hour's) demand. Production cost at each generator is assumed linear. Production is constrained above and below at each generator, and start-up costs are varied as described below. The hope is that this simple model captures some of the essence of the unit commitment problem without the detail that puts intuitive qualitative understanding beyond reach.

How might the cost of starting-up a generator depend on the time it has been down? We shall examine three extreme cost models which span the different possible relationships between down-time and start-up cost. In the "cold-start" model start-up cost is a positive quantity independent of time down— every start is like every other. In the "hot-start" model cost is proportional to time down— as though the longer the boiler is allowed to cool, the more fuel must be burned to bring it back to operating temperature. In the "ice-breaker" model, start-up (like a winter shipping channel in Lake Superior) is virtually costless for a time, after which it becomes extremely costly.[5]

The production costs of a generator can properly be said to be "intertemporal" if the cost of production in a given period depends on the levels of production in other periods. In this sense the cold-start model gives rise to intertemporal production costs: a unit's cost

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[4] In the papers that do impose constraints on minimum spinning reserves, the analyses often strike me as half-hearted. Why are these reserves required? To meet uncertain future demands. But future demand is usually assumed known so why ...?

[5] In all models the cost of start-up after *zero* down-time is assumed to be zero. This means that start-ups are always fresh starts.

today depends on whether the unit was up or down yesterday. The hot-start model, on the other hand, is really *not* intertemporal. Production today saves the same amount of current or future start-up cost independently of the levels of production in other periods. Thus if we think of start-up costs as being paid piecemeal in each period of downtime, intemporality vanishes. The ice-breaker model is intertemporal only if operated foolishly.[6]

In the next two sections we examine optimal unit commitment for sets of generators all of which are characterized in common by either the hot-start model or the cold-start model.

1. The hot-start model

For generator i the fuel cost of producing power p_i per unit time is $b_i + c_i p_i$. The start-up cost D_i of generator i is exactly proportional to the time t_i that it has been shut down: $D_i = t_i d_i$. Shutting down a generator *is* useful because if up it must be operated at a level $p_i \geq \underline{p}_i$. Output is top-constrained as well: $p_i \leq \overline{p}_i$.

The start-up cost feature would seem to deny us the opportunity of simply asking how best to produce a specified output. The best way to meet demand today generally depends on which generators (or "units") were up yesterday and what demand will arise tomorrow. But with down-time proportional start-up costs[7] the best commitment decisions today are independent of *both* yesterday's commitments and tomorrow's demands. Start-up costs for generator i are simply paid piecemeal in amount d_i (the fuel cost of keeping the water hot) in each period of downtime.

For given demand x in a period the unit commitment problem might be posed as follows. Choose a non-negative n-vector p of production levels for n generators and a binary n-vector u assigning up- or down-status to minimize

$$pc' + (1 - u)d'$$

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[6] Paradoxically, the models where start-up cost does depend on down-time are *not* intertemporal and the one where it doesn't is.

[7] And a zero interest rate, as we shall assume in this section only

subject to the constraints

$$u_i \underline{p}_i \leq p_i \leq u_i \overline{p}_i$$

and

$$\sum_i p_i \geq x.$$

This formulation is not satisfactory in two respects. It assesses charges for maintaining boiler temperature even to units that are assigned zero production in *all* periods. Second, it makes no use of the convenient truth that keeping the water hot in up periods must be about the same as keeping it hot in down periods. If d is to be charged in all periods for those generators used sometime, and in no periods for those generators used never, then we may eliminate d from the one-period optimization. We reformulate the problem as follows.

Denote the set of all generators by K and subsets of K by H. Let p_{ix}^H denote the production assigned to unit i in a period with total demand x when H is the subset of available generators. Now for each $H \subseteq K$ find u_{ix}^H and p_{ix}^H to minimize

(1) $$\sum_x \sum_i c_i p_{ix}^H$$

subject to

(2) $$u_{ix}^H \underline{p}_i \leq p_{ix}^H \leq u_{ix}^H \overline{p}_i$$

for al i and x and, for all x,

(3) $$\sum_i p_{ix}^H \geq x.$$

The relevant product of this large array of very simple one-period optimizations is the matrix A with elements

$$A_{x,H} \equiv \sum_i c_i p_{i,x}^H.$$

It remains only to find that unit subset H which is optimal against a given steady-state probability distribution $q(x)$ of demand. In other words, determine the least element of the vector

$$\sum_x q(x) A_{x,H} + \sum_{i \in H} d_i.$$

A small numerical example for a given H is exhibited in Appendix 1.

2. (s,S)-unit commitment in the cold-start model. Now suppose that starting up unit i costs a fixed amount D_i *independent* of the length of time i has been down. The decision problem is now roughly as follows. At the beginning of a period we inherit a binary vector u of unit commitments from last period and we observe a new (randomly drawn) demand x for the current period. We must choose a new unit commitment vector v and a production vector p consistent with v. The difference from the situation in Section 1 is that today's costs critically depend on the inherited commitment vector u *and* today's commitment decision v critically affects the costs that will be incurred tomorrow. Moreover, we do not know tomorrow's demand.

There *is* an easy part of the problem. Once v is chosen, p follows by a simple optimization. Define cost function $C(v, x)$ as follows[8]:

$$C(v, x) \equiv \min_p \sum_i c_i p_i \tag{4}$$

subject to

$$v_i \underline{p}_i \leq p_i \leq v_i \overline{p}_i \tag{5}$$

for all i, and

$$\sum_i p_i \geq x. \tag{6}$$

---

[8] Notice the difference from minimization problem (1-3). There, H indicated generators that were available if desired. Here v indicates generators that *must* be used

With that simplification, the commitment function we seek can be written as $f(u,x) = v$ or, dividing this multi-valued function into parts, $f_i(u_1,..,u_n,x) = v_i$ for $i = (1,..,n)$. What form is this function, if optimal in minimizing expected costs, likely to take?

First of all, is f_i likely to be independent of u_j for $j \neq i$? That is, can we write $f_i(u_1,..,u_n,x)$ as $f_i(u_i,x)$? If j is down, it is not implausible that we should be more reluctant to shut down i than if j were up— we may need this capacity tomorrow. Despite this concern we shall assume— short of a proof— that the optimal f does under certain conditions[9] exhibit this independence property. With this assumption we need only examine the behavior of the functions $f_i(u_i,x)$. For given i, if demand is sufficiently high, say $x > S_i$ for some value S_i, unit i will optimally be up ($v_i = 1$) whether $u_i = 1$ or $u_i = 0$. Similarly, if demand is sufficiently low, say $x < s_i$ for some value s_i, unit i will optimally be down independently of the value of u_i. Between these values commitment of i does depend on u_i. The assertion here is that in this demand interval commitment will not change; that is to say, $s_i \leq x \leq S_i$ implies $v_i = u_i$. The interpretation of the rule is thus: an uncommitted unit i remains down until demand exceeds a critical level S_i at which time it is started up. It remains up until demand falls below a critical level $s_i \leq S_i$ when it is shut down. It remains down until ... *etc.* The analogy to inventory theory is obvious. So is the motivation: Once committed, stay reluctantly committed; once down, stay reluctantly down. Operational savings are sacrificed to avoid start-up costs.

Next we must examine the relationships among the (s_i, S_i) pairs. An n-unit system will be called *well-behaved* if units can be indexed to make the following two properties true:

$$u_i = 1 \Longrightarrow u_j = 1 \text{ for } j < i$$

and

$$u_i = 0 \Longrightarrow u_j = 0 \text{ for } j > i.$$

[9] Independence clearly does fail to hold under certain circumstances that can only arise if the system is in a state unreachable by optimal policy. Suppose generators i and j are close substitutes and i is superior to j. By some accident of policy j is up and i is down. Clearly we should be less ready to start up i than if j were down and more ready to shut down j than if i were up.

In a well-behaved system, in other words, units are shut-down in LIFO fashion, and started up in LOFI fashion. The start-up and shut-down triggers then satisfy $S_i \leq S_j \Longleftrightarrow s_i \leq s_j$. We assume that parameter values are such that the optimal system is well-behaved.[10]

The functional equation describing optimal unit commitment can be written as follows. Let $F(u,x)$ be the expected present value, under optimal operation, of the future cost stream on the occasion of inheriting an up-vector u from yesterday and facing a demand of x today. (β is the one-period discount factor.)

$$F(u,x) \equiv \min_{p,v,w} [cp' + dw' + \beta\, \mathrm{E}_y(F(v,y)|x)] \tag{7}$$

subject to

$$v \times \underline{p} \leq p \leq v \times \overline{p} \tag{8}$$

$$\sum p \geq x \tag{9}$$

$$w \geq v - u. \tag{10}$$

with both v and w constrained to be binary variables.

To begin with, we do not know the ordering of units— call it the *grand merit order*[11]— indicating the optimal progression of start-ups up as demand increases. We start therefore with the more modest goal of determining the $(s,S)-$triggers optimal for a given arbitrary ordering $o(i)$ of units:

$$o(i) < o(j) \Longrightarrow S_i \leq S_j \quad \text{and} \quad s_i \leq s_j.$$

[10] If s exhibits a different ordering from S we have a phenomenon analogous to hysteresis. The shut-down sequence is not the reverse of the start-up sequence. Can this happen under an optimal commitment rule? Results from the hotstart model suggest that, with low start-up costs,indeed it can. Conjecture: A sufficient condition for a system to be well-behaved is $c_i \leq c_j \Longrightarrow c_i\underline{p}_i \leq c_j\underline{p}_j$ and $D_i \geq D_j$.

[11] Generally not the same as the more familiar operating-cost merit order

Since no start-up will optimally take place until needed, the top triggers are easily determined for the given order:

$$S_i = \sum_{j \ni o(j) < o(i)} \overline{p}_j. \tag{11}$$

Let b_{xy} denote the conditional probability of demand y tomorrow given demand x today; demand will be assumed to take only a finite number N of integral values. With these transition probabilities in hand we can address the problem of determining s_i. But first some preliminaries: For any state (u, x) and a fixed start-up order, operating cost in the current period is given by the function $C(u, x)$. In a well-behaved system the state vector u can be written more compactly: let $U_i \equiv (1_1, 1_2, .., 1_i, 0_{i+1}, .., 0_n)$. Next, for fixed i define $k(x)$ to be that index for which

$$\begin{cases} s_{k(x)} \leq x < s_{k(x)+1} & \text{when} \quad x < s_i, \\ s_{k(x)} \leq x \leq S_{k(x)+1} & \text{when} \quad s_i \leq x \leq S_{i+1} \\ S_{k(x)} < x \leq S_{k(x)+1} & \text{when} \quad S_{i+1} < x. \end{cases} \tag{12}$$

To simplify notation still further, let the generators be indexed in their order (perhaps not optimal) of start-up. Let (s, S) be an arbitrary trigger vector and (s^*, S^*) the trigger vector optimal for the given start-up order.

With these notational simplifications we can define a function $G_s(U_i, x)$ by

$$G_s(U_i, x) = C(U_{k(x)}, x) + \beta \, \mathrm{E}_y[G_s(U_{k(x)}, y)|x] + \begin{cases} 0 & \text{if} \quad x \leq S_{i+1} \\ \sum_{j=i+1}^{j=k(x)} D_j & \text{if} \quad x > S_{i+1} \end{cases}. \tag{13}$$

Since S^* is known (*i.e.*, $S_i = \sum_{j<i} \overline{p_j}$) we have the identity $G_{s^*}(U_i, x) \equiv F(U_i, x)$: functional equation (7) can be written without the "min" operator.

Nearly all of this detail cancels out of the expression for the important difference $\Delta_s(U_i, x) \equiv G_s(U_i, x) - G_s(U_{i-1}, x)$:

$$\Delta_s(U_i, x) = \begin{cases} 0 \quad \text{if} \quad x < s_i \\ \\ C(U_i, x) - C(U_{i-1}, x) + \beta \, \mathrm{E}_y[\Delta_s(U_i, y)|x] \\ \quad \text{if} \quad s_i \leq x \leq S_i \\ \\ -D_i \quad \text{if} \quad x > S_i. \end{cases} \tag{14}$$

Now fix i and suppose that $s_j = s_j^*$ for all $j \neq i$. Next examine the behavior of $\Delta_s(U_i, s)$ as s_i alone is varied.

If $s_i > s_i^*$ generator i gets shut down when demand drops just below s. This is too quick: inheriting U_i instead of U_{i-1} reduces expected costs and expanding the demand interval over which this advantage prevails will reduce costs further. Exactly the reverse holds when $s_i < s_i^*$. For generator i then

$$\Delta_{s^*}(U_i, s) \begin{Bmatrix} \leq \\ > \end{Bmatrix} 0 \quad \text{as} \quad s_i \begin{Bmatrix} \geq \\ < \end{Bmatrix} s_i^*. \tag{15}$$

To make use of (15) we must expand (14) with the aid of the demand transition matrix B. Again, for fixed i (and with that excuse, dropping some of the i-subscripts), define $B_m = \|b_{ij}\|$ with i and $j = s_i, .., S_i$ and $B_h = \|b_{ij}\|$ with $i = s_i, .., S_i$ and $j = S_i, .., N$. These two submatrices of B represent, respectively, the transitions of demand from closed "middle" interval $[s_i, S_i]$ to middle interval and from middle interval to half-open "high" interval $(S_i, .., N]$.[12]

Let $z_i \equiv C(U_i, x) - C(U_{i-1}, x)$ for $s_i \leq x \leq S_i$ (Possible because z is constant in this middle interval.) and let $\sigma_i \equiv (1, 0, 0, ..)$ of appropriate length. At $x = s_i$ equation (14) can now be written

$$\begin{aligned} \Delta_s(U_i, s) &= z_i + \sigma\Big(\sum_{t=1}^{\infty} \beta^t B_m^{t-1}\Big)(B_m(z, z, .., z)' - B_h(D, D, .., D)') \\ &= z_i + \sigma\beta(I - \beta B_m)^{-1}(B_m(z, z, .., z)' - B_h(D, D, .., D)'). \end{aligned} \tag{16}$$

The use of (16) to compute s^* is straighforward.[13] The cost difference z_i is usually equal to $\underline{p}_i(c_i - \max_{j<i} c_j)$, thus requiring no optimizing computation. The main difficulty, of course, is that the procedures described here must be repeated for *each* start-up order of

[12] It is perhaps worth noting that powers of B_m represent multi-stage transitions that have never departed from the middle demand interval.

[13] A numerical example is given in Appendix 2.

the set of K generators to learn which is best. That is not the end of the story, however. Some orderings are better candidates than others *ab initio*. Early start-ups are likely to be those with high start-up costs and low operating costs; late start-ups those with low $\underline{p}$'s.

3. A One-Stage Auction with "Groves" Rewards

Suppose each unit in a set of cold-start generators submits a 4-vector bid to contribute supply to a system that must meet a random pattern of demand over a sequence of T periods. Bidder i submits a not-necessarily truthful cost-parameter bid $(c_i, D_i, \underline{p}_i, \overline{p}_i)$. Winning bidders are selected according to the procedures outlined in Section 2.

The expected total of ostensible costs suffered by a set K of bidders in supplying this random demand is $R(K) \equiv \mathrm{E}_{x,u}((1-\beta^T)F(u,x)$ where the[14] joint probability distribution of u and x is taken to be the stationary distribution. The justification for this stationarity assumption is that T is large and that the auction is supposed to take place well in advance of actual commitment, so that who is up or down when business starts is a matter of chance.

Since most of the discussion that follows does not call upon the details of Section 2, we employ a different and simpler notation. Let total cost $R(K)$ be partitioned in two parts, costs incurred by Bidder i and the sum of those incurred by the other bidders in K: thus, resp., $R(K) \equiv C = C_i + C_{-i}$.

The arguments of the C functions are cost parameters (true or false) and commitment assignments which, in turn, are functions of cost-parameter bids. Let a vector of bids be denoted $b = (b_{-i}, b_i)$, with *true* bids, $\hat{b}$, $\hat{b}_{-i}$, $\hat{b}_i$ distinguished by hats. This optimal commitment assignment function, called P^*, has the obvious property,

$$C[P^*(\hat{b}), (\hat{b})] \leq C[P^*(b), (\hat{b})]. \tag{17}$$

In words hardly worth stating, the true cost of a commitment pattern properly designed

[14] "the" is a bit strong, since there is obvious circularity here. The distribution depends on the bid pattern.

for a false cost parameter vector cannot be less than the cost of one properly designed for the true cost parameter vector.

The total *ostensible* cost of an optimally assigned set b of arbitrary bids is $C[P^*(b), b]$, while the *true* cost of this same set of bids is $C[P^*(b), \hat{b}]$. Now suppose cost bids b are submitted by generators in a competitive auction for commitment assignments. Further suppose that $b = (\hat{b}_{-i}, b_i)$, that is, that all bids except Bidder i's are frozen at true values. How should Bidder i formulate his bid? His best choice in this situation depends on the scheme for rewarding winners.

Under a Groves reward scheme [15] Bidder i receives a *gross* return consisting of the difference between the social cost C^0 incurred by the system *without* him less the generally smaller cost incurred by his competitors when the system is optimized against the set of all bids including his own. In short:

$$C^0 - C_{-i}[P^*(\hat{b}_{-i}, b_i), (\hat{b}_{-i}, b_i)]. \tag{18}$$

His *net* return under the scheme is (18) minus his own true costs of meeting this assignment. His problem then is to choose b_i to maximize

$$C^0 - C_{-i}[P^*(\hat{b}_{-i}, b_i), (\hat{b}_{-i}, b_i)] - C_i[P^*(\hat{b}_{-i}, b_i), (\hat{b}_{-i}, \hat{b}_i)]. \tag{19}$$

Since $C_{-i}[P^*(\hat{b}_{-i}, b_i), (\hat{b}_{-i}, b_i)] = C_{-i}[P^*(\hat{b}_{-i}, b_i), (\hat{b}_{-i}, \hat{b}_i)]$ (19) becomes

$$C^0 - C[P^*(\hat{b}_{-i}, b_i), (\hat{b}_{-i}, \hat{b}_i)]. \tag{20}$$

By definition of P^* the second term in (20) is minimized at $b_i = \hat{b}_i$. *With Groves rewards, truth-telling is a Nash equilibrium.*

Another system might reward bidders in terms of realized outcomes instead of expectations. Commitment rules (*i.e.*, (s, S) triggers) would remain as before, but an unusual demand

[15] Groves and Ledyard (1977)

pattern might cause a bidder's contribution to be negative! The Groves incentive feature still holds, however, and bidders would make their offers on the basis of expected realized payoffs instead of (as in the previous case, and to put it crudely) expected expected payoffs.

The important aspect of this auction proposal is that even in a context as rife with intertemporal costs as the cold-start model, the need for multi-staged bidding does not appear to be compelling. With informative bid design, and evaluation and payoff schemes that encourage true cost revelation, perhaps the obvious and troubling problems associated with repeated bid revision can be avoided.

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APPENDIX 1. An Example of Optimized Hotstarts

Table 1 gives the input parameters and, for a range of demands, the optimal commitment outputs and total costs for a simple three-generator hotstart system. The results— easily verified by inspection[16]— are interesting. Notice that operating-cost merit order is *not* followed as demand increases. Unit 2 is started up at low demands, shut down at middle-level demands, started up again at higher demands and then, in favor of Unit 3, shut down a second time at still higher demands. Is this an exotic case?

Table 1. Outputs and Costs for a 3-Generator Hotstart System

```
HOTSTART.OUT


INPUTS
For each Generator:
              c  pmin  pmax
              1     2     3
   1          4     7    10
   2          8     2    10
   3          7     4    10

OUTPUTS
Commitments at Each Level of Demand & Total Cost
   1          0     2     0        16
   2          0     2     0        16
   3          0     3     0        24
   4          7     0     0        28
   5          7     0     0        28
   6          7     0     0        28
   7          7     0     0        28
   8          8     0     0        32
   9          9     0     0        36
  10         10     0     0        40
  11          9     2     0        52
  12         10     2     0        56
  13         10     3     0        64
  14         10     0     4        68
  15         10     0     5        75
  16         10     0     6        82
  17         10     0     7        89
  18         10     0     8        96
```

[16] Which suggests, perhaps, that the computation procedure described in Section 2 is not very efficient! The solution here is generated by the integer-programming solver CPLEX under control of the algebraic modeling language AMPL.

APPENDIX 2. A Simulation of Optimized Coldstarts

The MATLAB program "MARKOV" listed below simulates the performance of M coldstart generators over T periods, with optimally chosen up- and down- triggers, but with an arbitrarily chosen grand merit order and an arbitrarily chosen initial state vector. Down-triggers are computed by the function "SHUT", also listed. Inputs must be entered directly into the text listing.

Calling variable COST yields a cost history for the run, and calling variable ACT yields an action history. Triggers and average start-up and operating costs are output each run. Table 2 below, for instance, displays part of the action history of a run with the parameters for the four generators in the MARKOV listing. The optimal triggers are $s = (0, 0, 5, 21)$ and $S = (0, 10, 20, 30)$.

```
% "MARKOV"--- A MATLAB COLDSTART SIMULATION PROGRAM

% INPUT DATA:=============================================================
  T=500;                         % Length of simulation: T periods.
  N=40;                         % Demand values range over 1,..,N.
  beta=.9;                      % Discount factor

  AA=[  3   4   5   6
      600 600 500 400
        2   3   5   7
       10  10  10  10];         % Each column is a generator:
                                %  c, D, pmin, pmax

  meritorder=[1 2 3 4 ];        % Arbitrary:  Experiment to find the best.

% Note:  The Markov transition demand probabilities are fixed in the
%         program, and generated (quite arbitrarily, but choose your own!)
%         by the MATLAB binomial distribution function BINORND.

% END OF INPUT DATA ======================================================

cDpP=AA(:,meritorder);

c   =cDpP(1,:);                 % Vector of operating costs per MW-period
D   =cDpP(2,:);                 % Vector of start-up costs
pmin=cDpP(3,:);                 % Vector of min operating levels
pmax=cDpP(4,:);                 % Vector of max operating levels

M=length(c);


mu=(1/(N+2))*(1:N+1);           % To set up the "binornd" function below.

x=floor(N/2);
in=floor(M/2);
u=[ones(1,in) zeros(1,M-in)];   % Initial state: half up, half down.
```

```
                                  % Choose another if you wish.


S=cumsum(pmax);
S=[0 S(1:M-1)];                   % Start-up triggers

z=pmin.*(c-[0 c(1:M-1)]);         % The "pmin penalty" z on p.9
ZDSNB=[z;                         % Array inputs to the "shut" function.
       D;
       S;
       N*ones(1,M);
       beta*ones(1,M)];

s=[];                             % Use shut to compute shut-down triggers.
for i=1:M,
  ss=shut(ZDSNB(:,i));
  s=[s ss];
end;
[s;S]                             % Print triggers.

Cost=[];Act=[];Y=[];
upcost=0; opcost=0; cost=0;

for t=1:T,                        % Each round simulates one period.
 y=binornd(N,mu(x+1));            % Random demand 1,..,N
                                  %   when yester-period demand was x.
 Y=[Y y];
 v=(1-(y<s)).*u;                         % Effect of s
 v=min(1,v+(y>S));                       %  plus effect of S
                                         %  = Who is now up?
 w=max(0,v-u);                           % New start-ups

 yy=y-(pmin*v');
 cs=cumsum(pmax-pmin)<yy;
 last=max(0,yy-cs*(pmax-pmin)');
 kk=sum(cs)+1;
 pstar=(pmin.*v)+ [cs.*(pmax-pmin)];
 pstar(kk)=pstar(kk)+last;               % New operating levels

 upcost=upcost+w*D';
 opcost=opcost+c*pstar';
 cost=upcost+opcost;                     % Period costs

 out=[y  upcost  opcost   cost];
 Cost=[Cost;out];                        % Array cost history
 act=[y  v pstar];
 Act=[Act;act];                          % Array action history
 x=y;                                    % Reset for next round.
 u=v;                                    % Reset for next round.
end
Cost;                                    % Cost history
Act                                      % Action history

[upcost opcost cost]/T                   % Period cost averages
                                         %  Near to the present value
                                         %  of the text model if discount
                                         %  parameter Beta is close to unity
                                         %  and initial state is not too
                                         %  unusual....

```

```

% FUNCTION "SHUT" COMPUTES SHUT-DOWN TRIGGER s
%  This function is used in "MARKOV" simulation.

function [s]=shut(zdsnb)

z=zdsnb(1);                                   % "pmin penalty"--
                                              %   See definition, text p.9
D=zdsnb(2);                                   % Start-up cost
S=zdsnb(3);                                   % Start-up trigger
N=zdsnb(4);                                   % Demand values 1,..,N
beta=zdsnb(5);                                % Discount factor

mu=(1/(N+2))*(1:N+1);
Tb=[];
for j=1:N,
  tb=binopdf((0:N-1),(N-1),mu(j));            % Binomial densities
  Tb=[Tb;tb];
end
Tb;                                           % Transition matrix

Delta=[];
for s=1:S,
  Bm=Tb((s:S),:);
  Bh=Bm(:,(S+1:N));                           % "High-region" probs
  Bm=Bm(:,(s:S));                             % "Middle-region" probs
  m=S-s+1;
  A=inv(eye(m)-beta*Bm);
  deltam=z+beta*A*(z*sum(Bm')-D*sum(Bh'))';   % Eqn. (16) in text, p.9
  delta=deltam(1);
  Delta=[Delta;delta];
end

s=sum((Delta>0));                   % Output:  Soln. of (16)=0.
```

Table 2. Part of the Action History of a Run

x	u				p			
34	1	1	1	1	10	10	7	7
33	1	1	1	1	10	10	6	7
25	1	1	1	1	10	3	5	7
22	1	1	1	1	7	3	5	7
20	1	1	1	0	10	5	5	0
19	1	1	1	0	10	4	5	0
29	1	1	1	0	10	10	9	0
33	1	1	1	1	10	10	6	7
29	1	1	1	1	10	7	5	7
37	1	1	1	1	10	10	10	7
33	1	1	1	1	10	10	6	7
30	1	1	1	1	10	8	5	7
26	1	1	1	1	10	4	5	7
23	1	1	1	1	8	3	5	7
25	1	1	1	1	10	3	5	7
24	1	1	1	1	9	3	5	7
20	1	1	1	0	10	5	5	0
21	1	1	1	0	10	6	5	0
25	1	1	1	0	10	10	5	0
25	1	1	1	0	10	10	5	0
18	1	1	1	0	10	3	5	0
10	1	1	1	0	2	3	5	0
9	1	1	1	0	2	3	5	0
8	1	1	1	0	2	3	5	0
32	1	1	1	1	10	10	5	7
28	1	1	1	1	10	6	5	7
21	1	1	1	1	6	3	5	7
20	1	1	1	0	10	5	5	0
19	1	1	1	0	10	4	5	0
13	1	1	1	0	5	3	5	0
8	1	1	1	0	2	3	5	0
12	1	1	1	0	4	3	5	0
7	1	1	1	0	2	3	5	0
4	1	1	1	0	2	3	5	0
2	1	1	0	0	2	3	0	0
2	1	1	0	0	2	3	0	0
12	1	1	0	0	9	3	0	0
14	1	1	0	0	10	4	0	0
11	1	1	0	0	8	3	0	0
18	1	1	0	0	10	8	0	0
20	1	1	0	0	10	10	0	0
21	1	1	1	0	10	6	5	0
20	1	1	1	0	10	5	5	0
19	1	1	1	0	10	4	5	0