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Priority Network Access Pricing for Electric Power

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Abstract

We propose a priority-pricing scheme for zonal access to the electric power grid that is uniform across all buses in a zone. The Independent System Operator (ISO) charges bulk power traders a per unit *ex ante* transmission access fee. The zonal access fee serves as an access insurance premium that entitles a bulk power trader to either physical injection of one unit of energy or a compensation payment. The access fee per MWh depends on the injection zone and a self-selected strike price that serves as an insurance "deductible" that determines the scheduling priority of the insured transaction and the compensation level in case of curtailment. Inter-zonal transactions are charged (or credited) with an additional *ex post* congestion fee equal to the differences in zonal spot prices. The compensation for curtailed transactions equals the difference between the realized zonal spot price and the selected strike price (deductible level). The ISO manages congestion so as to minimize net compensation

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payments and thus, curtailment probabilities increase with strike price and for any particular strike price may vary from bus to bus. We calculate the rational expectations equilibrium for three-, four- and six-node systems and demonstrate that the efficiency losses of the proposed second best scheme relative to the efficient dispatch solutions are modest.

1 Introduction

Transmission pricing and congestion management protocols are basic ingredients of any restructuring scheme aimed at promoting open access and competition in electricity markets. The Federal Energy Regulatory Commission (FERC) has recognized the crucial role of open access to transmission networks in Orders 888 and 889, which provide general principles for the pricing and utilization of scarce transmission capacity. One of the basic trade-offs involved in implementing FERC's open access ruling is choosing between economic efficiency and the simplicity of pricing and congestion management protocols. While it is generally agreed upon that transmission pricing should provide economic signals that will induce efficient use of the transmission grid, it is not clear how precise such signals must be in order to capture most of the economic benefits from efficient congestion management. It is important to design a mechanism for regulating network access that is simple to implement, facilitates energy trading and will promote efficient network utilization.

Two extreme approaches on this spectrum are the Contract Network/Nodal Pricing approach (Hogan [8]) on one hand and the so called "postage stamp" approach on the other hand. In the nodal pricing approach, congestion management is performed through a central optimal dispatch, while transmission charges are determined *ex post* and set to the nodal spot price differences (i.e. the market opportunity cost associated with using a particular transmission line). Under the assumption of perfect information (regarding generation costs) and abstraction of intertemporal aspects of the production costs and constraints this approach is "first-best" i.e. it produces the economic dispatch solution. It has been argued, however, that the claimed efficiency of the nodal pricing approach is based on unrealistic assumptions, the implementation of the idealized nodal pricing paradigm is overly complex and it relies on a highly centralized market structure that inhibits competition and customer choice. Furthermore, the *ex post* determination of the transmission prices is a severe obstacle to efficient bilateral energy trading. (see Wu, Varaiya, Spiller and Oren [13]). The postage stamp approach, on the other hand, imposes a uniform charge on each unit of electricity shipped regardless of anything else (zonal differentiation has also been proposed). The simplicity of the postage stamp approach is compelling and it makes it easy for energy traders to incorporate transmission costs into their trading decisions. Unfortunately even with zonal differentiation this approach does not provide correct economic signals for transmission network usage and for congestion management. Nor does it provide locational economic signals for generation investments.

An alternative to nodal pricing, which in equilibrium can also achieve the first best outcome, was proposed by Chao and Peck [4]. It is based on parallel markets for link based transmission capacity rights and energy trading under a set of trading rules imposed by an Independent System Operator (ISO). The trading rules specify the transmission capacity rights required to support bilateral energy trades between any two buses[1] and are adjusted continuously to reflect changing system conditions. The decentralization in this approach and its reliance on market forces rather than on a central planning paradigm is attractive. However, its implementation would require a highly sophisticated level of electronic markets and information technology. Wilson [12] has demonstrated yet another way to achieve the first best solution by implementing a priority insurance scheme where the insurance premium varies for each pair of nodes. Neither of the above alternatives to nodal pricing offers a compelling improvement in terms of simplicity which is the primary objective of this paper.

We propose a priority insurance framework for assigning access privileges to the electricity transmission network where the premium or access fee is only differentiated according to the self-selected level of coverage but does not vary across buses within a set defined as a congestion zone. Instead, the probability of curtailment associated with each coverage level varies across buses and is endogenously derived from the congestion management protocol employed by an ISO, seeking to minimize net compensation to curtailed transactions. Specifically, in our proposed insurance scheme the premium per insured MWh in a particular zone is a function of a customer selected strike price (or a strike price function) that is equivalent to a "deductible" (or a "deductible" function) in common insurance schemes. In case of curtailment due to congestion a unit insurance entitles its holder to compensation in the amount of the forgone spot market revenue net of the selected deductible. In financial terms the curtailment compensation equals the forgone option value which is the difference between the realized zonal spot price and the selected strike price. Because the premium function

[1] A bus in an electric power grid can be thought of as a node in the transmission network.

is constrained to be uniform across each zone (as opposed to being different for each pair of nodes in Wilson [12]), the resulting equilibrium will only achieve a second best dispatch from the point of view of short-term efficiency. However, the general direction of the market signals facilitate efficient use of scarce network resources by inducing transactions that have higher opportunity values or that impact more congestion prone segments of the grid to seek higher levels of insurance (lower strike prices) in order to obtain higher scheduling priorities at their respective buses. Furthermore, the opportunity to share curtailment risk (by selecting a deductible above marginal cost) at injection nodes that do not impact congestion allows higher profit margins at such nodes thus providing the correct locational signals for generation investment. Although in general our scheme is a second-best approach, it yields the same first best outcome as [4], [8], and [12] do in the limiting case where each node is treated as a zone.

While the term ISO is often identified with a nonprofit entity as implemented in several regions in the US (California, PJM, New York Power Pool and New England), we do not subscribe to the nonprofit restriction and use the term ISO to describe an operator of the transmission grid that is independent of any generation or consumption entity. From the implementation point of view, our proposed scheme should work well with a TransCo framework where the ISO owns transmission lines. In this case, a Performance Based Regulation (PBR) approach will provide the ISO incentives to properly manage the priority access insurance scheme. Revenue requirements for transmission asset owners are met by having a three-part tariff: access and energy charges for loads, and insurance premium for generators. Such a PBR scheme can be designed so that the TransCo will not benefit from increased congestion but will benefit from increased transaction volume on the transmission grid thus providing proper incentives for transmission investment (e.g. see Awerbuch, Crew and Kleindorfer [1]).

The rest of the paper is organized as follows. We present the formulation for both cases of a single spot market and multiple zonal spot markets in section two; in section three, we demonstrate how this scheme is implemented through numerical examples and evaluate the efficiency losses; finally, we conclude with some observations and remarks.

2 A priority insurance mechanism

We consider a market design where the network is partitioned into a few congestion zones and consumers in each zone face a uniform zonal spot price for electricity (e.g., the California and Nordpool electricity markets). The transmission system is operated by an ISO that collects transmission service fees and is charged with efficient congestion management. However, our proposed transmission pricing scheme and congestion management protocol are new. For the purpose of this paper we formally define a zone as a subset of nodes sharing a common spot market (See Figure 1). All zones are mutually exclusive and collectively exhaustive. In our model, we assume that the transmission network has a fixed transmission capacity configuration and there is no uncertainty as to the availability of the transmission capacity[2]. In each zone i, there exists a single zonal spot price $S_i \equiv S(\omega)$ contingent upon a random state of the world, ω, which is given exogenously. The fluctuation of S_i reflects the randomness in the supply and demand conditions. The set of demand nodes where the spot markets are located is denoted by N_D and the set of supply nodes is denoted by N_S.

An unexpected hot summer day would cause a surge in demand for electricity, which naturally results in a high value of S_i and increased usage of the transmission network, possibly causing congestion. In such cases, the ISO needs to have an effective and efficient mechanism to allocate the limited transmission capacity to network users.

Our scheme offers bulk power traders wishing to engage in physical bilateral transactions[3] a priority differentiated transmission network access tariff specific to the zone in which power is injected. In addition bilateral transactions across different zones are subject to an *ex post* congestion charge (or credit) that equals the spot price difference between the corresponding zones. It is assumed that curtailed transactions are settled either financially or through the purchase of replacement power and that the settlement price equals the spot price at the buyer's zone.

Under the above framework, physical access to the transmission network by a generator producing power at marginal cost c per MWh can be valued as a financial "Call" option with strike price c in the zonal spot market corresponding to the injection node. Such an option is exercised only when the zonal spot price S_i exceeds the strike price and it yields

[2] The presence of uncertainty in transmission capacity does not affect the implementation of our priority insurance scheme. It only complicates the computation of the rational expectations equilibrium.

[3] We stress on "physical bilateral transactions" here because financial transactions may take place without accessing the transmission network.

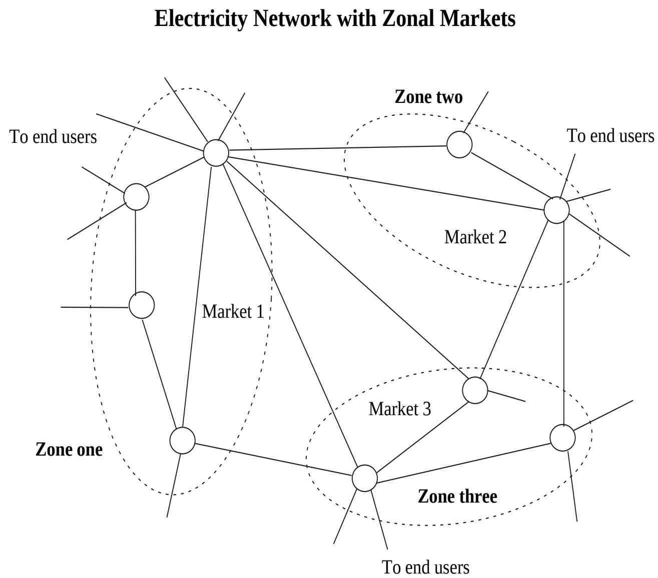

Figure 1: An electricity network with several spot markets

the difference $S_i \Leftrightarrow c$. Hence, the actuarial value of the option is $E_{S_i}[Max(0, S_i \Leftrightarrow c)]$ with expectation taken over the random zonal spot price. Motivated by this observation, we examine a per MWh *ex ante* transmission access charge in the form of an option insurance premium in our transmission pricing scheme. Specifically, the premium $X_i(c)$ in zone i can be chosen to be the option value corresponding to the zonal spot price forecast and a self-selected strike price c determining the curtailment compensation[4]. This payment would entitle a generator (or trader) to either physical access to the grid or a compensation payment that is equal to the forgone option value that equals the difference between the realized zonal spot price and the self-selected strike price. The ISO would then relieve congestion so as to minimize compensation payments to curtailed transactions net of the *ex post* interzonal congestion payments. In the remainder of this paper we use the above financial interpretation of the compensation. However, it may be intuitively useful to keep in mind that the strike price in this scheme plays the same role as a "deductible" in home owner's and automobile insurance in defining the level of risk-sharing by the insured. In this alternative interpretation the compensation is simply the damage (i.e., the forgone spot

[4]Generally speaking, the insurance premium function can be any arbitrary monotone function and different choices of the premium function have different implications on economic efficiency as we shall see in the later sections.

market revenue) due to curtailment less the deductible.

If each transmission network user[5] were to select a strike price that reveals its true marginal generation cost (i.e., fully insure the curtailment loss) then the above scheme would result in economic dispatch or least cost displacement. Furthermore, network users would be indifferent between physical access or compensation and would accrue zero profit whereas all the gains from producing at a cost below the spot price would go to the ISO (and ultimately to the transmission assets/rights owners).

The simplicity of this approach comes from the fact that we use a single transmission access tariff that depends only on the strike price irrespective of the injection node within a zone. However, because of that simplification, users may have an incentive to underinsure their transactions by selecting strike prices that are higher than their true marginal costs. In doing so they would estimate the probability of being curtailed and choose a strike price that will maximize their expected profits. Self-selected strike prices will depend on the true marginal cost and the probability of being curtailed at the particular injection node. In general, low marginal cost and high probability of curtailment will induce the selection of a lower strike price i.e., higher insurance level and higher service priority. Thus, the economic signal for congestion management is in the right direction although not exact.

The proposed mechanism can be described as a three-stage process (Figure 2).

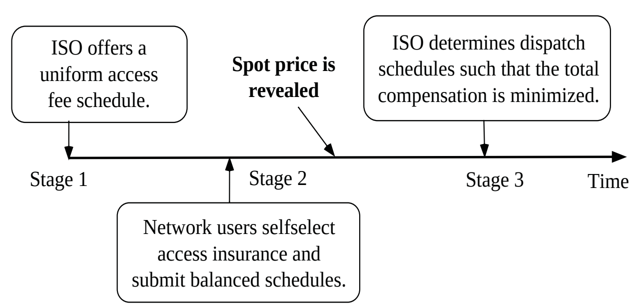

Figure 2: Timeline of the priority insurance scheme

Stage one The ISO posts a single insurance schedule $\{c, X_i(c)\}$ in each zone i, where $X_i(c)$ is the premium paid for insurance level c, allowing network users to insure network

[5]From here on, we use the terms "network user" and "transaction unit" interchangeably. Although we describe our priority insurance scheme for insuring network access from the perspective of the generators' side, this scheme is equally applicable to load serving entities' side.

access rights for their transaction units (multiple units can be insured at different levels)

It is assumed to be common knowledge that, when the spot price, S_i, is revealed, the ISO will manage the network congestion based on the criterion of minimizing total compensation payments net of interzonal congestion rent receipts. The implication of this assumption is that network users will form rational expectations about the locational service quality associated with a particular level of insurance at each node. The locational service quality is characterized in terms of the set of spot price contingencies under which transmission access at a specific bus is granted to a transaction unit insured at level c.

Stage two Before the random zonal spot prices are revealed, network users self-select an insurance level on each contracted unit in their schedules so as to maximize their expected profits. They do not need to specify the specific transaction nodes when purchasing their insurance. However, in a multi-zonal case the injection zones need to be revealed at this stage.

The spot price revelation in this time line may be interpreted as an accurate short-term spot price forecast employed by the network users to form their preferred schedule. This would be a more realistic interpretation when the reference settlement prices are the real time spot prices for imbalances.

Stage three At the third and final stage, network users submit their preferred schedules specifying injection nodes and selected insurance level for each transaction unit to the ISO. The ISO then grants transmission access or curtails submitted schedules so as to minimize total compensation payments net of *ex post* congestion revenues for interzonal transactions. The curtailed transactions are paid the difference between their revealed opportunity costs and the zonal spot price corresponding to the injection node.

We next layout the formulation in both the single spot market case and the multiple zonal spot markets case. In the following formulations and the remainder of this paper we use a lossless direct-current flow (DC-flow) model to approximate the transmission constraints. The formulation can be generalized, however, to account for losses and reactive power and voltage constraints.

2.1 Single spot market

When there exists only one spot market in a network, the ISO simply imposes one insurance premium schedule $X(c)$ (**stage one**), which is a decreasing function of the strike price c, for the entire network.

2.1.1 A network user's self-selection problem (stage two)

Given the ISO's insurance premium function $X(c)$, suppose a network user at node i subscribes to insurance level c for a transaction injecting at node i with true generation cost v. By purchasing the insurance, the user expects the transaction unit with insured cost c to get network access with probability $p_i(c,s)$ when the spot price S falls in the contingency set, which is a function of the insurance level c, $\Omega_i(c) \subset (0,+\infty)$ and be curtailed when the spot price S falls in $\overline{\Omega}_i(c)$, the complement set of $\Omega_i(c)$[6]. The reason for having a service probability function $p_i(c,s)$ in the contingency set $\Omega_i(c)$ is to incorporate the general case with bunching transactions at some insurance level c. Such "bunching" phenomena may be due to customer selection or the fact that the ISO offers only discrete levels of insurance for administrative reasons. When not all the access requests with the same insurance level c at node i can be accommodated due to transmission capacity constraints, the ISO randomly rations the network access among these units with the same insurance level thus yielding a probability of $p_i(c,s)$ for being served for a given s in the contingency set $\Omega_i(c)$. In the case of no "bunching" at a given insurance level c at node i, we have $(p_i(c,s) \equiv 1 : s \in \Omega_i(c))$. With the rational expectations $\{\Omega_i(c), (p_i(c,s) : s \in \Omega_i(c))\}$, the network user chooses the optimal c so as to maximize expected profit. Namely, the network user at node i would solve the following problem to get the optimal insurance level for a type i transaction unit with true opportunity cost v.

$$
(NU1) \quad c_i^*(v) = \arg\max_{c} \int_{\Omega_i(c)} [p_i(c,s)\cdot(s \Leftrightarrow v)^+ + (1 \Leftrightarrow p_i(c,s))\cdot(s \Leftrightarrow c)^+]dG(s) + \int_{\overline{\Omega}_i(c)} (s \Leftrightarrow c)^+ dG(s) \Leftrightarrow X(c) \tag{1}
$$

where $G(\cdot)$ is the cumulative distribution function of the random variable S; and $(s \Leftrightarrow v)^+$ denotes $\max(s \Leftrightarrow v, 0)$ from here on.

[6]As for an example of $\Omega_i(c)$, in our three-node numerical example in Section 3, $\Omega_i(c)$ takes the interval form of $[v, S_i(c)]$ with end point being $S_i(c) = k_{i1} - k_{i2}c$, a linear function of the insurance level c, and with k_{i1} and k_{i2} being two parameters to be determined.

2.1.2 The ISO problem (stage three)

After the random spot price is revealed (or accurately predicted), all network users submit their usage requests as well as their insured cost (or insurance level) c for each request. By aggregating the requested transactions according to their injection nodes and insurance levels the ISO ends up with insured cost curves $\widetilde{D}_i(c)$ for each i where $\widetilde{D}_i(c)$ is a monotonically increasing function of c representing the number of MW competing for injection at node i that are insured at strike price c or lower. In general the function $\widetilde{D}_i(c)$ is upper semicontinuous with discontinuities representing "bunching" of transactions at some strike prices and it may have flat spots representing ranges of strike prices that no one chooses. As mentioned before, these phenomena may result from customer selection or the restrictions on allowed insurance choices imposed by the ISO. We implicitly assume in this formulation an unlimited supply of replacement power (part of which can be curtailed demand) at the zonal spot price. When the network is congested, the ISO relieves the congestion by curtailing transactions such that the total insurance compensation payment is minimized. That is, for a revealed spot price S, we assume that the ISO solves the following minimization problem subject to transmission constraints.

$$
\begin{aligned}
&(ISO1) \\
&\min_{\{q_i : i \in N_S \cup N_D\}} \quad \sum_{i \in N_S} \int_{q_i}^{\widetilde{D}_i(s)} [s \Leftrightarrow \widetilde{v}_i(q)] dq \\
&s.t. \quad \sum_{i \in N_D} q_i \Leftrightarrow \sum_{j \in N_S} q_j = 0 \\
&\qquad q_i = \sum_{j \neq i} q_{ij} \\
&\qquad | q_{ij}(q_1, q_2, \cdots, q_{n-1}) | \leq C_{ij}, 1 \leq i < j \leq n \\
&\qquad q_i \geq 0 \quad \forall i \in N_S \cup N_D.
\end{aligned} \tag{2}
$$

where $\widetilde{v}_i(\cdot)$ is the inverse function of $\widetilde{D}_i(\cdot)$[7]; q_i is the net amount of power injected or ejected at node i; $q_{ij}(q_1, q_2, \cdots, q_{n-1})$ is the power flow function on line (i, j) (see [4], [8] and references therein for how to compute the power flow functions); and C_{ij} is the available capacity of line (i, j).

Bunching of transaction units at some insurance levels may result in non-uniqueness of the solution to the ISO minimum compensation problem and non-uniqueness of the implementation (i.e. who gets curtailed). This happens when there is bunching at the strike price

[7]Since the function $\widetilde{D}_i(\cdot)$ may have flat spots, we define $\widetilde{v}_i(\cdot)$ as the lower semicontinuous inverse function: $\widetilde{v}_i(q) = \min\{v : \widetilde{D}_i(v) \geq q\}$.

that separates the set of dispatched transactions from the set of curtailed transactions. To address such cases, we complement the minimum compensation criterion with a tie breaking rule that determines how limited network capacity for injection at each node should be rationed among transactions with identical insurance levels. The tie breaking rule is simply random rationing. When the ties occur at strike prices that entail compensation, the minimum compensation criterion will guarantee that transmission capacity is fully utilized. A special case occurs when bunching forms at a sufficiently high strike price for which the insurance premium is zero and the curtailment compensation is zero[8] at some node while the minimum compensation criterion yields free injection at this particular node. In such a case, the access capacity is rationed among all access requests that are not eligilbe for curtailment compensation through random rationing (unless all the requests can be accommodated).

With above rules, the ISO obtains a compensation-minimizing dispatch schedule $(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$. For every realized spot price $S = s$, there exists a corresponding $c_i(s)$ being the marginal insurance level (i.e. the highest c) granted transmission access (i.e., allowed to inject power) at node i.

Definition 1 *The above priority insurance mechanism is* **coherent** *in an electricity network if there exists an insurance premium function $X(c)$ and rational expectations of a set of dispatch contingencies and dispatch probability functions $\{\Omega_i(c), (p_i(c, s) : s \in \Omega_i(c))$ for all transactions injecting at node $i\}$ (which implicitly define $\{c_i(s), i \in N\}$) such that*

a) *$\{\widetilde{D}_i(c)$ for all $i\}$ are the distribution curves of the insured costs of all transaction units resulting from the network users' self-selection problem $(NU1)$;*

b) *$(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$ is a solution to $(ISO1)$ given $\{\widetilde{D}_i(c)$ for all $i\}$ for every revealed spot price s, and in the cases where the ISO can not accommodate all the access requests having the same insurance level, it employs random rationing as a tie breaker;*

c) $q_i^*(s) = \int_0^{c_i(s)} p_i(c, s) d\widetilde{D}_i(c)$ *for all i, s.*

We will later show in a more general setup of multiple spot markets that if every transaction unit reveals its true cost by purchasing insurance with a strike price which is equal to the true cost then our priority insurance scheme results in the economic dispatch (first best)

[8]The level of such a strike price depends on the choice of the insurance premium function and the spot price distribution.

solutions. However, network users in general have incentives to underinsure their access rights with the aforementioned choice of the insurance premium function. Our objective is to identify the *coherent* priority insurance schemes, attempt to characterize the scheme with the smallest possible deadweight efficiency loss due to imperfect contracting, and estimate those losses.

2.2 Multiple spot markets: zonal pricing

When there exist multiple zonal spot markets and the network is partitioned into several zones, the formulation is somewhat different. In this case, the ISO offers one insurance premium schedule $X_m(c)$ in each zone m. The ISO charges no *ex post* fee for transactions within one zone but imposes an **additional** *ex post* congestion fee (or counterflow credit) of $S_m \Leftrightarrow S_n$ per unit for transactions going from zone n to zone m (this is similar to Hogan [8]), where S_m denotes the random spot price in zone m.

2.2.1 The network user self-selection problem

Like in the single spot market case, a network user choosing to purchase insurance level c for one unit injected at node i of zone m, expects physical access with a probability of $p_i(c, \overline{s})$ when the realized zonal spot price vector $\overline{s} \equiv (s_1, s_2, \cdots, s_k)$ falls in the spot price contingency set $\Omega_i(c) \subset \mathcal{R}^k_{++}$. Thus a network user chooses the optimal c such that the expected profit is maximized. The optimal c for a transaction unit injected at node i belonging to zone $m(i)$ with true cost v is determined by solving the following problem:

$$
\begin{aligned}
&(NU2) \\
&c_i^*(v) = \arg\max_c \int_{\Omega_i(c)} [p_i(c,\overline{s}) \cdot (s_{m(i)} \Leftrightarrow v)^+ + (1 \Leftrightarrow p_i(c,\overline{s})) \cdot (s_{m(i)} \Leftrightarrow c)^+] dG \\
&\qquad\qquad + \int_{\overline{\Omega}_i(c)} (s_{m(i)} \Leftrightarrow c)^+ dG \Leftrightarrow X_{m(i)}(c)
\end{aligned}
\tag{3}
$$

where $\Omega_i(c)$ is the region of spot price contingencies under which the insurance level c would secure access to the network for a transaction unit injected at node i with probability $p_i(c, \overline{s})$; $\overline{\Omega}_i(c)$ is the complement of $\Omega_i(c)$; and $G \equiv G(s_1, \cdots, s_k)$ is the joint cumulative distribution function of the random spot prices $\{S_1, S_2, \cdots, S_k\}$.

In practice, we may offer a set of discrete insurance levels $\{c_1, c_2, \cdots, c_k\}$ and the corresponding set of premia $\{x_1, x_2, \cdots, x_k\}$. If the number of discrete levels is small we may wish to customize them to each zone. We will illustrate the merits of such an approach in an example.

2.2.2 The ISO problem

By aggregating all submitted insurance levels c, the ISO ends up with curtailment supply curves $\widetilde{D}_i(c)$ at each node i describing the number of MW competing for injection at node i insured at strike price c or lower. When the network is congested, the ISO relieves the congestion by curtailing transactions so as to minimize the total compensation payments. Namely, the ISO solves the following minimization problem subject to transmission Constraints. Again, in the case where the ISO can not accommodate all the usage requests having the same insurance level, it employs random rationing as a tie breaker.

$$
\begin{array}{lll}
(ISO2) & & \\
\min\limits_{\{q_i : i \in N_s \cup N_D\}} & \sum\limits_{i \in N_S} \int_{q_i}^{\widetilde{D}_i(s_{m(i)})} [s_{m(i)} \Leftrightarrow \widetilde{v}_i(q)] dq \Leftrightarrow \frac{1}{k} \sum\limits_{1 \le m < n \le k} (s_m \Leftrightarrow s_n)\left(\sum\limits_{j \in Z_m} q_j \Leftrightarrow \sum\limits_{j' \in Z_n} q_{j'} \right) & \\
s.t. & \sum\limits_{i \in N_D} q_i \Leftrightarrow \sum\limits_{j \in N_S} q_j = 0 & (4) \\
 & q_i = \sum\limits_{j \neq i} q_{ij} \quad i = 1, 2, \cdots, n. & \\
 & \mid q_{ij}(q_1, \cdots, q_{n-1}) \mid \le C_{ij} \quad 1 \le i < j \le n & \\
 & q_i \ge 0 \quad \forall i \in N_S \cup N_D. &
\end{array}
$$

where $\widetilde{v}_i(q) \equiv \widetilde{D}_i^{-1}(q) = \min\{v : \widetilde{D}_i(v) \ge q\}$; k is the number of zones in the network; Z_m denotes the node set of zone m; $m(i)$ denotes the zone to which node i belongs; and q_i is the net amount of power injected or ejected at node i. Again we augment the minimum compensation criterion with the following tie breaking rule: if multiple transactions are bunched at any marginal insurance level, then random rationing is employed to break the ties for access curtailment such that either all access requests are met or all available capacity are utilized. Hence, the ISO has a compensation-minimizing dispatch schedule $(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$ for every realized zonal spot price vector $(s_1, s_2, \cdots, s_m)$. There exists again a corresponding $c_i(s_1, s_2, \cdots, s_m)$ at node i, which is the insurance level purchased by the marginal transaction unit granted network access at node i for a revealed zonal spot price vector $(s_1, s_2, \cdots, s_m)$. If all network users truthfully reveal their marginal production costs by purchasing insurance $c^*(v) = v$, then the ISO's compensation-minimizing schedule is indeed the social welfare (gain from trade) maximizing schedule which is defined as follows.

Definition 2 *For a set of zonal spot prices* $(S_1, S_2, \cdots, S_k)$*, a dispatch schedule* $(q_1, q_2, \cdots, q_n)$ *is a social welfare maximizing (or, economic dispatch/first best) schedule if it is a solution to the (ED) problem.*

$$
\begin{array}{lll}
(ED) & & \\
\max\limits_{\{q_i : i \in N_S \cup N_D\}} & \sum\limits_{i \in N_D} q_i \cdot S_{m(i)} - \sum\limits_{i \in N_S} \int_0^{q_i} v_i(q) dq & \\
s.t. & \sum\limits_{i \in N_D} q_i - \sum\limits_{j \in N_S} q_j = 0 & \\
 & q_i = \sum\limits_{j \neq i} q_{ij} \quad i = 1, 2, \cdots, n. & \\
 & | \, q_{ij}(q_1, q_2, \cdots, q_{n-1}) \, | \leq C_{ij} \quad 1 \leq i < j \leq n & \\
 & q_i \geq 0 \quad \forall i \in N_S \cup N_D. &
\end{array}
\tag{5}
$$

where N_D *and* N_S *denote the demand node set and the supply node set, respectively; and* $D_i(v)$ *and* $v_i(q) \equiv D_i^{-1}(q)$ *are the true supply cost function and the true inverse supply cost function at supply node* i*, respectively.*

We summarize the above as a proposition and provide the proof in the appendix.

Proposition 1 *Suppose all network users purchase insurance with strike price revealing their true costs, i.e.* $c_i^*(v) = v$*. Then we have* $\widetilde{D}_i(c^*(v)) = D_i(v)$ *where* $D_i(v)$ *is the true cost curve at node* i*, and the solutions* $(q_1^*(s), q_2^*(s), \cdots, q_n^*(s))$ *of the ISO problems (*$ISO1$ *&* $ISO2$*) are also the corresponding social welfare maximizing solutions.*

Proof. See the Appendix. ∎

The concept of *coherent* insurance scheme in the multiple spot markets case is similarly defined as in **Definition 1** with $(NU2)$ replacing $(NU1)$ and $(ISO2)$ replacing $(ISO1)$.

Remark 1 *The existence of a* **coherent** *insurance scheme is not difficult to demonstrate. Suppose the insurance premium function* $X(c)$ *offered by the ISO is a very large positive constant such that no one would purchase any insurance. Then the ISO ends up with maximizing interzonal ex post revenue and resolves intrazonal congestion by randomly rationing network access among all requests. By definition, this large positive constant premium along with the scheme of maximizing interzonal revenue and the random rationing as a tie-breaker for access curtailment is a coherent insurance scheme.*

2.3 The choice of premium function X(c)

It is important to note that a proper choice of the insurance premium function $X(c)$ by the ISO is key to the performance of our scheme in terms of short-term economic efficiency. The premium function serves a dual purpose. On one hand, it provides the self-selection incentives so that different premium functions may lead to different insurance purchase distributions with different social welfare implications. At the same time the premium provides a source of revenue that would be needed to finance insurance compensation to curtailed load or to serve as a source of revenue for transmission asset owners. The optimal choice of the premium function depends on the specific objective, whether it is efficiency, revenue maximization for the transmission owners or some other criteria. In this paper we will not deal with the optimization of the premium function but rather explore the implications of special cases of such functions. In particular we examine a special case where the ISO chooses $X(c)$ as the expected benefit accrued to a transaction unit, with true cost $v = c$, from physical access to the grid. Such a premium function can be computed based on spot price forecast or be obtained implicitly by auctioning off transmission access insurance contracts with specified denominations of c. In such an auction risk neutral network users with no market power will bid their perceived actuarial value of the network access contracts and the resulting premium function $X(c)$ will reflect market consensus. Such a premium function is by no means optimal. In fact, it is heavily biased in favor of transmission ownership and under truthful revelation of generation cost it would award all the gains from trade to the transmission owners. Yet, it has some interesting properties that will be discussed below and it presents a useful benchmark for the efficiency properties of the proposed approach which will be explored via examples. In the following proposition we show that, under the above premium function, no transaction unit would have any incentive to overinsure its access to the network. Namely, the optimal solution, $c^*(v)$, to the self-selection problem is always no less than the true cost v. One of the implications of this result is that there is no adverse selection of revealed injection node at **stage three** where users submit their preferred schedules. If a user were to overinsure, there might be an incentive to reveal a injection request at a congestion prone node within a zone in order to receive curtailment compensation when the user would have curtailed supply voluntarily due to low spot price realization. But with underinsurance compensation is never paid when the users' true costs exceed the spot price and hence there is no incentive for misrepresenting the injection node.

Proposition 2 *If the ISO chooses*

$$X_m(c) = E_{S_m}(Max(S_m \Leftrightarrow c, 0)) \tag{6}$$

as the insurance premium function in each zone m, then $c^*(v) > v$ for any transaction unit with true cost v which can achieve a positive objective function value in solving the self-selection problem ($NU1$ or $NU2$), where $c^*(v)$ is the optimal solution to problem ($NU1$ or $NU2$) for the transaction unit with true cost v.

Proof. Consider a transaction unit with true cost v which can achieve a positive objective function value in solving problem *NU2*. It is sufficient to consider only those insurance levels having a positive probability of getting dispatched. If the unit is overinsured, i.e. $c < v$, then the following is true,

$$\begin{aligned}
0 &= E_{S_m}(Max(S_m - c, 0)) - X_m(c) \qquad \text{(By the definition of } X_m(c)\text{)} \\
&= \int_{\Omega_i(c)} (s_{m(i)} - c)^+ dG(s_1, s_2, \cdots, s_k) + \int_{\overline{\Omega}_i(c)} (s_{m(i)} - c)^+ dG(s_1, \cdots, s_k) - X_{m(i)}(c) \\
&= \int_{\Omega_i(c)} [p_i(c, \overline{s}) \cdot (s_{m(i)} - c)^+ + (1 - p_i(c, \overline{s})) \cdot (s_{m(i)} - c)^+] dG(s_1, \cdots, s_k) \\
&\qquad + \int_{\overline{\Omega}_i(c)} (s_{m(i)} - c)^+ dG(s_1, \cdots, s_k) - X_{m(i)}(c) \\
&> \int_{\Omega_i(c)} [p_i(c, \overline{s}) \cdot (s_{m(i)} - v)^+ + (1 - p_i(c, \overline{s})) \cdot (s_{m(i)} - c)^+] dG(s_1, \cdots, s_k) \\
&\qquad + \int_{\overline{\Omega}_i(c)} (s_{m(i)} - c)^+ dG(s_1, \cdots, s_k) - X_{m(i)}(c) \\
&\text{(since } (s_{m(i)} - c)^+ > (s_{m(i)} - v)^+ \text{ and the probability measure of } \Omega_i(c) \text{ is positive)}
\end{aligned}$$

The expression to the right of the last inequality sign is the objective function in the self-selection problem (NU2). Since this objective can achieve value zero under true insurance $c = v$, it follows that $c^*(v) \geq v$. ∎

Remark 2 *This result can be understood as a "winner's curse" result under a pay-your-bid pricing scheme. With the presence of network externality, the "winner's curse" usually leads to inefficient allocation of scarce transmission capacity due to the distortion in the offered supply functions at all supply nodes.*

In the next section, we shall demonstrate that the first best solutions can be achieved as possible equilibria of our priority insurance mechanism if a different functional form of the insurance premium function $X(c)$ is employed.

2.4 Convergence to social welfare maximization

Suppose a different insurance premium schedule is indeed offered at each node therefore resulting in a different dispatch contingency set $\Omega_i(c)$ and a corresponding dispatch probability function $(p_i(c, \overline{s}) : \overline{s} \in \Omega_i(c))$ at each node i, then we can find a premium function such that our proposed priority insurance scheme yields first best solutions.

In order to establish the limiting behavior of our approach in going from zones to nodes we first need to specify conditions on zonal aggregation so that our underlying assumption of exogenous random zonal spot prices will make sense at all levels of aggregation. As indicated earlier we attribute the randomness of the zonal prices to random demand shocks. This requires that every zone at which an exogenous spot price is specified have at least one demand node. Zones that do not contain a demand node will only be allowed to contain a single supply node and the spot prices in such zones will be endogenously determined. The endogenous market prices, denoted by S_j, at every supply node j serve as settlement prices for transactions in the respective single-node zones as well as for the computation of the insurance compensation to curtailed injections. We shall demonstrate that for a set of properly chosen $\{S_j, j \in N_S\}$ there exists a *coherent* priority insurance scheme achieving the first best solutions in equilibrium. First we characterize the appropriate set of $\{S_j, j \in N_S\}$. For each $\omega \in \Omega$, there exists a solution $\tilde{q}(\omega) \equiv (\tilde{q}_1(\omega), \tilde{q}_2(\omega), \cdots, \tilde{q}_n(\omega))$ to the (ED) problem (5) described in Definition 2. For simplicity, we assume that the true supply functions $\{D_j(v), j \in N_S\}$ are continuous and nowhere flat. With $D_j^{-1}(q)$ (or, $v_j(q)$) being the true inverse supply function at a supply node $j \in N_S$, we define the spot market price S_j to be

$$S_j(\omega) = D_j^{-1}(\tilde{q}_j(\omega)) \qquad \forall \omega \in \Omega. \tag{7}$$

Namely, the endogenous zonal spot prices assigned to isolated supply nodes are defined as the corresponding marginal costs of supply at these nodes under optimal dispatch, given all the exogenous spot price realizations at all zones containing demand nodes.

As the number of zones approaches the number of nodes, the ISO ends up offering a possibly different insurance premium schedule at each node. Suppose that the ISO offers a node-specific insurance scheme $\{c, X_j(c)\}$ at every node $j \in N_S$. Under each contingency $\omega \in \Omega$, a transaction unit with insurance c at any supply node j may request network access. If the network access is granted by the ISO, then the unit realizes a benefit of $\max(S_j(\omega) \Leftrightarrow v, 0)$. If the access request is denied, then the transaction unit is paid an amount of $\max(S_j(\omega) \Leftrightarrow c, 0)$ as the insurance compensation by the ISO. Given $\omega \in \Omega$, the

ISO's objective is to minimize the total insurance payments less the *ex post* cross-zone fees subject to transmission constraints, i.e.

$$
\begin{array}{lll}
\min\limits_{\{q_i: i \in N_S \cup N_D\}} & \sum\limits_{j \in N_S} \int_{q_j}^{\widetilde{D}_j(s_j)} [s_j \Leftrightarrow \widetilde{v}_j(q)] dq \Leftrightarrow \frac{1}{n} \sum\limits_{1 \leq l < m \leq n} (s_l \Leftrightarrow s_m)(q_l \Leftrightarrow q_m) & \\
s.t. & \sum_{i \in N_D} q_i \Leftrightarrow \sum_{j \in N_S} q_j = 0 & \\
 & q_i = \sum_{j \neq i} q_{ij} \quad i = 1, 2, \cdots, n. & (8) \\
 & \mid q_{ij}(q_1, \cdots, q_{n-1}) \mid \leq C_{ij} \quad 1 \leq i < j \leq n & \\
 & q_i \geq 0 \quad \forall i \in N_S \cup N_D. &
\end{array}
$$

where $\{S_j, j \in N_S\}$ is defined in (7). We conjecture that the dispatch contingency set $\Omega_j(c)$ associated with the insurance level c at node j is given by $\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}$ and the dispatch probability function is $(p_j(c, \overline{s}) \equiv 1 : \overline{s} \in \Omega_j(c))$. We next show that the set of endogenous spot prices $\{S_j, j \in N_S\}$ defined in (7), the insurance schedule $\{c, X(c) \equiv 0\}$, and the dispatch contingency sets and probability functions $\{\{\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}, (p_j(c, \overline{s}) \equiv 1 : \overline{s} \in \Omega_j(c))\}, j \in N_S\}$ form a rational expectations equilibrium.

Lemma 1 *Given $\{S_j, j \in N_S\}$ as defined in (7), $\{\{\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}, (p_j(c, \overline{s}) \equiv 1 : \overline{s} \in \Omega_j(c))\}, j \in N_S\}$ and $\{c, X(c) \equiv 0\}$, then $c^*(v) = v$ is an optimal solution to the self-selection problem (3) for a transaction unit with true cost v at any node $j \in N_S$.*

Proof. Given $\{\{\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}, (p_j(c, \overline{s}) \equiv 1 : \overline{s} \in \Omega_j(c))\}, j \in N_S\}$. Consider a transaction unit with true cost v at any node $j \in N_S$.

For any $c > v$, the objective function value of (3) is

$$
\begin{aligned}
U(c) &\equiv \int_{\Omega_j(c)} \max(s_j \Leftrightarrow v, 0) dG(s_1, \cdots, s_n) + \int_{\overline{\Omega}_j(c)} \max(s_j \Leftrightarrow c, 0) dG(s_1, \cdots, s_n) \Leftrightarrow X_j(c) \\
&= \int_{\{\omega : S_j(\omega) \geq c\}} (s_j \Leftrightarrow v)^+ dG(s_1, \cdots, s_n) + \int_{\{\omega : S_j(\omega) < c\}} (s_j \Leftrightarrow c)^+ dG(s_1, \cdots, s_n) \Leftrightarrow 0 \\
&= \int_c^{\infty} (s_j \Leftrightarrow v) dG(s_j)
\end{aligned}
$$

The first derivative of $U(c)$ is

$$
\begin{aligned}
U'(c) &= \frac{d}{dc} \int_c^{\infty} (s_j \Leftrightarrow v) dG(s_j) \\
&= (v \Leftrightarrow c) g(c)
\end{aligned}
$$

where $g(s_j)$ is the density function of $G(s_j)$. The fact of $g(c) > 0$ implies that $U'(c) < 0$ for $c > v$, i.e. $U(c)$ is strictly decreasing in c for $c > v$. Therefore,

$$U(v) > U(c) \qquad \forall c > v$$

For any $c \leq v$, the objective function value of (3) is

$$\begin{aligned}
U(c) &= \int_{\Omega_j(c)} \max(s_j \Leftrightarrow v, 0) dG(s_1, \cdots, s_n) + \int_{\overline{\Omega_j(c)}} \max(s_j \Leftrightarrow c, 0) dG(s_1, \cdots, s_n) \Leftrightarrow X_j(c) \\
&= \int_{\{\omega : S_j(\omega) \geq c\}} (s_j \Leftrightarrow v)^+ dG(s_1, \cdots, s_n) + \int_{\{\omega : S_j(\omega) < c\}} (s_j \Leftrightarrow c)^+ dG(s_1, \cdots, s_n) \Leftrightarrow 0 \\
&= \int_c^v (s_j \Leftrightarrow v)^+ dG(s_j) + \int_v^\infty (s_j \Leftrightarrow v) dG(s_j) \\
&= U(v)
\end{aligned}$$

Therefore, $c^*(v) = v$ is an optimal solution (although not unique) to the self-selection problem (3) for a transaction unit with true cost v at node j. ∎

According to Lemma 1, truth-telling is one optimal solution to the network user self-selection problems given the rational expectations of $\{S_j, j \in N_S\}$ and $\{\{\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}, (p_j(c, \overline{s}) \equiv 1 : \overline{s} \in \Omega_j(c))\}, j \in N_S\}$. When all the network users (or, transaction units) truthfully reveal their costs, the ISO ends up with the whole set of true cost functions $\{v_j(q), j \in N_S\}$. For any contingency $\omega \in \Omega$, given the rational expectations of $\{S_j, j \in N_S\}$ and $\{\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}, j \in N_S\}$, the amount of network access requests is $\tilde{q}_j(\omega) = D_j(S_j(\omega))$ at node $j \in N_S$. Since $\tilde{q}(\omega) \equiv (\tilde{q}_1(\omega), \tilde{q}_2(\omega), \cdots, \tilde{q}_n(\omega))$ is an optimal solution to the (ED) problem (5), by Proposition 1, it is also an optimal solution to the ISO insurance compensation minimization problem (8). Hence, the set of endogenous spot prices $\{S_j, j \in N_S\}$ defined in (7), the insurance schedule $\{c, X(c) \equiv 0\}$, and the dispatch contingency sets and probability functions $\{\{\Omega_j(c) = \{\omega : S_j(\omega) \geq c\}, (p_j(c, \overline{s}) \equiv 1 : \overline{s} \in \Omega_j(c))\}, j \in N_S\}$ form a rational expectations equilibrium. This completes the proof of the following proposition.

Proposition 3 *If each node of a transmission network is considered as a zone, then there exist both a set of endogenous spot prices and a coherent priority insurance scheme which yields the first best solutions.*

The above result shows that as the number of zones approaches the number of nodes our insurance scheme will approach the nodal pricing first best result. In order to achieve this result within our framework the premium function has to be reduced to zero which eliminates the source of revenue for payment of compensation. However, the equilibrium derived above is such that generators reveal their true supply costs, the ISO solves the economic dispatch problem whereas the spot prices at all the supply nodes are endogenously set to the corresponding marginal supply cost at each node under economic dispatch. Hence under that equilibrium there will not be curtailed injection with values of c exceeding the spot prices at the supply nodes and we end up with a degenerate case where compensation payments are zero. The net revenue collected by the ISO amounts to the *ex post* nodal price differences under economic dispatch which is exactly the congestion rents under Hogan's nodal pricing approach.

3 Numerical examples

One expects that the optimal premium function lies somewhere between the zero premium case which with full nodal desegregation yields the first best solution and the case of a premium function charging network users for the full actuarial value of transmission access benefits. Unfortunately, when nodes are aggregated into zones the degenerate zero premium function that works so well in the nodal case will not produce the needed incentives to elicit self-selection that could guide the ISO in rationing scarce transmission resources. Hence, we will use the latter premium function that errs in favor of incentives (versus efficiency) as a benchmark for the efficiency losses of the proposed second best approach. While general bounds on such losses would be desirable we were not able to obtain such general results. Hence we resort to simple, but by no means trivial, examples to explore the efficiency properties of *coherent* priority insurance schemes under zonal aggregation. In this section, we take a classical three-node network (Figure 3) with one spot market to show how a *coherent* priority insurance scheme is obtained. We then compute the efficiency loss of the particular scheme with respect to the economic dispatch solution. As previously mentioned, we use a DC-flow approximation and assume no losses in all our examples[9]. In the specific three-node network, each transaction is uniquely characterized by its injection node since we only consider one net demand node. As an example in the case of multiple spot markets, we present

[9]When losses or voltage constraints are considered, it is just a matter of incorporating them into the constraints of the problems of (ED) and (ISO1&2). This only complicates the calculation of the rational expectations equilibrium. The implementation of our scheme remains the same.

a four-node network with two spot markets (two zones) and explore the efficiency properties when users' choices are restricted to one and two discrete levels of insurance in each zone.

3.1 Single spot market: three-node network

Consider a three-node network with transmission line capacity

$$(C_{12}, C_{13}, C_{23}) = (136MW, 300MW, 254MW)$$

and equal admittance of 1. Node 3 is the location of the spot market with uniformly dis-

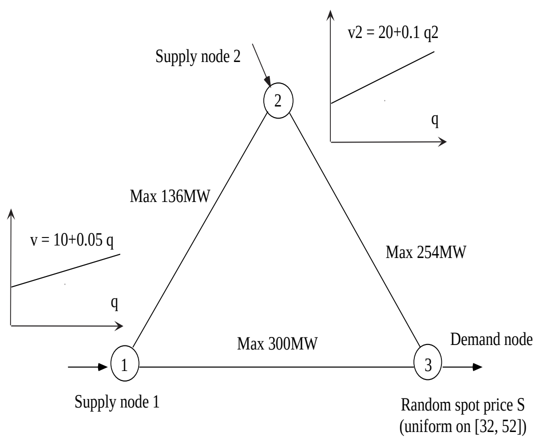

Figure 3: A three-node network

tributed random spot price $S \sim U(32, 52)$ and the cumulative distribution function of S is:

$$G(s) \quad = \quad \begin{cases} 0 & , \quad s \leq 32 \\ \dfrac{s \Leftrightarrow 32}{20} & , \quad 32 < s \leq 52 \\ 1 & , \quad s > 52 \end{cases} \tag{9}$$

We first compute the economic dispatch (first best) solution for each realization of the spot price S and the expected social welfare (gain from trade) of the first best solution. A social planner's objective of maximizing social welfare is equivalent to minimizing the shaded areas representing the displacement costs, as depicted in Figure 4[10]. Therefore, a social welfare

[10]If there is no transmission constraint, then the area of the shaded regions is zero for the social welfare maximizing dispatch solutions.

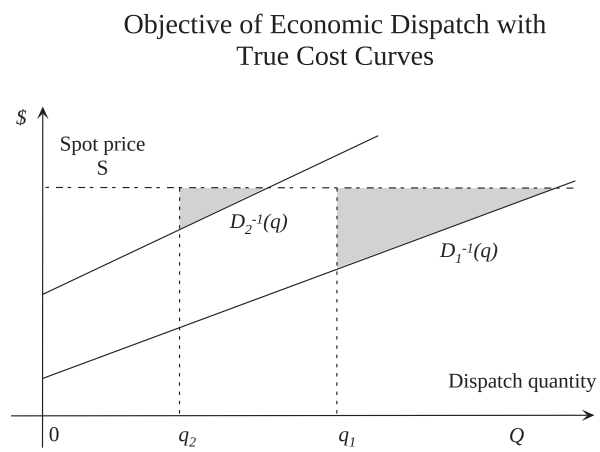

Figure 4: Objective of economic dispatch

maximizing ISO solves the following problem to obtain the economic dispatch for a realized $S(\omega) = s$:

$$\begin{aligned}
&SW(s) \equiv \min_{(q_1,q_2,q_3)} \sum_{i=1}^{2} \int_{q_i}^{D_i(s)} [s \Leftrightarrow v_i(q)]dq \\
&s.t. \sum_{i=1}^{2} q_i \Leftrightarrow q_3 = 0 \\
&\begin{pmatrix} \Leftrightarrow 136 \\ \Leftrightarrow 254 \\ \Leftrightarrow 300 \end{pmatrix} \leq \begin{pmatrix} \frac{1}{3} & \Leftrightarrow \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \\ \Leftrightarrow \frac{2}{3} & \Leftrightarrow \frac{1}{3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \leq \begin{pmatrix} 136 \\ 254 \\ 300 \end{pmatrix} \\
&q_i \geq 0, \quad i = 1, 2, 3.
\end{aligned} \tag{10}$$

where

$$\begin{pmatrix} 1/3 & \Leftrightarrow 1/3 \\ 1/3 & 2/3 \\ \Leftrightarrow 2/3 & \Leftrightarrow 1/3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

represents the power-flow functions on the three transmission lines. The solution is given by

$$\begin{cases} \widehat{q}_1 = \frac{20}{9}(210 \Leftrightarrow s) \\ \widehat{q}_2 = \frac{20}{9}(2s \Leftrightarrow 15) \end{cases}, \quad 32 \leq s \leq 52 \tag{11}$$

And the expected social welfare is $E[SW] = 10697$.

We now turn to the computation of a *coherent* priority insurance scheme where the ISO posts the insurance premium function $X(c)$ in the form of (6). The economic interpretation of $X(c)$ is that it equals the expected benefit accrued to a transaction with true unit cost

c receiving physical access to the network and hence avoiding a settlement cost at the spot market price. This premium can also be interpreted as the actuarial value of a financial "call option" with strike price c with respect to the underlying spot market.

$$\begin{aligned} X(c) &= E_S[\max(S \Leftrightarrow c, 0)] \\ &= \begin{cases} 42 \Leftrightarrow c & , \quad 0 \leq c \leq 32 \\ \frac{1}{40}(52 \Leftrightarrow c)^2 & , \quad 32 < c \leq 52 \\ 0 & , \quad c > 52 \end{cases} \end{aligned} \tag{12}$$

where the cumulative distribution function of S is given by (9). We conjecture that a network

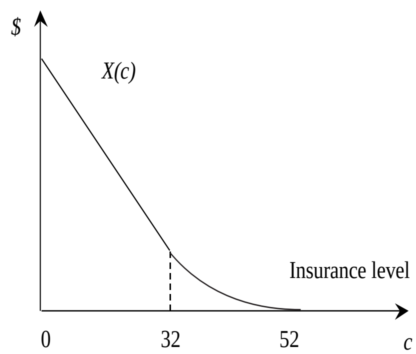

Figure 5: Insurance premium function

user who selects insurance level (or insured cost) c for a transaction injected at node 1 expects the transaction unit to get access for $S \in [\max(32, v), S_1(c)]$ where $S_1(c) = k_1 \Leftrightarrow k_2 c$ with k_1 and k_2 being parameters and $p_1(c, s) \equiv 1$ for $s \in [\max(32, v), S_1(c)]$. The degrees of freedom in computing the rational expectations equilibrium allow us to parameterize the contingency set under which access is provided in terms of a two parameter linear function defining $S_1(c)$[11]. Then the optimal insurance c chosen by a transaction unit with true cost v injected at node 1 is determined by the self-selection problem.

$$\begin{aligned} c_1^*(v) &= \arg\max_c \int_{\max(v,32)}^{S_1(c)} (s \Leftrightarrow v) dG(s) + \int_{S_1(c)}^{52} (s \Leftrightarrow c) dG(s) \Leftrightarrow X(c) \\ &= \begin{cases} \underline{c}_1 & , \quad v \leq v_1' \\ \frac{(k_1 - 32) + k_2 v}{2k_2} & , \quad v_1' < v \leq v_1'' \\ \overline{c}_1 & , \quad v > v_1'' \end{cases} \end{aligned} \tag{13}$$

[11]We constrain ourselves to compute for the rational expectations equilibrium in the form of linear functions. There may exist equilibrium of other functional forms.

where $\underline{c}_1/\overline{c}_1$ is the lowest/highest insurance level that a transaction unit would optimally purchase; and v_1'/v_1'' is the true cost of the marginal transaction unit which self-selects the insurance $\underline{c}_1/\overline{c}_1$. Similarly, we conjecture the spot price interval for which a unit transaction with insurance level c injecting at node 2 gets access to be $[S_2(c), 52]$ where $S_2(c) = k_3 + k_4 c$ with k_3 and k_4 being parameters and $p_2(c,s) \equiv 1$ for $s \in [S_2(c), 52]$. Then the optimal insurance level for a transaction unit with true cost v injected at node 2 is given by the following solution.

$$\begin{aligned} c_2^*(v) &= \arg\max_c \int_{\max(c,32)}^{S_2(c)} (s \Leftrightarrow c) dG(s) + \int_{S_2(c)}^{52} (s \Leftrightarrow v) dG(s) \Leftrightarrow X(c) \\ &= \begin{cases} \underline{c}_2 & , \quad v \le v_2' \\ \frac{(52-k_3)+k_4 v}{2k_4} & , \quad v_2' < v \le v_2'' \\ \overline{c}_2 & , \quad v > v_2'' \end{cases} \end{aligned} \tag{14}$$

where $\underline{c}_2/\overline{c}_2$ and v_2'/v_2'' are the counterparts of $\underline{c}_1/\overline{c}_1$ and v_1'/v_1''. For each realization of S, we have the marginal insurance levels c_1 and c_2 being granted network access at node 1 and 2, respectively, such that $S = k_1 \Leftrightarrow k_2 c_1(S) = k_3 + k_4 c_2(S)$. The resulting inverse insurance distribution curves $\widetilde{D}_i^{-1}(q)$ $(i = 1, 2)$ indicate the insurance level c at which there are no more than q transaction units having insurance level higher than c. The shapes of $\widetilde{D}_i^{-1}(q)$ $(i = 1, 2)$ are illustrated in Figure 6. When the random spot price is revealed, network users

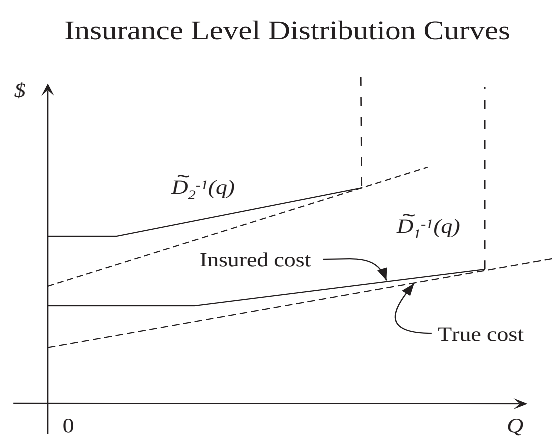

Figure 6: Insured cost distribution curves

submit their usage requests along with their insurance levels. Therefore, the above insurance distribution curves $\widetilde{D}_i(c)$ $(i = 1, 2)$ are revealed to the ISO. In case of network congestion, the ISO determines dispatch schedules based on the criterion of minimizing total curtailment compensation payments.

$$\begin{aligned}
& IP(s) \equiv \min_{(q_1,q_2,q_3)} \sum_{i=1}^{2} \int_{q_i}^{\widetilde{D}_i(s)} [s \Leftrightarrow \widetilde{D}_i^{-1}(q)]dq \\
& s.t.\ q_1 + q_2 \Leftrightarrow q_3 = 0 \\
& \begin{pmatrix} \Leftrightarrow 136 \\ \Leftrightarrow 254 \\ \Leftrightarrow 300 \end{pmatrix} \leq \begin{pmatrix} \frac{1}{3} & \Leftrightarrow\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \\ \Leftrightarrow\frac{2}{3} & \Leftrightarrow\frac{1}{3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \leq \begin{pmatrix} 136 \\ 254 \\ 300 \end{pmatrix} \\
& q_i \geq 0, \quad i = 1,2,3.
\end{aligned} \tag{15}$$

The solution of (15) is

$$\begin{cases} q_1^* = \frac{20}{k_2}(30 + k_1 \Leftrightarrow 10k_2 \Leftrightarrow 2s) \\ q_2^* = \frac{10}{k}(2s \Leftrightarrow k_3 \Leftrightarrow 20k_4 \Leftrightarrow 29) \end{cases}, \quad 32 \leq s \leq 52 \tag{16}$$

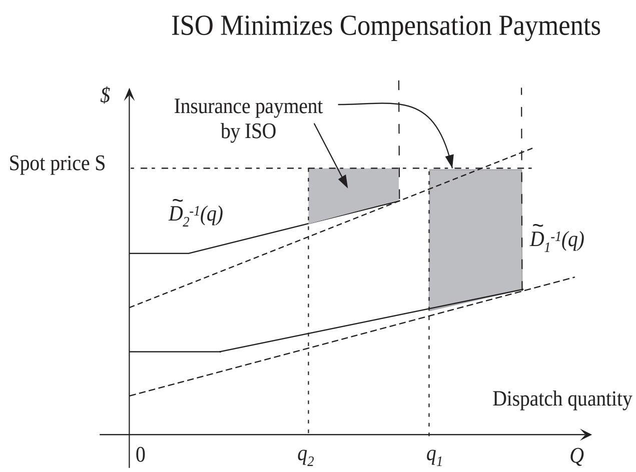

Figure 7: Objective of the ISO's minimization problem

Figure 7 gives a graphic representation of the ISO's objective of minimizing total insurance payments based on the revealed insured cost distribution curves.

Invoking the equilibrium condition c) in **Definition 1**, we can solve for the free parameters of the rational expectations equilibrium to obtain $\{k_1 = 317.78, k_2 = 9, k_3 = \Leftrightarrow 39.72, k_4 = 2.25\}$. The resulting network access contingencies corresponding to the rational expectations equilibrium are characterized by the boundary functions $\{S_i(c), i = 1,2\}$, given by:

$$\left\{\begin{array}{l} S_1(c) = \left\{\begin{array}{ll} \Leftrightarrow 9c + 317.78 & \text{for } c \in [29.53, 31.75] \\ 32 & o.w. \end{array}\right. \\ S_2(c) = \left\{\begin{array}{ll} 2.25c \Leftrightarrow 39.72 & \text{for } c \in [31.88, 40.77] \\ 52 & o.w. \end{array}\right. \end{array}\right. \quad (17)$$

The spot price contingency sets under which network access is granted to each insurance level at the two supply nodes are illustrated in Figure 8. We substitute the solution $\{k_i, i = 1, 2, 3, 4\}$ into (16) and get the induced dispatch schedules $\{(q_1^*(s), q_2^*(s)), 32 \leq s \leq 52\}$ under the above priority insurance scheme . The expected social welfare of the induced schedules is $E[SW^*] = 10593$. This amounts to only 0.974% efficiency loss. For this simple example, our calculation shows that the efficiency loss associated with the minimum compensation dispatch solution under the priority insurance scheme is rather small as compared to the first best solution. Figure 9 illustrates a comparison between the economic dispatch (first best) solution and the minimum compensation (second best) solution for every realization of the spot price S. The x-axis and y-axis represent the quantities of injected power at node 1 (q_1) and node 2 (q_2), respectively. The shaded region consists of all the feasible dispatch solutions of (q_1, q_2) subject to transmission constraints[12].

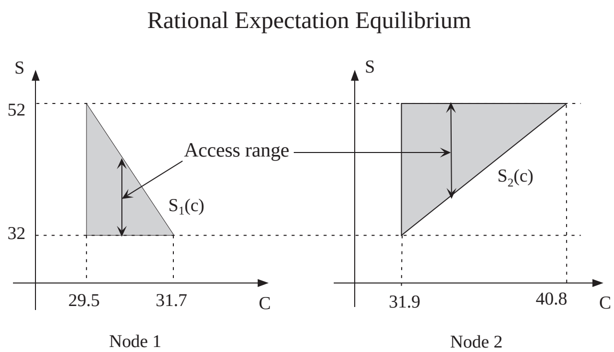

Figure 8: Rational expectation of network access price interval

To check the robustness of the above result we performed a modest sensitivity analysis calculating the efficiency loss for slightly varied different sets of parameters. Basically we vary the fixed costs of the true cost distribution curves so as to change the difference between

[12]The fact that both the first best solutions and the second best solutions lie on the boundary line of $2q_1 + q_2 = 900$ indicates that line 1-3 is congested under both the economic dispatch and the insurance compensation minimizing dispatch.

First Best .vs. Second Best

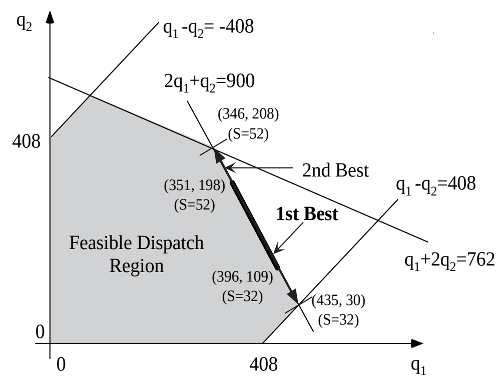

Figure 9: The comparison between 1st best and 2nd best solutions

true supply functions at the different supply nodes. The computation indicates that the efficiency losses are still of similar magnitudes (see Table 1).

Parameter set	$\begin{cases} v_1 = 9 + 0.05q_1 \\ v_2 = 22.5 + 0.1q_2 \end{cases}$	$\begin{cases} v_1 = 10 + 0.05q_1 \\ v_2 = 20 + 0.1q_2 \end{cases}$	$\begin{cases} v_1 = 10 + 0.05q_1 \\ v_2 = 23 + 0.1q_2 \end{cases}$
Efficiency loss	0.972 %	0.974 %	1.01 %

Table 1: Sensitivity of Efficiency Loss

In the above calculation we assumed that customers have a continuum of choices for the strike-price. In a realistic setting, as in the case of typical automobile insurance, one would expect a limited number of discrete options for the level of deductible (strike-price). One would expect that such restrictions may further increase the efficiency losses of the proposed scheme yet an important question is "what is the magnitude of such losses?". In the following section we explore this question within the context of somewhat more complex examples with four and six nodes, two congestion zones and up to two insurance levels in each zone. Unfortunately, calculating the equilibrium for the continuous strike-price selection for these examples is computationally prohibitive.

3.2 Multiple spot markets: 4-node and 6-node networks

We now turn to the multiple zonal spot markets case. Consider a 4-node network with two spot markets and two supply nodes as shown in Figure 10. Node 1 and 4 belong to zone

one while node 2 and 3 belong to zone two. The link 3-4 connecting the two zones is the only congested link with line-flow capacity of 80MW. All lines are of equal impedance of one. We assume the spot prices in zone 1 and zone 2 are jointly uniformly distributed over

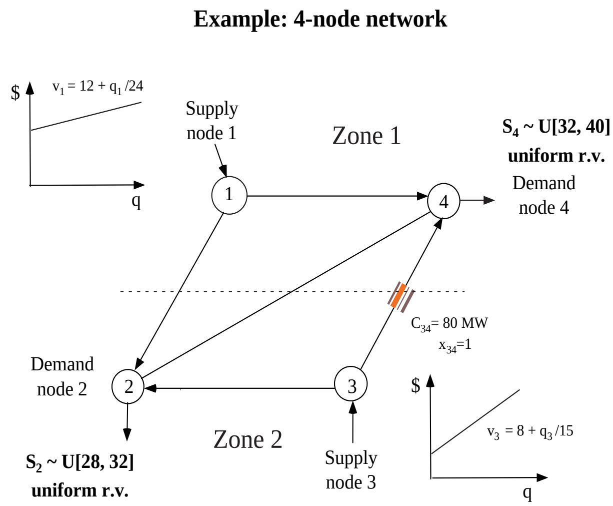

Figure 10: An example of four-node network

interval $[32, 40] \times [28, 32]$. The marginal distributions are $S_4 \sim U[32, 40]$ and $S_2 \sim U[28, 32]$, respectively. The true marginal cost functions at node 1 and node 3 are:

$$\begin{cases} v_1 = 12 + q_1/24 \\ v_3 = 8 + q_3/15 \end{cases}$$

The economic dispatch (first best) solutions for any given spot prices (s_2, s_4) are given by

$$\begin{cases} \widehat{q}_1 = 12(s_2 + s_4 \Leftrightarrow 24) \\ \widehat{q}_2 = \dfrac{\Leftrightarrow 3056 + 399 s_2 \Leftrightarrow 201 s_4}{4} \\ \widehat{q}_3 = \dfrac{15(5 s_2 \Leftrightarrow 3 s_4 \Leftrightarrow 16)}{2} \\ \widehat{q}_4 = \dfrac{1424 \Leftrightarrow 201 s_2 + 159 s_4}{4} \end{cases}$$

The expected social welfare of the first best solutions is $E_S[SW] = 8652$.

We consider the simplest situation where the ISO offers a one-level insurance scheme in the two zones, i.e. a network user's choice of insurance level is restricted to $\{c^1, x_i^1\}$ or $\{\infty, 0\}$

in zone i $(i = 1, 2)$. The premia x_i^1 are given by $X_i(c^1)$ $(i = 1, 3)$ where

$$
\begin{aligned}
X_1(c) &= E_{S_4}[\max(S_4 \Leftrightarrow c, 0)] \\
&= \begin{cases} 36 \Leftrightarrow c & , \quad c \leq 32 \\ \frac{1}{16}(40 \Leftrightarrow c)^2 & , \quad 32 < c \leq 40 \\ 0 & , \quad c > 40 \end{cases}
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
X_3(c) &= E_{S_2}[\max(S_2 \Leftrightarrow c, 0)] \\
&= \begin{cases} 30 \Leftrightarrow c & , \quad c \leq 28 \\ \frac{1}{8}(32 \Leftrightarrow c)^2 & , \quad 28 < c \leq 32 \\ 0 & , \quad c > 32 \end{cases}
\end{aligned}
\tag{19}
$$

The network users' self-selection problem amounts to the individual rationality condition, namely, $c_i^*(v) = c^1$ $(i = 1, 2)$ if and only if the expected benefit of purchasing c^1 is no less than 0. The ISO minimizing insurance compensation problem is

$$
\begin{aligned}
\min_{\{q_i; q_i^\alpha\}} \quad & \sum_{i=1,3} \sum_{\alpha=0}^{1} p_i^\alpha (\overline{q}_i^\alpha(s) \Leftrightarrow q_i^\alpha) \Leftrightarrow (s_4 \Leftrightarrow s_2)(q_3 \Leftrightarrow q_2) \\
s.t. \quad & q_1 + q_3 \Leftrightarrow q_2 \Leftrightarrow q_4 = 0 \\
& q_i = \sum_{\alpha=0}^{1} q_i^\alpha \quad (i = 1, 3) \\
& \mid q_{ij}(q_1, q_2, q_3) \mid \leq C_{ij} \\
& q_i^\alpha \leq \overline{q}_i^\alpha(s) \\
& q_i \geq 0, \; q_i^\alpha \geq 0
\end{aligned}
\tag{20}
$$

where

q_i^1 is the number of access requests with insurance at node i $(i = 1, 3)$

q_i^0 is the number of uninsured access requests at node i $(i = 1, 3)$

$\overline{q}_i^1(s)$ is the total number of access requests with insurance at node i $(i = 1, 3)$

$\overline{q}_i^0(s)$ is the total number of uninsured access requests at node i $(i = 1, 3)$

$p_1^1 = \max(s_4 \Leftrightarrow c^1, 0)$, $p_3^1 = \max(s_2 \Leftrightarrow c^1, 0)$ are the insurance payments

$p_i^0 = 0$ $(i = 1, 3)$

The term $(s_4 \Leftrightarrow s_2)(q_3 \Leftrightarrow q_2)$ in the objective function represents the *ex post* interzonal revenue (or cost) to the ISO since $(q_3 \Leftrightarrow q_2)$ is the net export (or import, depending on the sign of $(q_3 \Leftrightarrow q_2)$) from zone 2 to zone 1. By varying the insurance level c^1, we calculated several equilibrium solutions and found that the social welfare efficiency losses are not very sensitive to the choice of insurance level c^1. Taking $c^1 = 28.5$, the solutions to the ISO problem is

$$\begin{cases} q_1 = 396MW, & q_2 = 0MW \\ q_3 = 48.8MW, & q_4 = 444.8MW; \quad (5s_2 \Leftrightarrow 3s_4 \leq 57) \\ & \\ q_1 = 396MW, & q_2 = 646.75MW \\ q_3 = 307.5MW, & q_4 = 56.75MW; \quad (5s_2 \Leftrightarrow 3s_4 > 57) \end{cases}$$

which yield an expected social welfare of 7254. The calculation accounts for random rationing among insured access requests when transmission constraints prohibit scheduling of all such requests. The corresponding efficiency loss is equal to 16.15% which is roughly the smallest efficiency loss achievable with one insurance level.

We next consider a two-single-level insurance scheme with one level in each zone, i.e. network users' selections are restricted to $\{c_i^1, x_i^1\}$ or $\{\infty, 0\}$ in zone i $(i = 1, 2)$ and $c_1^1 \neq c_2^1$. The ISO insurance compensation minimization problem becomes:

$$\begin{aligned} \min_{\{q_i; q_i^\alpha\}} \quad & \sum_{i=1,3} \sum_{\alpha=0}^{1} p_i^\alpha(\overline{q}_i^\alpha(s) \Leftrightarrow q_i^\alpha) \Leftrightarrow (s_4 \Leftrightarrow s_2)(q_3 \Leftrightarrow q_2) \\ s.t. \quad & q_1 + q_3 \Leftrightarrow q_2 \Leftrightarrow q_4 = 0 \\ & q_i = \sum_{\alpha=0}^{1} q_i^\alpha \quad (i = 1, 3) \\ & \mid q_{ij}(q_1, q_2, q_3) \mid \leq C_{ij} \\ & q_i^\alpha \leq \overline{q}_i^\alpha(s) \\ & q_i \geq 0, \ q_i^\alpha \geq 0 \end{aligned} \tag{21}$$

where

q_i^1 is the number of access requests with insurance at node i $(i = 1, 3)$

q_i^0 is the number of uninsured access requests at node i $(i = 1, 3)$

$\overline{q}_i^1(s)$ is the total number of access requests with insurance at node i $(i = 1, 3)$

$\overline{q}_i^0(s)$ is the total number of uninsured access requests at node i $(i = 1, 3)$

$p_1^1 = \max(s_4 \Leftrightarrow c^1, 0)$, $p_3^1 = \max(s_2 \Leftrightarrow c^1, 0)$ are the insurance payments

$p_i^0 = 0$ $(i = 1, 3)$

For the instance of $c_1^1 = 30$ and $c_2^1 = 21$, we have $\{\overline{q}_1^1(s) = 432MW, \overline{q}_3^1(s) = 195MW\}$. The solutions to the ISO compensation minimization problem are:

$$\begin{cases} q_1 = 432MW, \quad q_2 = 0MW \\ q_3 = 41.6MW, \quad q_4 = 473.6MW; \quad (5s_2 \Leftrightarrow 3s_4 \leq 42) \\ \\ q_1 = 432MW, \quad q_2 = 383.5MW \\ q_3 = 195MW, \quad q_4 = 243.5MW; \quad (5s_2 \Leftrightarrow 3s_4 > 42) \end{cases}$$

The above dispatch schedules yield an expected social welfare of 8156.3 which amounts to an efficiency loss of 5.7% as compared to the expected social welfare of economic dispatch solutions. Note that given the rational expectations about the ISO minimum compensation dispatch over the corresponding spot price contingency regions, the expected benefits for a transaction unit with true cost v purchasing c_1^1 and c_2^1 are $(c_1^1 \Leftrightarrow v)$ and $91(c_2^1 \Leftrightarrow v)/150$, respectively. Therefore the marginal insurance-purchasing units at node 1 and node 2 have true costs of $v_1^* = c_1^1 = 30$ and $v_3^* = c_2^1 = 21$, respectively. By adding one more insurance level in each zone in the previous two-single-level insurance example, e.g. taking $c_1^1 = 30$ $c_1^2 = 31.5$ and $c_2^1 = 21$ $c_2^2 = 27$, we reduce the efficiency loss from 5.7% to 4.4%.

We further examine our scheme on a 6-node network which is similar to the example used in Chao and Peck [2]. A 6-node network divided into two interconnected zones is shown in Figure 11. Zone I consists of nodes 1, 2 and 3, and zone II consists of nodes 4, 5 and 6. Suppose that congestion only occurs on lines 1-6 and 4-6. The physical transmission

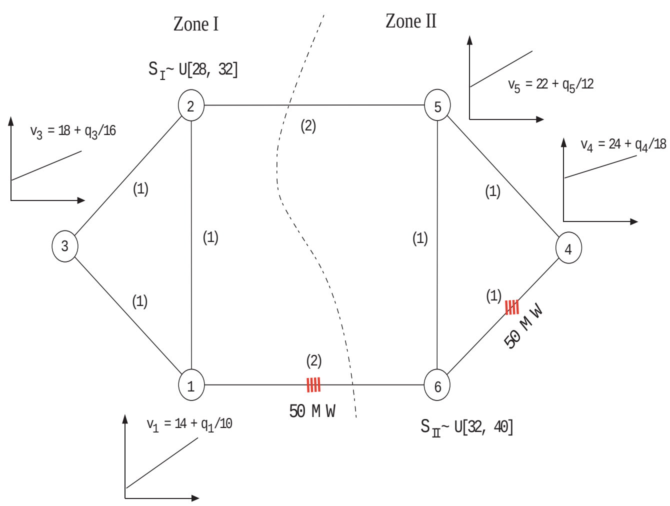

Figure 11: An example of six-node network

capacities for these two lines are 50 MW each. The line resistances are given by the numbers in parentheses.

We assume that nodes 1, 3, 4 and 5 are the supply nodes, and nodes 2 and 6 are the demand nodes. The marginal cost functions at these nodes are summarized in Table 2. Again, the spot prices in zone 1 and zone 2 are assumed to be jointly uniformly distributed

Node	Supply Function
1	$14 + q/10$
3	$18 + q/16$
4	$24 + q/18$
5	$22 + q/12$

Table 2: Supply functions

over interval $[28, 32] \times [32, 40]$. The respective marginal distributions are $S_6 \sim U[32, 40]$ and $S_2 \sim U[28, 32]$. Let the ISO offer a two-level priority insurance scheme in each zone where the insurance levels are $c_1^1 = 26.8 \;\; c_1^2 = 30$ and $c_2^1 = 26.8 \quad c_2^2 = 31$. The insurance premiums are given by (6). As we compute for the expected social welfare loss due to the ISO minimizing total insurance compensation, it is 4.58% of the expected social welfare of the first best solutions. We shall note that these insurance levels were not optimized to achieve the minimum efficiency loss. It is reasonable to expect that optimizing the insurance levels and adding more levels of insurance in each zone will further reduce the efficiency loss to an acceptable level.

4 Conclusion

At the intuitive level, our scheme can be viewed as a hybrid of priority insurance and a postage stamp approach. The different levels of insurance characterized by the revealed opportunity costs may be interpreted as postage stamps with different priorities. These priorities allow for network users' self-selection which in turn provides economic signals for the efficient rationing of scarce transmission resources. With a single zone in a transmission network, since we constrain the admissible insurance schemes to be uniform, we cannot expect a first best solution. Moreover, for the option-value based insurance premium function which is analogous to a pay-your-bid pricing scheme, efficiency losses also result from the "winner's curse" effect. However, if we partition a network into more zones and allow more different insurance premium schedules to be offered in different functional forms, the efficiency gains can be improved. The limiting case, where the insurance scheme is node specific, is equivalent to a nodal pricing approach. In essence, our scheme takes out part of the time and locational "price variability" present in a nodal pricing scheme and allows "quantity variability" in the form of uncertain access at a given price. Stable prices with a measure of uncertainty in service quality is a prevalent practice in most service industries. What is important to realize is that the proposed pricing scheme and the corresponding congestion management protocols are quite simple. The mathematical complexity is in attempting to simulate the market equilibrium. In reality that part is performed by the market itself.

Another important point to be made concerns the magnitude of the efficiency losses we have observed in our examples. Our calculations for the discrete priority levels resulted in efficiency losses ranging from 3-5% which are quite significant if such losses were indeed persistent. Fortunately that is not the case. These losses were calculated under the assumption that congestion exists within the zone. In reality, zonal boundaries are defined so that intrazonal congestion is rare. Typically, congestion may be present within a zone for at most 200 hours per year (about 2.5% of the time) which under a worse case scenario may represent 10% of net annual social surplus. Hence, a 5% efficiency loss during periods of intrazonal congestion would average to under 0.5% annual efficiency loss. This estimate is consistent with recent results by Green [7] who calculated efficiency losses due to zonal aggregation (with no intrazonal priority pricing) in the UK system and estimated these losses at under 1% of social welfare.

Appendix

Proof of Proposition 1

Proof. Notice that the (ED) problem and the $(ISO2)$ have the same set of constraints. It is therefore sufficient to show that the objectives of the two problems are equivalent provided $\widetilde{D}_i(c^*(v)) = D_i(v)$. Let $v_i(q)$ denote $D_i^{-1}(q)$.

$$
\begin{aligned}
& \max \sum_{i \in N_D} q_i \cdot s_{m(i)} \Leftrightarrow \sum_{i \in N_S} \int_0^{q_i} v_i(q)dq \qquad \text{(objective of the } (ED) \text{ problem)} \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot s_{m(i)} \Leftrightarrow \sum_{i \in N_S} \int_0^{D_i(s_{m(i)})} v_i(q)dq + \sum_{i \in N_S} \int_{q_i}^{D_i(s_{m(i)})} v_i(q)dq \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot s_{m(i)} + \sum_{i \in N_S} (D_i(s_{m(i)}) \Leftrightarrow q_i)s_{m(i)} \\
& \quad \Leftrightarrow \sum_{i \in N_S} (D_i(s_{m(i)}) \Leftrightarrow q_i)s_{m(i)} + \sum_{i \in N_S} \int_{q_i}^{D_i(s_{m(i)})} v_i(q)dq \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot s_{m(i)} + \sum_{i \in N_S} (D_i(s_{m(i)}) \Leftrightarrow q_i)s_{m(i)} \\
& \quad \Leftrightarrow \sum_{i \in N_S} \int_{q_i}^{D_i(s_{m(i)})} [s_{m(i)} \Leftrightarrow v_i(q)]dq \\
\Leftrightarrow \quad & \max \sum_{i \in N_D} q_i \cdot s_{m(i)} \Leftrightarrow \sum_{i' \in N_S} q_{i'} \cdot s_{m(i')} \Leftrightarrow IP \\
\Leftrightarrow \quad & \max \sum_{j=1}^{k} Q_j \cdot s_j \Leftrightarrow IP \\
\Leftrightarrow \quad & \min IP \Leftrightarrow \sum_{j=1}^{k} Q_j \cdot s_j \\
\Leftrightarrow \quad & \min IP \Leftrightarrow \frac{1}{k} \sum_{1 \le l < j \le k} (Q_l \Leftrightarrow Q_j)(s_l \Leftrightarrow s_j) \qquad \text{(objective of the } (ISO2) \text{ problem)}
\end{aligned}
$$

where

$$
\begin{aligned}
Q_k \quad & \equiv \quad \sum_{i \in Z_k \cap N_D} q_i \Leftrightarrow \sum_{j \in Z_k \cap N_S} q_j \text{ with } Z_k \text{ denoting the node set of zone } k. \\
IP \quad & \equiv \quad \sum_{i \in N_S} \int_{q_i}^{D_i(s_{m(i)})} [s_{m(i)} \Leftrightarrow v_i(q)]dq
\end{aligned}
$$

The last equivalent relationship utilizes the fact that $\sum_{i=1}^{k} Q_i = 0$. ∎

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