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Stochastic Models of Energy Commodity Prices and Their Applications: Mean-reversion with Jumps and Spikes

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Stochastic Models of Energy Commodity Prices and Their Applications: Mean-reversion with Jumps and Spikes

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Abstract

I propose several mean-reversion jump-diffusion models to describe spot prices of energy commodities that may be very costly to store. I incorporate multiple jumps, regime-switching and stochastic volatility into these models in order to capture the salient features of energy commodity prices due to physical characteristics of energy commodities. Prices of various energy commodity derivatives are derived under each model using the Fourier transform methods. In the context of deregulated electric power industry, I construct a real options approach to value physical assets such as generation and transmission facilities. The implications of modeling assumptions to the valuation of real assets are also examined.

I Introduction

Most of the existing literatures on modeling commodity prices deal with commodities which are storable. This is mainly because, until a few years ago, there had not been any traded commodities which were difficult to store. However, the presumption of all traded commodities being storable is no longer valid as electricity became a traded commodity in recent years.

Energy commodity markets grow rapidly as the restructuring of electricity supply industries is spreading in the United States and around the world. The volume of trade for electricity reported by US power marketers has increased almost fifty folds from 27 million MWh in 1995 to 1,195 million MWh in 1997 (Source: Edison Electric Institute). The global trend of electricity market reforms inevitably exposes the portfolios of generating assets and various supply contracts held by traditional electric power utility companies to market price risks. It has so far changed and will continue to change not only the way a utility company operates and manages its physical assets such as power plants, but also the way a utility company values and selects potential investment projects.

In general, risk management and asset valuation needs require in-depth understanding and sophisticated modeling of commodity spot prices. Primarily motivated by the surging demands for risk management and asset valuation due to deregulation in the $200 billion US electricity industry, I investigate the modeling of energy commodity prices in the cases where the underlying commodities may be very costly to store.

Among all the energy commodities, electricity poses the biggest challenge for researchers and practitioners to model its price behaviors. A distinguishing characteristic of electricity is that it can not be stored or inventoried economically once generated. Moreover, electricity supply and demand in a bulk electric power network has to be balanced continuously so as to prevent the network from collapsing. Since the supply and demand shocks can not be smoothed by inventories, electricity spot prices are volatile. In the summer of 1998, wholesale prices of electricity fluctuated between $0/MWh and $7000/MWh in the Midwest of US. It is not uncommon to see a 150% implied volatility in traded electricity options. Figure (1)

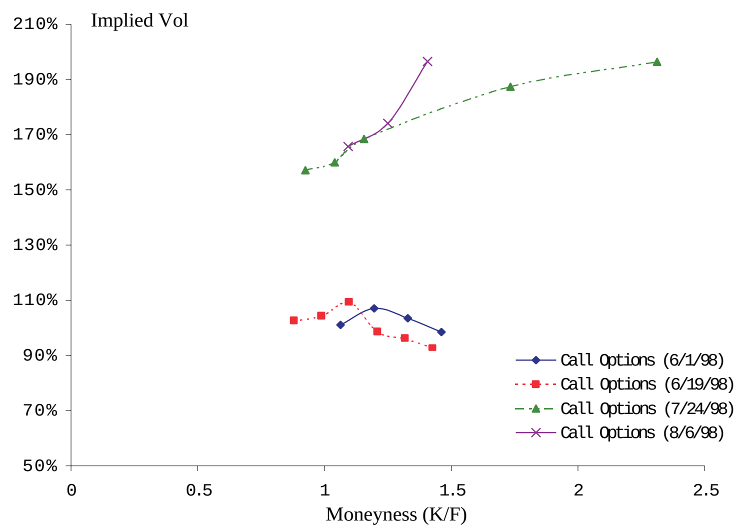

Figure 1: Implied Volatility of Call (Sept.) Options at Cinergy

plots the implied volatility of electricity call options in Eastern US across different strike prices at different points in time. On top of the tremendous levels of volatility, the highly seasonal patterns of electricity prices also complicate the modeling issues.

There have been few studies on modeling electricity prices since electricity markets only came into existence a few years ago in US. Schwartz (1997), and Miltersen and Schwartz (1998) are two of the most recent papers which concern the modeling of commodity spot prices. They investigate several stochastic models for commodity spot prices and perform an empirical analysis based on copper, gold and crude oil price data. They find that stochastic convenience yields could explain the term structure of forward prices and demonstrate the implications to hedging and real asset valuation by different models. Hilliard and Reis (1998) consider the effects of jumps and other factors in the spot price on the pricing of commodity futures, forwards, and futures options. One particular finding of theirs is that the jump in the spot price does not affect forward or futures prices. However, as I will illustrate later, this may not be true if the underlying commodity is almost non-storable such as electricity. Kaminski (1997) as well as Barz and Johnson (1998) are two papers on modeling electricity prices. Kaminski (1997) point out the needs of introducing jumps and stochastic volatility in modeling electricity prices. The Monte-Carlo simulation is used for electricity derivative pricing under the jump-diffusion price models. Barz and Johnson (1998) suggest the inadequacy of the Geometric Brownian motion and mean-reverting process in modeling electricity spot prices. With the objective of reflecting the key characteristics of electricity prices, they offer a price model which combined a mean-reverting process with a single jump process. However, they do not provide analytic results regarding derivative valuation under their proposed price model.

In this paper, I examine a broader class of stochastic models which can be used to model behaviors in commodity prices including jump, stochastic volatility, as well as stochastic convenience yield. I feel that models with jumps and stochastic volatility are particularly suitable for modeling the price processes of nearly non-storable commodities.

While some energy commodities, such as crude oil, may be properly modeled as traded securities, the nonstorability of electricity makes such an approach inappropriate. However, we can always view the spot price of a commodity as a state variable or a function of several state variables. All the physical contracts/financial derivatives on this commodity are therefore contingent claims on the state variables. I specify the spot price processes of energy commodities as affine jump-diffusion processes[1] which are introduced in Duffie and Kan (1996). Affine jump-diffusion processes are flexible enough to allow me to capture the special characteristics of commodity prices such as mean-reversion, seasonality, and spikes[2]. More importantly, I am able to compute the prices of various energy commodity derivatives under the assumed underlying affine jump-diffusion price processes by applying the transform analysis developed in Duffie, Pan and Singleton (1998). I consider not only the usual affine jump-diffusion models but also a regime-switching mean-reversion jump-diffusion model. The regime-switching model is used to capture the systematic alternations between "abnormal" and "normal" equilibrium states of supply and demand for a commodity.

The remainder of this paper is organized as follows. In the next section, I propose three alternative models for energy commodity prices and compute the transform functions needed for contingent claim pricing. In section III, I present illustrative examples of the models specified in section II and derive the pricing formulae of several energy commodity derivatives. The comparisons of the prices of energy commodity derivatives under different models are shown. I provide a heuristic method for estimating the model parameters by matching moment conditions using historical spot price data and calibrating the parameters to prices of traded options. In section IV, I construct a real options approach to value real assets such as generation and transmission facilities in the context of deregulated electricity industry. The implications of modeling assumptions to the valuation of real assets are also examined. Finally, I conclude in section V.

II Mean-Reverting Jump-Diffusion Models

The most noticeable price behavior of energy commodities is mean-reverting. When the price of a commodity is high, its supply tends to increase thus putting a downward pressure on the price; when the spot price is low, the supply of the commodity tends to decrease thus providing an upward lift to the price.

Another salient feature of energy commodity prices is the presence of price jumps and spikes. This is particularly prominent in the case where massive storage of a commodity is not economically viable and demand exhibits low elasticity. A perfect example would be electricity which is almost non-storable. Figure (2) shows the historical on-peak electricity spot prices in Texas (ERCOT) and at the California and Oregon border (COB). The jumpy behavior in electricity spot prices is mainly attributed to the fact that a typical regional aggregate supply function of electricity almost always has a kink at certain capacity level and the supply curve has a steep upward slope beyond that capacity level. Figure (3) represents a snap shot of the marginal cost curve of the electricity supply resource stack in western US.

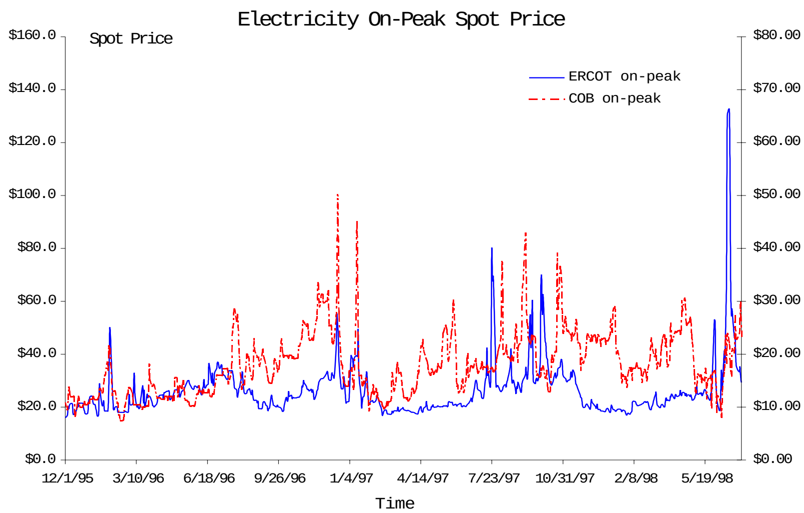

Figure 2: Electricity Historical Spot Prices

In a competitive market, electricity prices are determined by the intersection points of the aggregate demand and supply functions (the solid curves in Figure (3)). A forced outage of a major power plant or sudden surging demand would either shift the supply curve to the left or lift up the demand curve (the dashed curve in Figure (3)) therefore causing a price jump.

When the contingency making the spot price to jump high is short-term in nature, the high price will quickly fall back down to the normal range as the contingency disappears therefore causing a spike in the commodity price process. In the summer of 1998, we observed the spot price of electricity in Eastern and Midwestern US skyrocketing from \$50/MWh to \$7000/MWh because of the unexpected unavailability of some power generation plants and congestion on key transmission lines. Within a couple of days the price fell back to the \$50/MWh range as the lost generation and transmission capacity was restored. Electricity prices may also exhibit regime-switching jumps, caused by weather patterns and varying precipitation, in markets where the majority of installed electricity supply capacity is hydro power such as in the Nord Power Pool and the Victoria Power Pool.

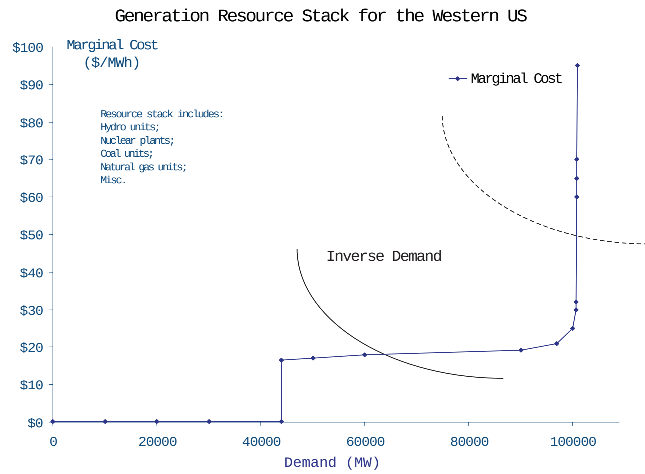

Figure 3: Generation Stack for Electricity in a Region

Although the mean reversion is well studied, there has been little work examining the implications by jumps and spikes to risk management and asset valuation.

In this paper, I examine the following three types of mean-reverting jump-diffusion models for modeling energy commodity spot prices.

1. Mean-reverting jump-diffusion process with deterministic volatility.
2. Mean-reverting jump-diffusion process with regime-switching.
3. Mean-reverting jump-diffusion process with stochastic volatility.

I consider two types of jumps in all of the above models. While our analytical approach could handle multiple types of jumps, I feel that, with properly chosen jump intensities, two types of jumps suffice in mimicking the jumps and/or spikes in the energy commodity price processes. The case of one type of jump is included as a special case when the intensity of type-2 jump is set to zero.

In addition to the commodity price process under consideration, I also jointly specify another factor process which can be correlated with the underlying commodity price. This additional factor could be the price of another commodity, or something else, such as the aggregate physical demand of the underlying commodity. In the case of electricity, the additional factor can be used to describe the spot price of the generating fuel such as natural gas. A jointly specified price process of the generating fuel is essential for risk management involving cross commodity risks between electricity and the fuel. There are empirical evidences demonstrating a positive correlation between electricity prices and the generating fuel prices in certain geographic regions during certain time periods of a year. In all models the risk free interest rate, r, is assumed to be deterministic.

A Model 1: A mean-reverting deterministic volatility process with two types of jumps

I start with specifying the spot price of an energy commodity as a mean-reverting jump-diffusion process with two types of jumps. Let the factor process X_t in (1) denote $\ln S_t^e$, where S_t^e is the price of the underlying energy commodity, e.g. electricity. Y_t is the other factor process which, in the case of modeling electricity spot price, can be used to specify the logarithm of the spot price of a generating fuel, *e.g.* $Y_t = \ln S_t^g$ where S_t^g is the spot price of natural gas. In this formulation, I have type-1 jump representing the upward jumps and type-2 jump representing the downward jumps. By setting the intensity functions of the jump processes in a proper way, we can mimic the spikes in the price process of the underlying energy commodity. Suppose the state vector process $(X_t, Y_t)'$ given by (1) is under the true measure and the risk premia associated with all state variables are linear functions of state variables. Then the state vector process has the same form as that of (1) under the risk-neutral measure, but with different coefficients. For the ease of pricing derivatives in section III, I choose to directly specify the state vector process under the risk-neutral measure from here on with the assumption that the risk premia associated with all state variables are linear functions of state variables.

Assume that, under regularity conditions, X_t and Y_t are strong solutions to the following stochastic differential equation (SDE) under the risk-neutral measure Q,

$$d\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \kappa_1(t)(\theta_1(t) - X_t) \\ \kappa_2(t)(\theta_2(t) - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1(t) & 0 \\ \rho(t)\sigma_2(t) & \sqrt{1-\rho(t)^2}\sigma_2(t) \end{pmatrix} dW_t + \sum_{i=1}^{2} \Delta Z_t^i \tag{1}$$

where $\kappa_1(t)$ and $\kappa_2(t)$ are the mean-reverting coefficients; $\theta_1(t)$ and $\theta_2(t)$ are the long term means; $\sigma_1(t)$ and $\sigma_2(t)$ are instantaneous volatility rates of X and Y; W_t is a $\mathcal{F}_t$-adapted standard Brownian motion under Q in $\Re^2$; Z^j is a compound Poisson process in $\Re^2$ with the Poisson arrival intensity being $\lambda_j(t)$ ($j = 1, 2$). ΔZ^j denotes the random jump size in $\Re^2$ with distribution function $v^j(z)$ ($j = 1, 2$). Let $\phi_J^j(c_1, c_2, t) \equiv \int_{R^2} \exp(c \cdot z) dv^j(z)$ be the transform function of the jump size distribution of type-j jumps ($j = 1, 2$).

The transform function Define the generalized transform function as

$$\varphi(u, X_t, Y_t, t, T) \equiv E^Q[e^{-r(T-t)} \exp(u_1 X_T + u_2 Y_T) \mid \mathcal{F}_t]$$

for any fixed time T where $u \equiv (u_1, u_2) \in C^2$. The transform function φ is well-defined at a given u under technical regularity conditions on $\kappa_*(t)$, $\theta_*(t)$, $\sigma_*(t)$, $\lambda_*(t)$, and $\phi_J^*(c_1, c_2, t)$.

Under the regularity conditions (e.g. see Duffie, Pan and Singleton (1998)), $\varphi \cdot e^{-rt}$ is a martingale under the risk-neutral measure Q since it is a $\mathcal{F}_t-$conditional expectation of a single random variable $\exp(u_1 X_T + u_2 Y_T)$. Therefore the drift term of $\varphi \cdot e^{-rt}$ is zero. Applying Ito's lemma for complex function, we observe that φ needs to satisfy the following fundamental partial differential equation (PDE)

$$\mathcal{D}f - rf = 0 \tag{2}$$

where

$$\mathcal{D}f \equiv \partial_t f + \mu_X \cdot \partial_X f + \frac{1}{2}tr(\partial_X^2 f \Sigma\Sigma^T) + \sum_{j=1}^{2}\lambda_j(t)\int_{R^2}[f(X_t + \Delta Z_t^j, t) - f(X_t, t)]dv_j(z)$$

The solution to the PDE (2) is given by (see Appendix A.1 for details)

$$\varphi(u, X_t, Y_t, t, T) = \exp(\alpha(t,u) + \beta_1(t,u)X_t + \beta_2(t,u)Y_t)$$

where

$$\begin{aligned}
\beta_1(t,[u_1,u_2]') &= u_1 \exp(-\textstyle\int_t^T \kappa_1(s)ds) \\
\beta_2(t,[u_1,u_2]') &= u_2 \exp(-\textstyle\int_t^T \kappa_2(s)ds) \\
\alpha(t,u) &= \int_t^T (\sum_{i=1}^{2}[\kappa_i(s)\theta_i(s)\beta_i(s,u) + \frac{1}{2}\sigma_i^2(s)\beta_i^2(s,u)] + \rho(s)\sigma_1(s)\sigma_2(s)\beta_1(s,u)\beta_2(s,u) \\
&\quad -r + \sum_{j=1}^{2}\lambda_j(s)(\phi_J^j(\beta_1(s,u),\beta_2(s,u),s) - 1))ds
\end{aligned} \tag{3}$$

B Model 2: A regime-switching mean-reverting process with two types of jumps

To motivate this model, I consider modeling the electricity prices in which case the forced outages of generation plants or unexpected contingencies in transmission networks often result in abnormally high spot prices for a short time period and then a quick price fall-back. In order to capture the phenomena of spot prices switching between "high" and "normal" states, I extend model 1 to a Markov regime-switching model which I describe in detail below.

Let U_t be a continuous-time two-state Markov chain

$$dU_t = 1_{U_t=0} \cdot \delta(U_t)dN_t^{(0)} + 1_{U_t=1} \cdot \delta(U_t)dN_t^{(1)} \tag{4}$$

where $N_t^{(i)}$ is a Poisson process with arrival intensity $\lambda^{(i)}$ $(i = 0, 1)$ and $\delta(0) = -\delta(1) = 1$. I next define the corresponding compensated continuous-time Markov chain $M(t)$ as

$$dM_t = -\lambda(U_t)\delta(U_t)dt + dU_t \tag{5}$$

The joint specification of electricity and the generating fuel price processes under the risk-neutral measure Q is given by:

$$\begin{aligned} d\begin{pmatrix} X_t \\ Y_t \end{pmatrix} &= \begin{pmatrix} \kappa_1(t)(\theta_1(t) - X_t) \\ \kappa_2(t)(\theta_2(t) - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1(t) & 0 \\ \rho(t)\sigma_2(t) & \sqrt{1 - \rho(t)^2}\sigma_2(t) \end{pmatrix} dW_t \\ &\quad + \sum_{j=1}^{2} \Delta Z_t^j + \iota(U_{t-})dM_t \end{aligned} \tag{6}$$

where W_t is a $\mathcal{F}_t$-adapted standard Brownian motion under Q in $\Re^2$; $\{\iota(i) \equiv (\iota_1(i), \iota_2(i))'; i = 0, 1\}$ denotes the sizes of the random jumps in state variables when regime-switching occurs. $\phi_{\iota(i)}(c_1, c_2, t) \equiv \int_{R^2} \exp(c \cdot z) dv_{\iota(i)}(z)$ is the transform function of the regime-jump size distribution $v_{\iota(i)}$ $(i = 1, 2)$. Z^j, ΔZ^j and $\phi_J^j(c_1, c_2, t)$ are similarly defined as those in Model 1. Strong solutions to (6) exist under regularity conditions.

The transform function Let $F^i(\overline{x}, t)$ $(i = 0, 1)$ denote

$$E[e^{-r(T-t)} \exp(u_1 X_T + u_2 Y_T) | X_t = x, Y_t = y, U_t = i]$$

where U_t is the Markov regime state variable. The infinitesimal generator $\mathcal{D}$ of F^i is given by

$$\begin{aligned} \mathcal{D}F^0(\overline{x}, t) &= dF^0(\overline{x}, t) + \lambda^{(0)} \textstyle\int_{R^2} [F^1(\overline{x} + \iota(0), t) - F^0(\overline{x}, t)] dv_t(\iota(0)) \\ \mathcal{D}F^1(\overline{x}, t) &= dF^1(\overline{x}, t) + \lambda^{(1)} \textstyle\int_{R^2} [F^0(\overline{x} + \iota(1), t) - F^1(\overline{x}, t)] dv_t(\iota(1)) \end{aligned} \tag{7}$$

where

$$
\begin{aligned}
dF^i(\overline{x},t) &= F^i_t + F^i_{\overline{x}} \cdot \mu^i(\overline{x},t) + \frac{1}{2} tr[F^i_{\overline{x}\overline{x}} \sigma^i(\overline{x},t)\sigma^i(\overline{x},t)^T] \\
&\quad + \sum_{j=1}^{2} \lambda^i_j(\overline{x},t) \int_{R^2} [F^i(\overline{x}+z,t) - F^i(\overline{x},t)] d\upsilon_{j,t}(z) \\
(i &= 0,1 \text{ is the regime state variable})
\end{aligned}
$$

The fundamental PDEs satisfied by the transform functions $F^i(\overline{x},t)$ $(i=0,1)$ are

$$
\mathcal{D}F^i(x,y,t) - rF^i(x,y,t) = 0 (i=0,1) \tag{8}
$$

The solutions to (8) assume the following forms

$$
\begin{aligned}
F^0(x,y,t) &= \exp(\alpha_0(t) + \beta_1(t)x + \beta_2(t)y) \qquad (9) \\
F^1(x,y,t) &= \exp(\alpha_1(t) + \beta_1(t)x + \beta_2(t)y)
\end{aligned}
$$

where $\alpha_0(t)$, $\alpha_1(t)$, $\beta_0(t)$, and $\beta_1(t)$ are solutions to a system of ordinary differential equations (ODEs) specified in Appendix A.2.

C Model 3: A mean-reverting stochastic volatility process with two types of jumps

I consider a three-factor affine jump-diffusion process with two types of jumps in this model (10). Once again, I motivate the model in the setting of modeling the electricity prices. Consider X_t and Y_t to be the logarithm of the spot prices of electricity and a generating fuel, e.g. natural gas, respectively. V_t represents the stochastic volatility factor. There are empirical evidences alluding to the fact that the volatility of electricity price is high when the aggregate demand is high and vice versa. Therefore, V_t can be thought as a factor which is proportional to the regional aggregate demand process for electricity. Jumps may appear in both X_t and V_t since weather conditions such as unusual heat waves usually cause simultaneous jumps in both the electricity price and the aggregate load. The state vector process $(X_t, V_t, Y_t)'$ is specified by (10). Under proper regularity conditions, there exists a Markov process which is the strong solution to the following SDEs under the risk-neutral measure Q.

$$d\begin{pmatrix} X_t \\ V_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \kappa_1(t)(\theta_1(t) - X_t) \\ \kappa_V(t)(\theta_V(t) - V_t) \\ \kappa_2(t)(\theta_2(t) - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sqrt{V_t} & 0 & 0 \\ \rho_1(t)\sigma_2(t)\sqrt{V_t} & \sqrt{1-\rho_1^2(t)}\sigma_2(t)\sqrt{V_t} & 0 \\ \rho_2(t)\sigma_3(t)\sqrt{V_t} & 0 & \sigma_3(t) \end{pmatrix} dW_t + \sum_{i=1}^{2} \Delta Z_t^i \tag{10}$$

where W is a $\mathcal{F}_t$-adapted standard Brownian motion under Q in $\Re^3$; Z^j is a compound Poisson process in $\Re^3$ with the Poisson arrival intensity being $\lambda^j(X_t, V_t, Y_t, t)$ $(j = 1, 2)$. I model the spiky behavior by assuming that the intensity function of type-1 jumps is only a function of time t, denoted by $\lambda^{(1)}(t)$, and the intensity of type-2 jumps is a function of V_t, i.e. $\lambda^{(2)}(V_t, t) = \lambda_2(t)V_t$. Let $\phi_J^j(c_1, c_2, c_3, t) \equiv \int_{R^2} \exp(c \cdot z) dv^j(z)$ denote the transform function of the jump-size distribution of type-j jumps, $v^j(z)$, $(j = 1, 2)$.

The transform function Following similar arguments to those used in Model 1, we know that the transform function

$$\varphi(u, X_t, V_t, Y_t, t, T) \equiv E^Q[e^{-r(T-t)} \exp(u_1 X_T + u_2 V_T + u_3 Y_T) \mid \mathcal{F}_t]$$

is of form

$$\varphi(u, X_t, V_t, Y_t, t, T) = \exp(\alpha(t, u) + \beta_1(t, u)X_T + \beta_2(t, u)V_T + \beta_3(t, u)Y_T) \tag{11}$$

where $\alpha(u,t)$ and $\beta(u,t) \equiv (\beta_1(u,t), \beta_2(u,t), \beta_3(u,t))'$ are solutions to a system of ODEs specified in Appendix A.3.

III Pricing of Energy Commodity Derivatives

Having specified the mean-reverting jump-diffusion price models and demonstrated how to compute the generalized transform functions of the state vector at any given time T, the prices of European-type contingent claims on the underlying energy commodity under the proposed models can then be obtained through the inversion of the transform functions. Suppose $\overline{X}_t$ is a state vector in R^n and $u \in C^n$ and the generalized transform function is given by

$$\begin{aligned} \varphi(u, \overline{X}_t, t, T) &\equiv E^Q[e^{-r(T-t)} \exp(u \cdot \overline{X}_T) | \mathcal{F}_t] \\ &= \exp[\alpha(t,u) + \beta(t,u) \cdot \overline{X}_t] \end{aligned} \tag{12}$$

Let $G(v, X_t, Y_t, t, T; \overline{a}, \overline{b})$ denote the time-t price of a contingent claim with payoff $\exp(\overline{a} \cdot \overline{X}_T)$ when $\overline{b} \cdot \overline{X}_T \leq v$ is true at time T where $\overline{a}$, $\overline{b}$ are vectors in R^n and $v \in R^1$, then we have (see Duffie, Pan and Singleton (1998) for a formal proof):

$$\begin{aligned} G(v, \overline{X}_t, t, T; \overline{a}, \overline{b}) &= E^Q[e^{-r(T-t)} \exp(\overline{a} \cdot \overline{X}_T) \mathbf{1}_{\overline{b} \cdot \overline{X}_T \leq v} \mid \mathcal{F}_t] \\ &= \frac{\varphi(\overline{a}, \overline{X}_t, t, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\varphi(\overline{a} + iw\overline{b}, \overline{X}_t, t, T) e^{-iwv}]}{w} dw \end{aligned} \tag{13}$$

For properly chosen v, $\overline{a}$, and $\overline{b}$, $G(v, X_t, Y_t, t, T; \overline{a}, \overline{b})$ serves as building blocks in pricing contingent claims such as forwards/futures, call/put options, and cross-commodity spread options. To illustrate the points, I take some concrete examples of the models proposed in section II and compute the prices of several commonly traded energy commodity derivatives. Specifically, model Ia is a special case of model I ($I = 1, 2, 3$). Closed-form solutions of the derivative securities (up to the Fourier inversion) are provided whenever available.

A Illustrative Models

The illustrative models presented here are obtained by setting the parameters to be constants in the three general models. The jumps appear in the primary commodity price and the volatility processes (Model 3a) only. The jump sizes are distributed as independent exponential random variables in R^n thus having the following transform function:

$$\phi_J^j(\overline{c}, t) \equiv \prod_{k=1}^{n} \frac{1}{1 - \mu_j^k c_k} \tag{14}$$

The simulated price paths under the three illustrative models are shown in Figure (4) for

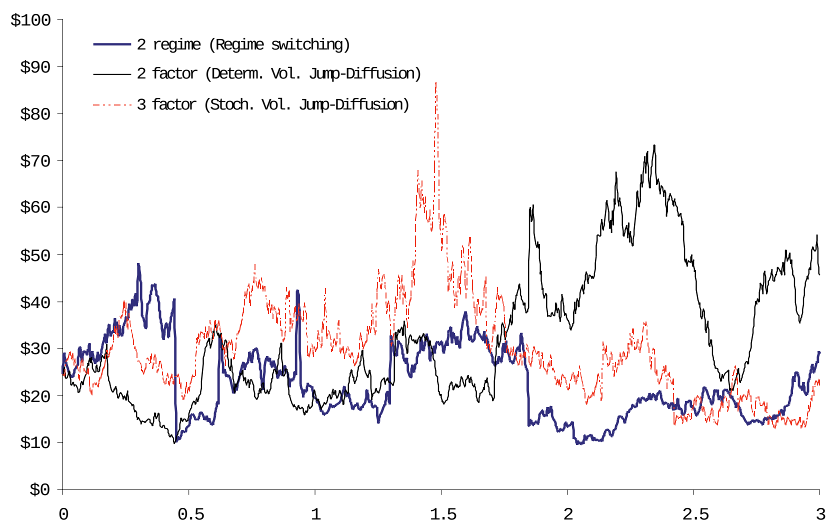

Figure 4: Simulated Spot Prices under the Three Models

parameters given in Table (1). The x-axis represents the simulation time horizon in years while the y-axis represents the commodity price level.

A.1 Model 1a

Model 1a (15) is a special case of (1) with all parameters being constants. The jumps are in the logarithm of the primary commodity spot price, X_t. The sizes of type-j jumps ($j = 1$, 2) are exponentially distributed with mean μ_J^j. The transform function of the jump-size distribution is $\phi_J^j(c_1, c_2, t) \equiv \dfrac{1}{1-\mu_J^j c_1}$ $(j = 1, 2)$.

$$d\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \kappa_1(\theta_1 - X_t) \\ \kappa_2(\theta_2 - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1 & 0 \\ \rho_1\sigma_2 & \sqrt{1-\rho_1^2}\sigma_2 \end{pmatrix} dW_t + \sum_{i=1}^{2} \Delta Z_t^i \tag{15}$$

where W is a $\mathcal{F}_t$-adapted standard Brownian motion in $\Re^2$.

The transform function φ_{1a} The closed-form solution of the transform function can be written out explicitly for this model.

$$\varphi_{1a}(u, X_t, Y_t, t, T) = \exp[\alpha(\tau) + \beta_1(\tau)X_t + \beta_2(\tau)Y_t] \tag{16}$$

where $\tau = T - t$. By solving the ordinary differential equations in (35) with all parameters being constants, we get the following,

$$\begin{aligned} \beta_1(\tau, u_1) &= u_1 \exp(-\kappa_1\tau) \\ \beta_2(\tau, u_2) &= u_2 \exp(-\kappa_2\tau) \\ \alpha(\tau, u) &= -r\tau - \sum_{j=1}^{2} \frac{\lambda_J^j}{\kappa_1} \ln \frac{u_1\mu_J^j - 1}{u_1\mu_J^j \exp(-\kappa_1\tau) - 1} + \frac{a_1\sigma_1^2 u_1^2}{4\kappa_1} + \frac{a_2\sigma_2^2 u_2^2}{4\kappa_2} \\ &\quad + u_1\theta_1(1 - \exp(-\kappa_1\tau)) + u_2\theta_2(1 - \exp(-\kappa_2\tau)) \\ &\quad + \frac{u_1u_2\rho_1\sigma_1\sigma_2(1 - \exp(-(\kappa_1 + \kappa_2)\tau))}{\kappa_1 + \kappa_2} \end{aligned} \tag{17}$$

with $a_1 = 1 - \exp(-2\kappa_1\tau)$ and $a_2 = 1 - \exp(-2\kappa_2\tau)$.

A.2 Model 2a

Model 2a (19) is a regime-switching model with the regime-jumps appearing only in the primary commodity price process. In the electricity markets, this is suitable for modeling the occasional price spikes in the electricity spot prices caused by forced outages of the major power generation plants or line contingency in transmission networks. For simplicity, I assume that there are no jumps within each regime.

$$
\begin{aligned}
d\begin{pmatrix} X_t \\ Y_t \end{pmatrix} &= \begin{pmatrix} \kappa_1(\theta_1 - X_t) \\ \kappa_2(\theta_2 - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1 & 0 \\ \rho_1\sigma_2 & \sqrt{1-\rho_1^2}\sigma_2 \end{pmatrix} dW_t \qquad (18) \\
&\quad + \iota(U_{t-})dM_t \qquad (19)
\end{aligned}
$$

where W is a $\mathcal{F}_t$-adapted standard Brownian motion in $\Re^2$. U_t is the regime state process as defined in (4). The sizes of regime-jumps are assumed to be distributed as independent exponential random variables and the transform functions of the regime-jump sizes are $\phi_\iota(c_1, c_2, t) \equiv \dfrac{1}{1-\mu_\iota c_1}$ $(\iota = 0, 1)$ where $\mu_0 \geq 0$ (upward jumps) and $\mu_1 \leq 0$ (downward jumps).

The transform function φ_{2a} For this model the transform function φ_{2a} can not be solved in closed-form completely. We have

$$
\begin{aligned}
\varphi_{2a}^0(x,y,t) &= \exp(\alpha_0(t) + \beta_1(t)x + \beta_2(t)y) \qquad (20) \\
\varphi_{2a}^1(x,y,t) &= \exp(\alpha_1(t) + \beta_1(t)x + \beta_2(t)y)
\end{aligned}
$$

where $\beta(t) \equiv \beta(t,u) \equiv (\beta_1(t,u), \beta_2(t,u))'$ has the closed-form solution of

$$
\begin{aligned}
\beta_1(\tau, u_1) &= u_1 \exp(-\kappa_1 \tau) \\
\beta_2(\tau, u_2) &= u_2 \exp(-\kappa_2 \tau)
\end{aligned}
$$

but $\alpha(t) \equiv \alpha(t,u) \equiv (\alpha_0(t,u), \alpha_1(t,u))'$ needs to be numerically computed from

$$
\frac{d}{dt}\begin{pmatrix} \alpha_0(t) \\ \alpha_1(t) \end{pmatrix} = -\begin{pmatrix} A_1(\beta(t),t) + \lambda^{(0)}[\dfrac{\exp(\alpha_1(t)-\alpha_0(t))}{1-\mu_0\beta_1(t,u_1)} - 1] \\ A_1(\beta(t),t) + \lambda^{(1)}[\dfrac{\exp(\alpha_0(t)-\alpha_1(t))}{1-\mu_1\beta_1(t,u_1)} - 1] \end{pmatrix}
$$

$$\begin{pmatrix} \alpha_0(0,u) \\ \alpha_1(0,u) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

with

$$A_1(\beta(t),t) = -r + \sum_{i=1}^{2}[\kappa_i\theta_i\beta_i + \frac{1}{2}\sigma_i^2\beta_i^2] - \rho_1\sigma_1\sigma_2\beta_1\beta_2$$

A.3 Model 3a

Model 3a (21) is a stochastic volatility model in which the type-1 jumps are simultaneous jumps in the commodity spot price and volatility processes, and the type-2 jumps are in the commodity spot price only. All parameters are constants.

$$d\begin{pmatrix} X_t \\ V_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \kappa_1(\theta_1 - X_t) \\ \kappa_V(\theta_V - V_t) \\ \kappa_2(\theta_2 - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sqrt{V_t} & 0 & 0 \\ \rho_1\sigma_2\sqrt{V_t} & \sqrt{1-\rho_1^2}\sigma_2\sqrt{V_t} & 0 \\ \rho_2\sigma_3\sqrt{V_t} & 0 & \sigma_3 \end{pmatrix} dW_t + \sum_{i=1}^{2}\Delta Z_t^i \quad (21)$$

where W is a $\mathcal{F}_t$-adapted standard Brownian motion in $\Re^3$; Z^i $(i = 1,2)$ is a compound Poisson process in $\Re^3$. The Poisson arrival intensity functions are $\lambda^1(X_t, V_t, Y_t, t) = \lambda_1$ and $\lambda^2(X_t, V_t, Y_t, t) = \lambda_2 V_t$. The transform functions of the jump-size distributions are

$$\begin{aligned} \phi_J^1(c_1,c_2,c_3,t) &\equiv \frac{1}{(1-\mu_1^1 c_1)(1-\mu_1^2 c_2)} \\ \phi_J^2(c_1,c_2,c_3,t) &\equiv \frac{1}{1-\mu_2^1 c_1} \end{aligned}$$

where μ_J^k is the mean size of the type-J $(J = 1,2)$ jump in factor k $(k = 1,2)$.

The transform function φ_{3a} From section II, we know that φ_{3a} is of form

$$\varphi_{3a}(u, X_t, V_t, Y_t, t, T) = \exp(\alpha(t,u) + \beta_1(t,u)X_T + \beta_2(t,u)V_T + \beta_3(t,u)Y_T)$$

Similar to model 2a, the transform function φ_{3a} does not have a closed-form solution. I numerically solve for both $\alpha(t,u)$ and $\beta(t,u) \equiv [\beta_1(t,u), \beta_2(t,u), \beta_3(t,u)]'$ from the following ordinary differential equations (ODEs)

$$\begin{aligned} \frac{d}{dt}\beta(t,u) + B(\beta(t,u),t) = 0, \qquad \beta(0,u) = u \\ \frac{d}{dt}\alpha(t,u) + A(\beta(t,u),t) = 0, \qquad \alpha(0,u) = 0 \end{aligned} \tag{22}$$

with

$$\begin{aligned} A(\beta,t) &= -r + \sum_{i=1}^{3} \kappa_i\theta_i\beta_i(t,u) + \tfrac{1}{2}\beta_3^2(t)\sigma_3^2 + \lambda_1[\frac{1}{(1-\mu_1^1\beta_1(t,u))(1-\mu_1^2\beta_2(t,u))} - 1] \\ B(\beta,t) &= \begin{pmatrix} \kappa_1\beta_1(t,u) + \lambda_{21}(\frac{1}{1-\mu_2^1\beta_1(t,u)} - 1) \\ \kappa_2\beta_2(t,u) + \lambda_{22}(\frac{1}{1-\mu_2^1\beta_1(t,u)} - 1) + \frac{1}{2}B_1(\beta,t) \\ \kappa_3\beta_3(t,u) \end{pmatrix} \end{aligned} \tag{23}$$

and

$$\begin{aligned} B_1(\beta,t) \;=\; & \beta_1(t,u)(\beta_1(t,u) + \beta_2(t,u)\rho_1\sigma_2 + \beta_3(t,u)\rho_2\sigma_3) \\ & +\beta_2(t,u)(\beta_1(t,u)\rho_1\sigma_2 + \beta_2(t,u)\sigma_2^2 + \beta_3(t,u)\rho_1\rho_2\sigma_2\sigma_3) \\ & +\beta_3(t,u)(\beta_1(t,u)\rho_2\sigma_3 + \beta_2(t,u)\rho_1\rho_2\sigma_2\sigma_3 + \beta_3(t,u)\rho_2^2\sigma_3^2) \end{aligned}$$

B Energy Commodity Derivatives

In this subsection, I derive the pricing formulae for the futures/forwards, calls, spark spread options, and locational spread options. The derivative prices are calculated using the parameters given in Table (1). I show the comparisons of the derivative prices under different models as well.

B.1 Futures/Forward Price

A futures (forward) contract promising to deliver one unit of commodity S^i at a future time T for a price of F has the following payoff at time T

$$\text{Payoff} = S_T^i - F$$

Since no initial payment is required to enter into a futures contract, the futures price F is given by

$$F(S_t^i, t, T) = E^Q[S_T^i \mid \mathcal{F}_t]$$

Rewrite the above expression as

$$\begin{aligned} F(S_t^i, t, T) &= E^Q[S_T^i \mid \mathcal{F}_t] \\ &= e^{r\tau} E^Q[e^{-r\tau} \cdot \exp(X_T^i) \mid \mathcal{F}_t] \end{aligned}$$

We therefore have

$$F(S_t^i, t, T) = e^{r\tau} \cdot \varphi(\overline{e}_i^T, \overline{X}_t, \tau) \tag{24}$$

where $\tau = T - t$; $\varphi(u, \overline{X}_t, \tau)$ is the transform function given by (12); $\overline{e}_i$ is the vector with i^{th} component being 1 and all other components being 0.

Futures price (model 1a) Recall the transform function φ_{1a} is obtained in (16) and (17). By setting $u = [1, 0]'$ in φ_{1a}, we get

$$\varphi_{1a}([1,0]', X_t, Y_t, \tau) = \exp[X_t \exp(-\kappa_1 \tau) - r\tau + \frac{a_1 \sigma_1^2}{4\kappa_1} + \theta_1(1 - \exp(-\kappa_1 \tau)) - j(\tau)]$$

where $\tau = T - t$, $X_t = \ln(S_t)$, $a_1 = 1 - \exp(-2\kappa_1 \tau)$ and $j(\tau) = \sum_{j=1}^{2} \frac{\lambda_J^j}{\kappa_1} \ln \frac{\mu_J^j - 1}{\mu_J^j \exp(-\kappa_1 \tau) - 1}$. Therefore by (24), we have the following proposition.

Proposition 1 *In* **Model 1a**, *the futures price of commodity S_t at time t with delivery time T is*

$$\begin{aligned} F(S_t,t,T) &= e^{r\tau}\cdot\varphi_{1a}([1,0]',X_t,Y_t,\tau) \\ &= \exp[X_t\exp(-\kappa_1\tau)+\frac{a_1\sigma_1^2}{4\kappa_1}+\theta_1(1-\exp(-\kappa_1\tau))-j(\tau)] \end{aligned} \tag{25}$$

where $\tau = T-t$, $X_t=\ln(S_t)$, $a_1 = 1-\exp(-2\kappa_1\tau)$ *and* $j(\tau)=\sum_{j=1}^{2}\frac{\lambda_J^j}{\kappa_1}\ln\frac{\mu_J^j-1}{\mu_J^j\exp(-\kappa_1\tau)-1}$.

Note that the futures price in this model is simply the scaled-up futures price in the Ornstein-Uhlenbeck mean-reversion model with the scaling factor being $\exp(-j(\tau))$. If we interpret the spikes in the commodity price process as upward jumps followed shortly by downward jumps of similar sizes, then over a long time horizon both the frequencies and the average sizes of the upward jumps and the downward jumps are roughly the same, i.e. $\lambda_J^1=\lambda_J^2$ and $\mu_J^1\approx-\mu_J^2$. One might intuitively think that the up and down jumps would offset each other's effect in the futures price. What (25) tells us is that this intuition is not quite right and indeed, in the case where $\lambda_J^1=\lambda_J^2$ and $\mu_J^1=-\mu_J^2$, the futures price is definitely higher than that corresponding to the no-jump case.

Futures price (model 2a) The futures price in model 2a is

$$F(S_t,t,T)=e^{r\tau}\cdot\varphi_{2a}^i([1,0]',X_t,Y_t,\tau)\quad(i=0,1)$$

where i is the Markov regime state variable; $\tau=T-t$ and the transform functions φ_{2a}^i are computed in (20).

Futures price (model 3a) The futures price in model 3a is

$$F(S_t,t,T)=e^{r\tau}\cdot\varphi_{3a}([1,0,0]',X_t,V_t,Y_t,\tau)$$

where $\tau=T-t$ and the transform function φ_{3a} is given in (A.3).

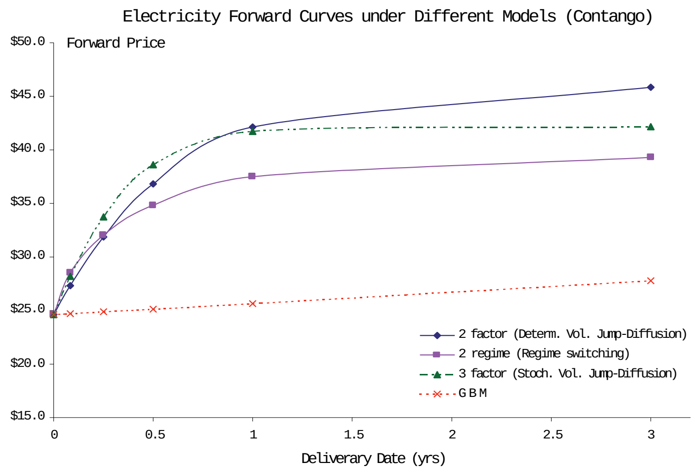

Figure 5: Forward Curves under Different Models (Contango)

Forward curves Using the parameters in Table (1) for modeling the electricity spot price in the Eastern region of US (Cinergy to be specific), I obtain forward curves at Cinergy under each of the three illustrative models. The jointly specified factor process is the spot price of natural gas at Henry Hub. For the initial values of $S_e = \$24.63$, $S_g = \$2.105$, $V = 0.5$, $U = 0$ and $r = 4\%$, Figure (5) illustrates the three forward curves of electricity which are all in contango form since the initial value S_e is lower than the long-term mean value. Figure (6) plots three electricity forward curves in backwardation when the initial electricity price S_e is set to be \$40 which is higher than the long-term mean value. The forward curves under the Geometric Brownian motion (GBM) price model are also shown in the two figures. Under the GBM price model, the forward prices always exhibit a fixed rate of growth.

B.2 Call Option

A "plain vanilla" European call option on commodity S^i with strike price K has the payoff of

$$C(S_T^i, K, T, T) = \max(S_T^i - K, 0)$$

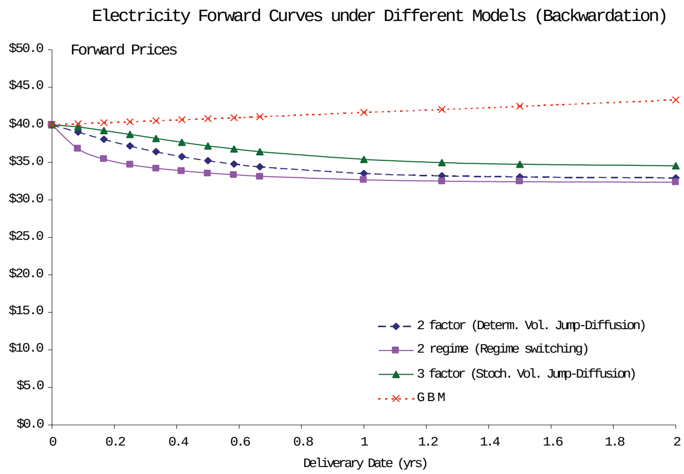

Figure 6: Forward Curves under Different Models (Backwardation)

at maturity time T . The price of the call option is given by

$$\begin{aligned} C(S_t^i, K, t, T) &= E^Q[e^{-r(T-t)} \max(S_T^i - K, 0) \mid \mathcal{F}_t] \\ &= E^Q[e^{-r\tau} \exp(X_T^i) 1_{X_T^i \geq \ln K} \mid \mathcal{F}_t] - K \cdot E^Q[e^{-r\tau} 1_{X_T^i \geq \ln K} \mid \mathcal{F}_t] \\ &= G_1 - K \cdot G_2 \end{aligned} \tag{26}$$

where $\tau = T - t$ and G_1, G_2 are obtained by setting $\{a = \overline{e}_i \ , b = -\overline{e}_i, \ v = -\ln K\}$ and $\{a = \overline{0} \ , b = -\overline{e}_i, \ v = -\ln K\}$ in (13), respectively.

$$\begin{aligned} G_1 &= E^Q[e^{-r\tau} \exp(X_T) 1_{X_T \geq \ln K} \mid \mathcal{F}_t] \\ &= \frac{F_t^i e^{-r\tau}}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\varphi([1 - w \cdot i, 0], \overline{X}_t, \tau) \exp(i \cdot w \ln K)]}{w} dw \\ &= F_t^i e^{-r\tau} (\frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\varphi([1 - w \cdot i, 0], \overline{X}_t, \tau) \exp(r\tau + i \cdot w \ln K)]}{w F_t^i} dw) \end{aligned} \tag{27}$$

where $F_t^i = e^{r\tau} \cdot \varphi(\overline{e}_i^T, \overline{X}_t, \tau)$ is the time-t forward price of commodity S^i with delivery time T.

$$G_2 = E^Q[e^{-r\tau} 1_{X_T^i \geq \ln K} \mid \mathcal{F}_t]$$

$$
\begin{aligned}
&= \frac{\varphi(\overline{0}, \overline{X}_t, t, T)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im[\varphi(i \cdot w\overline{e}_i, \overline{X}_t, t, T)e^{i \cdot w \ln K}]}{w} dw \\
&= e^{-r\tau}\left(\frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im[\varphi(i \cdot w\overline{e}_i, \overline{X}_t, t, T)e^{r \cdot \tau + i \cdot w \ln K}]}{w} dw\right) \qquad (28)
\end{aligned}
$$

Call option price Substituting φ_{1a}, φ_{2a}, and φ_{3a} into (27) and (28) we have the call option price given by (26) under **Model 1a**, **2a** and **3a**, respectively.

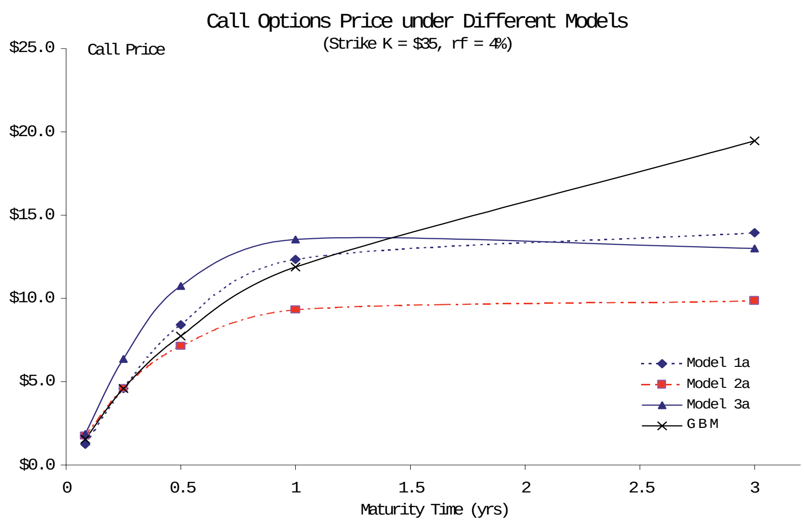

Figure 7: Call Options Price under Different Models

Volatility smile Figure (7) plots the call option values with different maturity time under different models. The call values under a Geometric Brownian motion (GBM) model are also plotted for comparison purpose. Note that, as maturity time increases, the mean-reversion effects in all three models cause the value of a call option to converge to a long-term value which is different from the spot price of the underlying. Figure (8) illustrates the implied volatility curves under the three illustrative models with the set of parameters given in Table (1).

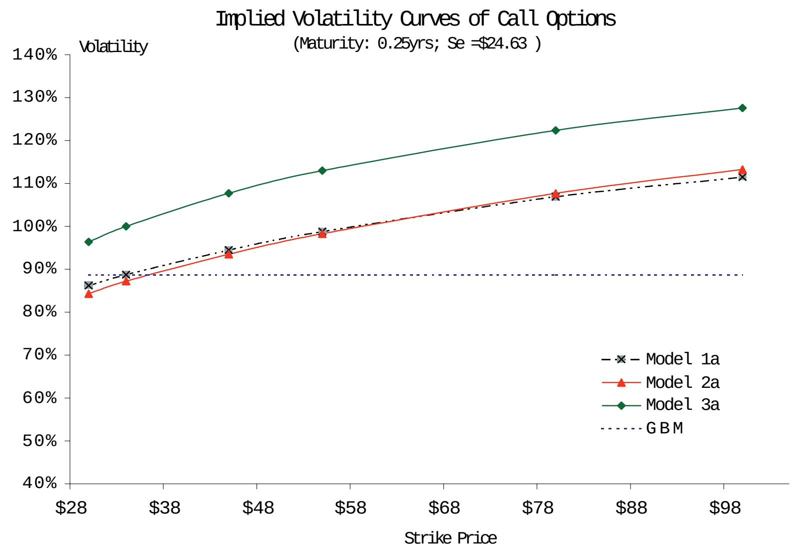

Figure 8: Volatility-Smile under Different Models

B.3 Cross Commodity Spread Option

In energy commodity markets, cross commodity derivatives play crucial roles in risk management. Crack spread options in crude oil markets as well as the spark spread and locational spread options in electricity markets are good examples. Deng, Johnson and Sogomonian (1998) illustrated how the spark spread options, which are derivatives on electricity and the fossil fuels used to generate electricity, can have various applications in risk management for utility companies and power marketers. Moreover, such options are essential in asset valuation for fossil fuel electricity generation plants. A European *spark spread call (SSC)* option pays off the positive part of the difference between the electricity spot price and the generating fuel cost at the time of maturity. Its payoff function is:

$$SSC(S_T^e, S_T^g, H, T) = \max(S_T^e - H \cdot S_T^g, 0)$$

where S_T^e and S_T^g are the prices of electricity and the generating fuel, respectively; the constant H, termed *strike heat rate*, represents the number of units of generating fuel contracted to generate one unit of electricity.

Another type of cross commodity option called locational spread option was also introduced in Deng, Johnson and Sogomonian (1998). A locational spread option pays off the positive part of the price difference between the prices of the underlying commodity at two different delivery points. In the context of electricity markets, locational spread options serve the purposes of hedging the transmission risks and they can also be used to value transmission expansion projects as shown in Deng, Johnson and Sogomonian (1998). The time T payoff of a European *locational spread call* option is

$$LSC(S_T^a, S_T^b, L, T) = \max(S_T^b - L \cdot S_T^a, 0)$$

where S_T^a and S_T^b are the time-T commodity prices at location a and b. The constant L is a loss factor reflecting the transportation/transmission losses or costs associated with shipping one unit of the commodity from location a to b.

Observing the similar payoff structures of the above two spread options, I define a general *cross-commodity spread call* option as an option with the following payoff at maturity time T,

$$CSC(S_T^1, S_T^2, K, T) = \max(S_T^1 - K \cdot S_T^2, 0)$$

where S_T^i is the spot price of commodity i ($i = 1, 2$) and K is a scaling constant associated with the spot price of commodity two. The interpretation of K is different in different examples. For instance, K represents the strike heat rate H in a spark spread option, and it represents the loss factor L in a locational spread option.

The time-t value of a European cross-commodity spread call option on two commodities is given by

$$
\begin{aligned}
CSC(S_t^1, S_t^2, K, t) &= E^Q[e^{-r(T-t)} \max(S_T^1 - K \cdot S_T^2, 0) \mid \mathcal{F}_t] \\
&= E^Q[e^{-r\tau} \exp(X_T^1) 1_{S_T^1 - K \cdot S_T^2 \geq 0} \mid \mathcal{F}_t] - E^Q[e^{-r\tau} K \exp(X_T^2) 1_{S_T^1 - K \cdot S_T^2 \geq 0} \mid \mathcal{F}_t] \\
&= G_1 - G_2 \qquad (29)
\end{aligned}
$$

where

$$
\begin{aligned}
G_1 &= G(0, \ln S_t^1, \ln(K \cdot S_t^2), t, T; [1, 0, \cdots, 0]', [-1, 1, 0, \cdots, 0]') \\
G_2 &= G(0, \ln S_t^1, \ln(K \cdot S_t^2), t, T; [0, 1, 0, \cdots, 0]', [-1, 1, 0, \cdots, 0]')
\end{aligned} \qquad (30)
$$

and recall that

$$
G(v, \overline{X}_t, t, T; \overline{a}, \overline{b}) = \frac{\varphi(\overline{a}, \overline{X}_t, t, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\varphi(\overline{a} + iw\overline{b}, \overline{X}_t, t, T) e^{-iwv}]}{w} dw
$$

The cross-commodity spread call option pricing formula (29) generalizes the exchange option pricing result of Margrabe (1978) to cases where the underlying state variables follow the proposed mean-reversion jump-diffusion processes.

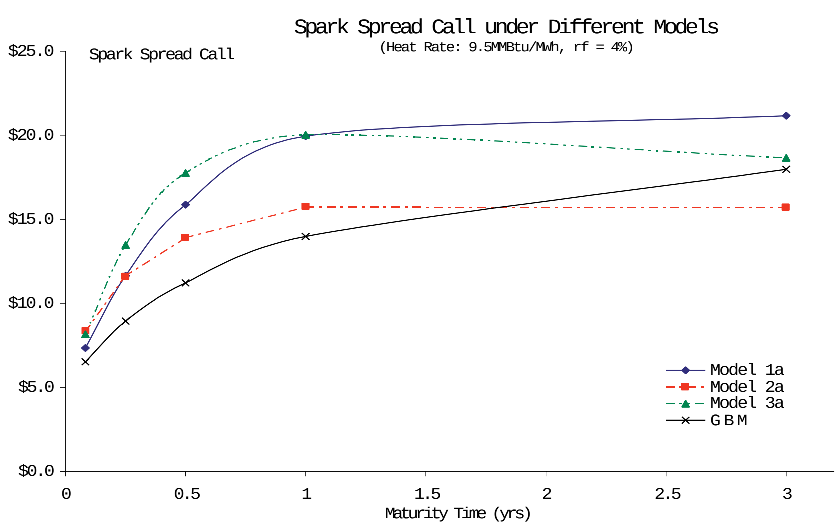

Figure 9: Spark Spread Call Price under Different Models

Cross-commodity spread call option price under each of the three models are obtained by substituting φ_{1a}, φ_{2a}, and φ_{3a} into (29) and (30).

The spark spread call option value with strike heat rate $H = 9.5$ for different maturity time is shown in Figure (9). Again, the spark spread call option value converges to the current spot price under the GBM price model. However, under the mean-reversion jump-diffusion price models, it converges to a long-term value which is most likely to be depending on fundamental characteristics of supply and demand.

	Model 1a	Model 2a	Model 3a
κ_1	1.70	1.37	2.17
κ_2	1.80	1.80	3.50
κ_3	N/A	N/A	1.80
θ_1	3.40	3.30	3.20
θ_2	0.87	0.87	0.85
θ_3	N/A	N/A	0.87
σ_1	0.74	0.80	N/A
σ_2	0.34	0.34	0.80
σ_3	N/A	N/A	0.54
ρ_1	0.20	0.20	0.25
ρ_2	N/A	N/A	0.20
λ_1	6.08	6.42	6.43
μ_{11}	0.19	0.26	0.23
μ_{12}	N/A	N/A	0.22
λ_2	7.00	8.20	5.00
μ_{21}	-0.11	-0.20	-0.14

Table 1: Parameters for the Illustrative Models

C Parameter Estimation

In this subsection, I provide a heuristic method for estimating the model parameters using the electricity price data. As an illustration, I pick **Model 1a** and derive the moment conditions from the transform function of the unconditional distribution of the underlying price return. I assume that the risk premium associated with the factor X is proportional to X, i.e. the risk premium is of form $\xi_X \cdot X$. For simplicity, I further assume the risk premia associated with the jumps are zero. As noted earlier, the price processes under the true measure are of the same forms as (15). In particular,

$$\begin{aligned}\kappa_i &= \kappa_i^* + \xi_i \\ \theta_i &= \frac{\kappa_i^* \cdot \theta_i^*}{\kappa_i^* + \xi_i} \\ \lambda_j &= \lambda_j^* \\ \mu_j &= \mu_j^*\end{aligned}$$

I then use the electricity and natural gas spot and futures price series to get the estimates for the model parameters under the true measure and the risk premia by matching moment conditions as well as the futures prices. The following proposition provides the unconditional mean, variance and skewness of the logarithm of the electricity price in **Model 1a**.

Proposition 2 *In* **Model 1a**, *let $X_\infty \equiv \lim_{t\to\infty} X_t$ denote the unconditional distribution of X_t where $X_t = \ln S_t^e$. If $E[|X_\infty|^n] < \infty$, then the mean, variance and skewness of X_∞ are*

$$\begin{aligned}\textit{mean} &= \theta_1 + \frac{\lambda_1\mu_1 + \lambda_2\mu_2}{\kappa_1} \\ \textit{variance} &= \frac{\sigma_1^2}{2\kappa_1} + \frac{\lambda_1\mu_1^2 + \lambda_2\mu_2^2}{\kappa_1} \\ \textit{skewness} &= \frac{4\sqrt{2\kappa_1}(\lambda_1\mu_1^3 + \lambda_2\mu_2^3)}{(\sigma_1^2 + 2\lambda_1\mu_1^2 + 2\lambda_2\mu_2^2)^{\frac{3}{2}}}\end{aligned}$$

Proof. In the transform function φ_{1a} as defined by (16) and (17), let $u = i \cdot w$ and $t \to \infty$, I obtain the characteristic function of X_∞ to be

$$\Phi_{X_\infty}(w) = \exp[i \cdot w\theta_1 - \frac{w^2\sigma_1^2}{4\kappa_1} - \sum_{j=1}^{2} \frac{\lambda_J^j}{\kappa_1} \ln(1 - i \cdot w\mu_J^j)]$$

If $E[|X_\infty|^n] < \infty$, then the n^{th} moment of X_∞ is given by

$$E[X_\infty^n] = (-i)^n \left.\frac{d^n}{dw^n}\Phi_{X_\infty}(w)\right|_{w=0}$$

In particular,

$$E[X_\infty] = (-i)\,\frac{d}{dw}\Phi_{X_\infty}(w)\Big|_{w=0} = \theta_1 + \frac{\lambda_1\mu_1 + \lambda_2\mu_2}{\kappa_1}$$

The formulae for variance and skewness are obtained in the same fashion. ∎

As shown in the proof of the Proposition (2), we can derive as many moment conditions as we desire for X_∞ from the characteristic function of X_∞ for estimating the model parameters of **Model 1a**. The parameters in **Model 2a** and **Model 3a** are obtained by minimizing the mean squared errors of the traded options prices.

IV Real Options Valuation of Capacity

In this section, I construct a real options approach for the valuation of installed capacity or real assets. For the ease of exposition, I take the deregulated electric power industry as an example to illustrate how to use derivative securities to value generation and transmission facilities. This section follows the methodology proposed in Deng, Johnson and Sogomonian (1998) but here the valuation is based on assumptions about the electricity spot price processes instead of the futures price processes. I also examine the implications of modeling assumptions to capacity valuation.

For an electric power generating asset which is used to transform some fuel into electricity, its economic value is determined by the spread between the price of electricity and the fuel that is used to generate electricity. The amount of fuel that a particular generation asset requires to generate a given amount of electricity depends on the asset's efficiency. This efficiency is summarized by the asset's **operating heat rate**, which is defined as the number of British thermal units (Btus) of the input fuel (measured in millions) required to generate one megawatt hour (MWh) of electricity. Thus the lower the operating heat rate, the more efficient the facility. The right to operate a generation asset with operating heat rate H that uses generating fuel g is clearly given by the value of a spark spread option with "strike" heat rate H written on generating fuel g because they yield the same payoff. Similarly, the value of a transmission asset that connects location 1 to location 2 is equal to the sum of the value of the locational spread options to buy electricity at location 1 and sell it at location 2 as well as the value of the options to buy electricity at location 2 and sell it a location 1 (in both cases, less the appropriate transmission costs).

This equivalence between the value of appropriately defined spark and locational spread options and the right to operate a generation or a transmission asset can be readily used to value such assets. I demonstrate this approach by developing a simple spark spread based model for valuing a fossil fuel generation asset. Once established, I fit the model and use it to generate estimates of the value of several gas-fired plants that have recently been sold. The accuracy of the model is then evaluated by comparing the estimates constructed to the prices at which the assets were actually sold.

In the analysis I make the following simplifying assumptions about the operating characteristics of the generation asset under consideration:

Assumption 1 Ramp-ups and ramp-downs of the generation facility can be done with very little advance notice.

Assumption 2 The facility's operation (e.g. start-up/shutdown costs) and maintenance costs are constants.

These assumptions are reasonable, since for a typical combined cycle gas turbine cogeneration plant the response time (ramp up/down) is several hours and the variable costs (e.g. operation and maintenance) are generally stable over time.

To construct a spark spread based estimate of the value of a generation asset, I estimate the value of the right to operate the asset over its remaining useful life. This value can be found by integrating the value of the spark spread options over the remaining life of the asset. Specifically,

Definition 1 *Let one unit of the* **time-t capacity right** *of a fossil fuel fired electric power plant represent the right to convert K_H units of generating fuel into one unit of electricity by using the plant at time t, where K_H is the plant's operating heat rate.*

The payoff of one unit of time-t capacity right is $\max(S_t^e - K_H S_t^g, 0)$, where S_t^e and S_t^g are the spot prices of electricity and the generating fuel at time t, respectively. Denote the value of one unit of the time-t capacity right by $u(t)$.

For a natural gas fired power plant, the value of $u(t)$ is given by the corresponding spark spread call option on electricity and natural gas with a strike heat rate of K_H. However, for a coal-fired power plant, it often has long-term coal supply contracts which guarantee the supply of coal at a predetermined price c. Therefore the payoff of one unit of time-t capacity right for a coal plant degenerates to that of a call option with strike price $K_H \cdot c$. In this case $u(t)$ is equal to the value of a call option.

Definition 2 *Denote the* **virtual value of one unit of generating capacity** *of a fossil fuel power plant by V. Then V is obtained through integrating the value of one unit of the plant's time-t capacity right over the remaining life $[0, T]$ of the power plant, i.e. $V = \int_0^T u(t)dt$.*

The virtual value of one unit of transmission capacity connecting locations i and j (i.e. nodes i and j) in a radial electricity transmission network[3]can be similarly defined as that of generating capacity.

Definition 3 *Let V_{i-j} denote the* **virtual value of one unit of transmission capacity** *connecting locations i and j. Then V_{i-j} consists of two parts: one part corresponds to the value of transmitting electricity from location i to location j and the other part corresponds to the value of transmitting electricity from location j to location i. Namely,*

$$V_{i-j} = \int_0^T u_1(t)dt + \int_0^T u_2(t)dt \tag{31}$$

where $u_1(t)$ is the value of a locational spread option from i to j with maturity time t (i.e. time- t payoff is $\max(S_t^j - L_{ij}S_t^i, 0)$);[4] *and $u_2(t)$ is the value of a locational spread option from j to i with maturity time t (i.e. time- t payoff is $\max(S_t^i - L_{ij}S_t^j, 0)$).*

Under **Assumption 1** and **2**, the virtual value less the present value of the future O&M costs is very close to the value of one unit capacity. Since the O&M costs are assumed to be constants and they do not vary with model parameters, I set them to be zero. For numerical computations, the virtual values in Definition 2 and 3 are approximated by the following discrete formula:

$$V \approx \sum_{i=1}^{n} u(t_i)(t_i - t_{i-1}) \tag{32}$$

where $t_1, t_2, \cdots, t_n$ are n maturity times satisfying $0 \equiv t_0 < t_1 < \cdots < t_{n-1} < t_n \equiv T$. The larger the number n is, the more accurate the approximation is in (32).

In what follows I will investigate the sensitivity of capacity value with respect to model parameters under the setting of **Model 1a**. I calculate the virtual capacity value for a hypothetical gas-fired power plant using spark spread valuation with parameters given in Table (2). The operating life-time of the power plant is assumed to be 15 years.

κ_1	4	κ_2	3
θ_1	3.15	θ_2	0.8706
σ_1	0.7481	σ_2	0.8693
ρ	0.3		
λ_1	2	λ_2	2
μ_1	0.1934	μ_2	-0.1934
S_1^0	21.7	S_2^0	3.16

Table 2: Parameters (Model 1a) for Capacity Valuation

I first examine how the presence of jumps in commodity prices affects the capacity value. Figure (10) plots the capacity value of a fictitious natural gas fired power plant, for the operating heat rate ranging from 7,000 MMBtu/MWh to 15,000 MMBtu/MWh, both with and without jumps ($\lambda_1 = \lambda_2 = 0$) in the electricity price process. The axis on the left is for the capacity value and the axis on the right is for the difference in capacity value of the

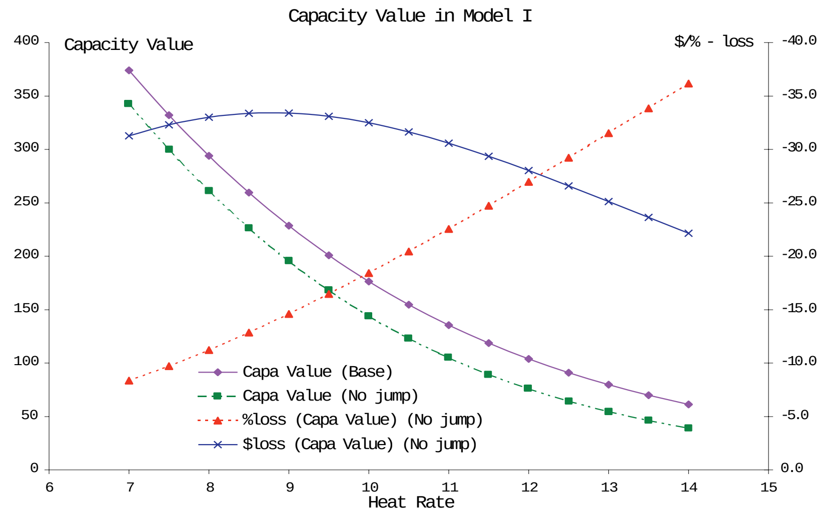

Figure 10: Capacity Value with/without Jumps (Model 1a)

two cases. The solid curve with diamonds represents the capacity value using parameters in Table (2) which is the base for the comparison. Figure (10) illustrates the change in capacity value in absolute dollar term and in percentage term on the right axis as a result of setting the jump intensities to zero in **Model 1a**. We can see that the less efficient a power plant the more portion of its capacity value attributed to the jumps in the spot price process. The presence of jumps makes up as much as 35% of the capacity value of the very inefficient power plants. We note that the capacity values with jumps (the solid curve with diamonds) computed here will also serve as the basis for the subsequent comparative static analysis on capacity value with respect to the changes in other model parameters.

In Figure (11), I plot the changes in capacity value due to a 10% increase or a 10% decrease in the electricity price volatility parameter σ_1, respectively. The solid curves represent the dollar value change with respect to the base case in Figure (10) on the left-side axis and the dotted curves show the percentage change with respect to the base case on the right-side axis. It is clear that the change in volatility causes greater dollar value changes for efficient plants than for inefficient plants.

The sensitivity of capacity value with respect to the correlation coefficient between the

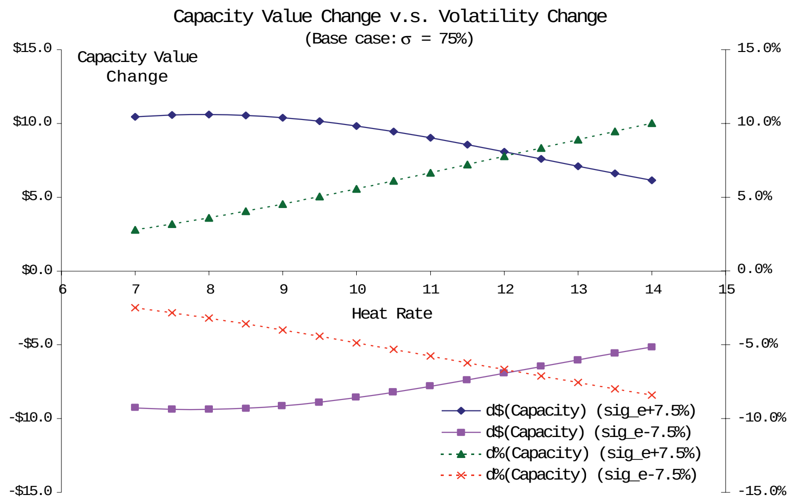

Figure 11: Sensitivity of Capacity Value to Electricity Spot Volatility (Model 1a)

generating fuel price and the electricity price is illustrated in Figure (12). While it is still true that the percentage change of capacity value varies monotonically with respect to the efficiency measure (operating heat rate) of a power plant, the largest dollar value change does not occur to the most efficient plant but to the plant with heat rate close to the market implied heat rate[5].

To compare the theoretical valuation of generation capacity with the market valuation, I plot a capacity value curve in Figure (13) using the NYMEX electricity futures prices at Palo Verde on 10/15/97 and the parameters obtained by fitting **Model 1a** to the NYMEX electricity futures prices at Pale Verde and the natural gas futures prices at Henry Hub. Figure (13) also plots the capacity value curve obtained by using the discounted cash flow (DCF) method for comparison purpose[6]. At the heat rate level of 9500, the capacity value of a natural gas power plant is around \$200/kW under **Model 1a**. The discounted cash flow method predicts a value of \$29/kW. To put things in perspective, I take a look at the four gas-fired power plants which Southern California Edison recently sold to Houston Industries. Unfortunately, not all of the individual plant dollar investments have been made public yet. As a proxy I use the total investment made by Houston Industries (\$237 million

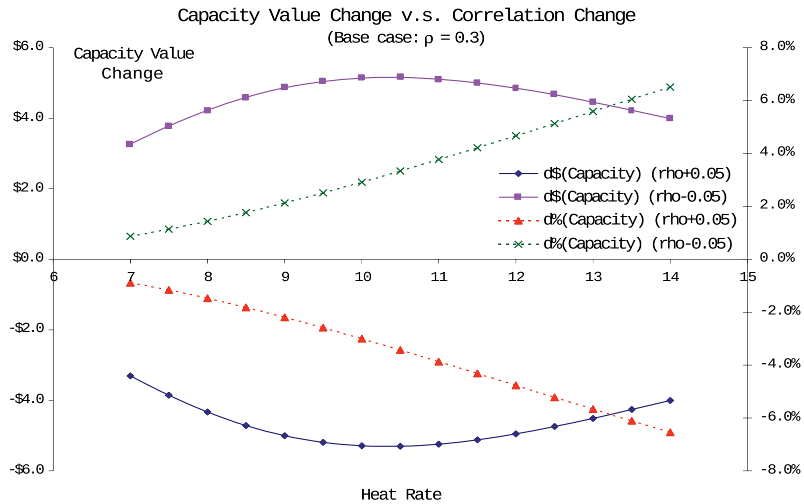

Figure 12: Sensitivity of Capacity Value to the Correlation Coefficient(Model 1a)

to purchase four plants - Coolwater, Ellwood, Etiwanda and Mandalay), divided by the total number of megawatts (MW) of capacity (2172MW) to get approximately \$110,000 per MW (or \$110/kW) of capacity. However, the Coolwater Plant[7], in Daggett, California, is the most efficient (with an average heat rate of 9,500) of the four plants in the package and thus should have a higher value per MW. I therefore assume that the implied market value for Coolwater could range from \$110,000 to \$220,000 per MW, or equivalently, \$110/kW to \$220/kW. Our real options based estimate of the capacity value explains the market valuation much better than does the discounted cash flow estimate.

V Conclusion

In this paper, I propose three types of mean-reversion jump-diffusion models for modeling energy commodity spot prices with jumps and spikes. I demonstrate how the prices of the energy commodity derivatives can be obtained by means of transform analysis. The market anticipation of jumps and spikes in the electricity spot price processes explains the enormous implied volatility observed from market prices of traded electricity options. Under

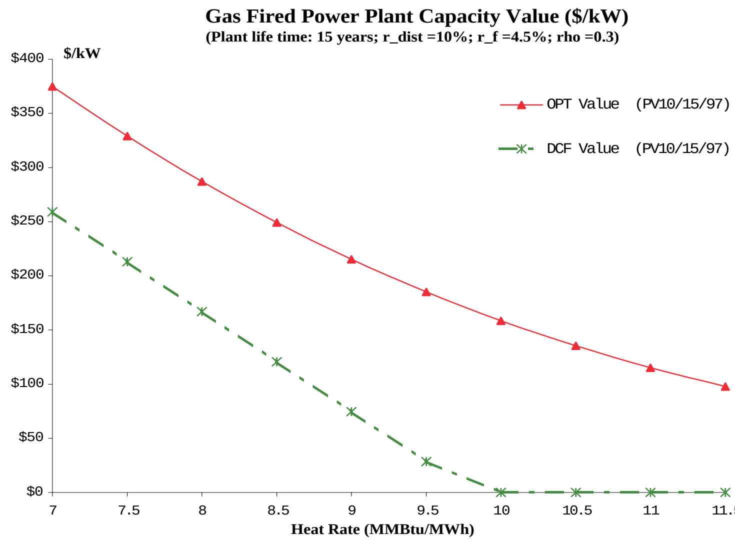

Figure 13: Value of Capacity: Spark Spread approach vs. DCF approach

the proposed mean-reversion jump-diffusion spot price models, as opposed to the commonly used Geometric Brownian motion model, the values of short-maturity out-of–the-money options approximate market prices very well. I construct a real options approach to value real assets through utilizing properly defined energy commodity derivatives. The implication of jumps and spikes to capacity valuation in the context of electric power industry is that, in the near-term when the effects of jumps and spikes are significant, even a very inefficient power plant can be quite valuable. This might explain why recently sold power plants fetched hefty premia over book value. I also mentioned briefly how I can fit the models using electricity price data but a formal parameter estimation was out of the scope of this work. As for future research, I feel that it is important to develop an efficient econometric model to perform a rigorous parameter estimation as more electricity market data becomes available.

Footnotes

1. For simplicity, I model the commodity price itself as a state variable. The extension of modeling the commodity price as an exponential-affine function of state variables is straightforward.

2. A "Spike" refers to an upward jump followed shortly by a downward jump.

3. A radial electricity transmission network is a network in which no parallel paths exist between any two nodes.

4. Market implied heat rate is defined to be the ratio of spot price of electricity to the spot price of the generating fuel.

5. Recall that L_{ij} is the transimission loss factor associated with the transmission of one unit of electricity from location i to j.

6. The capacity value curve is based on spark spread options on monthly on-peak (23x16) electricity and natural gas. The correlation coefficient of gas-to-electricity is assumed to be 0.3. The discount rate for the DCF approach is 10%. I assume a power plant will be operated for 15 years and there is no O&M costs.

7. The Coolwater Plant is made up of four units. Two 256MW Combined Cycle Gas Turbines plus a steam turbine; and two conventional turbines with capacity 65MW and 81MW each. Some re-power work has been done on the larger units.

Appendix

A Transform Functions in the General Models

A.1 The Transform Function in Model 1

It is conjectured that, under regularity conditions, the solution to the PDE (2) takes the form of

$$\varphi(u, X_t, Y_t, t, T) = \exp[\alpha(t,u) + \beta_1(t,u)X_t + \beta_2(t,u)Y_t] \tag{33}$$

Substituting (33) into (2) yields the following complex-valued ordinary differential equations for α and $\beta \equiv [\beta_1 \; \beta_2]'$

$$\begin{aligned} &\frac{d}{dt}\beta(t,u) + B(\beta(t,u),t) = 0, \qquad \beta(0,u) = u \\ &\frac{d}{dt}\alpha(t,u) + A(\beta(t,u),t) = 0, \qquad \alpha(0,u) = 0 \end{aligned} \tag{34}$$

with $A(\cdot,\cdot) : C^2 \times R \to C^1$ and $B(\cdot,\cdot) : C^2 \times R \to C^2$ being

$$\begin{aligned} A(\beta,t) &= \sum_{i=1}^{2}[\kappa_i\theta_i\beta_i + \tfrac{1}{2}\sigma_i^2\beta_i^2] + \rho\sigma_1\sigma_2\beta_1\beta_2 - r + \sum_{j=1}^{2}\lambda_j(t)(\phi_J^j(\beta_1,\beta_2,t) - 1) \\ B(\beta,t) &= \begin{pmatrix} \kappa_1\beta_1 \\ \kappa_2\beta_2 \end{pmatrix} \end{aligned} \tag{35}$$

Now, we integrate α and β out with the corresponding initial conditions to get the solution (3).

A.2 The Transform Function in Model 2

Substituting (9) into (8) with $\alpha(t) \equiv \alpha(t,u) \equiv (\alpha_0(t,u), \alpha_1(t,u))'$ and $\beta(t) \equiv \beta(t,u) \equiv (\beta_1(t,u), \beta_2(t,u))'$, we get the following ordinary differential equations (ODEs)

$$\begin{aligned} &\frac{d}{dt}\beta(t,u) + B(\beta(t,u),t) = 0, && \beta(0,u) = u \\ &\frac{d}{dt}\alpha(t,u) + A(\alpha(t,u),\beta(t,u),t) = 0, && \alpha(0,u) = 0 \end{aligned} \tag{36}$$

where

$$\begin{aligned} A(\alpha(t),\beta(t),t) &= \begin{pmatrix} A_1(\beta(t),t) + \lambda^{(0)}[\exp(\alpha_1(t)-\alpha_0(t))\phi_{\iota(0)}(\beta(t),t) - 1] \\ A_1(\beta(t),t) + \lambda^{(1)}[\exp(\alpha_0(t)-\alpha_1(t))\phi_{\iota(1)}(\beta(t),t) - 1] \end{pmatrix} \\ B(\beta(t),t) &= \begin{pmatrix} \kappa_1(t)\beta_1(t) \\ \kappa_2(t)\beta_2(t) \end{pmatrix} \end{aligned} \tag{37}$$

and

$$A_1(\beta(t),t) = \sum_{i=1}^{2}[\kappa_i\theta_i\beta_i + \frac{1}{2}\sigma_i^2\beta_i^2] - \rho\sigma_1\sigma_2\beta_1\beta_2 - r + \sum_{j=1}^{2}\lambda_j(\phi_J^j(\beta,t) - 1)$$

$\phi_J^j(\beta,t)$ and $\phi_{\iota(\cdot)}(\beta(t),t)$ are transform functions of the random variables representing jump size in state variables within a regime and associated with the regime-switching, respectively. When ODEs (36) do not have closed-form solution, we need to numerically solve (36) in order to obtain the value of the transform function.

A.3 The Transform Function in Model 3

The system of ODEs determining $\alpha(u,t)$ and $\beta(u,t) \equiv (\beta_1(u,t), \beta_2(u,t), \beta_3(u,t))'$ in the transform function (11) are

$$\begin{aligned} &\frac{d}{dt}\beta(t,u) + B(\beta(t,u),t) = 0, && \beta(0,u) = u \\ &\frac{d}{dt}\alpha(t,u) + A(\beta(t,u),t) = 0, && \alpha(0,u) = 0 \end{aligned} \tag{38}$$

with $A(\cdot,\cdot) : C^3 \times R \rightarrow C^1$ and $B(\cdot,\cdot) : C^3 \times R \rightarrow C^3$ being

$$A(\beta,t) = -r + \sum_{i=1}^{3} \kappa_i \theta_i \beta_i + \tfrac{1}{2}\beta_3^2 \sigma_3^2(t) + \lambda_1(t)(\phi_J^{(1)}(\beta,t) - 1)$$
$$B(\beta,t) = \begin{pmatrix} \kappa_1(t)\beta_1(t,u) \\ \kappa_2(t)\beta_2(t,u) + \lambda_2(t)(\phi_J^{(2)}(\beta,t) - 1) + \tfrac{1}{2}B_1(\beta,t) \\ \kappa_3(t)\beta_3(t,u) \end{pmatrix} \tag{39}$$

and

$$\begin{aligned} B_1(\beta,t) &= \beta_1(t,u)(\beta_1(t,u) + \beta_2(t,u)\rho_1(t)\sigma_2(t) + \beta_3(t,u)\rho_2(t)\sigma_3(t)) \\ &\quad +\beta_2(t,u)(\beta_1(t,u)\rho_1(t)\sigma_2(t) + \beta_2(t,u)\sigma_2^2(t) + \beta_3(t,u)\rho_1(t)\rho_2(t)\sigma_2(t)\sigma_3(t)) \\ &\quad +\beta_3(t,u)(\beta_1(t,u)\rho_2(t)\sigma_3(t) + \beta_2(t,u)\rho_1(t)\rho_2(t)\sigma_2(t)\sigma_3(t) + \beta_3(t,u)\rho_2^2(t)\sigma_3^2(t)) \end{aligned}$$

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